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BUSINESS CYCLE ACCOUNTING

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We propose a simple method to help researchers develop quantitative models of economic fluctuations. The method rests on the insight that many models are equivalent to a prototype growth model with time-varying wedges that resemble productivity, labor and investment taxes, and government consumption. Wedges that correspond to these variables—*efficiency*, *labor*, *investment*, and *government consumption wedges*—are measured and then fed back into the model so as to assess the fraction of various fluctuations they account for. Applying this method to U.S. data for the Great Depression and the 1982 recession reveals that the efficiency and labor wedges together account for essentially all of the fluctuations; the investment wedge plays a decidedly tertiary role, and the government consumption wedge plays none. Analyses of the entire postwar period and alternative model specifications support these results. Models with frictions manifested primarily as investment wedges are thus not promising for the study of U.S. business cycles.

KEYWORDS: Great Depression, sticky wages, sticky prices, financial frictions, productivity decline, capacity utilization, equivalence theorems.

IN BUILDING DETAILED, QUANTITATIVE MODELS of economic fluctuations, researchers face hard choices about where to introduce frictions into their models to allow the models to generate business cycle fluctuations similar to those in the data. Here we propose a simple method to guide these choices, and we demonstrate how to use it.

Our method has two components: an equivalence result and an accounting procedure. The *equivalence result* is that a large class of models, including models with various types of frictions, is equivalent to a prototype model with various types of time-varying wedges that distort the equilibrium decisions of agents operating in otherwise competitive markets. At face value, these wedges look like time-varying productivity, labor income taxes, investment taxes, and government consumption. We thus label the wedges *efficiency wedges*, *labor wedges*, *investment wedges*, and *government consumption wedges*.

The *accounting procedure* also has two components. It begins by measuring the wedges, using data together with the equilibrium conditions of a prototype model. The measured wedge values are then fed back into the prototype model, one at a time and in combinations, so as to assess how much of the observed movements of output, labor, and investment can be attributed to each wedge, separately and in combinations. By construction, all four wedges account for all of these observed movements. This accounting procedure leads us to label our method *business cycle accounting*.

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To demonstrate how the accounting procedure works, we apply it to two actual U.S. business cycle episodes: the most extreme in U.S. history, the Great Depression (1929–1939), and a downturn less severe and more like those seen since World War II, the 1982 recession. For the Great Depression period, we find that, in combination, the efficiency and labor wedges produce declines in output, labor, and investment from 1929 to 1933 only slightly more severe than in the data. These two wedges also account fairly well for the behavior of those variables in the recovery. Over the entire Depression period, however, the investment wedge actually drives output the wrong way, leading to an increase in output during much of the 1930s. Thus, the investment wedge cannot account for either the long, deep downturn or the subsequent slow recovery. Our analysis of the more typical 1982 U.S. recession produces essentially the same results for the efficiency and labor wedges in combination. Here the investment wedge plays essentially no role. In both episodes, the government consumption wedge plays virtually no role.

We extend our analysis to the entire postwar period by developing some summary statistics for 1959–2004. The statistics we focus on are the output fluctuations induced by each wedge alone and the correlations between those fluctuations and those actually in the data. Our findings from these statistics suggest that over the entire postwar period, the investment wedge plays a somewhat larger role in business cycle fluctuations than in the 1982 recession, but its role is substantially smaller than that of either the labor or efficiency wedges.

We begin the demonstration of our proposed method by establishing equivalence results that link the four wedges to detailed models. We start with detailed model economies in which technologies and preferences are similar to those in a benchmark prototype economy, and we show that frictions in the detailed economies manifest themselves as wedges in the prototype economy. We show that an economy in which the technology is constant but input-financing frictions vary over time is equivalent to a growth model with efficiency wedges. We show that an economy with sticky wages and monetary shocks, like that of Bordo, Erceg, and Evans (2000), is equivalent to a growth model with labor wedges. In the Appendix, we show that an economy with the type of credit market frictions considered by Bernanke, Gertler, and Gilchrist (1999) is equivalent to a growth model with investment wedges. Also in the Appendix, we show that an open economy model with fluctuating borrowing and lending is equivalent to a prototype (closed-economy) model with government consumption wedges. In the working paper version of this paper (Chari, Kehoe, and McGrattan (2004)), we also showed that an economy with the type of credit market frictions considered by Carlstrom and Fuerst (1997) is equivalent to a growth model with investment wedges, and that an economy with unions and antitrust policy shocks, like that of Cole and Ohanian (2004), is equivalent to a growth model with labor wedges.

Similar equivalence results can be established when technology and preferences in detailed economies are very different from those in the prototype economy. In such situations, the prototype economy can have wedges even if the detailed economies have no frictions. We show how wedges in the benchmark prototype economy can be decomposed into a part due to frictions and a part due to differences in technology and preferences by constructing alternative prototype economies that have technologies and preferences similar to those in the detailed economy.

Our quantitative findings suggest that financial frictions that manifest themselves primarily as investment wedges did not play a primary role in the Great Depression or postwar recessions. Such financial frictions play a prominent role in the models of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997), and Bernanke, Gertler, and Gilchrist (1999). More promising, our findings suggest, are models in which the underlying frictions manifest themselves as efficiency and labor wedges. One such model is the input-financing friction model described here in which financial frictions manifest themselves primarily as efficiency wedges. This model is consistent with the views of Bernanke (1983) on the importance of financial frictions. Also promising are sticky-wage models with monetary shocks, such as that of Bordo, Erceg, and Evans (2000), and models with monopoly power, such as that of Cole and Ohanian (2004) in which the underlying frictions manifest themselves primarily as labor wedges. In general, this application of our method suggests that successful future work will likely include mechanisms in which efficiency and labor wedges have a primary role and the investment wedge has, at best, a tertiary role. We view this finding as our key substantive contribution.

In our quantitative work, we also analyze some detailed economies with quite different technology and preferences than those in our benchmark prototype economy. These include variable instead of fixed capital utilization, different labor supply elasticities, and costs of adjusting investment. For these alternative detailed economies, we decompose the benchmark prototype wedges into their two sources—frictions and specification differences—by constructing alternative prototype economies that are equivalent to the detailed economies and so can measure the part of the wedges due to frictions. We find that with regard to the investment wedge's role in the business cycle, frictions driving that wedge are unchanged by different labor supply elasticities and worsened by variable capital utilization—with the latter specification, for example, the investment wedge boosts output even more during the Great Depression than it did in the benchmark economy. With investment adjustment costs, the frictions driving investment wedges do at least depress output during the downturns, but only modestly. Altogether, these analyses reinforce our conclusion that the investment wedge plays a decidedly tertiary role in business cycle fluctuations.

Our business cycle accounting method is intended to shed light on promising classes of mechanisms through which primitive shocks lead to economic fluctuations. It is not intended to identify the primitive sources of shocks. Many economists think, for example, that monetary shocks drove the U.S. Great Depression, but these economists disagree about the details of the driving mechanism. Our analysis suggests that models in which financial frictions show up primarily as investment wedges are not promising while models in which financial frictions show up as efficiency or labor wedges may well be. Thus, we conclude that researchers interested in developing models in which monetary shocks lead to the Great Depression should focus on detailed models in which financial frictions manifest themselves as efficiency and labor wedges.

Other economists, including Cole and Ohanian (1999, 2004) and Prescott (1999), emphasize nonmonetary factors behind the Great Depression, downplaying the importance of money and banking shocks. For such economists, our analysis guides them to promising models, like that of Cole and Ohanian (2004), in which fluctuations in the power of unions and cartels lead to labor wedges, and other models in which poor government policies lead to efficiency wedges.

In terms of method, the equivalence result provides the logical foundation for the way our accounting procedure uses the measured wedges. At a mechanical level, the wedges represent deviations in the prototype model's first-order conditions and in its relationship between inputs and outputs. One interpretation of these deviations, of course, is that they are simply errors, so that their size indicates the goodness-of-fit of the model. Under that interpretation, however, feeding the measured wedges back into the model makes no sense. Our equivalence result leads to a more economically useful interpretation of the deviations by linking them directly to classes of models; that link provides the rationale for feeding the measured wedges back into the model.

Also in terms of method, the accounting procedure goes beyond simply plotting the wedges. Such plots, by themselves, are not useful in evaluating the quantitative importance of competing mechanisms of business cycles because they tell us little about the equilibrium responses to the wedges. Feeding the measured wedges back into the prototype model and measuring the model's resulting equilibrium responses is what allows us to discriminate between competing mechanisms.

Finally, in terms of method, our decomposition of business cycle fluctuations is quite different from traditional decompositions. Those decompositions attempt to isolate the effects of (so-called) primitive shocks on equilibrium outcomes by making identifying assumptions, typically zero–one restrictions on variables and shocks. The problem with the traditional approach is that finding identifying assumptions that apply to a broad class of detailed models is hard. Hence, this approach is not useful in pointing researchers toward classes of promising models. Our approach, in contrast, can be applied to a broad class of detailed models. Our equivalence results, which provide a mapping from wedges to frictions in particular detailed models, play the role of the identifying assumptions in the traditional approach. This mapping is detailed-model specific and is the key to interpreting the properties of the wedges we document. For any detailed model of interest, researchers can use the mapping that is relevant for their model to learn whether it is promising. In this sense, our approach, while being purposefully less ambitious than the traditional approach, is much more flexible than that approach.

Our accounting procedure is intended to be a useful first step in guiding the construction of detailed models with various frictions to help researchers decide which frictions are quantitatively important to business cycle fluctuations. The procedure is not a way to test particular detailed models. If a detailed model is at hand, then it makes sense to confront that model directly with the data. Nevertheless, our procedure is useful in analyzing models with many frictions. For example, some researchers, such as Bernanke, Gertler, and Gilchrist (1999) and Christiano, Gust, and Roldos (2004), have argued that the data are well accounted for by models that include a host of frictions (such as credit market frictions, sticky wages, and sticky prices). Our analysis suggests that the features of these models that primarily lead to investment wedges can be dropped with only a modest effect on the models' ability to account for the data.

Our work here is related to a vast business cycle literature that we discuss in detail after we describe and apply our new method.

1. DEMONSTRATING THE EQUIVALENCE RESULT

Here we show how various detailed models that have underlying distortions are equivalent to a prototype growth model that has one or more wedges.

1.1. *The Benchmark Prototype Economy*

The *benchmark* prototype economy that we use later in our accounting procedure is a stochastic growth model. In each period t, the economy experiences one of finitely many events s_t, which index the shocks. We denote by $s^t = (s_0, \ldots, s_t)$ the history of events up through and including period t, and often refer to s^t as the *state*. The probability, as of period 0, of any particular history s^t is $\pi_t(s^t)$. The initial realization s_0 is given. The economy has four exogenous stochastic variables, all of which are functions of the underlying random variable s^t: the *efficiency wedge* $A_t(s^t)$, the *labor wedge* $1 - \tau_{lt}(s^t)$, the *investment wedge* $1/[1 + \tau_{xt}(s^t)]$, and the *government consumption wedge* $g_t(s^t)$.

In the model, consumers maximize expected utility over per capita consumption c_t and per capita labor l_t,

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \pi_t(s^t) U(c_t(s^t), l_t(s^t)) N_t,$$

subject to the budget constraint

$$\begin{aligned} &c_t + [1 + \tau_{xt}(s^t)] x_t(s^t) \\ &\quad = [1 - \tau_{lt}(s^t)] w_t(s^t) l_t(s^t) + r_t(s^t) k_t(s^{t-1}) + T_t(s^t) \end{aligned}$$

and the capital accumulation law

$$(1) \qquad (1+\gamma_n)k_{t+1}(s^t) = (1-\delta)k_t(s^{t-1}) + x_t(s^t),$$

where $k_t(s^{t-1})$ denotes the per capita capital stock, $x_t(s^t)$ is per capita investment, $w_t(s^t)$ is the wage rate, $r_t(s^t)$ is the rental rate on capital, β is the discount factor, δ is the depreciation rate of capital, N_t is the population with growth rate equal to $1+\gamma_n$, and $T_t(s^t)$ is per capita lump-sum transfers.

The production function is $A(s^t)F(k_t(s^{t-1}), (1+\gamma)^t l_t(s^t))$, where $1+\gamma$ is the rate of labor-augmenting technical progress, which is assumed to be a constant. Firms maximize profits given by $A_t(s^t)F(k_t(s^{t-1}), (1+\gamma)^t l_t(s^t)) - r_t(s^t)k_t(s^{t-1}) - w_t(s^t)l_t(s^t)$.

The equilibrium of this benchmark prototype economy is summarized by the resource constraint

$$(2) \qquad c_t(s^t) + x_t(s^t) + g_t(s^t) = y_t(s^t),$$

where $y_t(s^t)$ denotes per capita output, together with

$$(3) \qquad y_t(s^t) = A_t(s^t)F(k_t(s^{t-1}), (1+\gamma)^t l_t(s^t)),$$

$$(4) \qquad -\frac{U_{lt}(s^t)}{U_{ct}(s^t)} = [1-\tau_{lt}(s^t)]A_t(s^t)(1+\gamma)^t F_{lt},$$

and

$$(5) \qquad \begin{aligned} &U_{ct}(s^t)[1+\tau_{xt}(s^t)] \\ &\quad = \beta \sum_{s^{t+1}} \pi_t(s^{t+1}|s^t) U_{ct+1}(s^{t+1}) \\ &\qquad \times \left\{ A_{t+1}(s^{t+1})F_{kt+1}(s^{t+1}) + (1-\delta)[1+\tau_{xt+1}(s^{t+1})] \right\}, \end{aligned}$$

where, here and throughout, notations like U_{ct}, U_{lt}, F_{lt}, and F_{kt} denote the derivatives of the utility function and the production function with respect to their arguments, and $\pi_t(s^{t+1}|s^t)$ denotes the conditional probability $\pi_t(s^{t+1})/\pi_t(s^t)$. We assume that $g_t(s^t)$ fluctuates around a trend of $(1+\gamma)^t$.

Notice that in this benchmark prototype economy, the efficiency wedge resembles a blueprint technology parameter, and the labor wedge and the investment wedge resemble tax rates on labor income and investment. Other more elaborate models could be considered, such as models with other kinds of frictions that look like taxes on consumption or on capital income. Consumption taxes induce a wedge between the consumption–leisure marginal rate of substitution and the marginal product of labor in the same way as do labor income taxes. Such taxes, if they are time-varying, also distort the intertemporal margins in (5). Capital income taxes induce a wedge between the intertemporal marginal rate of substitution and the marginal product of capital that is only slightly different from the distortion induced by a tax on investment. We experimented with intertemporal distortions that resemble capital income taxes rather than investment taxes and found that our substantive conclusions are unaffected. (For details, see Chari, Kehoe, and McGrattan (2006), hereafter referred to as the *technical appendix*.)

We emphasize that each of the wedges represents the overall distortion to the relevant equilibrium condition of the model. For example, distortions both to labor supply affecting consumers and to labor demand affecting firms distort the static first-order condition (4). Our labor wedge represents the sum of these distortions. Thus, our method identifies the overall wedge induced by both distortions and does not identify each separately. Likewise, liquidity constraints on consumers distort the consumer's intertemporal Euler equation, while investment financing frictions on firms distort the firm's intertemporal Euler equation. Our method combines the Euler equations for the consumer and the firm, and, therefore, identifies only the overall wedge in the combined Euler equation given by (5). We focus on the overall wedges because what matters in determining business cycle fluctuations is the overall wedges, not each distortion separately.

1.2. *The Mapping—From Frictions to Wedges*

Now we illustrate the mapping between detailed economies and prototype economies for two types of wedges. We show that input-financing frictions in a detailed economy map into efficiency wedges in our prototype economy. Sticky wages in a monetary economy map into our prototype (real) economy with labor wedges. In the Appendix, we show as well that investment-financing frictions map into investment wedges and that fluctuations in net exports in an open economy map into government consumption wedges in our prototype (closed) economy. In general, our approach is to show that the frictions associated with specific economic environments manifest themselves as distortions in first-order conditions and resource constraints in a growth model. We refer to these distortions as *wedges*.

We choose simple models so as to illustrate how the detailed models map into the prototypes. Because many models map into the same configuration of wedges, identifying one particular configuration does not uniquely identify a model; rather, it identifies a whole class of models consistent with that configuration. In this sense, our method does not uniquely determine the model that is most promising to analyze business cycle fluctuations. It does, however, guide researchers to focus on the key margins that need to be distorted so as to capture the nature of the fluctuations.

A. *Efficiency wedges*

In many economies, underlying frictions either within or across firms cause factor inputs to be used inefficiently. These frictions in an underlying economy often show up as aggregate productivity shocks in a prototype economy similar to our benchmark economy. Schmitz (2005) presented an interesting example of within-firm frictions that resulted from work rules that lower measured productivity at the firm level. Lagos (2006) studied how labor market policies lead to misallocations of labor across firms and, thus, to lower aggregate productivity. Chu (2001) and Restuccia and Rogerson (2003) showed how government policies at the levels of plants and establishments lead to lower aggregate productivity.

