Art. # 1083, 10 pages, doi: 10.15700/saje.v35n3a1083

Can improving teachers' knowledge of mathematics lead to gains in learners' attainment in Mathematics?

Craig Pournara
Wits School of Education, Faculty of Humanities, University of the Witwatersrand, South Africa
craig.pournara@wits.ac.za
Jeremy Hodgen
School of Education, University of Nottingham, United Kingdom
Jill Adler and Vasen Pillay
Wits School of Education, Faculty of Humanities, University of the Witwatersrand, South Africa

It is wellknown that the majority of South African learners achieve extremely poorly in Mathematics. Many claim that one of the causes of this poor attainment is teachers' weak knowledge of mathematics, and propose that improving teachers' mathematical knowledge would improve learner attainment. However, the evidence-base for this proposed solution is currently relatively weak. We report on a quasi-experimental study examining the learning gains of Grade 10 learners from five secondary schools in the Johannesburg area, whose teachers participated in a year-long professional development course aimed at improving the teachers' knowledge of mathematics for teaching. Statistical analyses of pre- and post-test results show that the intervention group of learners (N = 586) taught by teachers who had participated in the professional development (N = 14) outperformed a matched control group of learners (N = 217) taught by teachers in the same schools (N = 7). An effect size of d = 0.17 for the intervention group is equivalent to two months' additional progress. While the learning gains are small, they are statistically significant. These findings provide empirical support for claims that attending to teachers' mathematical knowledge can impact learners' attainment. Suggestions are made regarding the form and substance of such professional development.

Keywords: learning gains; mathematics teacher knowledge; professional development

Introduction

Across the world there are attempts to improve teachers' mathematical knowledge in order to raise learner attainment. In South Africa, despite many years of mathematics professional development programmes aimed at redressing the devastating effects of apartheid schooling and apartheid teacher education, there is little evidence to show we have made much progress at the level of the learner. Claims of lack of impact are typically based on results of summative national and international assessments, such as the Trends in International Mathematics and Science Study (TIMSS), Programme for International Student Assessment (PISA), Southern Africa Consortium for Monitoring Education Quality (SACMEQ), the Annual National Assessments (ANA), and the National Senior Certificate (NSC) exams. This lack of impact is often attributed, at least in part, to teachers' poor mathematical knowledge (Carnoy, Chisholm, Addy, Arends, Baloyi, Irving, Raab, Reeves, Sapire & Sorto, 2011; Taylor, N & Taylor, S 2012) and there are instances where it has been shown that teachers don't know well enough the mathematics their learners need to learn (Bansilal, Brijlall & Mkhwanazi, 2014; Carnoy et al., 2011; Taylor, N & Taylor, S 2012).

A central goal of the Wits Maths Connect Secondary Project[i] (WMCS) is to develop models of professional development for secondary mathematics teachers that strengthen teachers' relationship to mathematics, and that ultimately lead to learning gains at all levels of secondary schooling. This requires deliberate attention to conceptualising, designing and implementing a professional development programme and then researching its impact on learner attainment. The process of researching a development initiative such as this is dependent on a carefully-conceptualised research design and the collection of robust evidence. In this paper we report on the initial stages of such a programme of research and development, and the evidence collected thus far for the impact of the professional development courses on learning gains.

A key decision in the research design is to reconsider how the impact of teachers' knowledge on learner attainment is measured. Learner results on national assessments are typically reported in terms of pass rates (*exempli gratia* (e.g.) NSC results) or in terms of average marks (e.g. ANAs). Yet in a context where the majority of learners are mathematically under-prepared for their current grade, pass rates and average marks are not appropriate measures by means of which to investigate change. We propose that, given the current education context in the country, learning gains is a more robust measure of change in learner performance, particularly when seeking to investigate links between teacher knowledge and learner attainment. In this study, we refer to 'learning gains' as changes in learners' scores in a pre-test/post-test design over one academic year. In this way, we are able, to some extent, to attribute learning gains to the teaching that learners receive in that year.

There is little evidence that mathematics professional development programmes in South Africa are having an impact on learner attainment. Furthermore, there are not yet adequate frameworks for evaluating the impact of professional development in the country. We investigated whether the professional development courses offered by WMCS constituted an intervention worth pursuing. We thus sought indicative rather than conclusive results to make a case for the continuation (or alternatively the termination) of the professional development courses.

The results reported here show that learners taught by teachers who participated in the professional development programme outperformed learners taught by teachers who did not participate in the programme. These indicative results suggest that the courses offered by WMCS are worth pursuing further, but that a more rigorous investigation into their impact on learner attainment is an essential future step. In addition, a secondary objective was to show that more rigorous evaluations of educational interventions using quasi-experimental designs are possible in the context of South African schools.

Teacher Knowledge and Learner Attainment

Following Shulman and colleagues' initial conceptualisation of teacher knowledge (Shulman, 1986, 1987), a great deal of work has been done on teachers' mathematical knowledge for teaching. There is widespread agreement that the knowledge teachers require for teaching Mathematics is more than sound content knowledge of mathematics itself. While some (e.g. Krauss, Baumert & Blum, 2008) refer to this additional knowledge as pedagogical content knowledge (PCK) following Shulman, others (e.g. Ball, Thames & Phelps, 2008) have attempted to disaggregate both content knowledge (CK), or subject matter knowledge (SMK), and PCK further. However, there is some lack of clarity about the boundaries between CK/SMK and PCK. So, while we find the two terms useful for emphasising different aspects of teacher knowledge, we argue that they are problematic when used as analytical constructs. We believe it is more productive to consider an amalgam of mathematical and teaching knowledge, and so we use the term "mathematics-for-teaching" (MfT) (Adler, 2005; Adler & Davis, 2006) to encompass both subject content knowledge and mathematics-specific pedagogical knowledge.

