Mathematics education in the spotlight: its purpose and some implications

Abstract

This paper argues for a re-framing of the purposes of mathematics education for the twenty-first century that combines apparently divergent philosophical approaches, arguing that the consequent empowerment should as a matter of individual equity (as well as benefit to wider society) be available to all young people. It suggests that the global mathematics attainment 'spotlight', and the English policy context in particular, offer both opportunities and constraints for the development of such a high quality mathematics education, and discusses the challenging implications for curriculum, and for the nature of teacher expertise, and particularly subject-specific expertise, needed.

Key words: mathematics education, human flourishing, powerful knowledge, qualification, socialisation, subjectification

Introduction

There is a global focus on mathematics attainment as a gateway to economic progress, with the leaders of many western nations anxious to see improved results in international performance indicators such as PISA or TIMSS (Baird et al, 2011). In parallel, there is a widespread re-focusing of mathematics curricular aims to better combine content (facts, procedures), deep conceptual understanding, and mathematical processes - and so enable fluency, mathematical reasoning and mathematical problem-solving, thought to be prerequisites for effective mathematical functioning in the twenty-first century. This is *hard*, and not being achieved at scale (Eurydice 2011, Spillane 2004). These twin foci serve together to create a high-stakes mathematics education environment.

In England, one feature of this has been growing awareness of 'curriculum coherence' (Schmidt & Prawat 2006): the alignment of curriculum, assessment, teacher development, resources, etc, so that contradictions are not set up, and professionals not subject to contradictory incentives and targets. Here I analyse in particular relationships between mathematics-related curriculum intentions (where by 'curriculum' I mean all planned school-related experiences) and the development of appropriate teacher expertise, where at present I restrict the argument to human 'teachers'.

I suggest the global policy context offers opportunities for a re-appraisal of the role of mathematics within the school curriculum, and of the consequent mathematics-specific capacity (Golding 2017a) needed by teachers of mathematics. Such an evaluation builds on agreed purposes for education, and here I harness Biesta's (2015) framework to suggest that the apparently divergent approaches adopted by Reiss and White (2013) and by Young and Muller (2013) have complementary lenses to offer – but that the implications of their combination for curriculum, and so for teacher expertise, are profound, and demanding. I discuss in particular the systemic and systematic teacher education needed to achieve such nuanced purposes. Parts of the paper are mathematically technical, but the shape of the argument stands without engagement with those.

What is mathematics?

The argument depends on some shared conception of mathematics as a discipline. In common with many, but by no means all, mathematicians, I understand the discipline to be concerned with the exploration of, and connections between, patterns that often arise from the external world, their abstraction and their relationships as established through reasoning. Crisan (2017) offers an accessible exegesis of mathematics as ubiquitous culturally-embedded activity: every human society we know about has developed a mathematical lens on the world, harnessing that lens for their flourishing. I develop these ideas further in Golding (2017b), and draw on that here. For an inspirational treatment from a philosophical point of view, I suggest Francis Su's (2017) Presidential address to the American Mathematical Society.

Mathematics is therefore part of our cultural heritage, concerned with concepts, but working with culturally-developed tools and processes. We are by nature both curious and inventive, and mathematical epistemology features dualities of discovery and creation: its objects include both those embedded in a physical world framed by Tegmark (2007) as completely mathematical in its structure, and discursive constructs with no existence independent of humans, as described by e.g. Adler and Sfard (2017). It is in that sense that mathematical truth is always provisional, open-ended and relational, situated, social and contextualised.

Then how is mathematical knowledge established? Answers to such questions have implications for the way mathematics should be learned and taught. Many would argue that what we have in western society is one, or some, possible mathematical accounts: elsewhere, globally and historically, are further accounts. In Lakatos' (1976) terms I adopt that fallibilist, rather than absolutist, position. For mathematics to be a shared endeavour, there need to be shared ways of working and of establishing new knowledge in the field: a shared epistemology and syntax. How those are applied when working with novice mathematicians will also be influenced by one's beliefs about how young people (and others) learn.

