Optimal System Decomposition for Distributed Fault Detection: Insights and Numerical Results ⋆

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Abstract: The paper deals with the problem of defining the optimal topology for a distributed fault detection architecture. A partition-based distributed fault detection method is considered based on the model of the system. The system is decomposed into subsystems and each subsystem is monitored by one local diagnoser. Non-overlapping decompositions are considered. A novel cost function is proposed to measure the detectability properties of a distributed fault detection method, depending on the topology of the detection framework. Different objective functions are taken into account and compared in order to analyze the influence of the decomposition on fault detection performance. Preliminary numerical results show that the minimization of the coupling between subsystems could not be always the best choice for the fault detection performance, and that the proposed cost function minimization allows the reduction of the detection time with the considered fault detection method.

Keywords: Fault detection, Distributed systems, Optimal decomposition, Topology

1. INTRODUCTION

The work presented in this paper is motivated by the growing interest towards monitoring methods for large-scale systems using distributed architectures (see, for instance Stanković et al. (2010); Shames et al. (2011); Zhang and Zhang (2012); Reppa et al. (2015); Lan and Patton (2016); Blanke et al. (2016); Gupta and Puig (2016); Davoodi et al. (2016); Riverso et al. (2016); Boem et al. (2017)). As it is well known, the advantages of adopting a distributed architecture are scalability, robustness and reduction of computational costs. The adoption of distributed approaches commonly implies the definition of a partition of the original system into subsystems, where each subsystem is monitored by a local agent, the Local Fault Diagnoser (LFD). More specifically, the term *system decomposition* (Šiljak (1978)) refers to the clustering of state, input, and output system variables into subsets, i.e. the subsystems. Since each LFD is devoted to monitor a subsystem, the decomposition of the overall system defines the topology of the diagnosis architecture. In the distributed fault detection frameworks recently proposed in the literature, the decomposition in subsystems is a given element of the problem. However, in many cases the choice of the decomposition is arbitrary. Therefore, some important questions arise: does the decomposition influence the fault detection performance? If so, is it possible to determine what the best decomposition is? The objective of this paper is to answer these questions.

In the computer science community, the problem of graph decomposition has been widely investigated. Many graph partitioning algorithms have been proposed (Karypis and Kumar (1998); Schloegel et al. (2000); Hager et al. (2013); Buluç et al. (2016)), mainly with the goal of minimizing the weights on the cuts of the graph.

Furthermore, the decomposition problem has been extensively studied in the field of decentralized and distributed control (Siljak (2011); Ocampo-Martinez et al. (2011); Anderson and Papachristodoulou (2012); Motee and Sayyar-Rodsari (2003); Langwen and Jingcheng (2012)), presenting algorithms both for overlapping and non-overlapping decompositions. However, in the Fault Detection and Isolation (FDI) context, it still represents a topic under research and the decision is often based either on the physical structure of the system or on traditional approaches tailored for control applications, such as the minimization of the coupling (Kyriacou et al. (2017)). Other recent works have considered the decomposition problem in the context of FDI, but not focusing on the detection performance. In Bregon et al. (2014), a decentralized fault diagnosis task using structural model decomposition is considered, but an event-based method is implemented in a qualitative approach. In Staroswiecki and Amani (2014), the topology of the information pattern is studied in order to allow fault-tolerant control reconfiguration. In Grbovic et al. (2012), the decomposition is designed using the Sparse Principal Component Analysis algorithm, but the

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proposed decentralized fault detection architecture is a data-driven approach, while our method is a model-based one. In these works, in general, the solution represents a compromise between the desirable reduction of the computation cost (leading to a large number of small subsystems) and a reasonably low communication cost (preferring a small number of subsystems). But another crucial aspect is represented by the performance in detecting a fault and in terms of presence of false alarms.

The aim of this work is to study the decomposition problem specifically for the fault detection task. The goal is to understand how the system decomposition and the adoption of distributed approaches can influence the detectability performance. An interesting and challenging research problem is how to define a suitable objective function able to represent the performance of the fault detection task.

The main contributions of the paper are: the analysis and comparison of different objective functions for the optimal decomposition problem; the definition of a metric to measure the detectability properties of a distributed model-based fault detection method; the analysis of the influence of the system decomposition on the fault detection performance. In particular we base our study on the novel fault detection approach proposed in Boem et al. (2016), formulated for non-overlapping decompositions, and we present some numerical results obtained applying it on several use-cases after decomposing the systems according to different decompositions. We show that the minimization of the coupling among subsystems, a metric usually used in control applications, does not imply in general the optimization of the detectability goal, and then it may lead to suboptimal solutions when used for distributed fault detection. Moreover, we show that the minimization of the proposed function cost is able to reduce the fault detection time with the considered distributed fault detection method.

