

# Scour influence on the fatigue life of operational monopile-supported offshore wind turbines

Ramtin Rezaei<sup>a</sup>, Philippe Duffour<sup>a</sup>, Paul Fromme<sup>b</sup>

<sup>a</sup>Department of Civil, Environmental & Geomatic Engineering, UCL, London, WC1E 6BT, UK

<sup>b</sup>Department of Mechanical Engineering, UCL, London, WC1E 6BT, UK

Contact Author: [p.duffour@ucl.ac.uk](mailto:p.duffour@ucl.ac.uk)

## Abstract

Offshore wind turbines (OWTs) supported on monopiles are an important source for renewable energy. Their fatigue life is governed by the environmental loads and the dynamic behaviour, depending on the support stiffness and thus soil-structure interaction. The effects of scour on the short and long-term responses of the NREL 5MW wind turbine under operational conditions have been analysed using a Finite Element (FE) beam model with Winkler springs to model soil-structure interaction. It was found that as a result of scour the modal properties of the wind turbine do not change significantly. However, the maximum bending moment in the monopile increases, leading to a significant reduction in fatigue life. Backfilling the scour hole can recover the fatigue life, depending mostly on the depth after backfilling. An approximate fatigue analysis method is proposed, based on the full time-domain analysis for one scour depth, predicting with good accuracy the fatigue life for different scour depths from the quasi-static changes in the bending moment.

**Keywords:** OWT; Fatigue life; Scour and backfilling; Monopile, Soil-structure interaction; FE beam model; Wind energy; Renewable energy

# 1 Introduction

Offshore wind energy is a fast-growing industry worldwide. Almost 90% of the largest offshore wind farms are located in Europe with most turbines supported on monopile foundations [1,2]. To ensure that wind energy remains competitive, it is essential to ensure that offshore wind turbines (OWT) are as cost-effective and efficient as possible. Fatigue is one of the key design driver for OWTs. The soil profile around the monopile and consequently the lateral stiffness of the structure is known to significantly influence the vibration due to environmental and rotor loads, and thus fatigue damage [3,4]. Scour changes the embedded length of the monopile and therefore the lateral stiffness of the system. A depth of  $1.3D_{pile}$  (where  $D_{pile}$  is the diameter of the pile) was suggested as the maximum equilibrium scour depth for current-dominated regimes [5,6]. This value is also recommended by DNV [7] for the design of offshore monopiles. However, on-site measurements were compared with design values [8] and it was shown that a scour depth of  $1.3D_{pile}$  can be conservative and that cost-savings could be achieved by adopting a lower scour level. Whitehouse et al. studied the backfilling process around monopiles using measurements from various locations in the North and Irish Seas [9]. In some cases, they reported higher scour depths than in reference [8]. A periodic variation of the scour depth depending on the time of the year was also observed, but measured data was too limited to draw firm conclusions [9]. Sørensen et al. studied the relative density of backfilling material through laboratory tests and found a range of 65% to 80% relative density with respect to the original soil [10]. Values were obtained from only one set of laboratory tests and lower densities may exist. Without scour protection, scour was found to extend laterally in the range of  $4-5D_{pile}$ . Even with scour protection, secondary scour can still occur around the pile [11].

Scour also decreases the soil resistance as it reduces the overburden pressure on the deeper soil layers [12,13]. Structures loaded laterally experience larger moments and displacements due to scour. Mostafa [13] studied the effects of scour width and depth on a pile loaded statically and laterally at the top. It was reported that pile head displacement and maximum bending moment increase by 155% and 200%, respectively, for a large scour depth of  $3D_{pile}$ . Softer soils were found to be more susceptible to scour [14,15]. The changes in natural frequencies of monopile-supported OWTs as a result of scour have been studied [14–18]. Various existing models of wind turbines have been investigated and the maximum reduction of the first natural frequency was up to 5% for the recommended design scour depth of  $1.3D_{pile}$ . Damgaard et al. [19,20] and Sørensen and Ibsen [17] studied the variations of the first natural frequency due to backfilling and found that backfilling of the scour hole recovers the modal properties of the OWT almost regardless of the backfilled material density [17,18,20].

Scour affects the dynamics of OWTs at different levels. As it increases the free length of the tower/monopile, the lateral stiffness and the natural frequencies of the OWT structure decreases. The mode shapes and material soil damping change, albeit only very slightly [18]. As a result of changes in the dynamic response, the fatigue damage in the monopile is affected. Offshore wind turbines are typically designed for a minimum service life of 20 years [7,21]. Fatigue damage due to scour around a turbine in parked conditions has been studied using simulations carried out on a fully coupled model implemented in HAWC [18]. Compared to the scenario

1 without scour, an increase of up to 40% in the fatigue damage was reported for scour levels of up to  $1.3D_{pile}$ ,  
2 kept constant throughout the turbine life. Tempel [22] studied the effects of larger levels of scour (up to  
3  $2.5D_{pile}$ ) on the fatigue damage of monopiles using a frequency-domain approach and found that this caused  
4 significant increases of up to 100% in the fatigue damage. Simple fatigue checks were conducted for various  
5 levels of scour and it was suggested that considering backfilling processes in the fatigue design of monopiles  
6 could reduce the stress demand by up to 37% [17].

7 In summary, scour has been shown to influence the fatigue life of offshore wind turbines and the effect of  
8 scour and backfilling on long-term damage in a parked wind turbine has been researched [17,18,23]. Due to  
9 the complexities associated with the fatigue analysis of offshore wind turbines, simplified methods have been  
10 proposed for this analysis [24–29]. However, the quantification of the scour influence on the fatigue life of an  
11 operational wind turbine and specific simplified methods for scour effects on fatigue analysis remain  
12 outstanding.