Until the mid-eighties, teachers' subject knowledge was only measured indirectly, and through proxy measures, such as state certification, number of Mathematics or Mathematics Education courses taken, and years of teaching Mathematics (Even, 1993). While it has been argued that such proxy measures are neither good measures of teachers' knowledge (Ball, Bass & Hill, 2004), nor good predictors of learner attainment (e.g. Hill, Ball & Schilling, 2008), there is some evidence of their predictive power in secondary Mathematics. For example, Darling-Hammond (2000) and Goldhaber and Brewer (2000) found a positive relationship between state certification and learning gains in the United States (US). Monk (1994) found a positive relationship between number of Mathematics courses taken and student achievement, although the effects were very small.

We suggest these proxy measures may hold some applicability for contexts such as South Africa, where teachers' mathematical knowledge bases are generally poor, and where, based on anecdotal evidence from our project schools, too many teachers who are teaching Mathematics at lower secondary level, have little, if any, training as Mathematics teachers. In such cases the number of post-school Mathematics courses taken does matter, and may be a predictor, albeit a poor one, of learner attainment. So, while Hill, Rowan and Ball (2005) argue that some research findings on learning gains from the Global South (e.g. Harbison & Hanushek, 1992, Mullens, Murnane & Willett, 1996) may not generalise to the US, we likewise suggest that the dismissive stance of some in the Global North to proxy measures may be inappropriate in the Global South. That said, we agree that proxy measures alone are insufficient as measures of teachers' mathematical knowledge.

Attempts to make use of more direct measures of teacher knowledge have taken different forms. Some have tested teachers on the same/similar level content as their learners. For example, Harbison and Hanushek (1992) administered the same test to Grade Four learners and their teachers in rural Brazil, and found that teachers' scores were a strong predictor of learners' scores. Working in Belize, Mullens et al. (1996) found that teachers' scores on the national primary school-leaving examination for Mathematics were a good predictor of the Mathematics scores of their Grade Three learners. In South Africa, the SACMEQ III study was extended to include testing of Grade Six teachers' mathematical knowledge on items typical of Grade Six level (and lower), where 15 items were common to both the teacher and learner tests. Taylor, N and Taylor, S (2012) report that teachers and learners performed well on only two simple items but that both teachers and learners performed poorly on eight items. This suggests that Grade Six teachers do not know much of the mathematics they teach well enough to teach it.

In the US and Germany, large research projects have developed sophisticated measures that attempt to disaggregate different components of teacher knowledge (Hill et al., 2008, Krauss et al., 2008). In the Study of Instructional Improvement, Hill et al. (2005) found that the mathematical-knowledge-for-teaching of Grade One and Grade Three teachers was a stronger predictor of learner attainment than were proxy measures, such as number of courses taken in Mathematics or Mathematics Methodology, or years of teaching experience, or average daily length of maths lessons. In Germany, Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand

and Tsai (2010) found that teachers' scores on SMK[ii] and PCK items were strongly linked to the mathematics teacher preparation they had received, with those preparing to teach at higher academic levels in schools outperforming their counterparts on both SMK and PCK. They argue that teachers' PCK is a better predictor of learner progress than SMK. While both studies found associations between teacher knowledge and learner attainment, neither study investigated the way in which interventions on teacher knowledge impact learner attainment.

Teachers' knowledge matters in all learning contexts. However, it matters more in contexts of poverty and low achievement. Nye, Konstantopoulos and Hedges (2004) found variance in learning gains attributable to teaching to be higher in low socio-economic status (SES) schools, and Krauss et al. (2008) have shown that differences in teachers' PCK had a larger impact in low-SES and low-achievement contexts in Germany.

Professional Development and Learner Attainment

The problem of how professional development can be designed to improve learners' attainment is not confined to developing countries like South Africa. A decade ago, a survey of the international literature found the field to be dominated by small-scale qualitative studies (Adler, Ball, Krainer, Lin & Novotna, 2005). More recently, in a review of the literature Gersten, Taylor, Keys, Rolfhus and Newman-Gonchar (2014) identified 643 studies of professional development relating to school Mathematics. Of these, only five met the 'What Works Clearinghouse' evidence standards. Of these five, only two found positive effects on learners' attainment, and only one of these five studies (Sample McMeeking, Orsi & Cobb, 2012) reported the effects of an intervention focused on teachers' mathematical knowledge. Sample McMeeking et al. (2012) report the effects of a study in which middle school teachers[iii] in the US participated in one or two university summer courses in Mathematics, lasting two to three weeks. The courses consisted of 80% mathematics content and 20% mathematics pedagogy. They found a discernible effect size on learner attainment for those teachers who had attended two courses, but not for those who had attended only one course. This effect size is reported by Gersten et al. (2014) as 0.20.[iv]

We move now to describe the content, structure and approaches of the WMCS professional development courses, and thus to describe what MfT was offered to teachers.