It is worth noting that, to the best of the authors' knowledge, it is the first time that the system decomposition problem is analyzed for non-overlapping frameworks, specifically for the distributed fault detection purposes, taking the detection performance into account. In Boem et al. (2015) preliminary results are presented for the definition of the optimal FDI topology taking the detectability properties into account, but focusing on specific faults and state trajectories, in the case of overlapping decompositions.

Notation. Given a stochastic variable x, we represent as $\mathbb{E}[x]$ its expected value, and as $\mathrm{Var}[x]$ its variance.

2. PROBLEM FORMULATION

Let us consider a discrete-time large-scale linear system Σ, modeled as

$$\begin{aligned} \Sigma \ : \ x(k+1) &= Ax(k) + w(k), \\ y(k) &= Cx(k) + v(k), \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $y \in \mathbb{R}^p$ and $u \in \mathbb{R}^m$ are the state, output and input vectors, and $w \in \mathbb{R}^n$ and $v \in \mathbb{R}^m$ are the process and measurement noises. We assume w and v are zero-mean white noises with covariance matrices Q and R, respectively. In order to implement a distributed model-based fault detection framework, it is necessary to decompose the system into $M \geq 1$ subsystems Σ_i, each with a local state vector $x_i \in \mathbb{R}^{n_i}$, a local output vector $y_i \in \mathbb{R}^{p_i}$ and a local input vector $u_i \in \mathbb{R}^{m_i}$. The local vectors are obtained by clustering the components of the monolithic system vectors x, y and u. The decomposition has to cover the whole original system and can be overlapping or non-overlapping (Šiljak (1978)) depending on whether or not some state variables are shared among different subsystems. We will consider non-overlapping decompositions.

Once the decomposition problem is solved, the monitored large-scale system Σ can be modeled by M interconnected subsystems, where each subsystem Σ_i, with $i = 1, \dots, M$, is described by the following equations:

$$\begin{aligned} \Sigma_i \ : \ x_i(k+1) &= A_{ii}x_i(k) + \sum_{j \neq i} A_{ij}x_j(k) + w_i(k), \\ y_i(k) &= C_i x_i(k) + v_i(k), \end{aligned} \tag{2}$$

where $x_i(k), w_i(k) \in \mathbb{R}^{n_i}$ and $y_i(k), v_i(k) \in \mathbb{R}^{p_i}$. We assume that $w_i(k)$ and $v_i(k)$ are zero-mean white noises, $\mathbb{E}\{w_i(k)w_j^\top(k)\} = Q_i\delta_{ij}$, $\mathbb{E}\{v_i(k)v_j(k)\} = R_i\delta_{ij}$ (with $R_i > 0$), and $\mathbb{E}\{w_i(k)v_j^\top(h)\} = 0$ for all $i, j = 1, \dots, M$ and $h, k \geq 0$. In the above notation, δ_{ij} is the Kronecher delta function.

The goal of the paper is to understand the influence of the decomposition on the fault detection performance and what is the best choice among all the possible decompositions of the system Σ. To do this, we now briefly introduce a specific distributed model-based fault detection method we will use to evaluate the role of the decomposition.

3. MODEL-BASED DISTRIBUTED FAULT DETECTION APPROACH

We briefly describe the distributed model-based fault detection method presented in Boem et al. (2016). In the considered approach, the local diagnosers monitor each subsystem by communicating with neighboring subsystems and estimating the local state. To estimate the local state vector each diagnoser locally implements a Luenberger observer:

$$\begin{aligned} \hat{x}_i(k+1) &= \sum_{j \in \mathcal{N}_i} \{A_{ij}\hat{x}_j(k) + L_{ij}[y_j(k) - C_j\hat{x}_j(k)]\} \\ \hat{y}_i(k) &= C_i\hat{x}_i(k), \end{aligned} \tag{3}$$

where $\mathcal{N}_i$ denotes the set of predecessors of subsystem i defined as $\mathcal{N}_i = \{j \,|\, A_{ij} \neq 0\}$. We also define $\mathcal{S}_i$, the set of successors of subsystem i, as $\mathcal{S}_i = \{j \,|\, i \in \mathcal{N}_j\}$. Note that i is in general included in $\mathcal{S}_i$ and $\mathcal{N}_i$. For later use, we also define the set of strict neighbors and successors $\tilde{\mathcal{N}}_i = \mathcal{N}_i \backslash \{i\}$ and $\tilde{\mathcal{S}}_i = \mathcal{S}_i \backslash \{i\}$, respectively.

