



12 **Abstract:**

13 Monocrystalline silicon wafers are widely used in the photovoltaic industry for solar panels  
14 with high conversion efficiency. Guided ultrasonic waves offer the potential to efficiently detect  
15 micro-cracks in the thin wafers. Previous studies of ultrasonic wave propagation in silicon  
16 focused on effects of material anisotropy on bulk ultrasonic waves, but the dependence of the  
17 wave propagation characteristics on the material anisotropy is not well understood for Lamb  
18 waves. The phase slowness and beam skewing of the two fundamental Lamb wave modes  $A_0$   
19 and  $S_0$  were investigated. Experimental measurements using contact wedge transducer  
20 excitation and laser measurement were conducted. Good agreement was found between the  
21 theoretically calculated angular dependency of the phase slowness and measurements for  
22 different propagation directions relative to the crystal orientation. Significant wave skew and  
23 beam widening was observed experimentally due to the anisotropy, especially for the  $S_0$  mode.  
24 Explicit Finite Element (FE) simulations were conducted to visualize and quantify the guided  
25 wave beam skew. Good agreement was found for the  $A_0$  mode, but a systematic discrepancy  
26 was observed for the  $S_0$  mode. These effects need to be considered for the non-destructive  
27 testing of wafers using guided waves.

28

29 **Keywords:**

30 Non-contact Laser Measurement, Guided Ultrasonic Waves, Anisotropy, Wave Beam Skew

31

32 **PACS:**

33 43.35.Zc, 43.20.Mv, 43.35.Cg

34 **I. Introduction**

35 Solar photovoltaics has become an important source for renewable electricity production. The  
36 development of silicon solar cell modules relies on the production of thin wafers with high  
37 conversion efficiency. In practice, the minimum thickness during production is limited by the  
38 wafer breakage rates<sup>1</sup>. The cutting process induces micro-cracks on the wafer surface. Based  
39 on their sensitivity for micro-crack detection, acoustic and ultrasonic methods have been  
40 considered for in-line monitoring of the wafers during the manufacturing processes. Scanning  
41 acoustic microscopy at frequencies up to 250 MHz was used for the detection of surface cracks  
42 as small as  $10\ \mu\text{m}$ <sup>2</sup>. Belyaev et al.<sup>3</sup> proposed a resonance ultrasonic vibration technique for fast  
43 crack detection in silicon wafers. Frequency shift and bandwidth changes of longitudinal  
44 vibration modes were used as indicators of millimeter size cracks. Guided wave approaches  
45 were proposed for rapid inspection of silicon wafers<sup>4</sup>. Chakrapani et al.<sup>5</sup> used air-coupled  
46 transducers in pitch-catch configuration to generate the fundamental antisymmetric Lamb wave  
47 mode  $A_0$  at 200 kHz in  $200\ \mu\text{m}$  thick mono- and polycrystalline silicon wafers and detect cracks.  
48 Laser generated Lamb waves were propagated in  $525\ \mu\text{m}$  thick monocrystalline silicon wafers  
49 to detect and estimate the size of 5 mm long artificial cracks<sup>6</sup>.

50 Guided ultrasonic waves can propagate over long distances for the inspection of large structures  
51 such as plates or pipes<sup>7-9</sup>. High frequency guided wave methods were employed to detect  
52 surface cracks in isotropic metallic plates<sup>10</sup> and perform in-situ monitoring of fatigue crack  
53 growth<sup>11</sup>. The propagation of guided ultrasonic waves in anisotropic composite structures has  
54 been investigated for aerospace and wind turbine applications. Chapuis et al.<sup>12</sup> investigated the  
55 energy radiation of Lamb waves in a thin fiber reinforced composite plate for an axisymmetric  
56 source and proposed a far field approximation using Green's function. Numerical and  
57 experimental results showed a significant direction-dependent focusing effect of the Lamb  
58 modes. Karmazin et al.<sup>13</sup> presented a far-field asymptotic solution for Lamb waves generated

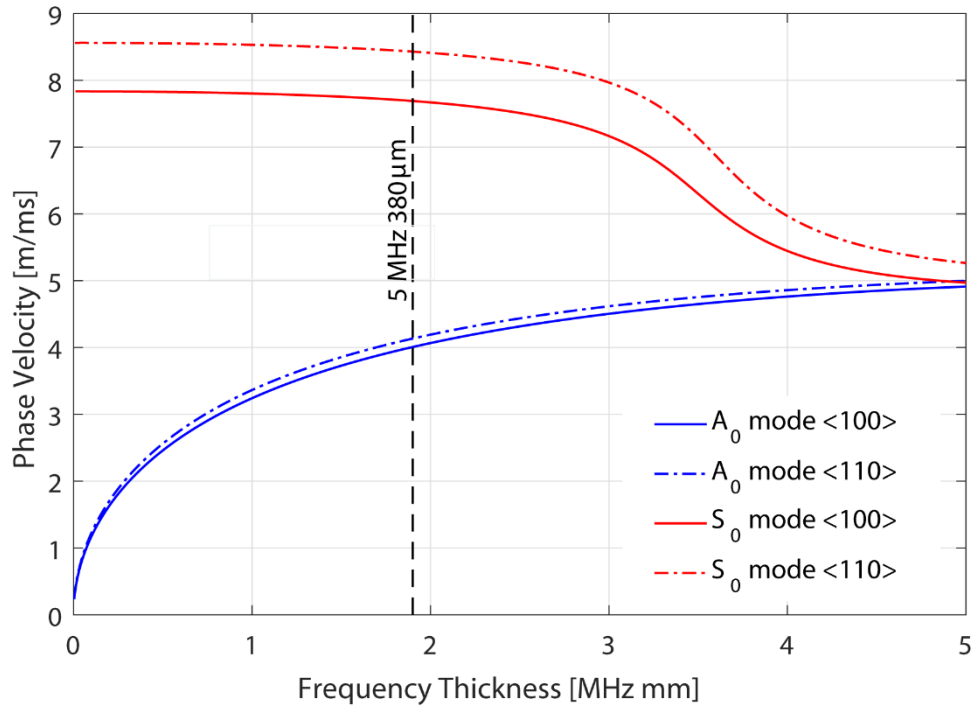
59 in a layered anisotropic plate by a surface-bonded source of finite size. Leleux et al.<sup>14</sup> developed  
60 a multi-element matrix ultrasonic probe to inspect large composite plate components in pulse-  
61 echo mode from one single position and detect delamination and impact damage. The modal  
62 selectivity of the probe and the directivity of the generated ultrasonic field were discussed. For  
63 an incident ultrasonic beam on an anisotropic multilayered structure, Potel et al.<sup>15</sup> demonstrated  
64 that the Lamb wave beam generated in the plate can deviate with respect to the sagittal plane  
65 of excitation towards the stiffer direction of the anisotropic structure.

66 The propagation of ultrasonic waves in anisotropic materials is characterized by different  
67 representative curves (two-dimensional) or surfaces (three-dimensional)<sup>16</sup>. For bulk waves, the  
68 phase velocity surface and the slowness surface (inverse of phase velocity) can be calculated  
69 by solving the Christoffel equation as a function of the wave normal. The values of these wave  
70 parameters give an estimation of the material stiffness directional dependency. The ray or group  
71 velocity surface is more difficult to calculate since analytical relations cannot be found in all  
72 directions. The direction of the group velocity vector is normal to the slowness surface at that  
73 point and the angle between wave vector and the acoustic ray is characterized by the skew  
74 angle<sup>17</sup>. The direction dependency of the group velocity for bulk wave propagation in thick  
75 silicon discs has been investigated theoretically and experimentally, pointing out the  
76 complicated cusps and folding of the ray surface<sup>18</sup>. Audoin et al.<sup>19</sup> presented an approach to  
77 recover the stiffness coefficients of anisotropic media from the measured group velocity  
78 surface, implementing a signal processing technique to measure the arrival time of each  
79 generated wave. Analytical equations relating group velocity to stiffness constants were  
80 proposed for arbitrary planes in transversely isotropic materials<sup>20</sup>. Reverdy and Audoin  
81 developed a phase velocity based method allowing for the determination of the elastic constants  
82 of anisotropic materials by means of laser ultrasonic techniques<sup>21</sup>. Maris investigated the  
83 phenomenon of energy focusing due to elastic anisotropy for bulk wave propagation in crystals

84 and proposed an explicit expression to evaluate the enhancement of energy flow in the principal  
85 directions<sup>22</sup>.

