Financial Frictions and Fluctuations in Volatility

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The US Great Recession featured a large decline in output and labor, tighter financial conditions, and a large increase in firm growth dispersion. We build a model in which increased volatility at the firm level generates a downturn and worsened credit conditions. The key idea is that hiring inputs is risky because financial frictions limit firms' ability to insure against shocks. An increase in volatility induces firms to reduce their inputs to reduce such risk. Our model can generate most of the decline in output and labor in the Great Recession and the observed increase in firms' interest rate spreads.

During the recent US financial crisis, the economy experienced a severe contraction in economic activity and a tightening of financial conditions. At the micro level, the crisis was accompanied by large increases in

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the cross-sectional dispersion of firm growth rates (Bloom et al. 2018). At the macro level, it was accompanied by a large decline in labor and output. During the crisis, financial conditions tightened in that firms' credit spreads increased and both debt purchases and equity payouts decreased, but aggregate total-factor productivity (TFP) fell only slightly. Motivated by these observations, we build a quantitative general equilibrium model with heterogeneous firms and financial frictions in which increases in volatility at the firm level lead to increases in the cross-sectional dispersion of firm growth rates, a worsening of financial conditions, and decreases in aggregate labor and output with small movements in measured TFP.

The key idea in the model is that hiring inputs to produce output is a risky endeavor. Firms must hire inputs and take on the financial obligations to pay for them before they receive the revenues from their sales. In this context, any idiosyncratic shock that occurs between the hiring of inputs and the receipt of revenues generated by those inputs makes hiring inputs risky. When financial markets are incomplete, in that firms have only debt contracts to insure against such shocks, they must bear this risk. This risk has real consequences if, when firms cannot meet their financial obligations, they must experience a costly default. In the model, an increase in uncertainty arising from an increase in the volatility of idiosyncratic productivity shocks makes the revenues from any given amount of labor more volatile and the probability of a default more likely. In equilibrium, an increase in volatility leads firms to pull back on their hiring of inputs.

We quantify our model and ask, Can an increase in the volatility of firm-level idiosyncratic productivity shocks that generates the observed increase in the cross-sectional dispersion in the recent recession lead to a sizable contraction in aggregate economic activity and tighter financial conditions? We find that the answer is yes. Our model can generate most of the decline in output and employment seen in the Great Recession of 2007–9. Our model can also generate increases in firm credit spreads, as well as reductions in debt purchases and equity payouts, comparable to those observed in the data. More generally, we find that the model generates labor fluctuations that are large relative to those in output, similar to the relationship in the data: the fluctuations in labor in both the model and the data are about 30 percent more volatile than those in output. The ability to generate such a pattern has been a major goal of the business cycle literature. Underlying these aggregate macro predictions are a rich set of micro predictions. An important part of

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the evidence we give in favor of our mechanism is to compare the model to firm-level data and show that it generates data consistent with the distributions and covariates of firm spreads, leverage, debt purchases, and equity payouts.

Our model has a continuum of heterogeneous firms that produce differentiated products. The productivity of these firms is subject to idiosyncratic shocks. The volatility of these shocks is stochastically time varying, and these *volatility shocks* are the only aggregate shocks in the economy.

The model has three key ingredients. First, firms hire their inputs—here, labor—and produce before they know their idiosyncratic shocks. That hiring labor is a risky investment is a hallmark of quantitative search and matching models but is missing from most simple macroeconomic models that have, essentially, static labor choices. Here we capture that feature in a simple way: firms commit to hiring labor before they experience idiosyncratic shocks. Second, financial markets are incomplete, in that firms have access only to state-uncontingent debt and can default on it. Firms face interest rate schedules for borrowing that reflect their default probabilities and are increasing in their borrowing and labor choices and depend on all shocks. Third, motivated by the work of Jensen (1986), we introduce an incentive problem in that managers can divert free cash flow to projects with private benefits at the expense of firms. This incentive problem creates an *agency friction* that makes it optimal for firms to constrain the free cash flow. This constraint makes the firm less able to self-insure against shocks.

Given these ingredients, when firms choose their inputs, they face a trade-off between the expected return from hiring workers and the risk of default. As firms increase their labor, they increase the expected return conditional on not defaulting, but they also increase the probability of default because of the increase in total wages that have to be paid after their idiosyncratic shocks are realized. For a given variance of idiosyncratic productivity shocks, firms choose labor to balance the increase in expected return against the costs from increasing default probabilities. The increase in the probability of default has two costs: it raises both the probability of liquidation and the interest rate firms pay on their borrowing. These effects constitute an extra cost of increasing labor and thus increase the wedge between the marginal product of labor and the wage, distorting the firm's optimal labor choice.

When the variance of the idiosyncratic productivity shocks increases at a given level of labor and borrowing, the probability of default rises and the interest rate schedule tightens, both of which increase the distortions for labor and borrowing. Firms become more cautious and respond by decreasing labor and borrowing. In equilibrium, default probabilities and credit spreads increase. At the aggregate level, these firm-level responses imply that when volatility increases, aggregate output and employment both fall, debt purchases are reduced, and credit spreads increase.

The result that firms decrease employment when the variance of idiosyncratic productivity shocks increases depends critically on our assumptions of incomplete financial markets and agency frictions. If firms had access to complete financial markets, an increase in the volatility of persistent productivity shocks would actually lead to an increase in aggregate employment. Firms would simply restructure the pattern of payments across states so that they would never default, and resources would be disproportionately reallocated to the most productive firms, causing a boom.[1]

Consider next the role played by agency frictions. In our model, firms have a precautionary motive to self-insure by maintaining a buffer stock of unused credit. If incentives to self-insure are sufficiently strong, firms build up such a large buffer stock that they can greatly dampen fluctuations in labor. Our agency frictions limit the incentives to build up such a large buffer stock.

We are motivated to introduce such frictions by a large literature in finance that argues that there are substantial agency costs of maintaining a large buffer stock of unused credit, and these agency costs help to explain why firms typically have large amounts of debt. In particular, Jensen (1986) argues that, in practice, if firms retain a large buffer, managers use these funds in ways that benefit their private interests rather than shareholder interests. Since shareholders understand these incentives, they give the firms incentives to pay out funds immediately rather than retain them. We model this *Jensen effect* by assuming that managers can divert the buffer stock of unused credit to projects that benefit them at the expense of the firm. In the presence of such a friction, the firm finds it optimal to limit the size of the buffer stock and maintain high debt levels.

In our quantitative analysis, we discipline the key shocks in the model, namely, the idiosyncratic firm productivity shock process, including those governing the aggregate volatility shocks, by ensuring that the model produces the observed time variation in the cross-sectional dispersion of the growth rate of sales. The external validation of the shocks and the model consists of working out the implications for real and financial data that our model was not designed to fit at both the macro and micro levels.

We validate the model by using both an event analysis of the Great Recession and the first and second moments witnessed over the past three

[1] This reallocation effect is referred to as the Oi-Hartman-Abel effect, based on the work of Oi (1961), Hartman (1972), and Abel (1983).

decades. In terms of aggregate data, we show that the model generates time paths for spreads, debt repurchases, and equity payouts similar to those observed during the Great Recession and second moments for these variables for the longer sample. In terms of micro data, we show that the model generates firm-level distributions of spreads, sales growth, leverage, debt purchases, and equity payouts, as well as firm-level correlations of these variables with leverage, that resemble those in the data.

We view our model as providing a new mechanism that links increases in firm-level volatility to downturns. To keep the model simple, we have abstracted from additional forces that would lead it to generate a slow recovery, as has been observed following the Great Recession. In so doing, we follow the spirit of much of the work on the Great Depression, including Cole and Ohanian (2004), that divides the analysis of the downturn and recovery into mechanisms that generate the sharp downturn and mechanisms that generate a slow recovery.

Related literature.—Our work is motivated in part by the evidence of Bloom et al. (2018) that uses detailed census micro data to document that the dispersion of plant-level shocks to TFP is strongly countercyclical, rising steeply in recessions. As in Bloom et al. (2018), we treat the volatility of idiosyncratic shocks as a primitive aggregate shock. Bloom et al. (2018) discuss a recent literature in which volatility endogenously increases in recessions and conclude that recessions are driven by a combination of negative first-moment and positive second-moment shocks, with causality likely to run in both directions.

Although we share with Bloom et al. (2018) the idea that volatility shocks drive aggregate fluctuations, our motivations differ sharply. Our model formalizes the popular narrative that financial distress played a critical role in generating the severity of the downturn in the Great Recession, whereas Bloom et al. (2018) consider a model with perfect financial markets that, by construction, has no financial distress. Instead, their work focuses on the interaction between volatility shocks and adjustment costs in generating the type of second moments seen in postwar business cycles.

To see how greatly our mechanisms differ, we consider the same increase in volatility witnessed in the Great Recession in a version of our model with perfect financial markets. With such markets, in contrast to both the data and the predictions of our model, not only would there be no financial distress, as measured, say, by increased spreads, but also there would be a boom in output. Finally, we differ from Bloom et al. (2018) in that we include external-validation exercises showing that the empirical implications of the model are consistent with micro and macro financial data.

Schaal (2017) endogenizes adjustment costs to labor by incorporating time-varying volatility shocks into a search model of the labor market.

He shows that while an increase in idiosyncratic volatility leads to an increase in unemployment, it actually leads to an increase in output. A major success of his model is that a given drop in aggregate TFP generates an increase in unemployment larger than that in Shimer's (2005) search model. In contrast to our work, however, this framework cannot account for any of the downturn in output during the Great Recession from the observed increase in volatility.

As in our work, Christiano, Motto, and Rostagno (2014) and Gilchrist, Sim, and Zakrajsek (2014) explore the business cycle implications of volatility shocks in environments with financial frictions. While both of these studies are complementary to ours, we focus on different issues. Christiano et al. (2014) show that, in a dynamic stochastic general equilibrium model with nominal rigidities and financial frictions, volatility shocks to the quality of capital account for a significant portion of the fluctuations in output. In contrast to our work, they focus solely on aggregate implications and abstract from any features of the micro data, including the observed high persistence in firm-level productivity shocks and the patterns of real and financial outcomes at the firm level, such as sales growth and spreads. Gilchrist et al. (2014) have a frictionless labor market and instead focus on the dynamics of investment. Differently from us, they abstract from any feature that can generate the large observed labor wedge in the Great Recession documented by Brinca et al. (2016).[2]

Our work is also related to studies on heterogeneous firms and financial frictions. For example, Cooley and Quadrini (2001) develop a model of heterogeneous firms with incomplete financial markets and default risk and explore its implications for the dynamics of firm investment growth and exit. In other work, Cooley, Marimon, and Quadrini (2004) find in a general equilibrium setting that limited enforceability of financial contracts amplifies the effects of technology shocks on output.

Finally, several researchers, including Buera, Kaboski, and Shin (2011), Buera and Shin (2013), and Midrigan and Xu (2014), have used heterogeneous firm models without aggregate shocks to help account for the relation between financial frictions and the level of development.

A recent literature, linking financial frictions and business cycles, has developed quantitative business cycle models in which the exogenous shock directly shifts a parameter of the credit constraint. (See, e.g., the work of Jermann and Quadrini [2012], Midrigan and Philippon [2016], Guerrieri and Lorenzoni [2017], and Perri and Quadrini [2018].) As do we, this literature aims at generating business cycle fluctuations without large fluctuations in aggregate productivity. One difference is that in our

[2] In some recent work, Zeke (2016) finds that shocks to skewness and default costs amplify our mechanisms and interact with volatility shocks in producing large aggregate effects on output.

model, the tightening of the credit constraint is endogenously linked to our volatility shocks, measured from firm-level data, while in this literature the shock to the credit constraint is exogenous and chosen only on the basis of aggregate data. Khan and Thomas (2013) have exogenous shocks directly to the collateral constraint in a model with heterogeneous firms subject to investment adjustment costs. They find that a shock that tightens the collateral constraint can generate a long-lasting recession. Their model differs from ours in that they abstract from any labor market frictions, and in their model credit spreads are zero. Our work is complementary to this literature.

As shown in Eggertsson and Krugman (2012), financial shocks can lead to reductions in aggregate demand when coupled with sticky prices. In a quantitative model, Midrigan and Philippon (2016) find that shocks that directly tighten credit constraints in an economy with sticky prices can account for under half of the employment decline during the Great Recession when the economy hits the zero lower bound. We think of our work as complementary to theirs. Adding sticky prices and a zero lower-bound constraint to our model would simply amplify our effects.

I. Our Mechanism in a Simple Example

Before we turn to our full model, we construct a simple example to illustrate our mechanism in its starkest and most intuitive form. Specifically, we show how, in the presence of financial frictions, fluctuations in the volatility of idiosyncratic shocks give rise to distortions that generate fluctuations in labor. To do so, we compare the optimal labor choice of firms in two environments: one in which they can fully insure against shocks and one in which they cannot insure at all.

Consider a model with a continuum of firms that solve one-period problems. Firms begin with some debt obligations b, produce using the technology $y = \ell^{\alpha}$, and maximize equity payouts, which must be nonnegative. They choose the amount of labor input ℓ to hire before the idiosyncratic shock z for this product is realized. These shocks are drawn from a continuous distribution $\pi_z(z)$ with volatility σ_z. The demand for a given firm's product is given by

$$y = \left(\frac{z}{p}\right)^{\eta} Y, \tag{1}$$

where Y is aggregate output. As we discuss below, the shock z can be interpreted as a productivity shock. At the end of the period, after shock z is realized, a firm chooses the price p for its product and sells it. If a firm has sufficient revenues from these sales, it pays equity holders its revenues net of its wage bill $w\ell$ and debt obligations. This firm also receives a continuation value V, here simply modeled as a positive constant. If the firm cannot pay its wage bill and debt, it defaults, equity payouts are zero, and the firm also receives a continuation value of zero.

Consider, first, what happens when financial markets are complete. Imagine that a firm chooses the state-contingent pattern of repayments $b(z)$ to meet its total debt obligations b and, hence, faces the constraint

$$\int_0^\infty b(z)\pi_z(z)\,dz = b. \tag{2}$$

The firm chooses labor and state-contingent debt to solve the following problem:

$$\max_{\ell,b(z)}\int_0^\infty [p(z,\ell)\ell^\alpha - w\ell - b(z)]\pi_z(z)\,dz + V,$$

subject to equation (2) and the nonnegative equity payout condition

$$p(z,\ell)\ell^\alpha - w\ell - b(z) \ge 0, \tag{3}$$

where $p(z,\ell) = zY^{1/\eta}\ell^{-\alpha/\eta}$ is the price the firm sets to sell all of its output and is derived from equation (1) and $y = \ell^\alpha$. Assume that the initial debt b is small enough that it can be paid for by the average profits of the firm across states. Hence, with complete financial markets, the firm can guarantee positive cash flows in every state by using state-contingent debt $b(z)$, and the equity payout constraint is not binding.

With complete markets, the firm's optimal labor choice ℓ^* is such that the expected marginal product of labor is a constant markup over the wage

$$E(p(z,\ell^*))\alpha(\ell^*)^{\alpha-1} = \frac{\eta}{\eta-1}w. \tag{4}$$

Since $p(z,\ell)$ is linear in z, this first-order condition implies that with complete financial markets, fluctuations in the volatility of the idiosyncratic shock z that do not affect its mean will have no impact on a firm's labor choice.

Now consider what happens when financial markets are not complete. The existing debt is state uncontingent, so firms have no way to insure against idiosyncratic shocks. Here, firms with large employment have to default and exit when they experience low-productivity shocks, since their cash flow is insufficient to cover the wage bill plus debt repayments. Effectively, the firm chooses its labor input ℓ as well as a cutoff productivity $\hat{z}$ below which it defaults, where for any ℓ, $\hat{z}$ is the lowest z such that $p(z,\ell)\ell^\alpha \ge w\ell + b$, where $p(z,\ell)$ is described above. Thus, the firm solves the following problem:

$$\max_{\ell,\hat{z}}\int_{\hat{z}}^\infty [p(z,\ell)z\ell^\alpha - w\ell - b]\pi_z(z)\,dz + \int_{\hat{z}}^\infty V\pi_z(z)\,dz,$$

subject to the constraint $p(\hat{z},\ell)\ell^\alpha - w\ell - b = 0$. This constraint defines the cutoff productivity $\hat{z}$ below which the firm defaults, because for any

$z < \hat{z}$, the firm would have negative equity payouts. The cutoff $\hat{z}$ is increasing in labor because as labor ℓ is increased, the wage bill $w\ell$ increases by more than the revenues $p(z, \ell)\ell^{\alpha}$. The larger the level of labor ℓ, the larger the probability of default for the firm.

In this environment, the optimal choice of labor does not simply maximize expected profits, as it does with complete financial markets. Here, the firm balances the marginal increase in profits from an increase in ℓ with the increased costs arising from a higher probability of default that such an increase entails. The choice of ℓ^* satisfies

$$E(p(z, \ell^*)|z \geq \hat{z})\alpha(\ell^*)^{\alpha-1} = \frac{\eta}{\eta - 1}\left(w + V\frac{\pi_z(\hat{z})}{1 - \Pi_z(\hat{z})}\frac{d\hat{z}}{d\ell^*}\right), \tag{5}$$

where $p(\hat{z}, \ell^*)(\ell^*)^{\alpha} - w\ell^* - b = 0$ and $\Pi_z(z)$ is the distribution function associated with the density $\pi_z(z)$.