The Transition Maths Intervention

Most mathematics professional development programmes in South Africa can be described as taking either a repair approach or a conceptual approach to the mathematics in their offerings. Repair approaches focus on teachers redoing school Mathematics in the same ways as their learners would learn it. Here, teachers rehearse the steps necessary to solve typical tasks from the school curriculum. Conceptual approaches frequently work from the assumption that teachers' mathematical knowledge is procedural, and thus inadequate, and that interventions should provide them with a deep conceptual understanding to complement their procedural knowledge. Both approaches have limitations. A repair approach tends to position teachers as school learners, which stands in stark contrast to generally-held principles of professional development (e.g. Clarke, 1994), and does not go beyond a narrow knowledge of the mathematics of the curriculum to address mathematics for teaching more broadly. Conceptual approaches focus extensively on developing conceptual insight, often through extended problem-solving tasks. While we value conceptual insight and challenging tasks, our concern is that often such programmes adopt an exclusively conceptual approach with little regard for the role of procedures or procedural fluency (Kilpatrick, Swafford & Findell, 2001) in mathematical proficiency, and the place of procedures in typical tasks in secondary school Mathematics. Much of school Mathematics is characterised by applying familiar procedures, and it is thus important to deal with such features of school Mathematics in professional development, and to do so in ways that are principled, and thus constructive for teachers and learners.

The Transition Maths (TM) courses form the backbone of the professional development work of WMCS, and were designed with the assumption that focusing on teachers' MfT leads to better teaching, which ultimately translates into increased learner attainment. We thus assume a direct effect on teacher knowledge, and an indirect and delayed effect on learner attainment. The courses focus on mathematics content (75%) and aspects of mathematics teaching (25%), and thus are structured in a similar ratio to Sample McMeeking et al.'s (2012) programme. Each course comprises eight two-day contact sessions over a year, with independent work between these sessions, which includes tutorials on the mathematics content, and tasks related to teaching.

While the courses have distinct foci, both focus on learning MfT through revisiting known mathematics and learning new mathematics (Pournara, 2013). The goals of revisiting are to deepen teachers' grasp of the content, frequently by exploring extreme cases or by problematising aspects that may be taken for granted rather than redoing to improve procedural fluency. Revisiting builds on, strengthens and extends teachers' existing knowledge of the mathematics at hand. Whilst revisiting tasks are structured around 'known' mathematics, the activity focuses on issues such as making connections between different representations, and between different sections of the curriculum.

In TM1, we revisit curriculum content of Grades Eight to 10, and for these teachers we consider the content of Grades 11 and 12 as 'new maths'. In TM2 we revisit the content of Grades 11 and 12, and then extend this beyond the school curriculum to some aspects of tertiary mathematics. In both courses we treat new content in the school curriculum as new mathematics. We show the distinction between revisiting known mathematics and learning new mathematics through the topic of functions, a key concept in both the school curriculum and advanced mathematics.

In TM1, we begin with a process orientation to function (Sfard, 1991), emphasising the catchphrase "graphs come from points, and points come from the relationship between inputs and outputs". We then extend to a structural view of function (Sfard, 1991), with a focus on transformations of functions. These approaches are reinforced by working with multiple representations of functions. We begin with the familiar (to teachers) linear and basic quadratic function, and extend to other functions in the Further Education and Training (FET) curriculum and beyond. In TM2, we build further on a structural view of function as we extend to more advanced functions, including inverses, and beyond the FET curriculum with work on algebra of functions, and piecewise functions. In both courses we work with the square-root function ($f(x) = a\sqrt{x-b}+c$), which is not included in the school curriculum, but which exemplifies key aspects of function, such as domain, range and inverse in powerful ways.

In both courses, we pay attention to the key procedures that learners are required to learn, such as factorisation, solving equations and proving riders. Our emphasis is on studying the routine as a set of logically derived steps, rather than a cue-based exercise in manipulating symbols.

With regard to aspects of teaching, we structure our intervention around the notion of *mathematical discourse in instruction* (Adler & Ronda, In press; Adler & Venkat, 2014). We operationalise this through a focus on aspects that are typical of teaching, irrespective of pedagogy, *videlicet* (viz.) choosing and using examples; providing explanations and justifications; and learner activity. A key strength of this approach is that it focuses on issues that are sufficiently close to teachers' current practice as to be possible to implement. We work from the assumption that better teaching is characterised by more thoughtful selections of examples and tasks, and by mathematical explanations that focus explicitly on the mathematical object (e.g. concept or procedure) that the teacher intends the learners to learn. This is achieved through attention to the appropriate use of mathematical terminology, by means of a range of relevant representations. We focus on opportunities for learner participation that go beyond single-word responses, completing teachers' sentences, and copying procedures from the chalkboard. We work on these three components with teachers in the TM courses, examining records of practice using these notions.

Methods

We used a quasi-experimental design to assess the effect of the TM intervention on learner attainment. In this section, we describe the sampling, design and content of the test, and the analytic methods that were used.

