PhD Thesis

ASSESSING PROCESS DESIGN WITH REGARD TO MPC PERFORMANCE USING A NOVEL MULTI-MODEL PREDICTION METHOD

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Declaration

I, Flavio Augusto Martins Strutzel, confirm that the work presented in this Thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the Thesis.

Signature:______________________________

Date: 21/11/2017

Abstract

Model Predictive Control (MPC) is nowadays ubiquitous in the chemical industry and offers significant advantages over standard feedback controllers. Notwithstanding, projects of new plants are still being carried out without assessing how key design decisions, e.g., selection of production route, plant layout and equipment, will affect future MPC performance. The problem addressed in this Thesis is comparing the economic benefits available for different flowsheets through the use of MPC, and thus determining if certain design choices favour or hinder expected profitability. The Economic MPC Optimisation (EMOP) index is presented to measure how disturbances and restrictions affect the MPC's ability to deliver better control and optimisation.

To the author's knowledge, the EMOP index is the first integrated design and control methodology to address the problem of zone constrained MPC with economic optimisation capabilities (today's standard in the chemical industry). This approach assumes the availability of a set of linear state-space models valid within the desired control zone, which is defined by the upper and lower bounds of each controlled and manipulated variable. Process economics provides the basis for the analysis. The index needs to be minimised in order to find the most profitable steady state within the zone constraints towards which the MPC is expected to direct the process. An analysis of the effects of disturbances on the index illustrates how they may reduce profitability by restricting the ability of an MPC to reach dynamic equilibrium near process constraints, which in turn increases product quality giveaway and costs. Hence the index monetises the required control effort.

Since linear models were used to predict the dynamic behaviour of chemical processes, which often exhibit significant nonlinearity, this Thesis also includes a new multi-model prediction method. This new method, called Simultaneous Multi-Linear Prediction (SMLP), presents a more accurate output prediction than the use of single linear models, keeping at the same time much of their numerical advantages and their relative ease of obtainment. Comparing the SMLP to existing multi-model approaches, the main novelty is that it is built by defining and updating multiple states simultaneously, thus eliminating the need for partitioning the state-input space into regions and associating with each region a different state update equation. Each state's contribution to the overall output is obtained according to the relative distance between their identification point, i.e., the set of operating conditions at which an approximation of the nonlinear model is obtained, and the current operating point, in addition to a set of parameters obtained through regression analysis.

Additionally, the SMLP is built upon data obtained from step response models that can be obtained by commercial, black-box dynamic simulators. These state-of-the-art simulators are the industry's standard for designing large-scale plants, the focus of this Thesis. Building an SMLP system yields an approximation of the nonlinear model, whose full set of equations is not of the user's knowledge. The resulting system can be used for predictive control schemes or integrated process design and control. Applying the SMLP to optimisation problems with linear restrictions results in convex problems that are easy to solve. The issue of model uncertainty was also addressed for the EMOP index and SMLP systems. Due to the impact of uncertainty, the index may be defined as a numeric interval instead of a single number, within which the true value lies.

A case of study consisting of four alternative designs for a realistically sized crude oil atmospheric distillation plant is provided in order to demonstrate the joint use and applicability of both the EMOP index and the SMLP. In addition, a comparison between the EMOP index and a competing methodology is presented that is based on a case study consisting of the activated sludge process of a wastewater treatment plant.

Keywords

Integrated Process Design and Control, Simultaneous Process Design and Control, Model Predictive Control, MPC, Zone Constrained MPC, Zone Control, Controllability Analysis, Crude Oil Distillation, Linear Hybrid Systems, Multi-model MPC, Activated Sludge Wastewater Treatment.

Impact Statement

This Thesis presents two main novel elements: a new integrated process design and control approach; and a new method for approximating nonlinear systems as a collection of linear state-space models.

The first contribution, called EMOP index methodology, can be used as a decision-making tool during the design phase of new chemical, petrochemical and oil refining units. It provides a performance ranking of candidate designs based on their expected operating expenses (OPEX) in a number of production scenarios. The main case study presented in this Thesis, which studied realistic designs for a crude oil distillation unit, demonstrated that the selection of a suboptimal flowsheet can increase OPEX from 2% to 55% relative to the optimal flowsheet. Even the lower range of this figure translates into expressive amounts of money being wasted, especially given the long life-cycles of the chemical industry's projects. Design teams working for project companies or their clients should apply the EMOP index because it is the first Controllability Analysis method adequate for the special case of zone constrained model predictive control (MPC), the chemical industry's de facto standard for advanced control systems. Also, like any method of Controllability Analysis, the EMOP index can be used to discover serious controllability issues in the early design phase, which has the potential of saving millions of dollars by avoiding delays in the project completion to implement corrections, or avoiding a lifetime of troubled operation, if corrections are not implemented. Unlike most methods, however, EMOP can provide an estimative for the losses such controllability issues can create. Inside academia, this work can have an impact by inspiring research on new methods for zone constrained MPC, as well as new efforts to monetise controllability.

The second contribution is a linear state-space formulation capable of accurately representing inherently nonlinear processes without incurring in the mathematical disadvantages of using nonlinear models. Called Simultaneous Multi-Linear Prediction (SMLP), this formulation can be applied with MPC, yielding more precise control actions by reducing the prediction error. Nonetheless, the advantage of providing a better approximation of nonlinear models is even more relevant when the prediction is used for integrated process design and control. In this field, even small accuracy gains are extremely important and may impact the layout choice for a new plant. A comparison between a PieceWise Affine system (a standard multi-model formulation) and the SMLP showed that the later provided an accuracy gain of 44.86%, using the nonlinear model as a reference. Another advantage is the economy of both computational time and engineering man-hours required as compared to developing a nonlinear, rigorous first-principles or hybrid model for a complex industrial process. SMLP may inspire further developments in the Linear Hybrid Systems framework, and automation suppliers may embed SMLP in their MPC solutions.

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Notation List

Roman alphabet

Symbol	**Definition**	**Unit**
$\overline{\overline{\boldsymbol{C_d}}}$	State-space disturbance matrix along the prediction horizon	
$\overline{\overline{\boldsymbol{C_u}}}$	State-space input matrix along the control horizon	
$\overline{\boldsymbol{X}}_{\boldsymbol{m_\vartheta}}$	Target for state change of the subsystem ϑ	
$\overline{\boldsymbol{X}}_{\boldsymbol{M}}$	Target for state change of the SMLP system	
$\overline{\boldsymbol{x}}_{\boldsymbol{\vartheta,1}}$	Initial state of the subsystem ϑ	
$\overline{\boldsymbol{x}}_{\boldsymbol{\vartheta,2}}$	Target state of the subsystem ϑ	
$\overline{\boldsymbol{y}}_{\boldsymbol{ave}}$	average value of the CV with quality threshold through the prediction horizon	
$\overline{\boldsymbol{y}}_{\boldsymbol{qual}}$	Value of CV quality threshold value for which the price variation occurs	
$\boldsymbol{A}_{\boldsymbol{m_\vartheta}}$	State-space model system matrix of sub-model ϑ	
A_s	Settler area	m^2
$\boldsymbol{B}_{\boldsymbol{m_\vartheta}}$	State-space model input matrix of sub-model ϑ	
$\boldsymbol{C}_{\boldsymbol{m_\vartheta}}$	State-space model output matrix of sub-model ϑ	
c_1	Dissolved oxygen (DO) concentration at the output of the aeration tanks input flow	mg/L
c_s	Saturation oxygen (DO) concentration in the aeration tanks	$\frac{\mathrm{mg}}{\mathrm{L}}$
$\boldsymbol{D}_{\boldsymbol{m_\vartheta}}$	State-space model disturbance matrix of sub-model ϑ	
$\boldsymbol{D_K}$	Space of possible disturbance values	
$\boldsymbol{d^m}$	Measured vector of disturbance variables (DVs)	
$e_{s_n}^{\vartheta}$	Absolute value of prediction error of sub-model ϑ relative to step n	
$E_{i,k}$	Error relative to controlled variable i at each instant k	
E_{lin}^{ϑ}	Linearisation error for a single sub-model ϑ	
f_{k1}	Aeration factor	

f_{kd}	Yield coefficient between biomass endogenous and substrate contribution to the medium	L^{-1}
$\boldsymbol{G_d}$	Transfer function matrix describing the effects of disturbance variables	
$\boldsymbol{g_\vartheta}$	Degradation function of sub-model ϑ	
I_{EMOP}	Economic MPC Optimisation index	
$I_{EMOP-BM}$	Lower bound (best-case uncertainty realisation) of the EMOP index	
$I_{EMOP-UN}$	Uncertain interval of the Economic MPC Optimisation index	
$I_{EMOP-WM}$	Upper bound (worst-case uncertainty realisation) of the EMOP index	
J_{CV}	Contribution of control zone violations to the EMOP objective function	
J_k	Basic EMOP objective function value at time k	
J_{MV}	Contribution of MV economic optimisation to the EMOP objective function	
J_q	Total opportunity cost due to quality thresholds	
J_{sat}	Auxiliary component of the EMOP index use to drive CVs away from saturation	
K_{01}	Yield coefficient between the cellular growth and the oxygen consumption rate	
K_c	Kinetic coefficient of biomass decay by biological waste	$\frac{L}{h}$
K_d	Kinetic coefficient of biomass decay by endogenous metabolism	$\frac{L}{h}$
K_{la}	Mass transfer coefficient in aeration process oxygen uptake rate	h^{-1}
K_s	Saturation constant	$\frac{mg}{L}$
l_b	Height of the second layer of the secondary clarifiers	m
l_d	Height of the first layer of the secondary clarifiers	m
l_r	Height of the third layer of the secondary clarifiers	m
M_ϑ^{vote}	Number of votes of partition ϑ of the PWA model implementation	
n_p	Number of plants to be assessed	
N_s	Number of steps used for model identification	

n_u	Number of manipulated variables	
n_y	Number of controlled variables	
$\boldsymbol{\widetilde{P}}$	Neighbourhood of the nominal plant	
$\boldsymbol{P_{qual}}$	Price of the most premium product variant	
$\boldsymbol{P_{reg}}$	Regular price of a product with quality threshold	US$
$\boldsymbol{Q_{m_\vartheta}}$	Observability matrix of the subsystem ϑ	
$\boldsymbol{q_i}$	Influent flow	$\mathrm{m^3/h}$
$\boldsymbol{q_p}$	Purge flow	$\mathrm{m^3/h}$
$\boldsymbol{q_r}$	Recycle flow	
$\boldsymbol{q_r}$	Recycle flow	$\mathrm{m^3/h}$
$\boldsymbol{R_{m_\vartheta}}$	Reachability matrix of the subsystem ϑ	
$\boldsymbol{Rev_\theta^k}$	Plant θ's operational revenue at initial OP (time k)	
$\boldsymbol{R_M}$	Reachability matrix of the SMLP system	
s_1	Substrate (COD) concentration at the output of the aeration tanks	$\frac{\mathrm{mg}}{\mathrm{L}}$
s_i	Substrate concentration at the influent	$\frac{\mathrm{mg}}{\mathrm{L}}$
s_{ir}	Bioreactor inlet substrate concentration	$\frac{\mathrm{mg}}{\mathrm{L}}$
$\boldsymbol{SL_1}$	Soft-Landing matrix for the first order derivatives	
$\boldsymbol{SL_2}$	Soft-Landing matrix for the second order derivatives	
T_C	Ethylene critical temperature	
$\boldsymbol{T_{EP}}$	Error penalization matrix parameter vector for overshoot rejection prioritisation	
$\boldsymbol{T_{lower}}$	Parameter matrix for error penalization (lower bound)	
$\boldsymbol{T_{SL1}}$	Parameter vectors for first order derivatives Soft-Landing matrix	
$\boldsymbol{T_{SL2}}$	Parameter vectors for second-order derivatives Soft-Landing matrix	
$\boldsymbol{T_{upper}}$	Parameter matrix for error penalization (upper bound)	
u_1	Feed volumetric flow rate	
u_2	Ethylene concentration in the feed	
$\boldsymbol{U_K}$	Space of possible manipulated variable values	
$\boldsymbol{u_{max}}$	Vector of minimum values for manipulated variables	
$\boldsymbol{u_{min}}$	Vector of maximum values for manipulated variables	

$\boldsymbol{U_s}$	Left singular vector matrix	
$\boldsymbol{V_1}$	Bioreactor volume	m^3
$\boldsymbol{V_{max}}$	Vector of economic optimisation weights for MVs (maximisation)	
$\boldsymbol{V_{min}}$	Vector of economic optimisation weights for MVs (minimisation)	
$\boldsymbol{v_{prod}}$	Output volume of the product with quality thresholds	
$\boldsymbol{v_s(x_b)}$	Settling rate function of the activated sludge in the settler depending on x_b	$\frac{g}{m^2} \cdot h$
$\boldsymbol{v_s(x_d)}$	Settling rate function of the activated sludge in the settler depending on x_d	$\frac{g}{m^2} \cdot h$
$\boldsymbol{W_{lower}}$	Weight vector for the lower bound (minimum value) of y	
$\boldsymbol{w_p}$	Weight defining acceptable dynamic responses	
$\boldsymbol{W_{upper}}$	Weight vector for the upper bound (maximum value) of y	
x_1	Biomass concentration at the output of the aeration tanks	$\frac{mg}{L}$
x_1	Dimensionless gas density	
x_2	Ethylene concentration	
x_3	Ethylene oxide concentration	
x_4	Reactor temperature	
x_b	Biomass concentration in the settler second layer	$\frac{mg}{L}$
x_d	Biomass concentration at the surface of the settler	$\frac{mg}{L}$
x_i	Biomass concentration at the influent	$\frac{mg}{L}$
x_{ir}	Bioreactor inlet biomass concentration	$\frac{mg}{L}$
x_k	State vector at time k	
x_r	Biomass concentration at the bottom of the settler	$\frac{mg}{L}$
y_{p_i}	i'th output pole vector	
y_c	Yield coefficient between cellular growth and substrate elimination	
$\boldsymbol{y'_k}$	Uncertain model prediction vector for controlled variables at time k	
$\boldsymbol{y^-_{sat}}$	Vector of controlled variables saturated at the lower bound	
$\boldsymbol{y^+_{sat}}$	Vector of controlled variables saturated at the upper bound	

$\boldsymbol{y}_k^{BM}$	Updated initial CVs for the best-case model (step 3 of the uncertainty procedure)	
$\boldsymbol{y}_k^{WM}$	Updated initial CVs for the worst-case model (step 3 of the uncertainty procedure)	
$\boldsymbol{y}_{max}$	Vector of maximum values for controlled variables	
$\boldsymbol{y}_{max}^*$	Vector of maximum values for the sensor measuring controlled variables	
$\boldsymbol{y}_{min}$	Vector of minimum values for controlled variables	
$\boldsymbol{y}_{min}^*$	Vector of minimum values for the sensor measuring controlled variables	
$\boldsymbol{y}^{nl}$	Output of the nonlinear model (real plant or simulation package)	
$\boldsymbol{y}^{sp}$	Vector of reference signals (setpoints)	
$\boldsymbol{y}^{m}$	Measured vector of controlled variables (CVs)	
$\boldsymbol{A}$	State-space model system matrix	
$\boldsymbol{A}$	Vector of parameters of the ethylene oxide process model	
ar	Settling rate experimental parameter	$\frac{\mathrm{L}}{\mathrm{mg}}$
$\boldsymbol{B}$	State-space model input matrix	
$\boldsymbol{B}$	Vector of parameters of the ethylene oxide process model	
$\boldsymbol{C}$	State-space model output matrix	
c	Bioreactor input flow	m^3/h
$\boldsymbol{D}$	State-space model disturbance matrix	
$\boldsymbol{d}$	Vector of disturbance variables (DVs)	
$\boldsymbol{d}$	Vector of design variables	
$\boldsymbol{E}$	Vector of error	
$\boldsymbol{EP}$	Error penalization matrix	
$\boldsymbol{f}$	Vector of inequalities	
$\boldsymbol{g}$	Constraints that represent feasible operation (eg physical constraints, specifications)	
$\boldsymbol{G}$	Transfer function matrix describing the effects of manipulated variables	
$\boldsymbol{G}$	Process gain matrix	
$\boldsymbol{h}$	First principles model, ie, heat and material balances	
$\boldsymbol{I}$	Identity matrix	

k	Current time discrete time	
$\boldsymbol{K}$	One-degree-of-freedom proportional controller	
m	Number of time increments of the control horizon	
$\boldsymbol{n}$	Measurement noise	
nr	Settling rate experimental parameter	
OUR	Oxygen uptake rate	$\frac{\mathrm{mg}}{\mathrm{L}} \cdot \mathrm{h}$
p	prediction horizon	
$\boldsymbol{P}$	**Plant**	
$\boldsymbol{P}$	Positive definite solution of the Riccati equation	
q	Bioreactor input flow	
$\boldsymbol{Q}$	LQR weight	
$\boldsymbol{R}$	Input suppression factor	
$\boldsymbol{s}$	Laplace variable	
$\boldsymbol{S}$	Sensitivity Function	
t	Time	s or h
$\boldsymbol{T}$	Complementary Sensitivity Function	
$\boldsymbol{u}$	Absolute value of the vector of manipulated variables (MVs)	
$\boldsymbol{V}$	Right singular vector matrix	
$\boldsymbol{V}$	A locally positive function	
Y	Yield of ethylene oxide	
$\boldsymbol{y}$	Vector of controlled variables (CVs)	

Greek Alphabet

Symbol	Definition	Unit
α	Auxiliary variable that denoting time	
$\boldsymbol{\beta}_{y,d}^{-}$	Minimum negative relative model mismatch relative to DVs	
$\boldsymbol{\beta}_{y,d}^{+}$	Maximum positive relative model mismatch relative to DVs	
$\boldsymbol{\beta}_{y,u}^{-}$	Minimum negative relative model mismatch relative to MVs	
$\boldsymbol{\beta}_{y,u}^{+}$	Maximum positive relative model mismatch relative to MVs	
$\boldsymbol{B}_{d}^{-}$	DV negative uncertainty matrix	
$\boldsymbol{B}_{d}^{+}$	DV positive uncertainty matrix	
$\boldsymbol{B}_{u}^{-}$	MV negative uncertainty matrix	
$\boldsymbol{B}_{u}^{+}$	MV positive uncertainty matrix	
$\boldsymbol{\gamma}$	Vector of parameters of the ethylene oxide process model	
$\gamma_{y_i d_j}$	Realisation of the uncertainty between y_I and d_j	
$\gamma_{y_i u_j}$	Realisation of the uncertainty between y_I and u_j	
$\boldsymbol{\Gamma}_{\Delta d_k}$	Matrix generated by the product between the DVs set its bounded uncertainty realisation	
$\boldsymbol{\Gamma}_{\Delta u_k}$	Matrix generated by the product between the MVs set its bounded uncertainty realisation	
$\boldsymbol{\delta}$	State of the ASP model	
Δ	Sampling period	
$\Delta\boldsymbol{d}'$	Interval of disturbance magnitude of the uncertain model	
$\Delta\boldsymbol{d_k}$	DV vector at k	
$\Delta\boldsymbol{d_K}$	Matrix of DV values along the prediction horizon as planned at k	
$\Delta\boldsymbol{P}^{qual}$	Added value, ie, product price difference between premium priced and regular product	
$\Delta\boldsymbol{u}$	MV movement, control action	
$\Delta\boldsymbol{u}'$	Interval of control action magnitude of the uncertain model	
$\Delta\boldsymbol{u_k}$	MV vector at k	
$\Delta\boldsymbol{u_K}$	Matrix of MV values along the control horizon as planned at k	
$\Delta y_{y_i,u_i}^{amp}$	Norm of the amplitude of the dynamic response of the multi-linear system to a step at the end of the prediction horizon, for a certain CV/MV couple	

Δy_{s_n}	steady-state response amplitude of plant data relative to the n[th] step	
$\Delta \boldsymbol{y}'_{\boldsymbol{BM}}$	Prediction mismatch between the nominal model and the best-case model	
$\Delta y_i^{d'}$	Change in the steady-state output prediction caused by a bounded realisation of DV uncertainty	
Δy_i^{u}	Output change caused by an MV movement Δu_j	
$\Delta y_i^{d'}$	Change in the steady-state output prediction caused by a bounded realisation of DV uncertainty	
$\Delta y_i^{u'}$	Change in the steady-state output prediction caused by a bounded realisation of MV uncertainty	
$\Delta \boldsymbol{y}_{\boldsymbol{p}}^{\boldsymbol{\vartheta}}$	Model mismatch value at the end of the prediction horizon	
$\Delta \bar{\boldsymbol{y}}_{\boldsymbol{qual}}$	Difference between the key CV quality threshold value for which the price variation occurs, $\bar{y}_{qual}$, and the average value of the key CV through the prediction horizon, $\bar{y}_{ave}$	
$\Delta \boldsymbol{y}'_{\boldsymbol{WM}}$	Prediction mismatch between the nominal model and the worst-case model	
$\boldsymbol{\Delta\theta^+}$	Expected deviations of uncertain parameters in the positive direction	
$\boldsymbol{\Delta\theta^-}$	Expected deviations of uncertain parameters in the negative direction	
ε_ϑ	Weight of sub-model ϑ in the main prediction	
$\boldsymbol{\theta}$	Uncertain parameters	
$\boldsymbol{\theta^L}$	Lower bound on uncertain parameters	
$\boldsymbol{\theta^N}$	Nominal value of the uncertain parameters	
$\boldsymbol{\theta^U}$	Upper bound uncertain parameters	
$\boldsymbol{\vartheta}$	Auxiliary binary variable	
$\boldsymbol{\lambda}$	Vector of eigenvalues	
$\boldsymbol{\Lambda}$	Relative gain array	
$\boldsymbol{\mu}$	Auxiliary binary variable	
$\boldsymbol{\mu_{max}}$	Maximum growth rate of the microorganisms	h^{-1}
$\boldsymbol{v^{prod}}$	Volume produced of product with quality threshold	
$\boldsymbol{\Pi}$	Set of possible plants	
$\boldsymbol{\bar{\sigma}}$	Singular value	

$\boldsymbol{\sigma}$	Vector of singular values
$\boldsymbol{\Sigma}$	Diagonal matrix of singular values
τ	Time
$\boldsymbol{\tau_{\vartheta}}$	Distance coefficient of model ϑ
$\boldsymbol{T}$	Range of uncertain parameters
$\boldsymbol{\varphi_{n}^{\vartheta}}$	Relative prediction error of sub-model ϑ relative to the n^{th} step
$\boldsymbol{\varphi_{y_i,d_l}^{\vartheta}}$	Model mismatch between the simultaneous multi-linear prediction system and sub-model ϑ concerning d_I and y_i
$\boldsymbol{\varphi_{y_i,u_i}^{\vartheta}}$	Model mismatch between the simultaneous multi-linear prediction system and sub-model ϑ concerning u_I and y_i
$\boldsymbol{\chi}$	Flexibility function
$\boldsymbol{\psi}$	Feasibility function
ω	Frequency

Acronyms List

Acronym	Definition
BIBO	Bounded-Input Bounded-Output
BWA	Analytical Bounds Worst-case Approach
CLF	Control Lyapunov Function
CV	Controlled variable
DAE	Differential Equations
DC	Disturbance Cost
DCN	Disturbance Condition Number
DIC	Decentralized Integral Controllable
DMC	Dynamic Matrix Control
DOI	Dynamic Operability Index
EMOP	Economic MPC Optimisation index
EMPC	Economic MPC
GDP	Generalised Disjunctive Programming
GPC	Generalized Predictive Control
HEN	Heat Exchanger Network
HWA	Hybrid Worst-Case Approach
IC	Integral Controllable
ICI	Controllable with Integrity
ICPS	Integrated control and process synthesis
IHMPC	Infinite Horizon Model predictive control
IMC	Internal Model Control
IPDCF	Integrated process design and control framework
ISE	Integrated Squared Error
KKT	Karush-Kuhn-Tucker
LCA	Life Cycle Assessment
LDMC	Linear Dynamic Matrix Control
LQR	Linear–Quadratic Regulator
MAC	Model Algorithmic Control
MIC	Morari Indexes of Integral Controllability
MIDO	Mixed-Integer Dynamic Optimisation

MIMO	Multiple Inputs Multiple Outputs
MINLP	Mixed-Integer Nonlinear Problem
MIOCP	Mixed-Integer Optimal Control Problem
MPC	Model predictive control
MV	Manipulated variables
NLP	Nonlinear Optimisation Problem
NMPC	Nonlinear Model Predictive Control
OCI	Output Controllability Index
OP	Operating point
OPEX	Operating expenses
PID	Proportional–integral–derivative
PWA	PieceWise Affine
PWARX	PieceWise Autoregressive Exogenous
QDMC	Quadratic Dynamic Matrix Control
QP	Quadratic program
RHPT	Right Half-Plane Transmission
RTO	Real-Time Optimisation
SARX	Switched Autoregressive Exogenous
SISO	Single-Input Single-Output
SMLP	Simultaneous Multi-Linear Prediction
SP	Setpoint
SQP	Sequential Quadratic Programming
SVA	Structured Singular Value Analysis

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1 Introduction and Motivation

1.1 The Integrated Process Design and Control Framework

Optimisation methodologies are applied to several areas of chemical engineering to solve a wide range of problems. Among these, two of the most relevant are process control and optimal process design. Both require careful consideration at the design phase of chemical engineering projects.

The design of a new chemical plant, or process synthesis, involves projecting process equipment and assembling them in the correct layout to meet production goals (carry out a chemical reaction, separation process, etc.). During the design phase, the project engineer should optimise the correlation between return on capital and invested capital. Sometimes achieving higher efficiency and smaller operational costs may offset a larger initial investment, and such trade-offs are fundamental to the process synthesis problem.

On the other hand, the design of a control system involves analysing the dynamic behaviour of a certain plant and selecting a convenient control structure and a set of tuning parameters to achieve the desired performance. The control system must be able to reject disturbances successfully, and to keep stable the key variables. The unstable operation could lead to safety and environmental constraint violations and reduced profits. Different control structures need to be tested for any given process flowsheet, and their performance must be evaluated and benchmarked to provide input to the decision process.

The traditional approach has been solving these problems separately, in a sequential approach. In industrial projects, they are often carried out by different groups of professionals with different expertise. Usually, the first step is the creation of a flowsheet by the process design team to meet production requirements, in which the process route, equipment layout, products and raw materials are defined based on an optimal set of steady-state conditions. Next, the control design team evaluates process dynamics in order to assess whether or not the desired set of operating conditions can be achieved and maintained. The strictly necessary modifications to this end are then proposed, i.e., at this phase changes are normally kept at the bare minimum with a view to avoiding delays and friction between the teams. This situation is far from ideal because even if control and optimal process design may often be regarded as separated areas in chemical engineering practice, in reality, they are deeply related. Process design decisions have a very significant impact on control system performance, which in turn is extremely important to guarantee stable and profitable operation.

In the chemical industry, the goal of any chemical plant is to produce products that fulfil all specifications while obtaining maximum revenue with minimum cost. To reach such a goal it is necessary to provide the plant with a correctly engineered control system, which must possess a convenient set of controlled and manipulated variables, clearly defined control objectives and convenient tuning parameters. The operation must be stable and optimised, and the plants must be operated as flexibly as possible to adapt satisfactorily to changes in the process such as varying product demand and specifications, and oscillation in feed composition, flowrate, pressure and temperature. In such a context, the application of appropriate process control strategies allows for the successful operation of the plants improving profitability by increasing product throughput and yield of higher valued products and by decreasing energy consumption and pollution. They also help process automation, which reduces operational costs.

Many recent works published in control theory have focused on the development of new algorithms, but we believe this field has matured, and larger gains may be achieved by switching the focus back to the process itself, and especially its design phase. Assuming that the control system is well engineered, the limitations on its ability to control and optimise chemical plants are mostly related to the plant's characteristics. Ultimately, the degree to which controlled variables can keep at their desired values in the face of disturbances and saturation of control elements is defined by process dynamics, which, in turn, is reflected in the plant's model.

For example, plant equipment can be undersized from a control performance perspective leading to intrinsically poor control performance. Or an exceedingly small feed drum may imply in the impossibility of properly controlling the feed flow rate. To avoid uncontrollable plants, a common solution has been enlarging equipment to ensure a stable dynamic behaviour. Adding these overdesign factors certainly improves control response, but if these are exaggerated, they may also lead to the specification of unnecessarily expensive designs. In brief, a trade-off normally exists between capital cost savings and the robustness of the plant design.

How can the correct equipment size be known? How to select the best plant layout? The integrated process design and control framework (IPDCF) has been suggested as an attractive alternative to overcome the issues associated with the sequential approach traditionally used in the design of industrial process units (Sharifzadeh, 2013). It consists of solving, iteratively or simultaneously, both the control and flowsheet design optimisation problems while adding stability concerns as restrictions. In this way, a systematic analysis of plant dynamics is incorporated into the process design procedure to obtain a compromise solution between profitability and smooth and stable operation.

Significant progress has been made in the IPDCF, but untapped opportunities for contributions remain. The present work is an addition to the IPDCF aiming at providing a new analytical tool to assess plant design. The goal of this project is to address the limitations possessed by currently available methodologies in some situations that will be discussed in the next Sections.

1.2 A Classification for Integrated Process Design and Control Methods

The IPDCF is based on the fact that the achievable dynamic performance is a property of the plant and inherent to process design. The performance will depend on aspects of the plant units and their configuration, creating both unit and system holdups and sensitivities, and on the type of control exercised (Morari, 1983a,b; Edgar, 2004). The integrated design philosophy contemplates the important trade-off between profitability and controllability, incorporating the assessment of dynamic behaviour in the initial steps of process design. Predicting whether dynamic behaviour requirements are met as early as possible in the design phase is greatly advantageous since this information unlocks economic benefits and improved plant operation. Consequently, there has been a growing recognition of the need to consider the controllability and resiliency of a chemical process during design stage (Pistikopoulos and Van Schijndel, 1999).

According to Lewin (1999), IPDCF methods can be classified into two main classes: (i) methods which enable screening alternative designs for controllability, henceforth referred as Controllability Analysis; and (ii) methods which integrate the design of the process and the control system, henceforth referred as Integrated Control and Process Synthesis (ICPS). The fundamentals of both classes are now going to be provided, but further details can be found in the Literature Review, Chapter 2.

The first class focuses on plant controllability. Roughly, the concept of controllability denotes the ability to control the main variables of a process unit around their desirable values using only certain admissible manipulations. The exact definition varies within the framework or the type of models applied. Controllability is a concept that arises from the analysis of the fundamental limitations to the performance of control systems, which were first studied in a systematic way in Morari (1983a,b) making use of the perfect control concept. These studies gave birth to a series of indicators for the evaluation of open and closed-loop controllability, allowing comparison and classification of flowsheets regarding operational characteristics. Some measures using this concept include resiliency indices (Morari, 1983b), disturbance condition numbers (Skogestad and Morari, 1987) and the relative disturbance array (Stanley, Marino-Galarraga and McAvoy, 1985).

While this class of methods has the advantage of being easily integrated into traditional design procedures, the indices are often calculated based on either steady state or linear dynamic models, introducing significant approximations and reducing the rigour of the analysis. Also, the relation between the indices and the closed-loop performance may be unclear, which becomes evident by the large number of papers where the authors verify their findings through closed-loop dynamic simulations (Lewin, 1999). Controllability Analysis received a large number of contributions in the decades of 1980s and 1990s, but fewer in recent years. Formal definitions of the measures and methods of Controllability Analysis are given in Sections 2.1.1 and 2.1.2.

The second class, ICPS, consists of solving mass and energy balances for the flowsheet, sizing equipment and evaluating control performance at the same time for a certain flowsheet. The simultaneous optimisation of both the process and the control scheme is parameterised by means of a so-called superstructure in which dynamic performance requirements are included as constraints to optimal design. The aim is replacing the methods for early process synthesis traditionally used to obtain flowsheets, which rely on heuristic methods and simulation, by a single layer optimisation problem with embedded control structures. Alternative designs can be compared based on, for example, the Integrated Squared Error (ISE) for specific disturbance scenarios (Schweiger and Floudas, 1999), evaluating the system's dynamic feasibility and stability.

Methods of this class can be numerically intensive, limiting their applicability to small or medium scale case studies and requiring specific assumptions on the control system to be used. However, aided by the ever-increasing availability of computing power, the ICPS has received several contributions in recent years, and there is already a sizable body of work continuously expanding its practical applicability. In most of these works, a correlation between key sizing/layout parameters and control performance is established resulting in a cost function for which an optimisation solver will attempt to find the global solution. Structural changes in the process design and the control structure can be incorporated by adding integer decisions in the analysis by solving an intensive MINLP (Mixed Integer Nonlinear Programming) formulation. However, the addition of integer variables increases the problem's dimensionality, turning it even more expensive regarding required computing power and time. A downside to ICPS is that, if the optimisation is too radical, flowsheets thus designed may depend on the good functioning of the control system to be stable. Problems with sensors and control elements such as valves are bigger issues for these precisely designed plants which, by design, have little room available for control system malfunctioning. ICPS is discussed in further detail in Chapter 2 for classic feedback control, and in Section 2.3 for Model Predictive Control.

1.3 Model Predictive Control and the Monetisation of Control Performance

One way of monetising control performance is to evaluate the profitability or OPEX (Operating expenses) of the industrial unit with the control system activated, then deactivate the system and evaluate it again, and then subtract the latter figure from the former. The difference observed, i.e, the increase in profit (or reduction of expenses), can be explained by the reduced variability of the operating conditions that arise from the actions of the control system when the system successfully controlled.

But how does reduced variability leads to higher revenue? It allows the operating team to drive the process closer to restrictions, enabling the reduction of the product quality giveaway. The giveaway gap is the difference between the quality of the product and the quality specification. It means that the manufacturer produces products of better quality than needed, which has an effect on the cost (lower yield, more energy, higher temperature, more reflux, etc.). Restrictions related to safety also restrict profitability, e.g., the maximum temperature and pressure admissible by a chemical reactor nay restrict conversion. Hence a well-engineered control system is able to maximise operating revenue of the plant through the expansion of the range of feasible and stable operating points (OP), which can be sustained without producing off-spec products or compromising safety.

Any configuration of the control system is able defines the sustainable OP range up to a certain degree, but some types of controllers yield broader ranges than others and for the same control system configuration, the tuning parameters being used also affect the OP range. Most of the methodologies developed for the IPDCF of chemical processes have feedback controllers such as PID (proportional–integral–derivative) embedded in the analysis. PIDs are SISO (Single-Input Single-Output) feedback controllers, which normally operate with a single setpoint (SP), and are standard in the chemical industry as well as numerous other applications. Ideally, SPs are set at each variable’s economic optimal OP, as defined by the project team.

In practice, for multivariable problems, this approach introduces a well-known conflict between control goals. The issue arises as each PID tries to keep its controlled variable (CV) at the required SP without regard to disturbing other variables. Since most systems do not possess enough degrees of freedom, either due to the lack of manipulated variables (MV) or saturation of control elements, meeting all control goals simultaneously is frequently impossible (González and Odloak, 2009). Advanced “decoupling” techniques can help lessen the problem, but nevertheless, interactions between controllers inevitably impose limits to feasible OPs. In short, one should look at the bigger picture and consider the problem of interactions between SISO controllers to define optimal SPs.

Model Predictive Control (MPC) is a more powerful kind of control structure that has matured for almost four decades of development in which it has been widely implemented and recognised. MPC control schemes are popular solutions to meet the control requirements of complex chemical processes due to their capacity for handling multivariable systems with the inverse response, as well as time delayed and highly nonlinear systems. MPC is a far more suitable for use with MIMO (Multiple Inputs Multiple Outputs) systems with strong interactions since it controls simultaneously all variables and minimises the global error according to control goal prioritisation (Morari and Lee, 1999). More information concerning MPC is provided in Section 2.2.

The MPC packages may replace PID controllers entirely in some processes, but they are more commonly encountered operating in conjunction with them. In this arrangement, MPCs normally occupy a higher position in the control hierarchy and provide SPs to PIDs (Scattolini, 2009). Moreover, by introducing economic goals in their objective functions or integrating with a Real-Time Optimisation (RTO) layer, some MPC schemes are designed to drive the plant as close as possible to the economically optimal operating point (Limon et al., 2013). Fig. 1 presents the standard hierarchy of control systems found in many industrial operations (Zanin et al., 2002), which includes process instrumentation (sensor and valves), the regulatory control system, i.e., basic control loops (PIDs), and the advanced control systems such as MPC and RTO.

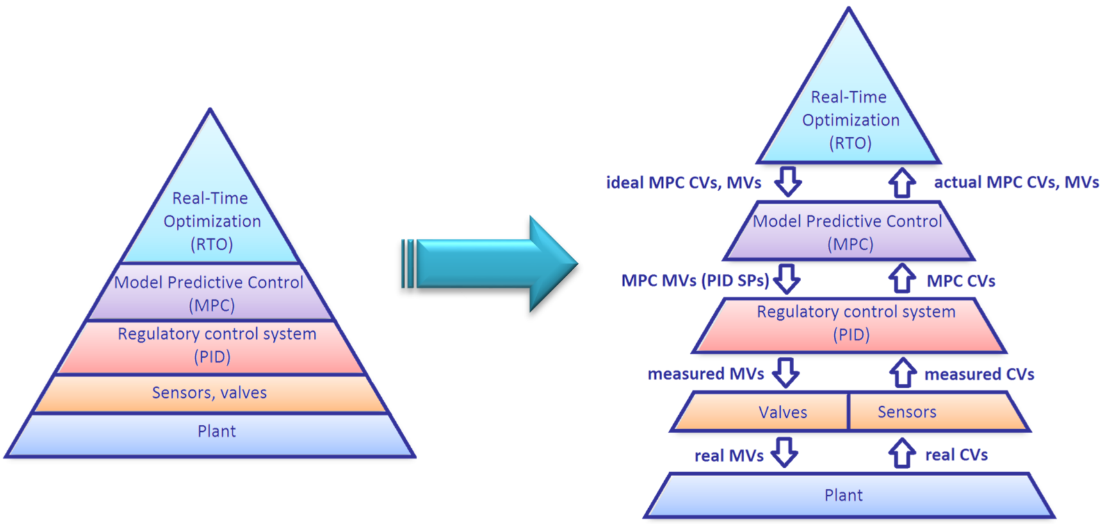

Fig. 1 – The typical hierarchy of control systems.

In IPDCF methodology presented in this work, the structure of control systems presented in Fig. 1 is assumed to be present in the future industrial unit, which means that the control system being engineered should be able to drive the plant to the optimal OP. To this end, it is especially important in this scheme the presence of the MPC layer, which may have by itself economic optimisation capabilities or work in conjunction with an RTO system - it does not matter the exact layout. Fig. 2 presents a simplified scheme to illustrate the economic benefits adding such an MPC to the usual regulatory control. With only the regulatory control system activated, the operating team must set the operating conditions at a safe distance from key restrictions, according to the observed process variability. When the MPC is activated in conjunction with the regulatory control, variability is further reduced and thus it becomes safe to drive the process closer to restrictions.

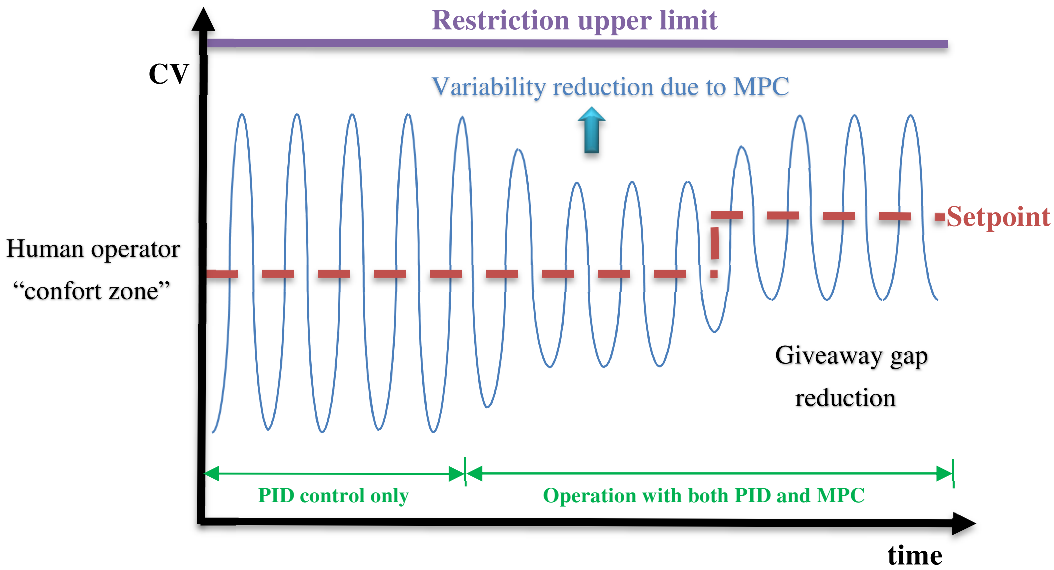

Fig. 2 – MPC can be used to minimise the quality giveaway.

Hence MPC increases control performance monetisation by increasing plant revenue. Because of such interesting characteristics, incorporating MPC in the IPDCF is a desirable development which has sought after by a number of researchers, whose results are discussed in Section 2.3. Some of these works share with this Thesis the goal of optimising flowsheets jointly for both MPC and feedback controllers. This motivation is explored further in Section 1.4.

1.4 Project Motivation

For the foreseeable future, some limitations will exist to the complexity of problems that can be solved through Integrated Control and Process Synthesis (ICPS). To avoid these limitations, which are mainly the complexity of the optimisation problem and the long computational time to solve it, and the significant engineering effort required for process modelling (see the discussion in Chapter 2), this Thesis makes a contribution to the Controllability Analysis framework, albeit in an innovative way. Our goal is to provide tools to be used alongside the usual heuristic methods for early process synthesis, addressing some questions still unanswered by currently available Controllability Analysis methods.

Normalised stability indexes such as controllability and resiliency are not directly linked to revenue and profitability, which is a point for improvement. Stability is of course required but by itself, considered in from process economics, it might be a poor guide for selecting the flowsheet. Analysis of dynamic behaviour can be used to assess what operational point results in the optimal operational return while meeting the minimum stability requirements. Generally speaking, control systems must enable secure and stable process operation, environmental compliance, on spec product quality and economic optimality. Since these goals are deeply related to plant revenue and can often be translated into monetary values, monetising control performance is possible and arguably should be one of the objectives of Controllability Analysis. For this reason, it would be desirable to have an approach relating explicitly the process dynamics and the amount of the control effort to operational revenue.

The speed with which a controller can return the process to the original operating region is less important than the ability to operate close to the restrictions on the controlled variables since it is a well-known fact that economically optimum OPs usually lie on constraint intersections (Narraway and Perkins, 1993). Nowadays it is not such a strong assumption that, if sensors and control elements work properly, every one of the available world-class MPC packages will be able to drive the process to this optimal OP (Angeli, 2012) since they possess economic optimisation capabilities. But if we can assume this as a fact, how should the design of chemical processes be affected? This Thesis provides a new methodology with embedded MPC that can be used to select the best among a number of candidate process designs for any given continuous or semi-continuous process. The following questions need to be addressed to accomplish this goal:

- Within the range of all possible operating points or conditions available for a given plant, what is the most profitable?
- Is the path from an arbitrary initial operating point to this desired point feasible? Does it violate operational constraints? To what extent? Are these violations acceptable?
- Can the optimal operating point be sustained by the said plant?
- Given a number of different plant designs, how does each plant's optimal operating point compares based on revenue?
- Given a set of controlled and manipulated variables and their bounds, what effect has a certain plant layout modification on monetised MPC performance?
- How changes in product specifications affect plant revenue? Should they lead to further changes in the layout?

Hence, the problem being addressed is finding the optimal feasible operating point that exists within the range of possible conditions for a certain flowsheet while assuming MPC. To this end, we must also consider control goal definitions of the embedded MPC structure, which affect this problem. While the optimal operating point analysis will be similar to that of PIDs if the embedded MPC has fixed SPs, MPC can also operate with flexible control goals known as "control zones". The use of control zones changes how the optimal feasible operating point of a flowsheet is defined in comparison to a fixed SP approach. In this case, a variable can move inside its control zone without penalisation (see the description of this MPC formulation in Section 2.2.2). Fig. 3 presents a diagram showing that the interaction of control and process design can affect not only the optimum operating point but also the desired trajectory to reach it from an initial state.

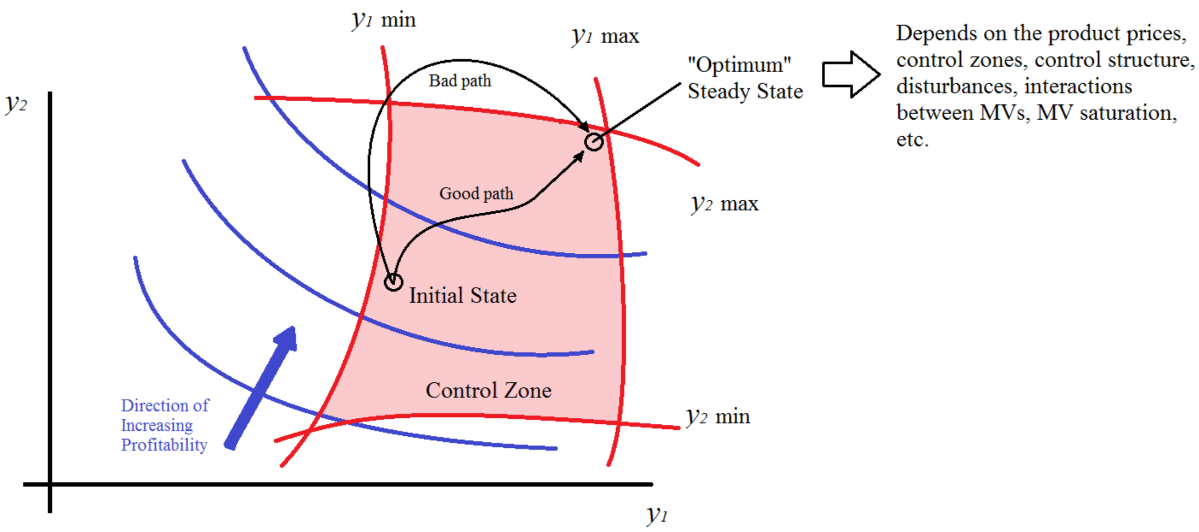

Fig. 3 – Searching for the best path to the optimal operating point for a control zone system with two controlled variables.

Zone constrained MPC has become the advanced control method of choice in the chemical industries such petroleum refining and petrochemical processes. Being a very useful variant of MPC, it is time for a methodology addressing it to be included in the IPDCF. Hence, a method to assess dynamic behaviour valid for any zone constrained MPC is presented in this Thesis. It consists of evaluating in a number of scenarios the Economic MPC Optimisation (EMOP) index of each alternative plant design. The EMOP index is an assessment tool based on a monetised measure of the required control effort, enabling flowsheet benchmark.

1.5 Thesis Organisation

This Thesis is organised into eight Chapters. This introduction was the first Chapter. Chapter 2 is the Literature Review, which contains a comprehensive review of the IPDCF (Integrated Process Design and Control Framework), a short review of MPC and a review of existing IPDCF strategies that embed MPC structures. Chapter 3 details the EMOP index methodology, a new IPDCF method; Chapter 4 contains the

Simultaneous Multi-Linear Prediction (SMLP), a multiple state-space model approach developed to improve the accuracy of the EMOP index by reducing nonlinearity-related error. In Chapter 5, an extension is presented to address the issue of model uncertainty. Chapter 6 features a case of study concerning the assessment of four possible layouts for a crude oil distillation unit for which the full methodology is applied. The results obtained for this case study are discussed and interpreted. Chapter 7 compares the EMOP index methodology to another integrated process design and control method by applying them to the same case study, the active sludge of a wastewater treatment plant. In Section 7.2, the same model is used to benchmark the SMLP in comparison to other multi-model approaches. Final conclusions are presented in Chapter 8.

2 Literature Review

This Literature Review Chapter of this PhD Thesis presents a comprehensive survey of the Integrated Process and Control Design Framework.

2.1 Review of Integrated Process and Control Design Methodologies

The main contribution of this Thesis can be classified as part of the Integrated Process and Control Design Framework, and for this reason, a general view of it is presented in this Chapter. Relevant concepts are presented in this Chapter intending to provide a theoretical background for the EMOP index methodology. This Literature Review is organised in following items:

1. Concepts and Measures of Controllability;
2. Process-Oriented Methods for Controllability Analysis;
3. Integrated Process Design and Control Framework - Methods of Integrated Control and Process Synthesis;
4. Review of Model Predictive Control;
5. Review of Integrated Process Design and Model Predictive Control Methodologies;
6. The Linear Hybrid Systems framework;
7. Conclusions from the Literature Review.

Key controllability concepts and measures are discussed in the first item, as well as the fundamental limitations to control performance. The second item focuses on Controllability Analysis methods that deal with complications such as systems with recycles, systems with steady-state multiplicity and methods that integrate rigorous modelling and passivity theory. The third item presents methods with embedded Controllability Analysis which also present integrated design optimisation problems. Such integrated problems have design variables to affecting feedback control structures, plant layout and sizing parameters. These first three items make up the bulk of the IPDCF. Being numerous and sharing fewer similarities to the EMOP methodology, these works are only briefly discussed, with a view to the completeness of this review, through Section 2.1.

The fifth item is discussed in Section 2.3 and consists of integrated design and control papers with embedded MPC. Since there are few papers in this category and these are more closely related to this PhD project, an elaborate discussion is provided for each one of them. Item four consists of a quick introduction to MPC, which is presented in Section 2.2.

2.1.1 Concepts and Measures of Controllability

This Section introduces a series of efforts made over the years towards measuring controllability. A plant is controllable if there exists a controller (connecting plant measurements and plant inputs) that yields acceptable performance for all expected plant variations (Skogestad and Postlethwaite, 1996). According to this view, controllability is independent of the controller, being a property of the plant (or process) alone. Thus, it can only be modified by changing the plant itself, that is, by (plant) design changes. Such changes may include:

- Changing the apparatus itself, e.g. type, size, layout, etc.;
- Relocating sensors and actuators;
- Adding new equipment to dampen disturbances;
- Adding extra sensors;
- Signal Processing, e.g., signal filter;
- Adding extra actuators;
- Changing the control objectives.

A plethora of methods were proposed over the years with the purpose of evaluating a plant's sensitivity to disturbances, which can be controller-independent (open loop analysis) or not (closed loop analysis). 'Controllability Analysis' currently covers to the assessment of flowsheet properties such as State Controllability, resiliency, flexibility, operability, switchability and stability, among others, all of which will be reviewed in the next subsections. These measures are often summarised in the form of indexes that show the effects of perturbations on controlled variables and operational constraints and their propagation through the process.

Let us start this discussion on methods of Controllability Analysis defining the "Input-Output" controllability. The evaluation of controllability was first introduced by Ziegler and Nichols (1943). Skogestad and Postlethwaite (1996) provided the following definition for Input-Output controllability:

Definition 2.1.1.1 Input-output controllability *is the ability to achieve acceptable control performance; that is, to keep the outputs (*$\boldsymbol{y}$*) within specified bounds or displacements from their references (*$\boldsymbol{y^{sp}}$*), in spite of unknown but bounded variations, such as disturbances (*$\boldsymbol{d}$*) and plant changes (including uncertainty), using available inputs (*$\boldsymbol{u}$*) and available measurements (*$\boldsymbol{y^m}$ *or* $\boldsymbol{d^m}$*).*

This seminal idea implies that the process performance depends on the availability of both measured and manipulated variables. Input-Output Controllability can also be described as the ability of an external input (the vector of controlled variables) to move the output from any initial condition to any final condition in a finite time interval. Note that being controllable, in this narrow definition of controllability, does not mean that once a state is reached, that state can be maintained. It merely means that said state can be reached within a reasonable timeframe. Later methodologies for investigating the open-loop, input-output controllability of a process include Biss and Perkins (1993).

The concept of "input-output" controllability, insufficient as it is to guarantee proper closed-loop dynamic behaviour, paved the way to a broader theoretical background and more restricted definitions. For instance, Integral controllability is another, more rigorous, criterium. According to Campo and Morari (1994), a plant with a given control configuration is Integral Controllable (IC) if it is stable with all controllers operating and suitably tuned and remains stable when the gains of all controllers are detuned simultaneously by the same factor $\varepsilon \in [0,1]$. If multiloop SISO controllers are used, integral controllability is an important criterion in the variable pairing. Yu and Luyben (1986) eliminated pairings with negative Morari Indexes of Integral Controllability (MIC) to ensure integral controllability. The MICs are the eigenvalues of the $\mathrm{G}^{+}(0)$ matrix (the plant steady-state gain matrix with the signs adjusted so that all diagonal elements have positive signs). If all of the individual loops are integrally controllable, a negative value of any of the eigenvalues of $\mathrm{G}^{+}(0)$ means that the variable pairing will produce an unstable closed-loop system if each loop is detuned at an arbitrary rate. It should be noticed that for 3 x 3 or higher order systems, there are instances for which no variable pairing will give MICs that are all positive.

Campo and Morari (1994) also defined systems which are Controllable with Integrity (ICI), i.e., a system that is IC, and remains IC, if any number and combination of controllers are taken out of service (set on manual mode).

Even if more restrictive, ICI is not yet the ideal degree of controllability. Usually, it is desired to design systems that reach the classification of Decentralized Integral Controllable (DIC). The system is DIC if it is ICI and remains stable when any number of controllers are detuned by individual factors $\varepsilon_i \in [0,1], i = 1, \dots, n_c$, where n_c is the number of active controllers. Besides being the most demanding property, DIC is also the one most difficult to ascertain because of the complexity of the problem (Skogestad and Morari, 1988).

Also, Wolff et al. (1992), Wolff et al. (1994) and Wolff (1994) propose procedures to evaluate the inherent control properties of chemical plants using analytic tools. Wolff et al. (1992) present a method to assess linear controllability, combining different controllability measures (RGA, Section 2.1.1.7, DCN, 2.1.1.6, among others) that complement each other for an enhanced understanding of the process behaviour. In Wolff et al. (1994) a systematic study of the operability and decentralised control system design of the total plant is presented. It involves the optimised selection of manipulated and controlled variables, and flexibility and Controllability Analysis using linear controllability indices.

The Controllability Analysis based on dynamic models may be carried out by computing some indicator of the evolution of model outputs throughout a predefined time horizon. A typical technique is to obtain the integral of the square control error (ISE) using the dynamic nonlinear model. Some examples of the use of this index can be found in Schweiger and Floudas (1997), Bansal et al. (1998), Asteasuain et al. (2006, 2007) and Revollar et al. (2010a). Also in Flores-Tlacuahuac and Biegler (2007), where aside from the ISE, they use additionally the time to steady state. In Exler et al. (2008) a set of performance indexes, including the ISE and other open loop measures, is evaluated for the activated sludge process, as well as pumping energy and the aeration energy in the system (the same process used as case study in Chapter 7).

2.1.1.1 State Controllability

Over the years the rise of the state-space framework enabled further controllability definitions. The state of a deterministic system is the set of values of all the system's state variables (those variables characterised by dynamic equations), that completely describes the system at any given time. In particular, if the states at present and all current and future values of the control actions are known, no additional information on the system is needed to predict future conditions.

The term 'State Controllability', is a concept first introduced by Kalman (1960) that describes the ability of an external input to move the internal state of a system from an initial state to any other final state in a finite time interval. A system is state-controllable if there exists an input which can achieve the desired state in a given time (Ogata, 1997). Skogestad and Postlethwaite (1996) provided a formal definition for State Controllability:

Definition 2.1.1.1.1 State Controllability. *The dynamical system* $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$, *or equivalently the pair* $(\boldsymbol{A}, \boldsymbol{B})$, *is said to be state controllable if, for any initial state* $\boldsymbol{x}(\boldsymbol{0}) = \boldsymbol{x_0}$, *any time* $t_1 > 0$ *and any final state* $\boldsymbol{x_1}$, *there exists an input* $\boldsymbol{u}(\boldsymbol{t})$ *such that* $\boldsymbol{x}(\boldsymbol{t_1}) = \boldsymbol{x_1}$. *Otherwise the system is said to be state uncontrollable.*

Rosenbrock (1970) gives a thorough discussion of the issues of State Controllability and also defines the term 'functional controllability', defined as the capability of a dynamical system of having its output changed to reproduce certain trajectories belonging to given a class output sub-spaces. This concept turns out to be related to the property of 'invertibility' of the process model, as investigated in Morari (1983a,b). Additionally, some particular concepts of controllability and observability for nonlinear systems in state-space are developed in Hermann and Krener (1977). A system is State Observable if we can obtain the value of all individual states by measuring the output $y(t)$ over some time period. According to Skogestad and Postlethwaite (1996), State Observability can be defined as follows:

Definition 2.1.1.1.2 State Observability. *The dynamical system* $\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu}$, $\boldsymbol{y} = \boldsymbol{Cx}$ *(or the pair* $(\boldsymbol{A}, \boldsymbol{B})$*) is said to be State Observable if, for any time* $t_1 > 0$, *the initial state* $\boldsymbol{x}(\boldsymbol{0}) = \boldsymbol{x_0}$ *can be determined from the time history of the input* $\boldsymbol{u}(\boldsymbol{t})$ *and the output* $\boldsymbol{y}(\boldsymbol{t})$ *in the interval* $[0;\ t_1]$. *Otherwise the system, or* $(\boldsymbol{A}, \boldsymbol{C})$, *is said to be state unobservable. Let* λ_i *be the i'th eigenvalue of* $\boldsymbol{A}$, t_i *be the corresponding eigenvector,* $\boldsymbol{A}t_i = \boldsymbol{A}\lambda_i$, *and* $y_{p_i} = \boldsymbol{C}t_i$ *the i'th output pole vector. Then the system* $(\boldsymbol{A}, \boldsymbol{C})$ *is State Observable if and only if* $y_{p_i} \neq 0, \forall i$.

In words, a system is State Observable if and only if all its output pole vectors are nonzero. Some works are found in the literature where the methodology to integrate design and control focuses in analysing the State Controllability of nonlinear systems. The works by Ochoa (2005) and Ochoa and Alvarez (2005), are interesting contributions where the integrated design is carried out to ensure the local controllability of input affine, nonlinear systems, by means of some metrics for practical controllability based on state-space theory. They concern different aspects of the process, such as the available degrees of freedom for control, the rank of the local controllability matrix, the system invertibility, and the range of available control actions and the existence of a linear reachability trajectory. These indices are examined to address problems such as misleading interactions between inputs and states, wrong selection of manipulated variables or final control elements and physical restrictions of the states, which preclude the assurance of practical controllability. The procedure uses the phenomenological model of the process and selects the manipulated variables and the best structure (pairing) for control. It also includes the determination of the available operation range for the input variables and the selection of perturbations tolerances under different scenarios. The method addresses the plant optimisation as a function of the investment and operation costs while including the evaluation of the controllability metrics and considering the restrictions imposed by them. At last, the control system is designed to suit the optimal plant, knowing that its controllability is assured at the desired operating point. Authors present an ammonium-water separation process with a reactor-flash-exchanger plant, as a design example.

An extension of this work to undertake the integrated design of coupled systems is found in Munoz et al. (2008), where a methodology is proposed to verify the controllability of coupled systems based on the computation of the accessibility distribution and the controllable/non-controllable states decomposition. In Alvarez (2008) the Hankel Matrix is proposed as controllability measure. In Lamanna et al. (2009) the state-space practical Controllability Analysis is used to impose restrictions in the integrated design of a sulfidation tower by integrated-optimisation methods. Calderón et al. (2012a) propose the redesign for a wastewater treatment plant based on the results of the nonlinear State Controllability Analysis of the system. The set theory is used to check the controllability limits of the system including disturbances limits and constraints on control inputs. In Calderón et al. (2012b) a comparison between differential geometric and set-theoretical (randomised algorithms) methods to consider the nonlinear State Controllability is presented. A detailed description of the methodology to assess nonlinear State Controllability in the integrated design framework can be found in Alvarez (2012).

2.1.1.2 Dynamic Resiliency - Perfect Control and its Limitations

Morari (1983a, 1983b) suggested making the problem of controllability assessment independent of the controller selection problem. This is done by finding a plant's best achievable closed-loop control performance for all possible constant parameter linear controllers. This target, the upper bound on the achievable closed-loop performance, is defined as the plant's dynamic resilience. Thus, "dynamic resilience" is an expression of the plant's inherent limitation on the closed-loop system's dynamic response which is not biased by specific choices of controllers. Furthermore, dynamic Resiliency is concerned with fast and smooth changeover and recovery from process disturbance, with the ultimate goal of determining the inherent dynamic characteristic of a plant independently of the selection of a particular controller or another.

The higher the Dynamic Resilience of a plant the closer it is to be a perfectly controllable plant, i.e., a plant for which perfect control is achievable. Perfect control can be defined as a series of control actions such that all the outputs are held at their nominal values in the steady state, i.e., no offset occurs. Hence, a perfect controller ensures total disturbance rejection. Let us consider a linear transfer function model of the form given by Eq. 1:

$$\boldsymbol{y(s)} = \boldsymbol{G(s)u(s)} + \boldsymbol{G_d(s)d(s)} \qquad \text{Eq. 1}$$

where $\boldsymbol{u}$ is the vector of manipulated variables (MVs), $\boldsymbol{d}$ is the vector of disturbance variables (DVs), $\boldsymbol{y}$ is the vector of controlled variables (CVs) and $\boldsymbol{G}$ and $\boldsymbol{G_d}$ are transfer function matrices describing respectively the effects of MVs and DVs. All these parameters of Eq. 1 are Laplace transforms in the s-domain, where s is a complex number frequency parameter. The objective is to minimise the error, $\boldsymbol{E} = \boldsymbol{y} - \boldsymbol{y^{sp}}$, where $\boldsymbol{y^{sp}}$ is the vector of reference signals (setpoints). Perfect control is theoretically possible when we have at least as MVs inputs as CVs. Mathematically, the set of perfect control actions is defined as the solution to the following problem:

$$\boldsymbol{U_{min}} = \max_{\boldsymbol{d}} \min_{\boldsymbol{u}} \|\boldsymbol{u}\| \;\; \boldsymbol{s.t.} \;\; \boldsymbol{Gu} + \boldsymbol{G_d d} = \boldsymbol{0} \qquad \text{Eq. 2}$$

Where omitted the Laplace transfer function notation. For square plants (same number of inputs and outputs) the perfect control problem of Eq. 2 has a unique solution given by Eq. 3:

$$\boldsymbol{u} = -\boldsymbol{G^{-1}G_d d} \qquad \text{Eq. 3}$$

Eq. 3 represents a perfect feedforward controller, assuming d is measurable. An important fact is that perfect control requires the controller to somehow generate an inverse of G. According to Morari (1983a), perfect control cannot be achieved if:

- $\boldsymbol{G}$ contains Right-Half-Plane-Transmission zeros (since then $\boldsymbol{G^{-1}}$ is unstable and has an inverse response, see Section 2.1.1.4);
- $\boldsymbol{G}$ contain time delays (since then $\boldsymbol{G^{-1}}$ contains a prediction);
- $\boldsymbol{G}$ has more poles than zeros (since then $\boldsymbol{G^{-1}}$ is unrealisable);
- $\boldsymbol{G}$ is uncertain (since then the exact $\boldsymbol{G^{-1}}$ cannot be obtained);
- Input vector $\boldsymbol{u}(\boldsymbol{s})$ reaches saturation.

Since one or more of those restrictions are always present for industrial plants, perfect control is not physically realisable (Skogestad and Wolff, 1992). Controllability Analysis involves using qualitative and quantitative methods to assess how controllable is a plant, i.e., how close its achievable control performance is to perfect control.

Let us address the issue of model uncertainty. $\boldsymbol{G}$ may be uncertain due to a series of factors. For instance, frequently the exact values of parameters that are inputted to the model are unknown and cannot be exactly inferred. Another kind of uncertainty is the parametric variability, that comes from the variability of input variables of the model. Structural uncertainty may also be presented, which is also known as model inadequacy, model bias, or model discrepancy. It depends on how accurately a mathematical model describes the true system for a real-life situation, because models are almost always only approximations to reality. Algorithmic uncertainty, also known as numerical uncertainty or discrete uncertainty, arises from numerical errors and numerical approximations per implementation of the computer model, whose implementation is necessary when models are too complicated to solve exactly (a frequent situation in chemical engineering). Experimental uncertainty, also known as observation error, this comes from the variability of measurements, which is inevitable and can be noticed by repeating a measurement for many times using the same settings for all inputs/variables. Details concerning the plethora of uncertainty definitions can be found in Iman and Helton (1988). An adequate definition of model uncertainty for the purpose of process control was provided in Skogestad and Morari (1986):

> **Definition 2.1.1.2.1 Model uncertainty.** *Assuming that the plant $\boldsymbol{P}$ is linear and time invariant, but that its exact mathematical description is unknown. However, it is known to be in a specified "neighbourhood" of the "nominal" system, whose mathematical "model" $\widetilde{\boldsymbol{P}}$ is available. This neighborhood will be denoted the uncertainty "set"; it defines the set of possible plants $\boldsymbol{\Pi}$. In some cases the uncertainty set $\boldsymbol{\Pi}$ may include a finite number of plants.*

However, in most cases we will define $\boldsymbol{\Pi}$ in terms of norm-bounded perturbations on $\boldsymbol{P}$, and the set $\boldsymbol{\Pi}$ becomes infinite.

Skogestad and Morari (1986) studied the effects of model uncertainty on achievable control performance, namely the Dynamic Resilience of the plant. They define "Performance" as the quality of the closed-loop response, which is typically related to the error signal for a standard feedback controller such as the one in Fig. 4.

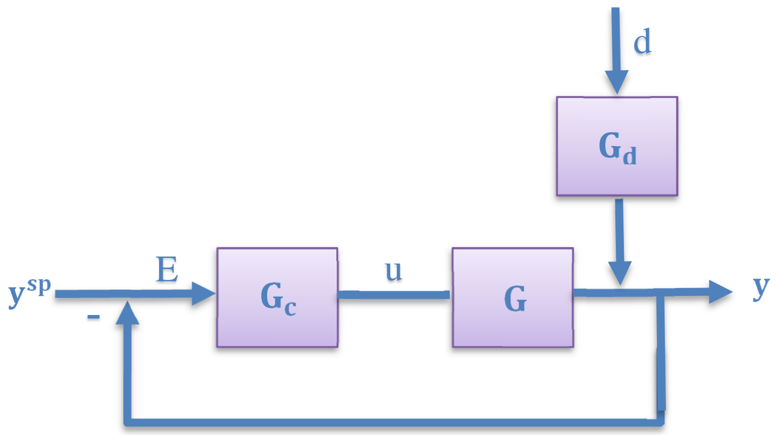

Fig. 4 – Feedback system with controller $\boldsymbol{G_c}$, disturbance $\boldsymbol{G_d}$ and plant $\boldsymbol{G}$.

The error signal $(\boldsymbol{E})$ should be small for the expected disturbances $(\boldsymbol{d})$ and reference signals $(\boldsymbol{y^{sp}})$. For a one-degree-of-freedom proportional controller given by $\boldsymbol{G_c = K(s)}$, whose input is $\boldsymbol{y^{sp} - y^m = y^{sp} - y - n}$, where $\boldsymbol{y^m}$ is the measured output and $\boldsymbol{n}$ is the measurement noise, the input to the plant becomes:

$$\boldsymbol{u = K(s)(y^{sp} - y - n)} \qquad \text{Eq. 4}$$

Replacing Eq. 4 in Eq. 1 and omitting the Laplace transfer function notation the feedback control system becomes:

$$\boldsymbol{y = GK(y^{sp} - y - n) + G_d d} \qquad \text{Eq. 5}$$

Eq. 4 can be rearranged to yield:

$$\boldsymbol{(I + GK)y = GKy^{sp} + G_d d - GKn} \qquad \text{Eq. 6}$$

Hence the closed-loop response is:

$$\boldsymbol{y = \underbrace{(I + GK)^{-1}GK}_{T}\, y^{sp} + \underbrace{(I + GK)^{-1}}_{S}\, G_d d - \underbrace{(I + GK)^{-1}GK}_{T}\, n} \qquad \text{Eq. 7}$$

Where S is the Sensitivity Function and T is the Complementary Sensitivity Function.

The control error is

$$\boldsymbol{E} = \boldsymbol{y} - \boldsymbol{y}^{sp} = -\boldsymbol{S}\boldsymbol{y}^{sp} + \boldsymbol{S}\boldsymbol{G}_d\boldsymbol{d} - \boldsymbol{T}\boldsymbol{n} \qquad \text{Eq. 8}$$

In Eq. 8 we have used the fact that $\boldsymbol{T} - \boldsymbol{I} = -\boldsymbol{S}$. The corresponding plant signal is provided by

$$\boldsymbol{u} = \boldsymbol{K}\boldsymbol{S}\boldsymbol{y}^{sp} - \boldsymbol{K}\boldsymbol{S}\boldsymbol{G}_d\boldsymbol{d} - \boldsymbol{K}\boldsymbol{S}\boldsymbol{n} \qquad \text{Eq. 9}$$

When the measurement noise can be neglected the Sensitivity Function ($\boldsymbol{S}$) describes the relationship between $\boldsymbol{y}^{sp}$, $\boldsymbol{d}$ and $\boldsymbol{E}$. The term Sensitivity Function is natural because $\boldsymbol{S}$ gives the sensitivity reduction afforded by feedback.

$$\boldsymbol{E} = \boldsymbol{y} - \boldsymbol{y}^{sp} = \boldsymbol{S}(\boldsymbol{G}_d\boldsymbol{d} - \boldsymbol{y}^{sp}) \qquad \text{Eq. 10}$$

To have "good" performance, $\boldsymbol{S}$ has to be "small". The magnitude of $\boldsymbol{S}$ may be measured using the singular value $\overline{\boldsymbol{\sigma}}$. At a given frequency $\boldsymbol{\omega}$, $\overline{\boldsymbol{\sigma}}\big(\boldsymbol{S}(\boldsymbol{j\omega})\big)$ represents the "worst" amplification ($\|\boldsymbol{E}\|_2/\|\boldsymbol{G}_d\boldsymbol{d} - \boldsymbol{y}^{sp}\|_2$) of $(\boldsymbol{G}_d\boldsymbol{d} - \boldsymbol{y}^{sp})$. By "worst" we mean that $(\boldsymbol{G}_d\boldsymbol{d} - \boldsymbol{y}^{sp})$ is in the direction giving rise to the largest amplification. A typical performance specification is provided by Eq. 11:

$$\boldsymbol{E}\overline{\boldsymbol{\sigma}}(\boldsymbol{S}) \leq \left|\boldsymbol{w}_p\right|^{-1} \qquad \text{Eq. 11}$$

where $\boldsymbol{w}_p(\boldsymbol{s})$ is a weight which is used to define what dynamic responses are acceptable. Therefore, the Sensitivity Function is a measure of the Dynamic Resilience of a plant.

Weitz and Lewin (1996) proposed a procedure for Dynamic Resilience analysis that relies on a modelling strategy which makes use of linear approximations, which are obtained from the simulation of the steady-state flowsheet. Karafyllis and Kokossis (2002) introduced Disturbance Resiliency Index (DRI) as a measure for the integration of design and control. The measure reflects on the ability of the process to reject disturbances and prevent saturation in the manipulated variables. The measure is defined mathematically, and a set of properties and theorems are proved to enable its use. For a large number of systems and networks, the application of the theory yields analytical expressions one can study and analyse. In other cases, it yields bounding expressions that one can embed in optimisation formulations and mathematical models. The measure quantifies the disturbance resiliency properties of a process. Compared to the works of Halemane and Grossmann (1983) and Grossmann, Halemane and Swaney (1983), it has the advantage of a clear extension to the dynamical properties of the system.

2.1.1.3 Feasibility and Flexibility Analysis

Flexibility is the ability to obtain feasible steady-state operation at a number of given operating points, i.e. over a range of uncertain conditions. These uncertain conditions can be defined from expected variations in raw material and process performance.

The flexibility analysis involves two important problems: the feasibility test and the quantification of the inherent flexibility of a process (Grossmann and Morari, 1984). The feasibility test problem determines the existence of at least one set of manipulated variables that can be selected during plant operation, such that, for every possible realisation of the uncertain parameters all the process constraints are satisfied (Halemane and Grossmann, 1983).

Let us see how these problems we can be systematically addressed through mathematical formulations developed by Grossmann and co-workers (Grossmann et al., 1983; Grossmann and Straub, 1991). We will consider simple vertex solution methods as well as an active set method, which does not necessarily have to examine all the vertex points or even assume that critical points correspond to vertices. The basic model assumed for the flexibility analysis involves the following vectors of variables and parameters: $\boldsymbol{d}$, the vector of design variables corresponding to the structure and equipment sizes of the plant; $\boldsymbol{x}$ the state variables that define the system (e.g. flows, temperatures); $\boldsymbol{u}$ the control variables that can be adjusted during operation (e.g. flows, utility loads), $\boldsymbol{\theta}$ the uncertain parameters (e.g. inlet conditions, reaction rate constants). Eq. 12 represents a first principles model, i.e., heat and material balances:

$$\boldsymbol{h}(\boldsymbol{d}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta}) = 0 \qquad \text{Eq. 12}$$

where by definition $\dim\{\boldsymbol{h}\} = \dim\{\boldsymbol{x}\}$. The constraints that represent feasible operation (e.g. physical constraints, specifications) are given by Eq. 13:

$$\boldsymbol{g}(\boldsymbol{d}, \boldsymbol{x}, \boldsymbol{u}, \boldsymbol{\theta}) \leq 0 \qquad \text{Eq. 13}$$

Although in principle flexibility can be analysed directly in terms of Eq. 12 and Eq. 13, for simplicity in this discussion the state variables from Eq. 12 are eliminated. In this way, the state variables become an implicit function of $\boldsymbol{d}$, $\boldsymbol{u}$ and $\boldsymbol{\theta}$. That is, $\boldsymbol{x} = \boldsymbol{x}(\boldsymbol{d}, \boldsymbol{u}, \boldsymbol{\theta})$. Eq. 13 thus becomes:

$$\boldsymbol{g}(\boldsymbol{d},\boldsymbol{x}(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta}),\boldsymbol{u},\boldsymbol{\theta}) = \boldsymbol{f}(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta}) \leq 0 \qquad \text{Eq. 14}$$

Hence, the feasibility of operation of a design d operating at a given value of the uncertain $\boldsymbol{\theta}$ parameters is determined by establishing whether by proper adjustment of the control variables $\boldsymbol{u}$ each inequality $f_j(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta}),\ \mathrm{j} \in \mathbf{J}$ is less or equal to zero.

Now the mathematical formulations for both the flexibility test problem (Halemane and Grossmann, 1983) is presented. Assume we are given a nominal value of the uncertain parameters $\boldsymbol{\theta^N}$, as well as expected deviations $\boldsymbol{\Delta\theta^+}$, $\boldsymbol{\Delta\theta^-}$, in the positive and negative directions. This then implies that the uncertain parameters $\boldsymbol{\theta}$ have the upper bound as defined in Eq. 15 and lower bound as in Eq. 16:

$$\boldsymbol{\theta^U} = \boldsymbol{\theta^N} + \boldsymbol{\Delta\theta^+} \qquad \text{Eq. 15}$$

$$\boldsymbol{\theta^L} = \boldsymbol{\theta^N} - \boldsymbol{\Delta\theta^-} \qquad \text{Eq. 16}$$

The flexibility test problem (Halemane and Grossmann, 1983) for a given design d consists in determining whether by proper adjustment of the controls u the inequalities $f_j(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta}),\ j \in \boldsymbol{J}$ hold for all $\boldsymbol{\theta} \in \boldsymbol{T} = \{\boldsymbol{\theta}: \boldsymbol{\theta^L} \leq \boldsymbol{\theta} \leq \boldsymbol{\theta^U}\}$. To answer this question, we first consider whether for fixed value of $\boldsymbol{\theta}$ the controls $\boldsymbol{u}$ can be adjusted to meet the constraints $f_j(\boldsymbol{d},\boldsymbol{z},\boldsymbol{\theta})$. This can be accomplished if we select the controls u to minimize the largest f_j that is:

$$\boldsymbol{\psi}(\boldsymbol{d},\boldsymbol{\theta}) = \min_{\boldsymbol{z}} \max_{j \in \boldsymbol{J}} \{f_j(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta})\} \qquad \text{Eq. 17}$$

Where $\boldsymbol{\psi}(\boldsymbol{d},\boldsymbol{\theta})$ is defined as the feasibility function. If $\boldsymbol{\psi}(\boldsymbol{d},\boldsymbol{\theta}) \leq 0$, we can have feasible operation; $(\boldsymbol{d},\boldsymbol{\theta}) > 0$, there is infeasible operation even if we do our best in trying to adjust the control variables $\boldsymbol{u}$. If $\boldsymbol{\psi}(\boldsymbol{d},\boldsymbol{\theta}) = 0$ it also means that we are on the boundary of the region of operation. The problem is given by Eq. 17 can be stated as a standard optimization problem (LP or NLP) by defining a scalar variable z, such that:

$$\begin{aligned} \boldsymbol{\psi}(\boldsymbol{d},\boldsymbol{\theta}) = \min_{\boldsymbol{u},\boldsymbol{z}} \quad & \boldsymbol{u} \\ s.t. \quad & f_j(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta}) \leq z \qquad \forall\, j \in J \end{aligned} \qquad \text{Eq. 18}$$

In order to determine whether we can attain operation in the parameter range of interest, $\theta \in T = \{\theta: \theta^L \leq \theta \leq \theta^U\}$, we need to establish whether $\psi(d,\theta) \leq 0$ for all $\theta \in T$. This is also equivalent to stating whether the maximum value of is less or equal than zero in the range of θ. Hence, the flexibility test problem can be formulated as shown in Eq. 19:

$$\chi(\boldsymbol{d}) = \max_{\boldsymbol{\theta}\in\boldsymbol{T}}\{\boldsymbol{f}_j(\boldsymbol{d},\boldsymbol{\theta})\}$$ Eq. 19

Where $\boldsymbol{\chi}(\boldsymbol{d})$ corresponds to the flexibility function of design $\boldsymbol{d}$ over the range $\boldsymbol{T}$. If $\boldsymbol{\chi}(\boldsymbol{d}) \leq 0$, it then means that feasible operation can be attained over the parameter range T, and shown in the item a, Fig. 5. However, if $\boldsymbol{\chi}(\boldsymbol{d}) > 0$ it means that at least for part of the range of $\boldsymbol{T}$, feasible operation cannot be achieved (see item b, Fig. 5). Also, the value of $\boldsymbol{\theta}$ that is determined in Eq. 19 can be regarded as critical for the parameter range $\boldsymbol{T}$ since it is the one where the feasibility of operation is the smallest ($\boldsymbol{\chi}(\boldsymbol{d}) \leq 0$) or where maximum constraint violation occurs ($\boldsymbol{\chi}(\boldsymbol{d}) > 0$).

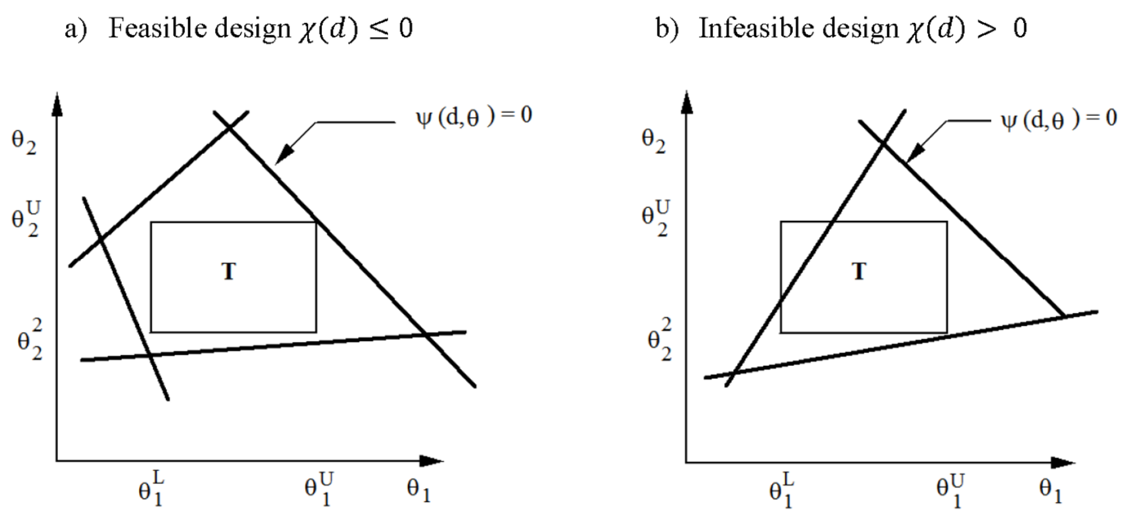

Fig. 5 – Regions of feasible operation for feasible and infeasible design (flexibility test problem).

Finally, by substituting Eq. 17 in Eq. 19, the general mathematical formulation of the flexibility test problem yields Eq. 20:

$$\boldsymbol{\chi}(\boldsymbol{d}) = \max_{\boldsymbol{\theta}\in\boldsymbol{T}} \min_{\boldsymbol{z}} \max_{\boldsymbol{j}\in\boldsymbol{J}} \; f_j(\boldsymbol{d},\boldsymbol{u},\boldsymbol{\theta})$$ Eq. 20

Works based on these principles include the iterative methodology which has been proposed in Bandoni et al. (1994) and Bahri et al. (1996) in order to analyse the steady-state flexibility of a chemical plant. In this method, to ensure the feasible operation of the plant, the optimum point is moved to somewhere inside the feasible region (called back-off point). Therefore, having disturbances in the system will not cause any constraint violation. The amount of back-off can be used as a measure of the flexibility of the plant.

Bahri, Bandoni and Romagnoli (1997) propose an integrated flexibility and Controllability Analysis based on dynamic mixed-integer nonlinear programming that consists of a two-stage problem for each iteration of the algorithm. These two stages are called outer and inner loops, and each loop has been formulated as a semi-infinite optimisation problem. Outer loops give the optimal operating conditions for a given set of disturbances. In the inner loops, the feasibility of the operating conditions proposed in the last outer loop is tested. In this stage, the disturbance realisations that produce most constraint violations are found (if any) and passed to the next outer loop. In this way, the authors claim, the effect of disturbances on the process design and operation, as well as its ideal performance, under a variety of control schemes can be estimated. The method is illustrated using a mini-integrated plant as a case study.

Bansal et al. (2000b) propose a parametric programming framework for feasibility test, flexibility index, design optimization, and stochastic flexibility of linear systems, providing an explicit information about the dependence of the system flexibility on the values of the design variables. A case study of a double-effect distillation system (where the overhead vapour from one column is used to supply the reboiler heat for another column operating at a lower pressure), for which a dual approach is described: in the first, the steady-state process design and the control system were optimised sequentially; potential operability bottlenecks are identified. In the second approach, the process design and the control system are optimised simultaneously leading to increased economic benefit. Bansal et al. (2002a) generalise and unify this approach for the flexibility analysis and design of nonlinear systems. Recent works dealing with flexibility evaluation are Lima et al. (2010a,b), Chang et al. (2009) and Adi and Chang (2011).

2.1.1.4 Operability and Switchability Analysis

Operability is the ability of the plant to provide acceptable static and dynamic operational performance. Operability includes flexibility, switchability, controllability and several other issues. Switchability is the ability to switch between operating points. For service plants, fast switching may be desirable to minimise loss of product and energy consumption. Although controllability and flexibility are strongly related concepts (Grossmann and Morari, 1984), controllability deals with the dynamic operation, and it is a measure of the achievable dynamic performance, while flexibility is focused on the steady-state operation and it is the capability to handle alternate operating conditions.

Dynamic feasibility is a commonly used term that encompasses both a static aspect, which is incorporated into flexibility and a dynamic aspect, which is part of switchability. The feasibility analysis problem is to determine if a given design can feasibly operate over the considered range of uncertainty.

In Mohideen et al. (1996a) dynamic feasibility analysis is included in the integrated design problem, verifying the operation and control constraints all over the uncertainty range of the parameters in the established time horizon.

Bahri et al. (1996a, 1997) propose the dynamic operability analysis within the process synthesis and control structure selection problem. This analysis includes aspects as stability, controllability and flexibility; its objective is to optimise the process economy subject to feasible regulatory dynamics. Stands out the use of the backwards margin based on the dynamic nonlinear model. It relates the economic aspects with the operability, by fixing the distance between the optimal steady-state operating point and the dynamic operating point of the plant. They also consider the dynamic feasibility and indexes as the ISE and the steady-state time. In Ekawati and Bahri (2003) this analysis is completed by introducing the Output Controllability Index, OCI (Vinson and Georgakis, 2000).

In Novak Pintaric and Kravanja (2004) flexibility and static operability analysis are introduced in the problem formulation by determining in a first stage, the optimal flexible structure and optimal oversizing of the process units that guarantee feasibility of the design for a fixed degree of flexibility. In a second stage, the structural alternatives and additional manipulative variables are included in the mathematical model to introduce additional degrees of freedom for efficient control. Malcolm et al. (2007) contemplates the process and control design over a set of uncertain parameters by solving the steady-state flexibility test and the dynamic flexibility test.

Vinson and Georgakis (2000, 2002) define the Output Controllability Index (OCI) or Operability Index (OI) which is a steady state and nonlinear measure of the ability of a design to reach all points of the desired output space and to reject the expected disturbances using input actions not exceeding the available input space. It has been proven to be effective for both linear and nonlinear processes. Its extension to a dynamic analysis called Dynamic Operability Index (DOI) is presented in Uzturk and Georgakis (2002). These operability analysis tools are exploited in Subramanian et al. (2001) for examining the inherent steady state operability of continuous processes, using as an example a CSTR system. They propose an approach that further extends the original OI formulation to include nonsquare systems, distinguishing different categories for process outputs: (1) set-point controlled, with outputs to be controlled at a desired value, and (2) set-interval controlled, with outputs to be controlled within the desired range. In Georgakis et al. (2003) a similar methodology is presented. An extension of the operability analysis for plantwide systems is applied to the Tennessee Eastman process in Subramanian and Georgakis (2005).

Psarris and Floudas (1991) claim that the location of Right-Half-Plane-Transmission (RHPT) zeros is essential for the assessment of dynamic operability since if RHPT zeros are close to the origin of the complex plane severe limitations are imposed to the closed-loop performance of multivariable systems. The authors claim that in multivariable systems with time delays, infinite transmission zeros may be present which makes their numerical determination and therefore the assessment of dynamic operability is very difficult. The methodology presented in Psarris and Floudas (1991) provides a tool for assessment of dynamic operability of MIMO delay systems through the characterisation of the transmission zeros by using the theory of the distribution of roots of quasi-polynomials to identify systems with infinite RHPT zeros.

Other studies which develop strategies to incorporate controllability and operability insights into the practice of process design include Grossmann and Morari (1984), Grossmann and Floudas (1987), Grossmann and Straub (1991) and Dimitriadis and Pistikopoulos (1995).

2.1.1.5 Disturbance Cost Index

Lewin (1996) presented the Disturbance Cost index, DC, a frequency-dependent resiliency index defined as the control effort required in rejecting the worst-case disturbance, intended to provide guidance as part of a modern process design methodology. As most indexes, the DC represents a measure of the control effort (which can be roughly defined as how much the controller needs to move manipulated variables compared to their ranges) required to reject the worst-case disturbance, normalised to enable comparison between different plants. The authors justified the introduction of this index by discussing the incapability of the previously mentioned measures for quantifying the disturbance resiliency properties of a linear process.

Let us assuming perfect disturbance rejection by the control system and a linear dynamic model for the process. The norm of the actuator response computed, $\|u\|$, is a function of the disturbance direction. The relative cost of rejecting a particular disturbance d can be computed as a function of its direction, .i.e., a quantitative measure of the control effort to reject a given disturbance vector is the Euclidean norm. Hence, this norm, $\|u\|$, is defined as the Disturbance Cost. The DC index was extended by Solovyev and Lewin (2002) to address nonlinear, state-space models.

Lewin and Bogle (1996) applied the DC index to perform the selection of the optimal operating point for a continuous industrial polymerization reactor. For each operating point, the DC contours were plotted as functions of disturbance frequency and direction, yielding a graphical representation of the space of disturbance realisations the control loop is able to reject.

A measure similar to the DC has been proposed in Narraway et al. (1991), which presents a method to evaluate the impact of disturbances on plant economic performance in alternative process structures or alternative control schemes for a given process. The best operating point in the absence of disturbances is obtained by nonlinear steady-state optimisation, and frequency response analysis of a linearized plant dynamic model is used to estimate the effects of disturbances on this ideal performance under a variety of control strategies. A modification of this method is presented in Narraway and Perkins (1993). In this work, they provide a measure of the best achievable economic performance as the amount that the operating point must be backed off from constraints active at the optimal operating point to accommodate the effects of disturbances. The back-off idea is also used to measure the effect of dynamical performance on economics because the required back off represent the necessary extra cost to ensure that none of the operating constraints which affect controllability is violated. Perfect control is assumed and integer programming techniques are used for screening the potential control structures which are then all subjected to controllability analyses or are used as control structures for nonlinear dynamic economic analysis.

2.1.1.6 Disturbance Condition Number

The effectiveness of disturbance suppression in a multivariable control system can depend strongly on the direction of the disturbance. Among the indexes proposed to assess the resilience of chemical plants subject to disturbances, there is the method for quantifying the effect of disturbance direction on closed-loop performance presented in Skogestad and Morari (1987). There is a bound on the magnitude was obtained for the worst-case relative gain by studying the direction of a disturbance, resulting in the introduction of the Disturbance Condition Number (DCN). For a particular plant, the DCN indicates the magnitude of the MV input magnitude necessary in order to compensate for the effect of a disturbance of unitary magnitude in comparison to a disturbance whose direction is the best possible. The "best" disturbance direction is the one requiring the least action by the MVs. The Disturbance Condition Number is only a good indication of which set-point vector will be difficult to track (Lewin, 1996).

2.1.1.7 Additional Controllability measures – the Relative Gain Array and Singular-Value Decomposition

The singular values of a matrix are a measure of how close the matrix is to be "singular", i.e., to have a determinant that is zero. The N singular values of a real NxN matrix A are defined as the square root of the eigenvalues of the matrix formed by multiplying the original matrix by its transpose:

$$\sigma_{i[A]} = \sqrt{\lambda_{i[A^T A]}}, \quad i = 1, \dots, N \qquad \text{Eq. 21}$$

The minimum singular value was introduced as a controllability index by Skogestad and Morari (1987), who suggested that a smaller minimum singular value implies that large input magnitudes may be needed, and such plants are undesirable. The SVD on G and G_d is useful for examining which manipulated input combinations have the largest effect and which disturbances give the largest output variations. It can also be used to predict directional sensitivity of a process. The process gain matrix is decomposed into three matrices, as shown in Eq. 22:

$$\boldsymbol{G} = \boldsymbol{U_s \Sigma V^T} \qquad \text{Eq. 22}$$

where U_s is the left singular vector matrix, Σ the diagonal matrix of singular values, ordered, and V the right singular vector matrix. The left and right singular vector matrices are both orthonormal matrices, i.e., each column of the matrix is orthogonal to all other columns, and the columns each are unit length. The diagonal singular value matrix is ordered so that the largest singular value is in the (1, 1) position. Note that the standard notation for SVD is to use U to represent the left singular vector matrix. When performing singular value analysis, it is important to scale the inputs and outputs to cover the same range.

"Condition number" is another important controllability index based on singular value analysis. It is defined as the ratio between the largest singular value and the smallest nonzero singular value. Plants with large condition number are called ill-conditioned and require widely different input magnitudes depending on the direction of the desired output (i.e. the plant is sensitive to unstructured input uncertain) (McAvoy and Braatz, 2003). Note that the condition number is scaling dependent.

SVD can be used to assess just how well an algebraic process control problem is posed and whether any sensitivity problems can be expected when it is solved. Early application of SVD to process control systems was carried out by researchers at the University of Tennessee (Downs and Moore, 1981; Moore, 1986). Lau et al. (1985) used SVD to design multivariable control systems. Grosdidier et al. (1985) established a quantitative relationship between the condition number of a process gain matrix and its relative gain. Skogestad and Morari (1987) derived conditions for robust stability that involved the condition number. A number of useful results on applying SVD to process control problems are collected by Skogestad and Postlethwaite (1996). This includes the result that saturation of the manipulated variable is a potential problem if the minimum singular value of the process gain matrix is less than 1. They also show that the multivariable effects of input uncertainties are small for processes with small condition numbers. McAvoy and Braatz (2003) used SVD to analyse a process gain matrix, focusing on the case where the process gain matrix has a large maximum singular value, with its minimum singular value being either large or small. It is shown that the closed-loop control of such processes can result in poor transient performance as a result of valve accuracy considerations, even if the condition number is small and the minimum singular value is large, which would indicate no performance limitations according to existing controllability criteria.

Another widely used controllability measure is the RGA which was introduced by Bristol, 1966. For instance, consider a plant defined by a square MIMO model $\boldsymbol{G(s)}$, as shown in Eq. 6:

$$\boldsymbol{y(s)} = \boldsymbol{G(s)u(s)} \qquad \text{Eq. 23}$$

The relative gain array is defined as the ratio of process gain as seen by a given controller with all other loops open to the process gain as seen by a given controller with all other loops closed and can be computed by Eq. 7:

$$\boldsymbol{\Lambda(s)} = \boldsymbol{G(s)} \otimes \left(\boldsymbol{G^{-1}(s)}\right)^{\boldsymbol{T}} \qquad \text{Eq. 24}$$

where the $\otimes$ symbol indicates element-by-element multiplication (Schur product). An important property of the RGA is that it is scaling independently. The elements in the RGA can be numbers that vary from very large negative values to very large positive values. The larger the values of the elements of RGA, the more sensitive the transfer function will be in relation to errors. The closer the number is to 1, the less difference closing the other loop makes on the loop being considered. For interactive plants which do not have large RGA elements, a decoupler may be useful. In particular, this applies to the case where the RGA-elements vary in magnitude with frequency, and it may be difficult to find a good pairing for decentralised control (Skogestad and Hovd, 1990). The problem with pairing in order of avoiding interaction is that the interaction is not necessarily always the undesirable thing. Therefore, the use of the RGA to decide how to pair variables is not an effective tool for Controllability Analysis. In Barton et al. (1992), the controllability of plant designs previously obtained by economic optimisation of stationary models is evaluated, the steady-state Relative Gain Array is used to determine the best input-output pairings, and the limitations to the functional controllability are analysed. Then, the designs are modified in order to improve their deficiencies.

The RGA and the determinant of the gain matrix provide useful information about integral controllability and integrity (e.g., failure tolerance), which are important issues in decentralised control. The RGA also gives information about robustness with respect to modelling errors and input uncertainty. Based on these interesting properties, Häggblom (2008) used the RGA to consider model uncertainty explicitly and thus investigate the integral controllability and integrity of uncertain systems.

2.1.2 Process-Oriented Methods for Controllability Analysis

Several approaches where systematic actions are taken to improve some controllability measures of plant performance and economic indicators have appeared in the literature. They screen preliminary alternatives either by constraining some controllability indicators or by optimising them, allowing the process or control design to be carried out accordingly. Since the stated optimisation problems allow for the consideration of process and control specifications as well as constraints, these methods are used for accommodating decision variables within a unique integrated-optimisation framework to solve both process design and the control-systems design (Perkins and Walsh, 1996).