So what relationship should any 'mathematics' experienced in classrooms have to the mathematics studied in universities, or used outside academia? In Golding (2017b), intended to be accessible to the non-specialist, I argue that school mathematics can, and should, be an near-authentic sub-discipline of mathematics – and that for example academic mathematics, and the mathematics of the mature user of the discipline, comprise further, overlapping, sub-disciplines. One implication of that view is that with such an understanding, school students are indeed novice mathematicians, and one aim of school mathematics is to further draw them into a mathematics sub-culture. For Bernstein (2000) too, the school subject is a recontextualisation of the parent discipline, with organising epistemic coherence stemming from that discipline, even if for a different purpose, but that too begs questions of operationalisation. Note that already, above, we have alternative, or perhaps complementary, lenses on mathematics as discovered or invented – and this dualism is reflected in perspectives on the purposes of education as student-centred, coming to understand and harness the (natural and social) world, and with 'learning' equivalent to 'comprehension', or world-centred, disrupting and de-centering the student: both could be argued to support human flourishing. I therefore turn next to consideration of the purpose of education, and the implications of that for mathematics.

The purpose of education and the role of mathematics within that

I argue that the purpose of socially-sanctioned (and funded) school education is to nurture young people's range of constructive potential and induct them into the culture of the society, so that they both mature into fulfilled and well-rounded adults, and are able to contribute to and constructively critique society. It exposes young people to that which is not easily accessible elsewhere, inducting them into social ways of being. In so doing, it supports Reiss and White's (2013) 'aims-based' curriculum, that argues for a central curriculum design principle of the achievement of human (individual and societal) flourishing. The notion of human 'flourishing' is not uncontested, but here I adopt a neo-Aristotelian view of *eudaimonia* consistent with Reiss and White's and for which meaning and purpose are prerequisites. However, such a purpose should also introduce them, inter alia, to Young and Muller's (2013) 'powerful knowledge' – by particular social means: there is in principle no fundamental contradiction here, although I shall show there are tensions inherent in achieving both.

Young and Muller (2013) argue that the school curriculum, whatever else it does, should endow young people with disciplinary 'powerful knowledge' – and that historically, they have been short-changed by the rationing of the school curriculum to that which preserves social hierarchies – and limits individual potential. 'Powerful knowledge' here is used as a 'master discourse' (Ball, 2010): an organising language that offers a framing principle for actions and values, presented as an unexceptionable good and so often serving to preclude dissent. Young and Muller develop an argument for the inclusion of a range of discipline-embedded knowledge, each offering access to aspects of human flourishing. However, they do not offer effective criteria for deciding *which* disciplinary knowledge is to be considered 'powerful'. For mathematics, as in the argument below focused on times-table knowledge, I suggest genuine powerful knowledge centres on deep and robust conceptual understanding – and familiarity with mathematically valued ways of working with those. Here, the 'value' stems from the discipline – as developed for 21st-century cultures. To harness those concepts young people of course *also* need to develop a repertoire of facts, skills, and processes that they can use as tools. At the end of their article (2013, p 247) Young and Muller I suggest overplay their argument and refer to 'powerful knowledge' as not only a necessary, but a sufficient, basis for curriculum design.

I suggest that Biesta's (2015) framework of three domains of educational purpose (and function) allows a reconciliation of Young and Muller's position with Reiss and White's. He argues education has purposes of *qualification*, understood either in a narrow sense, for example becoming qualified to perform a certain task or job, or in a much wider sense, such that young people are well-prepared to thrive in modern, complex societies. It is also about *socialisation*, initiating children and young people into existing traditions, cultures, ways of doing and ways of being. That socialisation also happens unconsciously through teachers' and students' enculturated ways of being (thus also contributing to the reproduction of material and social inequalities). Any educational activity, further, impacts on the qualities of the person: Biesta frames this as *subjectification*, as it concerns processes of being/becoming a flourishing human subject - and importantly, both Biesta's (2015) and Young and Muller's (2013) uses of 'flourishing' are consistent with Reiss and White's (2013). These are three qualitatively different domains with regard to which we need to state and justify what we seek our students to achieve. A 'good' education requires attention to all three, and although not always easily separable, they are sometimes in tension.