Sample

We worked in 11 secondary schools in the greater Johannesburg area, with six schools located in townships and four in suburban areas. The townhip schools are no-fee schools, while those in the suburbs may be described as low-fee schools, although they generally struggle to collect these fees. Most of the schools have been classified as "under-performing" or "priority schools" at some stage in the life of the project. Consequently, they have been subjected to increased bureaucratic control from the provincial education department, which includes the requirement that they write externally-set examinations twice a year. Over the duration of the project, the Mathematics pass rates in some schools have fluctuated considerably. There have been substantial demographic shifts in recent years in the suburban schools, and so the vast majority of learners in these schools are black.[v]

The test was conducted in five project schools during the 2013 academic year. The selection of schools was purposeful, to include fee-paying and non-fee-paying schools, as well as those where teachers teaching Grade 10 Mathematics in 2013 included TM participants and non-TM participants. In addition, based on our previous experiences of the challenges in collecting learner test data in the project schools, it was important to select schools where teachers were committed to running the pre- and post-tests, and thus supporting our research.

Twenty-one teachers participated in the study, 14 of whom were TM teachers, while seven were non-TM teachers. The selection of all teachers was based on their teaching of at least one Grade 10 Maths class in one of the five selected schools and their willingness to participate in the study. We were fortunate that all Grade 10 Maths teachers in the five schools agreed to participate in the study. It is important to acknowledge here that the provincial department offered catch up lessons for learners, and workshops for teachers at various points in time, and further, that teachers participated in various activities related to their professional lives. With respect to learners, all learners across all schools had opportunity to attend catch-up lessons. With respect to teachers, we cannot consider the WMCS intervention as somehow divorced from this wider contact, where some TM teachers and non-TM teachers in different schools might participate in teacher support activities. However, it was clear from our interactions with school leadership that the WMCS professional development was considered to be the dominant initiative amongst Maths teachers in the school.

The timing of teachers' participation in the courses is important in relation to the timing of the pre- and post-tests. Six teachers completed the TM1 course in 2012, whilst another four teachers enrolled for the TM1 course at the beginning of 2013, and completed it at the end of 2013. Four teachers participated in the TM2 course, which ran from July 2012 to June 2013. Thus, there were eight teachers enrolled in a course for at least part of the data collection period. This is worth noting because research (e.g. Clarke, 1994) suggests that the impact of professional development programmes on teachers' classroom mathematics practice is delayed.

The pre-test was written by 882 learners, while only 803 learners wrote both pre- and post-test. We analysed only the scripts of those learners who had written both tests in order to compare learning gains. In total, 586 learners (73%) were taught by TM teachers, with 217 (27%) learners in the control group, taught by non-TM teachers.[vi] We refer to those taught by TM teachers as TM learners, and to the control group as non-TM learners. The breakdown across TM1 and TM2 is shown in Table 1.

Table 1 Numbers of learners in each Transition Maths group and the control group, who wrote pre-test and post-test.

	N
TM1	392
TM2	194
Control	217
TOTAL	803

The Grade 10 Learning Gains Test

The Grade 10 Learning Gains Test was designed by the project team as a test of key aspects of the Grade 10 algebra, functions and geometry curriculum, using typical curriculum items. The total mark for the test was 76. Algebra content (51% of marks) included simplification, substitution, factorisation, equations and linear patterns. Functions content (36%) included function notation, properties of linear functions, quadratic functions and related transformations. Particular emphasis was placed on moving between different representations of functions. Geometry (13%) dealt with triangles, quadrilaterals, and congruency, with one question requiring formal proof. A selection of 'look-alike' questions is provided in Appendix A. The look-alikes are necessary to ensure confidentiality of the test items. They are questions that are close, but not identical, to those given in the actual test. For example, if a test item appeared as "factorise fully: $am^2 - 3am$" then a look-alike might appear as: "factorise fully: $kp^3 - 2kp$".

The test was designed to contain items with a range of difficulty. For example, sample questions 1, 4a and 5 would be considered easy questions at Grade 10 level, since they deal with Grade Nine content. Questions 3, 4b, 4c and 4d tested typical content introduced in Grade 10. Question 2 is a relatively difficult example of a quadratic equation at Grade 10 level. Items were revised through an iterative process so as to reduce the complexity of easier items, so that, for example, fewer concepts/procedures were tested in a single question.

The test was designed to be administered during a typical maths lesson (approximately one hour), in order to reduce interference in the teaching schedule.

A Rasch analysis was used to assess the validity of the test (Hodgen, Pillay, Adler & Pournara, 2014). In brief, this analysis showed that the test was fit for the purpose of comparing the learning gains between the TM groups and the control group of learners. The test performed well on dimensionality tests. Almost all items provided an excellent fit to the Rasch model, with occasional misfit well below the level that would degrade measurement. However, there were relatively few easier items, leading to poorer discrimination amongst the lowest attaining learners. As a result, the test may underestimate the gains made by the lower attaining learners.

Analysis

In comparing the overall changes in mean scores, we used both descriptive and inferential statistics for both TM groups, together and separately. We report comparisons using overall changes in the mean score. Using SPSS 22.0, regression was used to compare differences between learners in classes taught by TM teachers and the control. We report the TM group as a whole, together with the comparison of each TM group to the control. In order to calculate a meaningful effect size, we calculated Cohen's *d*, and then interpreted this value following Higgins, Kokotsaki and Coe (2012) in terms of additional months of progress, beyond the progress that might be expected of learners without the intervention.