Here we develop a detailed economy with input-financing frictions and use it to make two points. This economy illustrates the general idea that frictions that lead to inefficient factor utilization map into efficiency wedges in a prototype economy. Beyond that, however, the economy also demonstrates that financial frictions can show up as efficiency wedges rather than as investment wedges. In our detailed economy, financing frictions lead some firms to pay higher interest rates for working capital than do other firms. Thus, these frictions lead to an inefficient allocation of inputs across firms.

i. *A detailed economy with input-financing frictions.* Consider a simple detailed economy with financing frictions that distort the allocation of intermediate inputs across two types of firms. Both types of firms must borrow to pay for an intermediate input in advance of production. One type of firm is more financially constrained in the sense that it pays a higher interest rate on borrowing than does the other type. We think of these frictions as capturing the idea that some firms, such as small firms, often have difficulty borrowing. One motivation for the higher interest rate faced by the financially constrained firms is that moral hazard problems are more severe for small firms.

Specifically, consider the following economy. Aggregate gross output q_t is a combination of the gross output q_{it} from the economy's two sectors, indexed $i = 1, 2$, where 1 indicates the sector of firms that are more financially constrained and 2 denotes the sector of firms that are less financially constrained. The sectors' gross output is combined according to

$$q_t = q_{1t}^{\phi} q_{2t}^{1-\phi}, \tag{6}$$

where $0 < \phi < 1$. The representative producer of the gross output q_t chooses q_{1t} and q_{2t} to solve this problem,

$$\max q_t - p_{1t}q_{1t} - p_{2t}q_{2t},$$

subject to (6), where p_{it} is the price of the output of sector i.

The resource constraint for gross output in this economy is

$$c_t + k_{t+1} + m_{1t} + m_{2t} = q_t + (1 - \delta)k_t, \tag{7}$$

where c_t is consumption, k_t is the capital stock, and m_{1t} and m_{2t} are intermediate goods used in sectors 1 and 2, respectively. Final output, given by $y_t = q_t - m_{1t} - m_{2t}$, is gross output less the intermediate goods used.

The gross output of each sector i, q_{it}, is made from intermediate goods m_{it} and a composite value-added good z_{it} according to

$$q_{it} = m_{it}^{\theta} z_{it}^{1-\theta}, \tag{8}$$

where $0 < \theta < 1$. The composite value-added good is produced from capital k_t and labor l_t according to

$$z_{1t} + z_{2t} = z_t = F(k_t, l_t). \tag{9}$$

The producer of gross output of sector i chooses the composite good z_{it} and the intermediate good m_{it} to solve this problem,

$$\max p_{it} q_{it} - v_t z_{it} - R_{it} m_{it},$$

subject to (8). Here v_t is the price of the composite good and R_{it} is the gross within-period interest rate paid on borrowing by firms in sector i. If firms in sector 1 are more financially constrained than those in sector 2, then $R_{1t} > R_{2t}$. Let $R_{it} = R_t(1+\tau_{it})$, where R_t is the rate consumers earn within period t and τ_{it} measures the within-period spread, induced by financing constraints, between the rate paid to consumers who save and the rate paid by firms in sector i. Because consumers do not discount utility within the period, $R_t = 1$.

In this economy, the representative producer of the composite good z_t chooses k_t and l_t to solve this problem,

$$\max v_t z_t - w_t l_t - r_t k_t$$

subject to (9), where w_t is the wage rate and r_t is the rental rate on capital.

Consumers solve this problem,

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t, l_t), \tag{10}$$

subject to

$$c_t + k_{t+1} = r_t k_t + w_t l_t + (1-\delta) k_t + T_t,$$

where $l_t = l_{1t} + l_{2t}$ is the economy's total labor supply and $T_t = R_t \sum_i \tau_{it} m_{it}$ denotes lump-sum transfers. Here we assume that the financing frictions act like distorting taxes and the proceeds are rebated to consumers. If, instead, we assumed that these frictions represent, say, lost gross output, then we would adjust the economy's resource constraint (7) appropriately.

ii. *The associated prototype economy with efficiency wedges.* Now consider a version of the benchmark prototype economy that will have the same aggregate allocations as the input-financing frictions economy just detailed. This prototype economy is identical to our benchmark prototype except that the new prototype economy has an investment wedge that resembles a tax on capital income rather than a tax on investment. Here the government consumption wedge is set equal to zero.

Now the consumer's budget constraint is

$$c_t + k_{t+1} = (1-\tau_{kt})r_t k_t + (1-\tau_{lt})w_t l_t + (1-\delta)k_t + T_t \tag{11}$$

and the efficiency wedge is

$$A_t = \kappa(a_{1t}^{1-\phi}a_{2t}^{\phi})^{\theta/(1-\theta)}[1-\theta(a_{1t}+a_{2t})], \tag{12}$$

where $a_{1t} = \phi/(1+\tau_{1t})$, $a_{2t} = (1-\phi)/(1+\tau_{2t})$, $\kappa = [\phi^{\phi}(1-\phi)^{1-\phi}\theta^{\theta}]^{1/(1-\theta)}$, and τ_{1t} and τ_{2t} are the interest rate spreads in the detailed economy.

Comparing the first-order conditions in the detailed economy with input-financing frictions to those of the associated prototype economy with efficiency wedges leads immediately to the following proposition:

PROPOSITION 1: *Consider a prototype economy that has resource constraint* (2) *and consumer budget constraint* (11) *that has exogenous processes for the efficiency wedge A_t given in* (12), *the labor wedge given by*

$$\frac{1}{1-\tau_{lt}} = \frac{1}{1-\theta}\left[1-\theta\left(\frac{\phi}{1+\tau_{1t}^*}+\frac{1-\phi}{1+\tau_{2t}^*}\right)\right], \tag{13}$$

and the investment wedge given by $\tau_{kt} = \tau_{lt}$, where τ_{1t}^* and τ_{2t}^* are the interest rate spreads from the detailed economy with input-financing frictions. Then the equilibrium allocations for aggregate variables in the detailed economy are equilibrium allocations in this prototype economy.

Consider the following special case of Proposition 1 in which only the efficiency wedge fluctuates. Specifically, suppose that in the detailed economy the interest rate spreads τ_{1t} and τ_{2t} fluctuate over time, but in such a way that the weighted average of these spreads,

$$a_{1t} + a_{2t} = \frac{\phi}{1+\tau_{1t}} + \frac{1-\phi}{1+\tau_{2t}}, \tag{14}$$

is constant while $a_{1t}^{1-\phi}a_{2t}^{\phi}$ fluctuates. Then from (13) we see that the labor and investment wedges are constant, and from (12) we see that the efficiency wedge fluctuates. In this case, on average, financing frictions are unchanged, but relative distortions fluctuate. An outside observer who attempted to fit the data generated by the detailed economy with input-financing frictions to the prototype economy would identify the fluctuations in relative distortions with fluctuations in technology and would see no fluctuations in either the labor wedge $1-\tau_{lt}$ or the investment wedge τ_{kt}. In particular, periods in which the relative distortions increase would be misinterpreted as periods of technological regress.

B. *Labor wedges*

Now we show that a monetary economy with sticky wages is equivalent to a (real) prototype economy with labor wedges. In the detailed economy, the shocks are to monetary policy, while in the prototype economy, the shocks are to the labor wedge.

i. *A detailed economy with sticky wages.* Consider a monetary economy populated by a large number of identical, infinitely lived consumers. The economy consists of a competitive final goods producer and a continuum of monopolistically competitive unions that set their nominal wages in advance of the realization of shocks to the economy. Each union represents all consumers who supply a specific type of labor.

In each period t, the commodities in this economy are a consumption–capital good, money, and a continuum of differentiated types of labor, indexed by $j \in [0,1]$. The technology for producing final goods from capital and a labor aggregate at history, or state, s^t has constant returns to scale and is given by $y(s^t)=F(k(s^{t-1}),l(s^t))$, where $y(s^t)$ is output of the final good, $k(s^{t-1})$ is capital, and

$$l(s^t)=\left[\int l(j,s^t)^v\,dj\right]^{1/v} \tag{15}$$

is an aggregate of the differentiated types of labor $l(j,s^t)$.

The final goods producer in this economy behaves competitively. This producer has some initial capital stock $k(s^{-1})$ and accumulates capital according to $k(s^t)=(1-\delta)k(s^{t-1})+x(s^t)$, where $x(s^t)$ is investment. The present discounted value of profits for this producer is

$$\sum_{t=0}^{\infty}\sum_{s^t} Q(s^t)[P(s^t)y(s^t)-P(s^t)x(s^t)-W(s^{t-1})l(s^t)], \tag{16}$$

where $Q(s^t)$ is the price of a dollar at s^t in an abstract unit of account, $P(s^t)$ is the dollar price of final goods at s^t, and $W(s^{t-1})$ is the aggregate nominal wage at s^t, which depends on only s^{t-1} because of wage stickiness.

The producer's problem can be stated in two parts. First, the producer chooses sequences for capital $k(s^{t-1})$, investment $x(s^t)$, and aggregate labor

$l(s^t)$ so as to maximize (16) given the production function and the capital accumulation law. The first-order conditions can be summarized by

$$P(s^t)F_l(s^t) = W(s^{t-1}) \tag{17}$$

and

$$Q(s^t)P(s^t) = \sum_{s_{t+1}} Q(s^{t+1})P(s^{t+1})[F_k(s^{t+1}) + 1 - \delta]. \tag{18}$$

Second, for any given amount of aggregate labor $l(s^t)$, the producer's demand for each type of differentiated labor is given by the solution to

$$\min_{\substack{\{l(j,s^t)\}\\ j\in[0,1]}} \int W(j, s^{t-1})l(j, s^t)\, dj \tag{19}$$

subject to (15); here $W(j, s^{t-1})$ is the nominal wage for differentiated labor of type j. Nominal wages are set by unions before the realization of the event in period t; thus, wages depend on, at most, s^{t-1}. The demand for labor of type j by the final goods producer is

$$l^d(j, s^t) = \left[\frac{W(s^{t-1})}{W(j, s^{t-1})}\right]^{1/(1-v)} l(s^t), \tag{20}$$

where $W(s^{t-1}) \equiv [\int W(j, s^{t-1})^{v/(v-1)}\, dj]^{(v-1)/v}$ is the aggregate nominal wage. The minimized value in (19) is, thus, $W(s^{t-1})l(s^t)$.

In this economy, consumers can be thought of as being organized into a continuum of unions indexed by j. Each union consists of all the consumers in the economy with labor of type j. Each union realizes that it faces a downward-sloping demand for its type of labor, given by (20). In each period, the new wages are set before the realization of the economy's current shocks.

The preferences of a representative consumer in the jth union are

$$\sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi_t(s^t)\big[U(c(j, s^t), l(j, s^t)) + V(M(j, s^t)/P(s^t))\big], \tag{21}$$

where $c(j, s^t)$, $l(j, s^t)$, and $M(j, s^t)$ are the consumption, labor supply, and money holdings of this consumer, and $P(s^t)$ is the economy's overall price level. Note that the utility function is separable in real balances. This economy has complete markets for state-contingent nominal claims. The asset structure is represented by a set of complete, contingent, one-period nominal bonds. Let $B(j, s^{t+1})$ denote the consumers' holdings of such a bond purchased in period t at history s^t, with payoffs contingent on some particular event s_{t+1}

in $t+1$, where $s^{t+1}=(s^t, s_{t+1})$. One unit of this bond pays one dollar in period $t+1$ if the particular event s_{t+1} occurs and pays zero otherwise. Let $Q(s^{t+1}|s^t)$ denote the dollar price of this bond in period t at history s^t, where $Q(s^{t+1}|s^t)=Q(s^{t+1})/Q(s^t)$.

The problem of the jth union is to maximize (21) subject to the budget constraint

$$P(s^t)c(j,s^t)+M(j,s^t)+\sum_{s_{t+1}}Q(s^{t+1}|s^t)B(j,s^{t+1})$$
$$\le W(j,s^{t-1})l(j,s^t)+M(j,s^{t-1})+B(j,s^t)+P(s^t)T(s^t)+D(s^t),$$

the constraint $l(j,s^t)=l^d(j,s^t)$, and the borrowing constraint $B(s^{t+1})\ge -P(s^t)\overline{b}$, where $l^d(j,s^t)$ is given by (20). Here $T(s^t)$ denotes transfers and the positive constant $\overline{b}$ constrains the amount of real borrowing by the union. Also, $D(s^t)=P(s^t)y(s^t)-P(s^t)x(s^t)-W(s^{t-1})l(s^t)$ are the dividends paid by the firms. The initial conditions $M(j,s^{-1})$ and $B(j,s^0)$ are given and assumed to be the same for all j. Notice that in this problem, the union chooses the wage and agrees to supply whatever labor is demanded at that wage.

The first-order conditions for this problem can be summarized by

$$\frac{V_m(j,s^t)}{P(s^t)}-\frac{U_c(j,s^t)}{P(s^t)}+\beta\sum_{s_{t+1}}\pi(s^{t+1}|s^t)\frac{U_c(j,s^{t+1})}{P(s^{t+1})}=0, \tag{22}$$

$$Q(s^t|s^{t-1})=\beta\pi_t(s^t|s^{t-1})\frac{U_c(j,s^t)}{U_c(j,s^{t-1})}\frac{P(s^{t-1})}{P(s^t)}, \tag{23}$$

and

$$W(j,s^{t-1})=-\frac{\sum_{s^t}Q(s^t)P(s^t)U_l(j,s^t)/U_c(j,s^t)l^d(j,s^t)}{v\sum_{s^t}Q(s^t)l^d(j,s^t)}. \tag{24}$$

Here $\pi_t(s^{t+1}|s^t)=\pi_t(s^{t+1})/\pi_t(s^t)$ is the conditional probability of s^{t+1} given s^t. Notice that in a steady state, (24) reduces to $W/P=(1/v)(-U_l/U_c)$, so that real wages are set as a markup over the marginal rate of substitution between labor and consumption. Given the symmetry among the unions, all of them choose the same consumption, labor, money balances, bond holdings, and wages, which are denoted simply by $c(s^t)$, $l(s^t)$, $M(s^t)$, $B(s^{t+1})$, and $W(s^t)$.

Consider next the specification of the money supply process and the market-clearing conditions for this sticky-wage economy. The nominal money supply process is given by $M(s^t)=\mu(s^t)M(s^{t-1})$, where $\mu(s^t)$ is a stochastic process. New money balances are distributed to consumers in a lump-sum fashion by having nominal transfers satisfy $P(s^t)T(s^t)=M(s^t)-M(s^{t-1})$. The resource constraint for this economy is $c(s^t)+k(s^t)=y(s^t)+(1-\delta)k(s^{t-1})$. Bond market clearing requires that $B(s^{t+1})=0$.

ii. *The associated prototype economy with labor wedges.* Consider now a real prototype economy with labor wedges and the production function for final goods given above in the detailed economy with sticky wages. The representative firm maximizes (16) subject to the capital accumulation law given above. The first-order conditions can be summarized by (17) and (18). The representative consumer maximizes

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi_t(s^t)U(c(s^t),l(s^t))$$

subject to the budget constraint

$$c(s^t)+\sum_{s_{t+1}}q(s^{t+1}|s^t)b(s^{t+1})$$
$$\leq[1-\tau_l(s^t)]w(s^t)l(s^t)+b(s^t)+v(s^t)+d(s^t)$$

with $w(s^t)$ replacing $W(s^{t-1})/P(s^t)$ and $q(s^{t+1}/s^t)$ replacing $Q(s^{t+1})P(^{st+1})/Q(s^t)P(s^t)$ and a bound on real bond holdings, where the lowercase letters q, b, w, v, and d denote the real values of bond prices, debt, wages, lump-sum transfers, and dividends. Here the first-order condition for bonds is identical to that in (23) once symmetry has been imposed with $q(s^t/s^{t-1})$ replacing $Q(s^t/s^{t-1})P(s^t)/P(s^{t-1})$. The first-order condition for labor is given by

$$-\frac{U_l(s^t)}{U_c(s^t)}=[1-\tau_l(s^t)]w(s^t).$$

Consider an equilibrium of the sticky-wage economy for some given stochastic process $M^*(s^t)$ on money supply. Denote all of the allocations and prices in this equilibrium with asterisks. Then the following proposition can be easily established:

PROPOSITION 2: *Consider the prototype economy just described with labor wedges given by*

$$(25)\qquad 1-\tau_l(s^t)=-\frac{U_l^*(s^t)}{U_c^*(s^t)}\frac{1}{F_l^*(s^t)},$$

where $U_l^*(s^t)$, $U_c^*(s^t)$, and $F_l^*(s^t)$ are evaluated at the equilibrium of the sticky-wage economy and where real transfers are equal to the real value of transfers in the sticky-wage economy adjusted for the interest cost of holding money. Then the equilibrium allocations and prices in the sticky-wage economy are the same as those in the prototype economy.