Biesta points to common quite narrow definitions of 'learning' and of 'effectiveness' of some enactments currently adopted, leading to a distortion of these three core purposes, but within those argues (2015, 10) that in addition to a technical judgement about the effectiveness of our actions and arrangements, there is a need for judgement (*phronesis*) about the educational desirability of our actions and arrangements – implicitly, for both individual and society. Good teachers therefore need not only appropriate knowledge and skills, and induction into an appropriate professional culture, but *educational virtuosity*, that is, embodied educational wisdom: the ability to make wise educational judgements about what is to be done, about what is educationally desirable within that socially-sanctioned framework.

So what potential tensions are inherent in an attempt to go beyond a simplistic, and binary, view of student as *either* subject *or* object to accommodate support for both 'powerful knowledge' and 'individual and social flourishing'? I have pointed to an aspiration that young people should both be inducted into society and culture (mathematical and wider) so as to allow full participation, and develop their own meaning-making and values so as to be equipped to constructively critique those cultures. I would want them to develop autonomy as individuals and embryonic mathematicians, as well as agency as citizens and users of mathematics. In curriculum terms, this needs design for learning as well as direct teaching with wisdom that leads into a shared world as well as shared knowledge. However, the balance needed to achieve that requires the exercise of a deep phronesis – systemically and on the part of individual teachers - that is both sensitive to, and often constrained by, characteristics of the students as individuals and groups, and the prevailing (mathematical and wider) context and culture. In short, there are no easy routes to achieving such aspirations.

In Biesta's (2015) terms, many English stakeholders are currently perceived to value mathematics education for (narrow) qualification purposes: primarily utilitarian, offering access to techno-scientific literacies and so a gateway to economic prosperity for individual and society, though as mathematics pervades a wide spectrum of our lives it has the potential to empower much more widely (Smith, 2017). For these ends, but also for wider 'qualification', there is a growing awareness that young people should have a *broad mathematics education* that equips them less directly, but *potently*: skills of mathematical problem posing and solving, and a critical appreciation of the use of mathematical approaches in society (leading to social empowerment through mathematics). Recent curriculum document changes in England, as elsewhere, pay at least some nominal attention to that. Such changes support experiencing mathematics not only as discovered and transmitted, but also as invented – although in many cases curriculum coherence still has to be achieved.

I have already pointed to mathematics as part of our cultural heritage, and given also this fundamental role in twenty-first-century economies, it is a matter of both social justice and equity that all young people should be able to access a mathematics education that supports strong mathematical participation and progression. As part of their *socialisation* then, young people should, at an appropriate level, understand the nature of mathematical activity – the syntax and epistemology of the discipline, the valued ways of working in the mathematics sub-culture, their surprises, frustrations and joys – if they are to fully participate in 21st-century culture and society. Culturally, I would argue, in common with Ernest (2000, 12):

> Learners should gain a qualitative or intuitive understanding of some of the big ideas of mathematics such as pattern, symmetry, structure, proof, paradox, recursion, randomness, chaos, infinity. Mathematics contains many of the deepest, most powerful and exciting ideas created by humankind. These extend our thinking and imaging power, as well as providing the scientific equivalent of poetry, offering noble, aesthetic, and even spiritual experiences.

Implications for curriculum

What sort of curriculum is implicit in such approaches? Biesta (2015) points to a current equating of 'learning' with meaning-making that is rather limiting, though Ofsted (2012) showed that in mathematics classrooms in England even that was not being achieved. In the long-term, though, students cannot come to use mathematical facts and skills with integrity if they have no known meaning for them: as above, meaning and purpose are necessary, if not sufficient, for human flourishing. So deep conceptual understanding is of value in mathematics education, and over-use of a master discourse such as 'powerful knowledge' can occlude the relative and differential powers of different sorts of knowledge within a discipline. Further, over-emphasis on a reductive 'learnification' (Biesta, 2015) can compromise the plurality on which a dialogic education supporting socialisation, depends. I give two such examples. First, rote knowledge of times tables facts is necessary for efficient mathematical functioning – but table knowledge is much more powerful if it's recognised as applicable in a range of (external and mathematical) situations, representative of a process that has an associated inverse process, and so on. Secondly, acquaintance with the use of Roman numerals gives (conceptual) access to mathematical aspects of a culture pervasive (among the powerful) in western civilisations until quite recently, but of *greater* power is a grasp of the limitations of that number system in relation to others based on place value, since those offer access to appreciation of the potency of a concise and precise language for mathematics. There are other routes to such knowledge, but one point for my subsequent argument is that there is discipline-informed judgement needed to discern and harness such distinctions.