86 Lamb wave propagation in anisotropic plate structures can be described by the same  
87 characteristic parameters. Group velocity curves in thin anisotropic, carbon fiber-reinforced  
88 epoxy laminates with thicknesses of approximately 150  $\mu\text{m}$  were measured using a point-source  
89 point-receiver configuration and compared to theoretical curves<sup>23</sup>. As the ratio of wavelength  
90 to specimen thickness was large, the Lamb wave propagation was described in terms of in-plane  
91 longitudinal and shear membrane waves, considering plane wave propagation. The influence of  
92 the anisotropy on Zero Group Velocity (ZGV) Lamb modes was investigated in  
93 monocrystalline, 525  $\mu\text{m}$  thick silicon wafers using a line laser source with a spectrum limited  
94 to 18 MHz<sup>24</sup>. The results demonstrated the amplitude and frequency dependency of the ZGV  
95 modes as a function of the excitation orientation and a deviation of the acoustic beam for  
96 directions between the principal crystallographic axes. Slowness surface measurements of leaky  
97 Lamb waves for silicon wafers immersed in water have been performed<sup>25</sup>.

98 This contribution presents a systematic analysis of the effect of anisotropy on Lamb wave  
99 propagation in monocrystalline silicon wafers by means of measurements and Finite Element  
100 (FE) simulations. The fundamental antisymmetric and symmetric Lamb modes  $A_0$  and  $S_0$  were  
101 generated in monocrystalline silicon wafers using angle beam transducers and the ultrasonic  
102 field was measured by means of laser interferometry. The phase slowness curve was measured  
103 in the far field of the transducer and compared with theoretical values. The guided wave beam  
104 skew was investigated experimentally and compared to FE results and theoretical predictions.

105 **II. Theoretical background**

106

107 Fig. 1 (color online): Phase velocity dispersion diagram for  $A_0$  and  $S_0$  Lamb wave modes,  
 108 propagation in silicon wafer in <100> (solid) and <110> (dash-dotted) crystal directions;  
 109 measurements conducted at 5 MHz center frequency in 380  $\mu\text{m}$  thick wafer (vertical dashed).  
 110

110

111 Monocrystalline silicon is an orthotropic material characterized by three cubic symmetric  
 112 stiffness constants. Standard values were taken from literature<sup>26</sup> as  $C_{11} = 165.7$  GPa,  $C_{12} = 63.9$   
 113 GPa, and  $C_{44} = 79.6$  GPa. Stiffness constants can decrease from 1% - 3% for higher levels of  
 114 doping. However, this effect is usually ignored for engineering calculations<sup>26</sup>. The constants  
 115 were determined for <100> crystal orientation, corresponding to an azimuth angle  $0^\circ$ . In  
 116 contrast to other anisotropic engineering materials, silicon has a  $45^\circ$  symmetry of the material  
 117 properties, with a stiffness variation in the order of 15% between the principal <110> direction  
 118 with the highest stiffness and the principal <100> direction with the lowest stiffness<sup>26</sup>.  
 119 Theoretical calculation of the phase velocity of the fundamental Lamb modes as a function of  
 120 the angular orientation of the silicon crystal was performed for a 380  $\mu\text{m}$  thick silicon wafer

121 using Disperse<sup>27</sup> and is shown in Fig. 1. The phase velocity at 5 MHz for the antisymmetric  $A_0$   
122 mode changes by about 3% between crystal directions, and the phase velocity of the symmetric  
123  $S_0$  mode by about 10%. The wavelength  $\lambda$  for the  $A_0$  mode (approximately 0.8 mm) and for the  
124  $S_0$  mode (approximately 1.6 mm) change by the same percentage between propagation  
125 directions. Often the phase slowness, the inverse of the phase velocity, is used to display the  
126 propagation velocity variation in function of the crystallographic orientation. The wave skew  
127 for any phase direction can be visualized as the normal direction to the phase slowness curve<sup>17</sup>.

128

### 129 **III. Experiments**

130 Polished monocrystalline (001)-cut silicon wafers were used to measure the phase slowness and  
131 beam skew of the fundamental Lamb wave modes  $A_0$  and  $S_0$ . The P-type silicon specimens  
132 were boron doped and had a diameter of 100 mm (4 inch) and nominal thickness of 380  $\mu\text{m}$ .  
133 Within the (001) plane of the wafer, the material properties are identical every 90°. The  
134 principal direction with the lowest stiffness [100] was labelled as 0°, the principal direction  
135 with highest stiffness [110] as 45°, with 90° corresponding to the [010] principal direction with  
136 lowest stiffness (identical to [100] direction). The wave modes were excited using a commercial  
137 piezoelectric transducer with 5 MHz center frequency (Harisonic ABM0504 5MHz). The  
138 transducer was attached to custom-made Nylon wedges (8.5 mm width) with the angle adapted  
139 to excite selectively the  $A_0$  mode (41° wedge angle) or the  $S_0$  mode (19° wedge angle) according  
140 to Snell's law and nominal phase velocity values for propagation in the  $\langle 110 \rangle$  direction. Both  
141 the silicon wafer and transducer were fixed in custom-made holders to achieve accurate  
142 positioning and defined contact pressure between the wedge and the wafer. The angle beam  
143 wedge transducer was coupled to the wafer using liquid ultrasonic couplant, with excess  
144 couplant on the wafer surface removed. The wedge was pressed down with a defined force of  
145 approximately 10N by setting the compression length of a spring, calibrated in advance. The

146 contact force was selected in a force range where limited variation of the guided wave amplitude  
147 was observed, but low enough to limit the risk of wafer breakage. For measurements in different  
148 directions relative to the crystallographic orientation, the wafer holder allowed angular  
149 adjustments with an accuracy of approximately  $1^\circ$ .

150 The excitation signal was defined as a sinusoidal toneburst with 12 cycles and 5 MHz center  
151 frequency using an arbitrary function generator (Agilent 33220A). The signal was amplified to  
152 approximately 400 V<sub>pp</sub> using a power amplifier (RF 1020L) and applied to the piezoelectric  
153 transducer. This allows good control over the frequency content of the excited narrowband  
154 guided wave pulse to reduce dispersion with limited pulse time length. The out-of-plane  
155 displacement was measured using a non-contact laser interferometer (Polytec OFV-5000 and  
156 OFV 505). Measured time series were band-pass (2 to 7 MHz) filtered (KH 3945), averaged 50  
157 times, and recorded using a digital storage oscilloscope (LeCroy 9304). The laser interferometer  
158 was fixed to a 2 axis scanning rig and moved parallel to the wafer, keeping the laser beam  
159 perpendicular to the polished wafer surface.