The proof of this proposition is immediate from comparing the first-order conditions, the budget constraints, and the resource constraints for the prototype economy with labor wedges to those of the detailed economy with sticky wages. The key idea is that distortions in the sticky-wage economy between the marginal product of labor implicit in (24) and the marginal rate of substitution between leisure and consumption are perfectly captured by the labor wedges (25) in the prototype economy.

2. THE ACCOUNTING PROCEDURE

Having established our equivalence result, we now describe our accounting procedure at a conceptual level and discuss a Markovian implementation of it.

Our procedure is to conduct experiments that isolate the marginal effect of each wedge as well as the marginal effects of combinations of these wedges on aggregate variables. In the experiment in which we isolate the marginal effect of the efficiency wedge, for example, we hold the other wedges fixed at some constant values in all periods. In conducting this experiment, we ensure that the probability distribution of the efficiency wedge coincides with that in the prototype economy. In effect, we ensure that agents' expectations of how the efficiency wedge will evolve are the same as in the prototype economy. For each experiment, we compare the properties of the resulting equilibria to those of the prototype economy. These comparisons, together with our equivalence results, allow us to identify promising classes of detailed economies.

2.1. *The Accounting Procedure at a Conceptual Level*

Suppose for now that the stochastic process $\pi_t(s^t)$ and the realizations of the state s^t in some particular episode are known. Recall that the prototype economy has one underlying (vector-valued) random variable, the state s^t, which has a probability of $\pi_t(s^t)$. All of the other stochastic variables, including the four wedges—the efficiency wedge $A_t(s^t)$, the labor wedge $1-\tau_{lt}(s^t)$, the investment wedge $1/[1+\tau_{xt}(s^t)]$, and the government consumption wedge $g_t(s^t)$—are simply functions of this random variable. Hence, when the state s^t is known, so are the wedges.

To evaluate the effects of just the efficiency wedge, for example, we consider an economy, referred to as an *efficiency wedge alone economy*, with the same underlying state s^t, the same probability $\pi_t(s^t)$, and the same function $A_t(s^t)$ for the efficiency wedge as in the prototype economy, but in which the other three wedges are set to constants, that is, $\tau_{lt}(s^t)=\bar{\tau}_l$, $\tau_{xt}(s^t)=\bar{\tau}_x$, and $g_t(s^t)=\bar{g}$. Note that this construction ensures that the probability distribution of the efficiency wedge in this economy is identical to that in the prototype economy.

For the efficiency wedge alone economy, we then compute the equilibrium outcomes associated with the realizations of the state s^t in a particular episode and compare these outcomes to those of the economy with all four wedges. We find this comparison to be of particular interest because, in our applications, the realizations s^t are such that the economy with all four wedges exactly reproduces the data on output, labor, investment, and consumption.

In a similar manner, we define the *labor wedge alone economy*, the *investment wedge alone economy*, and the *government consumption wedge alone economy,* as well as economies with a combination of wedges such as the *efficiency and labor wedge economy*.

2.2. *A Markovian Implementation*

So far we have described our procedure under the assumption that we know the stochastic process $\pi_t(s^t)$ and that we can observe the state s^t. In practice, of course, we need either to specify the stochastic process a priori or to use data to estimate it, and we need to uncover the state s^t from the data. Here we describe a set of assumptions that makes these efforts easy. Then we describe in detail the three steps involved in implementing our procedure.

We assume that the state s^t follows a Markov process of the form $\pi(s_t|s_{t-1})$ and that the wedges in period t can be used to uncover the event s_t uniquely, in the sense that the mapping from the event s_t to the wedges $(A_t, \tau_{lt}, \tau_{xt}, g_t)$ is one to one and onto. Given this assumption, without loss of generality, let the underlying event $s_t = (s_{At}, s_{lt}, s_{xt}, s_{gt})$, and let $A_t(s^t) = s_{At}$, $\tau_{lt}(s^t) = s_{lt}$, $\tau_{xt}(s^t) = s_{xt}$, and $g_t(s^t) = s_{gt}$. Note that we have effectively assumed that agents use only past wedges to forecast future wedges and that the wedges in period t are sufficient statistics for the event in period t.

The first step in our procedure is to use data on y_t, l_t, x_t, and g_t from an actual economy to estimate the parameters of the Markov process $\pi(s_t|s_{t-1})$. We can do so using a variety of methods, including the maximum likelihood procedure described below.

The second step in our procedure is to uncover the event s_t by measuring the realized wedges. We measure the government consumption wedge directly from the data as the sum of government spending and net exports. To obtain the values of the other three wedges, we use the data and the model's decision rules. With y_t^d, l_t^d, x_t^d, g_t^d, and k_0^d denoting the data, and $y(s_t, k_t)$, $l(s_t, k_t)$, and $x(s_t, k_t)$ denoting the decision rules of the model, the realized wedge series s_t^d solves

$$(26) \qquad y_t^d = y(s_t^d, k_t), \quad l_t^d = l(s_t^d, k_t), \quad \text{and} \quad x_t^d = x(s_t^d, k_t),$$

with $k_{t+1} = (1 - \delta)k_t + x_t^d$, $k_0 = k_0^d$, and $g_t = g_t^d$. Note that we construct a series for the capital stock using the capital accumulation law (1), data on investment x_t, and an initial choice of capital stock k_0. In effect, we solve for the three unknown elements of the vector s_t using the three equations (3)–(5) and thereby uncover the state. We use the associated values for the wedges in our experiments.

Note that the four wedges account for all of the movement in output, labor, investment, and government consumption, in that if we feed the four wedges into the three decision rules in (26) and use $g_t(s_t^d) = s_{gt}$ along with the law of motion for capital, we simply recover the original data.

Note also that in measuring the realized wedges, the estimated stochastic process plays a role in measuring only the investment wedge. To see that the stochastic process does not play a role in measuring the efficiency and labor wedges, note that these wedges can equivalently be directly calculated from (3) and (4) without computing the equilibrium of the model. In contrast, calculating the investment wedge requires computing the equilibrium of the model because the right side of (5) has expectations over future values of consumption, the capital stock, the wedges, and so on. The equilibrium of the model depends on these expectations and, therefore, on the stochastic process driving the wedges.

The third step in our procedure is to conduct experiments to isolate the marginal effects of the wedges. To do that, we allow a subset of the wedges to fluctuate as they do in the data while the others are set to constants. To evaluate the effects of the efficiency wedge, we compute the decision rules for the efficiency wedge alone economy, denoted $y^e(s_t, k_t)$, $l^e(s_t, k_t)$, and $x^e(s_t, k_t)$, in which $A_t(s^t) = s_{At}$, $\tau_{lt}(s^t) = \bar{\tau}_l$, $\tau_{xt}(s^t) = \bar{\tau}_x$, and $g_t(s^t) = \bar{g}$. Starting from k_0^d, we then use s_t^d, the decision rules, and the capital accumulation law to compute the realized sequence of output, labor, and investment, y_t^e, l_t^e, and x_t^e, which we call the *efficiency wedge components* of output, labor, and investment. We compare these components to output, labor, and investment in the data. Other components are computed and compared similarly.

Notice that in this experiment we computed the decision rules for an economy in which only one wedge fluctuates and the others are set to be constants in all events. The fluctuations in the one wedge are driven by fluctuations in a 4 dimensional state s_t.

Notice also that our experiments are designed to separate out the direct effect and the forecasting effect of fluctuations in wedges. As a wedge fluctuates, it directly affects either budget constraints or resource constraints. This fluctuation also affects the forecasts of that wedge as well as of other wedges in the future. Our experiments are designed so that when we hold a particular wedge constant, we eliminate the direct effect of that wedge, but we retain its forecasting effect on the other wedges. By doing so, we ensure that expectations of the fluctuating wedges are identical to those in the prototype economy.

Here we focus on one simple way to specify the expectations of agents: assume they simply use past values of the wedges to forecast future values. An extension of our Markovian procedure is to use past endogenous variables, such as output, investment, consumption, and perhaps even asset prices such as stock market values, in addition to past wedges to forecast future wedges. Another approach is simply to specify these expectations directly, as we did in our earlier work (Chari, Kehoe, and McGrattan (2002)) and then conduct a variety of experiments to determine how the results change as the specification is changed.

3. APPLYING THE ACCOUNTING APPLICATION

Now we demonstrate how to apply our accounting procedure to two U.S. business cycle episodes: the Great Depression and the postwar recession of 1982. We then extend our analysis to the entire postwar period. (In the technical appendix, we describe in detail our data sources, parameter choices, computational methods, and estimation procedures.)

3.1. *Details of the Application*

To apply our accounting procedure, we use functional forms and parameter values that are familiar from the business cycle literature. We assume that the production function has the form $F(k,l) = k^{\alpha}l^{1-\alpha}$ and the utility function has the form $U(c,l) = \log c + \psi \log(1-l)$. We choose the capital share $\alpha = .35$ and the time allocation parameter $\psi = 2.24$. We choose the depreciation rate δ, the discount factor β, and growth rates γ and γ_n so that, on an annualized basis, depreciation is 4.64%, the rate of time preference is 3%, the population growth rate is 1.5%, and the growth of technology is 1.6%.

To estimate the stochastic process for the state, we first specify a vector autoregressive AR(1) process for the event $s_t = (s_{At}, s_{lt}, s_{xt}, s_{gt})$ of the form

$$s_{t+1} = P_0 + P s_t + \varepsilon_{t+1}, \tag{27}$$

where the shock ε_t is independent and identically distributed over time and is distributed normally with mean zero and covariance matrix V. To ensure that our estimate of V is positive semidefinite, we estimate the lower triangular matrix Q, where $V = QQ'$. The matrix Q has no structural interpretation. (In Section 5, we elaborate on the contrast between our decomposition and more traditional decompositions that impose structural interpretations on Q.)

We then use a standard maximum likelihood procedure to estimate the parameters P_0, P, and V of the vector AR(1) process for the wedges. In doing so, we use the log-linear decision rules of the prototype economy and data on output, labor, investment, and the sum of government consumption and net exports.

For our Great Depression experiments, we proceed as follows. We discretize the process (27) and simulate the economy using nonlinear decision rules from a finite-element method. We use nonlinear decision rules in these experiments because the shocks are so large that, for a given stochastic process, the linear decision rules are a poor approximation to the nonlinear decision rules. Of course, we would rather have used the nonlinear decision rules to estimate the parameters of the vector AR(1) process. We do not do so because this exercise is computationally demanding. Instead we experiment by varying the parameters of the vector AR(1) process and find that our results are very similar across these experiments.

For our postwar experiments, we use the log-linear decision rules and the continuous state process (27).

To implement our accounting procedure, we must first adjust the data to make them consistent with the theory. In particular, we adjust the U.S. data on output and its components to remove sales taxes and to add the service flow for consumer durables. For the pre-World War II period, we remove military compensation as well. We estimate separate sets of parameters for the stochastic process for wedges (27) for each of our two historical episodes. The other parameters are the same in the two episodes. (See our technical appendix for our rationale for this decision.) The stochastic process parameters for the Great Depression analysis are estimated using annual data for 1901–1940; those for analysis after World War II use quarterly data for 1959:1–2004:3. In the Great Depression analysis, we impose the additional restriction that the covariance between the shocks to the government consumption wedge and those to the other wedges is zero. This restriction avoids having the large movements in government consumption associated with World War I dominate the estimation of the stochastic process.

Table I displays the resulting estimated values for the parameters of the coefficient matrices, P and Q, and the associated confidence bands for our two historical data periods. The stochastic process (27) with these values will be used by agents in our economy to form their expectations about future wedges.

3.2. *Findings*

Now we describe the results of applying our procedure to two historical U.S. business cycle episodes. In the Great Depression, the efficiency and labor wedges play a central role for all variables considered. In the 1982 recession, the efficiency wedge plays a central role for output and investment, while the labor wedge plays a central role for labor. The government consumption wedge plays no role in either period; most strikingly, neither does the investment wedge.

In reporting our findings, we remove a trend of 1.6% from output, investment, and the government consumption wedge. Both output and labor are normalized to equal 100 in the base periods: 1929 for the Great Depression and 1979:1 for the 1982 recession. In both of these historical episodes, investment (detrended) is divided by the base period level of output. Because the government consumption component accounts for virtually none of the fluctuations in output, labor, and investment, we discuss the government consumption wedge and its components only in our technical appendix. Here we focus primarily on the fluctuations due to the efficiency, labor, and investment wedges.

TABLE I

PARAMETERS OF THE VECTOR AR(1) STOCHASTIC PROCESS IN TWO HISTORICAL EPISODES[a]
(ESTIMATED USING MAXIMUM LIKELIHOOD WITH U.S. DATA[b])

A. Annual Data, 1901–1940

Coefficient Matrix P on Lagged States

.732 (.470, .856)	.0521 (−.0364, .142)	−.317 (−.716, .130)	0
−.150 (−.339, .0504)	1.04 (.908, 1.10)	.390 (−.0751, .782)	0
−.0114 (−.384, .260)	−.0197 (−.262, .126)	.0731 (−.363, .296)	0
0	0	0	.750 (.424, .814)

Coefficient Matrix Q, Where $V = QQ'$

.0575 (.0440, .0666)	0	0	0
−.00561 (−.0216, .00952)	.0555 (.0378, .0643)	0	0
.000299 (−.0308, .0230)	−.000253 (−.0167, .0121)	.0369 (.0194, .0489)	0
0	0	0	.221 (.145, .276)

Means of states = [.541 (.503, .591), −.190 (−.271, −.0867), .286 (.216, .364), −2.79 (−2.95, −2.55)]

B. Quarterly Data, 1959:1–2004:3

Coefficient Matrix P on Lagged States

.980 (.944, .984)	−.0138 (−.0192, .00222)	−.0117 (−.0129, −.00605)	.0192 (.0125, .0259)
−.0330 (−.0396, −.0061)	.956 (.920, .959)	−.0451 (−.0512, −.0286)	.0569 (.0473, .0677)
−.0702 (−.1087, −.0672)	−.0460 (−.0612, −.0304)	.896 (.879, .907)	.104 (.0817, .112)
.00481 (−.0278, .0116)	−.00811 (−.0158, .0157)	.0488 (.0371, .0643)	.971 (.954, .974)

Coefficient Matrix Q, Where $V = QQ'$

.0116 (.0105, .0126)	0	0	0
.00141 (.000462, .00232)	.00644 (.00567, .00695)	0	0
−.0105 (−.0141, −.00779)	.00103 (−.00278, .00266)	.0158 (.0133, .0190)	0
−.000575 (−.00219, .00132)	.00611 (.00383, .00760)	.0142 (.0121, .0154)	.00458 (.00386, .00554)

Means of states = [−.0239 (−.0301, −.0137), .328 (.322, .336), .483 (.473, .495), −1.53 (−1.55, −1.52)]

[a]To ensure stationarity, we add to the likelihood function a penalty term proportional to $\max(|\lambda_{\max}| - .995, 0)^2$, where $\lambda_{\max}$ is the maximal eigenvalue of P. Numbers in parentheses are 90% confidence intervals for a bootstrapped distribution with 500 replications. To ensure that the variance–covariance matrix V is positive semidefinite, we estimate Q rather than $V = QQ'$.

[b]For the sources of basic data, see Chari, Kehoe, and McGrattan (2006).

A. *The Great Depression*

Our findings for the period 1929–1939, which includes the Great Depression, are displayed in Figures 1–4. In sum, we find that the efficiency and labor wedges account for essentially all of the movements of output, labor, and investment in the Depression period and that the investment wedge actually drives output the wrong way.

In Figure 1, we display actual U.S. output along with the three measured wedges for that period: the efficiency wedge A, the labor wedge $(1-\tau_l)$, and the investment wedge $1/(1+\tau_x)$. We see that the underlying distortions revealed by the three wedges have different patterns. The distortions that manifest themselves as efficiency and labor wedges become substantially worse between 1929 and 1933. By 1939, the efficiency wedge has returned to the 1929 trend level, but the labor wedge has not. Over the period, the investment wedge fluctuates, but investment decisions are generally less distorted, in the sense that τ_x is smaller between 1932 and 1939 than it is in 1929. Note that this investment wedge pattern does not square with models of business cycles in which financial frictions increase in downturns and decrease in recoveries.