Results

In Figure 1 we present a graphical comparison of the TM groups with the control. For the TM groups as a whole, and for TM1, the TM learners' initial attainment was below that of the control, whereas their attainment after a year's teaching by the TM teachers was above that of the control. The attainment of TM2 learners on the pre-test was below that of the control and, while the gap in attainment narrows over time, their attainment in the post-test was still slightly below the control group.

Table 2 shows pre- and post-test results for the TM group as a whole. The TM group gains are greater than those of the control, although the variation of the scores and gains is considerable. In Table 3, we disaggregate the results for TM1 and TM2. Both show gains over the control, although the gain for TM1 is much larger. However, the variation in TM1 gains is quite large, which suggests that some learners, or some TM1-teachers' classes, did better than others.

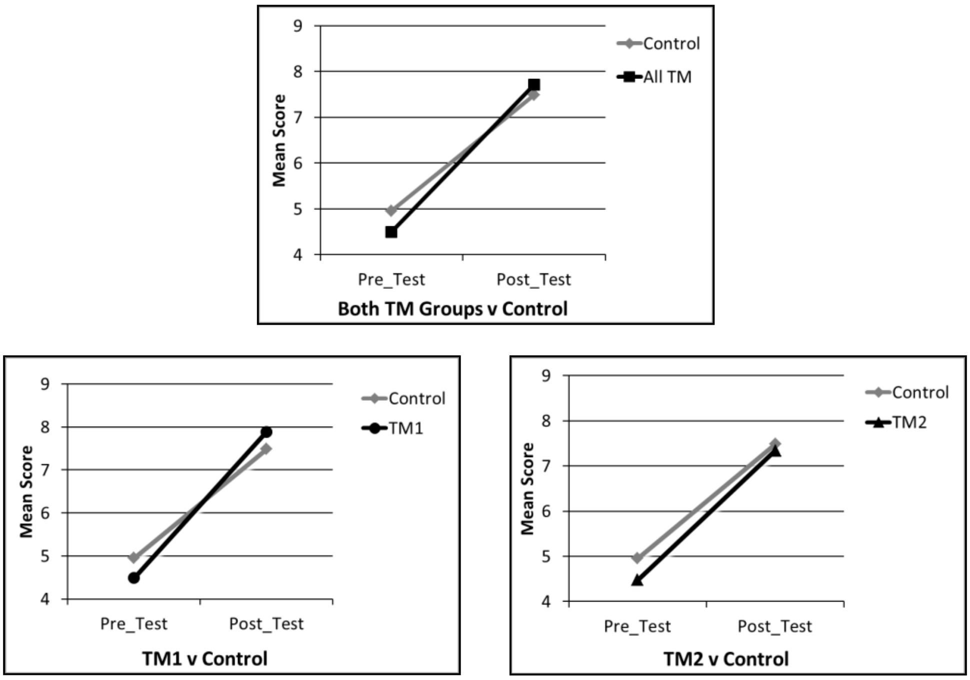


Figure 1 Mean pre- and post-test scores for the TM groups as a whole and for each of TM1 and TM2 separately

Table 2 Mean pre- and post-test scores, standard deviations (*SD*) and gains for the TM groups as a whole

		Pre-test score		Post-test score		Gain	
	N	Mean	*SD*	Mean	*SD*	Mean	*SD*
TM1 and TM2	586	4.49	3.40	7.71	5.12	3.22	3.78
Control	217	4.95	3.63	7.49	5.01	2.54	3.28

Table 3 Mean pre- and post-test scores, and gains for the TM1 and TM2 classes separately

		Pre-test score		Post-test score		Gain	
	N	Mean	*SD*	Mean	*SD*	Mean	*SD*
TM1	392	4.49	3.35	7.89	5.32	3.40	4.03
TM2	194	4.48	3.51	7.34	4.70	2.86	3.20
Control	217	4.95	3.63	7.49	5.01	2.54	3.28

It is important to note that both groups start from a very low base and that the gains for both groups are relatively small. For example, the average pre-test percentage score for the combined TM group was 5.9%, and this improved to 10.1% on the post-test. This indicates that the majority of learners had not grasped the content of Grade 10 Mathematics in algebra, function and geometry. We discuss this further in the conclusion.

The gains for the TM group were significantly greater than the control for the TM group as a whole (mean difference, 0.68; *SE*, 0.29; t (801) = 2.33; p = 0.020).[vii] When considered separately, the gains for the TM1 group were also significant (mean difference, 0.86; *SE*, 0.32; t (607) = 2.68; p = 0.008). However, the gains for the TM2 group were not significantly greater than the control group, although the gains were positive (mean difference, 0.31; *SE*, 0.32; t (409) = 0.97; p = 0.331).

In Table 4, we show the results of a linear regression for the entire sample, with post-test scores as the dependent variable. In this analysis, pre-test scores are treated as an independent variable in order to control for prior attainment. Learner participation in TM was treated as a dummy independent variable. In Table 5, we show similar regressions for the TM1 and TM2 learners, considered separately. As would be expected, the results are similar to the *t*-tests reported above. In each case, the raw effect (the unstandardised TM coefficient) is slightly greater than the mean differences reported above, although in practical terms, the effects are very similar. It can be seen from the standardised coefficients that the effect of participation in a TM class is small in comparison to prior attainment (as measured by pre-test scores).

