Journal of Physics: Conference Series

PAPER • OPEN ACCESS

New theory of stellar convection without the mixing-length parameter: new stellar atmosphere model

To cite this article: Stefano Pasetto *et al* 2018 *J. Phys.: Conf. Ser.* **940** 012020

IOP Conf. Series: Journal of Physics: Conf. Series **940** (2018) 012020 doi:10.1088/1742-6596/940/1/012020

New theory of stellar convection without the mixing-length parameter: new stellar atmosphere model

Stefano Pasetto[1,2], Cesare Chiosi[3], Mark Cropper[2], Eva K. Grebel[4]

[1] Observatories of the Carnegie Institution of Washington, 813 Santa Barbara, Street, Pasadena, CA 91101, USA

[2] Mullard Space Science Laboratory, University College London, Holmbury , St. Mary, Dorking, Surrey RH5 6NT, United Kingdom

[3] Dept. of Physics & Astronomy "Galileo Galilei", University of Padua, Vicolo dell'Osservatorio, 5, 35141 Padova PD, Italy

[4]Astronomisches Rechen-Institut, Zentrum für Astronomie der Universität Heidelberg, Mönchhofstr 12-14, 69120 Heidelberg, Germany

E-mail: spasetto@carnegiescience.edu

Abstract. Stellar convection is usually described by the mixing-length theory, which makes use of the mixing-length scale factor to express the convective flux, velocity, and temperature gradients of the convective elements and stellar medium. The mixing-length scale is proportional to the local pressure scale height of the star, and the proportionality factor (i.e. the mixing-length parameter) is determined by comparing the stellar models to some calibrator, i.e. the Sun. No strong arguments exist to suggest that the mixing-length parameter is the same in all stars and all evolutionary phases and because of this, all stellar models in the literature are hampered by this basic uncertainty.

In a recent paper [1] we presented a new theory that does not require the mixing length parameter. Our self-consistent analytical formulation of stellar convection determines all the properties of stellar convection as a function of the physical behavior of the convective elements themselves and the surrounding medium. The new theory of stellar convection is formulated starting from a conventional solution of the Navier-Stokes/Euler equations expressed in a non-inertial reference frame co-moving with the convective elements. The motion of stellar convective cells inside convective-unstable layers is fully determined by a new system of equations for convection in a non-local and time-dependent formalism.

The predictions of the new theory are compared with those from the standard mixing-length paradigm with positive results for atmosphere models of the Sun and all the stars in the Hertzsprung-Russell diagram.

1. Introduction

The transfer of energy by convection is of paramount importance in all the stars. High-mass stars, roughly for masses $M > 1.3M_{\odot}$ contain fully convective cores, all stars $M \in [0.1, 100]\, M_{\odot}$ have outer convective envelopes, and finally stars smaller in mass than $M < 0.3M_{\odot}$ are fully convective. Despite its great importance, a satisfactory treatment of stellar convection in stars is still open to debate and a self-consistent treatment of the physics of convective energy transfer is still missing. The ideal goal would be to obtain a set of self-consistent equations, i.e. a set of equations resulting from physical assumptions without ad-hoc parameters to be determined by means of suitable calibrators (usually the Sun).

The most successful theory dealing with the external convection is the mixing-length theory (MLT) developed long ago by [2] and [3]. The MLT is the paradigm reference to which every new theory has to be compared with, because of its success over decades in which it has been used. In the MLT, the convective elements are supposed to travel a mean-free-path l_m [4]. l_m which is assumed to be proportional to the natural distance scale h_P, given by the pressure stratification of the star, the proportionality constant being the mixing-length (ML) parameter Λ_m, defined implicitly as $l_m \equiv \Lambda_m h_P$. The parameter Λ_m is derived from comparing the theoretical luminosity, radius and effective temperature of a stellar model for the Sun to its observational values.

In a recent paper [1], we developed the first theory of stellar convection in which the solar properties are reproduced without making use of free parameters. In the following we will refer to this theory as the scale-free convection (SFC) theory. In this approach the authors obtained a solution for the equations governing stellar atmospheres that self-consistently predict the energy transport, luminosities, radii and effective temperatures all along the evolutionary sequence of a star.