In Figure 2, we plot the 1929–1939 data for U.S. output, labor, and investment along with the model's predictions for those variables when the model includes just one wedge. In terms of the data, note that labor declines 27%

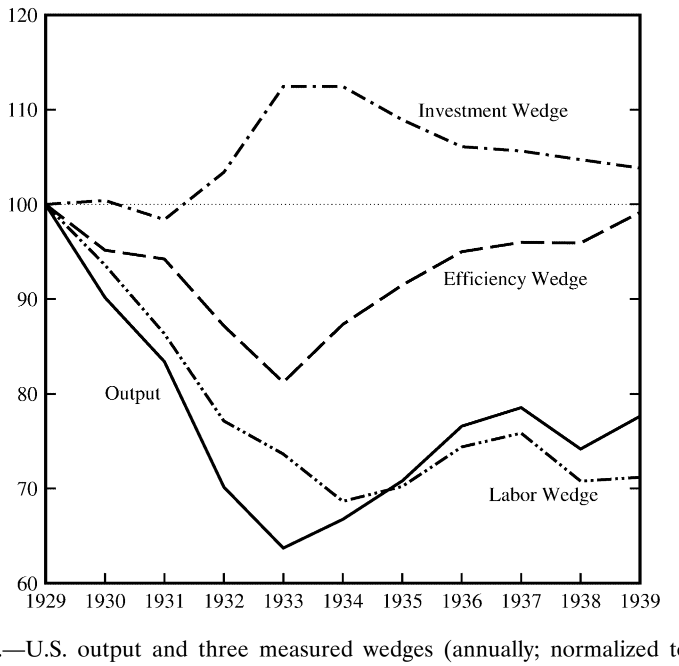

FIGURE 1.—U.S. output and three measured wedges (annually; normalized to equal 100 in 1929).

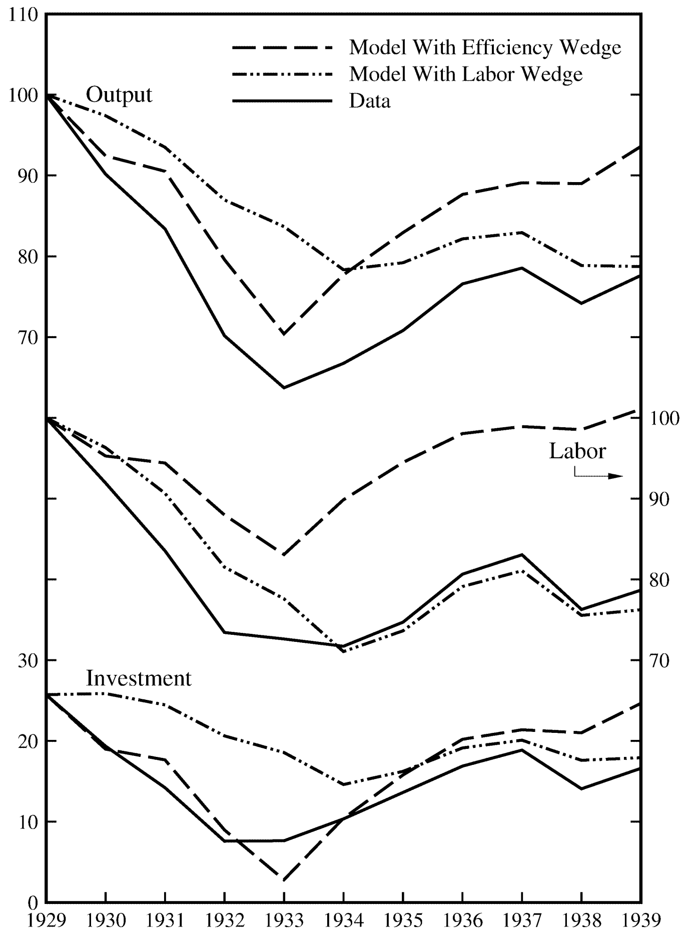

FIGURE 2.—Data and predictions of the models with just one wedge.

from 1929 to 1933 and stays relatively low for the rest of the decade. Investment also declines sharply from 1929 to 1933, but partially recovers by the end of the decade. Interestingly, in an algebraic sense, about half of output's 36% fall from 1929 to 1933 is due to the decline in investment.

In terms of the model, we start by assessing the separate contributions of the three wedges.

Consider first the contribution of the efficiency wedge. In Figure 2, we see that with this wedge alone, the model predicts that output declines less than it actually does in the data and that it recovers more rapidly. For example, by 1933, predicted output falls about 30%, while U.S. output falls about 36%. Thus, the efficiency wedge accounts for over 80% of the decline of output in the data. By 1939, predicted output is only about 6% below trend rather than the observed 22%. As can also be seen in Figure 2, the reason for this predicted rapid recovery is that the efficiency wedge accounts for only a small part of the observed movements in labor in the data. By 1933, the fall in predicted investment is similar to but somewhat greater than that in the data; it recovers faster, however.

Consider next the contributions of the labor wedge. In Figure 2, we see that with this wedge alone, the model predicts output due to the labor wedge to fall by 1933 a little less than half as much as output falls in the data: 16% vs. 36%. By 1939, however, the labor wedge model's predicted output completely captures the slow recovery: it predicts output falling 21%, approximately as much as output does that year in the data. This model captures the slow output recovery because predicted labor due to the labor wedge also captures the sluggishness in labor after 1933 remarkably well. The associated prediction for investment is a decline, but not the actual sharp decline from 1929 to 1933.

Summarizing Figure 2, we can say that the efficiency wedge accounts for over three-quarters of output's downturn during the Great Depression but misses its slow recovery, while the labor wedge accounts for about one-half of this downturn and essentially all of the slow recovery.

Now consider the investment wedge. In Figure 3, we again plot the data for output, labor, and investment, but this time along with the contributions to those variables that the model predicts are due to the investment wedge alone. This figure demonstrates that the investment wedge's contributions completely miss the observed movements in all three variables. The investment wedge actually leads output to rise by about 9% by 1933.

Together, then, Figures 2 and 3 suggest that the efficiency and labor wedges account for essentially all of the movements of output, labor, and investment in the Depression period and that the investment wedge accounts for almost none. This suggestion is confirmed by Figure 4, where we plot the combined contribution from the efficiency, labor, and (insignificant) government consumption wedges (labeled Model With No Investment Wedge). As can be seen from the figure, essentially all of the fluctuations in output, labor, and investment can be accounted for by movements in the efficiency and labor wedges. For comparison, we also plot the combined contribution due to the labor, investment, and government consumption wedges (labeled Model With No Efficiency Wedge). This combination does not do well. In fact, comparing Figures 2 and 4, we see that the model with this combination is further from the data than the model with the labor wedge component alone.

One issue of possible concern with our findings about the role of the investment wedge is that measuring it is subtler than measuring the other wedges. Recall that measurement of this wedge depends on the details of the stochastic process that governs the wedges, whereas the size of the other wedges can be inferred from static equilibrium conditions. To address this concern, we conduct an additional experiment intended to give the model with no efficiency wedge the best chance to account for the data.

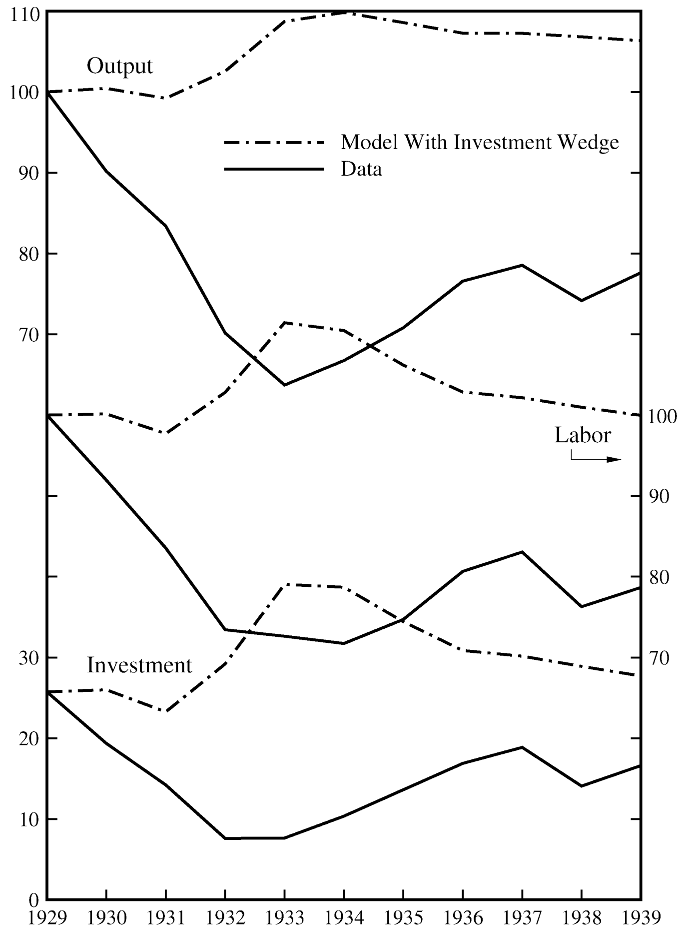

FIGURE 3.—Data and predictions of the model with just the investment wedge.

In this experiment, we choose the investment wedge to be as large as it needs to be for investment in the model to be as close as possible to investment in the data, and we set the other wedges to be constants. Predictions of this model, which we call the *Model With Maximum Investment Wedge*, turn out to match the behavior of consumption in the data poorly. For example, from 1929 to 1933, consumption in the model rises more than 8% relative to trend, while consumption in the data declines about 28%. (For details, see the technical appendix.) We label this poor performance the *consumption anomaly* of the investment wedge model.

Altogether, these findings lead us to conclude that distortions that manifest themselves primarily as investment wedges played essentially no useful role in the U.S. Great Depression.

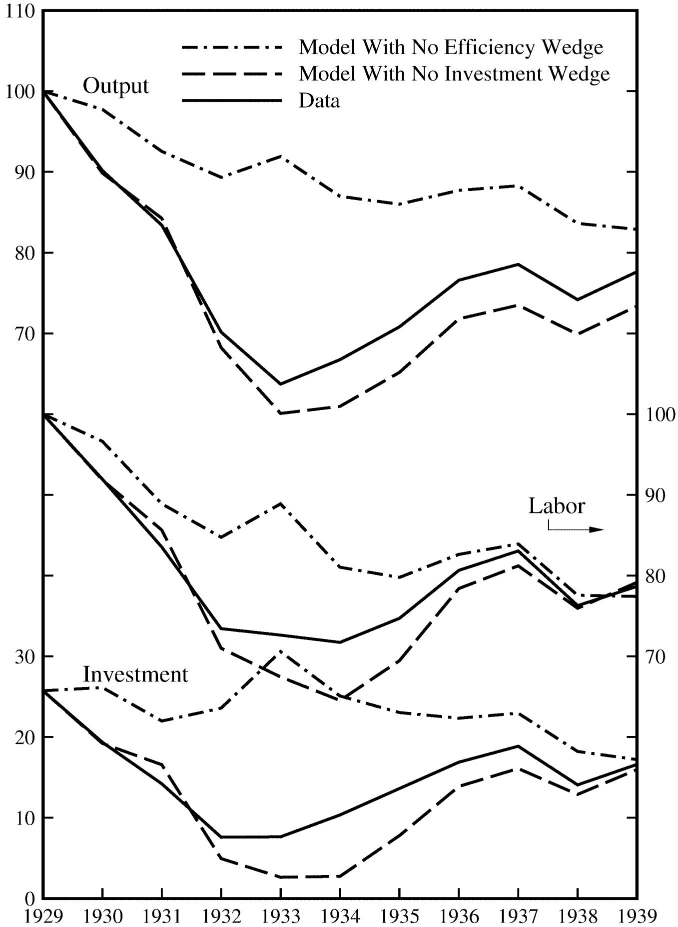

FIGURE 4.—Data and predictions of the models with all wedges but one.

B. *The 1982 recession*

Now we apply our accounting procedure to a more typical U.S. business cycle: the recession of 1982. Here we get basically the same results as with the earlier period: the efficiency and labor wedges play primary roles in the business cycle fluctuations, and the investment wedge plays essentially none.

We start here, as we did in the Great Depression analysis, by displaying actual U.S. output over the entire business cycle period (here, 1979–1985) along with the three measured wedges for that period. In Figure 5, we see that output falls nearly 10% relative to trend between 1979 and 1982, and by 1985 is back up to about 1% below trend. We also see that the efficiency wedge falls

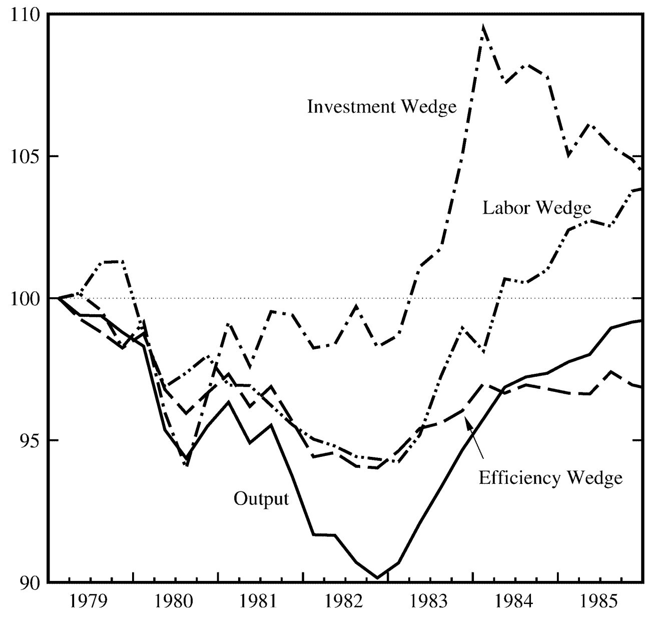

FIGURE 5.—U.S. output and three measured wedges (quarterly, 1979:1–1985:4; normalized to equal 100 in 1979:1).

between 1979 and 1982, and by 1985 is still a little more than 3% below trend. The labor wedge also worsens from 1979 to 1982, but it improves substantially by 1985. The investment wedge, meanwhile, fluctuates until 1983 and improves thereafter.

An analysis of the effects of the wedges separately for the 1979–1985 period is shown in Figures 6 and 7. In Figure 6, we see that the model with the efficiency wedge alone produces a decline in output from 1979 to 1982 of 6%, which is about 60% of the actual decline in that period. Here output recovers a bit more slowly than in the data, but seems to otherwise generally parallel the data's movements. The model with the labor wedge alone produces a decline in output from 1979 to 1982 of only about 3%. In Figure 7, we see that the model with just the investment wedge produces essentially no fluctuations in output.

Now we examine how well a combination of wedges reproduces the data for the 1982 recession period just as we did for the Depression period. In Figure 8, we plot the movements in output, labor, and investment during 1979–1985 due to two combinations of wedges. One is the combined effects of the efficiency, labor, and (insignificant) government consumption components (labeled Model With No Investment Wedge). In terms of output, this combination mimics the decline in output until 1982 extremely well and produces a slightly shallower recovery than in the data. The other is the combination of

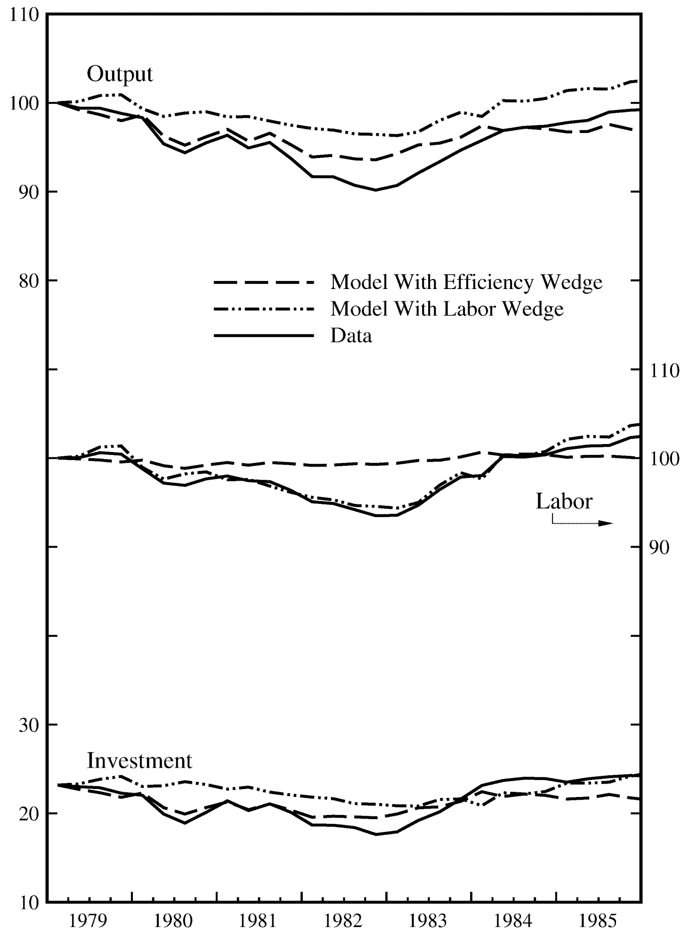

FIGURE 6.—Data and predictions of the models with just one wedge.

the labor, investment, and government components (labeled Model With No Efficiency Wedge), which produces a modest decline in output relative to the data. In Figures 6, 7, and 8, we see clearly that in the model with no efficiency wedge, the labor wedge accounts for essentially all of the decline and the investment wedge accounts for essentially none.