When financial markets are incomplete and firms face default risk, the choice of ℓ equates the effective marginal product of labor in the states in which the firm does not default to the marginal costs arising from increasing labor, which includes the wage and the loss in future value. This loss in future value arising from default risk and encoded in the second term on the right-hand side of condition (5) distorts the firm's first-order condition and increases the wedge between the marginal product of labor and the wage (see Chari, Kehoe, and McGrattan [2007] for a discussion of the labor wedge).

In contrast to what happens with complete financial markets, here fluctuations in the volatility of idiosyncratic shocks do affect the first-order condition of labor. Increases in volatility typically increase the hazard rate $\pi_z(\hat{z})/(1 - \Pi_z(\hat{z}))$, which in turn leads to a larger distortion and a smaller labor input for any given wage w and aggregate output Y. More precisely, in the appendix, we assume that z is lognormally distributed, with $E(z) = 1$ and $\mathrm{Var}(\log z) = \sigma_z^2$. We show that if the value of continuation V is sufficiently large, then a mean-preserving spread, namely, an increase in σ_z, leads to a decrease in labor ℓ. In this case, an increase in volatility increases the risk of default and gives firms incentives to lower this risk by reducing their labor input.

Note that the first-order condition (5) shares some features with that for the choice of capital in standard costly state verification models, as in Bernanke, Gertler, and Gilchrist (1999). In particular, the lost resources from default in our example play a role similar to that of the lost resources from monitoring in the costly state verification framework.

II. Model

We now turn to our general model, namely, a dynamic open-economy model that incorporates financial frictions and variations in the volatility of idiosyncratic shocks at the firm level. The economy has continuums of final-goods firms, intermediate-goods firms, and households. Each household, or *family*, consists of a continuum of workers and managers that insures its members. The final-goods firms are competitive and have a technology that converts intermediate goods into a final good. This technology is subject to idiosyncratic shocks that affect the productivity of intermediate goods used to produce final goods. The volatility of these shocks is stochastically time varying, and these volatility shocks are the only aggregate shocks in the economy.

The intermediate-goods firms are monopolistically competitive and use labor to produce differentiated products. They can borrow only state-uncontingent debt and are allowed to default on both their debt and payments to workers. If they default, they exit the market with zero value. New firms enter to replace defaulting firms that exit. Households have preferences over consumption and leisure, provide labor services to intermediate-goods firms, and own all firms.

At the end of any given period, firms decide how many workers to hire for the next period and how much to borrow, while households decide how much labor to supply to the market for the next period. In the beginning of the next period, aggregate and idiosyncratic shocks are realized. Intermediate-goods firms set their prices, produce, sell their products to final-goods firms, choose whether to pay their existing debts and their wage bill, and distribute equity payouts. The final-goods firms buy the intermediate goods and produce. Potential new firms decide whether to enter the market. Households consume and receive payments on their assets.

A. *Final- and Intermediate-Goods Firms*

The final good is traded on world markets and has a price of one. The final good Y_t is produced from a fixed variety of nontraded intermediate goods $i \in [0, 1]$ via the technology

$$Y_t \leq \left(\int z_t(i) y_t(i)^{(\eta-1)/\eta} di \right)^{\eta/(\eta-1)}, \tag{6}$$

where the elasticity of demand η is greater than one. Final-goods firms choose the intermediate goods $\{y_t(i)\}$ to solve

$$\max_{\{y_t(i)\}} Y_t - \int p_t(i) y_t(i)\, di, \tag{7}$$

subject to constraint (6), where $p_t(i)$ is the price of good i relative to the price of the final good. This problem yields that the demand $y_t(i)$ for good i is

$$y_t(i) = \left(\frac{z_t(i)}{p_t(i)}\right)^\eta Y_t. \tag{8}$$

A measure μ_f of intermediate-goods firms produce differentiated goods that are subject to idiosyncratic productivity shocks z_t that follow a Markov process with transition function $\pi_z(z_t|z_{t-1}, \sigma_{t-1})$, where σ_{t-1} is an aggregate shock to the standard deviation of the idiosyncratic productivity shocks. The aggregate shock σ_t follows a Markov process with transition function $\pi_\sigma(\sigma_t|\sigma_{t-1})$. Firms are also subject to idiosyncratic revenue shocks κ_t that have a distribution $\Phi(\kappa)$ and are independent over time. These firms are monopolistically competitive and produce output y_t, using technology $y_t = \ell_t^\alpha \ell_{mt}^\theta$, where ℓ_t is the input of workers, ℓ_{mt} is the input of a single manager, and $0 < \alpha < 1$. Since each active firm uses one manager, we simply impose $\ell_{mt} = 1$ from now on.

Here, since the final-goods production function has no value added but rather simply combines the intermediate goods, we can reinterpret our setup as follows. The aggregator of final goods is $Y_t = (\int \tilde{y}_t(i)^{(\eta-1)/\eta}\, di)^{\eta/(\eta-1)}$, and each final good i produces $\tilde{y}_t(i) = z_t^{\eta/(\eta-1)} \ell_t^\alpha \ell_{mt}^\theta$ units of good i with associated demand function $\tilde{y}_t(i) = \tilde{p}_t(i)^{-\eta} Y_t$ and price $\tilde{p}_t(i) = z_t^{\eta/(1-\eta)} p_t(i)$. Here, when measured in logs, the TFP of a firm is proportional to z_t. This alternative interpretation is useful to keep in mind when using the data to help set the parameters for z_t.

After all shocks are realized, each firm decides on the price $p_t(i)$ of its product and whether to repay or default. Since firms face demand curves with an elasticity larger than one, they always choose prices to sell all of their output, and, hence, we can set $p_t(i) = z_t(i)(Y_t/\ell_t^\alpha(i))^{1/\eta}$ and eliminate prices as a choice variable from now on. Firms that default exit, whereas firms that continue pay their debt b_t, choose borrowing b_{t+1} and labor input ℓ_{t+1} at the end of period t, paying the associated wage bill only after they produce. Debt pays off b_{t+1} at $t+1$, as long as a firm chooses not to default at $t+1$, and gives the firm $q_t b_{t+1}$ at t where, as we show below, the bond price q_t is a function that reflects the compensation for the loss in case of default.

Firms pay their equity holders their revenues net of production costs and net payments on debt. Equity payouts d_t are restricted to be nonnegative and satisfy the *nonnegative equity payout* condition

$$d_t = p_t \ell_t^\alpha - w_t \ell_t - w_{mt} - \kappa_t - b_t + q_t b_{t+1} \geq 0, \tag{9}$$

where w_t is the wage of workers and w_{mt} is the wage of managers. Firms use variations in equity payouts to help buffer shocks. It will turn out that this motive leads equity payouts to be procyclical, as they are in the data.

It is convenient for the recursive formulation to define the cash on hand x_t as

$$x_t = p_t \ell_t^\alpha - w_t \ell_t - w_{mt} - \kappa_t - b_t. \tag{10}$$

The *idiosyncratic state* of a firm, (z_t, x_t), records the current idiosyncratic shock z_t and its cash on hand x_t, whereas the *aggregate state* $S_t = (\sigma_t, \Upsilon_t)$ records the current aggregate shock σ_t and the measure Υ_t over idiosyncratic states, which satisfies $\int d\Upsilon_t(z_t, x_t) = \mu_f$. It is permissible to index a firm by its idiosyncratic state (z_t, x_t) rather than by its index i because all intermediate-goods firms with the same idiosyncratic state make the same decisions.

We provide a brief overview of the firm's problem before we formally describe it. The firm's value is the discounted value of its stream of equity payouts. In each period, the firm chooses equity payouts, the default decision, borrowing, and next period's labor. The firm has a budget constraint, a nonnegativity condition on equity payouts, and an agency friction constraint derived from the manager's incentive problem. Firms default only when their value is less than or equal to zero. Since equity payouts are nonnegative, the firm's value is always nonnegative. Since the firm will never default if it can pay positive equity payouts in the current period, it follows that the firm will default only if there is no feasible choice for it that leads to nonnegative equity payouts; that is, it defaults only if its budget set is empty.

1. Financial Frictions

We turn now to discussing the bond price and the default decision that determines it and then turn to the agency friction. The bond price $q_t = q(S_t, z_t, \ell_{t+1}, b_{t+1})$ reflects the compensation for the loss in case of default and depends on the current aggregate state S_t, the firm's current idiosyncratic shock z_t, and two decisions of the firm—its labor input ℓ_{t+1} and its borrowing level b_{t+1}. To derive when firms default, let $M(S_t, z_t)$ be the *maximal borrowing*, namely, the largest amount a firm can borrow, given the bond price schedule q, that is,

$$M(S_t, z_t) = \max_{\{\ell_{t+1}, b_{t+1}\}} q(S_t, z_t, \ell_{t+1}, b_{t+1}) b_{t+1}, \tag{11}$$

and let $\bar{\ell}(S_t, z_t)$ and $\bar{b}(S_t, z_t)$ be the labor and debt plan, respectively, associated with this maximal borrowing. Let $\kappa_{t+1}^* = \kappa^*(S_t, S_{t+1}, z_{t+1}, \ell_{t+1}, b_{t+1})$ be the highest level of the revenue shock, such that if at this level a firm borrows this maximal amount, it can just satisfy the nonnegative equity payout condition. From condition (9), this *cutoff level* of revenue shock satisfies

$$\begin{aligned}\kappa^*_{t+1} &\equiv \kappa^*(S_t, S_{t+1}, z_{t+1}, \ell_{t+1}, b_{t+1}) \\ &= p_{t+1}\ell^\alpha_{t+1} - w_{t+1}\ell_{t+1} - w_{\mathrm{m}t+1} - b_{t+1} + M(S_{t+1}, z_{t+1}),\end{aligned} \tag{12}$$

where

$$p_{t+1} = z_{t+1}\left(\frac{Y_{t+1}}{\ell^\alpha_{t+1}}\right)^{1/\eta}. \tag{13}$$

In equation (12), the wages for workers, $w_{t+1} = w(S_t)$, and managers, $w_{\mathrm{m}t+1} = w_{\mathrm{m}}(S_t)$, depend on the aggregate state S_t because they are determined at the end of period t. In equation (13), aggregate output $Y_{t+1} = Y(S_t)$ also depends on the aggregate state S_t, because it is based on choices made at the end of period t and the distribution of idiosyncratic productivity shocks at $t + 1$, which is known at the end of period t.

The default decision thus has a cutoff form: repay in period $t + 1$ if the revenue shock $\kappa \le \kappa^*_{t+1}$, which occurs with probability $\Phi(\kappa^*_{t+1})$, and default otherwise. Hence, the bond price schedule that ensures that lenders break even is defined by

$$q(S_t, z_t, \ell_{t+1}, b_{t+1}) = \beta \int \pi_\sigma(\sigma_{t+1}|\sigma_t)\pi_z(z_{t+1}|z_t, \sigma_t)\Phi(\kappa^*_{t+1})\, d\sigma_{t+1}\, dz_{t+1}, \tag{14}$$

where β is the discount factor of risk-neutral international intermediaries and the aggregate state S_t evolves according to its transition function described below. We define the *firm credit spread*, or simply *spread*, to be

$$\frac{1}{q(S_t, z_t, \ell_{t+1}, b_{t+1})} - \frac{1}{\beta}, \tag{15}$$

which is the difference between the interest rate on a firm's defaultable bond and the interest rate on default-free bonds charged by international intermediaries.

All firms, even those that default, choose prices and produce. Defaulting firms with enough revenues to cover their wage bill, namely, those with $p_t\ell^\alpha_t - w_t\ell_t - w_{\mathrm{m}t} - \kappa_t \ge 0$, pay this wage bill in full, and those with insufficient revenues to cover current wages pay out all of their revenues to labor. Defaulting firms can pay their wage bill in full if[3]

$$\kappa_t \le \bar\kappa(S_t, z_{t+1}, \ell_{t+1}, b_{t+1}) = p_{t+1}(S_t, z_{t+1}, \ell_{t+1})\ell^\alpha_{t+1} - w(S_t)\ell_{t+1} - w_{\mathrm{m}t+1}. \tag{16}$$

Some defaulting firms have revenues that are greater than the wage bill but less than that needed to pay their debt. We assume that such revenues are lost to bankruptcy costs.[4]

[3] Note that $\bar\kappa_t$ can be either smaller or larger than κ^*_t because defaulting firms cannot borrow.

[4] In our quantitative model, these bankruptcy costs upon default are about 2 percent of firm value, an amount that falls in the range of estimates for the direct bankruptcy costs of Warner (1977) and Altman (1984) of 1–6 percent.

Consider next the agency friction. This friction captures the tensions between shareholders and managers discussed by Jensen (1986). The idea is that if the plans of the firm do not exhaust most of the credit available to the firm, then managers are tempted to access this unused credit and use the resulting funds to benefit their private interests. When shareholders choose their borrowing, they understand the incentives of managers to divert unused credit. This agency friction will end up implying a constraint on the maximum amount of unused credit or, equivalently, a constraint on the minimum amount of borrowing.

To set up the agency friction, we develop the problems of managers. We assume there are a large number of potential managers who can work in intermediate-goods firms with wage $w_{\mathrm{m}t}$ or can use a backyard technology to produce $\bar{w}_{\mathrm{m}}$ units of final goods each period. Given the large number of potential managers, competition implies that managers earn $w_{\mathrm{m}t} = \bar{w}_{\mathrm{m}}$. These managers belong to families that consist of a continuum of managers and workers who insure its members against idiosyncratic risk. In period t, each family values the contribution of any member to the family's total income in period $t + j$ with the stochastic discount factor $Q_{t,t+j} = Q_{t,t+j}(S_{t+j}|S_t)$.

We break each period t into two stages: *day* and *night*. During the day, the manager's actions are observable to the owners, and they can ensure that the manager carries out their will. The manager receives funds $q_t b_{t+1}$ from the financial intermediary based on the shareholders' plan (ℓ_{t+1}, b_{t+1}). These funds flow into the firm during the day and, hence, become outside of the manager's grasp: they are used to pay period t wages, old debt, costs from revenue shocks, and equity payouts net of the receipts from sales and satisfy condition (9).

At night, the manager's actions are unobserved by the owners. If a firm leaves too much unused credit, the manager of that firm will make the following deviation. After the firm borrows $q_t b_{t+1}$, the manager returns to the financial intermediary with an altered plan, $\bar{\ell}(S_t, z_t)$, $\bar{b}(S_t, z_t)$, which gives the maximal borrowing $M(S_t, z_t)$. The financial intermediary gives the deviating manager the funds requested, net of the original loan amount $q_t b_{t+1}$, which is used to immediately pay it off. The manager uses the additional funds to hire workers $\ell_{s,t+1}$ for a side project chosen to just exhaust the unused credit at the current wages, such that $\ell_{s,t+1}$ satisfies

$$M(S_t, z_t) - q_t b_{t+1} = w_{t+1} \ell_{s,t+1}. \tag{17}$$

At time $t + 1$, this side project produces a good with a stochastic current period payoff for the manager of

$$p_{s,t+1} \lambda \ell_{s,t+1}^{\alpha}, \tag{18}$$

where the fraction λ determines the profitability of this side project and $p_{s,t+1}$ satisfies equation (13). The output from this side project is a private benefit to the manager and has no value to the firm.

The next morning, the $\bar{\ell}(S_t, z_t)$ workers from the deviating manager's plan show up and produce. The owners find out that the deviating manager has altered their plan, increased the debt of the firm to $\bar{b}(S_t, z_t)$, and absconded with the incremental funds in equation (17). The owners then fire the deviating manager, and the firm faces the new nonnegative equity payout condition

$$p_{t+1}\bar{\ell}^{\alpha}(S_t, z_t) - w_{t+1}\bar{\ell}^{\alpha}(S_t, z_t) - \bar{w}_{\mathrm{m}} - \kappa_{t+1} - \bar{b}(S_t, z_t) + q_{t+1}b_{t+2} \geq 0. \tag{19}$$

Note that, in effect, the deviating manager has increased the future indebtedness of the firm and, instead of paying out the incremental funds raised to equity holders, has diverted these funds to generate a private benefit.

