

Statistical methods for detecting admixture

Pongsakorn Wangkumhang & Garrett Hellenthal

University College London Genetics Institute (UGI), Department of Genetics, Evolution and Environment, University College London, London, United Kingdom

The increasing availability of large-scale autosomal genetic variation data sampled from world-wide geographic areas, coupled with advances in the statistical methodology to analyse these data, is showcasing the power of DNA as a major tool to gain insights into the demographic history of humans and other organisms. Here we review statistical techniques that shed light on a specific aspect of demography: the detection and description of admixture events where two or more genetically distinct groups intermixed at one or more times in the past. In particular we give an overview of some of the widely-used methods to identify and describe admixture events using autosomal DNA from unrelated individuals, with a particular focus on analysing biallelic Single-Nucleotide-Polymorphisms (SNP) markers.

While Y-chromosome and mitochondrial (mtDNA) have been extremely valuable in studying sex-biased admixture, where one of the admixing groups contributes a disproportionate number of males or females [1], current widely-used approaches focus on analysing autosomal DNA as it contains many thousands of times more (sex-averaged) information than Y/mtDNA. As examples, we apply some of these approaches to two sets of simulated data from [2] containing 474,491 autosomal SNPs (Figure 1). Each set consists of 20 simulated individuals, descending from a pulse of admixture between two sources occurring 30 generations ago. These sources are African and European in the first set, contributing $\approx 80\%$ and $\approx 20\%$ of the DNA, respectively, while the sources are Central-South-Asian and European in the second set, each contributing $\approx 50\%$ of the DNA. Following the procedure in [3], these simulations used DNA from 21 present-day Yoruban individuals from Nigeria, 21 present-day Brahui individuals from Pakistan and 28 present-day individuals from France as the African, Central-South-Asian and European admixing sources, respectively. In analyses below, we also include surrogate individuals representing each of these continental admixing sources, in particular using the genomes of 22 Mandenka from Africa, 21 Balochi from Central South Asia, and 23 British/Irish from Europe.

Signatures of admixture

Two of the most widely-used techniques for describing population structure and demography are principal components analysis (PCA) and clustering algorithms. While the genetic patterns captured by each approach may be attributable to factors other than admixture, they can each highlight individuals who descend from multiple intermixing source groups.

Data dimension reduction techniques

PCA is an algebraic technique that applies eigenvector decomposition to reduce high-dimensional data to a small number of independent variables, termed principal components, that explain a large proportion of variation in the complete data. Typically PCA is applied to the genome-wide genotype data of multiple individuals, and the top few principal components are plotted to visualize genetic distance among the individuals [4, 5, 6, 7, 8, 9]. For example, Figure 1b plots the first two principal components from the PCA program EIGENSTRAT [6] applied to our simulated data and the surrogate individuals. Note that surrogate individuals from the three continental groups fall into different corners of the plot, while each set of admixed individuals

fall between the two continental groups comprising their ancestry in a manner roughly consistent with admixture proportions. However, caution is strongly warranted against concluding admixture wherever such a pattern is observed, as PCA projections can depend on sample size, SNP ascertainment, and features of demography besides admixture [9, 10].

To better interpret these patterns, recent approaches have related these concepts to isolation-by-distance models that assume genetic similarity decreases with geographic distance between samples. For example, SpaceMix [11] highlights populations whose allele frequencies are more correlated than expected when assuming genetic similarity decays exponentially with distance, while EEMS [12] relates genetic similarity to the distance among individuals' geographic locations (e.g. birthplace or sampling information) assuming migration occurs between neighboring demes via a stepping stone model [13]. Each approach then attempts to identify genetic associations beyond that explained by these simple distance models, which may indicate long-distance admixture.

Clustering algorithms

Clustering algorithms attempt to classify individuals into a discrete number of groups based on associations among their genetic patterns. Popular algorithms for doing so include STRUCTURE [14], ADMIXTURE [15], FRAPPE [16] and related models [17, 18]. While these techniques assume loci are unlinked, STRUCTURE version 2.0 [19] models linkage among loci caused by the inheritance of blocks of DNA from different admixing sources (as in Fig 1a), while still ignoring correlations among tightly linked loci (though see fineSTRUCTURE [20]). Typically users select a number of clusters K (though K can also be estimated, e.g. [17, 20]), and many of these programs then determine the proportion of each individual's DNA derived from K inferred clusters. Therefore, it can be tempting to interpret the K inferred clusters as K ancestral source groups that potentially intermixed, so that individuals whose DNA is assigned to multiple clusters descend from historical admixture events. Under this interpretation, these programs also estimate the proportion of DNA contributed by each source. Figure 1c provides results from ADMIXTURE applied to the simulated and surrogate data using $K = 2 - 5$. Here ADMIXTURE accurately describes sources and proportions of admixture in the two simulated groups at $K = 3$, but does not always characterize this admixture well at other values of K , with cross-validation choosing $K = 2$ as the best-fitting value. Therefore, as in PCA, multiple disparate demographic histories can lead to identical clustering [14, 21], warranting caution against concluding admixture when individuals are classified as mixtures of different clusters or – conversely – concluding no admixture when individuals are assigned to a single cluster.