Then, for fault detection purposes, each local diagnoser computes a *local residual signal*

$$r_i(k) := y_i(k) - \hat{y}_i(k)$$

and uses it, together with a properly designed threshold, to monitor the corresponding subsystem. Given $\alpha > 1$ and taking advantage of the Chebishev inequality, for each l-th component $r_{i,l}$ of the residual r_i we can write

$$\Pr(\mathbb{E}[r_{i,l}] - \alpha\sqrt{\mathtt{Var}[r_{i,l}]} \leq r_{i,l} \leq \mathbb{E}[r_{i,l}] + \alpha\sqrt{\mathtt{Var}[r_{i,l}]}) \geq 1 - \frac{1}{\alpha^2}.$$

We define component-wise the time-varying threshold

$$\bar{r}_{i,l}(k) = \alpha\sqrt{\mathtt{Var}[r_{i,l}(k)]}. \tag{4}$$

Therefore, since $\mathbb{E}[r_i(k)] = 0$ for all k, in healthy conditions

$$|r_i(k)| \leq \bar{r}_i(k),$$

with a probability greater than $1 - \frac{1}{\alpha^2}$.

In order to compute the fault detection thresholds $\bar{r}_i(k)$, we analyze the local residual that can be written as:

$$r_i(k) = C_i e_i(k) + v_i(k),$$

where $e_i(k) = x_i(k) - \hat{x}_i(k)$ is the local estimation error. Hence, considering Eq. (4), we would need the estimation error covariance matrix $\Pi(k)$, but this matrix cannot be computed in a distributed way. In Boem et al. (2016), the authors overcome this limitation by proposing a suitable upper bound for the covariance matrix of the estimation error which can be updated in a distributed way.

The upper bound $B(k)$ for the estimation error covariance matrix $\Pi(k)$ is a block diagonal matrix. Each diagnoser updates the related time-varying block matrix $B_i(k)$ according to:

$$B_i(k+1) = \sum_{j \in \mathcal{N}_i} \Big[(\tilde{A}_{ij} - L_{ij}\tilde{C}_j)B_j(k)(\tilde{A}_{ij} - L_{ij}\tilde{C}_j)^\top + L_{ij}\tilde{R}_j L_{ij}^\top\Big] + Q_i\,, \tag{5}$$

where, for all $i, j = 1, \ldots, M$, $\tilde{A}_{ij} = \sqrt{\varsigma_j}A_{ij}$, $\tilde{C}_i = \sqrt{\varsigma_i}C_i$, and $\tilde{R}_i = \varsigma_i R_i$, and $\varsigma_i = |\mathcal{S}_i|$.

In Boem et al. (2016) it is proven that $B_i(k)$ is an upper bound to $\Pi_i(k)$, for all $i = 1, \ldots, M$ and for all $k \geq 1$. Furthermore it is proven that this upper bound converges, given some conditions on L_{ij}. The proposed bound can be used for the computation of the local thresholds as:

$$\bar{r}_{i,l}(k) = \alpha\sqrt{\left[C_i B_i(k) C_i^\top + R_i\right]_{ll}}, \tag{6}$$

where $[M]_{ij}$ denotes the (i,j)-th element of matrix M. In the following, for analysis purposes, we will assume $C_i = I$ for all $i = 1, \ldots, M$.

4. OPTIMAL DECOMPOSITION AND OBJECTIVE FUNCTIONS

Based on the fault detection method presented in the previous section, we now propose a novel cost function to define the optimal decomposition problem. This allows us to measure the performance of the fault detection method in terms of detectability, that is, the ability of a method to detect a fault. In order to be robust towards the presence of uncertainties and noise, and to reduce the presence of false-alarms, a certain level of conservativeness is introduced in the method by the detection threshold. As illustrated in the previous section, the fault detection thresholds for each residual component depend on the variances of the associated estimation errors, as evident from (5), which depend on the specific chosen decomposition. In order to allow a distributed framework, instead of the actual covariance matrix $\Pi(k)$ we consider the upper bound $B(k)$. Hence, the thresholds depend on the elements on the diagonal of $B(k)$. Given the same residual signal, we would like the threshold values to be the lowest possible in order to have narrow detection intervals, and consequently a faster detection, but continuing to guarantee the same false-alarm rate, which is lower than $\frac{1}{\alpha^2}$ in the proposed approach by exploiting the Chebishev inequality. Consequently, we consider as an objective function to be minimized for the choice of the best decomposition, the following quantity:

$$tr(\bar{B}) = \sum_i (\bar{b}_{ii}), \tag{7}$$

where $\bar{B}$ is the steady-state value of matrix B. The choice of the steady state matrix allows the computation of an index that does not depend on the time and on the specific trajectories of the system. Moreover, the convergence properties of matrix B are presented in Boem et al. (2016) and we observed in simulation that the decompositions ranking by trace remains essentially unchanged after the first time-step. Thus, the trace of the steady state value of matrix B can be used as a metric for the choice of the best decomposition in terms of detectability.