After being fired, the manager regains the ability to work in the market or use the backyard technology with probability θ. A manager will not divert unused credit if the diversion payoff is sufficiently small relative to the wage, in that

$$E_t Q_{t,t+1}(\lambda p_{s,t+1}\ell_{s,t+1}^{\alpha}) + E_t\theta\sum_{j=2}^{\infty} Q_{t,t+j}\bar{w}_{\mathrm{m}} \leq E_t\sum_{j=1}^{\infty} Q_{t,t+j}\bar{w}_{\mathrm{m}}, \tag{20}$$

where we recall that the manager values goods at $t + j$ using the stochastic discount factor $Q_{t,t+j}$ of the family. The left side of this constraint captures both the current-period payoff of diverting funds and the present value of payoffs after being fired, while the right side is the present value of wages if the manager never diverts funds. To prevent diversion, firms must have a small enough amount of unused credit so that the value of the side project the manager can undertake is sufficiently small, which means choosing borrowing to be sufficiently high so that the agency friction constraint

$$q(S_t, z_t, \ell_{t+1}, b_{t+1})b_{t+1} \geq M(S_t, z_t) - F_{\mathrm{m}}(S_t, z_t) \tag{21}$$

holds. Here $F_{\mathrm{m}}(S_t, z_t)$ is the maximum amount of unused credit, or free cash flow, that prevents diversion and is obtained by substituting condition (17) into constraint (20), using equation (13), and rearranging to get

$$F_{\mathrm{m}}(S_t, z_t) = \left[\frac{E_t Q_{t,t+1}\bar{w}_{\mathrm{m}} + (1-\theta)E_t\sum_{j=2}^{\infty} Q_{t,t+j}\bar{w}_{\mathrm{m}}}{E_t Q_{t,t+1}\lambda z_{t+1}(Y_{t+1})^{1/\eta}}\right]^{\eta/\alpha(\eta-1)} w_{t+1}, \tag{22}$$

where $w_{t+1} = w(S_t)$ and $Y_{t+1} = Y(S_t)$. This maximum cash flow depends on the side-project technology, the manager's wage, the probability of a fired manager regaining a job, and the wage rate of workers.

The agency friction constraint plays an important role in our model. In the model, the combination of uncontingent debt and the nonnegative equity payout condition restricts the ability of the firm to choose the size of employment to maximize expected profits. That restriction gives firms an incentive to build up a large buffer stock of unused credit, which would allow the firm to self-insure against idiosyncratic shocks. The agency friction constraint makes building up such a buffer stock unattractive.

Most dynamic models of financial frictions face a similar issue. The financial frictions, by themselves, make internal finance through retained earnings more attractive than external finance. Absent some other force, firms build up their savings and circumvent these frictions. In the literature, the forces used include finite lifetimes (Bernanke et al. 1999; Gertler and Kiyotaki 2011), impatient entrepreneurs (Kiyotaki and Moore 1997), and the tax benefits of debt (Jermann and Quadrini 2012). For a survey of these forces and the role they play, see Quadrini (2011).

2. The Firm's Recursive Problem

Consider now the problem of an incumbent firm. Let $V(S_t, z_t, x_t)$ denote the value of the firm after shocks are realized in period t. The value of such a firm is

$$V(S_t, z_t, x_t) = 0$$

for any state (S_t, z_t, x_t) such that the budget set is empty, in that even if it borrows the maximal amount, it cannot make nonnegative equity payouts, that is, $d_t = x_t + M(S_t, z_t) < 0$. For all other states (S_t, z_t, x_t), the budget set is nonempty, and firms continue their operations and choose labor ℓ_{t+1}, new borrowing b_{t+1}, and equity payouts d_t to solve

$$V(S_t, z_t, x_t) = \max_{\{\ell_{t+1}, b_{t+1}, d_t\}} d_t + \int Q_{t,t+1}(S_{t+1}|S_t)\pi_z(z_{t+1}|z_t, \sigma_t) \left[\int^{\kappa^*_{t+1}} V(S_{t+1}, z_{t+1}, x_{t+1})\, d\Phi(\kappa)\right] d\sigma_{t+1}\, dz_{t+1},$$

subject to the nonnegative equity payout condition

$$d_t = x_t + q_t b_{t+1} \geq 0 \tag{23}$$

and the agency friction constraint (21), where κ^*_{t+1} and q_t and are given in equations (12) and (14), respectively. Cash on hand tomorrow is given by

$$x_{t+1} = p_{t+1}\ell_{t+1}^{\alpha} - w_{t+1}\ell_{t+1} - \bar{w}_{\mathrm{m}} - b_{t+1} - \kappa_{t+1}, \tag{24}$$

where p_{t+1} is from equation (13), wages for workers $w_{t+1} = w(S_t)$, and output $Y_{t+1} = Y(S_t)$. The law of motion for aggregate states $S_{t+1} = (\Upsilon_{t+1},\sigma_{t+1})$ has

$$\Upsilon_{t+1} = H(\sigma_{t+1}, S_t), \tag{25}$$

where σ_{t+1} follows the Markov chain $\pi_\sigma(\sigma_{t+1}|\sigma_t)$. This problem gives the decision rules for labor $\ell_{t+1} = \ell(S_t, z_t, x_t)$, borrowing $b_{t+1} = b(S_t, z_t, x_t)$, and equity payouts $d_t = d(S_t, z_t, x_t)$.

Our model features an aggregate volatility shock σ_t and two idiosyncratic shocks: the serially correlated productivity shocks z_t and the independently and identically distributed (i.i.d.) revenue shocks κ_t. When the volatility of the productivity shocks increases, the borrowing schedule of firms worsens, and as we expand on below, firms reduce both their labor and their borrowing. The revenue shock helps the model generate the level of spreads in the data. To understand why these shocks generate spreads, note that firms have a limited ability to prepare against these shocks besides keeping debt low. With agency frictions, however, keeping debt very low is costly. Hence, when a large additive shock hits, firms default, and the anticipation of such default generates spreads. Since these revenue shocks are i.i.d., they do not have much effect on either the cyclicality or the standard deviation of the median spread, nor do they affect business cycle properties, which instead are determined mostly by the interaction between the aggregate volatility shocks and the productivity shocks.

Now consider firm entry. The model has a continuum of potential entering firms every period, each of which draws an entry cost ω from a lognormal distribution with mean $\bar{\omega}$ and standard deviation σ_ω with density $\psi(\omega)$ and cumulative distribution function $\Psi(\omega)$. To enter, firms have to pay an entry cost ω in period t and decide on the labor input ℓ_{t+1}^{e} for the following period. An entering firm must borrow to pay the entry cost and current equity payouts.

Firms that enter in period t draw their idiosyncratic productivity z_{t+1} in period $t+1$ according to the *entry density* $\pi_z^{\mathrm{e}}(z_{t+1}, \sigma_t)$. For simplicity, we assume that this entry density is given by one of the rows of the Markov chain governing incumbents' idiosyncratic shocks, so that $\pi_z^{\mathrm{e}}(z_{t+1}, \sigma_t) = \pi_z(z_{t+1}|z_{\mathrm{e}}, \sigma_t)$ where z_{e} simply indexes the initial density from which entrants draw their shocks. Hence, lenders treat all entrants symmetrically and lend them up to $M(S_t, z_{\mathrm{e}})$. We assume that from the measure of potential entrants with entry costs smaller than the maximal borrowing, namely, those with $\omega \leq M(S_t, z_{\mathrm{e}})$, a subset is chosen randomly, so that the measure of entering firms equals the measure of exiting firms. All such firms have an incentive to enter. An entering firm at t solves the same problem as an incumbent firm with $x = -\omega$ and $z = z_e$, where, again, z_e simply indexes the density from which entrants draw their initial shock z_{t+1}.

B. Families

There are a large number of identical households, or families, with a measure μ_w of workers and μ_m of managers that satisfy $\mu_w + \mu_m = 1$ and $\mu_m > \mu_f$. At the beginning of period t, each family sends a mass μ_w of workers to the market, each of whom supplies L_t units of labor at *effective wage* $W_t = W(S_{t-1})$. This wage is determined before the shock σ_t is realized and, hence, is a function of the aggregate state S_{t-1}. The family also sends a mass μ_m as managers, each of whom works one unit and earns $\bar{w}_m$: μ_f of them work as managers and $\mu_m - \mu_f$ of them work in home production. Families know the measure Υ_t over the idiosyncratic states of firms but do not see individual firms. The family supplies total labor of workers $\mu_w L_t$ to an anonymous market, and then this labor is distributed across firms according to their relative demands. In particular, families cannot choose to which firms they send their measure of workers. The family equalizes consumption of all family members in each period. The period utility of the family is $U(C_t) - \mu_w G(L_t) - \mu_m G(1)$, where from now on we drop the irrelevant constant $\mu_m G(1)$.

After the aggregate shock σ_t and the idiosyncratic shocks are realized, the families choose their consumption C_t and state-contingent asset holdings $A_{t+1} = \{A_{t+1}(\sigma_{t+1})\}$, get paid the effective wage W_t for workers and $\bar{w}_m$ for managers, and receive aggregate equity payouts D_t from their ownership of the intermediate-goods firms.

The state of a family is their vector of assets A_t and the beginning-of-period state S_{t-1}. The recursive problem for families is the following:

$$V^h(A_t, S_{t-1})$$
$$= \max_{L_t}\left\{\int_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) \max_{C_t(\sigma_t),\{A_{t+1}(\sigma_{t+1})\}} \left[U(C_t) - \mu_w G(L_t) + \beta V^h(A_{t+1}, S_t)\right] d\sigma_t\right\},$$

subject to their budget constraint for each σ_t,

$$C_t(\sigma_t) + \int_{\sigma_{t+1}} Q_{t,t+1}(S_{t+1}|S_t)A_{t+1}(\sigma_{t+1})\, d\sigma_{t+1}$$
$$= \mu_w W_t(S_{t-1})L_t + \mu_m \bar{w}_m + A_t(\sigma_t) + D_t(\sigma_t, S_{t-1}),$$

where $S_{t+1} = (\Upsilon_{t+1}, \sigma_{t+1})$ and $\Upsilon_{t+1} = H(\sigma_{t+1}, S_t)$.

Our open economy is small in the world economy, so that the one-period-ahead state-contingent prices are given by

$$Q_{t,t+1}(S_{t+1}|S_t) = \beta\pi_\sigma(\sigma_{t+1}|\sigma_t), \tag{26}$$

and $Q_{t,t+j}(S_{t+j}|S_t) = \beta^j \pi_\sigma^j(\sigma_{t+j}|\sigma_t)$ for any $j > 1$, where $\pi_\sigma^j(\sigma_{t+j}|\sigma_t)$ is the conditional probability in period $t + j$ of σ_{t+j}, given σ_t induced by the Markov chain.

In the budget constraint, $\mu_w W_t(S_{t-1})L_t$ is the total wage payments to the measure of labor $\mu_w L_t$. Aggregate equity payouts $D_t(\sigma_t, S_{t-1})$ are determined after the shock σ_t is realized and, hence, are functions of the aggregate state S_{t-1} and the shock σ_t. The first-order condition for the labor of workers is

$$\frac{\int_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1})\mu_w G'(L_t)\, d\sigma_t}{\int_{\sigma_t} \pi_\sigma(\sigma_t|\sigma_{t-1}) U'(C_t(\sigma_t))\, d\sigma_t} = W_t(S_{t-1}). \tag{27}$$

Using the envelope condition and equation (26), the first-order condition for consumption implies the *risk-sharing condition,* $U'(C_t(\sigma_t)) = U'(C_{t+1}(\sigma_{t+1}))$.

Consider next the relation between effective wage W_t for workers and their promised wage w_t. The aggregate wage payments that families receive from all firms is $\mu_w W_t L_t$, whereas w_t is the promised wages that an individual firm offers but may not pay. A given firm pays the full promised wages w_t if $\kappa < \kappa_t^*$ or $\kappa < \bar{\kappa}_t$. We denote the corresponding full repayment of wages set

$$\Omega_R(S_{t-1}, S_t, z_t, x_{t-1}, z_{t-1}) = \{\kappa : \kappa \le \kappa_t^* \text{ or } \kappa \le \bar{\kappa}_t\},$$

where κ_t^* and $\bar{\kappa}_t$ are given in equations (12) and (16), respectively, where we evaluate $\ell_t = \ell(S_{t-1}, z_{t-1}, x_{t-1})$ and $b_t = b(S_{t-1}, z_{t-1}, x_{t-1})$. Let Ω_D be the default set in which firms pay less than the full face value of wages. The aggregate wage payments at t that a family receives from firms,

$$\mu_w W(S_{t-1})L_t(S_{t-1}) = \int \pi_\sigma(\sigma_t|\sigma_{t-1})\pi_z(z_t|z_{t-1}, \sigma_{t-1}) \left[\int_{\Omega_R} w_t \ell_t d\Phi(\kappa) + \int_{\Omega_D} \max\{p_t \ell_t^\alpha - \bar{w}_m - \kappa, 0\}\, d\Phi(\kappa)\right] d\Upsilon_{t-1}\, d\sigma_t\, dz_t.$$

Here the family understands that when it supplies a measure of workers to the market in which each firm is promising a face value of wage w_t, once default has been taken into account, the effective wage will be only $W_t < w_t$.[5]

The aggregate equity payout that families receive each period is the sum of all the equity payouts from intermediate-goods firms, so that

[5] In our quantitative model, firms almost always pay their wage bill, so that $W_t = 0.9999 w_t$. Here we do not consider firms defaulting on their payments to managers because, in our quantitative model with bounded supports on shocks, firms always have enough revenues, net of the revenue shock κ_t, to pay the wages of managers in full.

$$D_t(\sigma_t, S_{t-1}) = \int d(x, z, \sigma_t, H(\sigma_t, S_{t-1}))\, dH(\sigma_t, S_{t-1}). \tag{28}$$

The family's problem gives the decision rule for labor, $L_t(S_{t-1})$, and the decision rules for consumption and asset holdings, $C_t(\sigma_t, S_{t-1})$ and $A_{t+1}(\sigma_{t+1}|\sigma_t, S_{t-1})$, respectively.

C. *Equilibrium*

Here we specify the equilibrium conditions in our model for aggregates in $t + 1$. Market clearing in the labor market requires that the amount of labor demanded by firms equals the amount of labor supplied by families,

$$\int \ell_{t+1}(S_t, z_t, x_t)\, d\Upsilon_t(z_t, x_t) = \mu_{\mathrm{w}} L_{t+1}(S_t). \tag{29}$$

Output satisfies

$$Y(S_t) = \left[\iint_{z_{t+1}, x_t, z_t} \pi_z(z_{t+1}|z_t, \sigma_t) z_{t+1} y_{t+1}(S_t, z_t, x_t)^{(\eta-1)/\eta}\, d\Upsilon_t(z_t, x_t)\right]^{\eta/(\eta-1)}, \tag{30}$$

where $y_{t+1} = \ell_{t+1}^{\alpha}$. The measure of exiting firms at $t + 1$ when the aggregate shock is σ_{t+1} is

$$E^X_{t+1}(\sigma_{t+1}, S_t) = \int_{z_{t+1}, x_t, z_t} \int_{\kappa \ge \kappa^*_{t+1}} \pi_z(z_{t+1}|z_t, \sigma_t)\, d\Phi(\kappa)\, d\Upsilon_t(z_t, x_t).$$

The transition function for the measure of firms is $\Upsilon_{t+1} = H(\sigma_{t+1}, S_t)$, which consists of incumbent firms that do not default at time $t + 1$ and new entrant firms and is implicitly defined by

$$\begin{aligned} H(x_{t+1}, z_{t+1}; \sigma_{t+1}, S_t) = {} & \int \Lambda(x_{t+1}, z_{t+1}, x_t, z_t|\sigma_{t+1}, S_t)\, d\Upsilon_t(z_t, x_t) \\ & + E^X_{t+1}(\sigma_{t+1}, S_t) \frac{\psi(-x_{t+1}) I_{\{z_{t+1}=z_e, -x_{t+1} \le M(\sigma_{t+1}, H(\sigma_{t+1}, S_t), z_e)\}}}{\displaystyle\int_{\omega \le M(\sigma_{t+1}, H(\sigma_{t+1}, S_t), z_e)} d\Psi(\omega)}. \end{aligned}$$

The first term in the transition function comes from the incumbents. To understand this term, note that when the aggregate state in period t is S_t and the aggregate shock in period $t + 1$ is σ_{t+1}, the probability that an incumbent firm with some state (z_t, x_t) transits to state (z_{t+1}, x_{t+1}) is given by Λ. Here $\Lambda(x_{t+1}, z_{t+1}, x_t, z_t|\sigma_{t+1}, S_t) = \pi_z(z_{t+1}|z_t, \sigma_t)\phi(\kappa_{t+1})$ if, at that state (z_t, x_t), the decision rules $\ell_{t+1} = \ell(S_t, z_t, x_t)$ and $b_{t+1} = b(S_t, z_t, x_t)$, together with the given κ_{t+1}, produce the particular level of cash on hand x_{t+1}, so that κ_{t+1} satisfies

$$\kappa_{t+1} = z_{t+1} Y(S_t)^{1/\eta} \ell_{t+1}^{\alpha[(\eta-1)/\eta]} - w(S_t)\ell_{t+1} - \bar{w}_{\mathrm{m}} - b_{t+1} - x_{t+1}$$

and $\kappa_{t+1} \le \kappa^*(S_t, \sigma_{t+1}, H(S_t, \sigma_{t+1}), z_{t+1}, \ell_{t+1}, b_{t+1})$, so the firm does not default. If any of these conditions do not hold, then $\Lambda(x_{t+1}, z_{t+1}, x_t, z_t | \sigma_{t+1}, S_t) = 0$.