Identifying admixture

f statistics for admixture testing

If population C descends from an admixture event between two populations A and B , the allele frequency p_C of a variant at a locus (e.g. a SNP) in C typically should fall between the frequencies p_A, p_B of that variant in populations A and B , respectively. Therefore, to test for admixture, Reich et al [22] propose the *f*3 statistic, defined as an unbiased estimator for $(p_C - p_A)(p_C - p_B)$ averaged across all loci. This statistic is significantly negative if population C descends from an admixture event between two sources related to A and B , i.e. if p_C falls between p_A and p_B , with significance assessed using jack-knife re-sampling based on dividing the genome into independent regions (e.g. chromosomes). Applying ADMIXTOOLS to calculate *f*3 statistics for the two simulated populations of Figure 1 using the given surrogates finds strong evidence of admixture in both (Z -scores < -30). However, the authors note that the *f*3 statistic

may no longer be negative if C has experienced a high degree of drift following admixture, e.g. due to a bottleneck event, as this can cause p_C to no longer often fall between p_A and p_B [23]. A related test is the f_4 statistic, which estimates the average of $(p_A - p_B)(p_C - p_D)$ across loci using the sample allele frequencies from four populations A - D [22] to formally test whether (A, B) and (C, D) form clades. If the f_4 statistic is significantly positive (respectively negative), then A (respectively B) is more related ancestrally to C and/or B (respectively A) is more related to D than expected, possibly indicating admixture or incorrect clades. Ratios of f_4 statistics can also be used to infer proportions of admixture from each source, assuming the tree topology among populations is known [22, 23].

Tree-based inference

Expanding upon these ideas, a topology detailing the order of splits among a set of n populations can be constructed based on correlations among those populations' allele frequencies. For example, qpGraph [22, 23] scores how well observed f statistics fit those predicted by a user-specified bifurcating tree relating the n populations, which may contain multiple migration events. The authors caution that n should be small in practice. MixMapper [24] extends qpGraph to a larger number of populations, by first inferring which populations are unadmixed using f_3 statistics, then building a bifurcating tree using these putatively unadmixed populations, and finally adding admixed populations onto this tree. TREEMIX [25] is a related model that builds bifurcating trees relating a large number of groups and then adds links between branches to indicate admixture. TREEMIX uses a multivariate normal distribution to relate observed allele frequencies among populations, incorporating work by [26, 27], with the mean and covariance of this distribution defined by the (unknown) tree branch lengths (measuring drift) and admixture links among populations. Figure 1d shows the inferred tree topology and admixture events when applying TREEMIX to the simulated data and surrogate individuals, correctly characterising the admixture in both cases. While useful for describing relations among populations and unearthing migration events, a major limitation of these approaches is that the unknown complexity of genetic relationships among large numbers of populations necessitates exploring an enormous search space encompassing the possible tree topologies and potential migration events. Therefore, while these approaches can provide solutions that fit patterns in allele frequency correlation across populations, it is difficult to assess the extent to which other equally-likely solutions exist [28].

Dating admixture

The block-like manner in which autosomal DNA is inherited potentially enables identifying when admixture occurred. For convenience, many approaches assume two or more ancestral sources intermixed over a short time interval (e.g. one generation), often referred to as a “pulse” of admixture, after which individuals in the admixed population randomly mate for subsequent generations. In this setting, within the admixed genomes r generations after this admixture event, the probability that two loci were inherited from the same ancestor at the time of admixture decays exponentially with rate r per Morgan [19]. Thus fitting an exponential rate of decay to the association between loci attributable to admixture can be used to determine r . Violations of this simplistic model, e.g. continuous migration, can be diagnosed by studying whether a single exponential decay function is not a good fit to the data or by mathematically formulating the expected pattern of decay under more complex models. In contrast to e.g. f_3 statistics, typically only admixture within a few hundred generations can be detected using these approaches, since beyond this segment sizes will have decreased to levels not distinguishable

from levels of noise. Another potential issue is how well an exponential function captures certain admixture scenarios [29].