160

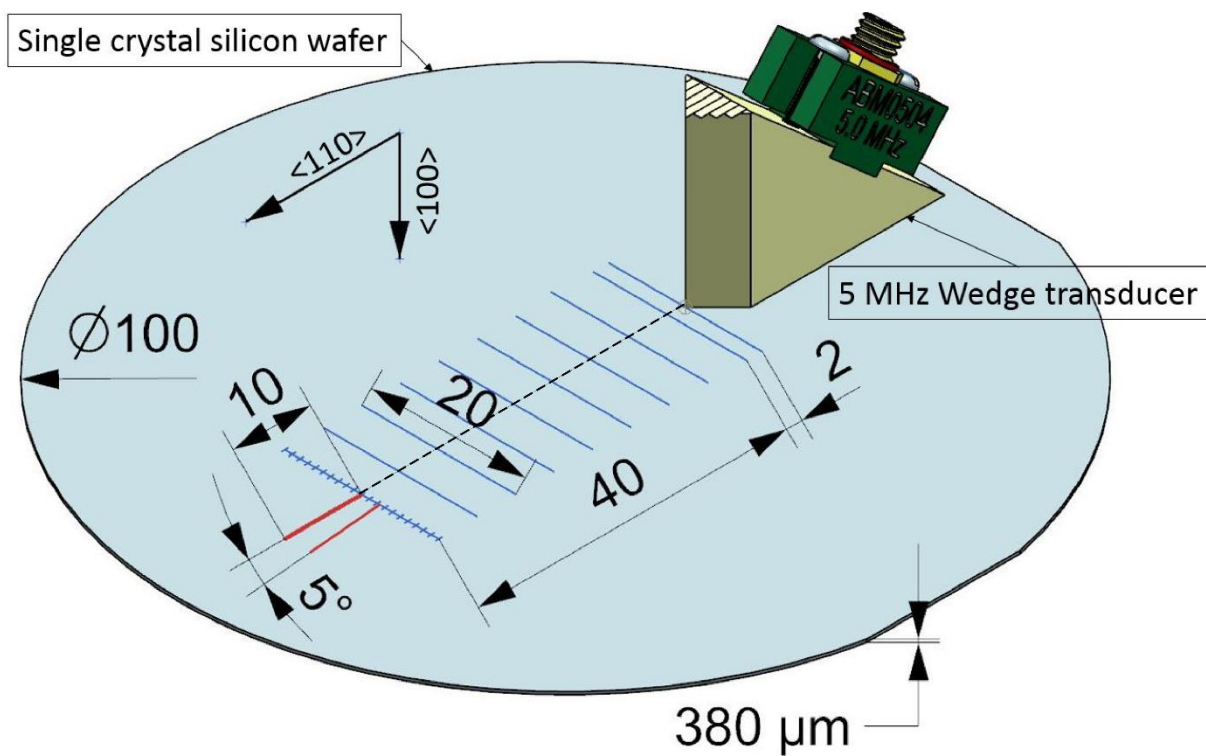
161 The phase velocity was measured in the transducer far field on the center line of the wedge, at  
162 a distance of 42 mm from the angle beam transducer. The measurements were performed over  
163 a straight line of 10 mm with measurement steps every 0.2 mm, in total 51 measurement points  
164 (Fig. 2). The silicon wafer was rotated using the holder in  $5^\circ$  angular steps from the [100] ( $0^\circ$ )  
165 crystal direction via the [110] ( $45^\circ$ ) direction to the [010] ( $90^\circ$ ) crystallographic direction and  
166 measurements taken. Hilbert transform was applied to the time trace for each measurement  
167 point to extract the arrival time of the signal peak and calculate the group velocity  $c_g$ . Based on  
168 the group velocity, the time trace was windowed for each spatial step to contain only the  
169 incident wave mode of interest. Fast Fourier Transform (FFT) was used to extract the phase  
170 value of the time trace for every measurement point. After removal of  $2\pi$  phase jumps, the



171 phase variation as a function of the spatial measurement step was fitted with a linear function  
172 to calculate the phase velocity  $c_p$  for each propagation direction.

173 For the beam skew experiments the laser measurement was performed over a two-dimensional  
174 area, recording the guided wave signal on 9 parallel lines (5 mm steps) perpendicular to the  
175 expected beam propagation direction (excitation center line) over a length of 40 mm in front of  
176 the transducer. Each line had 21 steps of 1 mm (Fig. 2). For each measurement point the  
177 amplitude of the guided wave pulse was extracted after time windowing using FFT and  
178 recorded. Beam skew experiments were conducted for both fundamental Lamb wave modes for  
179 4 orientations of the silicon wafer ( $0^\circ$ ,  $15^\circ$ ,  $30^\circ$ ,  $45^\circ$ ).

180



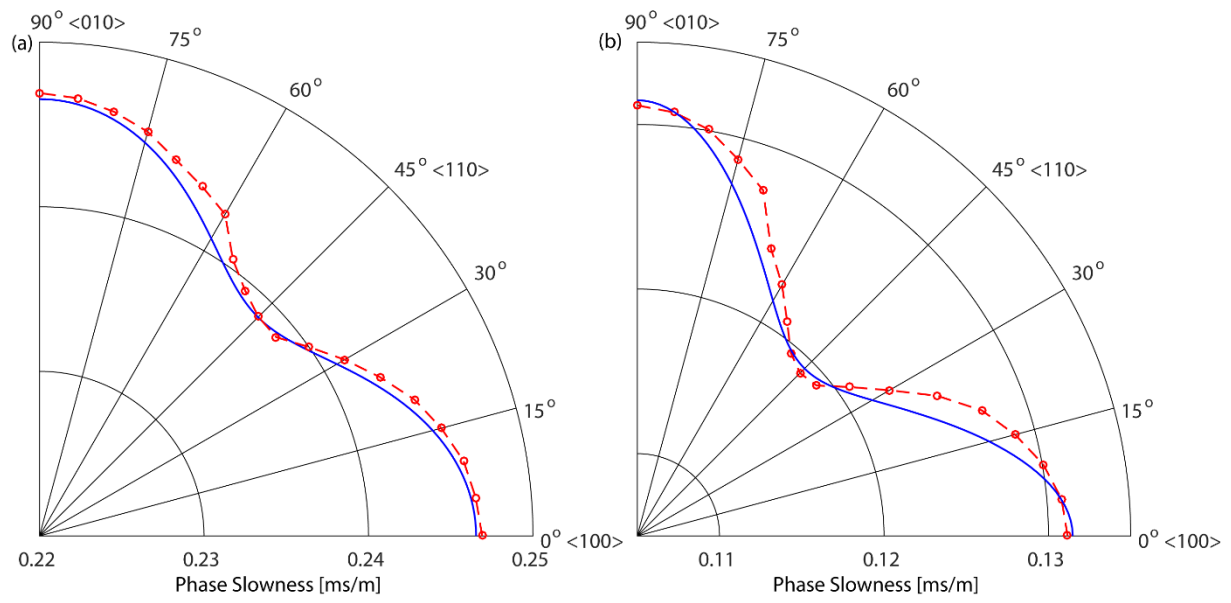
181

182 Fig. 2 (color online): Schematic representation of the experimental setup showing (100) silicon  
183 wafer (diameter 100 mm, thickness 380  $\mu\text{m}$ ), angle beam wedge transducer, laser measurement  
184 area for beam skew (40 mm x 20 mm) and line for phase velocity (10 mm).

185 **IV. Phase slowness**

186 The theoretical phase slowness curves were predicted using Disperse<sup>27</sup> for the nominal  
187 orthotropic material properties of silicon and compared to the measured values (Fig. 3). For the  
188  $A_0$  mode a 3% variation with propagation angle was predicted. The measured values match the  
189 theoretical values well and show the expected symmetry to the  $45^\circ$  orientation. For the  $S_0$  mode  
190 the variation of the phase slowness with orientation is predicted theoretically to be about 10%  
191 and matched from the measurement results in the  $0^\circ$ ,  $45^\circ$  and  $90^\circ$  directions. Therefore, the  
192 silicon wafer material properties and the relative change in stiffness with crystallographic  
193 orientation lie within the range of 1-3% of the nominal properties stated in literature<sup>26</sup>.  
194 However, a small but systematic offset in the non-principal directions was found, with higher  
195 experimental slowness (lower phase velocity) values than theoretically predicted. For these  
196 directions, a significant wave skew of the  $S_0$  mode was observed, leading to low amplitude in  
197 the far field of the transducer along the center line where phase velocity was measured. It should  
198 be noted that the experimental setup for the  $S_0$  mode excitation, with the transducer wedge  
199 width approximately 3 times the wavelength, does not exactly match the theoretical  
200 assumptions for plane wave generation, implications of which will be discussed further.