The second term in the transition function comes from new entrants. To understand this term, note that the probability that a new entrant's state is (z_{t+1}, x_{t+1}) equals the density of the entry cost $\psi(-x_{t+1})$ at $\omega = -x_{t+1}$, conditional on both $z_{t+1} = z_{\mathrm{e}}$ and that this entry cost is less than the borrowing limit, so that $-x_{t+1} \le M(\sigma_{t+1}, H(\sigma_{t+1}, S_t), z_{\mathrm{e}})$. The term in the denominator scales the total measure of new entrants so that it equals the total measure of exiting firms.

Given the initial measure Υ_0 and an initial aggregate shock σ_0, an *equilibrium* consists of policy and value functions of intermediate-goods firms $\{d(S_t, z_t, x_t), b(S_t, z_t, x_t), \ell(S_t, z_t, x_t), V(S_t, z_t, x_t)\}$; families $C(\sigma_t, S_{t-1})$, $L(S_{t-1})$, and $A(\sigma_{t+1}|\sigma_t, S_{t-1})$; the wage rate $w(S_{t-1})$, the effective wage rate $W(S_{t-1})$, and state-contingent prices $Q_{t,t+1}(S_{t+1}|S_t)$; bond price schedules $q(S_t, z_t, \ell_{t+1}, b_{t+1})$; and the evolution of aggregate states Υ_t governed by the transition function $H(\sigma_t, S_{t-1})$, such that for all t, (1) the policy and value functions of intermediate-goods firms satisfy their optimization problem, (2) family decisions are optimal, (3) the bond price schedule satisfies the break-even condition and the state-contingent prices satisfy equation (26), (4) the labor market clears, and (5) the evolution of the measure of firms is consistent with the policy functions of firms, families, and shocks.

D. *Characterizing Firms' Decisions*

Here we show how the intuition from our simple example extends to our model and characterize some properties of firms' decisions.

Consider the firm's first-order conditions for labor ℓ' and new borrowing b'. The first-order condition with respect to labor is a generalization of the corresponding first-order condition (5) from our simple example, namely,

$$\alpha(\ell')^{\alpha-1}\int_{s'\in R}\pi(s'|z,\sigma)p'\,ds'$$

$$= \frac{\eta}{\eta-1}\left\{w' + \frac{E_{z',\sigma'}V'^{*}\phi(\kappa^*)(-\partial\kappa^*/\partial\ell) + [(1+\gamma+\mu)/\beta](-\partial q/\partial\ell')b'}{E_{z',\sigma'}\int^{\kappa^*}(1+\gamma')\,d\Phi(\kappa')}\right\}, \tag{31}$$

where $s' = \{z', \sigma', \kappa'\}$, p' is given by equation (13), and the probability density $\pi(s'|z,\sigma)$ is defined by

$$\pi(s'|z,\sigma) = \frac{\pi_\sigma(\sigma'|\sigma)\pi_z(z'|z,\sigma)\phi(\kappa')(1+\gamma')}{E_{z',\sigma'}\int^{\kappa^*}(1+\gamma')\,d\Phi(\kappa')},$$

the repayment set is $R = \{(z', \sigma', \kappa') : \kappa \leq \kappa^*(S, S', z', \ell', b')\}$, and the multipliers γ and μ are associated with the nonnegative equity payout condition (9) and the agency friction constraint (21).

The optimal labor choice equates the weighted expected marginal benefit of labor to expected marginal cost times a markup. This expected benefit, given by the left-hand side of equation (31), is calculated using the "distorted" probability density $\pi(s'|z, \sigma)$. This benefit weights the marginal product in future states, taking into account two forces. First, it puts weight only on states in which the firm repays the debt tomorrow, because whenever the firm defaults, its shareholders receive zero. Second, it puts more weight on states in which the nonnegative equity payout condition tomorrow is binding, in that $\gamma' > 0$. Here $1 + \gamma'$ is the shadow price of cash on hand and reflects the marginal value of internal funds to a firm.

The expected marginal cost of labor, given by the right-hand side of equation (31), equals the marked-up value of the wage and a wedge. The first term in the numerator of this wedge is the loss in value from default incurred from hiring an additional unit of labor and is similar to the wedge in the simple example. This term is proportional to $V'^*\phi(\kappa^*)(-\partial\kappa^*/\partial\ell)$, where $V'^*\phi(\kappa^*)$ is the firm's future value evaluated at the default cutoff weighted by the probability of the cutoff and $-\partial\kappa^*/\partial\ell$ captures how the cutoff changes with labor. Since the cutoff decreases with labor, at least for low values of z, this first term is generally positive and acts like a tax on labor.

The second term in the numerator of this wedge, which was not present in the simple example, comes from the decrease in the bond price from hiring an extra unit of labor. The denominator of this wedge is the expected value of the shadow price in nondefault states.

It is useful to contrast these first-order conditions to those that would arise in a version of the model with complete markets and no agency frictions. In that version, the first-order condition for labor would be

$$\alpha(\ell')^{\alpha-1}\int_{z'}\pi_z(z'|z,\sigma)p'\,dz' = \frac{\eta}{\eta-1}w', \tag{32}$$

where p' is given in equation (13), which is the same as in our simple example (eq. [5]). As in that example, labor is chosen statically by the firm so as to equate the current marginal product of labor to a markup over the current wage.

In the simple example, we abstracted from new borrowing; here we do not. The first-order condition with respect to new borrowing is

$$(1 + \gamma + \mu)\left(q + b'\frac{\partial q}{\partial b'}\right) = \beta E_{z',\sigma'}\int^{\kappa^*}(1 + \gamma')\, d\Phi(\kappa') + \beta E_{z',\sigma'}V'^*\phi(\kappa^*). \quad (33)$$

The optimal level of new borrowing equates the effective marginal benefit of new borrowing to the expected marginal cost. Borrowing one more unit gives a direct increase in current resources of q and leads to a fall in the price of existing debt, giving a total change in current resources of $q + b'\partial q/\partial b'$. On the margin, these resources help relax both the nonnegative equity payout condition and the agency friction constraint and, hence, are valued at the sum of the multipliers on these constraints. This marginal borrowing relaxes the agency constraint because issuing more debt leaves less unused credit that the manager can use for a side project.

The marginal cost of borrowing, given by the right-hand side of this condition, consists of two terms. The first term reflects the cost of repaying but is relevant only in repayment states and is weighted by the shadow price of cash on hand in those states, namely, $1 + \gamma'$. The second term is the loss in value from default.

We can characterize in more detail firms' decision rules as a function of cash on hand. In figure 1, we show how the value of borrowing qb', equity payouts d, and the multiplier for equity payouts γ all vary with cash on hand. While these patterns hold in general, we choose to illustrate them for a firm from our quantitative model discussed below. The following lemma formalizes these patterns.

LEMMA. For $x < -M(S_t, z_t)$, the firm defaults. For $x \geq -M(S_t, z_t)$, there exists a cutoff level of cash on hand, $\hat{x}(S_t, z_t)$, such that for $x < \hat{x}$, the nonnegative equity payout constraint is binding, $\gamma > 0$, and the value of borrowing qb' increases one for one as cash on hand decreases, whereas for $x \geq \hat{x}$, the nonnegative equity payout constraint is slack, $\gamma = 0$, and the bond price, labor, and borrowing do not vary with cash on hand, whereas equity payouts increase one for one with cash on hand.

Proof. From the definition of $M(S_t, z_t)$, for any level of $x < -M(S_t, z_t)$, the budget set is empty and the firm necessarily defaults. For $x \geq -M(S_t, x_t)$, we construct the cutoff level of cash on hand by solving a relaxed version of the firm's problem for the optimal levels of new borrowing and labor in which we drop the nonnegative equity payout constraint for the current period only, namely,

$$\tilde{V}(S_t, z_t, x_t) = x_t + \left\{\max_{\{\ell_{t+1}, b_{t+1}\}} q_t b_{t+1} + \beta\int \pi_\sigma(\sigma_{t+1}|\sigma_t)\pi_z(z_{t+1}|z_t, \sigma_t) \right.$$
$$\left. \times \left[\int^{\kappa^*_{t+1}} V(S_{t+1}, z_{t+1}, x_{t+1})\, d\Phi(\kappa)\right] d\sigma_{t+1}\, dz_{t+1}\right\},$$

subject to the agency friction constraint (21), where the cash on hand tomorrow x_{t+1} is given in equation (24) and the aggregate state evolves

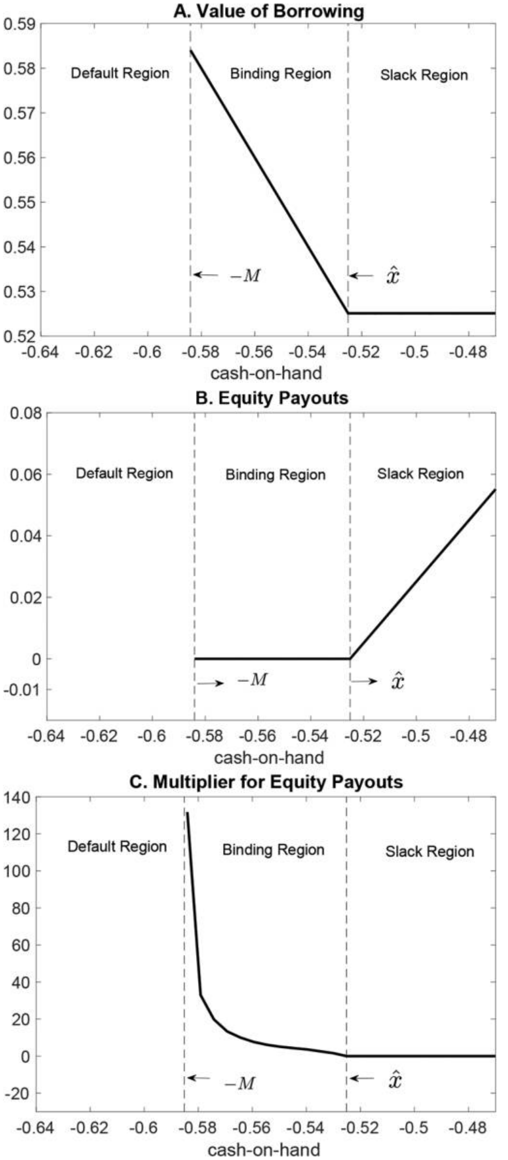

FIG. 1.—Firm decision rules

according to the state evolution equation. Note that cash on hand x_t enters simply as an additive constant in the objective function and not in any constraint. Hence, the *relaxed solution* does not vary with x_t and has the form $\hat{\ell}(S_t, z_t)$ and $\hat{b}(S_t, z_t)$, so that the associated bond price $\hat{q} = q(S_t, z_t, \hat{\ell}, \hat{b})$ and the value of borrowing, denoted $\hat{q}\hat{b}$, also do not vary with

x_t. The *cutoff level* of cash on hand is defined by $-\hat{x}(S_t, z_t) \equiv \hat{q}\hat{b}$. For a level of cash on hand below this cutoff level, the nonnegative equity constraint binds, and the firm chooses its borrowing level so that equity payouts are zero. For cash on hand above this cutoff level, the optimal level of borrowing does not vary with x and is given by the solution to the relaxed problem $\hat{q}\hat{b}$. Because the associated equity payouts satisfy $d = x + \hat{q}\hat{b}$, they increase one for one with x. Clearly, the multiplier on the nonnegative equity payout constraint $\gamma(S_t, z_t, x_t) = 0$ if $x_t \geq \hat{x}$ and $\gamma(S_t, z_t, x_t) > 0$ if $x_t < \hat{x}$. QED

III. Quantitative Analysis

We begin with a description of the data we use, discuss our parameterization, and describe how we choose parameters using a moment-matching exercise. Since our model is highly nonlinear and has occasionally binding constraints, we explain our algorithm in some detail.

We then explore the workings of our model, starting at the firm level. We begin with an analysis of spreads and decision rules and how these shift with aggregate volatility. We study the impulse responses for a firm's labor in response to an increase in aggregate volatility. We illustrate the importance of the financial structure by contrasting the response of a firm in our baseline model to one of a firm with frictionless financial markets. We then compare firm-level statistics in the model and the data.

We then turn to the model's predictions for aggregate variables. We begin with business cycle moments and then show that the model can account for many of the patterns of aggregates during the Great Recession.

A. *Data*

We use a combination of quarterly aggregate data from the national income and product accounts (NIPA), the Bureau of Labor Statistics (BLS), the Federal Reserve's flow-of-funds accounts, Moody's, and firm-level data from Compustat since 1985. From NIPA we use gross domestic product (GDP), and from BLS we use hours. From the flow of funds we use information on equity and debt for the nonfinancial corporate sector to construct our aggregate measures for debt purchases and equity payouts. From Compustat we construct five firm-level series: sales growth, leverage, equity payouts, debt purchases, and spreads.

Consider first the firm-level series from Compustat. As in Bloom (2009), we restrict the sample for firms to those with at least 100 quarters of observations since 1970. We define sales *growth* for each firm as $(s_{it} - s_{it-4})/0.5(s_{it} + s_{it-4})$ where s_{it} is the nominal sales for firm i at the t deflated by the consumer price index. We follow Davis and Haltiwanger (1992) in defining growth as being relative to the average level in order to have a measure that is less sensitive to extreme values of sales. We follow Bloom

(2009) in computing growth rates across four quarters to help eliminate the strong seasonality evident in the data. Using the panel data on firm growth rates, we construct the time series of the interquartile range (IQR) of sales growth across firms for each quarter. We define *leverage* as total firm debt, defined as the sum of short-term and long-term debt, divided by average sales, which is the average of sales over the past eight quarters expressed in annual terms. We define *equity payouts* as the average across the previous four quarters of the ratio of the sum of dividends and net equity repurchases to average sales. We define *debt purchases* as the average across the previous four quarters of the ratio of the change in total firm debt to average sales. To construct the spread for a given firm, we use Compustat to obtain the credit rating for each firm in each quarter and then proxy the firm's spread, using Moody's spread for that credit rating in the given period.

Consider next the aggregate measures for debt purchases and equity payouts from the flow of funds. We use data from the nonfinancial corporate sector, and, in contrast to the firm-level definitions, we define debt purchases and equity payouts relative to GDP rather than sales. We use the NIPA data for GDP. For more details, see the appendix.

B. *Parameterization and Quantification*

Here we discuss how we parameterize preferences and technologies and choose the parameters of the model.

1. Parameterization

We assume that the utility function has the additively separable form

$$U(C) - \mu_{\mathrm{w}} G(L) = \frac{C^{1-\sigma}}{1-\sigma} - \chi \frac{L^{1+\nu}}{1+\nu}, \tag{34}$$

where χ captures both the share of workers and the weight on the disutility of labor. Consider next the parameterization of the Markov processes over idiosyncratic shocks and aggregate shocks to volatility. We want the parameterization to allow for an increase in the volatility of the idiosyncratic productivity shock z while keeping fixed the mean level of this shock. We assume a discrete process for idiosyncratic productivity shocks that approximates the autoregressive process,

$$\log z_t = \mu_t + \rho_z \log z_{t-1} + \sigma_{t-1}\varepsilon_t, \tag{35}$$

where the innovations $\varepsilon_t \sim N(0,1)$ are independent across firms. We choose $\mu_t = -\sigma_{t-1}^2/2$ so as to keep the mean level of z across firms unchanged as σ_{t-1} varies. We assume that the volatility shock σ_t takes on two values, a high value, σ_{H}, and a low value, σ_{L}, with transition probabilities determined by the probabilities of remaining in the high- and low-volatility states, p_{HH} and p_{LL}.

The revenue shock κ_t is assumed to be normal, with mean $\bar{\kappa}$ and standard deviation σ_κ. Note that in the definition of equity payouts, equation (9), the manager's wage and the revenue shock enter symmetrically, so that only the sum, $\bar{w}_{\mathrm{m}} + \kappa_t$, matters for decisions. From the definition of free cash flow in equation (22), we see that in the agency friction constraint, only the ratio $\tilde{\lambda} \equiv \lambda(1 - \beta)/[(1 - \beta\theta)\bar{w}_{\mathrm{m}}]$ matters. Hence, we parameterize only σ_κ, $\bar{w}_{\mathrm{m}} + \bar{\kappa}$, and $\tilde{\lambda}$ and refer to $\tilde{\lambda}$ as the *agency friction.*

We divide the parameters into two groups. We use existing studies to assign some parameters and use a moment-matching exercise to assign others.

2. Assigned Parameters

The assigned parameters are $\Theta_{\mathrm{A}} = \{\beta, \nu, \sigma, \chi, \alpha, \eta, \rho_z\}$. Many of these parameters are fairly standard, and we choose them to reflect commonly used values. The model is quarterly. In terms of preferences, we set the discount factor $\beta = 0.99$ so that the annual interest rate is 4 percent, and we set $\nu = 0.50$, which implies a labor elasticity of 2. This elasticity is in the range of elasticities used in macroeconomic work, as reported by Rogerson and Wallenius (2009). We also redo our experiments with $\nu = 1$, which implies a labor elasticity of 1, and find similar results. We normalize χ so that average hours per worker are one. We set $\sigma = 2$, a common estimate in the business cycle literature. However, given the risk-sharing condition and the separable utility, this parameter matters little for fluctuations.