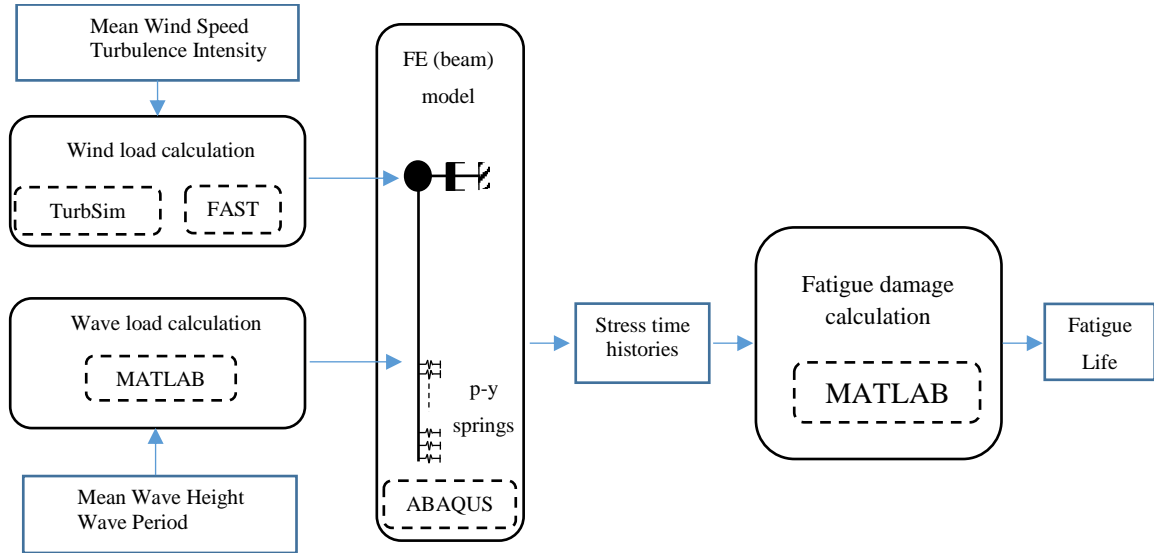
13 This paper investigates systematically the influence of scour on a reference 5MW offshore wind turbine, using  
14 FE time-domain dynamic decoupled simulations. The turbine is assumed to be in operation throughout and  
15 each scour depth investigated is considered constant throughout the OWT service life. The simulation results  
16 are combined using environmental state data to predict the actual fatigue life of the system. Full fatigue life is  
17 a key indicator for designers, but rarely calculated in the published literature. Furthermore, a novel and efficient  
18 simplified fatigue analysis method is proposed based on these findings, which might be of interest to the  
19 offshore wind industry to reduce the computational effort to account for scour effects in fatigue analyses. The  
20 methodology is described in section 2. Section 3.1 discusses the short-term effects of scour (changes in modal  
21 properties). In section 3.2, long-term effects (variation of fatigue life with scour depth and backfilling) are  
22 studied. A simplified fatigue analysis method is proposed in section 3.3, followed by conclusions in section 4.

## 23 **2 Methodology**

### 24 **2.1 Modelling approach**

25 This paper investigates the fatigue life of a 5MW wind turbine model with properties provided by the US  
26 National Renewable Energy Laboratory (NREL) [30]. Various studies have been based on this wind turbine  
27 as a significant amount of data is available for it [31–37]. In this study, different software packages were used  
28 for the fatigue analysis, as shown in Fig. 1. The pre-processing module TurbSim was used to model the  
29 turbulent incoming wind field as an input to the aero-elastic software FAST [38]. FAST is a coupled aero-  
30 elastic package that provided the resultant rotor thrust time histories from the blade-wind interaction. The rotor  
31 thrust time histories were obtained from FAST, constraining the tower and monopile motion to allow for the  
32 separate modelling of aerodynamic damping. Wave load time histories were generated separately in Matlab  
33 based on the JONSWAP spectrum [40]. These were used as inputs to an FE (ABAQUS) model of the tower,  
34 monopile, and soil-structure interaction. The two separately calculated aerodynamic and wave load time  
35 histories were applied as point loads at their respective heights in the FE model, in line with previous literature.

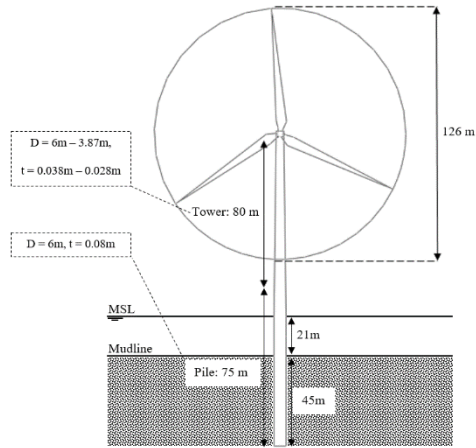
1 A separate ABAQUS model was developed because FAST has limited capabilities for modelling soil-structure  
 2 interaction and only allows for a basic structural model of the tower/monopile. Time domain dynamic analyses  
 3 were carried out for varying environmental conditions. FE stress time histories were rainflow-counted to  
 4 calculate the fatigue life of the wind turbine at every scour depth.



**Figure 1.** Schematic of software packages used in the fatigue life analysis.

## 5 2.2 Geometry and properties of the OWT

6 The NREL 5MW wind turbine is schematically shown in Fig. 2. The rotor diameter is 126m, and the hub  
 7 height at 92m above mean sea level. The monopile embedded depth is 45m and the water depth is 21m. The  
 8 steel pile section has an elastic modulus of 210GPa, a Poisson's ratio of 0.3 and a density of 7850 kg/m<sup>3</sup>, while  
 9 a higher density ( $\rho=8500\text{kg/m}^3$ ) was used for the tower section to account for the mass of secondary steel [30].  
 10 The cut-in and cut-out speeds of the turbine are 3m/s and 25m/s, respectively, with the rated rotor speed at  
 11 12.1rpm. The first natural frequency of the wind turbine is 0.25Hz and lies between the 1P (rotor) and 3P  
 12 (blade passing) frequency ranges. The rotor blades are 62.7m long, with properties documented in the report  
 13 by Lindenburg [39].



**Figure 2.** Reference 5MW wind turbine dimensions. D stands for the diameter of the pile or tower as per label.

### 2.3 Numerical simulation

Linear, 0.5m-long Timoshenko beam elements were used for the FE modelling of the tower and monopile in ABAQUS. The dynamic response of the system was studied using one-hour implicit time-domain simulations with a 0.1s time increment. Four hundred seconds were added at the start of the load time series and the corresponding simulation data was later discarded to avoid any potential initial transient effects. The model included a total damping ratio of 6%. This is comprised of 4% aerodynamic applied as a viscous dashpot at the tower top and 2% for combined hydrodynamic (0.2%), structural (1%) and soil (0.8%) damping, applied as Rayleigh damping and calibrated using the first two bending mode natural frequencies. The hysteretic component of the soil damping is reduced slightly by scour as layers of dissipating material are removed [18]. Soil damping ratio is usually considered to be less than 1%. Therefore, removing top soil for up to 10% of the embedded length of the pile can be considered to have only a negligible effect on the overall damping.

Table 1 shows the soil profile data used in this paper for the reference wind turbine at its planned location [40]. The soil profile is comprised of medium-dense to dense sands with the top layer formed from loose sand.

**Table 1.** Soil profile for proposed location off Netherlands coast, modified according to data from [40].

Depth	Soil layer	$\gamma_{\text{sat}}$ (kN/m <sup>3</sup> )	$\phi^{\circ}$
0m to -15m	Loose sand	17	27.5
-15m to -20m	Firm clay	19	20
-20m to -25m	Fine to medium sand	19	32.5
-25m to -65m	Fine to medium sand	19	35

The lateral support of the soil was modelled using the p-y curves recommended by DNV [7]. A spacing of 1.5m was used for the soil profile discretisation (30 springs), based on results from a preliminary study (not shown). The bottom of the monopile was vertically supported on a roller due to the limited influence of vertical

1 soil resistance on the lateral and bending responses of the wind turbine structure. Following a preliminary  
2 sensitivity study of the static response and dynamic properties of the wind turbine, the fairly shallow clay layer  
3 was replaced by the medium-dense sand with similar overall stiffness that lies underneath.