2. A mixing-length free set of equation for stellar atmospheres

The ideas at the base of the SFC theory are in principle simple. Let us think for example of the upward motion of a convective element. The evolution of a single convective cell can be considered as the sum of the upward motion and the expansion. In the MLT only the upward motion is considered. The free-parameter of the MLT stems indeed from the assumptions made to describe the upward motion of the convective elements. Therefore, the only logical alternative in developing a new theory is to consider the expansion of the convective as the main driver of the whole process. To make the upward motion ineffective it is enough to write the equations describing the motion of a convective element in a reference frame co-moving with it. In such a case, all equations are referred to convective element and the latter is at rest. [1] name S_1 this comoving reference, to distinguish it from the inertial reference frame centered on the star and named S_0. In [1] the hydrodynamic equations have been integrated accounting for the non-inertial apparent forces that arise in the treatment of any physical system evolving in S_1. Under the assumption that viscous terms are much smaller than the inertial ones and the magnetic field is negligible, the potential flow approximation can be adopted and suitably formulated in S_1 (mathematical formulation in S_1 is slightly more complicated). In order to keep the equations analytically treatable, [1] limit the analysis to the linear regime. If we limit ourselves to the subsonic regime of the stellar convection, the velocity of a convective element, $\mathbf{v}$, will be much smaller that its expansion rate, $\left\|\frac{d\xi}{dt}\right\| \equiv \left\|\dot{\xi}\right\|$, where ξ is the size of the convective elements. Therefore, a linear theory on the small parameter $\varepsilon \equiv \frac{\|\mathbf{v}\|}{\|\dot{\xi}\|} \ll 1$ can be developed. In particular, within the framework of this linear approximation, in the equations governing the evolution of the expansion rate of a convective element, the role of the inertia of the fluid displaced by the motion of the convective element turns out to be important. In contrast, this term has always been neglected in the literature and the evolution of the convective elements was always studied only in relation to its vertical motion (this led indeed to the problem of the mixing-length scale).

As a result of this approach, we obtain a new system of equations for the energy transfer as a function of the radiative plus conductive flux $\varphi_{\mathrm{rad|cnd}}$, the convective flux φ_{cnv}, the average temperature over pressure gradient $\nabla_e \equiv \left|\frac{d\ln T}{d\ln P}\right|_e$ of the element, and the stellar gradient ∇. Moreover, two extra variables, the mean velocity $\bar{v}$ and the mean size $\bar{\xi}_e$ of the convective elements are obtained as a result of the solution of the system. All these physical quantities are a function of the pressure P, temperature T, density ρ, specific heat at constant pressure c_p, adiabatic gradient of temperature over pressure $\nabla_{\rm ad} \equiv \left|\frac{d\ln T}{d\ln P}\right|_{\rm ad}$, radiative gradient $\nabla_{\rm rad}$, molecular weight gradient $\nabla_\mu \equiv \frac{d\ln\mu}{d\ln P}$, the gravity g, the opacity κ. Finally, all these quantities are a function of the position inside the star and time.

The general form of the system of equations obtained in [1] is:

$$\begin{cases} \varphi_{\rm rad/cnd} &=& \frac{4ac}{3}\frac{T^4}{\kappa h_P \rho}\nabla \\ \varphi_{\rm rad/cnd}+\varphi_{\rm cnv} &=& \frac{4ac}{3}\frac{T^4}{\kappa h_P \rho}\nabla_{\rm rad} \\ \bar{v}^2 &=& \frac{\nabla-\nabla_e-\frac{\varphi}{\delta}\nabla_\mu}{\frac{3h_P}{2\delta\bar{v}\tau}+\left(\nabla_e+2\nabla-\frac{\varphi}{2\delta}\nabla_\mu\right)}\bar{\xi}_e g \\ \varphi_{\rm cnv} &=& \rho c_P T\left(\nabla-\nabla_e\right)\frac{\bar{v}^2\tau}{h_P} \\ \frac{\nabla_e-\nabla_{\rm ad}}{\nabla-\nabla_e} &=& \frac{4acT^3}{\kappa\rho^2 c_P}\frac{\tau}{\bar{\xi}_e^2} \\ \bar{\xi}_e &=& \frac{g}{4}\frac{\nabla-\nabla_e-\frac{\varphi}{\delta}\nabla_\mu}{\frac{3h_P}{2\delta\bar{v}\tau}+\left(\nabla_e+2\nabla-\frac{\varphi}{2\delta}\nabla_\mu\right)}\bar{\chi}, \end{cases} \tag{1}$$