Consider the parameters governing production. For the intermediate-goods production function, we set the parameter α equal to the labor share of 0.70 and think of there being two other fixed factors, managerial input and capital, which receive a share of 0.30. For the final-goods production function, we choose the elasticity-of-substitution parameter $\eta = 5.75$ so as to generate a markup of about 20 percent, which is in the range estimated by Basu and Fernald (1997). We choose the serial correlation of the firm-level productivity shock $\rho_z = 0.91$. This value is consistent with the estimates of Foster, Haltiwanger, and Syverson (2008) for measures of their traditional TFP index, which measures the dollar value of output deflated by a four-digit industry-level deflator.

3. Parameters from Moment Matching

The 10 parameters set in the moment-matching exercise are

$$\Theta_{\mathrm{M}} = \left\{\sigma_{\mathrm{H}}, \sigma_{\mathrm{L}}, p_{\mathrm{HH}}, p_{\mathrm{LL}}, \bar{\kappa} + \bar{w}_{\mathrm{m}}, \sigma_\kappa, \tilde{\lambda}, z_{\mathrm{e}}, \bar{\omega}, \sigma_\omega\right\}.$$

We target 10 moments. The first four are the mean, standard deviation, autocorrelation, and skewness of the IQR of sales growth. The next three are the median firm spread and its standard deviation and the median firm leverage. To calculate these medians, we first calculate for each period the median spread and leverage in the cross section and then report the medians of the constructed time series. Likewise, the standard deviation of the median spread is the standard deviation of the cross-sectional medians. The final three are the mean productivity and mean employment of entrants relative to incumbents, as reported by Lee and Mukoyama (2015), and an average leverage of entrants equal to that of incumbents.

Our model is highly nonlinear, and all parameters affect all the moments. Nevertheless, some parameters are more important for certain statistics. The mean IQR is largely driven by the mean volatility shock σ_t. The IQR standard deviation is determined largely by the distance between σ_{L} and σ_{H}, and the IQR autocorrelation is determined by the levels of the transition probabilities p_{LL} and p_{HH} of these shocks. The IQR skewness is controlled by the difference in these transition probabilities. In our calibration, p_{LL} is sufficiently larger than p_{HH} so that, on average, high-volatility shocks are realized relatively infrequently. This leads to skewness because the resulting IQR reflects the disproportionate probability that is put on the low-volatility shocks. The median spread and its standard deviation are affected by the standard deviation of the revenue shocks and the agency friction. Holding fixed the magnitude of the agency friction, the larger is the standard deviation of the revenue shock, the larger is the median spread. The median leverage is also largely determined by the mean revenue shock and the agency friction. Holding fixed the mean revenue shock, the larger is the agency friction, the larger is the median leverage. The relative productivity, employment, and leverage of entrants are determined by z_{e}, $\bar{\omega}$, and σ_{ω}, respectively.

The parameters we use are reported in table 1. In table 2, we report the target moments in the data and the model. Overall, the model produces similar statistics for the IQR, spreads, and leverage.

C. *Algorithm*

Here we provide an overview of the algorithm we use to solve the model and relegate the detailed description to the online appendix.

To solve its problem, each firm needs to forecast next period's wage $w(S)$ and output $Y(S)$, and it needs a transition law for the aggregate state. In practice, it is infeasible to include the entire measure Υ in the state. Instead, we follow a version of Krusell and Smith (1998) to approximate the forecasting rules for the firm. We do so by approximating the measure of firms Υ with lags of aggregate shocks, $(\sigma_{-1}, \sigma_{-2}, \sigma_{-3}, k)$, where k

TABLE 1
Parameter Values

Parameter	Values	Definition/Source
Parameters from moment matching:		
Volatility levels	$\sigma_H = .12$, $\sigma_L = .09$	IQR mean and standard deviation
Volatility transition	$p_{HH} = .84$, $p_{LL} = .94$	IQR autocorrelation and skewness
Revenue shock process	$\bar{\kappa} + \bar{w}_m = .005$, $\sigma_\kappa = .036$	Spread median and standard deviation
Agency friction	$\tilde{\lambda} = .079$	Leverage median
Entry	$z_e/\bar{z} = .64$, $\bar{\omega} = 1$, $\sigma_\omega = 2$	New entrants: productivity, labor, and leverage
Assigned parameters:		
Persistent z	$\rho_z = .9$	Foster et al. 2008
Labor elasticity	$\nu = .5$	Rogerson and Wallenius 2009
Labor share	$\alpha = .7$	National accounts
Markup	$\eta/(\eta - 1) = 1.21$	Basu and Fernald 1997
Discount rate	$\beta = .99$	Annual interest rate 4%
Curvature	$\sigma = 2$	Standard business cycle models

records how many periods the aggregate shocks have been unchanged.[6] Here $k = 1, \ldots, \bar{k}$, and $\bar{k}$ is the upper bound on this number of periods. In a slight abuse of notation, we use $S = (\sigma, \sigma_{-1}, \sigma_{-2}, \sigma_{-3}, k)$ in the rest of this description of the algorithm to denote our approximation to the aggregate state. The law of motion of our approximation to the aggregate state is given by $H(\sigma', S) = (\sigma', \sigma, \sigma_{-1}, \sigma_{-2}, k')$, with $k' = k + 1$ if $\sigma' = \sigma = \sigma_{-1} = \sigma_{-2}$ and 0 otherwise.[7]

We start with an initial guess of two arrays for the aggregate wages, $w^0(S)$, and output, $Y^0(S)$, referred to as *aggregate rules*. We then solve the model with two loops, an inner and an outer.

In the inner loop, taking as given the current set of aggregate rules, we first solve for the bond price schedule by iterating on the borrowing limit $M(S, z)$ in equation (11), the default cutoff $\kappa^*(S, S', z', \ell', b')$ in equation (12), and the bond price $q(S, z, \ell', b')$ in equation (14) until convergence. Given the resulting bond price schedule, we then iteratively solve each firm's optimization problem, using a combination of policy function and value function iteration until convergence. In the iterations, we also iterate on a set of arrays of grids $\{X(S, z)\}$ where the set of points

[6] To understand why we structured the state space as we did, note that at a mechanical level, the number of states in S is 32. We experimented with many ways of defining these 32 states. The way we did it is far superior, e.g., to letting $S = (\sigma, \sigma_{-1}, \sigma_{-2}, \sigma_{-3}, \sigma_{-4})$, which also has 32 states. The reason seems to be twofold: our method allows us to drop many unimportant low-probability strings and to go farther back in time with the same number of states in a way that captures the slow running up or running down of debt.

[7] To help motivate this approach to approximating the state, note that for any initial distribution Υ_0, after sufficiently many periods, the distribution over Υ_t becomes independent of this initial distribution and instead depends only on the history of aggregate shocks $(\sigma_0, \ldots, \sigma_t)$. We think of our approximation as simply a truncation of that history.

TABLE 2
Moment-Matching Exercise

Moments	Data	Model
Mean IQR (%)	21	18
Standard deviation of IQR (%)	3.5	3.6
Autocorrelation IQR (%)	88	87
Skewness IQR	.8	.9
Median spread (%)	1.5	2.8
Standard deviation of median spread (%)	1.1	.9
Median leverage (%)	26	29
Relative entrant:		
Productivity (%)	75	67
Labor (%)	60	50

$X(S, z) = \{x_1, \ldots, x_N\}$ varies with (S, z). We begin with an initial guess on the array of grids $\{X^0(S, z)\}$, the multiplier function on the nonnegative equity payout condition $\{\gamma^0(S, z, x)\}$, and the value function $\{V^0(S, z, x)\}$, where the multiplier function and value function are defined for all values of x in a range $[-M(S, z), \infty]$. For each iteration n, given the array of grids, the multipliers, and the value function from the previous iteration, we solve for the updated array of grids $\{X^{n+1}(S, z)\}$, multiplier function $\{\gamma^{n+1}(S, z, x)\}$, and value function $\{V^{n+1}(S, z, x)\}$ in two steps. In these steps, we use the result that for all cash-on-hand levels x greater than some cutoff level $\hat{x}(S, z)$, the nonnegative equity payout condition is not binding and the decision rules for labor and debt do not vary with cash on hand x. We refer to the associated values of labor and debt as the *nonbinding levels of labor and debt* and denote them $\hat{\ell}(S, z)$ and $\hat{b}(S, z)$, respectively.

In particular, given the multipliers $\{\gamma^n(S, z, x)\}$ and the value function $\{V^n(S, z, x)\}$ in the first step, we solve for these nonbinding levels. To do so, we solve a relaxed problem in which we drop both the nonnegative equity payout constraint and the agency friction constraint and then check whether the constructed tentative solutions satisfy the agency friction condition. If they do, then we set the nonbinding levels of labor and debt equal to the tentative solutions. If they do not, we impose that the agency friction constraint binds and define these nonbinding levels to be the resulting solution. We then define the cutoff level

$$\hat{x}(S, z) = -q\big(S, z, \hat{\ell}(S, z), \hat{b}(S, z)\big)\hat{b}(S, z)$$

and construct the new grid by setting $x_1 = -M(S, z)$ and $x_N = \hat{x}(S, z)$. In the second step, we solve for the decisions and multipliers at intermediate points, using the firm first-order conditions and the nonnegative equity payout condition. Finally, we update the value function. We iterate on these steps until the grids, the multipliers, and the value functions converge.

In the outer loop, taking as given the converged decisions from the inner loop, we start with a measure of firms $\Upsilon_0(z, x)$ and simulate the economy for T periods. In each period t, we record firms' labor choices $\{\ell_{t+1}(z, x)\}$, borrowing $\{b_{t+1}(z, x)\}$, and default decisions $\{\iota_t(z, x)\}$ as well as wages and aggregate output. We then project the simulated values for wages and output on a set of dummy variables corresponding to the state S. We use the fitted values as the new aggregate rules $w(S) = w^{n+1}(S)$ and $Y(S) = Y^{n+1}(S)$.

Given the new guesses for the aggregate rules, we then go back to the inner loop and first iterate on the bond price schedule to convergence and then, using the new bond price schedule, iterate on the grids, multipliers, and value functions to convergence. Then, given these converged values, we simulate the economy and construct new guesses for aggregate rules. We then repeat the procedure until the arrays of aggregate output and wages converge.

D. *Firm-Level Decisions and Responses*

We begin by studying firm spread schedules, decision rules, and responses to an aggregate shock.

1. Spread Schedules

The bond price schedule that a given firm faces, $q(S_t, z_t, \ell', b')$, depends on the aggregate state S_t, the firm's idiosyncratic shock z_t, and the firm's choice of labor and borrowing. The bond price schedule maps into a *spread schedule* that firms face on their borrowing, given by

$$\mathrm{spr}(S_t, z_t, \ell', b') = \frac{1}{q(S_t, z_t, \ell', b')} - \frac{1}{\beta}.$$

In figure 2, we display the spread schedules for different levels of labor and volatility. In figure 2*A*, the aggregate state, denoted S^{L}, has aggregate shock σ_{L} and a measure of firm-level states Υ that emerges after a long sequence of low-volatility shocks. We choose the idiosyncratic shock z to be the median level of this distribution. The lines in figure 2*A* show how the resulting spread schedule varies with borrowing for two different levels of labor, that is, the lines are the function $\mathrm{spr}(S^{\mathrm{L}}, z, \ell', \cdot)$ evaluated at two levels of ℓ'.

As this figure shows, if a firm chooses higher levels of borrowing, it faces higher spreads. The reason is simply that for a given level of labor, the higher the level of debt, the greater the tendency for firms to default. The lines in the figure also show that if a firm chooses a higher level of labor, it faces higher spreads. The logic behind this feature is more subtle:

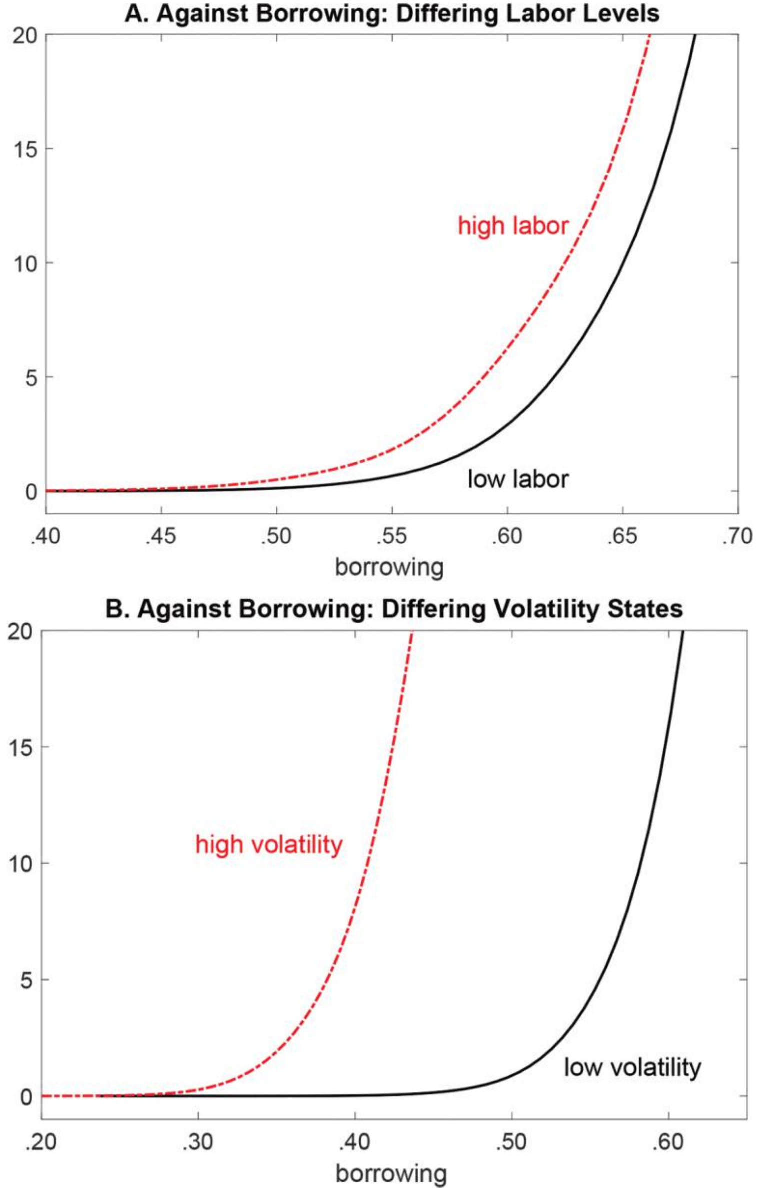

FIG. 2.—Spread schedules

a higher level of labor is associated with higher spreads because firms default more in the low-z states, and, on the margin, a higher level of labor tends to decrease profits in such z states because of the larger wage bill. Hence, hiring more labor increases the default probability and drives up the interest rate paid by firms.

Figure 2*B* shows how the spread schedule differs in two aggregate states: the low-volatility state S^{L} described above and the corresponding high-volatility state S^{H} that has aggregate shock σ_{H} and a measure of firm-level states Υ that emerges after a long sequence of high-volatility shocks. The lines in figure 2*B* represent the functions $\mathrm{spr}(S^{\mathrm{L}}, z, \ell', \cdot)$ and $\mathrm{spr}(S^{\mathrm{H}}, z, \ell', \cdot)$. Here both the idiosyncratic shock z and the level of labor are at their medians. Spreads in the high-volatility state S^{H} are higher than those in the low-volatility state S^{L} for any level of borrowing, reflecting a higher probability that a firm will default. The firm is more likely to default in S^{H} than in S^{L}, both because the probability of low idiosyncratic productivity shocks is higher and because the ability to borrow in the future is more restricted.

2. Decision Rules

Consider next the firm's decision rules. In figure 1, discussed above, we graphed the value of borrowing qb', equity payouts d, and the multiplier for equity payouts γ as a function of cash on hand for a firm at the median level of idiosyncratic shock z and at the low-volatility aggregate state S^{L}. In figure 3, for the same firm, we graph the firm's choices of new labor ℓ' and the equilibrium spread at its optimal choices.

As highlighted in the lemma, for cash on hand x above the cutoff $\hat{x}$, labor and the equilibrium spread do not vary with cash on hand, and the multiplier for equity payout is zero. For cash on hand below this cutoff, the multiplier increases as cash on hand decreases, thus restricting the plan for labor. Indeed, from figure 3*A* we see that as x is decreased below this cutoff level, labor at first decreases and then starts increasing. The decreasing part is straightforward: as cash on hand decreases today, the firm has to borrow more to increase qb', and the current multiplier γ increases. This increase in the multiplier increases the shadow price of labor and thus the wedge in equation (31). The firm responds by decreasing its labor to decrease the spread and reduce the wedge. The increasing part is more subtle. As cash on hand decreases sufficiently, the new borrowing necessary to meet the nonnegative equity payout condition increases so much that the default rate and the spread increase rapidly. Hence, conditional on repaying, the idiosyncratic shock z is higher, and so the relevant marginal product of labor, given by the left-hand side of equation (31), increases. The firm responds by increasing its level of labor accordingly.