4 Local scour was modelled by removing from the soil profile, the number of top springs necessary to produce  
5 scour depths  $d_i$  of up to  $1.5D_{pile}$ . The ultimate lateral resistance of soil ( $p_u$ ) was reduced down to  $6D_{pile}$  depth  
6 below the original seabed to include the effects of overburden pressure reduction [17]. When backfilling was  
7 investigated, springs with a p-y curve modified according to the material density and reduced angle of friction  
8 were used. A given scour depth was considered to apply for the whole life of the structure. Throughout this  
9 study, the scenario used as baseline for comparison was the design case without scour.

10 A set of 22 environmental states (ES) were adopted from available data for the planned location [23], as  
11 described in Table A1 (Appendix A). The wind speeds range from 4m/s to 24m/s, grouped into 11 bins 2m/s  
12 wide. Following [23], each wind speed was correlated to 2 different sea states defined as significant wave  
13 height paired with a zero-crossing period bin. The significant wave height bins were 0.5m wide, ranging from  
14 0.5m to 3.0m, and the zero-crossing periods ranging from 3s to 5s were clustered into 1s bins. The probability  
15 of occurrence of each sea state, also shown in Table A1 was obtained empirically from the scatter diagram  
16 presented in [23]. Wind and wave misalignment can decrease the fatigue life of OWTs by increasing the side-  
17 side vibration, which is less damped. However according to Nehal [41] the misalignment between wind and  
18 wave at this particular location is minor, so the aerodynamic and hydrodynamic loads were considered to be  
19 aligned. The Kaimal spectrum [42] was used to model wind turbulence. For the calculation of rotor thrust in  
20 FAST, a constant pitch angle corresponding to the mean wind speed considered was used, following the  
21 relationship provided by [30]. The tower, monopile and foundation flexibilities were removed to allow  
22 complete control of the aerodynamic damping in ABAQUS when the rotor thrusts were applied to the tower  
23 top [21,33]. The hydrodynamic loads from the wave and current actions were calculated using Morison's  
24 equation together with linear wave theory and Wheeler stretching for the JONSWAP spectra, commonly used  
25 for the North Sea [43]. The resultant hydrodynamic pressure was applied as a lateral force at the mean sea  
26 level. The hydrodynamic mass was added to the submerged part of the monopile, increasing the monopile  
27 density as suggested by [44]. The wind and wave loads were calculated using 0.1s time increments to match  
28 the FE time steps.

## 29 2.4 Fatigue life calculation

30 The bilinear S-N curve class E, proposed by DNV [45] for the transition of weld to base material on the outside  
31 of girth welds in monopiles, was used for the calculation of fatigue damage. The parameters defining this S-N  
32 curve are shown in Table 2, where  $N$  refers to the number of cycles to failure,  $\log(\bar{a})$  corresponds to the intercept  
33 of the  $\log(N)$  axis,  $m$  is the negative inverse slope of the curve, and  $SCF$  is the stress concentration factor.

**Table 2.** Bilinear S-N curve class E, parameters according to DNV [45].

$N \leq 10^6$		$N \geq 10^6$		Thickness component	Hot-spot
Log( $\bar{a}_1$ )	$m_1$	Log( $\bar{a}_2$ )	$m_2$	k	SCF
11.61	3.0	15.35	5.0	0.2	1.13

1

2 The fatigue damage was calculated along the monopile depth to consider the maximum stress at hypothetical  
3 welding points for varying scour depths. For each environmental state, the output stress time-histories from  
4 ABAQUS were rainflow counted in MATLAB and the damage calculated by adding the damages caused by  
5 each stress bin. As each environmental state has a different probability of occurrence (table A1), the total  
6 fatigue damage was obtained by summing each damage contribution according to the Palmgren-Miner (PM)  
7 sum rule.

### 8 3 Results and discussion

#### 9 3.1 Short-term effects of scour

10 Figure 3 shows the relative changes in natural frequencies as a result of scour. The first natural frequency  
11 decreases by approximately 2.2%, while the second natural frequency shows a maximum reduction of 3.6%  
12 as a result of  $1.5D_{pile}$  local scour depth. The natural frequencies decrease nonlinearly with increasing scour  
13 depth for low depths and then fairly linearly for scour depths above  $0.75D_{pile}$ . It should be noted that for the  
14 recommended design scour depth of  $1.3D_{pile}$ , the reduction of the first natural frequency is below 2%. The  
15 changes are in agreement with previous results [22,46], but are too small to be reliably attributed to scour  
16 should frequency measurements be used for scour detection. The influence of scour on the mode shapes was  
17 investigated, but found to be small.

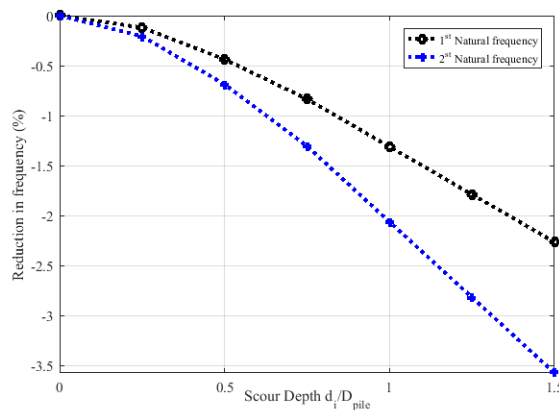
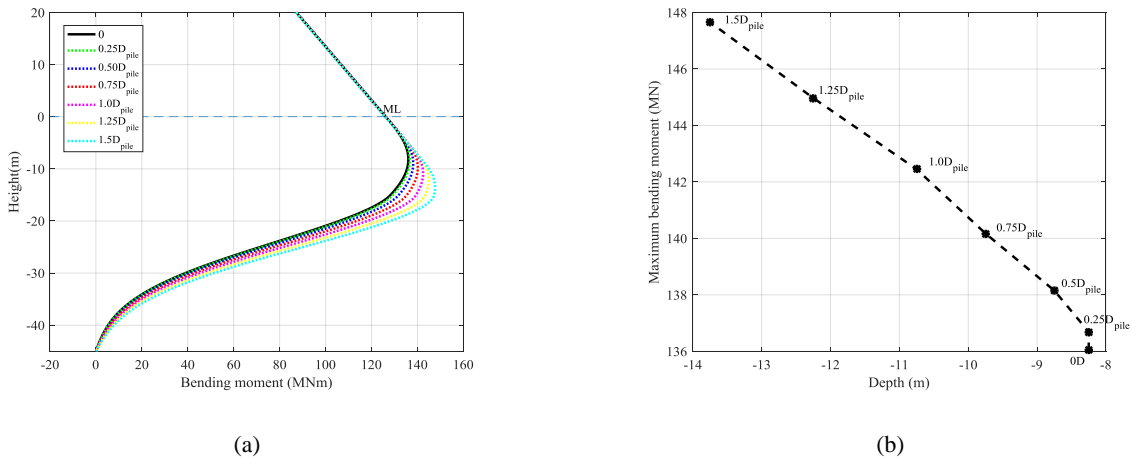


Figure 3. Relative change of natural frequencies due to scour.

