Essays on Macroeconomics and Firm Financing

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Declaration

I, Benjamin Ming Kit Hemingway, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

Abstract

This thesis consists of three chapters on firm financing and how issues related to firm financing may impact on the macroeconomy.

In the second chapter, the role of collateral in debt contracts is explored within an environment where banks also face regulatory solvency constraints. I model a credit market with imperfect information and aggregate uncertainty. Here collateral plays a dual role. First, it can help mitigate the adverse selection problem by acting as a screening device. Second it also helps the bank satisfy any regulatory constraint by reducing the loss given default that the bank suffers in bad aggregate states. As the regulatory constraint becomes more strict, collateral may become less effective as a screening device, highlighted by the possibility of pooling equilibria existing.

The third chapter builds a model of SME loan applications that is consistent with existing survey data. Specifically, it captures several observable features of the loan market. By explicitly modelling the loan application phase, I am able to justify why firms apply for loans and are still subsequently rejected. This chapter also provides a theoretical contribution in that there is the possibility, in a model without asymmetric information, of 'pure credit rationing' where observationally equivalent firms are granted a loan with while others are not.

The fourth chapter, investigates how creditor and debtor rights in the case of firm insolvency impact on the equilibrium outcomes in a firm dynamics model. Two insolvency regimes are compared, a creditor-friendly regime such as the UK and a debtor-friendly regime such as the US. Debtor-friendly regimes are shown to be more costly in the steady-state, leading to larger spreads on firm debt. The model dynamics find a response to productivity shocks that are largely consistent with the UK and the US following the financial crisis.

Impact Statement

A key topic for policy makers, especially since the Financial Crisis of 2007-2008, is firm financing; questions include whether firms have sufficient access to finance, what determines how and to what extent firms are financed and what happens if firms cannot repay their debt. These questions are of interest to policy makers in and of themselves but of greater importance is the link between firm financing and the performance of the economy as a whole. This thesis address some key issues of firm financing that are of particular importance to policy makers and contributes to the academic debate in these areas.

Chapter 2, titled *Banking regulation and collateral screening in a model of information asymmetry*, explores the role of collateral in debt contracts within an environment where banks also face regulatory solvency constraints. This is especially relevant for policy makers as following the Financial Crisis, there has been an increased focus on bank regulation. In particular, stress-tests such as the Supervisory Capital Assessment Program (SCAP) and the subsequent Comprehensive Capital Analysis and Review (CCAR) in the US and similar programs in other countries, have been found to negatively impact lending to firms by Acharya et al. (2018) and others. Chapter 2 complements this literature through a theoretical adverse selection model that proves the existence of pooling equilibria should banking regulation become sufficiently strict.

Chapter 3, titled *A Model of Credit Rationing in SME Loan Applications*, addresses the determinants of a firm's access to finance. Existing models are inconsistent with survey data that distinguishes between applications for bank loans and the outcome of those loan applications. In the data, we observe firms that choose not to apply for loans because they think they will be rejected, while other firms apply, but are still rejected outright. By explicitly modelling the loan application phase and the loan decision phase, I am able to capture the key features of the data.

Chapter 4, titled *Macroeconomic implications of insolvency regimes*, addresses the question of what happens when a firm defaults on its loan obligations and to what extent this matters. Specifically, I investigate how creditor and debtor rights in the case of firm insolvency impact on the equilibrium outcomes. In particular, in an application to the financial crisis, I show that labour productivity falls more sharply in the creditor-friendly regime while employment does not. In addition, this chapter suggests a possible explanation for the different employment and labour productivity response in the UK and US since the financial crisis.

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Chapter 1

Introduction

What determines whether firms get sufficient financing and how does the availability of financing impact firm decision making? Since the Financial Crisis of 2007-2008, these questions have become increasingly important to the understanding of the economy. This thesis consists of three chapters on firm financing and how issues related to firm financing may impact on the macroeconomy.