I would argue, then, that for Biesta's (2015) '*subjectification*' to be achieved within a mathematics domain, students should come to recognise mathematics both as a tool with which to engage with the world, and as a source of personal and social enrichment, cultural heritage and wonder. Further, they need the positive affective resources to harness that education confidently (self-efficacy in relation to an appropriate degree of mathematical functioning, resilience, collaborative and learning dispositions…). It follows that Young and Muller's 'powerful knowledge' in mathematics, if it is to be more than a slogan, transcends facts and procedural skills to include knowledge of valued ways of harnessing imagination and creativity to deductive reasoning, pattern-making, analysis and synthesis. It can often spring from modelling of situations in the external world and might be applied to solve problems in those contexts, but also includes the mathematically-internal study of the implications of models and mathematical structures. To the extent such knowledge is held socially and contextually, the means of accomplishing this need to include the social.

Young people should therefore not be restricted to learning about concepts and facts, skills and processes, though they need all those: they need also to understand that the discipline of mathematics itself is the authority. The messiness and debate and choices of the discipline are often hidden unnecessarily. Further, it is not just the content of the curriculum that is important: if they are to function confidently and effectively mathematically, young people need to experience, in sustained ways, valued mathematical ways of working, and that is part of Biesta's (2015) 'subjectification'.

Cuoco et al (1996) frame such ways of working as 'mathematical habits of mind' saying mathematicians are 'pattern sniffers, experimenters, describers, tinkerers, inventors, visualisers, conjecturers, guessers'. For a classroom, Burton (2004) quotes a poster translating this as 'have imaginative ideas, ask questions, make mistakes and use them to learn new things, are organised and systematic, describe, explain and discuss their work, look for patterns and connections, and keep going when it is difficult.' Details vary across the mathematics literature, but the point is made that authentic mathematical activity undertaken by genuine mathematics novices is a far cry from that seen in many English classrooms (Ofsted, 2012), where in too many cases, supported by predictable and routine assessments, the teacher (or the textbook) has been seen as the source of mathematical authority and the students' job is to reproduce demonstrated approaches to solving standard exercises.

School mathematics education, then, should aim to support increasing participation in the human endeavour of mathematics in an authentic relationship with the discipline (though necessarily restricted by the institutional constraints of schooling). As such, it has contributions to make - for every young person - to each of Biesta's purposes of 'qualification', 'socialisation' and 'subjectification'. However, the teacher of mathematics also has a moral purpose wider than induction into the discipline: he is also the teacher of the whole young person and, for Reiss and White (2013), has responsibility for enabling his/her wider flourishing, as well as developing the student's capacity to support the flourishing of others. I shall show below that the implications of this for teacher education are challenging: the teacher too needs to develop appropriate professional qualification, socialisation and subjectification.

Expertise for teaching mathematics

It is an attractive argument that the education of the young person (or, more typically, group of perhaps thirty young people) is too important to be left to the discretion of the individual teacher – but there is a balance to be made, and Biesta (2015) convincingly calls for a collective, but non-identical, practice. I suggest that high-level articulation of shared purposes for education in terms of qualification, socialisation *and* subjectification, and consequent decisions about educational curriculum structures are the province of government, though drawing substantially on culturally-embedded phronesis. Medium level curriculum decisions should, given the stated purposes of education, lie with education and disciplinary, including pedagogic, community experts. The role of a knowledgeable and (widely) effective professional teacher is then to enact those with phronesis (practical wisdom) in a particular context, for the educational benefit of students, fulfilling the range of purposes of education in a balanced way that will change as time and context change. Such a position is not currently fully supported in English policy, though it is in some other developed jurisdictions. I demonstrate below that large parts of the related detail are situated, dependent on context-specific and professional subject-related knowledge, both of which reside in the expertise of teachers rather than government.