18 Scour increases the free length of the wind turbine support structure. In monopiles, this leads to an increase in  
19 the bending moment in the structure below the seabed, which is typically the critical fatigue location for  
20 monopile-supported offshore wind turbines [28]. Figure 4(a) shows the change in bending moment in the

1 monopile for a combination of 1MN notional static wind and wave loads. As a result of scour, the peak bending  
 2 moment increases and its location shifts downwards. The maximum considered scour depth of  $1.5D_{pile}$  causes  
 3 an approximately 9% increase in the maximum bending moment in the monopile.



**Figure 4.** (a) Bending moment in the monopile (mudline is marked at zero depth) and (b) variation of maximum bending moment with its location below mudline due to varying scour depths.

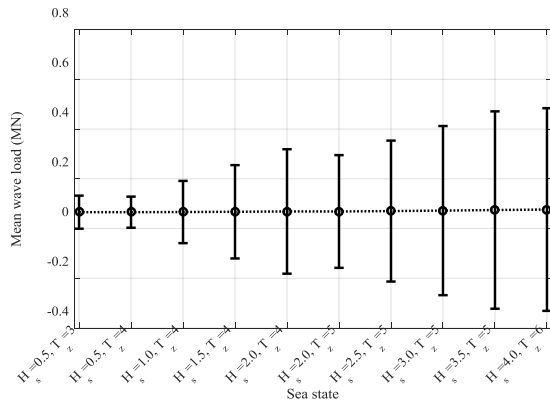
4 Figure 4(b) shows the maximum bending moment in the monopile against its location below the mudline for  
 5 varying scour depths. The location of maximum stress shifts by almost 6m for a scour depth of  $1.5D_{pile}$ . The  
 6 location of maximum stress can also vary depending on the respective environmental load amplitudes. In the  
 7 preliminary design stage, the exact weld locations in the monopile are uncertain. Because of this uncertainty,  
 8 the fatigue life was calculated at three depths ( $z_1 = -8\text{m}$ ,  $z_2 = -10.5\text{m}$ , and  $z_3 = -12\text{m}$ ) with high stress amplitudes,  
 9 where the weld could be located. These were chosen to include the maximum bending moment for all scour  
 10 depths.

## 11 3.2 Fatigue analysis – long-term effects of scour

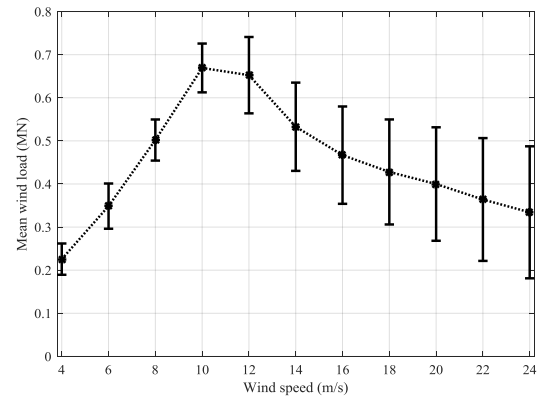
### 12 3.2.1 Environmental loads

13 In fatigue analysis, the mean and standard deviation of the aerodynamic and hydrodynamic loads can be useful  
 14 to characterise their static and dynamic (varying) components. Figure 5(a) shows that the static component of  
 15 the wave load increases only slightly with wave period and height, whereas the dynamic component increases  
 16 significantly. Figure 5(b) shows that when the wind turbine is in operation, the mean wind load peaks at the  
 17 rated wind speed and then decreases (as blades are increasingly feathered), whereas the standard deviation  
 18 shows a continuous increase with the wind speed due to turbulence. As aerodynamic loads have a significantly  
 19 higher lever arm from the mudline, they tend to dominate the quasi-static bending moment below mudline.





(a)



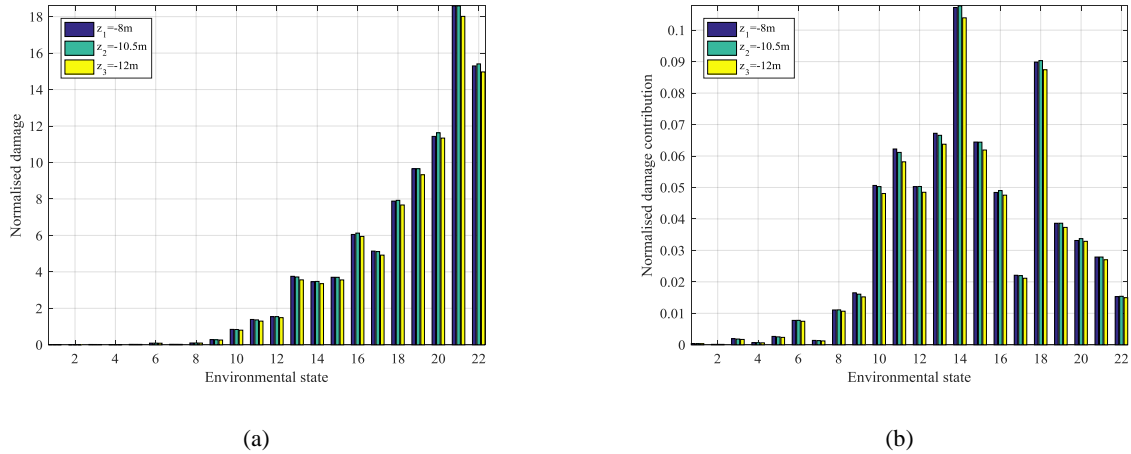
(b)

**Figure 5.** Mean and standard deviation of (a) wave load for different sea states and (b) wind load for varying wind speed.

1

### 2 3.2.2 No scour

3 The fatigue damage in the OWT was first studied at the selected locations when no scour is present. Figure  
 4 6(a) shows the fatigue damage for each environmental state (section 2.3) at the three chosen depths, normalised  
 5 against the damage corresponding to a fatigue life of 20 years. The lower environmental states (up to #8) lead  
 6 to limited fatigue damage compared to the damage caused by the higher environmental states with higher wind  
 7 speeds and wave heights. This mirrors the observation made from Fig. 5: as the dynamic component of the  
 8 loads increases, so does the fatigue damage. Small variations are observed at the considered depths, depending  
 9 on the resulting bending moment of wind and wave loads and different dynamic amplification. Normalised  
 10 fatigue damages at  $z_2$  are slightly higher (by up to 1%) than at  $z_1$  for higher wind speeds ( $V_w=18\text{m/s}$ ,  
 11 environmental states 16 and above) as the maximum bending moment for the case without scour occurs at a  
 12 depth of about 8–10 meters. Figure 6(b) shows the normalised damage contribution of each environmental  
 13 state, weighted by their respective probability of occurrence. The combined contribution of fatigue damage  
 14 caused by the environmental states 10 to 18 accounts for more than 70% of the overall fatigue damage in the  
 15 monopile.