201



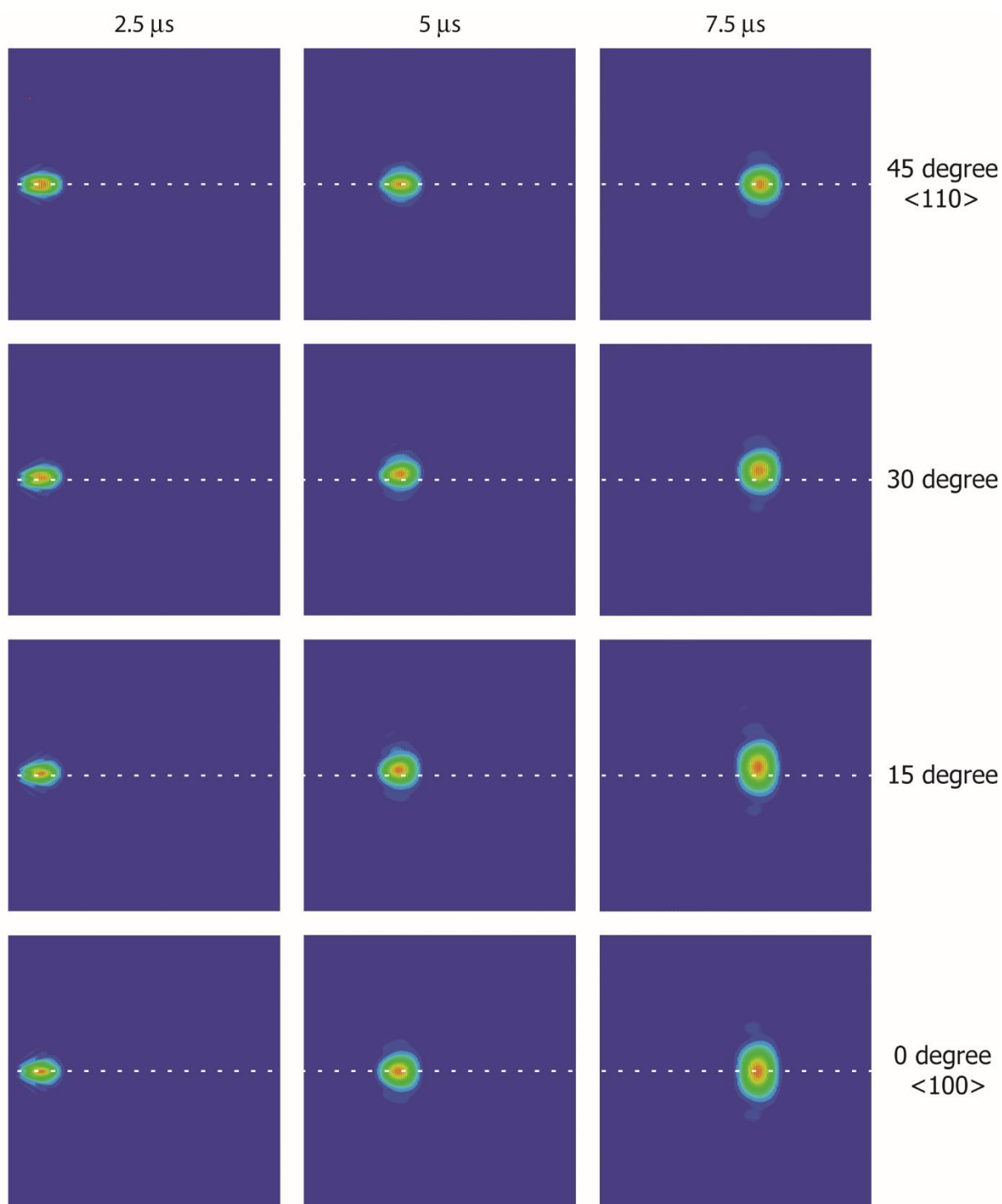
202

203 Fig. 3 (color online): Comparison of phase slowness curves at 5 MHz in 380  $\mu\text{m}$  thick  
 204 monocrystalline silicon wafer; theory (solid) and measurement (dashed, circles): (a)  $A_0$  mode;  
 205 (b)  $S_0$  mode.

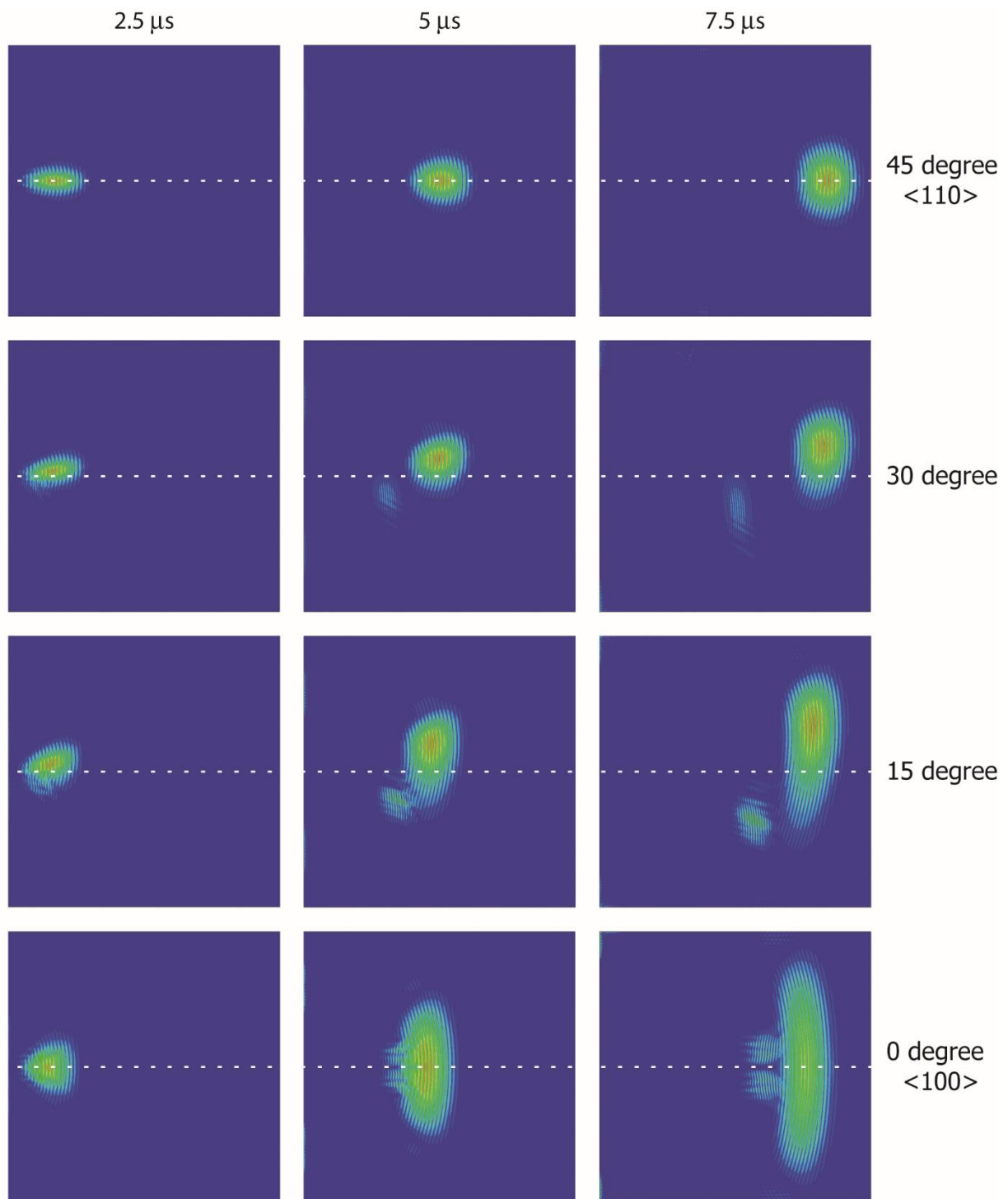
## 206 **V. Finite Element Simulations**