This objective function has been compared with other fundamental cost functions in order to highlight possible correlations. In particular we considered:

(1) Communication cost, expressed in terms of number of state variables that must be exchanged at each time-step between the LFDs:

$$C_{comm} = \sum_{i \in M} \sum_{j \in \tilde{\mathcal{N}}_i} (n_j). \tag{8}$$

(2) Computational cost, expressed in terms of number of operations - mainly products - the agent monitoring the most computationally expensive subsystem has to compute for estimation, evaluation of the residual and the threshold (under the assumption that C is an identity matrix):

$$C_{comp} = \mathtt{max}_{i \in M} \sum_{j \in \mathcal{N}_i} (5n_i + 2n_i n_j + 4n_i n_j^2). \tag{9}$$

(3) Coupling or level of interaction between subsystems, which is an objective function traditionally used for the determination of the best decomposition for control applications, and it is expressed as the sum of the weights on cut edges of the system structural graph:

$$C_{coupling} = \sum_{i \in M} \sum_{j \in \tilde{\mathcal{N}}_i} \sum_r \sum_s (a_{ij_{rs}}), \tag{10}$$

where $a_{ij_{rs}}$ is the (r,s) element of matrix A_{ij}, with $i \neq j$.

We are now going to evaluate these objective functions for all the possible decompositions in some use-cases.

5. NUMERICAL RESULTS AND DISCUSSION

In order to better understand the effect of decomposing a system into subsystems on the distributed fault detection performance, we have applied the distributed approach described in Section 3 to several systems. For each system, we evaluated the previously defined cost functions for all the possible decompositions.

All the possible decompositions have been determined combining permutation and partitions of the dimension n of the system state, excluding redundancies. In Table 1 the number of decompositions for several systems dimensions is reported. It is worth noting that this kind of analysis would not be feasible for real large-scale systems, since the number of possible decompositions grows exponentially with the state dimension. This analysis is used in this paper in order to evaluate which objectives functions are suitable in order to determine the best decomposition for fault detection purposes.

Table 1. Number of decompositions N for different state vector dimensions n

n	5	6	7	8	9	10
N	52	203	877	4140	21147	115975

The notation we use to describe a decomposition is composed of two strings:

- a *permutation string* with the indices of the state variables, written in the order they have to be considered;
- a *division string* with the dimension (number of state variables) of each subsystem, referring to the order of the permutation string.

Then, for example, the permutation string "14725836" and the division string "332" refer to a decomposition in which the state variables with index 1, 4 and 7 are grouped together in one subsystem of dimension 3; the state variables with index 2, 5 and 8 are grouped in a second subsystem of dimension 3, and finally state variables 3 and 6 are included in a third subsystem of dimension 2.

5.1 Application examples

We now present some use-cases we have considered for analysis in this paper. Other examples have been also analyzed in Gei (2017), but, due to length limits, these have been selected for their significance:

- A reduced version of the system studied in Kyriacou et al. (2017) concerning the distributed contaminant detection in intelligent buildings. More specifically, we consider a portion of the floor plan comprising the rooms from 1 to 8. In Kyriacou et al. (2017), the authors select the decomposition based on the minimization of the coupling. Hence, it is of interest to determine whether or not this choice leads in turn to the maximization of detectability, i.e. the minimization of the detection threshold values.
- A tanks system composed of 8 tanks as in Figure 1.
- A cascade tanks system as the one shown in Figure 2.
- A tanks system as the one shown in Figure 3, which has a hierarchical structure being described by a lower triangular discrete-time state matrix.

The considered tanks systems are variations of the quadruple-tanks process described in Shneiderman and Palmor (2010). A complete description of the models and

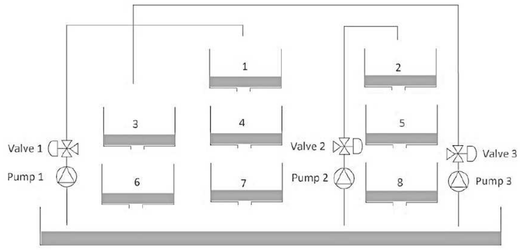

Fig. 1. Decentralised tanks system composed of 8 tanks.