Addressing this, and complementary to Biesta's (2015) reclamation of education from 'learnification', Fenstermacher (1986, 39-40) says teaching includes

> 'instructing the learner on the procedures and demands of the studenting role, selecting the material to be learned, adapting that material so that it is appropriate to the level of the learner, constructing the most appropriate opportunities for the learner to gain access to the content (...), monitoring and appraising the student's progress, and serving the learner as one of the primary sources of knowledge and skill'.

But here we come to one of the biggest challenges associated with trying to establish principled approaches to mathematics education. Teaching for the building up of powerful knowledge, given the range of these roles and more, is *hard*. It is clear that enacting much of the above requires discipline-specific knowledge – minimally, of the culture-specific school sub-discipline at an appropriate level, of the disciplinary language, organisation, approaches and ways of establishing knowledge – its epistemology and syntax, shared values and ways of working. Teachers have to understand its recontextualisation for the classroom and the ways of re-presenting mathematics so as to make it accessible to young people. They have to know the ways students interact with mathematics, the potential barriers to that, and the systemic disciplinary expectations as communicated in curriculum documents and assessments. Teachers' knowing, of doing, and of being all have discipline-specific elements if they are to make the potential of this school mathematics available for learning. As described above, that is not the sum of the demands on teachers of mathematics, of course, for they are not just teachers of mathematics, but of whole young people. Each of these roles entails *judgement (phronesis)* in its enactment: good teaching is inherently situated and context-dependent.

So what subject-specific knowledge do teachers of mathematics need? In a mathematics classroom, authority derives from at least the parent discipline, knowledge of the subject for teaching, and the teacher's pedagogical knowledge, including their subject-specific pedagogical knowledge: that knowledge, held at a deep level, is needed for 'curriculum-making' (Lambert and Biddulph, 2015), that is, the transformation and synthesis of the written intended curriculum into a coherent and meaningful classroom-enacted whole – in ways which are consistent with the values of the professional community and its standards of integrity and wise judgement.

What is the nature of the requisite discipline-specific knowledge? Ball et al (2008) show teacher effectiveness appears to have no simple relationship with for example the number of discipline-related college courses taken, though there is clearly a need for a confidence and deep grasp of mathematics beyond the level currently being taught. Ma (1999), though, demonstrates the need for 'profound understanding of elementary mathematics'. It is knowledge which differs from that of a numerate layman, or professional user of the discipline, but also differs profoundly from that of an academic mathematician. Further, we have seen above that it includes discipline-specific appreciation of the purpose and role of mathematics education in contributing to the qualification, socialisation and subjectification of the student.

For example, a Primary teacher whose children have a reasonable grasp of whole numbers has then to teach that ¾ is part of a whole (and which whole). It is also 3 lots of a quarter of one or more identical wholes, 3 identical wholes shared equally among 4, an operator (as in ¾ *of* a pile of Smarties), a number in its own right, with its own position on a number line, a ratio, an equivalence class of fractions… All these concepts have to be planned for, introduced and worked with at appropriate times, with a range of appropriate representations, and in culturally and contextually appropriate ways, and young people supported in making connections between these different conceptualisations, until, in the long term, they compress that understanding into a single, but multi-faceted, idea of '3/4' that they can use as a tool within and beyond mathematics. (For the academic mathematician, in contrast, '3/4' is one of the 'rationals' defined from the whole numbers in an abstract, algebraic, way: the related two sub-disciplines require different knowledge.) Further, the same Primary teacher in the melee of the classroom moment, has to comprehend, evaluate, and decide how to deal with pupils' alternative understandings as they develop, responding in ways which build not only that student but others' mathematical ways of participating meaningfully in society, their socialisation into mathematics and wider cultures, and the wider subjectification of the student. That is no small task, even for a mathematics specialist, but Primary teachers in England typically teach across the curriculum: the consequent holistic knowledge of each young person is highly valued culturally (Alexander, 2010), but might also be argued to contribute to that subjectification.