207 The propagation of the guided wave modes in the silicon wafers was simulated using three-  
208 dimensional (3D) Finite Element (FE) models to visualize and better understand the beam skew  
209 due to the material anisotropy. The model of a 380  $\mu\text{m}$  thick silicon plate (size: 60 mm x 60  
210 mm) was implemented with approximately 11.5 million linear brick elements ( $\Delta x = \Delta y = 50$   
211  $\mu\text{m}$ ,  $\Delta z = 47.5 \mu\text{m}$ ) in ABAQUS Explicit, similar to the model described in <sup>28</sup>. Explicit time  
212 integration was used, and the element size and time step were chosen to adhere to the usual  
213 stability criteria<sup>29</sup>. The element size was chosen small enough compared to the wavelength,  
214 using at least 16 elements per wavelength. The orthotropic material properties were specified  
215 using the nominal material properties from literature stated above<sup>26</sup>. For simulations of  
216 different excitation orientations relative to the principal axis, the material properties were  
217 specified with a rotated axis system, keeping the geometry the same. Line excitation of  
218 respectively the  $A_0$  and  $S_0$  Lamb wave modes was introduced at one edge of the plate.  
219 Approximating the effective transducer width used for the experiments, all nodes over a width  
220 of 5 mm at the center of the plate thickness were excited either using in-plane ( $S_0$  mode) or  
221 out-of-plane ( $A_0$  mode) force. The amplitude over the 5 mm excitation width was set to vary  
222 from 0.5 at the sides to 1 at the center using a Hanning distribution. The excitation pulse was  
223 set as a 12 cycle toneburst with a center frequency  $f$  of 5 MHz to match the experiments. The  
224 amplitude for both modes was monitored as the out-of-plane displacement at the top surface  
225 nodes on points in a rectangular area of 40 mm (width step: 1 mm) by 50 mm (length step: 2.5  
226 mm) in front of the excitation to capture the propagation and skew of the guided wave mode  
227 pulses. The time trace at each monitoring node was time gated to remove reflections from the  
228 plate edges and other modes. Fast Fourier Transform (FFT) was used to extract the amplitude  
229 at the center frequency of 5 MHz for each monitoring node.

230 **VI. Wave beam skew visualization**



231  
 232 Fig. 4 (color online): Time snapshots of  $A_0$  mode wave propagation, FE simulation at 3 times  
 233 (2.5, 5.0, 7.5  $\mu\text{s}$ ) for different crystallographic orientations relative to line excitation at edge of  
 234 plate; 60 mm by 60 mm area shown; dashed line: excitation center line.



235

236 Fig. 5 (color online): Time snapshots of  $S_0$  mode wave propagation, FE simulation at 3 times

237 (2.5, 5.0, 7.5  $\mu\text{s}$ ) for different crystallographic orientations relative to line excitation at edge of

238 plate; 60 mm by 60 mm area shown; dashed line: excitation center line.

239

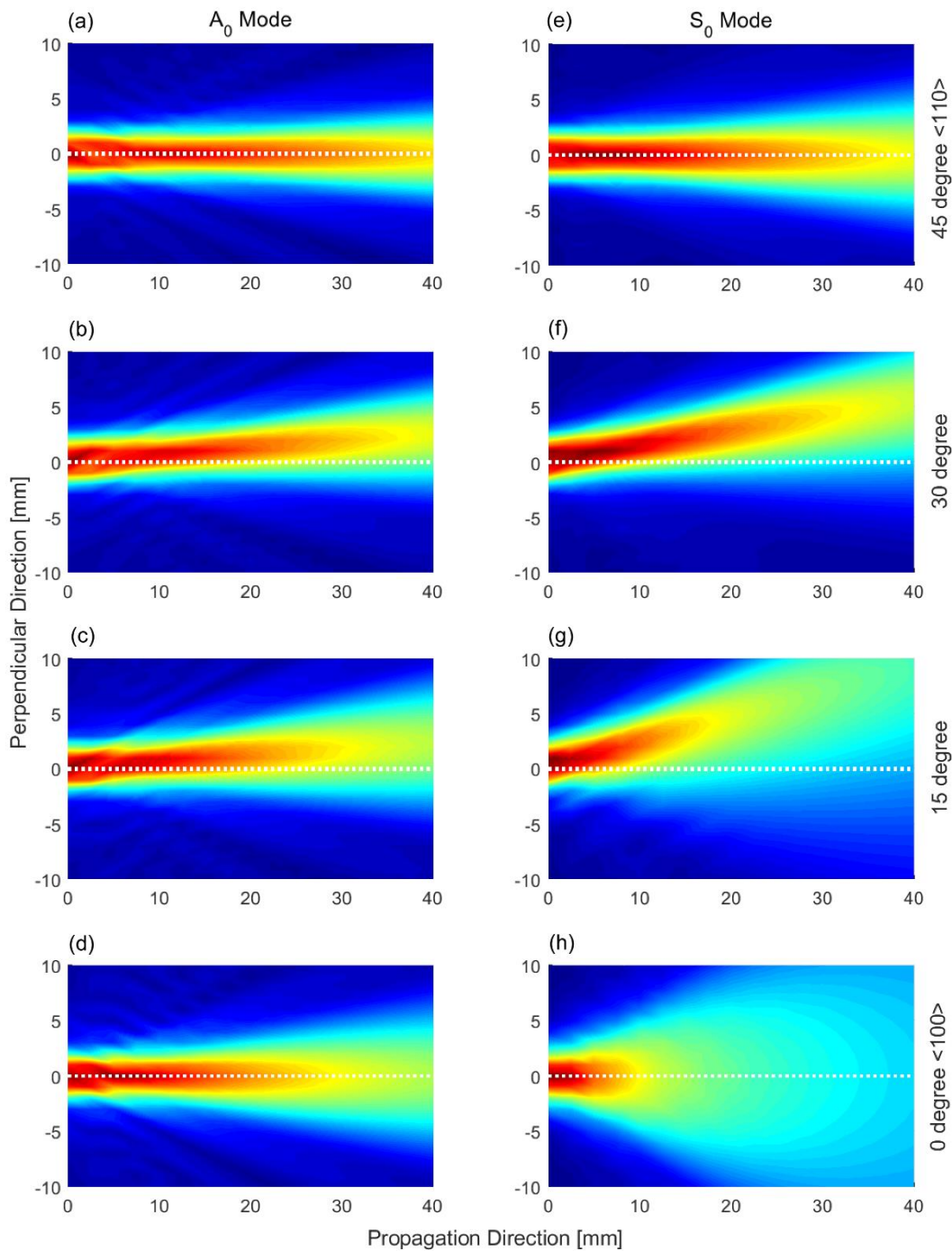
240 Figure 4 shows time snapshots of the simulated  $A_0$  mode propagation for different  
241 crystallographic orientations relative to the line excitation at the left edge of the plate. As the 5  
242 mm line excitation is relatively long compared to the wavelength of the  $A_0$  mode  
243 (approximately 0.8 mm), a rather narrow wave pulse is excited. For the two principal  
244 crystallographic directions in the  $\langle 110 \rangle$  ( $45^\circ$ ) and  $\langle 100 \rangle$  ( $0^\circ$ ) crystal orientations, the wave  
245 front remains perpendicular to the excitation line (wave propagation along dashed excitation  
246 center line). For the  $30^\circ$  and  $15^\circ$  directions, the excitation center line is not aligned with the  
247 material symmetry axes and a slight wave skew can be observed relative to the dashed line. The  
248 effect is small for the  $A_0$  mode, as the variation in the theoretical phase slowness is only about  
249 3%, giving a maximum theoretically predicted wave skew angle of  $3.6^\circ$ . Comparing the time  
250 snapshots at  $7.5 \mu\text{s}$ , it can be observed that the pulse spreads more in the width direction as the  
251 crystallographic orientation moves away from the  $\langle 110 \rangle$  direction with the highest stiffness.