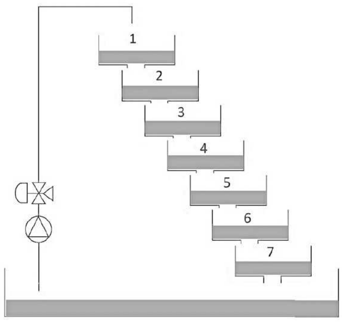

Fig. 2. Cascade tanks system.

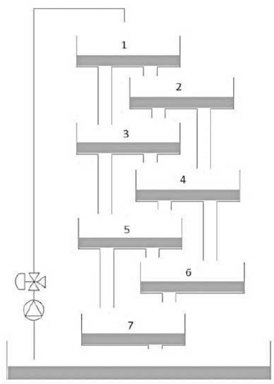

Fig. 3. Hierarchical tanks system.

the parameters values can be find in Gei (2017). We simulate these systems with covariance matrices $Q = R = 0.000001\ I$.

5.2 Comparison between different cost functions

In Figure 4, 5 and 6 we can see the value of detectability cost function (7), as well as the computational cost, the communication cost and the coupling cost function, all normalized between 0 and 1, for all the possible decompositions for the building model, the eight tanks model and the cascade tanks model, respectively. The objective of the decomposition optimization problem is to minimize all these cost functions. In order to show the results, the decompositions have been ordered as follows:

- decompositions with the same number of subsystems are gathered together and the groups are separated by the vertical dashed lines;
- inside each group the results are ordered according to the increasing value of cost function (7).

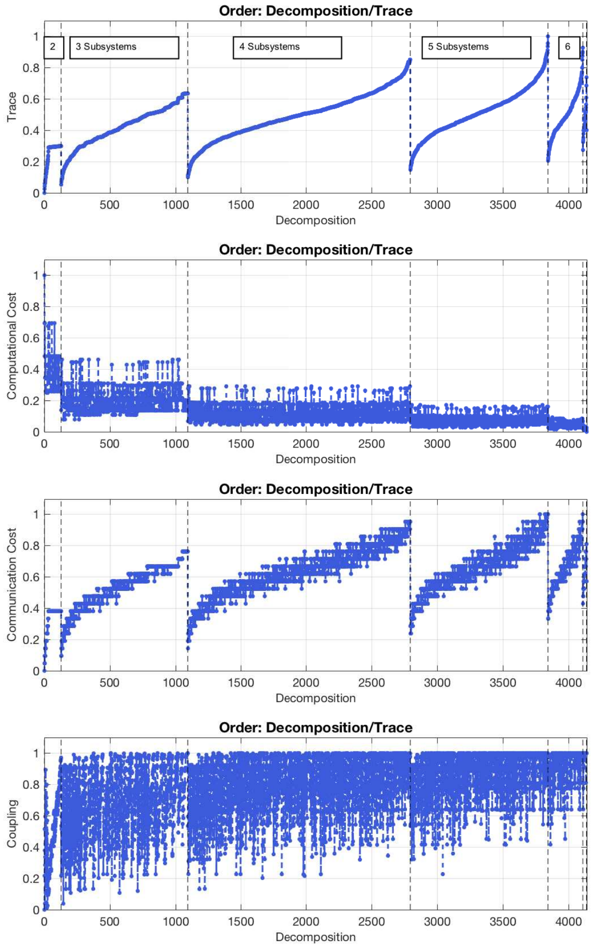

Fig. 4. Building model: value of the cost functions for different decompositions, ordered by the number of subsystems and the detectability cost function (7).