Among the opening contributors for controllability assessment and its incorporation into process synthesis and the selection of the control structure, Morari and Stephanopoulos (1980) discuss the structural design of alternative regulatory control schemes to satisfy a posed objective. They use structural models to describe the interactions among the units of a plant and the physicochemical phenomena occurring in the various units. They discuss the relevance of controllability and observability in the synthesis of control structures and use modified versions to develop all the alternative feasible regulatory structures in an algorithmic fashion. Various examples are presented to illustrate the developed concepts and strategies, including the application of the overall synthesis method to an integrated chemical plant.

Marselle et al. (1982) present a heat exchanger network synthesis technique that takes into account aspects of flexibility and resiliency, leading to networks flexible to changes in the plant operating conditions. The method involves the structural and parametric design of the network and the synthesis of the regulatory control structure. The objective is to find the structure able to operate feasibly in a specific range of uncertain parameters while achieving the maximum energy recovery. Saboo and Morari (1984) develop a rigorous synthesis technique based on the fundamental properties for maximum energy recovery in heat exchanger systems which leads to networks that can handle specific inlet temperature variations and also guarantee maximum energy recovery. In Morari et al. (1985) these techniques are extended to the synthesis of the heat exchanger network and the control structure for a sequence of two exothermic open-loop unstable continuous stirred tank reactors.

Modern chemical plants are highly integrated and interconnected which invariably introduce a dynamic coupling between the process units. Material and energy recycle affects process performance leading to complex dynamic behaviours, such as inverse response, open loop instability and unexpected behaviour. Among the several authors who proposed strategies to quantify the impact of such effects on controllability, we may highlight Denn and Lavie (1982), Morud and Skogestad (1994), McAvoy and Miller (1999), Jacobsen (1997), Dimian et al. (1997), Semino and Giuliani (1997), Bildea and Dimian (2003), Lakshminarayanan (2004) and Bildea et al. (2004).

Zheng and Mahajanam (1999) have pointed out that there are very few indices available which establish a direct relationship between cost and controllability. They propose an index to quantify the cost associated with dynamic controllability of a process with a given control structure, focusing on the additional surge volume (or overdesign) required to achieve the control objectives. Such cost/controllability index is used to quantify the cost associated with dynamic controllability. Establishing a relationship between capital costs and controllability as a way to monetise control performance is an interesting idea, and closely related to the motivation of this PhD project. However, here we place focus instead on operating costs and plant revenue.

Along similar lines, Zheng et al. (1999) propose a hierarchical procedure where alternative plantwide control systems are synthesised and compared regarding economics. They describe the design procedure for an existing plant (a simple reactor-separator-recycle system) and also show how the interesting problem of determining the optimum surge capacities of a process can be addressed through simple modifications.

We may also highlight works such as Stephanopoulos et al. (1979), Morari et al. (1980), Morari et al. (1987) and the series "Design of Resilient Processing Plants" (Lenhoff and Morari, 1982; Marselle et al., 1982; Morari, 1983; Saboo and Morari, 1984; Holt and Morari, 1985a,b; Morari et al., 1985; Saboo et al., 1985; Skogestad and Morari, 1987).

2.1.2.1 Controllability Analysis of Systems with Recycles

Luyben and co-workers present a series of papers devoted to the study of dynamics and control of recycling systems in chemical processes (Luyben, 1993a, 1993b, 1993c, 1994, 1999; Tyreus and Luyben, 1993). The special dynamic behaviour of recycling systems, identified in the works just mentioned, are important in the development of process design methodologies, in the subsequent works of the authors. Particularly, Elliott and Luyben (1995) present a capacity-based economic approach which allows comparing and screening quantitatively conceptual plant designs assessing both, steady-state economics and dynamic controllability of the process.

The alternative plant designs are evaluated considering their ability to maximise annual profit in the presence of their associated peak disturbances. The method deals explicitly with the impact of product quality variability on plant profits, considering the losses generated in the fraction of the time that the product is outside the limits of desired specifications. A reactor/stripper recycle system is considered as a case study. The methodology is applied in the design of a complex recycle system consisting of one reactor and two distillation columns in Elliott and Luyben (1996). In this case study, the approach is used to design parameter alternatives, conceptual design flowsheet alternatives, and control structure alternatives for the system.

Luyben and Luyben (1996) deal with the plantwide design and control of a complex process containing two reaction steps, three distillation columns, two recycle flows and six chemical components. A heuristic design procedure and a nonlinear optimisation are used to determine an approximate economically optimal steady-state design; the sensitivity to design parameters and specifications is evaluated and control strategies are developed using guidelines from previous plantwide control studies. In Luyben (2000), the trade-off between the reactor size, recycle flow rate and reactor inlet temperature of a gas-phase reactor/recycle plant in the steady-state design is studied, as well as the economic impact of inert components in the feed stream. In a second step, alternative control structures are evaluated and basic control strategies are applied in the presence of large disturbances. Reyes and Luyben (2000) present a similar study for an irreversible reaction system with a reactor feed preheating system (feed-effluent heat exchanger and furnace) where the steady-state economics and the dynamic controllability of this dual-recycled system are compared with those of single-recycle processes. Both Reyes and Luyben (2001a) and Reyes and Luyben (2001b) focused on processes with realistic separation systems (a distillation column) for gas-phase tubular reactors with a liquid recycle and with a dual recycle system.

Other contributions are found in the works of Cheng and Yu (2003) and Kiss et al. (2005). The former explores the dynamics of simple recycle plants under different process designs using different control structures. The recycle dynamics is evaluated using transfer-function-based linear analysis and also validated using rigorous nonlinear simulation; finally, implications to control structure design are specified for different levels of reactor conversions. Kiss et al. (2005) address the design of recycling systems involving multiple reactions. They use the mass balance model of the plant to capture the interaction between units and to predict the main pattern of behaviour. After choosing the method of controlling the plantwide material balance, the nonlinear analysis reveals regions of infeasibility, high sensitivity, state multiplicity, and instability.

2.1.2.2 Controllability Analysis Based on Steady-State Multiplicity Analysis

Input multiplicities occur when more than one set of manipulated variables (MVs) can produce the desired steady-state output (CVs). Multiplicities are related to the stability and desirability of any intended loop pairing of MVs and CVs. The danger under the existence of input multiplicities is the undetected transition from one steady-state to another. It is thus a good policy to examine the steady-state behaviour to detect multiplicities since the possibility of an undetected transition to an unanticipated, and perhaps economically undesirable, the steady-state operating condition is best eliminated at the system design stage (Koppel, 1982).

Some interesting works are focused on integrating operability criteria into chemical reactors design based on the steady state multiplicity analysis. Several preliminary results by Russo and Bequette (1995, 1996, 1997, 1998) use the bifurcation based approach to study the behaviour of CSTRs showing that the infeasible operation regions that affect open loop and closed loop performance can be avoided with some parameter modifications in the design stage. More recently, Altimari and Bildea (2009) tackle the integrated design and control of plantwide systems. Their methodology evaluates the steady-state multiplicity and allows selecting possible flowsheets and admissible control structures regarding feasibility. Guidelines are derived which enable the selection of reactor parameters in a way that guarantees wide margins of plant operating conditions from infeasibility boundaries. While thorough analysis is provided on the interaction between reactor design and plant operability, no connection to process economics is discussed. In the current project, we wish to link the idea of steady-state multiplicity to operating cost and revenue.

The influence of input/output multiplicity on stability and non-minimum phase behaviour of chemical reaction systems is studied in Yuan et al. (2009), Yuan et al. (2011), Wang et al. (2011) and Wang et al. (2013). With a focus on inherently safer designs, their study reveals how the essential properties of a process change with variations in its operating conditions. A systematic framework that includes multiplicity and phase behaviour together with open loop stability analysis over the entire feasible operation region of plantwide processes is presented in Yuan et al. (2012b).

Yuan et al. (2009), address a strategy for classifying the process operating region into distinct zones at the early stage of process design, based on stability/instability and minimum/non-minimum phase behaviour analysis. Ma et al. (2010) presented an approach using continuation and optimisation methods for modifying a process design to avoid the issues caused by input multiplicity. Wang et al. (2011) conclude that stability and phase behaviour should be analysed considering the overall system rather than individual units because those properties may differ from the global system. Yuan et al. (2011) present a methodology that explores the open and closed-loop controllability of the liquid-phase catalytic oxidation of toluene. They evaluate set-point tracking and disturbances rejection in various sub-regions with different controllability characteristics.

2.1.2.3 Controllability Analysis Based on Phenomenological Models and the Passivity Theory

Put together in this section are the procedures where the phenomenological knowledge of the process is used to distinguish the designs with best dynamic performance, using sensibility analysis of thermodynamic properties or, specifically, the passivity theory.

In fact, several works can be found in the literature that takes advantage of mathematical models and thermodynamic properties of a process to improve process synthesis, integrating sensitivity to perturbations and other control aspects. In Gani et al. (1997), different process flowsheets and equipment design parameters are generated through simulations using simple or rigorous models of the process, analysing at every step different process features, and including environmental aspects and controllability. In Russel et al. (2002), more emphasis is given to the analysis of the process model as a preliminary solution step for integration of design and control problems. In Li et al. (2003), a systematic sensitivity analysis of the process model is developed to select the best control structure. In Ramirez Jimenez and Gani (2007a,b), a model based analysis methodology for the integrated design and control is presented, using first-principles phenomenological models of different complexities to identify the interactions between the process and design variables. Parametric sensitivity analysis is performed to determine the control structure.

In Hamid et al. (2010) the simultaneous process and control system design of a process is addressed by the reverse design algorithm approach. The formulation of the integrated optimisation process design and process control problem is decomposed in four subproblems easier to solve. The search space is reduced by considering thermodynamic and feasibility aspects, the concepts of the attainable region (AR) and driving force (DF) are used to locate the optimal process-controller design in terms of the optimal condition of operation from design and control viewpoints. The AR concept is used to find the optimal (design target) values of the process variables for any reaction system. The DF concept is used in this methodology to find the optimal (design target) values of the process variables for separation systems. The final selection and verification are performed according to the value of the objective function. Alvarado-Morales et al. (2010) extend this methodology, proposing a framework that combines the simultaneous process design and controller design methodology and the process-group contribution (PGC) methodology. A process flowsheet can be described by means of a set of process-groups bonded together to represent the structure. The PGC methodology has been used to generate and test feasible design candidates based on the principles of the group-contribution approach used in chemical property estimation. It is applied to the bioethanol production process; however, this is a general framework that can be applied to different processes.

A category of process control algorithms based on thermodynamic-models was proposed in Ydstie and Viswanath (1994) and incorporated in the Integrated Process and Control Design Framework by Meeuse et al. (2000, 2001), Meeuse (2002) and Meeuse and Grievink (2002). These works embed Controllability Analysis within the process synthesis, assessing sensitivity to perturbations by means of the passivity theory. A passive component may be either a component that consumes (but does not produce) energy (thermodynamic passivity) or a component that is incapable of power gain (incremental passivity). Passivity is then a property that can be used to demonstrate that passive passivity systems will be stable under specific criteria. The passivity systems are a class of processes that dissipate certain types of physical or virtual energy, defined by Lyapunov-like functions. The authors use the passivity framework, linked to process thermodynamics, in process input-output Controllability Analysis. This approach allows for studying the stability of distributed systems and the selection of the manipulated and measured variables pairing alternatives that ensure stability and efficient plant operation by relating the entropy production sensibility of the plant with its sensibility to perturbations. Specifically, in Meeuse (2002) and Meeuse and Grievink (2004), controllability conditions are incorporated in the process synthesis by considering thermodynamic aspects of the process, to derive some design guidelines.

2.1.3 Integrated Process Design and Control Framework - Methods of Integrated Control and Process Synthesis

This section will focus on the available methods of Integrated Control and Process Synthesis, the second class of methods defined in Lewin (1999). Works that fall into this classification are used to generate process flowsheets, embedding Controllability Analysis in the process synthesis. Different operability and sensitivity qualities are and explored to determine plantwide process structure, and also to provide insight into control system design.

The dynamic performance measures are introduced in the process design, originating a single optimisation scenario which may additionally contain the tuning of the controllers and even the selection of the control structure. The formulation of the optimisation problem contains decision variables, objective functions and constraints related to economics as well as operating and control performance aspects. Thus, this approach provides the possibility of carrying out at once the process and the control system design by solving the optimisation problem, providing the plant design that best satisfies the compromise between economic and control aspects and all the criterion considered in the problem formulation.

Pioneering works that introduced the idea of integrating the process design and the controllability issues in a comprehensive optimisation problem were those by Lenhoff and Morari (1982), Palazoglu and Arkun (1986) and Georgiou and Floudas (1990), among others. In Lenhoff and Morari (1982) an optimisation based design approach considering economic and dynamic aspects simultaneously is proposed, taking into account process structural decisions, parametric changes and the control structure selection which leads to a multiobjective optimisation problem. Georgiou and Floudas (1990) developed a systematic framework for control system synthesis. They used the generic rank of a process structural matrix as an index of structural controllability to select the best process configuration, computed by solving an integer-linear optimisation problem.

Perkins and Walsh (1996) pointed out the notable trend towards the use of optimisation as a tool for the integration of process design and process control, which was enabled by advances in computational hardware and optimisation methods and driven by the need to place control design decisions on the same basis as process design decisions.

2.1.3.1 The Integrated Control and Process Synthesis Problem

A complete formulation of the integrated design of a process includes in addition to the determination of the plant dimensions and operating conditions, the selection of the plant topology (process synthesis) and the selection of the control structure (input-output pairing and control scheme). When the process synthesis is considered, the optimisation problem is posed based on a superstructure containing all the possible alternatives of the process (algorithmic synthesis or automatic synthesis), aimed to find the optimal flowsheet in the economic and controllable sense. The selection of the control system configuration can also be embedded in a superstructure. This formulation involves continuous variables, representing the dimensions and operating conditions, and discrete variables, related to the process/controller structure.

As both discrete and continuous variables are embedded into this problem, it can be formulated naturally as a mixed-integer dynamic optimisation (MIDO) problem. The integer variables take care of discrete decisions (i.e. flowsheet structure, the number of control loops, the number of distillation columns, etc.), while the continuous variables are normally related to design variables (i.e. flows, temperatures, composition, etc.). As many chemical processes feature nonlinear behaviour around optimal design regions, it is likely that the MIDO problem gives rise to a nonlinear optimisation formulation.

Walsh and Perkins (1996) and Narraway and Perkins (1993) were among the first to consider general mathematical programming techniques for the simultaneous synthesis and control problem using dynamic process models, proposing methods that possess an explicit economic component was presented for optimal plant design involving a classical feedback control structure. In practice, similar approaches involve attributing a cost to a control performance measure such as the integral error, e.g., ISE or IAE, performing a worst-case design optimisation for tuning one or more PID controllers for optimal system response and then varying a number of equipment design parameters, then repeating the process until the global solution is found. This avoids equipment oversizing and thus decreases costs. Dimitriadis and Pistikopoulos (1995) applied the ideas reported in Halemane and Grossmann (1983) to systems described by sets of differential and algebraic equations.

Different formulations of the integrated design including the process synthesis and the selection of the control structure are found in the literature. Luyben and Floudas (1994a) present a general formulation of the problem considering a superstructure for the process synthesis that includes all possible design alternatives of interest and open-loop steady-State Controllability measures. Mohideen et al. (1996a) propose a unified process synthesis optimisation framework for obtaining process designs together with the control structure and controller design under uncertainty. The objective is to design the process and the required control scheme at a minimum total annualised cost which comprises investment and operating costs including controller costs. It results in an optimum set of design variables, the best selection/pairing of controlled-manipulated variables and the optimal values of the controller parameters. Similar formulations and solution procedures were reported in Bahri, Bandoni and Romagnoli (1997) and Schweiger and Floudas (1999). The main drawback of these methodologies stems from the fact that the combined design and control problem, a mixed-integer dynamic optimisation (MIDO) problem, is solved as a mixed integer nonlinear programming (MINLP) problem by transforming the system of differential and algebraic equations into algebraic equations by using either full discretisation or integration. In the former case, the size of the resulting MINLP is explosive, and in the latter case, extensive dual information is needed to formulate a (mixed integer linear) master problem that is ever increasing in size.

The Mixed-Integer Dynamic Optimisation algorithms reported in the literature are based on complete discretisation (Mohideen et al., 1996a), on an adjoint-based approach (Sakizlis et al., 2001), and on an outer approximation (Bansal et al., 2003). None of these methods are guaranteed to find the exact global optimum of the underlying optimisation problem. Therefore, the need for new global mixed-integer dynamic optimisation algorithms becomes extremely important in preventing the generation of economically unfavourable designs.

Some other works addressing the complete integrated design problem involving closed-loop behaviour analysis into the optimisation are Mohideen et al. (1996b), Bahri et al. (1996a), Bansal et al. (2000a), Kookos and Perkins (2001), Ekawati (2003), Flores-Tlacuahuac and Biegler (2007) and Revollar et al. (2012). Angira (2005), Angira and Alladwar (2007), Babu and Angira (2006), Angira and Babu (2006) used evolutionary solution methods for solving the nonlinear, mixed integer nonlinear and dynamic optimisation problems encountered in chemical engineering. But these techniques are not applied and tested on Mixed-Integer Dynamic Optimisation problems that arise from the simultaneous process design and control. Sánchez-Sánchez and Ricardez-Sandoval (2013a), Trainor et al. (2013) and Sharifzadeh and Thornhill (2013) also develop fully integrated process synthesis and control formulations.

A number of works carry out the integrated design considering only the process synthesis and the determination of the optimal plant dimensions, operating conditions and even the controller parameters: Schweiger and Floudas (1997), Bahri et al. (1997), Gutierrez (2000), Sakizlis et al. (2003, 2004), Malcolm et al. (2007), Revollar et al. (2008) and the recent contributions of Revollar et al. (2010a), Revollar (2011) and Sánchez-Sánchez and Ricardez-Sandoval (2013b). Some other works focus on process dimensioning and determination of optimal operating conditions including the selection of the control structure and controller tuning: Narraway and Perkins (1994), Asteasuain et al. (2005, 2006, 2007), Patel et al. (2007) and Flores-Tlacuahuac and Biegler (2008).

Other works considering fixed structures are Lenhoff and Morari (1982), Palazoglu and Arkun (1986), Luyben and Floudas (1994b), Gutiérrez and Vega (2000), Blanco and Bandoni (2003), Chawankul et al. (2007), Miranda et al. (2008), Grosch et al.

(2008), Kim and Linninger (2010), Francisco et al. (2011) and Ricardez-Sandoval (2012). Large scale systems are addressed in recent works as Exler et al. (2008), Moon et al. (2011), Ricardez-Sandoval et al. (2009c, 2010, 2011) and Munoz et al. (2012).

2.1.3.2 Integrated Control and Process Synthesis with Guaranteed Robust Performance

A fundamental question about control algorithms is its robustness to model uncertainty and noise. When we say that a control system is robust we mean that stability is maintained and that the performance specifications are met for a specified range of model variations and a class of noise signals (uncertainty range). To be meaningful, any statement about "robustness" of a particular control algorithm must make reference to a specific uncertainty range as well as specific stability and performance criteria.

The robust control framework makes use of a terminal cost that is also a Control Lyapunov Function for the system. Control Lyapunov Functions (CLFs) are an extension of standard Lyapunov functions and were originally introduced by Sontag (1983). CLF is a Lyapunov function $V(x)$ for a system with control inputs. They allow the constructive design of controllers and the Lyapunov function that proves their stability. The ordinary Lyapunov function is used to test whether a dynamical system is stable (more restrictively, asymptotically stable). That is, whether the system starting in a state $x \neq 0$ in some domain D will remain in D, or for asymptotic stability will eventually return to $x = 0$. A CLF, however, is used to test whether a system is feedback stabilizable, that is whether for any state x there exists a control $u(x, t)$ such that the system can be brought to the zero state by applying a control action u. Consider a nonlinear control system $\dot{x} = f(x, u)$, where $x \in \mathbb{R}^{n_x}$ and $u \in \mathbb{R}^{n_u}$. The following definition is valid.

Definition 2.1.3.2.1 Control Lyapunov Function. *A locally positive function* $\boldsymbol{V} : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{+}$ *is called a Control Lyapunov Function (CLF) for a control system if Eq. 25 is valid.*

$$\forall \boldsymbol{x} \neq 0, \exists \boldsymbol{u} \quad inf\left(\frac{\partial V}{\partial x} f(\boldsymbol{x}, \boldsymbol{u})\right) < 0 \qquad \textit{Eq. 25}$$

Eq. 25 is a key condition; it says that for each state $\boldsymbol{x}$ we can find a control $\boldsymbol{u}$ that will reduce the magnitude of $\boldsymbol{V}$. Intuitively, if in each state we can always find a way to reduce the $\boldsymbol{V}$, we should eventually be able to bring the $\boldsymbol{V}$ to zero, that is to bring the system to a stop. This is made rigorous by the following result:

> **Artstein's theorem.** The dynamical system has a differentiable control-Lyapunov function if and only if there exists a regular stabilizing feedback $\boldsymbol{u}(\boldsymbol{x})$.

A complete treatment is given in Krstic (1995). In general, it is difficult to find a CLF for a given system. However, for many classes of systems, there are specialized methods that can be used. One of the simplest is to use the Jacobian linearization of the system around the desired equilibrium point and generate a CLF by solving a Linear–Quadratic Regulator (LQR) problem. The problem of minimizing a quadratic performance index, Eq. 26, subject to $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$ and $\boldsymbol{x}(\boldsymbol{0}) = \boldsymbol{x_0}$, results in finding the positive definite solution of the Riccati equation, P, Eq. 27:

$$J = \int_0^{\infty}\left(\boldsymbol{x}^{\boldsymbol{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) + \boldsymbol{u}^{\boldsymbol{T}}\boldsymbol{R}\boldsymbol{u}(t)\right)dt \qquad \text{Eq. 26}$$

$$\boldsymbol{A}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-\boldsymbol{1}}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P} + \boldsymbol{Q} = 0 \qquad \text{Eq. 27}$$

where $\boldsymbol{R}$ is the factor of input suppression and $\boldsymbol{Q}$ is the LQR weight. The optimal control action is given by Eq. 28:

$$\boldsymbol{u} = -\boldsymbol{R}^{-\boldsymbol{1}}\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{P}\,\boldsymbol{x} \qquad \text{Eq. 28}$$

where $\boldsymbol{V} = \boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x}$ is a CLF for the system. In the case of the nonlinear system $\dot{\boldsymbol{x}} = f(\boldsymbol{x}, \boldsymbol{u})$, *A* and *B* are taken as:

$$\boldsymbol{A} = \left.\frac{\partial f(\boldsymbol{x},\boldsymbol{u})}{\partial \boldsymbol{x}}\right|_{(0,0)} \qquad \text{Eq. 29}$$

$$\boldsymbol{B} = \left.\frac{\partial f(\boldsymbol{x},\boldsymbol{u})}{\partial \boldsymbol{u}}\right|_{(0,0)} \qquad \text{Eq. 30}$$

The CLF $\boldsymbol{V}(x) = \boldsymbol{x}^{\boldsymbol{T}}\boldsymbol{P}\boldsymbol{x}$ is valid in a region around the equilibrium $(0,0)$. More complicated methods for finding control Lyapunov functions are often required and many techniques have been developed. An overview of some of these methods can be found in Jadbabaie (2001).

In the recent years, robust approaches using CLFs have been introduced in the integrated process design and control formulations. They consider the uncertainties existing in real processes to provide robustness properties to the obtained plants and the worst-case variability. In these approaches, the process nonlinear dynamic model is represented as uncertain models that can be used to calculate bounds on the variables that are involved in the objective function and the constraints of the problem under consideration (Ricardez-Sandoval et al., 2010).

In many works, the effects of uncertainties and perturbations are ignored or else very simple perturbations profiles are considered (Narraway and Perkins, 1994; Schweiger and Floudas, 1997; Bahri, 1996; Kookos and Perkins, 2001). Nevertheless, in Bandoni et al. (1994) an algorithm for the worst-case is presented, to compute the maximum variation of the uncertain parameters that can take place without impairing the feasibility of the process. Another group of publications can be found, focused on studying the effects of different settings of perturbations and parameter uncertainties on the process economics and dynamic performance (Mohideen et al., 1996a, 1996b; Bahri et al., 1996b, 1997; Bansal et al., 2000a; Asteasuain et al., 2007).

In Chawankul et al. (2007) robust integrated design has been developed, particularly quantifying the uncertainties as a family of linear models around the nominal model. These uncertain models have been typically used in robust control, and they have also been used for integrated design in Francisco et al. (2011). However, most of the robust integrated design methods consider parametric uncertainty. In Moon et al. (2011) some uncertain scenarios are considered varying process parameters. In Munoz et al. (2012), an extension of the normal vector method is developed to consider simultaneously disturbances and uncertain model and process parameters. Ricardez-Sandoval et al. (2009a) consider model parametric uncertainty that is translated to an uncertain state-space model. In this study, uncertain dynamic models of the process, obtained from the nonlinear dynamic closed-loop process model, were used to estimate analytical bounds on the worst-case variability and the process feasibility constraints. The bounds were computed using a formulation introduced by the authors based on a Structured Singular Value analysis named 'Analytical Bounds Worst-case Approach' (BWA). Later this approach was extended to a robust Finite Impulse Response model with uncertain parameters (Ricardez-Sandoval et al., 2009b, 2009c, 2010). In Ricardez-Sandoval et al. (2011), the uncertainty of process physical parameters was addressed. Sánchez-Sánchez and Ricardez-Sandoval (2013a,b) include process synthesis and control structure decisions, but using again the uncertain Finite Impulse Response model.

As for the treatment of disturbances, Chawankul et al. (2007) only consider sinusoidal time-varying disturbances, and Gerhard et al. (2005) and Monnigmann and Marquardt (2005) are also limited to particular disturbances. Other works consider a general form of the disturbances, by means of their maximal magnitude. Particularly, Ricardez-Sandoval et al. (2008, 2009a, 2009b, 2009c, 2010, 2011) assume general disturbances of bounded magnitude, carefully calculating the worst-case disturbance. Francisco et al. (2011) also consider the maximal magnitude of the disturbances based on the actual weather profiles.

In Monnigmann and Marquardt (2005) a method is proposed that establishes robust measures based on a minimal distance between the uncertain parameter space region and the critical boundaries. Later, in Grosch et al. (2008), constraints are imposed simultaneously on time-domain performance indicators and the asymptotic dynamic process behaviour while optimising the steady-state profit of the plant, accounting for the effect of uncertainty in both, design and model parameters. This approach is difficult to apply in the presence of more than one disturbance, then, to overcome its disadvantages. Munoz et al. (2012) use an extension of the normal vector approach proposed in Monnigmann and Marquardt (2002) to consider the simultaneously robust asymptotic stability of steady states despite parametric uncertainty and robust feasibility of the transient behaviour despite disturbances.

In particular, several articles by Ricardez-Sandoval and co-workers present a robust approach based methodology that performs the simultaneous design and control under disturbances and process model parameters uncertainties. In Chawankul et al. (2007) a measure of the closed loop output performance is introduced based on the output widest variability caused by model uncertainties and constraints related to the robust stability of the plant imposed. Furthermore, this performance index is added to the objective function as a cost associated with the variability. In Ricardez-Sandoval et al. (2008, 2009a, 2009b) a new technique is presented to assess the flexibility, stability and controllability of a process. In this method, the infinite time horizon bounds are estimated for the worst-case scenarios, enforcing process feasibility constraints by using the Structured Singular Value Analysis (SVA), avoiding expensive dynamic optimisations. This methodology is improved in Ricardez-Sandoval et al. (2009c) to reduce the computational requirements of the method towards its application to large-scale processes; the methodology is referred as the Analytical Bounds Worst-case Approach (BWA). However, a disadvantage of this approach is the conservatism resulting from the use of analytical bounds. In Ricardez-Sandoval et al. (2010) a method named Hybrid Worst-Case Approach (HWA) is proposed. It combines the analytical calculation of the worst-case disturbance and dynamic simulations using the mechanistic closed-loop process model to calculate variability. It is expected to reduce the conservatism in the final design at the expense of additional computational time in the calculations. Ricardez-Sandoval et al. (2011) have expanded hybrid worst-case approach considering time-varying disturbances and parametric model uncertainties, making it suitable for application to large-scale systems.

In Sánchez-Sánchez and Ricardez-Sandoval (2013a) is presented a method for optimal process synthesis and control structure selection that simultaneously evaluates dynamic flexibility and dynamic feasibility in the presence of the worst-case (critical) time-trajectories in the disturbances. Furthermore, a robust stability test based on Quadratic Lyapunov theory is included in this methodology to ensure that the optimal design is asymptotically stable for any of the magnitude-bounded perturbations considered in the analysis. The disturbances are treated as stochastic time-discrete unmeasured inputs. The work of Trainor et al. (2013) adopts this methodology for the design of a ternary distillation system treating disturbances as random time-dependent bounded perturbations. It presents a methodology for the optimal process and control design of dynamic systems under uncertainty that incorporates robust feasibility and stability analyses formulated as convex mathematical problems. This approach is computationally attractive since it does not require the solution of a MINLP. A norm-bounded metric based on Structured Singular Value (SSV) analysis is employed to estimate the worst-case deviation in the process constraints in the presence of critical realisations in the disturbances.

In Gutierrez et al. (2013) an integrated design methodology focused on the selection of an optimal control structure is addressed by adding a communication cost function within the overall cost function. Different control structures composed of centralised and fully decentralised predictive controllers are considered in the analysis. A cost function related to the worst-case closed-loop variability is calculated using analytical bounds derived from tests used for robust control design.

In Matallana et al. (2011) a design methodology based on the optimisation of the domain of attraction is proposed. The idea is to simultaneously ensure asymptotic stability and an optimum domain of attraction of the resulting operating point in a certain sense. The approach consists of maximising the radius of a ball in the state's space within which negative definiteness of the time derivative of a quadratic type Lyapunov function can be ensured.

In Francisco et al. (2011) and Francisco (2011) norm based indexes for controllability are considered. They allow for including robust performance conditions within the integrated design procedure by using a polyhedral uncertainty region, limited by multiple linearized models. The multiobjective problem stated include investment, operating costs, and dynamical indexes based on the weighted sum of some norms of different closed-loop transfer functions of the system.

2.1.3.3 Probabilistic Based Integrated Control and Process Synthesis Methods

Some of the recent works presented in the literature for optimal design considers a stochastic or probabilistic-based approach. Most of the design procedures ensure the appropriate process performance in the presence of uncertainties and disturbances focusing on the worst-case scenario given by the critical realisations in the disturbances and the uncertain system's parameters that produce the largest deviations in the controlled variables, demanding major control efforts to maintain the desired operating conditions. This is called the worst-case process variability (Bahakim and Ricardez-Sandoval, 2014). The overestimation of the uncertainties, typical in process design methodologies, leads to conservative design decisions resulting in an unnecessary deterioration of the objective function, In such sense, probabilistic programming is a promising solution for solving optimisation problems under uncertainty in the process industry (Li et al., 2008) allowing to take into account the probability of occurrence of the worst-case variability in the process variables.

Few works have introduced such considerations in the integrated design formulation. Ricardez-Sandoval (2012) introduces a distribution analysis on the worst-case variability in the integrated design framework. The worst-case variability is approximated by normal distribution functions to estimate the largest variability expected for the process variables at a user-defined probability limit. Thus, the user can rank the goals of the design according to its particular criterion. The worst-case variability estimates are used to evaluate the process constraints, the system's dynamic performance and the system's cost function enabling the assessment of the optimal process design by assigning different probability levels to the process variables used to evaluate the process constraints and the process economics. In Bahakim and Ricardez-Sandoval (2014) an optimisation framework for achieving a feasible and stable optimal process design in the presence of stochastic disturbances while using advanced model-based control scheme is proposed.

2.1.3.4 Examining the Control Structures used in Integrated Control and Process Synthesis

In several formulations of the integrated optimisation of process design and control the controller parameters are introduced as decision variables in the optimisation, while in others they are tuned empirically. Some formulations focus in the analysis of the open loop system to obtain an optimal and controllable design for any possible controller, as in Luyben and Floudas (1994a), Grosch et al. (2008), Matallana et al. (2011) and Guerra et al. (2012). In some works, the notion of perfect control is assumed in the optimisation formulation avoiding the complexity associated with the controllers' evaluation. Sharifzadeh and Thornhill (2012) propose a simplified optimisation framework with a multiobjective function taking advantage of the perfect control concept, which is the best performance that a given control structure can achieve. Later this approach is introduced in the integrated design formulation in Sharifzadeh and Thornhill (2013). Perfect control is also supposed in Narraway and Perkins (1993, 1994) and Blanco and Bandoni (2003).

The usual type of controller included in most of the integrated optimisation based formulations independently of the scope of the problem is the feedback decentralized PI or PID (Narraway et al., 1991; Walsh and Perkins, 1994; Bahri, 1996; Schweiger and Floudas, 1997; Bansal et al., 2002b; Exler et al., 2008; Grosch et al., 2008; Ricardez-Sandoval et al., 2011; Sánchez-Sánchez and Ricardez-Sandoval, 2013a,b; Gutierrez et al., 2013; Trainor et al., 2013; Ricardez-Sandoval, 2012). An early step towards the application of advanced control schemes is observed in Kookos and Perkins (2001) where a multivariable PI is implemented. Asteasuain et al. (2006) combines a scheme of feedback PI and feedforward multivariable control, while Asteasuain et al. (2007) use a PI multivariable controller and a relation control scheme is used. The parameters of the PI controller are considered decision variables in the optimisation problem. Nevertheless, in Bahri (1996) and Bahri et al. (1997) pre-designed PI controllers that are more fine-tuned after the procedure, in Dominguez et al. (2009) the PID IMC tuning method (Skogestad, 2003) is used to include the controller design within the integrated design framework.

In Patel et al. (2007) and Miranda et al. (2008), optimal controllers are considered. Malcolm et al. (2007) and Moon et al. (2011) use Linear Quadratic Regulators (LQR). Finally, Lu et al. (2010) consider a fuzzy-model-based controller which estimate the process behaviour and derive fuzzy rules to guarantee stability, robustness and feasibility.

Focusing on the synthesis of a control system structure, Lin et al. (1991) establish the concept of Output Structural Controllability (OSC) and derive a condition to ensure Output Structural Controllability of a process explaining how to use it for the selection of the control schemes in chemical plants. Later, Hopkins et al. (1998) make use of this index (OSC) for integrating process design and control in the process and control structure synthesis. Also, Lee et al. (2001) study the structural controllability concept about the propagation paths of the perturbations. They use only the structural digraph of the plant and their relative order matrices, without knowledge of other process details, to select the best flowsheets and discard non-controllable alternatives.

2.1.3.5 Multi-Objective Control and Process Synthesis Problems

The mathematical formulation of the optimisation depends on the scope of the problem, the techniques used for introducing the quantification of controllability and other properties related to dynamic performance, the control scheme and the treatment of disturbances and uncertainties. The multi-objective nature of the integrated process and control design can be addressed using an optimisation problem with different cost functions, or problems with just one objective function based on economic aspects and constraints related to dynamic performance indices.

Palazoglu and Arkun (1986) formulate a multi-objective optimisation using robustness indices as constraints to quantify the dynamic operability which is illustrated by solving design and operability problems of a CSTRs system. In Luyben and Floudas (1994a) a Mixed-Integer Nonlinear Problem (MINLP), multi-objective programming problem is posed, where economic objectives and some linear controllability indexes are optimised. In Schweiger and Floudas (1997) the Mixed-Integer Optimal Control Problem (MIOCP) is simplified into a Mixed-Integer Nonlinear Problem with Differential Equations (MINLP/DAE). Imposing different limits to the constraints, Pareto curves can be developed to reveal compromise solutions.

Blanco and Bandoni (2003) introduce controllability measures in this type of formulation using the eigenvalues optimisation theory. Matallana et al. (2011) maximise the region of asymptotic stability of the equilibrium point, which results in a bi-level optimisation problem with non-differentiable inner subproblems, which is solved using a stochastic (derivative free) algorithm in the outer level. Sharifzadeh and Thornhill (2012) propose a simplified optimisation framework with a multiobjective function taking advantage of the perfect control concept which is extended in Sharifzadeh and Thornhill

(2013) introducing the inversely controlled process model which results in a dynamic optimisation formulation that is solved by sequential integration and by full discretisation.

Alhammadi and Romagnoli (2004) proposed an integrated framework for plantwide control and dynamic modelling that incorporated not only the usual metrics such as controllability and economic performance but also environmental performance and energy integration, resulting in a multi-objective optimisation problem. The impact of energy integration is evaluated by embedding thermal pinch analysis to examine the trade-off over a number of Heat Exchanger Network (HEN) designs. In addition, an environmental performance measure is added to the optimisation problem by adding the Life Cycle Assessment (LCA) index to the cost function. The solution is obtained by a sequential approach that consists of four steps: economic and environmental optimisation; HEN design optimisation; Controllability Analysis; and plant-wide control and dynamics modelling. The method was applied to a large scale Vinyl Chloride Monomer plant.

In Asteasuain et al. (2006), the optimisation based simultaneous design and control of a polymerisation reactor translates into a multi-objective, Mixed-Integer Dynamic Optimisation Problem (MIDO). The two objectives are an economic function with the investment and operation costs, and a dynamic index similar to the ISE-related to the product quality. The problem is solved by the application of a decomposition algorithm where there is a master mixed-integer, the MINLP and an associated dynamic optimisation problem.

Miranda et al. (2008) formulate the problem focusing on the application of optimal control theory, relying on Pontryagin's minimum principle. The Euler–Lagrange equations are derived from the underlying optimisation problem which is then solved by using a discretisation technique.

Malcolm et al. (2007) and Moon et al. (2011) propose a new mathematical methodology to reduce the combinatorial complexity of multi-objective integrated design and control by embedding control for specific process designs. The optimal design problem is solved using the Nelder–Mead simplex method. Other alternative optimisation formulations and methods have been applied successfully to solve the complex integrated design problem, for instance, multi-objective formulations are successfully solved with stochastic optimisation methods based on genetic algorithms in Revollar et al. (2010b, 2010c).

2.1.3.6 Formulations with an economic objective function and controllability constraints

In these works, different formulations of the optimisation problem are considered, introducing the controllability issues or dynamic performance indices as constraints. Although it is not equivalent to multi-objective formulations, it may simplify the optimisation problems, once the particular bounds have been carefully selected.

In Bahri (1996) the economics of the process is optimised and feasible regulatory dynamics is ensured by means of constraints on the dynamic operability conditions. The problem is solved with the application of a two-level iterative algorithm. On the first level the structure, dimensions and operating conditions are obtained through an MINLP. On the second level, the feasibility of the solution is examined using the resolution of the associated NLP problems. This methodology is also applied in Bahri et al. (1996a) and Bahri et al. (1997), while in Ekawati and Bahri (2003) it is enlarged by adding a new controllability index to perform the dynamic operability analysis.

Mohideen et al. (1996a) propose a general formulation containing the total annual cost as the minimizing function, subject to the constraints associated with (a) the differential and algebraic equations of the process model, (b) the feasibility of the operation, (c) the trajectory and (d) the variability of the process due to perturbations and uncertainties. This formulation results into a MIDO. The proposed algorithm for its resolution requires the decomposition in two subproblems and the application of an iterative procedure, starting with the determination of the optimal process design and control structure to end with the evaluation of the feasibility of the process operation throughout the possible range of perturbations and uncertainties. This framework is also adopted in the works of Bansal et al. (2002b), Sakizlis et al. (2003) and Sakizlis et al. (2004).

Kookos and Perkins (2001) propose a decomposition algorithm, based on upper and lower bounds on the economic performance of the flowsheet. The lower bounds are generated by solving the optimisation problem involving flowsheet layout and control structure, while a restricted dynamic optimisation problem with fixed layout yields the upper bounds and time-invariant design parameters. The bounds generated get progressively tighter as the method iterates, eventually providing the optimal design.

In Flores-Tlacuahuac and Biegler (2007) an algorithm based on the transformation of a MIDO problem into a MINLP program is proposed. Three MINLP formulations are developed and evaluated: a nonconvex formulation, the conventional Big-M formulation and Generalised Disjunctive Programming (GDP).

In Chawankul et al. (2007) the variability of the controlled output is included in the objective function, imposing constraints on the manipulated variables to improve disturbance rejection and to ensure robust stability. In this work, the nonlinear plant is represented by a family of linear models.

Asteasuain et al. (2007) is an extension of Asteasuain et al. (2006) adding uncertainties and perturbations while using only one objective function related to the product quality. A two-level optimisation algorithm is applied to solve the problem. An initial set of uncertain parameters is considered and then extended up to the complete dominion of uncertainty to find the maximum violation of the operation constraints.

It is important to note that, it is quite difficult to disconnect the formulation of the integrated optimisation problems from the solution approaches. Note that some common approaches result in Nonlinear Optimisation Problems (NLP), MINLP and MIDO. Nevertheless, some algorithms have been developed to solve the MIDO problem and can be classified depending on the reformulation of the original MIDO problem into an MINLP problem or a bi-level optimisation problem (Sakizlis et al., 2004; Hamid, 2011).

Moreover, another classification can be made taking into account the optimisation methods applied for the resolution of the integrated design problem. Thus, the optimisation strategies basically can be deterministic methods or alternative methods such as stochastic and hybrid algorithms (Egea et al., 2007). For instance, in Exler et al. (2008), Lamanna et al. (2009), Francisco et al. (2009), Revollar et al. (2010a) and Revollar et al. (2012), stochastic methods as tabu search, simulated annealing and genetic algorithms are applied to solving different problems.

2.1.4 Conclusions from the Review of Integrated Process Design and Control Framework

This section provided a review of Integrated Process Design and Control methods. The main definitions and methods were presented with a view to enabling a holistic approach when making important design choices such as plant layout and equipment sizing. The reader should now understand that process design and control design are strongly correlated, and an integrated optimisation problem has advantages such as increased robustness and better design from an economic standpoint.

Also, the reader is now familiar with the numerous integrated methodologies, which may be roughly classified as either part of the sequential or the simultaneous frameworks. In the sequential framework, often called Controllability Analysis, the process design is fixed, and its properties are evaluated to assess whether or not a good control performance can be obtained. In the simultaneous framework, here called Integrated Control and Process Synthesis, the process design is not fixed. Key parameters and decision variables are varied by optimisation algorithms, whose objective functions have embedded some measure of controllability. When the simultaneous framework is applied, the optimal layout can be obtained which reaches a compromise between profitability and stability. Arguably, Integrated Control and Process Synthesis is the ideal approach for designing new plants; but if for any reason a situation presents itself where its use is infeasible, then Controllability Analysis must be carried out to avoid uncontrollable plants to be designed.

A key weakness of the IPDCF is that no single method is adequate to all processes since none can address all possible future control issues. Several classes are available, each encompassing dozens of methods. Only expert knowledge of the process and extensive simulation of its dynamic behaviour can help determine which method provides the optimal flowsheet and control system. At least a couple distinct IPDCF methods should be tested and benchmarked during the design phase so that the engineering team can be confident in the choices made. Another weakness is the possibility of an overly ambitious reduction of oversizing coefficients of key equipment. Plants thus designed may become unstable in actual operation due to unpredictable factors.

2.2 Review of Model Predictive Control

According to Skogestad and Postlethwaite (1996), while controllability is a property of the flowsheet and thus independent of the controller, it is dependent on the definition of control objectives. Indeed, in this report, we make the case that using Model Predictive Control (MPC), and especially Zone Constrained MPC, fundamentally changes the way control objectives are defined and thus controllability should be measured accordingly. Since the use of MPC structures is assumed and it is key for the hypothesis developed here, a short review of this class of control algorithms will now be presented.

MPC algorithms (also referred to as Receding Horizon Control and Moving Horizon Optimal Control) make use of explicit process models to predict the future response of a plant. It is widely regarded by both industry and academia as an effective means to deal with multivariable constrained control problems. The MPC algorithm attempts to optimise future plant behaviour at each control interval by defining an optimised sequence of future manipulated variable adjustments. It controls a subset of future points in time for each output and compares them with the chosen reference trajectory. After the optimal sequence of moves for each input is found by an optimisation solver, the first input is then sent to the process, and at the next control interval, the calculation is repeated. One of MPC's strength lies in its use of step response data, which are physically intuitive, and in the fact that it can handle hard constraints explicitly through on-line optimisation. At the dynamic optimisation level, an MPC controller must compute a set of input adjustments that will drive the process to the desired steady-state operating point without violating constraints. The constraints to the MPC control problem can be hard or soft, ranked in order of priority, and the models may be both phenomenological or result of the impulse response of plant.

MPC was first developed to meet the control needs of power plants and petroleum refineries, but nowadays it can now be found in a wide variety of application areas including several chemical processes, food processing, automotive, and aerospace industry.

Although the appearing of receding horizon control ideas which later lead to MPC can be traced back to the work of Kalman (1960) in the early 1960s, the real breakthrough in MPC theory and also the first industrial application were made by Charles R. Cutler, at the time an employee of Shell Oil Company in Houston, Texas. He proposed the Dynamic Matrix Control (DMC) algorithm (Cutler and Ramaker, 1979). The original DMC was later expanded to multivariable control with constraints (Cutler, 1983), and to use Linear programming techniques based solvers (LDMC) (Morshedi et al. 1985), and finally to make use of quadratic programming (QDMC) solvers in order to quickly find the optimal solution for linear systems (Garcia, Morshedi, 1986). This last paper was given some detail now because the authors showed how the DMC objective function could be re-written in the form of a standard QP, which was an important breakthrough.

The original DMC algorithm provided excellent control of unconstrained multivariable processes. Constraint handling, however, was still somewhat ad hoc. This weakness was addressed by posing the DMC algorithm as a quadratic program (QP) in which input and output constraints appear explicitly, and a quadratic performance objective is evaluated over a finite prediction horizon. Future plant output behaviour specified by trying to follow the SP as closely as possible subject to a move suppression term. The optimal inputs were the solution to the quadratic program. QP itself is one of the simplest possible optimisation problems since the Hessian of the QP is positive definite for linear plants, and thus the resulting optimisation problem is convex.

The technical success meant that essentially all vendors had adopted a DMC-like approach in their commercial MPC packages and this fact had a significant impact on process control industry. There is probably not a single major oil company in the world, where DMC or a similar product is not employed. Cutler later left Shell to start his company which was later purchased by Aspen Technology, that no offers the DMC+ package.

But other early implementations of MPC were proposed, including Model Algorithmic Control (MAC) (Rouhani, Mehra, 1982), and Internal Model Control (IMC) (Garcia and Morari, 1982), and the first comprehensive exposition of Generalized Predictive Control (GPC) (Clarke et al. 1987), all of which also have demonstrated their effectiveness in industrial applications.

Let us now make a quick comparison between DMC and GPC. At first sight, their concepts are similar, but the goals behind the development of DMC and GPC are not the same. DMC was conceived to use a time-domain model (finite impulse or step response model) in order to handle constrained, multivariable control problems which are typical for the oil and some chemical industries. Before MPC became popular this problem was handled by feedback controllers improved by selectors, overrides, decouplers, time-delay compensators, etc.

GPC was intended to offer a new adaptive control alternative, and stochastic aspects played a key role in GPC, while the original DMC formulation was completely deterministic. The GPC approach is awkward for multivariable constrained systems which are much more commonly encountered in the oil and chemical industries than situations where adaptive control is needed (Rashid, Moses 2000).

IMC was probably as technically capable as DMC, and its implementation was very similar, but it never enjoyed the same kind of popularity. However, some of its ideas, such as the robustness filter recommended by IMC theory were later used in the development of robust MPC (Morari and Zafiriou, 1989). This was another breakthrough that opened the way to numerous works that propose guaranteed stability up to this day. Robust MPC is a subfield of robust control, whose main motivation for researchers has been the development of robust control theories that can guarantee closed-loop stability in the presence of modelling errors. Using models with polytopic uncertainty, this set of techniques is based on the use of Lyapunov functions. Stability is guaranteed by means of enforcing that the MPC cost function is a strictly decreasing Lyapunov function for the closed-loop. The receding horizon state feedback control law robustly asymptotically stabilises the closed-loop system (see Section 2.1.3.2).

2.2.1 Robust Model Predictive Control and Techniques to Enforce Stability

Below we briefly review some of the popular techniques used in the literature to "enforce" closed loop stability. A robust control system can account for bounded disturbances while still ensuring that state constraints are met.

Morari and co-workers were responsible for the early efforts into adapting the robust control framework to include MPC schemes. Works such as Campo and Morari (1987), Kothare, Balakrishnan and Morari (1996) and Bemporad and Morari (1999) provided the basis for robust model predictive control by formulating the cost functions as Lyapunov equations, resulting in the nominal stability of the closed control loop. Lee, Morari and Garcia (1994) is also a very relevant paper which helped establish state-space models as the standard for modern MPC formulations. Finally, MPC surveys such as Garcia, Prett and Morari (1989) and Morari and Lee (1999) provided interesting and useful overviews on robust MPC theory for beginners as well as experienced researchers.

The seminal idea that prompted the establishment of the robust MPC framework is the set of "Terminal Constraint" techniques, an approach firstly proposed in Kwon and Pearson (1977). According to Bemporad and Morari (1999), this set of techniques can be divided into two main classes. In the first class, the objective function is defined in such a way that it corresponds to a Lyapunov function. The second explicitly requires that the difference between current state and reference shrinks in some norm through the prediction horizon.

The main drawback of using terminal constraints is that the control effort required to steer the state to the reference can be large, especially if a short control horizon is used, and therefore feasibility is a critical issue. Feasibility is limited to the "domain of attraction" of the closed-loop (MPC+plant) is defined as the set of initial states that can be steered to the reference values. This is a problem because the range of operating points where a plant is expected to operate may be considerably larger than the set of initial states which steerable to the reference in an arbitrary number of steps. Also, the speed of control actions can be negatively affected because of the artificial terminal constraint. A variation of the terminal constraint idea has been proposed where only the unstable modes are forced to zero at the end of the horizon (Rawlings and Muske, 1993). This mitigates some of the mentioned problems.

Let us briefly discuss other important contributions to the robust MPC field. Nominally stable MPC algorithms that use terminal constraints include: Infinite Output Prediction Horizon (Keerthi and Gilbert, 1988; Rawlings and Muske, 1993; Zheng and Morari 1995); the terminal Weighting Matrix (Kwon et al. 1983; Kwon and Byun 1989); Invariant Terminal Set (Scokaert and Rawlings 1996); Contraction Constraint (Polak and Yang 1993a,b; Zheng 1995).

Mhaskar, El-Farra and Christofides (2005) presented a Lyapunov-based model predictive control framework for switched nonlinear systems (models that switch at prescribed times). Mhaskar, El-Farra and Christofides (2006) consider the problem of the stabilisation of nonlinear systems subject to state and control constraints. An auxiliary Lyapunov-based analytical bounded control design is proposed to characterise a "stability region" of the MPC and also provide a feasible initial guess to the optimisation problem. It is also worth citing Heidarinejad, Liu and Christofides (2012) which proposes an Economic MPC (EMPC) of nonlinear process systems using Lyapunov techniques, and Liu, de la Peña and Christofides (2009) and Christofides et al. (2013) which introduce the concept of Distributed MPC, a robust MPC scheme which establishes communication between several different MPC controllers in order to achieve better closed-loop control performance. Specifically, each of the distributed algorithms comprehends both a local controller, who optimises a local (non-cooperative) cost function and also a global controller which handles a global (cooperative) cost function that refers to other distributed algorithms.

Allgöwer and co-workers contributed to several widely cited works on robust MPC theory. Among them we could highlight Chen and Allgower (1998), Allgöwer et al. (1999) and Findeisen and Allgöwer (2002), all of which helped establish the field of Nonlinear Model Predictive Control (NMPC) taking advantage of better nonlinear optimisation algorithms and increased computational power available in the late 1990's and early 2000's. NMPC is making use of nonlinear state-space models do better predict the process outputs and provide more efficient control actions, but it also results in a control problem for which is much harder to find the globally optimal solution.

Odloak and co-workers contributed to several works on Infinite horizon MPC (IHMPC), including González, Perez and Odloak (2009) and Rodrigues and Odloak

(2003a), which expanded IHMPC to integrating processes, and González and Odloak (2009), which introduced a Zone Constrained IHMPC (see Section 2.2.2). We may define IHMPC as an inherently robust MPC framework for constrained linear systems, which makes use of an infinite prediction horizon by replacing the infinite horizon objective by a finite one after defining a penalty weight matrix (terminal cost) at the end of the input horizon. The terminal weight is obtained from the solution of a discrete-time Lyapunov equation. Stability is guaranteed as long as the related optimisation problem is feasible. Also, features were added that enabled the IHMPC to handle common industrial problems such as zone constrained MPC and integrating processes, as well adapting IHMPC for use with models with polytopic uncertainty (Rodrigues and Odloak, 2003b), and providing robust integration with real-time optimisation packages (Alvarez et al., 2009).

Although terminal constraint techniques are very popular and effective for guaranteeing closed-loop MPC stability, some alternative robust MPC schemes were proposed. Some of them are outlined in Table 1.

Table 1 – Alternative robust MPC schemes.

Min-Max MPC (Scokaert and Mayne, 1998)	In this formulation, the optimisation is performed on all possible evolutions of the disturbance. This is the optimal solution to linear robust control problems; however, it carries a high computational cost.
Constraint Tightening MPC (Richards and How, 2006)	Here the state constraints are enlarged by a given margin so that a trajectory can be guaranteed to be found in any evolution of disturbance.
Tube MPC (Langson et al. 2004)	This uses an independent nominal model of the system and uses a feedback controller to ensure the actual state converges to the nominal state. The amount of separation required from the state constraints is determined by the robust positively invariant (RPI) set, which is the set of all possible state deviations that may be introduced by a disturbance with the feedback controller.
Multi-Stage MPC (Lucia et al. 2013)	This uses a scenario-tree formulation by approximating the uncertainty space with a set of samples, and the approach is non-conservative because it takes into account that the measurement information is available at every time stages in the prediction and the decisions at every stage can be different and can act as recourse to counteract the effects of uncertainties. The drawback of the approach, however, is that the size of the problem grows exponentially with the number of uncertainties and the prediction horizon.

2.2.2 Zone Constrained Model Predictive Control

Zone constrained MPC was briefly cited in the introductory Section and, being a key element of this project, it shall now be described in further detail. Zone constrained MPC is not a unique class of MPC algorithms, but rather an alternative way of defining the control objectives which can be applied to any MPC scheme.

In the 'perfect' model predictive control framework the goal is the complete rejection of the disturbances, and thus it is required that the output variables return to and remain in the original state before the end of the prediction horizon. But it does not matter how advanced or robust a controller may be if the processes do not possess an equal or superior number of unsaturated manipulated variables compared to the number controlled variables, a solution for the perfect control problem does not exist.

So in most control applications, these restrictions apply and thus perfect control is not attainable, but even so, all controlled variables need to be kept within certain limits. This is the reason why most commercial MPC controllers in the chemical industry operate with a variation of partial control often denominated "zone control" or "zone constraints", in which every controlled variable has maximum and minimum desired values, so that the control problem is not to keep each one at a fixed set-point but instead to keep them all inside the zones bounded by their maximum and minimum values.

However, the zone constrained MPC will not be able to keep all controlled variables within their desired control zones all of the time due to the lack of degrees of freedom, and then some restrictions are eventually violated. This may happen because there may be disturbances acting in the process, or the zones that were defined are too narrow, or perhaps the inputs to the process are already saturated. The zone control restrictions to the MPC problem are sometimes called "soft constraints" since they can be eventually violated. Likewise, every manipulated variable also has its required maximum and minimum values. The MPC controller cannot violate these restrictions; hence they are called a set of "hard constraints". These manipulated variable restrictions are associated with physical constraints. For example, a control valve cannot have percentage opening outside the 0-100% range, or the operating temperature of a reactor must obey its metallurgical limit, and a plant feed flow cannot be negative and so on. On the other hand, the soft constraints are usually related to product or process specifications and are defined by process dynamics and thus cannot be set to a specific value forthwith. Please note that classical set-point MPC is just a special case of zone control MPC, in which output maximum and minimum limits are equal. Zone control MPC also may encompass any kind economic or robust MPC, being, in reality, the most generic MPC definition.

In fact, virtually all MPC packages commercialised globally are a combination of zone constrained MPC and economic MPC, and some of them also claim to have robust performance. Among these packages, we can mention Honeywell™ MPC, Shell-Yokogawa Exa-SMOC™, Emerson DeltaV™ Predict and AspenTech DMCplus™ as offering simultaneously both process (zone) control and economic optimisation, in a combination that has become a standard feature for industrial control applications to perform. Schemes with guaranteed stability are not as popular in the industry since they result in unnecessarily slow control actions (as stated by Bemporad and Morari (1999), Robust MPC control actions may be excessively conservative).

Whereas most recent academic research has been focusing on robust MPC, comparatively fewer contributions have been made for optimising zone constrained MPC. Some examples of research concerning zone control are found in González and Odloak (2009), Grosman et al. (2010), Luo et al. (2012) and Zhang et al. (2011). There is also a more sizeable bibliography of works covering the integration of economic optimisation and MPC control, such as Porfírio and Odloak (2011), Gouvêa and Odloak (1998) and Adetola and Guay (2010).

In zone control each controlled variable has a minimum and maximum desired variable but some of these constraints may have more importance than others and, for this reason, when defining the MPC control problem it is common practice to assign each controlled variable a weight value, which establishes the relative priority each bound will have in the solution. For example, constraints to the process, the equipment and environmental safety normally have precedence over those concerning product specifications. Henceforth these weight values that relate the comparative importance of each controlled variable will be denominated $W_{i,upper}$ and $W_{i,lower}$, meaning the weights respectively for the upper (maximum value) and lower (minimum value) bounds of controlled variable y_i, where $i = 1, \dots, n_y$, and n_y is the number of controlled variables.

Let us now present a quick example to further clarify Zone Constrained MPC. For instance, consider now a system controlled by an MPC controller that consists of two controlled variables, y_1 and y_2, a single manipulated variable, u_1, and a single disturbance, d_1. Also, consider that y_1 has a higher weight in the control problem than y_2, so that $W_{1,upper} = W_{1,lower} = 2$ and $W_{2,upper} = W_{2,lower} = 0.5$, and that the zone constraints bounding the controlled variables are $y_{1,max} = y_{2,max} = 2$ and $y_{1,min} = y_{2,min} = -2$. Let it be assumed that the models defined by Eq. 31 to Eq. 34 describe the interaction between the controlled variables, the manipulated variable and the disturbance:

$$G_{y_1,u_1}(s) = \frac{-1}{3s^2+s+1} \quad \text{Eq. 31}$$

$$G_{y_2,u_1}(s) = \frac{-1}{2s+1} \quad \text{Eq. 32}$$

$$G_{y_1,d_1}(s) = \frac{1}{s+1} \quad \text{Eq. 33}$$

$$G_{y_2,d_1}(s) = \frac{-0.5}{s+1} \quad \text{Eq. 34}$$

Now consider that this system is subject to a disturbance $d_1(s) = \frac{3}{s}$. If no control action is taken, y_1 will increase until it violates its upper bound while y_2 decreases, but without leaving its desired control zone. In order to keep higher priority y_1 within bounds, decreasing its value to the maximum limit, $y_{1,max} = 2$ then the minimum control input necessary is $u_1(s) = \frac{1}{s}$. However, this movement in the manipulated variable will further decrease y_2, which will thus violate its lower bound, $y_{2,min} = -2$. In this case, the zone constrained MPC controller will prioritise the variable whose deviation from bounds or error has the largest impact on its objective function, which means keeping y_1 in its control zone at the expense of the less important y_2, since it doesn't have enough degrees of freedom to control both. However, the controller should make the minimum movement necessary to maintain y_1 within its control zone, minimising the error due to violating y_2 restriction.

Let us assume that the controller takes 10 seconds to react to the d_1. In this case, the system response is the one shown in figure 6. Please note that the control action is very steep in this example and an MPC controller can be tuned to provide a more smooth response. However, this has been ignored to simplify the analysis.

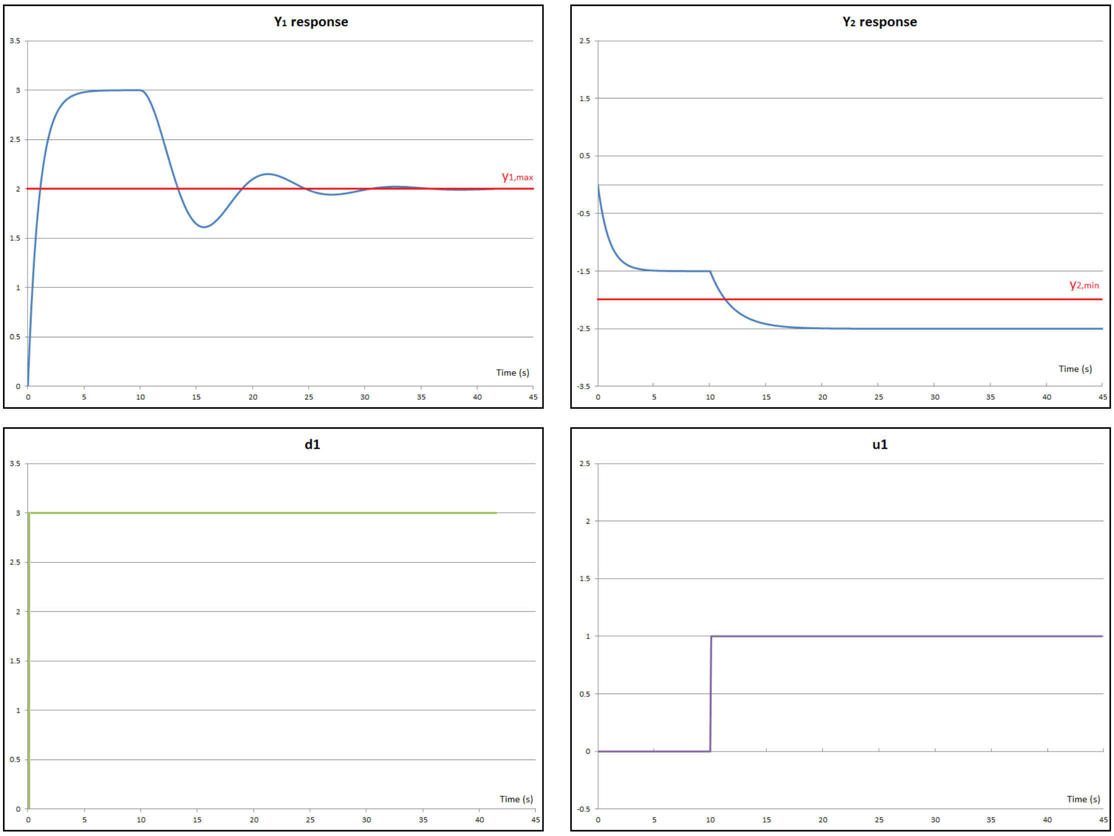

Fig. 6 – How an MPC controller handles the zone control problem.

Fig. 7 presents a diagram showing how a zone constrained MPC calculates the error (E_i) along the whole trajectory prediction:

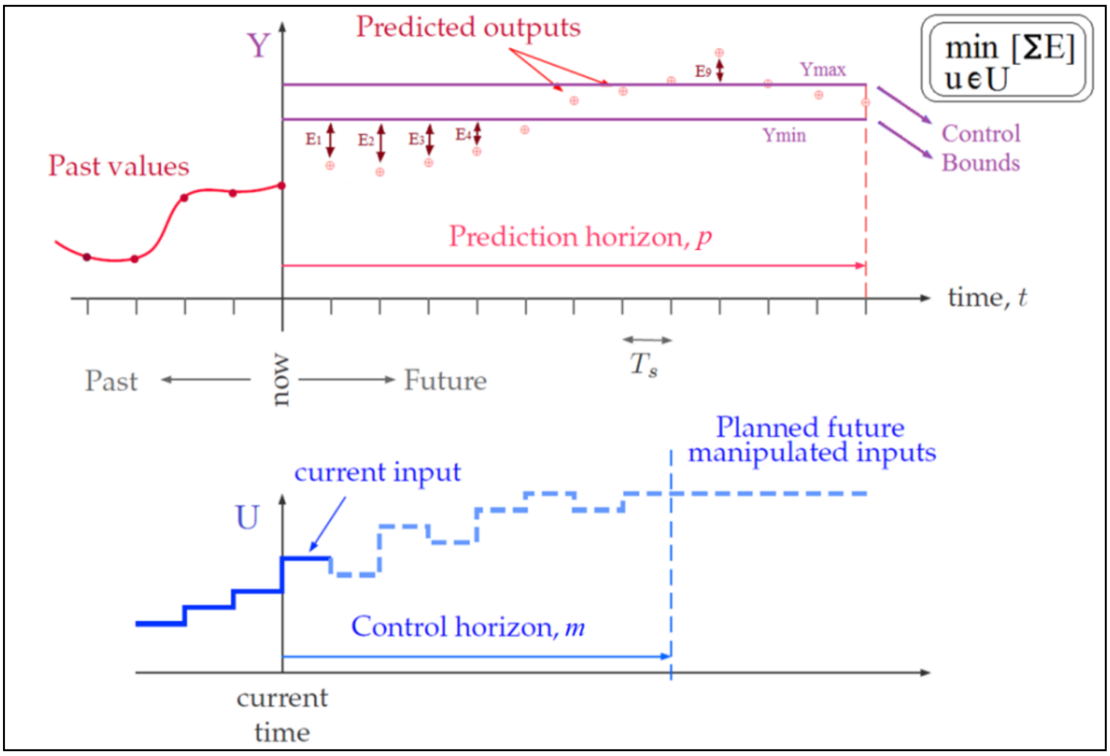

Fig. 7 – Error calculation for Zone Constrained MPC.

Eq. 35 defines the error related to vector of controlled variables at each instant k:

$$E_k = [y_{min} - y_k] \cdot W_{lower} + [y_k - y_{max}] \cdot W_{upper} \quad \text{Eq. 35}$$

A key point here is that the definition of the control zone is related to process constraints, not to the controller; maximum and minimum values for each variable are usually provided by the operator through MPC control user interface. Safety concerns and desired product specifications are constraints to possible solutions by bounding the control zones.

2.2.3 Interfaces of industrial MPC implementations

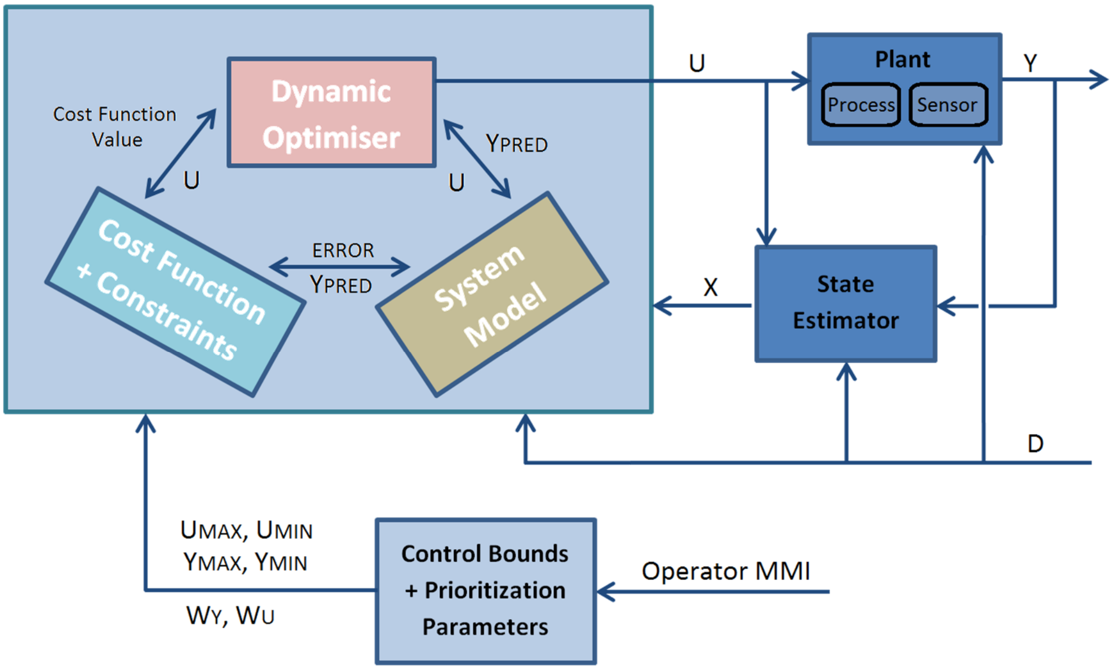

Fig. 8 – Interfaces of an industrial MPC implementation.

Industrial implementations of MPC schemes present several interfaces and components. Fig. 8 shows the interactions between the main elements inside the blue box: the model, which provides the output prediction; the cost function or algorithm, which evaluates the prediction according to the control goal priorities; and the optimiser or solver which searches for the optimal set of control action through the input-space. Human operators provide limits for MVs and CVs as well as the direction and priority (weight parameters) of economic optimisation, setting the cost function parameters and restrictions for the solver to use during its search for the optimal solution. Sensors located in the process site measure properties such as temperature, pressure, level and flow rate from the relevant streams. These measurements are filtered and converted to vectors of appropriate form (states) to enable future output prediction and bias correction. This conversion is performed by a state estimator, also known as state observer, which is a system that provides an estimate of the internal state of a given real system, from measurements of the input and output of the real system. An estimator is required when using state-space models since in most practical cases, the real state of the system cannot be determined by direct observation. Instead, indirect effects of the internal state are observed by way of the system outputs. If a system is observable, it is possible to fully reconstruct the system state from its output measurements using the state observer (Definition 2.1.1.1.2). The Kalman filter, proposed in Kalman (1960) is the most commonly employed state estimator. All layers of the MPC implementation are equally important for a successful operation.

2.2.4 Conclusions from the Model Predictive Control Review

A brief review of Model Predictive Control was presented in order to bring the reader's knowledge of to the level required to, firstly, understand some key choices made concerning the EMOP methodology development; secondly, allowing a full understanding of the review of Integrated Process Design and MPC Methodologies in the next Section.

So the reader should now have a basic understanding of the MPC fundamentals and be aware now that it is the most popular 'advanced control' structure in the industry, and also the one that has been receiving more academic contributions. Particularly relevant in recent years are the contributions mostly focused in guaranteeing closed-loop robust performance, which was examined in Section 2.2.1.

Section 2.2.2 presented 'Zone Constrained MPC', which is the form in which MPC control problem objectives are usually defined in industrial applications. The reader should now understand that the usual approach of fixed SPs or reference values is a special case of 'Zone Control', itself a broader definition of MPC control goals. Additionally, it is important to emphasise that most algorithms can be adapted as 'Zone Constrained' MPC, including those with guaranteed robust performance.

2.3 Review of Integrated Process Design and Model Predictive Control Methodologies

Adding MPC to the Integrated Process Design and Control framework is a very challenging task which has been only recently received the publication of several results. The purpose of this Section is to present a comprehensive survey of such works.

2.3.1 Embedding MPC in Flowsheet Analysis

As discussed in the Literature Review of Methods of Integrated Control and Process Synthesis, Chapter 2, there are several interesting papers where the integrated design methodology is applied to determinate the optimal design and control structure for a given process. Most of them undertake challenging issues in the integrated design framework such as alternative procedures to evaluate controllability, uncertainties handling techniques, the inclusion of different control strategies or address a complex application. An important aspect of these works on integrated design is the type of controllers and control strategies considered. Given how necessary and widespread considered conventional feedback controllers such as PI or PID are for the control of continuous processes such as those found in the chemical industry, it is only natural that the bulk of simultaneous design and control methodologies developed for chemical processes use this kind of control structure in their analysis, and these works were covered in Section 2.1. The works that deal with advanced control structures, such as the hugely popular Model Predictive Control, are yet few. All of them, to the author's knowledge, are discussed in this Section.

Until now, MPC schemes appear seldom in the framework's literature because the application of advanced control strategies in the integrated design framework is limited by the complexity of the resulting optimisation problems. However, the availability of improved computational resources allowing more powerful optimisation and computing methods, together with mature Controllability Analysis tools and advanced control technologies, provide the necessary driving force to address advanced control techniques, which introduce significant improvements in the process dynamic performance, particularly in the multivariable cases. MPC has become the advanced control method of choice in the chemical industries such oil refining for mixture separation, reactors and product blending, and for this reason, its insertion in the integrated design framework is a desirable development.

Integrated design of the chemical processes and MPC controller problem consists of simultaneously determining the plant and MPC controller parameters together with a steady state working point, while the investment and operating costs are minimised. Applied in the design stage of a plant design, the MPC control has a higher potential for reducing the necessity for oversizing than the usual feedback controllers like the PID controller, especially for systems with large delays, nonlinear response, and MIMO system with considerable loop interactions.

Perhaps Brengel and Seider (1992) was the first work to propose advanced strategies to extend the integrated design approach to use a nonlinear MPC control. The nonlinear analysis was carried to obtain more economical designs that are flexible and controllable in regimes characterised by greater sensitivities to modelling errors (process/model mismatch) and changes in set-points, and in the rejection of disturbances are more difficult to achieve. Through the applications of nonlinear programming for multi-objective design, operations and control optimisation, they argue that it should be possible to reduce the occurrence of overdesign in the process industries. The MPC control is formulated as an NLP that includes a differential-algebraic by the state-space model for the process. An interesting Section of this paper is the one which approaches "coordinated optimisations". The NLP solver uses the process model to evaluate the design objective function as well as the controllability of the proposed design, as the design optimisation proceeds. The idea is to simulate several disturbance scenarios and to penalise the design objective for poor controllability, which results in decreased profitability due to off-spec production. This was achieved by combining the two NLPs for design and control such that the results of each one affect the other. A nonlinear fermentation process was used as a study case and a coordinated optimisation strategy to solve the simultaneous problem is proposed, where the economic objective function is penalised by deficient controllability. This translates into a bi-level programming problem (BPP) which is later on simplified to obtain a solution, which can be classified as a multi-objective formulation (see Section 2.1.3.5). This work was published before robust MPC became the norm is thus no attention is paid to guaranteed closed-loop stability.

Loeblein and Perkins (1999) introduce a methodology for analysing the economic performance of different structures of an integrated MPC and on-line optimisation system. The tuning of non-constrained MPC and the evaluation of its performance within the optimisation framework are performed. The variance of the constrained variables of a closed-loop system subject to stochastic disturbances is calculated and then the necessary amount of back-off from the constraints due to disturbance realisations is analysed and later related to process economics. A simulated case study of a fluid-catalytic cracker is used to illustrate the methodology, but it consisted only of the riser-regenerator section. Monte Carlo simulations were used to confirm method predictions. Although process simulation was available, the magnitude of model uncertainty was just assumed. Also, no guidance was provided on how to obtain the uncertainty parameters.

Contributions made in recent years include Sakizlis et al. (2003) and Sakizlis et al. (2004), which presented an extension of the process and control design framework that incorporates parametric model-based predictive controllers. Applying parametric programming for the controller derivation, the authors removed the need for solving an optimisation problem online by giving rise to a closed-form controller structure. Parametric programming algorithms derive the explicit mapping of the optimal control actions in the space of the state measurements. Thus, a simple explicit state feedback controller was derived that moves off-line the embedded on-line control optimisation and preserves all the beneficial features of MPC. The solution consists of a set of affine control functions in terms of the states and a set of regions where these functions are valid. This mapping of the manipulated inputs in the state space constitutes a feedback parametric control law for the system. The authors applied this strategy to a typical Benzene/Toluene distillation column, proving significant economic and operability benefits. The embedded MPC structure added terminal constraints to guarantee Lyapunov closed-loop stability, which is missing in some similar works.

Baker and Swartz (2006) discussed the advantages of integrated design and the importance of accounting for actuator saturation, which could lead to suboptimal designs. In order to consider actuator saturation effects in the integrated design and control, which results in model discontinuities, and to avoid potential difficulties with a sequential solution approach in which the integration of the model differential-algebraic equation system is separated from the optimisation, they followed a simultaneous solution approach in which the actuator saturation is handled through mixed-integer constraints. The authors also considered integrated design and control with constrained model predictive control (MPC). The resulting problem shares the characteristic of model discontinuity with the actuator saturation problem above, but is more complex, since the control calculation involves the solution of a quadratic programming (QP) problem at every sampling period. When embedded within a design optimisation problem, this results in a multi-level optimisation problem. The authors proposed a simultaneous solution approach in which the MPC optimisation subproblems are replaced by their Karush-Kuhn-Tucker (KKT) optimality conditions. This results in a single-level optimisation problem with complementarity constraints. An interior point algorithm designed for mathematical programs with complementarity constraints was found to solve the problem more reliably and significantly faster than a mixed-integer quadratic programming formulation. However, it is not made clear by the authors how the process/model mismatch could be handled using the methodology.

An approach to deal with process/model mismatch is given in Chawankul et al. (2007). A variability cost is attributed each CV and the sum of capital and operating costs are added into a single objective function. The MPC internal model is a nominal linear model with parameter uncertainty, resulting in a robust model that represents the nonlinear process by a family of linear models. This approach avoided nonlinear dynamic simulations, offering computational advantages. The worst-case variability was quantified and its associated economic cost was calculated and referred to as the robust variability cost. This integrated method was applied to design a multi-component distillation column. A downside is that this methodology uses as process model a dynamic matrix of response coefficients. This approach is not as flexible and rigorous as state-space models, and thus it has for long been abandoned by most of the academia (Lundström et al., 1995; Lee, Morari and Garcia, 1994).

Francisco et al. (2011), whose methodology is intended to provide simultaneously the plant dimensions, the control parameters and a steady state working point using an IHMPC formulation with a terminal penalty and a model uncertainty approach for robustness. The optimisation problem is a multi-objective nonlinear constrained optimisation problem, including capital and operating costs and controllability indexes. Differently, from the previous works reviewed here, the authors used an MPC formulation which operates over an infinite horizon in order to guarantee stability. This IHMPC was implemented with a terminal penalty and a multi-model approach for robust performance. As it is often the case in integrated design papers, the methodology presented made use of norm-based indexes to assess controllability. The optimisation problem is stated as a multi-objective nonlinear constrained optimisation problem. The objective functions include investment, operating costs and dynamical controllability indexes based on the weighted sum of some norms of different closed-loop transfer functions of the system. The paper illustrated the application of the proposed methodology with the design of the activated sludge process of a wastewater treatment plant. A comparison between this procedure and the Economic MPC Optimisation index is presented in Section 7.2. However, performing this comparison was made somewhat difficult by the fact that Francisco et al. (2011) presented several controllability indexes without discussing how they should be prioritised, or how the different measures are related to each other.

Sánchez-Sánchez and Ricardez-Sandoval (2013a) presented a new integration of process flowsheet and control design methodology that incorporates MPC strategy in the robust analysis (see Section 2.1.4.2 for other robust methodologies). It consists of several layers of optimisation problems: dynamic flexibility analysis, a robust dynamic feasibility analysis, a nominal stability analysis, and a robust asymptotic stability analysis. The robust dynamic feasibility test implements a norm-bounded analysis that computes the critical realisations in the disturbances that produce the worst-case variability in the outputs. Likewise, process asymptotic stability is enforced by adding a formal asymptotic stability test. Those layers enable the specification of an optimal design that remains feasible and asymptotically stable despite critical realisations in the disturbances. The methodology incorporates structural decisions in the analysis for the selection of an optimal process flowsheet while formulating the analysis as a convex problem for which efficient numerical algorithms exist. The simultaneous process flowsheet and MPC design method were tested on a system of Continuous Stirred Tank Reactors. It is worth noting that the MPC formulation embedded in the analysis is perhaps over-simplistic since no effort was made to ensure the stability of the closed loop. Sakizlis et al. (2004) have a better approach in this particular case.

An updated method was presented by Bahakim and Ricardez-Sandoval (2014), involving the identification of an internal MPC model and solving an optimisation problem at each time step in which the MPC algorithm rejected stochastic-based worst-case disturbances. The control performance was added to a design cost function that also included the capital costs derived from equipment sizing parameters. The authors discuss the idea behind most of the optimisation -based approaches for simultaneous design and control, which is to determine the disturbances that produce the largest deviations in key controlled variables and therefore demand significant efforts from the control system to maintain the process within specifications in the presence of these conditions. Often termed worst-case scenario, this strategy is used by the simultaneous design and control methodologies to evaluate a design cost function. The several process constraints have to be considered in the analysis as well in order to contemplate the worst-case scenario in a safe and acceptable fashion without violating the critical operating restrictions of the system. An issue that this approach has is the challenge presented to the user in providing adequate parameters for the worst-variability distribution function. The assumption that this advanced degree of process knowledge is available is quite strong.

In order to avoid an unnecessary level of conservatism and thus incur in expensive process designs, Bahakim and Ricardez-Sandoval consider in their methodology how often the largest (worst-case) variability are going to occur and consider the level of significance of each violation. The authors considered in their methodology how often the worst-case would occur and the level of significance of constraints violations, arguing that it is not reasonable to overdesign the plant with increased costs due to extremely rare situations. The MPC controller presented in this paper uses a discrete linear state-space model that changes by each iteration as certain design parameters change. The model is also affected by the nominal (steady-state) conditions of the manipulated variables and the process set points. That means the linear MPC model needs to be identified (re-calculated) at each optimisation step. Stochastic disturbances were used and analysis was performed to verify their effect on the constrained variables. A probabilistic-based approach was employed to evaluate the process constraints; while a closed-loop nonlinear process model was simulated using multiple stochastic realisations of the disturbances. The worst-case (largest) deviation observed in any constraint for a particular realisation in the disturbances is called the stochastic-based worst-case variability (SB-WCV).

The review presented in this Section is summarised in Table 2, which displays a comparison between the papers concerning numerical methodologies, control structures, case studies and modelling strategy used by each of them.

Table 2 – Paper comparison of integrated process design and MPC methodologies.

	Numerical Methodology	MPC Structure	Case Study Process	Case Study Scale / Complexity	Model
Brengel and Seider (1992)	NLP (omotopy-continuation method)	Nonlinear Economic	Fermentation Plant	Small/Low	Phenomelological
Loeblein and Perkins (1999)	Quadratic programming (QP)	Linear Unconstrained	Fluid-Catalytic Cracker	Small/Low (linear model, 3 outputs, 2 inputs)	Hybrid Model
Sakizlis et al. (2004)	Mixed-Integer Nonlinear Programming (MINLP)	Parametric	Binary Distillation	Small/Low	Phenomenological
Baker and Swartz (2006)	NLP (Interior Point Algorithm)	Linear	CSTR	Small/Low	Phenomenological
Chawankul et al. (2007)	Linear Programming (LP)	Linear	Binary Distillation	Small/Low	Phenomenological/Step test
Francisco et al. (2011)	Sequential Quadratic Programming (SQP) and Multi-Objective Goal Attainment Optimisation	Infinite Horizon	Activated Sludge Process (Wastewater Treatment)	Small/Low	Phenomenological
Sanchez-Sanchez and Ricardez-Sandoval (2013)	Mixed-Integer Nonlinear Programming (MINLP) and Quadratic Programming (QP)	Linear Constrained	CSTR	Small/Low	Phenomenological
Bahakim and Ricardez-Sandoval (2014)	Global Optimisation (Genetic Algorithms)	Linear	Wastewater plant	Small/Low	Phenomenological

2.3.2 Conclusions from the review of integrated process design and MPC methodologies

MPC control has a higher potential for reducing the necessity for oversizing than the usual feedback controllers like the PID controller, especially for systems with large delays, nonlinear response, and MIMO system with considerable loop interactions. While it is desirable to incorporate MPC in the simultaneous design and control methodology, significant computational challenges that arise from the methodologies already proposed with this goal, e.g., the need to identify an internal MPC model and solve an optimisation problem (which may or not be convex) at each time step and simulating the resulting system. This is an issue that the works reviewed in this Section could not entirely overcome with the possible exception of Sakizlis et al. (2004), which proposes a different approach using parametric MPC that does not require dynamic simulations.

Even without embedding MPC, finding the global solution for plantwide optimisation can be already a challenging task as NLP and MINLP problems are notoriously hard to solve (Sakizlis et al., 2004). Even without accounting for control performance of the resulting flowsheet, plantwide optimisation is not commonly employed in most projects of chemical and petrochemical plants due to the complexity of the resulting problems and the inherent difficulty of achieving a global solution for several thousands of nonlinear and often discontinuous equations. Hence for industrial applications, plantwide optimisation has often been rejected in favour of sequential design strategies. This framework consists of defining the design of each single equipment as a segregated optimisation problem, and using one subsystem's solution to provide next one's input conditions, and thus reducing the number of variables and computing time. This approach currently works better for the large problem and will remain to do so for the foreseeable future.

The existing framework concerning integrated process design and MPC methodologies is subject to these same restrictions applicable to standalone plantwide optimisation, but when MPC is embedded in the objective function, these become more limiting. Therefore, in addition to not being effective under circumstances where plantwide optimisation is unsuitable, the body of work covered in this Section adds another layer of numerical and modelling complexity. As stated by Ricardez-Sandoval et al. (2009), the algorithmic framework involving MPC is computationally demanding even when a small number of process units are considered. Of course, it is true that the works discussed in this Section were undoubtedly successful in addressing ideally behaved processes such as binary distillation columns of chemically similar solvents and other separation processes of mixtures presenting near-ideal behaviour, i.e., systems whose deviation from Raoult's law can be ignored, or non-ideal solutions to which Raoult's law applies and fugacity and activity coefficients can be easily calculated. For a process that can be easily and satisfactory modelled such as these, the available integrated MPC and design problem framework was proven to be adequate.

Additionally, the range of problems to which these procedures may be applied is further restricted by the use of phenomenological models by all works reviewed here. While very general and elegant, this kind of model is not adequate for all kinds of processes.

For example, difficulties arise when dealing with petroleum fractions: each subsystem usually has well over one hundred non-ideal hypothetical components; and the severe operating conditions mean that the behaviour of gases, solutions and mixtures is also non-ideal; multiphase flow is very common and hard to model adequately; equipment designs are intricate. Some petroleum refining processes such as delayed coker cannot be satisfyingly modelled phenomenologically, and in this cases, design teams use commercial process simulators whose models rely on statistical information, e.g., neural networks or hybrid models, which are closed-source intellectual property. That being the case, the user is not provided with the set of equations being used. For those reasons, and the time and engineering effort required for rigorous modelling is always very large and, unless models are linearised, even with the optimisation solvers and processing power available at the time of writing, it is doubtful that a solution could be found in reasonable time. To avoid these issues, the state-space models used by the EMOP method are obtained through the classical approach of assessing the dynamic response to step increments in the MVs and DVs, as discussed in Chapter 3.

Another problem with the methodologies discussed here is that they always assume that a certain MPC formulation will be used, instead of being valid for any generic MPC algorithm. Furthermore, the control objectives were defined as SPs and not as control zones, which are more general formulations (see Section 2.2.2 for details). Given the usefulness and popularity of these control schemes, this is restrictive and an important omission.

Briefly, not all process design necessities and use cases are covered by the body of work presented in this Section. There is an important gap. Precisely the kind of process that is hard to model and numerically demanding would benefit greatly from improved design, which could guarantee longer operating times and greater profitability for capital-intensive industrial operations. The EMOP method addresses this challenge.

2.4 The Linear Hybrid Systems framework

This Thesis presents a novel method for the linear approximation of nonlinear systems using multiple linear models, called Simultaneous Multi-Linear Prediction (SMLP), which is detailed in Section 4. For this reason, it is necessary to provide some information about the state-of-art multi-model approaches that are currently available alternatives as to the SMLP. Such approaches can be classified into the Linear Hybrid Systems framework, presented in this Section.

A hybrid system is a collection of digital programs interacting with each other and with an analogue environment. Each logic state of the digital part of the hybrid system acts on the analogue part inducing a different operational mode. On the other hand, the evolution of the analogue part triggers switches in the states of the digital part. Many physical phenomena admit a natural hybrid description, such as control valve saturation; digital controllers embedded in a continuous process, which act on on/off valves; process equipment switching; switching on and off the electrical motor of pumps and compressors, and many others complications that demand adequate modelling. Moreover, some of the linear hybrid classes can be used to reduce the prediction error induced by process nonlinearity, a goal that the current work shares. Christophersen (2006) defined a general class of Linear Hybrid Systems that includes the following classes:

- Mixed logical dynamical systems;
- Linear complementary systems;
- Max-min-plus-scaling systems;
- Polyhedral piecewise affine systems or piecewise affine systems;

Bemporad and Morari (1999) introduced the mixed logical dynamical systems. In this framework, auxiliary variables are used to transform logical facts involving continuous variables into linear inequalities. These auxiliary variables are incorporated into the state-space and used to update the state vector, expressing relations that describe interdependent physical laws, logic rules, and operating constraints. Saturation functions, discrete inputs, qualitative outputs, bilinear systems and finite state machines can be modelled by this class, which also provides a multi-model approach by expressing nonlinear dynamic systems through combinational logic as piecewise linear time-invariant dynamic systems.

Another hybrid system class was introduced in the seminal paper Heemels et al. (2000), which used ideas from the theory of the linear complementarity systems (LCS).

LCS is a hybrid dynamical system defined by a linear ordinary differential equation (ODE) involving an algebraic variable that is required to be a solution of a finite-dimensional linear complementarity problem (LCP) (Cottle et al., 1992). The LCS is defined by a finite number of smooth ODEs, called modes, with transitions between the modes occurring along a state trajectory. One way to understand LCS is viewing it as a class of dynamical systems that switches between several operating modes. Within each mode, an LCS behaves like a linear system.

Heemels et al. (2000) used LCP for the mode selection process. As for the determination of jumps, it was based on linear system theory, more specifically the geometric theory of linear systems. To obtain a solution of a complementarity system, the associated jumps of the state variables have to be specified, i.e., the conditions under which a transition from one given mode to another given mode will take place must be precisely defined. The state spaces corresponding to different modes are not necessarily of the same dimension, but they are embedded in one encompassing space. Therefore, state trajectories may exhibit discontinuities when a mode switch takes place, as acknowledged by the authors.

De Schutter and Van Den Boom (2001) introduced a hybrid system class called max-min-plus-scaling systems in which discrete event systems that can be modelled using the operations maximisation, minimization, addition and scalar multiplication. These systems are extensions of max-plus linear systems (Baccelli et al., 1992; Cuninghame-Green, 1979), which can be used to model discrete event systems with synchronisation but no choice. Introducing choice led to the appearance of the minimum operation, resulting in the max-min-plus systems. A further extension was obtained by adding scalar multiplication. This yielded max-min-plus-scaling (MMPS) systems, which is shown by the authors to encompass several other classes of discrete event systems such as max-plus-linear systems, bilinear max-plus systems, polynomial max-plus systems, separated max-min-plus systems and regular max-min-plus systems.

Finally, Piecewise affine approximations are a class of Linear Hybrid Systems that are used for approximating nonlinear systems using a multi-model approach (Heemels et al., 2001). They can capture nonlinearities by partitioning the state-input space into regions and associating with each region a different affine state update equation (Sontag, 1981). This paradigm is a powerful modelling tool that can capture general nonlinearities (e.g. by local approximation), constraints, saturations, switches, discrete inputs and states, and other hybrid modelling phenomena in dynamical systems.

PWA systems can approximate nonlinear systems via multiple linearisations at different operating points (OP) (Sontag, 1981; Ferrari-Trecate et al., 2003; Roll et al., 2004). Space is partitioned through a series of linear approximations. As the plant operating point moves throughout the state-input space, the state update equation is changed according to the linear model valid locally, as shown in Fig. 9. The larger the number of partitions the closer the approximations become to the nonlinear model.

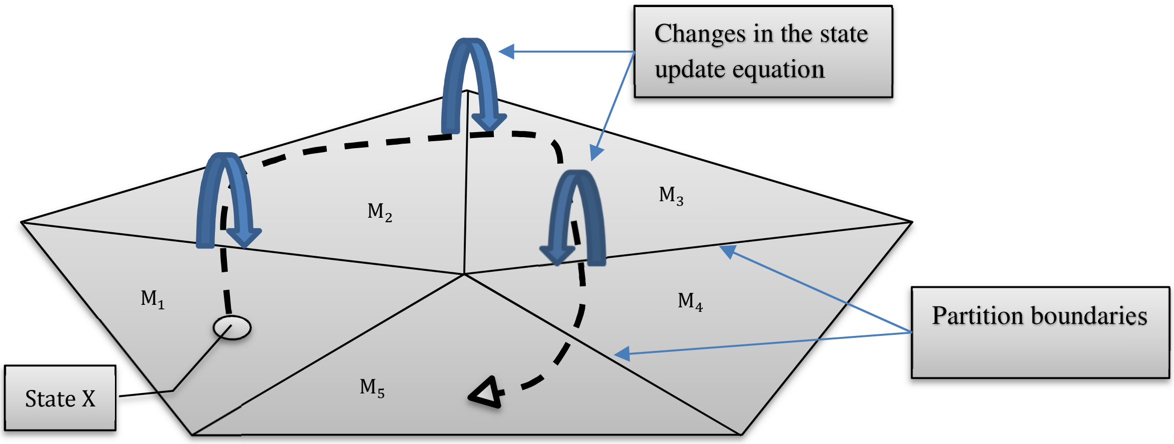

Fig. 9 – The operating point moves through the boundaries of a PWA system.

A Literature Review of switched and PWA models is presented in Roll (2003), and a tutorial paper in Paoletti et al. (2007). A survey on switched and piecewise affine system identification is presented in Garulli et al. (2012), from which the main approaches on the topic were presented. These main types of approaches were classified as optimisation-based, algebraic and recursive methods for Switched Autoregressive Exogenous (SARX) models, optimisation-based and clustering-based methods for PieceWise ARX (PWARX) models, and batch and recursive methods for state-space systems. According to Garulli et al. (2012), a common feature of the approaches, which is present in works such as Vidal et al. (2003), Roll et al. (2004), Ferrari-Trecate et al. (2003), Nakada et al. (2005), Juloski et al. (2005) and Bemporad et al. (2005), is that they lead to suboptimal solutions to the problem of inferring a PWARX model from data, while keeping an affordable computational burden.

In Section 4 a few shortcomings of PWA systems while used to represent model nonlinearity are discussed. These shortcomings motivated the development of the SMLP method, and in that Section it is discussed how it addressed them.

2.5 Conclusions from the Literature Review

Considering the discussion presented in this Chapter, it becomes clear that the most obvious gap in the existing Integrated Control and Process Synthesis (ICPS) body of work is the lack of an MPC-embedded method suitable for large-scale plants. This is in part caused by the increased complexity of the MPC control problem as compared to relatively simple feedback control structures, resulting in numerical and modelling challenges. Another issue is that while feedback controllers consist mostly in standardised PIDs, hundreds of distinct MPC algorithms exist, and each of the ICPS methods with embedded MPC discussed in Section 2.3 uses a different algorithm in their cost functions and simulations. Since the closed loop behaviour is being considered, the conclusions obtained by these methods are not valid for other algorithms, and even for different selections of tuning parameters. Furthermore, the closed loop analysis results in an optimisation problem composed of several layers that can be rapidly become intractable for larger systems, which are the main subjects of this work. Another gap currently found in the ICPS framework is the lack of a method with control goals defined as zones. Therefore, it is desirable to devise a methodology capable of fillings these gaps while tackling the challenge of a high computational demand. With this in mind, the Economic MPC Optimisation index is introduced in the next Chapter.

