Predictive Mathematical Models for the Spread and Treatment of Hyperoxia-induced Photoreceptor Degeneration in Retinitis Pigmentosa IOVS — Supplementary Material

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Figure S1: Simulation results showing exemplar initial oxygen and photoreceptor profiles (see Eq. (7)). The initial oxygen profile is the steady-state distribution corresponding to the initial photoreceptor profile. Panel (**a**) shows healthy oxygen and photoreceptor profiles without photoreceptor degeneration ($F(\theta, \phi) = 1$). Panel (**b**) shows oxygen and photoreceptor profiles where photoreceptors have been removed from an annular region with inner and outer boundaries (θ_1, θ_2) = (0.4, 0.6) × Θ (rad) ($F(\theta, \phi) = \frac{1}{2}(\tanh(S(\theta_1 - \theta)) + \tanh(S(\theta - \theta_2)) + 2)$). Panel (**c**) shows oxygen and photoreceptor profiles where photoreceptors have been removed from a disc-shaped region with centre eccentricity $\theta_c = 0.3 \times \Theta$ (rad) and radius parameter $\psi = 0.05 \times \Theta$ (rad) ($F(\theta, \phi) = \frac{1}{2}(\tanh(\hat{S}((\theta - \theta_c)^2 + (\phi - \hat{\phi}_c)^2 \sin^2(\theta) - \psi^2)) + 1$)). See Tables 2 and 3 for the remaining parameter values.



Figure S2: Simulation results showing how retinal degeneration progresses following the loss of an annulus of photoreceptors. Heat maps show the photoreceptor distribution in the initial- (left) and steady-states (right), while θ_1 (rad) and θ_2 (rad) are the initial eccentricities of the inner and outer boundaries of the degenerate annulus respectively. (a) $(\theta_1, \theta_2) = (0.4, 0.6) \times \Theta$ (rad): the degenerate annulus does not expand, (b) $(\theta_1, \theta_2) = (0.08, 0.4) \times \Theta$ (rad): the degenerate annulus expands centrally only, (c) $(\theta_1, \theta_2) = (0.6, 0.9) \times \Theta$ (rad): the degenerate annulus expands peripherally only and (d) $(\theta_1, \theta_2) = (0.08, 0.9) \times \Theta$ (rad): the degenerate annulus expands both centrally and peripherally. See Tables 2 and 3 for the remaining parameter values.