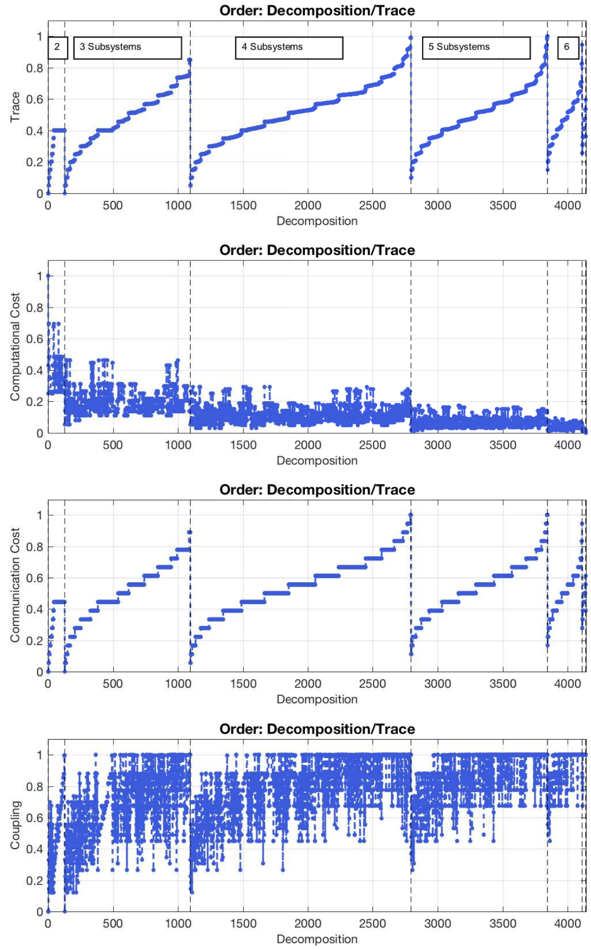

Fig. 5. Eight Tanks model: value of the cost functions for different decompositions, ordered by the number of subsystems and the detectability cost function (7).

From the Figures, we can notice there is a clear correlation between the trace of B objective function, representing the detectability cost, and the communication cost. The same cannot be said about the coupling cost function; in fact, there seems to be almost no correlation between the coupling and the detectability cost functions. This suggests that selecting the decomposition according to the coupling cost function does not guarantee the minimization of the thresholds value and hence a faster detection.

We can also notice in this examples that the trace assumes a wide range of values inside each group characterized by the same number of subsystems and delimited by dashed lines. Furthermore we can see that there are equally good decompositions with different numbers of subsystems. As intuitively obvious, the computation cost decreases with the increase of the number of subsystems, for all cases.

From the results we obtained (not visible from the Figures), we can give some indication to the choice of the best decomposition according to the minimization of the detectability cost function (7)). As a first observation, we noted that in the case of the eight tanks model (Figure 1), where the system has a structure composed by three not interconnected parts, the results show that it is preferred to decompose the system into its natural components/subsystems, but we could also gather together two or more 'natural components' so that they are monitored by the same agent, and this would not downgrade the performance of the fault detection. As a second observation, we saw that in general, for systems with a hierarchical structure like the one in Figure 3, it is better to separate into smaller subsystems the state variables which have more influence on the dynamics of the others.

Furthermore, in order to analyze the significance of the chosen cost function (7) for detectability, we also considered other options. In fact, the trace of matrix B, is not the only one available and might not be the best one for our purposes. The problem is that we would like to minimize all the components on the diagonal of B, finding the decomposition which guarantees the lowest threshold for every single residual. The trace is the most intuitive metric and it represents a simple solution to this multi-objective

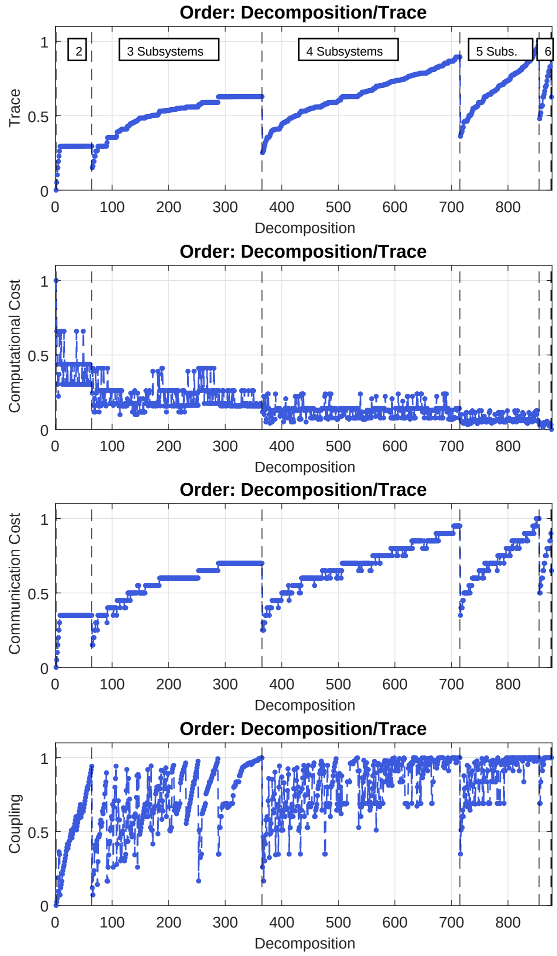

Fig. 6. Cascade Tanks model: value of the cost functions for different decompositions, ordered by the number of subsystems and the detectability cost function (7).