3 Assessing Plant Design for MPC Performance

The starting point of the Economic MPC Optimisation index methodology is a set of previously generated candidate designs, which are kept fixed during the analysis. Since the approach adopted is sequential, one could classify the EMOP methodology as part of the Controllability Analysis framework (Sections 2.1.1 and 2.1.2) rather than ICPS (Section 2.1.3). At the same time, the sequential approach opens an interesting possibility: using the EMOP index in addition to any ICPS procedure in order to test and validate the flowsheets generated (as done in Section 7.2 for the activated sludge process). Another relevant choice made during the design of the EMOP methodology was to deal only with open loop dynamic behaviour and make no assumptions towards closed-loop performance since these cannot be generalised across different MPC algorithms. The main drawback of dealing solely with the open-loop is the need of all plants to be bounded-input, bounded-output stable while closed-loop methods can design plants with less rigorous degrees of stability.

For the goal of embedding MPC in the IPDCF while dealing with large-scale, complex systems, segregating the problems of optimising capital expenditures (CAPEX) and operating expenses (OPEX) was the sensible choice. Restricting the analysis to OPEX greatly simplifies modelling and simulation and avoids the use of decision variables and the need for mixed-integer programming. This is the first reason why the candidate flowsheets were fixed; the second reason is to enable the use of commercial simulation packages to provide the models instead of first-principle models for most usage scenarios. The EMOP index makes use of multiple linear state-space models obtained through step-test model identification, a classic approach, using identification data provided by any given dynamic simulator. Although first-principle models can be used for simple processes, model identification is recommended for complex systems instead of iteratively solving mass and energy balances for each cost function evaluation.

Using both a sequential approach and step response models greatly reduces the numerical complexity of the resulting problem, enabling the analysis of larger systems than similar methodologies. Even if the models thus obtained are linear, the effect of process nonlinearity in the analysis can be mitigated by the use of multiples models (Chapter 4) and by bounding results according to model uncertainty (Chapter 5).

One could argue that such simulation packages could provide input directly to the index evaluation, but this is not practical for three reasons: firstly, each cost function evaluation would require a new simulation, and the total time would require for thousands of evaluations would be too long; secondly, the EMOP algorithm software does not run in the same environment, and establishing a link between the dynamic simulator and the solver can be complicated, requiring the use of communication protocols and additional coding; thirdly and more importantly, commercial simulation packages act as black-boxed which do not present their internal models to the user, but provide only numerical output. As such, assessing properties such as stability for these plants it would be impossible without using the numerical output to identify the models.

Since the flowsheets are fixed, the EMOP index can only be used to answer two key questions: what candidate flowsheets meet the dynamic restrictions of the process? Among these, what candidate flowsheet is the most profitable?

To this end, the main deliverables of this Thesis can be thus summarised:

- A Controllability Analysis methodology to assess the dynamic behaviour of flowsheets with regard to MPC performance which is directly linked to process economics;
- Two different case studies to demonstrate the use of the methodology, one of which is a large-scale process with embedded feedback control structures;
- A new multi-model approach called Simultaneous Multi-Linear Prediction (SMLP), developed to reduce nonlinearity-related error and thus improve the accuracy of the EMOP index while keeping the advantages of the use of linear models;

Zone Constrained MPC is assumed. The EMOP methodology favours solutions that have smooth transitions to the final state, and also penalises violations of zone constraints, making sure that the dynamic trajectory leading to the optimised steady-state is feasible. Also, restrictions concerning manipulated variables such as their maximum rate of change and maximum and minimum values are incorporated in the analysis.

Fig. 10 provides a workflow showing the order by which the methodologies presented in this Thesis should be performed, including the EMOP index and SMLP modelling.

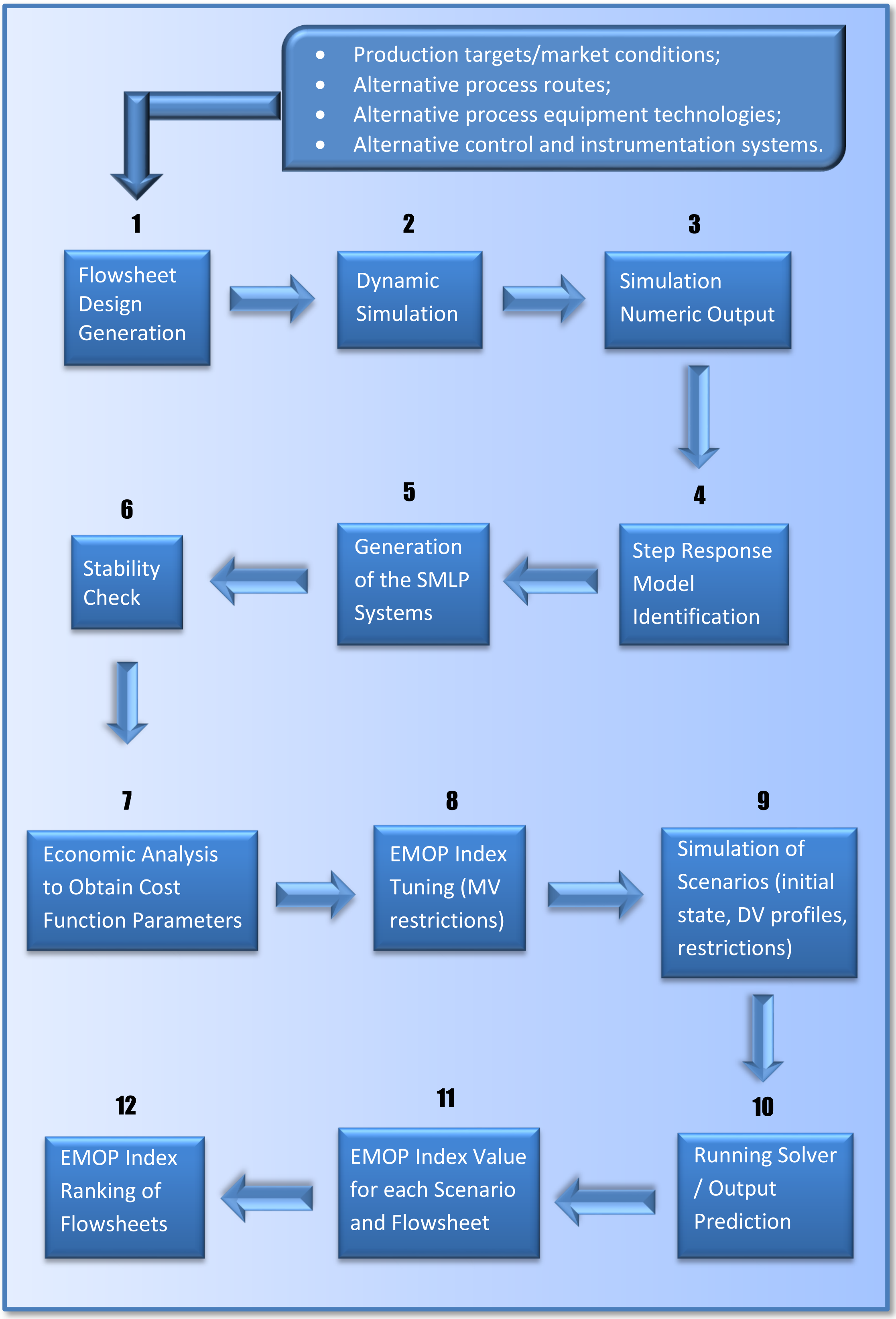

Fig. 10 – Workflow of the joint use of the EMOP index and the SMLP.

Now each step shown in Fig. 10 is discussed in detail. The whole of these procedures compose the EMOP index method which is one of the main deliverables of this Thesis.

Preliminary research

Preliminary research normally includes research of market conditions and the definition of the product being marketed; product prices and production targets; research of alternative process routes, equipment technologies; energy and raw materials costs; but seldom involves alternatives for control systems and instrumentation. This a standard procedure in process engineering.

Step 1 – Flowsheet Design Generation

In this step, the candidate flowsheets are defined based on the findings of the preliminary research. It is assumed that operation is continuous for all plants and that control zones will be defined for each plant.

At this point, the design team has usually picked out one of the alternative process routes and has some idea of the layout to be chosen. Differences between flowsheets might be subtle, such as different embedded regulatory control loop structures, or they might be moderate, e.g., different equipment layout, sizing and specifications for operating conditions. However, it is possible the information yielded by preliminary research was not enough to decide upon a single process route. In this case, considerable differences among flowsheets may arise, with production routes and layouts altogether different. In this case, the reader should consult Section 3.9.

Step 2 – Dynamic Simulation or Rigorous Modelling

Dynamic simulations of all flowsheets defined in Step 1 may be carried out in a commercial simulation package such as Petro-Sim®, UniSim®, Hysys®, etc. As previously discussed in this Chapter, these state-of-the-art simulators will support our integrated design and control approach, which is intended for large-scale, complex processes for which the full set of equations is unavailable. However, nothing prevents a first principles model to be used if the process is relatively simple. The dynamic simulations or first-principles model shall be referred in this Thesis as the nonlinear model.

Step 3 – Simulation Numeric Output (Optional)

If dynamic simulation was used, attention must be paid to the consistency of the data generated. Its numeric output must be validated through a sufficiently broad range of operating conditions. Output and input data must be stored in convenient data formats such as .csv or .xlsx to be handled in the software environment containing the optimisation solver, e.g., MATLAB®, GAMS®, etc. In this Thesis, the simulation data is presented as a series of linear transfer functions in the Laplace domain describing the relation between each CV, MV and DV, since this format is compact and convenient.

Step 4 – Step Response Model Identification (Optional)

Linearising the nonlinear model brings advantages when using optimisation solvers to find the optimal operating point, resulting always in convex problems for which fast conversion can be obtained. In this Thesis, each linearisation or approximation of the nonlinear model is called a sub-model.

Each sub-model is to be obtained through model identification on a set of data generated by step increments on MVs and DVs. Alternatively, the relay response could be used. Tests may be performed on a dynamic simulator or on an experimental pilot plant, or even on a real industrial unit, for the sake of the methodology, it is indifferent. Let us refer to the data thus obtained as the nonlinear model data, from which the sub-models are derived and validated. The initial state from which a test is performed will be referred as an identification point (IP) or epicentre, and it will be considered the epicentre of the resulting sub-model. For each input/output couple of the MIMO system, different values for the sub-model's order should be tested, and the order adopted is the one that minimises the residue between the nonlinear model data and the sub-model being considered. This procedure avoids model overfitting or underfitting, both of which can have catastrophic results especially when the operating point (OP) is removed from the vicinity of the model's epicentre. In this Thesis, the identification data used was different from the validation data, and we consider this to be a good practice.

Step 5 – Generation of the SMLP Systems (Optional)

This step is optional as the user may also use the nonlinear model, a single linear model or any other multi-model approach to predict plant behaviour.

The SMLP is a method for obtaining a prediction for the future values of an output vector. Three main elements are used to provide this prediction: the initial state, the input profiles and the set of sub-models (linear approximations of a nonlinear model) identified at distinct states, yielded by Step 4. Hence the SMLP consists of the representation of a flowsheet as a collection of linear state-space models, which are combined to yield an overall output prediction.

The greater the number of sub-models, the better the resilience of the resulting SMLP system with regard to process nonlinearity. The location of the epicentres of the sub-models is key to the SMLP method. The control engineer should select epicentres that are representative of the most common or critical operating conditions. In similar fashion to the sub-model validation of Step 4, the SMLP system should be validated against a test data set. Described in detail in Chapter 4 of this Thesis, the SMLP method has 3 variants. These variants present a trade-off between the accuracy displayed while emulating the nonlinear model and the time and effort required to assemble the system.

Step 6 – Stability Check

Being an open-loop reachability problem, the EMOP index only makes sense for Bounded-Input Bounded-Output (BIBO) stable flowsheets. We are interested in the optimal steady-state which is unbiased by the upper layer (MPC) control scheme. This optimal OP cannot be obtained for unbounded responses. The classical stability criteria for MIMO systems is detailed by Lyapunov (1892) (see Section 2.1.3.2). If an SMLP system is being used, the reader is encouraged to perform the stability check presented in Section 4.1.6.

Step 7 – Economic Analysis to Obtain Cost Function Parameters

This step provides the link between process economics and dynamics by evaluating the impact of plant dynamic behaviour and disturbances on revenue and OPEX. Basically, each variable receives a price tag based on product prices, quality, and costs. This step is discussed in Sections 3.3, 3.4, 3.7 and 3.9 as the cost function is constructed step by step. While there is no universal approach to perform this Step, since it depends on the process and control goals considered, the main case study of this Thesis which is shown in Chapter 6 can serve as a guideline.

Step 8 – EMOP Index Tuning (MV restrictions)

The EMOP index is an open-loop analysis intended to find each's flowsheet optimal OP. There is, of course, a transient between the initial state and the optimal OP and, just like an MPC, the OP is reached by manipulating process inputs. However, the optimisation problem is reduced by the lack of feedback. In MPC the prediction horizon keeps being shifted forward and for this reason, MPC is sometimes called receding horizon control. The EMOP uses a fixed horizon instead of a receding horizon. Since no measurement noise and no unmeasured disturbances are present, states can be calculated from the state-space model (Section 3.1 and Appendix) without the need of using state estimators.

There is, however, the need to ensure a smooth transition and no bound violation between initial and optimal OPs, and this subject is covered in Sections 3.5, 3.6 and 3.8. The control bounds on MVs usually reflect safety concerns. They may be related to equipment restrictions such as the maximum pressure and temperature tolerated by vessels, the minimum flow rate and the maximum head of pumps and compressors, etc. They may also refer to limits on feed, reflux or output flow rates.

Step 9 – Simulation of Scenarios (initial state, DV profiles, restrictions)

Process knowledge is paramount for the correct implementation of the EMOP method. This knowledge ideally encompasses the most commonly encountered disturbances encountered for the flowsheets being assessed, e.g., changing feed composition and production targets, changes in operating conditions such as variate bounds in temperature, pressure, product quality, etc. These scenarios consist of a range of operating conditions that reflect situations that are either very commonly experienced or uncommon but critical.

Step 10 – Running Solver / Output Prediction

Based on the scenarios defined in Step 9, the solver searches for the series of control actions leading to the OP of minimal OPEX/maximum revenue. Ideally, the same scenarios are evaluated for every flowsheet, i.e., the same changes in operating conditions are going to happen to all flowsheets. If this is unpractical due to existing differences in the flowsheets, the reader should use the method described in section 3.9.

While the SMLP system is linear, nonlinear restrictions and variate parameters introduce nonlinearities into the optimisation problem. The solver to be used must be able to handle such complications. For the case studies presented in this Thesis, we found out that the Interior-Point Algorithm with Analytic Hessian provided by MATLAB® 2016b (fmincon) was able to yield the desired results.

Step 11 – EMOP Index Value for each Scenario and Flowsheet

The gap between the EMOP indexes evaluated at the final and initial OPs yields a monetised measure of Controllability based on dynamic behaviour differences between flowsheets. This value is closely related to expected differences in OPEX between plants. The basic mathematical formula for the EMOP index evaluation is provided in Section 3.4. The version incorporating smoothness concerns is presented in Section 3.5, and the version with variate product prices is given in Section 3.7.

Step 12 – EMOP Index Ranking of Flowsheets

The indexes yielded by each scenario must be combined into a single index that describes the overall quality of dynamic response for a certain flowsheet. A weighted geometric average is suggested. The ranking of overall EMOP indexes shows the most favourable among the set of candidate flowsheets.

3.1 A State-Space Methodology

Most MPC schemes make use of linear state-space models, which can be identified with relative ease through step tests of the manipulated inputs. The prediction yielded by space-state models can also be used to evaluate the EMOP index of a flowsheet. Lee et al. (1994) proposed a generic linear state-space model representation in the incremental form as defined in Eq. 36 and Eq. 37:

$$\boldsymbol{y_k} = \boldsymbol{Cx_k} \qquad \text{Eq. 36}$$

$$\boldsymbol{x_{k+1}} = \boldsymbol{Ax_k} + \boldsymbol{B}\Delta\boldsymbol{u_k} + D\Delta\boldsymbol{d_k} \qquad \text{Eq. 37}$$

where $\boldsymbol{y} \in \mathbb{R}^{n_y}$ is the vector of process outputs, $\boldsymbol{x} \in \mathbb{R}^{n_x}$ is the state vector, $\boldsymbol{u} \in \mathbb{R}^{n_u}$ is the control input vector, $\boldsymbol{d} \in \mathbb{R}^{n_d}$ is the disturbance vector and $\Delta\boldsymbol{u_k}$ and $\Delta\boldsymbol{d_k}$ are the matrices of MVs and DVs increments defined respectively by Eq. 38 and Eq. 39:

$$\Delta \boldsymbol{u_k} = \begin{bmatrix} \boldsymbol{u_{k+1}} - \boldsymbol{u_k} \\ \boldsymbol{u_{k+2}} - \boldsymbol{u_{k+1}} \\ \vdots \\ \boldsymbol{u_{k+m}} - \boldsymbol{u_{k+m-1}} \end{bmatrix} \qquad \text{Eq. 38}$$

$$\Delta \boldsymbol{d_k} = \begin{bmatrix} \boldsymbol{d_{k+1}} - \boldsymbol{d_k} \\ \boldsymbol{d_{k+2}} - \boldsymbol{d_{k+1}} \\ \vdots \\ \boldsymbol{d_{k+p}} - \boldsymbol{d_{k+p-1}} \end{bmatrix} \qquad \text{Eq. 39}$$

where p is the prediction horizon and m is the number of time increments of the control horizon, which is also the number of control actions performed. Here we assume that the number of DV movements is equal to the prediction horizon.

The model formulation defined by Eq. 36 and Eq. 37 makes use of deviation variables and represents a process flowsheet that is assumed to be fixed during the analysis (this approach does not aim to replace early stage process synthesis usually based on steady-state information). Any steady state corresponds to a point where $\Delta \boldsymbol{u_k} = \boldsymbol{0}$ and there is no need to know the explicit value of $\boldsymbol{u}$ at the steady state corresponding to a particular output SP. The matrices of the linear state-space model can be identified with relative ease through step tests of the manipulated inputs. Obtaining models from the analytical step response is a model identification procedure that has been widely used in the MPC framework as an option to phenomenological modelling.

This state-space representation may be applied to time delayed stable, unstable and integrating systems as long as matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$ and $\boldsymbol{D}$ can be defined to properly represent these complications. The state-space model formulation as proposed in Strutzel et al. (2013) meets these needs and for this reason was selected for use in this work. More details and further developments about this kind of model representation, designated output prediction oriented model (OPOM), which was first proposed by Rodrigues and Odloak (2000), can be found in Martins et al. (2013), in Santoro and Odloak (2012) and González et al. (2007).

Based on this model, we set the goal of obtaining a method to assess how promising a given flowsheet is with regard to zone constrained MPC performance, i.e., a method to assess which plant is better placed to accommodate DVs (Disturbance Variables) while being optimised by an MPC. To this end, obtaining the output vector at the end of the prediction horizon (p) is going to be useful in later Sections. At an arbitrary time instant k, it is possible to predict the values of the process outputs at $k + p$ using the following procedure described by Eq. 40:

$$\boldsymbol{y}_{\boldsymbol{k+1}} = \boldsymbol{C}\boldsymbol{x}_{\boldsymbol{k+1}} = \boldsymbol{C}\boldsymbol{A}\boldsymbol{x}_{\boldsymbol{k}} + \boldsymbol{C}\boldsymbol{B}\Delta\boldsymbol{u}_{\boldsymbol{k}} + \boldsymbol{C}\boldsymbol{D}\Delta\boldsymbol{d}_{\boldsymbol{k}}$$

$$\boldsymbol{y}_{\boldsymbol{k+2}} = \boldsymbol{C}\boldsymbol{x}_{\boldsymbol{k+2}} = \boldsymbol{C}\boldsymbol{A}\boldsymbol{x}_{\boldsymbol{k+1}} + \boldsymbol{C}\boldsymbol{B}\Delta\boldsymbol{u}_{\boldsymbol{k+1}} + \boldsymbol{C}\boldsymbol{D}\Delta\boldsymbol{d}_{\boldsymbol{k+1}}$$

$$= \boldsymbol{C}\boldsymbol{A}^{\boldsymbol{2}}\boldsymbol{x}_{\boldsymbol{k}} + [\boldsymbol{CAB}\ \boldsymbol{CB}]\begin{bmatrix}\Delta\boldsymbol{u}_{\boldsymbol{k}}\\ \Delta\boldsymbol{u}_{\boldsymbol{k+1}}\end{bmatrix} + [\boldsymbol{CAD}\ \boldsymbol{CD}]\begin{bmatrix}\Delta\boldsymbol{d}_{\boldsymbol{k}}\\ \Delta\boldsymbol{d}_{\boldsymbol{k+1}}\end{bmatrix}$$

$$\vdots$$

$$\boldsymbol{y}_{\boldsymbol{k+p}} = \boldsymbol{C}\boldsymbol{A}^{\boldsymbol{p}}\boldsymbol{x}_{\boldsymbol{k}} + \\ [\boldsymbol{C}\boldsymbol{A}^{\boldsymbol{p-1}}\boldsymbol{B}\ \boldsymbol{C}\boldsymbol{A}^{\boldsymbol{p-2}}\boldsymbol{B} \ldots\ \boldsymbol{C}\boldsymbol{A}^{\boldsymbol{p-m}}\boldsymbol{B}][\Delta\boldsymbol{u}_{\boldsymbol{k}}\ \Delta\boldsymbol{u}_{\boldsymbol{k+1}} \ldots\ \Delta\boldsymbol{u}_{\boldsymbol{k+m-1}}]^T + \\ [\boldsymbol{C}\boldsymbol{A}^{\boldsymbol{p-1}}\boldsymbol{D}\ \boldsymbol{C}\boldsymbol{A}^{\boldsymbol{p-2}}\boldsymbol{D} \ldots\ \boldsymbol{C}\boldsymbol{A}^{\boldsymbol{2}}\boldsymbol{D}]\big[\Delta\boldsymbol{d}_{\boldsymbol{k}}\ \Delta\boldsymbol{d}_{\boldsymbol{k+1}} \ldots\ \Delta\boldsymbol{d}_{\boldsymbol{k+p-3}}\big]^T \qquad \text{Eq. 40}$$

3.2 Index for Control Bound Violations

The weight values for zone constrained MPC will be denominated $W_{i,upper}$ and $W_{i,lower}$ meaning respectively the weights for the upper (maximum value) and lower (minimum value) bounds of the controlled variable y_i, where $i = 1, \ldots, n_y$, and n_y is the number of controlled variables. Initially, let us consider the steady-state achieved by the MPC where the plant will operate for much of the time. The questions of smoothness of the transient response shall be dealt with in Section 3.5, but for now, let us just assume that given enough time the plant will reach steady-state after a series of control and optimisation actions. If only the last instant in the prediction is considered, a new problem arises in which the goal is to minimise the sum of the predicted deviations from the control zone for each process output multiplied by its weight. A cost function for this control problem can be defined as defined in Eq. 41 to Eq. 44:

$$J_{CV_{k+p}} = \left|\boldsymbol{y_{min}} - \boldsymbol{y_{k+p}}\right| \cdot \boldsymbol{W_{lower}} + \left|\boldsymbol{y_{k+p}} - \boldsymbol{y_{max}}\right| \cdot \boldsymbol{W_{upper}} \qquad \text{Eq. 41}$$

$$if\ \boldsymbol{y_{min}} \leq \boldsymbol{y_{k+p}} \leq \boldsymbol{y_{max}} \ \Rightarrow \boldsymbol{W_{upper}} = \boldsymbol{W_{lower}} = \boldsymbol{0} \qquad \text{Eq. 42}$$

$$if\ \boldsymbol{y_{min}} > \boldsymbol{y_{k+p}} \ \Rightarrow \boldsymbol{W_{lower}} > \boldsymbol{0} \qquad \text{Eq. 43}$$

$$if\ \boldsymbol{y_{k+p}} > \boldsymbol{y_{max}} \ \Rightarrow \boldsymbol{W_{upper}} > \boldsymbol{0} \qquad \text{Eq. 44}$$

In this problem, it is of special interest to know if there is a set of manipulated variables that leads the system to a state where all outputs are within their zone constraints at the end of the prediction horizon. For this ideally bounded state, we have $J_{CV_{k+p}} = 0$. Alternatively, if there is no ideal solution for Eq. 41, the optimisation is going to find the final state that minimises the violation of the control bounds, resulting in the minimal $J_{CV_{k+p}}$.

3.3 An Economic Optimisation Index

MPC controllers found in the chemical industry frequently possess economic optimisation functions in addition to the control capabilities. Also, recent research has shown that process economics can be optimised directly in the dynamic control problem, which can take advantage of potentially higher profit transients to give a superior economic performance. Examples of this approach include Amrit et al. (2013) and Strutzel et al. (2013). The optimisation is performed by changing the manipulated process inputs when the process finds itself within its control bounds, and degrees of freedom are available to be employed in optimisation tasks. In oil refining processes, MVs are often related to process energy cost. For instance, it may be necessary to burn more natural gas in the fired heater to increase the temperature of the feed stream to a reactor. If the feed temperature is an MV increasing it has a negative impact on process profitability, which depends on the price of natural gas. Another example would be diesel production in an atmospheric crude oil distillation column. In this process, it is often possible to improve the quality of the diesel by reducing its output and, consequently, increasing the atmospheric residue output. However, diesel has a much higher commercial value, so if the flow rate of diesel is a manipulated variable, it is positively correlated to profitability. Other MVs are not strongly correlated to energy costs or product prices and can be altered freely. In inorganic processes, the goal is often to maximise chemical reaction conversion and the relation between MV and costs may be less obvious.

It is thus interesting to define for each MV whether it is positively or negatively related to profitability, and to what degree. Let us now define two sets of optimisation weights, $V_{j,min}$ and $V_{j,max}$, where $j = 1, \dots, n_u$, where n_u is the number of MVs, which illustrate the optimisation direction and relative priority among the various MVs for economic purposes. If a given MV is positively correlated to profitability and at the present moment is not being employed for control actions, it should stay as close as possible to its maximum limit or upper bound. Likewise, if it is negatively related to profitability, it should stay close to its minimum limit or lower bound. In the single layer MPC control scheme, the optimisation weights V_{min} *and* V_{max} are very small compared to the control zone weights W_{upper} and W_{lower} in the cost function, and optimisation is performed without hindering the control objectives. Ferramosca et al. (2014) provide a formal proof of convergence for such an approach.

A simplified optimisation cost function may then be established yielding Eq. 45, which relates the distance between MVs and their bounds at the prediction's end.

$$J_{MV_{k+p}} = |\boldsymbol{u_{k+p}} - \boldsymbol{u_{min}}| \cdot \boldsymbol{V_{min}} + |\boldsymbol{u_{max}} - \boldsymbol{u_{k+p}}| \cdot \boldsymbol{V_{max}} \qquad \text{Eq. 45}$$

MVs are subject to:

$$\boldsymbol{u_{min}} \leq \boldsymbol{u_{k+p}} \leq \boldsymbol{u_{max}} \qquad \text{Eq. 46}$$

where:

$$\boldsymbol{u_{k+p}} = \sum_{j=1}^{m} \Delta \boldsymbol{u_{k+j}} \qquad \text{Eq. 47}$$

Concerning the MVs, it is desired to determine how far they stand from the hard constraints because, depending on the direction of optimisation, this distance denotes how much room there is for optimisation. The optimisation weights denoting the direction of optimisation or increased profit are subject to Eq. 48, Eq. 49 and Eq. 50:

$$V_{j,min} \geq 0 \qquad \text{Eq. 48}$$

$$V_{j,max} \geq 0 \qquad \text{Eq. 49}$$

$$V_{j,min} \cdot V_{j,max} = 0 \qquad \text{Eq. 50}$$

where $j = 1, \dots, n_u$. This set of restrictions was included to guarantee that a single economic optimisation direction exists for each variable if there is any: if for an MV of index j, $V_{j,min} = V_{j,max} = 0$, then the variable doesn't have any optimisation direction, being neutral for profitability.

3.4 The Simplified Economic MPC Optimisation Index (EMOP)

Adding Eq. 41 and Eq. 45 in a single cost function yields the basic form of the EMOP index:

$$J_{k+p} = J_{CV_{k+p}} + J_{MV_{k+p}}$$

$$= |\boldsymbol{y_{min}} - \boldsymbol{y_{k+p}}| \cdot \boldsymbol{W_{lower}} + |\boldsymbol{y_{k+p}} - \boldsymbol{y_{max}}| \cdot \boldsymbol{W_{upper}} + \quad \text{Eq. 51}$$

$$|\boldsymbol{u_{k+p}} - \boldsymbol{u_{min}}| \cdot \boldsymbol{V_{min}} + |\boldsymbol{u_{max}} - \boldsymbol{u_{k+p}}| \cdot \boldsymbol{V_{max}}$$

Solutions are subject to Eq. 42 to Eq. 44, Eq. 46, and Eq. 48 to Eq. 50. The prediction of $\boldsymbol{y}$ at the time instant $k+p$ is given by Eq. 40, which can be further simplified by defining the following matrices:

$$\overline{\overline{\boldsymbol{C_u}}} = [\boldsymbol{CA^{p-1}B}\ \boldsymbol{CA^{p-2}B} \dots \boldsymbol{CA^{p-m}B}] \quad \text{Eq. 52}$$

$$\overline{\overline{\boldsymbol{C_d}}} = [\boldsymbol{CA^{p-1}D}\ \boldsymbol{CA^{p-2}D} \dots \boldsymbol{CA^{2}D}] \quad \text{Eq. 53}$$

$$\Delta\boldsymbol{u_K} = [\Delta\boldsymbol{u_k}\ \Delta\boldsymbol{u_{k+1}} \dots \Delta\boldsymbol{u_{k+m-1}}]^T \quad \text{Eq. 54}$$

$$\Delta\boldsymbol{d_K} = \left[\Delta\boldsymbol{d_k}\ \Delta\boldsymbol{d_{k+1}} \dots \Delta\boldsymbol{d_{k+p-3}}\right]^T \quad \text{Eq. 55}$$

Replacing these new terms in Eq. 40 yields:

$$\boldsymbol{y_{k+p}} = \boldsymbol{CA^p x_k} + \overline{\overline{\boldsymbol{C_u}}}\,\Delta\boldsymbol{u_K} + \overline{\overline{\boldsymbol{C_d}}}\,\Delta\boldsymbol{d_K} \quad \text{Eq. 56}$$

The vector u_{k+p} may be calculated as shown in Eq. 57:

$$\boldsymbol{u_{k+p}} = \sum_{j=1}^{m} \Delta\boldsymbol{u_{k+j}} = \boldsymbol{I_{m.n_u}} \cdot \Delta\boldsymbol{u_K} \quad \text{Eq. 57}$$

where:

$$\mathbf{I_{m.nu}} = \underbrace{\begin{bmatrix} 1 & \cdots & 0 & 1 & \cdots & 0 & & 1 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 & \cdots & 1 & & 0 & \cdots & 1 \end{bmatrix}}_{m.n_u} \Bigg\} n_u \qquad \text{Eq. 58}$$

Applying Eq. 56 and Eq. 57 in the cost function Eq. 51, and then rewriting the resulting equation in vector form, yields Eq. 59, which is a more functional form for the EMOP index. The definition below is valid:

Definition 3.4.1 The Simplified Economic MPC Optimisation index. *Let J_{k+p} be an economics-based control performance index of a plant, which increases in value to penalise zone control bound violations at a time instant $(k+p)$, where k is the initial time, p is the prediction horizon. Let $\boldsymbol{W_{lower}} \in \mathbb{R}^{n_y}$ and $\boldsymbol{W_{upper}} \in \mathbb{R}^{n_y}$ be respectively the rate of increase of J_{k+p} due to lower control zone and upper control zone bound violations, and n_y be the number of CVs. Let there also be increases of J_{k+p} proportional to the gap between MVs and their optimal values at $k+p$. The intensity with which the gaps are penalised is proportional to weights $\boldsymbol{V_{min}} \in \mathbb{R}^{n_u}$ and $\boldsymbol{V_{max}} \in \mathbb{R}^{n_u}$, which define the direction (maximisation or minimisation) and relative priority of economic optimisation, where n_u be the number of MVs. Let $\Delta\boldsymbol{u_k} \in \mathbb{R}^{n_u\, x\, m}$ be a matrix of MV increments or control actions used, where the number of actions (control horizon) is m. Let $\Delta\boldsymbol{d_k} \in \mathbb{R}^{n_d\, x\, m}$ be a matrix of DV increments or disturbances realisations, where the number of DVs is d. Let $\overline{\overline{\boldsymbol{C_u}}} \in \mathbb{R}^{n_u\, x\, m}$ and $\overline{\overline{\boldsymbol{C_d}}} \in \mathbb{R}^{n_d\, x\, m}$ be the matrices defining the steady-state gains of the dynamic responses to $\Delta\boldsymbol{u_k}$ and $\Delta\boldsymbol{d_k}$. Let $\boldsymbol{I_{m.nu}} \in \mathbb{R}^{m.n_u}$ be the result of the concatenation of m identity matrices of n_u dimension. Let and $\boldsymbol{y_{max}} \in \mathbb{R}^{n_y}$ and $\boldsymbol{y_{min}} \in \mathbb{R}^{n_y}$ be vectors denoting the upper and lower control zone bounds defined for the CV set. An evaluation of the Economic MPC Optimisation (EMOP) index yields J_{k+p}, as defined in Eq. 59, relative to a set of values provided for $\Delta\boldsymbol{u_k}$ and $\Delta\boldsymbol{d_k}$. Let U_K and D_K be respectively the spaces of possible realisations of Δu_k and Δd_k. For a plant described by the state-space model defined by Eq. 36 and Eq. 37, subject to a set of disturbance variables (DVs), $\Delta\boldsymbol{d_K} \in \boldsymbol{D_K}$, the basic form of the EMOP index is defined by Eq. 59. The methodology consists in solving the optimisation problem of Eq. 60 to find $\Delta\boldsymbol{u_k} \in \boldsymbol{D_U}$ that minimises J_{k+p},* *subject to the restrictions posed by Eq. 42 to Eq. 44, Eq. 46, and Eq. 48 to Eq. 50.*

$$J_{k+p} = \left|\boldsymbol{y_{min}} - \boldsymbol{CA^p x_k} - \overline{\overline{\boldsymbol{C_u}}}\, \Delta \boldsymbol{u_K} - \overline{\overline{\boldsymbol{C_d}}}\, \Delta \boldsymbol{d_K}\right| \cdot \boldsymbol{W_{lower}}$$
$$+\left|\boldsymbol{CA^p x_k} + \overline{\overline{\boldsymbol{C_u}}}\, \Delta u_K + \overline{\overline{\boldsymbol{C_d}}}\, \Delta \boldsymbol{d_K} - \boldsymbol{y_{max}}\right| \cdot \boldsymbol{W_{upper}} \qquad \text{Eq. 59}$$
$$+\left|\boldsymbol{I_{m.n_u}} \cdot \Delta \boldsymbol{u_K} - \boldsymbol{u_{min}}\right| \cdot \boldsymbol{V_{min}} + \left|\boldsymbol{u_{max}} - \boldsymbol{I_{m.n_u}} \cdot \Delta \boldsymbol{u_K}\right| \cdot \boldsymbol{V_{max}}$$

$$I_{EMOP} = \min_{\Delta \boldsymbol{u_K} \in \boldsymbol{U}} (\boldsymbol{J_{k+p}}) \qquad \text{Eq. 60}$$

3.5 Ensuring Viable Solutions

The economic cost function defined in Eq. 60 guarantees that the final state will be as close as possible to the economically optimal state without violating the MPC constraints but does not consider the transient response. Now we shall modify the cost function to ensure that the transition to the final state is as smooth as possible. To achieve this, we now introduce two new parameters in the cost function that will penalise steep changes in the final predicted values for the states, favouring smooth curves for the controlled variables at the end of the prediction.

These new terms shall be called “Soft-Landing” matrices and will be inversely proportional respectively to the first and second order derivatives of the controlled variables at instant $k + p$.

Definition 3.5.1 First-Order and Second-Order Soft-Landing matrices. *Let $\boldsymbol{T_{SL1}} \in \mathbb{R}^{n_y}$ and $\boldsymbol{T_{SL2}} \in \mathbb{R}^{n_y}$ be parameter vectors that define the priority of stablishing steady-state for each CV at the end of the prediction horizon. Let $\boldsymbol{y^*_{max}} \in \mathbb{R}^{n_y}$ and $\boldsymbol{y^*_{min}} \in \mathbb{R}^{n_y}$ be vectors containing the maximum and minimum sensor ranges limits for each CV, Δt be the sampling period and $\boldsymbol{I_{ny}} \in \mathbb{R}^{n_y \, x \, n_y}$ be the identity matrix of n_y dimensions, where n_y is the number of CVs. The first-order and second-order Soft-Landing matrices $\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$ for the complete form of the EMOP index are defined respectively by Eq. 61 and Eq. 62.*

$$\boldsymbol{SL_1} = \boldsymbol{I_{ny}} - \boldsymbol{T_{SL1}}\left[\frac{diag|y_{k+p}-y_{k+p-1}|}{\Delta t} \cdot diag|\boldsymbol{y^*_{max}} - \boldsymbol{y^*_{min}}|^{-1}\right] \quad \text{Eq. 61}$$

$$\boldsymbol{SL_2} = \boldsymbol{I_{ny}} - \boldsymbol{T_{SL2}}\left[\frac{diag\left|\begin{matrix}[y_{k+p}-y_{k+p-1}]\\-[y_{k+p-1}-y_{k+p-2}]\end{matrix}\right|}{\Delta t^2} \cdot diag|\boldsymbol{y^*_{max}} - \boldsymbol{y^*_{min}}|^{-1}\right] \quad \text{Eq. 62}$$

The larger the vectors of first and second order CV derivatives at the end of prediction horizon ($k+p$) are, the closer $\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$ are from being null matrices of infinite determinant values. For null derivatives, $\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$ are equal to identity matrices of equivalent dimensions. The rates by which $\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$ grow detached from identity matrices as the derivatives increase are given by $\boldsymbol{T_{SL1}}$ and $\boldsymbol{T_{SL2}}$, that define the how strong sharp CV moves should be rejected for each variable. The Soft-Landing matrices will be relevant mostly if the prediction horizon, p, is small and the system doesn't have enough time to stabilise.

By multiplying the economic cost function by the inverses of the Soft-Landing matrices, its value will increase proportionally to the slope of the final output prediction. Higher values favour flatter curves at the expense of a more aggressive approach. Values for $\boldsymbol{T_{SL1}}$ and $\boldsymbol{T_{SL2}}$ must be assigned so that $\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$ are contained in the unit circle. Tuning guidance for the Soft-Landing matrices is provided in Section 3.6.

The first order derivative is the difference between the controlled variable's predictions at $k+p$ and $k+p-1$, which can be obtained using Eq. 63:

$$\begin{aligned}
&\boldsymbol{y_{k+p}} - \boldsymbol{y_{k+p-1}} = \\
&\boldsymbol{CA^p x_k} + [\boldsymbol{CA^{p-1}B}\ \boldsymbol{CA^{p-2}B} \ldots \boldsymbol{CA^{p-m}B}][\Delta\boldsymbol{u_k}\ \Delta\boldsymbol{u_{k+1}} \ldots \Delta\boldsymbol{u_{k+m-1}}]^T \\
&+[\boldsymbol{CA^{p-1}D}\ \boldsymbol{CA^{p-2}D} \ldots \boldsymbol{CA^2D}]\left[\Delta\boldsymbol{d_k}\ \Delta\boldsymbol{d_{k+1}} \ldots \Delta\boldsymbol{d_{k+p-3}}\right]^T \\
&-\boldsymbol{CA^{p-1}x_k} \\
&-[\boldsymbol{CA^{p-2}B}\ \boldsymbol{CA^{p-3}B} \ldots \boldsymbol{CA^{p-m-1}B}][\Delta\boldsymbol{u_k}\ \Delta\boldsymbol{u_{k+1}} \ldots \Delta\boldsymbol{u_{k+m-1}}]^T \\
&-[\boldsymbol{CA^{p-2}D}\ \boldsymbol{CA^{p-3}D} \ldots \boldsymbol{CAD}]\left[\Delta\boldsymbol{d_k}\ \Delta\boldsymbol{d_{k+1}} \ldots \Delta\boldsymbol{d_{k+p-3}}\right]^T = \\
&\left[\boldsymbol{I_{ny}} - \boldsymbol{A^{-1}}\right] \cdot \\
&\left\{\begin{matrix}\boldsymbol{CA^p x_k} + \\ [\boldsymbol{CA^{p-1}B}\ \boldsymbol{CA^{p-2}B} \ldots \boldsymbol{CA^{p-m}B}][\Delta\boldsymbol{u_k}\ \Delta\boldsymbol{u_{k+1}} \ldots \Delta\boldsymbol{u_{k+m-1}}]^T \\ +[\boldsymbol{CA^{p-1}D}\ \boldsymbol{CA^{p-2}D} \ldots \boldsymbol{CA^2D}]\left[\Delta\boldsymbol{d_k}\ \Delta\boldsymbol{d_{k+1}} \ldots \Delta\boldsymbol{d_{k+p-3}}\right]^T\end{matrix}\right\}
\end{aligned} \quad \text{Eq. 63}$$

Replacing Eq. 63 and the matrices defined in Eq. 56, the first-order Soft-Landing matrix becomes Eq. 64:

$$\boldsymbol{SL_1} = \boldsymbol{I_{ny}} - \boldsymbol{T_{SL1}}\left[diag\ \frac{|[\boldsymbol{I_{ny}}-\boldsymbol{A}^{-1}][\boldsymbol{CA^p x_k}+\overline{\overline{\boldsymbol{C_u}}}\,\Delta\boldsymbol{u_K}+\overline{\overline{\boldsymbol{C_d}}}\,\Delta\boldsymbol{d_K}]|}{\Delta t}\cdot diag|\boldsymbol{y_{max}}-\boldsymbol{y_{min}}|^{-1}\right] \quad \text{Eq. 64}$$

A similar procedure may be used to obtain the SL_2, the term related to the second order derivative of the controlled variables at $k + p$, which is the difference between the slope at $k + p$ and the slope at $k + p - 1$. The second order derivative may be calculated as follows:

$$[\boldsymbol{y_{k+p}} - \boldsymbol{y_{k+p-1}}] - [\boldsymbol{y_{k+p-1}} - \boldsymbol{y_{k+p-2}}] = \boldsymbol{y_{k+p}} - 2\boldsymbol{y_{k+p-1}} + \boldsymbol{y_{k+p-2}}$$

$$= [\boldsymbol{I_{ny}} - 2\boldsymbol{A^{-1}} + \boldsymbol{A^{-2}}]\cdot \left\{\begin{matrix} \boldsymbol{CA^p x_k} + \\ [\boldsymbol{CA^{p-1}B}\ \boldsymbol{CA^{p-2}B} \dots \boldsymbol{CA^{p-m}B}][\Delta\boldsymbol{u_k}\ \Delta\boldsymbol{u_{k+1}} \dots \Delta\boldsymbol{u_{k+m-1}}]^T \\ +[\boldsymbol{CA^{p-1}D}\ \boldsymbol{CA^{p-2}D} \dots \boldsymbol{CA^2D}][\Delta\boldsymbol{d_k}\ \Delta\boldsymbol{d_{k+1}} \dots \Delta\boldsymbol{d_{k+p-3}}]^T \end{matrix}\right\} \quad \text{Eq. 65}$$

$$= [\boldsymbol{I_{ny}} - 2\boldsymbol{A^{-1}} + \boldsymbol{A^{-2}}][\boldsymbol{CA^p x_k} + \overline{\overline{\boldsymbol{C_u}}}\,\Delta\boldsymbol{u_K} + \overline{\overline{\boldsymbol{C_d}}}\,\Delta\boldsymbol{d_K}]$$

Hence Eq. 63 becomes:

$$\boldsymbol{SL_2} = \boldsymbol{I_{ny}} - \boldsymbol{T_{SL2}}\left[\begin{matrix} diag\ \frac{|[\boldsymbol{I_{ny}}-2\boldsymbol{A}^{-1}+\boldsymbol{A}^{-2}][\boldsymbol{CA^p x_k}+\overline{\overline{\boldsymbol{C_u}}}\,\Delta\boldsymbol{u_K}+\overline{\overline{\boldsymbol{C_d}}}\,\Delta\boldsymbol{d_K}]|}{\Delta t^2} \\ \cdot\, diag|\boldsymbol{y_{max}}-\boldsymbol{y_{min}}|^{-1} \end{matrix}\right] \quad \text{Eq. 66}$$

The Soft-Landing matrices are incorporated into the cost function, which means that if one or more destabilising sequences of control actions do exist when under evaluation they would cause the cost function value to explode and thus these sequences would be ignored by the solver. If no smooth solution is available at all the final solution would have a high value that should alert the control engineer.

Besides guaranteeing a smooth transition to steady-state at the prediction's end, we shall now introduce a new term in the cost function that will penalise temporary violations of the control bounds that may occur during the trajectory between the initial state and the optimal final state. This matrix, shall be denoted the "error penalisation" matrix (EP) and it will increase the cost function value if any of the controlled variables stay out of their control zone.

Definition 3.5.2 Error-Penalisation matrix. *Let $\boldsymbol{T_{EP}} \in \mathbb{R}^{n_y}$ be a parameter vector that define the priority of rejecting control zone bound violations for each CV, α be an auxiliary variable that denotes time during the transient, whose value varies from $\alpha = k$ (beginning of the prediction horizon) to $\alpha = k + p$ (end of the prediction horizon), $\boldsymbol{y_\alpha} \in \mathbb{R}^{n_y}$ be the vector of controlled variables at time alpha, and $\boldsymbol{y_{max}} \in \mathbb{R}^{n_y}$ and $\boldsymbol{y_{min}} \in \mathbb{R}^{n_y}$ be vectors denoting the upper and lower control zone bounds, $\boldsymbol{T_{upper,\alpha}} \in \mathbb{R}^{n_y}$ and $\boldsymbol{T_{lower,\alpha}} \in \mathbb{R}^{n_y}$ be parameter vectors that define, respectively, the active upper and lower control zone bounds at time alpha, where n_y is the number of CVs. The Error-Penalisation matrix $\boldsymbol{EP} \in \mathbb{R}^{n_y \, x \, n_y}$ is defined by Eq. 67.*

$$\boldsymbol{EP} = \boldsymbol{I_{ny}} - \boldsymbol{T_{EP}} \sum_{\alpha=k}^{k+p} \left\{ [diag(p|\boldsymbol{y_{max}} - \boldsymbol{y_{min}}|)]^{-1} \cdot diag \begin{pmatrix} |\boldsymbol{y_\alpha} - \boldsymbol{y_{max}}| \cdot \boldsymbol{T_{upper,\alpha}} \\ +|\boldsymbol{y_{min}} - \boldsymbol{y_\alpha}| \cdot \boldsymbol{T_{lower,\alpha}} \end{pmatrix} \right\} \quad \text{Eq. 67}$$

So $\boldsymbol{y_\alpha}$ is compared to $\boldsymbol{y_{max}}$ and $\boldsymbol{y_{min}}$ throughout the transient to assess if any CV left its control zone. When obtaining solutions to the EMOP optimisation problem, the restrictions to $\boldsymbol{T_{upper}}$ and $\boldsymbol{T_{lower}}$ shown in Eq. 68 to Eq. 70 must be included.

$$if \ y_{i,min} \le y_{k+p,i,\alpha} \le y_{i,max} \Rightarrow T_{i,upper} = T_{i,lower} = 0 \quad \text{Eq. 68}$$

$$if \ y_{i,min} > y_{k+p,i,\alpha} \Rightarrow T_{i,lower} > 0 \quad \text{Eq. 69}$$

$$if \ y_{k+p,i,\alpha} > y_{i,max} \Rightarrow T_{i,upper} > 0 \quad \text{Eq. 70}$$

These restrictions guarantee that $\boldsymbol{EP}$ will decrease if, during the transient, any controlled variable overshoots. If that happens, the solution will be penalised even if the final state is within its control zone. Matrices $\boldsymbol{T_{lower}}$, $\boldsymbol{T_{upper}}$ and $\boldsymbol{T_{EP}}$ define how strongly overshooting will be rejected, but they must be small enough to guarantee that $\boldsymbol{EP} > \boldsymbol{0}$ through all the transient response.

Finally, we can reach the complete form for the EMOP index by multiplying the cost function given by Eq. 59 by the inverse of the determinants of the Soft-Landing and the error penalisation matrices, $\boldsymbol{SL_1}$, $\boldsymbol{SL_2}$ and $\boldsymbol{EP}$.

Definition 3.5.3 The Economic MPC Optimisation index. *In the complete form of the EMOP index, the optimisation problem of Eq. 60 is replaced by the one defined by Eq. 71, which includes the effects of* $\boldsymbol{SL_1}$, $\boldsymbol{SL_2}$ and $\boldsymbol{EP}$.

$$I_{EMOP} = \min_{\Delta \boldsymbol{u_K} \in \boldsymbol{U}} \left(|\boldsymbol{SL_1}|^{-1} |\boldsymbol{SL_2}|^{-1} \left[J_{k+p} + |\boldsymbol{EP}^{-1}| \right] \right) \quad \text{Eq. 71}$$

The desired outcome is that, for the selected plant design, the impact of matrices $\boldsymbol{SL_1}$, $\boldsymbol{SL_2}$ and $\boldsymbol{EP}$ on the Economic MPC Optimisation index will be very small. Should these matrices affect the EMOP index in a remarkable way, it would be a sign that either their tuning is too aggressive or that the flowsheet is rather inappropriate. In a typical use scenario, the optimal value is mostly a consequence of the control actions necessary to offset the effects of DVs, changes in the control zones or changes in economics parameters. Hence, the standard index formulation of Eq. 71 should cause the optimisation solver to discard solutions which present significant overshooting. A good solution for the EMOP problem guarantees that eventual violations of control zones are quick enough to keep the effects of the EP matrix relatively small.

If the plant is stable and if the prediction horizon p is sufficiently larger than the control horizon m, control actions will already have taken full effect by the end of the prediction, $k + p$, and hence $\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$ will be nearly identity matrices. In turn, this means the EMOP index refers to a steady state. If, however, if p is small the index cost function will increase, and the solution will be penalised heavily.

3.6 EMOP Index Interpretation and Tuning

So how can we interpret the Economic MPC Optimisation index? The index is indirectly related to properties such as controllability, flexibility, operability, feasibility and switch ability, but it provides results directly related to the economics of the process. Specifically, the EMOP index is measured by monetary units such as US dollars and is strongly linked to plant operating revenue. The feedback from this methodology enables the control engineer to validate changes in the flowsheets and assess their impact on controllability and profitability.

What is being measured by the index? It quantifies the effects of key process design choices in the capacity of the MPC controller to avoid economic losses, providing a measure of how much a plant can be optimised while keeping controlled variables within the bounds of the zone control. The first two terms in Eq. 59 penalise any plant whose optimal steady-state lies outside the bounds defined by zone control for one or more CVs. Typical MPC CVs be classified into two categories:

- Product specifications, e.g., the purity grade of a chemical product;
- Operation specifications, e.g., combustion chamber pressure of a fired heater.

Products specifications are directly related to process economics since off-spec products have lower market value or even no value at all. Operation specifications are normally indirectly related to process economics, acting as restrictions to process optimisation. One exception would be catalyst temperature, which is related to deactivation rate and thus can be linked to monetary loss.

Manipulated variables sometimes can also be linked to manufacturing costs or revenue. Some examples of MVs with a strong relation to process economics include:

- Feed flow rate;
- Product output;
- Energy supply, e.g., electricity, natural gas, etc.;
- Product recycles;
- Pumparound Duty (energy saved due to feed preheating).

The third and fourth terms in Eq. 59 penalise the gap between optimal MV value and the final steady-state value and may be interpreted as being the MV opportunity cost. The steady state is key in the analysis since it is where the plant is expected to operate during most of the time. However, it is also necessary to evaluate how the plant behaves during state transitions. It is of special importance to ascertain that environmental and safety CVs do not overshoot during the transient, as even small violations can have a significant negative impact. Eq. 67 was added to the index to penalise constraint violations as it is driven to the optimal state. Eq. 61 and Eq. 62 were added to promote control actions that result in smooth plant responses. If the Soft-Landing matrices ($\boldsymbol{SL_1}$ and $\boldsymbol{SL_2}$) and the error penalization matrix ($\boldsymbol{EP}$) are small this will cause the EMOP index to increase, reflecting a poor dynamic response. The magnitude of this penalisation depends on decisions made by the project team concerning which CVs must be prioritised and what harm can be brought by eventual constraint violations.

The vectors $\boldsymbol{T_{SL1}}$, $\boldsymbol{T_{SL2}}$, $\boldsymbol{T_{EP}}$, $\boldsymbol{T_{upper}}$ and $\boldsymbol{T_{lower}}$ all require manual tuning. Their values should reflect prioritisation among controlled variables as well the desired balance between steady-state optimisation and penalisation for eventual issues in the transient behaviour. The larger the values, the greater the penalties for lack of a smooth transition to the final state and violations of the control zones. The exact values that should be assigned depending on the number of variables (a larger number of variables increases their cumulative effect) and the desired penalty. For example, one could tune $\boldsymbol{T_{EP}}$, $\boldsymbol{T_{upper}}$ and $\boldsymbol{T_{lower}}$ in such a way that $|\boldsymbol{EP}|^{-1} = J_{k+p} \cdot [0.5/n_y]$, if a single variable stays unbounded for 50% of the transient. If in addition to that another variable is unbounded for 75% of the transient, $|\boldsymbol{EP}|^{-1} = J_{k+p} \cdot [(0.5 + 0.75)/n_y]$, and so on. Another possibility is setting $\boldsymbol{T_{SL1}}$ and $\boldsymbol{T_{SL2}}$ in such a way as to provide Soft-Landing matrices that follow, approximately, $|\boldsymbol{SL_1}|^{-1} = J_{k+p} \cdot \left(\prod \sqrt[n_y]{1 + \boldsymbol{y'_{k+p}}/\boldsymbol{y_{k+p}}} - 1\right)$ and $|\boldsymbol{SL_2}|^{-1} = J_{k+p} \cdot \left(\prod \sqrt[n_y]{1 + \boldsymbol{y''_{k+p}}/\boldsymbol{y_{k+p}}} - 1\right)$. While these are useful guidelines, the tuning choices ultimately depend on the control engineer's judgement about the correct balance between steady-state and transient performances, both of which are important elements of the analysis. The concept of "optimal tuning" does not apply here: what is necessary is that the selection of tuning parameters reflects adequately the criteria by which process performance is going to be judged. The parameters must be the same for all flowsheets resulting in the use of the same criterion.

3.7 Including price variations in the EMOP cost function

In some processes, the market prices of one or more products are related to key CVs. Just to name two relevant examples, petrol (gasoline) and diesel oil are priced in the global market according to, respectively, their Octane Number and Cetane Index. Also, numerous inorganic chemical processes have their products priced according to their degree of purity. Therefore, the possibility of producing premium as opposed to regularly priced products should be incorporated into the EMOP index's cost function. To accomplish that, a new term is added to the cost function for each product price variation. This new term is a function of a new variable, $\Delta\overline{\boldsymbol{y}}_{\boldsymbol{qual}}$, defined as the difference between the key CV quality threshold value for which the price variation occurs, $\overline{\boldsymbol{y}}_{\boldsymbol{qual}}$, and the average value of the key CV through the prediction horizon, $\overline{\boldsymbol{y}}_{\boldsymbol{ave}}$, as shown in Eq. 72 and Eq. 73:

$$if\ (\overline{\boldsymbol{y}}_{\boldsymbol{qual}} - \overline{\boldsymbol{y}}_{\boldsymbol{ave}}) \geq \boldsymbol{0},\ \Delta\overline{\boldsymbol{y}}_{\boldsymbol{qual}} = (\overline{\boldsymbol{y}}_{\boldsymbol{qual}} - \overline{\boldsymbol{y}}_{\boldsymbol{ave}}) \quad \text{Eq. 72}$$

$$if\ (\overline{\boldsymbol{y}}_{\boldsymbol{qual}} - \overline{\boldsymbol{y}}_{\boldsymbol{ave}}) < \boldsymbol{0},\ \Delta\overline{\boldsymbol{y}}_{\boldsymbol{qual}} = \boldsymbol{0} \quad \text{Eq. 73}$$

So, the added cost function value, $\mathrm{j_q}$, is defined as:

$$\boldsymbol{j_q} = \Delta\overline{\boldsymbol{y}}_{\boldsymbol{qual}} \cdot \frac{\Delta \boldsymbol{P}_{\boldsymbol{qual}}}{v_{prod}} \cdot \boldsymbol{v}_{\boldsymbol{prod}} \qquad \text{Eq. 74}$$

Where $\Delta\boldsymbol{P}_{\boldsymbol{qual}}$ is the added value, i.e., product price difference between premium priced and regular product, and $\boldsymbol{v}_{\boldsymbol{prod}}$ is the volume produced of said product. If there are more than one quality threshold for a single CV, adding the price increments of all the consecutive thresholds to the price of the regular product, $\boldsymbol{P}_{\boldsymbol{reg}}$, should yield the price of the most premium variant, $\boldsymbol{P}_{\boldsymbol{qual,nq}}$, as shown in Eq. 75:

$$\boldsymbol{P}_{\boldsymbol{qual,1}} = \boldsymbol{P}_{\boldsymbol{reg}} + \Delta\boldsymbol{P}_{\boldsymbol{qual,1}}$$

$$\vdots$$

$$\boldsymbol{P}_{\boldsymbol{qual,nq}} = \boldsymbol{P}_{\boldsymbol{reg}} + \sum_{\boldsymbol{j=1}}^{\boldsymbol{nq}} \Delta\boldsymbol{P}_{\boldsymbol{qual,j}} \qquad \text{Eq. 75}$$

Where nq is the number of quality thresholds of the product being considered. For processes with several CVs with quality thresholds, it is useful to aggregate all added cost function values in a single term, as per Eq. 76:

$$J_q = \sum_{i=1}^{ny} \sum_{j=1}^{nq,i} \Delta\overline{y}_{qual,i} \cdot \frac{\Delta P_{qual,i,j}}{v_{prod,1}} \cdot v_{prod,i} \qquad \text{Eq. 76}$$

Where J_q is total opportunity cost due to quality thresholds. This way, J_q becomes null if only the most premium priced product variants are being produced. Adding to Eq. 71, the EMOP index becomes:

$$I_{EMOP} = \min_{\Delta u_K \in U} \left(|\boldsymbol{SL_1}|^{-1} |\boldsymbol{SL_2}|^{-1} \left[J_{k+p} + J_q + |\boldsymbol{EP}^{-1}| \right] \right) \qquad \text{Eq. 77}$$

3.8 Exploring the Relation between the Regulatory Control Layer, MPC Layer and the EMOP Index

The approach presented in this Thesis is based on the premise that DVs are known and estimated a priori and follow a given time-dependent profile. While this may seem restrictive, this choice reflects the nature of the dual layer control strategy used, where the MPC is used for economic optimisation, and feedback controllers deal with fast disturbances.

Also, that in many chemical processes often the MPC layer DVs are known and planned ahead of time by operating staff. Examples of MPC layer DVs include events such as changes in the feed composition and product specifications, programmed equipment shutdowns, tank switches, and changes in pipeline alignment. Here we compare for each plant how planned one-off occurrences, which are very frequent and impact process profitability. DVs that may be modelled stochastically, such as observational noise, are normally dealt with by the regulatory control layer, which is much faster. The control engineer must keep in mind that the regulatory layer is part of the plant which is being evaluated and modifying it changes the model and analysis results.

This Thesis does not concern the issue of the selection of the lower level control structure, i.e., selecting MVs and CVs for regulatory control and dealing with their interconnections. This important subject has already been investigated in Kookos and Perkins (2001) and Kookos and Perkins (2012), among others (a review of methods for input/output selection is presented in De Wal and Jager, 2001). For the EMOP index method, it is enough to assume that the plant is Integral Controllable and the embedded control structures of each flowsheet were correctly engineered and tested, yielding controllable plants. If this assumption is correct, the tuning of the lower level control structure will not affect the EMOP index significantly.

For instance, in the case study provided in Section 4, the linear models that define each plant are closed-loop models involving multiloop feedback controllers. The sample time used for the state-space models was 10 minutes while PID controller sample time was set at 1 second at the simulation. The large difference in speed between MPC and PID variables greatly diminishes the index sensitivity to the regulatory control structure. Tests showed that the PIDs could bring its CVs back to their SPs easily and rapidly when the process was disturbed. Also, these PIDs proved themselves easy to tune and had excellent performance for all plants. And finally, the issue of feedback control structures for crude oil distillation has been adequately tackled in sources such as Luyben (2013) and Brambilla (2014), providing templates that could be embedded in the flowsheets. So, in this case, the effect of the regulatory control layer was small enough to be safely ignored.

However, this may not always be the case. If there is no clarity over the selection of control structure, or if one or more flowsheets are not Integral Controllable, then the design and tuning of the lower layer may have an impact on the EMOP index. A bad score may help to detect, for example, if the regulatory control is tuned too slowly or aggressively, or if its actuators run at their saturation limits, etc., but should such issues be detected, the flowsheet should be discarded.

3.9 Using the EMOP index to compare plants with radically different layouts

So far in this work, we focused on comparing flowsheets which share main characteristics and have enough similarity to possess the same MPC structure, i.e., the same set of CVs and MVs. Additionally, in our study case in Chapter 6, the same control zones were used for all plants, and all of them started the simulation at the same (null) state. Having the same control zones, optimisation weights and initial states greatly facilitates the flowsheet assessment. The effects of comparatively small design differences can be thus evaluated. But there may be situations where we desire to compare plants for which these assumptions are not accurate. During the early stages of chemical process design, it may be necessary to choose between the different routes and technologies available to produce a certain product. For instance, some different routes for methanol production exist, including synthesis from oil, natural gas or coal. It is unlikely that these candidate flowsheets will share the same MPC structure.

So here a new parameter is introduced in the EMOP index as a means to enable comparison between plants that have each its own control bound definitions and starting point, MVs, CVs and parameters. This extended EMOP index can be used to compare completely different plants.

The EMOP index is a measure of monetisation of the control effort, and it calculates changes in operating revenue caused by control actions and disturbances. One basic assumption until now, besides shared MPC structure, is that all flowsheet designs have the same expected operating revenue at the initial operating point. For instance, all crude oil distillation plants described in Chapter 4 have the feed flowrate, the same oil mix, and produce the same products at the start of the simulation. Therefore, all plants began the simulation with the same operating revenue.

This basic assumption will not, of course, be correct for designs that use different production routes, raw materials, MPC control structures, etc. So, in this case, the distinct initial operating revenue of each plant must be account for. Let us subtract operational revenue (total sales minus costs) at an initial time (k) from the EMOP index definition, Eq. 77, yielding Eq. 78:

$$I_{EMOP,\ P_\theta} = \min_{\Delta \boldsymbol{u}_{K_\theta} \in \boldsymbol{U}_\theta, P_\theta} \left(\left|\boldsymbol{SL}_{\boldsymbol{1},\boldsymbol{\theta}}\right|^{-1} \left|\boldsymbol{SL}_{\boldsymbol{2},\boldsymbol{\theta}}\right|^{-1} \left[J_{k+p,\theta} + J_{q,\theta} + \left|\boldsymbol{EP}_{\boldsymbol{\theta}}^{-\boldsymbol{1}}\right| \right] \right) - Rev_\theta^k \qquad \text{Eq. 78}$$

Where $\theta = 1, \ldots, n_p$, and n_p is the number of plants to be assessed. As before, the first term in Eq. 78 represents the changes in operating revenue during the transient and the new second term represents the starting point for revenue for plant θ. It is important to pay attention to ensure consistency in the time units used for Rev, which should be the same as in the first term. As before, lower values are better and, with this new definition, the EMOP index can now present negative values. Note that all terms of Eq. 78 are defined exclusively for plant θ, whose control problem no longer needs to be identical to those of the other plants.

A difficulty that arises from this extended index of Eq. 78 is that each plant now has its own arbitrary set of tuning parameters, $\boldsymbol{T}_{\boldsymbol{SL1}_{\boldsymbol{\theta}}}$, $\boldsymbol{T}_{\boldsymbol{SL2}_{\boldsymbol{\theta}}}$ and $\boldsymbol{T}_{\boldsymbol{EP}_{\boldsymbol{\theta}}}$. These arbitrary definitions can determinate the outcome of flowsheet performance raking if not enough care is taken. This problem can be mitigated by two ways: the first method is to ignoring transient response by setting the parameters vectors $\boldsymbol{T}_{\boldsymbol{SL1}_{\boldsymbol{\theta}}} = \boldsymbol{T}_{\boldsymbol{SL2}_{\boldsymbol{\theta}}} = \boldsymbol{T}_{\boldsymbol{EP}_{\boldsymbol{\theta}}} = \boldsymbol{0}$, yielding $\boldsymbol{SL}_{\boldsymbol{1},\boldsymbol{\theta}} = \boldsymbol{SL}_{\boldsymbol{2},\boldsymbol{\theta}} = \boldsymbol{EP}_{\boldsymbol{\theta}} = \boldsymbol{I}$; the disadvantage proceeding like this is that a manual inspection of the transient of each solution is required. Another alternative is setting the tuning parameters $\boldsymbol{T}_{\boldsymbol{SL1}_{\boldsymbol{\theta}}}, \boldsymbol{T}_{\boldsymbol{SL2}_{\boldsymbol{\theta}}}$ and $\boldsymbol{T}_{\boldsymbol{EP}_{\boldsymbol{\theta}}}$ aggressively. It may sound counterintuitive that aggressive tuning can reduce the effects of the Soft-Landing and error penalisation matrices, but it should lead the optimisation algorithm to avoid transient responses with nonzero first and second CVs derivatives, and to ensure that no control bound violation occurs during the transient. If the solver is successful in obtaining such a set of control actions, it yields that $\boldsymbol{SL}_{\boldsymbol{1},\boldsymbol{\theta}} = \boldsymbol{SL}_{\boldsymbol{2},\boldsymbol{\theta}} = \boldsymbol{EP}_{\boldsymbol{\theta}} = \boldsymbol{I}$. If it fails to do so, the cost function reaches nearly infinite values very quickly. The disadvantage is that solution thus obtained may be excessively conservative.

Both these tuning alternatives guarantee that all viable solutions will have no impact from $\boldsymbol{SL}_{\boldsymbol{1},\boldsymbol{\theta}}, \boldsymbol{SL}_{\boldsymbol{2},\boldsymbol{\theta}}$ and $\boldsymbol{EP}_{\boldsymbol{\theta}}$, and the indexes obtained from Eq. 78 become readily comparable between different process routes and plant layouts. However, the issue of weighting scenarios becomes critical. For flowsheets generated by the same production routes one can use the same scenarios and the same weights to average the EMOP indexes, and no unfair advantage is given to a plant. In this case weighting between scenarios can be defined based on a qualitative perception of what are their relative importance. However, assessing alternative production routes require scenario weighting to be probabilistic based. Knowledge of the rate of occurrence of each scenario existing for each process route is required to avoid a biased assessment of the designs.

In order to help the reader visualise the effect of aggressive tuning, let us consider the first-order soft-landing matrix ($\boldsymbol{SL_1}$) for system with 9 CVs. Let us also assume that the first-order derivative obtained at the end of prediction horizon is equal to an identity matrix. From Eq. 71, we have that the EMOP index ratio of increase due nonzero at time $k+p$ is given by $|\boldsymbol{SL_1}|^{-1}$, which can be obtained from Eq. 61:

$$|\boldsymbol{SL_1}|^{-1} = \left| \boldsymbol{I_9} - \boldsymbol{T_{SL1}} \underbrace{\left[\frac{diag|y_{k+p} - y_{k+p-1}|}{\Delta t} \cdot diag|\boldsymbol{y^*_{max}} - \boldsymbol{y^*_{min}}|^{-1} \right]}_{\boldsymbol{I_9}} \right|^{-1} \quad \text{Eq. 79}$$

$$= |\boldsymbol{I_9} - \boldsymbol{T_{SL1}}|^{-1}$$

Let us define $\boldsymbol{T_{SL1}} = \boldsymbol{I_9}.T_{SL1}$ and plot $|\boldsymbol{SL_1}|^{-1}$ as a function of T_{SL1} for this system. If T_{SL1} assumes the values 0.00, 0.05, 0.10, 0.15, 0.20, 0.25 and 0.30, $|\boldsymbol{SL_1}|^{-1}$ increases nearly exponentially, as shown in Fig. 11. Therefore, the if we set the tuning parameter T_{SL1} to an aggressive value such as 0.3, the solver will have a great incentive to avoid all solutions with nonzero first-order derivative at $k+p$, or the index will rise sharply. If we have aggressive tuning for all plants, transient dependent responses which are affected by tuning parameters will likely be discarded regardless of their process route, leading to evaluation based solely on the final steady-state.

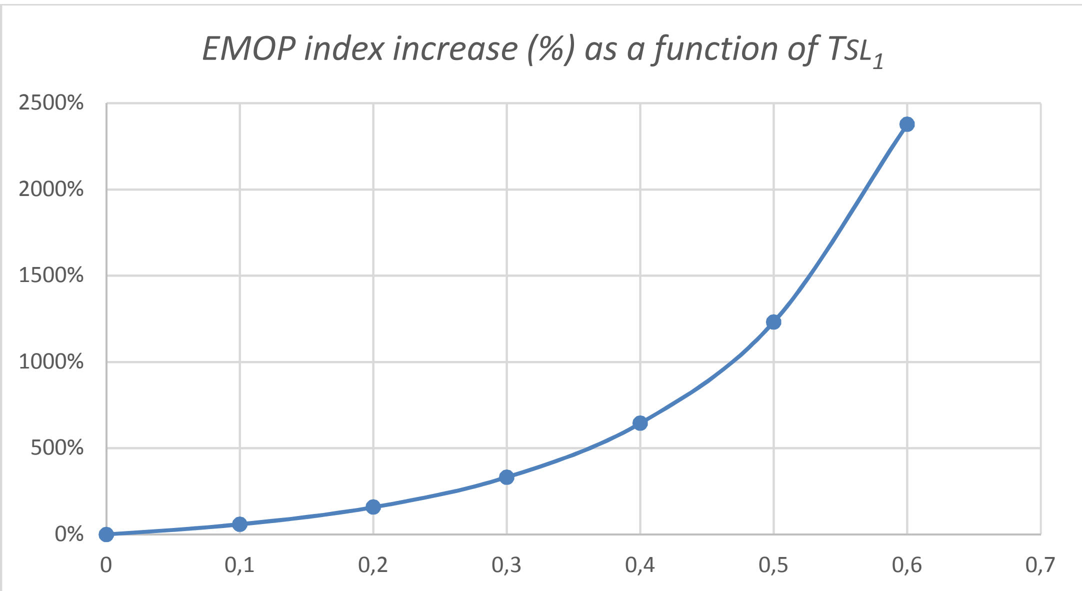

Fig. 11 – EMOP index increase due to first-order soft-landing matrix for a system with 9 CVs and unitary first-order derivative at the end of prediction.

3.10 Conclusions concerning the EMOP index

In brief, the EMOP index methodology can be considered as a state reachability problem that deals with zone restrictions, disturbances, smoothness of the transient response and process economics. Details of the concept of state reachability can be found in Vidyasagar (2002). As a way of recapping the contents of this Chapter, let us now outline the EMOP index's main features:

- The EMOP index measures the impact of process dynamics in the OPEX;
- To this end, zone constrained MPC with economic capabilities is assumed;
- The optimised EMOP index determines what are the most profitable and yet reachable state, and does so evaluating just how much room for optimisation is available. The lower the index, the better the state reachability for a given plant;
- The index is used to assess how well-suited a flowsheet is to accommodate disturbances while being and controlled and optimised by any MPC, allowing comparison between different process plants;
- The required control effort and the achievable control performance are measured in the face of a set of disturbances and zones constraints. For any adequate plant design, at least one set of control actions exists that successfully rejects disturbances while economically optimising the process;
- Control performance is monetised by measuring the revenue changes caused by the control actions adopted and bound violations of environmental restrictions and product specifications, i.e., the index must account for the eventual economic losses due to the necessary control actions;
- The index indicates the best achievable control performance from a broad economic and stability standpoint. As it is always the case for Controllability Analysis, it only depends on the plant's own characteristics and control goals, independently of the MPC algorithm and tuning parameters that will be eventually used to control these plants;
- The index provides a workable solution for assessing control performance and its economic ramifications for moderately complex plants (or a key section of a highly complex process unit, i.e., reactional section, distillation column, heat exchanging section, etc.);

- The EMOP index ranking of the candidate plant designs shows how they compare in terms of resilience to disturbances, controllability, product quality giveaway and costs.

Considering the contributions above, one could argue that the EMOP index differs significantly from the existent body of work of integrated process design and control methods. Its innovation also comes from the fact that it is not intended to replace other methods, but to be used alongside them as a decision-making tool.

4 The simultaneous multi-linear prediction

The assessment of control performance of large-scale systems uses data from commercial, black-box nonlinear dynamic simulation packages that are popular in the chemical industry. These state-of-the-art simulators are the tools of choice of most process design teams. We use the integrated design and control approach for large-scale, complex processes for which the full set of equations is unavailable.

Data from one such simulator was used in Strutzel and Bogle (2016) to identify linear state-space models that predicted the dynamic behaviour of each flowsheet. This is a better option than using the nonlinear simulation package directly since that is very computationally intensive. However, using linear models to predict the behaviour of nonlinear plants is inevitably going to result in some error. This problem was addressed by defining the EMOP index as an interval (bounded by best and worst scenarios) within which the true controllability index must be contained.

In this Thesis, an alternative approach for reducing nonlinearity-introduced error is presented while keeping all the advantages of using linear models. This solution, a new multi-model state-space approach called Simultaneous Multi-Linear Prediction (SMLP), is described in the following Sections. The main idea is to ensure that the evaluation of the index and the subsequent ranking of flowsheets are as accurate as possible within a reasonable timeframe, thus rendering the EMOP index more resilient to process nonlinearity. While not the focus of this Thesis, the SMLP can also be used for process control purposes and for MPC in particular, since its open loop stability can be guaranteed as we will show.

A major part of MPC's appeal in industry stems from the use of a linear finite step response model of the process and a quadratic objective function. When MPC is employed on processes with significant nonlinearity, the application of a linear model-based controller may have to be limited to relatively small operating regions. Specifically, since the computations are entirely based on the model prediction, the accuracy of the model has a significant effect on the performance of the closed-loop system (Gopinath et al, 1995). Hence, the capabilities of MPC will degrade as the operating level moves away from its original design level of operation (Dougherty and Cooper, 2003). Enhancing prediction accuracy will increase closed-loop stability and help avoid overdamped or underdamped responses.

MPC for SMLP also has one advantage over MPC with nonlinear models (NMPC): while NMPC may involve non-convex optimisation problems for which local minima can found but nothing can be said about the global minimum (Kantner and Primbs, 1997), the use of an SMLP prediction provides a convex cost function for which the global minimum can be easily found without the need for relaxation.

As discussed in Section 2.4, PieceWise Affine (PWA) systems are popular methods for obtaining multi-model linear approximations of nonlinear systems. Nevertheless, PWA approximations have some issues that keep them from being the ideal solution for use with integrated process design and control approaches: firstly, the availability of the full set of equations of the nonlinear model is a requirement for existing PWA methods. This is not going to be the case if most of the commercial simulation packages are to be used.

While it is relatively straightforward to employ a standard PWA approach to identify step response models, a good approximation of the nonlinear simulation package cannot be guaranteed. The clustering technique (Ferrari-Trecate et al., 2003) and the "point-to-point" method (Lowe and Zohdy, 2010) which are used to identify optimal partitions for PWA systems depend on the availability of a full set of differential equations describing the process. Alternatively, an explicit expression for probability density of the data can be used (Nakada et al. 2005). If this condition is not met, then any partition definition will necessarily be arbitrary since it will be dependent on the starting point of the identification data. Both the choice of linearisation points and boundaries will not be optimal, and the PWA identification problem becomes finding the best fit to the available data. In this situation, the only way one could ensure that a modified PWA scheme would provide an adequate approximation would be through the use of numerous models valid for small regions, if data is available from several starting points.

This leads to the second issue: it is desirable to have a multi-model class capable of reliable and accurate prediction by using only a small number of sub-models. If the necessary number of partitions is high, the process of data-based model identification is very laborious and time-demanding. This is true when the data is provided by dynamic simulators but even more so when we desire to control an existing chemical plant.

Thirdly, the discontinuities at the boundaries of the regions can generate inconsistencies, with sudden changes in the output predictions that might introduce error in the plant assessment. For these reasons, the SMLP is presented as an alternative to PWA and other Linear Hybrid Systems.

4.1.1 Generating sub-models

The simultaneous multi-linear prediction is a method for obtaining a prediction for the future values of an output vector. Three main elements are used to provide this prediction: the initial state, the input profiles and a set of sub-models (linear approximations of a nonlinear model) identified at distinct states.

Each sub-model is to be obtained through model identification on a set of data generated by step increments on MVs and DVs. Alternatively, the relay response could be used. Tests may be performed on a dynamic simulator or on an experimental pilot plant, or even on a real industrial unit, for the sake of the methodology it is indifferent. Let us refer to the data thus obtained as the nonlinear model data, from which the sub-models are derived and validated. The initial state from which a test is performed will be referred as an identification point (IP), and it will be considered the epicentre of the resulting sub-model. The greater the number of IPs/sub-models, the better the resilience of the resulting SMLP system with regard to process nonlinearity. The control engineer should select IPs that are representative of the most common or critical operating conditions.

For each input/output couple, several values for the order of each sub-model should be tested, and the order adopted is that which minimises the numerical residue between the nonlinear model data and the sub-model being considered for all epicentres/IPs, not only the one being identified. This procedure avoids model overfitting or underfitting, which can have catastrophic results especially when the operating point (OP) is out of the vicinity of the model's IP. The model validation should be performed against a test data set for the SMLP system output and the output of each of its sub-models individually. In this Thesis, the identification data used was different from the validation data, and we consider this to be a good practice. The SMLP's parameters are chosen in order to reflect a range of conditions and thus the SMLP system is expected to exhibit smaller numerical residue in relation to both the identification and validation data than any of its constituent sub-models.

4.1.2 Introducing the Simultaneous Multi-Linear Prediction (SMLP)

In this Section, the novel multiple state-space model method, Simultaneous Multi-Linear Prediction (SMLP), is presented. Its role is to reduce the nonlinearity-related error and avoid discontinuities between approximated regions. The SMLP avoids discontinuities by avoiding partitions and changes in the state update equation through the whole space of feasible operating points. Instead of a single state vector as found in Linear Hybrid Systems such as PWA, the SMLP uses multiple sub-state vectors calculated simultaneously, one for each linear state-space sub-model. Note that since these model matrices do not share the same state vector, they need not possess the same dimensions (a requirement for PWA systems). In the SMLP, all sub-states to some degree contribute to the main output at all times, as shown in Fig. 12:

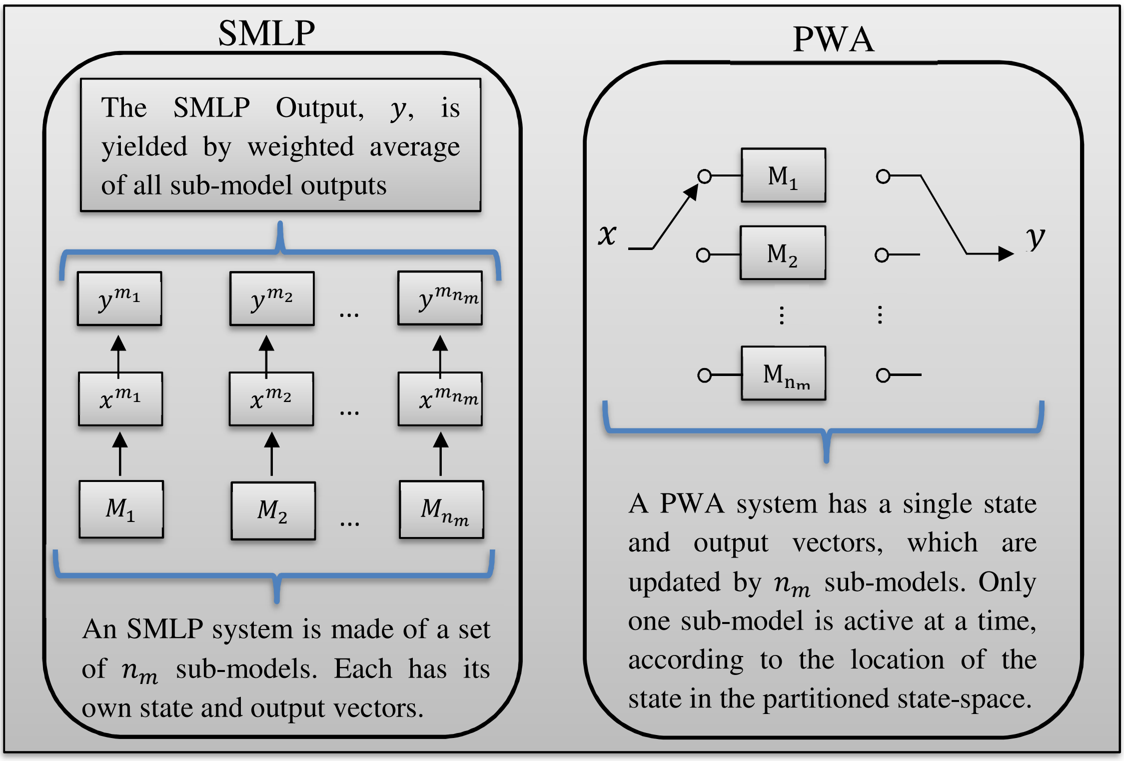

Fig. 12 – The output prediction can be generated by multiple simultaneous states, each one with its own update rule (SMLP system), or by a single state which continuously changes update rule (PWA system).

Each sub-state creates an output rate of change vector, and the result of the weighted addition of these vectors will generate the main output. The contribution ratio of each component is defined by a weight parameter, itself a function of three factors: the current position of the operating point; the points where each sub-model was originally identified (known as identification points or IPs); and a set of parameters obtained through linear regression to minimise the gap between the SMLP output and the simulator.

The SMLP does not assume that one particular sub-model has priority over others within an arbitrary partition of the input space. Without being restricted to any arbitrary bounds, the weight parameters change freely inside the whole space with a view to minimising the nonlinearity error. Besides avoiding discontinuities, eliminating partitions may provide a gain in accuracy since the search for optimal parameters becomes an unconstrained optimisation problem.

Three variants of the SMLP methodology have been devised by considering two different definitions of operating point (or OP) and two options for parametrisation obtained through regression analysis. In the first SMLP variant, the OP is the output (CV) vector (y), whereas for the other two variants the OP is the input vector ($[\mathrm{u}\ \mathrm{d}]$) combining manipulated and disturbance variables (MVs and DVs). The first definition is appropriate when the CVs can be known precisely and updated regularly, and when there are no time delays or non-stationary (integrating) variables such as levels. On the other hand, OP can always be expressed as an MV vector since MVs are defined by the control system and are thus readily available. Measured disturbance variables (DVs) should also be incorporated in the OP.

As for the parametrisations, the two options consist of either having a single set of parameters for each multivariable sub-model, which will be called MIMO parametrisation or having a separated set for each pair of output/input, called SISO parametrisation, which may yield an improved prediction. A key disadvantage of the SISO parametrisation as compared to the MIMO parametrisation is the increased time required to perform the regression analysis, as it is individually done for each CV/MV or CV/DV pair. This trade-off is significant if the number of variables is high.

4.1.3 SMLP Method 1 – the MIMO parametrisation

The Simultaneous Multi-Linear Prediction (SMLP) is a method for obtaining a prediction for the future values of an output vector. Three main elements are used to provide this prediction: the initial state, the input profiles and a set of sub-models (linear approximations of a nonlinear model) identified at distinct states.

Let us first outline the SMLP method for the special case of an output vector with two variables, $\boldsymbol{y_1}$ and $\boldsymbol{y_2}$, which are to be controlled for a defined set of disturbances. Consider that, for a given flowsheet, model identification was performed at two identification points, $\boldsymbol{IP_1} = \left[y_{1,IP_1}, y_{2,IP_1}\right]$ and $\boldsymbol{IP_2} = \left[y_{1,IP_2}, y_{2,IP_2}\right]$, yielding respectively sub-models $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$ that are Taylor approximations of the nonlinear model.

The SMLP formulation makes use of a standard linear state-space formulation presented in Lee et al. (1994). The sub-models $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$ are systems of equations defined, respectively, by the state-space matrices $\boldsymbol{A_{m_1}}$, $\boldsymbol{B_{m_1}}$, $\boldsymbol{C_{m_1}}$ and $\boldsymbol{D_{m_1}}$ and $\boldsymbol{A_{m_2}}$, $\boldsymbol{B_{m_2}}$, $\boldsymbol{C_{m_2}}$ and $\boldsymbol{D_{m_2}}$, as well as by state vectors of appropriate dimensions, $\boldsymbol{x_k^{m_1}}$ and $\boldsymbol{x_k^{m_2}}$, where k is the time at initial conditions. The state vectors $\boldsymbol{x_k^{m_1}}$ and $\boldsymbol{x_k^{m_2}}$ are henceforth called simultaneous sub-states since they evolve independently from each other, being subject to the same set of control actions ($\boldsymbol{u_k}, \boldsymbol{u_{k+1}}, \dots, \boldsymbol{u_{k+m}}$, where m is the number of control actions) through the control horizon, and also to the same set of disturbances ($\boldsymbol{d_k}, \boldsymbol{d_{k+1}}, \dots, \boldsymbol{d_{k+p}}$, where p is the length of the prediction) through the entirety of the prediction horizon. Hence, each sub-model has a sub-state update equation in the incremental form, as shown in Eq. 80 and Eq. 81:

$$\boldsymbol{x_{k+1}^{m_1}} = \boldsymbol{A_{m_1}}\boldsymbol{x_k^{m_1}} + \boldsymbol{B_{m_1}}\Delta\boldsymbol{u_k} + \boldsymbol{D_{m_1}}\Delta\boldsymbol{d_k} \quad \text{Eq. 80}$$

$$\boldsymbol{x_{k+1}^{m_2}} = \boldsymbol{A_{m_2}}\boldsymbol{x_k^{m_2}} + \boldsymbol{B_{m_2}}\Delta\boldsymbol{u_k} + \boldsymbol{D_{m_2}}\Delta\boldsymbol{d_k} \quad \text{Eq. 81}$$

Where $\Delta\boldsymbol{u_k} = [\boldsymbol{u_k} - \boldsymbol{u_{k-1}}]^T$ and $\Delta\boldsymbol{d_k} = [\boldsymbol{u_k} - \boldsymbol{u_{k-1}}]^T$. Therefore, even though the inputs are the same for both sub-models, different values for next-instant sub-states, $\boldsymbol{x_{k+1}^{m_1}}$ and $\boldsymbol{x_{k+1}^{m_2}}$, are obtained as well as different output predictions, respectively $\boldsymbol{y_k^{m_1}}$ and $\boldsymbol{y_k^{m_2}}$ for $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$, as shown in Eq. 82 and Eq. 83:

$$\boldsymbol{y_k^{m_1}} = \boldsymbol{C_{m_1}}\boldsymbol{x_k^{m_1}} \quad \text{Eq. 82}$$

$$\boldsymbol{y_k^{m_2}} = \boldsymbol{C_{m_2}}\boldsymbol{x_k^{m_2}} \quad \text{Eq. 83}$$

Let us define the operating point (OP) at time k as being the output vector of the multi-model system, i.e., $\boldsymbol{OP_k} = [y_{1,k}, y_{2,k}]$. Note that usually $\boldsymbol{OP_k} \neq \boldsymbol{y_k^{m_1}} \neq \boldsymbol{y_k^{m_2}}$. Due to nonlinearity effects, the model $\boldsymbol{M_1}$ is expected to get less accurate the further the operating point $\boldsymbol{OP}$ is removed from the first identification point $\boldsymbol{IP_1}$. In other words, if the plant is operated close to an identification point, its modelling error will probably be small. This should be valid as a general trend even if untrue at some points. So, it makes sense that the weights of $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$ are roughly inversely proportional to, respectively, the distance in the Euclidean n_y-space between points $\boldsymbol{IP_1}$ and $\boldsymbol{OP_k}$ ($\overline{\boldsymbol{IP_1OP_k}}$) and between $\boldsymbol{IP_2}$ and $\boldsymbol{OP_k}$, $(\overline{\boldsymbol{IP_2OP_k}})$, where n_y is the number of CVs ($n_y = 2$, in this case). This concept is shown graphically in Fig. 13:

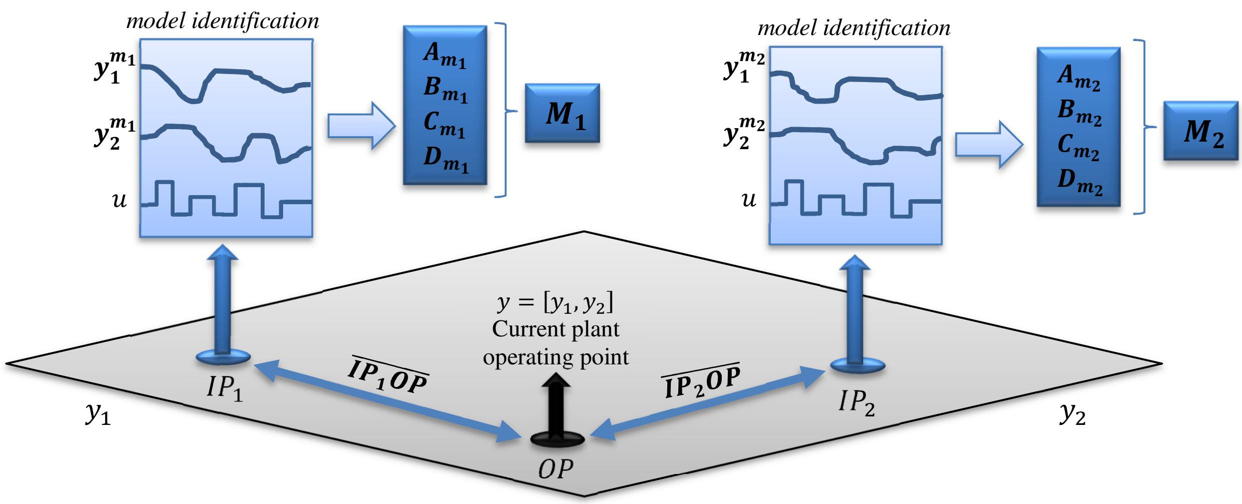

Fig. 13 – The vectors connecting the operating point ($\boldsymbol{OP}$) and $\boldsymbol{IP_1}$ and $\boldsymbol{IP_2}$.

Please note that obtaining more than one model for the same $\boldsymbol{IP}$ would make no sense, so we always assume $\boldsymbol{IP_1} \neq \boldsymbol{IP_2}$. We define distance coefficients, τ_1 and τ_2 , to represent the normalised proximity between $\boldsymbol{OP_k}$, $\boldsymbol{IP_1}$ and $\boldsymbol{IP_2}$.

$$\tau_1 = \frac{\sum_{\omega=1}^{2}(\|\overline{\boldsymbol{IP_1OP_k}}\|_\omega + \|\overline{\boldsymbol{IP_2OP_k}}\|_\omega)/\Delta \boldsymbol{y}_{\boldsymbol{\omega}}^*}{\sum_{\omega=1}^{2}\|\overline{\boldsymbol{IP_1OP_k}}\|_\omega/\Delta \boldsymbol{y}_{\boldsymbol{\omega}}^*} \qquad \text{Eq. 84}$$

$$\tau_2 = \frac{\sum_{\omega=1}^{2}(\|\overline{\boldsymbol{IP_1OP_k}}\|_\omega + \|\overline{\boldsymbol{IP_2OP_k}}\|_\omega)/\Delta \boldsymbol{y}_{\boldsymbol{\omega}}^*}{\sum_{\omega=1}^{2}\|\overline{\boldsymbol{IP_2OP_k}}\|_\omega/\Delta \boldsymbol{y}_{\boldsymbol{\omega}}^*} \qquad \text{Eq. 85}$$

where $\Delta \boldsymbol{y}^* = \boldsymbol{y}_{\boldsymbol{max}}^* - \boldsymbol{y}_{\boldsymbol{min}}^*$ is the sensor range of the outputs and $\|\overline{\boldsymbol{IP_1OP_k}}\|$ and $\|\overline{\boldsymbol{IP_2OP_k}}\|$ are the Euclidean norms of, respectively, $\overline{\boldsymbol{IP_1OP_k}}$ and $\overline{\boldsymbol{IP_1OP_k}}$. The numerators of Eq. 84 and Eq. 85 both contain the sum of the Euclidean norms of the distances between the current operating point, $\boldsymbol{OP_k}$, and the identification points of both $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$, i.e., $\overline{\boldsymbol{IP_1OP_k}}$ and $\overline{\boldsymbol{IP_2OP_k}}$, for each CV ω, where $\omega = 1, 2$. On the other hand, the denominators of Eq. 84 and Eq. 85 contain, respectively, the sum of Euclidean norms of $\overline{\boldsymbol{IP_1OP_k}}$ and $\overline{\boldsymbol{IP_2OP_k}}$ for each CV ω, where $\omega = 1, 2$.

In exploring the dynamic performance using these linearised models the SMLP method will search across the space covered by the two sub-models. As it crosses the boundaries instead of switching between models or upgrade equations here we calculate and update simultaneously the two output vectors and use a weighted arithmetic average to provide the combined model output prediction, $\boldsymbol{y_{k+1}}$. The weights of $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$, denoted respectively as ε_1 and ε_2, are functions of the distance coefficients, τ_1 and τ_2.

The format and coefficients of these functions should be selected as a means of minimising the error of the multi-model prediction as compared to a set of reference trajectories.