where a is the radiation-density constant, and c the speed of light and for the purposes of this paper $\bar{\chi}$ is a function of time linking size to velocity, i.e. a monotonic linear map (a bijection) between time, velocity and size of the convective elements (see Appendix A of [1] for more details). In addition to this we need a numerical procedure to solve all the above equations together with their boundary conditions. We adopt the code for stellar models written by [5] and largely modified and updated by the Padua group: [6] with semi-convection, [7] with ballistic convective overshoot from the core, [8] with envelope overshoot, [9], [10] and [11] with turbulent diffusion, finally the many revision and improvements described in [12], [13], [14]. In the future we will implement the new theory of convection also in the twin-code developed independently by [15]. The SFC theory of [1] and the classical MLT are run in parallel so that comparison is possible.

3. Results: the model matching the Sun

We present here a comparison between the standard MLT and the SFC theory. The results are obtained from solving the system of Eq.1 for each layer of a stellar atmosphere governed by the equations considered in the previous section. We consider the stellar track of [14] best fitting the present position of the Sun on the HRD e.g., $\log_{10}\{L/L_\odot, T_{\rm eff}\} \cong \{0.000, 3.762\}$ with standard chemical composition $\{X, Y\} = \{0.71, 0.27\}$. The results are shown in Fig.1. In the same plot we show also the predictions of the MLT with $\Lambda_m = 1.65$ (the MLT is according to the version presented in [4], so that comparison between SFC theory and MLT is possible. Both the temperature gradients ∇ and ∇_e and fluxes $\varphi_{\rm rad|cnd}$ and $\varphi_{\rm cnv}$ predicted by SFC theory and MLT are in mutual agreement over an impressive range in pressure of almost ten orders of magnitude.

4. Conclusions

We have presented here the first results of the integration of stellar atmospheres with SFC theory developed in [1]. We have set up a numerical code to systematically integrate as function of time and position the equations presented in [1] and have run it in parallel with the standard MLT. All the results achieved by MLT are successfully recovered by the SFC theory without making use of any adjustable free-parameter. We argue that the new theory despite its linear formulation is able to capture the essence of the convection in the stellar atmospheric layers in a simple manner. Furthermore, the SFC theory has a predictive potential that descriptive analysis of numerical simulations still miss. To be able to generate numerical simulations with millions of degrees of freedoms does not automatically mean that we fully understand them. An

IOP Conf. Series: Journal of Physics: Conf. Series **940** (2018) 012020 doi:10.1088/1742-6596/940/1/012020