optimization problem, as it minimizes the sum of all the threshold components. However it might lead to unbalanced solutions with high thresholds for some components and low thresholds for others. Another possible metric is the maximum value on the diagonal of B. This gives us the guarantee that the maximum thresholds component is minimized. However, this cost function would treat the same way two decompositions which have equal maximum value on the diagonal but one has all the elements on the diagonal equal to that maximum and the other one has the other elements really low. We hence understand that the choice is not univocal and trivial. In order to compare the two detectability metrics, we show in Figure 7 and 8 the trends of the two cost functions for the building model and the eight tanks model. In these plots the values are not normalized between zero and one and this allows us to see what the achievable improvement in each size group is. The trends are similar but the maximum value on the diagonal of B takes a more discrete set of values, thus not making easy the choice between different decompositions. For this reason, the trace might represent a more informative metric.

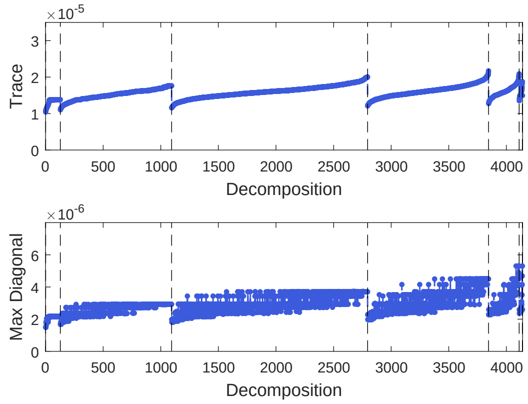

Fig. 7. Building model: comparison between trace and maximum value of the diagonal of B.

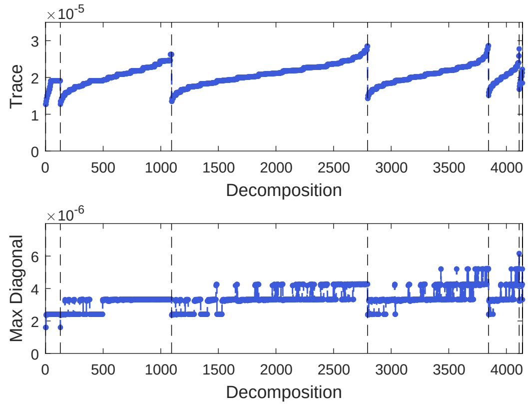

Fig. 8. Eight tanks model: comparison between trace and maximum value of the diagonal of B.

5.3 Relation between system decomposition and detection time

The simulation results presented in the previous subsection showed that the decomposition has an influence on the bound matrix B of the covariance matrix Π. We saw in Section 3 that the elements on the diagonal of B are used to compute the thresholds for all the components of the residual vectors. This means that the lower these elements on the diagonal of B, the narrower and less conservative the thresholds are, maintaining the same maximum false-alarm rate. Also the choice of smaller α would make the detection thresholds narrower but this would come at the expenses of a higher false-alarm rate.

Intuitively, a less conservative bound leads in turn to a faster detection of a fault. In this section, we analyze what influence the decomposition can have on detection time and we show that the minimization of cost function (7), that is the trace of bound B, implies a reduction of the detection time. To do this, we simulate some faults in the use-cases previously presented in this section and we implement the distributed fault detection method in Section 3, considering different decompositions of the system. In particular, we report the results for the eight tanks model and the hierarchical tanks model, where the considered faults are leakages in one of the tanks. We then compare the detection instant for different decompositions.

Table 2. Average value and standard deviation of the detection time

Decomposition	Average Detection Time [s]	Standard Deviation [s]
Decomposition: "14725836" Division: "332"	135	13
Decomposition: "34567812" Division: "332"	387	189

Table 3. Average value and standard deviation of the detection time

Decomposition	Average Detection Time [s]	Standard Deviation [s]
Decomposition: "4567123" Division: "4111"	134	23
Decomposition: "2345167" Division: "4111"	337	158