With this in mind, we introduce “degradation functions”, $\boldsymbol{g_1}$ and $\boldsymbol{g_2}$, composed of positive, nonzero functions of the distance coefficients. They serve as nonlinear weighting parameters that will later be used to provide the averaged weights (ε_1 and ε_2) of $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$ in the main prediction. It is desirable that the degradation functions become roughly proportional to, respectively, τ_1 and τ_2, and with this in mind $\boldsymbol{g_1}$ and $\boldsymbol{g_2}$ may assume any format the user considers to be appropriate. Here a 4th order polynomial with non-negative coefficients was used, as seen in Eq. 86 and Eq. 87. This format is adequate since it avoids negative numbers and the averaged weight “degradation” decreases monotonically as the $\boldsymbol{OP}$ approaches a given $\boldsymbol{IP}$. Hence, a sub-model is dominant over others as a component of the SMLP in the proximities of its $\boldsymbol{IP}$, e.g., $\boldsymbol{g_1}(\tau_1) \gg \boldsymbol{g_2}(\tau_2)$ when $\boldsymbol{OP_k} \to \boldsymbol{IP_1}$, and $\boldsymbol{g_2}(\tau_2) \gg \boldsymbol{g_1}(\tau_1)$ when $\boldsymbol{OP_k} \to \boldsymbol{IP_2}$ (note that $\tau_1 \to \infty$ and $\tau_2 \to 1$ when $\overline{\boldsymbol{IP_1 OP_k}} \to [0,0]$. Similarly, $\tau_2 \to \infty$ and $\tau_1 \to 1$ when $\overline{\boldsymbol{IP_2 OP_k}} \to [0,0]$).

$$\boldsymbol{g_1}(\tau_1) = g_{1,4}\tau_1^4 + g_{1,3}\tau_1^3 + g_{1,2}\tau_1^2 + g_{1,1}\tau_1 + g_{1,0} \qquad \text{Eq. 86}$$

$$\boldsymbol{g_2}(\tau_2) = g_{2,4}\tau_2^4 + g_{2,3}\tau_2^3 + g_{2,2}\tau_2^2 + g_{2,1}\tau_2 + g_{2,0} \qquad \text{Eq. 87}$$

By defining the degradation functions as a 4th order polynomial, a nonlinear component was added to the output update equation composed by a set of linear models. This number of coefficients was proved in later sections to be high enough to correctly capture the nonlinearity of the plants presented later in this paper. However, finding a suitable format for the degradation function is a trial and error process, and there is no way to predict which one is going to yield minimum model mismatch. In brief, the format of the degradation functions has to be decided on a case-by-case basis.

While searching for the optimised degradation functions one should follow standard procedures of data analysis, especially with regard to avoiding the overfitting phenomenon. Overfit occurs when a model is too strongly tailored to the particularities of the training set and generalizes poorly to new data. The "classic" way to avoid this is to use three groups of datasets - a training set, a test set, and a validation set. Coefficients are obtained using the training set; the best form of the equation is found using the test set, and the validation set is used to test for over-fitting (Gareth, 2013). Another method commonly used to avoid overfitting is to insert a Tikhonov regularisation term, which aims at measuring the complexity of the function. The higher the complexity, the higher the regularisation term will be (Tikhonov, 1963).

These standard procedures are well known, and the reader is strongly encouraged to deploy them while applying the SMLP for engineering purposes. However, for the sake of making the paper simpler and shorter, only a 2nd and a 4th order polynomials were tested as degradation functions, and only a training dataset was used. Better results were obtained for the 4th order polynomial, which is assumed to be optimal. Fig. 14 shows a possible visual interpretation of functions $\boldsymbol{g_1}$ and $\boldsymbol{g_2}$:

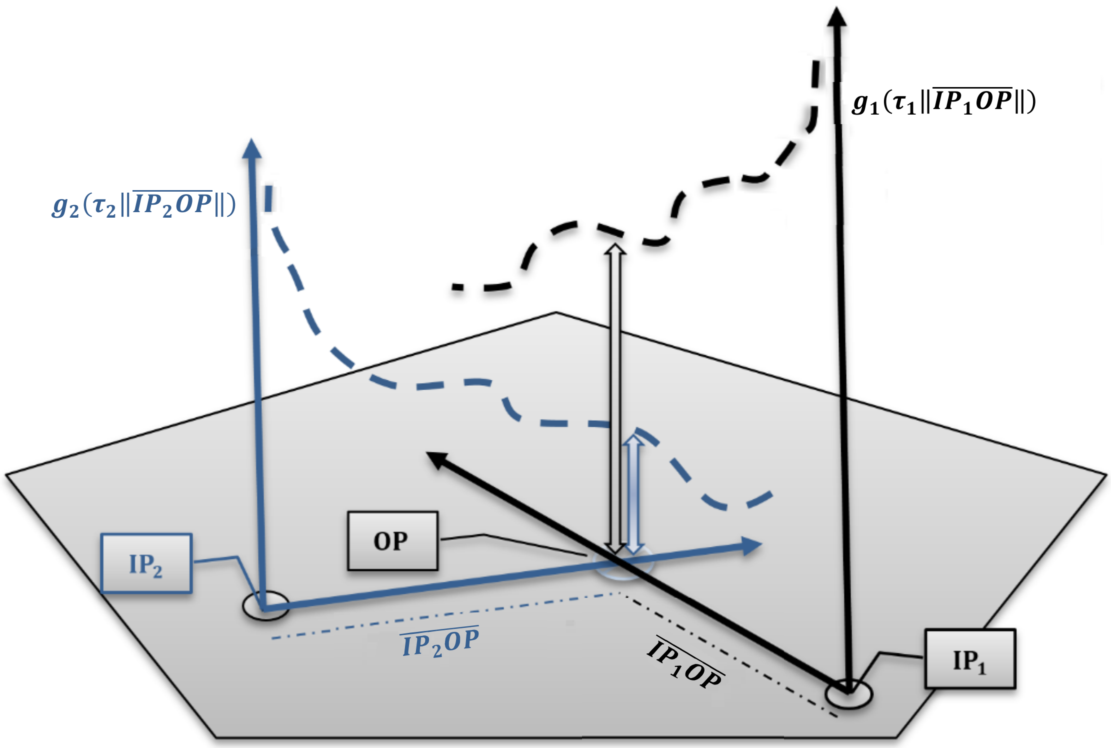

Fig. 14 – The "degradation functions", $\boldsymbol{g_1}$ and $\boldsymbol{g_2}$, are functions of the "distance coefficients" $\boldsymbol{\tau_1}$ and $\boldsymbol{\tau_2}$.

The averaged weights for the arithmetic average between sub-models, ε_1 and ε_2 are given by the proportion of $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$ in the sum of the degradation functions, as per Eq. 88 and Eq. 89:

$$\varepsilon_1 = \frac{g_1(\tau_1)}{g_1(\tau_1)+g_2(\tau_2)} \quad \text{Eq. 88}$$

$$\varepsilon_2 = \frac{g_2(\tau_2)}{g_1(\tau_1)+g_2(\tau_2)} \quad \text{Eq. 89}$$

Note that $\varepsilon_1 + \varepsilon_2 = 1$. Finally, ε_1 and ε_2 are used in Eq. 90 to calculate the SMLP output at $(k + p)$. The SMLP output, which may also be called "main output" or "multi-model output" is normally the vector of CVs, but may sometimes contain other, noncontrolled variables that we wish to monitor. It must contain exactly the same variables as the sub-model outputs, i.e., $\dim(\boldsymbol{y_k}) = \dim(\boldsymbol{y_k^{m_1}}) = \dim(\boldsymbol{y_k^{m_2}})$.

$$[\boldsymbol{y_{k+1}} - \boldsymbol{y_k}] = \left[\varepsilon_{1,k+1}\left[\boldsymbol{y_{k+1}^{m_1}} - \boldsymbol{y_k^{m_1}}\right] \quad \varepsilon_{2,k+1}\left[\boldsymbol{y_{k+1}^{m_2}} - \boldsymbol{y_k^{m_2}}\right]\right]^T \qquad \text{Eq. 90}$$

Note that the rate of change of y_{k+1} is given by adding fractions ($\varepsilon_{1,k+1}$ and $\varepsilon_{2,k+1}$) of the rates of change of $\boldsymbol{y_{k+1}^{m_1}}$ and $\boldsymbol{y_{k+1}^{m_2}}$, and no discontinuity is introduced in the prediction.

Now we will extend the multi-model approach to any number n_m of linear sub-models and any number n_y of CVs, i.e., $\boldsymbol{OP_k} = \left[y_{1,k}, \dots, y_{n_y,k}\right]$, $\boldsymbol{IP_1} = \left[y_{1,IP_1}, \dots, y_{n_y,IP_1}\right], \dots, \boldsymbol{IP_{n_m}} = \left[y_{1,IP_{n_m}}, \dots, y_{n_y,IP_{n_m}}\right]$. As before, there will be as many simultaneous sub-states and outputs as the number of sub-models, as shown in Eq. 91 and Eq. 92:

$$\boldsymbol{x_{k+1}^{m_1}} = \boldsymbol{A_{m_1}}\boldsymbol{x_k^{m_1}} + \boldsymbol{B_{m_1}}\Delta\boldsymbol{u_k} + \boldsymbol{D_{m_1}}\Delta\boldsymbol{d_k}$$

$$\vdots$$

$$\boldsymbol{x_{k+1}^{m_{n_m}}} = \boldsymbol{A_{m_{n_m}}}\boldsymbol{x_k^{m_{n_m}}} + \boldsymbol{B_{m_{n_m}}}\Delta\boldsymbol{u_k} + \boldsymbol{D_{m_{n_m}}}\Delta\boldsymbol{d_k} \qquad \text{Eq. 91}$$

$$\boldsymbol{y_k^{m_1}} = \boldsymbol{C_{m_1}}\boldsymbol{x_k^{m_1}}$$

$$\vdots$$

$$\boldsymbol{y_k^{m_{n_m}}} = \boldsymbol{C_{m_{n_m}}}\boldsymbol{x_k^{m_{n_m}}} \qquad \text{Eq. 92}$$

Eq. 93 may be used to obtain the distance coefficients, τ_ϑ:

$$\tau_\vartheta = \frac{\sum_{\varphi=1}^{n_m}\sum_{\omega=1}^{n_y}\left\|\overline{\boldsymbol{IP_\varphi OP_k}}\right\|_\omega / \Delta y_\omega^*}{\sum_{\omega=1}^{n_y}\left\|\overline{\boldsymbol{IP_\vartheta OP_k}}\right\|_\omega / \Delta y_\omega^*}, \ \vartheta = 1, \dots, n_m \qquad \text{Eq. 93}$$

Note that the demominator of Eq. 93 contains the sum of the Euclidean norms of the distances between the operating point $\boldsymbol{OP_k}$ and identification point $\boldsymbol{IP_\vartheta}$ ($\overline{\boldsymbol{IP_\vartheta OP_k}}$) of a particular sub-model $\boldsymbol{M_\vartheta}$, for each CV ω, where $\omega = 1, \dots, n_y$. The numerator, however, contains the sum of the sum of Euclidean norms of the distances between $\boldsymbol{OP_k}$ and $\boldsymbol{IP_\varphi}$ ($\overline{\boldsymbol{IP_\varphi OP_k}}$) for each CV ω, where $\omega = 1, \dots, n_y$, and each sub-model $\boldsymbol{M_\varphi}$, where $\varphi = 1, \dots, n_m$. The degradation function of sub-model $\boldsymbol{M_\vartheta}$ is given by Eq. 94:

$$\boldsymbol{g_\vartheta}(\tau_\vartheta) = g_{\vartheta,4}\tau_1^4 + g_{\vartheta,3}\tau_1^3 + g_{\vartheta,2}\tau_1^2 + g_{\vartheta,1}\tau_1 + g_{\vartheta,0},\ \vartheta = 1,\dots,n_m \qquad \text{Eq. 94}$$

The weight of sub-model $\boldsymbol{M_\vartheta}$ in the SMLP output prediction, ε_ϑ, is then defined by Eq. 95:

$$\varepsilon_\vartheta = \frac{g_\vartheta(\tau_\vartheta)}{\sum_{\varphi=1}^{n_m} g_\varphi(\tau_\varphi)},\ \vartheta = 1,\dots,n_m \qquad \text{Eq. 95}$$

Note that $\sum_{\vartheta=1}^{n_m} \varepsilon_\vartheta = 1$. The SMLP output prediction can now be obtained through the use of Eq. 96:

$$[\boldsymbol{y_{k+1}} - \boldsymbol{y_k}] = \left[\sum_{\vartheta=1}^{n_m} \varepsilon_{\vartheta,k+1}\left[\boldsymbol{y_{k+1}^{m_{n_\vartheta}}} - \boldsymbol{y_k^{m_{n_\vartheta}}}\right]\right] \qquad \text{Eq. 96}$$

A workflow showing the flux of data in this first variant of the SMLP method is presented in Fig. 15:

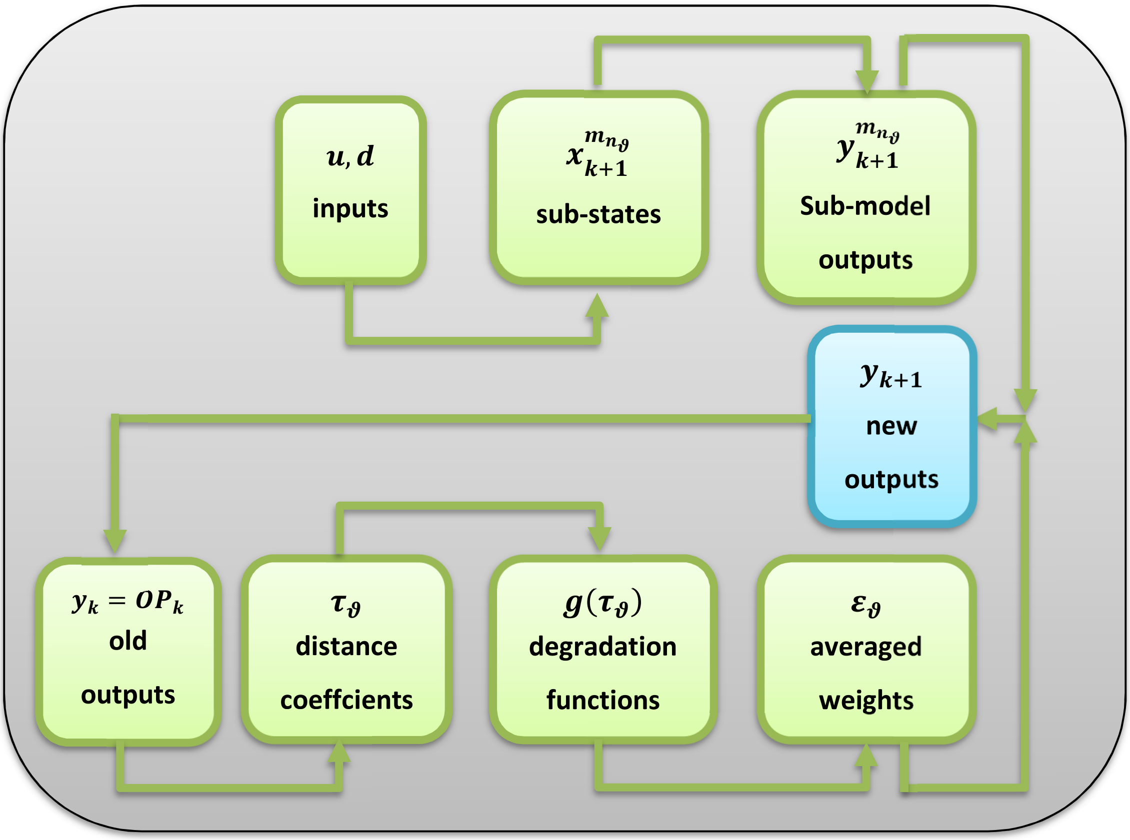

Fig. 15 – Data workflow of the first SMLP method. The distance coefficients are functions of the output, and the new prediction is provided by the multiplication of the sub-model outputs and the averaged weights.

In order to obtain the coefficients of the degradation functions g, we proceed with a regression analysis to minimise the normalised square error between the nonlinear model data (obtained from the commercial dynamic simulation package) and the SMLP, using the same inputs, through the entire prediction horizon. We recommend using several random trajectories and different starting points to properly identify the optimal constants through a range of scenarios. For this first method, a SMLP system can be defined as follows:

Definition 4.1.3.1 Simultaneous Multi-Linear Prediction system with MIMO parametrisation (OP defined as the output vector). *Let there be a set of* n_m *sub-models, where each sub-model* $\boldsymbol{M_\vartheta}$ *is defined by the state-space matrices* $\boldsymbol{A_{m_\vartheta}}$, $\boldsymbol{B_{m_\vartheta}}$, $\boldsymbol{C_{m_\vartheta}}$ *and* $\boldsymbol{D_{m_\vartheta}}$, *a sub-state vector* $\boldsymbol{x_k^{m_\vartheta}}$ *and an identification point* $\boldsymbol{IP}_\vartheta = \left[y_{1,IP_\vartheta}, \dots, y_{n_y,IP_\vartheta}\right]$, *where* $\vartheta = 1, \dots, n_m$, n_y *is the number of CVs and* k *is an arbitrary initial time. Let the sub-state vectors for the next instant,* $\boldsymbol{x_{k+1}^{m_\vartheta}}$, *be provided by Eq. 91 and Eq. 92. Let the vector of distance coefficients of sub-model* $\boldsymbol{M_\vartheta}$, $\boldsymbol{\tau_\vartheta} \in \mathbb{R}^{n_m}$, *be yielded by the ratio between* $\sum_{\varphi=1,\dots,n_m} \overline{\boldsymbol{IP_\varphi OP_k}}$ *and* $\overline{\boldsymbol{IP_\vartheta OP_k}}$ *normalised for each CV, as defined in Eq. 93, where is the* $\boldsymbol{OP_k}$ *operating point at time k, here defined as* $\boldsymbol{OP_k} = \left[y_{1,k}, \dots, y_{n_y,k}\right]$, *and* y_k *is the main or multi-model output prediction at time k. Let the degradation function of sub-model* $\boldsymbol{M_\vartheta}$, $\boldsymbol{g_\vartheta}(\boldsymbol{\tau_\vartheta})$, *be defined by a set of non-negative coefficients* $g_{\vartheta,4}$, $g_{\vartheta,3}$, $g_{\vartheta,2}$, $g_{\vartheta,1}$ *and* $g_{\vartheta,0}$. *Let the weight of the sub-model* $\boldsymbol{M_\vartheta}$ *in the main output prediction for each CV,* ε_ϑ, *be defined as the ratio between* $\boldsymbol{g_\vartheta}(\boldsymbol{\tau_\vartheta})$ *and* $\sum_{\varphi=1}^{n_m} \boldsymbol{g_\varphi}(\boldsymbol{\tau_\varphi})$. *For such a system,* $\boldsymbol{y_{k+1}}$ *can be obtained through the use of Eq. 96, and the degradation function coefficients are obtained by solving the regression analysis problem given by Eq. 97 for a number* n_t *of trajectories.*

$$\min_{g_{\vartheta,4}, g_{\vartheta,3}, g_{\vartheta,2}, g_{\vartheta,1}, g_{\vartheta,0}} \sum_\vartheta \sum_\eta \sum_\alpha \left(\left[\boldsymbol{y_{\eta,\alpha}^{nl}} - \boldsymbol{y_{\eta,\alpha}^{m_{n_\vartheta}}} \right] diag|\boldsymbol{y_{max}^*} - \boldsymbol{y_{min}^*}|^{-1} \right)^2 \qquad \text{Eq. 97}$$

Where $\boldsymbol{y^{nl}}$ is the training dataset provided by the dynamic simulator concerning the trajectories being tested, $\vartheta = 1, \dots, n_m$; $\alpha = 1, \dots, p$; $\eta = 1, \dots, n_t$, and n_t is the number of reference trajectories that makes up the validation data set.

Hence, the regression analysis parameters assume values that reduce as much as possible the prediction mismatch between the commercial dynamic simulation package and the multi-model prediction. This provides us with a high degree of information about the process, opening the "black box" of the commercial simulator and providing a proper basis for the EMOP analysis. The regression analysis problem of Eq. 97 is demanding computationally for large MIMO systems and large datasets, so it cannot be done continuously. If should be performed again every time changes occur to the plant, alongside with sub-model identification, at least.

As a reference to measure the performance of the SMLP, let us define in a similar way the linearisation error for a single sub-model $\boldsymbol{M_\vartheta}$. The traditional single model approach is the equivalent of setting the degradation function of sub-model $\boldsymbol{M_\vartheta}$ to $\varepsilon_\vartheta = 1$, thus ignoring the other sub-models, yielding Eq. 98:

$$\underset{\varepsilon_\vartheta=1}{E_{lin}^{\vartheta}} = \sum_\eta \sum_\alpha \left(\left[\boldsymbol{y_{\eta,\alpha}^{nl}} - \boldsymbol{y_{\eta,\alpha}^{m_{n_\vartheta}}} \right] diag|\boldsymbol{y_{max}^*} - \boldsymbol{y_{min}^*}|^{-1} \right)^2 \qquad \text{Eq. 98}$$

The formulation presented in this section for an output-tracking SMLP system was defined with a view to avoiding the disadvantages of PWA systems presented in section 2.4. The goal of the SMLP is enabling a good approximation of a nonlinear plant using a small number of sub-models obtained at arbitrarily defined identification points, while altogether avoiding the need of partitioning the state-space and the inherent discontinuities thus introduced at the boundaries. Key to this formulation is the use of "degradation functions", that are defined to be continuous over the entire state-space, in order to shift smoothly the weight of each sub-model in the SMLP output. The most important step of the SMLP methodology is performing a regression analysis (Eq. 97) to find adequate format and parameters for the degradation functions. Having an optimised set of degradation functions results that, at any given point of the state-space, the SMLP output will tend to follow the sub-models that perform best in that region, while ignoring sub-models that perform poorly there, yielding a closer approximation to the nonlinear plant through the state-space covered by the test and validation data. The more comprehensive the data, the wider the range of states where the prediction is optimised.

An infinite number of states can be associated with a single OP. For this reason, the degradation functions are defined over the OP space instead of over the space-state: obtaining optimal coefficients for a degradation function defined over the space-state would very likely be impossible given the issue of state multiplicity. Moreover, OP concept is more intuitive, being widely applied in the chemical industry at large. In industrial operations, operators refer to operating points, not states.

4.1.4 SMLP Method 2 – Operating point (OP) defined as the input vector

In the previous Section, we measured the distance between OP and IPs using the output values, which are linearly related to the sub-states. Now another possibility is presented: defining $\boldsymbol{OP}$ and $\boldsymbol{IPs}$ as sets of MVs and DVs. In this case, $\boldsymbol{OP_k} = [u_{1,k}, \dots, u_{n_u,k}, d_{1,k}, \dots, d_{n_d,k}]$, $\boldsymbol{IP_1} = [u_{1,IP_1}, \dots, u_{n_u,IP_1}, d_{1,IP_1}, \dots, d_{n_d,IP_1}]$,…, $\boldsymbol{IP_{nm}} = \left[u_{1,IP_{nm}}, \dots, uy_{n_u,IP_{nm}}, d_{1,IP_{nm}}, \dots, d_{n_d,IP_{nm}}\right]$, where n_d is the number of DVs. Eq. 93 is thus updated to provide the new distance coefficients, τ_ϑ:

$$\tau_\vartheta = \frac{\sum_{\varphi=1}^{nm} \sum_{\omega}^{nu+nd} \left\| \overline{\boldsymbol{IP_\varphi OP_k}} \right\|_\omega / [\Delta\boldsymbol{u}\ \Delta\boldsymbol{d}]_\omega}{\sum_{\omega}^{nu+nd} \| \overline{\boldsymbol{IP_\vartheta OP_k}} \|_\omega / [\Delta\boldsymbol{u}\ \Delta\boldsymbol{d}]_\omega} \qquad \text{Eq. 99}$$

Where $\Delta\boldsymbol{u} = \boldsymbol{u_{max}} - \boldsymbol{u_{min}}$, $u_{j,max} > u_{j,min}$, $j = 1, \dots, n_u$, $\Delta\boldsymbol{d} = \boldsymbol{d_{max}} - \boldsymbol{d_{min}}$, $d_{l,max} > d_{l,min}$, $l = 1, \dots, n_d$, and the Euclidean norm of the IP/OP distance is now normalised by the MV and DV ranges. The rest of the procedure to obtain the SMLP remains the same, but the parameters defining the degradation functions, $\boldsymbol{g_\vartheta}(\tau_\vartheta)$, are different and thus another regression analysis has to be performed. Evidently, the coefficients for methods 1 and 2 are not interchangeable even if the regression analysis is defined by Eq. 97 for both. Fig. 16 shows a workflow of the data flux in this second variant of the SMLP method.

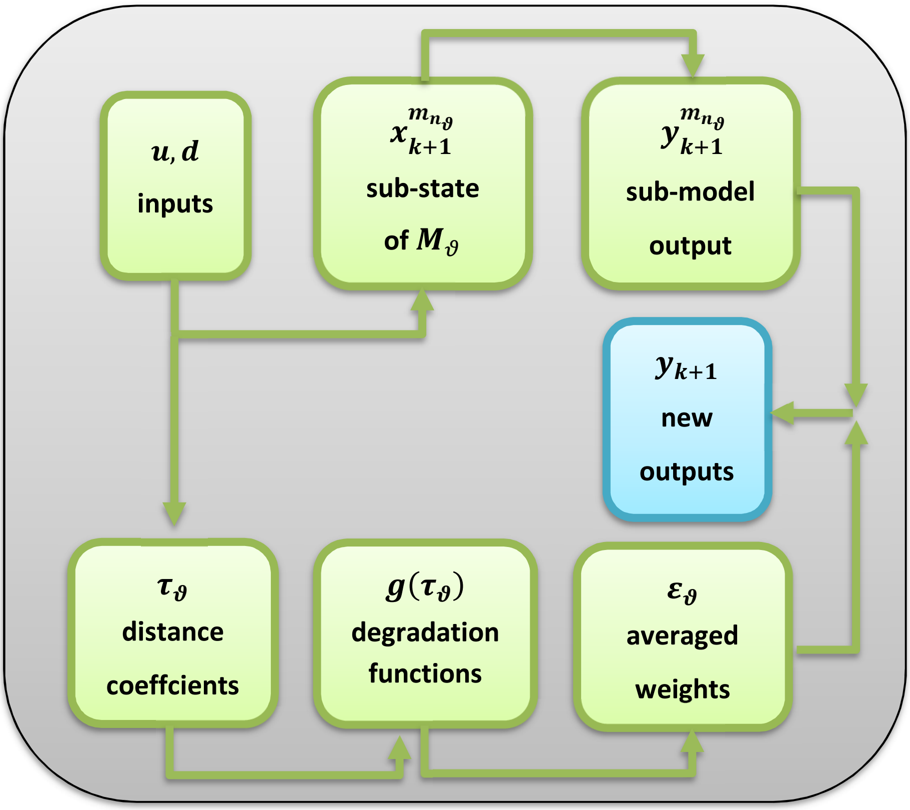

Fig. 16 – Data workflow of the second SMLP method. The distance coefficients are now functions of the input vector instead of the output vector.

For this second method, a SMLP system can be defined as follows:

Definition 4.1.4.1 Simultaneous Multi-Linear Prediction system with MIMO parametrisation (OP defined as the input vector). *Let there be a set of* n_m *sub-models, where each sub-model* $\boldsymbol{M_\vartheta}$ *is defined by the state-space matrices* $\boldsymbol{A}_{\boldsymbol{m}_\vartheta}$, $\boldsymbol{B}_{\boldsymbol{m}_\vartheta}$, $\boldsymbol{C}_{\boldsymbol{m}_\vartheta}$ *and* $\boldsymbol{D}_{\boldsymbol{m}_\vartheta}$, *a sub-state vector* $\boldsymbol{x}_{\boldsymbol{k}}^{\boldsymbol{m}_\vartheta}$ *and an identification point* $\boldsymbol{IP}_\vartheta = [u_{1,IP_\vartheta}, \ldots, u_{n_u,IP_\vartheta}, d_{1,IP_\vartheta}, \ldots, d_{n_d,IP_\vartheta}]$, *where* $\vartheta = 1, \ldots, n_m$, n_u *is the number of MVs,* n_d *is the number of DVs and* k *is an arbitrary initial time. Let the sub-state vectors for the next instant,* $\boldsymbol{x}_{\boldsymbol{k+1}}^{\boldsymbol{m}_\vartheta}$, *be provided by Eq. 91 and Eq. 92. Let the vector of distance coefficients of sub-model* $\boldsymbol{M_\vartheta}$, $\boldsymbol{\tau_\vartheta} \in \mathbb{R}^{n_m}$, *be yielded by the ratio between* $\sum_{\varphi=1,\ldots,n_m} \overline{\boldsymbol{IP_\varphi OP_k}}$ *and* $\overline{\boldsymbol{IP_\vartheta OP_k}}$ *normalised for each MV, as defined in Eq. 99, where is the* $\boldsymbol{OP_k}$ *operating point at time k, here defined as* $\boldsymbol{OP_k} = [u_{1,k}, \ldots, u_{n_u,k}, d_{1,k}, \ldots, d_{n_d,k}]$. *Let the degradation function of sub-model* $\boldsymbol{M}_\vartheta$, $\boldsymbol{g_\vartheta}(\boldsymbol{\tau_\vartheta})$, *be defined by a set of non-negative coefficients* $g_{\vartheta,4}$, $g_{\vartheta,3}$, $g_{\vartheta,2}$, $g_{\vartheta,1}$ *and* $g_{\vartheta,0}$. *Let the weight of the sub-model* $\boldsymbol{M_\vartheta}$ *in the main output prediction for each CV,* ε_ϑ, *be defined as the ratio between* $\boldsymbol{g_\vartheta}(\boldsymbol{\tau_\vartheta})$ *and* $\sum_{\varphi=1}^{n_m} \boldsymbol{g_\varphi}(\boldsymbol{\tau_\varphi})$. *For such a system, the main or multi-model output prediction at time* $k+1$, $\boldsymbol{y_{k+1}}$ *can be obtained through the use of Eq. 96, and the degradation function coefficients are obtained by solving the regression analysis problem given by Eq. 97 for a number* n_t *of trajectories.*

4.1.5 SMLP Method 3 – the SISO parametrisation

In previous Sections, the SMLP was based on the assumption that we can calculate a single weight ε_ϑ for each of the linear sub-model $\boldsymbol{M_\vartheta}$, where $\vartheta = 1, \ldots, n_m$, which dictates the overall contribution of model ϑ. In this case, the contribution to the prediction calculation is, in relative terms, the same for all CVs and inputs. Let us return to the initial example of Section 3.1, where the multi-linear prediction was provided by two sub-models, $\boldsymbol{M_1}$ and $\boldsymbol{M_2}$, and we had two CVs, y_1 and y_2. If, for instance, $\varepsilon_1 > \varepsilon_2$, then sub-model $\boldsymbol{M_1}$ is the main contributor for the prediction calculation of both y_1 and y_2. However, maybe at current OP sub-model $\boldsymbol{M_1}$ is adequate to calculate y_1, while sub-model $\boldsymbol{M_2}$ is better suited to provide y_2, or vice-versa. In this case, there may be some accuracy gain by defining a different ε to each MV, for each sub-model. Let us carry out a few changes in the multi-linear prediction to reflect this concept. Eq. 99 is modified to provide the new distance coefficients, $\tau_{u_j}^{\vartheta}$ and $\tau_{d_l}^{\vartheta}$, where $j = 1, \dots, n_u$, $l = 1, \dots, n_d$ and $\vartheta = 1, \dots, n_m$.

$$\tau_{u_1}^{\vartheta} = \frac{\sum_{\varphi=1}^{nm} \left\| \overline{IP_{u_1,\varphi} OP_{u_1}} \right\| / \Delta u_1}{\left\| \overline{IP_{u_1,\vartheta} OP_{u_1}} \right\| / \Delta u_1}$$

$$\vdots$$

$$\tau_{u_{n_u}}^{\vartheta} = \frac{\sum_{\varphi=1}^{nm} \left\| \overline{IP_{u_{n_u},\varphi} OP_{u_{n_u}}} \right\| / \Delta u_{n_u}}{\left\| \overline{IP_{u_{n_u},\vartheta} OP_{u_{n_u}}} \right\| / \Delta u_{n_u}} \qquad \text{Eq. 100}$$

$$\tau_{d_1}^{\vartheta} = \frac{\sum_{\varphi=1}^{nm} \left\| \overline{IP_{d_1,\varphi} OP_{d_1}} \right\| / \Delta d_1}{\left\| \overline{IP_{d_1,\vartheta} OP_{d_1}} \right\| / \Delta d_1}$$

$$\vdots$$

$$\tau_{d_{n_d}}^{\vartheta} = \frac{\sum_{\varphi=1}^{nm} \left\| \overline{IP_{d_{n_d},\varphi} OP_{d_{n_d}}} \right\| / \Delta d_{n_d}}{\left\| \overline{IP_{d_{n_d},\vartheta} OP_{d_{n_d}}} \right\| / \Delta d_{n_d}} \qquad \text{Eq. 101}$$

The degradation functions should now reflect variable by variable input tracking, and there will now be one set of functions for the MVs, $\boldsymbol{g}_{\boldsymbol{u_j}}^{\boldsymbol{\vartheta}}$, where $j = 1, \dots, n_u$, and another set for the DVs, $\boldsymbol{g}_{\boldsymbol{d_l}}^{\boldsymbol{\vartheta}}$, where $l = 1, \dots, n_d$, and $\vartheta = 1, \dots, n_m$ for both sets. As before, these functions work as nonlinear weight parameters defining the weight of each sub-model $\boldsymbol{M_\vartheta}$ in the prediction of each CV, but now the weight is also distinct for each input. Once more a 4th order polynomial with non-negative coefficients is used as the format of the functions, as per Eq. 102 and Eq. 103:

$$\boldsymbol{g}_{\boldsymbol{u_j}}^{\boldsymbol{\vartheta}}\left(\tau_{u_j}^{\vartheta}\right) = g_{u_j}^{\vartheta,4} {\tau_{u_j}^{\vartheta}}^4 + g_{u_j}^{\vartheta,3} {\tau_{u_j}^{\vartheta}}^3 + g_{u_j}^{\vartheta,2} {\tau_{u_j}^{\vartheta}}^2 + g_{u_j}^{\vartheta,1} \tau_{u_j}^{\vartheta} + g_{u_j}^{\vartheta,0} \qquad \text{Eq. 102}$$

$$\boldsymbol{g}_{\boldsymbol{d_l}}^{\boldsymbol{\vartheta}}\left(\tau_{d_l}^{\vartheta}\right) = g_{d_l}^{\vartheta,4} {\tau_{d_l}^{\vartheta}}^4 + g_{d_l}^{\vartheta,3} {\tau_{d_l}^{\vartheta}}^3 + g_{d_l}^{\vartheta,2} {\tau_{d_l}^{\vartheta}}^2 + g_{d_l}^{\vartheta,1} \tau_{d_l}^{\vartheta} + g_{d_l}^{\vartheta,0} \qquad \text{Eq. 103}$$

Eq. 95 is modified to reflect the fact that the weight parameters are in this case defined MV by MV and DV by DV, yielding Eq. 104 and Eq. 105:

$$\varepsilon_{u_j}^{\vartheta} = \frac{g_{u_j}^{\vartheta}\left(\tau_{u_j,\vartheta}\right)}{\sum_{\varphi=1}^{nm} g_{u_j}^{\varphi}\left(\tau_{u_j,\varphi}\right)} \qquad \text{Eq. 104}$$

$$\varepsilon_{d_l}^{\vartheta} = \frac{g_{d_l}^{\vartheta}\left(\tau_{d_l,\vartheta}\right)}{\sum_{\varphi=1}^{nm} g_{d_l}^{\varphi}\left(\tau_{d_l,\varphi}\right)} \qquad \text{Eq. 105}$$

The simultaneous sub-states are obtained from each linear sub-model $\boldsymbol{M_\vartheta}$, where $\vartheta = 1, \dots, n_m$, and the weights $\boldsymbol{\varepsilon_d}$ and $\boldsymbol{\varepsilon_u}$, as shown in Eq. 106 to Eq. 108

$$x_{k+1}^{m_1} = A_{m_1} x_k^{m_1} + B_{m_1} \varepsilon_{u,k}^1 \Delta u_k + D_{m_1} \varepsilon_{d,k}^1 \Delta d_k$$

$$\vdots$$

$$x_{k+1}^{m_{nm}} = A_{m_{nm}} x_k^{m_{nm}} + B_{m_{nm}} \varepsilon_{u,k}^{nm} \Delta u_k + D_{m_{nm}} \varepsilon_{d,k}^{nm} \Delta d_k \quad \text{Eq. 106}$$

$$y_k^{m_1} = C_{m_1} x_k^{m_1}$$

$$\vdots$$

$$y_k^{m_{nm}} = C_{m_{nm}} x_k^{m_{nm}} \quad \text{Eq. 107}$$

$$y_k = y_k^{m_1} + \cdots + y_k^{m_{nm}} \quad \text{Eq. 108}$$

Note that when variable by variable input tracking is being performed, we are not applying the full inputs, $\Delta \boldsymbol{u}$ and $\Delta \boldsymbol{d}$, to all the linear sub-models, as in Sections 3.1 and 3.2. Instead, we split the inputs among the sub-models (Eq. 106) using $\boldsymbol{\varepsilon_d}$ and $\boldsymbol{\varepsilon_u}$, and sum the full resulting outputs (Eq. 108). The approach presented in this Section, henceforth called SMLP method 3, uses a larger number of regression analysis coefficients $(3 \cdot (n_u + n_d) \cdot n_m)$ as compared to method 1 and 2 $(3 \cdot n_m)$, and this extra information most likely increases the prediction's accuracy. Fig. 17 presents the data workflow of this third variant of the SMLP method.

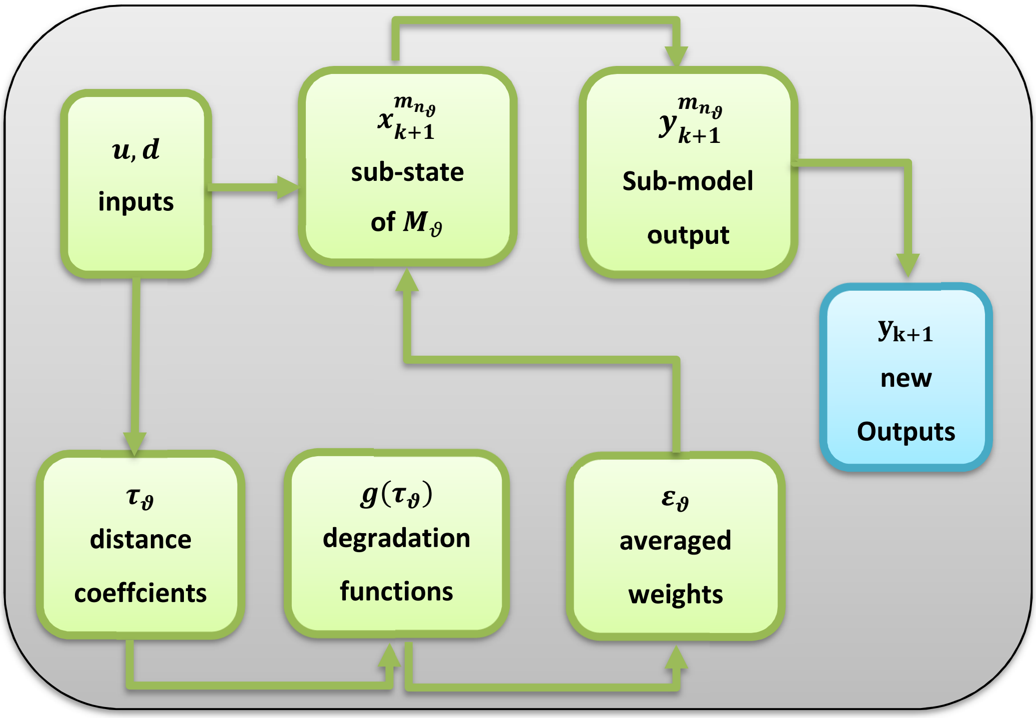

Fig. 17 – Data workflow of the third SMLP method. The averaged weights are now used to obtain the new sub-states.

A new regression analysis problem arises to obtain the coefficients of the functions $\boldsymbol{g}_{\boldsymbol{u_j},\boldsymbol{\vartheta}}(\tau_\vartheta)$ and $\boldsymbol{g}_{\boldsymbol{d_l},\boldsymbol{\vartheta}}(\tau_\vartheta)$. As before, we wish to minimise the square error between the nonlinear model and the SMLP, using a random trajectory, through the entire prediction horizon.

For this third method, a SMLP system is defined as follows:

Definition 4.1.5.1 Simultaneous Multi-Linear Prediction system with SISO parametrisation (OP defined as the input vector). *Let there be a set of n_m sub-models, where each sub-model $\boldsymbol{M_\vartheta}$ is defined by the state-space matrices $\boldsymbol{A_{m_\vartheta}}$, $\boldsymbol{B_{m_\vartheta}}$, $\boldsymbol{C_{m_\vartheta}}$ and $\boldsymbol{D_{m_\vartheta}}$, a sub-state vector $\boldsymbol{x_k^{m_\vartheta}}$ and an identification point $\boldsymbol{IP}_\vartheta = [u_{1,IP_\vartheta}, \dots, u_{n_u,IP_\vartheta}, d_{1,IP_\vartheta}, \dots, d_{n_d,IP_\vartheta}]$, where $\vartheta = 1, \dots, n_m$, n_u is the number of MVs, n_d is the number of DVs and k is an arbitrary initial time. Let $\tau_{u_j}^\vartheta$ be the distance coefficient of sub-model $\boldsymbol{M_\vartheta}$ related to the manipulated variable j, and $\tau_{d_l}^\vartheta$ be the distance coefficient of sub-model $\boldsymbol{M_\vartheta}$ related to the disturbance variable l, where $j = 1, \dots, n_u$, $l = 1, \dots, n_d$ and $\vartheta = 1, \dots, n_m$. Let $\tau_{u_j}^\vartheta$ be yielded by the ratio between $\sum_{\varphi=1}^{n_m} \left\| \overline{IP_{u_j,\varphi} OP_{u_j}} \right\|$ and $\left\| \overline{IP_{u_j,\vartheta} OP_{u_j}} \right\|$ normalised for each MV, and $\tau_{d_l}^\vartheta$ be yielded by the ratio between $\sum_{\varphi=1}^{n_m} \left\| \overline{IP_{d_l,\varphi} OP_{k,d_l}} \right\|$ and $\left\| \overline{IP_{d_l,\vartheta} OP_{d_l}} \right\|$ normalised for each DV, as defined in Eq. 100 and Eq. 101, where is the $\boldsymbol{OP_k}$ operating point at time k, here defined as $\boldsymbol{OP_k} = [u_{1,k}, \dots, u_{n_u,k}, d_{1,k}, \dots, d_{n_d,k}]$. Let the degradation functions of sub-model $\boldsymbol{M_\vartheta}$ related respectively to u_j and d_l, $\boldsymbol{g_{u_j}^\vartheta}\left(\tau_{u_j}^\vartheta\right)$ and $\boldsymbol{g_{d_l}^\vartheta}\left(\tau_{d_l}^\vartheta\right)$, be defined by the sets of non-negative coefficients $g_{u_j}^{\vartheta,4}$, $g_{u_j}^{\vartheta,3}$, $g_{u_j}^{\vartheta,2}$, $g_{u_j}^{\vartheta,1}$ and $g_{u_j}^{\vartheta,0}$ and $g_{d_l}^{\vartheta,4}$, $g_{d_l}^{\vartheta,3}$, $g_{d_l}^{\vartheta,2}$, $g_{d_l}^{\vartheta,1}$ and $g_{d_l}^{\vartheta,0}$. Let the weights of the sub-model $\boldsymbol{M_\vartheta}$ in the main output prediction for u_j and d_l, $\varepsilon_{u_j}^\vartheta$ and $\varepsilon_{d_l}^\vartheta$, be defined respectively as the ratio between $g_{u_j}^\vartheta\left(\tau_{u_j,\vartheta}\right)$ and $\sum_{\varphi=1}^{n_m} g_{u_j}^\varphi\left(\tau_{u_j,\varphi}\right)$ and the ratio between $g_{d_l}^\vartheta\left(\tau_{d_l,\vartheta}\right)$ and $\sum_{\varphi=1}^{n_m} g_{d_l}^\varphi\left(\tau_{d_l,\varphi}\right)$. For such a system, the sub-state vectors for the next instant, $\boldsymbol{x_{k+1}^\vartheta}$, can be obtained through the use of Eq. 106 and Eq. 107. The main or multi-model output prediction at time $k+1$, $\boldsymbol{y_{k+1}}$, is given by Eq. 108, and the coefficients of the degradation functions are obtained by solving the regression analysis problem given by Eq. 109. This* *problem is defined for the variable by variable input tracking over a number n_t of trajectories.*

$$\min_{\substack{g_{u_j}^{\vartheta,4},g_{u_j}^{\vartheta,3},g_{u_j}^{\vartheta,2},g_{u_j}^{\vartheta,1},g_{u_j}^{\vartheta,0}\\ g_{d_l}^{\vartheta,4},g_{d_l}^{\vartheta,3},g_{d_l}^{\vartheta,2},g_{d_l}^{\vartheta,1},g_{d_l}^{\vartheta,0}}} \sum_{\vartheta}\sum_{\eta}\sum_{\alpha}\left(\left[\boldsymbol{y}_{\boldsymbol{\eta},\boldsymbol{\alpha}}^{\boldsymbol{nl}}-\boldsymbol{y}_{\boldsymbol{\eta},\boldsymbol{\alpha}}^{\boldsymbol{m}_{n_\vartheta}}\right] diag|\boldsymbol{y}_{\boldsymbol{max}}^{*}-\boldsymbol{y}_{\boldsymbol{min}}^{*}|^{-1}\right)^2 \quad \text{Eq. 109}$$

For Eq. 109, we have as before $j = 1, \dots, n_u$, $l = 1, \dots, n_d$, $\vartheta = 1, \dots, n_m$, $\alpha = 1, \dots, p$; $\eta = 1, \dots, n_t$.

4.1.6 The stability of an SMLP system

Concerning the stability of a SLMP system, the following theorem is valid:

Theorem 4.1.6.1 the stability of an SMLP system. *An SMLP system is Lyapunov stable if and only if composed only of stable sub-models.*

Proof. *An SMLP system consists of the sum of a finite number of dynamic responses relative to each sub-model, each of which is multiplied by a non-negative scalar, ε. If all sub-models are stable, the dynamic responses that make up the SMLP are going to be bound, and it is self-evident that the sum of a finite number of bounded dynamic responses is also bounded. The SMLP cannot make any of the individual responses unbounded. For instance, let us consider model state matrix $\boldsymbol{A}_{\boldsymbol{m}_\vartheta}$ of sub-model $\boldsymbol{M}_{\boldsymbol{\vartheta}}$. From Lyapunov (1892), for continuous-time state-space formulations, if a model matrix $\boldsymbol{A}_{\boldsymbol{m}_\vartheta}$ has eigenvalues λ_ϑ with negative real parts, the system is BIBO (Bounded-Input Bounded-Output) stable. We have from the theorem of ESMM (Eigenvalues of a Scalar Multiple of a Matrix) that the eigenvalues $\varepsilon_\vartheta\lambda_\vartheta$ of matrix $\left[\varepsilon_\vartheta\boldsymbol{A}_{\boldsymbol{m}_\vartheta}\right]$ will also have negative real parts since $\varepsilon_\vartheta \in [0,1]$. For discrete-time formulations, a model is asymptotically (Schur) stable if and only if all the eigenvalues of its state matrix A have a magnitude less than one, i.e. lie inside the unit circle. Assuming that the sub-model $\boldsymbol{M}_{\boldsymbol{\vartheta}}$ is stable and thus $|\lambda_\vartheta| \in [0,1]$, it results in $\varepsilon_\vartheta\lambda_\vartheta \in [0,1]$, and therefore the response remains BIBO stable.*

Being Lyapunov Stable is a requirement for all flowsheets being evaluated through the use of the EMOP index (Strutzel and Bogle, 2016).

4.1.7 Reachability of an SMLP system

It is desirable to know if any given value for $\boldsymbol{x}_{\boldsymbol{k}}^{\boldsymbol{m}_{n_\vartheta}}$ can be reached, i.e., if the system $(\boldsymbol{A}_{\boldsymbol{m}_\vartheta}, \boldsymbol{B}_{\boldsymbol{m}_\vartheta})$ is completely reachable. Hence, let us focus on the problem, defined in Eq. 110, of determining a sequence of m incremental control actions transferring the sub-state vector from $\boldsymbol{x}_{\boldsymbol{k}}^{\boldsymbol{m}_{n_\vartheta}} = \overline{\boldsymbol{x}}_{\vartheta,1}$ to $\boldsymbol{x}_{\boldsymbol{k+m}}^{\boldsymbol{m}_{n_\vartheta}} = \overline{\boldsymbol{x}}_{\vartheta,2}$.

$$\underbrace{\overline{\boldsymbol{x}}_{\vartheta,2} - \boldsymbol{A}_{\boldsymbol{m}_\vartheta}^{m}\, \overline{\boldsymbol{x}}_{\vartheta,1}}_{\overline{X}_{m_\vartheta}} = \underbrace{\left[\boldsymbol{B}_{\boldsymbol{m}_\vartheta}\;\; \boldsymbol{A}_{\boldsymbol{m}_\vartheta}\boldsymbol{B}_{\boldsymbol{m}_\vartheta} \dots \boldsymbol{A}_{\boldsymbol{m}_\vartheta}^{m-1}\boldsymbol{B}_{\boldsymbol{m}_\vartheta}\right]}_{R_{m_\vartheta}} \underbrace{\left[\Delta\boldsymbol{u}_{k}\; \Delta\boldsymbol{u}_{k+1} \dots \Delta\boldsymbol{u}_{k+m-1}\right]^{T}}_{\Delta u_K{}^{T}} \qquad \text{Eq. 110}$$

This is equivalent to solving the linear system of Eq. 111 with respect to Δu_K:

$$\boldsymbol{R}_{\boldsymbol{m}_\vartheta}\, \Delta\boldsymbol{u}_{\boldsymbol{K}}{}^{\boldsymbol{T}} = \overline{\boldsymbol{X}}_{\boldsymbol{m}_\vartheta} \qquad \text{Eq. 111}$$

The matrix $\boldsymbol{R}_{\boldsymbol{m}_\vartheta} \in \mathbb{R}^{\boldsymbol{d}_{A,\vartheta}\, x\, m \cdot \boldsymbol{n}_{\boldsymbol{u}}}$ is called the reachability matrix of the subsystem. A solution $\Delta\boldsymbol{u}_{\boldsymbol{K}}$ to Eq. 111 exists if and only if $\overline{\boldsymbol{X}}_{\boldsymbol{m}_\vartheta} \in \boldsymbol{Im}(\boldsymbol{R}_{\boldsymbol{m}_\vartheta})$, where $\boldsymbol{Im}(\boldsymbol{R}_{\boldsymbol{m}_\vartheta})$ is the set of states that are reachable from the initial state $\overline{\boldsymbol{x}}_{\vartheta,1}$. According to the Rouché-Capelli theorem, this is true if $rank(\boldsymbol{R}_{\boldsymbol{m}_\vartheta}\, \overline{\boldsymbol{X}}_{\boldsymbol{m}_\vartheta}) = rank(\boldsymbol{R}_{\boldsymbol{m}_\vartheta})$ and hence the following well-known theorem is valid:

> ***Theorem the complete reachability of a system (Kalman et al., 1969).*** *The subsystem $(\boldsymbol{A}_{\boldsymbol{m}_\vartheta}, \boldsymbol{B}_{\boldsymbol{m}_\vartheta})$ is completely reachable if and only its reachability matrix is full rank, i.e., the largest rank possible for a matrix of the same dimensions*

The full rank value is the lesser of the number of rows and columns and in this case $rank(\boldsymbol{R}_{\boldsymbol{m}_\vartheta}) = m \cdot n_u$, where $\vartheta = 1, \dots, n_m$. A proof of the theorem is presented in Kalman et al. (1969). Let us now approach the multi-model reachability problem. The reachability matrices and the desired states of each sub-model can be concatenated yielding Eq. 112, the multi-model version of the system of Eq. 111. Note that the solution $\Delta\boldsymbol{u}_{\boldsymbol{K}}$ is the same as for Eq. 111.

$$\underbrace{\begin{bmatrix} \boldsymbol{R}_{\boldsymbol{m}_1} \\ \vdots \\ \boldsymbol{R}_{\boldsymbol{m}_{n_m}} \end{bmatrix}}_{R_M} \Delta\boldsymbol{u}_{\boldsymbol{K}}{}^{T} = \underbrace{\begin{bmatrix} \overline{\boldsymbol{X}}_{\boldsymbol{m}_1} \\ \vdots \\ \overline{\boldsymbol{X}}_{\boldsymbol{m}_{n_m}} \end{bmatrix}}_{\overline{X}_M} \qquad \text{Eq. 112}$$

The theorem bellow is valid for SMLP systems:

Theorem 4.1.7.1 the complete reachability of an SMLP system. *An SMLP system is completely reachable if and only if composed solely of completely reachable sub-models.*

Proof. *If an SMLP system is composed solely of completely reachable sub-models, the reachability matrices of each sub-model* $\boldsymbol{M_\vartheta}$, $\boldsymbol{R_{m_\vartheta}}$, *have full rank and the reachability matrix of the SMLP system,* $\boldsymbol{R_M}$, *also has full rank. Since* $\boldsymbol{R_M}$ *is the concatenation of the individual* $\boldsymbol{R_{m_\vartheta}}$, *where* $\vartheta = 1, \dots, n_m$, *it results in* $rank(\boldsymbol{R_M}) = rank\left(\boldsymbol{R_{m_\vartheta}}\right) = m \cdot n_u$. *Likewise,* $\boldsymbol{R_M}$ *will be rank deficient and the SMLP system will not be completely reachable if one or more sub-systems do not have full rank.*

A completely reachable SMLP will be able to achieve any output value in the unrestricted control problem. In addition to that, it is shown in Hautus (1972) that controllability is a weaker condition than reachability, and also that controllable systems are stabilisable. Therefore, if an SMLP is completely reachable it is also controllable and stabilisable.

4.1.8 Filtering sub-state changes

Due to the unavoidable presence of measurement noise and unmeasured disturbances in industrial, closed-loop MPC applications, the control engineer should implement a state-estimator, such as a Kalman Filter, for each sub-state of a SMLP system. The usual tuning considerations on this subject apply (Gelb, 1974). Please note that the regression analysis problems given by Eq. 97 and Eq. 109 are affected by the state-estimator tuning, and thus re-identification of the degradation functions is necessary if any of the estimators is changed. In this paper, state-estimators were not required since it was assumed that no measurement noise and no unmeasured disturbances were present in the case-studies.

5 Quantifying the effects of model uncertainty in the joint use of the EMOP index and the SMLP

When performing Controllability Analysis, the evaluation of plant performance should be robust to uncertainty in the model parameters. In this Section model uncertainty is embedded in the EMOP Index for the special case of the SMLP. Even if the SMLP approach is suitable to represent process nonlinearity by successful in removing most of the linearisation error, the remaining error may be still enough to potentially lead to the wrong comparison of flowsheets in some cases, so the case for embedding model uncertainty is strong.

We shall now define a model uncertainty measure suitable to represent process nonlinearity. Since simulation is the source of process data in the current work, and for this reason, there is no sensor related issues, model nonlinearity is the major source of modelling error (numerical error being the remaining possibility). We are interested in knowing the magnitude of uncertainty in the steady-state output change in response to an input such as a control action, as shown in Fig. 18:

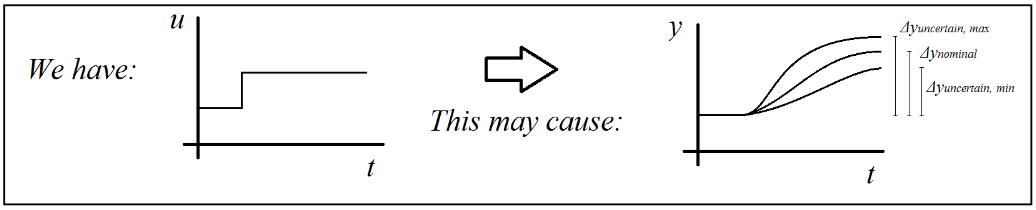

Fig. 18 – Uncertainty in the magnitude of the steady-state output change, given a certain input change.

So how can we measure the uncertainty of an SMLP system describing the dynamic behaviour of an output/input pair? An interesting possibility is comparing the dynamic response of nonlinear model to that of the SMLP system as well as that of each one of the n_m sub-models that compose it. For each linear sub-model ϑ composing the system, let us consider the output response, y_i^{ϑ}, to a unitary step change in a certain input, u_j or d_l, where $i = 1, \dots, n_y$, $j = 1, \dots, n_u$ and $l = 1, \dots, n_d$, $\vartheta = 1, \dots, n_m$, and $\boldsymbol{y}_i^{\boldsymbol{nl}}$ is the nonlinear system's output. Similarly, let us also consider the SMLP system's response, which we shall denote as y_i^{ϑ}, $\vartheta = 0$.

Whereas the multi-linear system can better represent the flowsheet at most states, as it is shown in Section 6 for the crude oil study case, the linear sub-models are assumed to be accurate when close to their identification points. Thus, the maximum model mismatch between $\boldsymbol{y}_i^{nl}$ and $\boldsymbol{y}_i^{\vartheta}$ during the prediction horizon is representative of the uncertainty of the multi-linear system's prediction (where $\vartheta = 0, \ldots, n_m$, meaning we consider the largest mismatch among the outputs of all sub-models plus the main SMLP prediction). For instance, consider a multi-linear SMLP system of composed of 2 linear sub-models. Given the dynamic response to a unity step shown in Fig. 4 below, the model mismatch at the end of the prediction horizon for each sub-model will be Δy_p^1 and Δy_p^2, and the model mismatch for the SMLP system will be Δy_p^0.

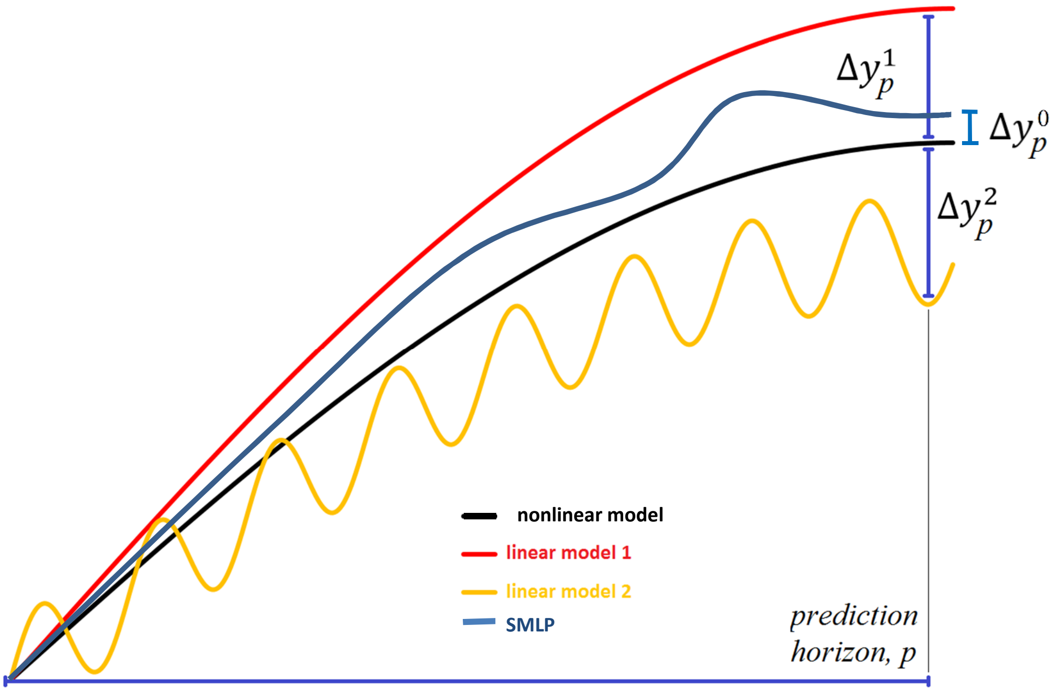

Fig. 19 – Model mismatch between sub-models 1, 2 and 3.

Note that $\Delta y_p^0 > 0$, $\Delta y_p^1 > 0$ and $\Delta y_p^2 < 0$, so the mismatch value can be positive or negative. Let $\Delta y_{y_i,u_j}^{amp}$ be the norm of the amplitude of the dynamic response of the multi-linear system to a step at the end of the prediction horizon, for a certain CV/MV couple. Concerning u_j and y_i, the relative model mismatch between the nonlinear model and the sub-model ϑ is given by Eq. 113:

$$\varphi_{y_i,u_j}^{\vartheta} = \Delta y_p^{\vartheta} / \Delta y_{y_i,u_j}^{amp} \qquad \text{Eq. 113}$$

Similarly, the relative model mismatch between the nonlinear model and sub-model ϑ is given by Eq. 114 for a certain CV/DV couple:

$$\varphi_{y_i,d_l}^{\vartheta} = \Delta y_p^{\vartheta} / \Delta y_{y_i,d_l}^{amp} \qquad \text{Eq. 114}$$

It is important to consider that if the output/input pair is weakly related $\left(\Delta y_{y_i,d_l}^{amp} < 0.01 \text{ or } \varphi_{y_i,u_j}^{\vartheta} < 0.01\right)$, the denominators in Eq. 113 and Eq. 114 will be small and φ might become very large. In this case, this large uncertainty is meaningless and unhelpful to the analysis, and thus the control engineer should ignore uncertainty for this *y/u* or *y/d* pair by setting $\varphi = 0$. It is important to notice that sub-models can be biased to a direction, e.g., providing always smaller changes in output prediction than plant data, consistently resulting in a negative model error. Or, alternatively, prediction error may be randomly distributed for different steps. For this reason, β_{y_i,u_j}^{-} and β_{y_i,u_j}^{+}, respectively the minimum negative relative mismatch and the maximum positive relative mismatch between y_i and u_j, are defined Eq. 115 and Eq. 116:

$$\beta_{y_i,u_j}^{-} = \min_{\vartheta=1,\ldots,n_m} \mu_{\vartheta}\, \varphi_{y_i,u_i}^{\vartheta}, \qquad i = 1, \ldots, n_y, \; j = 1, \ldots, n_u \qquad \text{Eq. 115}$$

$$\beta_{y_i,u_j}^{+} = \max_{\vartheta=1,\ldots,n_m} \psi_{\vartheta}\, \varphi_{y_i,u_i}^{\vartheta}, \qquad i = 1, \ldots, n_y, \; j = 1, \ldots, n_u \qquad \text{Eq. 116}$$

where $\mu_{\vartheta} = 1$ and $\psi_{\vartheta} = 0$ if $\varphi_{y_I,u_i}^{\vartheta} < 0$, $\mu_{\vartheta} = 0$ and $\psi_{\vartheta} = 1$ if $\varphi_{y_I,u_i}^{\vartheta} > 0$. Parameters β_{y_i,u_j}^{-} and β_{y_i,u_j}^{+} may be understood as fractions of the nominal multi-model response. Similarly, β_{y_i,d_j}^{-} and β_{y_i,d_j}^{+} are respectively the minimum negative and the maximum positive relative model mismatch between y_i and the DV d_j, as defined in Eq. 117 and Eq. 118:

$$\beta_{y_i,d_l}^{-} = \min_{\vartheta=1,\ldots,nm} \mu_{\vartheta}\, \varphi_{y_i,d_l}^{\vartheta}, \qquad i = 1, \ldots, n_y, \; l = 1, \ldots, n_d \qquad \text{Eq. 117}$$

$$\beta_{y_i,d_l}^{+} = \max_{\vartheta=1,\ldots,nm} \psi_{\vartheta}\, \varphi_{y_i,d_l}^{\vartheta}, \qquad i = 1, \ldots, n_y, \; l = 1, \ldots, n_d \qquad \text{Eq. 118}$$

The control engineer should test several operating points (OPs) throughout the control zone and in the vicinity using different step amplitudes to obtain reliable sub-models and to ensure that the magnitude of the resulting uncertainty parameter is representative of nonlinearity effects. Let us define a set of matrices B_u^{+}, B_u^{-}, B_d^{+} and B_d^{-} in Eq. 119 to Eq. 122, which are going to store all model prediction error data from step tests obtained for each possible coupling of *y/u* or *y/d*:

$$\boldsymbol{B_u^+} = \begin{bmatrix} \beta^+_{y_1,u_1} & \cdots & \beta^+_{y_{ny},u_1} \\ \vdots & \ddots & \vdots \\ \beta^+_{y_1,u_{nu}} & \cdots & \beta^+_{y_{ny},u_{nu}} \end{bmatrix} \quad \text{Eq. 119}$$

$$\boldsymbol{B_u^-} = \begin{bmatrix} \beta^-_{y_1,u_1} & \cdots & \beta^-_{y_{ny},u_1} \\ \vdots & \ddots & \vdots \\ \beta^-_{y_1,u_{nu}} & \cdots & \beta^-_{y_{ny},u_{nu}} \end{bmatrix} \quad \text{Eq. 120}$$

$$\boldsymbol{B_d^+} = \begin{bmatrix} \beta^+_{y_1,d_1} & \cdots & \beta^+_{y_{ny},d_1} \\ \vdots & \ddots & \vdots \\ \beta^+_{y_1,d_{nd}} & \cdots & \beta^+_{y_{ny},d_{nd}} \end{bmatrix} \quad \text{Eq. 121}$$

$$\boldsymbol{B_d^-} = \begin{bmatrix} \beta^-_{y_1,d_1} & \cdots & \beta^-_{y_{ny},d_1} \\ \vdots & \ddots & \vdots \\ \beta^-_{y_1,d_{nd}} & \cdots & \beta^-_{y_{ny},d_{nd}} \end{bmatrix} \quad \text{Eq. 122}$$

The uncertainty related to an output change, Δy_i^u, which arises due to an input movement, Δu_j, may be characterised by the model mismatch. The change in the steady-state output prediction caused by a bounded realisation of MV uncertainty, $\Delta y_i^{u'}$, is contained inside the interval defined by the nominal model prediction, Δy_i^u, multiplied by $1+\beta^-_{y_I,u_j}$ and $1+\beta^+_{y_I,u_j}$, as shown in Eq. 123:

$$\Delta y_i^{u'} \in \left[\Delta y_i \cdot \left(1+\beta^-_{y_i,u_j}\right), \Delta y_i \cdot \left(1+\beta^+_{y_i,u_j}\right)\right], \quad i=1,\dots,n_y,\ j=1,\dots,n_u \quad \text{Eq. 123}$$

Likewise, for disturbances we have Eq. 124:

$$\Delta y_i^{d'} \in \left[\Delta y_i \cdot \left(1+\beta^-_{y_i,d_j}\right), \Delta y_i \cdot \left(1+\beta^+_{y_i,d_j}\right)\right], \quad i=1,\dots,n_y,\ j=1,\dots,n_d \quad \text{Eq. 124}$$

For the calculation procedure presented in the Section, it is also useful to define the uncertainty related to input changes. For instance, imagine we are able to measure plant output, and we observe a certain dynamic response, but for some reason, the input causing the change in the process cannot be measured. We may use the SMLP system to estimate the unknown magnitude of the MV (or disturbance) change, which will be contained in the value interval $\Delta u_j'$ (or $\Delta d_j'$), in a similar way to $\Delta y_i^{u'}$. This concept is shown in Fig. 20:

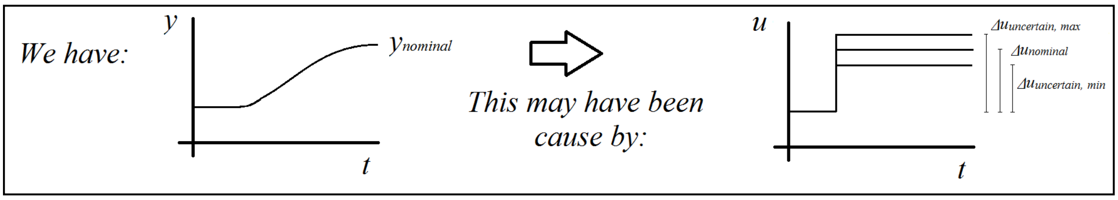

Fig. 20 – Uncertainty in the magnitude of an unknown input change, given a certain output change.

Now this concept of uncertainty is clearly defined, and we have the understanding that it describes input and output changes $(\Delta u, \Delta d, \Delta y)$, not absolute values (u, d, y). Given this uncertainty definition, if a linear continuous-time state-space model is being used, there is a relation of proportionality between $\frac{\Delta y_i^{u'}}{\Delta y_i^u}$ and $\frac{\Delta u_j'}{\Delta u_j}$, and also between $\frac{\Delta y_i^{d'}}{\Delta y_i^d}$ and $\frac{\Delta d_j'}{\Delta d_j}$. Due to model linearity, if we multiply an MV change, Δu_j, by any real number contained in $\left[1 + \beta_{y_i,u_j}^-, 1 + \beta_{y_i,u_j}^+\right]$, the resulting output change will be contained inside Δy_i^u. Similarly, multiplying a disturbance change, Δd_j, by any real number contained in $\left[1 + \beta_{y_i,d_j}^-, 1 + \beta_{y_i,d_j}^+\right]$, the resulting output change will be contained inside Δy_i^d. We use this result to obtain Eq. 125 and Eq. 126, which are respectively the interval of control action magnitude of the uncertain model, $\Delta u_j'$, and the interval of disturbance magnitude of the uncertain model, $\Delta d_j'$:

$$\Delta u_j' \in \left[\Delta u_j \cdot \left(1 + \beta_{y_i,u_j}^-\right), \Delta u_j \cdot \left(1 + \beta_{y_i,u_j}^+\right)\right], \quad i = 1, \dots, n_y, \ j = 1, \dots, n_u \qquad \text{Eq. 125}$$

$$\Delta d_j' \in \left[\Delta d_j \cdot \left(1 + \beta_{y_i,d_j}^-\right), \Delta d_i \cdot \left(1 + \beta_{y_i,d_j}^+\right)\right], \quad i = 1, \dots, n_y, \ j = 1, \dots, n_d \qquad \text{Eq. 126}$$

Let us define two new variables, the uncertainty realisation parameters γ_{y_i,u_j} and γ_{y_i,d_j}, which are real numbers that can assume any value inside the limits defined by, respectively, $\left[1 + \beta_{y_i,u_j}^-, 1 + \beta_{y_i,u_j}^+\right]$ and $\left[1 + \beta_{y_i,d_j}^-, 1 + \beta_{y_i,d_j}^+\right]$, as shown in Eq. 127 and Eq. 128:

$$\gamma_{y_i u_j} \in \left[1+\beta^-_{y_i,u_j}, 1+\beta^+_{y_i,u_j}\right] \quad \text{Eq. 127}$$

$$\gamma_{y_i d_j} \in \left[1+\beta^-_{y_i,d_j}, 1+\beta^+_{y_i,d_j}\right] \quad \text{Eq. 128}$$

Eq. 106 to Eq. 108, which calculate the output prediction of the nominal SMLP model for method 3, can be modified to incorporate these new variables, obtaining new output trajectories as the uncertainty parameters assume different values. To do that, instead of applying the original MV change Δu_k at time k, we apply $\gamma_{y_i u_j}\Delta u_k$, where $i = 1, \dots ny, j = 1, \dots nu$. At time $k+1$ we apply $\gamma_{y_i u_j}\Delta u_{k+1}$ instead of Δu_{k+1}, and so forth until the end of the control horizon, $k+m$. The same approach can be used for disturbances. We replace Δd_k for $\gamma_{y_i d_j}\Delta d_k$ at time k, where $i = 1, \dots n_y$, $j = 1, \dots n_d$, and then replace Δd_{k+1} for $\gamma_{y_i d_j}\Delta d_{k+1}$ at time $k+1$, and proceed likewise all the way until the end of the prediction horizon, $k+p$. These substitutions yield Eq. 129 and Eq. 130, enabling us to calculate a new output prediction of the uncertain model, $\boldsymbol{y}'$, where $\boldsymbol{y}' \in (\Delta\boldsymbol{y^u} \cup \Delta\boldsymbol{y^d})$.

$$\boldsymbol{y}^{\vartheta\,\prime}_{k+1} = \boldsymbol{C}_{m_1}\boldsymbol{x}^{\vartheta}_{k+1} = \boldsymbol{C}_{m_\vartheta}\boldsymbol{A}_{m_\vartheta}\boldsymbol{x}^{\vartheta}_{k} + diag\left(\boldsymbol{C}_{m_\vartheta}\boldsymbol{B}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{u,k}\,\boldsymbol{\Gamma}_{\Delta u_k}\right) + diag\left(\boldsymbol{C}_{m_\vartheta}\boldsymbol{D}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{d,k}\,\boldsymbol{\Gamma}_{\Delta d_k}\right)$$

$$\boldsymbol{y}^{\vartheta\,\prime}_{k+2} = \boldsymbol{C}_{m_\vartheta}\boldsymbol{x}^{\vartheta}_{k+2} = \boldsymbol{C}_{m_\vartheta}\boldsymbol{A}_{m_\vartheta}\boldsymbol{x}^{\vartheta}_{k+1} + diag\left(\boldsymbol{C}_{m_\vartheta}\boldsymbol{B}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{u,k+1}\,\boldsymbol{\Gamma}_{\Delta u_{k+1}}\right)$$
$$+diag\left(\boldsymbol{C}_{m_\vartheta}\boldsymbol{D}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{d,k+1}\,\boldsymbol{\Gamma}_{\Delta d_{k+1}}\right)$$

$$= \boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^2_{m_\vartheta}\boldsymbol{x}^{\vartheta}_{k} + diag\left(\begin{bmatrix}\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}_{m_\vartheta}\boldsymbol{B}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{u,k} & \boldsymbol{C}_{m_\vartheta}\boldsymbol{B}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{u,k+1}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Gamma}_{\Delta u_k}\\ \boldsymbol{\Gamma}_{\Delta u_{k+1}}\end{bmatrix}\right) +$$
$$diag\left(\begin{bmatrix}\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}_{m_\vartheta}\boldsymbol{D}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{d,k} & \boldsymbol{C}_{m_\vartheta}\boldsymbol{D}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{d,k+1}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Gamma}_{\Delta d_k}\\ \boldsymbol{\Gamma}_{\Delta d_{k+1}}\end{bmatrix}\right)$$

$$\vdots$$

$$\boldsymbol{y}^{\vartheta\,\prime}_{k+p} = \boldsymbol{C}_{m_1}\boldsymbol{A}^{p}_{m_1}\boldsymbol{x}^{1}_{k} + \quad \text{Eq. 129}$$
$$diag\left(\begin{bmatrix}\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^{p-1}_{m_\vartheta}\boldsymbol{B}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{u,k}\;\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^{p-2}_{m_\vartheta}\boldsymbol{B}_{m_1}\boldsymbol{\varepsilon}^{\vartheta}_{u,k+1}\;\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^{p-m}_{m_\vartheta}\boldsymbol{B}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{u,k+m}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Gamma}_{\Delta u_k}\\ \boldsymbol{\Gamma}_{\Delta u_{k+1}}\\ \vdots\\ \boldsymbol{\Gamma}_{\Delta u_{k+m-1}}\end{bmatrix}\right) +$$
$$diag\left(\begin{bmatrix}\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^{p-1}_{m_\vartheta}\boldsymbol{D}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{d,k}\;\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^{p-2}_{m_\vartheta}\boldsymbol{D}_{m_1}\boldsymbol{\varepsilon}^{\vartheta}_{d,k+1}\;\boldsymbol{C}_{m_\vartheta}\boldsymbol{A}^{p-m}_{m_\vartheta}\boldsymbol{D}_{m_\vartheta}\boldsymbol{\varepsilon}^{\vartheta}_{d,k+m}\end{bmatrix}\begin{bmatrix}\boldsymbol{\Gamma}_{\Delta u_k}\\ \boldsymbol{\Gamma}_{\Delta u_{k+1}}\\ \vdots\\ \boldsymbol{\Gamma}_{\Delta u_{k+m-1}}\end{bmatrix}\right)$$

$$\boldsymbol{y}'_{k+1} = \sum_{\vartheta=1}^{m_m} \boldsymbol{y}^{\vartheta\,\prime}_{k+1} \quad \text{Eq. 130}$$

where:

$$\boldsymbol{\Gamma}_{\Delta u_{k+\sigma},\sigma=0,\ldots,m-1} = \begin{bmatrix} \gamma_{y_1u_1}\Delta u_{k+\sigma,1} & \gamma_{y_2u_1}\Delta u_{k+\sigma,1} & \cdots & \gamma_{y_{n_y}u_1}\Delta u_{k+\sigma,1} \\ \gamma_{y_1u_2}\Delta u_{k+\sigma,2} & \gamma_{y_2u_2}\Delta u_{k+\sigma,2} & \cdots & \gamma_{y_{n_y}u_2}\Delta u_{k+\sigma,2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{y_1u_{n_u}}\Delta u_{k+\sigma,n_u} & \gamma_{y_2u_{nu}}\Delta u_{k+\sigma,n_u} & \cdots & \gamma_{y_{n_y}u_{n_u}}\Delta u_{k+\sigma,n_u} \end{bmatrix} \quad \text{Eq. 131}$$

$$\boldsymbol{\Gamma}_{\Delta d_{k+\sigma},\sigma=0,\ldots,p-3} = \begin{bmatrix} \gamma_{y_1d_1}\Delta d_{k+\sigma,1} & \gamma_{y_2d_1}\Delta d_{k+\sigma,1} & \cdots & \gamma_{y_{n_y}d_1}\Delta d_{k+\sigma,1} \\ \gamma_{y_1d_2}\Delta d_{k+\sigma,2} & \gamma_{y_2d_2}\Delta d_{k+\sigma,2} & \cdots & \gamma_{y_{n_y}d_2}\Delta d_{k+\sigma,2} \\ \vdots & \vdots & \ddots & \vdots \\ \gamma_{y_1d_{n_d}}\Delta d_{k+\sigma,n_d} & \gamma_{y_2d_{n_d}}\Delta d_{k+\sigma,nd} & \cdots & \gamma_{y_{n_y}d_{n_d}}\Delta d_{k+\sigma,nd} \end{bmatrix} \quad \text{Eq. 132}$$

Where the matrices $\boldsymbol{\Gamma}_{\Delta u_k}$ and $\boldsymbol{\Gamma}_{\Delta d_k}$ are generated by the product between the sets of MVs and DVs and their respective bounded uncertainty realisations. The EMOP index procedure will be now updated to incorporate model uncertainty. Since Eq. 130 may be used to obtain the feasible intervals of predictions, enabling us to calculate the "worst" and "best" possible model indices. This new optimisation problem has three layers. The first step of the new procedure is to solve the nominal model problem described by standard EMOP index, obtaining the optimised output prediction provided by the nominal model. The second step is to use Eq. 130 to solve two new optimisation problems in which the model uncertainty parameters vary within their intervals (Eq. 127 and Eq. 128) in order to maximise (worst-case) or minimise (best case) the index cost function, while keeping the same set of optimised MVs from the first step, thus obtaining new CV values, $\boldsymbol{y}'$. Let us define the difference between the uncertain (best and worst-cases) and nominal model's prediction as $\Delta\boldsymbol{y}'_{BM} = \boldsymbol{y}'_{BM} - \boldsymbol{y}$ and $\Delta\boldsymbol{y}'_{WM} = \boldsymbol{y}'_{WM} - \boldsymbol{y}$, which represents the bounded uncertainty of plant response related to process nonlinearity. The third and final step consists of solving the nominal problem again twice, but with two new starting points defined by Eq. 133 and Eq. 134:

$$\boldsymbol{y}_k^{BM} = \boldsymbol{y}_k + \Delta\boldsymbol{y}'_{BM,k+p} \quad \text{Eq. 133}$$

$$\boldsymbol{y}_k^{WM} = \boldsymbol{y}_k + \Delta\boldsymbol{y}'_{WM,k+p} \quad \text{Eq. 134}$$

The starting state provided by Eq. 133 is more favourable than the original, leading to a lower index value, while the state provided by Eq. 134 is less favourable, hence leading to a higher index value. The control engineer may interpret this third step as being the necessary correction in the control actions an MPC would take when perceiving the prediction error between expected and real plant behaviour. The bounds of the EMOP index interval, obtained using the uncertainty definition and the 3-step procedure presented in this Section, are given by Eq. 135 and Eq. 136:

$$I_{EMOP-WM} = \min_{\Delta u_{K3},\, y_k^{WM}} \left\{ \max_{\gamma_{y,u}, \gamma_{y,d}, y} \left[\min_{\Delta u_{K1}, y} \left(|SL_1|^{-1} |SL_2|^{-1} \left[J_{k+p} + J_q + |EP^{-1}| \right] \right) \right] \right\} \quad \text{Eq. 135}$$

$$I_{EMOP-BM} = \min_{\Delta u_{K3},\, y_k^{BM}} \left\{ \min_{\gamma_{y,u}, \gamma_{y,d}, y} \left[\min_{\Delta u_{K1}, y} \left(|SL_1|^{-1} |SL_2|^{-1} \left[J_{k+p} + J_q + |EP^{-1}| \right] \right) \right] \right\} \quad \text{Eq. 136}$$

where $\Delta u_{K1} \in U$, $\Delta u_{K3} \in U$. The difference between the nominal model index value found at step 1, and the value determined using Eq. 135 represents the increased control effort required for the worst model case. The lower control effort required by the best model case, provided by Eq. 136, reduces the index since the new starting point provides additional degrees of freedom for economic optimisation. The nominal model value is expected to be contained within the interval established by the best and worst-cases which are the limiting cases representing the largest possible performance deviation from the nominal model. While the shape of the distribution function is not known for the model parameters, it is likely that the real model is much closer to the nominal model than to the extreme best and worst-case models. By incorporating model uncertainty, the analysis now provides for each flowsheet an index interval instead of a single value, bounding the expected MPC and optimisation performances of each plant.

Since model uncertainty and nonlinearity are closely related, Eq. 135 provides the worst-case scenario which predicts the maximum damage that model nonlinearity effects can cause to the process. This worst-case may be either due to a poor plant response or the impossibility of meeting specifications. Similarly, Eq. 136 provides the maximum eventual benefits that could be brought by nonlinearity effects. The nonlinearity of plant behaviour is bounded if this extended method is used. we can use the difference between best-case and worst-case scenarios, $|I_{EMOP-WM} - I_{EMOP-BM}|$, to define the EMOP index confidence interval, as shown in Eq. 137.

$$I_{EMOP-UN} \in \left[I_{EMOP} + \left| \frac{I_{EMOP-WM} - I_{EMOP-BM}}{2} \right|, I_{EMOP} + \left| \frac{I_{EMOP-BM} - I_{EMOP-WM}}{2} \right| \right] \quad \text{Eq. 137}$$

It is important to notice that if the problem has nonlinear parameters V and W, which is the case for the crude oil distillation case study, the nominal model $\mathrm{I_{EMOP}}$ is not directly comparable to $\mathrm{I_{EMOP-WM}}$ and $\mathrm{I_{EMOP-BM}}$. In this situation, no proof can be provided that Eq. 137 is valid. Nevertheless, unless the variation in V and W between steps 1 and 3 is very large, the interval defined by Eq. 137 will be a reasonable estimation of the confidence interval.

Due to the use of zone control MPC, an additional term should be added in some cases to Eq. 136. When one or more CV is saturated or close to saturation at the end of the first step (nominal model evaluation), i.e., close or equal to their upper or lower boundaries at time p, the best scenario evaluation should open some slack between said CVs and their active control bound. This has the effect of penalising (increasing) the index during step 1, but open opportunities to further lower the index during step 3, enabling the solver to find the best solution. Let $\boldsymbol{y_{sat}^{+}}$ be the vector of CVs saturated at $\boldsymbol{y_{max}}$ and $\boldsymbol{y_{sat}^{-}}$ be the vector of CVs saturated at $\boldsymbol{y_{min}}$, where $y_{sat,i}^{+} = 0$ if y_i is not saturated at the upper bound and $y_{sat,i}^{-} = 0$ if y_i is not saturated at the lower bound. Let us add a term J_{sat} to Eq. 136 to drive these CVs away from saturation. This term is defined in Eq. 138 and is only active during step 2.

$$J_{sat} = \boldsymbol{W_{sat,i}^{-}}\, \boldsymbol{y_{sat}^{-}} + \boldsymbol{W_{sat,j}^{+}}\, \boldsymbol{y_{sat}^{+}} \qquad \text{Eq. 138}$$

Where $\boldsymbol{W_{sat,i}^{-}}$ and $\boldsymbol{W_{sat,j}^{+}}$ are negative and positive real numbers which act as optimisation weights. The control engineer should not set these weights at values large enough to unbind all saturated CVs, driving them further inside the desirable control zone. Adding Eq. 138 to the cost function of in Eq. 77, yields Eq. 139:

$$I_{EMOP-BM} = \min_{\Delta \boldsymbol{u_{K3}},\, \boldsymbol{y_k^{BM}}} \left\{ \min_{\gamma_{y,u}, \gamma_{y,d}, \boldsymbol{y}} \left[\min_{\Delta \boldsymbol{u_{K1}}, \boldsymbol{y}} \left(|\boldsymbol{SL_1}|^{-1} |\boldsymbol{SL_2}|^{-1} \left[J_{k+p} + J_q + |\boldsymbol{EP}^{-1}| + J_{sat} \right] \right) \right] \right\} \qquad \text{Eq. 139}$$

Fig. 21 illustrates the need for the inclusion of an anti-saturation parameter in Step 2 of the EMOP index evaluation for uncertain models. As can be seen, the extra term corrects the lack of penalisation for CV movement inside the control zone, creating slack for additional optimisation during Step 3.

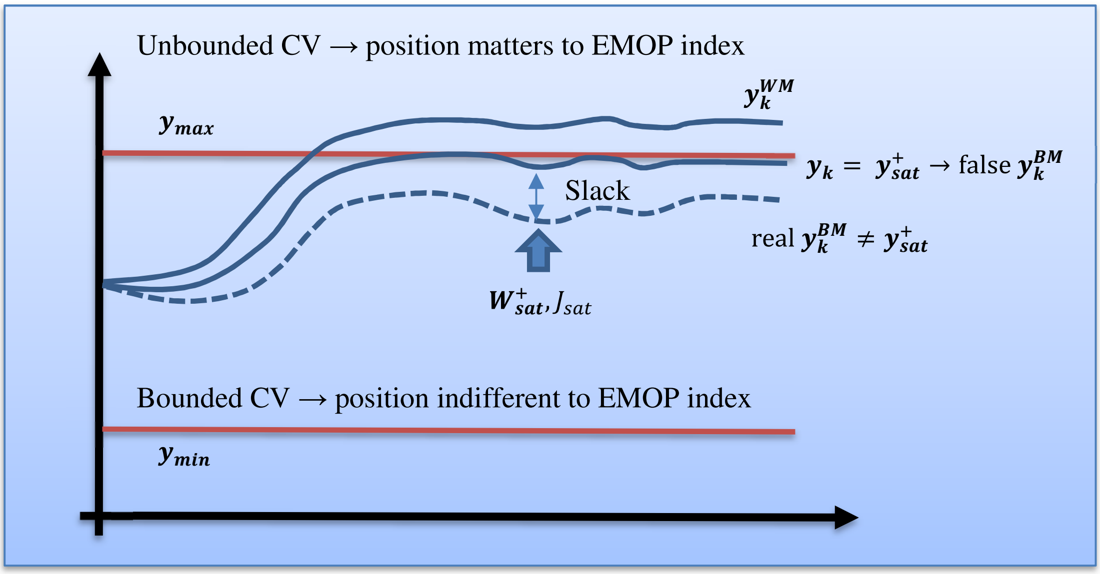

Fig. 21 – The inclusion of an anti-saturation parameter in Step 2 of the EMOP index evaluation for uncertain models.

5.1.1 An alternative method to modelling uncertainty – the model identification error-based approach

As an alternative to the model mismatch-based uncertainty approach, one could consider instead that uncertainty arises from the inherent modelling error resulting from the state-space model identification. As discussed in the appendix, the state-space sub-models used in this Thesis are derived from transfer functions, which in turn are identified from reference data from a commercial, nonlinear simulation package. Thus, one may consider that the state-space model of each CV/MV pair inherited the same level of uncertainty of the transfer function from which it was obtained.

Let us assume that, for each input/output pair, the test used to identify the linear model consists of a series of $n = 1, \ldots, N_s$ steps. After the nominal sub-model is identified, we may validate it by comparing its response to the identification data, thereby obtaining the sub-model prediction error related to each one of the steps performed. The error value can be positive or negative, and so can step response amplitude. The relative error, φ_n^{ϑ}, which relates to the nth step the absolute value of prediction error of sub-model ϑ to the response amplitude of plant data, can be obtained using Eq. 140:

$$\varphi_n^{\vartheta} = \frac{(\Delta y_{s_n} - \Delta y_{s_n}^{\vartheta})}{\Delta y_{s_n}} = \frac{e_{s_n}^{\vartheta}}{\Delta y_{s_n}} \qquad \text{Eq. 140}$$

where Δy_{s_n} is the steady-state response amplitude of plant data related to the nth step; $\Delta y_{s_n}^{\vartheta}$ is the steady-state response amplitude of the nominal sub-model ϑ related to the n^th^ step; and $e_{s_n}^{\vartheta}$ is the steady-state absolute value of the model error, also related to the n^th^ step and sub-model ϑ. Figure 7 illustrates this concept for a model identification test consisting of $N_s = 3$ steps:

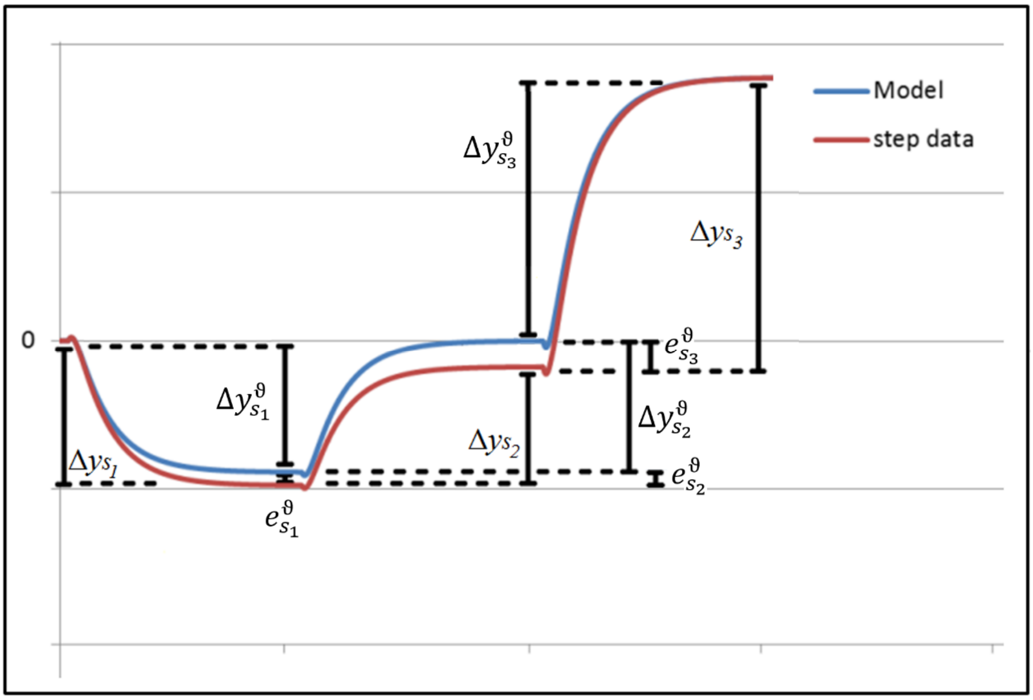

Fig. 22 – Representation of model error for a step test.

It is important to consider that if the output/input pair is weakly related, the denominator in Eq. 140 is small and φ_n^{ϑ} might become too large (>1). In this case, the control engineer should ignore uncertainty for this y/u pair and set $\varphi_n^{\vartheta} = 0$ for $n = 1, \dots, N_s$. It is important to notice that models can be biased to a particular direction, e.g., providing always smaller changes in output prediction than plant data, consistently resulting in a negative model error. Or, alternatively, prediction error may be randomly distributed for different steps. For this uncertainty definition, Eq. 141 and Eq. 142 define β_{y_i,u_j}^{-} and β_{y_i,u_j}^{+}, respectively the minimum negative relative error and the maximum positive relative error between y_I and u_j:

$$\beta_{y_i,u_j}^{-} = \min_{\substack{\vartheta=1,\dots,n_m \\ n=1,\dots,N_s}} \mu_n^{\vartheta}\, \varphi_n^{\vartheta,y_i,u_i}, \qquad i = 1, \dots, n_y, \; j = 1, \dots, n_u \qquad \text{Eq. 141}$$

$$\beta_{y_i,u_j}^{+} = \max_{\substack{\vartheta=1,\dots,n_m \\ n=1,\dots,N_s}} \psi_n^{\vartheta}\, \varphi_n^{\vartheta,y_i,u_i}, \qquad i = 1, \dots, n_y, \; j = 1, \dots, n_u \qquad \text{Eq. 142}$$

where $\mu_{\vartheta} = 1$ and $\psi_{\vartheta} = 0$ if $\varphi_{y_I,u_i}^{\vartheta} < 0$, $\mu_{\vartheta} = 0$ and $\psi_{\vartheta} = 1$ if $\varphi_{y_I,u_i}^{\vartheta} > 0$. The same procedure may be adopted to yield disturbance related uncertainty:

$$\beta^{-}_{y_i,d_j} = \min_{\substack{\vartheta=1,\dots,n_m \\ n=1,\dots,N_S}} \mu^{\vartheta}_{n}\, \varphi^{\vartheta,y_i,u_i}_{n}, \qquad i = 1,\dots,n_y,\ \ j = 1,\dots,n_d \qquad \text{Eq. 143}$$

$$\beta^{+}_{y_i,d_j} = \max_{\substack{\vartheta=1,\dots,n_m \\ n=1,\dots,N_S}} \psi^{\vartheta}_{n}\, \varphi^{\vartheta,y_i,u_i}_{n}, \qquad i = 1,\dots,n_y,\ \ j = 1,\dots,n_d \qquad \text{Eq. 144}$$

From this point, the procedure is similar as in the earlier Section. This approach was used in Strutzel and Bogle (2016), but it was later replaced with the model mismatch approach since the later was proved to capture the nonlinearity effects more efficiently. Matrices $\boldsymbol{\beta}^{-}_{\boldsymbol{y},\boldsymbol{u}}$, $\boldsymbol{\beta}^{+}_{\boldsymbol{y},\boldsymbol{u}}$, $\boldsymbol{\beta}^{-}_{\boldsymbol{y},\boldsymbol{d}}$ and $\boldsymbol{\beta}^{+}_{\boldsymbol{y},\boldsymbol{d}}$ seemed to be undersized by the model identification error approach presented in this Section.

This happens because nonlinearity can be better observed in the change of the operating point from which model identification is performed, rather than in the steps tests themselves. The step inputs cannot be large enough to reveal the whole model uncertainty under penalty of destabilising the control system and even the dynamic simulation itself.

6 Case of Study - EMOP Index for an Oil Distillation Process Unit

To demonstrate how the EMOP index may be applied to provide solutions to industrial scale problems, three possible designs for a crude oil distillation plant shall be presented. Also, a common set of controlled and manipulated variables for the MPC control problem will be defined. As the index is related to the best possible solution for this control problem, its value will indicate which plant can be better controlled by a well-tuned MPC controller. The index will be evaluated for two different scenarios and four distinct designs.

6.1 Describing the Control Problem

The models of the plants presented here were obtained through dynamic simulation using Honeywell's UniSim® software. They have a very similar design, but present key differences. These differences represent significant design decisions that the project engineers have to make through the process of specifying the layout and dimensions of a chemical plant. The distillation plants are rather simple and have a typical configuration. The problem has 36 components, eight local PI controllers, 21 subsystems and the column has 29 trays. This is considerably larger than the examples reported above. The base case, or plant 1, can be seen Fig. 23.