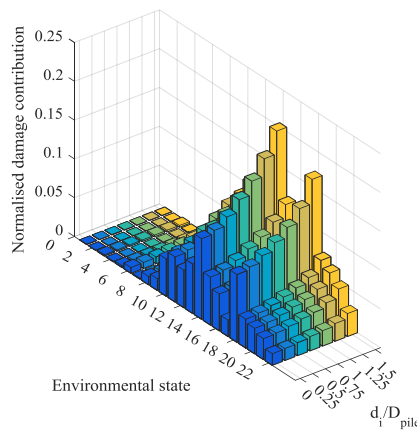


**Figure 6.** (a) Normalised fatigue damage and (b) normalised fatigue damage contributions of the environmental states at  $z_1$  (corresponding to the maximum stress in the no scour scenario),  $z_2$ , and  $z_3$ .

1 Summing the fatigue damages from all environmental states gives a fatigue life of approximately 28 years at  
 2 weld location at  $z_1 = -8\text{m}$  and at  $z_2 = -10.5\text{m}$ . The predicted fatigue life at  $z_3 = -12\text{m}$  closer to 29 years. This  
 3 fatigue life result was obtained assuming no wind-wave misalignment and no idling time for wind speeds in  
 4 the operational range.

### 5 3.2.3 Scour with no backfilling

6 Figure 7 shows the change in the normalised fatigue damage contribution for every environmental state due to  
 7 scour depths of up to  $1.5D_{\text{pile}}$  at the depth of  $z_3 = -12\text{m}$ , compared to the no scour scenario.

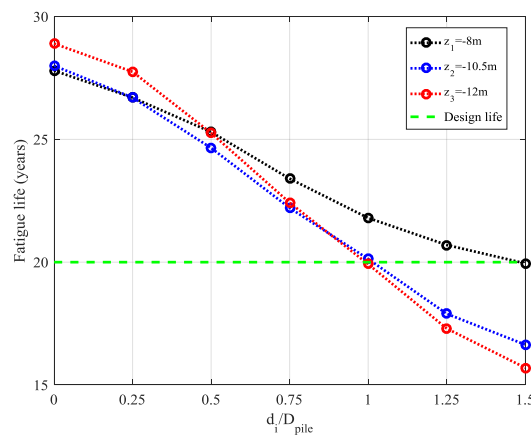


**Figure 7.** Comparison of the scour effects on the fatigue damage contributions of the environmental states at  $z_3 = -12\text{m}$ .

8 The fatigue damage contributions of each environmental state show a similar increase for scour depths of  
 9  $0.25D_{\text{pile}}$  to  $1.0D_{\text{pile}}$ . A higher increase is observed at  $1.25D_{\text{pile}}$  and  $1.5D_{\text{pile}}$ . As the location of the maximum  
 10 stress shifts downwards with increased scour depth (Fig. 4b), at  $z_3 = -12\text{m}$  a maximum increase of 102% in the  
 11 normalised damage contributions is observed for a scour depth of  $1.5D_{\text{pile}}$ . At  $z_1 = -8\text{m}$  and  $z_2 = -10.5\text{m}$ , the  
 12 variations in fatigue damage contributions show a similar increase for small to moderate scour depths (up to  
 13  $1.0D_{\text{pile}}$ ). However, for higher scour levels ( $>1.0D_{\text{pile}}$ ), a smaller increase was observed as the maximum stress

1 is located further downwards. The largest change in the normalised fatigue damage contributions for different  
 2 scour depths was approximately 40% and 75% at  $z_1 = -8\text{m}$  and  $z_2 = -10.5\text{m}$ , respectively. The combined effect  
 3 of the increase in bending moment and the shift in the location of maximum bending moment resulted in the  
 4 highest increase at  $z_3 = -12\text{m}$  and the lowest variation at  $z_1 = -8\text{m}$ .

5 Figure 8 shows the predicted fatigue life of the wind turbine at the considered weld locations for different  
 6 scour levels. The typical design life of 20 years is marked as a reference. The predicted fatigue life at  $z_1 = -8\text{m}$ ,  
 7 which is the critical location for the no scour scenario, reduces by 29% for a scour depth of  $1.5D_{\text{pile}}$ , to the  
 8 design life of 20 years. However, the downward shift in the location of maximum bending moment as a result  
 9 of scour leads to a higher rate of fatigue life reduction at the considered lower weld locations. At these two  
 10 locations, the fatigue life reduces to below the design life for a scour depth of  $1.0D_{\text{pile}}$ . At  $z_2 = -10.5\text{m}$  and  $z_3 =$   
 11  $-12\text{m}$ , the predicted fatigue life of approximately 18 years is 2 years shorter than the design life for a scour  
 12 depth of  $1.25D_{\text{pile}}$ . This demonstrates how important the consideration of bending moment shift is for the  
 13 fatigue limit state design of monopile-supported offshore wind turbines.

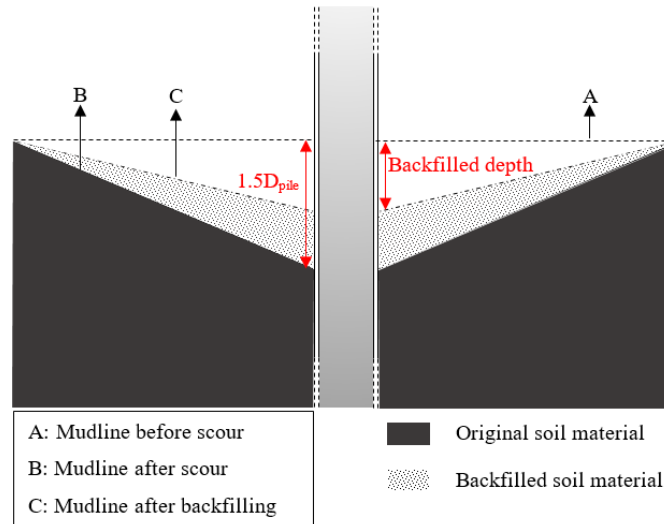


**Figure 8.** Fatigue life prediction for variations of scour depth around the wind turbine monopile at  $z_1$ ,  $z_2$  and  $z_3$ .

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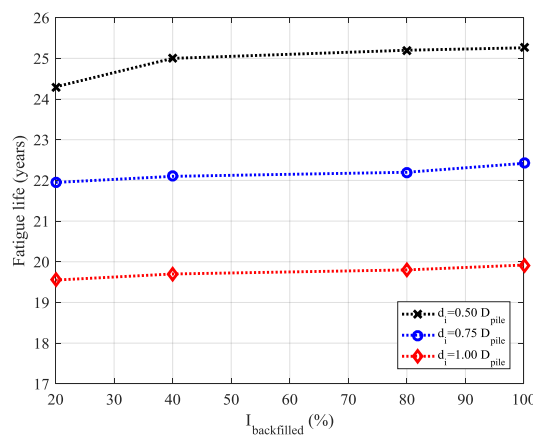
### 15 3.2.4 Scour with backfilled material

16 The influence of backfilling of the scour hole on the modal properties and fatigue life were investigated. The  
 17 scour depth before backfilling was assumed to be  $1.5D_{\text{pile}}$  and various heights and densities of the backfilled  
 18 material were considered (Fig. 9). Both the reduced material stiffness related to the density and overburden  
 19 pressure were accounted for in the p-y curves modelling the soil-structure interaction.