252 Figure 5 shows the time snapshots of the simulations for the  $S_0$  mode at the same times as for  
253 the  $A_0$  mode in Fig. 3. As the group velocity of the  $S_0$  mode is higher than for the  $A_0$  mode, the  
254 pulses have propagated farther. A slight excitation of the SH mode and an edge guided wave  
255 were observed for the non-principal orientations due to the mode coupling, and can be seen as  
256 the secondary, slower wave pulse for the  $30^\circ$  and  $15^\circ$  direction cases. For the  $\langle 110 \rangle$  crystal  
257 direction, the wave pulse propagates along the excitation center line with only limited pulse  
258 spreading. For the  $S_0$  mode, the excitation line length is about 3 times the wavelength (compared  
259 to 6 times  $\lambda$  for the  $A_0$  mode), so increased pulse spreading is to be expected. When the  
260 excitation direction is not aligned with one of the principal axes of the crystal ( $30^\circ$  and  $15^\circ$   
261 orientations), a significant wave skew can be observed, as the phase slowness varies by about  
262 10% for the  $S_0$  mode. Moving away from the  $\langle 110 \rangle$  direction with the highest stiffness, it can  
263 be observed that the pulse spreads perpendicular to the wave propagation direction. This effect  
264 can be seen to be the strongest in the  $\langle 100 \rangle$  orientation, which is a principal axis of the

265 anisotropy, but has the lowest stiffness and thus phase velocity. For this direction, no wave  
266 skew, but a slower group velocity and significantly wider pulse spreading than for the  $\langle 110 \rangle$   
267 orientation are observed. The significant widening of the wave pulse has not been previously  
268 reported in literature. The changes of up to 10% in the phase slowness and thus wavelength  
269 relative to the excitation aperture cannot explain this significant change in the beam spread  
270 angle, which can thus be attributed to anisotropic wave propagation effects further described  
271 below, with the largest effect occurring along a principal axis with a minimum of the guided  
272 wave velocity ( $\langle 100 \rangle$ ).

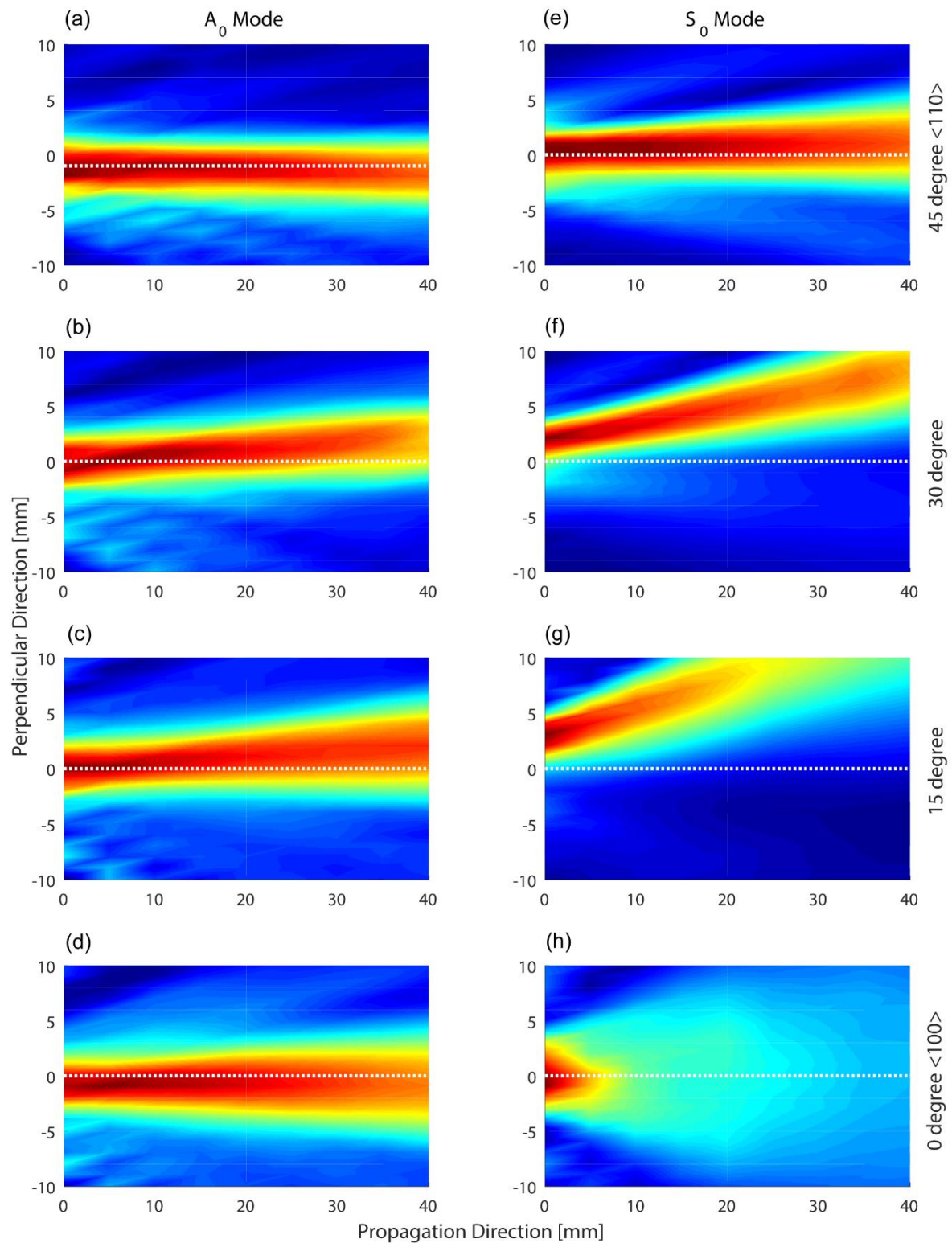
273 As described above, the displacement on a rectangular grid was recorded and the amplitude at  
274 each point of the grid evaluated to quantify the excited wave beam. These are shown in Fig. 6  
275 for the FE simulations of the  $A_0$  and  $S_0$  modes. For the  $A_0$  mode a strong beam can be observed  
276 for all crystallographic orientations, with a small wave skew for the  $15^\circ$  and  $30^\circ$  directions and  
277 a slightly larger beam widening for the  $0^\circ$  direction. The wave skew for the  $S_0$  mode for the  $15^\circ$   
278 and  $30^\circ$  crystallographic orientations is significantly higher. The resulting amplitude field is  
279 characterized by an asymmetry of the wave beam with different amplitude drop gradients in the  
280 positive and negative perpendicular direction, as can be seen for example in Fig. 6g. The beam  
281 widening for the  $S_0$  mode can be clearly seen, with a significantly larger amplitude decrease for  
282 the  $\langle 100 \rangle$  orientation than for the  $\langle 110 \rangle$  direction (compare Fig. 6h and Fig. 6e). The observed  
283 wave skew and beam widening effects can be related to the energy focusing effect discussed in  
284 <sup>12</sup> and <sup>14</sup> for composite plates. Based on the shape of the slowness curve for the  $S_0$  mode, the  
285 energy is expected to be essentially focused in the  $\langle 110 \rangle$  direction. The FE results can be  
286 compared to the experimental results shown in Fig. 7. Overall a good match can be seen with  
287 the same overall effects. For the measurement results, the wave fields for the  $\langle 110 \rangle$  and  $\langle 100 \rangle$   
288 directions are not perfectly symmetric, due to uncertainties for the angle of the experimental  
289 setup.





290

291 Fig. 6 (color online): FE simulation of beam skewing; evaluation of amplitude (FFT) at each  
 292 monitoring location for  $A_0$  and  $S_0$  mode, different crystallographic orientations relative to line  
 293 excitation.