For the eight tanks model we simulate a leakage in tank 1. We consider an incipient fault evolving as $(1-e^{-(t-t_0)})a_h$, being $t_0 = 100$ s the fault time and $a_h = 2.5\%a_1$, where $a_1 = 5.1\ cm^2$ is the cross-section of the outlet hole causing the leakage. In Figure 9, we compare the detection performance with two different decompositions composed of 3 subsystems: the first is characterized by the minimum value of the proposed detectability cost function, while the second is another decomposition that might be chosen for physical reasons grouping the tanks situated at the same level, characterized by a bigger detectability cost value. The threshold parameter α is chosen equal to 3. In Figure 9 we analyze the behavior in a time frame of 700 s. We can see the evolution of the residual corresponding to the first state variable and the related threshold for the considered decompositions. The decomposition minimizing the detectability cost function allows for a faster fault detection. We repeated the simulation 1000 times to evaluate average and standard deviation of the detection time (instant at which the residual crosses one of the two threshold). The average detection times and standard deviations are shown in Table 2. The difference between the average detection instants of the two decompositions is quite significant, representing almost four minutes of detection delay, proving how important can the decomposition choice be on the performance of the fault detection.

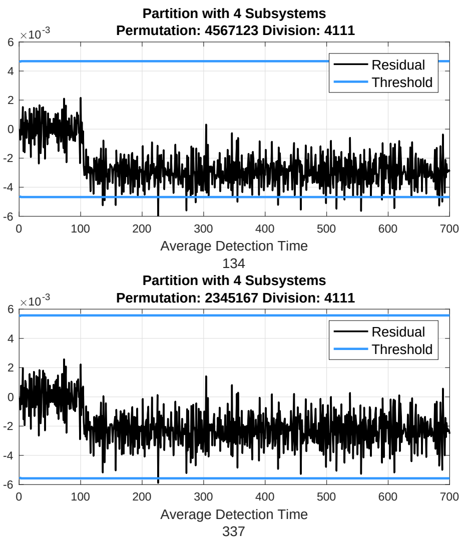

Fig. 9. Eight Tanks model: leakage in tank 1. Residual and threshold for state variable 1, with two different decompositions.

For the hierarchical tanks model, we simulated a leakage in tank 4. We consider an incipient fault evolving as $(1 - e^{-(t-t_0)})a_h$, being $t_0 = 100$ s the fault time and $a_h = 1.3\%a_4$, where $a_4 = 11.5\ cm^2$ is the cross-section of the outlet hole. We set $\alpha = 3$. The considered decompositions are "4567123" "4111" and "2345167" "4111", where the first is the one, with four subsystems, that minimizes the detectability cost function, while the second is the one, with four subsystems, that minimizes the coupling cost function. We repeated the simulation 1000 times. In Figure 10 we can see the evolution of the residual corresponding to the fourth state variable and the related threshold for one of the simulations and both decompositions. The average detection times and their standard deviations are shown in Table 3. Also in this case we find a significant difference in terms of detection time, as it exceeds three minutes, showing that the minimization of the detectability cost function allows to obtain better detection performance.

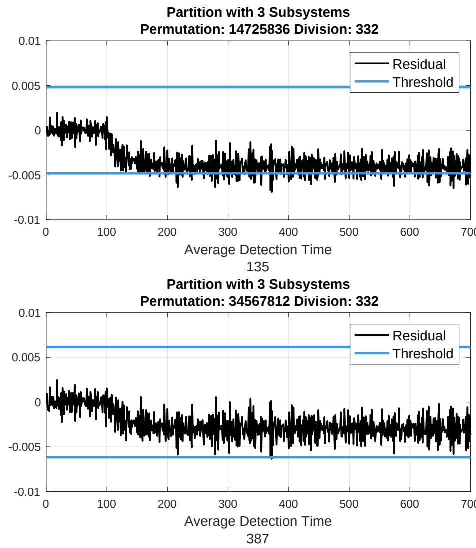

Fig. 10. Hierarchical Tanks model: leakage in tank 4. Residual and threshold for state variable 4, with two different decompositions.

From Figures 9 and 10, we can see that also the residual evolves differently according to different decompositions. First results seem to show that the minimization of the trace of the bound B makes the residual more sensitive to the fault. This is another aspect that will be analyzed in future works.

6. CONCLUDING REMARKS

In this preliminary paper, we analyzed the effect the system decomposition has on model-based distributed fault detection. The decomposition problem has been addressed in the past for control and estimation applications, however a solution tailored for the fault detection problem has not been provided yet. A cost function to measure the detection performance of the FD method has been proposed, and compared to other possible cost functions. As a future work, we are going to investigate the design of a synthesis method to compute the optimal decomposition with respect to the proposed detectability cost function and we will consider real large-scale systems examples.

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