In Table 6, we show the effect sizes for the TM group as a whole, and for TM1 and TM2 separately. Whilst all the effects are positive, the greatest gains were made by the TM1 group, where the effect size of 0.21 is equivalent to three months' additional progress (Higgins et al., 2012).

Table 4 Summary of regression analysis for the entire sample of students in comparison to the control. Dependent variable: post-test score. Independent variables: 'was learner in a TM class?' and pre-test score. Standard error for unstandardised coefficients shown in brackets.

	Unstandardised coefficients		Standardised coefficients		
Intervention group	Was learner in a TM class?	Pre-test score	Teacher did TM course?	Pre-test score	
	B	B	β	β	*p*
Both TM groups	0.688 (0.291)	1.024 (0.037)	0.060	0.698	0.018

Table 5 Summary of separate regression analyses for the TM1 and TM2 in comparison to the control. Dependent variable: post-test score. Independent variables: 'was learner in a TM class?' and pre-test score. Standard error for unstandardised coefficients shown in brackets.

	Unstandardised coefficients		Standardised coefficients		
Intervention group	Was learner in a TM class?	Pre-test score	Teacher did TM course?	Pre-test score	
	B	B	β	β	*p*
TM1	0.874 (0.321)	1.038 (0.044)	0.080	0.689	0.007
TM2	0.320 (0.321)	1.017 (0.045)	0.033	0.747	0.320

Table 6 Gains, pooled standard deviations and effect sizes (*d*) for the TM-classes as a whole and for TM1 and TM2 separately[viii]

	Gain	Pooled *SD*	*d*	Months' progress	Description
All TM	0.627	3.653	0.17	2	Low
TM1	0.803	3.783	0.21	3	Medium
TM2	0.285	3.242	0.08	1	Low

Discussion

These results indicate a small but statistically significant effect of $d = 0.17$ on learning gains for the TM intervention as a whole. According to Higgins et al. (2012), this is a small effect size, and equivalent to a gain of two months' additional progress. Although the effect is small, it is an indirect effect, which indicates that the TM intervention had an effect on learner attainment, even though the intervention was principally directed at increasing teacher knowledge, rather than learner attainment.

The effect for the TM1 intervention compared to the control is a medium effect of $d = 0.21$, which is a medium effect size, and equivalent to a gain of three months additional progress (Higgins et al., 2012).

On the other hand, the TM2 intervention, which was directed at more advanced mathematical content knowledge, shows a much smaller effect that is not statistically significant. However, this shows that the TM2's focus on advanced teacher knowledge beyond the Grade 10 curriculum has a positive, if modest, effect on learner attainment equivalent to around one month.

The differences in the gains between the TM1-group and the TM2-group are not easily accounted for, and given the small number of TM2 teachers in the sample, any suggestions must be made with extreme caution. One possible reason is that the TM2 course did not pay much attention to Grade 10 mathematical content, nor to the teaching of this content. By contrast, in TM1, there was a great deal of attention to Grade 10 content, and some attention to the associated teaching issues.

It is important to note a number of limitations to these results, in particular that the sample of teachers was small ($N = 21$), and that the variation in the gains made by learners was large, where some learners made much larger gains than others, even within the same class. In addition, the control group were from similar classes in the same schools as the intervention group, and we have controlled for prior attainment in our analysis. However, whilst we are reasonably confident that the intervention and control classes are similar, this does not constitute a rigorously matched sample. Hence, the results should be treated as indicative, rather than conclusive. Furthermore, we did not gather data on how teachers taught. We assume that non-TM teachers taught in similar ways to their previous teaching. While this is a reasonable assumption, the inability to confirm this nevertheless constitutes a potential limitation of the study.

Conclusion

In this paper, we have reported the impact of a professional development programme on learners' attainment in mathematics. While we treat our results as indicative rather than conclusive, this study makes several important contributions. Firstly, we have provided evidence that working on teachers' MfT through the TM courses led to learning gains amongst Grade 10 learners. While these gains were small, the effect size is equivalent to two months of additional progress, and similar in magnitude to that reported by Sample McMeeking et al. (2012). This gives us confidence to pursue the model of professional development linked to the TM courses. Our confidence is boosted by the fact that the effects of professional development on learning gains are always secondary effects, and tend to lag behind the completion of teachers' participation in professional development. It is therefore particularly encouraging that we have obtained effects so soon after teachers' participation in the TM courses. Secondly, we have demonstrated that the notion of learning gains is a more productive way of investigating improvements in the system than comparing results, across years, of summative or one-off assessments such as TIMSS, NSC or the ANA. Thirdly, the methodology of the study provides a productive means by which to associate learning with teaching in ways that have not yet been attempted in mathematics education in South Africa. Since previous studies of learner attainment have been divorced from specific teachers, it has not been possible to explore direct links between learning and teaching. Furthermore, our study shows the potential for rigorous evaluation of professional development interventions in the South African context. This, too, gives us confidence to pursue this line of research further.