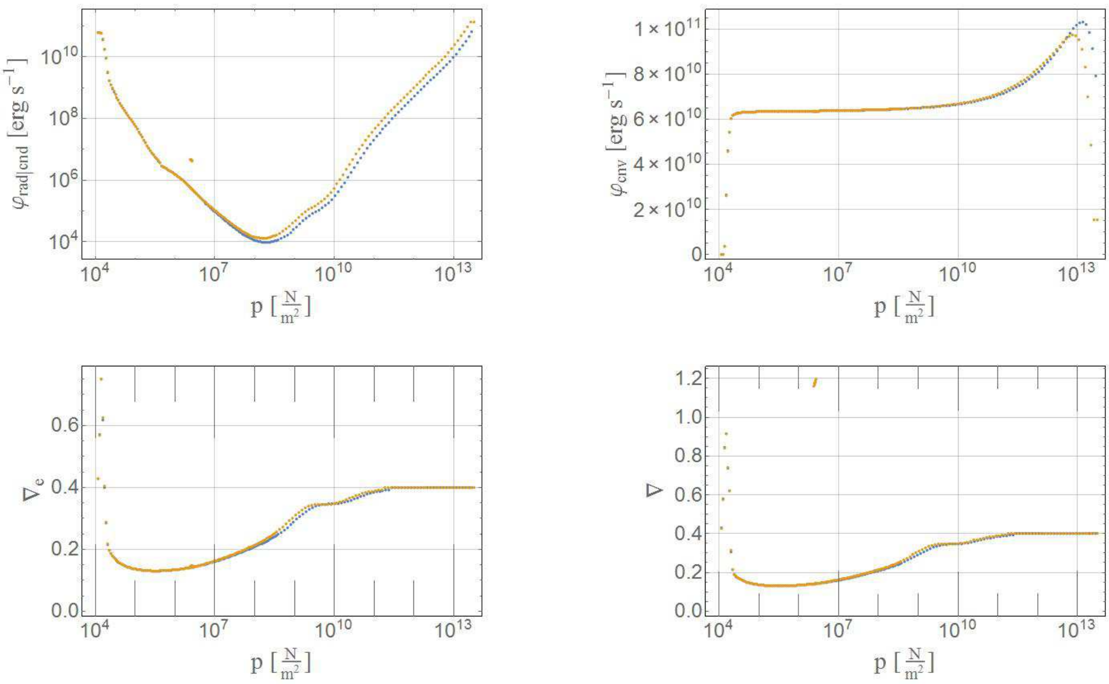

Figure 1. Solar fluxes and temperature gradient profiles for the internal pressure stratification of the star. The upper panels show the expectation for $\varphi_{\mathrm{rad|cnd}}$ on the left and φ_{cnv} on the right. Yellow refers to our theory, blue to the MLT.

emblematic example of these problems has recently been discussed by [17] where the authors fail to close the equations suggested by their hydrodynamic simulations. Their ultimate goal is to search 1D theory based on 3D simulations passing through a 2D intermediate stage. We dare to claim here that a 1D, parameter-free theory already exists [1] and that this is the right trail to follow.

References

[1] Pasetto S., Chiosi C., Cropper M., Grebel E. K., 2014, MNRAS, 445, 3592
[2] Biermann L., 1951, Zeitschrift fur Astrophysik, 28, 304
[3] Bohm-Vitense E., 1958, Zeitschrift fur Astrophysik, 46, 108
[4] Kippenhahn R., Weigert A., Weiss A., 2012, Stellar Structure and Evolution, Cambridge Press.
[5] Hofmeister E., Kippenhahn R., Weigert A., 1964, Zeitschrift fur Astrophysik, 59, 215
[6] Chiosi C., Summa C., 1970, Ap&SS, 8, 478
[7] Bressan A. G., Chiosi C., Bertelli G., 1981, A&A, 102, 25
[8] Alongi M., Bertelli G., Bressan A., Chiosi C., 1991, A&A, 244, 95
[9] Deng L., Bressan A., Chiosi C., 1996a, A&A, 313, 145
[10] Deng L., Bressan A., Chiosi C., 1996b, A&A, 313, 159
[11] Salasnich B., Bressan A., Chiosi C., 1999, A&A, 342, 131
[12] Bertelli G., Bressan A., Chiosi C., Ng Y. K., Ortolani S., 1994, Memorie della Societ Astronomia Italiana, 65, 689
[13] Bertelli G., Bressan A., Chiosi C., Ng Y. K., Ortolani S., 1995, A&A, 301, 381
[14] Bertelli G., Nasi E., 2001, AJ, 121, 1013
[15] Bertelli G., Nasi E., Girardi L., Chiosi C., Zoccali M., Gallart C., 2003, AJ, 125, 770
[16] Weiss, A., Schlattl, H., 2008, ApSS, 316, 99
[17] Tuteja G. S., Khattar D., Chakraborty B. B., Bansal S., 2010, Int. J. Contemp. Math. Sciences, 5, 1065
[18] Arnett W. D., Meakin C., Viallet M., Campbell S. W., Lattanzio J., Mocak M., 2015, ApJ, 809, 30