The process simulated has a realistically drafted layout for a medium-sized crude oil distillation process unit. The distillation column generates five different product streams (Naphtha, Kerosene, Light Diesel, Heavy Diesel and Residue). The Kerosene, Light Diesel, Heavy Diesel and Residue product streams are used to preheat the crude oil feed from 25 °C to about 220 °C in two series of heat exchangers, yielding high energetic efficiency. After the first series, the oil reaches an adequate temperature to enter the desalter drum where salt is removed from the oil. After passing through the second series of heat exchangers, the pre-heated oil enters a fired heater where its temperature is increased to 320-380 °C. The hot crude is then fed to the distillation column where the product streams are obtained. The cold light diesel and cold heavy diesel streams are mixed to generate the “Pool Diesel” stream, whose properties will be used to evaluate the cost index.

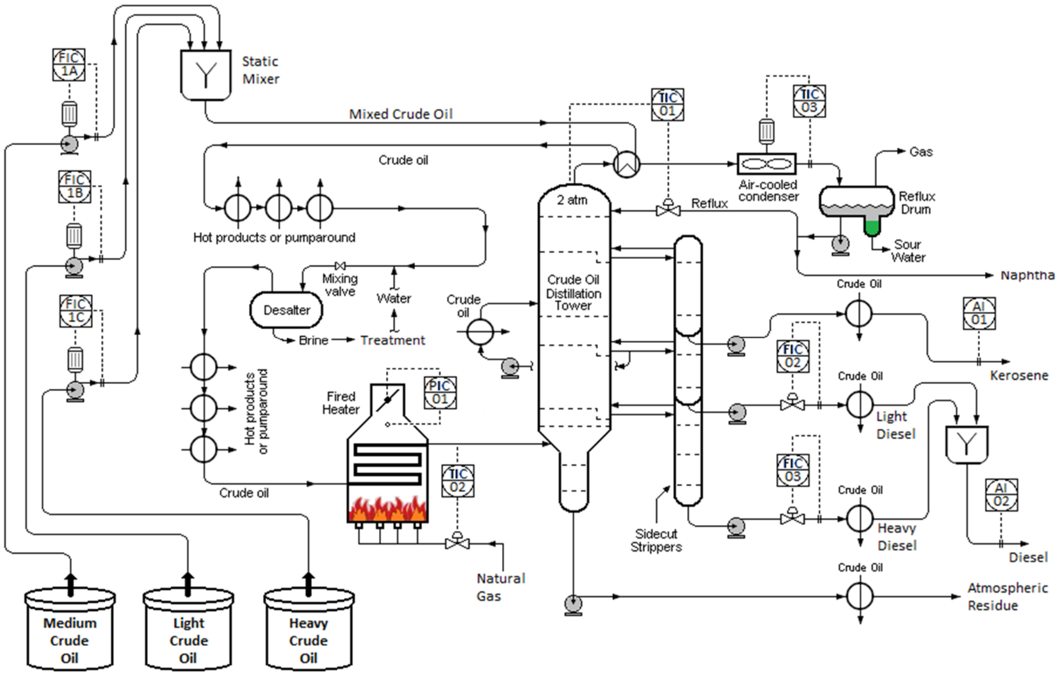

Fig. 23 – Plant 1 – Simplified Process Flowsheet.

There are three different types of crude oil available for processing in the three distillations units: medium (29.0 °API), light (32.3 °API) and heavy (26.2 °API) crude oils. The light oil provides better yields of the more valuable naphtha, diesel and kerosene and lower yield of the less desired atmospheric residue. However, it is the most expensive. The heavy oil provides poor yields of the lighter, more valuable products but on the other hand, it is considerably cheaper. Table 3 provides costs for the crude oils and the prices for the products, which will be used in the simulation to calculate the profitability of the process. Diesel has different prices depending on its Cetane Index (or CI) since a premium is charged for high-performance fuel. Higher CI fuels provided benefits such as quicker starting, quieter operation, and improved fuel efficiency, among others.

Table 3 – Crude Oil Costs and Product Prices.

Stream	US$/m^3	Price
Medium Crude Oil	US$/m^3	430.06
Heavy Crude Oil	US$/m^3	398.21
Light Crude Oil	US$/m^3	471.92
Naphtha	US$/m^3	484.84
Kerosene	US$/m^3	557.83
Diesel - CI 46	US$/m^3	551.21
Diesel - CI 48	US$/m^3	564.48
Diesel - CI 50	US$/m^3	583.88
Residue	US$/m^3	446.66

The optimisation problem for Plant 1, which is the same for Plants 2, 3 and 4, consists of maximising the share of Heavy Crude Oil in the feed while minimising the share of Light Crude Oil, bringing costs down, and at the same time increasing the yield of higher priced Diesel and Kerosene in the products. It is necessary to guarantee that the properties of kerosene and diesel are within specifications to ensure profitability. Hence, these product specifications act as restrictions for profit maximisation. Table 4 shows a list of the controlled variables whose limits must be enforced by an MPC controller and must be considered while evaluating the EMOP index. The values below are true specifications for the fuels marketed in the European Union.

Table 4 – Description and limits for the controlled variables.

	Controlled Variables Description	Unit	Maximum	Minimum
y_1	Cetane Index DIESEL		-	44
y_2	Flash Point DIESEL	C	-	55
y_3	ASTM D86 DIESEL 65%	C	-	250
y_4	ASTM D86 DIESEL 85%	C	350	-
y_5	ASTM D86 DIESEL 95%	C	370	-
y_6	Freezing Point DIESEL	C	-15	-
y_7	Density (15 C) DIESEL	kg/m^3	860	820
y_8	ASTM D86 KEROSENE 100%	C	300	-
y_9	Flash Point KEROSENE	C	-	38
y_{10}	Density (15 C) KEROSENE	kg/m^3	840	775
y_{11}	Freezing Point KEROSENE	C	-47	-

The manipulated variables available to the MPC controller and their limits can be found in Table 5. The plant has some PID feedback controllers, and the plant state-space model is a closed loop model. In a classic two-layer control framework, the MPC manipulated variables are the PID controllers' set points.

Table 5 – Description and limits for the manipulated variables. *Total diesel production (sum of u_3 and u_4) must be at least 110 m^3/h. **The sum of u_5, u_6 and u_7 must be equal to 800 m^3/h, keeping the total feed flow constant.

	Manipulated Variables Description	Unit	Maximum	Minimum
u_1	Temperature 01 tray TIC01.SP	C	70	40
u_2	Temperature Fired Heater TIC02(B).SP	C	380	320
u_3	Light Diesel Output FC02.SP	m^3/h	270 (*)	0
u_4	Heavy Diesel Output FC03.SP	m^3/h	65 (*)	0
u_5	Medium Crude Flow Rate FC01A.SP	m^3/h	800 (**)	0
u_6	Light Crude Flow Rate FC01B.SP	m^3/h	800 (**)	0
u_7	Heavy Crude Flow Rate FC01C.SP	m^3/h	800 (**)	0

A recycle of slop will be used as a measured DV. Processes such as atmospheric or vacuum distillation produce several main cuts as well as slop cuts. Slop oil is the collective term for mixtures of heavy fractions of oil, chemicals and water derived from a wide variety of sources in refineries or oil fields, often forming emulsions. For example, in a vacuum distillation unit, the slop oil and water are separated by gravity in the vacuum drum. It is also formed when tank waggons and oil tanks are cleaned and during maintenance work or in unforeseen oil accidents. Slop oil formation can be reduced but cannot be avoided, and the need to dispose of it results in one of the largest challenges in the everyday operation of an oil refinery.

The slop cuts produced during the operation of oil refining are conventionally stored in large oil lagoons or tanks to receive chemical treatment so as to enable them to be recycled to process units such as fluid catalytic cracking or, very often, atmospheric distillation units. Therefore, slop oil must be incorporated into the process feed from time to time. In the distillation unit simulated in this work, it is possible to treat the recycle of slop oil as a DV and measure the impact of changes in its flow rate in the controlled variables. The same set of variables was defined for all three plants, and a state space model for every pair of input and output has been identified through step tests carried out by dynamic simulation. These models are shown in the appendix.

Plant 2 is essentially the same process as plant 1, but has two product tanks which collect respectively the kerosene and pool diesel output streams. The kerosene tank has a volume of 616 m^3 and the diesel tank, 1692 m^3. This implies a residence time of 10 hours for both the kerosene and diesel streams if flow rates remain at their steady-state values. In plant 2, instead of being concerned about the properties of distillation column side streams of diesel and kerosene as in plant 1, it is desired to control the properties of the diesel and kerosene streams exiting the product tanks. Plant 2 is presented in Fig. 24. The virtual analysers AI01 and AI02 are placed in new positions, i.e., after the diesel and kerosene product tanks instead of after the column.

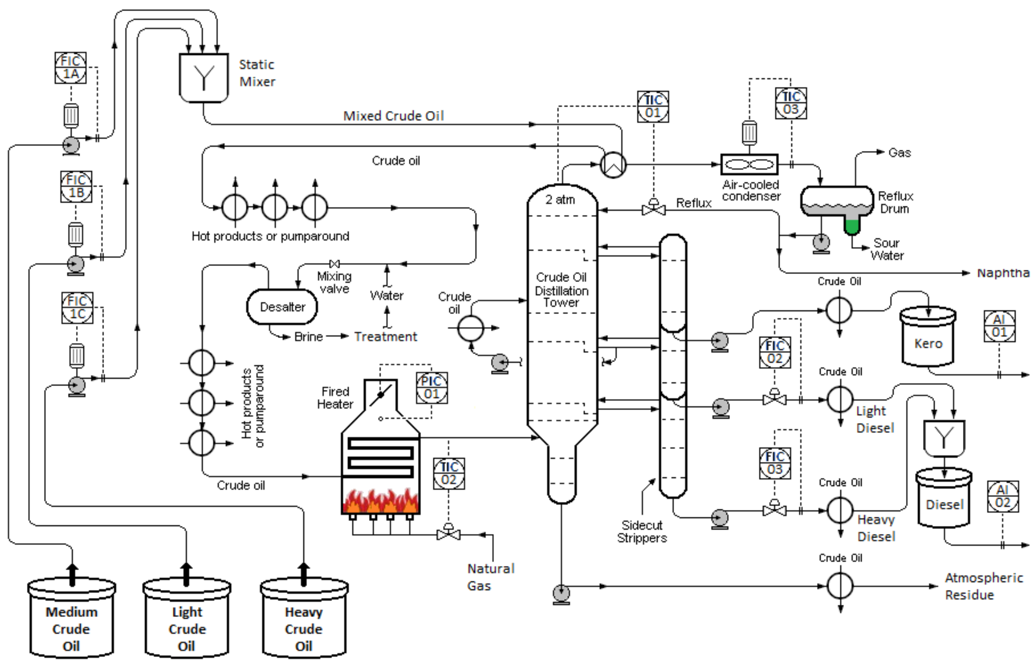

Fig. 24 – Plant 2 – Simplified Process Flowsheet – Plant with Product Tanks.

Plant 3 and 4 are also very similar to plant 1, but with distillations columns of remarkably different dimensions. Plant 3's column is of increased size compared to plants 1 and 2's, while Plant 4's has a smaller column accompanied by a pre-flash drum that removes the lighter fractions such as C1-C4 gases and light naphtha and an extra fired heater. This extra fired heater ensures the feed has an appropriate relation between gas and liquid phases. Plant 3's column may be considered to be slightly oversized for the nominal feed flow rate of 800 m^3/h, and the interesting point here is the slower dynamic response provided by a larger column. Plant 4's pre-flash drum has a volume of 12.56 m^3. The differences in the sizing parameters of the columns in plant 1, 2, 3 and 4 (column height and number of trays are the same) are shown in Table 6:

Table 6 – Column Sizing Parameters for Plants 1, 2, 3 and 4.

	Plant 1 and 2	**Plant 3**	**Plant 4**
Column Diameter (m)	13.7	15	11.62
Kerosene Stripper Diameter (m)	1.2	1.5	1.2
Light Diesel Stripper Diameter (m)	3.0	3.75	3.0
Heavy Diesel Stripper Diameter (m)	1.5	1.75	1.5
Tray Space (m)	0.60	0.70	0.51
Tray Volume (m^3)	88.45	123.7	52.1
Weir Height (mm)	50	65	42.40
Weir Length (m)	10.0	14.0	7.9
Downcomer Volume (m^3)	0.08836	0.1	0.08836
Internal Type	Sieve	Bubble Cap	Sieve

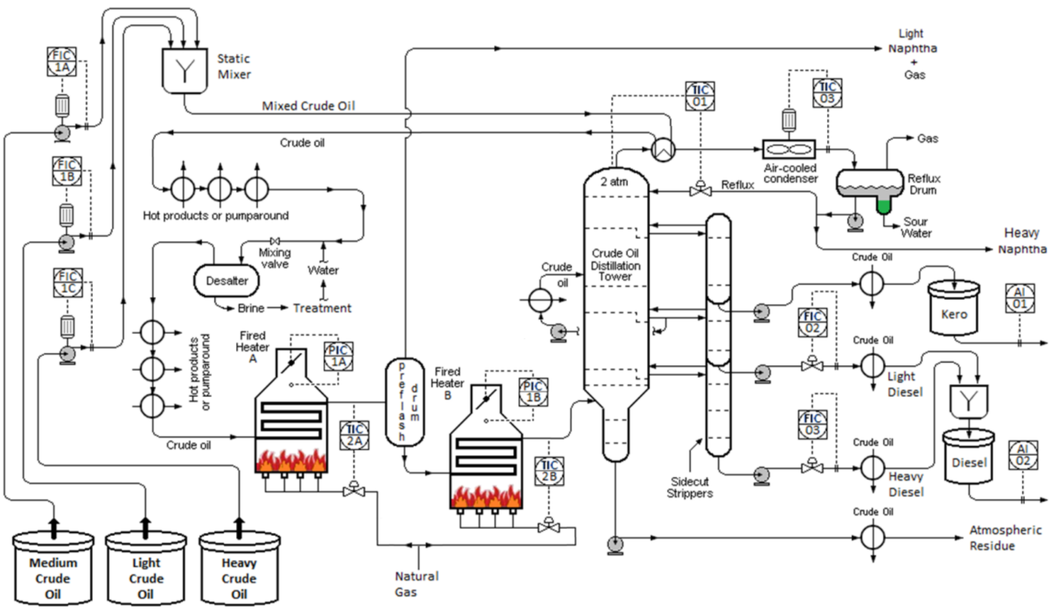

Fig. 25 – Plant 4 – Simplified Process Flowsheet – Plant with Pre-flash Drum.

6.2 Measuring the Economic Impact of the Control Effort

In the case of an oil distillation unit, it is necessary to burn more natural gas in the fired heater to increase the temperature of the feed stream to the column. Therefore, since the feed temperature is a manipulated variable increasing it has a negative impact on process profitability, which depends on the price of natural gas. Thus, u_2 is negatively correlated to profitability.

The MPC controller also should be able to maintain the diesel output at the maximum value that guarantees a specified product, without reducing output unnecessarily. The commercial value of the residue stream is much lower than that of the diesel, thus transferring hydrocarbons from the diesel to residue decreases revenue. Therefore, u_4 is positively correlated to profitability.

Finally, the composition of the feed is also defined by the MPC controller. It can manipulate the flow rate of Light, Medium and Heavy oil crudes to keep specifications within constraints, decreasing the volume of heavy oil and increasing light oil when necessary. Once more, there is a trade-off between profitability and product specifications, because lighter crudes usually generate better products but also cost more. The medium crude will make the bulk of the feed and has properties in between those of heavy and light oil.

6.3 Defining parameters for the measurement of a monetised control cost

As discussed in Sections 3.2 and 3.3, each of the process inputs and outputs requires a weighting parameter to which the analysis is highly sensitive. In this Section, adequate values for these shall be defined using market prices for the crude oil feed, product streams, energy costs and data from the simulation.

u_1 **– Temperature 01 tray (TIC01.SP)**

Concerning the distillation column top temperature control, there is an energetic cost to decreasing it due to the fact that more cooling water will be spent to increase the reflux flow rate. Calculating the cost of generating cooling water is a complex task, but it can all nevertheless be assumed that this cost is insignificant compared the other costs involved and therefore will be considered equal to zero. Thus, it is assumed that $V_{1,min} = V_{1,max} = 0$.

u_2 **– Temperature Fired Heater (TIC02.SP / TIC02B.SP)**

It is possible to establish a relation between the crude oil temperature at the fired heater outlet and the heat duty for plant 1,2 and 3. At a fixed feed flow of 800 m^3/h, the simulation provides the values found in Table 7:

Table 7 – Energy consumption by the fired heater.

T (C)	Q (Mcal/h)
330.07	61,066
334.11	62,406
335.13	62,676
336.10	62,966
340.00	64,332

where T is the fired heater outlet temperature, and Q is the heat duty. From the values above, we obtain the increase in energy consumption related to the increase in outlet temperature, as shown in Table 8:

Table 8 – Increase in energy consumption by the fired heater.

ΔT (C)	ΔQ (Mcal/h)	ΔQ/ΔT (Mcal/(C.h))
4.04	1,339.8	331.4
1.02	270.8	265.5
0.97	289.8	299.2
3.90	1,365.5	349.9

Clearly, the energy cost is dependent on the starting temperature, and the relationship is nonlinear. It also depends on the feed flow to the column. However, for control purposes, the values are close enough, and an average value can be used without any compromises to control performance. The average of ΔQ/ΔT = 311.5 Mcal/(C.h) shall be considered at any feed flow rate. Considering the Lower Heating Value (LHV) of natural gas equal to 8.747 Mcal/m^3 and a natural gas price of 0.33 US$/$m^3$, the cost related to u_2 can be calculated as follows:

$$V_{2,min} = \frac{\Delta Q/\Delta T}{LHV} P_{NG} = \frac{311.5\ Mcal/C.h}{8.747\ Mcal/m^3} 0.33 \frac{US\$}{m^3} = 11.75 \frac{US\$}{C.h} \quad \text{Eq. 145}$$

$$V_{2,max} = 0 \quad \text{Eq. 146}$$

For plant 4 u_2 is defined as the temperature at the fired heater B outlet. Because the light components are separated from the feed at the pre-flash drum, the feed properties and, thus the heat exchange coefficients, are slightly different. However, for the sake of simplicity, this difference will be ignored since its effects are very small.

u_3 / u_4 – Light Diesel Output (FC02.SP) and Heavy Diesel Output (FC03.SP)

In this example, the light and heavy diesel streams are combined to produce the “pool diesel”. The kerosene output and diesel output are placed sequentially in the boiling point curve and therefore transferring hydrocarbons between these streams is part of normal of everyday operations. However, the UniSim® simulation used in this example considers a fixed kerosene output in order for the simulator to be able to solve the fluid flow dynamic equations. However, there is a variable flux between the diesel output and the residue output and thus the optimisation gain is defined as the price difference between these streams:

$$V_{3,max} = V_{4,max} = P_{diesel} - P_{residue} = 551.21 - 470.17 = 81.04 \frac{\text{US\$}}{\text{m}^3} \qquad \text{Eq. 147}$$

$$V_{3,min} = V_{4,min} = 0 \qquad \text{Eq. 148}$$

As discussed at the beginning of this Section, increasing heavy diesel output reduces the residue output and thus increases profitability. In this case study the separation between light and heavy diesel output has no commercial importance, but in most refineries, these hydrocarbon streams have different destinations, such as being fed to different hydrotreating or hydrocracking process units, and therefore they may have different commercial values. These possibilities are not considered here.

u_5 – Medium Crude Oil Feed Flow Rate (FC01A.SP)

Given a set of flow rate values for each of the crude oils that compose the feed to the distillation column, at any given time the average price of the oil processed is given by:

$$P_{ave} = \frac{452.69\ \text{u}_5 + 496.76\ \text{u}_6 + 419.17\ \text{u}_7}{\text{u}_5 + \text{u}_6 + \text{u}_7} \frac{\text{US\$}}{\text{m}^3} \qquad \text{Eq. 149}$$

The difference between average oil price and the medium crude will provide the optimisation coefficient.

$$\Delta P_{medium} = P_{medium} - P_{ave} \qquad \text{Eq. 150}$$

Therefore, the following rule may be defined to obtain $V_{5,min}$ and $V_{5,max}$:

$$\text{if } \Delta P_{medium} > 0 \quad \Rightarrow \quad V_{5,max} = 0, \quad V_{5,min} = \Delta P_{medium} \qquad \text{Eq. 151}$$

$$\text{if } \Delta P_{medium} < 0 \quad \Rightarrow \quad V_{5,max} = -\Delta P_{medium}, \quad V_{5,min} = 0 \qquad \text{Eq. 152}$$

$$\text{if } \Delta P_{medium} = 0 \quad \Rightarrow \quad V_{5,max} = V_{5,min} = 0 \qquad \text{Eq. 153}$$

In the third case, all feed is already entirely composed of medium crude and average oil price doesn't change with changes in u_5.

u_6**– Light Crude Oil Feed Flow Rate (FC01B.SP)**

In a similar manner to the approach used for u_5, the difference between average oil price and the medium crude will provide the optimisation coefficient $V_{6,min}$, and since the light oil is the most expensive, the maximisation coefficient will always be zero.

$$V_{6,min} = \Delta P_{light} = P_{light} - P_{ave} \quad \text{Eq. 154}$$

$$V_{6,max} = 0 \quad \text{Eq. 155}$$

u_7 **– Heavy Crude Oil Feed Flow Rate (FC01C.SP)**

In a similar manner to the approach used for u_5 and u_6, the difference between an average oil price and the medium crude will provide the optimisation coefficient $V_{7,max}$, and since the light oil is the most expensive, the minimisation coefficient will always be zero:

$$V_{7,max} = \Delta P_{heavy} = P_{ave} - P_{heavy} \quad \text{Eq. 156}$$

$$V_{7,min} = 0 \quad \text{Eq. 157}$$

d **– Disturbance slop recycle**

In the operation of oil refineries, slop streams require a complex and expensive previous treatment to be able to be incorporated into the feed and reprocessed. In this case study, however, this cost will be ignored. Such simplification will not change the EMOP index problem because the slop recycle is a disturbance, and hence the MPC is not able to fix anyway. Also, it is important to notice that the slop stream does not have a price tag, meaning that it does not need to be purchased. Concerning the EMOP index, the hydrocarbons recovered from slop are available for “free”. Therefore, they cause the index to decrease in comparison with the first scenario once they replace crude oil in the feed. At the same time, since slop is composed mostly of very heavy, difficult to process oil cuts it diminishes the maximum quantity of cheap heavy oil that can be processed and increases the energy consumption, and in its turn, these effects increase the index. This discussion is to show that the relative values of the index are consequential to plant assessment, not the absolute. The difference between the indexes of each plant is key to evaluate performance.

Controlled Variables

To avoid having controlled variables out of their control zones due to the optimisation efforts, the parameters W_{upper} and W_{lower} must have high enough values that the cost generated by one or more process outputs outside their control zone is much higher than the cost due to optimisation. From an economic perspective, since product specifications are requirements for the product to be saleable, the optimisation weight for each controlled value can be defined as the value of the relevant product stream as provided in Table 3. Since y_1 to y_7 are related to diesel specifications, the cost of violating their restrictions will be equal to the diesel price multiplied by diesel output, which is defined as the sum of u_3 and u_4. In this case study the possibility, of selling diesel and kerosene as fuel oil is not being considered, and such a procedure is also very unlikely in industrial operations.

$$551.21\frac{\text{US\$}}{\text{m}^3}(u_3+u_4)\frac{\text{m}^3}{(10\text{ min})}=551.21(u_3+u_4)\frac{\text{US\$}}{(10\text{ min})} \qquad \text{Eq. 158}$$

Similarly, y_8 to y_{11} are related to kerosene specifications, and their weight in the optimisation problem will be equal to the kerosene price multiplied by kerosene output, which is kept fixed at 61.29 m^3/h, or 10.215 m^3/(10 min).

$$557.83\frac{\text{US\$}}{\text{m}^3}10.215\frac{\text{m}^3}{(10\text{ min})}=5698.23\frac{\text{US\$}}{(10\text{ min})} \qquad \text{Eq. 159}$$

Hence, the set of rules below was adopted for defining the weights of each controlled variable in the cost function:

$$\text{if } y_{i,min} \leq y_{k+p,i} \leq y_{i,max} \quad \Rightarrow \quad W_{i,lower} = W_{i,upper} = 0 \qquad \text{Eq. 160}$$

$$\text{if } y_{i,min} > y_{k+p,i} \quad \Rightarrow \quad W_{i,lower} = 551.21(u_3+u_4), \ \text{i} = 1,2,3,4,5,6,7 \qquad \text{Eq. 161}$$

$$\text{if } y_{k+p,i} > y_{i,max} \quad \Rightarrow \quad W_{i,upper} = 551.21(u_3+u_4), \ \text{i} = 1,2,3,4,5,6,7 \qquad \text{Eq. 162}$$

$$\text{if } y_{i,min} > y_{k+p,i} \quad \Rightarrow \quad W_{i,lower} = 5698.23, \ \text{i} = 8,9,10,11 \qquad \text{Eq. 163}$$

$$\text{if } y_{k+p,i} > y_{i,max} \quad \Rightarrow \quad W_{i,upper} = 5698.23, \ \text{i} = 8,9,10,11 \qquad \text{Eq. 164}$$

6.4 Results and Discussion

In this Section, the results obtained through the application of the EMOP to assessing alternative distillation plant designs are presented.

6.4.1 Applying the SMLP approach for the Crude Oil distillation

The output predictions for the four crude oil distillation plants were no longer provided by single linear state-space models, as has been done in Strutzel and Bogle (2016), but by SMLP systems composed of three sub-models each. Defining the IPs for each plant is the first step to building an SMLP system. Each IP may be defined as an output set for use with SMLP method 1 or, alternatively, as an input set for use with SMLP methods 2 and 3. Table 9 provides the IPs as input sets, which are common for all the four plant designs:

Table 9 – Input-based IPs identification points 1, 2 and 3.

	$u_1(\mathrm{C})$	$u_2(\mathrm{C})$	$u_3\left(\frac{\mathrm{m}^3}{\mathrm{h}}\right)$	$u_4\left(\frac{\mathrm{m}^3}{\mathrm{h}}\right)$	$u_5\left(\frac{\mathrm{m}^3}{\mathrm{h}}\right)$	$u_6\left(\frac{\mathrm{m}^3}{\mathrm{h}}\right)$	$u_7\left(\frac{\mathrm{m}^3}{\mathrm{h}}\right)$	$d_1\left(\frac{\mathrm{m}^3}{\mathrm{h}}\right)$
$\mathbf{IP_1}$	44.4	341.1	139.5	29.7	550.0	230.0	15.0	5.0
$\mathbf{IP_2}$	47.4	342.8	134.0	29.9	470.0	210.0	95.0	25.0
$\mathbf{IP_3}$	50.0	344.5	128.5	40.0	400.0	200.0	155.0	45.0

But since the designs are all different, note that for each flowsheet the same inputs will result in a different output, so the output-based IPs are not equal between plants, even if the input-based IPs are. The resulting IPs as output vectors are presented in Table 10:

Table 10 – Output-based IPs Identification points 1, 2 and 3.

	plant 1			**plant 2**			**plant 3**			**plant 4**		
	IP_1	IP_2	IP_3	IP_1	IP_2	IP_3	IP_1	IP_2	IP_3	IP_1	IP_2	IP_3
y_1	40.98	41.76	42.82	39.67	40.35	40.87	40.97	41.76	42.81	41.16	41.97	41.56
y_2	82.95	87.11	90.40	74.07	77.82	81.14	82.96	87.10	90.41	72.28	80.06	76.17
y_3	290.38	302.33	312.99	290.37	302.33	312.94	290.35	302.46	312.95	297.51	313.86	305.69
y_4	321.67	335.92	350.71	321.65	335.91	350.64	321.61	336.15	350.63	343.17	365.80	354.48
y_5	343.91	360.31	378.59	343.89	360.30	378.55	343.83	360.59	378.49	378.29	412.22	395.26
y_6	-24.83	-20.22	-16.22	-26.83	-22.30	-19.11	-24.84	-20.17	-16.23	-24.40	-18.60	-21.50
y_7	826.35	830.75	835.42	826.35	830.75	835.42	826.36	830.74	835.43	831.84	837.91	834.88
y_8	250.13	257.50	269.52	250.13	257.50	269.52	250.08	257.53	269.47	218.92	237.40	228.16
y_9	49.93	56.69	63.15	49.92	56.69	63.11	49.93	56.74	63.14	9.51	17.68	13.59
y_{10}	812.98	817.71	820.49	812.98	817.71	820.49	812.97	817.73	820.49	791.32	799.70	795.51
y_{11}	-68.45	-62.17	-56.81	-68.46	-62.30	-56.72	-68.46	-62.11	-56.71	-76.82	-74.66	-75.74

Now that IPs are fully defined for all plants, it is necessary to generate reference trajectories for the regression analysis. When generating an SMLP system, it is recommended that one performs a comprehensive regression analysis in order to obtain adequate constants for the degradation functions. As per Eq. 97 and Eq. 109, model error related to the nonlinear dynamic simulator must be minimised by testing several trajectories in order to identify the optimal constants. The three SMLP procedures detailed in Section 4 were tested and the results are presented in this Section. Due to space restrictions, only three trajectories were used to generate the regression analysis problems for each plant. Those trajectories are generated by Eq. 165, Eq. 166 and Eq. 167, through a control horizon of p = 60 (time unit is 10 minutes).

$$\boldsymbol{y}_{1,k}^{nl} = \boldsymbol{IP_1} + [3\ \ -6\ \ 27\ \ -6.5\ \ 80\ \ -80\ \ 26.7\ \ -4]\sin\left(\frac{\pi(k-1)}{20}\right) \quad \text{Eq. 165}$$

$$\boldsymbol{y}_{2,k}^{nl} = \boldsymbol{IP_2} + [-3\ \ 6\ \ -27\ \ 6.5\ \ -80\ \ 80\ \ -80\ \ 4]\sin\left(\frac{\pi(k-1)}{20}\right) \quad \text{Eq. 166}$$

$$\boldsymbol{y}_{3,k}^{nl} = \boldsymbol{IP_3} + [1.5\ \ -3\ \ 13.5\ \ -3.25\ \ 40\ \ -40\ \ 40\ \ -2]\sin\left(\frac{\pi(k-1)}{10}\right) \quad \text{Eq. 167}$$

where $\boldsymbol{IP_1}$, $\boldsymbol{IP_2}$ and $\boldsymbol{IP_3}$ are each sub-model' input-based IPs. Now we are going to present the results from regression analysis problems presented in Section 4 to obtain parameters for the degradation functions. The analysis consists of minimising the sum of the normalised squared prediction error (due to linearisation) for all variables through a prediction horizon of $\mathrm{p} = 60$, for each one of the 3 reference trajectories, as defined by Eq. 97 (methods 1 and 2) and Eq. 109 (method 3). Table 11 contains the optimised objective function values for plants 1, 2, 3 and 4 using the 3 SMLP methods.

At first, the regression analysis was performed using 2nd order degradation functions (keeping $g_{\vartheta,4} = g_{\vartheta,3} = 0$, and allowing $g_{\vartheta,2}$, $g_{\vartheta,1}$ and $g_{\vartheta,0}$ to assume any real value in the interval $0 < \mathrm{g} \leq 10^7$). The goal was to assess which of the three method variants had better performance and if the difference was significant.

Table 11 – Residual from regression analysis for the SMLP problem.

Total normalised squared error	Plant 1	Plant 2	Plant 3	Plant 4
Linear sub-model 1 $(\boldsymbol{IP_1})$	10.35	17.34	22.83	2.39
Linear sub-model 2 $(\boldsymbol{IP_2})$	32.42	8.87	12.05	6.82
Linear sub-model 3 $(\boldsymbol{IP_3})$	18.24	9.63	32.36	9.08
SMLP Method 1 – 2nd order	6.40	5.46	4.96	2.23
SMLP Method 2 – 2nd order	6.79	3.74	5.05	2.28
SMLP Method 3 – 2nd order	2.31	1.68	2.78	1.34
SMLP Method 1 – 4th order	5.89	3.50	4.87	2.21
SMLP Method 2 – 4th order	6.17	3.74	5.05	2.23
SMLP Method 3 – 4th order	2.31	1.59	2.78	1.33

As can be seen in Table 11, which contains the residue for each method variant as defined in Eq. 98, all methods performed better than any of the single linear sub-models, for each of the 4 plants. Additionally, method 3 provided the lowest prediction error for all plants, significantly reducing nonlinearity-induced error.

Another regression analysis was carried out, this time using 4th order degradation functions. The aim was to assess if additional parameters would reduce the error further, as compared to using a 2nd order polynomial. The results are shown in the last three lines of Table 11. The use of the 4th order polynomial resulted in significant accuracy gains for the SMLP Method 1, especially for plants 1 and 2, for which the residual was reduced respectively in 8% and 36%. Regarding the SMLP Method 2, a modest gain obtained for plants 1 and 4, of 9% and 2% residual reduction respectively. Finally, for the SMLP Method 3, a small improvement in accuracy could be gained for plant 2, of 5% residual reduction. We expect very small further error reduction to be available by using yet larger order degradation functions (or by the use of more complex functions). For this case study, a 4th order seems to be the sensible choice, presenting a good compromise between performance and time required for the regression analysis.

It is also evident that the single model prediction error (as compared to the nonlinear dynamic simulator) varies significantly among the different linear approximations. This demonstrates that Eq. 98 is useful to provide insight concerning the low performing linear models among each plant's set. Without comparison to reference trajectories, gaining this insight may be difficult. For instance, it was found out that "fit to model" data is a poor predictor of linearisation error for this case study, since the step test model identification provided a good fit to model data (always over 80%, reaching up to 99.9% in some cases) for the models of all input/output pairs. Thus, the significant prediction error in comparison to the nonlinear dynamic simulator is mostly due to process nonlinearity, not the standard step test identification procedure that provided state-space model matrices A, B, C and D.

6.4.2 Scenario 1 – Simultaneous control and optimisation while handling a measured disturbance

The four different plants were given the same starting point, i.e. the same initial values for the controlled and manipulated variables. Although we recommend several distinct starting points to be tested for thorough analysis, due to lack of space our analysis will be carried out using the single one: the system's origin. Initial values for all variables can be found in Table 12 and Table 13:

Table 12 – Initial values for process outputs.

	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
Case 1	46.00	81.76	288.81	319.42	341.63	-25.47	826.79	251.11	49.79	812.88	-68.38

Table 13 – Initial values for process inputs.

	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
Case 1	42.64	335.13	135.75	25.71	550.00	250.00	0.00	0.00

In this first scenario, all variables are at first within their control zones, but the plants are going to be disturbed by an increase in the slop feed to the process, which is raised from 0 to 40 m^3/h. The rate of increase will be of $24 \frac{(m^3/\mathrm{h})}{\mathrm{h}}$, so the value is achieved in 1h40m.

Optimisation was carried out with the restriction of constant total feed flow and the set of parameters introduced in Section 6.3 for plants 1, 2, 3 and 4, and using a control horizon $m = 10$, a prediction horizon $p = 60$ (10 hours) and $\boldsymbol{T_{SL1}}$ and $\boldsymbol{T_{SL2}}$ equal to unitary vectors multiplied by scalar 5. The weights $\boldsymbol{T_{lower}}$ and $\boldsymbol{T_{upper}}$ are unitary vectors when variables are outside their control zones and null when they are within. The vector $\boldsymbol{T_{EP}}$ was defined as being equal a vector containing the product prices related to each variable: $\boldsymbol{T_{EP}} = [551.21; 551.21; 551.21; 551.21; 551.21; 551.21; 551.21; 557.83; 557.83; 557.83; 557.83]$. Sensor ranges are given by $\boldsymbol{y^*_{max}} = [94; 143; 353; 394; 428; 35; 874; 318; 106; 848; -16]$, $\boldsymbol{y^*_{min}} = [-4; 17; 214; 264; 300; -75; 834; 169; -6; 783; -137]$. Given the starting point for case 1, the MVs Table 14 (only final values are shown) were found to be the best available for each plant, resulting in the CVs values for each plant provided in Table 15.

Table 14 – Inputs for the scenario 1. $*u_3 + u_4 \geq 110$; $**u_5 + u_6 + u_7 + d_1 = 800$.

	Manipulated Variables Description	Unit	Max. Value	Min. Value	Initial Value	Final Value			
						Plant 1	Plant 2	Plant 3	Plant 4
u_1	Temperature 01 tray TIC01.SP	C	70.0	40.0	42.64	42.95	41.57	43.76	45.71
u_2	Temperature Fired Heater TIC02.SP	C	380.0	320.0	335.13	334.09	330.61	339.96	336.01
u_3	Light Diesel Output FC02.SP	m3/h	270.0	0.0 *	135.75	135.78	130.54	130.72	126.24
u_4	Heavy Diesel Output FC03.SP	m3/h	65.0	0.0 *	25.71	26.65	21.44	29.33	30.81
u_5	Medium Crude Flow Rate FC01A.SP	m3/h	800 **	0.0	550.00	584.98	539.61	461.83	601.27
u_6	Light Crude Flow Rate FC01B.SP	m3/h	800 **	0.0	250.00	143.82	219.48	175.43	80.52
u_7	Heavy Crude Flow Rate FC01C.SP	m3/h	800 **	0.0	0.00	31.20	0.90	122.74	78.21
d_1	Slop Oil Feed Recycling Flow Rate	m3/h	0.00	0.00	0.00	40.00	40.00	40.00	40.00

Table 15 – Output predictions for the scenario 1.

	Controlled Variables Description	Unit	Min. Value	Max. Value	Initial Value	Final Value			
						Plant 1	Plant 2	Plant 3	Plant 4
y_1	Cetane Index DIESEL		46.0	-	46.00	**50.00**	46.55	50.67	49.86
y_2	Flash Point DIESEL	C	55.0	-	81.76	88.35	84.96	93.42	109.22
y_3	ASTM D86 DIESEL 65%	C	250.0	-	288.81	313.34	303.50	326.36	313.24
y_4	ASTM D86 DIESEL 85%	C	-	350.0	319.42	343.63	334.11	340.11	342.98
y_5	ASTM D86 DIESEL 95%	C	-	370.0	341.63	368.99	**369.97**	**370.00**	363.07
y_6	Freezing Point DIESEL	C	-	-15.0	-25.47	-25.60	-21.94	**-15.01**	**-15.00**
y_7	Density (15 C) DIESEL	kg/m³	820.0	860.0	826.79	828.54	829.96	832.99	834.33
y_8	ASTM D86 KEROSENE 100%	C	-	300.0	251.11	255.23	265.27	269.45	279.38
y_9	Flash Point KEROSENE	C	38.0	-	49.79	55.44	64.27	71.79	54.96
y_{10}	Density (15 C) KEROSENE	kg/m³	775.0	840.0	812.88	817.47	815.31	824.07	826.24
y_{11}	Freezing Point KEROSENE	C	-	-47.0	-68.38	-69.12	-64.88	-49.78	-66.95

For these sets of process inputs, the optimised cost function values for each plant are provided in Table 16.

Table 16 – Cost function values for each plant – scenario 1.

	Plant 1	Plant 2	Plant 3	Plant 4
EMOP Index Scenario 1	11819	24465	10758	12139

All plants managed to keep CVs within control bounds, so the difference in index values is explained by two factors: the mix of crude oil fed into the plant, and also by the quality (Cetane index) of diesel oil produced. Increasing the percentage of light oil makes it easier to meet quality requirements but increase costs, while adding more heavy oil has the contrary effect: it decreases the quality while cutting raw materials cost. Lower values for EMOP index are better and thus plant 3 presented the best performance through the use a large amount of heavy crude. It was closely followed by plant 1, which processed less heavy crude but also less light crude. Both plants 1 and 3 produced premium priced diesel with 50 Cetane Index (CI), which improved the EMOP index as compared to plants 2 and 4. Plant 4's performance was not as good but still acceptable. It used little light oil and a significant amount of heavy oil, but its diesel output did not meet the 50 CI bound, thus it would need to be marketed instead as 48 CI. The CV y_5 reached the upper bound of its control zone for plants 2 and 3, preventing further optimisation, whereas y_6 limited gains for plants 3 and 4. Plant 1 was limited by the control effort needed to attain the upper-quality threshold of $y_1 > 50$.

Plant 2 had the worst results, producing low priced 46 CI diesel, processing almost no heavy oil while requiring a large volume of pricey light oil. Since plants 1 and 2 are identical except for the location of the analysers, we can presume the reason for plants 2's higher index is poor controllability, which led to conservative control actions. The diesel and kerosene product tanks modified the dynamics of the product quality variables, rendering them slow to respond to MV changes. When assessing designs for which huge time delays such as this cannot be avoided, the EMOP parameters should be tuned to de-emphasise the transient response, focusing instead on achieving an optimal steady state.

6.4.3 Scenario 2 – Price changes

In this second scenario, changes were made to the crude oil and diesel prices and the process optimisation is resumed at the same state where each plant was at the end of scenario 1, meaning that each plant has a different starting point. How favourable this new starting point depends on the flowsheets themselves and their performance in the first scenario. The sale prices of all diesel oil variants were increased in 5%, and the costs of Medium and Light crudes were increased in 4% and 2% respectively, whereas the cheap heavy Crude was lowered in 11%. The new values are provided in Table 17:

Table 17 – New Crude Oil Costs and Product Prices.

Stream	US$/m³	Price
Medium Crude Oil	US$/m³	447.26
Heavy Crude Oil	US$/m³	354.41
Light Crude Oil	US$/m³	481.36
Diesel - CI 46	US$/m³	578.77
Diesel - CI 48	US$/m³	592.70
Diesel - CI 50	US$/m³	613.07

The vector T_{EP} is thus updated to the new Diesel - CI 46 price: $\boldsymbol{T_{EP}} = [578.77; 578.77; 578.77; 578.77; 578.77; 578.77; 578.77; 557.83; 557.83; 557.83; 557.83]$. Also, the process is to be disturbed by a decrease in the slop feed to the process from 40 to 20 $\mathrm{m^3/h}$, and the decrease rate will be of $12\frac{(\mathrm{m^3/h})}{\mathrm{h}}$. Once again, the final flow is achieved in 1h40m. Optimisation was carried out with keeping all remaining parameters constant, providing the MVs in Table 18 and the CVs in Table 19:

Table 18 – Inputs for the scenario 1. *$u_3 + u_4 \geq 110$; **$u_5 + u_6 + u_7 + d_1 = 800$; *** Initial MVs are the final values in Table 14.

	Manipulated Variables Description	Unit	Max. Value	Min. Value	Initial Value	Final Value			
						Plant 1	Plant 2	Plant 3	Plant 4
u_1	Temperature 01 tray TIC01.SP	C	70.0	40.0	***	43.05	57.91	44.73	44.97
u_2	Temperature Fired Heater TIC02.SP	C	380.0	320.0	***	335.26	338.17	339.48	334.79
u_3	Light Diesel Output FC02.SP	m³/h	270.0	0.0 *	***	138.77	98.01	128.42	128.68
u_4	Heavy Diesel Output FC03.SP	m³/h	65.0	0.0 *	***	27.42	12.80	28.63	31.84
u_5	Medium Crude Flow Rate FC01A.SP	m³/h	800 **	0.0	***	584.95	499.15	465.77	636.30
u_6	Light Crude Flow Rate FC01B.SP	m³/h	800 **	0.0	***	143.67	189.28	166.19	58.96
u_7	Heavy Crude Flow Rate FC01C.SP	m³/h	800 **	0.0	***	51.39	91.57	148.04	84.75
d_1	Slop Oil Feed Recycling Flow Rate	m³/h	0.00	0.00	***	20.00	20.00	20.00	20.00

Table 19 – Output predictions for the scenario 2. * Initial CVs are the final values in Table 15.

	Controlled Variables Description	Unit	Min. Value	Max. Value	Initial Value	Final Value			
						Plant 1	Plant 2	Plant 3	Plant 4
y_1	Cetane Index DIESEL		46.0	-	*	**49.99**	48.00	**50.00**	**50.00**
y_2	Flash Point DIESEL	C	55.0	-	*	89.02	83.00	94.16	108.73
y_3	ASTM D86 DIESEL 65%	C	250.0	-	*	314.32	296.18	326.41	312.50
y_4	ASTM D86 DIESEL 85%	C	-	350.0	*	345.06	320.50	341.60	340.62
y_5	ASTM D86 DIESEL 95%	C	-	370.0	*	**369.95**	**369.99**	369.78	357.36
y_6	Freezing Point DIESEL	C	-	-15.0	*	-25.12	-27.46	**-15.02**	**-15.08**
y_7	Density (15 C) DIESEL	kg/m³	820.0	860.0	*	827.80	843.46	834.04	833.71
y_8	ASTM D86 KEROSENE 100%	C	-	300.0	*	254.30	285.92	272.60	278.56
y_9	Flash Point KEROSENE	C	38.0	-	*	54.95	64.65	72.71	52.59
y_{10}	Density (15 C) KEROSENE	kg/m³	775.0	840.0	*	817.73	824.22	825.46	825.10
y_{11}	Freezing Point KEROSENE	C	-	-47.0	*	-69.23	-57.33	-47.50	-67.34

For these sets of process inputs, the optimal cost function values for each plant are provided in Table 20.

Table 20 – Cost function values for each plant – scenario 1.

	Plant 1	Plant 2	Plant 3	Plant 4
EMOP Index Scenario 2	16958	23885	15392	15675

Again, all CVs were kept within control zones, and plant 3 presented the best performance through the use a large amount of heavy crude and production of 50 CI diesel. Plant 4 also managed to produce 50 CI diesel this time, and used little of the expensive light oil, significantly improved its performance and thus coming in second. Plant 1 reached the 50 CI threshold and but processed very little heavy oil. Plant 2 once more was the last in performance and expected profitability, again providing a 46 CI diesel output. It is important to notice that y_1 acted as an active restriction for plants 1, 2 and 4, while y_5 acted as an active restriction for plants 1 and 2, and y_6 acted as a restriction for plants 3 and 4.

6.4.4 Selecting the Best Plant

The results obtained in both scenarios must be considered together in order to draw a consistent conclusion about which plant has the better characteristics when it comes to MPC zone control. The average index used in this Section was defined as the geometric mean of each plant's indices multiplied by the weight for each scenario, which was defined as 1 for both of them.

Table 21 – Average index for each plant.

	Plant 1	Plant 2	Plant 3	Plant 4	Case Weight
Case 1	11819	24465	10758	12139	1
Case 2	16958	23885	15392	15675	1
Geometric Mean	14157	24173	12868	13794	

As can be seen in Table 21, plant 3 had the best overall results in this application of the EMOP index and should provide better MPC controllability than the alternative flowsheets. However, plant 1 and 4 reasonably close results. In a comprehensive analysis, including capital and maintenance costs, one of them might prove itself the sensible choice. Plants 2 is an inadequate design. The choice of instrument location for plants 2 is obviously very poor, which had an adverse impact on model dynamics and control performance, and it should be avoided due to poor controllability.

Comparing these results to those obtained whilst assessing the same flowsheets in Strutzel and Bogle (2016), which used predictions provided by single linear models obtained for the same plants, a striking difference appears: the index of plant 4, which was the worst performer in the previous analysis, now stands between plant 1 and 3. The quality of the assessment done previously for this flowsheet was poor due to its strongly nonlinear gain. A linear model cannot represent this. By contrast, representing plant 4 as an SMLP system yielded a more accurate EMOP index, showing the advantages of this multi-model formulation for use with integrated process design and control.

It should be noted that while here only two scenarios were tested for a single disturbance, for the design of industrial plants many more possibilities should be investigated for a comprehensive analysis.

6.4.5 Effect of Soft-Landing matrices and error penalization matrices

The influence of the Soft-Landing matrices in the results was small, as can be seen from Table 22 which presents the rate by which they increased the EMOP index.

Table 22 – Effect of SL matrices.

$\lvert SL_1\rvert^{-1}\lvert SL_2\rvert^{-1}$	Plant 1	Plant 2	Plant 3	Plant 4
Case 1	1.0067	1.0048	1.0074	1.0070
Case 2	1.0069	1.0048	1.0074	1.0072

This is as expected because the system is stable and the prediction horizon is sufficiently large. However, if not enough time is given to the plant to settle, the impact of SL_1 and SL_2 may be significant and the solutions will be greatly penalized. Figure 13 shows the index increase for smaller values of p:

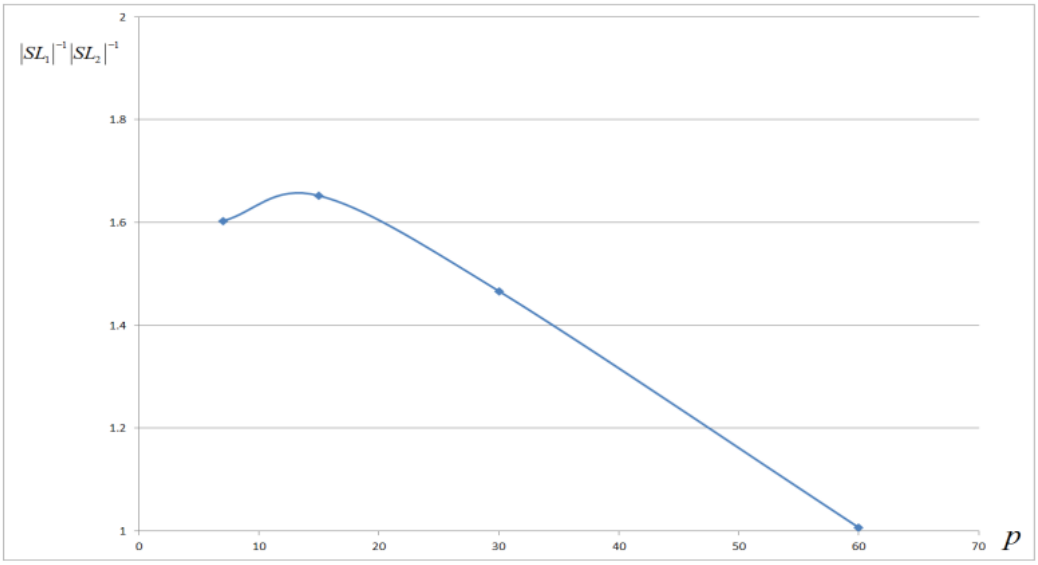

Fig. 26 – Effect of Soft-Landing matrices ($\lvert \boldsymbol{SL_1}\rvert^{-1}\lvert \boldsymbol{SL_2}\rvert^{-1}$) for plant 1, scenario 1, for various values of p.

Since this work is a steady-state focused analysis, the inclusion of SL matrices can be considered a cautionary measure to ensure validity of the results and force the optimisation algorithm to disregard oscillatory or overshooting solutions if possible. However, in some design cases the speed of dynamic response may be key and thus it may be necessary to assess the system thoroughly by testing several different values for p.

The identity matrix was always used for the error penalization matrix - which means no bound violations are permitted during the transients – with the exception of plant 2 in scenario 2 where $|\boldsymbol{EP}| = 1.0104$ to allow $y_1 < y_{min,1}$ over the whole predicted range.

6.4.6 Optimisation algorithm and computational cost

The optimisation problem was solved using the interior-point routine available in the optimisation toolbox in MATLAB. It is also worth noting that the problem has nonlinear constraints and required a solver able to deal with disjunctive programming. The results proved to be sensitive to the initial point chosen, and the optimisation algorithm often reached a local minimum instead of a global one. To avoid this shortcoming an iterative "genetic" algorithm strategy was implemented consisting of using the previous solution the new starting point, but randomly modifying it so as to implement "mutations", then evaluating the cost function and storing the new solution if it was better than the at least one of the 15 best previous ones. In this case, the worst solution among the set of 15 was dropped and replaced by the new, better solution. The random mutations were constrained to be within ±15% of the acceptable range for each manipulated variable, and 15000 iterations were performed for each plant and case studied. The fact that, in both scenarios, one or more CVs correlated to product quality were saturated at the end of the prediction horizon seems to suggest that the solutions presented are global. Saturated CVs mean no slack left for further optimisation since the use of a cheaper mix of raw materials or less energy would result in off-spec products with zero market value. While we hold the belief that global solutions have been obtained, no formal proof can be provided due to the nonlinear nature of the optimisation problem (introduced by varying weights and restrictions).

The same method was used as well for regression analysis problem given by Eq. 97 and Eq. 109, so the best fit could be obtained for the parameters of the degradation functions. At the time of writing, even if linear models are used, solving the economic MPC optimisation index involves a significant computational cost when dealing with complex systems. For example, the problem described in Section 6 was solved using MATLAB® R2016b running on a 3GHz quad-core Intel Xeon E5-1607 CPU, of which 50% of its capacity was available to the solver. In this condition, each iteration of the method took between 5 and 15 seconds, so a typical simulation would take 40h or so. However, the computational demand can be lowered by using smaller control or prediction horizons, m and p. Using $m = 1$ reduced the computing time used by each iteration to only 2-4 seconds. Setting m as high as possible is recommended, since it enables additional degrees of freedom for the algorithm, which in turn leads to less conservative control actions. On the other hand, k needs only to be high enough as to provide the entire transient prediction.

Using models obtained through step test model identification instead of mass and energy balances reduced the number of variables of the optimisation problem from around 30000 (500 variables of UniSim® simulation, multiplied by $p = 60$) to only 762 $\left(n_u \cdot m + \left(n_y + n_d\right) \cdot p\right)$.

6.4.7 Model uncertainty - obtaining a EMOP index interval for one of the designs

In order to demonstrate the uncertain model methodology of Chapter 5, let us present the EMOP index interval of plant 3, scenario 1 (due to space limitations model uncertainty will not be evaluated for all scenarios and flowsheets). The uncertainty matrices $\boldsymbol{B}_{\boldsymbol{u}}^{+}$, $\boldsymbol{B}_{\boldsymbol{u}}^{-}$, $\boldsymbol{B}_{\boldsymbol{d}}^{+}$ and $\boldsymbol{B}_{\boldsymbol{d}}^{-}$ for this flowsheet can be found in the appendix (Table 59 to Table 62). Applying the uncertain model method, it is possible to obtain an index interval inside which the real flowsheet will be contained. In order to keep y_1, y_5 and y_5 from getting saturated during step 2, Eq. 138 and Eq. 139 were used instead of the standard Eq. 136. The anti-saturation parameters used were $W_{sat,5}^{-} = W_{sat,6}^{-} = 100$ and $W_{sat,1}^{+} = -100$. The results yielded by best and worst flowsheets within model uncertainty limits are presented in Table 23, Table 24 and Table 25, together with those of the nominal model.

Table 23 – Plant 3 - uncertain model's CVs.

Plant 3 Case 2	**Step 1**	**Step 2**		**Step 3**	
	Nominal Model	Worst Case	Best Case	Worst Case	Best Case
$\boldsymbol{y_1}$	50.67	50.60	50.70	50.73	50.83
$\boldsymbol{y_2}$	93.42	93.97	92.73	94.37	93.44
$\boldsymbol{y_3}$	326.36	323.61	328.97	324.39	330.84
$\boldsymbol{y_4}$	340.11	345.51	331.26	349.62	336.10
$\boldsymbol{y_5}$	**370.00**	**371.85**	368.40	**370.00**	**369.99**
$\boldsymbol{y_6}$	**-15.01**	-17.09	-17.16	-15.28	**-15.01**
$\boldsymbol{y_7}$	832.99	833.10	832.77	832.69	832.98
$\boldsymbol{y_8}$	269.45	269.18	269.71	271.04	271.50
$\boldsymbol{y_9}$	71.79	72.01	71.16	72.40	71.93
$\boldsymbol{y_{10}}$	824.07	824.20	823.98	824.02	824.31
$\boldsymbol{y_{11}}$	-49.78	-49.51	-50.17	-50.43	-50.12

Table 24 – Plant 3 uncertain model's MVs.

Plant 3 Case 2	Step 1/2	Step 3	
	Nominal Model	Worst Case	Best Case
$\boldsymbol{u_1}$	43.76	43.99	43.94
$\boldsymbol{u_2}$	339.96	340.87	339.65
$\boldsymbol{u_3}$	130.72	133.06	132.14
$\boldsymbol{u_4}$	29.33	30.07	30.73
$\boldsymbol{u_5}$	461.83	447.86	447.26
$\boldsymbol{u_6}$	175.43	184.94	178.79
$\boldsymbol{u_7}$	122.74	127.20	133.95
$\boldsymbol{d_1}$	40.00	40.00	40.00

Table 25 – Plant 3 uncertain model's EMOP index.

Plant 3 scenario 1	Nominal Model	Best Model	Worst Model
Step 1	10758	-	-
Step 2	-	13813	175500
Step 3	-	9470	9395

Table 25 shows that the EMOP index is very sensitive to uncertainty in step 2, causing $y_5 > y_{5,max}$ $(= 370)$. This error resulted in substandard, unfit to be sold diesel being produced, causing the index to explode from 10758 (nominal model, $\mathrm{I_{EMOP}}$) to 175500 (worst model, $I_{EMOP-WM}$). However, this deviation was easily corrected by the additional set of control actions defined in step 3, bringing $I_{EMOP-WM}$ down to 9470. Note that the use of the additional anti-saturation parameter J_{sat}, defined in Eq. 138, caused $I_{EMOP-BM} > I_{EMOP}$ after step 2. The difference between $I_{EMOP-WM}$ and $I_{EMOP-BM}$ is quantifies the maximum negative and maximum positive impact of that can be caused by model uncertainty, as defined in Chapter 5, for this case study. As can be seen, after step 3 this difference becomes rather small, showing that the index is resilient to this level of uncertainty. It is also important to notice that both $I_{EMOP-WM}$ and $I_{EMOP-BM}$ are smaller than the nominal model I_{EMOP}. This happens because the effect of the Soft-Landing matrices $(|\boldsymbol{SL_1}|^{-1}|\boldsymbol{SL_2}|^{-1})$ was reduced from 0.62% in step 1 to 0.03% in step 3, and $\mathrm{V_{5,max}}$ was reduced from 2.18 to 0.75, and $V_{6,max}$ was reduced from 7.49 to 6.06, due to the changes in the initial composition of the crude oil feed between step 1 and 3. The EMOP index interval for plant 3, scenario 1 can be obtained through Eq. 137, yielding $I_{EMOP-UN,plant\ 3} \in [10720,10796]$.

6.4.8 Conclusions from the Crude Oil Distillation Study Case

This Section illustrated the EMOP index through a case study consisting of high complexity, the large-scale process consisting of four alternative designs for an oil distillation plant. It is useful to assess how the complexity of each of these flowsheets compares to those found in other contributions to the integrated process design and control framework (IPDCF). Key metrics were compiled in Table 26 for a number of such studies that addressed the problem of large-scale systems, describing the inherent complexity of the case study provided in each paper, as well a short description of the process. Measures provided in Table 26 include the number of components and PI controllers are provided. A higher number of these will increase model complexity and the need for extra computational power. The number of subsystems means the number of process equipment such as mixers, heat exchangers, etc. If one of this equipment is a distillation column with multiple trays, the number of trays is provided. While all systems addressed by works above are genuinely large-scale, oil refining processes, such as the set of crude oil distillation plants studied here, presents a particular challenge.

Table 26 – Complexity comparison between study cases.

Paper	Case study	Complexity
Bansal et al. (2002)	benzene/toluene binary distillation	2 components 3 PI controllers 4 subsystems + 30 trays
Bansal et al. (2000)	double-effect binary distillation (methanol/water)	2 components 3 PI controllers 7 subsystems (don’t disclose how many trays)
Trainor et al. (2013)	ternary distillation (Toluene, Hexane, Heptane)	3 components 2 PI controllers 6 subsystems + 28 trays
Ricardez-Sandoval et al. (2011)	Tennessee Eastman process	8 components 8 PI controllers 5 subsystems
Alvarado-Morales et al. (2010)	bioethanol production	16 components 2 PI controllers 7 subsystems
Strutzel and Bogle (2016)	crude oil distillation	36 components 8 PI controllers 21 subsystems+ 29 trays

The phenomenological models used by the papers presented in this Chapter are adequate for separation processes of mixtures presenting near-ideal behaviour, i.e., where deviation from Raoult’s law can be ignored, or mixtures of chemically similar solvents, or non-ideal solutions to which Raoult’s law applies and fugacity and activity coefficients can be easily calculated. But difficulties arise when dealing with petroleum fractions:

each subsystem usually has dozens of non-ideal hypothetical components; severe operating conditions mean that the behaviour of gases, solutions and mixtures is also non-ideal; multiphase flow is very common and hard to model adequately; equipment designs are intricate. The best simulators for this kind of process do not provide their set of equations, which are closed-source intellectual property. For all these reasons, and the time and engineering effort required for rigorous modelling is always very large and, unless models are linearised, even with the optimisation solvers and processing power available at the time of writing it is doubtful that a solution could be found in reasonable time. This Thesis aims to offer an alternative Controllability Analysis approach that is better suited for the plant-wide design of oil refining processes, and also to include the use of Model Predictive Control as the main control strategy. With this goal in mind, the use of commercial simulation packages for modelling was very successful as none of the IPDCF case studies in the benchmark matches the complexity and scale of the problem presented in this Section.

7 Comparison between two Integrated Process Design and Control Methodologies

In this Section, the Economic MPC Optimisation index is applied to the case study provided in Francisco et al. (2011), with the goal of enabling comparison between it and another methodology available in open literature. Both works s can be classified as Integrated Process Design and Control methods specially designed for flowsheets controlled by MPC schemes.

However, they possess different characteristics and goals. As detailed in Section 2.3.1, Francisco et al. (2011) present a synthesis method for the obtaining of new flowsheet designs by evaluating MPC performance. It combines controllability and economic indexes that are evaluated at each iteration as key plant parameters are changed. The EMOP index is used to assess the optimal operating point of a given flowsheet and finding the optimal trajectory to reach it, which is restricted by the control zones of the MPC scheme. The purpose of this comparison is determining whether the differences between methods can lead to different plant assessment.

To ensure the equivalence of models while comparing the methods, a phenomenological nonlinear state-space model will be used to represent several designs for the activated sludge process (ASP) of a wastewater treatment plant. Unlike the case study of the atmospheric distillation plant of Chapter 6, the numerical complexity is low for obtaining the dynamic response of the ASP. Thus, another goal is assessing the computational demands posed by the use of a small-scale nonlinear model, as compared to the large-scale linear model described in Chapter 6.

7.1 Description of an Activated Sludge Wastewater Treatment Plant

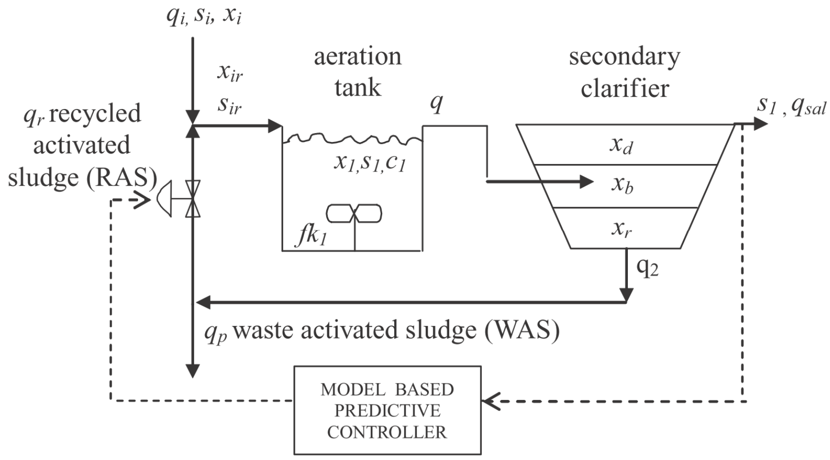

Fig. 27 – Plant and controller layout for the ASP for substrate elimination.

A wastewater treatment plant is used to process sewage and return clean to a water body, and the activated sludge process (ASP) is a very important part of the cleaning procedure. The water treatment comprises the following basic steps, though in this Chapter only (b) and (c) are considered:

a) The primary treatment is dedicated to the removal of gross solids, sand, oil and grease. A primary sedimentation is the last step of this stage. This process removes up to 50% of the total polluting sewage load.
b) The secondary treatment is the ASP. The mixed outlet stream from the primary sedimentation tanks is passed to the reactor. There, the aerobic action of a mixture of microorganisms is used to reduce the substrate concentration in the water. A bacterial culture degrades the organic substrate converting it into inorganic products, more biomass and water. The dissolved oxygen required is provided by a set of aeration turbines.
c) Clarification. The effluent is feed into clarification tanks, where the activated sludge and clean water are separated. After this, the water contains approximately 10% of the waste material, and the water is discharged to the river. Between 25% and 100% of the settled activated sludge is recycled to re-inoculate the reactor.

7.1.1 Mathematical model of the ASP for substrate removal

Now a simple model of ASP only for organic matter (substrate) elimination shall be described. The plant and controller layout can be seen in Fig. 27, comprising a bioreactor and a settler for clarification. The mathematical model assumes perfectly mixed tanks and is based on mass balances presented in Moreno et al. (1992).

7.1.1.1 Mass balance for the aeration tanks

The rate of change of the biomass, organic substrate and dissolved oxygen concentrations are given by:

$$\frac{dx_1}{dt} = \mu_{max} y_c \frac{s_1 x_1}{(K_s + s_1)} - K_d \frac{{x_1}^2}{s_1} - K_d x_1 + \frac{q}{V_1}(x_{ir} - x_1) \quad \text{Eq. 168}$$

$$\frac{ds_1}{dt} = -\mu_{max} \frac{s_1 x_1}{(K_s + s_1)} - f_{kd} K_d \frac{{x_1}^2}{s_1} + f_{kd} K_c x_1 + \frac{q}{V_1}(s_{ir} - s_1) \quad \text{Eq. 169}$$

$$\frac{dc_1}{dt} = K_{la} f_{k1}(c_s - c_1) - OUR - \frac{q}{V_1} c_1 \quad \text{Eq. 170}$$

where x_1, s_1 and c_1 are the biomass, substrate (Chemical Oxygen Demand, COD) and dissolved oxygen (DO) concentration at the output of the aeration tanks (mg/l); x_{ir} and s_{ir} are respectively the inlet biomass and substrate (mg/l). μ_{max} is the maximum growth rate of the microorganisms, q is the inlet flow (m^3/h), K_d is the kinetic coefficient of biomass decay by endogenous metabolism (1/h), K_c is the kinetic coefficient of biomass decay by biological waste, V_1 is the total useful volume for the six aeration tanks (m^3), y_c is the yield coefficient between cellular growth and substrate elimination, f_{kd} is the yield coefficient between biomass endogenous and substrate contribution to the medium, c_s is the DO concentration at saturation, K_{la} is the mass transfer coefficient, f_{k1} is the aeration factor which depends on the number and speed of working turbines, OUR is the oxygen uptake rate and K_s is the saturation constant.

For the biomass rate of change, the first term describes the biomass growth according to the Monod model, the second describes cell death, the third describes the biological waste, and the final term quantifies the dilution effects. For the rate of consumption of organic substrate, the first term expresses the decrease of the substrate through the activity of the biomass (Monod model), the second and third ones describe the transformation part of the dead biomass and biological waste into organic substrate, and the last term is the difference between the input and output substrate mass flow.

For the dissolved oxygen concentration, the Eq. 169 follows the classic literature: the first term is the rate of oxygen transferred to the water, the second describes the rate of oxygen used by the microorganisms (uptake rate), and the final term quantifies the dilution effects. Algebraic equations for x_{ir} and s_{ir} are expressed as mass balances:

$$x_{ir} = \frac{x_i q_i + x_r q_r}{q} \quad \text{Eq. 171}$$

$$s_{ir} = \frac{s_i q_i + s_1 q_r}{q} \quad \text{Eq. 172}$$

where x_i, s_i are the biomass and substrate at the influent, q_i is the input flow to the process. x_r and q_r are the recycle concentrations and flow rate. The equation for oxygen uptake rate is:

$$OUR = -K_{01}\mu_{max}\frac{x_1 s_1}{(K_s + s_1)} \quad \text{Eq. 173}$$

where K_{01} is the yield coefficient between the cellular growth and the oxygen consumption rate.

7.1.1.2 Mass balance for the secondary clarifiers (settlers)

The operation of these elements is described by mass balance equations and one expression for the settling of activated sludge. The model considers the difference in settling rates between layers of increasing biomass concentration.

This model attempts to capture the dynamic behaviour of the clarifiers:

$$A_s l_d \frac{dx_d}{dt} = q_{sal}x_b - q_{sal}x_d - A_s v_s(x_d) \quad \text{Eq. 174}$$

$$A_s l_b \frac{dx_b}{dt} = qx_1 - q_{sal}x_b - q_2 x_b + A_s v_s(x_d) - A_s v_s(x_b) \quad \text{Eq. 175}$$

$$A_s l_r \frac{dx_r}{dt} = q_2 x_b - q_2 x_r - A_s v_s(x_b) \quad \text{Eq. 176}$$

where x_d is the biomass concentration at the surface of the settler leaving the plant, q_{sal} is the flow of clean water at the output of the settler, x_b is the biomass concentration in the second layer, q_2 is the activated sludge total recycling flow, x_r is the biomass concentration at the bottom of the settler, v_s is the settling rate of the activated sludge, A_s is the area of the settler, and l_d, l_b, l_r are the height of the first, second and third layer, respectively (Fig. *27*). Note that the settler input flow q enters to the unit at the second layer level. The settling rate parameters are fitted to a curve provided by pilot plants:

$$v_s(x_b) = nr\ x_b e^{(-ar\ x_b)} \qquad \text{Eq. 177}$$

$$v_s(x_d) = nr\ x_d e^{(-ar\ x_d)} \qquad \text{Eq. 178}$$

The relations between the different flow rates are:

$$q = q_i + q_r \qquad \text{Eq. 179}$$

$$q_{sal} = q_i - q_p \qquad \text{Eq. 180}$$

$$q_2 = q_r + q_p \qquad \text{Eq. 181}$$

where q_p is the purge flow. The control of this process aims to keep the substrate at the output (s_1) below the legal requirement value despite the large variations of the flow rate (q_i) and the substrate concentration of the incoming water (s_i). The disturbances vector is: d $= (s_i, q_i)$. The recycling flow (q_r) is the manipulated variable, and the controlled output is the substrate (s_1) in the reactor: $u(k) = q_r$; $y(k) = s_1$. Biomass (x_1) in the reactor is a bounded variable. Table 27 provides a symbol list for the variables and parameters of the activated sludge process, as well as their units and values when convenient.

Table 27 – Operational, biological and physical parameters for the selected activated sludge process.

Symbol	Parameter	Unit	Value
μ_{max}	Maximum growth rate of the microorganisms	h^{-1}	0.1824
y_c	Yield coefficient between cellular growth and substrate elimination		0.5948
f_{kd}	Yield coefficient between biomass endogenous and substrate contribution to the medium	L^{-1}	0.2
K_d	Kinetic coefficient of biomass decay by endogenous metabolism	$\frac{L}{h}$	5.5e^{-5}
K_s	Saturation constant	$\frac{mg}{L}$	300
K_c	Kinetic coefficient of biomass decay by biological waste	$\frac{L}{h}$	1.333e^{-4}
c_s	Saturation oxygen (DO) concentration in the aeration tanks	$\frac{mg}{L}$	8
K_{la}	Mass transfer coefficient in aeration process oxygen uptake rate	h^{-1}	0.7
OUR	Oxygen uptake rate	$\frac{mg}{L} \cdot h$	(Variable)
K_{01}	Yield coefficient between the cellular growth and the oxygen consumption rate		0.0001
x_i	Biomass concentration at the influent	$\frac{mg}{L}$	(Variable)
s_i	Substrate concentration at the influent	$\frac{mg}{L}$	(Variable)
q_i	Influent flow	m^3/h	(Variable)
x_1	Biomass concentration at the output of the aeration tanks	$\frac{mg}{L}$	(Variable)
s_1	Substrate (COD) concentration at the output of the aeration tanks	$\frac{mg}{L}$	(Variable)
c_1	Dissolved oxygen (DO) concentration at the output of the aeration tanks input flow	$\frac{mg}{L}$	(Variable)
c	Bioreactor input flow	m^3/h	(Variable)
q_r	Recycle flow	m^3/h	(Variable)
q_p	Purge flow	m^3/h	(Variable)
x_{ir}	Bioreactor inlet biomass concentration	$\frac{mg}{L}$	(Variable)
s_{ir}	Bioreactor inlet substrate concentration	$\frac{mg}{L}$	(Variable)
f_{k1}	Aeration factor		0.15039
V_1	Bioreactor volume	m^3	(Depends on flowsheet)
A_s	Settler area	m^2	(Depends on flowsheet)
x_d	Biomass concentration at the surface of the settler	$\frac{mg}{L}$	(Variable)
x_b	Biomass concentration in the settler second layer	$\frac{mg}{L}$	(Variable)
x_r	Biomass concentration at the bottom of the settler	$\frac{mg}{L}$	(Variable)
$v_s(x_d)$	Settling rate function of the activated sludge in the settler depending on x_d	$\frac{g}{m^2} \cdot h$	(Variable)
$v_s(x_b)$	Settling rate function of the activated sludge in the settler depending on x_b	$\frac{g}{m^2} \cdot h$	(Variable)
nr	Settling rate experimental parameter		3.1563
ar	Settling rate experimental parameter	$\frac{L}{mg}$	-7.8567e^{-04}

7.1.2 Results from the Integrated Design (ID) Methodology (Francisco et al., 2011)

Present the full methodology developed in Francisco et al. (2011) is beyond the scope of this comparison. Here we are only interested in the results obtained, especially how the control performance of each plant was evaluated.

Each flowsheet was the result of a multi-objective nonlinear constrained optimisation problem, the objective function of which includes investment, operating costs, and controllability. The performance of each plant is assessed using an infinite horizon MPC simulation, implemented with a terminal penalty to guarantee stability. The simulation results are used to calculate a series of controllability indexes.

The control structure used is extremely simple and consists of SISO (single input single output) MPC, where the controlled variable is the Substrate (COD) concentration at the output of the aeration tanks, s_1, and the manipulated variable is the Recycle flow, q_r. Two disturbances were considered: the substrate concentration of the incoming water, s_i, and the plant flow rate, q_i. The control objective is that s_1 remains below 100 $\frac{\text{mg}}{\text{L}}$, which is a set point defined by environmental regulation.

The flowsheets are then compared with each other regarding some key performance metrics which were defined by the authors. In brief, the key metrics are: Index of disturbance rejection $\|w_p \cdot S_0 \cdot R_{d0}\|$; a numerical value that needs to have magnitude lower than 1 in order ensure disturbance rejection. Small values are desired; The control sensitivity M_0 (the transfer function between the worst-case disturbances and the control signals when the SP is set to zero). $\|M_0\|$ should be lesser than certain limits to avoid saturations and to keep the control system in the linear region. Small values are preferable; MPC controller weight, R. Lower values are better, as higher values increase costs; Maximum controlled variable deviation from SP $\left(\max \left|s_1 - s_1^{sp}\right|\right)$; Flowsheet costs (capital expenditure).

Table 28 below provides the data for the key metrics proposed in Francisco et al. (2011). The results obtained will be later compared with the Economic MPC Optimisation index for each plant. All designs are solutions to the same optimisation problem which was solved through the use of different mathematical algorithms and procedures.

Table 28 – Candidate designs for the Active Sludge Process (Francisco et al., 2011).

Flowsheet	A	B	C	D	E	F
$\boldsymbol{R}$ controller weight	0.00737	0.00694	0.00665	0.00575	0.00696	0.00724
$\boldsymbol{V_1}$ (m^3)	3628	3616.5	3796.3	3604.3	3632.1	3605.4
$\boldsymbol{A_s}$ (m^2)	2449.4	2459.4	2308.9	2452	2432	2452.1
$\boldsymbol{max\ \left\|s_1 - s_1^{sp}\right\|}$	13.98	13.88	13.66	13.68	13.97	14.09
$\boldsymbol{\left\|w_p \cdot S_0 \cdot R_{d0}\right\|}$	0.909	0.879	0.849	0.789	0.881	0.903
$\boldsymbol{\|M_0\|}$	3403.3	3500	3532.5	3808.4	3498.6	3443.4
Cost	0.142	0.1428	0.14395	0.14194	0.14198	0.14194

In Francisco et al. (2011) the authors refrain from pointing out the best among the plants generated by its methodology, highlighting instead where each one performed well or badly.

Therefore, a straightforward comparison between this method and the EMOP is not possible. An alternative to help to enable this comparison is to define a global measure of each plant's performance using the indexes of Table 28. It is straightforward to group all the indexes into a single one, which we shall refer to as "global index" (GI). Like the EMOP index, we wish lower values of this global measure to signify better general controllability and better designs. Therefore, the cost, $\left\|w_p \cdot S_0 \cdot R_{d0}\right\|$, $\|M_0\|$, R must be directly proportional to the global index, while max $\left|s_1 - s_1^{sp}\right|$ must be inversely proportional. Thus, GI can be obtained as shown in Eq. 182:

$$GI = Cost \cdot \left\|w_p \cdot S_0 \cdot R_{d0}\right\| \cdot \|M_0\| \cdot R \cdot max\ \left|s_1 - s_1^{sp}\right| \quad \text{Eq. 182}$$

This global index is interesting because it assembles all the parameters used in Francisco et al. (2011) to assess plant performance for the purpose of enabling a direct comparison with the EMOP index. Applying Eq. 182 for each of the flowsheets presented in Table 28 yields the results presented in Table 29:

Table 29 – Global Index for the ASP candidate designs.

Flowsheet	A	B	C	D	E	F
***R* controller weight**	45.26	42.32	39.22	33.55	42.55	45.02

As can be seen, by this metric plant D had the best performance while A had the worst.

7.1.3 Nonlinear State-Space Model for the ASP

Before applying the EMOP index methodology to the active sludge process, it is necessary first to obtain a state-space model of the process. Here we use a nonlinear state-space formulation detailed in Eq. 183 and Eq. 184 to provide the output prediction:

$$\dot{\boldsymbol{\delta}} = \boldsymbol{A\delta} + \boldsymbol{Bu} + \boldsymbol{Dd} \quad \text{Eq. 183}$$

$$\boldsymbol{y} = \boldsymbol{C\delta} \quad \text{Eq. 184}$$

where the letter $\boldsymbol{\delta}$ was used to denote the states instead of $\boldsymbol{x}$ to avoid confusion with some ASP variables. Let us now define the states, MVs and DVs of the MPC problem. It is important to point out that the variable scheme used in Francisco et al.

(2011) is arguably not ideal, and the case study does not have the flexibility expected from an industrial process. For example, there is no feed tank, and thus the feed flow rate cannot be lowered to ensure on spec plant effluent, as it is common in situations such as this where heavy penalties can be imposed on companies who fail to meet environmental standards. For this reason, the plant flow rate will be considered a disturbance, whose variability may undermine the plant control goals. Additionally, there is no control of the purge flow rate in the original case study. In the next Section, we shall examine how the inclusion of purge flow as an MV enables better controllability, as the flowsheets shall be evaluated for both the cases of the fixed and variable purge.

So, unlike the control problem defined in Francisco et al. (2011), which considers only the recycling flow rate, $\mathrm{q_r}$, as an MV, we shall also set the purge flow rate, $\mathrm{q_p}$, also as an MV. Table 30, Table 31 and Table 32 present respectively the model states, manipulated variables and disturbances to be used in the EMOP problem. The single controlled variable is $\mathrm{y} = \delta_2$ (substrate chemical oxygen demand).

Table 30 – States of the ASP model.

		The ASP Model States
$\boldsymbol{\delta_1}$	x_1	biomass COD (Chemical Oxygen Demand) ($\frac{\mathrm{mg}}{\mathrm{L}}$)
$\boldsymbol{\delta_2}$	s_1	substrate COD (Chemical Oxygen Demand) ($\frac{\mathrm{mg}}{\mathrm{L}}$)
$\boldsymbol{\delta_3}$	c_1	dissolved oxygen (DO) concentration at the output of the aeration tanks ($\frac{\mathrm{mg}}{\mathrm{L}}$)
$\boldsymbol{\delta_4}$	x_d	biomass concentration at the surface of the settler ($\frac{\mathrm{mg}}{\mathrm{L}}$)
$\boldsymbol{\delta_5}$	x_b	biomass concentration in the second layer ($\frac{\mathrm{mg}}{\mathrm{L}}$)
$\boldsymbol{\delta_6}$	x_r	biomass concentration at the bottom of the settler ($\frac{\mathrm{mg}}{\mathrm{L}}$)

Table 31 – ASP Process inputs.

		The ASP Model Inputs
$\boldsymbol{u_1}$	q_r	recycling flow rate (m^3/h)
$\boldsymbol{u_2}$	q_p	purge flow rate (m^3/h)

Table 32 – ASP process disturbances.

		The ASP Model Disturbances
$\boldsymbol{d_1}$	s_i	substrate concentration of the incoming water (mg/L)
$\boldsymbol{d_2}$	q_i	plant flow rate (m^3/h)

Therefore, the state array $\boldsymbol{\delta} = [\delta_1, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6]$, the MV array $\boldsymbol{u} = [u_1, u_2]$ and the disturbance array $\boldsymbol{u} = [d_1, d_2]$ are defined.

Combining Eq. 168 to Eq. 181 and replacing the variables defined in Table 30 to Table 32, yields the final form of the mass balance differential equations for the aeration tank and secondary clarifier, as presented in Eq. 185 to Eq. 190.

$$f_{\delta_1} = \frac{d\delta_1}{dt} = \mu_{max} y_c \frac{\delta_2 \delta_1}{(K_s+\delta_2)} - K_d \frac{{\delta_1}^2}{\delta_2} - K_c \delta_1 + \frac{1}{V_1}(x_i d_2 + \delta_6 u_1 - \delta_1 d_2 - \delta_1 u_1) \quad \text{Eq. 185}$$

$$f_{\delta_2} = \frac{d\delta_2}{dt} = -\mu_{max} \frac{\delta_2 \delta_1}{(K_s+\delta_2)} + f_{kd} \delta_1 \left(K_d \frac{\delta_1}{\delta_2} + \ K_c\right) + \frac{d_2}{V_1}(d_1 - \delta_2) \quad \text{Eq. 186}$$

$$f_{\delta_3} = \frac{d\delta_3}{dt} = K_{la} f_{k1}(c_s - \delta_3) + K_{01} \mu_{max} \frac{\delta_2 \delta_1}{(K_s+\delta_2)} - \frac{\delta_3}{V_1}(d_2 + u_1) \quad \text{Eq. 187}$$

$$f_{\delta_4} = \frac{d\delta_4}{dt} = \frac{d_2 \delta_5}{A_s l_d} - \frac{u_2 \delta_5}{A_s l_d} - \frac{d_2 \delta_4}{A_s l_d} + \frac{u_2 \delta_4}{A_s l_d} - \frac{nr\, \delta_4\, e^{(-ar\, \delta_4)}}{l_d} \quad \text{Eq. 188}$$

$$f_{\delta_5} = \frac{d\delta_5}{dt} = \frac{d_2 \delta_1}{A_s l_b} + \frac{u_1 \delta_1}{A_s l_b} - \frac{d_2 \delta_5}{A_s l_b} - \frac{u_1 \delta_5}{A_s l_b} + \frac{nr\, \delta_4\, e^{(-ar\, \delta_4)}}{l_b} - \frac{nr\, \delta_5 e^{(-ar\, \delta_5)}}{l_b} \quad \text{Eq. 189}$$

$$f_{\delta_6} = \frac{d\delta_6}{dt} = \frac{u_1 \delta_5}{A_s l_r} + \frac{u_2 \delta_5}{A_s l_r} - \frac{u_1 \delta_6}{A_s l_r} - \frac{u_2 \delta_6}{A_s l_r} + \frac{nr\, \delta_5 e^{(-ar\, \delta_5)}}{l_r} \quad \text{Eq. 190}$$

The nonlinear state-space model matrices can be easily obtained by rearranging the differential equations of the ASP process, yielding the system described in Eq. 191 to Eq. 194:

$$\boldsymbol{A} = \begin{bmatrix} \left[\mu_{max} y_c \frac{\delta_2}{(K_s+\delta_2)} - K_d \frac{\delta_1}{\delta_2} - K_c\right] \\ \left[-\mu_{max} \frac{\delta_1}{(K_s+\delta_2)} + \frac{f_{kd}\delta_1}{\delta_2}\left(K_d \frac{\delta_1}{\delta_2} + \ K_c\right)\right] \\ \left[K_{la} f_{k1}\left(\frac{c_s}{\delta_3} - 1\right) + \frac{K_{01}\mu_{max}}{\delta_3} \frac{\delta_2 \delta_1}{(K_s+\delta_2)}\right] \\ \left[-\frac{nr\, e^{(-ar\, \delta_4)}}{l_d}\right] \\ \left[\frac{nr\, \delta_4 e^{(-ar\, \delta_4)}}{\delta_5 l_b} - \frac{nr\, e^{(-ar\, \delta_5)}}{l_b}\right] \\ \left[\frac{nr\, \delta_5\, e^{(-ar\, \delta_5)}}{\delta_6 l_r}\right] \end{bmatrix} \quad \text{Eq. 191}$$

$$\boldsymbol{B} = \begin{bmatrix} \left[\frac{\delta_6 - \delta_1}{V_1}\right] & [0] \\ [0] & [0] \\ \left[-\frac{\delta_3}{V_1}\right] & [0] \\ [0] & \left[\frac{\delta_5 - \delta_4}{A_s l_d}\right] \\ \left[\frac{\delta_1 - \delta_5}{A_s l_b}\right] & [0] \\ \left[\frac{\delta_5 - \delta_6}{A_s l_r}\right] & \left[\frac{\delta_5 - \delta_6}{A_s l_r}\right] \end{bmatrix} \quad \text{Eq. 192}$$

$$\boldsymbol{C} = diag[0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$$ Eq. 193

$$\boldsymbol{D} = \begin{bmatrix} [0] & \left[\frac{x_i - \delta_1}{V_1}\right] \\ \left[\frac{q_i}{V_1}\right] & \left[-\frac{\delta_2}{V_1}\right] \\ [0] & \left[-\frac{\delta_3}{V_1}\right] \\ [0] & \left[\frac{\delta_5 - \delta_4}{A_s l_d}\right] \\ [0] & \left[\frac{\delta_1 - \delta_5}{A_s l_b}\right] \\ [0] & [0] \end{bmatrix}$$ Eq. 194

7.1.4 Applying the Economic MPC Optimisation index to the ASP

The EMOP index will now be calculated for each ASP flowsheet provided by Francisco et al. (2011). Let us consider a case study in which s_i and q_i obey the patterns defined by Eq. 195 and Eq. 196:

$$s_i(t) = 0.4 + 0.05\, log(t)$$ Eq. 195

$$q_i(t) = 1150 + 100\, sin(t)$$ Eq. 196

The prediction horizon considered is 100 hours ($\text{p} = 100$). Eq. 195 means that s_i will increase value from 0.4 to 0.5 mg/L during the prediction horizon. Meanwhile, q_i will oscillate around 1150 m^3/h, as per Eq. 196.

ASP process economics were not deeply discussed in Francisco et al. (2011). The operational costs are considered to be proportional to the sum of the recycle and purge flow rates, q_2, and a gain equal to 1 was used in the objective function. Let us assume that the penalty for constraint violation is 100 times larger than the benefit of reducing q_2. Based on these assumptions, we obtain the necessary economic optimisation parameters for the application of the EMOP index: $W_{lower} = 0$, $W_{upper} = 100$, $\boldsymbol{V_{min}} = [1 \;\; 1]$, $\boldsymbol{V_{max}} = [0 \;\; 0]$. Other parameters used include: control horizon equal to prediction horizon ($m = 100$); $\boldsymbol{T_{SL1}}$ and $\boldsymbol{T_{SL2}}$ equal to unitary vectors multiplied by scalar 0.005; the weights $T_{lower} = T_{upper} = 1$ when y is outside its control zone and null when it is within.

Here two scenarios will be explored: in the first one the system considered is a SISO system in which the controller can very only $\boldsymbol{q_r}$, while $\boldsymbol{q_p}$ is kept at the fixed value of 115 m^3/h, the same procedure discussed in Francisco et al. (2011). In the second scenario, both $\boldsymbol{q_r}$ and $\boldsymbol{q_p}$ are available to the controller as MVs, and the system becomes

1x2 (one output, two inputs), which should be helpful by adding a freedom degree. The EMOP index was calculated for these two scenarios, and the results are displayed in Table 33:

Table 33 – EMOP index for the alternative ASP flowsheets.

	Index Value for each ASP Plant					
	A	B	C	D	E	F
1x1	2.5634	2.6770	2.5215	2.4357	2.5633	2.0045
1x2	1.7773	1.6518	1.5397	1.6406	1.4706	1.5689
Average	2.1345	2.1028	1.9704	1.9990	1.9415	1.7734

So now it is possible to verify if the methodology described in Francisco et al. (2011) and the procedure proposed in this project provide comparable results. Let us carry out a comparison of the Global Index (Eq. 182) and the EMOP index for each ASP plant by plotting the results against each other in Fig. 28:

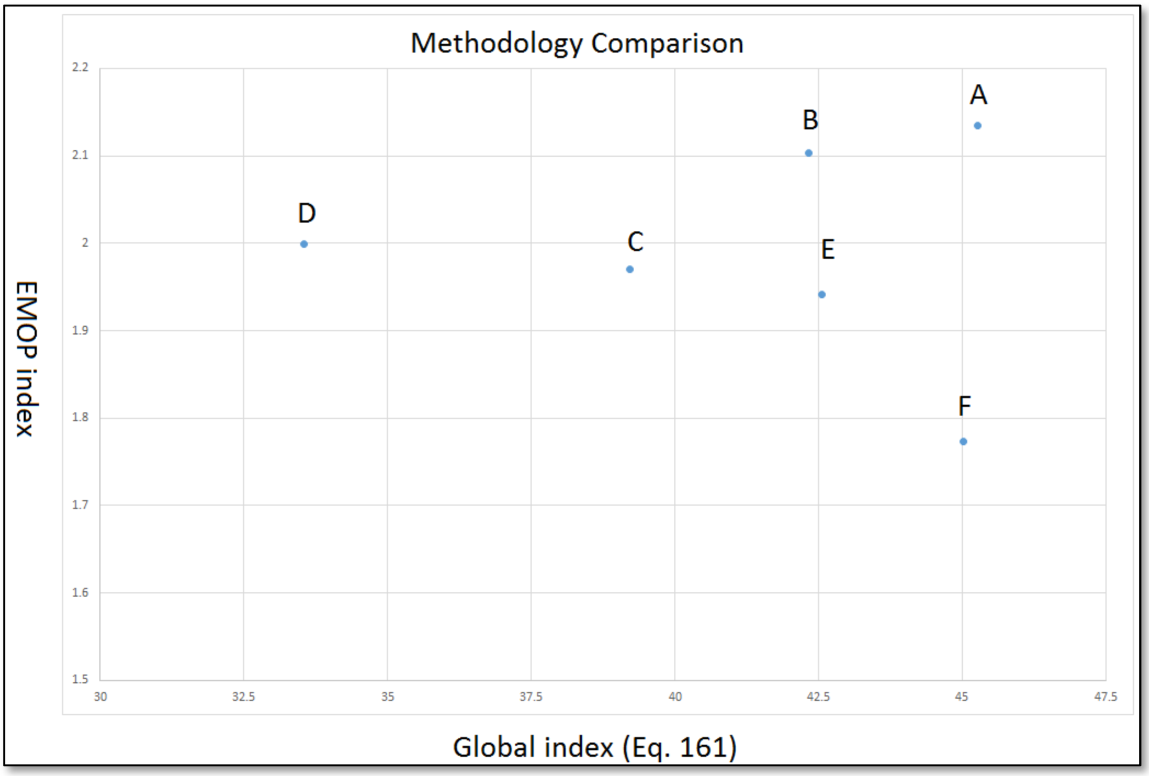

Fig. 28 – Economic MPC Optimisation index versus global index defined in Eq. 182.

As can be seen in Fig. 28, there no correlation between the results provided by both indexes. The lack of a well-defined relation is by no means a surprise: it simply confirms that each methodology is measuring controllability in a different way. Indeed, throughout this work, the case was made that it is not obvious that a plant with better input-output (or state) controllability is the most sensible choice from an economic perspective.

For instance, take the dynamic response provided by plant A in the 1x2 case, which is presented in Fig. 29 and Fig. 30. As one can see, there are control zone violations in the first 28 hours of operation, as the oxygen demand rises quickly, but later the plant stabilises below the upper bound. In this process, what is desired of an MPC is that purge and reflux (q_r and q_p) both remain in the lowest values that do not result in deterioration of output quality.

The EMOP index measures the trade-off between eventual (and sometimes unavoidable) constraint violations and optimised steady-state operation. The variability observed after the CV enters the control zone is of no importance to economical purposes, but it is relevant to the controllability measures provided in Francisco et al. (2011). Also, the degree to which q_r and q_p can be decreased is also of no consequence to some traditional controllability measures. What can be concluded by the comparison performed in this Section is that the Economic MPC Optimisation index is a good complement to the approaches currently available in the open literature since it properly monetises control performance and plant's dynamic behaviour, leading to the selection of the best design. It is also noteworthy that since the plants being compared are very similar, with almost identical dimensions, the difference between their EMOP indexes is small, and therefore their performance can be considered roughly similar. Since this optimisation problem is nonlinear, the index values found refer to the best local minimum available. Better solutions may be found by different solvers or different initial estimates for the solutions.

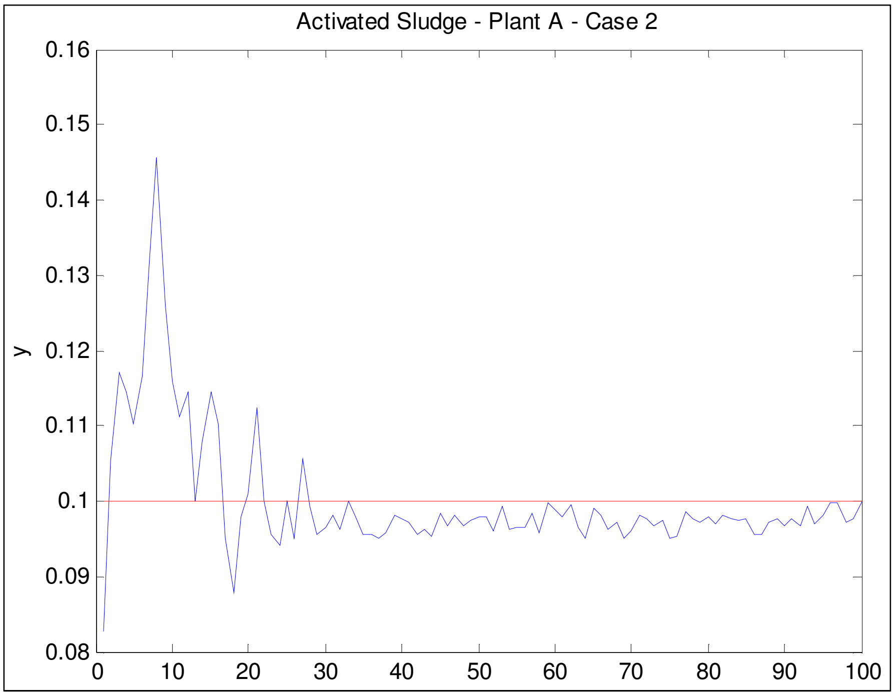

Fig. 29 – Dynamic Response Plant A – Controlled Variables.