3.3. *Extending the Analysis to the Entire Postwar Period*

So far we have analyzed the wedges and their contributions for specific episodes. The findings for both episodes suggest that frictions in detailed models, which manifest themselves as investment wedges in the benchmark prototype economy, play, at best, a tertiary role in accounting for business cycle

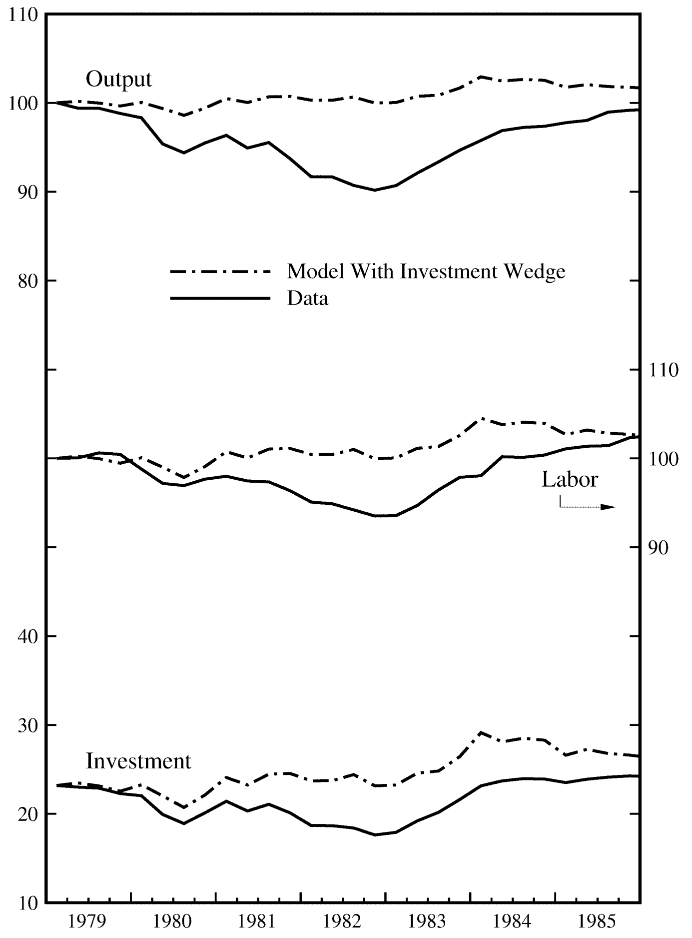

FIGURE 7.—Data and predictions of the model with just the investment wedge.

fluctuations. Do our findings apply beyond those particular episodes? We attempt to extend our analysis to the entire postwar period by developing some summary statistics for the period from 1959:1 through 2004:3 using HP-filtered data. We first consider the standard deviations of the wedges relative to output as well as correlations of the wedges with each other and with output at various leads and lags. We then consider the standard deviations and the cross correlations of output due to each wedge. These statistics summarize salient features of the wedges and their role in output fluctuations for the entire postwar sample. We think of the wedge statistics as analogs of our plots of the wedges and the output statistics as analogs of our plots of output due to just

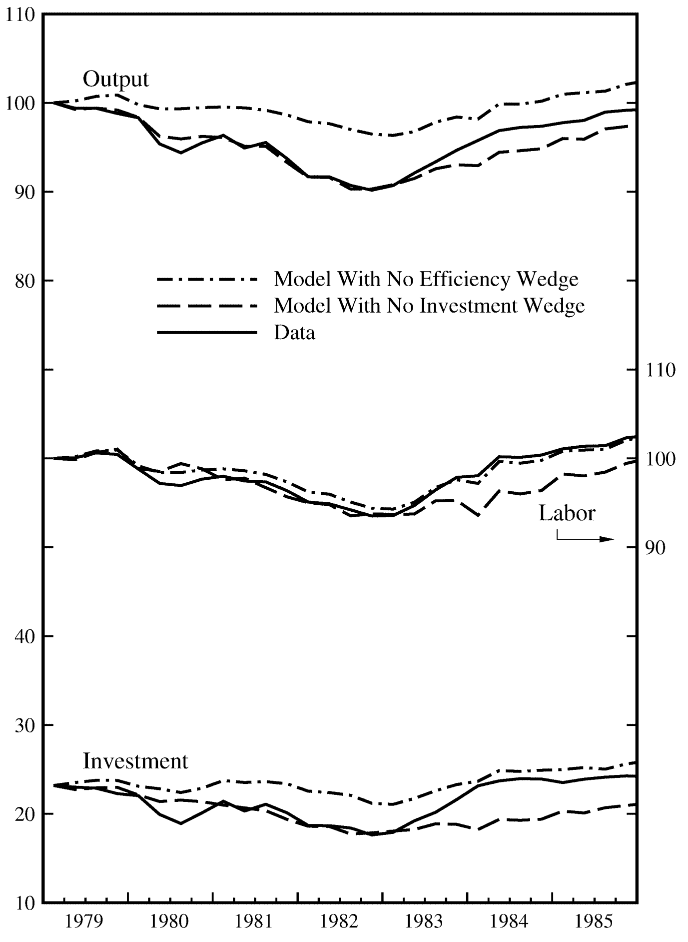

FIGURE 8.—Data and predictions of the models with all wedges but one.

one wedge.[2] The results suggest that our earlier findings do hold up, at least in a relative sense: the investment wedge seems to play a larger role over the entire postwar period than in the 1982 recession, but its effects are still quite modest compared to those of the other wedges.

In Tables II and III, we display standard deviations and cross correlations calculated using HP-filtered data for the postwar period. Panel A of Table II shows that the efficiency, labor, and investment wedges are positively correlated with

[2]In Chari, Kehoe, and McGrattan (2004), we applied a spectral method to determine the contributions of the wedges based on the population properties of the stochastic process generated by the model. We did this for both periods and found that the investment wedge plays only a modest role in both.

TABLE II

PROPERTIES OF THE WEDGES, 1959:1–2004:3[a]

A. Summary Statistics

Wedges	Standard Deviation Relative to Output	Cross Correlation of Wedge with Output at Lag $k =$ −2	−1	0	1	2
Efficiency	.63	.65	.76	.85	.60	.35
Labor	.92	.52	.65	.71	.73	.68
Investment	1.18	.44	.48	.47	.30	.09
Government consumption	1.51	−.42	−.42	−.33	−.24	−.11

B. Cross Correlations

Wedges (X, Y)	Cross Correlation of X with Y at Lag $k =$ −2	−1	0	1	2
Efficiency, labor	.57	.48	.30	.28	.16
Efficiency, investment	.31	.46	.61	.47	.35
Efficiency, government consumption	−.27	−.33	−.34	−.35	−.31
Labor, investment	−.07	.11	.18	.37	.46
Labor, government consumption	−.02	−.22	−.38	−.47	−.50
Investment, government consumption	−.60	−.73	−.88	−.70	−.51

[a]Series are first logged and detrended using the HP filter.

output, both contemporaneously and for several leads and lags. In contrast, the government consumption wedge is somewhat negatively correlated with output, both contemporaneously and for several leads and lags. (Note that the government consumption wedge is the sum of government consumption and net exports, and that net exports are negatively correlated with output.) Panel B of Table II shows that the cross correlations of the efficiency, labor, and investment wedges are generally positive.

Table III summarizes various statistics of the movements of output over this period due to each wedge. Consider panel A and focus first on the output fluctuations due to the efficiency wedge. Table III shows that output movements due to this wedge have a standard deviation that is 73% of that of output in the data. These movements are highly positively correlated with output in the data, both contemporaneously and for several leads and lags. These statistics are consistent with our episodic analysis of the 1982 recession, which showed that the efficiency wedge can account for about 60% of the actual decline in output during that period and comoves highly with it.

Consider next the role of the other wedges in the entire postwar period. Return to Table III. In panel A, again, we see that output due to the labor wedge alone fluctuates almost 60% as much as does output in the data and is posi-

TABLE III

PROPERTIES OF THE OUTPUT COMPONENTS, 1959:1–2004:3[a]

A. Summary Statistics

Output Components	Standard Deviation Relative to Output	Cross Correlation of Wedge with Output at Lag $k =$ −2	−1	0	1	2
Efficiency	.73	.65	.75	.83	.57	.31
Labor	.59	.44	.59	.68	.74	.74
Investment	.31	.33	.37	.40	.25	.07
Government consumption	.40	−.45	−.45	−.39	−.25	−.08

B. Cross Correlations

Output Components (X, Y)	Cross Correlation of X with Y at Lag $k =$ −2	−1	0	1	2
Efficiency, labor	.54	.41	.18	.15	.04
Efficiency, investment	.30	.44	.60	.40	.28
Efficiency, government consumption	−.34	−.45	−.56	−.48	−.39
Labor, investment	−.17	−.03	−.03	.20	.29
Labor, government consumption	.14	−.03	−.13	−.31	−.40
Investment, government consumption	−.49	−.63	−.87	−.66	−.48

[a] Series are first logged and detrended using the HP filter.

tively correlated with it. Output due to the investment wedge alone fluctuates less than one-third as much as output in the data and is somewhat positively correlated with it. Finally, output due to the government consumption wedge alone fluctuates about 40% as much as output in the data and is somewhat negatively correlated with it. In panel B of Table III, we see that output movements due to the efficiency and labor wedges as well as the efficiency and investment wedges are positively correlated, and that the cross correlations of output movements due to the other wedges are mostly essentially zero or negative.

All of our analyses using business cycle accounting thus seem to lead to the same conclusion: to study business cycles, the most promising detailed models to explore are those in which frictions manifest themselves primarily as efficiency or labor wedges, not as investment wedges.

4. INTERPRETING WEDGES WITH ALTERNATIVE TECHNOLOGY OR PREFERENCE SPECIFICATIONS

In detailed economies with technology and preferences similar to those in our benchmark prototype economy, the equivalence propositions proved thus far provide a mapping between frictions in those detailed economies and wedges in the prototype economy. Here we construct a similar mapping when technology or preferences differ in the two types of economies. We then ask if this alternative mapping changes our substantive conclusion that financial frictions, which manifest themselves primarily as investment wedges, are unlikely to play a primary role in accounting for business cycles. We find that it does not.

When detailed economies have technology or preferences different from the benchmark economy's, wedges in the benchmark economy can be viewed as arising from two sources: frictions in the detailed economy and differences in the specification of technology or preferences. Although researchers could simply use results from our benchmark prototype economy to draw inferences about promising classes of models, drawing such inferences is easier with an alternative approach. Basically, we decompose the wedges into their two sources. To do that, construct an alternative prototype economy with technology and preferences that do coincide with those in the detailed economy, and repeat the business cycle accounting procedure with those two economies. The part of the wedges in the benchmark prototype economy due to frictions, then, will be the wedges in the alternative prototype economy, while the remainder will be due to specification differences.

Here we use this approach to explore alternative prototype economies with technology and preference specifications chosen because of their popularity in the literature. These alternative specifications include variable instead of fixed capital utilization, different labor supply elasticities, and varying levels of costs to adjusting investment.

Two of these changes offer no help to investment wedges. Adding variable capital utilization to the analysis shifts the relative contributions of the efficiency and labor wedges to output's fluctuations—decreasing the efficiency wedge's contribution and increasing the labor wedge's—but this alternative specification leaves the investment wedge's contribution definitely in third place. Adding different labor elasticities to the analysis offers no help either.

The third specification change seems to give investment wedges a slightly larger role, but still not a primary one. With investment adjustment costs added to the analysis, the investment wedge in the benchmark prototype economy depends on both the investment wedge and the marginal cost of investment in the alternative prototype economy. We find that even if the investment wedge is constant in the benchmark economy, it will worsen during recessions and improve during booms in the alternative economy. With our measured wedges, this finding suggests that with large enough adjustment costs, investment wedges in detailed economies could play a significant role in business cycle fluctuations.

To study this possibility, we investigate the effects of two parameter values for adjustment costs: one at the level used by Bernanke, Gertler, and Gilchrist (1999), the *BGG level*, and one we consider *extreme*, at four times that level. For the Great Depression period, we find that for both adjustment cost levels, investment wedges play only a minor role. For the 1982 recession period, we find that these wedges play a very small role with BGG level costs and a somewhat larger but still modest role with the much higher costs. These findings suggest that researchers who think adjustment costs are extremely high may want to include in their models financial frictions that manifest themselves as investment wedges. Such models are not likely to do well, however, unless they also include other frictions that play the primary role in business cycle fluctuations.

4.1. *Details of Alternative Specifications*

A. *Variable capital utilization*

We begin with an extreme view about the amount of variability in capital utilization.

Our specification of the technology, which allows for variable capital utilization, follows the work of Kydland and Prescott (1988) and Hornstein and Prescott (1993). We assume that the production function is now

$$y = A(kh)^{\alpha}(nh)^{1-\alpha}, \tag{28}$$

where n is the number of workers employed and h is the length (or hours) of the workweek. The labor input is, then, $l = nh$.

In the data, we measure only the labor input l and the capital stock k. We do not directly measure h or n. The benchmark specification for the production function can be interpreted as assuming that all of the observed variation in measured labor input l is in the number of workers and that the workweek h is constant. Under this interpretation, our benchmark specification with *fixed capital utilization* correctly measures the efficiency wedge (up to the constant h).

Now we investigate the opposite extreme: Assume that the number of workers n is constant and that all the variation in labor is from the workweek h. Under this *variable capital utilization* specification, the services of capital kh are proportional to the product of the stock k and the labor input l, so that variations in the labor input induce variations in the flow of capital services. Thus, the capital utilization rate is proportional to the labor input l, and the efficiency wedge is proportional to y/k^{α}.

Consider an alternative prototype economy, denoted *economy* 2, that is identical to a deterministic version of our benchmark prototype economy, denoted *economy* 1, except that the production function is now given by $y = Ak^{\alpha}l$. Let the sequence of wedges and the equilibrium outcomes in the two economies be $(A_{it}, \tau_{lit}, \tau_{xit})$ and $(y_{it}, c_{it}, l_{it}, x_{it})$ for $i = 1, 2$. We then have the next proposition:

PROPOSITION 3: *If the sequence of wedges for an alternative prototype economy* 2 *are related to the wedges in the prototype economy* 1 *by* $A_{2t} = A_{1t}l_{1t}^{-\alpha}$,

$1 - \tau_{l2t} = (1-\alpha)(1-\tau_{l1t})$, *and* $\tau_{x2t} = \tau_{x1t}$, *then the equilibrium outcomes for the two economies coincide*.

PROOF: We prove this proposition by showing that the equilibrium conditions of economy 2 are satisfied at the equilibrium outcomes of economy 1. Because $y_{1t} = A_{1t}k_{1t}^{\alpha}l_{1t}^{1-\alpha}$, using the definition of A_{2t}, we have that $y_{1t} = A_{2t}k_{1t}^{\alpha}l_{1t}$. The first-order condition for labor in economy 1 is

$$-\frac{U_{lt}(c_{1t}, l_{1t})}{U_{ct}(c_{1t}, l_{1t})} = (1-\tau_{l1t})\frac{(1-\alpha)y_{1t}}{l_{1t}}.$$

Using the definition of τ_{l2t}, we have that

$$-\frac{U_{lt}(c_{1t}, l_{1t})}{U_{ct}(c_{1t}, l_{1t})} = (1-\tau_{l2t})\frac{y_{1t}}{l_{1t}}.$$

The rest of the equations that govern the equilibrium are unaffected. *Q.E.D.*

Note that even if the efficiency wedge in the alternative prototype economy does not fluctuate, the associated efficiency wedge in the prototype economy will. Proposition 3 also implies that if τ_{x1t} is a constant, so that the contribution of the investment wedge to fluctuations in economy 1 is zero, then τ_{x2t} is also a constant; hence, the contribution of the investment wedge to fluctuations in economy 2 is also zero. Extending this proposition to a stochastic environment is immediate.

Now suppose that we are interested in detailed economies with variable capital utilization. We use the alternative prototype economy to ask whether this change affects our substantive conclusions. In answering this question, we reestimate the parameters of the stochastic process for the underlying state. (For details, see the technical appendix.)

Variable capital utilization can induce significant changes in the measured efficiency wedge. To see these changes, in Figure 9, we plot the measured efficiency wedges for these two specifications of capital utilization during the Great Depression period (with it *fixed* in the benchmark economy and *variable* now). Clearly, when capital utilization is variable rather than fixed, the efficiency wedge falls less and recovers more by 1939. (For the other wedges, see the technical appendix.)

In Figure 10, we plot the data and the predicted output due to the efficiency and labor wedges for the 1930s when the model includes variable capital utilization. Comparing Figures 10 and 2, we see that with the remeasured efficiency wedge, the labor wedge plays a much larger role in accounting for the output downturn and slow recovery, and the efficiency wedge plays a much smaller role.