**Figure 9.** Diagram of the backfilling of the scour hole ( $1.5D_{pile}$ ) with respect to the original (zero scour) and scoured mudline levels.

1 A modal analysis of the backfilled systems showed that the density of the backfilled material has only a small  
 2 effect on the first natural frequency, with a maximum difference of 1% for densities as low as 20% compared  
 3 to the original soil. Similar observations were made in literature [18, 15]. Figure 10 shows the predicted fatigue  
 4 life for backfilling of the  $1.5D_{pile}$  scour hole to  $0.5D_{pile}$ ,  $0.75D_{pile}$  and  $1.0D_{pile}$  depths and with different backfilled  
 5 material densities. The material density ratio  $I_{backfilled}=100\%$  is the reference scenario where the original soil  
 6 density was used for each scour depth. It can be observed that the main factor influencing the fatigue life is the  
 7 effective scour depth, i.e., the height of the backfill, with only smaller changes due to the material density and  
 8 thus stiffness for values considered here.



**Figure 10.** Fatigue life prediction for various backfill heights and material densities from a scour depth of  $1.5D_{pile}$ , measured at  $z_s = -12m$ .

9 The fatigue life (compared to scour with a depth of  $0.5D_{pile}$ ) decreases by a maximum of one year when a  
 10  $1.5D_{pile}$  scour hole is backfilled to  $0.5D_{pile}$ , with a material density which is 20% ( $I_{backfilled}=20\%$ ) of that of the  
 11 original material ( $I_{backfilled}=100\%$ ), and thus the influence of backfilled material density can be regarded as very

1 limited. It should be noted that for this study the top soil at the location in the North Sea was taken as loose  
 2 sand (i.e. a rather soft soil), and that larger differences might be found at locations with stiffer top soils.

3 As discussed in section 1, scour and backfilling may occur periodically depending on the offshore conditions.  
 4 As the main driver on fatigue life is the effective depth, and the dependence between the two is broadly linear  
 5 (Fig. 8), the effect of backfilling can be factored in by linear interpolation of the fatigue life for different scour  
 6 depths, considering the service life percentages at each depth. Another practical application of these results  
 7 could be the prediction of fatigue life recovery by artificial backfilling of the scour hole after significant scour  
 8 is detected. Fatigue life extension and thus economic benefits for wind energy generation can then be  
 9 quantified.

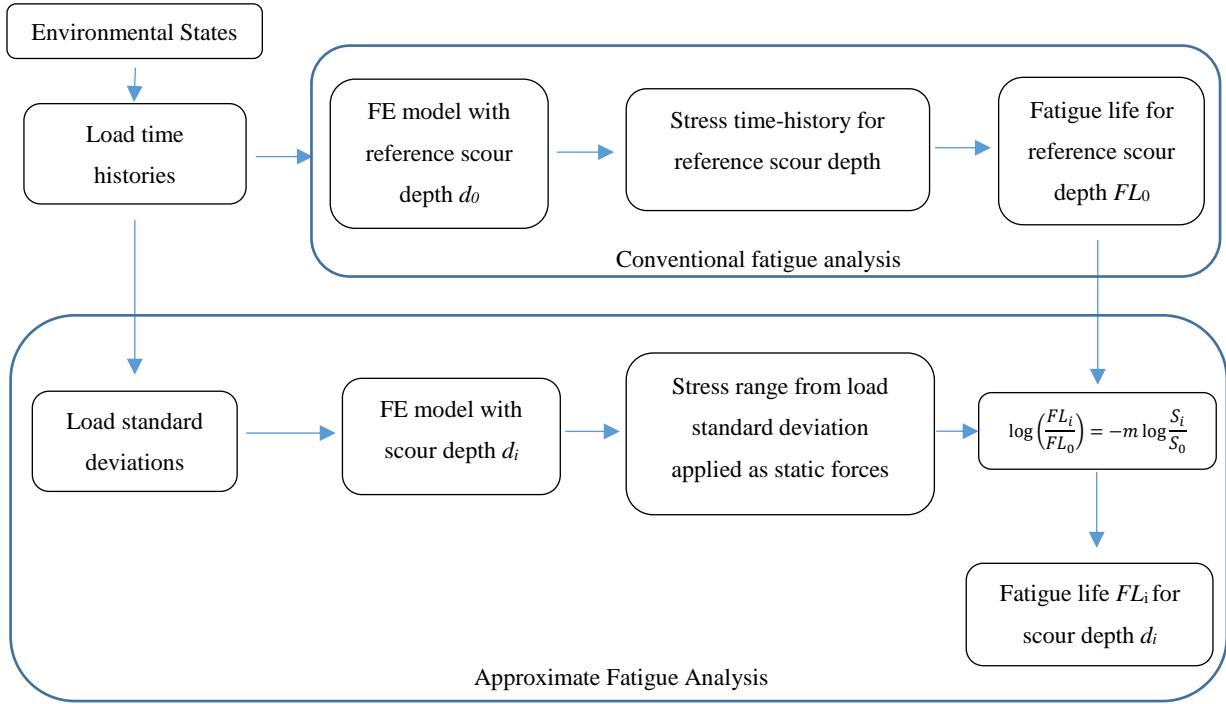
### 10 3.3 Approximate fatigue life prediction method

11 In this section, a simplified fatigue analysis method is developed, allowing the effect of scour on fatigue life  
 12 for a monopile-supported wind turbine in operation to be assessed with limited additional computational effort,  
 13 based on the full fatigue life calculation of the system at one reference scour depth. In a design situation, this  
 14 reference scour depth would probably be the maximum scour considered, but here we use the zero scour case  
 15 for consistency with the previous sections. The nonlinearity of the soil springs is assumed to be small enough  
 16 that the global behaviour of the system can be considered linear. For the typical frequency content of the  
 17 environmental loads, the dynamic response of the structure can be considered to be well approximated by that  
 18 of a single degree of freedom (SDoF) system, whose natural frequency  $\omega_1$  and damping ratio  $\zeta_1$  match that of  
 19 the first mode of the system. The response of a linear system can be modelled as the superposition of modal  
 20 responses to the wind and wave loads. As their spectral density is concentrated below the first natural frequency  
 21 of the OWT system, the system vibration can be approximated reasonably well by a SDoF system with a  
 22 natural frequency matched to the first mode of the OWT system. The displacement response of this SDoF  
 23 system with stiffness  $k$  to a harmonic force of mean  $F_m$ , amplitude  $F_a$ , and frequency  $\Omega$  is given by Eq. (1),  
 24 where the static and dynamic parts of the response have been labelled.

$$u(t) = \underbrace{\frac{F_m}{k}}_{\text{Static response}} + \underbrace{\frac{\overbrace{F_a/k}^{\text{Quasi-static}}}{\underbrace{\sqrt{\left[1 - \left(\frac{\Omega}{\omega_1}\right)^2\right]^2 + \left[2\zeta_1 \left(\frac{\Omega}{\omega_1}\right)\right]^2}}_{\text{Dynamic response}}}}_{\text{Dynamic amplification}}, \quad (1)$$

25 It was shown that the first natural frequency of the wind turbine changes only by a maximum of 2.2% for a  
 26 scour depth of  $1.5D_{\text{pile}}$ . However, the maximum bending moment in the monopile (and therefore the amplitude  
 27 of the response) showed a significant difference in magnitude and location as a result of the increased free  
 28 length of the structure and the changes in the stiffness of the surrounding soil. Therefore, the effect of scour  
 29 on the response can primarily be attributed to the quasi-static term in Eq. (1). Accordingly, an approximate  
 30 method schematically presented in Fig. 11 has been developed that uses the time-domain fatigue analysis of a

1 reference scour case to predict efficiently the fatigue life at different scour depths. It requires the static analysis  
 2 of the OWT for each scour depth and the appropriate S-N curves, but no additional time-history simulation.