Nevertheless, we note two issues that must be taken into account in further research. The first is a caveat, and relates to the teacher sample. It is possible that teachers who chose to participate in the TM courses were more motivated than their colleagues who chose not to participate, and this may lead to bias in the sample. In future research it will be important to attempt to set up randomised teacher samples. This is no easy task, partly because the need for a control group is key for purposes of comparison, and also because having a control group within the school to some extent controls for contextual factors that impact teachers' work. The second issue concerns the low levels of performance in both the pre- and post-tests, which show that, despite the improvements, the majority of learners in the study were not adequately prepared for Grade 10 Mathematics, and that by the end of their Grade 10 year, were not adequately prepared for Grade 11 Mathematics. This should not detract from the improvements made by TM teachers, but it does reflect the low base from which teachers are required to work. It is important to acknowledge the additional demands this places on teachers, and future research needs to investigate how professional development programmes might support teachers in addressing learners' under-preparedness in mathematics.

Acknowledgements

This work is based on the research of the Wits Maths Connect Secondary Project at the University of the Witwatersrand, supported by the FirstRand Foundation Mathematics Education Chairs Initiative and the Department of Science and Technology, and administered by the National Research Foundation. Any opinions, findings and conclusions or recommendations expressed in this material are those of the authors, and do not necessarily reflect the views of the institutions named above.

Notes

i. The Wits Maths Connect Secondary Project is one of the FirstRand Foundation-National Research Foundation Mathematics Chair projects, led by Jill Adler.

ii. In this study the authors refer to CK rather than SMK.

iii. Middle school in the US caters for learners aged between 10 and 14 years old.

iv. Gersten et al. (2014) report effect sizes using Hedges's g, a similar measure to Cohen's d used later in this paper, but corrected for bias.

v. We use the term "black" as a generic term for the apartheid race classifications to include African, Indian and Coloured.

vi. As is commonplace in any intervention, we experienced learner "drop-out" from post-test and teacher movement during the year, but this was relatively small. Across the schools there were also three new teachers who were allocated Grade 10 Mathematics during the course of the year.

vii. The Levene's test suggests that the variances for both TM groups together and for the TM1 group alone are different to the control. A "variances not assumed" t-test indicated lower p values of 0.013 (All TM) and 0.05 (TM1). In the paper we report the slightly more conservative result of the standard t-test. A non-parametric test (Mann-Whitney U) was also conducted in both these cases. This showed a significant p value of 0.022 (All TM) and 0.014 (TM1).

viii. These effect sizes are calculated using the slightly more conservative mean differences rather than regression coefficients.

References

Adler J 2005. Mathematics for teaching: What is it and why is it important that we talk about it? *Pythagoras*, 62:2-11. Available at http://pythagoras.org.za/index.php/pythagoras/article/viewFile/109/112. Accessed 5 August 2015.

Adler J, Ball D, Krainer K, Lin FL & Novotna J 2005. Reflections on an emerging field: Researching mathematics teacher education. *Educational Studies in Mathematics*, 60(2):359-381. doi: 10.1007/s10649-005-5072-6

Adler J & Davis Z 2006. Opening another black box: Researching mathematics for teaching in mathematics teacher education. *Journal for Research in Mathematics Education*, 37(4):270-296. doi: 10.2307/30034851

Adler J & Ronda E In press. A framework for mathematics discourse in instruction: Documenting shifts in practice. *African Journal for Research in Mathematics Science and Technology Education*.

Adler J & Venkat H 2014. Teachers' mathematical discourse in instruction: Focus on examples and explanations. In H Venkat, M Rollnick, J Loughran & M Askew (eds). *Exploring mathematics and science teachers' knowledge: Windows into teacher thinking*. Abingdon, UK: Routledge.

Ball D, Bass H & Hill H 2004. Knowing and using mathematical knowledge in teaching: Learning what matters. In A Buffler & R Laugksch (eds). *Proceedings of the twelfth annual conference of the Southern African Association for Research in Mathematics, Science and Technology Education (SAARMSTE)*. Cape Town: SAARMSTE.

Ball DL, Thames MH & Phelps G 2008. Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5):389-407. doi: 10.1177/0022487108324554

Bansilal S, Brijlall D & Mkhwanazi T 2014. An exploration of the common content knowledge of high school mathematics teachers. *Perspectives in Education*, 32(1):34-50.

Baumert J, Kunter M, Blum W, Brunner M, Voss T, Jordan A, Klusmann U, Krauss S, Neubrand M & Tsai YM 2010. Teachers' mathematical knowledge, cognitive activation in the classroom, and student progress. *American Educational Research Journal*, 47(1):133-180. doi: 10.3102/0002831209345157

Carnoy M, Chisholm L, Addy N, Arends F, Baloyi H, Irving M, Raab E, Reeves C, Sapire I & Sorto A 2011. *The process of learning in South Africa: The quality of mathematics teaching in North West Province.* Technical report. Pretoria, SA/Stanford, CA: Human Sciences Research Council (HSRC)/School of Education, Stanford University.

Clarke DM 1994. Ten key principles from research for the professional development of mathematics teachers. In DB Aichele & AF Coxford (eds). *Professional development for teachers of mathematics: 1994 Yearbook.* Reston, VA: National Council of Teachers of Mathematics (NCTM).

Darling-Hammond L 2000. Teacher quality and student achievement: A review of state policy evidence. *Education Policy Analysis Archives*, 8(1):1-44.