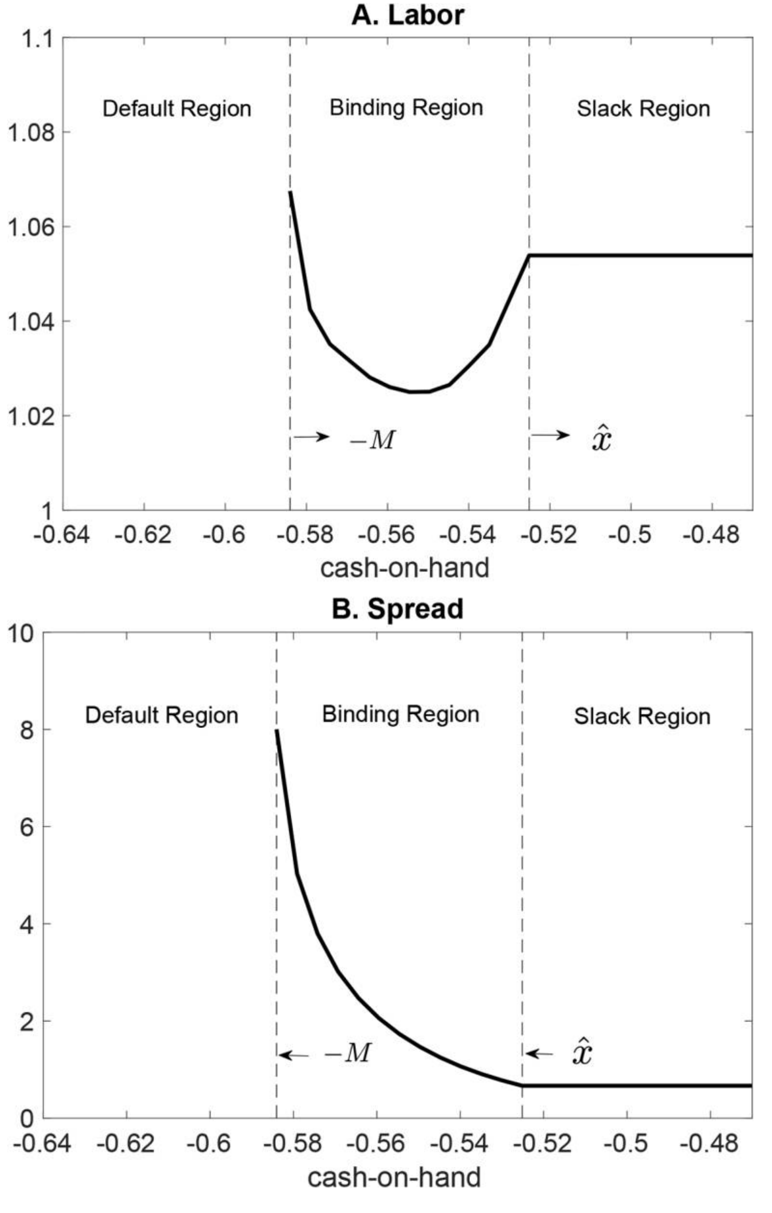

FIG. 3.—Firm decision rules

For the spread, note from figure 3*B* that, below the cutoff $\hat{x}$, the spread increases as x decreases. Briefly, the quantitative impact of the increased borrowing on spreads outweighs the effect from changing labor, so the spread increases.

3. Decision Rules and Aggregate Volatility

Consider next how decision rules differ across the low- and high-volatility states, S^L and S^H. Figure 4 compares the decision rules for the value of borrowing, equity payouts, labor, and equilibrium spread at these optimal choices for these aggregate states. As we did above, in each panel for the idiosyncratic shock, we consider a value of z at the median level of the distribution. We see that in the high-volatility state, the decision rules for the value of borrowing, equity payouts, and labor are shifted down and the

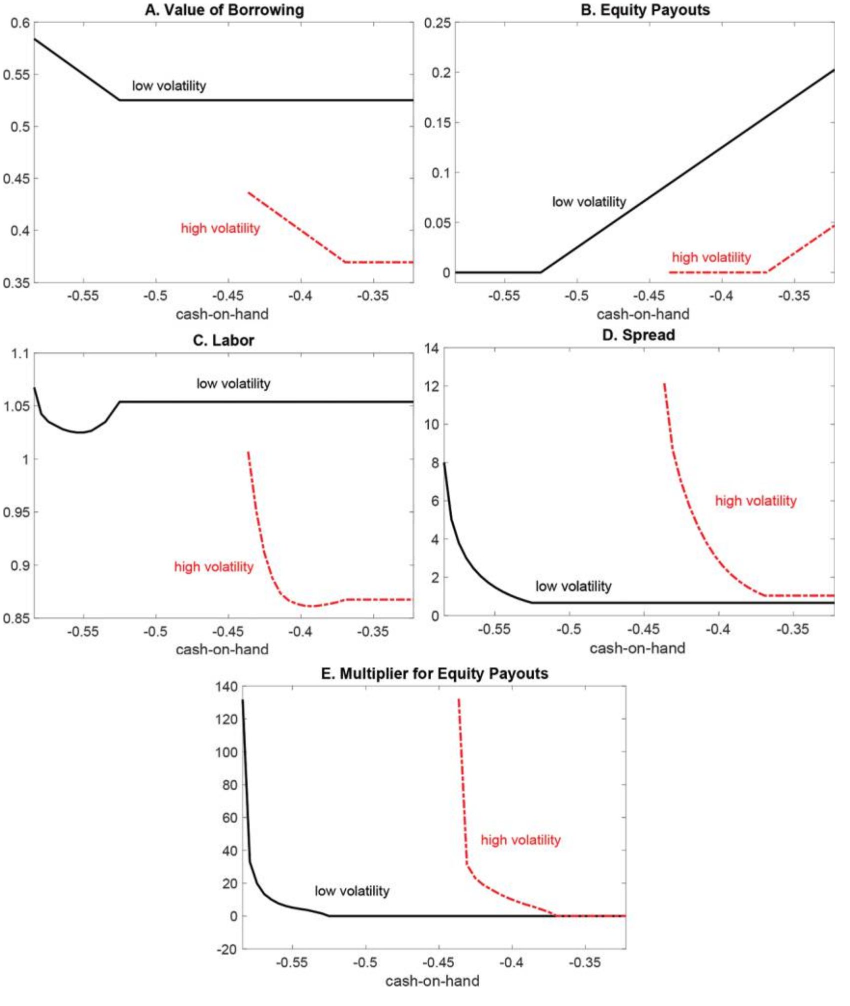

Fig. 4.—Firm decision rules

equilibrium spread is shifted up relative to those in the low-volatility state. The intuition is that the increase in volatility endogenously increases the wedge in the first-order condition for labor described above, so that the firm becomes more cautious in hiring labor. This caution also extends to the value of borrowing and equity payouts. The increase in volatility induces firms to reduce their level of borrowing and equity payouts. Nevertheless, the equilibrium spread increases because, as described above, the spread schedule is more restricted when volatility is high.

The change in decisions for the value of borrowing, equity payouts, and labor can be thought of as coming from two parts. The first is the partial-equilibrium effect, namely, for a given level of wages and aggregate output, firms tend to decrease their value of borrowing, equity payouts, and labor for precautionary reasons. The second is the general equilibrium effect, namely, as volatility increases, wages fall and aggregate output falls. The lower wages induce firms to hire more labor, whereas the lower aggregate output induces firms to hire less labor. Quantitatively, the wage effect dominates, so the general equilibrium effect tends to dampen the drop in labor relative to the partial equilibrium effect.

4. Impulse Responses for a Firm's Labor

We want to contrast the firm's response for labor with and without frictions. We focus on the impulse response for labor for a firm with the median z and κ as volatility switches from low to high. Along this impulse response, we keep the level of both z and κ at their median levels. The responses of this firm are driven by three factors that are exogenous to its choices: the change in the probability of future levels of z, which are drawn from a more dispersed distribution than under low volatility; the change in the wage and aggregate demand; and the resulting change in the schedule for borrowing that it faces.

Specifically, we suppose that the aggregate state in period 0 is S^{L} and that in period 1 the economy switches to the high-volatility state and stays there throughout the experiment, eventually ending in S^{H}.

A firm in the baseline model.—Consider first a firm in the baseline model. In figure 5, we see that, on impact, the firm decreases its labor by 4 percent, and that, after four periods, labor drops by a total of about 8 percent and stays persistently at a depressed level. The firm becomes cautious in its hiring decisions for two reasons that are driven by the increased volatility. First, the firm now fears receiving a very low idiosyncratic shock z at which, at its original level of labor, it will have to default. Second, spread schedules tighten, and firms understand that if they do indeed receive a very low idiosyncratic productivity shock, they will be unable to borrow as easily as they could when volatility was low. This shift in the spread schedule thus reinforces the tendency of firms to be cautious in hiring.

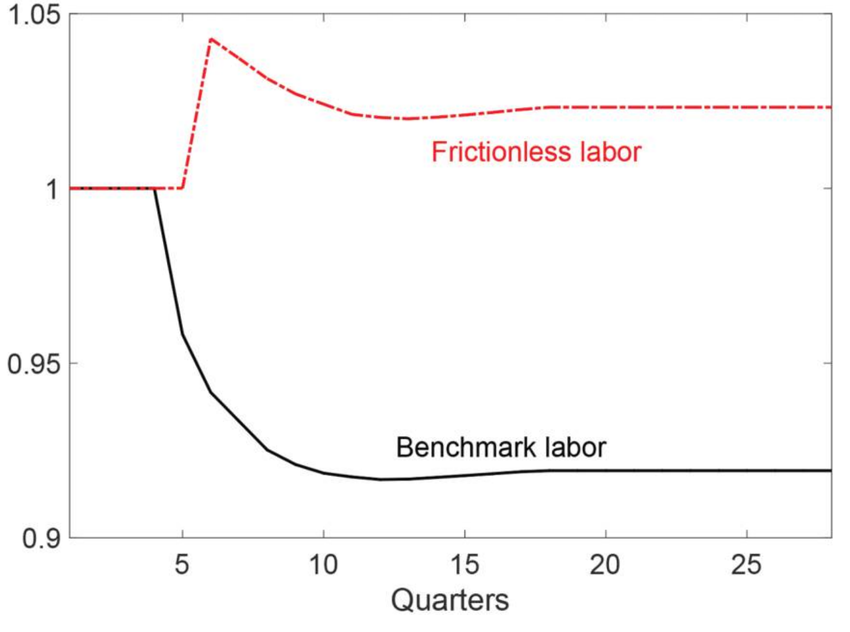

FIG. 5.—Firm labor impulse responses

In general equilibrium, since this increase in volatility leads overall employment to fall, it also leads to a fall in wages and a fall in aggregate demand, in the sense that the Y_t term in equation (8) falls, so that the demand schedule facing each firm shifts inward.

A firm with frictionless financial markets.—To isolate the firm-level effects from the general equilibrium effects, we suppose that a single firm operates without frictions in the midst of an economy in which all other firms face the frictions in the baseline model. This lack of frictions is modeled by allowing this firm to borrow using complete markets and assuming that there is no agency friction, so that labor satisfies equation (32). The upshot of these assumptions is that this frictionless firm faces the same aggregate wages and demand schedule as do the firms in our baseline model.

In figure 5, we compare the impulse response of labor for this frictionless firm to the corresponding impulse response for a firm in our baseline model. We normalize the initial values of labor in the two economies to be equal. (Absent this normalization, the level of labor for the frictionless firm is about 30 percent higher than that of the firm in the baseline model.) As this figure makes clear, such a frictionless firm actually increases its labor when volatility increases. The effects are threefold: the fall in wages increases this firm's incentives to hire workers, the inward shift in the demand for its product reduces these incentives, and the lack of frictions implies that the firm can insure against all the increase in idiosyncratic risk from the more dispersed distribution of productivity shocks. On net, the wage effect dominates, and the firm hires more workers.

Note that a firm in our baseline model also faces a net positive effect from the general equilibrium forces, dominated by the fall in wages, but the frictions that make the firm cautious outweigh this effect.

E. Firm Moments

Before we present the model's aggregate implications, we show that the model can produce the broad patterns in firm-level statistics. Our earlier moment-matching exercise ensured that the model was consistent with some basic features of firms' financial conditions, including median spread, median leverage, and the dispersion of sales growth. Here we take a closer look at firm-level statistics in the model and the data.

Table 3 presents some moments of the cross-sectional distribution of firms. Consider first the spreads. In each period, we compute the spread at the first, second, and third quartiles in the distribution of spreads and then consider the time series median of spreads at each quartile. In table 3, we see that in the model, the spreads are a bit higher and more dispersed than they are in the data. For example, the median of the spread at the 50th percentile is 2.8 in the model and 1.5 in the data, whereas the median spread at the 75th percentile is 6.3 in the model and 2.8 in the data.

Consider next the distribution of the growth of sales. In the model, we abstracted from any force that leads to trend growth in a firm's sales, such

TABLE 3
Firm Distributions

	Percentile		
	25th	50th	75th
Data (%):			
Spread	1	1.5	2.8
Growth	−9	0	11
Leverage	9	26	62
Debt Purchases	−9	0	18
Equity Payouts	−4	0	12
Model (%):			
Spread	1.1	2.8	6.3
Growth	−7	0	9
Leverage	25	29	33
Debt Purchases	−14	0	16
Equity Payouts	−19	0	23

Note.—In the data and the model, for each variable and quarter, we calculate the 25th, 50th, and 75th percentiles across firms. Then we report the median of each time series. Growth and dividends are reported relative to the median 50th percentile. Data are from Compustat. See variable definitions in the appendix.

as an upward drift in the size of z. Because of this abstraction, by construction the median of the 50th percentile of firms' growth in the model is zero. To make the statistics in the data comparable to those in the model, we subtract the median of the 50th percentile of firms' growth. We see that the model does a good job of replicating this distribution.

In terms of leverage, the median of the 50th percentile in the model is similar to that in the data (29 in the model and 26 in the data), but the model's distribution is more compressed than that in the data. The distribution of debt purchases in the model is also roughly similar to that in the data.

Finally, consider the equity payouts. In the model, we have firms with decreasing returns in the variable input, namely, labor, which can be thought of as arising from a fixed factor such as land or a fixed capital stock. Thus, the equity payouts to agents outside the firm should be thought of as the sum of payments to the fixed factor and the payments to the owners of the firm. The data are much more complicated: firms rent some of their capital and land, pay for some of it with debt, and retain part of their earnings inside the firm. The different ways of structuring payments will affect the median payouts of the firms. To avoid the issue of differing medians in the model and the data, in both we subtract out the median equity-payouts-to-sales ratios. We see that the model gives a dispersion in the equity-payouts-to-sales ratio wider than that in the data.

Consider next the correlations of firm-level variables with leverage displayed in table 4. For a given variable, such as spreads, we compute the correlation of each firm's spread with its leverage over time and report the median of these correlations across firms. The correlation between spreads and leverage is positive in both the model and the data. This correlation arises because firms that have low cash on hand tend to have high spreads and high leverage. Firms tend to have high spreads when they have low cash on hand because they need to borrow more to make nonnegative payouts to equity and, hence, have a higher risk of default. Firms tend to have high leverage when they have low cash on hand because higher debt decreases cash on hand one for one, as seen from

TABLE 4
Firm Correlations (Median Correlation with Leverage, %)

	Data	Model
Spread	11	20
Growth	9	28
Debt Purchases	45	59
Equity Payouts	−5	13

Note.—For the data and the model, we compute for each firm the correlation between its spread, growth, debt purchases, or equity payouts and its leverage across time. We report the median correlation across firms. Data are from Compustat. See variable definitions in the appendix.

the definition (eq. [10]). The correlation between growth and leverage is also positive in the model and the data. In the model, firms with high growth are those that receive relatively high productivity shocks. The increase in productivity allows firms to borrow more at the same rate. This effect induces firms to take on more debt and, thus, higher leverage.

Next, the correlation between debt purchases and leverage is also positive in both the model and the data. In the model, firms with low cash on hand have higher leverage, as explained, and tend to borrow more. Finally, equity payouts are nearly uncorrelated with leverage in both the model and the data. In the model, two opposing forces are at work. One force is that firms with low cash on hand and high leverage tend to have low equity payouts. The opposing force is that firms with high current productivity tend to have high leverage and high equity payouts. These two forces tend to cancel out each other and lead to small correlations between equity payouts and leverage.