**Figure 11.** Schematic flow chart of the simplified fatigue analysis method to assess scour effects.

3 The approximate analysis proceeds as follows. The full fatigue life calculation requires the wind and wave  
 4 load time histories for each environmental state  $k$ . The standard deviations of these time series are calculated  
 5 and used as a proxy for the amplitude of the dynamic forces ( $F_a$  in Eq. (1)). Applying these as static forces on  
 6 the FE model of the turbine for various scour depths  $d_i$  yields the quasi-static stress amplitude  $S_i^k$ . Denoting  
 7  $S_0^k$  the standard deviation of the stress time history obtained for the reference scour depth (obtained the same  
 8 way) and dividing  $S_i^k$  by  $S_0^k$  gives a stress ratio. As these ratios are slightly different for each environmental  
 9 state, the weighted average of the stress ratios ( $\overline{R_S}$ ) is calculated using the probability of occurrence of each  
 10 environmental state, following the damage calculation procedure.

11 In the next step, the definition of the fatigue life and S-N curves are used to express the ratio of fatigue life in  
 12 terms of the weighted stress ratios at different scour depths. The fatigue life ( $FL$ ) of the wind turbine is defined  
 13 by:

$$FL = \frac{T_{sim}}{n_s/N_s}, \quad (2)$$

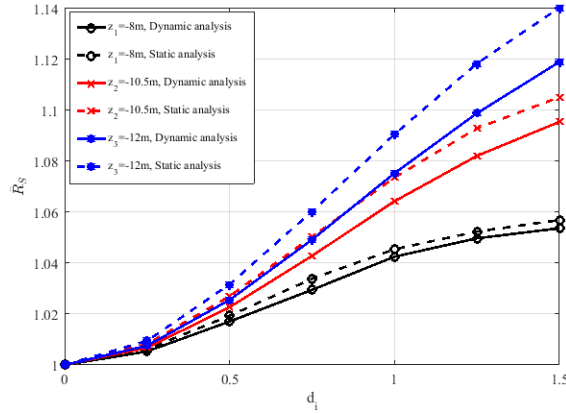
14 where  $T_{sim}$  is the simulation time,  $n_s$  is the number of stress cycles in the simulation and  $N_s$  is the number of  
 15 cycles to failure for the given stress range. For two harmonic loads with the same force frequency but different

1 magnitudes, the number of cycles in the simulation and the simulation time are the same for both stress ranges.  
 2 Thus, the ratio of fatigue lives ( $FL_i$ ) is given as:

$$\log\left(\frac{FL_2}{FL_1}\right) = \log\left(\frac{N_{S_2}}{N_{S_1}}\right) = -m(\log S_2 - \log S_1) = -m\log\left(\frac{S_2}{S_1}\right) = -m\log(\overline{R_S}), \quad (3)$$

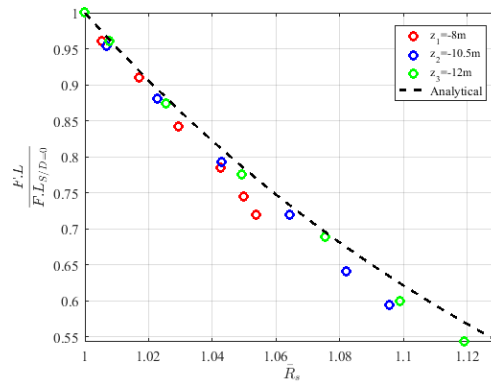
3 where  $S_{1/2}$  are the stress ranges and  $m$  is the inverse of the slope of the S-N curve. A preliminary assessment  
 4 of the stresses in the monopile showed that the majority of stress cycles are in the range that corresponds to  
 5 the part of the bi-linear S-N curve where  $m=5$  (Table 2). These stress cycles also contribute to the majority of  
 6 the fatigue damage in the monopile. Accordingly, the slope of fatigue life and stress ratios relationship in log-  
 7 log scale was considered as a constant  $m=5$  to simplify the fatigue life comparisons. Therefore Eq. (3) is  
 8 applicable to problems where the bulk of both stress ranges belong to the same branch of the S-N curves or  
 9 the cases where a linear S-N curve is assumed instead of the bi-linear S-N curve.

10 The results obtained from full fatigue analyses discussed in section 3.2 allow the validity of the approximate  
 11 method and some of its underlying assumptions to be verified. Figure 12 shows the weighted stress ratios  
 12 obtained as just described compared to the standard deviation of the dynamic stress ratios obtained through  
 13 full time domain analyses for the same scour depths. As can be observed, both curves show a similar ascending  
 14 trend with scour depth. However, the statically acquired stress ratios show a slight overestimation compared  
 15 to the stress ratios obtained from the time-domain simulations, which increases with scour depth. This can be  
 16 a result of fluctuations in the location of maximum bending moment in the time-domain simulations and the  
 17 more pronounced role of the dynamic amplification factor, due to changes in the natural frequency, at higher  
 18 scour depths. Overall this shows that the static stress ratios are a suitable substitute in fatigue damage  
 19 calculation for the full dynamic stress ratios, as the maximum difference between the absolute values of the  
 20 static and dynamic stress ratios is less than 2% measured at  $z_3=-12\text{m}$ , while at  $z_1=-8\text{m}$ , this deviation is less  
 21 than 1%.



**Figure 12.** Comparison of the weighted average of the quasi-static stress ratios with the dynamic ratios for various scour depths at  $z_1$ ,  $z_2$  and  $z_3$ .