Teaching for mathematics – at any level – therefore requires a great breadth and depth of subject-related knowledge – and much more - if it is to be effective. One widely-used theorisation of necessary mathematics-specific knowledge is Ball et al's (2008) 'egg', which builds on Shulman's (1987) generic typology of teacher knowledge. This divides the subject specific knowledge needed into *subject knowledge* and *subject pedagogic knowledge*. The former encompasses not only what the mathematically competent adult commonly needs, but deep knowledge of its different conceptualisations, where those might be encountered, and the relationship between them, as in the example of 'knowing' ¾ above, as well as the links of each idea to others within or beyond mathematics. Pedagogically, the teacher also needs to know where those different conceptualisations sit within the curriculum, the progressions to and from those (as structured by the discipline), how concepts might meaningfully and constructively be re-presented to students, in what contexts, how students, and particularly the students in this class, might typically understand them in 'different' ways, as well as students' affective needs in relation to mathematics, how to elicit related developing understanding and harness that constructively, and how to support a grasp of the related mathematically-valued connections both within and beyond mathematics.

An alternative, higher-level theorisation analysing disciplinary knowledge that is exposed in classrooms comes from Rowland et al (2005). From classroom observations, they show that in teaching mathematics, teachers draw on mathematical '*Foundation*' knowledge - of mathematics, students in relation to mathematics, curriculum, etc; on '*Transformation*' knowledge – transformation of the mathematics to a form where students can access the underlying motivations, structures and warrants; '*Connection*' knowledge – within and beyond mathematics, of the authentic use of mathematical thinking and of utilitarian applications; and on '*Contingency*' knowledge of how to field questions, and variety of responses, some of which will be based on challenging and non-standard conceptualisations of emerging ideas - and in mathematically constructive ways. Again, what is seen to be necessary is a rich and deeply connected discipline-embedded network that can be drawn on with *phronesis* to further the range of purposes of education. Although in both cases the categories have 'fuzzy' boundaries when operationalised, they serve to point us to complex aspects of teachers' subject-specific knowledge that are critical to good teaching – as well as highly situated and contextualised. These conceptualisations therefore have implications for teacher initial and continuing development – but only acquire their full meaning and import when applied in relation to the underlying valued purposes of education.

The development of mathematics teacher expertise

I argue in Golding (2015) that such expertise needs specialised development – including in initial and early career teacher education. Teacherly knowledge is of a dual nature, comprising both theoretical and practical: not only do teachers of mathematics, for example, have to understand the multi-faceted nature of ¾, but they have to be able to operationalise education for that with a specific group of children who bring particular mathematical and whole selves to the classroom, and will respond differentially to particular personal and mathematical approaches and mathematical explanations and embodiments. Education for such complexity requires expertise in teacher development in each of the theoretical (mathematics education, teacher education, child psychology…) and the classroom domains – and the development of practical wisdom. Schools do not typically house experts in teacher theoretical education, but they do house experts in classroom teaching for their own context. Very often, then, effective initial teacher education can be achieved through strong partnerships between university education departments and schools, and models of teacherly phronesis might be found in either. The former, typically staffed by experienced teachers further resourced to develop expertise in teacher education, can also provide time and knowledgeable support for developing deep informed reflection, building of goal-related dispositions, knowledge of the evidence base, and the opening up of possibilities that are often not available in schools, which by their nature are focused on the education of young people. From the earlier discussion, it is clear that for all teachers of mathematics, substantial parts of this early development need to be subject-specific.