Techniques to infer local ancestry tracts

In theory, within each admixed individual contiguous segments (or “tracts”) of DNA inherited from the original admixing sources can be identified by matching the admixed individual’s genetic variation patterns to that of a set of surrogates meant to represent the source groups (Figure 1a). The sizes of these inferred tracts are then compared to those expected under different models of migration, to determine the best-fitting date(s) of admixture and the proportions of ancestry inherited from each admixing source [30, 31]. Several different algorithms have been proposed to do this matching (e.g. [32, 33, 3, 34, 35, 36, 37, 38, 39, 40]) that have different advantages, such as the number of putative admixing sources allowed, whether the data must be pre-phased, whether tightly linked haplotype information is directly modeled, etc, with the details of many summarized in [38].

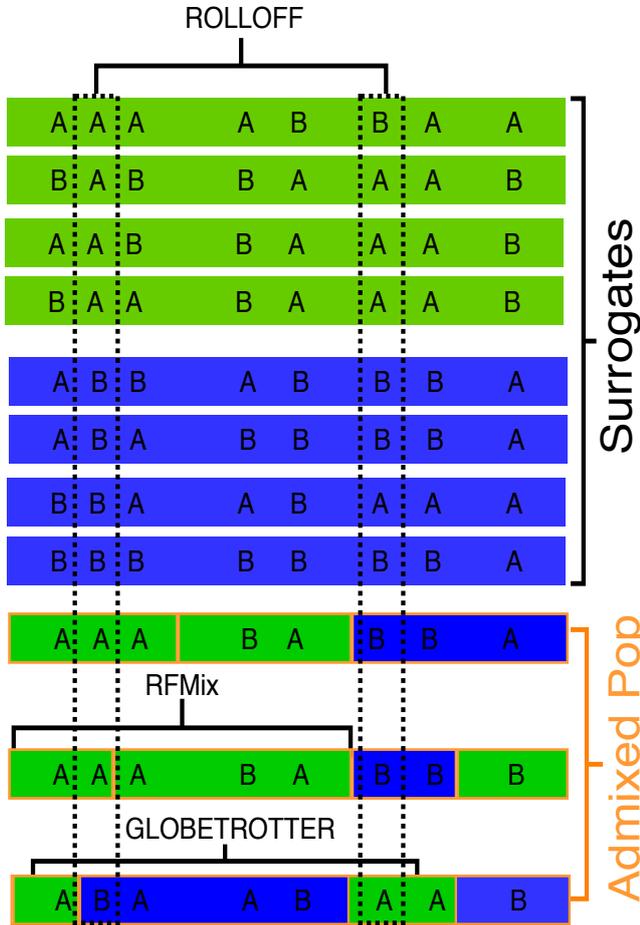
As admixing sources must be distinguishable using small numbers of SNPs in local region, these approaches are only applicable to recent admixture between genetically diverged sources, such as Latin and African American human data, where tracts are both longer and easier to classify. As an illustration of this, Figure 1e,h summarises results from applying RFMix [39], which uses linear discriminant analysis to match segments in admixed individuals to those from reference populations, to the simulated and surrogate data from [2]. There is much better agreement between the expected exponential decay and the observed tract length distribution inferred by RFMix for the Yoruba-French simulation relative to the French-Brahui simulation, because the two admixing sources are considerably more genetically similar in the latter.

Techniques using linkage disequilibrium decay patterns

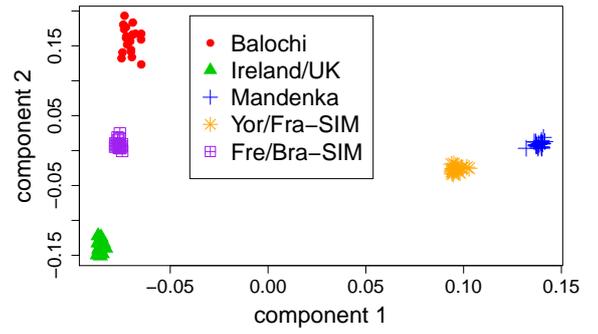
Other approaches avoid directly inferring local tracts of DNA by instead measuring how the association among loci, i.e. linkage disequilibrium (LD), attributable to admixture decays with increasing genetic distance. For example, ROLLOFF [41, 42, 23] and ALDER [43] measure the decay in LD versus genetic distance among pairs of biallelic loci (e.g. SNPs), weighted by each locus’s ability to distinguish between surrogates of the original admixing populations. The weighted covariance (or correlation) between pairs of SNPs in the admixed populations should decay at rate r per Morgan, and the amplitude of these decay curves can be used to infer the proportion of admixture [43]. To potentially increase power, GLOBETROTTER [2] instead measures the association among pairs of haplotype segments in the admixed individuals, rather than pairs of SNPs. Here each haplotype segment is defined by the surrogate population it is most closely related to ancestrally, as inferred using CHROMOPAINTER [20] on pre-phased data. ROLLOFF and ALDER infer the best surrogates for each source by finding the best model fit out of all possible pairings of available surrogates, while GLOBETROTTER infers the genetic make-up of each source as a mixture of DNA from all potential surrogate groups (i.e. without requiring pre-specification of surrogates). Both can identify multiple admixture events at different times [44, 2], and GLOBETROTTER can also infer whether >2 sources intermixed at approximately the same time.