In Figure 11, we plot the three data series again, this time with the predictions of the variable capital utilization model with just the investment wedge.

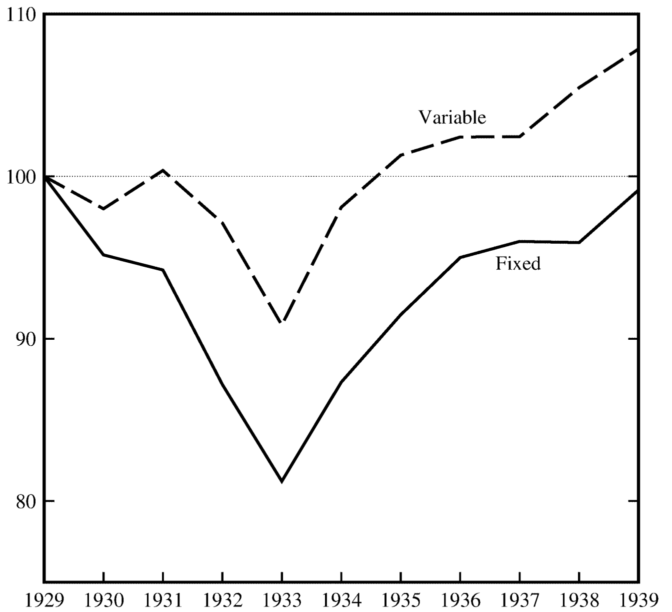

FIGURE 9.—Varying the capital utilization specification during the Great Depression period. Measured efficiency wedges for two capital utilization specifications.

Comparing this to Figure 3, we see that with variable capital utilization, the investment wedge drives output the wrong way to an even greater extent than in the benchmark economy.

In Figure 12, we compare the contributions of the sum of the efficiency and labor wedges for the two specifications of capital utilization (fixed and variable) during the Great Depression period. The figure shows that these contributions are quite similar. Although remeasuring the efficiency wedge changes the relative contributions of the two wedges, it clearly has little effect on their combined contribution.

Overall, then, taking account of variable capital utilization strengthens our finding that in the Great Depression period, the efficiency and labor wedges play a primary role and investment wedges do not.

B. *Different labor supply elasticities*

Now we consider the effects on our results of changing the elasticity of labor supply. We assume in our benchmark model that preferences are logarithmic in both consumption and leisure. Consider now an alternative prototype economy with a different elasticity of labor supply. We show that a result analogous to that in Proposition 3 holds: allowing for different labor supply elasticities changes the size of the measured labor wedge but not that of the measured investment wedge. Therefore, if the contribution of the investment wedge is

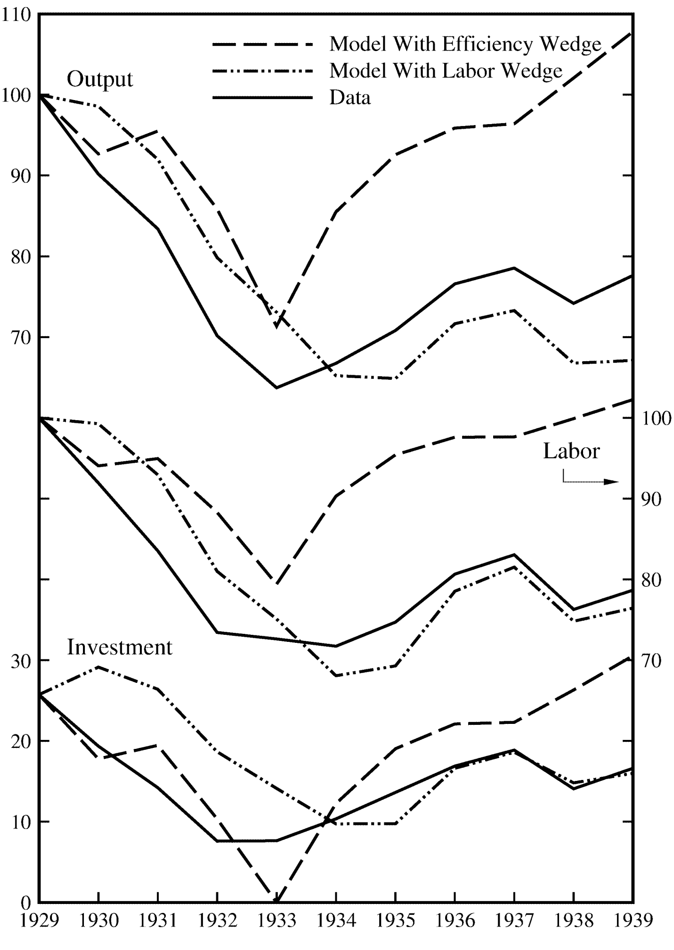

FIGURE 10.—Data and predictions of the models with variable capital utilization and just one wedge.

zero in the benchmark prototype economy, it is also zero in an economy with a different labor supply elasticity.

To see that, consider an alternative prototype economy that is identical to a deterministic version of our benchmark model except that now the utility function is given by $U(c) + V_2(1 - l)$. Denote the utility function in our benchmark prototype economy (economy 1) by $U(c) + V_1(1 - l)$. Clearly, by varying the function V_2, we can generate a wide range of alternative labor supply elasticities.

Let the sequence of wedges and the equilibrium outcomes in the two economies be $(A_{it}, \tau_{lit}, \tau_{xit})$ and $(y_{it}, c_{it}, l_{it}, x_{it})$ for $i = 1, 2$. We then have the next proposition:

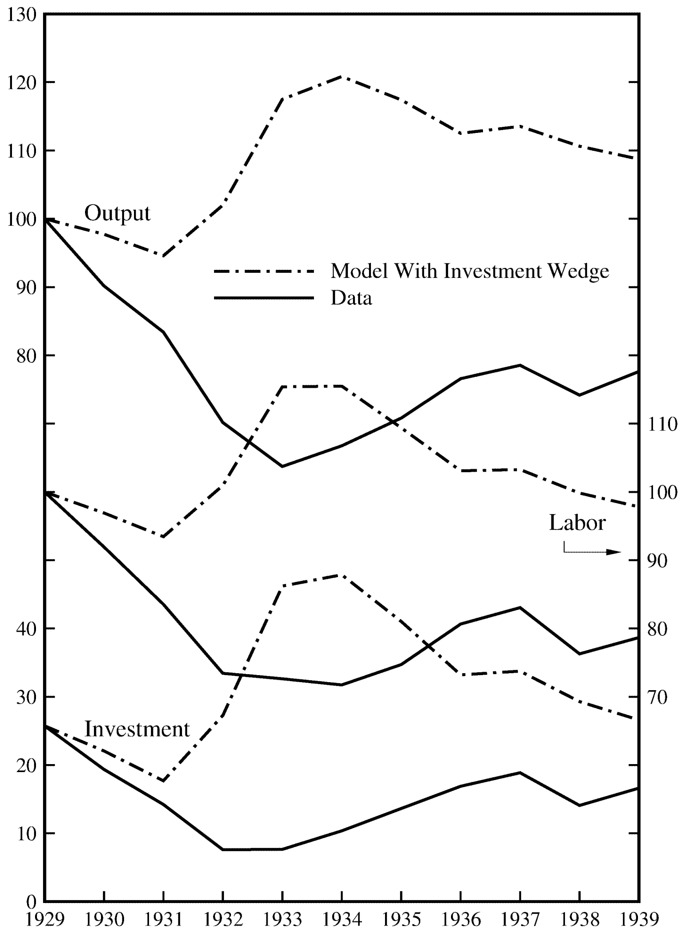

FIGURE 11.—Data and predictions of the model with variable capital utilization and just the investment wedge.

PROPOSITION 4: *If the sequence of wedges for the alternative prototype economy, economy* 2, *is given by*

$$1 - \tau_{l2t} = (1 - \tau_{l1t})\frac{V_2'(1 - l_{1t})}{V_1'(1 - l_{1t})},$$

and if $A_{2t} = A_{1t}$ *and* $\tau_{x2t} = \tau_{x1t}$, *then the equilibrium outcomes for the two economies coincide*.

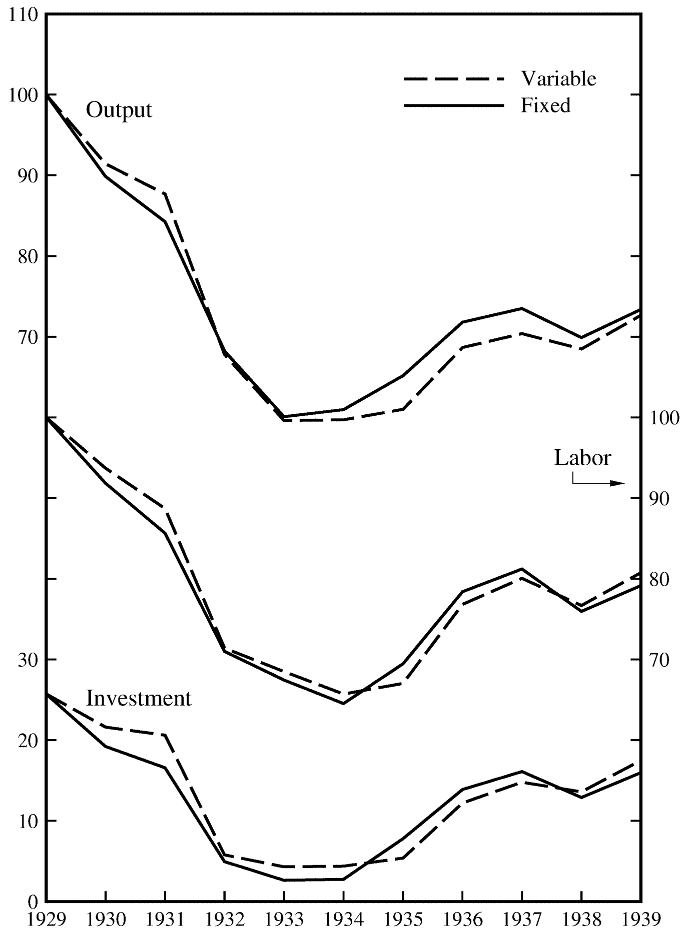

FIGURE 12.—Predictions of the models with fixed and variable capital utilization and with all but the investment wedge.

PROOF: We prove this proposition by showing that the equilibrium conditions of economy 2 are satisfied at the equilibrium outcomes of economy 1. The first-order condition for labor input in economy 1 is

$$-\frac{V_1'(1-l_{1t})}{U'(c_{1t})} = (1-\tau_{l1t})\frac{(1-\alpha)y_{1t}}{l_{1t}}.$$

Using the definition of τ_{l2t}, we have that

$$-\frac{V_2'(1-l_{1t})}{U'(c_{1t})} = (1-\tau_{l2t})\frac{(1-\alpha)y_{1t}}{l_{1t}},$$

so that the first-order condition for labor in economy 2 is satisfied. The rest of the equations that govern the equilibrium are unaffected. *Q.E.D.*

Note that here even if the labor wedge does not fluctuate in our benchmark prototype economy, it typically will in the alternative prototype economy. Note also that the investment wedges are the same in both economies. Thus, if the investment wedge is constant in one economy, it is constant in the other, and the contribution of the investment wedge to fluctuations is zero in both economies. Extending this proposition to a stochastic environment is immediate.

C. *Investment adjustment costs*

Now we consider a third alternative prototype economy, this one with investment adjustment costs. These costs can be interpreted as standing in for one of two features of detailed economies. One is that the detailed economies have adjustment costs in converting output into installed capital. Another interpretation is that the detailed model does not have adjustment costs, but that financial frictions manifest themselves as adjustment costs in the alternative prototype economy (as in Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997)).

In this alternative prototype economy, the only difference from the benchmark prototype economy is that the capital accumulation law is no longer (1), but rather is

$$(29) \qquad (1+\gamma_n)k_{t+1} = (1-\delta)k_t + x_t - \phi\left(\frac{x_t}{k_t}\right)k_t,$$

where ϕ represents the per unit cost of adjusting the capital stock. In the macroeconomic literature, a commonly used functional form for the adjustment costs ϕ is

$$(30) \qquad \phi\left(\frac{x}{k}\right) = \frac{a}{2}\left(\frac{x}{k} - b\right)^2,$$

where $b = \delta + \gamma + \gamma_n$ is the steady-state value of the investment–capital ratio.

To set up the analog of Propositions 3 and 4, let the wedges in the benchmark prototype economy, economy 1, and the alternative prototype economy, economy 2, be $(A_{it}, \tau_{lit}, \tau_{xit})$ and $(y_{it}, c_{it}, l_{it}, x_{it})$ for $i = 1, 2$. For simplicity, let τ_{x2t} and g_{2t} be identically zero. The proof of the following proposition is immediate.

PROPOSITION 5: *If the sequence of wedges for the alternative prototype economy, economy* 2, *is given by* $A_{2t} = A_{1t}$, $\tau_{l2t} = \tau_{l1t}$, τ_{x1t} implicitly defined by

$$(31) \qquad \frac{A_{1t+1}F_{kt+1} + (1-\delta)(1+\tau_{x1t+1})}{1+\tau_{x1t}}$$

$$= \frac{A_{2t+1}F_{kt+1} + (1-\delta+\phi_t - x_{2t+1}\phi'_{t+1}/k_{2t+1})(1-\phi'_{t+1})^{-1}}{(1-\phi'_t)^{-1}},$$

and $g_{1t} = \phi_t k_{2t}$, *then the equilibrium outcomes for the two economies coincide.*

To understand the investment wedge τ_{x1t}, note that if adjustment costs are given by (30), then the term $\phi_t - x_{t+1}\phi'_{t+1}/k_{t+1}$ in (31) equals $a[(x/k)^2 - b^2]/2$, which is an order of magnitude smaller than $\phi'_t = a[x/k - b]$. Setting this term to zero gives the approximation

$$(32) \qquad 1 + \tau_{x1t} = \frac{1}{1-\phi'_t}.$$

From (32) and the convexity of ϕ, we see that τ_{x1} is increasing in x/k and is zero when x/k is at its steady-state value. Hence, in recessions, when x/k is relatively low, τ_{x1} is negative; in booms, when x/k is relatively high, τ_{x1} is positive. In this sense, even when the alternative prototype economy has no investment wedges, τ_{x1} will be countercyclical in the benchmark prototype economy, so investment distortions will be smaller in recessions.

Note, more generally, that when investment wedges in the alternative prototype economy are nonzero, the analog of (32) is

$$(33) \qquad 1 + \tau_{x1t} = \frac{1+\tau_{x2t}}{1-\phi'_t}.$$

We can also use the equivalence map in (33) in the reverse direction. Imagine that the data are generated from a detailed economy with no adjustment costs and no frictions. The benchmark prototype economy will have $\tau_{x1t} = 0$. Suppose a researcher considers an alternative prototype economy with adjustment costs. This researcher will find that $1 + \tau_{x2t} = 1 - \phi'_t$, so that the investment wedge will be procyclical even though the detailed economy has no frictions.

More generally, a researcher who incorrectly specifies too high a level of adjustment costs in the alternative prototype economy will infer that investment wedges play a much larger role than they actually do. To get some intuition for this result, consider two alternative prototype economies A and B, both of which have investment wedges and adjustment costs. The analog of our approximation (33) is $(1 + \tau_{xAt})/(1 - \phi'_{At}) = (1 + \tau_{xBt})/(1 - \phi'_{Bt})$, where ϕ_{it} denotes adjustment costs in economy $i = A, B$. Straightforward algebra establishes that specifying too high a level of adjustment costs makes investment wedges seem worse in recessions than they actually are.