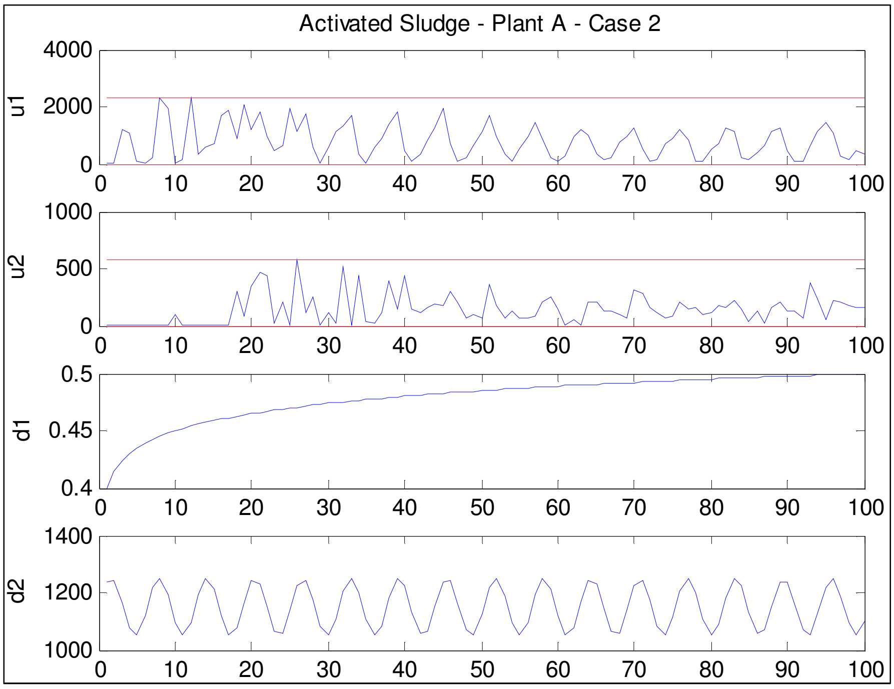

Fig. 30 – Dynamic Response Plant A – Manipulated Variables and Disturbances.

7.1.5 Computational cost of the ASP case study

As the model dimensions grow, the number of calculations involved in solving the Economic MPC Optimisation Index increases exponentially. Therefore, having a smaller system such as the ASP process described in this Section lowers considerably computational demand. In turn, this enables the use of much higher prediction and control horizons, p and m, proving a more detailed solution.

The same computer used for the crude oil distillation case study could solve the EMOP index problem for the ASP with each iteration for the problem taking around 150s. For this case, purge flow was used as MVs and the prediction and control horizons were set at $p = m = 100$. Using $p = 60$ and $m = 6$ instead has brought the iteration time to only 4s, approximately.

7.2 Using the ASP as a case study to benchmark the Simultaneous Multi-Linear Prediction (SMLP)

It is desirable to know if the SLMP can approximate nonlinear models more accurately than the commonly employed piecewise affine (PWA) systems, i.e., if SLMP can further reduce error due to linearisation under frequent circumstances. With this goal in mind, the activated sludge process (ASP) case study will be used to benchmark the SLMP, by means of comparing its prediction to those generated by the nonlinear model presented in this Chapter and by a standard PWA approach. The fact that the set of ODEs is available for the ASP, and that the model is relatively simple, makes it convenient to perform such a comparison.

The SMLP method is a particularly interesting option to consider when the nonlinear model is unknown and the linear models must be obtained through model identification at arbitrary states. To emulate this typical use scenario, the model matrices (Eq. 199 to Eq. 202) were evaluated at three distinct, arbitrary states, yielding three linear models. These models were used to generate a SMLP and a PWA system. Since these systems consist of the same collection of linearised models and were provided with the same initial operating point (OP), MVs and DVs, differences in the predictions are the direct and sole result of differences in the methodologies themselves.

The PWA partition rules i.e., the boundaries of the active region of each linearisation, were defined by the arithmetic mean between the model's linearisation states. Due to the multivariable nature of this control problem, a conflict may arise when selecting the active model (the one used to update the state): a certain OP might lie inside the validity region of different models at the same time if multiple variables are considered. Therefore, the PWA model selection will be given by a voting system: each variable casts a vote according to which space partition it is currently within, and the model with more votes is going to be selected. In the case of a tie, the current model has the preference if it is one the models with the same number of votes; otherwise, a random selection of the new active model occurs.

7.2.1 A linearised state-space model for the ASP

The linearisation of the model presented in Eq. 199 to Eq. 202 is the starting point of the process of obtaining an SMLP system for the ASP. The system was linearised at n_m different points and the resulting system was assembled in a state-space format, as shown in Eq. 197 and Eq. 198. The linearised matrices are given in Eq. 199 to Eq. 202.

$$\boldsymbol{\delta}_{\boldsymbol{k+1}}^{\boldsymbol{m}_{\boldsymbol{n}_\vartheta}} = \boldsymbol{A}_{\boldsymbol{m}_\vartheta}\boldsymbol{\delta}_{\boldsymbol{k}}^{\boldsymbol{m}_{\boldsymbol{n}_\vartheta}} + \boldsymbol{B}_{\boldsymbol{m}_\vartheta}\Delta\boldsymbol{u}_{\boldsymbol{k}} + \boldsymbol{D}_{\boldsymbol{m}_\vartheta}\Delta\boldsymbol{d}_{\boldsymbol{k}}, \vartheta = 1, \dots, n_m \qquad \text{Eq. 197}$$

$$\boldsymbol{y}_{\boldsymbol{\vartheta}}^{\boldsymbol{k}} = \boldsymbol{C}_{\boldsymbol{m}_\vartheta}\, \boldsymbol{\delta}_{\boldsymbol{k}}^{\boldsymbol{m}_{\boldsymbol{n}_\vartheta}},\ \vartheta = 1, \dots, n_m \qquad \text{Eq. 198}$$

$$\boldsymbol{A}_{\boldsymbol{m}_\vartheta} = \begin{bmatrix} \frac{df_{\boldsymbol{\delta}_1}}{d\boldsymbol{\delta}_1} & \frac{df_{\boldsymbol{\delta}_1}}{d\boldsymbol{\delta}_2} & \frac{df_{\boldsymbol{\delta}_1}}{d\boldsymbol{\delta}_3} & \frac{df_{\boldsymbol{\delta}_1}}{d\boldsymbol{\delta}_4} & \frac{df_{\boldsymbol{\delta}_1}}{d\boldsymbol{\delta}_5} & \frac{df_{\boldsymbol{\delta}_1}}{d\boldsymbol{\delta}_6} \\ \frac{df_{\boldsymbol{\delta}_2}}{d\boldsymbol{\delta}_1} & \frac{df_{\boldsymbol{\delta}_2}}{d\boldsymbol{\delta}_2} & \frac{df_{\boldsymbol{\delta}_2}}{d\boldsymbol{\delta}_3} & \frac{df_{\boldsymbol{\delta}_2}}{d\boldsymbol{\delta}_4} & \frac{df_{\boldsymbol{\delta}_2}}{d\boldsymbol{\delta}_5} & \frac{df_{\boldsymbol{\delta}_2}}{d\boldsymbol{\delta}_6} \\ \frac{df_{\boldsymbol{\delta}_3}}{d\boldsymbol{\delta}_1} & \frac{df_{\boldsymbol{\delta}_3}}{d\boldsymbol{\delta}_2} & \frac{df_{\boldsymbol{\delta}_3}}{d\boldsymbol{\delta}_3} & \frac{df_{\boldsymbol{\delta}_3}}{d\boldsymbol{\delta}_4} & \frac{df_{\boldsymbol{\delta}_3}}{d\boldsymbol{\delta}_5} & \frac{df_{\boldsymbol{\delta}_3}}{d\boldsymbol{\delta}_6} \\ \frac{df_{\boldsymbol{\delta}_4}}{d\boldsymbol{\delta}_1} & \frac{df_{\boldsymbol{\delta}_4}}{d\boldsymbol{\delta}_2} & \frac{df_{\boldsymbol{\delta}_4}}{d\boldsymbol{\delta}_3} & \frac{df_{\boldsymbol{\delta}_4}}{d\boldsymbol{\delta}_4} & \frac{df_{\boldsymbol{\delta}_4}}{d\boldsymbol{\delta}_5} & \frac{df_{\boldsymbol{\delta}_4}}{d\boldsymbol{\delta}_6} \\ \frac{df_{\boldsymbol{\delta}_5}}{d\boldsymbol{\delta}_1} & \frac{df_{\boldsymbol{\delta}_5}}{d\boldsymbol{\delta}_2} & \frac{df_{\boldsymbol{\delta}_5}}{d\boldsymbol{\delta}_3} & \frac{df_{\boldsymbol{\delta}_5}}{d\boldsymbol{\delta}_4} & \frac{df_{\boldsymbol{\delta}_5}}{d\boldsymbol{\delta}_5} & \frac{df_{\boldsymbol{\delta}_5}}{d\boldsymbol{\delta}_6} \\ \frac{df_{\boldsymbol{\delta}_6}}{d\boldsymbol{\delta}_1} & \frac{df_{\boldsymbol{\delta}_6}}{d\boldsymbol{\delta}_2} & \frac{df_{\boldsymbol{\delta}_6}}{d\boldsymbol{\delta}_3} & \frac{df_{\boldsymbol{\delta}_6}}{d\boldsymbol{\delta}_4} & \frac{df_{\boldsymbol{\delta}_6}}{d\boldsymbol{\delta}_5} & \frac{df_{\boldsymbol{\delta}_6}}{d\boldsymbol{\delta}_6} \end{bmatrix} = \qquad \text{Eq. 199}$$

$$\begin{bmatrix} \begin{bmatrix} \frac{\mu_{max}y_c s_1}{(K_s+s_1)} - 2K_d\frac{x_1}{s_1} \\ -K_c - \frac{(q_i+q_r)}{V_1} \end{bmatrix} & \begin{bmatrix} \frac{\mu_{max}y_c x_1 K_s}{(K_s+s_1)^2} \\ +K_d\left(\frac{x_1}{s_1}\right)^2 \end{bmatrix} & [0] & [0] & [0] & \left[\frac{q_r}{V_1}\right] \\ \begin{bmatrix} \frac{-\mu_{max}s_1}{(K_s+s_1)} + \\ \frac{2f_{kd}x_1K_d}{s_1} + f_{kd}K_c \end{bmatrix} & \begin{bmatrix} \frac{-\mu_{max}x_1K_s}{(K_s+s_1)^2} \\ -f_{kd}K_d\left(\frac{x_1}{s_1}\right)^2 - \frac{q_i}{V_1} \end{bmatrix} & [0] & [0] & [0] & [0] \\ \left[\frac{K_{01}\mu_{max}s_1}{(K_s+s_1)}\right] & \left[\frac{K_{01}\mu_{max}x_1K_s}{(K_s+s_1)^2}\right] & \begin{bmatrix} -K_{la}f_{k1} \\ -\frac{(q_i+q_r)}{V_1} \end{bmatrix} & [0] & [0] & [0] \\ [0] & [0] & [0] & \begin{bmatrix} \frac{q_p-q_i}{A_s l_d} + \\ \frac{nr}{l_d}\left(arx_d e^{(-ar\,x_d)} - e^{(-ar\,x_d)}\right) \end{bmatrix} & \left[\frac{q_i-q_p}{A_s l_d}\right] & [0] \\ \left[\frac{q_i+q_r}{A_s l_b}\right] & [0] & [0] & \left[\frac{nr}{l_d}\left(e^{(-ar\,x_d)} - arx_d e^{(-ar\,x_d)}\right)\right] & \begin{bmatrix} \frac{nr}{l_b}\left(arx_b e^{(-ar\,x_b)} - e^{(-ar\,x_b)}\right) \\ -\frac{(q_i+q_r)}{A_s l_b} \end{bmatrix} & [0] \\ [0] & [0] & [0] & [0] & \begin{bmatrix} \frac{q_r+q_p}{A_s l_r} + \\ \frac{nr}{l_r}\left(e^{(-ar\,x_b)} - arx_b e^{(-ar\,x_b)}\right) \end{bmatrix} & \left[-\frac{(q_r+q_p)}{A_s l_r}\right] \end{bmatrix}$$

$$\boldsymbol{B}_{\boldsymbol{m}_{\vartheta}} = \begin{bmatrix} \frac{df_{\delta_1}}{du_1} & \frac{df_{\delta_1}}{du_2} \\ \frac{df_{\delta_2}}{du_1} & \frac{df_{\delta_2}}{du_2} \\ \frac{df_{\delta_3}}{du_1} & \frac{df_{\delta_3}}{du_2} \\ \frac{df_{\delta_4}}{du_1} & \frac{df_{\delta_4}}{du_2} \\ \frac{df_{\delta_5}}{du_1} & \frac{df_{\delta_5}}{du_2} \\ \frac{df_{\delta_6}}{du_1} & \frac{df_{\delta_6}}{du_2} \end{bmatrix} = \begin{bmatrix} \left[\frac{\delta_6-\delta_1}{V_1}\right] & [0] \\ [0] & [0] \\ \left[-\frac{\delta_3}{V_1}\right] & [0] \\ [0] & \left[\frac{\delta_4-\delta_5}{A_s l_d}\right] \\ \left[\frac{\delta_1-\delta_5}{A_s l_b}\right] & [0] \\ \left[\frac{\delta_5-\delta_6}{A_s l_r}\right] & \left[\frac{\delta_5-\delta_6}{A_s l_r}\right] \end{bmatrix} \quad \text{Eq. 200}$$

$$\boldsymbol{C}_{\boldsymbol{m}_{\vartheta}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{Eq. 201}$$

$$\boldsymbol{D}_{\boldsymbol{m}_{\vartheta}} = \begin{bmatrix} \frac{df_{\delta_1}}{dd_1} & \frac{df_{\delta_1}}{dd_2} \\ \frac{df_{\delta_2}}{dd_1} & \frac{df_{\delta_2}}{dd_2} \\ \frac{df_{\delta_3}}{dd_1} & \frac{df_{\delta_3}}{dd_2} \\ \frac{df_{\delta_4}}{dd_1} & \frac{df_{\delta_4}}{dd_2} \\ \frac{df_{\delta_5}}{dd_1} & \frac{df_{\delta_5}}{dd_2} \\ \frac{df_{\delta_6}}{dd_1} & \frac{df_{\delta_6}}{dd_2} \end{bmatrix} = \begin{bmatrix} [0] & \left[\frac{x_i-\delta_1}{V_1}\right] \\ \left[\frac{d_2}{V_1}\right] & \left[\frac{d_1-\delta_2}{V_1}\right] \\ [0] & \left[-\frac{\delta_3}{V_1}\right] \\ [0] & \left[\frac{\delta_5-\delta_4}{A_s l_d}\right] \\ [0] & \left[\frac{\delta_1-\delta_5}{A_s l_b}\right] \\ [0] & [0] \end{bmatrix} \quad \text{Eq. 202}$$

7.2.2 PWA and SMLP representations of the ASP

The operating point (***OP***), the identification points (***IPs***) and the partitions for the PWA system will be defined, respectively, as the input vectors (MVs + DVs) and input ranges. Table 34 provides the IPs values used to obtain the models:

Table 34 – Input-based IPs identification points 1, 2 and 3.

	$\boldsymbol{u_1}(\mathbf{m^3/h})$	$\boldsymbol{u_2}(\mathbf{m^3/h})$	$\boldsymbol{d_1}\left(\frac{\mathbf{mg}}{\mathbf{L}}\right)$	$\boldsymbol{d_2}\left(\frac{\mathbf{m^3}}{\mathbf{h}}\right)$
$\boldsymbol{IP_1}$	170	1020	1	1700
$\boldsymbol{IP_2}$	720	45	0.70	900
$\boldsymbol{IP_3}$	400	400	4.100	1000

The PWA partition definitions and voting rules are provided below in Eq. 203 to Eq. 206. Variables M_1^{vote}, M_2^{vote} and M_3^{vote} signify the number of votes cast to each model according to the position of each variable. They are set to zero each time the prediction is computed.

$$\begin{aligned}
&\text{if } u_1 \leq 285 \rightarrow M_1^{vote} = M_1^{vote} + 1;\\
&\text{if } 285 < u_1 \leq 560 \rightarrow M_3^{vote} = M_3^{vote} + 1;\\
&\text{if } 560 < u_1 \rightarrow M_2^{vote} = M_2^{vote} + 1;
\end{aligned} \qquad \text{Eq. 203}$$

$$\begin{aligned}
&\text{if } \mathrm{u}_2 \leq 222.5 \rightarrow M_2^{vote} = M_2^{vote} + 1;\\
&\text{if } 222.5 < u_2 \leq 710 \rightarrow M_3^{vote} = M_3^{vote} + 1;\\
&\text{if } 710 < u_2 \rightarrow M_1^{vote} = M_1^{vote} + 1;
\end{aligned} \qquad \text{Eq. 204}$$

$$\begin{aligned}
&\text{if } \mathrm{d}_1 \leq 0.85 \rightarrow M_2^{vote} = M_2^{vote} + 1;\\
&\text{if } 0.85 < \mathrm{d}_1 \leq 2.55 \rightarrow M_1^{vote} = M_1^{vote} + 1;\\
&\text{if } 2.55 < \mathrm{d}_1 \rightarrow M_3^{vote} = M_3^{vote} + 1;
\end{aligned} \qquad \text{Eq. 205}$$

$$\begin{aligned}
&\text{if } \mathrm{d}_2 \leq 950 \rightarrow M_2^{vote} = M_2^{vote} + 1;\\
&\text{if } 950 < d_2 \leq 1350 \rightarrow M_3^{vote} = M_3^{vote} + 1;\\
&\text{if } 1350 < d_2 \rightarrow M_1^{vote} = M_1^{vote} + 1;
\end{aligned} \qquad \text{Eq. 206}$$

As for the SMLP system, SMLP method 2 was chosen to represent the plant, and thus the same IPs defined in Table 34 can be used. A second order polynomial was used as an attraction function, and regression analysis was performed to obtain its parameters, thus minimising the error between the nonlinear model and the SMLP system.

7.2.3 Comparison of results

The linearised models of plants contained in Table 28 have eigenvalues out of the unit circle and therefore are not BIBO stable. A better visual comparison between the PWA and the SMLP systems can be obtained if the plant reaches a steady-state after the input changes, and for this reason, a new plant was created which is BIBO stable. This new plant is based on plant A but with different heights for the first, second and third layers of the settlers (and $l_d = 6$ m, $l_b = 3$ m, $l_r = 4.5$ m).

The prediction of the single controlled variable of the ASP, the chemical oxygen demand (COD), was obtained from the nonlinear model as well as from PWA and the SMLP systems. An initial OP distinct from the three linearisation points was selected

($\boldsymbol{OP_k} = [345, 115, 0.40, 1150]$), and also a rule is provided for describing the OP as a function of time, which is presented in Eq. 207, yielding a series of control moves and disturbances:

$$\boldsymbol{OP_{k+\alpha}} = \begin{bmatrix} OP_{1,k} \cdot \exp(-0.002\alpha) \\ OP_{2,k} \cdot \exp(-0.002\alpha)^{-1} \\ OP_{3,k} + 10 \cdot [1 - \exp(-0.1\alpha)] \\ OP_{4,k} \cdot (1 + \sin(\alpha)/10) \end{bmatrix}, \alpha = 1, \dots, \mathrm{p} \qquad \text{Eq. 207}$$

Comparing side-by-side the predictions shown in Fig. 31, the SMLP prediction was considerably more accurate and further reduced the error arising from process nonlinearity. One can also see the significant discontinuities caused by sudden changes in the state update rule for the PWA system, and also the SMLP approximation is more accurate most of the time. Using the nonlinear model as a reference, the integration of error over the period of 100 minutes yielded 138.32 $\mathrm{mg} \cdot \frac{\mathrm{min}}{\mathrm{L}}$ and 76.27 $\mathrm{mg} \cdot \frac{\mathrm{min}}{\mathrm{L}}$ for the PWA and the SMLP systems respectively. This translates into a 44.86% error additional reduction provided by the SMLP method over the PWA in this use scenario.

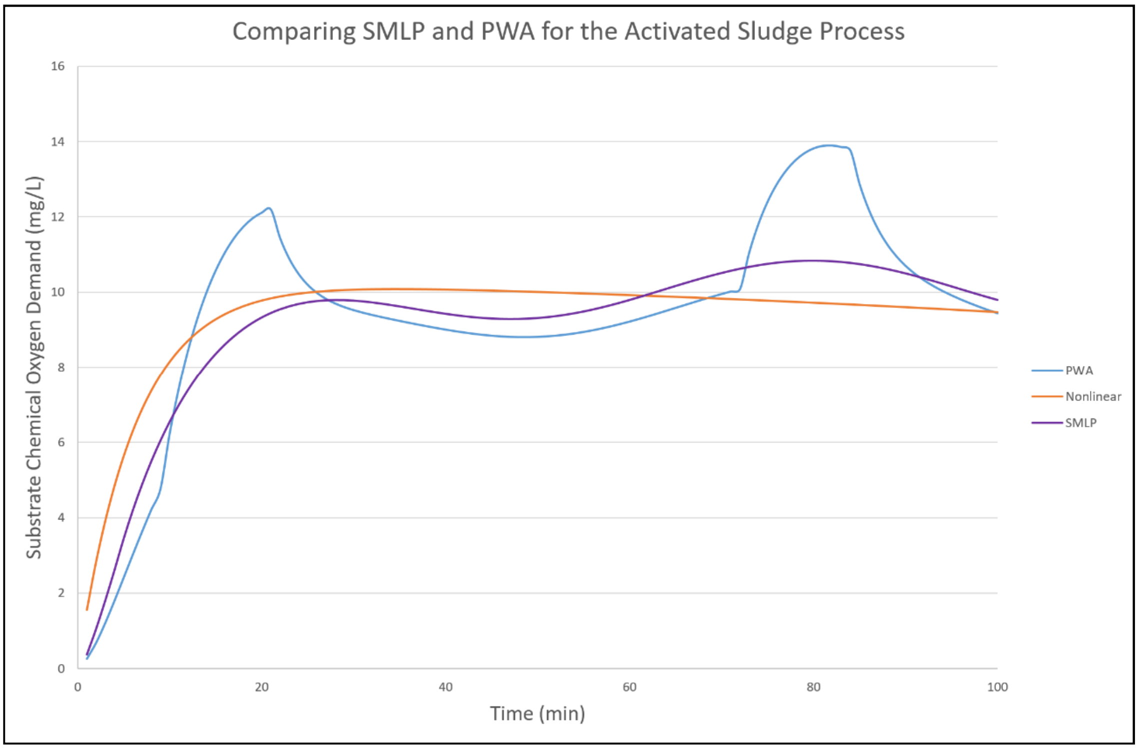

Fig. 31 – Predictions generated by the SMLP and PWA systems and by the ASP nonlinear model.

7.3 Final considerations about the Simultaneous Multi-Linear Prediction method

The Simultaneous Multi-Linear Prediction (SMLP) method becomes an interesting option when it is desired to obtain linear approximations of an unknown model relying just on its numerical output. Moreover, the SMLP is recommended when the number of available linearisations (sub-models) is limited but the expected operational range is considerably wide. Thus, the SMLP brings performance improvements in some frequent and important situations: when commercial, closed source dynamic simulation packages are to be used for designing new plants, or when an existing plant is to be modelled for MPC. The models show that the SMLP is able to greatly reduce the inherent error incurred when using linearised models. It was shown that SMLP systems can be applied in the fields of control, dynamic analysis and, as demonstrated in this Thesis, integrated process design and control.

The main alternative to using an SMLP system would be a multi-model PWA system. But for the typical SMLP use scenario, this may not be as good an option. The issue is that switching the state update rules based on arbitrary, fixed thresholds may not improve the quality of the multi-model prediction. This is true because a linear sub-model of a piecewise affine system might not be more accurate than the rest in the entirety of its region, i.e., there is no guarantee of obtaining an optimal partition. But even optimal partitions can sometimes be a problem. Sudden and possibly drastic model transitions may happen as the OP crosses a threshold. This issue is normally minimised by the introduction of restrictions to obtain smoother transitions between the region boundaries, but this reduces the fidelity of the approximation. Avoiding this trade-off, the use of SMLP shifts the priority among sub-models continuously, without the need for boundaries as a means to achieve a more faithful reproduction of the nonlinear model. No threshold or partition rule is used and changes in the output update equation are likely to be smoother. The SMLP regression analysis is unconstrained and solely focused on reducing model mismatch. Also, the SMLP method does not require the use of mixed-integer linear programming (MILP), allowing the use of faster optimisation algorithms.

SMLP systems can be used to evaluate expected flowsheet control performance for integrated process design and control. In the case study provided, an SMLP system was assembled to predict the dynamic behaviour of a crude oil distillation plant. This provided the basis successfully for an application of the Economic MPC optimisation (EMOP) index presented in Strutzel and Bogle (2016). Hence, by using the EMOP index together with the SMLP, the former became more precise and accurate in its analysis of the economic impact of the MPC control effort.

8 Conclusions

This Thesis presents a novel methodology for integrated design and control of chemical processes, the Economic MPC Optimisation (EMOP) index. It is used to economically assessing flowsheet designs from the standpoint of zone constrained MPC with single layer economic optimisation, whose consequences on optimal plant design had not yet been explored at the time of writing. It is also a suitable ‘Controllability Analysis’ approach to address industrial-scale, highly complexes systems, with special emphasis on petroleum refining processes.

The EMOP index uses knowledge of process economics to indicate the differences in expected revenue (or operating expenses) between candidate process designs. For this goal, the weighting parameters in the cost function in EMOP must be representative of the real operating conditions, and a careful selection of operation scenarios must be done based on expected disturbances and price changes of raw materials, products and energy. Assessing the effects of measured, modelled disturbances such as feed composition changes is an important part of the methodology, being precisely the kind of disturbance MPC controllers can successfully deal with. Hence, defining the scenarios for the EMOP index evaluation is a key aspect of the analysis, requiring knowledge of both the challenges faced during operation and the marketplace.

The fact that the EMOP cost function slightly resembles MPC cost function may be confusing to some readers. Hence, it is important to point out that the closed-loop behaviour was not investigated in this Thesis, which instead dealt with the issue of state reachability constrained by zone control and disturbances, where the optimal steady-state is defined by process economics. The option of not investigating closed-loop behaviour was based on the desire of avoiding the generality loss that would occur if concerns such as the choice of MPC algorithm and the selection of tuning parameters were incorporated in the EMOP method. It is usually the flowsheet design rather than the MPC algorithm that limits economic performance, and hence the focus was placed solely on the open-loop behaviour. Hence, the EMOP isolates the contribution of each flowsheet’s characteristics to an expected economic performance from other facts. A downside of this decision is that some advances in MPC theory, such as guaranteed closed-loop stability, could not be incorporated into the EMOP methodology. Assessing closed-loop steady-states instead of open-loop could lead to the selection of plants which are not stable but are stabilisable or controllable. The optimal operating points for such plants are likely to lie closer to the active restrictions, and profitability could thus be higher, but the operation could become impractical even with small malfunctions of the control system.

The other unique contribution presented in this Thesis is the Simultaneous Multi-Linear Prediction (SMLP) method, a representation of dynamic systems composed of multiple linear state-space models that can approximate nonlinear models or existing plants. Strong evidence was given that the SMLP can reduce the error greatly inherently possessed by linearisations while keeping its advantages. An important feature of SMLP systems is that they can be built without being based on an explicit set of equations based on first-principles. Only the numerical output of the nonlinear model (or real-world plant) is required, differentiating the SMLP from competing approaches.

A comparison between a PieceWise Affine system (a standard multi-model formulation) and the SMLP showed that the later provided an accuracy gain of 44.86%, using the nonlinear model as a reference. Future work could involve the inclusion of logical variables in the SMLP state update equations, which is an element found in the formulations belonging to the linear hybrid framework. This would be useful to represent complications such as saturation functions, discrete inputs, qualitative outputs, bilinear systems and finite state machines, which are important in many processes.

It is also important to discuss openly the weaknesses of the EMOP/SMLP framework. First of all, the large-scale case study presented in Chapter 6 was based on a simulation that ran on a commercial, closed-source software. Such simulators are expensive, and often the full license required to enable all the software capabilities is not available. Furthermore, it is already a daunting task to assemble a working steady-state simulation for a full plant and obtain consistent results. Using the dynamic mode greatly complicates matters and, to the author's knowledge, many design teams are satisfied with a good steady-state simulation. The main incentive behind the development of dynamic models in recent years has been providing training platforms for operators. In this case, the dynamic simulation is linked to the control system and its user interface, generating a high-fidelity environment where the operators can be prepared to perform their duties. This opens up an interesting possibility for the EMOP index and the SMLP, which is to build the models alongside the training platform, so the costs of dynamic modelling are shared between these two activities.

Another issue encountered while building SMLP systems was a considerable time necessary to perform the step identification and generate linear sub-models at several IPs. A semi-automatic routine was later developed to this end, but all sub-models had to be inspected for consistency by a person, one by one. During this process, it was found out that a model can have an excellent fit to data scores, i.e., its output may match the identification data perfectly, and still be far from accurate, providing very poor results with any other input.

In my opinion, future work in the IPDCF should prioritise the multivariable control problem, with the aim of yielding global solutions that take every variable into account. Also present in future work should be the concern for the monetisation of control performance, i.e., providing a clear estimate of the gains brought on by improved design. To this end, embedding zone-constrained MPC control goal formulations is a natural choice, especially since it is the de facto standard in the chemical industry. The ever-growing computing power and software tools available to researchers will enable increasingly complex first-principles or hybrid models to be built. Such models can be used to investigate robust closed-loop performance in face of challenges such as steady-state multiplicity, recycles, and interactions between control layers, and integration with real-time optimisation algorithms. A plant thus projected will be truly optimised both from the economic, operability and sustainability standpoints. I believe the way forward should include partnerships and deep cooperation between academic institutions, automation/process software vendors, projecting and chemical companies, which has not been happening frequently enough. There is much to be gain from such cooperation since hardly any of these players alone has all the pieces necessary to solve the puzzle of designing the interconnected and smart chemical plant of the 21st century.

Future work concerning the SMLP may involve uniting it with the Linear Hybrid Systems framework. The update equations of the SMLP's multiple simultaneous sub-states may be adapted to present switching behaviour, with partitions valid over regions of the input space. Also, the update equations may be formulated with logical variables to represent other complications of the chemical process. Also, the degradation functions may be changed to ascribe some weight to past OP coordinates. This could act as a filter to the rate of change between sub-model weights in the main output prediction.

References

Adetola V, Guay M. Integration of real time optimization and model predictive control. Journal of Process Control 2010;20:125–33.

Adi VSK, Chang CT. Two-tier search strategy to identify nominal operating conditions for maximum flexibility. Ind Eng Chem Res 2011;50:10707–16.

Alfani F, Carberry JJ. Chim. Ind. (Milan) 1970;52:1192.

Alhammadi H, Romagnoli J. Process Design and Operation: Incorporating Environmental, Profitability, Heat Integration and Controllability Considerations. The Integration of Process Design and Control 2004;17:264-306.

Allgöwer F, Badgwell T, Qin J, Rawlings J, Wright S. Nonlinear predictive control and moving horizon estimation – An introductory overview. P. Frank (ed.), Advances in Control, Highlights of ECC'99, Springer, London, 391–449.

Altimari P, Bildea C. Integrated design and control of plantwide systems coupling exothermic and endothermic reactions. Comput Chem Eng 2009;33:911–23.

Alvarado-Morales M, Hamid M, Sin G, Gernaey K, Woodley J, Gani R. A model-based methodology for simultaneous design and control of a bioethanol production process. Comput Chem Eng 2010;23(12):2043–61.

Alvarez H. Introducción al diseno simultáneo de proceso y control. Colombia: Académica Espanola; 2012.

Alvarez L. Tesis de Maestría en Ingeniería Química Metodología para el diseno˜ de control total de planta Tesis de Maestría en Ingeniería Química. Universidad Nacional de Colombia-Medellín; 2008.

Alvarez LA, Francischinelli EM, Santoro BF, Odloak D. Stable model predictive control for integrating systems with optimizing targets. Ind Eng Chem Res 2009;48(20):9141-50.

Angeli D, Amrit R, Rawlings JB. On Average Performance and Stability of Economic Model Predictive Control. IEEE Trans Autom Control 2012;57(7):1615–26.

Angira R, Alladwar S. Optimisation of dynamic systems: A trigonometric differential evolution approach. Comput Chem Eng 2007;31:1055–63.

Angira R, Babu BV. Optimisation of process synthesis and design problems: A modified differential evolution approach. Chem Eng Sci 2006;61:4707–21.

Angira R. Evolutionary computation for optimisation of selected nonlinear chemical processes [PhD Thesis]. Birla Institute of Technology & Science 2005, Pilani.

Asteasuain M, Bandoni A, Sarmoria C, Brandolin A. Simultaneous process and control system design for grade transition in styrene polymerization. Chem Eng Sci 2006;61:3362–78.

Asteasuain M, Brandolin A, Sarmoria C, Bandoni A. Integrated process and control system design of polymerization reactors under uncertainty. Optimal grade transition operation. In: 2nd Mercosur congress on chemical engineering. 4th Mercosur congress on process systems engineering; 2005.

Asteasuain M, Sarmoria C, Brandolin A, Bandoni A. Integration of control aspects and uncertainty in the process design of polymerization reactors. Chem Eng J 2007;131:135–44.

Babu BV, Angira R. Modified differential evolution for optimisation of nonlinear chemical processes. Comput Chem Eng 2006;30:989–1002.

Baccelli F, Cohen G, Olsder G, Quadrat J. Synchronization and Linearity. Wiley, 1992.

Bahakim S, Ricardez-Sandoval L. Simultaneous design and MPC-based control for dynamic systems under uncertainty: a stochastic approach. Comput Chem Eng 2014;63(17):66–81.

Bahri P, Bandoni A, Romagnoli J. Effect of disturbances in optimising control: the steady state open-loop back-off problem. AIChE J 1996b;42:983–94

Bahri P. PhD Thesis A new integrated approach for operability analysis of chemical plants PhD Thesis. Sydney: University of Sydney; 1996.

Bahri PA, Bandoni JA, Barton GW, Romagnoli JA. Back-off calculations in optimising control: a dynamic approach. Comput Chem Eng 1995;19:699-708.

Bahri PA, Bandoni JA, Romagnoli JA. Integrated flexibility and Controllability Analysis in design of chemical processes. AIChE J 1997;43:997–1015.

Bahri PA, Bandoni JA, Romagnoli JA. Operability assessment in chemical plants. Comput Chem Eng 1996a;S20:S787–92.

Baker R, Swartz C. Interior point solution of integrated plant and control design problems with embedded MPC. In: AIChE annual meeting; 2006. p. 12–7.

Bandoni JA, Romagnoli JA, Barton G. On optimizing control and the effect of disturbances: calculation of the open loop backoffs. Comput Chem Eng 1994;18S:105S–9S.

Bansal V, Perkins J, Pistikopoulos E. A case study in simultaneous design and control using rigorous, mixed-integer dynamic optimisation models. Ind Eng Chem Res 2002b;41(4):760–78.

Bansal V, Perkins J, Pistikopoulos E. Flexibility analysis and design of dynamic processes with stochastic parameters. Comput Chem Eng 1998;22S:S817–20.

Bansal V, Perkins JD, Pistikopoulos E, Ross R, van Schijndel JMG. Simulta-neous design and control optimisation under uncertainty. Comput Chem Eng 2000a;24:261–6.

Bansal V, Perkins JD, Pistikopoulos E. Flexibility analysis and design of linear systems by parametric programming. AIChE J 2000b;46:335.

Bansal V, Perkins JD, Pistikopoulos E. Flexibility analysis and design using a parametric programming framework. AIChE J 2002a;48(12):2851–68.

Bansal V, Perkins JD, Pistikopoulos EN. A case study in simultaneous design and control using rigorous, mixed-integer dynamic optimisation models. Ind Eng Chem Res 2002;41:760–78.

Bansal V, Sakizlis V, Ross R, Perkins JD, Pistikopoulos EN. New algorithms for mixed integer dynamic optimisation. Comput Chem Eng 2003;27:647–68.

Barton G, Padley M, Perkins J. Incorporating operability measures into the process synthesis stage of design. In: Proceedings of IFAC workshop on interactions between process design and process control. London: Pergamon Press; 1992. p. 95–8.

Bemporad A, Garulli A, Paoletti S, Vicino A. A bounded-error approach to piecewise affine system identification. IEEE Trans Autom Control 2005;50(10):1567–80.

Bemporad A, Morari M. Robust model predictive control: A survey, Lecture Notes in Control and Information Sciences 1999;245:207–226.

Bemporad A, Morari M: Control of systems integrating logic, dynamics, and constraints. Automatica 1999, 35(3):407–27.

Bildea C, Dimian A, Cruz S, Iedema P. Design of tubular reactors in recycle systems. Comput Chem Eng 2004;28(1–2):63–72.

Bildea C, Dimian A. Fixing flow rates in recycle systems: Luyben's rule revisited. Ind Eng Chem Res 2003;42:4578.

Biss D, Perkins JD. Application of input-output Controllability Analysis to chemical processes. Proc ECC 1993, Groningen, Netherlands, 1056-1059.

Blanco A, Bandoni A. Interaction between process design and process operability of chemical processes: an eigenvalue optimisation approach. Comput Chem Eng 2003;27(8–9):1291–301.

Brambilla A. Distillation Control & Optimization: Operation Fundamentals through Software Control. McGraw-Hill 2014. ISBN-13: 978-0071820684.

Brengel DD, Seider W. Coordinated design and control optimization of nonlinear processes, Chem Eng Com 1992;16:861–86.

Bristol EH. On a new measure of interaction for multivariable process control. IEEE Trans Autom Control 1966;11:186–93.

Calderón C, Alzate A, Gómez L, Alvarez H. State Controllability Analysis and re-design for a wastewater treatment plant. In: Mediterranean conference on control and automation; 2012a.

Calderón C, Gómez L, Alvarez H. Nonlinear state space controllability: set theory vs differential geometry. In: Congreso Latinoamericano de Control Automático; 2012b.

Campo PJ, Morari M. Achievable closed-loop properties of systems under decentralized control: Conditions involving the steady-state gain. IEEE Trans. Autom. Control 1994;39:932– 43.

Campo PJ, Morari M. Robust model predictive control. Proc American Contr Conf 1987;2:1021–26.

Chang C, Li B, Liou C. Development of a generalized mixed integer nonlinear pro-gramming model for assessing and improving the operational flexibility of water network designs. Ind Eng Chem Res 2009;48(7):3496–504.

Chawankul N, Ricardez-Sandoval LA, Budman H, Douglas PL. Integration of Design and Control: A Robust Control Approach Using MPC. The Canadian Journal of Chemical Engineering 2007;85(4):433–46.

Chen H, Allgower F. A quasi-infinite horizon nonlinear model predictive control scheme with guaranteed stability. Automatica 1998;34(10):1205–17.

Cheng Y, Yu Ch. Effects of process design on recycle dynamics and its implication to control structure selection. Ind Eng Chem Res 2003;42:4348–65.

Christofides PD, Scattolini R, de la Peña DM, Liu J. Distributed model predictive control: A tutorial review and future research directions. Comput Chem Eng 2013;51:21–41.

Christophersen FJ. Optimal Control and Analysis for Constrained Piecewise Affine Systems. Dissertation ETH Zurich No. 16807.

Cottle RW, Pang JS, Stone RE. The Linear Complementarity Problem, Academic Press, Cambridge, 1992.

Cuninghame-Green R. Minimax Algebra. Springer, 1979.

Cutler CR. Dynamic Matrix Control: An Optimal Multivariable Control Algorithm with Consraints. Ph.D. Thesis, U. of Houston 1983.

Cutler CR, Ramaker BL. Dynamic Matrix Control-A Computer Control Algorithm. AIChE 86th National Meeting 1979, Houston, TX.

De Schutter B, Van den Boom T. On model predictive control for max-min-plus-scaling discrete event systems. Automatica 2001, 37(7):1049–56.

De Wal MV, Jager BD. A review of methods for input/output selection. Automatica 2001;37:487–510.

Denn MM, Lavie R. Dynamics of plants with recycle. Chem Eng J 1982;24:55–9. Dimian A, Groenendijk A, Kersten S, Iedema P. Effect of recycle interactions on

Dimian AC, Groenendijk AJ, Kersten SRA, Iedema PD. Effect of Recycle Interactions on Dynamics and Control of Complex Plants. Comput Chem Eng 1997;21:291–96.

Dimitriadis V, Pistikopoulos E. Flexibility analysis of dynamic systems. Ind Eng Chem Res 1995;34(12):1451–4462.

Dominguez D, Revollar S, Francisco M, Lamanna R, Vega P. Simultaneous synthesis and integrated design of chemical processes using IMC PID tuning methods. In: International conference on chemical and process engineering (IcheaP); 2009.

Dougherty D, Cooper D. A practical multiple model adaptive strategy for single-loop MPC. Control Engineering Practice 2003:11:141–159.

Downs J, Moore C. Steady-State Gain Analysis for Azeotropic Distillation. Proceedings of the Joint American Control Conference, 1981; Paper WP-7.

Downs JJ, Vogel EF. A plant wide industrial process control problem. American Institute of Chemical Engineering Annual mtg. Chicago, USA, 1990.

E. Sontag, “Nonlinear regulation: The piecewise linear approach,” IEEE Trans Autom Control, vol. 26, no. 2, pp. 346–358, 1981.

Edgar TF. Control and operations: when does controllability equal profitability? Comput Chem Eng 2004;29:41–49.

Egea JA, Vries D, Alonso A, Banga JR. Global optimisation for integrated design and control of computationally expensive process models. Ind Eng Chem Res 2007;46:9148–57.

Ekawati E, Bahri P. Integration of output controllability index within dynamic operability framework in process system design. J Process Contr 2003;13: 717–27.

Ekawati E. PhD Thesis The development of systematic controllability assessment for process control designs PhD Thesis. School of Engineering Science, Murdoch University; 2003.

Elliott T, Luyben WL. Capacity-based approach for the quantitative assessment of process controllability during the conceptual design stage. Ind Eng Chem Res 1995;34:3907–15.

Elliott T, Luyben WL. Quantitative assessment of controllability during the design of a ternary system with two recycle streams. Ind Eng Chem Res 1996;35: 3470–9.

Exler O, Antelo LT, Egea JA, Alonso AA, Banga JR. A tabu search-based algorithm for mixed-integer nonlinear problems and its application to integrated process and control system design. Comput Chem Eng 2008;32:1877–91.

Farina L, Rinaldi S. Positive Linear Systems: Theory and Applications. Wiley 2000.

Ferrari-Trecate G, Muselli M, Liberati D, Morari M. A clustering technique for the identification of piecewise affine systems. Automatica 2003, 39(2):205–17.

Findeisen R, Allgöwer F. An introduction to nonlinear model predictive control. 21st Benelux Meeting on Systems and Control 11, 119-141, 2002.

Flores-Tlacuahuac A, Biegler L. Integrated control and process design during optimal polymer grade transition operations. Comput Chem Eng 2008;32: 2823–37.

Flores-Tlacuahuac A, Biegler L. Simultaneous mixed-integer dynamic optimisation for integrated design and control. Comput Chem Eng 2007;31:588–600.

Francisco M, Revollar S, Vega P, Lamanna R. Simultaneous synthesis, design and control of processes using model predictive control. In: Proceedings of international symposium on advanced control of chemical processes (Adchem); 2009.

Francisco M, Vega P, Alvarez H. Robust integrated design of process with terminal penalty model predictive controllers. Ind Eng Chem Res 2011;89:1011−24.

Francisco M. PhD Tesis Diseno simultaneo de procesos y sistemas de control predictivo mediante índices de controlabilidad basados en normas PhD Tesis. Spain: Universidad de Salamanca; 2011.

Gani R, Hytoft G, Jaksland C, Jensen A. An integrated computer aided system for integrated design of chemical processes. Comput Chem Eng 1997;21:1135–46.

Garcia CE, Morari M. Internal model control. A unifying review and some new results. Ind Eng Chem Process Des Dev 1982;21(2):308–23.

Garcia CE, Prett DM, Morari M. Model predictive control: theory and practice – a survey. Automatica 1989;25(3):335-48.

Garcia EG, Morshedi AM. Quadratic Programming Solution of Dynamic Matrix Control (QDMC). Chem Eng Com 1986;46(1-3):73-87.

Garulli A, Paoletti S, Vicino A. A survey on switched and piecewise affine system identification. 16th IFAC Symposium on System Identification 2012; The International Federation of Automatic Control.

Gareth J. An Introduction to Statistical Learning: with Applications in R. Springer, 2013:176. ISBN 978-1461471370.

Georgakis C, Uzturk D, Subramanian S, Vinson D. On the operability of continuous processes. Control Eng Pract 2003;11:859–69.

Georgiou A, Floudas C. Structural properties of large scale systems. Int J Control 1990;51(1):169–87.

Gerhard J, Monnigmann M, Marquardt W. Constructive nonlinear dynamics foundations and application to robust nonlinear control. In: Meurer T, Graichen K, Gilles ED, editors. Control and observer design for nonlinear finite and infinite dimensional systems, vol. 322. Berlin: Springer; 2005. p. 165–82.

González AH, Odloak D. A stable MPC with zone control. Journal of Process Control 2009;19(1):110–122.

Gonzalez AH, Perez JM, Odloak D. Infinite horizon MPC with non-minimal state space feedback. J Proc Control 2009;19:473–81.

Gopinath, R., Bequette, B. W., Roy, R. J., Kaufman, H., & Yu, C. Issues in the design ofa multirate model-based controller for a nonlinear drug infusion system. Biotechnology Progress 1995;11:318–32.

Gouvêa MT, Odloak D. One-layer real time optimization of LPG production in the FCC unit: Procedure, advantages and disadvantages. Comput Chem Eng 1998;22:191–98.

Grosch R, Mönnigmann M, Marquardt W. Integrated design and control for robust performance: application to an MSMPR crystallizer. J Process Contr 2008;18:173–88.

Grosdidier P, Morari M, Holt BR. Closed-Loop properties from Steady-State Gain Information. Ind Eng Chem Fundam 1985;24:221–35.

Grosman B, Dassau E, Zisser HC, Jovanovič L, Doyle III FJ. Zone Model Predictive Control: A Strategy to Minimize Hyper- and Hypoglycemic Events. Journal of Diabetes Science and Technology 2010;4(4).

Grossmann IE, Floudas C. Active constraint strategy for flexibility analysis of chemical processes. Comput Chem Eng 1987;11(6):675–93.

Grossmann IE, Halemane KP, Swaney RE. Optimization Strategies for Flexible Chemical Processes. Comput Chem Eng 1983;7(4):439–62.

Grossmann IE, Morari M. Operability, resiliency and flexibility – process design objectives for a changing world. In: Westerberg AW, Chien HH, editors. Proceedings of 2nd international conference on foundations of computer-aided process design; 1984. p. 931.

Grossmann IE, Straub DA. Recent Developments in the Evaluation and Optimization of Flexible Chemical Processes. In: Puigjaner L, Espuna A, editors. Computer-oriented process engineering, vol. 10. Amsterdam: Elsevier; 1991. p. 49–59.

Guerra I, Lamanna R, Revollar S. An activated-sludge-process application of integrated design and predictive control with instantaneous linearization. In: IEEE Mediterranean conference on control and automation; 2012.

Gutierrez G, Ricardez-Sandoval LA, Budman H, Prada C. An MPC-based control structure selection approach for simultaneous process and control design. Comput Chem Eng 2013., http://dx.doi.org/10.1016/j.compchemeng.2013.08.014.

Gutiérrez G, Vega P. Integrated design of activated sludge process taking into account the closed loop controllability. In: Proceedings of the ESCAPE, 10; 2000. p. 63–9.

Gutierrez G. PhD Tesis Diseno˜ integrado y síntesis de procesos aplicada al proceso de fangos activados PhD Tesis. Spain: Universidad de Valladolid; 2000.

Halemane KP, Grossmann IE. Optimal process design under uncertainty. AIChE J 1983;29:425–33.

Hamid M, Kamaruddin bin Abd. Model-based integrated process design and controller design of chemical processes. Ph.D. Thesis; 2011.

Hamid M, Sin G, Gani R. Integration of process design and controller design for chemical processes using model based methodology. Comput Chem Eng 2010;34:683–99.

Hautus M. Controllability and stabilizability of sampled systems. IEEE Trans Autom Control 1972;17(4):528–531.

Heemels WPMH, De Schutter B, Bemporad A. Equivalence of hybrid dynamical models. Automatica 2001, 37(7):1085–91.

Heemels WPMH, Schumacher JM, Weiland S. Linear complementarity systems. SIAM J Appl Math 2000, 60(4):1234–69.

Heidarinejad M, Liu J, Christofides PD. Economic model predictive control of nonlinear process systems using Lyapunov techniques. AIChE Journal 2012;58(3):855-70.

Hermann R, Krener AJ. Nonlinear controllability and observability. IEEE Trans Autom Control 1977:5.

Holt B, Morari M. Design of resilient processing plants—V: The effect of deadtime on dynamic resilience. Chem Eng Sci 1985a;40(7):1229–37.

Holt B, Morari M. Design of resilient processing plants—VI. The effect of right-half-plane zeros on dynamic resilience. Chem Eng Sci 1985b;40(1):59–74.

Hopkins L, Lant P, Newell B. Output structural controllability: a tool for integrated process design and control. J Process Contr 1998;8(1):57–68.

Iman RL, Helton JC. An Investigation of Uncertainty and Sensitivity Analysis Techniques for Computer Models. Risk Analysis 1988;8(1):71–90.

Jacobsen E. Effect of recycle on the plant zero dynamics. Comput Chem Eng 1997;21(Suppl. 20):S279–84.

Jadbabaie A. Nonlinear Receding Horizon Control: A Control Lyapunov Function Approach. PhD thesis, California Institute of Technology, Control and Dynamical Systems, 2001.

Juloski ALj, Weiland S, Heemels WPMH. A Bayesian approach to identification of hybrid systems. IEEE Trans Autom Control 2005;50(10):1520–33.

Kalman RE, Falb PL. Arbib MA. Topic in Mathematical System Theory, MacGraw-Hill Book Company, New York, 1969.

Kalman RE. A new approach to linear filtering and prediction problems. Journal of Basic Engineering 1960;82 (1):35–45.

Kantner M, Primbs J. Nonlinear MPC Lower Bounds via Robust Simulation. Proceedings of the American Control Conference Albuquerque 1997, New Mexico.

Karafyllis I, Kokossis A. On a new measure for the integration of process design and control: The disturbance resiliency index. Chem Eng Sci 2002;57:873–86.

Keerthi SS, Gilbert EG. Optimal infinite-horizon feedback control laws for a general class of constrained discrete-time systems: stability and moving-horizon approximations. J. Opt. Theory and Applications 1988;57:265– 93.

Kim S, Linninger A. Integration of design and control for a large scale flowsheet. In: Pierucci S, Buzzi Ferraris G, editors. 20th European symposium on computer aided process engineering (ESCAPE-20). Ischia: Elsevier B.V.; 2010.

Kiss A, Bildea C, Dimian A, Iedema P. Design of recycle systems with parallel and consecutive reactions by nonlinear analysis. Ind Eng Chem Res 2005;44: 576–87.

Kookos IK, Perkins JD. An algorithm for simultaneous process design and control. Ind Eng Chem Res 2001;40:4079–88.

Kookos IK, Perkins JD. An algorithmic method for the selection of multivariable process control structures. J Proc Control 2002;12(1):85–99.

Kookos IK, Perkins JD. Heuristic-Based Mathematical Programming Framework for Control Structure Selection. Ind Eng Chem Res 2001;40 (9):2079–88.

Koppel LB. Input multiplicities in nonlinear multivariable control system. AIChE J 1982;28(6):935–45.

Kothare MV, Balakrishnan V, Morari M. Robust constrained model predictive control using linear matrix inequalities. Automatica 1996;32(10):1361-79.

Krstic M, I. Kanellakopoulos I, Kokotovic P. Nonlinear and Adaptive Control Design. Wiley, 1995.

Kurt E. Häggblom. Integral Controllability and Integrity for Uncertain Systems. IEEE. 2008 American Control Conference. Seattle, Washington, USA.

Kwon WH, Bruckstein AM, Kailath T. Stabilizing state-feedback design via the moving horizon method. Int. J. Control 1983;37(3):631–43.

Kwon WH, DG Byun. Receding horizon tracking control as a predictive control and its stability properties. Int. J. Control 1989;50(5):1807– 24.

Kwon WH, Pearson AE. A modified quadratic cost problem and feedback stabilization of a linear system. Automatic Control 1977;22(5):838–42.

Lakshminarayanan S. Recycle effect index: a measure to aid in control system design for recycle processes. Ind Eng Chem Res 2004;6:1499–511.

Lamanna R, Vega P, Revollar S, Alvarez H. Diseno˜ Simultáneo de Proceso y Control de una Torre Sulfitadora de Jugo de Cana˜ de Azúcar. Rev Iberoam Autom Inform Ind 2009;6(3):32–43.

Langson W, Chryssochoos I, Rakovic SV, Mayne DQ. Robust model predictive control using tubes. Automatica 2004;40(1):125–33.

Lau H, Alvarez J, Jensen K. Synthesis of Control Structure by Singular Value Analysis. AIChE J 1985;31:427.

Lee B, Kim Y, Shin D, Yoon E. A study of the evaluation of structural controllability in chemical processes. Comput Chem Eng 2001;25:85–95.

Lee JH, Morari M, Garcia CE. State-space interpretation of model predictive control. Automatica 1994;30(4):707-17.

Lenhoff A, Morari M. Design of resilient processing plants—I. Process design under consideration of dynamic aspects. Chem Eng Sci 1982;37:245–58.

Lewin DR. A simple tool for disturbance resiliency diagnosis and feed-forward control design. Comput Chem Eng 1996;20(1):13–25.

Lewin DR. Interaction of design and control. Proceedings of 7th Mediterranean conference on control and automation (MED 99), Hafia, Israel, 1999.

Li H, Gani R, Jørgensen S. Process-insights-based control structuring of an integrated distillation pilot plant. Ind Eng Chem Res 2003;42:4620–7.

Li P, Arellano-Garcia H, Wozny G. Chance constrained programming approach to process optimisation under uncertainty. Comput Chem Eng 2008;32(1–2): 25–45.

Lima F, Georgakis C, Smith J, Schnelle P, Vinson D. Operability-based determination of feasible control constraints for several high-dimensional nonsquare industrial processes. AIChE J 2010a;56(5):1249–61.

Lima F, Jia Z, Ierapetritou M, Georgakis C. Similarities and differences between the concepts of operability and flexibility: the steady-state case. AIChE J 2010b;56:702–16.

Lin X, Tade M, Newell R. Output structural controllability condition for the syn-Thesis of control systems for chemical processes. Int J Syst Sci 1991;22(1): 107–32.

Liu J, Muñoz de la Peña D, Christofides PD. Distributed model predictive control of nonlinear process systems. AIChE Journal 2009;55(5):1171-84.

Loeblein C, Perkins J. Structural design for on-line process optimisation: I. Dynamic economics of MPC. AIChE J 1999;45(5):1018–29.

Lowe GK, Zohdy MA. Modeling nonlinear systems using multiple piecewise linear equations. Nonlinear Analysis: Modelling and Control 2010;15(4):451–58.

Lu X, Li H, Duan J, Su D. Integrated design and control under uncertainty: a fuzzy modelling approach. Ind Eng Chem Res 2010;49:1312–24.

Lucia S, Finkler T, Engell S. Multi-stage nonlinear model predictive control applied to a semi-batch polymerization reactor under uncertainty. J Proc Cont 2013;23(9):1306–19.

Luo X, Zhou X, Wang S, Xu F. Zone Control with Model Predictive Control for Much Constrained States Process. Proceedings of the 31st Chinese Control Conference, July 25-27, 2012, Hefei, China.

Luyben ML, Floudas C. Analyzing the interaction of design and control–1. A multi-objective framework and application to binary distillation synthesis. Comput Chem Eng 1994a;18:933–69.

Luyben ML, Floudas C. Analyzing the interaction of design and control–2. Reactor–separator–recycle system. Comput Chem Eng 1994b;18:971–93.

Luyben ML, Luyben WL. Design and control of a complex process involving two reaction steps, three distillation columns, and two recycle streams. Ind Eng Chem Res 1996;34:3885–98.

Luyben WL, Pszalgowski KM, Schaefer MR, Siddons C. Design and control of conventional and reactive distillation processes for the production of butyl acetate. Ind Eng Chem Res 2004;43(25):8014-25.

Luyben WL. Design and control of gas-phase reactor/recycle processes with reversible exothermic reactions. Ind Eng Chem Res 2000;39:1529.

Luyben WL. Distillation Design and Control Using Aspen Simulation, 2nd Edition. Wiley 2013. ISBN: 978-1-118-41143-8.

Luyben WL. Dynamics and control of recycle systems. 1. Simple open-loop and closed-loop systems. Ind Eng Chem Res 1993a;32:466–75.

Luyben WL. Dynamics and control of recycle systems. 2. Comparison of alternative process designs. Ind Eng Chem Res 1993b;32:476–86.

Luyben WL. Dynamics and control of recycle systems. 3. Alternative process designs in a ternary system. Ind Eng Chem Res 1993c;32:1142–53.

Luyben WL. Effect of feed impurity on the design and control of a ternary two-recycle process. Ind Eng Chem Res 1999;38:3430–7.

Luyben WL. Snowball effects in reactor/separator processes with recycle. Ind Eng Chem Res 1994;33:299–305.

Lyapunov AM. The General Problem of the Stability of Motion (In Russian), Doctoral dissertation, Univ. Kharkov 1892 English translations: (1) Stability of Motion, Academic Press, New-York & London, 1966 (2) The General Problem of the Stability of Motion, (A. T. Fuller trans.) Taylor & Francis, London 1992.

Ma K, Valdés-González H, Bogle IDL. Process design in SISO systems with input multiplicity using bifurcation analysis and optimisation. J Proc Control 2010;20(3):241-47.

Mahdi Sharifzadeh. Integration of process design and control: A review. Chem Eng Res Des 2013; 91(12): 2515–49.

Malcolm A, Polan J, Zhang L, Ogunnaike BA, Linninger A. Integrating systems design and control using dynamic flexibility analysis. AIChE J 2007;53: 2048–61.

Marselle D, Morari M, Rudd D. Design of resilient processing plants. II. Design and control of energy management systems. Chem Eng Sci 1982;37:259–70.

Matallana L, Blanco A, Bandoni A. Nonlinear dynamic systems design based on the optimisation of the domain of attraction. Math Comput Model 2011;53(5–6):731–45.

McAvoy T, Braatz RD. Controllability of Processes with Large Singular Values. Ind Eng Chem Res 2003;42:6155–65.

McAvoy T, Miller R. Incorporating integrating variables into steady-state models for plantwide control analysis and design. Ind Eng Chem Res 1999;38:412–20.

Meeuse F, Deugd R, Kapteijn F, Verheijen P, Ypma S. Increasing the selectivity of the Fischer Tropsch process by periodic operation. Comput Aided Chem Eng 2001;9:699–704.

Meeuse F, Grievink J, Verheijen P, Vander M. Conceptual design of processes for structured products. In: Malone, Trainham, Carnahan, editors. Proceedings ESCAPE-5; 2000. p. 324–8.

Meeuse F, Grievink J. Optimum controllability of distributed systems based on non-equilibrium thermodynamics. In: Grievink J, Schijndel JV, editors. ESCAPE-12; 2002. p. 259–64.

Meeuse F, Grievink J. Thermodynamic controllability assessment in process synthe-sis. In: Seferlis P, Georgiadis MC, editors. The integration of process design and control. Amsterdam: Elsevier B.V.; 2004. p. 146–67.

Meeuse F. PhD Thesis On the design of chemical processes with improved control-lability characteristics PhD Thesis. Technische Universiteit Delft; 2002.

Mhaskar P, El-Farra NH, Christofides PD. Predictive control of switched nonlinear systems with scheduled mode transitions IEEE Trans Autom Control 2005:50(11):1670-80.

Mhaskar P, El-Farra NH, Christofides PD. Stabilization of nonlinear systems with state and control constraints using Lyapunov-based predictive control. Systems & Control Letters 2006;55(8):650-59.

Miranda M, Reneaume J, Meyer X, Meyer M, Szigetid F. Integrating process design and control: an application of optimal control to chemical processes. Chem Eng Process 2008;47(11):2004–18.

Mohideen MJ, Perkins JD, Pistikopoulos EN. Optimal design of dynamic systems under uncertainty. AIChE Journal 1996b;42(8):2251–72.

Mohideen MJ, Perkins JD, Pistikopoulos EN. Optimal synthesis and design of dynamic systems under uncertainty. Comput Chem Eng 1996a;20S:S895–900.

Monnigmann M, Marquardt W. Normal vectors on manifolds of critical points for parametric robustness of equilibrium solutions of ODE systems. J Nonlinear Sci 2002;12:85–112.

Monnigmann M, Marquardt W. Steady-state process optimisation with guaranteed robust stability and flexibility: application to HDA reaction section. Ind Eng Chem Res 2005;44:2737–53.

Moon J, Kim S, Linninger A. Integrated design and control under uncertainty: embedded control optimisation for plantwide processes. Comput Chem Eng 2011;35:1718–24.

Moore C. Application of SVD for the Design, Analysis, and Control of Industrial Processes. Proceedings of the American Control Conference, 1986; Paper TA-1.

Morari M, Arkun Y, Stephanopoulos G. Studies in the synthesis of control structures for chemical processes: Part I: Formulation of the problem. Process decomposition and the classification of the control tasks. Analysis of the optimizing control structures. AIChE J 1980;26(2):220–3.

Morari M, Grimm W, Oglesby M, Prosser I. Design of resilient processing plants—VII. Design of energy management system for unstable reactors—new insights. Chem Eng Sci 1985;40(2):187–98.

Morari M, Lee JH. Model predictive control. Past, present and future. Comput Chem Eng 1999;23:667-82.

Morari M, Stephanopoulos G. Studies in the synthesis of control structures for chemical processes: Part II: Structural aspects and the synthesis of alternative feasible control schemes. Analysis of the optimizing control structures. AIChE J 1980;26(2):232–46.

Morari M, Zafiriou E, Holt B. Design of resilient processing plants. New characteri-zation of the effect of RHP zeros. Chem Eng Sci 1987;42(10):2425–8.

Morari M, Zafiriou E. Robust process control. Prentice-Hall 1989.

Morari M. Design of resilient processing plants—III. A general framework for the assessment of dynamic resilience. Chem Eng Sci 1983a;38:1881.

Morari M. Flexibility and resiliency of process systems. Comput Chem Eng 1983b;7:423–437.

Morshedi AM, Cutler CR, Skovanek TA. Optimal Solution of Dynamic Matrix Control with Linear Programming Techniques (LDMC). Proc automatic Control Conf 1985;199.

Morud J, Skogestad S. The dynamic behavior of integrated plants. In: Symposium PSE'94; 1994. p. 913–8.

Munoz D, Alvarez H, Ochoa S. ¿Hacia dónde va la integración del diseno y el control del proceso? El papel de la controlabilidad de estado y el diseno bajo incer-tidumbre. In: XIII Congreso Latinoamericano de Control Automático/VI Congreso Venezolano de Automatización y Control; 2008.

Munoz D, Gerhard J, Marquardt W. A normal vector approach for integrated process and control design with uncertain model parameters and disturbances. Comput Chem Eng 2012;40:202–12.

Muske KR, Rawlings JB. Model predictive control with linear models. AIChE J 1993;39(2):262–87.

Nakada H, Takaba K, Katayama T. Identification of piecewise affine systems based on statistical clustering technique. Automatica, 41(5):905–913, 2005.

Narraway L, Perkins JD, Barton G. Interactions between process design and process control: economic analysis of process dynamics. J Process Contr 1991;1:243–50.

Narraway L, Perkins JD. Selection of process control structure based in economics. Comput Chem Eng 1994;S18:S511–5.

Narraway L, Perkins JD. Selection of process control structure based on linear dynamic economics. Ind Eng Chem Res 1993;32:2681–92.

Novak Pintaric Z, Kravanja Z. A strategy for MINLP synthesis of flexible and operable processes. Comput Chem Eng 2004;28(6–7):1105–19.

Ochoa S, Alvarez H. A methodology for the integration of process design and control in the state space. In: Proceedings of the 2nd Mercosur congress on chemical engineering and 4th Mercosur congress on process systems engineering; 2005.

Ochoa S. Master Thesis A methodology for the design-control integration in state-space Master Thesis. Medellín: National University of Colombia; 2005 [in Spanish].

Ogata K. Modern Control Engineering. Prentice Hall, 1997.

Palazoglu A, Arkun Y. A multiobjective approach to design chemical plants with robust dynamic operability characteristics. Comput Chem Eng 1986;10:567–75.

Paoletti S, Juloski A Lj. , G. Ferrari-Trecate, and R. Vidal. Identification of hybrid systems: A tutorial. European Journal of Control 2007;13(2–3):242–60.

Patel J, Uygun K, Huang Y. A path constrained method for integration of process design and control. Comput Chem Eng 2007;32:1373–84.

Perkins JD, Walsh SPK. Optimization as a tool for design/control integration. Comput Chem Eng 1996;20(4):315–23.

Pistikopoulos EN, Van Schijndel JMG. Towards the integration of process design, process control & process operability - current status & future trends. In Foundations of Computer-Aided Process Design, Snowmass, Colorado, USA, 1999.

Polak E, Yang TH. Moving horizon control of linear systems with input saturation and plant uncertainty– part 1. robustness. Int. J. Control 1993a;58(3):613–638.

Polak E, Yang TH. Moving horizon control of linear systems with input saturation and plant uncertainty– part 2. disturbance rejection and tracking. Int. J. Control 1993b;58(3):639–663.

Porfírio CR, Odloak D. Optimizing model predictive control of an industrial distillation column, Control Engineering Practice 2011;19(10):1137–46.

Psarris P, Floudas CA. Dynamic operability of MIMO systems with time delays and transmission zeroes. I. Assessment. Chem Eng Sci 1991;46(10):2691–707.

Ramirez Jimenez E, Gani R. Methodology for the design and analysis of reaction-separation systems with recycle. 1. The design perspective. Ind Eng Chem Res 2007a;46(24):8066–83.

Ramirez Jimenez E, Gani R. Methodology for the design and analysis of reaction-separation systems with recycle. 2. Design and control algorithms. Ind Eng Chem Res 2007b;46(24):8084–100.

Rashid MA, Moses OT. Nonlinear Model-based Process Control Applications in Petroleum Refining, Springer 2000.

Revollar S, Francisco M, Vega P, Lamanna R. Genetic algorithms for the synthesis and integrated design of processes using advanced control strategies. In: International symposium on distributed computing and artificial intelligence (DCAI'08). Lecture Notes on Computer Science 50/2009; 2008. p. 205–14.

Revollar S, Francisco M, Vega P, Lamanna R. Stochastic optimisation for the simultaneous synthesis and control system design of an activated sludge process. Latin Am Appl Res 2010a;40:137–46.

Revollar S, Lamanna R, Rodríguez A, Vega P, Francisco M. Integrated design methodology for improving the economics and dynamical performance of the activated sludge process. In: Ecotechnologies for wastewater treatment. Technical, envi-ronmental and economic challenges (ECOSTP); 2012.

Revollar S, Lamanna R, Vega P, Francisco M. Multiobjective genetic algorithms for the integrated design of chemical processes using advanced control techniques. In: Pierucci S, Buzzi Ferraris G, editors. Proceedings of 20th European symposium on computer aided process engineering (ESCAPE-20). Ischia: Elsevier B.V.; 2010b.

Revollar S, Rodríguez A, Lamanna R, Francisco M, Vega P. Multiobjective genetic algorithms for the simultaneous design and control of the activated sludge process. In: Proceedings of Congress of chemical and process engineering (CHISA 2010); 2010c.

Revollar S. PhD Tesis Algortimos genéticos en el diseno integrado de procesos quími-cos PhD Tesis. Venezuela: Universidad Simón Bolívar; 2011.

Reyes F, Luyben WL. Design and control of a gas-phase adiabatic tubular reactor process with liquid recycle. Ind Eng Chem Res 2001a;40(17):3762–74.

Reyes F, Luyben WL. Design and control of tubular reactor systems with both gas and liquid recycles. Ind Eng Chem Res 2001b;40(19):4089–101.

Reyes F, Luyben WL. Steady-state and dynamic effects of design alternatives in heat-exchanger/furnace/reactor processes. Ind Eng Chem Res 2000;39:3335.

Ricardez-Sandoval L, Budman H, Douglas P. Application of robust control tools to the simultaneous design and control of dynamic systems. Ind Eng Chem Res 2009b;48:813.

Ricardez-Sandoval L, Budman H, Douglas P. Simultaneous design and control of chemical processes with application to the Tennessee Eastman process. J Process Contr 2009c;19:1377–91.

Ricardez-Sandoval L, Budman H, Douglas P. Simultaneous design and control: a new approach and comparisons with existing methodologies. Ind Eng Chem Res 2010;49:2822–33.

Ricardez-Sandoval L, Budman HM, Douglas P. Simultaneous design and control of processes under uncertainty: a robust modelling approach. J Process Contr 2008;18:735–52.

Ricardez-Sandoval L, Douglas P, Budman H. A methodology for the simultaneous design and control of large-scale systems under process parameter uncertainty. Comput Chem Eng 2011;35(2):307–18.

Ricardez-Sandoval L. Optimal design and control of dynamic systems under uncertainty: a probabilistic approach. Comput Chem Eng 2012;43(10):91–107.

Ricardez-Sandoval LA, Budman H, Douglas P. Integration of Design and Control for Chemical Processes: A Review of the Literature and Some Recent Results. Annual Reviews in Control 2009;33(2):158-71.

Richards A, How J. Robust stable model predictive control with constraint tightening. Proceedings of the American Control Conference 2006.

Rodrigues MA, Odloak D. An infinite horizon model predictive control for stable and integrating processes. Comput Chem Eng 2003;27(8):1113-28.

Rodrigues MA, Odloak D. MPC for stable linear systems with model uncertainty. Automatica 2003;39(4):569-83.

Roll J, Bemporad A, Ljung L. Identification of piecewise affine systems via mixed-integer programming. Automatica 2004, 40:37–50.

Roll J, Bemporad A, Ljung L. Identification of piecewise affine systems via mixed-integer programming. Automatica2004, 40:37–50.

Roll J. Local and Piecewise Affine Approaches to System Identification. PhD Thesis, Department of Electrical Engineering, Link¨oping University, Linkoping, Sweden, 2003. http://www.control.isy.liu.se/publications/.

Rosenbrock HH. State space and multivariable theory. Wiley Interscience Division, 1970, London.

Rouhani R, Mehra RK. Model algorithmic control (MAC): basic theoretical properties. Automatica 1982;18(4):401-14.

Russel BM, Henriksen JP, Jørgensen SB, Gani R. Integration of design and control through model analysis. Comput Chem Eng 2002;26:213–25.

Russo L, Bequette B. Effect of process design on the open loop behaviour of a jacketed exothermic CSTR. Comput Chem Eng 1996;20:417–26.

Russo L, Bequette B. Impact of process design on the multiplicity behaviour of a jacketed exothermic CSTR. AIChE J 1995;41:135–47.

Russo L, Bequette B. Operability of chemical reactors: multiplicity behaviour of a jacketed styrene polymerization reactor. Chem Eng Sci 1998;53(1):27–45.

Russo L, Bequette B. Process design for operability: a styrene polymerization appli-cation. Comput Chem Eng 1997;21(Suppl.):S571–6.

Saboo AK, Morari M, Woodcock DC. Design of Resilient Processing Plants–VIII, A Resilience Index for Heat Exchanger Networks. Chem Eng Sci 1985, 40:1553–65.

Saboo AK, Morari M. Design of resilient processing plants-IV: some new results on heat exchange network synthesis. Chem Eng Sci 1984;39(3):579–92.

Sakizlis V, Bansal V, Ross R, Perkins JD, Pistikopoulos EN. An adjoint-based algorithm for mixed integer dynamic optimisation. R. Gani & S. Jorgensen (Eds.), ESCAPE-11 2001, Kolding, Denmark;9:273–78.

Sakizlis V, Perkins J, Pistikopoulos E. Parametric controllers in simultaneous process and control design optimisation. Ind Eng Chem Res 2003;42(20): 4545–63.

Sakizlis V, Perkins JD, Pistikopoulos EN. Recent advances in optimization-based simultaneous process and control design. Comput Chem Eng 2004;28:2069–86.

Sánchez-Sánchez K, Ricardez-Sandoval L. Simultaneous design and control under uncertainty using model predictive control. Ind Eng Chem Res 2013b;52(13):4815–33.

Sánchez-Sánchez KB, Ricardez-Sandoval LA. Simultaneous process synthesis and control design under uncertainty: A worst-case performance approach. AIChE J 2013;59(7):2497–2514.

Santoro BF, Odloak D, Closed-loop stable model predictive control of integrating systems with dead time. J Proc Control 2012;22:1209–18.

Schweiger C, Floudas C. Interaction of design and control: optimisation with dynamic models. In: Hager W, Pardalos P, editors. Optimal control: theory, algorithms and applications. Kluwer Academic Publishers; 1997. p. 388–435.

Schweiger C, Floudas C. Optimisation framework for the synthesis of chemical reactor networks. Ind Eng Chem Res 1999;38:744–66.

Scokaert PO, Mayne DQ. Min-max feedback model predictive control for constrained linear systems. IEEE Trans Autom Control 1998;43(8):1136–42.

Scokaert POM, Rawlings JB. Infinite horizon linear quadratic control with constraints. Proc IFAC 1996; Vol 7a-04 1. San Francisco, USA. 109–114.

Semino D, Giuliani G. Control configuration selection in recycle systems by steady state analysis. Comput Chem Eng 1997;21(Suppl. S273).

Sharifzadeh M, Thornhill NF. Integrated design and control using a dynamic inversely controlled process model. Comput Chem Eng 2013;48:121–34.

Sharifzadeh M, Thornhill NF. Optimal selection of control structure using a steady-state inversely controlled process model. Comput Chem Eng 2012;38:126–38.

Shirangi MG, Durlofsky LJ. Closed-Loop Field Development Under Uncertainty by Use of Optimization With Sample Validation. SPE Journal 2015; 20(5):908–22.

Skogestad S, Hovd M. Use of frequency dependent RGA for control structure selection. Proceedings of ACC 1990;2133–39. San Diego USA.

Skogestad S, Morari M. Effect of disturbance directions on closed loop performance. Ind Eng Chem Res 1987;26:2029–35.

Skogestad S, Morari M. Variable selection for decentralized control. AIChE Annual Meeting 1988, Paper 128c, Washington, DC.

Skogestad S, Postlethwaite I. Multivariable feedback control analysis and design. 2nd ed. New York: Wiley; 1996.

Skogestad S. Simple analytic rules for model reduction and PID controller tuning. J Process Contr 2003;13(4):291–309.

Solovyev BM, Lewin DR. A Steady-state Process Resiliency Index for nonlinear Processes: 1. Analysis, W. L. Luyben Edition of Ind Eng Chem Res 2002.

Sontag ED. A Lyapunov-like characterization of asymptotic controllability. SIAM Journal of Control and Optimization 1983;21:462–471.

Sontag ED. Nonlinear regulation: The piecewise linear approach. IEEE Trans Autom Control 1981, 26(2):346–58.

Stanley G, Marino-Galarraga M, and McAvoy TJ. Shortcut operability analysis. 1. The relative disturbance gain. Ind Eng Chem Process Des Dev 1985;24(4):1181–88.

Stephanopoulos G, Arkun Y, Morari M. A unified approach to the synthesis of control structures for complex chemical plants. Comput Chem Eng 1979;3(1–4):573.

Strutzel FAM, Bogle IDL. Assessing plant design with regard to MPC performance. Comput Chem Eng 2016;94:180–211.

Strutzel FAM, Odloak D, Zanin AC. Economic MPC of an Industrial Diesel Hydrotreating Plant. Proceedings of the IASTED International Conference. Intelligent Systems and Control 2013.

Subramanian S, Georgakis C. Methodology for the steady-state operability analysis of plantwide systems. Ind Eng Chem Res 2005;44:7770–86.

Subramanian S, Uzturk D, Georgakis C. An optimisation-based approach for the operability analysis of continuously stirred tank reactors. Ind Eng Chem Res 2001;40:4238–52.

Tikhonov AN. О решении некорректно поставленных задач и методе регуляризации". Doklady Akademii Nauk SSSR 1963. 151:501–504. Translated in "Solution of incorrectly formulated problems and the regularization method". Soviet Mathematics. 4: 1035–1038.

Trainor M, Giannakeas V, Kiss C, Ricardez-Sandoval L. Optimal process and control design under uncertainty: a methodology with robust feasibility and stability analyses. Chem Eng Sci 2013;104:1065–80.

Tyreus B, Luyben WL. Dynamics and control of recycle systems. 4. Ternary systems with one or two recycle streams. Ind Eng Chem Res 1993;32:1154–62.

Uzturk D, Georgakis C. Inherent dynamic operability of processes: general defini-tions and analysis of SISO cases. Ind Eng Chem Res 2002;41:421–32.

Vidal R, Soatto S, Ma Y, Sastry S. An algebraic geometric approach to the identification of a class of Linear Hybrid Systems. In Proc. 42nd IEEE Conference on Decision and Control 2003:167–172.

Vinson D, Georgakis C. A new measure of process output controllability. J Process Contr 2000;10(2–3):185–94.

Vinson D, Georgakis C. Inventory control structure independence of the process operability index. Ind Eng Chem Res 2002;41:3970–83.

Walsh S, Perkins J. Application of integrated process and control system design to waste water neutralisation. Comput Chem Eng 1994;18S:S183–7.

Walsh S, Perkins JD. Operability and control in process synthesis and design. In Anderson J. L. (Ed.), Advances in chemical engineering – Process synthesis. New York: Academic Press 1996;301–402.

Wang H, Yuan Z, Chen B, He X, Zhao J, Qiu T. Analysis of the stability and controlla-bility of chemical processes. Comput Chem Eng 2011;35(6):1101–9.

Wang H, Zhang N, Qiu T, Zhao J, He X, Chen B. A process design frame-work for considering the stability of steady state operating points and Hopf singularity points in chemical processes. Chem Eng Sci 2013;99(9): 252–64.

Weitz O, Lewin D. Dynamic controllability and resiliency diagnosis using steady state process flowsheet data. Comput Chem Eng 1996;20(4): 325–36.

Wolf EA, Skogestad S, Hovd M, Mathisen KW. A procedure for Controllability Analysis. IFAC workshop on integration between process design and process control, Imperial college, 1992, London.

Wolff E, Perkins J, Skogestad S. A procedure for operability analysis. In: ESCAPE-4; 1994.

Wolff E. PhD Thesis Studies on control of integrated plants PhD Thesis. University of Trondheim, The Norwegian Institute of Technology; 1994.

Ydstie BE, Viswanath K. From thermodynamics to process control, Proc. Symp. Proc. Systems Engineering, 1994, Kyongju, Korea.

Yu C, Luyben W L. Robustness with respect to integral controllability. Ind Eng Chem Res 1986;26(5):1043–45.

Yuan Z, Chen B, Zhao J. An overview on Controllability Analysis of chemical processes. AIChE Journal 2011; 57(5):1185–1201.

Yuan Z, Chen B, Zhao J. Controllability Analysis for the liquid-phase catalytic oxidation of toluene to benzoic acid. Chem Eng Sci 2011;66:5137–47.

Yuan Z, Wang H, Chen B, Zhao J. Operating zone segregation of chemical reaction systems based on stability and non-minimum phase behavior analysis. Chem Eng J 2009;155(1–2):304–11.

Yuan Z, Zhang N, Chen B, Zhao J. Systematic Controllability Analysis for chemical processes. AIChE J 2012b;58(10):3096–109.

Zheng A, M Morari. Stability of model predictive control with mixed constraints. Automatic Control 1995;40:1818–23.

Zheng A, Mahajanam RV, Douglas JM. Hierarchical procedure for plantwide control system synthesis. AIChE J 1999;45(6):1255–65.

Zheng A, Mahajanam. A quantitative controllability index. Ind Eng Res 1999;38(3):999–1006.

Zheng ZQ. Robust Control of Systems Subject to Constraints. Ph.D. dissertation. California Institute of Technology. Pasadena, CA, U.S.A. 1995.

Ziegler JG, Nichols NB. Process lags in automatic control circuits. Transactions ASME 1943;65:433–44.

Appendix

A State-Space Formulation for Time-Delayed, Integrating Systems

The state-space formulation proposed in Strutzel (2013), which is adequate for integrating systems with time delays and was used for this Thesis. It also has complexities such as the "funnel" definition of control zones, which are not relevant to the case study contemplated here. This formulation is part of the output prediction oriented model (OPOM) framework, where the model can be interpreted as a generalisation of the step response represented in the analytical form. More details about this class of models can be found in Martins et al. (2013) and González et al. (2007), Rodrigues and Odloak (2000), Santoro and Odloak (2012), and several others, which addressed the infinite horizon MPC problem among other developments. In this Appendix, a simplified version of this state-space framework is presented.

For this purpose, assume that the multivariable system has n_u inputs and n_y outputs from which n_i are integrating variables. Considering that for each pair $(y_i,\ u_j)$, one has a transfer function of the form:

$$\boldsymbol{G_{i,j}}(\boldsymbol{s}) = \frac{b_{i,j,0}+b_{i,j,1}s+\cdots+b_{i,j,n_b}s^{n_b}}{s(s-r_{i,j,1})(s-r_{i,j,2})\cdots(s-r_{i,j,n_a})}e^{-\theta_{i,j}s} \qquad \text{Eq. 208}$$

where it is assumed that the poles of $\boldsymbol{G_{i,j}}$ are non-repeated. The step response of the above transfer function can be represented as shown in Eq. 209:

$$\boldsymbol{S_{i,j}}(\boldsymbol{s}) = \frac{G_{i,j}(s)}{s} = \frac{d_{i,j}^{0}}{s}e^{-\theta_{i,j}s} + \frac{d_{i,j,1}^{d}}{s-r_{i,j,1}}e^{-\theta_{i,j}s} + \cdots + \frac{d_{i,j,n_a}^{d}}{s-r_{i,j,n_a}}e^{-\theta_{i,j}s} + \frac{d_{i,j}^{i}}{s^2}e^{-\theta_{i,j}s} \qquad \text{Eq. 209}$$

Assuming that Δt is the sampling time, Eq. 209 is equivalent to Eq. 210:

$$S_{i,j}(k\Delta t) = 0, if\ k\Delta t \le \theta_{i,j} \qquad \text{Eq. 210}$$

Where $S_{i,j}(k\Delta t)$ is defined in Eq. 211:

$$S_{i,j}(k\Delta t) = d_{i,j}^{0} + d_{i,j,1}^{d} e^{r_{i,j,1} k\Delta t - \theta_{i,j}} + \cdots + d_{i,j,na}^{d} e^{r_{i,j,n_a} k\Delta t - \theta_{i,j}} + d_{i,j}^{i}\left(k\Delta t - \theta_{i,j}\right),\ if\ k\Delta t > \theta_{i,j} \qquad \text{Eq. 211}$$

Following the OPOM approach, the multivariable system, the step response defined in Eq. 210 can be translated into the model:

$$\begin{bmatrix} \boldsymbol{x}_{k+1}^{s} \\ \boldsymbol{x}_{k+1}^{d} \\ \boldsymbol{x}_{k+1}^{i} \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_{ny} & \boldsymbol{0} & \Delta t \boldsymbol{I}_{ny}^{*} \\ \boldsymbol{0} & \boldsymbol{F} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{ny}^{*} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{k}^{s} \\ \boldsymbol{x}_{k}^{d} \\ \boldsymbol{x}_{k}^{i} \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_{0}^{s} \\ \boldsymbol{B}_{0}^{d} \\ \boldsymbol{B}_{0}^{i} \end{bmatrix} \Delta \boldsymbol{u}_{k} + \begin{bmatrix} \boldsymbol{B}_{1}^{s} \\ \boldsymbol{B}_{1}^{d} \\ \boldsymbol{B}_{1}^{i} \end{bmatrix} \Delta \boldsymbol{u}_{k-1} + \cdots + \begin{bmatrix} \boldsymbol{B}_{\theta_{max}}^{s} \\ \boldsymbol{B}_{\theta_{max}}^{d} \\ \boldsymbol{B}_{\theta_{max}}^{i} \end{bmatrix} \Delta \boldsymbol{u}_{k-\theta_{max}} \qquad \text{Eq. 212}$$

$$\boldsymbol{y}_{k} = \left[\boldsymbol{I}_{n_y}\ \ \boldsymbol{\Psi}\ \ \boldsymbol{0}_{n_y \cdot n_y} \right] \boldsymbol{x}_{k} \qquad \text{Eq. 213}$$

Where $\boldsymbol{x}^{s} \in \mathbb{R}^{n_y}$; $\boldsymbol{x}^{d} \in \mathbb{C}^{n_d}$, $\boldsymbol{n_d} = \boldsymbol{n_y}\, \boldsymbol{n_u}\, \boldsymbol{n_a}$; $\boldsymbol{x}^{i} \in \mathbb{R}^{n_y}$, $\boldsymbol{y} \in \mathbb{R}^{n_y}$ and $\boldsymbol{\theta_{max}}$ is the largest time delay between any input and any output.

The state vector component $\boldsymbol{x}_{k}^{s}$ corresponds to the integrating states introduced into the model through the adopted incremental form of the input. The state components $\boldsymbol{x}_{k}^{d}$ and $\boldsymbol{x}_{k}^{i}$ correspond respectively to the stable and integrating states of the original system, $\boldsymbol{I}_{n_y}^{*}$ is a diagonal matrix with ones in the entries corresponding to the integrating outputs and zeros in the remaining positions. If the stable poles of the system are non-repeated, matrix F can be represented as stated in Eq. 214

$$\boldsymbol{F} = diag(e^{r_{1,1,1}\Delta t} \cdots e^{r_{1,1,n_a}\Delta t} \cdots e^{r_{1,n_u,1}\Delta t} \cdots e^{r_{1,n_u,n_a}\Delta t} \cdots e^{r_{n_y,1,1}\Delta t} \cdots e^{r_{n_y,1,n_a}\Delta t} \cdots e^{r_{n_y,n_u,1}\Delta t} \cdots e^{r_{n_y,n_u,n_a}\Delta t}) \quad \mathbf{F} \in \mathbb{C}^{\mathbf{n_d} \cdot \mathbf{n_d}} \qquad \text{Eq. 214}$$

Matrices $\boldsymbol{B}_{l}^{s}$ and $\boldsymbol{B}_{l}^{i}$, with $l = 1, \dots, \theta_{max}$ are computed as shown in Eq. 215 and Eq. 216:

$$if\ l \neq \theta_{i,j}\ \rightarrow\ \boldsymbol{B}_{l}^{s} = 0;\ \boldsymbol{B}_{l}^{i} = 0 \qquad \text{Eq. 215}$$

$$if\ l = \theta_{i,j}\ \rightarrow\ [\boldsymbol{B}_{l}^{s}]_{i,j} = \boldsymbol{d}_{i,j}^{0} + \Delta t \boldsymbol{d}_{i,j}^{i};\ \left[\boldsymbol{B}_{l}^{i}\right]_{i,j} = \boldsymbol{d}_{i,j}^{i} \qquad \text{Eq. 216}$$

Construction of matrices $\boldsymbol{B}_{l}^{d}$ is a little more subtle. If there were no dead times (*l=0*) then $\boldsymbol{B}_{l}^{d} = \boldsymbol{D}^{d}\boldsymbol{F}\boldsymbol{N}$, where matrices $\boldsymbol{D}^{d}$ and $\boldsymbol{N}$ are computed as follows:

$$\boldsymbol{D} = diag\left(d^d_{1,1,1} \cdots d^d_{1,1,n_a} \cdots d^d_{1,n_u,1} \cdots d^d_{1,n_u,n_a} \cdots d^d_{n_y,1,1} \cdots d^d_{n_y,1,n_a} \cdots d^d_{ny,n_u,1} \cdots d^d_{n_y,n_u,n_a}\right), \quad \boldsymbol{D} \in \mathbb{C}^{n_d \cdot n_d}$$ Eq. 217

$$\boldsymbol{N} = \left.\begin{bmatrix} \boldsymbol{J} \\ \boldsymbol{J} \\ \vdots \\ \boldsymbol{J} \end{bmatrix}\right\} \boldsymbol{ny}, \boldsymbol{N} \in \mathbb{R}^{n_d \cdot n_u}$$ Eq. 218

$$\boldsymbol{J} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 1 & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}, \boldsymbol{J} \in \mathbb{R}^{n_u\, n_a \cdot n_u}$$ Eq. 219

Alternatively, if $l \neq 0$, then each matrix $\boldsymbol{B^d_l}$ would have the same dimension as $\boldsymbol{D^d F N}$ where those elements corresponding to transfer functions with dead time different from l are replaced with zeros.

Finally, the matrices $\boldsymbol{\Psi}$ and $\boldsymbol{\Phi}$ that appear in the output matrix C are given by Eq. 220 and Eq. 221:

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Phi} & & \boldsymbol{0} \\ & \ddots & \\ \boldsymbol{0} & & \boldsymbol{\Phi} \end{bmatrix}, \boldsymbol{\Psi} \in \mathbb{R}^{n_y \cdot n_d}$$ Eq. 220

$$\boldsymbol{\Phi} = [1 \cdots 1], \boldsymbol{\Phi} \in \mathbb{R}^{n_u\, n_a}$$ Eq. 221

The state of the OPOM state-space representation is defined in Eq. 222:

$$\boldsymbol{x_k} = \left[\boldsymbol{x^s_k}^{\boldsymbol{T}} \quad \boldsymbol{x^d_k}^{\boldsymbol{T}} \quad \boldsymbol{x^i_k}^{\boldsymbol{T}} \quad \boldsymbol{z_{1,k}}^{\boldsymbol{T}} \quad \boldsymbol{z_{2,k}}^{\boldsymbol{T}} \quad \cdots \quad \boldsymbol{z_{\theta_{max},k}}^{\boldsymbol{T}}\right]^{\boldsymbol{T}}$$ Eq. 222

The additional components of the state defined in Eq. 222, $\boldsymbol{z_1}, \ldots, \boldsymbol{z_{\theta_{max}}}$, have a clear physical interpretation as these components correspond to the past input moves, or $\boldsymbol{z_{j,k}} = \Delta \boldsymbol{u_{k-j}}$. The following matrices states are considered:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{I}_{n_y} & \boldsymbol{0} & \Delta t \boldsymbol{I}^*_{n_y} & \boldsymbol{B}^s_1 & \boldsymbol{B}^s_2 & \cdots & \boldsymbol{B}^s_{\theta_{max}-1} & \boldsymbol{B}^s_{\theta_{max}} \\ \boldsymbol{0} & \boldsymbol{F} & \boldsymbol{0} & \boldsymbol{B}^d_1 & \boldsymbol{B}^d_2 & \cdots & \boldsymbol{B}^d_{\theta_{max}-1} & \boldsymbol{B}^d_{\theta_{max}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}^*_{n_y} & \boldsymbol{B}^i_1 & \boldsymbol{B}^i_2 & \cdots & \boldsymbol{B}^i_{\theta_{max}-1} & \boldsymbol{B}^i_{\theta_{max}} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I}_{n_u} & \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{I}_{n_u} & \boldsymbol{0} \end{bmatrix} \quad \text{Eq. 223}$$

$$\frac{\boldsymbol{B}}{\boldsymbol{D}} = \begin{bmatrix} \boldsymbol{B}^s_0 \\ \boldsymbol{B}^d_0 \\ \boldsymbol{B}^i_0 \\ \boldsymbol{I}_{n_u} \\ \boldsymbol{0} \\ \vdots \\ \boldsymbol{0} \end{bmatrix} \quad \text{Eq. 224}$$

$$\boldsymbol{C} = \left[\boldsymbol{I}_{n_y} \;\; \boldsymbol{\Psi} \;\; \boldsymbol{0}_{n_y \cdot (n_y + n_u\,\theta_{max})}\right] \quad \text{Eq. 225}$$

where $\boldsymbol{x} \in \mathbb{C}^{\boldsymbol{n_x}}$, $\boldsymbol{n_x} = \boldsymbol{2n_y} + \boldsymbol{n_d} + \boldsymbol{n_u}\,\boldsymbol{\theta_{max}}$, $\boldsymbol{z_1}, \dots, \boldsymbol{z_{\theta_{max}}} \epsilon\, \mathbb{R}^{\boldsymbol{n_u}}$. The matrices describing respectively the effects of control actions, B, and the effects of disturbances, D, are formulated in an identical way as stated in Eq. 224.