294

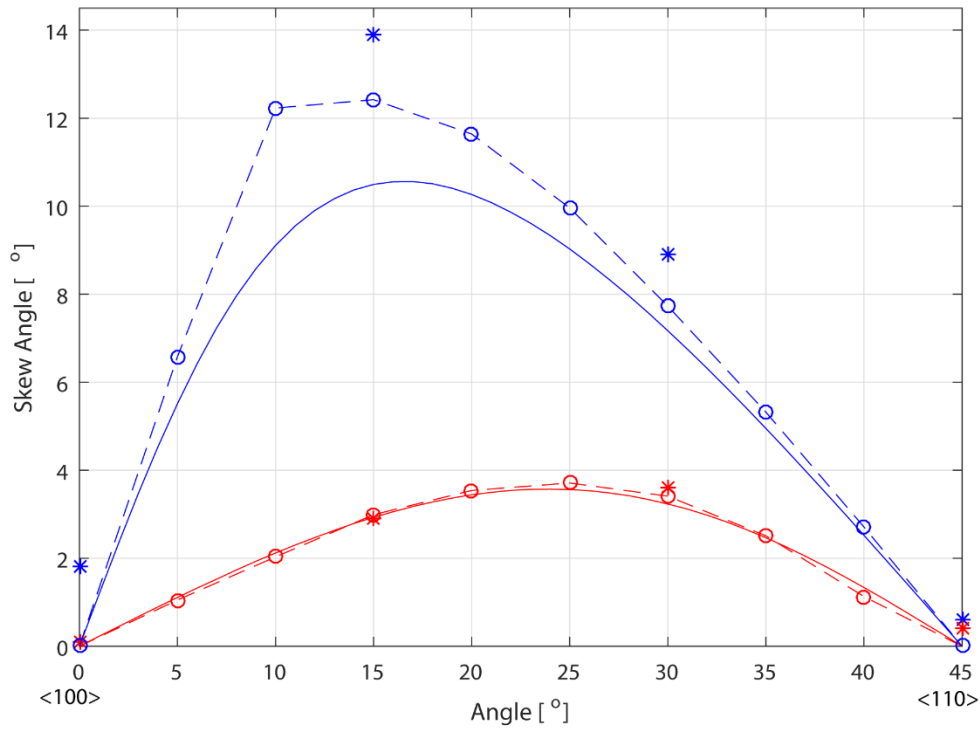
295 Fig. 7 (color online): Experimental measurement of beam skewing; evaluation of amplitude  
 296 (FFT) at each monitoring location for  $A_0$  and  $S_0$  mode, different crystallographic orientations  
 297 relative to line excitation.

298 **VII. Wave beam skew evaluation**

299 The magnitude and angular dependency of the wave beam skew for the two fundamental Lamb  
300 modes, predicted from theoretical calculations, are shown in Fig. 8 (solid line) and Table 1. The  
301 maximum skew angle is calculated as  $10.6^\circ$  for the  $S_0$  mode at an orientation angle of  $17^\circ$ , while  
302 for the  $A_0$  mode the maximum theoretical skew angle of  $3.6^\circ$  occurs at  $24^\circ$  orientation. The  
303 significantly larger wave skew for the  $S_0$  mode is in line with the larger variations of the phase  
304 slowness curves (Fig. 3). For the FE simulations and the experiments, the wave skew was  
305 evaluated for both Lamb wave modes and all relative crystal orientations (every  $5^\circ$  for FE  
306 simulations, every  $15^\circ$  for experiments). The wave skew angle was evaluated by computing the  
307 angle between the excitation center line and the trajectory of the acoustic ray with maximum  
308 field amplitude<sup>15</sup>. This approach assumes that the wave vector direction in the main acoustic  
309 beam is defined by the wedge orientation and is therefore parallel to the excitation center line.  
310 Depending on the excitation width, the acoustic ray with maximum amplitude can have a small  
311 offset from the acoustic ray with zero phase gradient (relative to the excitation center line),  
312 leading to an overestimation of the wave skew angle. For each measurement line perpendicular  
313 to the center line, the amplitude was interpolated to a step size of 0.1 mm using a low pass filter  
314 in Matlab to accurately evaluate small angles, especially for the  $A_0$  wave mode skew. The  
315 location of the amplitude maxima was extracted and a straight line fitted to obtain an  
316 approximation of the wave skew angle, discarding the first 5 mm in front of the excitation due  
317 to near field effects. Figure 8 shows the obtained wave skew angles for different  
318 crystallographic directions.

319 The experimental and FE simulation results for the  $A_0$  mode match very well with the  
320 theoretical predictions (Fig. 8, Table 1). For the  $A_0$  mode results, the skew angle is rather small  
321 and a strong beam was observed for all directions in Figs. 6 and 7. No wave skew was observed  
322 in the FE simulations for the principal  $0^\circ$  and  $45^\circ$  directions as the wave propagation is perfectly

323 symmetric. For the experimental results, slight asymmetries and beam skew were seen in the  
324 principal directions (Fig. 7a/d), and this was found to be up to  $0.4^\circ$  for the  $A_0$  mode (below  
325 experimental uncertainty of  $1^\circ$ ). For the  $S_0$  mode, both the experimental and FE simulation  
326 evaluation show a similar dependency of the skew angle as predicted theoretically, but with a  
327 consistent overestimation (Fig. 8, Table 1). No wave skew was observed in the FE simulations  
328 for the principal  $0^\circ$  and  $45^\circ$  directions, with experimental results showing a maximum wave  
329 skew angle of  $1.8^\circ$ . The evaluation of both experiments and FE simulations gave higher values  
330 than predicted for the non-principal directions, but the angular dependency matches well. A  
331 preliminary evaluation of the phase gradients obtained from additional FE simulations with a  
332 finer grid (not shown) showed small angular offsets in the main acoustic beam between the  
333 excitation center line and the wave vector (assumed to be parallel to the center line), leading to  
334 an overestimation of the skew angle. The phase gradient analysis also showed that, in the far-  
335 field, the wave propagation characteristics of the acoustic beam are closer to a point-like source  
336 than to plane wave propagation with parallel rays, which is in line with the short line source  
337 employed here. These elements would help to explain the observed systematic offset compared  
338 to theory. For the experiments, the spacing of the measurement grid and noise precluded such  
339 an evaluation, as the correction for  $2\pi$  jumps in the wave phase field was problematic.  
340 Therefore, a decision was taken to use the same evaluation criterion for both the experiments  
341 and FE simulations to allow comparison.



342

343 Figure 8 (color online): Comparison of wave skew angle  $A_0$  (red) and  $S_0$  (blue) modes: theory

344 (solid, every  $1^\circ$ ), FEA (circles, dashed, every  $5^\circ$ ), experiment (stars, every  $15^\circ$ ).

345 Table 1: Comparison wave skew angle  $A_0$  and  $S_0$  modes theory, FE, experiment (experimental

346 uncertainty order of  $1^\circ$ ).

Mode	Angle	Theory	FE	Experiment
$S_0$	$0^\circ$	$0^\circ$	$0^\circ$	$1.8^\circ$
	$15^\circ$	$10.5^\circ$	$12.4^\circ$	$13.9^\circ$
	$30^\circ$	$7.2^\circ$	$7.7^\circ$	$8.9^\circ$
	$45^\circ$	$0^\circ$	$0^\circ$	$0.6^\circ$
$A_0$	$0^\circ$	$0^\circ$	$0^\circ$	$0.1^\circ$
	$15^\circ$	$2.9^\circ$	$3.0^\circ$	$2.9^\circ$
	$30^\circ$	$3.2^\circ$	$3.4^\circ$	$3.6^\circ$
	$45^\circ$	$0^\circ$	$0^\circ$	$0.4^\circ$

347

348 **VIII. Conclusions**

349 The propagation of the fundamental Lamb wave modes in monocrystalline silicon wafers (380  
350  $\mu\text{m}$  thickness) was investigated. The anisotropic material properties lead to a variation of the  
351 phase velocity depending on the propagation direction relative to the crystallographic  
352 orientation. The phase velocity was measured experimentally using a customized angle beam  
353 transducer for the selective excitation of the guided wave modes, measuring the out-of-plane  
354 displacement with a non-contact laser vibrometer. Good agreement of the measured phase  
355 slowness curves with theoretical predictions was found. The larger variation of the phase  
356 slowness for the fundamental symmetric  $S_0$  mode leads to a significant wave skew angle for  
357 propagation in non-principal directions of the silicon crystal. The effect was observed  
358 experimentally and verified from FE simulations. Both the experimental and FE simulation  
359 evaluation show a similar angular dependency of the skew angle as predicted theoretically, but  
360 with a consistent overestimation. This discrepancy may result from the evaluation procedure of  
361 the skew angle, based on the acoustic ray with maximum amplitude, and from the short line  
362 sources used for the  $S_0$  mode excitation in the simulations and the measurements. In directions  
363 with lower stiffness, a significant beam widening was observed, that has not been reported  
364 previously in literature. For the  $A_0$  mode the wave skew effect was confirmed and good  
365 agreement between experimental measurements, FE simulations, and theoretical predictions  
366 was obtained. The wave skew and beam widening can lead to significant amplitude drop in the  
367 normal excitation direction and need to be considered for potential non-destructive testing  
368 applications for silicon wafers.