Now we consider a prototype economy with adjustment costs and ask whether frictions that manifest themselves as investment wedges in such a prototype economy play an important role. We find that they do not, either when they are set at the level chosen by Bernanke, Gertler, and Gilchrist (1999)

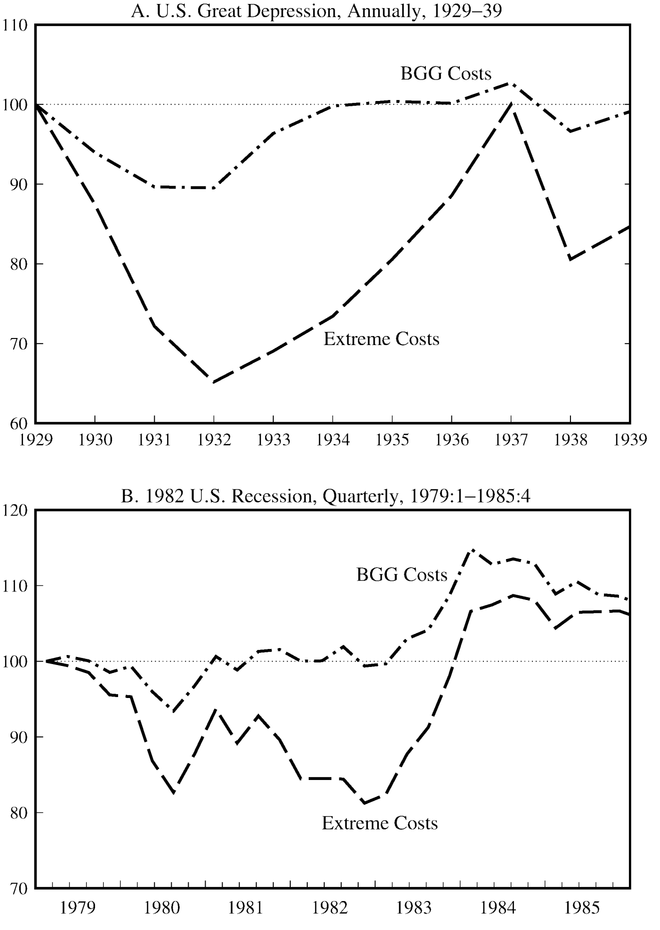

FIGURE 13.—Measured investment wedges for two adjustment cost specifications (normalized to equal 100 in 1929 or 1979:1).

or at four times that level. We begin by following Bernanke, Gertler, and Gilchrist (1999) in how we choose the value for the parameter a. Bernanke, Gertler, and Gilchrist (BGG) chose this parameter so that the elasticity, η, of the price of capital with respect to the investment–capital ratio is .25. In this setup, the price of capital $q = 1/(1-\phi')$, so that, evaluated at the steady state, $\eta = a(\delta + \gamma + \gamma_n)$. Given our other parameters, $a = 3.22$. In Figures 13 and 14, this parameterization is the model labeled BGG Costs. Bernanke, Gertler, and Gilchrist also argued that a reasonable range for the elasticity η is between 0 and .5, and that values much outside this range imply implausibly high adjustment costs. We consider an extreme case in which $\eta = 1$, so that $a = 12.88$, roughly four times the BGG level. In the figures, this parameterization is the

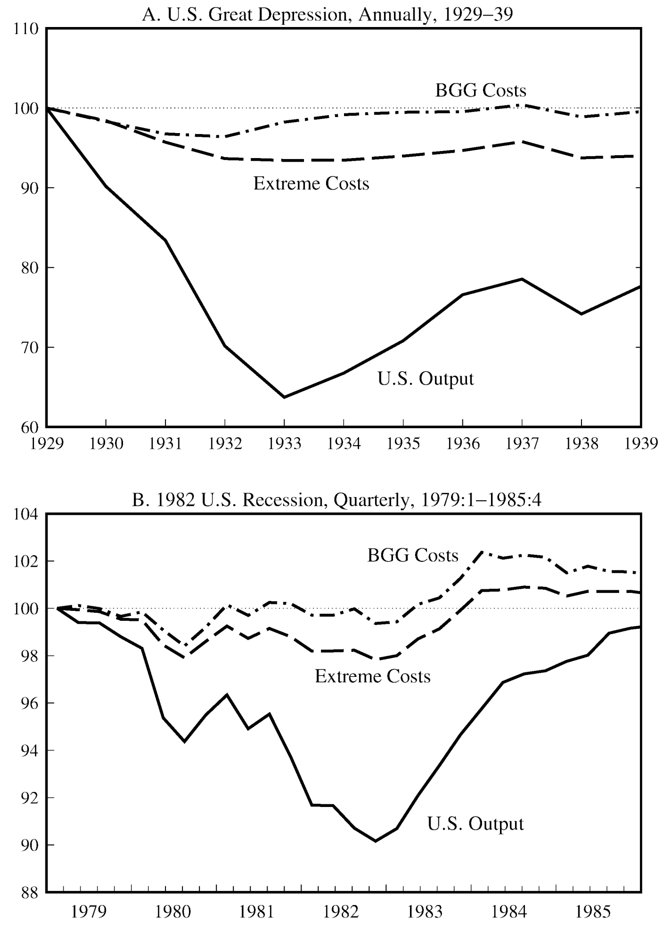

FIGURE 14.—U.S. output and predictions of model with alternative adjustment costs and just the investment wedge (normalized to equal 100 in 1929 or 1979:1).

model labeled Extreme Costs. For each setting of the parameter a, we reestimate the stochastic process for the state. (For the parameter values of the stochastic process, see the technical appendix.)

Comparing the investment wedges in Figure 1 and panel A of Figure 13, we see that introducing investment adjustment costs leads the investment wedge to worsen rather than improve in the early part of the Great Depression. In panel A of Figure 14 we see that this worsening produces a decline in output from 1929 to 1933. With the BGG adjustment costs, however, the decline is tiny (2% from 1929 to 1933). Even with the extreme adjustment costs, the decline is only 6.5% and, hence, accounts for only about one-sixth of the overall decline in output.[3] Moreover, with these extreme adjustment costs, the consumption anomaly associated with investment wedges is acute. For example, from 1929 to 1932, relative to trend, consumption in the data falls about 18%, while in the model it actually rises by more than 5%.

We are skeptical that detailed models with extreme adjustment costs are worth exploring. With extreme costs, given the observed levels of investment and the capital stock in the data, (30) implies that the resources lost due to adjustment costs as a fraction of output are nearly 7% in 1933. We share Bernanke, Gertler, and Gilchrist's (1999) concerns that costs of this magnitude are implausibly large. From 1929 to 1933, investment falls sharply, but the adjustment costs implied by (30) rise sharply. Why would firms incur adjustment costs simply by investing at positive rates below their steady-state value? The idea that managers incurred huge adjustment costs simply because they were watching their machines depreciate seems farfetched. Furthermore, the idea that from 1929 to 1933 investment fell sharply but adjustment costs rose sharply is inconsistent with interpreting these costs as arising from monitoring costs in an economy with financial frictions. Indeed, in such an economy, as investment activity falls, so do monitoring costs.

We performed similar experiments for the 1982 recession period with similar results. In panel B of Figure 13, we see that with BGG costs, the investment wedge fluctuates little through the end of 1982 and improves thereafter. With extreme costs, the investment wedge worsens until 1983 and improves thereafter. Panel B of Figure 14 shows that in the model with costs at the BGG level, the investment wedge plays essentially no role in output fluctuations. With costs at four times the BGG level, the panel shows that the investment wedge plays a bit larger but still modest role. Recall that, relative to trend, output in the data falls almost 10% from its peak to its trough. With extreme adjustment costs, output due to the investment wedge falls about 2.2%, so that the investment wedge appears to account for roughly one-fifth of the fall in output.

In our judgment, the investment wedge actually accounts for much less than one-fifth of the 1982 recession fall in output because we think extreme adjustment costs are implausible. Even aside from the Great Depression issues, we find models with extreme adjustment costs difficult to reconcile with data on plant level investment decisions. An extensive literature has documented that investment at the plant level has large spikes. Doms and Dunne (1994), for example, examined plant level data for 33,000 plants over 17 years. They showed that much of the growth in the capital stock of a plant is concentrated in a short period of time. In the year of highest investment, the capital stock grows 45%,

[3]In general, in our Great Depression experiments, linear methods perform poorly compared to our nonlinear method. With extreme adjustment costs, for example, linear methods produce large errors, on the order of 100%.

while in the years immediately before and after that, it grows less than 10%. Such behavior is clearly inconsistent with extreme adjustment costs.

Furthermore, the use of adjustment costs in macroeconomic analysis is controversial. Kydland and Prescott (1982), for example, have argued that models with adjustment costs like those in equation (30) are inconsistent with the data. Such models imply a static relationship between the investment–capital ratio and the relative price of investment goods to output (of the form $q = 1/[1 - \phi'(x/k)]$). This means that the elasticity of the investment–capital ratio with respect to the relative price is the same in the short run and the long run. Kydland and Prescott argued that this is not consistent with the data, where short-run elasticities are much smaller than long-run elasticities.

Finally, the finding that investment wedges play a modest role when adjustment costs are extreme is easy to reconcile with the view that financial frictions, which manifest themselves primarily as investment wedges, play a small role over the business cycle. This reconciliation uses the insight discussed above that a modeler who incorrectly specifies too high a level of adjustment costs will incorrectly find too large a role for investment wedges.

5. CONTRASTING OUR DECOMPOSITION WITH TRADITIONAL DECOMPOSITIONS

Our decomposition of business cycle fluctuations is intended to isolate the partial effects of each of the wedges on equilibrium outcomes, and in this sense, it is different from traditional decompositions. Those decompositions attempt to isolate the effects of (so-called) primitive shocks on equilibrium outcomes; ours does not. Isolating the effects of primitive shocks requires specifying a detailed model. Because our procedure precedes the specification of a detailed model, it obviously cannot be used to isolate the effects of primitive shocks. To clarify the distinction between our decomposition and a traditional one, here we describe a traditional decomposition and explain why we prefer ours.

The traditional decomposition attempts to isolate the effects of primitive shocks by "naming the innovations." Recall that in our stochastic process for the four wedges, (27), the innovations ε_{t+1} are allowed to be contemporaneously correlated with the covariance matrix V. Under the traditional decomposition, the primitive shocks, say, η_{t+1}, are assumed to be mean zero, to be contemporaneously uncorrelated with $E\eta_{t+1}\eta'_{t+1} = I$, and to lead to the same stochastic process for the wedges. Identifying these primitive shocks requires specifying a matrix R so that $R\eta_{t+1} = \varepsilon_{t+1}$ and $RR' = V$. Names are then made up for these shocks, including *money* shocks, *demand* shocks, *technology* shocks, and so on.

In the traditional method, then, given any sequence of realized wedges s_t and the specification of the matrix R, the associated realized values of $\eta_t = (\eta_{1t}, \eta_{2t}, \eta_{3t}, \eta_{4t})'$ are computed. The movements in, say, output, are then decomposed into the movements due to each one of these primitive shocks as follows. Let $s_t(\eta_1) = (\log A_t(\eta_1), \tau_{lt}(\eta_1), \tau_{xt}(\eta_1), \log g_t(\eta_1))$ denote the realized values of the four wedges when the primitive shock sequence $\eta_t = (\eta_{1t}, 0, 0, 0)$ is fed into

$$s_{t+1} = P_0 + Ps_t + R\eta_{t+1}.$$

(Note that when the covariance matrix V is not diagonal, movements in a primitive shock η_{1t} will lead to movements in more than one wedge.) The predicted value of, say, output due to η_1 is then computed from the decision rules of the model according to $y_t(\eta_1) = y(k_t(\eta_1), s_t(\eta_1))$, where $k_t(\eta_1)$ is computed recursively using the decision rule for investment, the initial capital stock, and the capital accumulation law. The values $s_t(\eta_2)$, $y_t(\eta_2)$, and so forth are computed in a similar way.

Notice that in this traditional decomposition method, the realized value of each wedge is simply the sum of the parts due to η_1, η_2, and so on, in that

$$\log A_t = \sum_i \log A_t(\eta_i), \quad \tau_{lt} = \sum_i \tau_{lt}(\eta_i),$$
$$\tau_{xt} = \sum_i \tau_{xt}(\eta_i), \quad \log g_t = \sum_i \log g_t(\eta_i).$$

In this sense, the traditional decomposition attempts to decompose each of the four wedges into four component parts, each of which is due to a primitive shock.

Our decomposition is purposefully less ambitious. It includes only the effect of the movements of each total wedge, not any of its subparts. The advantage of our decomposition is that it is invariant to R, so that we do not need to make identifying assumptions implicit in the specification of the matrix R, nor do we need to make up names for the shocks. The invariance makes our method valuable. The problem with the traditional approach is that finding identifying assumptions that apply to a broad class of detailed models is very difficult. Hence, this approach is not useful to point researchers toward classes of promising models. In a sense, the traditional approach puts the cart before the horse: the decomposition requires specifying the matrix R, but doing that requires a detailed model. Our decomposition is useful precisely because it does not need to make up identifying assumptions to specify the matrix R. Our equivalence results have identifying assumptions built into them, so that results from the benchmark prototype economy can be used to uncover promising classes of detailed models.

6. REVIEWING THE RELATED LITERATURE

Our work here is related to the existing literature in terms of methodology and the interpretation of the wedges.

6.1. *Methodology*

Our basic method is to use restrictions from economic theory to back out wedges from the data, formulate stochastic processes for these wedges, and then put the wedges back into a quantitative general equilibrium model for an accounting exercise. This basic idea is at the heart of an enormous amount of work in the real business cycle theory literature. Prescott (1986), for example, asked what fraction of the variance of output can plausibly be attributed to productivity shocks, which we have referred to as the *efficiency wedge*. Subsequent studies have expanded this general equilibrium accounting exercise to include a wide variety of other shocks. (See, for example, the studies in Cooley's (1995) volume.)

An important difference between our method and others is that we back out the labor wedge and the investment wedge from the combined consumer and firm first-order conditions, while most of the recent business cycle literature uses direct measures of labor and investment shocks. Perhaps the most closely related precursor of our method is McGrattan's (1991); she used the equilibrium of her model to infer the implicit wedges. Ingram, Kocherlakota, and Savin (1997) advocated a similar approach.

6.2. *Wedge Interpretations*

The idea that taxes of various kinds distort the relationship between various marginal conditions is the cornerstone of public finance. Taxes are not the only well-known distortions; monopoly power by unions or firms is also commonly thought to produce a labor wedge. Additionally, the idea that a labor wedge is produced by sticky wages or sticky prices is the cornerstone of the new Keynesian approach to business cycles; see, for example, the survey by Rotemberg and Woodford (1999). One contribution of our work here is to show the precise mapping between the wedges and general equilibrium models with frictions.

Many studies have plotted one or more of the four wedges. The efficiency wedge has been extensively studied. (See, for example, Kehoe and Prescott (2002).) The labor wedge has also been studied. For example, Parkin (1988), Hall (1997), and Galí, Gertler, and López-Salido (2007) all graphed and interpreted the labor wedge for the postwar data. Parkin discussed how monetary shocks might drive this wedge, and Hall discussed how search frictions might drive it. Galí, Gertler, and López-Salido discussed a variety of interpretations of the labor wedge, as did Rotemberg and Woodford (1991, 1999). Mulligan (2002a, 2002b) plotted the labor wedge for the United States for much of the 20th century, including the Great Depression period. He interpreted movements in this wedge as arising from changes in labor market institutions and regulation, including features we discuss here. Cole and Ohanian (2002) plotted the labor wedge for the Great Depression and offered interpretations similar to ours. The investment wedge has been investigated by McGrattan (1991), Braun (1994), Carlstrom and Fuerst (1997), and Cooper and Ejarque (2000).

7. CONCLUSIONS AND EXTENSIONS

This study is aimed at applied theorists who are interested in building detailed, quantitative models of economic fluctuations. Once such theorists have chosen the primitive sources of shocks to economic activity, they need to choose the mechanisms through which the shocks lead to business cycle fluctuations. We have shown that these mechanisms can be summarized by their effects on four wedges in the standard growth model. Our business cycle accounting method can be used to judge which mechanisms are promising and which are not, thus helping theorists narrow their options. We view our method as an alternative to the use of structural vector autoregressions (VARs), which has also been advocated as a way to identify promising mechanisms. (Elsewhere, in Chari, Kehoe, and McGrattan (2005), we argued that structural VARs have deficiencies that limit their usefulness.)

Here we have demonstrated how our method works by applying it to two historical episodes—the Great Depression and the 1982 U.S. recession. We have found that efficiency and labor wedges, in combination, account for essentially all of the decline and recovery in these business cycles; investment wedges play, at best, a tertiary role. These results hold in summary statistics of the entire postwar period and in alternative specifications of the growth model. We have also found that when we maximize the contribution of the investment wedge, the models display a consumption anomaly: they tend to produce much smaller declines in consumption during downturns than occur in the data. These findings together imply that existing models of financial frictions in which the distortions primarily manifest themselves as investment wedges can account, at best, for only a small fraction of the fluctuations in the Great Depression or more typical U.S. downturns. This finding is our primary substantive contribution.

We have seen that if adjustment costs are extreme, investment wedges can play a larger but still modest role over the business cycle. A combination of microeconomic and macroeconomic observations argue against extreme adjustment costs, and we view them as a theoretical curiosity of no applied interest.

A useful extension of our work here would be to decompose our wedges into a portion that comes from explicit taxes imposed by governments and a portion that comes from frictions in detailed models. This decomposition would be particularly useful when explicit taxes vary significantly over the business cycle.

Researchers have argued—quite beyond the prominent model of Bernanke and Gertler (1989)—that frictions in financial markets are important for business cycle fluctuations. (See Bernanke (1983) and the motivation in Bernanke and Gertler (1989).) We stress that our findings do not contradict this idea. Indeed, we have shown that a detailed economy with input-financing frictions is equivalent to a prototype economy with efficiency wedges. In this sense, although existing models of financial frictions are not promising, new models in which financial frictions show up as efficiency and labor wedges are.