F. *Business Cycle Moments*

So far we have focused on firm-level moments. We are also interested in the moments of aggregate variables in our model over the business cycle. In table 5, we report for both the data and the model the standard deviations of output, labor, IQR, median spreads, aggregate debt purchases relative to output, and aggregate equity payouts relative to output, as well as the correlations of these variables with output. The output and labor

TABLE 5
BUSINESS CYCLES

	Data	Model
Standard deviations (%):		
Output	1.13	.97
Labor (rel Output)	1.26	1.31
IQR	3.50	3.62
Spread	1.10	.91
Debt Purchases/Output	2.51	2.83
Equity Payouts/Output	1.76	2.74
Correlation with Output (%):		
Labor	81	94
IQR	−30	−45
Spread	−28	−33
Debt Purchases/Output	75	21
Equity Payouts/Output	45	18

NOTE.—Data for output and labor are from NIPA, data for the IQRs of sales growth and spread are from Compustat and Moody's, and data for debt change and dividends are from the flow of funds. Output and labor are logged and HP filtered, with a smoothing parameter equal to 1,600. See details of the variable definitions in the data appendix.

series are logged and Hodrick-Prescott (HP)-filtered quarterly data from 1985:1 to 2013:1.

Table 5 shows that volatility shocks at the micro level lead output in the model to fluctuate nearly as much as that in the data. Since we are abstracting from all other shocks that contribute to fluctuations in output, we think of this result as showing that micro-level volatility shocks can potentially account for a sizable fraction of the volatility in aggregate output.

More interesting is the behavior of labor. Recall that one of the main problems of business cycle models with only productivity shocks is that they generate a much lower volatility of labor relative to output than is observed in the data. Here, instead, there is no such problem: the relative volatility of labor to output is very similar in the model and the data (1.31 in the model vs. 1.26 in the data). Moreover, as the lower part of table 5 shows, in both the model and the data, labor is highly correlated with output. These results on labor represent the primary success of the model for business cycle moments.

Consider now the statistics for the IQR. The standard deviation of the IQR is close in the model and the data by our calibration. The interesting result here is that the data show that, over the past 30 years, the correlation between the IQR and output is negative. The correlation is −0.30 in the data and −0.45 in the model. In comparing these numbers, it is useful to remember that if we add in other aggregate shocks that we have abstracted from, such as aggregate productivity shocks, this correlation will be weakened in the model and hence become closer to that in the data.

Turning to financial variables, we see that the volatility of the median spread, one of our calibration moments, is close in the model and the data. More interesting is that the model produces a key feature of the data: firm spreads are countercyclical. Specifically, in the model and the data, the median spread is negatively correlated with output: −0.33 in the model and −0.28 in the data. The volatility of the ratio of debt purchases to output in the model is 2.83, close to the corresponding value in the data of 2.51. The debt purchases ratio is positively correlated with output in the model (0.21), although less so than in the data (0.75). Equity payouts are somewhat more volatile in the model than in the data: the volatility of the ratio of equity payouts to output is 2.74 in the model and 1.76 in the data. This equity payout ratio is also positively correlated with output in the model (0.18), although somewhat less so than in the data (0.45).

Although we abstract from aggregate productivity shocks, our model generates modest movements in measured TFP because, with our financial frictions, labor is not efficiently allocated across firms. One way to illustrate the fluctuations in measured TFP is to define a measure of aggregate TFP as an outsider would, namely, $Y_t / L_t^{\alpha} K_t^{1-\alpha}$ with $K_t = \bar{K}$. We find that the correlations of TFP with output and labor are positive in the model, as in the data. In the model the correlations of TFP with output and labor are both 0.40, whereas in the data these correlations are 0.57 and 0.24, respectively. The fluctuations in measured TFP in the model are about a third as volatile as those in the data.

G. *The Great Recession of 2007–9*

We ask how much of the movement in aggregates in the recession of 2007–9 can be accounted for by our model. We show that our model can account for much of this movement.

In this experiment, we choose a sequence of volatility shocks so that the IQR of sales growth in the model reproduces the corresponding IQR path in the data. We think of this procedure as using the data and the model to back out the realized sequence of volatility shocks. We plot the aggregate series after detrending them with a linear trend and normalizing them by the first observation.

1. Baseline Model

In figure 6*A*, we show the IQR of sales growth for both the data and the model. As the figure shows, the IQR increased substantially during the recession, from 0.16 in 2007:3 to almost 0.34 in 2009:2. Note that the IQR reached its highest level since 1985 at the height of the Great Recession.

The model generates substantial declines in aggregate output and labor over this period. In figure 6*B*, we see that over the period 2007:3–2009:2, the model generates a decline in output of 9.5 percent, whereas in the data, output declines by 9.2 percent. Figure 6*C* shows that the dynamics of labor are similar to those of output: the model produces about a 9.7 percent decline in labor, whereas in the data, labor declines by about 8.7 percent. Thus, the model can account for essentially all of the contraction in output and labor that occurred in the Great Recession.

The Great Recession had sizable changes in financial variables. Consider first the median spread across firms. Figure 6*D* shows that in the data, the median spread increases by about 575 basis points by 2008:4, whereas in the model, it increases by about 480 basis points by 2009:2. Note that in the model, the peak of the spread occurs two quarters later than it does in the data. The reason is that in the data, the IQR is highest at the end of the recession, and in the model the spread is largest when the IQR is highest.

Figure 6*E* shows the pattern of aggregate debt purchases over output. By the end of the recession, debt purchases had fallen by 7.5 percent in the data and 9.5 percent in the model. This pattern of debt purchases

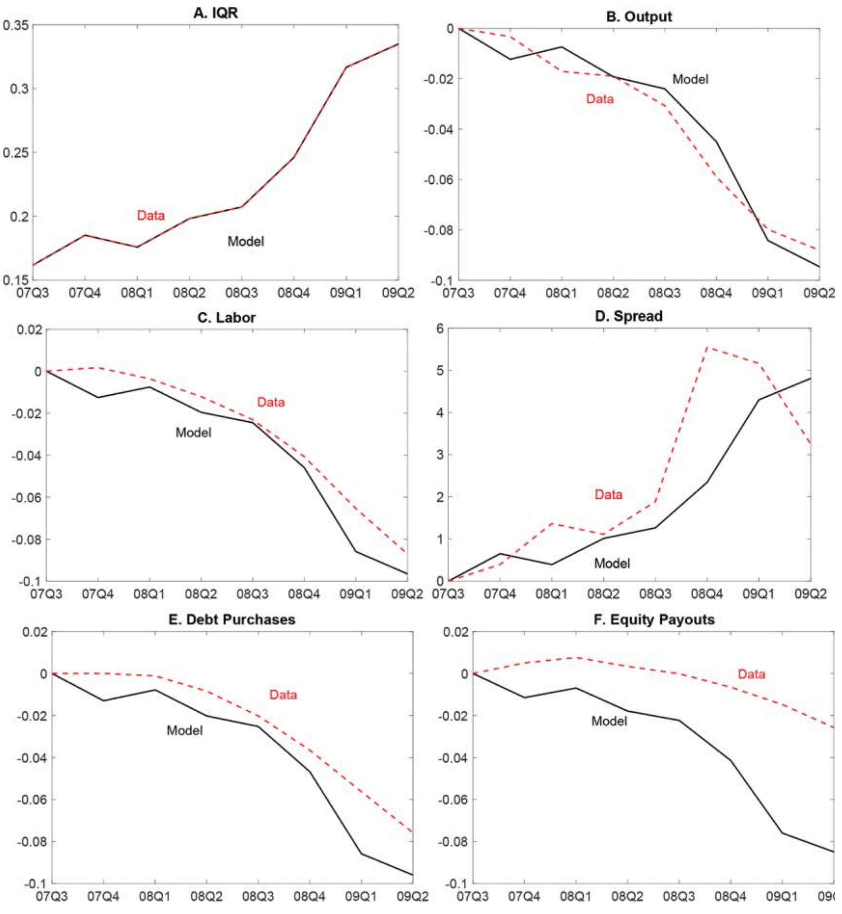

FIG. 6.—Great Recession event

implies that the outstanding level of firm debt slowly falls over the recession. Figure 6*F* plots equity payouts over output. In the data, equity payouts over output fall by about 3 percent by the end of the recession, whereas in the model they fall more, by about 8 percent.

Here we have focused on the Great Recession of 2007–9. We have not tried to account for the slow recovery following the end of the recession in 2009. As it stands, our model cannot account for the slow recovery. The reason is twofold. First, in the data, our measure of volatility, the IQR of sales growth, falls relatively quickly after 2009. Second, our model has a tight link between volatility and output, so that when volatility falls, output recovers. One reason for this tight connection is that agents know exactly when the volatility shifts. A more elaborate stochastic structure on information in which agents receive only noisy signals of the underlying aggregate shocks, such as in Kozlowski, Veldkamp, and Venkateswaran (2016), would allow the model to break this tight connection. Another reason is that we have abstracted from other mechanisms, such as adjustment costs in debt or in labor, search frictions, and so on, that stretch out the impact of shocks on aggregates. Finally, we have abstracted from other shocks, including policy uncertainty shocks, that Baker, Bloom, and Davis (2016) show actually increase further after the end of the Great Recession. While it is conceptually straightforward to extend the model to have a more elaborate information structure, various adjustment costs, search frictions, and more elaborate shocks, doing so is computationally infeasible for us.

2. Lower Labor Elasticity

So far we have assumed a Frisch labor supply elasticity of $1/\nu = 2$. Here we redo our Great Recession experiment with a lower labor elasticity of $1/\nu = 1$. When we change the labor supply elasticity, we do not adjust the other parameters in our moment-matching exercise, so this experiment should be thought of as a simple comparative-statics exercise. In figure 7, we see that the financial variables are affected little by this change. The main effects are that both output and labor fall less than they did in our baseline model. For example, in the baseline model, by the second quarter of 2009, output has fallen by 9.5 percent and labor has fallen by 9.7 percent, whereas with the lower elasticity, the corresponding falls are 7.4 and 6.7 percent, respectively.

In the appendix, we report all the statistics corresponding to those in tables 2–5 for this lower elasticity. The basic pattern is that with a lower labor elasticity, the financial variables change little, whereas output and labor become less volatile. For example, moving from the benchmark to the lower labor elasticity results in a drop in the volatility of output from 0.97 to 0.75 and a drop in the relative volatility of labor to output from 1.31 to 1.22.

3. Frictionless Financial Markets

To help isolate the quantitative role of frictions in our baseline model, it is useful to contrast the implications for output and labor for a version of our model with frictionless financial markets, namely, complete markets and no agency frictions. In contrast to our earlier study of a firm's impulse response, here we consider the full general equilibrium effects with endogenous wages and aggregate demand.

As figure 8 shows, with frictionless financial markets, output and labor both increase sharply when volatility rises: output increases by about

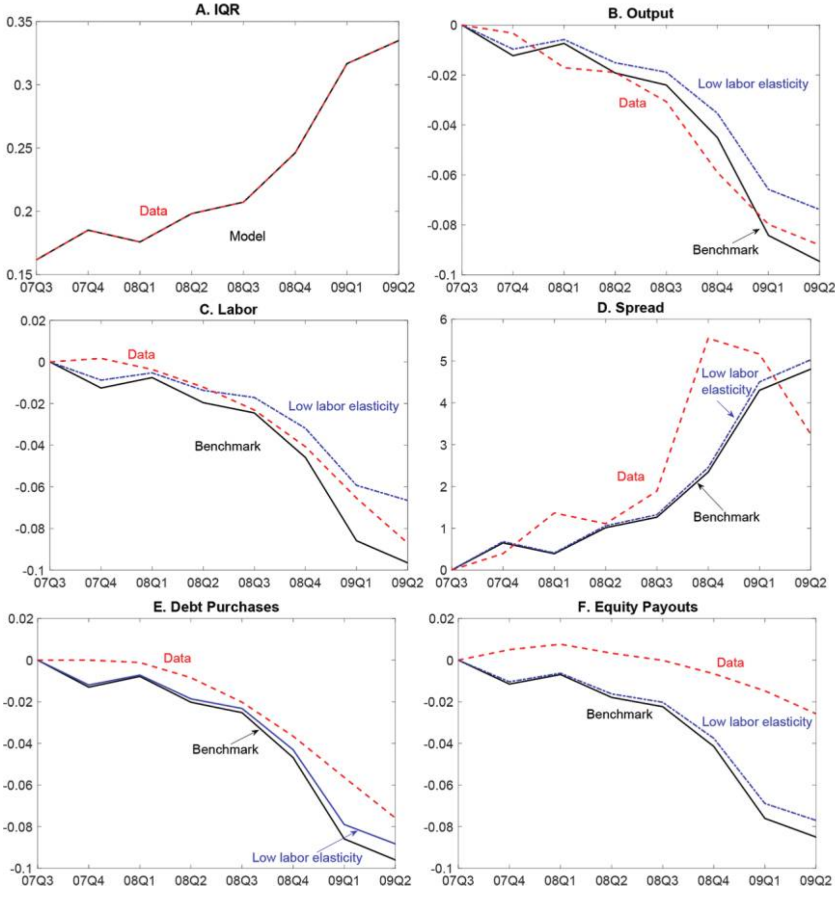

Fig. 7.—Great Recession event, lower labor elasticity

8 percent and labor by about 6 percent. The channel by which this increase takes place is referred to as the Oi-Hartman-Abel effect, based on the work of Oi (1961), Hartman (1972), and Abel (1983). The mechanism is that when the distribution of z spreads out and z is serially correlated, firms with high z tend to hire relatively more of the factor inputs. To understand why, consider a mean-preserving spread in the distribution of idiosyncratic productivity shocks. With a more spread-out distribution, a firm in the upper fraction of the distribution now has a higher level of productivity than it did under a less spread-out distribution. With serially correlated productivity shocks, a firm knows that if its productivity shock is high today, then its mean productivity shock tomorrow will also be high. All else equal, this force leads the firm to increase its labor.

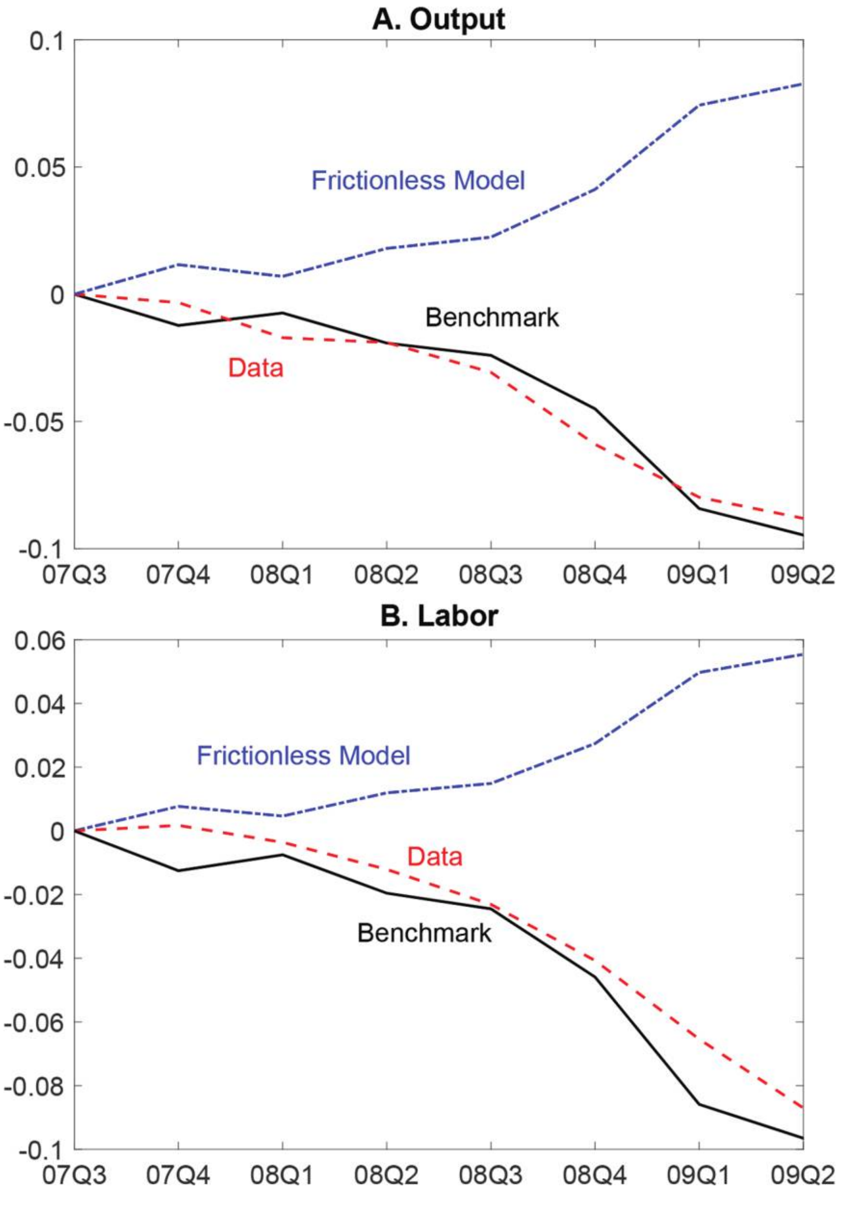

FIG. 8.—Great Recession event, frictionless financial markets

In our baseline model, these same Oi-Hartman-Abel forces are present, but to a much weaker degree because firms are unable to insure against the risk of a low realization of z. With financial frictions, if a firm in the upper fraction of the distribution sharply increases the amount of labor it hires, then, in the unfortunate circumstance that the realized level of z in the next period is actually very low, it will default. This inability to insure against the low realization of z shocks makes such a firm cautious and undoes the Oi-Hartman-Abel effect.