The discourse surrounding further teacher development has in recent years, and globally, been one of ‘teacher deficit’, and in England there is substantial evidence that in schools at present there is indeed a lack of appropriate mathematics knowledge for teaching that significantly limits the quality of mathematics learning available (e.g. Ofsted 2012), however worthy the curriculum intentions. However, there is some evidence that given appropriate subject-specific and generic foundations (the precise nature and extent of which are yet to be established), together with sufficient time, goal-related dispositions and appropriate access to expertise, some of which might be peer-collaborative, experienced teachers are often able to develop in new directions and depths in semi-autonomous ways (e.g. Chan et al, 2018; Golding, Smith and Blaylock, 2018).

Yet none of this is sufficient to ensure the development of a teachers’ practical wisdom or phronesis, which needs all of the above and more: as Biesta (2016) argues, competences are necessary, but not sufficient, for good teaching, which depends on effective harnessing of those competences for agreed valued domains of educational purposes. I argue, consistent with Biesta (2016), that true phronesis can only be developed with experience, together with professional example and sustained informed reflection. Again, there is a clear but different role apparent for early contextualised experiences in school and supported deep reflection in the university – both ideally supported by experienced, effective teachers able to model the exercise of professional phronesis. Where a system has, for whatever reason, limited capacity to model the range of such ‘professional virtue’, the challenges of developing effective (mathematics) education are of course exacerbated.

How does the policy context in England impinge on possibilities for mathematics education?

In a high-stakes policy environment, it is difficult for policymakers, end-users and those involved in mathematics education alike to step back and reflect in depth about what it is we are trying to achieve in mathematics education – and to expose the variety of aims and purposes assumed by different policy ‘players’ (Ball et al, 2011), so that those are held up to scrutiny and the possibilities for workable shared values debated. I would argue that reasonable performance on widely-valued international attainment measures should be expected from a wealthy nation – but questions should be asked about whether such measures together reflect the range of socially-valued mathematics education (and other) outcomes. TIMSS measures performance on curriculum-close mathematics questions, whereas PISA assesses solution of rather more contextualised tasks - yet political response rarely seems to be based on the nature of what is measured (Baird et al 2011). I would suggest that even good performance on both of these, together arguably supporting mathematically ‘powerful knowledge’ is not sufficient for either personal or societal flourishing.

Further, in England, teachers of mathematics, from Early Years upwards, are dealing with rapid and frequent curriculum changes. They are also working in high-stakes assessment environments whose focus, for mathematics at least, has recently been broadened but which still, by its nature, privileges short-term performance. One argument in favour of the high value given to academic disciplinary functioning (Gibb, 2017) is that it enables all young people to access the knowledge that, in a more liberal education environment, would be available only to the privileged. But a consequence of rapid and aspirational change is that teachers have very little time or energy available for deep reflection on, or considered preparation or development of, their teaching. It is not yet obvious that the combination is of net benefit to young people – though recent changes have yet to bed down.

It is clear, then, that if we want to achieve an authentically mathematically well-equipped population, for purposes of individuals' and society's flourishing, we certainly need supply and development of sufficient mathematically knowledgeable and pedagogically effective teachers, able to develop enacted curriculum in discipline-knowledgeable ways and exert professional judgment that builds up a balanced (wider) qualification, socialisation and subjectification for their context. There are both opportunities and constraints associated with this.

Mathematics education is currently valued by a range of stakeholders including policymakers. There is therefore a (relatively) good level of financial investment available, and an in-principle will to develop teacher education in ways that support that. There is also an opportunity to move away from a 'deficit' model of teacher continuing education and build on the undoubted potential of well-prepared teachers to drive their own (both subject-specific and generic) development, learning from their teaching (e.g. Chan et al 2018) and from opportunities to work collaboratively with expert others, supported by bespoke time and moderate funding (e.g. Golding, Smith and Blaylock, 2018).

On the other hand, while a high-stakes accountability regime and valuing of international performance measures of mathematical functioning and more *can* support maths education development, accompanying national assessments that do not fully support curriculum intentions serve to undermine any such broad and balanced curriculum provision (Neumann et al, 2017). Longstanding lack of teacher subject -specific expertise (Ofsted 2012), coupled with recruitment and retention challenges (National Audit Office 2016) can threaten the contextual, reflexive, profoundly *phronesis*-dependent and iterative elements of the work of good teaching.