Continuous-Time Transfer Functions for the Crude Oil Distillation Process

As discussed in the first part of this appendix, this work made use of the output prediction oriented model (OPOM), a state-space formulation that is built upon the transfer functions that define the interaction between each pair of manipulated/disturbance and controlled variable. In order to identify these models, a series of step tests were carried using the dynamic simulations of the four different designs of the crude oil distillation process presented in Chapter 6. Table 35 to Table 58 display parameters to be substituted in Eq. 226 as a means to represent each model as a 5th order transfer function, which can be easily converted to any state-space formulation.

$$\boldsymbol{G_{i,j}}(s) = \frac{b_{i,j,0}+b_{i,j,1}s+b_{i,j,2}s^2+b_{i,j,3}s^3+b_{i,j,4}s^4+b_{i,j,5}s^5}{a_{i,j,0}+a_{i,j,1}s+a_{i,j,2}s^2+a_{i,j,3}s^3+a_{i,j,4}s^4+a_{i,j,5}s^5} \qquad \text{Eq. 226}$$

Table 35 – Plant 1 – Sub-model 1 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0.0000827924	0
	b4	0.0090046489	0	-0.0000642898	0	0	0	0.0051925815	0
	b3	0.0043974221	0	-0.0970145448	0.0002514577	0	0.0112212326	-0.0000020317	0.0006379532
	b2	0.0005638163	-0.6594430621	0.0354375930	0.1170434304	0.0099090955	-0.0022919057	-0.0039022526	-0.0034168098
	b1	0.0000061370	6.1537399176	0.0000181738	0.0314538294	-0.0112287701	-0.0056052935	-0.0004885562	-0.0000818437
	b0	0.0000000934	-0.1649113672	0.0000027346	0.0022399915	-0.0007643234	-0.0000044667	-0.0000146137	-0.0000082694
y_2	b5	0	0	0	0	0	0	0	-0.0039176576
	b4	0.0747179135	0	0	-0.0000711789	0	0	0	0.0442060078
	b3	0.0816719547	-0.0532769731	0	0.7364941116	0.0790217947	1.4750338736	4.6963905563	-0.0282380084
	b2	0.0078314266	0.7789631714	0.1158548845	0.8395632971	-0.0331556249	-0.2509076518	-0.4239366006	-0.0132062440
	b1	0.0000783820	1.4515902382	0.1946674648	1.2313993214	-0.0237464687	-0.6868907059	-2.7430205912	-0.0003776090
	b0	0.0000022620	0.0604834238	0.0225121073	0.1265831132	-0.0010333426	-0.0654041668	-0.3381401040	-0.0000307580
y_3	b5	0	0	0	0	0.0006031578	0	0	-0.0078241560
	b4	-0.0010643203	0	0	0	0.1770685145	0.2292102963	0	0.1387827555
	b3	-0.0451047972	0	0	0	-0.0374658115	-0.0462469651	1.3875247314	-0.1056419195
	b2	0.3155489702	0.8002098341	0	0	-0.0873803269	-0.1071820022	-0.4942325967	-0.0255144588
	b1	0.3454852269	0.6807099310	1.6339371062	14.6682006006	-0.0075465301	-0.0088016359	-0.6833511239	-0.0008565464
	b0	0.0268754518	-0.0760942812	0.2388023195	2.0351357929	-0.0000103318	-0.0000021889	-0.0490273716	-0.0000573083
y_4	b5	0	0	0	0	0	0	0.0004709939	0
	b4	-0.0162231038	0	-0.2452035375	0.0004679805	0	0.1106209763	0.0752665477	0
	b3	0.0978435571	0	0.0669171922	0.8430420219	0.3091892740	-0.0038294545	-0.0302638315	0.0259084502
	b2	0.0093171805	0	0.1641553439	0.1979932971	-0.2402945056	-0.0758286215	-0.0357991060	-0.0638219403
	b1	0.0001607394	5.0075273851	0.0258918941	0.0082076933	-0.0863192117	-0.0083275461	-0.0015410310	-0.0017041691
	b0	0.0000148895	-0.6795222504	0.0008431281	0.0000186757	-0.0000986479	-0.0000073962	-0.0000315367	-0.0001405162
y_5	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0.0867369582	0	0	258.6169725597	0.0589507346	0.0683384722	0.0606853155	0.0324977233
	b2	0.0108980333	0.2998447194	0	70.4315334882	-0.0439624255	-0.0598330588	-0.0722236828	-0.0373699023
	b1	0.0001717923	-0.0351875261	-1.8879141230	0.1818354435	-0.0087983807	-0.0038011994	0.0001152627	-0.0008893935
	b0	0.0000210849	-0.0014306767	2.3445408666	0.0064477173	-0.0000150397	-0.0000410719	-0.0001240765	-0.0000931558
y_6	b5	0	0	0	0	0.0005307665	0	0	-0.0029482526
	b4	0.0821977635	0	-0.0001504349	0	0.0955779821	0	0	0.0663042250
	b3	0.0178273905	0.0381162756	-0.0597518725	0	-0.0242476520	0.7581758789	0.3249895310	-0.0458074657
	b2	0.0010297013	0.3497284145	0.0530862854	0	-0.0390703616	-0.4194856202	-0.0815733115	-0.0130573512
	b1	0.0000176315	0.1129314912	0.0710194257	19.0693044826	-0.0018749581	-0.1995891094	-0.1903307177	-0.0004125399
	b0	0.0000001491	-0.0127890170	0.0053773140	2.7287280009	-0.0000001333	-0.0007969403	-0.0140803468	-0.0000279777
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0	0	-3.0271358881	0	0.3880124442	0.4163597840	0	0.2775690202
	b2	0	0.1091487602	-5.9243531239	4.9786105812	0.0181331491	-0.0305611047	0.3260241842	-0.0109702671
	b1	0.1376325604	0.0658030622	-2.1194935057	0.7924741674	-0.0330846199	-0.0362997601	0.0099098431	0.0003656075
	b0	0.0255050957	-0.0138732779	-0.1342463138	0.0000251844	-0.0009484470	-0.0001324338	-0.0184660586	-0.0000559589
y_8	b5	-0.0121949090	0	0	-0.0015066432	0	0	0	0
	b4	0.0970648052	0	0	0.0000734494	0.0690251188	0	0	0
	b3	0.0035051418	0	-0.0042571857	0.0000724827	-0.0652502557	-0.0264413295	-0.0170304497	-0.0138963430
	b2	0.0001970770	0	0.0044588544	0.0000020141	-0.0021282216	-0.0051758494	-0.0019693558	-0.0002502991
	b1	0.0000063562	0.7102439397	0.0029591906	0.0000003236	-0.0007543690	-0.0000490512	-0.0000455681	-0.0000426759
	b0	0.0000000052	-0.1597287902	0.0022493446	0.0000000030	-0.0000351613	-0.0000094627	-0.0000051709	-0.0000007559
y_9	b5	0	0	-0.0000164821	0	0	0	0	0
	b4	0	0	-0.0010495568	0	0	0	0.0001548918	0
	b3	0	0	0.0012967152	0.0000115087	0.0309192381	0.0471477828	0.0223837834	0
	b2	0.0359652354	0.0671704385	0.0005591556	0.0000029886	-0.0508373933	-0.0700138198	-0.0239104759	-0.0050198665
	b1	0.0276181022	-0.0015624070	0.0000409578	0.0000000539	-0.0085309103	-0.0100855729	-0.0035153836	-0.0001168482
	b0	0.0019580329	-0.0006271116	0.0000001146	04	-0.0000044584	-0.0000020480	-0.0000460903	-0.0000124103
y_{10}	b5	0.0056735964	0	0.0000138869	-0.0000967890	0	0	0	0
	b4	0.0410511767	0	0.0005097213	0.0004647256	0.0001841934	0	0.0002185129	0
	b3	0.0295247698	-0.0192206172	0.0000136878	-0.0000080792	0.0347347974	0.4157241285	0.0321966102	0.0967770094
	b2	0.0004325970	0.0498177801	0.0000020999	0.0000031314	-0.0026645324	-0.2338815381	0.0024139064	-0.0431490673
	b1	0.0000189232	-0.0062600699	0.0000000387	-0.0000000149	-0.0141678732	-0.0022116008	-0.0110937285	-0.0008018156
	b0	0.0000002495	-0.0012845818	06	0.0000000019	-0.0004648858	-0.0002996656	-0.0005389941	-0.0001150853
y_{11}	b5	0	0	0	0	0	0.0003436242	0	0
	b4	0.0003965794	0	0	0.0000233070	0	0.0181322527	0.0137018772	0
	b3	0.0337817002	0	-0.0051585523	-0.0006363237	0.0237010712	-0.0077802518	-0.0025838221	0
	b2	0.0039805533	0	0.0060271977	-0.0009181902	-0.0149834729	-0.0203592838	-0.0176664947	-0.0111466432
	b1	0.0000548537	0.5327197971	0.0020309064	-0.0003318486	-0.0170900199	-0.0000024080	-0.0004839236	-0.0002498950
	b0	0.0000062141	-0.1692145439	0.0020933222	0.0002137905	-0.0000385986	-0.0000014032	-0.0000093485	-0.0000258462

Table 36 – Plant 1 – Sub-model 1 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	1	0	0	0	0	1	1
	a4	0.6376390753	57.1240909686	1	0	1	1	1.5122470740	0.8136305093
	a3	0.1322298355	156.7735836415	8.1357214750	1	1.5801584466	1.7196928081	1.5074339043	1.3362904921
	a2	0.0080263469	236.5827898243	0.9439806983	2.2100051025	3.0070639878	1.3360674485	0.3008729069	0.1622953301
	a1	0.0000950738	165.6208330689	0.0007965546	0.4713110802	0.5212112141	0.1228274930	0.0211192310	0.0067340413
	a0	0.0000010704	15.4518103400	0.0000924136	0.0273070331	0.0201899285	0.0000599817	0.0004773942	0.0002897067
y_2	a5	1	1	1	0	0	1	1	1
	a4	1.3278953420	3.6576647817	4.9012112744	1	1	16.6094350165	95.8401252103	1.8201627346
	a3	0.3752173026	8.9158923567	7.9942874779	11.4994150299	1.8341648229	31.1553933763	168.6592112869	1.2230239712
	a2	0.0202036315	9.8120268519	9.7774412065	13.9017159872	1.4103307078	29.0997296260	171.6892181336	0.1433695161
	a1	0.0002547788	6.5966726403	3.5632778260	15.5351561628	0.2035035379	6.2701698644	44.2651784920	0.0059863981
	a0	0.0000051845	0.7428053223	0.2723426722	1.3937089513	0.0063406477	0.3219933993	2.6568020301	0.0002481811
y_3	a5	0	1	1	0	1	1	1	1
	a4	1	2.4245475283	5.2755653801	1	1.9457762481	1.9961759251	11.7769250694	2.1087801542
	a3	2.0028746433	5.1984982118	11.5445251418	5.6238719390	1.7586052625	1.7818732706	20.9001916538	1.2119772528
	a2	2.7893292198	4.8294145910	17.1270741318	30.7544473828	0.4571749186	0.4526660670	17.5715500180	0.1446742887
	a1	0.8630318094	2.8794271345	7.0067920429	46.6570644154	0.0264278913	0.0249497504	4.2001278868	0.0060438319
	a0	0.0475172625	0.4609213712	0.6115198372	5.2150081579	0.0000336321	0.0000047723	0.2119332424	0.0002584772
y_4	a5	0	1	1	0	1	1	1	1
	a4	1	7.5881091615	2.9559319124	1	3.7898928598	1.7658249566	1.5997777071	1.0649173198
	a3	0.2635317565	21.8722984645	2.4740266887	2.1938582561	6.0117423820	1.5581603359	1.1360377891	1.9079339328
	a2	0.0148306566	26.1850741805	0.6039037884	0.3563116655	2.6625149784	0.3301554825	0.1637285866	0.2958316331
	a1	0.0004260715	19.2491806619	0.0516786169	0.0127994659	0.2295704910	0.0180718191	0.0058910766	0.0112649004
	a0	0.0000213632	2.1338495124	0.0013213614	0.0000304089	0.0002185642	0.0000145557	0.0001031183	0.0004790850
y_5	a5	1	1	0	1	0	0	1	0
	a4	1.6580025362	2.0006753461	1	457.0479648004	1	1	1.8909971144	1
	a3	0.3330722403	2.7495806366	8.1534543966	1075.9588635942	1.6878895950	1.6286853258	2.6566233896	1.5869053470
	a2	0.0194010926	1.4793237172	21.8234721865	98.8421596462	0.4024010878	0.2329158275	0.2373771443	0.1789559326
	a1	0.0006438550	0.1553686195	35.9909918166	0.3283061034	0.0234793039	0.0092526531	0.0045494059	0.0076433876
	a0	0.0000314555	0.0035939288	3.3698219597	0.0087679585	0.0000375938	0.0000873961	0.0004012124	0.0003145127
y_6	a5	1	1	0	0	1	1	1	1
	a4	0.6140711714	2.1960723463	1	1	2.0352841347	7.5141247204	5.9853857184	2.0737858045
	a3	0.0976140892	3.9822238468	2.4681022507	9.1676510426	1.8353589886	13.8377073705	11.9193206955	1.3893296425
	a2	0.0047715333	3.3912359210	2.7313277561	83.8700654331	0.4472053389	9.8533544748	10.5147557568	0.1919253203
	a1	0.0000823666	1.5590911264	0.8274170919	131.6245735233	0.0170416048	1.5246720417	2.9770516802	0.0074797246
	a0	0.0000005926	0.2284077604	0.0484101856	15.9715960487	0.0000026470	0.0054389259	0.1609326046	0.0003336819
y_7	a5	0	1	1	1	1	1	0	1
	a4	1	1.8148804636	6.0536403540	6.4241505644	5.3113139195	5.0512876743	1	2.0608536674
	a3	2.1020247489	3.8583457155	18.4650334338	36.0886567056	10.3013530737	9.6607909649	4.7869368711	6.8582307181
	a2	4.3823966491	2.9580136241	21.6225906701	56.6163820428	6.3610635551	5.3827757642	9.3967792883	1.3362625300
	a1	2.2205404909	1.6256626938	7.8854340393	5.8856372886	0.8128597136	0.5406899398	5.1618982900	0.0516441851
	a0	0.1936483460	0.1839608115	0.6364571386	0.0002308046	0.0176920243	0.0017990754	0.4815347622	0.0025668940
y_8	a5	1	1	0	1	1	1	1	1
	a4	0.1503165495	5.2914989943	1	0.0823363411	1.7612233230	0.7143893521	0.7118270698	0.6364857458
	a3	0.0060416343	9.4383170595	1.6288532678	0.0186780306	0.2808403785	0.1869629710	0.1427399489	0.0773446327
	a2	0.0003013495	12.1910527159	1.9415184315	0.0002493173	0.0284959740	0.0126794867	0.0091525279	0.0029083717
	a1	0.0000071973	7.0272021743	1.0849974820	0.0000465862	0.0030861613	0.0003410102	0.0003667913	0.0002271625
	a0	0.0000000114	0.6875865416	0.0913812231	0.0000000154	0.0000952984	0.0000209327	0.0000190806	0.0000029419
y_9	a5	0	1	1	1	1	1	0	0
	a4	0	2.6709005522	0.9036684355	0.2617045411	2.9727260851	2.9932566279	1	0
	a3	1	4.1595781982	2.0342735754	0.0489651777	6.7243287020	7.0669115231	4.0633355224	1
	a2	1.1685556864	3.4141065060	0.2971676921	0.0032226781	1.8625722509	1.8595146699	1.0704070385	0.0958438593
	a1	0.1850699637	0.6893360445	0.0116842004	0.0000348537	0.0859687417	0.0783596957	0.0565865968	0.0046538370
	a0	0.0060289535	0.0266493390	0.0000246267	0.0000002589	0	0.0000201669	0.0006022054	0.0001558030
y_{10}	a5	1	0	1	1	0	0	0	1
	a4	0.9306103428	1	0.1063412273	0.0267282066	1	1	1	3.3101140498
	a3	0.1221665947	1.6811218332	0.0078952170	0.0066844438	2.2590439479	16.9902911759	2.4091250004	7.3266801993
	a2	0.0012536788	1.9082785185	0.0003241362	0.0000928110	1.7318674230	2.4789318458	1.8934078219	0.9991431874
	a1	0.0000836952	0.4641571252	0.0000114926	0.0000092154	0.2176257211	0.0454442316	0.2651306812	0.0420559587
	a0	0.0000004220	0.0259298442	0	0.0000000334	0.0050433158	0.0027168395	0.0080547305	0.0017131294
y_{11}	a5	0	0	1	0	0	1	1	0
	a4	1	1	4.0983180413	1	1	1.8742551779	2.6825700945	1
	a3	0.3106013697	19.3228011827	8.7774769532	1.0770241763	3.0245839487	1.8409289498	2.5703082765	1.6621158799
	a2	0.0224269003	24.2754485938	9.0843086864	2.7237344215	2.0572526620	0.2449227627	0.4299471920	0.2816174992
	a1	0.0004886125	26.2075178899	6.1064968801	1.1386682736	0.2700825364	0.0001327012	0.0111093562	0.0102825282
	a0	0.0000328028	4.9283050708	0.7390441708	0.1384226495	0.0005657081	0.0000175345	0.0001905284	0.0005086611

Table 37 – Plant 1 – Sub-model 2 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0.0002146302	0	0	0.0001991927	0	0	0
	b4	0	-0.0091778720	0	-0.0877340801	0.0030313393	0.0022730715	0.0012874287	0.0029588065
	b3	-0.0079423550	0.0181795846	-0.0002508646	-0.0435629399	-0.0018012112	-0.0016158390	-0.0010001109	-0.0000777862
	b2	0.0095037786	-0.0008025337	-0.0249249493	-0.0046837892	0.0000052602	0.0000215192	-0.0000351598	-0.0015015828
	b1	0.0000194712	0.0000091555	-0.0003267190	-0.0000143808	-0.0000048748	-0.0000043698	-0.0000013400	0.0000000542
	b0	0.0000007962	-0.0000005514	0.0006744252	-0.0000005031	-0.0000000094	0.0000000167	-0.0000000534	-0.0000022689
y_2	b5	0	0	0	0	0	0	0	0
	b4	0.0521636527	0	-0.0001009798	0	0.0003659946	0.0004327485	0	0.0007363627
	b3	0.0060836313	0	-0.0065319674	0	0.0434373753	0.0594636373	29.9651601030	0.0362862506
	b2	0.0005396908	0.3349364887	0.0171041502	0	-0.0110023791	-0.0140148487	7.2259065302	-0.0132590808
	b1	0.0000033853	0.5725661320	0.0244778080	-18.5684789504	-0.0195015508	-0.0243091461	-26.7994206729	-0.0164533929
	b0	0.0000003148	0.0282268255	0.0024515306	-1.3873768420	-0.0021226183	-0.0026178983	-4.4093097111	-0.0015633454
y_3	b5	-0.0337646192	0	0	0	0.0006279500	0.0007721963	0.0007758228	0
	b4	0.2363204459	0	0	0	0.2172214216	0.2808631326	0.1634205421	0.0030942392
	b3	0.0225452558	0.2010943643	0	-0.0036898623	-0.0501463474	-0.0641572368	-0.0439888308	0.1756340215
	b2	0.0016099874	0.4804647470	0	-0.7247103167	-0.1083237102	-0.1312876888	-0.0853293027	-0.0662359789
	b1	0.0000029123	0.0068414809	1.7897630169	-0.2543273379	-0.0107644133	-0.0132143858	-0.0081850536	-0.0967420312
	b0	0.0000001300	-0.0069024056	0.2719221215	-0.0166320858	-0.0002140996	-0.0002729038	-0.0001252842	-0.0078371380
y_4	b5	0	0	0	0	0	0	0	0
	b4	0	-0.1336041481	0	0	0	0	-0.0000959287	0.1023303000
	b3	0.1414380112	0.3603402729	-0.6030535362	0	0.1277869614	0.1623408600	0.0966623292	-0.0289608721
	b2	0.0580529255	-0.0268997006	0.6295819702	0	-0.0727991469	-0.0899402405	-0.0585538252	-0.0706419973
	b1	0.0010176285	-0.0030226315	0.1416424460	-24.0670993774	-0.0437659767	-0.0519242744	-0.0355953678	-0.0086890873
	b0	0.0000033161	-0.0001232224	0.0002680490	-3.9779829636	-0.0007629059	-0.0007614144	-0.0006209541	-0.0001929708
y_5	b5	0	0	0	0	0	-0.0003280257	0	0
	b4	0	0.0046948424	0	0	0.0375018471	0.0929178364	0.0280719387	0
	b3	0.0707807272	-0.1129361573	0	0	0.0269282941	-0.0829216990	0.0164128009	0.0670042328
	b2	0.0022775607	0.2684966859	0	-2.0016005150	-0.0634511952	0.0002001833	-0.0486720999	-0.0983470215
	b1	0.0001089771	-0.0355565109	-1.6060185179	-0.8949813679	-0.0000028644	-0.0002049021	-0.0000000700	-0.0039557350
	b0	0.0000032533	-0.0010936047	1.9607818990	-0.0913878226	-0.0000025468	-0.0000001159	-0.0000067180	-0.0001594683
y_6	b5	0	0	0	0	0	0.0004505751	0	0.0014077985
	b4	0.0858024497	0	-0.0608132462	0.0000268360	0.1067491631	0.1456024414	0.0777383310	0.0874905741
	b3	0.1492274059	0	0.0469674439	-0.3840213399	-0.0307634743	-0.0464665621	-0.0253199179	-0.0306304452
	b2	0.0161710214	0.3157452976	0.0710748110	-0.6460479300	-0.0445654917	-0.0496766341	-0.0345228444	-0.0432067962
	b1	0.0004277372	0.5487360624	0.0050956458	-1.0102261358	-0.0029798129	-0.0048663910	-0.0022543584	-0.0035318038
	b0	0.0000004574	-0.0575535158	0.0000052445	-0.1210646585	0.0000000785	-0.0001584398	0.0000000192	-0.0000391020
y_7	b5	-0.0083946917	0	0	0	0	0.0008093867	0	0
	b4	0.0401243332	-0.0592688987	0	0	0	0.1136692777	0	0
	b3	0.0010344802	0.0820757314	-314.9523250879	0	0	-0.0076795437	0	0.3852055680
	b2	0.0000815129	0.0693023977	-1111.4994997105	0	0.3800259882	-0.0099875009	0.3384561951	0.2106610259
	b1	0.0000021456	-0.0137862773	-384.2158446832	-29.6418815565	-0.0126609687	-0.0000001350	0.0101257614	-0.0451727921
	b0	-0.0000000014	-0.0002087618	-23.2691026127	-4.4540756355	-0.0317558105	-0.0000002988	-0.0231595213	-0.0053058495
y_8	b5	0	0	0	-0.0000323204	0	0.0005355930	0	0
	b4	0.0063559044	-0.0074842651	0	0.0079870465	0.0107381233	0.0113991776	0	0
	b3	0.0684302768	0.0198918749	0.0003903671	-0.0006060402	-0.0127017484	-0.0134820848	0	-0.0014367760
	b2	0.0098427817	-0.0049155320	0.0004873674	-0.0005184105	-0.0001956947	-0.0007796724	0.0110529353	-0.0087370890
	b1	0.0003623304	0.0000424227	0.0000219739	-0.0000189613	-0.0000134171	-0.0000199797	-0.0196982484	-0.0000758577
	b0	0.0000010432	-0.0000156402	0.0000084517	-0.0000054193	-0.0000000739	-0.0000012031	-0.0020074994	-0.0000010334
y_9	b5	-0.0130697290	0	0	0	0	0	0	0
	b4	0.0851551426	-0.0146580685	-0.0031936663	0	0	0	0	0
	b3	0.0028532439	0.0563754993	0.0036821408	0	0	-0.0219922201	0	0
	b2	0.0001502676	-0.0073639632	0.0011757357	0.0004164096	0	-0.0002864484	-0.0237533887	-0.0299635544
	b1	0.0000055956	0.0000929425	0.0000040912	-0.0002031343	-0.5706961589	-0.0000154155	-0.0000004559	-0.0038388194
	b0	0.0000000119	-0.0000293714	-0.0000000367	-0.0000001145	-0.0671957832	-0.0000000917	-0.0000250836	-0.0001096460
y_{10}	b5	-0.0060572555	0	0	0	0	0.0000737797	0	0
	b4	0.0437713712	-0.0088165804	0	0	0.0002127268	0.0427194468	0.0000927648	0
	b3	0.0318135077	0.0319785020	0	0.0034857104	0.0443641067	0.0021497875	0.0368548069	0.0326595181
	b2	0.0022059135	-0.0081458033	0	-0.0012411413	-0.0071797241	-0.0261361530	-0.0027459713	-0.0177493951
	b1	0.0000054979	-0.0001382865	0.0010170757	-0.0000307257	-0.0148612980	-0.0013902795	-0.0098143257	-0.0056016050
	b0	0.0000002837	-0.0000386645	0.0000013186	-02	-0.0009267309	-0.0000194948	-0.0005826612	-0.0003276632
y_{11}	b5	0	0.0003831918	-0.0000330637	0	0	0	0.0002230434	0
	b4	0.1040332878	-0.0247073304	-0.0061841163	0	0.0383432016	0.0514251280	0.0269388996	0
	b3	0.0021919315	0.0822546796	0.0044392204	0.0135463363	-0.0288359031	-0.0381738374	-0.0201305491	0
	b2	0.0002618435	-0.0284917187	0.0017175958	0.0019412132	-0.0496629748	-0.0649313674	-0.0364963404	0
	b1	0.0000047342	0.0001897244	0.0015459534	0.0027038105	-0.0024824425	-0.0039339645	-0.0016246151	-0.2499667813
	b0	0.0000000154	-0.0000704215	0.0000501103	-0.0030142777	-0.0000487449	-0.0000947204	-0.0000284683	-0.0188611926

Table 38 – Plant 1 – Sub-model 2 – Transfer Function Denominator Parameters.

Pl	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	0	1	0	1	1	1	0	1
	a4	1	1.5863431790	0	2.1803382245	1.1653529350	0.9127108126	1	3.1465133506
	a3	1.9634013903	1.0929610333	1	0.8550422205	0.1062679781	0.0644747018	0.0818647411	1.3281985360
	a2	0.2262916483	0.0733621716	2.2108900231	0.0678752044	0.0032936354	0.0026376131	0.0053487386	0.1066712159
	a1	0.0000817782	0.0014374533	0.5148179606	0.0002395325	0.0002699555	0.0001767125	0.0001043429	0.0019922814
	a0	0.0000094198	0.0000325355	0.0331597387	0.0000068608	0.0000006742	0.0000000556	0.0000047653	0.0001530909
y_2	a5	1	1	0	1	0	0	1	0
	a4	0.3586605734	3.1435165448	1	18.1643362325	1	1	1087.4648679863	1
	a3	0.0347112457	7.2263765857	1.6189296032	76.2075014748	1.9132921918	2.0312439336	2154.7558528451	1.9421480424
	a2	0.0020026558	7.4947368044	2.1229981129	304.6166419178	1.7759833082	1.8457684467	2572.4713057740	1.7558052732
	a1	0.0000207490	4.8353542906	0.7153229996	388.3135855956	0.4209199792	0.4262556543	888.7696869794	0.3759866373
	a0	0.0000010856	0.5641451307	0.0503075212	25.7746010743	0.0244031913	0.0243706714	64.0157601313	0.0199645020
y_3	a5	1	1	1	0	1	1	1	0
	a4	0.3991593691	1.7094392482	5.4486123664	0	2.0415018007	2.1085185438	1.9612797413	1
	a3	0.0339379718	2.5318482096	11.4383418668	1	1.8850142499	1.9142321211	1.8118449836	2.0303986971
	a2	0.0018764020	1.6801529425	16.7909812311	2.2401194239	0.5085058024	0.5099233388	0.4876968877	1.8676529341
	a1	0.0000033498	0.4850132799	6.9549123331	0.6389431682	0.0358021003	0.0362296041	0.0329505560	0.4723729921
	a0	0.0000001762	0.0349938839	0.6188665237	0.0378474662	0.0006410903	0.0006694947	0.0004659655	0.0259577782
y_4	a5	0	1	1	0	0	0	0	1
	a4	1	1.2951765223	3.2520621379	1	1	1	1	1.9659315548
	a3	0.5326458606	1.2432125924	6.8929852163	6.0586652722	1.7125811687	1.7512344638	1.6462184814	1.6455653465
	a2	0.0585181001	0.2220385226	2.3841822849	33.0046149577	0.9902837377	0.9841650722	0.9763860107	0.3594607856
	a1	0.0008872903	0.0125548069	0.1901982596	50.9921842281	0.1098362138	0.1053159357	0.1097852944	0.0251860816
	a0	0.0000025378	0.0003109919	0.0003575290	5.2604727386	0.0016378032	0.0013455154	0.0016415341	0.0004649292
y_5	a5	1	0	0	0	1	1	1	1
	a4	0.7041379740	1	1	1	1.8129458714	1.6775030674	1.6906030371	2.3236528497
	a3	0.0986204748	1.6580126375	4.3884671837	3.6041274084	1.6356099496	0.1512081004	1.5311969514	2.9430452663
	a2	0.0028502793	1.0604933480	15.9254022438	8.1144306105	0.1370645541	0.0042303887	0.1292581932	0.3608217346
	a1	0.0001423993	0.1120001902	25.0038678638	1.8388353645	0.0000687994	0.0003671955	0.0002127275	0.0142521013
	a0	0.0000026649	0.0021941299	2.3630123503	0.1076137288	0.0000056511	0.0000002200	0.0000178802	0.0003829989
y_6	a5	1	1	1	0	1	1	1	1
	a4	1.7514711252	3.0623146116	2.3226288187	1	2.1744554220	2.3064448515	2.0455218086	2.2278636863
	a3	0.6583751275	6.5563052108	2.6612791463	4.2763828835	1.9851500854	2.0078676081	1.8833286610	2.1097605721
	a2	0.0555031246	7.0386792863	0.8124032351	7.0193830250	0.5122965154	0.5166456043	0.4898531725	0.5731863595
	a1	0.0012953632	4.6306050281	0.0453345896	6.5329211718	0.0256250119	0.0397586735	0.0242208776	0.0352600767
	a0	0.0000010900	0.9606460068	0.0000503355	0.6940991158	0	0.0011135936	0.0000003587	0.0003702289
y_7	a5	1	1	1	1	0	1	0	1
	a4	0.2512475731	2.4705772400	406.0004562291	10.8584288100	1	2.0690613263	1	4.6940318138
	a3	0.0071567393	2.4556371630	1961.7520011917	60.8840037455	4.6546437534	1.2860244374	4.5820985625	11.1476292586
	a2	0.0005175063	1.5189765894	3677.0068254992	239.6080670394	9.1526816090	0.1280276754	9.0889078817	10.4580667082
	a1	0.0000103205	0.1974073983	1399.2763018625	337.6633778572	5.3326736457	0.0000399174	5.3367394071	2.2731617217
	a0	0.0000000134	0.0024908786	111.8001507921	32.6985057066	0.5266523888	0.0000039719	0.5224473956	0.1144739578
y_8	a5	1	1	1	1	1	1	0	1
	a4	3.0504392263	1.3903808122	0.5582292966	29.0988633487	1.9834821136	1.8095909167	1	1.7812822383
	a3	0.7315342305	1.0577258493	1.2018544174	6.9624012105	0.2571980245	0.2945232188	3.4656779367	1.5055856751
	a2	0.0604398121	0.1210196845	0.1666989405	0.8278837601	0.0056938940	0.0137248879	4.3616914398	0.1692941205
	a1	0.0016968156	0.0038189527	0.0233805869	0.0682349377	0.0002428236	0.0004345511	0.8835584044	0.0013953628
	a0	0.0000032047	0.0003553788	0.0021261344	0.0039630920	0.0000013145	0.0000163452	0.0438712875	0.0000203861
y_9	a5	1	1	1	0	1	0	1	0
	a4	0.1690003618	1.0654944405	0.9340274267	0	12.4756378719	1	0.8049609463	1
	a3	0.0052687202	1.4208060977	2.0942286299	0	22.1117468233	0.0905405756	1.8674606401	1.7456078154
	a2	0.0002969044	0.1658473654	0.1612055062	1	32.4322828994	0.0014365377	0.1482006585	0.3868629456
	a1	0.0000059368	0.0074166788	0.0003490659	0.0813347336	6.9020534078	0.0000583804	0.0019038054	0.0239826861
	a0	0.0000000342	0.0004923439	0.0000032125	0	0.2817842012	0.0000001976	0.0001502978	0.0004738553
y_{10}	a5	1	1	0	0	0	1	0	0
	a4	0.9069574187	1.2500616859	1	1	1	2.8851670571	1	1
	a3	0.1474422506	1.5191853715	0.3602428086	5.7345674487	3.1613973878	2.4869349356	2.9291231117	2.8254065465
	a2	0.0064791250	0.2527339655	2.0173064550	1.0883232686	1.9914664151	0.3858883536	1.8231944977	0.9882077373
	a1	0.0000226059	0.0125770621	0.2526402824	0.0152910706	0.3000167079	0.0143891464	0.2656463577	0.1143734343
	a0	0.0000009730	0.0008372243	0	0.0000988037	0.0107585464	0.0001795438	0.0091519719	0.0039596981
y_{11}	a5	1	1	1	1	1	1	1	0
	a4	0.1689933991	1.3620224712	2.0773725498	4.7333583211	2.5330580504	2.6729051613	2.3452855084	1
	a3	0.0056213301	1.6458034930	2.7334846259	9.6850145044	2.2498199207	2.3448082179	2.1443793560	7.0162296952
	a2	0.0004205433	0.3138127425	1.7461871920	10.6349105993	0.3850463087	0.4196815266	0.3602769951	10.4909879450
	a1	0.0000066893	0.0040711895	0.2611855443	7.1200149308	0.0158511132	0.0204092881	0.0134133634	2.2441307690
	a0	0.0000000258	0.0007690826	0.0068166056	0.9016776896	0.0002714832	0.0004146653	0.0002086911	0.1067113398

Table 39 – Plant 1 – Sub-model 3 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	-0.0000304206	-0.0004811173	0	0	0	0	0
	b4	-0.0967598548	-0.0009884211	-0.0296067046	0	0.0024055083	0	0	0.0038449213
	b3	0.0757443938	0.0018699441	-0.0025507392	0	0.0012435071	0	0	-0.0018803664
	b2	0.0402694524	0.0037816404	0.0005873454	0.1872027740	-0.0004363791	0.0064371603	0.0014603898	-0.0007214263
	b1	0.0008788880	0.0011459035	-0.0000212824	0.3804417343	-0.0020497651	-0.0003911489	-0.0016085650	-0.0000010313
	b0	0.0000024289	-0.0000059921	0.0000070799	0.0611929572	-0.0000691372	-0.0041725982	-0.0000012559	-0.0000003004
y_2	b5	0	0	0	0	0	0	0	0
	b4	0	0	-0.0004595011	0	0.0009349578	0.0964977502	0.0004338471	0.0010607255
	b3	0.1701853788	0	-0.0154080337	0.2929186916	0.1098390664	-0.0330707891	0.0436993928	0.0543642719
	b2	0.0645609446	0.0785754810	0.0209652632	0.3446787536	-0.0402159523	-0.0325872920	-0.0149114427	-0.0195216732
	b1	0.0000126727	0.1500729702	0.0288375261	0.5291807252	-0.0424606605	-0.0026768588	-0.0218512442	-0.0247581780
	b0	0.0000240915	0.0095622703	0.0021209980	0.0649327586	-0.0034825132	-0.0000007496	-0.0021296606	-0.0021241143
y_3	b5	0	0	-0.0034423830	0	0	0	0	0.0028266091
	b4	0.2933262611	0	-0.2178252255	0.0012458682	0.0032479772	0.3126539383	0	0.2206005931
	b3	0.0037663145	-0.0073530113	0.1401979813	0.4167266571	0.3968248155	-0.0979865917	3.2373336680	-0.0953843127
	b2	0.0006276029	0.1388094874	0.1968575734	0.0859389794	-0.1491127943	-0.1453533498	-1.6801852762	-0.1096309346
	b1	0.0000084007	0.2324094432	0.0170033709	0.0002299544	-0.1860377963	-0.0105997862	-1.4104925956	-0.0076602082
	b0	0.0000000291	-0.0356017729	0.0000143683	0.0000466836	-0.0132415865	0.0000001275	-0.0882192990	-0.0000777037
y_4	b5	0.0056303574	0	0	0	0	0.0017757943	0.0010649518	0
	b4	-0.5768002099	0	0	0	0.0018503252	0.1356180743	0.0786049685	0
	b3	0.9627108377	0	-0.0047701229	0	0.1804136720	-0.0081378350	-0.0136045652	0.8790667934
	b2	0.1807511817	0.0851166922	-0.3508763309	0	-0.0339960560	-0.1089559456	-0.0710357052	-0.1041722242
	b1	0.0067432843	0.1696093864	0.0644862404	35.0254446464	-0.1419415046	-0.0094952184	-0.0059001702	-0.8932084302
	b0	0.0000094498	-0.0347332831	0.1771092640	7.4652959797	-0.0061583284	-0.0002858034	-0.0001489775	-0.0255336445
y_5	b5	0	0	0	0	0	0	0	0
	b4	-0.0997037821	0	-0.4500685627	0	0.0902973172	0	0.0412096157	0
	b3	0.1459416038	0	0.4059725295	0	0.0191763759	0.1771056001	0.0123993562	0
	b2	0.0334931150	0.0530276560	-0.0046429290	35.2806476778	-0.1161101580	-0.1920668770	-0.0584595608	0.2172711511
	b1	0.0000392396	0.1334069169	0.0047835183	16.4295202161	0.0000021175	-0.0111699596	-0.0062537222	-0.2777015389
	b0	0.0000025036	-0.0324804720	0.0000139824	0.1260652020	-0.0000014995	-0.0007044555	-0.0003545904	-0.0220038195
y_6	b5	0	0	0	0	0	0	0	0
	b4	-0.0004675831	0	0	0	0	0.0007157070	0.0004893828	0.0823001165
	b3	-0.0666103798	0	-0.2676786259	0	0.1618401644	0.1228331290	0.0708239799	-0.0313958109
	b2	0.2087545187	0.0472180377	0.1795643659	0	-0.0598624839	-0.0378395231	-0.0244844278	-0.0400162512
	b1	0.2074568881	0.0881291009	0.206113331	17.2795835259	-0.0652118665	-0.0511321408	-0.0335057212	-0.0046856523
	b0	0.0131703726	-0.0118095499	0.0001926803	2.0582675939	-0.0040324235	-0.0033729533	-0.0022635411	-0.0001764549
y_7	b5	0	0	0	0.0003960432	0	0	0	0
	b4	0	0	0	0.0636796773	0	0	0	0
	b3	0.0741208265	0	0	0.0173826029	0	0.5394954237	0.3635085128	0.4112716783
	b2	0.0328476819	0.0172579835	-2.6721977459	0.0002649429	0.7420823303	0.0107212801	0.0530503394	0.1673423923
	b1	0.0000045688	0.0216876943	-2.5838894295	0.0000227459	0.0001344630	-0.0739762568	-0.0364941213	-0.0623591554
	b0	0.0000089132	-0.0052030159	-0.2055267629	0.0000000139	-0.0794124225	-0.0029112182	-0.0013249714	-0.0023645576
y_8	b5	0	0	0	0	0	-0.0063925466	0	0
	b4	0	0	0	0.0001026033	0	0.4701152233	0	0
	b3	0	0.0049959836	0.0018195983	0.0000332704	0	-0.5212368032	0.0241700324	-0.0230110074
	b2	0.2411165069	0.0117398640	0.0000092617	0.0000004157	0.0866805382	0.0001641637	-0.0408932188	-0.0135836427
	b1	0.0287416034	-0.0056688548	0.0000061724	0.0000001260	-0.1258846718	-0.0012919484	-0.0000201526	-0.0003656413
	b0	0.0000004719	-0.0000551889	-0.0000000061	03	-0.0000749529	-0.0000014223	-0.0000010659	-0.0000170570
y_9	b5	0	0	0	0	0	0	0	0
	b4	0.0497344087	0	-0.0033056454	-0.0000125879	0	0	0	0
	b3	0.0058706050	0.0020056980	0.0023038402	-0.0006560784	-0.0198088948	0	0	0
	b2	0.0001555260	0.0040816319	0.0023817555	0.0001722519	-0.0041790486	-0.0278235870	-0.0164384305	-0.0686159728
	b1	0.0000124831	-0.0002274497	0.0007858726	0.0000132644	-0.0000527292	-0.0022620946	-0.0014284604	-0.1063029955
	b0	0.0000000658	-0.0000713210	0.0000351091	0.0000174236	-0.0000102258	-0.0000017045	-0.0000002121	-0.0086614055
y_{10}	b5	0	0	0	0	0	0.0002468965	0	0
	b4	0.0497222865	0	-0.0018456209	0	0.0771085143	0.0530115805	0	0
	b3	0.1624272265	0	0.0018350115	-0.0061221821	0.0007292580	-0.0056877641	12.9621447071	0.0360322429
	b2	0.0011747171	0.0420119747	0.0006960273	0.0017809492	-0.0327787949	-0.0252289037	20.1871287867	-0.0204498601
	b1	0.0001387676	-0.0186320257	0.0004168678	0.0000613299	-0.0013168704	-0.0012354086	-14.3512831331	-0.0039670023
	b0	0.0000007692	-0.0006685108	-0.0000000615	0.0000001408	-0.0000100117	-0.0000008527	-1.1353433423	-0.0001378550
y_{11}	b5	0	0	-0.0000934202	-0.0000183621	0	0.0005417753	0.0003286385	0
	b4	0.1365172836	-0.0000661001	-0.0046557073	-0.0018390046	0	0.0447057305	0.0237660745	0
	b3	0.0262371205	-0.0026124443	0.0034097977	-0.0000195326	0.0680361355	-0.0388019267	-0.0210196327	0
	b2	0.0014844508	0.0090516703	0.0023949468	-0.0000981210	-0.0643536292	-0.0657235635	-0.0372750394	-0.0262663223
	b1	0.0000175874	0.0063418806	0.0010802997	0.0001812689	-0.0837513303	-0.0024093602	-0.0016661070	-0.0405761206
	b0	0.0000009636	-0.0066301879	0.0003404349	0.0000072024	-0.0044569281	-0.0000115763	-0.0000292552	-0.0021460877

Table 40 – Plant 1 – Sub-model 3 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	1	1	0	1	0	0	1
	a4	22.3899483137	1.8265912935	2.2621052653	1	1.3686827472	1	0	2.2188077140
	a3	8.3232794084	2.4632781813	0.6522204290	6.2568572818	1.7391254410	2.6458118104	1	0.6119921057
	a2	0.7685965336	1.3732289193	0.0682333687	19.5679070116	0.8222238684	4.5861236843	1.0794972534	0.0459516726
	a1	0.0136561983	0.3389528802	0.0052685580	18.4285107472	0.0935949087	2.2162871662	0.1085460142	0.0002548693
	a0	0.0000334310	0.0249025025	0.0004067646	1.9822943182	0.0022716357	0.1908772561	0.0000733851	0.0000187628
y_2	a5	1	1	0	0	0	1	0	0
	a4	1.7900687557	3.1856999094	1	1	1	2.0984813991	1	1
	a3	1.2270688810	7.3488314417	1.4352434028	11.4776718308	1.8682055980	1.8098983768	1.7244453505	1.9264305955
	a2	0.1265876627	7.6708633142	1.8543864029	14.9137499280	1.6601539493	0.3577383636	1.6549080429	1.8028381211
	a1	0.0004472899	5.2179139993	0.5316387653	15.7616425268	0.3408810659	0.0182207835	0.3888063675	0.3816696142
	a0	0.0000459052	0.6880023792	0.0298542604	1.7179647670	0.0172865146	0.0000048123	0.0221496386	0.0196212800
y_3	a5	1	1	1	0	0	1	1	1
	a4	0.2179810997	2.7787750234	2.0469883594	1	1	1.9691972317	18.4321962481	2.0300083531
	a3	0.0048087394	6.1909271899	2.0348118471	2.0498828666	1.8502269999	1.7568554441	32.8891545138	1.8278414768
	a2	0.0004679240	6.1432122167	0.5249571299	0.3357303695	1.6411551689	0.4154127254	26.8727831046	0.4162581158
	a1	0.0000056830	4.1716362571	0.0320222242	0.0011146151	0.3811533771	0.0210711783	5.7092531710	0.0217820114
	a0	0.0000000252	0.7524431903	0.0000277954	0.0001822563	0.0191512119	0.0000006419	0.2631691110	0.0002094602
y_4	a5	1	1	0	0	0	1	1	1
	a4	4.3926740048	2.9554371944	0	1	1	1.7096616009	1.5753533015	8.9656148171
	a3	1.3497299191	5.6157649824	1	16.5747694418	1.6077899917	1.4845143836	1.3873363288	16.4311633450
	a2	0.1247732517	5.6083217038	2.3056953589	96.9952971583	1.3370636703	0.2816318963	0.2619603330	15.3285149203
	a1	0.0034979787	3.0264533364	1.6084581270	138.0003649608	0.2059178707	0.0174308556	0.0154048820	2.1937296088
	a0	0.0000050387	0.3477638403	0.1934240529	16.6866512432	0.0064517797	0.0004142638	0.0003196828	0.0497641779
y_5	a5	0	1	1	0	1	1	1	0
	a4	1	3.1978256166	3.3264923611	1	1.4909592006	2.1943011128	1.6394881605	1
	a3	0.2662668467	5.5948892543	0.4058937291	125.7762211227	1.2544136085	2.9439131469	1.4576661989	4.5127786490
	a2	0.0178015161	5.6700829611	0.0400721787	290.6294570245	0.1164251580	0.4531322357	0.2665183039	5.8082702559
	a1	0.0000293075	2.6256615633	0.0046148862	33.4964716501	0.0000178378	0.0262942903	0.0206585024	0.9739688662
	a0	0.0000015131	0.2398736313	0.0000131932	0.2437732444	0.0000016270	0.0009745806	0.0007241222	0.0409429484
y_6	a5	0	1	1	1	0	0	0	1
	a4	1	2.9687600653	5.1363763013	8.7468269673	1	1	1	2.2012783410
	a3	2.1986666535	6.5121772051	9.5874594894	50.1355839555	2.0532219181	2.1244872525	1.9717565629	2.1442537286
	a2	2.4118909634	6.9704355677	9.8230124027	175.6137535982	1.8805096470	1.9819517242	1.8837612807	0.6212532068
	a1	0.6603196982	4.8191217801	2.0158038919	243.1102517175	0.4646944714	0.5083553999	0.5015072532	0.0555289334
	a0	0.0315294019	1.0743592950	0.0018851559	25.0518332694	0.0219476851	0.0251166753	0.0255314784	0.0017656474
y_7	a5	1	1	0	1	0	1	1	1
	a4	1.2899678802	1.9184020491	1	2.1674124994	1	5.4389150316	4.6753639680	4.9282169350
	a3	0.8673357614	4.2417780091	4.4327748772	0.3095053000	4.8962259116	10.6412167244	9.2202257761	10.5263886655
	a2	0.0985143015	3.4368956134	10.8115252479	0.0055560015	9.4990757359	7.2055825498	6.3065116156	9.1990270845
	a1	0.0002230907	2.1871098925	8.4882439095	0.0003864341	5.9524896069	1.0459071934	0.9013319349	1.4996624341
	a0	0.0000252535	0.2744672723	1.0371296677	0	0.6349777324	0.0301310210	0.0236729795	0.0397025529
y_8	a5	0	1	0	1	0	1	0	1
	a4	1	1.9127912654	1	0.2404627238	0	12.7819786998	1	1.1868208477
	a3	1.7274488271	2.2769193141	0.1170897031	0.0234804562	1	1.4452951753	1.6959629572	0.5703454519
	a2	0.3601226890	1.2995303429	0.0044119139	0.0008934976	2.3675030916	0.0330306386	0.1908760472	0.0617919084
	a1	0.0198803570	0.1562441146	0.0003781282	0.0000698780	0.2714326500	0.0036430045	0.0000595262	0.0019087334
	a0	0	0.0013521820	0.0000000640	0.0000001020	0	0.0000015311	0.0000066599	0.0000608017
y_9	a5	1	1	1	0	1	1	1	1
	a4	0.2121624978	1.6506552801	2.6698689789	1	1.3344287230	1.3806923892	1.3202965809	6.3565688815
	a3	0.0133927061	2.4192547112	3.0940355058	1.1915448950	0.4445537426	2.5422022003	2.4650823992	12.8692140780
	a2	0.0004634203	1.5922231387	2.1636563884	1.8246936667	0.0292980631	0.4635913032	0.4746295994	13.5279587427
	a1	0.0000235113	0.3900713193	0.2998871567	0.3548812223	0.0011240103	0.0179252534	0.0189117869	2.2920221761
	a0	0.0000000469	0.0222601207	0.0093653809	0.0686911608	0.0000619218	0.0000155961	0.0000046286	0.0876309853
y_{10}	a5	1	1	1	0	1	1	1	0
	a4	3.1879133674	12.4770097957	1.7089162274	1	3.2420730370	3.2965395506	552.7986058614	1
	a3	0.3799787766	13.1655631524	2.1072350289	14.9867882816	2.5697749554	2.5590566726	1447.9103568379	2.9666748932
	a2	0.0046729179	14.7682934904	1.2821375169	5.7470030424	0.3601600697	0.3750967472	2405.7514833214	0.8408439430
	a1	0.0003195953	2.9155807504	0.1231764223	0.1517361862	0.0104679260	0.0118230676	459.7186921792	0.0716036185
	a0	0.0000012435	0.0792449635	0	0.0003111686	0.0000788441	0.0000104738	19.0051696861	0.0017808717
y_{11}	a5	1	0	1	1	0	1	1	0
	a4	0.3404266928	1	1.7923237081	1.5912334123	1	2.3830874281	2.1114267401	1
	a3	0.0379420946	2.1003810462	2.7165020411	2.2958893268	2.5670401905	2.3117101431	2.1562798640	1.8859937841
	a2	0.0015905250	2.7656634273	1.7377825637	1.2756813233	2.2835377288	0.3748704352	0.3678869607	1.8695312487
	a1	0.0000257336	1.8645354845	0.6011589015	0.2374295248	0.3953766413	0.0106934623	0.0135512492	0.3297399120
	a0	0.0000009356	0.3748473374	0.0559731116	0.0077789922	0.0145339932	0.0000527354	0.0002116879	0.0120333511

Table 41 – Plant 2 – Sub-model 1 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0	0.0000105592	0	0.0006485585	-0.0000530434	0	0	0
	b2	0.0002021277	0.0008053314	0.0001090997	0.0001103545	-0.0001276728	0	0	0
	b1	-0.0000008793	0.0000616034	0.0000001065	0.0000014156	-0.0000019503	0	0	0
	b0	0.0000000276	-0.0000022755	0.0000000037	0.0000002450	-0.0000000157	0	0	0
y_2	b5	0	0	0.0000333343	0	0	0	0	0
	b4	0	-0.0011797225	-0.0028658019	0	-0.0001033887	0	0	0
	b3	0	0.0020624122	-0.0013776906	0	0.0002552216	0	0	0
	b2	0.0003760169	0.0005599782	0.0047383111	0.0025884668	-0.0002887653	0	0	0
	b1	-0.0000012762	0.0000075953	-0.0000020041	-0.0000006613	-0.0000064880	0	0	0
	b0	0.0000000748	0.0000010915	0.0000016630	0.0000044051	-0.0000000470	0	0	0
y_3	b5	0	0	-0.0002605893	0	-0.0002379648	0	0	0
	b4	0	0	-0.0033864604	0	0.0012166461	0	0	0
	b3	0	0	0.0026076766	0.0031594354	-0.0005156922	0	0	0
	b2	0.0023543340	0.0049545352	0.0012502043	0.0019575861	-0.0004600914	0	0	0
	b1	-0.0000124809	-0.0001814582	0.0000049563	0.0000162856	-0.0000467351	0	0	0
	b0	0.0000004465	-0.0000315574	0.0000012144	0.0000000240	-0.0000001554	0	0	0
y_4	b5	0	0	0	0	0	0	0	0
	b4	0.0005437500	0	-0.0008883635	0	0.0007678622	0	0	0
	b3	0.0007419398	0	-0.0027730092	0.0176006554	-0.0006663475	0	0	0
	b2	0.0021915415	0.0078522676	0.0023060602	0.0055162226	-0.0000031747	0	0	0
	b1	-0.0000125387	-0.0006204302	-0.0000010161	0.0006891328	-0.0000014867	0	0	0
	b0	0.0000004024	-0.0000577645	0.0000015692	0.0000007035	-0.0000000037	0	0	0
y_5	b5	0	0.0002257902	0.0000747327	0	-0.0001335956	0	0	0
	b4	0.0010501639	-0.0006795354	-0.0012156465	0	0.0008051448	0	0	0
	b3	-0.0002100744	0.0022503538	0.0008367428	-0.0001970067	-0.0004396091	0	0	0
	b2	0.0007886041	-0.0002568101	0.0000331362	0.0026969878	-0.0000086461	0	0	0
	b1	-0.0000041693	0.0000001108	0.0000006100	0.0009504691	-0.0000020170	0	0	0
	b0	0.0000001352	-0.0000007408	0.0000000226	0.0000005902	0.0000000024	0	0	0
y_6	b5	0	0	0	0	0	0	0	0
	b4	-0.0010427388	0	-0.0004883568	0.0001517918	0	0	0	0
	b3	0.0015805409	0	0.0006906983	0.0011194126	0	0.0018713711	0	0
	b2	0.0013186353	0	0.0000903036	0.0039374246	0.0016684874	-0.0017119642	0.0017174627	-0.0034511624
	b1	-0.0000034714	0.0048981531	0.0000018591	0.0002826369	-0.0020450245	-0.0000662931	-0.0020575607	-0.0000434738
	b0	0.0000001334	-0.0004532158	0.0000000811	0.0000141302	-0.0000000070	-0.0000000993	-0.0000041105	-0.0000090459
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0.0665698364	0	0	0.1471002289	0.2888052733	0.5773483454	0	0.3820623195
	b2	0.0281275457	0.1501707398	-2.6051938439	0.0248314830	0.0121883336	-0.0438761340	0.2430387054	-0.0151120690
	b1	0.0000274984	0.0914268778	-2.6286378281	0.0000097100	-0.0244003082	-0.0498693842	0.0073465225	0.0005123089
	b0	0.0000082372	-0.0192359460	-0.2108396101	0.0000004808	-0.0006675078	-0.0001483825	-0.0137453794	-0.0000773529
y_8	b5	-0.0090656644	0	0.0000932625	-0.0020177510	-0.0058362569	0	0	0
	b4	0.0721577293	0	-0.0028639391	0.0000944744	0.0398799246	0	0	0
	b3	0.0026057135	0	0.0031608319	0.0001013188	-0.0302840739	-0.0196564171	-0.0235121585	-0.0103305061
	b2	0.0001465065	0	0.0021929233	0.0000028536	-0.0554138279	-0.0038477133	-0.0027188833	-0.0001860717
	b1	0.0000047252	0.5407470620	0.0013435382	0.0000004508	-0.0009972414	-0.0000364645	-0.0000629111	-0.0000317251
	b0	0.0000000039	-0.1212838147	0.0000392868	0.0000000042	-0.0000066728	-0.0000070346	-0.0000071390	-0.0000005620
y_9	b5	0	0.0000003985	0	0	0	0	0	0
	b4	0	-0.0000606421	0.0000084610	0	0	0.0001015675	0.0000309521	-0.0003126651
	b3	0	0.0002750513	0.0000030143	0	0	0.0000588567	-0.0001166646	-0.0001356060
	b2	0	-0.0000127488	0.0000000685	0	-0.0012679443	-0.0001214780	-0.0000524500	-0.0000512510
	b1	0.0083630022	-0.0000006064	0.0000000113	0.0000005850	0.0000245838	0.0000001386	-0.0000000675	-0.0000003677
	b0	0.0026028561	-0.0000001119	01	0.0000001041	-0.0001907248	-0.0000000289	-0.0000000179	-0.0000000963
y_{10}	b5	0	0	0	-0.0000784936	0	0	0.0001944190	0
	b4	0.0280751511	0.0000483327	0	0.0003485420	0.0002551417	0	0.3027194636	0
	b3	0.0058796909	-0.0157766891	0.0042810907	-0.0000068064	0.0479136684	0.2029774843	-0.0954799828	0.0719437826
	b2	0.0000969388	0.0387891436	0.0000236632	0.0000023508	-0.0036102989	-0.1178633076	-0.0239908233	-0.0320769069
	b1	0.0000063278	-0.0033572042	0.0001435389	-0.0000000115	-0.0195869412	-0.0006042024	-0.0001064624	-0.0005960677
	b0	0.0000000348	-0.0013626740	0.0000003119	0.0000000015	-0.0006445468	-0.0001476188	-0.0000068921	-0.0000855542
y_{11}	b5	0	-0.0000192161	0	0.0000053272	0.0001771473	0	0	0
	b4	0	0.0001549295	0.0000144473	0.0000122718	0.0000808719	0	-0.0000487828	0
	b3	0	0.0002944829	-0.0000078867	0.0000004165	-0.0001121576	0	0.0002000716	-0.0000227317
	b2	-0.0069739157	-0.0000916487	0.0000197535	0.0000000540	-0.0000009131	0	-0.0001960566	-0.0000837834
	b1	0.0018871296	-0.0000017362	-0.0000003185	06	-0.0000014968	-0.0002871898	-0.0000059933	-0.0000011680
	b0	-0.0000175362	-0.0000002145	0.0000071819	0	-0.0000000223	-0.0000012773	-0.0000000016	-0.0000002729

Table 42 – Plant 2 – Sub-model 1 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	0	1	0	1	1	0	0	0
	a4	1	1.3507773846	1	2.0505333586	2.5459409282	0	0	0
	a3	0.1389741430	1.1178069763	0.1278988268	0.2432815790	0.3156891595	0	0	0
	a2	0.0012515215	0.2175232032	0.0021193276	0.0081242947	0.0086520647	0	0	0
	a1	0.0000122512	0.0134499920	0.0000045124	0.0005295602	0.0000971112	0	0	0
	a0	0.0000001026	0.0001699655	0.0000000735	0.0000079519	0.0000005114	1	1	1
y_2	a5	0	1	1	0	0	0	0	0
	a4	1	1.0321925954	16.8526437466	1	1	0	0	0
	a3	0.2235842644	0.2887153565	7.0514227451	4.8607540969	0.2562881211	0	0	0
	a2	0.0017588627	0.0092619440	0.1160946870	0.0891436860	0.0080387517	0	0	0
	a1	0.0000445817	0.0005933967	0.0024769828	0.0082455499	0.0001178249	0	0	0
	a0	0.0000001417	0.0000102265	0.0000387033	0.0001483415	0.0000003869	1	1	1
y_3	a5	0	1	1	0	1	0	0	0
	a4	1	1.3256143351	0.8770312258	1	0.8030191512	0	0	0
	a3	0.3491007758	1.2309820553	0.1519942911	0.4178283212	0.2059811911	0	0	0
	a2	0.0023434779	0.3371775272	0.0034117767	0.0099103709	0.0161713210	0	0	0
	a1	0.0000482234	0.0198294963	0.0001494529	0.0000648156	0.0002609199	0	0	0
	a0	0.0000003046	0.0002560674	0.0000022327	0.0000001167	0.0000007662	1	1	1
y_4	a5	0	1	0	1	1	0	0	0
	a4	1	1.6024388138	1	1.7965021318	0.1648570086	0	0	0
	a3	0.1471268816	1.2565771703	0.1738836809	0.5288161072	0.0056337703	0	0	0
	a2	0.0009963385	0.2304979257	0.0032637295	0.0541375393	0.0003659525	0	0	0
	a1	0.0000188194	0.0108847753	0.0001191037	0.0008067949	0.0000060483	0	0	0
	a0	0.0000001111	0.0001337089	0.0000017663	0.0000012991	0.0000000097	1	1	1
y_5	a5	0	1	1	0	1	0	0	0
	a4	1	0.7430327125	0.1489796612	0	0.1485903016	0	0	0
	a3	0.0998199022	0.0852072696	0.0071800090	1	0.0087500236	0	0	0
	a2	0.0008420395	0.0044157477	0.0001622476	0.1204811684	0.0005913750	0	0	0
	a1	0.0000103375	0.0001862951	0.0000044619	0.0018161334	0.0000058419	0	0	0
	a0	0.0000000765	0.0000028488	0.0000000411	0.0000020726	0.0000000325	1	1	1
y_6	a5	0	0	1	0	1	1	1	1
	a4	1	1	0.6473935561	1	1.2903523520	2.4446258986	1.4683268549	10.4732031134
	a3	0.2653296007	1.5431967181	0.0773744616	1.5432376945	2.5683166616	1.0772813419	2.8901158270	3.7222645358
	a2	0.0031078612	1.3283514917	0.0026113749	0.1361410524	0.9198969436	0.0510432746	1.0832577634	0.1236758551
	a1	0.0000193241	0.3767487921	0.0000819485	0.0073189983	0.0148284997	0.0006577167	0.0195003172	0.0102044684
	a0	0.0000001729	0.0062836388	0.0000010081	0.0000951577	0.0000035217	0.0000004053	0.0000402803	0.0001431268
y_7	a5	1	1	0	0	1	1	0	1
	a4	1.3572130005	1.8097388978	1	1	5.3083619025	5.0581347472	1	2.0503248828
	a3	1.2342340899	3.8604394101	5.8838370829	2.2562370610	10.3018254782	9.6996882654	4.7959686546	6.8378238706
	a2	0.1577333624	2.9585351944	15.1889006110	0.2490035716	6.3185649178	5.3682685821	9.4236350956	1.3305939516
	a1	0.0003662304	1.6304808086	12.2535482201	0.0000581560	0.7981747255	0.5349731362	5.1716543055	0.0515429933
	a0	0.0000466952	0.1847430953	1.3430591345	0.0000064175	0.0167372794	0.0014379936	0.4822115293	0.0025696256
y_8	a5	1	1	1	1	1	1	1	1
	a4	0.1503165495	5.2958977956	1.4426653376	0.0819735316	1.6707276449	0.7143893521	0.7118270698	0.6364857458
	a3	0.0060416343	9.6051772750	1.9036848539	0.0189506739	1.2823644598	0.1869629710	0.1427399489	0.0773446327
	a2	0.0003013495	12.4041520034	0.8948105947	0.0002445433	0.1213568826	0.0126794867	0.0091525279	0.0029083717
	a1	0.0000071973	7.1742903199	0.0968367840	0.0000471779	0.0021935868	0.0003410102	0.0003667913	0.0002271625
	a0	0.0000000114	0.7023874291	0.0021394427	0.0000000076	0.0000103854	0.0000209327	0.0000190806	0.0000029419
y_9	a5	0	1	1	0	0	0	1	0
	a4	0	1.0520707432	0.0846599356	0	0	1	0.4973705586	1
	a3	0	0.2011216030	0.0059078738	1	0	0.1004218807	0.0404825688	0.0696313712
	a2	0	0.0135481797	0.0003194767	0.0682934679	1	0.0011761851	0.0006461549	0.0028875427
	a1	1	0.0005700614	0.0000081714	0.0126378766	0.1034187624	0.0000257065	0.0000139432	0.0001277360
	a0	0.0012442849	0.0000067138	0.0000000235	0.0000088941	0.0011842736	0.0000002356	0.0000001644	0.0000015950
y_{10}	a5	1	0	1	1	0	0	1	1
	a4	0.3508745334	1	10.3303617886	0.0266592523	1	1	12.1942436420	3.3101140498
	a3	0.0329630540	1.8072304391	1.3849122268	0.0067533245	2.2582640232	10.1444276417	4.0762547364	7.3266801993
	a2	0.0004869378	2.0463462435	0.3502970090	0.0000904611	1.7336864338	1.4145982715	0.2805338782	0.9991431874
	a1	0.0000342696	0.5705451687	0.0424946191	0.0000093553	0.2180957901	0.0213939878	0.0020121829	0.0420559587
	a0	0.0000000216	0.0371332563	0.0000018047	0.0000000323	0.0050649862	0.0015783916	0.0000766420	0.0017131294
y_{11}	a5	0	1	0	1	1	1	0	0
	a4	0	0.8374120844	1	0.0205744522	0.1556860330	1.7553458934	1	1
	a3	0	0.1807903308	0.1616483189	0.0041432406	0.0189629132	1.9200749947	0.2257513582	0.1612929715
	a2	1	0.0083958996	0.7696995569	0.0000493362	0.0021255042	0.3222545834	0.0088047129	0.0073685733
	a1	0.0022672061	0.0003959719	0.1208917214	0.0000030674	0.0000638827	0.0060404997	0.0000904162	0.0005383537
	a0	0.0000110931	0.0000057115	0.0018330126	0.0000000167	0.0000003994	0.0000226508	0.0000000207	0.0000069499

Table 43 – Plant 2 – Sub-model 2 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0	0.0000083959
	b4	0	0	0	0	0.0001646988	0.0001945853	0	0.0000245339
	b3	0	0.0004579769	-0.0000755727	0	-0.0000415313	-0.0000363037	0	-0.0000077387
	b2	0.0000266875	-0.0000120663	-0.0000852804	0.0013504814	0.0000027681	0.0000025809	0.0004399067	-0.0000030015
	b1	0.0000091466	0.0000010903	0.0000266649	-0.0002071391	-0.0000005413	-0.0000003578	-0.0002935855	-0.0000000350
	b0	-0.0000000437	-0.0000000313	-0.0000000342	0.0000024410	04	0.0000000020	-0.0000008720	-0.0000000022
y_2	b5	0	0	0	0	0.0000445325	0	0	0
	b4	0	0	0	0	0.0002779238	0.0002916357	0	0.0003070780
	b3	0.0010106250	-0.0015726386	0	0	-0.0001881113	0.0004773534	0.0001881203	0.0000832513
	b2	0.0012054628	0.0028013185	0.0005213455	-0.0190835358	-0.0000545334	-0.0003711473	0.0000794712	-0.0001363461
	b1	0.0002495583	0.0008950736	0.0001495288	-0.0026425631	-0.0000031566	-0.0000514493	-0.0001490255	-0.0000003114
	b0	0.0000004676	0.0000153172	-0.0000000312	-0.0000021044	-0.0000000027	-0.0000013860	-0.0000004961	-0.0000002835
y_3	b5	0	-0.0004267206	0	0	0	0	0	0
	b4	0	-0.0087536959	0	0	0.0001429595	0.0004666645	0.0001485433	-0.0004235691
	b3	0	0.0115364458	-0.0045768795	-0.0034364386	0.0012508687	0.0026747842	0.0009805454	0.0020048723
	b2	0	-0.0011981122	0.0059345573	-0.0020279844	-0.0011416812	-0.0025076291	-0.0009347483	-0.0019532109
	b1	-0.0035877559	0.0000207564	0.0013661854	-0.0000033794	-0.0000266400	-0.0000563681	-0.0000240001	-0.0001205239
	b0	0.0064160140	-0.0000040714	0.0000004825	-0.0000017302	-0.0000001717	-0.0000003426	-0.0000000903	-0.0000033023
y_4	b5	0	-0.0004195370	0	0	0	0	0	0
	b4	0	-0.0072930566	0	0	0	0	0	0
	b3	0	0.0083951082	0	-0.0205209457	0	0	0	0
	b2	0	-0.0011352139	-0.0146373318	-0.0045843759	0	0	0	-0.0008417440
	b1	-0.0045614631	0.0000023551	0.0163370273	-0.0000060247	-0.0010914736	-0.0006197362	-0.0009954657	0.0000003341
	b0	0.0047473950	-0.0000037782	0.0000209701	-0.0000015374	-0.0000004490	-0.0000000668	-0.0000005434	-0.0000013480
y_5	b5	0	-0.0005394863	0	0	0	0	0	0
	b4	0	-0.0016683556	0	0	0	0	0	0
	b3	0	0.0018487474	0	-0.0031201381	0.0016208241	0.0003430117	0.0014969053	0
	b2	0	-0.0002780772	0	-0.0011419668	-0.0014593569	-0.0005164719	-0.0014175673	0.0007745400
	b1	-0.0027651123	-0.0000023218	-0.0019280963	-0.0000045661	0.0000013119	0.0000004274	0.0000013405	-0.0009032513
	b0	0.0017894571	-0.0000009383	0.0011135411	-0.0000003297	-0.0000003316	-0.0000001217	-0.0000002536	-0.0000028607
y_6	b5	0	0	-0.0000211806	0	0	0	0	0
	b4	0	-0.0015753530	-0.0011157351	0	0	-0.0000468704	0	0
	b3	0	0.0023805120	0.0014797675	-0.0070229353	0	0.0008365529	0	0.0005070473
	b2	0.0037396289	-0.0002208452	0.0000065378	-0.0006138597	0.0008631190	-0.0006965313	0.0005662017	-0.0004538986
	b1	0.0001189712	0.0000042213	0.0000056547	-0.0000056215	-0.0012233663	-0.0000090619	-0.0004782242	0.0000013788
	b0	0.0000014600	-0.0000006401	02	-0.0000004919	-0.0000016283	-0.0000000361	-0.0000002515	-0.0000010479
y_7	b5	-0.0061526846	0	0	0	0	0	0	0
	b4	0.0296668498	0	0	0	0	0	0	0
	b3	0.0008082933	0	0	0	0	0	0	0.2734867646
	b2	0.0000636204	0.1321910413	-4.2785460923	-1.0846738670	0.5091044695	0.3581208846	0.4622904385	0.1156658241
	b1	0.0000017696	-0.0225098633	-4.3044072055	-1.9576299851	-0.0203533212	-0.0387203183	0.0129070084	-0.0354414360
	b0	07	-0.0002223130	-0.3355447832	-0.2722545938	-0.0417693541	-0.0324541603	-0.0314011268	-0.0012892473
y_8	b5	0	0	0	0.0000817090	0	0.0003981585	0	0
	b4	0.0050271593	-0.0055637836	0	0.0000344088	0.0148250023	0.0084741196	0	0
	b3	0.0497708158	0.0147875692	0.0002901979	-0.0000209241	-0.0175359739	-0.0100225475	0	-0.0010680956
	b2	0.0072378057	-0.0036541940	0.0003623077	-0.0000027475	-0.0002701752	-0.0005796065	0.0517751917	-0.0064951298
	b1	0.0002707512	0.0000315369	0.0000163354	-0.0000004381	-0.0000185236	-0.0000148529	-0.1099601828	-0.0000563924
	b0	0.0000007591	-0.0000116269	0.0000062830	-0.0000000115	-0.0000001020	-0.0000008944	-0.0089639132	-0.0000007683
y_9	b5	0	-0.0001188010	0	0	0	0.0012065733	0	0
	b4	0	-0.0240690388	0	0	0	0.0004719949	0	0
	b3	0	0.0200082325	0	0	0	-0.0004660992	0	0
	b2	-0.0002016495	-0.0028511082	0	-0.0000038731	-0.0002153636	-0.0000592822	-0.0001977354	-0.0002283097
	b1	0.0010289943	0.0000599742	0.0000166552	-0.0000000435	-0.0000460999	-0.0000003429	-0.0000350732	-0.0000004358
	b0	0.0013293070	-0.0000107140	-0.0000002102	-06	0.0000000132	-04	0.0000000127	-0.0000003504
y_{10}	b5	-0.0040354413	0	0	0	0	0.0000614155	0.0000286214	0
	b4	0.0322028014	-0.0068020531	0	0.0000355112	0	0.0362719151	11.6760861163	0.0001643220
	b3	0.0167444308	0.0239450464	0.0007411858	0.0025927586	0.3243103791	0.0019762885	-2.8697500175	0.0240872290
	b2	0.0011291981	-0.0061656389	0.0004712910	-0.0009114023	0.3059953575	-0.0222968342	-1.0800340567	-0.0136841675
	b1	0.0000027097	-0.0000845970	0.0000043797	-0.0000040963	-0.3848958333	-0.0011960293	-0.0436156677	-0.0035952072
	b0	0.0000001239	-0.0000294108	0.0000000318	0.0000001074	-0.0292012080	-0.0000163213	-0.0000436047	-0.0001918816
y_{11}	b5	0	0	0	0	0	0	0	0
	b4	0	0.0000789961	0	0	0	0	0	0.0001827257
	b3	0	0.0001541394	0.0000200309	0	0	0.0003744995	0	-0.0002969679
	b2	0	-0.0000991345	-0.0000001390	0	0	-0.0004501734	0	-0.0000682615
	b1	0.0012500148	0.0000001303	0.0000000686	-0.0000089236	-0.0005127704	-0.0000065300	-0.0005042485	-0.0000001914
	b0	0.0000008153	-0.0000003078	-05	-0.0000000353	0.0000001098	-0.0000000174	-0.0000000136	-0.0000000119

Table 44 – Plant 2 – Sub-model 2 – Transfer Function Denominator Parameters.

PI	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	1	0	0	1	1	0	1
	a4	0.9022658577	1.4167540291	0	0	0.1402274786	0.2397707429	1	0.3139954487
	a3	0.3173851327	0.1144181768	1	0	0.0156038975	0.0114872846	21.6057345836	0.0276853766
	a2	0.0255477291	0.0076090816	0.1329340032	1	0.0018798958	0.0021917784	2.5612733951	0.0008726327
	a1	0.0002253713	0.0002574887	0.0018370107	0.0017388599	0.0000294349	0.0000166053	0.0435563322	0.0000205078
	a0	0.0000002662	0.0000066296	0	0.0000747768	0.0000000080	0.0000001465	0.0001119346	0.0000003009
y_2	a5	0	0	1	1	1	0	0	0
	a4	1	1	1.4609958758	3.8032430285	0.5491104364	1	0	1
	a3	0.4897330091	1.0653609121	0.9353149427	13.3563647566	0.0825374409	0.3531747536	1	0.1732826433
	a2	0.0563708683	0.3286032327	0.1610617195	2.0530141748	0.0042803077	0.0346824855	0.2093974551	0.0053417270
	a1	0.0006991356	0.0112487072	0.0023945335	0.0328002825	0.0000540074	0.0010997108	0.0035888443	0.0003592680
	a0	0.0000071343	0.0000907810	0.0000000301	0.0000197011	0.0000000388	0.0000113428	0.0000142317	0.0000053242
y_3	a5	0	1	1	0	0	0	0	0
	a4	0	1.1493784228	3.8886158584	1	1	1	1	1
	a3	0	0.3309695649	2.2749239139	0.3922966508	0.3363296518	0.3252537675	0.3434637049	0.4009238735
	a2	1	0.0104316017	0.2865644933	0.0070564978	0.0114593500	0.0108665445	0.0122610604	0.0251294677
	a1	0.3449236252	0.0010461294	0.0042584574	0.0003343929	0.0001576989	0.0001448445	0.0001545607	0.0008157440
	a0	0.0048397043	0.0000129245	0.0000023299	0.0000052884	0.0000006354	0.0000005570	0.0000003900	0.0000074581
y_4	a5	0	1	0	1	1	1	1	1
	a4	0	1.0201588062	1	3.5714659176	0.7507242908	0.6735141682	0.8286698481	1.3977570748
	a3	0	0.1800965935	12.2595753515	0.5535729355	0.8632976847	0.7621399370	0.9532127408	0.1874560514
	a2	1	0.0072207071	1.9553151965	0.0088355212	0.1183576305	0.1026561292	0.1330434304	0.0049621452
	a1	0.2091556904	0.0005193761	0.0313827594	0.0001892288	0.0017253097	0.0014724350	0.0019524728	0.0002989867
	a0	0.0026829128	0.0000057891	0.0000371099	0.0000026043	0.0000017616	0.0000009751	0.0000023834	0.0000043654
y_5	a5	0	1	0	0	1	1	1	1
	a4	0	0.5687713341	0	1	1.4699109337	0.8346571866	1.7451058141	1.3745938232
	a3	0	0.0801955458	0	0.1270054779	0.1624665830	0.0893185832	0.1919656249	2.0351323311
	a2	1	0.0034060145	1	0.0024742197	0.0023563156	0.0012771814	0.0027367182	0.2143254141
	a1	0.1579902477	0.0002111567	0.1257493303	0.0000417821	0.0000380656	0.0000217722	0.0000348602	0.0035846856
	a0	0.0018123064	0.0000019125	0.0017827840	0.0000005723	0.0000004720	0.0000002623	0.0000004398	0.0000099388
y_6	a5	0	1	1	1	1	0	0	0
	a4	1	0.9698714976	0.5954991382	2.7477883929	1.3258623878	1	0	1
	a3	1.0553524202	0.3615643339	0.0180458683	0.2882374723	2.5555033053	0.4453838244	1	0.3692358911
	a2	0.0437484849	0.0105674664	0.0023318739	0.0061063638	0.9265907561	0.0122280387	0.4491529310	0.0083144703
	a1	0.0009137417	0.0010295630	0.0000377092	0.0002323759	0.0156587066	0.0001133455	0.0073182757	0.0008544210
	a0	0.0000024354	0.0000143565	0.0000000180	0.0000031408	0.0000252732	0.0000003001	0.0000069820	0.0000138852
y_7	a5	1	1	0	1	0	0	0	1
	a4	0.2516606967	3.0552345950	1	4.7678810673	1	1	1	4.4033571585
	a3	0.0075797767	3.3615651340	5.5873022701	17.9036346473	4.5314974527	4.6258713970	4.5222249435	10.1203516285
	a2	0.0005472978	2.8085427909	14.2614550266	37.2565227113	8.9445384355	9.0630227573	9.0057115879	8.9918088444
	a1	0.0000115863	0.4171279077	11.5592086229	29.3734958867	5.1306521332	5.0596839112	5.2583191917	1.4625895061
	a0	0.0000000266	0.0037049659	1.2631255000	2.6897968259	0.5015135716	0.4870742807	0.5131920208	0.0371892621
y_8	a5	1	1	1	1	1	1	1	1
	a4	2.9875467844	1.3903808122	0.5582292966	0.4969747432	1.9834821136	1.8095909167	6.3309963745	1.7812822383
	a3	0.7203395005	1.0577258493	1.2018544174	0.0783270383	0.2571980245	0.2945232188	14.3930697228	1.5055856751
	a2	0.0600511505	0.1210196845	0.1666989405	0.0098272301	0.0056938940	0.0137248879	17.4986753520	0.1692941205
	a1	0.0017076050	0.0038189527	0.0233805869	0.0005700999	0.0002428236	0.0004345511	3.2100470531	0.0013953628
	a0	0.0000030964	0.0003553788	0.0021261344	0.0000118567	0.0000013145	0.0000163452	0.1418164524	0.0000203861
y_9	a5	0	1	0	0	1	1	1	0
	a4	0	32.6992877304	0	1	1.2628323939	0.2147019182	1.4360957771	1
	a3	0	4.7137164960	1	0.0818784103	0.3226717796	0.0132708183	0.3421977710	0.1041241910
	a2	1	0.2033522271	0.1246992002	0.0019432189	0.0225012340	0.0002018663	0.0227273771	0.0026059392
	a1	0.1386706609	0.0172574502	0.0000159029	0.0000193616	0.0002415064	0.0000013049	0.0002427179	0.0001606628
	a0	0.0012259182	0.0002098152	0.0000019830	0.0000000088	0.0000008700	0.0000000016	0.0000007680	0.0000016400
y_{10}	a5	1	1	1	0	1	1	1	0
	a4	0.6778925912	1.2489268998	0.4669077588	1	10.1678115424	2.8874563752	456.6826204285	1
	a3	0.1034473339	1.5270514898	1.5154201059	5.6154548676	26.7957935483	2.4961804940	178.6062121511	2.7366984250
	a2	0.0044574198	0.2505126585	0.1507556211	0.9081818579	32.8319818511	0.3885616676	18.4290042430	0.8939480133
	a1	0.0000142108	0.0122712014	0.0014157508	0.0001173394	6.1339980248	0.0145190679	0.5019425324	0.0952741995
	a0	0.0000005982	0.0008566492	0.0000002274	0.0000153144	0.2456834863	0.0001769231	0.0004975308	0.0031165035
y_{11}	a5	1	1	1	0	1	0	1	1
	a4	0.9049849117	0.4975606977	0.1427663052	0	1.2602621676	1	1.5926543722	0.3722848156
	a3	0.6372676372	0.0997878349	0.0046206328	1	1.6580641686	0.1926480226	2.1277577405	0.0373841108
	a2	0.0849876385	0.0028738145	0.0005153596	0.1211725019	0.2558788680	0.0051261064	0.3310813409	0.0006396440
	a1	0.0013767782	0.0003055812	0.0000032551	0.0024834831	0.0037105051	0.0000466205	0.0048703332	0.0000075642
	a0	0.0000040080	0.0000039898	0.0000000085	0.0000015873	0.0000007325	0.0000000730	0.0000022506	0.0000000846

Table 45 – Plant 2 – Sub-model 3 – Transfer Function Numerator Parameters.

Pl	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0	0.0000175239	0	0	-0.0000333074	-0.0000372682	0.0000319584	0
	b2	0	0.0000005680	0	0	-0.0000392829	-0.0000228452	-0.0000190525	0.0010899294
	b1	0.0000665099	0.0000000178	0.0001262609	0.0000331833	-0.0000097290	-0.0000009110	-0.0000011602	-0.0002110223
	b0	-0.0000005132	-05	0.0000175729	-0.0000002939	-0.0000000461	-0.0000000067	0.0000000017	0.0000013831
y_2	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	-0.0016103813	0.0004405647	0	0	0	0	0	-0.0003027473
	b2	0.0019083898	-0.0000036305	0.0004840073	0.0058592297	0	-0.0008393909	0	-0.0000034523
	b1	0.0000090411	0.0000016609	0.0000071425	-0.0000074492	-0.0003246591	-0.0002703587	-0.0001866340	-0.0000047599
	b0	-0.0000000990	-0.0000000097	0.0000000210	0.0000000261	-0.0000004223	-0.0000008478	0.0000006028	0.0000000048
y_3	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	-0.0002043482
	b3	0	0	0.0052484237	0	0	0	0	-0.0022874121
	b2	0.0061340698	0	-0.0095451519	0	0	0	0	-0.0010612134
	b1	0.0000140237	-0.0000801431	0.0061282710	0.0089135247	-0.0021110274	-0.0020838636	-0.0047037902	-0.0000057284
	b0	-0.0000001858	-0.0000004924	-0.0000033103	0.0051078944	0.0000005953	-0.0000009606	-0.0000003227	-0.0000000687
y_4	b5	0	0	0	0	0	0	0	0.0008202328
	b4	-0.0030490538	0	0	0	-0.0002678044	0	0	0.0001039400
	b3	-0.0030681869	0	0	0	-0.0014278452	0	0	-0.0007550896
	b2	0.0029098675	-0.0000196715	0.0030428617	0	-0.0000045963	0	0	-0.0000441384
	b1	0.0000046170	-0.0000016641	0.0055855208	0.0076101457	-0.0000020413	-0.0009716500	-0.0011924207	-0.0000042160
	b0	-0.0000000679	-0.0000000313	0.0001668090	-0.0000106677	-0.0000000064	-0.0000005271	0.0000003736	-0.0000000104
y_5	b5	0	0	0	0	0	0	-0.0002883790	0
	b4	0	0	0	0	0.0023714099	0.0027427250	0.0009104693	0
	b3	0	0	0	0.0012924335	-0.0013684009	-0.0015221134	-0.0006148573	0.0017498474
	b2	-0.0156977637	0.0017547171	0.0026802565	0.0006677549	0.0000100803	-0.0000762867	-0.0000342308	-0.0007545139
	b1	0.0007754898	-0.0003634464	0.0000078206	0.0000115395	-0.0000047640	-0.0000047937	-0.0000039121	0.0000023857
	b0	0.0029331458	-0.0000018932	0.0000000138	-0.0000000302	0.0000000017	0.0000000020	-0.0000000012	-0.0000001466
y_6	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0	0	0	0	0	0	0.0002852926	0
	b2	0.0033199036	0.0015373411	-0.0035800828	0.0004254101	0	0	0.0001025960	0
	b1	0.0001184041	-0.0001885723	0.0020361844	0.0000226659	-0.0007031823	-0.0008220264	-0.0003432217	-0.0007390235
	b0	-0.0000003228	0.0000005544	0.0000024102	-0.0000003319	-0.0000000609	-0.0000000370	-0.0000002025	0.0000023095
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0.0970009299	0	0	0	0	0.7354469313	0.2528129462	0
	b2	0.0422178516	0.0224736172	-1.9154960249	0	1.2772605815	0.0526000159	0.0260166174	0.5305425938
	b1	0.0000453325	0.0277576844	-1.9601720792	25.3117557456	-0.0605915309	-0.1091498343	-0.0239855565	0.2047934264
	b0	0.0000115303	-0.0066741618	-0.1568666168	5.3411632263	-0.1313758473	-0.0060268188	-0.0005026206	-0.0859792667
y_8	b5	0	0	0	0	0	-0.0083261013	0	0
	b4	0.0718807243	0	0	0.0000748122	0	0.6090570143	0.1162237335	0
	b3	0.1855911945	0.0064947786	0.0023654778	0.0000211687	0	-0.6757156899	-0.1278380390	-0.0296744462
	b2	0.0248093522	0.0152618232	0.0000120402	0.0000002983	0.0693444306	0.0002008011	-0.0003207655	-0.0183699815
	b1	0.0001961484	-0.0073695112	0.0000080242	0.0000000788	-0.1007077374	-0.0016791518	-0.0004219872	-0.0004884064
	b0	0.0000019400	-0.0000717456	-0.0000000079	02	-0.0000599623	-0.0000019481	-0.0000017881	-0.0000229578
y_9	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0	0	0	0	0	-0.0001405224	0	0
	b2	-0.0275423287	0.0005879621	0.0000007831	0.0000000252	0	-0.0000556917	0	0
	b1	0.0047213447	-0.0000178545	0.0000000051	-0.0000000418	-0.0002381400	-0.0000034266	-0.0001410354	-0.0001006227
	b0	-0.0000441181	0.0000017793	04	0.0000000012	0.0000010185	0.0000000137	0.0000002612	0.0000008231
y_{10}	b5	0	-0.0000594176	0	0	0	0.0002954625	0	0
	b4	0.0664476649	-0.0013910804	-0.0023953517	0	0	0.0629222633	0.0000711968	0
	b3	0.2247552834	0.0045989169	0.0023891502	-0.0042855275	0.2203472006	-0.0070594978	0.0294013806	0.0348918527
	b2	0.0016409563	0.0027479811	0.0008943826	0.0012466645	0.1562178688	-0.0298472639	0.0002873347	-0.0086190178
	b1	0.0001930643	-0.0014772641	0.0005389002	0.0000429309	-0.2299438601	-0.0014515736	-0.0084886602	-0.0128737798
	b0	0.0000010602	-0.0004231430	-0.0000000678	0.0000000986	-0.0165988601	-0.0000014236	-0.0004700969	-0.0006437994
y_{11}	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0	0	0	0	0
	b3	0	0	0	-0.0001029896	0	0	0	-0.0005288331
	b2	0.0027001630	0.0014687961	0	-0.0000041810	0	0	0.0048142306	-0.0000023269
	b1	0.0000076554	-0.0003705849	01	0.0000014123	-0.0004753838	-0.0013369067	-0.0012253774	-0.0000019404
	b0	-0.0000000454	0.0000002433	0	-0.0000000094	-0.0000002590	0.0000002482	0.0000115119	-0.0000000017

Table 46 – Plant 2 – Sub-model 3 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	1	0	0	0	1	0	0
	a4	1.9437103443	1.0792119713	0	1	1	0.9072922849	1	0
	a3	0.7143443581	0.0707527664	0	2.1983888194	0.4251510121	0.1091559238	0.2020622332	0
	a2	0.0859927770	0.0078773966	1	0.5297253220	0.0329832869	0.0045115186	0.0102481912	1
	a1	0.0004979633	0.0000061654	0.1356894124	0.0031492844	0.0006812622	0.0000754165	0.0001052777	0.0036092864
	a0	0.0000009085	0.0000006791	0.0011941802	0.0000088090	0	0	0.0000001004	0.0000418789
y_2	a5	0	0	1	0	0	0	0	1
	a4	1	1	2.1075204692	1	0	0	0	0.1975822494
	a3	0.2564361342	0.0021601916	0.7658630905	13.4277390131	1	1	1	0.0205439005
	a2	0.0046579580	0.0044086048	0.0233969096	0.2240552976	0.1828531210	0.1123260894	0.2374377036	0.0029399411
	a1	0.0000207119	0.0000091743	0.0002014657	0.0000647941	0.0030779313	0.0021939279	0.0023543267	0.0000393287
	a0	0.0000000554	0.0000006354	0.0000002153	0.0000010811	0	0	0.0000046793	0.0000000188
y_3	a5	0	0	0	0	0	0	0	1
	a4	1	0	0	0	0	0	1	0.9925006063
	a3	0.2154916885	1	1	0	1	1	6.5221993089	0.1425551858
	a2	0.0038821826	0.0824912314	0.5471030839	0	0.2177880861	0.2189989745	1.2682328782	0.0030819964
	a1	0.0000073457	0.0020456550	0.0081355739	1	0.0032679128	0.0034723635	0.0201941630	0.0000197278
	a0	0.0000001312	0.0000006134	0.0000013831	0.0134701790	0.0000013529	0.0000018736	0.0000021755	0.0000001494
y_4	a5	0	1	0	1	1	0	0	1
	a4	1	0.2702288864	1	0.5666471409	0.1417926749	0	1	0.1952332486
	a3	0.1535808615	0.0430988270	1.8948298151	1.9893123147	0.0040484736	1	0.9194209174	0.0158637915
	a2	0.0025558601	0.0017667511	0.3679234928	0.8345452862	0.0002071486	0.1310004623	0.1310771296	0.0008773493
	a1	0.0000049981	0.0000647373	0.0132345705	0.0115816918	0.0000037175	0.0020809290	0.0018886121	0.0000138640
	a0	0.0000000815	0.0000000308	0.0001292340	0.0000031502	0.0000000095	0	0.0000003708	0
y_5	a5	0	0	1	0	1	1	1	0
	a4	0	1	1.0808078268	1	0.1356875409	0.1658560715	0.1905169869	1
	a3	0	1.3450114802	0.1294062948	0.1325142579	0.0053589168	0.0110564208	0.0156341499	0.1405678430
	a2	1	0.1387174401	0.0022962383	0.0041750403	0.0004739352	0.0004350384	0.0008724358	0.0017794477
	a1	0.1578057914	0.0030303748	0.0000010316	0.0000233801	0.0000065632	0.0000052534	0.0000115330	0.0000303582
	a0	0.0017228853	0.0000027633	0.0000000182	0.0000000597	0.0000000135	0.0000000063	0	0.0000003377
y_6	a5	1	0	0	0	1	0	0	0
	a4	1.2633682283	0	0	1	1.0862546737	0	0	0
	a3	0.4565080131	0	0	0.7281657003	1.1533039922	1	1	1
	a2	0.0215590591	1	1	0.0717378832	0.3623527309	0.3105109927	0.3582234572	0.3952098298
	a1	0.0001887205	0.0110687674	0.0177371785	0.0000433089	0.0058685140	0.0049962541	0.0061996472	0.0043662938
	a0	0.0000002882	0.0000139002	0.0000430692	0.0000008697	0.0000012371	0.0000012273	0	0.0000011461
y_7	a5	1	1	0	1	1	1	1	0
	a4	1.3029538655	1.9091158389	1	32.1874512866	4.1088033657	5.7423226156	4.5673784461	1
	a3	0.8604396958	4.2177240905	4.5948269484	187.1237466730	16.1937504630	11.2783989314	9.0509097484	4.3568320419
	a2	0.0980359743	3.4071816312	11.2353745117	875.9302122263	25.6972487651	8.1087497083	5.8644876839	9.7378446459
	a1	0.0003130028	2.1630282268	9.1694637496	1174.4912737547	14.7783594479	1.3136450563	0.7496808360	8.8452948280
	a0	0.0000250928	0.2708406040	1.1310118916	126.9067017526	1.4978805877	0.0480188375	0.0127866078	1.1226297471
y_8	a5	1	1	0	1	0	1	1	1
	a4	1.0339842524	1.9127912654	1	0.2230116372	0	12.7467919109	7.2549735463	1.1970907055
	a3	0.2284505932	2.2769193141	0.1170897031	0.0217619164	1	1.4410824244	0.8808461329	0.5890879389
	a2	0.0139786303	1.2995303429	0.0044119139	0.0008039078	2.3675030916	0.0330596082	0.0272952806	0.0641150113
	a1	0.0001146625	0.1562441146	0.0003781282	0.0000628793	0.2714326500	0.0036443081	0.0028805275	0.0019656608
	a0	0.0000009217	0.0013521820	0.0000000640	0.0000000852	0	0.0000017461	0.0000099237	0.0000629548
y_9	a5	0	0	1	1	0	1	0	0
	a4	0	0	0.1953875742	0.6736834115	0	0.4000706693	0	0
	a3	0	0	0.0078388615	0.2383293498	1	0.0553641531	1	1
	a2	1	1	0.0002745032	0.1205854426	0.1118368532	0.0024845421	0.1069916161	0.1156748743
	a1	0.0000000751	04	0.0000036622	0.0018937306	0.0008161224	0.0000184722	0.0011263857	0.0003763260
	a0	0.0000425295	0.0001452077	0.0000000042	0.0000002790	0.0000027146	0.0000000354	0.0000010150	0.0000048594
y_{10}	a5	1	1	1	0	1	1	0	0
	a4	3.3902397213	1.8661201094	1.7012287890	1	7.6019118506	3.2652756085	1	1
	a3	0.4043723931	2.5864514964	2.1031711520	14.9867882816	18.0062174622	2.5258945684	3.2326786915	2.5191818108
	a2	0.0050261507	1.6423202143	1.2714686993	5.7470030424	21.4910426112	0.3688696063	2.2926086962	1.4945664037
	a1	0.0003415775	0.4643899603	0.1226615814	0.1517361862	3.8478500278	0.0116130644	0.3336864832	0.2012598078
	a0	0.0000013223	0.0387077374	0	0.0003111686	0.1535906434	0.0000136045	0.0112262603	0.0063804478
y_{11}	a5	0	0	1	0	0	0	0	1
	a4	1	0	0.0434593880	0	0	1	0	0.1750310058
	a3	0.1479431059	0	0.0020290995	1	1	1.9631340720	0	0.0060747686
	a2	0.0027004884	1	0.0000348783	0.1658187045	0.1474516580	0.2991203847	1	0.0006895879
	a1	0.0000060077	0.0138249033	0.0000008874	0.0023489638	0.0022809940	0.0042941403	0.0007605556	0.0000085470
	a0	0.0000001080	0.0000060835	0	0.0000003977	0	0.0000000670	0.0000526818	0.0000000619

Table 47 – Plant 3 – Sub-model 1 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0	0
	b4	0.0088265377	0	0	0.1148066535	0	-0.0000617049	0.0056779129	0
	b3	0.0032902506	0	0	0.0774254574	0.0078000437	0.0114822949	-0.0012490954	0
	b2	0.0003150114	-0.0258105881	-0.0757127689	0.0082045612	-0.0019656391	-0.0026171062	-0.0031390142	-0.0029029759
	b1	0.0000041281	0.5849528973	0.0116802676	0.0000282916	-0.0038822835	-0.0048766530	-0.0000017349	-0.0000723765
	b0	0.0000000510	-0.0149272364	0.0061516246	0.0000006085	0.0000001888	-0.0000100091	-0.0000027660	-0.0000069582
y_2	b5	-0.0039096482	0	0	0.0000258205	0	0	0	-0.0038565107
	b4	0.0707643044	0	-0.1960958602	0.7428811725	0	0	0.0003487225	0.0642953837
	b3	0.0578356360	-0.0493317362	-0.1388095299	0.9875932485	1.1629408914	0.5514690706	0.0578688278	-0.0316719715
	b2	0.0068412453	0.5632949179	0.1393578490	1.2819935417	-0.3309061086	-0.2555883639	-0.0130848198	-0.0149671831
	b1	0.0000518072	1.0262823462	0.8795715778	0.1125098406	-0.3955333648	-0.1413999845	-0.0223051227	-0.0004341016
	b0	0.0000013232	0.0400236244	0.1017949219	0.0002946908	-0.0306112196	-0.0066498750	-0.0021445081	-0.0000348271
y_3	b5	0	0	0	0	0	0.0010956671	0.0002767085	-0.0079085427
	b4	-0.0009043812	0	-0.2033617575	0	0.1731163471	0.2145907014	0.1398816110	0.1602123021
	b3	-0.1103957089	0	0.1982914405	0	-0.0298955412	-0.0345505893	-0.0497696964	-0.1070915423
	b2	0.3100273231	0.5916043620	0.1365419591	0	-0.0806847628	-0.0977007066	-0.0502784387	-0.0264095833
	b1	0.3099043910	0.6165535354	0.0081760591	16.3121853737	-0.0069315319	-0.0073931353	-0.0026991857	-0.0009044360
	b0	0.0207475447	-0.0671774358	0.0000559529	1.9215007790	-0.0000037294	-0.0000008061	-0.0000020235	-0.0000590189
y_4	b5	0	0	0	0	-0.0003545851	0	0.0002887180	0
	b4	-0.0195094886	0	-0.2605066146	0	0.0801275690	0.0983953472	0.0697419224	0
	b3	0.0909114681	0	0.0467562611	0	-0.0034139430	0.0083311167	-0.0230844811	0.0321276965
	b2	0.0070711099	0	0.1390273359	0	-0.0564177330	-0.0710786167	-0.0330389274	-0.0642091967
	b1	0.0001939331	2.3949561661	0.0220481508	31.9946799923	-0.0040492612	-0.0077383904	-0.0021999788	-0.0017565333
	b0	0.0000147083	-0.3220977687	0.0008159501	4.6345899078	-0.0000051504	-0.0000016178	-0.0000543340	-0.0001414797
y_5	b5	0	0	0	0.0015950460	0	0	0	0
	b4	0	0	0	0.6325692292	-0.0002152645	0	0.0232248731	-0.0032027931
	b3	-1.4331843106	0	0	0.2048819219	0.0358501529	0	0.0202420286	0.0111445825
	b2	2.0498547018	0.2388432977	0	0.0169193848	0.0032407326	0	-0.0392489436	-0.0228388709
	b1	0.0000249920	-0.0271222814	-2.3662893349	0.0012400850	-0.0333401267	-0.0465852208	0.0000374170	-0.0005267533
	b0	0.0014582277	-0.0011019907	2.3908854272	0.0000479255	-0.0000261511	-0.0003934891	-0.0000650229	-0.0000585280
y_6	b5	0	0	0	0	0.0004255303	0	0	-0.0035621339
	b4	-0.0001993029	0	-0.0730081339	0	0.0947819198	0	0	0.6508748026
	b3	-0.0488922018	0.0362862725	0.0429753535	0	-0.0181301911	0.7958644884	1.8474594330	-0.3447143136
	b2	0.1524119450	0.2554372844	0.0685545186	0	-0.0380285671	-0.4378828407	-0.6884694848	-0.0181009459
	b1	0.1609509211	0.0670647409	0.0058467833	4.0285151031	-0.0021521860	-0.1684534920	-0.5956396808	-0.0012913937
	b0	0.0093404961	-0.0079507790	0.0001296385	0.9388559704	-0.0000027027	-0.0006366489	-0.0227983311	-0.0000490580
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	0.0003241525	0	0	0	0	0	0
	b3	-0.1306272230	-0.0480606686	-4.1243525373	0	148.1765787534	83.0834925148	0	0.2927647102
	b2	0.1911461735	0.0836295155	-6.1844755747	0	192.2670447375	160.3316323779	0.3376626862	-0.0090080920
	b1	0.6326307601	0.0138171154	-1.1851615513	4.8059581634	-24.4777095916	-26.4063505361	0.0046509577	0.0004088230
	b0	0.0926010159	-0.0045004903	-0.0517847126	0.6320786479	-7.6814565117	-7.9399927693	-0.0145975139	-0.0000544584
y_8	b5	0	0	0	0.0000919421	0	-0.0017356215	0	0.0125468543
	b4	0	0	0	-0.0058194907	0	0.0322926547	0	-0.0176851611
	b3	0	0	0.0018431750	0.0014470922	0	-0.0044007366	-0.0169669526	-0.0047684193
	b2	0	0	0.0005927737	-0.0000222052	0.0505149412	-0.0502010419	-0.0019167301	-0.0001406101
	b1	0.3536987244	0.5087488668	0.0000004105	0.0000028158	-0.0519237853	-0.0003217669	-0.0000345536	-0.0000134945
	b0	0.0182118185	-0.1263767806	0.0000000174	-0.0000000161	-0.0001548623	-0.0000032684	-0.0000038000	-0.0000002523
y_9	b5	0	0	0	0	0	0	0	0
	b4	0.0270166590	0	-0.0017986188	0	0	0	0	0
	b3	0.0051171588	0	0.0017198022	0	0	0.0442326090	0.0159780365	0
	b2	0.0002748816	0.0413796129	0.0005921534	0.0000459729	0.1412746290	-0.0672689292	-0.0287456332	-0.0049872295
	b1	0.0000113360	-0.0021155042	0.0000144183	0.0000001023	-0.1283318760	-0.0084394240	-0.0031337366	-0.0001281669
	b0	0.0000004667	-0.0003840880	0.0000000183	0.0000000066	-0.0129213440	-0.0000025141	-0.0000375047	-0.0000117683
y_{10}	b5	0	0	-0.0002329884	0	0.0001945052	0	0.0001500766	0
	b4	0.0346272897	0.0000291861	0.0004924264	0	3.1342867044	0.0764542171	0.0314577474	0
	b3	0.0048306266	-0.0130930416	0.0000046642	-0.0001709291	-1.3961641608	-0.0342246943	-0.0048573904	0
	b2	0.0001887476	0.0359678091	0.0000014958	0.0000917689	-0.1070289033	-0.0096250696	-0.0066050925	-0.0029198309
	b1	0.0000162736	-0.0050282540	-0.0000000032	0.0000003677	-0.0026111186	-0.0001302558	-0.0000046493	-0.0000686361
	b0	0.0000002659	-0.0012887783	01	0.0000000414	-0.0001881337	-0.0000060997	-0.0000027361	-0.0000081407
y_{11}	b5	0	0	0.0000018737	0	0	0	0	0.0018521551
	b4	0	0	-0.0022431971	-0.0006424568	0	0	0	-0.0067241188
	b3	0.1051089725	-0.0024466439	0.0013955060	0.0003785498	0.0219737666	0.0105784662	0.0245907406	-0.0006535329
	b2	0.1565109935	0.0221512126	0.0004965740	0.0000037643	-0.0125871686	-0.0327703927	-0.0433200762	-0.0000394042
	b1	0.0199789880	-0.0076288055	0.0006027662	0.0000008628	-0.0175910544	-0.0000057333	-0.0005842263	-0.0000016700
	b0	0.0000105473	-0.0002975147	0.0000005740	0.0000000043	-0.0000116030	-0.0000046244	-0.0000309957	-0.0000000149

Table 48 – Plant 3 – Sub-model 1 – Transfer Function Denominator Parameters.