Typically with these techniques, inferring proportions of admixture from each source is more challenging than inferring dates of admixture, with the former often suffering from a lack of identifiability (e.g. surrogate populations may themselves be admixed to an unknown degree). Furthermore, in the cases of continuous admixture or migration at several different times, inferred dates from these approaches may be biased towards the most recent date [43, 2]. Nonetheless, decay of LD due to admixture can still be usefully modeled by these approaches in

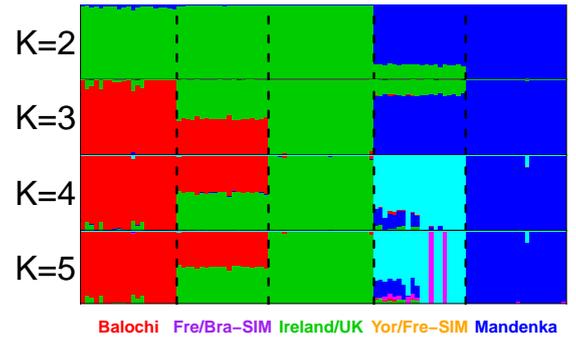
far more subtle cases relative to approaches that directly identify tracts. For example, note the considerably more accurate modeling of the true date in the French-Brahui simulation (Figure 1h-j), with GLOBETROTTER showing more precision than ROLLOFF in these simulations.



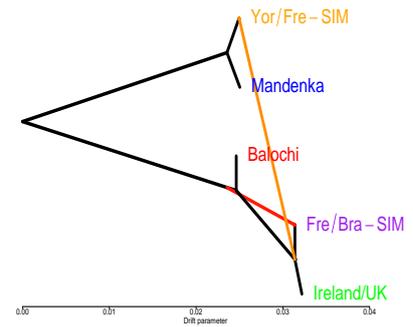
(a) admixture inference schematic



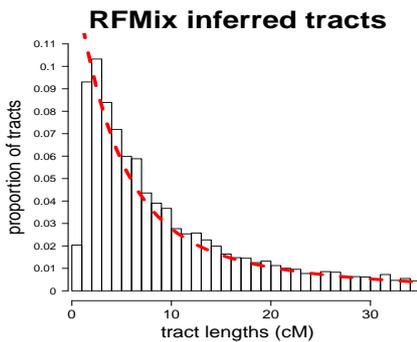
(b) EIGENSTRAT (PCA)