In parallel, English teacher education, both initial and continuing, is currently in a state of flux (e.g. Brooks, this volume) with fragmented and increasingly generic provision, often led by schools and sometimes with no input from Higher Education. So despite a range of threats to the health of a subject perceived to be important, development of mathematics knowledge for teaching is being marginalised. The arguments above make it clear that such a situation undermines both the attainment of an effective mathematics education, and the viability of a balanced, phronesis-rich, education that is balanced across purposes.

Moving to a point where we have a consensus about the needs of initial and continuing teacher development, though, assumes attainment of a shared view of the kind of teaching that is needed in schools. Those sustained conversations between professionals, policymakers and stakeholders about what we value in education, why, and how that might be achieved, are important. It would be naïve to suggest that previous teacher education systems were ideal: they operated in a national context with less competition for mathematics expertise, and fewer pressures of performativity on teacher educators.

We now, though, know far more than we did about how young people learn mathematics, and what teachers need to be able to support that: we need to capitalise on that knowledge in a coherent and long-term way, and to recognise the need for a theorisation of discipline-specific expertise in both subject and subject pedagogical arenas. And because we patently do not yet in England have a shared vision for education, let alone mathematics education, the present challenges offer every incentive to develop those in the interests of achieving individuals and societies that are genuinely flourishing.

Emerging English – and global - mathematics curricula for students 5-18 do, to a large extent, embody widely-held values of mathematics education communities. As we acquire evidence of what is working well and why, we should find ways to support the *evolution* of those curricula and the surrounding system (assessment, CPD, resources...) into a coherent whole, that values education for individual and societal flourishing without sacrificing the sorts of knowledge that keep open doors to social equity. A variety of initial teacher education models and moves to mixed models of teacher development challenge our assumptions about how teachers learn, when and how. That's healthy – provided it's also regularly and effectively evaluated, and changes made in response to that evaluation, so that we don't persist in perpetuating mediocrity wherever it occurs.

Conclusions

This paper suggests that the global mathematics attainment 'spotlight' offers both opportunities and constraints for the development of high quality mathematics education – and for debate about what that comprises. I use Biesta's (2015) framework for the purposes of education to argue that mathematics has a role in contributing to both Reiss and White's (2013) 'education for human flourishing' and Young and Muller's (2013) 'powerful knowledge' – although achieving a wise balance is demanding not on only policy makers but also on individual teachers. For mathematics, though I critique Young and Muller's use of the term, 'powerful knowledge' minimally takes the form of deep conceptual understanding. For any one concept this is complemented by a range of known facts and established skills that enable its effective harnessing for a variety of socially-valued and individually-enriching purposes, though it remains the case that different mathematics conceptual areas remain differentially 'powerful'. The details of the appropriate range and scope of conceptual access should remain contextually and socially determined, though with due regard to the global nature of the economy and society in which twenty-first century young people will live. The consequent mathematical – and wider - empowerment should as a matter of equity and social justice (as well as benefit to wider society) be available to all young people.

Such a mathematics education, though, only represents part of the responsibility of the teacher of mathematics, who should also look to the development and flourishing of the whole young person, exercised with appropriate *phronesis*. I discuss the consequent nature of mathematics teacher expertise, and particularly subject-specific expertise, needed to facilitate such education, arguing that the initial development of that to a well-evidenced threshold level should be supported by centres of teacher education expertise which, because of the dual theoretical and practical nature of that education, will often comprise partnerships of universities with schools and colleges. Beyond that initial threshold, I argue that teachers are often best served by opportunity to build on that initial expertise, with sufficient space and time for deep and collegially informed reflection, supported as appropriate by peer or external expertise, and so moving away from a model of a deficit in teacher capacity. I suggest the current 'high stakes' policy context in England and elsewhere offers an opportunity to prioritise a theorisation of subject-specific teacher expertise that would be situated in the professional rather than the policy domain, and to fund both a focused re-capacitating of the mathematics education teaching force, and a movement towards such a professional model of further learning.

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