IV. Conclusion

Many observers believe that the depth of the Great Recession was due to the interaction of shocks with financial frictions. We have formalized this idea in a model with heterogeneous firms that face default risk and time-varying volatility shocks. We find that fluctuations in the volatility of idiosyncratic productivity shocks lead to quantitatively sizable contractions in economic activity as well as a tightening in financial conditions. In the model, as in the Great Recession, we observe a large increase in the cross-sectional dispersion of firm growth rates and a large decline in aggregate labor and output, accompanied by a tightening in financial conditions, all of which manifests as increases in firm credit spreads and declines in debt purchases and equity payouts. Hence, we think of our model as a promising parable for the Great Recession of 2007–9.

A critical feature of our analysis is the use of micro firm-level data for both disciplining the parameterization of the model and checking many empirical predictions of the model mechanisms. We use firm-level data on time-varying volatility, credit spreads, and leverage to parameterize the volatility shocks, including the increase in volatility during the Great Recession, and the magnitude of the financial frictions in our model. We then show that the resulting model predictions for the distributions of firm growth rates, credit spreads, debt, and equity, as well as firm covariates among these variables, resemble the patterns of the micro firm-level data. Hence, the macro predictions of the model occur in a framework that is consistent with micro observations. We view this attempt to connect the macro and micro predictions to be a strength of the paper and a useful addition to the growing literature using heterogeneous firm models for business cycles—a literature that, unlike this paper, has not, with few exceptions, confronted the micro-level predictions of the models developed in that literature.

We think the quantitative framework developed in this paper, business cycles with firm-level default risk, can be used for studying other applications. One area of interest is financial regulation. The framework can be useful in studying the real implications of financial regulation that change firms' borrowing incentives. Another application is the monetary policy transmission to the real economy through changes in firms' financial conditions. Finally, as explored in Arellano, Bai, and Bocola (2017), the framework is useful for studying the connection between sovereign debt crises and firm default risk.

Appendix

This appendix contains four sections. Section A provides details for the comparative-statics exercise performed in the simple example. Section B discusses extending the model to allow firms to default on the wages for managers. Section C describes the firm-level and aggregate data. Finally, Section D reports the results for our model with a lower labor elasticity.

A. *Comparative-Statics Exercise for Volatility*

To illustrate the effects of increasing volatility on the labor choice of firms in the simple example of Section II, we consider the case in which $\ln(z)$ follows a normal distribution, $N(\mu, \sigma^2)$. We assume that $b = 0$ and use the demand function $p(z,\ell) = zY^{1/\eta}\ell^{-\alpha/\eta}$ and the threshold $p(\hat{z},\ell)\ell^{\alpha} - w\ell = 0$ to rewrite the first-order condition

$$E(p(z,\ell)|z \geq \hat{z})\alpha\ell^{\alpha-1} = \frac{\eta}{\eta-1}\left(w + V\frac{\pi_z(\hat{z})}{1-\Pi_z(\hat{z})}\frac{d\hat{z}}{d\ell}\right) \tag{A1}$$

as

$$E\left(z \geq \frac{w}{A}\ell^{1-\theta}\right)A\theta\ell^{\theta-1} - w = \frac{(1-\theta)wV}{A\ell^{\theta}}\frac{\pi_z((w/A)\ell^{1-\theta})}{1-\Pi_z((w/A)\ell^{1-\theta})}, \tag{A2}$$

where $\theta = \alpha[(\eta-1)/\eta]$ and $A = Y^{1/\eta}$. To express these distributions as standard normals, use the fact that $E(z) = e^{\mu+\sigma^2/2}$ and write condition (A2) as

$$e^{\mu+\sigma^2/2}\Phi\left(\frac{\mu+\sigma^2-\ln((w/A)\ell^{1-\theta})}{\sigma}\right)A\theta\ell^{\theta-1} - w = \frac{(1-\theta)wV}{A\ell^{\theta}}h\left(\frac{\ln((w/A)\ell^{1-\theta})-\mu}{\sigma}\right), \tag{A3}$$

where Φ and h are the cumulative distribution function and hazard, respectively, for the standard normal distribution.

We want to consider the effects of a mean-preserving spread of the distribution. To do so, we set $E(z) = 1$, which implies that $\mu = -\sigma^2/2$. The first-order condition (A3) becomes

$$\Phi\left(\frac{\sigma^2/2-\ln((w/A)\ell^{1-\theta})}{\sigma}\right)A\theta\ell^{\theta-1} - w = \frac{(1-\theta)wV}{A\ell^{\theta}}h\left(\frac{\ln((w/A)\ell^{1-\theta})+\sigma^2/2}{\sigma}\right). \tag{A4}$$

To evaluate how labor ℓ changes with volatility σ, we totally differentiate condition (A4) and get an expression for $d\ell/d\sigma$:

$$\frac{d\ell}{d\sigma} = \frac{[(1-\theta)wV/A\ell^{\theta}]h_{\sigma}(\cdot) - \Phi_{\sigma}(\cdot)\theta\ell^{\theta-1}}{[\Phi_{\ell}(\cdot)A\theta\ell^{\theta-1} + \Phi(\cdot)A\theta(\theta-1)\ell^{\theta-2}] + \theta[(1-\theta)wV/A\ell^{\theta+1}]h(\cdot) - [(1-\theta)wV/A\ell^{\theta}]h_{\ell}(\cdot)}. \tag{A5}$$

Using condition (A4) for the bottom of equation (A5), we find, after some simplification, that

$$\frac{d\ell}{d\sigma} = \frac{[(1-\theta)wV/A\ell^{\theta}]h'(x)(dx/d\sigma) - \theta\ell^{\theta-1}\phi(y)(dy/d\sigma)}{\phi(y)(dy/d\ell)A\theta\ell^{\theta-1} + \Phi(y)A\ell^{\theta-2}[\theta(2\theta-1)] - (\theta/\ell)w - \left[(1-\theta)wV/A\ell^{\theta}\right]h'(x)(dx/d\ell)}, \tag{A6}$$

where $x = (\ln((w/A)\ell^{1-\theta}) + \sigma^2/2)/\sigma$ and $y = (\sigma^2/2 - \ln((w/A)\ell^{1-\theta}))/\sigma$.

We will show that under the following assumption, this derivative is negative.

Assumption 1. $\theta < 1/2$ and $\{V, \sigma\}$ satisfy

$$V \geq \frac{A^{1/(1-\theta)}(\Phi(\sigma)\exp(\sigma^2/2)\theta - 1)}{h(0)(\exp(\sigma^2/2)w)^{\theta/(1-\theta)}(1-\theta)} \tag{A7}$$

for given A and w.

Proposition 1. Under assumption 1, $d\ell/d\sigma < 0$, so that labor declines as volatility increases.

Proof. Consider the expression in equation (A6). First, note that $dy/d\ell < 0$ and $dx/d\ell > 0$, and recall that the derivative of the hazard for a standard normal satisfies $h'(x) > 0$. Sufficient conditions for $d\ell/d\sigma < 0$ are that $\theta < 1/2$ and that $dy/d\sigma < 0$ and $dx/d\sigma > 0$, where

$$\frac{dy}{d\sigma} = \frac{1}{2} + \frac{\ln((w/A)\ell^{1-\theta})}{\sigma^2} \text{ and } \frac{dx}{d\sigma} = -\frac{\ln((w/A)\ell^{1-\theta})}{\sigma^2} + \frac{1}{2}. \tag{A8}$$

With these sufficient conditions, the bottom of equation (A6) is negative and the top is positive.

Now, since $dy/d\sigma < 0$ implies that $dx/d\sigma > 0$, we need only show that $dy/d\sigma < 0$. We first show that under assumption 1,

$$\ln\left(\frac{w}{A}\ell^{1-\theta}\right) \leq -\frac{\sigma^2}{2}, \tag{A9}$$

which will imply that the default probability $\delta = \Phi[(\ln((w/A)\ell^{1-\theta}) + \sigma^2/2)/\sigma] \leq 1/2$.

To show that condition (A9) holds, we need to show that the optimal labor is not too large, that is,

$$\frac{w}{A}\ell^{1-\theta} \leq \exp\left(-\frac{\sigma^2}{2}\right), \tag{A10}$$

or

$$\ell \leq \left(\frac{A}{w}\exp\left(-\frac{\sigma^2}{2}\right)\right)^{1/(1-\theta)}. \tag{A11}$$

Let $\bar{\ell} = [(A/W)\exp(-\sigma^2/2)]^{1/(1-\theta)}$.

Next, we show that when $\theta < 1/2$, $d\ell/dV < 0$. Totally differentiating the first-order condition and using $\theta < 1/2$ gives

$$\frac{d\ell}{dV} = \frac{[(1-\theta)w/A\ell^{\theta}]h(\cdot)}{\phi(y)(dy/d\ell)A\theta\ell^{\theta-1} + \Phi(y)A\ell^{\theta-2}[\theta(2\theta-1)] - (\theta/\ell)w - [(1-\alpha)wV/\ell^{\alpha}]h'(x)(dx/d\ell)} < 0. \tag{A12}$$

Note that $d\ell/dV < 0$ because the top of the numerator of this expression is positive and each term in the bottom is negative.

Thus, we can find a sufficiently high continuation value, denoted $V_{\min}$, so that for all $V \geq V_{\min}$ the optimal labor is sufficiently low; that is, $\ell < \bar{\ell}$. We define $V_{\min}$ to be the continuation value, so that the optimal labor is given by $\bar{\ell}$. After some manipulations, we find that

$$V_{\min} = \frac{A^{1/(1-\theta)}(\Phi(\sigma)\exp(\sigma^2/2)\theta - 1)}{h(0)(\exp(\sigma^2/2)w)^{\theta/(\theta-1)}(1-\theta)}.$$

Specifically, since $V \geq V_{\min}$, $dy/d\sigma < 0$, and hence $d\ell/d\sigma < 0$. QED

B. *Wages for Managers*

In the main text, we assumed that firms always pay the managers' wages. Here we allow firms to default on the managers' wages. Let $w_{mt}(S_{t-1})$ be the face value of wages offered to managers. Defaulting firms pay managers first, then workers, then debt. Hence, defaulting firms pay their managers in full if

$$\kappa_t \leq \bar{\kappa}_m(S_t, z_{t+1}, \ell_{t+1}, b_{t+1}) = p_{t+1}(S_t, z_{t+1}, \ell_{t+1})\ell_{t+1}^{\alpha} - w_{mt+1}(S_t), \tag{A13}$$

and they pay managers $\max\{p_t\ell_t^{\alpha} - \kappa, 0\}$ otherwise. The face value of wages offered to managers $w_{mt}(S_{t-1})$ adjusts with the aggregate state so that managers earn their value in home production $\bar{w}_m$. That is, the face value $w_{mt}(S_{t-1})$ that managers earn satisfies

$$\bar{w}_m = \int \pi_\sigma \pi_z \left[\int_{\kappa\in\Omega_{mR}} w_{mt}(S_{t-1})\Upsilon_{t-1}\, d\Phi(\kappa) + \int_{\kappa\in\Omega_{mD}} \max\{p_t\ell_t^{\alpha} - \kappa, 0\}\Upsilon_{t-1}\, d\Phi(\kappa)\right],$$

where $\pi_\sigma\pi_z = \pi_\sigma(\sigma_t|\sigma_{t-1})\pi_z(z_t|z_{t-1}, \sigma_{t-1})$, Ω_{mR} denotes the set of states such that the managers are paid their face value, so that

$$\Omega_{mR}(S_{t-1}, S_t, z_t, x_{t-1}, z_{t-1}) = \{\kappa : \kappa \leq \kappa_t^* \text{ or } \kappa \leq \bar{\kappa}_{mt}\},$$

and Ω_{mD} is the complementary set of states. In the quantitative model with bounded supports on shocks, the firms always repay in full the managers' wage, and $w_{mt}(S_{t-1}) = \bar{w}_m$.

C. *Data*

We describe in detail the firm-level and aggregate data as well as the definitions of all variables.

Firm-level data.—We use firm-level data on US publicly traded firms from the Compustat database. Our sample of firms is constructed as follows. We drop financial firms (SIC codes between 6000 and 6799) and public administration firms (SIC codes ≥ 9000). We also drop firm-quarter observations with negative sales. We keep firms with at least 100 quarters of observations since 1970:1. We use the observations since 1985:1 and have a resulting unbalanced panel with 2,258 firms.

Firm-level variables.—We construct the following variables:

- Spread. From Compustat we obtain for each firm and quarter its Standard and Poor's (S&P) credit rating. We translate the S&P credit rating to a Moody's credit rating, using a standard scale, as in Johnson (2003). From Moody's we obtain spread time series for each of the 17 credit ratings from Aaa to Caa. These data are monthly, covering the period from January 31, 1991, to April 30, 2013. We then proxy the firm's spread, using the average monthly Moody's spread for that credit rating for the given quarter.
- Sales Growth. To calculate the variable Sales Growth, we compute the ratio of change in sales relative to the corresponding quarter in the previous year to the average sales in those two quarters. For each firm, we compute $\text{growth}_t = (\text{saleq}_t - \text{saleq}_{t-4})/0.5(\text{saleq}_t + \text{saleq}_{t-4})$, where saleq is quarterly sales.
- Leverage. To calculate the variable Leverage, we first calculate a ratio of total debt, defined as the sum of current liabilities and long-term debt (dlcq + dlttq), to the average quarterly sales. The average of quarterly sales is taken over the eight previous quarters (including the current quarter). We then winsorize the ratio at the 1st and the 99th percentiles and divide it by 4.
- Debt Purchases. To calculate the variable Debt Purchases, we first calculate the ratio of the change in total debt relative to the corresponding quarter in the previous year to average quarterly sales. We then winsorize the ratio at the 1st and 99th percentiles.
- Equity Payouts. To calculate the variable Equity Payouts, we first calculate the equity payout, which is equal to the purchase of common and preferred stock (prstkcy), minus the sale of common and preferred stock (sstky), plus the total value of dividends paid (cshoq [common shares outstanding] × dvpspq [dividends per share paid]). We then calculate the ratio of the sum of equity payouts over the four previous quarters (including the current quarter) to the average quarterly sales and winsorize the ratio at the 1st and the 99th percentiles.

Aggregate variables.—We construct the following variables:

- Output: the HP-filtered log of real GDP (billions of chained 2009 dollars, seasonally adjusted; from NIPA) for the period 1985:1–2013:1.
- Employment: the HP-filtered log of total hours worked, from the BLS, for the period 1985:1–2013:1.
- IQR: the interquartile range of Sales Growth, from Compustat.
- Spread: median Spread across all firms, from Compustat.
- Debt Purchases/Output: the ratio of debt increases (seasonally adjusted, from flow of funds) to nominal GDP averaged across the four previous quarters (including the current quarter).

- Equity Payouts/Output: the ratio of Equity Payouts (seasonally adjusted, from flow of funds) to nominal GDP averaged across the four previous quarters (including the current quarter).
- Employment (rel Output in table 5): the ratio of the standard deviation of Employment (logged and HP filtered) to Output (logged and HP filtered).

D. *Results with Lower Labor Elasticity*

Here we show results with a labor elasticity of 1.

TABLE A1
Lower Labor Elasticity: Firm Distributions

	Percentile		
	25th	50th	75th
Data (%):			
Spread	1	1.5	2.8
Growth	−9	0	11
Leverage	9	26	62
Debt Purchases	−9	0	18
Equity Payouts	−4	0	12
Benchmark (%):			
Spread	1.1	2.8	6.3
Growth	−7	0	9
Leverage	25	29	33
Debt Purchases	−14	0	16
Equity Payouts	−19	0	23
Lower Elasticity (%):			
Spread	.9	2.6	6.0
Growth	−7	0	9
Leverage	25	29	33
Debt Purchases	−14	0	16
Equity Payouts	−13	0	23

Note.—See table 3 note.

TABLE A2
Lower Labor Elasticity: Firm Correlations
(Median Correlation with Leverage)

	Data	Benchmark	Lower Labor Elasticity
Spread	.11	.20	.20
Growth	.09	.28	.28
Debt Purchases	.45	.59	.60
Equity Payouts	−.05	.13	.13

Note.—See table 4 note.

TABLE A3
Lower Labor Elasticity: Business Cycles

	Data	Benchmark	Lower Labor Elasticity
Standard deviations (%):			
Output	1.13	.97	.75
Employment (rel Output)	1.26	1.31	1.22
IQR	3.50	3.62	3.71
Spread	1.10	.91	.97
Debt Purchases/Output	2.51	2.83	2.63
Equity Payouts/Output	1.76	2.74	2.60
Correlation with Output:			
Employment	.81	.94	.90
IQR	−.30	−.45	−.38
Spread	−.28	−.33	−.32
Debt Purchases/Output	.75	.21	.12
Equity Payouts/Output	.45	.18	.27

Note.—See table 5 note.

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