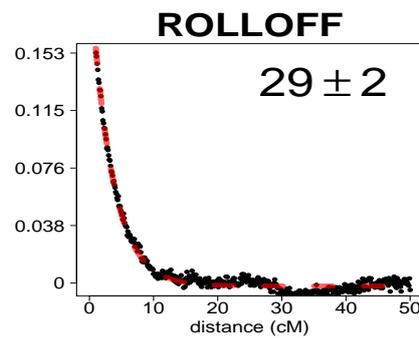
(c) ADMIXTURE



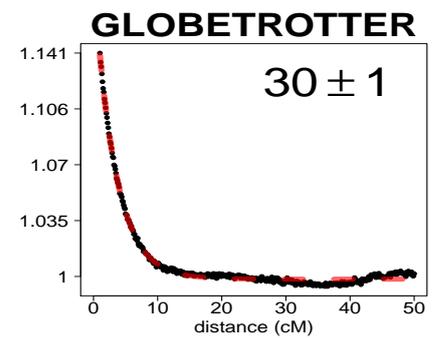
(d) TREEMIX



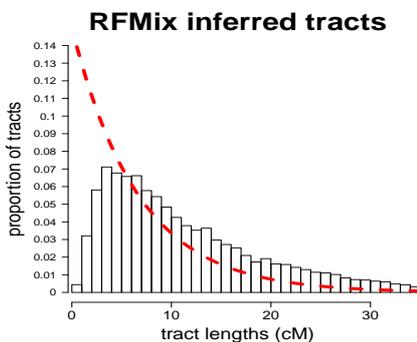
(e) Yoruba 80% / French 20%



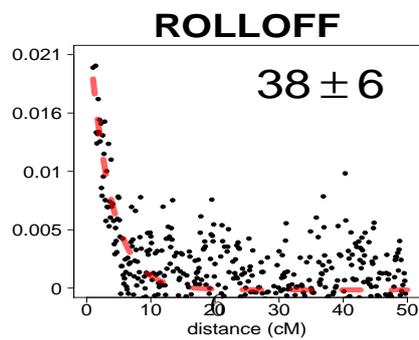
(f) Yoruba 80% / French 20%



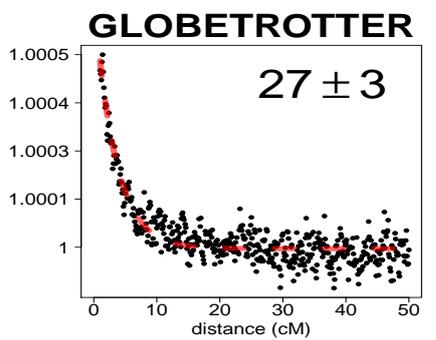
(g) Yoruba 80% / French 20%



(h) Brahui 50% / French 50%



(i) Brahui 50% / French 50%



(j) Brahui 50% / French 50%

Figure 1. Identifying admixture in simulated data from [2]. (a) Populations represented by blue/green surrogate groups mix r generations in the past, generating admixed haplotypes at bottom. A and B denote allele types at each biallelic SNP; orange bars separate segments in admixed chromosomes that exactly match a region of a depicted surrogate haplotype. (b) EIGENSTRAT (each dot an individual), (c) ADMIXTURE for cluster numbers $K = 2 - 5$ (columns = individuals, colors = clusters) and (d) TREEMIX (black lines = inferred topology; red/orange lines = migration edges) applied to allele frequency data from individuals simulated as mixtures of 50% French + 50% Brahui (“Fre/Bra-SIM”) or 80% Yoruba + 20% French (“Yor/Fre-SIM”), also incorporating (Balochi,Ireland/UK,Mandenka) as surrogates for (Brahui,French,Yoruba), respectively. (e)-(j) RFMix, ROLLOFF, and GLOBETROTTER applied to date admixture in these simulations, with each approach intuitively explained in (a) as measuring (e,h) the distribution of tract lengths matched to the same surrogate group, or the decay of LD (black dots) versus genetic distance between (f,i) SNPs differentiated between the two surrogates or (g,j) haplotype blocks matched to a surrogate (Ireland). Dashed red lines give the expected decay for the true proportions and true admixture date of 30 generations ago, and numbers in the top right of (f,g,i,j) give inferred dates and standard errors.

Conclusion and perspectives

Statistical advances have rapidly increased the amount of admixture information we can extract out of large-scale autosomal variation data, with admixture events within the last 4,000 years now characterized in most global populations [43, 2]. However, one notable limitation is that most approaches rely on using surrogates for the original (unknown) admixing sources, and it is unclear how accurate these surrogates may be. For example, often modern-day samples are used as surrogates despite themselves having recent admixture from other sources. While the rapid increase of reliable DNA extractions from ancient human remains, i.e. aDNA (e.g. [45]), eventually may provide more accurate surrogates to the original (possibly unadmixed) sources, it is unclear from their current sparsity whether enough representative aDNA can be obtained for many admixture events of interest. Such aDNA also opens the possibility for unearthing details of admixture further back in time, e.g. dating admixture within aDNA samples [46]. In addition, the increasing availability of large-scale genetic variation resources from modern individuals will enable more precise inference, such as better classification of continuous versus pulses of admixture and better ability to detect subtle admixture between genetically similar sources [47]. Furthermore forthcoming large-scale sequencing datasets that capture rare genetic variants will enable better identification of recent shared ancestry among individuals.

Finally, since the study of demographic processes such as admixture affects the whole genomes of individuals, one promising avenue of future work is using these detailed inferences as a null model when testing specific regions of the genome for evidence of selection. Recent examples include such extensions for PCA approaches [48] and for MixMapper [49], as well as using local admixture tract inference approaches to identify whether admixed populations contain more ancestry from particular sources than average in specific genomic regions [50, 40]. In this way these models not only enable understanding of our history, but may also point towards genetic regions facilitating our adaptation to disease pressures in a complimentary manner to on-going genome-wide association studies.

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