Even R 1993. Subject-matter knowledge and pedagogical content knowledge: Prospective secondary teachers and the function concept. *Journal for Research in Mathematics Education*, 24(2):94-116. doi: 10.2307/749215

Gersten R, Taylor MJ, Keys TD, Rolfhus E & Newman-Gonchar R 2014. *Summary of research on the effectiveness of math professional development approaches*. (REL 2014-010). Washington, DC: US Department of Education, Institute of Education Sciences, National Center for Education Evaluation and Regional Assistance, Regional Educational Laboratory Southeast. Available at http://ies.ed.gov/ncee/edlabs/regions/southeast/pdf/rel_2014010.pdf. Accessed 30 July 2015.

Goldhaber DD & Brewer DJ 2000. Does teacher certification matter? High school teacher certification status and student achievement. *Educational Evaluation and Policy Analysis*, 22(2):129-145. Available at http://epa.sagepub.com/content/22/2/129.full.pdf. Accessed 30 July 2015.

Harbison RW & Hanushek EA 1992. *Educational performance of the poor: Lessons from rural northeast Brazil.* Oxford: Oxford University Press.

Higgins S, Kokotsaki D & Coe R 2012. *The Teaching and Learning Toolkit*. London, UK: Education Endowment Foundation/The Sutton Trust. Available at https://educationendowmentfoundation.org.uk/uploads/pdf/Teaching_and_Learning_Toolkit_%28July_12%29.pdf. Accessed 30 July 2015.

Hill HC, Ball DL & Schilling SG 2008. Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4):372-400. Available at http://www.ugr.es/~pflores/2008_9/Master_Conocim/textos%20JP/%5B1%5D_Hill-Ball-Schilling-JRME2008-07.pdf. Accessed 30 July 2015.

Hill HC, Rowan B & Ball DL 2005. Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2):371-406. doi: 10.3102/00028312042002371

Hodgen J, Pillay V, Adler J & Pournara C 2014. *The validity of the Grade 10 Learning Gains Test*. Technical Report No. 1 for the Wits Maths Connect Secondary (WMCS) Project. Johannesburg: Wits Maths Connect Secondary Project.

Kilpatrick J, Swafford J & Findell B (eds.) 2001. *Adding it up: Helping children learn mathematics*. Washington, DC: National Academy Press.

Krauss S, Baumert J & Blum W 2008. Secondary mathematics teachers' pedagogical content knowledge and content knowledge: validation of the COACTIV constructs. *ZDM: The International Journal on Mathematics Education,* 40(5):873-892. doi: 10.1007/s11858-008-0141-9

Monk DH 1994. Subject area preparation of secondary mathematics and science teachers and student achievement. *Economics of Education Review*, 13(2):125-145. doi: 10.1016/0272-7757(94)90003-5

Mullens JE, Murnane RJ & Willett JB 1996. The contribution of training and subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize. *Comparative Education Review*, 40(2):139-157.

Nye B, Konstantopoulos S & Hedges LV 2004. How

large are teacher effects? *Educational Evaulation and Policy Analysis*, 26(3):237-257. Available at http://steinhardt.nyu.edu/scmsAdmin/uploads/002/834/127%20-%20Nye%20B%20%20Hedges%20L%20%20V%20%20%20Konstantopoulos%20S%20%20%282004%29.pdf. Accessed 31 July 2015.
Pournara C 2013. Mathematics-for-teaching in pre-service mathematics teacher education: The case of financial mathematics. Unpublished PhD thesis, Johannesburg: University of the Witwatersrand.
Sample McMeeking LB, Orsi R & Cobb RB 2012. Effects of a teacher professional development program on the mathematics achievement of middle school students. *Journal for Research in Mathematics Education*, 43(2):159-181. doi: 10.5951/jresematheduc.43.2.0159
Sfard A 1991. On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. *Educational Studies in Mathematics*, 22(1):1-36. doi: 10.1007/BF00302715
Shulman LS 1986. Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2):4-14.
Shulman LS 1987. Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review*, 57(1):1-22.
Taylor N & Taylor S 2012. Teacher knowledge and professional habitus. In N Taylor, S Van der Berg & T Mabogoane (eds). *What makes schools effective? Report of South Africa's national school effectiveness study*. Cape Town: Pearson.

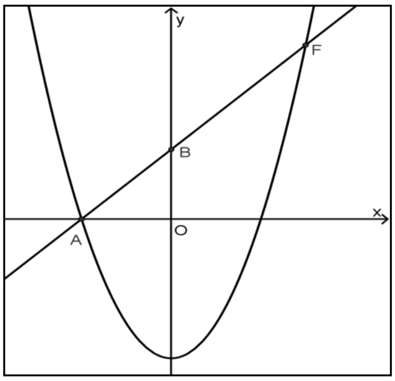
Appendix A

1) Factorise fully: $-4k + 20$

2) Solve for the unknown: $x(x+3) = 10$

3) Given $f(x) = 6 - x$, determine x if $f(x) = 14$

4) The diagram shows the graphs of $y = x + 2$ and $y = x^2 - 4$



a) Write down the coordinates of B.
b) Write down the minimum value of the parabola.
c) The 2 graphs intersect at A and F. Determine the coordinates of A and F.
d) Assume the graph of the parabola is translated 2 units down. Give the equation of the new graph.

5) Determine the size of x. Show how you obtained your answer.