PI	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	1	0	1	0	0	1	1
	a4	0.5420352999	8.0997348473	1	2.3869080177	1	1	1.4276071350	0.9009321133
	a3	0.0838202251	19.3943925899	3.0050675717	1.2303588213	1.7316577218	1.7515205325	1.1733907369	1.1983509221
	a2	0.0043612914	25.2111950271	7.4568623565	0.0950071648	1.2244885743	1.2245796060	0.0990416196	0.1346417397
	a1	0.0000587865	15.7308369225	2.2305534753	0.0003576191	0.0972009874	0.1019277635	0.0010388924	0.0057199256
	a0	0.0000005338	1.3031046440	0.1472555752	0.0000068000	0.0000082843	0.0001673819	0.0000855731	0.0002286027
y_2	a5	1	1	1	1	1	1	0	1
	a4	1.0451917605	3.2031584009	20.1809928713	11.4872832543	15.0081649321	6.1080777537	1	2.1473339271
	a3	0.2863526382	7.3152749558	29.8328307695	15.1796611678	28.7770606974	11.7200341127	1.8132397804	1.4128656129
	a2	0.0168520323	7.4044811660	42.1170592104	15.0295527146	22.5011722877	7.2475549629	1.5337793339	0.1576429838
	a1	0.0001565830	4.5760072507	15.8073351046	1.1703213263	4.0495433917	0.9666258010	0.3160632902	0.0066979271
	a0	0.0000027823	0.4637748046	1.1585537176	0.0030872546	0.1769979908	0.0306874812	0.0158200344	0.0002642918
y_3	a5	0	1	1	0	1	1	1	1
	a4	1	2.3086378016	2.6197167746	1	1.9467407210	1.9249428705	1.8284486520	2.2613972464
	a3	2.1511435309	4.8310667085	2.0764588783	5.8084314488	1.6519098957	1.6363781849	1.3820765884	1.2340026311
	a2	2.6993655636	4.2951520880	0.4342313827	31.9441054075	0.4100443925	0.3913131619	0.2783000212	0.1443269922
	a1	0.7258880651	2.5138165763	0.0203402531	47.9974534683	0.0227740954	0.0197615693	0.0110084894	0.0061252832
	a0	0.0344441703	0.3838913084	0.0001359016	4.6163321593	0.0000096091	0	0.0000091735	0.0002504049
y_4	a5	0	1	1	0	1	1	1	1
	a4	1	5.3598101911	2.8591708683	1	1.5949844551	1.6933235365	1.5803362173	1.1033836709
	a3	0.2251875118	12.4926620123	2.2417079420	6.0947788633	1.4110055096	1.5059902781	1.0926074734	1.9948612470
	a2	0.0115399278	13.8988482383	0.5201517755	48.6768134884	0.2426325792	0.3074331450	0.1709470658	0.2901370838
	a1	0.0004741857	9.5351062667	0.0439748981	75.3745687858	0.0100503835	0.0158161869	0.0083757216	0.0113091511
	a0	0.0000198659	0.9528042112	0.0012035139	6.7816941578	0.0000124724	0.0000015847	0.0001662860	0.0004545131
y_5	a5	0	1	0	1	0	1	1	0
	a4	1	1.8713547307	1	2.5629963412	1	1.6082723890	1.6469246769	1
	a3	35.9861829997	2.4200232367	8.0115818562	0.4110356330	1.6494422176	2.1445802043	1.5824570143	1.0549331742
	a2	2.8727792217	1.2142883512	22.9247121578	0.0336515259	1.1331198937	1.3306434961	0.1226738205	0.1036841544
	a1	0.0270069649	0.1171852194	38.8299892996	0.0021348753	0.0831071985	0.1058364062	0.0026080906	0.0046809970
	a0	0.0020173140	0.0026099516	3.2367233062	0.0000625864	0.0000542774	0.0007724304	0.0001979098	0.0001855032
y_6	a5	0	1	1	0	1	1	1	1
	a4	1	1.9408888239	2.5475515435	1	2.0667435717	7.7306510551	26.6684770842	12.6398839832
	a3	2.2054242323	3.2699327462	2.6900699928	4.7344257284	1.7776267628	14.2375588546	50.1852163500	5.6815009258
	a2	2.9235643324	2.4953574234	0.7930457057	16.8786855124	0.4307778915	8.9368405390	38.6659041382	0.2862004856
	a1	0.8636984246	1.0248401303	0.0542064478	28.2262187579	0.0185694369	1.2133687887	7.8153551784	0.0197788641
	a0	0.0387529528	0.1335654221	0.0010990883	5.1788204688	0.0000232408	0.0040521906	0.2445554744	0.0005477836
y_7	a5	1	0	1	0	1	1	0	1
	a4	7.3917026348	1	6.8246384925	1	1805.9596748276	948.3495199444	1	2.0734325016
	a3	16.9987542031	1.7392956975	19.3684508878	6.2878992224	4987.4841820789	3052.2536534611	4.7122255119	6.7594093878
	a2	23.8969649199	1.6479796097	20.0938014038	32.9526072471	6129.3282656364	4194.3895596366	9.1929229152	1.2516181780
	a1	9.1460099059	0.6404981404	4.3658855502	50.3521540302	2020.4876010679	1525.4928131253	4.4087552783	0.0506197671
	a0	0.6593463905	0.0561753656	0.2317566708	4.4008996768	136.8468743808	107.0143049482	0.3592429484	0.0023315967
y_8	a5	0	1	0	1	0	1	1	1
	a4	0	4.8703871565	1	1.9795342521	0	1.7283884398	0.7293039537	0.3090645789
	a3	1	9.2235405981	0.3178278661	0.0925333203	1	1.2818755606	0.1362030777	0.0273547056
	a2	3.9080105692	10.8683050812	0.0231679662	0.0048792004	1.3959563496	0.1097421928	0.0077285200	0.0012071585
	a1	0.5872469870	5.8224828287	0.0000137355	0.0001841375	0.1413582106	0.0008050927	0.0002734883	0.0000684287
	a0	0.0190596374	0.5087515264	0.0000009984	0.0000001778	0.0003862549	0.0000044551	0.0000127238	0.0000009220
y_9	a5	1	1	1	0	0	1	1	0
	a4	0.3054785764	2.3359906413	1.2335275727	1	0	3.0239267732	2.2490124256	0
	a3	0.0290146519	3.4431425918	2.3802456141	0.4877599682	1	7.1158006163	5.3164043879	1
	a2	0.0012740843	2.5009564719	0.1986109627	0.0319638579	16.9856424413	1.6514622181	1.0697407423	0.1004546643
	a1	0.0000588735	0.4486206167	0.0037346661	0.0000167656	3.4826251058	0.0618429212	0.0476154220	0.0046348653
	a0	0.0000013248	0.0152850975	0.0000000165	0.0000010975	0.1195352183	0.0000220420	0.0004628941	0.0001398355
y_{10}	a5	1	0	1	0	1	1	1	1
	a4	0.2745828135	1	0.1118098203	1	151.1456953548	3.7849948084	2.1979487995	1.4822657760
	a3	0.0221439790	1.6428972026	0.0045847365	0.2075031157	29.8362997861	1.2384676028	1.2308065859	0.6134784506
	a2	0.0011753068	1.6807687745	0.0002698688	0.0334946658	1.4454349779	0.0975105773	0.0982558705	0.0673643158
	a1	0.0000625557	0.4229118048	0.0000001560	0.0000824670	0.0528394850	0.0016340659	0.0005112244	0.0031395107
	a0	0.0000008729	0.0247006258	0.0000000091	0.0000131328	0.0019356836	0.0000525676	0.0000404456	0.0001139918
y_{11}	a5	1	0	1	1	0	1	1	1
	a4	3.0394706468	1	2.0805845617	1.3188140016	1	1.5831727509	3.1574527976	0.2425215160
	a3	5.9801864243	1.2716332870	2.6792576219	0.2627269411	3.0847056885	2.7941272468	5.8715913401	0.0189393657
	a2	1.4946073937	1.3222869137	1.6037916380	0.0031240341	2.1548732063	0.3706318470	0.9240439721	0.0011378699
	a1	0.0996855891	0.2747634656	0.1994905451	0.0005475502	0.2590855297	0.0004023455	0.0156097946	0.0000334105
	a0	0.0000887906	0.0080078656	0.0001283311	0.0000000980	0.0001393897	0.0000533396	0.0005962020	0.0000002805

Table 49 – Plant 3 – Sub-model 2 – Transfer Function Numerator Parameters.

Pl	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0	0
	b4	0	-0.0114852360	0	0	0	0	0	0
	b3	-0.0023079261	0.0164335978	-0.0200895799	0	-0.0004055095	0	0	0.0181282697
	b2	0.0034330665	-0.0007400408	0.0032745330	-0.1003064131	0.0000055145	-0.0001909173	-0.0008010328	-0.0072229269
	b1	-0.0000022186	0.0000079968	0.0000056640	-0.0121859332	-0.0000009959	-0.0000010139	-0.0000814506	-0.0005997186
	b0	0.0000004643	-0.0000006004	0.0000003346	0.0000031483	0.0000000049	-0.0000001336	0.0000000099	-0.0000415031
y_2	b5	0	0	0	0	0.0001807522	0	0	0
	b4	0.0118910160	0	0	0	0.1033978507	0	0.0003384502	0
	b3	0.0367370167	0	0	0	-0.0548384868	0.1505536789	0.0621832549	0.6615889000
	b2	0.0041981086	0	-0.0578403402	0	-0.0112477858	-0.0724879498	-0.0367419712	-0.2421566706
	b1	0.0000507331	1.9043147866	0.1133910438	-268.0677337434	-0.0000810825	-0.0153807854	-0.0072544176	-0.0635182366
	b0	0.0000003630	0.0932461342	0.0388239502	-182.6822578418	-0.0000001757	-0.0000001261	-0.0000001948	-0.0011890148
y_3	b5	0	-0.0013114288	-0.0012280469	0	0	0	0	0
	b4	0	-0.0407344766	-0.2003386419	0	0.2348672648	0.2815438819	0.1773960062	0
	b3	0	0.1027854980	0.2070622905	0	-0.1417372042	-0.1366884928	-0.1028141678	0
	b2	0.5890488437	0.2877992619	0.0634013217	0	-0.0474004927	-0.0787310859	-0.0445187150	0.4362402392
	b1	0.1027456076	-0.0092834763	0.0003721147	-4641.7907678691	-0.0030167615	-0.0052771691	-0.0028509980	-0.5250847724
	b0	0.0000590585	-0.0047585806	0.0001144129	-2321.4891731725	-0.0000370251	-0.0000319270	-0.0000225462	-0.0567903099
y_4	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	-0.9481559382	0.1527737729	0	0.1280244577	0
	b3	0.1159177449	0	0	-0.2849407076	-0.1020343762	0.1648350547	-0.0846065740	0
	b2	0.0022243464	0.3355772504	0.1051544571	-0.0253977703	-0.0199126161	-0.0738167036	-0.0157495007	0.4383647567
	b1	0.0000564409	-0.0389667947	0.0099810141	-0.0005948706	-0.0000000833	-0.0582736998	-0.0000006280	-0.4579870372
	b0	0.0000008737	-0.0043783719	0.0000692159	-0.0000428554	-0.0000001511	-0.0039496135	-0.0000002129	-0.0692368603
y_5	b5	0	0	0	0	0	0	0.0005210000	0
	b4	0	-0.0994365257	0.0004990659	-0.0027974626	0	0	0.0495148577	0
	b3	0	0.1868729469	-0.2500369572	-0.6397489267	0	0.0710277604	-0.0480584462	0.0497685957
	b2	0.0881384448	-0.0221563544	0.1808098062	-0.1850037621	0.0685838279	-0.0642340841	-0.0006756610	-0.0480902252
	b1	-0.0000810388	-0.0015063329	0.0010486460	-0.0103702393	-0.0587518169	-0.0021622788	-0.0003030340	-0.0024717538
	b0	0.0000214652	-0.0001630454	0.0001727514	-0.0000000442	-0.0008656014	-0.0000069189	-0.0000062280	-0.0000231190
y_6	b5	0	0	0	0	0	0	0	0.0004224611
	b4	0	0	0	0	0.1177128734	0	0.0900648011	0.0995328700
	b3	0.3222398244	0	0	-0.3058024990	-0.0310029078	0	-0.0413039318	-0.0336578910
	b2	0.0510489911	0.1865504605	-0.1562780715	-0.1499519708	-0.0470255178	0.1633647422	-0.0257349915	-0.0390018704
	b1	0.0000834190	-0.0107780654	0.1926911970	-0.0041383703	-0.0028027845	-0.1104021718	-0.0018712208	-0.0028855661
	b0	0.0000027520	-0.0015547693	0.0495014336	-0.0000127414	0.0000001237	-0.0129850840	-0.0000016240	-0.0000659441
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	-0.0233585654	0	0	0	0	0	0.0787489714
	b3	0	0.0536869346	0	0	1.9852945801	1.3825742162	0	0.0902797164
	b2	0.0553082773	-0.0044984571	-3.4375137306	0	-0.1738376749	-0.1717429734	5.5515058049	-0.0177177061
	b1	0.0189839789	-0.0012705472	-3.0720065887	-6.0688182519	-0.0757639938	-0.0678536843	-0.0526347885	0.0000962323
	b0	0.0000103471	-0.0000567210	-0.1973736792	-0.7376776812	-0.0002263665	-0.0002855750	-0.1917480314	-0.0000180310
y_8	b5	-0.0031915945	0.0001144851	0	0	0.0000678504	0	-0.0000048991	0
	b4	0.0178052469	-0.0029515215	0	0	0.0033807883	0	0.0034909828	0
	b3	0.0096293268	0.0089411649	0	0	-0.0062837031	0.0068151145	-0.0052999489	0
	b2	0.0005519233	-0.0032831042	0	-0.0001011497	-0.0001790236	-0.0101345745	-0.0002013563	-0.0101139640
	b1	0.0000018920	-0.0000024212	0.0006209544	-0.0000471890	-0.0000085668	0.0000190508	-0.0000097876	0.0000027954
	b0	0.0000001298	-0.0000105062	0.0000004228	-0.0000002420	-0.0000002434	-0.0000194770	-0.0000003732	-0.0000137105
y_9	b5	0	-0.0019066345	0	0	0	0	0	0
	b4	0	0.0160506820	0	0	0.0096258192	0	0.0059970572	0
	b3	0.4256979415	-0.0002309334	0	-0.0011503691	-0.0149514719	0	-0.0114628089	0
	b2	0.0418674900	-0.0004550253	0.5745911097	-0.0001716955	-0.0017204902	-0.0291000722	-0.0009102005	-0.1531010564
	b1	0.0001544442	-0.0000041360	0.1003061724	-0.0000014942	-0.0000471916	-0.0019376101	-0.0000154180	-0.0193701613
	b0	0.0000059923	-0.0000030648	0.0000167308	-0.0000000084	-0.0000000182	-0.0000013681	-0.0000000811	-0.0002240421
y_{10}	b5	0	0	0	0	0	0	0	0
	b4	0.0009385476	0	0	0	0	0	0	0
	b3	0.0353316364	0	0	-0.0016918754	-0.0044324562	4.4121816527	0	0.2290544383
	b2	0.0020826456	0.0096245123	0	-0.0001836556	-0.0007154779	-2.2816782801	0.5502544024	-0.1121062886
	b1	0.0000151242	-0.0046567497	0.0003037978	-0.0000095520	-0.0000284203	-0.1539641039	-0.2700462580	-0.0069326925
	b0	0.0000001925	-0.0000318107	-0.0000008120	-0.0000000171	-0.0000000439	-0.0000474691	-0.0002450115	-0.0000197075
y_{11}	b5	0	0	0	0	0	0	0	0
	b4	0.0907764945	-0.0030141848	0	0	0	0	0	0
	b3	0.0104081107	0.0315345810	0.0005351950	0.0048233699	0	0	0	0
	b2	0.0002632177	-0.0179642877	0.0007609547	-0.0025666591	0	0	0	0
	b1	0.0000183248	-0.0003301155	0.0000041637	-0.0000247112	-0.0502191488	-0.0654406655	-0.0387873745	-0.1513733548
	b0	0.0000001599	-0.0000592867	0.0000000859	-0.0000000645	-0.0004321930	-0.0023742147	-0.0001834978	-0.0025601818

Table 50 – Plant 3 – Sub-model 2 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	0	1	0	0	1	1	0	0
	a4	0	1.7259684747	1	0	0.3279144091	0.4688894218	0	1
	a3	1	1.1260394628	1.4849268612	1	0.0186549268	0.1283969563	1	8.6454335141
	a2	0.0624803316	0.0531176183	0.1485259632	2.0340335389	0.0008712436	0.0108422404	0.1124333551	1.1610790672
	a1	0.0000864841	0.0017547225	0.0000747711	0.1661605821	0.0000416539	0.0000870359	0.0046822606	0.0822447156
	a0	0.0000054025	0.0000201649	0.0000067989	0.0000031125	0.0000000539	0.0000071293	0.0000025833	0.0025990330
y_2	a5	0	1	0	0	1	0	0	0
	a4	1	7.5728156988	1	0	3.7112049665	1	1	1
	a3	0.2513917507	19.5850466537	4.8796669756	1	1.7896117013	4.0708320367	2.9659467266	19.0793252347
	a2	0.0166660767	23.3288115612	8.0944369828	1698.9120211027	0.1311250472	1.9892932469	1.4549352186	10.5895524291
	a1	0.0001780432	16.7660469181	5.9488759717	7144.0268613974	0.0009054390	0.1357568738	0.0996467773	0.9188214656
	a0	0.0000014830	1.7222133878	0.7533237887	3215.5911026662	0.0000020205	0.0000024215	0.0000076016	0.0144357970
y_3	a5	0	1	1	0	1	1	1	0
	a4	1	1.6833249419	2.0714062343	0	2.2492446529	2.2534504718	2.1719249042	1
	a3	2.6133735853	2.3054162122	1.2315876929	1	1.2419800792	1.4459528846	1.3093402459	4.3111380638
	a2	1.2597102590	1.3919552118	0.1407800628	3405.3155088275	0.1900901710	0.2649167355	0.2214249292	7.8080130436
	a1	0.1099838152	0.3576578743	0.0022065482	15389.6284493676	0.0091846794	0.0126633402	0.0104781508	2.8544726645
	a0	0	0.0224327837	0.0002459237	5031.2740133113	0.0001042294	0.0000734378	0.0000787397	0.1769685932
y_4	a5	0	1	1	1	1	0	1	0
	a4	1	2.1308549906	1.5539747622	2.0192431458	2.1282038618	1	2.1506927219	1
	a3	0.1200853459	2.6193615051	1.3892146228	0.4820623478	0.6612317669	2.1401587105	0.6524308528	5.3110624680
	a2	0.0024347497	1.6837082748	0.2611270230	0.0337355304	0.0406821123	1.1973227011	0.0396112363	10.3268359313
	a1	0.0000583918	0.2625732519	0.0132765866	0.0009747076	0.0000054761	0.1714696977	0.0000120451	2.7869795133
	a0	0.0000007392	0.0101822982	0.0000857916	0.0000535654	0.0000003014	0.0066737581	0.0000005139	0.1571999631
y_5	a5	0	1	0	0	0	0	1	0
	a4	1	1.7193729101	1	1	0	1	1.7708805305	1
	a3	1.1808955921	1.0856363618	2.7292406209	2.2364681621	1	1.6379165814	0.1637261397	1.6970780825
	a2	0.0725317278	0.1403677466	0.2268627719	0.3127798050	1.7379775428	0.1643908112	0.0136220648	0.1974406352
	a1	0.0002708010	0.0090832007	0.0040335893	0.0114904886	0.1464856031	0.0037835406	0.0009922651	0.0063898460
	a0	0.0000165779	0.0003042935	0.0001955784	0.0000001968	0.0017413420	0.0000114768	0.0000155046	0.0000522597
y_6	a5	1	1	0	0	1	0	1	1
	a4	3.9216268673	1.8481040582	1	1	2.6031563748	0	2.5023336579	2.4872027412
	a3	1.5966060996	2.7596054786	3.0261358025	2.2512197064	2.1738927610	1	1.7550131331	2.0885680008
	a2	0.1580762893	1.8704570330	6.2143318557	0.8384293208	0.5160396382	2.6415495300	0.3650426259	0.4942870607
	a1	0.0001645189	0.4939342959	3.5455696662	0.0223064373	0.0226904361	1.2255865369	0.0192397863	0.0289756406
	a0	0.0000145525	0.0240110503	0.4153945616	0.0000682973	0.0000006196	0.0861439240	0.0000166336	0.0005846857
y_7	a5	0	1	0	0	1	1	0	1
	a4	1	1.3488935314	1	1	17.3122511519	11.9976759084	1	1.9678649967
	a3	1.2997763883	1.1733262941	5.1916183331	5.9068121903	41.8751444681	26.6235248465	49.1040920453	2.8573263468
	a2	0.8915096884	0.3006765964	13.0625851308	40.9277553695	17.4890422688	11.5416867951	124.1685889750	0.4330159321
	a1	0.0902840295	0.0235156591	9.6365956223	64.7360749552	1.2180997818	0.8508099286	56.2837393233	0.0028619843
	a0	0	0.0006524709	0.8875345902	5.0876736612	0.0036442294	0.0034601223	4.0483156997	0.0004282429
y_8	a5	1	1	0	0	1	0	1	1
	a4	0.5878079185	1.1854260972	0	1	1.1996204165	1	1.3065952789	2.2661203315
	a3	0.0704426256	0.8319660677	1	1.2837134834	0.1424915878	1.5058758877	0.1710408392	2.0483910523
	a2	0.0024833276	0.0823302142	1.5646100368	0.5337851950	0.0043799176	0.1339736421	0.0065749799	0.1726748542
	a1	0.0000153561	0.0032674172	0.1492619064	0.0309051963	0.0001912812	0.0028599411	0.0003072318	0.0027759150
	a0	0.0000005146	0.0002249889	0.0000811123	0.0001526724	0.0000037326	0.0002508342	0.0000076023	0.0002299913
y_9	a5	1	1	0	1	0	0	0	1
	a4	6.9678913555	0.6308441336	1	5.4247181826	1	1	1	5.7274766296
	a3	1.1110514369	0.1892057190	429.7969223364	1.3000930052	0.1891926947	1.4827973571	0.1530954394	10.2868093798
	a2	0.0449668314	0.0132182148	232.2954426682	0.0667972895	0.0108780605	0.2129262449	0.0066323423	2.1245201115
	a1	0.0002187157	0.0010679454	11.4112771550	0.0004533462	0.0001853873	0.0059830272	0.0000865736	0.0966926808
	a0	0.0000062137	0.0000478641	0.0024048135	0.0000008222	0.0000003511	0.0000060309	0.0000006513	0.0008979680
y_{10}	a5	0	0	0	1	1	0	0	0
	a4	1	1	0	10.1405496473	0.7117908588	1	0	1
	a3	0.1553255978	0.7841784974	0	2.0525553745	0.1549288932	217.2563658822	1	15.1369873778
	a2	0.0060111252	0.8766096176	1	0.1527780926	0.0122859480	38.1725859208	43.8167285190	2.4489831736
	a1	0.0000428767	0.1025802201	0.0647889578	0.0052089611	0.0003157011	1.3085218261	4.1749030819	0.0822613150
	a0	0.0000005956	0.0006514557	0.0000127418	0.0000005845	0.0000004819	0.0004162519	02	0.0002399978
y_{11}	a5	1	1	0	0	0	0	0	0
	a4	0.2373350513	1.0985346141	0	1	1	1	1	1
	a3	0.0164699107	1.1454248512	1	5.3887026754	1.6002133768	1.7890719964	1.5671613641	4.2944189667
	a2	0.0005203813	0.2211082973	0.0885136946	0.7792755619	2.3833322273	2.4826808752	2.4017021532	7.2901869601
	a1	0.0000259881	0.0078820348	0.0007153449	0.0077667983	0.2868002099	0.3651672206	0.2811487098	0.9442051583
	a0	0.0000001783	0.0006046363	0.0000023497	0.0000156242	0.0021904437	0.0097407334	0.0011857124	0.0133891270

Table 51 – Plant 3 – Sub-model 3 – Transfer Function Numerator Parameters.

PI	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	-0.0007834414	0	0	0	0	0
	b4	-0.0338356492	0	-0.0315665952	0	-0.0000021439	-0.0000192419	0	0.0045153607
	b3	0.0301776976	0	-0.0053121674	0	0.0040476663	0.0032452578	0.0010091304	-0.0016282240
	b2	0.0140949862	0	0.0006274727	0	-0.0022995834	-0.0013931431	-0.0007928835	-0.0009795173
	b1	0.0002560498	0.1059939085	-0.0000548763	2.1151558888	-0.0006188477	-0.0005838876	-0.0008388003	-0.0000011262
	b0	0.0000008188	-0.0004768229	0.0000103432	0.3311729829	-0.0000224284	-0.0000080936	-0.0000005116	-0.0000005073
y_2	b5	-0.0184846410	0	-0.0002944467	0	0	0	0	0
	b4	0.0810384065	0	-0.0219186891	0	0.0009506876	0.0006431024	0	0
	b3	0.0088632961	-0.0091839383	0.0220363425	0.0004264313	0.1142628429	0.0904472615	0.0466980902	74.8080846124
	b2	0.0008055759	0.0628592683	0.0210985004	0.0642341737	-0.0347750970	-0.0214851714	-0.0136025204	-11.8632378735
	b1	0.0000014403	0.1265192953	0.0006871098	0.0220243148	-0.0354048641	-0.0283541830	-0.0180585215	-27.3714615130
	b0	0.0000000894	0.0072920013	0.0000019087	0.0004261021	-0.0028003152	-0.0024734394	-0.0016085733	-2.7552698413
y_3	b5	0	0	-0.0038665483	0	0	0	0	0
	b4	0	0	-0.2351553374	0.0000653202	0.3334750819	0.0023662572	0.0015624268	0.0039707882
	b3	0	0	0.1036103206	0.4022769915	-0.0748973720	0.2503254657	0.1508118546	0.2069935964
	b2	0.7610818160	0.0879853685	0.1651986422	0.0707971505	-0.1686508411	-0.0586246364	-0.0416713834	-0.0806495677
	b1	0.0009873382	0.1332497450	0.0141241480	0.0016807930	-0.0155037590	-0.1187766192	-0.0791482395	-0.0860416179
	b0	0.0000586507	-0.0192131380	0.0000099600	0.0000033189	-0.0000063465	-0.0088194219	-0.0059006502	-0.0047182138
y_4	b5	0	0	0	0	0	0	0.0011738672	0
	b4	0	0	-0.2670754044	0	0	0	0.0649046836	0.0985489683
	b3	0	0	-0.0254540389	0	366.5466737365	0.3576309291	-0.0045028046	-0.0262994496
	b2	0.4676111287	0.0567671619	0.1596314933	16.9692099343	128.2578911893	-0.1924386477	-0.0588573853	-0.0655091469
	b1	0.0305870794	0.1077690557	0.0000037001	3.1760631184	-438.3503112003	-0.1741303201	-0.0024768821	-0.0004834947
	b0	0.0000478667	-0.0207364407	0.0000486728	0.0075766919	-21.9426757804	-0.0065944953	-0.0000523056	-0.0001182751
y_5	b5	0	0	0	0	0	0	0	0
	b4	0	0	-0.0053917706	0.2905631499	0	0	0.0367873596	0
	b3	0	0	-0.2973991307	0.1573012703	0	0.1276513480	0.0086615083	0
	b2	0	0.0365243746	-0.0374266176	0.0171685033	0.1571108624	-0.1367863356	-0.0471219762	0.7919520356
	b1	0.1315696810	0.0880926956	0.2232469978	0.0000930574	-0.1858893549	-0.0059733543	-0.0003102722	-0.8466611454
	b0	0.0005817011	-0.0201821327	0.0003432590	0.0000097933	-0.0045550288	-0.0004777067	-0.0001379901	-0.0510918158
y_6	b5	0	0.0000707234	0	0	0.0010990922	0.0007250703	0.0005175091	0
	b4	0	-0.0021435431	-0.0806746116	0	0.1361960283	0.1043612171	0.0615058756	0
	b3	0	0.0006377914	0.0353710261	0	-0.0373723520	-0.0243133637	-0.0170871854	0.0833111531
	b2	0.0815953559	0.0289230789	0.0319658425	0	-0.0554827045	-0.0425548187	-0.0277869460	-0.0263668555
	b1	0.1864655531	0.0573189313	0.0002866695	514.8636293363	-0.0035866558	-0.0031687589	-0.0017197081	-0.0351883223
	b0	0.0115851820	-0.0072259817	0.0000802179	365.3251647928	-0.0000723344	-0.0000358835	-0.0000081742	-0.0021230027
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	0	0	0.0004502139	0.0012719269	0.0007935494	0	0
	b3	0.0500173719	0	0	0.0692417993	0.1612725505	0.1055173894	0	0.4069008106
	b2	0.0045825393	0.0132484841	-2.7496224761	0.0144507119	0.0243388723	0.0185549211	0.3704492862	0.1748114023
	b1	0.0000814015	0.0106539823	-2.0246407717	0.0000311329	-0.0154721412	-0.0133039617	0.0130572166	-0.0468989629
	b0	0.0000003379	-0.0025413087	-0.1399467106	0.0000059129	-0.0010789953	-0.0011855497	-0.0242764892	-0.0033795972
y_8	b5	-0.0061329133	0	0	0	0	0	0	0
	b4	0.0674823406	0	0	-0.0002233522	0.1161332399	0.0722334552	0.0336153509	0.0226864396
	b3	0.0909113303	0	0	0.0001473781	-0.1337467241	-0.0867857635	-0.0423134413	-0.0428496923
	b2	0.0049449922	0	0.0029813265	-0.0000020191	-0.0055539412	-0.0002096741	-0.0002188660	-0.0001005361
	b1	0.0000110758	0.0194225230	0.0000069287	0.0000003528	-0.0000686603	-0.0001883988	-0.0000794953	-0.0000720798
	b0	0.0000005686	-0.0082295989	0.0000002450	-0.0000000022	-0.0000024656	-0.0000004229	-0.0000004637	-0.0000003046
y_9	b5	0	0	-0.0000670460	0	0	0	0	0
	b4	0.0428659356	-0.0011640654	-0.0016707917	0	0	0	0	0
	b3	0.0044701796	0.0029307348	0.0020057569	0	0.0237937628	-0.0014162852	0.0087702761	-0.0058547035
	b2	0.0001204638	0.0004043665	0.0006396034	0.0007659521	-0.0621915416	-0.0163681898	-0.0260241408	-0.0135911429
	b1	0.0000099848	-0.0001087345	0.0000321955	0.0000411847	-0.0053259803	0.0000035590	-0.0022290695	-0.0025697199
	b0	0.0000000338	-0.0000015105	0.0000000827	0.0000000495	-0.0000036601	-0.0000189281	-0.0000009189	-0.0001108412
y_{10}	b5	0	0	-0.0001871928	0	0	0	0	-0.0002760305
	b4	0	0	-0.0008754998	-0.0000424609	0	0	0	0.0344216368
	b3	0.4770412366	0	0.0007616054	0.0001743711	0.0811476813	0.0472161638	0	-0.0192596998
	b2	0.0427129458	0.0047917692	0.0005645629	0.0000067874	-0.0048425156	-0.0022512850	0.0771350424	-0.0019015794
	b1	0.0010259441	-0.0021258065	0.0000509866	0.0000004324	-0.0232991587	-0.0195810535	-0.0363200351	-0.0000716104
	b0	0.0000021075	-0.0002939552	0.0000000442	0.0000000062	-0.0013160002	-0.0011003716	-0.0035566393	-0.0000025134
y_{11}	b5	-0.0074984476	0	-0.0000399708	0	0.0006513002	0	0.0004555181	0
	b4	0.1196562177	0	-0.0078493606	0	0.0695097696	0	0.0251706822	0
	b3	0.0059563206	0	0.0057130378	-0.0021894093	-0.0473057556	0.0391406608	-0.0180913643	-0.0142529372
	b2	0.0002889589	0.0152611939	0.0009538667	0.0012671029	-0.0767660486	-0.1121803292	-0.0300844740	-0.0397909026
	b1	0.0000007541	0.0005537411	0.0012427103	-0.0000023441	-0.0046033002	-0.0067681569	-0.0005399470	-0.0036641346
	b0	0.0000000272	-0.0061921005	-0.0000001013	0.0000001215	-0.0000015056	-0.0000105181	-0.0000292672	-0.0001114199

Table 52 – Plant 3 – Sub-model 3 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	1	1	0	0	0	1	1
	a4	10.0743618760	22.3195357228	2.3807299891	1	1	1	1.2426368266	2.8478862808
	a3	3.2669003890	37.5581722754	0.7558476485	10.9508581644	1.4467858531	1.5330178970	1.9584587876	0.8586887061
	a2	0.2654940855	51.8079960721	0.0827640983	73.7523822460	0.3538643391	0.4174437988	0.7402260218	0.0625464979
	a1	0.0039795489	23.6105478840	0.0076051303	106.3817633764	0.0298714886	0.0315086395	0.0565011183	0.0004421940
	a0	0.0000116848	2.0274519205	0.0005964805	10.7222616849	0.0007352679	0.0003637264	0.0000245216	0.0000314724
y_2	a5	1	1	1	0	0	0	0	1
	a4	0.3064254033	3.3279063067	1.4293206215	0	1	1	1	877.0097335184
	a3	0.0299090799	7.1557031627	1.6174547945	1	1.9140948352	2.0376476667	1.7796079593	2302.5053671870
	a2	0.0016507981	7.1003741540	0.3423169820	2.2021592050	1.5451026108	1.6809645969	1.5387310274	2122.1197711066
	a1	0.0000036660	4.4406647366	0.0097797255	0.6000390521	0.2939022199	0.3369089355	0.3262248811	478.3482993331
	a0	0.0000001985	0.5215519962	0.0000255622	0.0112123588	0.0138955259	0.0167325261	0.0167366249	25.5683792976
y_3	a5	1	1	1	0	1	0	0	0
	a4	1.2432355983	2.2878429309	2.0270504441	1	1.7883133121	1	1	1
	a3	2.7229958523	4.6901035782	1.8833749371	1.9429153827	1.5628362967	1.7839304864	1.6949184537	1.9194037364
	a2	0.5641778065	4.1244824828	0.4563927422	0.2849638219	0.3881395207	1.5310258606	1.4819354788	1.5884350013
	a1	0.0003168514	2.5428021684	0.0265948288	0.0065140682	0.0225717884	0.3530697158	0.3491997474	0.3173077429
	a0	0.0000638298	0.4085835373	0.0000193856	0.0000134297	0.0000080616	0.0175354047	0.0176110847	0.0127228591
y_4	a5	1	1	1	1	1	1	1	1
	a4	1.8176546621	2.5246325958	1.7728054806	4.2139327200	2888.1017115377	3.5316298469	1.4503840695	1.7439677312
	a3	2.4166150798	4.5521251244	1.5524816256	42.2623597151	4331.3758213331	5.3257134355	1.2346945258	1.2868243207
	a2	0.3995741933	4.0895213119	0.1753820414	65.7577245736	4421.8239154723	2.8045059616	0.1799405931	0.1418375468
	a1	0.0162002038	2.0435529055	0.0004765443	7.1659258228	688.9334662087	0.3488662655	0.0063822833	0.0031768415
	a0	0.0000158008	0.2082186948	0.0000536776	0.0171750961	22.9831825695	0.0095397551	0.0001130320	0.0002328264
y_5	a5	0	1	0	1	0	1	1	0
	a4	0	2.8209609881	1	2.5379894457	1	1.9535917224	1.5051624825	1
	a3	1	4.6684291569	2.9443368004	0.5682052084	1.8589399057	2.3570512483	1.1708846096	14.9612715849
	a2	0.8628689758	4.3036295632	2.3317549959	0.0346934996	2.2030809228	0.3057810875	0.1096919050	19.6177188069
	a1	0.0717501875	1.8493013335	0.2143933291	0.0003273922	0.2432045354	0.0165932925	0.0040901254	2.7837558904
	a0	0.0002858408	0.1493587927	0.0003350367	0.0000191659	0.0045366420	0.0006606032	0.0002827755	0.0951420273
y_6	a5	0	1	1	0	1	1	1	0
	a4	1	2.6525240745	1.9575012423	0	1.9076335660	2.0002483172	1.8629682761	1
	a3	1.5120611823	5.3008584329	1.7936829623	1	1.6856787667	1.7683788805	1.6734046610	2.1690679258
	a2	2.0616541162	5.1728427996	0.3285743034	2346.8953679495	0.4101603761	0.4476076169	0.4190578610	1.9663779107
	a1	0.6069485517	3.2679630391	0.0065131391	9581.9363781622	0.0216427093	0.0250991244	0.0198776847	0.4731496483
	a0	0.0277187089	0.6609924080	0.0007791340	4451.3530379269	0.0003959710	0.0002672234	0.0000930676	0.0212815222
y_7	a5	0	1	0	0	0	0	0	1
	a4	1	1.6550176194	1	1	1	1	1	4.6063138578
	a3	0.2484307556	3.2348714335	4.3351093896	2.2970187596	1.9415969650	1.9529978584	4.6394969994	10.0356624239
	a2	0.0157165059	2.3118883799	10.4245591662	0.2428666730	1.3642438970	1.4659156708	8.9055200257	8.5564226519
	a1	0.0002446194	1.2444913235	6.6850430803	0.0009448162	0.2179783555	0.2664712796	4.7599487810	1.4836220101
	a0	0.0000007919	0.1354913470	0.7099741284	0.0000997273	0.0085959504	0.0122783426	0.4365046410	0.0567851754
y_8	a5	1	0	1	0	1	1	1	1
	a4	0.7400256815	1	0.4380010397	1	2.9973293627	2.4032097350	1.9122620070	1.6508469249
	a3	0.0997378266	2.9765392495	1.6446002301	0.0599332589	0.4053510814	0.2597754241	0.2124547192	0.1613104735
	a2	0.0034288142	3.2691971293	0.1868047340	0.0029567707	0.0125427085	0.0052866947	0.0044522173	0.0035389580
	a1	0.0000113145	2.0357321967	0.0005748348	0.0001525765	0.0002070692	0.0005487653	0.0003911328	0.0002696587
	a0	0.0000003799	0.2036518082	0.0000156348	0.0000000899	0.0000044013	0.0000003166	0.0000016728	0.0000012668
y_9	a5	1	1	1	0	1	1	1	1
	a4	0.1838661485	1.2395827145	1.1087673084	1	1.8338607997	0.9436008867	1.7472743539	1.7043842367
	a3	0.0105930448	1.3779075374	1.8226076098	16.3617758085	4.6971194602	1.7607926589	4.2801811327	2.0247788214
	a2	0.0004103052	0.4540738570	0.2506308915	3.1126787162	0.8682593271	0.1417280586	0.8012335813	0.5126643830
	a1	0.0000183994	0.0438305128	0.0089262734	0.1797342025	0.0322600152	0.0019764066	0.0296732771	0.0439373167
	a0	0.0000000168	0.0004600290	0.0000192615	0.0001831791	0.0000277081	0.0001579702	0.0000166244	0.0011323786
y_{10}	a5	1	1	1	0	0	0	0	1
	a4	10.0303256873	2.2527884640	0.7882580906	1	1	1	1	2.8224679454
	a3	1.9397990796	3.0374012638	1.7260037312	0.3784133174	3.3957164524	3.0819720997	2.6836894774	0.5743383799
	a2	0.1124659418	2.3487844871	0.3131519759	0.0049896221	2.1726035878	2.2550391929	6.8856017745	0.0361704014
	a1	0.0021508744	0.5854890757	0.0152863809	0.0009263826	0.2965492054	0.3182377942	1.3797697118	0.0013404049
	a0	0.0000037509	0.0345366390	0.0000232236	0.0000026920	0.0097508915	0.0105023650	0.0599055159	0.0000322227
y_{11}	a5	1	1	1	1	1	1	1	1
	a4	0.1714470632	2.8163149122	2.4402934066	1.0261510926	2.6453699072	1.9090442353	2.1040198687	1.7556032618
	a3	0.0085089835	4.9443255372	2.6590151022	5.0872864567	2.3284920247	3.9495940648	1.9640064592	2.0503307718
	a2	0.0003015557	4.5024750324	1.7287019667	1.2064381705	0.3923757289	0.7626984824	0.2587946022	0.4111607399
	a1	0.0000008950	2.2964122956	0.2043876559	0.0000675194	0.0151765909	0.0295677865	0.0056739749	0.0266174254
	a0	0.0000000301	0.3523913633	0.0000001234	0.0000160119	0.0000042734	0.0000474080	0.0002143944	0.0006288629

Table 53 – Plant 4 – Sub-model 1 – Transfer Function Numerator Parameters.

P1	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	-0.0003398582	0	0	0	0	0
	b4	0	0	-0.0128412996	0	0	0	0	0
	b3	0	0.0026264473	-0.0037183714	0.0006134323	0.2000838754	0.2266069046	0	-0.0065871858
	b2	0.0061243144	0.0831890611	0.0031908585	0.1497157262	-0.0927013696	-0.0854313423	0.1540238991	0.0000465880
	b1	0.2244401151	0.0410823410	-0.0000067821	0.0774527353	-0.1905269933	-0.2119712587	-0.1016525640	0.0003774270
	b0	0.0819371965	-0.0172775025	0.0000055159	0.0074412121	-0.0082899544	-0.0140656791	-0.1409328227	0.0000310042
y_2	b5	0	0	-0.0003021472	0	0	0	0	0
	b4	0	0	-0.0512095361	0	0	0	0	0
	b3	0	0	0.2803122228	0.5417407438	0	0	2.4538636436	-0.0942408770
	b2	25.7075822962	0.7998142125	0.1452219134	0.4910952854	1.0061286592	1.2620750798	-0.9043532802	0.0244584000
	b1	55.7570703564	0.5160450721	0.3845366713	0.4607010355	-1.2777362802	-1.5163945644	-2.6360488410	0.0157011996
	b0	11.9648468751	-0.1907211448	0.0576317456	0.0003823091	-0.4580917922	-0.5068752753	-0.3408726849	0.0007352902
y_3	b5	0	0	0	0	0	0	0	0.0127278626
	b4	0	0	0	0	0	0	0	-0.0428284385
	b3	0	0.8034483315	0	2.7781586283	11.7045942151	0	1.7812271397	0.0025038550
	b2	0.4162919516	0.3276532755	2.0151904942	3.5419922850	6.3441837096	6.9348203850	-0.3094577362	-0.0161289950
	b1	7.6014871675	-0.4052022066	1.7057527905	2.8822344228	-22.6724407442	1.8900124851	-1.6807697765	-0.0034702875
	b0	1.0729820159	-0.0479669569	0.5269880492	0.4324463116	-0.4187732684	-12.5499752187	-3.6163840643	-0.0000593525
y_4	b5	0	0	-0.0014093235	-0.0000116922	0	0	0	0.0067977883
	b4	0	0	-0.1165209181	0.7575281282	0.0007740098	0	0	-0.0025527132
	b3	0	0.5351739296	0.3163992407	3.3189244621	0.1273603186	0.1280274375	330.7636535001	-0.0457772012
	b2	2.6421893073	-0.1371362948	0.1646189750	4.6100959103	0.6845088906	0.8322515018	379.6819497745	-0.0005222381
	b1	4.4165018718	-0.1306128933	0.0075591955	1.5666867430	-0.7769205868	-1.1016578108	-1029.7347565748	-0.0196991917
	b0	0.9612782594	-0.0016481139	0.0001466214	0.1715775790	-0.6867056662	-0.4602403714	-76.3813793547	-0.0002068662
y_5	b5	0	0	0	0	0	0	0	0.0063662791
	b4	0	-0.0016245740	-0.1860813615	0	0.0000237138	0	0	-0.0346477925
	b3	0.0053895371	0.0483617033	0.2140502433	0	0.2389612908	0	0	0.0040165453
	b2	-0.8376299223	0.1526180904	0.1505437602	864.9228577558	-0.3497851651	0	-0.1792667512	-0.0103420215
	b1	2.0313675005	-0.0978635515	0.0023302227	538.6023498569	-0.0000512058	-0.9029434218	-0.2581992596	-0.0003131471
	b0	0.7146459775	-0.0032493318	0.0000080778	33.2321870313	-0.0000042460	-0.0124842603	-0.0456436148	-0.0000346203
y_6	b5	0	0	0	0	0	-0.0000378755	0	0
	b4	-0.0023199932	0	0.0001513809	0	-0.0147384266	0.1525516448	0	0
	b3	0.3083462785	0.3604294067	0.1280724574	-0.0004423233	1.6638619913	1.6391517611	1.2095534113	0
	b2	-0.0498637629	0.3249959981	0.1188537356	0.5079953789	-0.5572261664	-0.2339821176	-0.7101241574	-0.0008121933
	b1	7.0275456640	-0.2098827211	0.1892933991	0.9604068347	-1.4067286888	-1.3697314009	-0.8339964915	-0.0000421838
	b0	4.6456963606	-0.0361146598	0.0343235710	1.5900767538	-1.0237564982	-0.8606980441	-0.7551138863	-0.0000011240
y_7	b5	-0.0090660443	0	0	0	0	0	0	0
	b4	0.1361154939	0	0	0.0001145976	0	0	0	0.0007311580
	b3	0.0022618497	0.1332832943	0	0.2100079488	0	0.3988102577	0.2078353239	2.1279349046
	b2	0.0003617123	-0.0798180321	-4.1525012819	0.4952729480	0.6536951406	-0.2925474588	-0.1455037857	0.2462434441
	b1	0.0000061067	-0.0029373397	-5.4899970700	0.8227220641	0.0720564407	-0.0527760191	-0.0108792681	0.0132675641
	b0	0.0000000244	-0.0000698705	-0.4665294065	0.1110047450	-0.4805551391	-0.0004983488	-0.0000340141	0.0003645931
y_8	b5	0	0	0.0000597834	0.0007357677	0	0	0	-0.0018952202
	b4	0	0	0.0379109711	-0.0096537698	0	0	0	-0.0209951969
	b3	17.9085383789	0.0502573669	-0.0281694100	0.0030610460	0	0	0.0550107877	0.0150684643
	b2	14.2989020973	-0.0378907289	0.0190454649	-0.0003417507	2.4344135378	4.5893289523	-0.1856982552	0.0137340047
	b1	0.0413756694	-0.0401359615	-0.0065323388	-0.0000026770	-6.8225717849	-11.9956767385	-0.0055775454	0.0014356325
	b0	0.0353942091	-0.0003006556	-0.0001084591	-0.0000000649	-1.1151799933	-2.1266618392	-0.0000327975	0.0000295554
y_9	b5	0	0	0.0000153605	0	0	0	0	-0.0016968762
	b4	0	0	0.0433638547	0	0	0	0	-0.0523684977
	b3	0	0.1216246751	-0.0315856378	-0.0208308522	-0.1258832777	-0.0004202275	-0.0155618198	0.0230860542
	b2	0	-0.1731245054	0.0228995092	0.0061303242	-0.0471205992	1.4952302455	-0.0085044686	0.0251735267
	b1	5.8587166577	-0.1040261685	-0.0107070135	-0.0006162257	-0.0040961430	-2.1842165275	-0.0007288477	0.0001947329
	b0	1.2572294049	-0.0042478468	0.0000639651	-0.0000083170	-0.0000162427	-0.1852276197	-0.0000000692	0.0000114825
y_{10}	b5	0	0	0.0001623035	0	0	0	0	-0.0018353810
	b4	0	0	0.0673089495	-0.0000705401	0	0	0	-0.0511717960
	b3	0	0	-0.0478286314	0.0364262148	-0.0006138581	0	0	0.0566271846
	b2	67.3228702090	0	0.0354885129	-0.0568154112	4.8151792017	7.2711146122	-0.1444461821	0.0265605461
	b1	65.1898780482	-0.3766336028	-0.0149266589	0.0251602309	-5.0707628558	-23.3401941305	-0.0943238049	0.0001725883
	b0	6.1194779458	-0.0295829676	0.0000269414	-0.0002475046	-0.2711509269	-1.0229894540	-0.0080360579	0.0000098536
y_{11}	b5	0	0	0.0000729156	0	0	-0.0001046413	0.0000790164	-0.0007400098
	b4	0	0.0009572892	0.0335610378	0	0	0.0420177578	-0.0089247789	-0.0458348978
	b3	34.7328738569	0.0360991818	-0.0204032260	-0.0123940750	0	0.0061682197	-0.0056313028	0.0217620767
	b2	149.7146353288	-0.0217138523	0.0177874122	0.0036153116	-0.1061529931	-0.0998908739	-0.0004151771	0.0061949148
	b1	211.9329117753	-0.0379328131	-0.0067663973	-0.0004301009	-0.0449717204	-0.1068282487	-0.0000038389	0.0059083117
	b0	7.9725150141	-0.0016385502	0.0000512682	0.0000001888	-0.0024735329	-0.0071522208	-0.0000000153	0.0002043971

Table 54 – Plant 4 – Sub-model 1 – Transfer Function Denominator Parameters.

Pl	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	0	1	1	0	1	1	0	1
	a4	1	1.4310356657	1.5300342720	0	7.5664111867	6.7517748643	1	1.1968113695
	a3	1.5051111115	7.5700842960	1.0963742323	1	12.7208676101	12.0716958067	7.5024400032	3.3962490304
	a2	7.4169785709	5.8684641201	0.1409186049	2.9016356768	44.1303902649	39.0577980873	12.1557370115	1.9926054656
	a1	7.7931002049	4.1895987456	0.0018661321	1.1071529227	8.0526663279	8.1744919072	43.7226058855	0.1817518562
	a0	0.7434371304	0.6012698262	0.0002357837	0.0906439202	0.2635752321	0.3622728472	5.6493230391	0.0072951879
y_2	a5	1	0	1	1	0	0	1	1
	a4	24.9192674822	1	1.4556148667	2.0640118257	1	1	9.6736345825	0.9444083596
	a3	39.0330191398	4.5616318581	7.4292535253	8.5341879776	2.9848768579	2.7533134574	16.8866948666	4.6431892700
	a2	170.4561979683	6.0458689893	5.7854794915	6.4780560205	8.7681958480	8.5748965244	61.6221913792	2.6698502243
	a1	135.1313777736	4.4245277880	4.3381063811	3.7353562555	16.4440816855	14.8825350401	24.9285950267	0.2794188537
	a0	11.6522745734	1.3548231433	0.4679873303	0.0030049472	1.9224797160	1.6806460932	1.8111710258	0.0089944397
y_3	a5	0	1	1	1	1	0	1	1
	a4	1	2.2235090102	1.5893511178	2.9477449893	17.3650717970	1	3.9764398975	0.6696306549
	a3	1.9198606205	4.1445082185	9.8664684038	9.9602199460	29.8171970838	8.5546248222	13.5568590279	1.3957643639
	a2	7.6364848442	2.5444676306	8.3635074670	8.9028131896	112.4565817953	16.8347496654	26.5294376475	0.4030660559
	a1	9.6414558468	1.2321880609	4.7155359059	5.1896611052	61.0371653104	52.6562397855	30.2499309752	0.1150427621
	a0	1.0392233352	0.1216400926	0.8153013200	0.6679085620	1.0658437748	26.5303460717	11.4829845836	0.0019393849
y_4	a5	1	1	1	1	0	0	1	1
	a4	2.0451027987	2.3869909926	1.5595698955	3.0931794755	1	1	1540.5467235021	1.0504385254
	a3	8.4148592758	2.0656313608	1.0638060472	8.2370461169	1.7717729840	1.3771657499	1692.8621395940	1.7257225360
	a2	11.5553478853	1.1132392951	0.1914516807	6.5540534108	7.3734374784	7.0621753652	9842.6140330749	0.7519770527
	a1	6.7932215235	0.1933147877	0.0078128713	1.8282749101	7.1975155934	4.8837683753	3173.3688805233	0.2381913576
	a0	0.7569482897	0.0023103117	0.0001390057	0.1616522463	1.2763815793	0.7068522318	173.6876366201	0.0023958705
y_5	a5	0	0	1	0	0	1	1	1
	a4	1	1	1.6536260237	1	1	1.5200038855	1.0248247799	0.6105907366
	a3	1.2717583562	0.8247667431	0.9099498559	561.6705535950	3.4260892405	6.6818758076	5.7851659695	0.4929077323
	a2	7.6643313092	0.6116011208	0.1103990847	2116.3690068822	0.4909558345	7.8799568053	4.0837687143	0.0887919200
	a1	4.5480104264	0.1007742127	0.0015616005	465.2998285062	0.0000610418	1.1516158362	1.0467712581	0.0040572789
	a0	0.4417052223	0.0025655637	0.0000052608	21.5096342472	0.0000087122	0.0141489770	0.0778441036	0.0002478887
y_6	a5	0	1	0	0	1	1	1	1
	a4	1	2.4722993181	1	0	5.5310082166	4.2915726868	5.1174385867	0.5455877402
	a3	4.8422964324	5.2838349997	2.5773991590	1	13.6855997346	11.7726089219	12.7690686626	0.4060343822
	a2	10.7849311103	3.8287974090	2.2889499243	3.2412553170	35.0939815038	25.8040893288	32.0646416746	0.1127666741
	a1	30.0232912374	2.0058289341	1.5437144273	7.3288534454	23.0308253221	17.2165748405	19.4851729110	0.0155963146
	a0	13.0157871611	0.3185325008	0.2054692391	6.0368993544	8.0518188585	5.5549924748	7.3843111007	0.0003694925
y_7	a5	1	1	0	0	0	1	1	0
	a4	0.6876230199	1.6756014279	1	1	1	1.2351969108	0.8781266014	1
	a3	0.0144901228	1.5191109203	7.5004446872	2.5382487046	4.4741645112	7.6073840964	7.2235453449	52.5914111217
	a2	0.0016588350	0.6240684587	20.1860222632	6.5936670137	10.4438953587	4.9782043358	3.0632089524	15.5564218341
	a1	0.0000284225	0.0198395409	22.2301944851	5.1310146112	24.6285626282	0.5104835662	0.1636455799	0.6417332982
	a0	0.0000001178	0.0004464652	3.3288044173	0.5144525226	5.5131859016	0.0046414989	0.0005307659	0.0324638587
y_8	a5	0	0	1	1	1	1	1	1
	a4	1	1	2.2834285098	0.7361934238	23.2917737232	32.9978236388	0.5227568859	1.0198573960
	a3	128.1096262889	0.8751229881	3.3655508539	0.6682385593	28.0097801648	35.8459226914	7.4915580779	1.6201772496
	a2	13.5265784174	0.5984408253	1.9815801894	0.1293683913	180.2463157736	248.0431429582	1.2235727340	0.2400528614
	a1	0.3135662275	0.1269642665	0.7018978009	0.0018210495	62.9416337527	95.4629104453	0.0316386306	0.0146339974
	a0	0.0331020382	0.0009331525	0.0556494005	0.0000137470	4.4360274053	6.9432224419	0.0001739910	0.0002517855
y_9	a5	0	1	1	1	1	0	0	1
	a4	1	2.8774817779	1.9975628151	1.8196117710	4.9313265380	0	1	1.2301641696
	a3	3.5201045868	3.7468674675	4.3108201672	3.2190756211	3.1124754464	1	0.6121829692	2.6663573679
	a2	11.3523377538	2.1082986490	2.1957117712	1.7278428156	0.7998668610	38.2894970079	0.1961658126	0.1833966924
	a1	25.0589286693	0.6859999002	1.0858653720	0.8101479707	0.0429804975	29.8038380497	0.0099224608	0.0025313115
	a0	1.4576923813	0.0433312827	0.0680591883	0.0635245170	0.0001539894	1.4326232980	0	0.0000768671
y_{10}	a5	1	1	1	0	0	1	1	1
	a4	41.2042469847	2.4807062576	2.1980315679	1	0	45.8230933608	0.8453182590	1.1850543188
	a3	63.8444072346	4.5163979514	4.1805006285	1.9839329158	1	46.1892874631	7.1370768490	2.1443668352
	a2	315.6983962985	3.2473817181	2.4893141183	5.3268249579	78.5738755642	337.7444740749	4.3791286011	0.2301741438
	a1	130.6710120510	1.5053378891	1.1305768600	2.2893753643	54.6137403074	186.6457409890	1.4793886529	0.0021791498
	a0	8.0034594481	0.2062010642	0.1290836933	1.6365915579	2.3555441659	6.8531784035	0.0946428037	0.0000843941
y_{11}	a5	1	0	1	1	1	1	1	1
	a4	260.8351402455	1	2.5764525272	1.8237906488	0.9793158125	1.4720493588	0.6900284371	2.2385459695
	a3	393.1079848084	1.3496743573	4.7770743975	3.2916129883	7.1836393602	7.7188944322	0.2690712825	2.8832684043
	a2	1842.8222284170	1.1241107640	3.3773118116	1.9257851593	4.9083162804	6.3283392895	0.0253552301	1.2886171481
	a1	1219.8069608713	0.4305083699	1.4473429964	0.9518143485	1.6308112659	2.9145371304	0.0002421833	0.2808015387
	a0	56.5483776673	0.0704681441	0.2634815830	0.1329023561	0.1174621380	0.2686299870	0.0000011356	0.0112495218

Table 55 – Plant 4 – Sub-model 2 – Transfer Function Numerator Parameters.

PI	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0	0	0	0	0.000020134	0
	b4	0	0	0	0	0.000398466	0	-0.000253251	0
	b3	0	0	-0.016552550	0	0.028233280	0.038338952	0.021505330	0
	b2	0	0.017324224	0.001243825	-0.286556622	-0.021709531	-0.019850048	-0.024328851	0.005571739
	b1	0.187354964	-0.000615082	0.000001331	-0.279151750	-0.006362923	-0.005460434	-0.007417424	-0.009454404
	b0	0.117539195	-0.000033825	0.000000067	-0.026286594	-0.016748395	-0.023513438	-0.012496691	-0.000058000
y_2	b5	0	0	-0.000992203	0	0	0	0	0
	b4	0	0	-0.044847668	-0.000650215	0	0	0	0
	b3	0	0	0.230055145	-0.087834093	3.365738095	3.840124179	0	0
	b2	-2.721084069	0.865450417	0.128942070	-0.280526177	-0.075041139	0.232888468	5.280777808	1.106749294
	b1	28.228443970	0.734234202	0.225632437	-0.355028683	-3.037570876	-3.414341381	-5.918539823	-2.439108668
	b0	9.380630893	-0.087626109	0.031206352	-0.137135776	-0.531898708	-0.568287580	-2.975087155	-0.942868497
y_3	b5	-0.000288070	0	-0.000708641	0	0	0	0	0
	b4	-8.404214968	0	0.344635989	-0.632772900	0	0	0	0
	b3	-2.201790633	0	0.201747030	-1.591418165	0	8.997036494	0	0
	b2	-18.619039587	2.493145331	0.163903268	-2.905762632	6.636939676	3.886581081	5.467507154	8.799422737
	b1	42.666357426	0.653827569	0.025912555	-1.229162998	0.611751935	-13.351718221	-2.946356692	-7.588377190
	b0	4.988559096	-0.742737930	0.000010826	-0.163368405	-8.712412784	-0.672290136	-10.620238264	-11.247295234
y_4	b5	0	0	0	0	0	0	0	0
	b4	0	-0.005075221	0	0	0	0	0.158245263	0
	b3	0	0.135902136	0.342530255	-2.286229542	2.374566997	0	0.433503171	38.802114210
	b2	-3.640525174	0.378274122	0.172379711	-1.003089457	2.817821496	1.200868568	-0.727727973	34.565391161
	b1	11.778151454	-0.210310766	0.002206556	-0.000377990	-6.851112672	-0.888637215	-0.046948568	-81.852410727
	b0	6.012223086	-0.003649170	0.000024255	-0.000016897	-0.182734989	-0.883163630	0.000000373	-4.184882375
y_5	b5	0	0	0	0	-0.013220103	0	-0.000124625	0
	b4	0	0	0	-0.017288467	-0.052489913	0	0.322091884	0.063592690
	b3	-1.481769722	0.080811039	0	-1.807610924	0.517819136	0	-0.172292828	0.451915138
	b2	3.952607982	0.252226840	-3.594827720	-1.010538566	-0.336570009	3.160829218	-0.015876245	-0.225946188
	b1	2.264843382	-0.158864805	8.187266069	-0.003043626	-0.270145255	-3.735341903	-0.228937292	-0.364031011
	b0	0.040528344	-0.000030858	1.727094588	-0.001242047	-0.080725222	-0.015407947	-0.035522311	-0.019985269
y_6	b5	0.000036940	0	-0.001449310	0	0	0.000484671	0	0
	b4	-2.604864330	0	-0.062398768	0	0	-0.033523762	0	0
	b3	-0.303295663	-0.094776080	0.256759991	-759.659696318	2.001165157	2.399680445	1.415256942	0
	b2	-4.834292297	1.196258859	0.164628817	-1923.836572512	-0.030360071	-0.283507337	-0.246597522	2.837830699
	b1	13.504113372	0.348560145	0.385646965	-4170.655896022	-1.152253231	-1.123757395	-0.730834621	-2.890640310
	b0	1.050338099	-0.294299778	0.055726739	-3233.361033751	-1.306927760	-1.343330355	-1.151170219	-3.346119749
y_7	b5	0	0	0	0.000067535	0	0	0	0
	b4	-0.007896849	0	0	-0.157239196	0	0	0	0
	b3	-0.013725628	0	-0.003744419	-0.392445952	1.811977317	1.376070441	19.587758301	150.059910434
	b2	-1.882524837	0.123266508	-1.045284110	-0.565424589	1.918348392	1.331058074	23.116935986	110.209136021
	b1	4.333998709	-0.055880291	-0.320476857	-0.074653627	-1.817935398	-1.427606894	-18.388019412	-91.844473398
	b0	1.493673276	-0.001530425	-0.013988412	-0.002501885	-0.176023772	-0.152423594	-1.557455026	-7.369667139
y_8	b5	0	0	-0.000428915	-0.000028577	0	0	0	0
	b4	0.003913878	0.045940768	0.025060668	0.008665487	0	0.004493123	0	0.043332970
	b3	0.851827750	-0.014504984	-0.012304510	-0.031069765	0	0.059106312	0.000348493	0.020428109
	b2	0.398792899	-0.041282418	0.007126912	0.047387631	0	0.065580814	2.054366923	-0.121658832
	b1	0.000895513	-0.002459604	-0.003049851	-0.016290545	-1.342656433	-0.167984245	-0.182365016	-0.018365256
	b0	0.000285409	-0.000082596	-0.000165065	0.003336871	-0.156209515	-0.023576287	-0.413165641	-0.000152034
y_9	b5	0	0	-0.006859859	0.001638864	0	0	0	0
	b4	0	0.002676969	0.003973951	-0.040680559	0	0	0	0
	b3	0.051253706	0.138678474	0.054254234	0.042974061	326.991042408	0	141.049515193	0
	b2	2.738075235	-0.079582091	-0.030788232	-0.018242320	-56.664784351	-0.387838936	-42.870578290	-0.671622415
	b1	0.683826683	-0.061156419	-0.002638117	0.003895792	-499.485142786	-0.029742435	-189.526441720	-0.488930380
	b0	0.000796265	-0.002280053	0.000007710	0.000039884	-27.770727946	-0.000511909	-8.776884638	-0.034666712
y_{10}	b5	0	0	-0.006732359	0.000910821	0	0	0	0
	b4	0	0	0.018792610	-0.039032741	0	0	0	0
	b3	0	0.255528325	0.026313518	0.039624143	177.506870197	264.668137945	164.708034361	0
	b2	1.942284664	-0.156057225	-0.015596000	-0.015924758	15.918746076	43.259711996	20.932624930	0
	b1	1.381763247	-0.209529624	-0.002359208	0.002966423	-268.569945534	-446.903410912	-229.275929849	0.122426515
	b0	0.131628415	-0.014868771	-0.000028744	0.000121203	-5.775022774	-12.201842980	-14.778815772	-0.307239978
y_{11}	b5	0	0	-0.002964140	0.001180279	0	0	0	0.000900129
	b4	0	0	0.020153004	-0.020010414	0	0	0.204262018	0.153827858
	b3	0	0	0.002735912	0.020599348	0	0	0.262111201	0.172951116
	b2	1.154000686	0.150186624	-0.002623776	-0.008345722	0.243644898	0.293257062	0.223318123	-0.026204126
	b1	0.723818592	-0.284093499	-0.001834165	0.001527119	-0.519365631	-0.621770998	-0.435800972	-0.419744912
	b0	0.046646092	-0.014998994	-0.000003053	0.000032351	-0.034543414	-0.045025430	-0.076908403	-0.067006412

Table 56 – Plant 4 – Sub-model 2 – Transfer Function Denominator Parameters.

Pl	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	0	0	0	0	1	1	1	0
	a4	1	1	1	1	2.155622771	2.292045623	2.194396518	1
	a3	3.112705714	1.211797550	1.166153784	3.216797448	7.033246742	7.355976303	6.915465744	0.234923141
	a2	5.513634274	0.910077825	0.229252356	9.015220505	9.602672884	10.324520267	9.741888632	4.890341142
	a1	14.083221322	0.153418179	0.000026048	5.119065197	9.007117007	10.342623082	8.387834685	0.684647662
	a0	1.475016439	0.003949474	0.000002862	0.414118977	1.052011364	1.212912491	0.973417887	0.004016623
y_2	a5	1	1	1	0	1	1	1	1
	a4	2.003156025	1.945718104	1.104274519	1	13.741847373	12.557777680	4.985178161	1.672233538
	a3	24.662804449	6.667920737	5.406778048	1.230573675	15.366329137	14.320925790	37.897862673	15.437903561
	a2	15.783969597	8.545869734	4.192370891	5.083401860	64.869613555	59.059294283	34.759068582	10.584322806
	a1	93.309507046	5.441361100	2.669133657	4.072815283	34.152310665	30.682401815	149.491873992	48.705481256
	a0	10.767272048	1.695931378	0.276905656	1.225848685	3.124382278	2.708199824	22.047387802	6.322521099
y_3	a5	1	1	1	1	0	1	1	1
	a4	4.955870468	1.986572197	1.068593216	1.616379951	1	10.920796114	2.806612229	2.679222531
	a3	13.336046546	6.196011709	0.689438394	5.423780968	9.607579631	13.143976953	21.094812254	23.204028129
	a2	24.609944494	7.817324311	0.301645075	5.041534199	11.057458044	50.842015624	21.862232836	21.915211090
	a1	34.757560236	4.154535691	0.034333286	1.855368213	44.168669867	26.923231545	74.783447623	85.062021864
	a0	3.054959883	1.583872961	0.000014361	0.216239693	18.722049159	1.190655137	27.905461667	27.748094370
y_4	a5	0	0	1	0	1	0	1	1
	a4	1	1	1.166908627	1	8.020087686	1	0.536539666	109.396988728
	a3	3.391412434	1.173679795	0.767850052	2.774069274	8.558611223	1.318882274	4.699888356	63.531957760
	a2	6.394509612	0.883613792	0.126241304	0.690722852	36.400510456	5.174995389	1.486276610	512.287623715
	a1	15.294699004	0.233412280	0.001571749	0.000181582	9.863035616	5.240532329	0.072743633	149.846918994
	a0	2.555749677	0.003636940	0.000016934	0.000016456	0.235461334	0.934549234	0.000001080	6.097566993
y_5	a5	1	0	0	0	1	1	1	1
	a4	1.301016207	1	1	1	0.724354657	3.798394618	2.496986046	0.826629819
	a3	5.296408464	0.692935628	18.866376620	2.315255356	4.905712923	6.123238315	3.070811636	4.825295716
	a2	5.140829659	0.477194755	23.335580512	0.387508363	2.481300331	17.182339615	2.068160772	2.960376220
	a1	0.696868021	0.065991634	8.022723762	0.003152861	0.740635696	2.653335174	0.511714037	0.497405241
	a0	0.010817876	0.000010386	0.668105426	0.000472305	0.067064380	0.010194783	0.036444460	0.018980043
y_6	a5	1	1	1	1	1	1	1	1
	a4	4.202775359	2.503292548	1.131162455	2716.306866191	6.980724350	6.608011208	6.290525186	2.443932869
	a3	12.523267258	6.687401559	5.534033024	6350.688468002	12.134868173	10.948043960	11.955749212	21.202119164
	a2	20.700137867	10.892444935	4.413139037	16938.621416734	34.765151769	32.399661240	31.800453182	20.674047777
	a1	31.905728429	5.229271418	3.117830420	24306.474970682	27.151362591	21.999757377	27.123803031	76.236225975
	a0	2.132306688	2.563354644	0.345760873	12286.834591147	10.139655632	8.617958846	10.973854991	29.534373292
y_7	a5	0	0	0	1	1	1	1	1
	a4	1	1	0	1.525212775	15.646184152	9.598950584	210.905985574	1087.225533277
	a3	5.303443402	1.238099881	1	4.925099060	13.809641105	10.396536529	124.225111871	607.830038956
	a2	7.135499723	1.154373051	3.461369891	2.983126397	72.686084063	44.158168840	992.182692469	5184.142638539
	a1	24.692130052	0.337458356	1.640723174	0.327587308	25.190845122	16.220695882	317.118821845	1534.185374406
	a0	3.725851691	0.007332189	0.137333984	0.009971212	1.538606432	1.066943676	17.481477999	81.668982358
y_8	a5	0	1	1	1	1	0	0	1
	a4	1	1.037804735	1.150751366	1.078926319	10.620238728	1	0	0.580672059
	a3	3.441571832	0.688836837	2.643104137	5.393704063	9.298893802	0.621872764	1	5.082205607
	a2	0.369885444	0.186403497	0.986804334	4.144072173	50.716211192	4.944957786	13.245044440	1.650342701
	a1	0.002404366	0.010100098	0.366145103	2.898745582	15.255611538	1.672380787	26.620598537	0.124406748
	a0	0.000257916	0.000296204	0.019747089	0.555018068	0.954170552	0.115265617	3.290234465	0.000974310
y_9	a5	0	0	1	1	1	1	1	1
	a4	0	1	0.574841581	0.874926421	2081.858566417	0.893071051	1154.852101751	3.099113873
	a3	1	1.332626820	4.669464364	3.811312319	1533.809723062	5.129771122	783.027362888	6.169419589
	a2	5.546797964	0.803273968	1.911749873	1.201724913	10397.960302253	2.820770453	5769.870189961	14.097644959
	a1	0.464500696	0.266577209	0.247826520	0.806155964	4553.865240577	0.175659851	2228.040392591	4.651629793
	a0	0.000337547	0.018576602	0.008903303	0.038780140	194.568471961	0.002756182	82.374301497	0.228184832
y_{10}	a5	0	1	1	1	1	1	1	0
	a4	1	2.940893902	0.471556527	1.019301986	1196.164677118	1454.086360598	1440.221265216	0
	a3	0.941133137	4.296061125	4.318490735	3.660794680	891.279795141	1139.879068232	1079.918394636	1
	a2	5.310158870	2.582248497	1.351856713	1.271797525	5979.952511618	7247.742733599	7296.571249305	0.698463212
	a1	2.041619389	0.971022250	0.269092432	0.873529212	2586.734072764	3399.874212893	3258.174704069	5.540629292
	a0	0.135227419	0.107055050	0.012041953	0.052488754	50.888398675	83.761975500	179.726397389	2.866195599
y_{11}	a5	0	1	1	1	0	0	1	1
	a4	1	1.820863098	0.537477653	0.775483477	1	1	6.035273542	4.817942895
	a3	1.025735249	5.724347585	4.016679105	4.026136496	2.792897121	2.609759253	9.202477395	8.121960570
	a2	5.422368081	4.470465139	1.142169181	1.234131858	5.968829790	5.911468106	28.146375803	22.605706207
	a1	2.338294367	2.273860728	0.372022602	1.103640421	12.498103080	11.563514825	13.615677999	10.457931488
	a0	0.159404936	0.413831050	0.030938263	0.076727245	0.984338095	1.005931680	2.864070074	1.941511649

Table 57 – Plant 4 – Sub-model 3 – Transfer Function Numerator Parameters.

Pl	N	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	b5	0	0	0.0003078410	0	0	0	0	0
	b4	0	0	-0.0027553646	0	0	0	0	0
	b3	-7.8813901300	0.0078812927	-0.0001749214	0	0.0596137512	0.0723026009	0.0606960333	0
	b2	4.9541328914	0.0000624732	-0.0000086551	0.0218501740	-0.1062614552	-0.1204673414	-0.0809391419	0.0598309539
	b1	12.3178130244	0.0000231634	-0.0000005514	-0.0002644975	-0.0554224031	-0.0322675066	-0.2136456849	-0.2964347494
	b0	2.7433177944	0.0000001944	-04	0.0000548358	-0.0043030076	-0.0023148852	-0.0245464575	-0.0530336587
y_2	b5	0	0	0.0025735342	0	0	0	-0.0001739439	0.0001820523
	b4	0	0	0.0006428653	0	0	0	0.0161939753	0.0652812839
	b3	-3.7259103435	0.2982538677	-0.0000044088	0	0.1452709981	0.3039379449	-0.0042641417	0.0959530254
	b2	6.4027315976	0.2661949832	0.0000064794	0	-0.4146535925	-0.3886443174	-0.3247159901	-0.3406375149
	b1	14.7385191563	0.0036803698	-0.0000000901	-0.0032970027	-0.7453754043	-0.9007970270	-0.5660436256	-0.6020060776
	b0	2.9664130199	0.0001614490	0.0000000039	0.5969388827	-0.1812132559	-0.2226720378	-0.1273590702	-0.1374126410
y_3	b5	0	0	0	0.2148379355	0	0	-0.0004022472	0
	b4	0	0	-0.0002274839	0.0455407838	0	0	0.1397452217	0
	b3	-7.2753955570	0	0.0567334501	0.0010508947	0.7260791905	1.3044547285	-0.0708552625	0
	b2	22.5162003030	0.0415266836	0.0002489750	0.0002443561	-1.7792689222	-1.8991308802	-0.6947026796	0.8062354068
	b1	24.7371142009	1.4667212491	0.0000831687	0.0000028039	-3.2460030814	-3.5168314433	-2.5361131097	-2.6345320116
	b0	3.1066433413	0.0934791171	0.0000004437	0.0000002505	-0.4826128679	-0.5534360120	-0.3852472905	-2.1009419485
y_4	b5	0	0	0	0	0	0	0	0
	b4	0	0.0578205476	0	0	0	0	0	0
	b3	-3.5374611605	0.3958542231	0	1.6057105067	1.0371295427	1.4716173222	0.6303762003	2.6292978387
	b2	18.3850803360	0.0062010162	0.0779134627	0.2578369604	-2.0558604152	-2.0905761374	-2.1607916820	-2.9160987990
	b1	15.5197100247	0.0034799171	-0.0001658711	0.0001858585	-3.9886381413	-3.9116724679	-4.0820321347	-8.0222565668
	b0	3.7189101007	0.0000221356	0.0000204785	0.0005729095	-0.4358631948	-0.3866020962	-0.4915071372	-0.9101977831
y_5	b5	0	0.0168763667	0	0	0	0	-0.0002019120	0
	b4	0	0.7138329887	0	0.6297708807	0.0043678842	0.0049621034	0.2414631185	0.0127874532
	b3	-2.3912330893	0.0896270025	0.0918977124	0.0016066214	0.1188198121	0.1576053729	0.1010573980	-0.1001860790
	b2	19.1918862637	0.6424556650	0.0011669429	0.0119001517	0.2279822084	0.5063255249	-0.6900756684	0.8830177020
	b1	12.8274346528	-0.0017544871	0.0000523001	-0.0001396960	-1.4187740071	-1.7790433460	-2.6598370573	-3.1916821448
	b0	3.9573114461	0.0009800918	0.0000005973	0.0000213813	-0.0086490951	0.0388360880	-0.0951124650	-0.0492664494
y_6	b5	0	0	0	0.0706733612	0	0	0	0
	b4	0	0	0	0.0207584516	0	0	0	0
	b3	0	1.1422873934	0	0.0003050577	0	0	0	0
	b2	-3.6793826383	0.2089576785	-0.0000017571	0.0001239599	0.5884360208	2.9370625725	-0.1395774466	0.2533465661
	b1	9.5841412199	0.1015553508	0.0000000923	0.0000007729	-2.7643235579	-7.1870654159	-1.7390703202	-2.6686161234
	b0	12.8287734424	0.0002450712	-0.0000000059	0.0000001551	-2.9266581400	-1.5511477030	-1.2772386764	-1.1216775777
y_7	b5	0	0	0	0	0	0	0	0
	b4	0	0	0.0013207961	0	0	0	0	0
	b3	-4.7369986868	1.5226125518	-5.4907178795	0.4342480658	1.5165193172	1.8952107035	1.1115668572	1.6515426592
	b2	27.2943342986	0.3971392293	0.3539705973	0.0050559156	-1.6550793034	-1.5428689075	-1.6544466951	-1.8591646953
	b1	19.8943984548	-1.0060839887	0.0037242815	0.0003102355	0.5430445077	0.3695229902	0.7206037151	0.1295402550
	b0	4.1049546164	-0.0195030845	0.0005297982	0.0000013332	-2.9919447908	-4.0657177262	-2.1374718336	-2.7249294431
y_8	b5	0	0	0	-0.0049913910	0	0	0	0
	b4	-0.2700879067	0	0.0004720191	0.0003199634	0.0400323534	0	0.0342744355	0
	b3	0.9082320035	0	-0.0001422605	-0.0000297072	0.0477446649	0	0.0204463981	0
	b2	0.3162546063	0.1193010041	-0.0000592973	-0.0000021678	-0.0181810912	187.2644745804	-0.0063905770	0.2616489506
	b1	0.0021617371	-0.1360649405	-0.0000002589	0.0000000069	-0.0839258531	-248.8398577541	-0.0527751717	-0.2986450039
	b0	0.0002243505	-0.0053275442	-0.0000000134	-0.0000000037	-0.0180215969	-48.7561413701	-0.0158245099	-0.0421932408
y_9	b5	0	0	0	0.0046545298	0	0	0	0
	b4	0	0	0.0080896454	-0.0092309452	0	0	0	0
	b3	-0.0796596767	0	0.0293368128	0.0014494796	-0.1097502565	0	28.0651836004	0
	b2	-0.4691883298	4.0346612533	-0.0100402948	-0.0000972361	-0.0013398234	0	-15.6285868917	-0.6161125285
	b1	3.1559813145	-4.3477479498	-0.0009828369	0.0000099379	-0.0000171260	-0.1523553064	-0.9176168419	-0.4609611627
	b0	0.4521325467	-0.1872372600	0.0000000784	-0.0000003611	-0.0000001877	-0.0071834667	-0.0000681906	-0.0238351651
y_{10}	b5	0	-0.0021910236	0	0.0022362523	0	0	0	0
	b4	0	-0.3223348117	-0.0004705139	-0.0103374293	0	0	0	0
	b3	0	0.8015688802	-1.4997430261	0.0007161044	0.4278241411	0	0	0
	b2	14.0470914044	-0.1805404152	-0.3886000768	-0.0000591954	-0.5096854962	0.6933844728	-0.2132064054	0.8640926443
	b1	17.2729521233	-0.6816666187	-1.3476226948	0.0000008382	-0.1182214281	-0.4783611514	-0.0850120317	-1.2478573533
	b0	2.4406479625	-0.0508723970	-0.0371398115	-0.0000003245	-0.0490667508	-0.7888206341	-0.0036959423	-2.6133627232
y_{11}	b5	0	0	-0.0015501843	0.0046944234	0	0	0.0001082916	0
	b4	0	0	0.0001870485	-0.0086720126	0	0	0.1553330975	0
	b3	0.2197181967	0.2921456187	0.0147135768	0.0013896454	193.3966314454	1.9846473991	0.3012837925	0.5720421718
	b2	7.9222512861	0.0975208248	-0.0058357207	-0.0000953928	97.5252492772	0.7142287131	0.1755985547	-0.3680282601
	b1	7.9048070491	-0.4004037227	-0.0002973473	0.0000083817	-284.2048200838	-3.1999825886	-0.4413844782	-0.9312461437
	b0	0.5098634519	-0.0191079371	-0.0000017146	-0.0000002380	-44.7447313267	-0.4709133226	-0.0719020770	-0.0542867495

Table 58 – Plant 4 – Sub-model 3 – Transfer Function Denominator Parameters.

P1	D	u_1	u_2	u_3	u_4	u_5	u_6	u_7	d_1
y_1	a5	1	0	1	0	1	1	1	1
	a4	37.668665745	0	0.563383387	0	4.455178287	3.688049610	7.749396274	2.336725830
	a3	134.204557841	1	0.024366163	1	8.534806442	7.556484148	15.693451990	16.871751274
	a2	279.702759619	0.022939333	0.002100357	0.455190671	22.952452702	18.387657446	43.320117501	16.212893712
	a1	643.913018172	0.002268946	0.000068558	0.002362961	8.004184520	3.887523362	39.876372645	58.451079053
	a0	73.419836351	0.000006760	0.000000426	0.001075598	0.506847132	0.227912390	3.509234987	7.010083536
y_2	a5	1	0	1	0	1	1	1	1
	a4	4.286621273	1	0.200976094	0	2.215338018	2.553324469	1.652427756	1.773432579
	a3	18.248501259	1.084668650	0.003532616	0	9.284081522	9.557856027	8.607355844	8.926285541
	a2	28.300127310	1.188123275	0.000687536	0	11.680736930	13.590339667	8.373988678	9.142441481
	a1	62.103420186	0.016096996	0.000001408	1	16.663449663	17.336737175	14.481107520	15.439404610
	a0	6.703565218	0.000717179	0.000000214	7.089618027	2.136584910	2.166548703	1.846136570	1.815114772
y_3	a5	1	0	0	1	1	1	1	0
	a4	3.402838222	0	1	0.062568677	3.283402112	3.482171523	2.047856039	1
	a3	16.184455468	0	0.625646622	0.007389174	10.407865492	10.335989214	9.433209394	2.759999778
	a2	22.200459702	1	0.001821821	0.000307773	18.038936011	19.113096212	11.029109315	9.972044635
	a1	52.416909394	2.343336600	0.000875953	0.000013227	20.591696654	19.720259454	18.200658731	14.621090363
	a0	5.966357252	0.114723121	0	0.000000357	3.706768374	3.711866802	3.389041116	18.488733643
y_4	a5	1	0	0	0	1	1	1	1
	a4	3.112517695	1	0	1	5.137619210	4.729366203	5.658405059	9.008899475
	a3	15.293459965	0.317809049	1	2.722601143	11.747680948	11.155861328	12.436312132	18.707307325
	a2	20.925458136	0.011876291	0.271517952	0.256859718	28.990779645	26.491224660	32.166832963	54.560751607
	a1	48.377320501	0.002776032	0.000217752	0.006737287	23.728583964	21.242393345	26.518527379	53.889790971
	a0	10.322528464	0.000013466	0.000059100	0.000629496	5.941111555	5.257763214	6.697151675	13.310593853
y_5	a5	1	1	1	1	0	0	1	0
	a4	3.096154531	1.543703170	0.967679843	0.348247728	1	1	3.405054086	1
	a3	14.558056605	1.371783393	0.171439966	0.037056660	1.292766412	1.449631574	8.840812629	2.698078259
	a2	20.691449649	0.360381693	0.002732196	0.005491505	6.272892912	6.387721851	18.117970711	7.264406920
	a1	44.320367796	0.001931615	0.000101306	0.000083638	5.552822059	6.430535897	12.175877320	14.290741602
	a0	11.329793932	0.000504910	0.000000959	0.000011228	1.378852587	1.591558724	2.836083777	3.041126321
y_6	a5	0	1	0	1	1	1	1	1
	a4	1	2.010202963	1	0.072771252	4.029819640	5.736085753	2.921012209	2.459827501
	a3	3.461511441	4.424298577	0.008840450	0.006873718	17.249503405	24.579073280	13.208387368	14.551588136
	a2	16.144929161	2.676601443	0.002208356	0.000448261	32.961443596	39.911145372	21.304178515	18.654346786
	a1	21.479348031	0.695241749	0.000017033	0.000011176	59.784249723	94.028144081	38.257906583	46.145302920
	a0	51.498646681	0	0.000000351	0.000000660	46.125727091	20.629605804	24.302745557	19.869711489
y_7	a5	1	1	0	1	1	1	1	1
	a4	3.361337463	1.747618431	1	13.972265975	3.655227730	3.589278046	3.633681923	3.585929911
	a3	14.900058698	5.516573169	208.067182511	2.038111149	11.324550062	11.591648429	10.976552001	12.770029337
	a2	22.028171701	4.037013357	55.504973213	0.031998263	23.478447776	23.526669971	23.015607979	24.234966675
	a1	45.446306970	2.474302120	0.913625214	0.001388982	25.562163263	27.309462885	23.638587815	33.601060352
	a0	9.221996153	0.047748225	0.057112469	0.000005479	19.122341631	20.710596164	17.654451564	21.317209892
y_8	a5	1	0	0	1	1	1	1	0
	a4	0.591213350	1	1	0.030394949	1.334456921	1659.969306617	1.223727258	0
	a3	5.603612589	1.849337696	0.297193806	0.013766139	6.136414478	1274.231409122	6.069977721	1
	a2	0.605796879	2.039899839	0.016581068	0.000063239	5.366157677	10222.324781857	4.723322908	16.627553321
	a1	0.006974788	0.697031648	0.000172939	0.000022751	1.864606163	4108.245551552	1.900667250	5.688014674
	a0	0.000415511	0.023252701	0.000002404	0.000000009	0.164084472	351.206066600	0.188019140	0.408291969
y_9	a5	0	1	1	1	1	0	0	1
	a4	0	7.326833344	0.803043921	0.265601101	1.696614366	0	1	2.311595354
	a3	1	18.241407367	4.379219787	0.133083650	0.852736611	1	184.243709888	6.581257273
	a2	0.422443672	22.135597617	2.429657416	0.004765020	0.009474567	1.710834717	188.718615281	11.723631983
	a1	5.584832416	12.844787639	0.300944446	0.001080910	0.000136546	1.002052730	8.979331762	3.978532262
	a0	0.405006451	1.388699473	0.012747927	0.000021351	0.000001470	0.039889655	0.000680401	0.163340474
y_{10}	a5	1	1	0	1	1	0	1	1
	a4	3.371889803	2.619358216	1	0.271291792	1.324955297	1	0.753506290	1.819314737
	a3	12.262307765	5.817827676	94.332872781	0.148615774	6.714073100	2.343479393	5.746548727	16.572745570
	a2	19.797144812	4.735189973	31.497626044	0.005688355	6.261194826	7.464494189	3.473570145	15.999312093
	a1	29.816743283	2.118478838	443.842622607	0.001188766	1.620592913	11.937326612	0.951857263	59.570839744
	a0	2.879553244	0.264912374	31.598793352	0	0.374857714	4.683826889	0.037119523	20.607526588
y_{11}	a5	1	1	1	1	1	1	1	1
	a4	3.156430497	2.322305418	1.000881395	0.509508418	1716.418109299	14.357112277	4.494295702	5.599683512
	a3	11.748316358	5.262510809	4.708795418	0.215475737	1593.147203104	19.145725936	9.010398336	11.412691571
	a2	18.215314651	4.683659685	3.569641698	0.012935528	10039.840046057	81.907369740	23.474111206	31.473773761
	a1	27.332045731	2.415123661	0.511523727	0.001574812	5025.628356411	43.947829464	10.759617008	22.531573106
	a0	1.976657761	0.415034344	0.015604028	0.000068750	1095.282711487	9.026746219	2.331043983	1.429690492

Table 59 – Crude oil distillation – Plant 1 – Positive Uncertainty Parameters for the MVs.

$\mathbf{B_u^+}$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
u_1	0.6607	0.3192	0.2963	0.5641	0.4278	0	0.0587	0.0134	0.6541	0.7849	0.5214
u_2	0	0.6799	0	0	0	0	0	0	0	0	0
u_3	0.3342	0.2225	0	0.1849	0.2351	0	0	0	0	0	0
u_4	0	0.2636	0.4556	0.4316	0.4755	0.5524	0.5927	0	0	0	0
u_5	0	0	0	0	0	0	0	0	0	0	0
u_6	0	0	0	0	0	0	0	0	0	0	0
u_7	0	0	0	0	0	0	0	0	0	0	0

Table 60 – Crude oil distillation – Plant 1 – Negative Uncertainty Parameters for the MVs.

$\mathbf{B_u^-}$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
u_1	0	-0.2492	-0.4951	-0.4412	-0.5007	-0.4597	-0.5825	-0.844	-0.3716	0	-0.7136
u_2	0	-0.7134	0	0	0	0	0	0	0	0	0
u_3	-0.3784	-0.2794	-0.3786	-0.1749	-0.1959	-0.2899	0	0	0	0	0
u_4	0	-0.3154	-0.7255	-0.7252	-0.7771	-0.7661	-0.2611	0	0	0	0
u_5	0	0	0	0	0	0	0	0	0	0	0
u_6	0	0	0	0	0	0	0	0	0	0	0
u_7	0	0	0	0	0	0	0	0	0	0	0

Table 61 – Crude oil distillation – Plant 1 – Positive Uncertainty Parameters for the DV.

$\boldsymbol{B_u^+}$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
d_1	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*

Table 62 – Crude oil distillation – Plant 1 – Negative Uncertainty Parameters for the DV.

$\boldsymbol{B_u^-}$	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}	y_{11}
d_1	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*