369

370 **Acknowledgements**

371 The authors would like to thank Prof. N. Saffari for the generous provision of experimental  
372 equipment.

373 **References**

- 374 <sup>1</sup> A. Luque and S. Hegedus, *Handbook of Photovoltaic Science and Engineering*, (Wiley,  
375 West Sussex, 2011), pp. 239-240.
- 376 <sup>2</sup> A. Belyaev, O. Polupan, S. Ostapenko, D.P. Hess, and J. P. Kalejs, “Resonance ultrasonic  
377 vibration diagnostics of elastic stress in full-size silicon wafers,” *Semicond. Sci. Technol.*  
378 **21**, 254–260 (2006).
- 379 <sup>3</sup> A. Belyaev, O. Polupan, W. Dallas, S. Ostapenko, D. Hess, and J. Wohlgemuth, “Crack  
380 detection and analyses using resonance ultrasonic vibrations in full-size crystalline silicon  
381 wafers,” *Appl. Phys. Lett.* **88**, 111907–111909 (2006).
- 382 <sup>4</sup> M.J. Padiyar, S.K. Chakrapani, C. V. Krishnamurthy, and K. Balasubramaniam, “Crack  
383 detection in polycrystalline silicon wafers using air-coupled ultrasonic guided waves,” in  
384 *Proceedings of the National Seminar & Exhibition on Non-Destructive Evaluation* (NDE  
385 2009, Dec. 10–12, 2009), pp. 341–345.
- 386 <sup>5</sup> S.K. Chakrapani, M.J. Padiyar, and K. Balasubramaniam, “Crack detection in full size Cz-  
387 silicon wafers using Lamb wave air coupled ultrasonic testing (LAC-UT),” *J. Nondestr.*  
388 *Eval.* **31**, 46–55 (2012).
- 389 <sup>6</sup> M.-K. Song, K.-Y. Jhang, “Crack Detection in Single-Crystalline Silicon Wafer Using  
390 Laser Generated Lamb Wave,” *Adv. Mater. Sci. Eng.* **2013**, 950791 (2013).
- 391 <sup>7</sup> D. E. Chimenti, “Guided waves in plates and their use in materials characterization,” *Appl.*  
392 *Mech. Rev.* **50**, 247-284 (1997).
- 393 <sup>8</sup> P. Cawley, M.J.S. Lowe, D.N. Alleyne, B. Pavlakovic, and P.D. Wilcox, “Practical long  
394 range guided wave testing: applications to pipes and rail,” *Mater. Eval.* **61**, 66-74 (2003).
- 395 <sup>9</sup> J.L. Rose, “Standing on the shoulders of giants: An example of guided wave inspection,”  
396 *Mater. Eval.* **60**, 53-59 (2002).

- 397 <sup>10</sup> B. Masserey and P. Fromme, “On the reflection of coupled Rayleigh-like waves at surface  
398 defects in plates,” *J. Acoust. Soc. Am.* **123**, 88-98 (2008).
- 399 <sup>11</sup> B. Masserey and P. Fromme, “In-situ monitoring of fatigue crack growth using high  
400 frequency guided waves,” *NDT&E Int.* **71**, 1-7 (2015).
- 401 <sup>12</sup> B. Chapuis, N. Terrien, D. Royer, “Excitation and focusing of Lamb waves in a  
402 multilayered anisotropic plate,” *J. Acoust. Soc. Am.* **127**, 198-203 (2010).
- 403 <sup>13</sup> A. Karmazin, E. Kirillova, W. Seemann and P. Syromyatnikov, “A study of time harmonic  
404 guided Lamb waves and their caustics in composite plates,” *Ultrasonics* **53**, 283-293  
405 (2013).
- 406 <sup>14</sup> A. Leleux, P. Micheau and M. Castaings, “Long Range Detection of Defects in Composite  
407 Plates Using Lamb Waves Generated and Detected by Ultrasonic Phased Array Probes,” *J.*  
408 *Nondestruct. Eval.* **32**, 200-214 (2013).
- 409 <sup>15</sup> C. Potel, S. Baly, J.-F. de Belleval, M. Lowe and P. Gagnol, “Deviation of a  
410 monochromatic Lamb wave beam in anisotropic multilayered media: asymptotic analysis,  
411 numerical and experimental results,” *IEEE Trans. Ultrason. Ferroel. Freq. Contr.* **52**, 987-  
412 1001 (2005).
- 413 <sup>16</sup> B.A. Auld, *Acoustic fields and waves in solids* (Wiley, New York, 1973), Vol. **1**, pp. 108-  
414 123.
- 415 <sup>17</sup> J.L. Rose, *Ultrasonic waves in solid media* (Cambridge University Press, Cambridge,  
416 1999), pp. 33-37.
- 417 <sup>18</sup> K.Y. Kim, K.C. Bretz, A.G. Every, W. Sachse, “Ultrasonic imaging of the group velocity  
418 surface about the cubic axis in silicon,” *J. Appl. Phys.* **79**, 1857-1863 (1996).
- 419 <sup>19</sup> B. Audoin, C. Bescond, M. Dechamps, “Measurement of stiffness coefficients of  
420 anisotropic materials from pointlike generation and detection of acoustic waves,” *J. Appl.*  
421 *Phys.* **80**, 3760-3771 (1996).



- 422 <sup>20</sup> A.D. Degtyar, S.I. Rokhlin, "Comparison of elastic constant determination in anisotropic  
423 materials from ultrasonic group and phase velocity data," *J. Acoust. Soc. Am.* **102**, 3458-  
424 3466 (1997).
- 425 <sup>21</sup> F. Reverdy, B. Audoin, "Elastic constants determination of anisotropic materials from  
426 phase velocities of acoustic waves generated and detected by lasers," *J. Acoust. Soc. Am.*  
427 **109**, 1965-1972 (2001).
- 428 <sup>22</sup> J. Maris, "Enhancement of Heat Pulses in Crystals due to Elastic Anisotropy," *J. Acoust.*  
429 *Soc. Am.* **50**, 812-818 (1971).
- 430 <sup>23</sup> M. Veidt, W. Sachse, "Ultrasonic point-source/point-receiver measurements in thin  
431 specimens," *J. Acoust. Soc. Am.* **96**, 2318-2326 (1994).
- 432 <sup>24</sup> C. Prada, D. Clorenec, T.W. Murray, D. Royer, "Influence of the anisotropy on zero-  
433 group velocity Lamb modes," *J. Acoust. Soc. Am.* **126**, 620-625 (2009).
- 434 <sup>25</sup> M. Youngjae, Y. Gyeongwon, K. Kyung-Min, R. Yuji, K. Young, "Comparison of  
435 slowness profiles of Lamb waves with elastic moduli and crystal structure in single  
436 crystalline silicon wafers," *J. Korean Soc. Nondestruct. Test.* **36** (2016).
- 437 <sup>26</sup> M.A. Hopcroft, W.D. Nix and T.W. Kenny, "What is the Young's Modulus of Silicon?," *J.*  
438 *Microelectromech. Syst.* **19**, 229-238 (2010).
- 439 <sup>27</sup> B. Pavlakovic, M. Lowe, D. Alleyne, and P. Cawley, "DISPERSE: A general purpose  
440 program for creating dispersion curves," in *Review of Progress in Quantitative*  
441 *Nondestructive Evaluation*, edited by D.O. Thompson and D.E. Chimenti (Plenum, New  
442 York, 1997), Vol. **16**, pp. 185-192.
- 443 <sup>28</sup> M. Pizzolato, B. Masserey, J.-L. Robyr and P. Fromme, "High frequency guided wave  
444 propagation in monocrystalline silicon wafers," in *Proceedings of SPIE*, edited by T.  
445 Kundu (SPIE, Washington, 2017), Vol. **10170**, 101702B.

- 446 <sup>29</sup> J. Virieux, “P-SV wave propagation in heterogeneous media: Velocity-stress finite-  
447 difference method,” *Geophysics* **51**, 889–901 (1986).