Our results suggest that future theoretical work should focus on developing models that lead to fluctuations in efficiency and labor wedges. Many existing models produce fluctuations in labor wedges. The challenging task is to develop detailed models in which primitive shocks lead to fluctuations in efficiency wedges as well.

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APPENDIX: THE MAPPING FOR TWO OTHER WEDGES

Here we demonstrate the mapping from two other detailed economies with frictions to two prototype economies with wedges. In the preceding text, we described the mapping for efficiency and labor wedges. In this appendix, we describe it for investment and government consumption wedges.

A1. *Investment Wedges Due to Financial Frictions*

We start with the mapping of financial frictions to investment wedges.

In Chari, Kehoe, and McGrattan (2004), we showed the equivalence between the Carlstrom and Fuerst (1997) model and a prototype economy. Here we focus on the financial frictions in the Bernanke, Gertler, and Gilchrist (1999) model and abstract from the monetary features of that model. Bernanke, Gertler, and Gilchrist began by deriving the optimal financial contracts between risk-neutral entrepreneurs and financial intermediaries in an environment with no aggregate uncertainty. These contracts resemble debt contracts (with default). Bernanke, Gertler, and Gilchrist tried to extend their derivation of optimal contracts to an economy with aggregate uncertainty. The contracts they considered are not optimal given the environment, because these contracts do not allow risk sharing between risk-averse consumers and risk-neutral entrepreneurs. Here we solve for optimal debt-like contracts that arbitrarily rule out such risk sharing, but even so, our first-order conditions differ from those of Bernanke, Gertler, and Gilchrist. (One reason for this difference is that our break-even constraint for the financial intermediary and our law of motion for entrepreneurial net worth differ from theirs. See equations (35) and (37) below.)

A1.1. *A detailed economy with financial frictions*

The Bernanke–Gertler–Gilchrist model has a continuum of risk-neutral entrepreneurs of mass L_e, a continuum of consumers of mass 1, and a representative firm. Output $y(s^t)$ is produced according to

$$y(s^t) = A(s^t)k^\alpha[l^\Omega L_e^{1-\Omega}]^{1-\alpha}, \tag{34}$$

where $A(s^t)$, k, and l denote the technology shock, the capital stock, and the labor supplied by consumers. The stochastic process for the technology shock is given by $\log A(s^{t+1}) = (1-\rho_A)\log A + \rho_A \log A(s^t) + \varepsilon_A(s^{t+1})$. Each entrepreneur supplies one unit of labor inelastically. The representative firm's maximization problem is to choose l, L_e, and k to maximize profits $A(s^t)k^\alpha[l^\Omega L_e^{1-\Omega}]^{1-\alpha} - w(s^t)l - w_e(s^t)L_e - r(s^t)k$, where $w(s^t)$, $w_e(s^t)$, and $r(s^t)$ denote the wage rate for consumers, the wage rate for entrepreneurs, and the rental rate on capital.

New capital goods can be produced only by entrepreneurs. Each entrepreneur owns a technology that transforms output and old capital at the end of any period into capital goods at the beginning of the following period. In each period t, each entrepreneur receives an idiosyncratic shock ω drawn from a distribution $F(\omega)$ with expected value 1. This shock is independent and identically distributed across entrepreneurs and time. The realization of ω is private information to the entrepreneur. An entrepreneur who buys $k(s^{t-1})$ units of goods in period $t-1$ produces $\omega k(s^{t-1})$ units of capital at the beginning of period t. These capital goods are sold at price $R_k(s^t) = r(s^t) + (1-\delta)$, where δ is the rate of depreciation of the capital goods.

Entrepreneurs finance the production of new capital goods partly with their own net worth, $n(s^{t-1})$, and partly with loans from financial intermediaries. These intermediaries offer contracts with the following cutoff form: in all idiosyncratic states at t in which $\omega \geq \bar\omega(s^t)$, the entrepreneur pays $\bar\omega(s^t)R_k(s^t)k(s^{t-1})$ and keeps $[\omega - \bar\omega(s^t)]R_k(s^t)k(s^{t-1})$. In all other states, the entrepreneur receives nothing, while the financial intermediary receives $\omega R_k(s^t)k(s^{t-1})$ net of monitoring costs $\mu\omega R_k(s^t)k(s^{t-1})$. We assume that financial intermediaries make zero profit in equilibrium.

Given an entrepreneur's net worth $n(s^{t-1})$, the contracting problem for a representative entrepreneur is to choose the cutoff $\bar\omega(s^t)$ for each state and the amount of goods invested $k(s^{t-1})$ to maximize the expected utility of the entrepreneurs,

$$\sum \pi(s^t|s^{t-1})\big\{\big[1-\Gamma(\bar\omega(s^t))\big]R_k(s^t)k(s^{t-1})\,dF(\omega)\big\},$$

subject to a break-even constraint for the intermediary,

$$\sum (s^t|s^{t-1})\big[\Gamma(\bar\omega(s^t)) - \mu G(\bar\omega(s^t))\big]R_k(s^t)k(s^{t-1}) = k(s^{t-1}) - n(s^{t-1}), \tag{35}$$

where $\Gamma(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega)\,d\omega + \bar{\omega}\int_{\bar{\omega}}^{\infty} f(\omega)\,d\omega$ and $G(\bar{\omega}) = \int_0^{\bar{\omega}} \omega f(\omega)\,d\omega$. Here $q(s^t|s^{t-1})$ denotes the price of consumption goods in state s^t in units of consumption goods in state s^{t-1}.

This formulation implies an aggregation result: The capital demand of the entrepreneurs is linear in their net worth, so that total demand for goods from them depends only on their total net worth. The solution to the contracting problem is characterized by the first-order conditions with respect to $k(s^{t-1})$ and $\bar{\omega}(s^t)$, which can be summarized by

$$\begin{aligned}(36)\qquad &\frac{\sum \pi(s^t|s^{t-1})[1-\Gamma(\bar{\omega}(s^t))]R_k(s^t)}{\sum \pi(s^t|s^{t-1})\Gamma'(\bar{\omega}(s^t))R_k(s^t)} \\ &\quad = \frac{\sum q(s^t|s^{t-1})[\Gamma(\bar{\omega}(s^t)) - \mu G(\bar{\omega}(s^t))]R_k(s^t) - 1}{\sum q(s^t|s^{t-1})[\Gamma'(\bar{\omega}(s^t)) - \mu G'(\bar{\omega}(s^t))]R_k(s^t)}\end{aligned}$$

and the break-even constraint (35).

In each period, a fraction γ of existing entrepreneurs dies and is replaced by a fraction γ of newborn entrepreneurs. At the beginning of each period, entrepreneurs learn whether they will die this period. We assume that γ is sufficiently small and the technology for producing investment goods is sufficiently productive so that entrepreneurs consume their net worth only when they are about to die. All entrepreneurs who are going to die consume their entire net worth and do not supply labor in the current period. (The newborn replacements do instead.) Those entrepreneurs who do not die save their wage income plus their income from producing capital goods. The aggregate income of the entrepreneurs is, then,

$$\int [\omega - \bar{\omega}(s^t)]R_k(s^t)k(s^{t-1})\,dF(\omega) + w_e(s^t)L_e,$$

where $k(s^{t-1})$ is the aggregate capital stock. The law of motion for aggregate net worth is given by

$$(37)\qquad n(s^t) = \gamma \int [\omega - \bar{\omega}(s^t)]R_k(s^t)k(s^{t-1})\,dF(\omega) + w_e(s^t)L_e.$$

Total consumption by entrepreneurs in any period is

$$(38)\qquad c_e(s^t) = (1-\gamma)\int [\omega - \bar{\omega}(s^t)]R_k(s^t)k(s^{t-1})\,dF(\omega).$$

Consumers maximize utility given by

$$(39)\qquad \sum_{t=0}^{\infty}\sum_{s^t} \beta^t \pi_t(s^t)U(c(s^t), l(s^t)),$$

where $c(s^t)$ denotes consumption in state s^t subject to

$$c(s^t) + \sum_{s_{t+1}} q(s^{t+1}|s^t)b(s^{t+1}) \leq w(s^t)l(s^t) + b(s^t) + T(s^t) \tag{40}$$

for $t = 0, 1, \ldots$ and to borrowing constraints, $b(s^{t+1}) \geq \bar{b}$, for some large negative number $\bar{b}$. Here $T(s^t)$ is lump-sum transfers. The initial condition $b(s^0)$ is given. Each of the bonds $b(s^{t+1})$ is a claim to one unit of consumption in state s^{t+1} and costs $q(s^{t+1}|s^t)$ dollars in state s^t. The first-order conditions for the consumer can be written as

$$-\frac{U_l(s^t)}{U_c(s^t)} = w(s^t) \tag{41}$$

and

$$q(s^{t+1}|s^t) = \beta\pi(s^{t+1}|s^t)\frac{U_c(s^{t+1})}{U_c(s^t)}. \tag{42}$$

The market-clearing condition for final goods is then

$$\begin{aligned} &c(s^t) + c_e(s^t) + \mu G(\bar{\omega}(s^t))R_k(s^t)k(s^{t-1}) + k(s^t) - (1-\delta)k(s^{t-1}) \\ &\quad = y(s^t). \end{aligned}$$

A1.2. *The associated prototype economy with investment wedges*

Now consider a prototype economy that is the same as our benchmark prototype economy except for the following changes. We assume that the production function is as in (34), with L_e interpreted as a fixed input rented from the government, and we assume that consumers are taxed on capital income but not on investment or labor income. With capital income taxes, the consumers' budget constraint is given by

$$\begin{aligned} &c(s^t) + k(s^t) - (1-\delta)k(s^{t-1}) \\ &\quad = w(s^t)l(s^t) + [1-\tau_k(s^t)]r(s^t)k(s^{t-1}) + \delta\tau_k(s^t)k(s^{t-1}) + T(s^t), \end{aligned}$$

where $\tau_k(s^t)$ is the tax rate on capital income. Here $\tau_k(s^t)$ plays the role of an investment wedge. The resource constraint for the prototype economy is as in the benchmark prototype economy. Let government consumption in the prototype economy be given by

$$g(s^t) = c_e^*(s^t) + \mu G(\bar{\omega}^*(s^t))R_k^*(s^t)k^*(s^{t-1}), \tag{43}$$

where asterisks denote the allocations in the detailed economy with investment frictions.

We now show how the tax on capital income in our prototype economy can be constructed from the detailed economy. First, the break-even constraint in the detailed economy can be rewritten as

$$\sum q^*(s^t|s^{t-1})\Big\{\big[\Gamma(\bar{\omega}^*(s^t)) - \mu G(\bar{\omega}^*(s^t))\big]R_k^*(s^t) + \frac{n^*(s^{t-1})}{k^*(s^{t-1})}\frac{U_c^*(s^{t-1})}{\beta\pi(s^t|s^{t-1})U_c^*(s^t)}\Big\} = 1,$$

where $q^*(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1})U_c^*(s^t)/U_c^*(s^{t-1})$. The intertemporal Euler equation in the prototype economy is

$$\sum q(s^t|s^{t-1})\big\{[1-\tau_k(s^t)]A(s^t)F_k(s^t) + (1-\delta)\big\} = 1,$$

where $q(s^t|s^{t-1}) = \beta\pi(s^t|s^{t-1})U_c(s^t)/U_c(s^{t-1})$. Let the tax rate on capital income $\tau_k(s^t)$ be such that

$$\begin{aligned}(44)\qquad &[1-\tau_k(s^t)][A(s^t)F_k^*(s^t) - \delta] + 1 \\ &\quad = \big[\Gamma(\bar{\omega}^*(s^t)) - \mu G(\bar{\omega}^*(s^t))\big]R_k^*(s^t) + \frac{n^*(s^{t-1})}{k^*(s^{t-1})}\frac{U_c^*(s^{t-1})}{\beta\pi(s^t|s^{t-1})U_c^*(s^t)}.\end{aligned}$$

Comparing first-order conditions for the two economies, we then have the next proposition:

PROPOSITION 6: *Consider the prototype economy just described, with government consumption given by* (43) *and capital income taxes given by* (44). *The aggregate equilibrium allocations for this prototype economy coincide with those of the detailed economy with financial frictions.*

A2. *Government Consumption Wedges Due to International Borrowing and Lending*

Now we develop a detailed economy with international borrowing and lending, and show that net exports in that economy are equivalent to a government consumption wedge in an associated prototype economy.

A2.1. *A detailed economy with international borrowing and lending*

Consider a model of a world economy with N countries and a single homogeneous good in each period. We use the same notation for uncertainty as before.

The representative consumer in country i has preferences

$$(45)\qquad \sum \beta^t\pi_t(s^t)U(c_i(s^t), l_i(s^t)),$$

where $c_i(s^t)$ and $l_i(s^t)$ denote consumption and labor. The consumer's budget constraint is

$$c_i(s^t) + b_i(s^t) + k_i(s^t) \leq F(k_i(s^{t-1}), l_i(s^t)) + (1-\delta)k_i(s^{t-1}) + \sum_{s^{t+1}} q(s^{t+1}/s^t)b_i(s^{t+1}), \tag{46}$$

where $b^i(s^{t+1})$ denotes the amount of state-contingent borrowing by the consumer in country i in period t, $q(s^{t+1}/s^t)$ denotes the corresponding state-contingent price, and $k_i(s^t)$ denotes the capital stock.

An *equilibrium for this detailed economy* is a set of allocations $(c_i(s^t), k_i(s^t), l_i(s^t), b_i(s^{t+1}))$ and prices $q(s^t/s^{t-1})$ such that these allocations both solve the consumer's problem in each country i and satisfy the world resource constraint:

$$\sum_{i=1}^{N}[c_i(s^t) + k_i(s^t)] \leq \sum_{i=1}^{N}\big[F(k_i(s^{t-1}), l_i(s^t)) + (1-\delta)k_i(s^{t-1})\big]. \tag{47}$$

Note that in this economy, the net exports of country i are given by

$$F(k_i(s^{t-1}), l_i(s^t)) - [k_i(s^t) - (1-\delta)k_i(s^{t-1})] - c_i(s^t).$$

A2.2. *The associated prototype economy with government consumption wedges*

Now consider a prototype economy of a single closed economy i with an exogenous stochastic variable, government consumption $g_i(s^t)$, which we call the *government consumption wedge*. In this economy, consumers maximize (45) subject to their budget constraint

$$c_i(s^t) + k_i(s^t) = w_i(s^t)l_i(s^t) + [r_i(s^t) + 1 - \delta]k_i(s^{t-1}) + T_i(s^t), \tag{48}$$

where $w_i(s^t)$, $r_i(s^t)$, and $T_i(s^t)$ are the wage rate, the capital rental rate, and lump-sum transfers. In each state s^t, firms choose k and l to maximize $F(k,l) - r_i(s^t)k - w_i(s^t)l$. The government's budget constraint is

$$g_i(s^t) + T_i(s^t) = 0. \tag{49}$$

The resource constraint for this economy is

$$c_i(s^t) + g_i(s^t) + k_i(s^t) = F(k_i(s^{t-1}), l_i(s^t)) + (1-\delta)k_i(s^{t-1}). \tag{50}$$

An *equilibrium of the prototype economy* is, then, a set of allocations $(c_i(s^t), k_i(s^t), l_i(s^t), g_i(s^t), T_i(s^t))$ and prices $(w_i(s^t), r_i(s^t))$ such that these allocations are optimal for consumers and firms, and the resource constraint is satisfied.

The following proposition shows that the government consumption wedge in this prototype economy consists of net exports in the original economy.

PROPOSITION 7: *Consider the equilibrium allocations* $(c_i^*(s^t), k_i^*(s^t), l_i^*(s^t), b_i^*(s^{t+1}))$ *for country* i *in the detailed economy. Let the government consumption wedge be*

$$g_i(s^t) = F(k_i^*(s^{t-1}), l_i^*(s^t)) - [k_i^*(s^t) - (1-\delta)k_i^*(s^{t-1})] - c_i^*(s^t), \tag{51}$$

let the wage and capital rental rates be $w_i(s^t) = F_{li}^*(s^t)$ *and* $r_i(s^t) = F_{ki}^*(s^t)$, *and let* $T_i(s^t)$ *be defined by* (49). *Then the allocations* $(c_i^*(s^t), k_i^*(s^t), l_i^*(s^t), g_i^*(s^t), T_i(s^t))$ *and the prices* $(w_i(s^t), r_i(s^t))$ *are an equilibrium for the prototype economy.*

The proof follows by noting that the first-order conditions are the same in the two economies and that, given the government consumption wedge (51), the consumer's budget constraint (46) in the detailed economy is equivalent to the resource constraint (50) in the prototype economy.

Note that for simplicity we have abstracted from government consumption in the detailed economy. If that economy had government consumption as well, then the government consumption wedge in the prototype economy would be the sum of net exports and government consumption in the detailed economy.

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