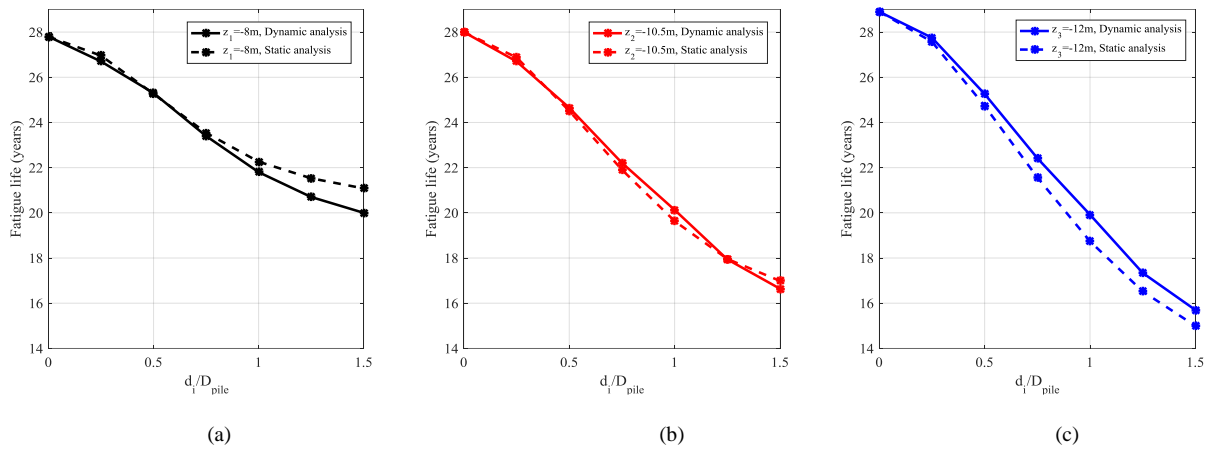
1 Figure 13 shows the relationship between the fatigue life and the weighted stress standard deviation ratios  
 2 obtained from the time-domain simulations at the three locations, as an equivalent representation of the stress  
 3 ratios. The dashed curve represents the analytical curve for the relationship between the fatigue life and stress  
 4 ratios shown in Eq. (3). Similar to the stress ratios, the fatigue life ratios were normalised with respect to the  
 5 no scour reference case. Good agreement is found between the analytical curve and the results from the time-  
 6 domain simulations with a slightly increased discrepancy for scour depths of  $1.25D_{pile}$  and  $1.5D_{pile}$  at all three  
 7 locations. This relates to the limits for the assumptions made in this study regarding the corresponding negative  
 8 inverse slope ( $m$ ) of the section of S-N curve for which the majority of damage occurs.



**Figure 13.** Changes in the fatigue life ratio with respect to the standard deviation ratio.

9 With the establishment of the close match between the static and dynamic stress response ratios, the fatigue  
 10 life of the wind turbine can be predicted from the analytical curve shown in Fig. 13 for different scour depths,  
 11 taking the respective static stress ratios into account. Figure 14 compares the fatigue life calculated by time-  
 12 domain dynamic simulations with the prediction of the fatigue life from the static analysis of the wind turbine.





**Figure 14.** Comparison of fatigue life ratios between the dynamic (simulations) and static (analytical) analysis results at (a)  $z_1 = -8\text{m}$ , (b)  $z_2 = -10.5\text{m}$  and (c)  $z_3 = -12\text{m}$ .

1 As can be observed, good agreement exists between the two predictions, with a difference of up to 5%. For  
 2 intermediate scour depths (i.e. up to  $1.0D_{\text{pile}}$ ), fatigue life ratios calculated at  $z_1 = -8\text{m}$  and  $z_2 = -10.5\text{m}$  show a  
 3 smaller difference, while the highest differences are at  $z_3 = -12\text{m}$ . However, it should be noted that the difference  
 4 between dynamic and static fatigue life ratios tend to grow with scour depth.

5 Consideration of scour uncertainties and backfilling process in the fatigue limit state design of offshore wind  
 6 turbines is a time-consuming task. With the approximate method, the fatigue life could be examined with much  
 7 less effort, using the fatigue life of the wind turbine at one scour level as a reference, and the static analysis of  
 8 the wind turbine for different scour depths, provided the modal properties of the wind turbine experience only  
 9 minor changes due to scour.

## 10 4 Conclusions

11 A study of the short and long-term influences of scour on fatigue life, performed using quasi-static, modal and  
 12 time-domain fatigue analyses for the NREL 5MW case study wind turbine model has been presented. Modal  
 13 analysis of the wind turbine support structure for various scour depths showed that the first natural frequency  
 14 of the wind turbine showed limited sensitivity to scour, especially for intermediate scour levels. The static  
 15 analysis showed that scour not only increases the maximum bending moment in the monopile by up to 9%,  
 16 but it also shifts the location of this maximum down, for instance by 6m down for the maximum considered  
 17 scour depth of  $1.5D_{\text{pile}}$ . However, the exact shift depends on the soil profile and the relative magnitude of the  
 18 loads. This phenomenon was found to be an important consequence of scour, as a variety of section details  
 19 (such as welds) could be affected as the maximum bending moment shifts within this range.

20 To account for the uncertainty in the position of the maximum bending moment, the fatigue life was  
 21 investigated at three different locations in the monopile. The fatigue analysis showed that scour influences the  
 22 fatigue damage in an offshore wind turbine significantly. This study demonstrated that a scour depth of  $1.5D_{\text{pile}}$   
 23 could result in a 45% reduction of the fatigue life of the wind turbine structure. This amount of change in the

1 fatigue life could lead to an unsafe level for the OWT. The backfilling of the scour hole can occur naturally or  
2 can be done artificially and was shown to have a significant influence on the fatigue life. The relative material  
3 density of the backfilled soil does not have a significant effect on the modal properties or fatigue life of the  
4 wind turbine, so that the backfilled depth can be treated as an effective scour depth for simplicity in practice.  
5 The fatigue demand on the structural details reduces in direct proportion of the backfilled period. This way the  
6 cost-effectiveness of artificial backfilling and consequent fatigue life extension can be quantified.

7 This study showed that scour mainly influences the response of the system in a quasi-static manner, which  
8 leads to the change in the fatigue life of the wind turbine. A comparison was made between the changes in the  
9 fatigue life from the dynamic analysis of the wind turbine in the time-domain against the results from a static  
10 analysis of the wind turbine, and a good match was found. Based on this finding, an approximate fatigue  
11 analysis method was proposed, requiring one full time-domain simulation as the reference and using the  
12 statically acquired stress ratios to calculate the fatigue life. It was argued that if the changes in the natural  
13 frequency of the wind turbine are small, this approach can be applied in practice to save a significant amount  
14 of computation time in analysing the effect for different scour and backfilling scenarios. Different scenarios  
15 and costs for fatigue life extension of OWTs could be assessed for the economic benefit to further reduce  
16 renewable energy generation costs.

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## APPENDIX A

**Table A1.** Environmental states, based on data from [22].

State	V <sub>w</sub> (m/s)	T <sub>Z</sub> (s)	H <sub>S</sub> (m)	P <sub>State</sub> (%)
1	4	3	0.5	3.95
2	4	4	0.5	3.21
3	6	3	0.5	11.17
4	6	4	0.5	7.22
5	8	3	0.5	11.45
6	8	4	1.0	8.68
7	10	3	0.5	5.31
8	10	4	1.0	11.33
9	12	4	1.0	5.86
10	12	4	1.5	6.00
11	14	4	1.5	4.48
12	14	5	2.0	3.26
13	16	4	2.0	1.79
14	16	5	2.5	3.10
15	18	5	2.5	1.74
16	18	5	3.0	0.80
17	20	5	2.5	0.43
18	20	5	3.0	1.14
19	22	5	3.0	0.40
20	22	6	4.0	0.29
21	24	5	3.5	0.15
22	24	6	4.0	0.10

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