In Chapter 2, titled *Banking regulation and collateral screening in a model of information asymmetry*, the role of collateral in debt contracts is explored within an environment where banks also face regulatory solvency constraints. Recently, regulators and policy makers have increased their focus on ensuring stability in the banking sector. Stress-tests such as the US Supervisory Capital Assessment Program (SCAP), are one tool which regulators can use to ensure bank solvency. I model a credit market with imperfect information and aggregate uncertainty. Here collateral plays a dual role. First, it can help mitigate the adverse selection problem by acting as a screening device. Second it also helps the bank satisfy any regulatory constraint by reducing the loss given default that the bank suffers in bad states of the world. As the regulatory constraint becomes more strict, collateral may become less effective as a screening device, highlighted by the possibility of pooling equilibria existing.

Chapter 3, titled *A Model of Credit Rationing in SME Loan Applications*, is concerned with building a modelling framework for the study of SME loan applications that is consistent with existing survey data. Specifically, it aims to capture several observable features of the loan market. First, firms choose whether they apply for a loan, and firms that do not apply for loans may not need a loan, or may think they will not obtain a loan. Second, a firm's loan application may not be successful. A firm may receive only part of the funding it had requested, or it may have its loan application outright. By explicitly modelling the loan application phase, I am able to justify why firms apply for loans and are still subsequently rejected. This chapter also provides a theoretical contribution in that there is the possibility, in a model without asymmetric information, of 'pure credit rationing' where observationally equivalent firms are granted a loan with while others are not. This result is made possible because of the additional loan application phase.

Chapter 4, titled *Macroeconomic implications of insolvency regimes*, investigates how creditor and debtor rights in the case of firm insolvency impact on the equilibrium outcomes in a firm dynamics model. I build a heterogeneous firm model with financial frictions where defaulting firms can enter insolvency and continue production or be liquidated and exit. Financial frictions impact firm production decisions and make capital relatively more costly than labour for borrowing constrained firms. As a result, financially constrained firms are less capital intensive and have a lower capital-to-labour ratio than unconstrained firms. Two insolvency regimes are compared, a creditor-friendly regime such as the UK and a debtor-friendly regime such as the US. Debtor-friendly regimes are shown to be more costly in the steady-state, leading to larger spreads on firm debt. The model dynamics find a response to productivity shocks that are largely consistent with the UK and the US following the financial crisis. I show that the model provides a precise account for the differential effects of productivity shocks across economies that differ in the credit/debtor rights. In particular, in an application to the financial crisis, I show that labour productivity falls more sharply in the creditor-friendly regime while employment does not. This paper suggests a possible explanation for the different employment and labour productivity response in the UK and US since the financial crisis.

Chapter 2

Banking regulation and collateral screening in a model of information asymmetry

2.1 Introduction

Following the Financial Crisis of 2007-2008, regulators and policy makers have increased their focus on ensuring stability in the banking sector. One key tool at the regulator's disposal is stress-testing, which has become more widely used by regulators since the financial crisis. The US led the way on stress-testing, with one of the first post-crisis stress-tests being the US Supervisory Capital Assessment Program (SCAP) which was conducted by the Federal Reserve early in 2009. The SCAP and its successor, the Comprehensive Capital Analysis and Review (CCAR) aim to ensure that the largest US banks have sufficient capital to survive the stress scenario. One important innovation of the SCAP and CCAR is that the results were publicly disclosed on a bank-by-bank basis. This creates a strong incentive for the banks to have sufficient policies and capital in place to ensure they are able to pass the stress-test.

There is some evidence to suggest that the use of these stress-tests by the Federal Reserve, and similar programs used by other banking regulators may have negative consequences. For example, Acharya et al. (2018), focussing on lending to large firms in the US, find that stress-tested banks reduce the quantity of loans supplied to firms and increase borrowing rates. Similarly, Cortés et al. (2018) document negative effects of stress-testing on small business loans. Specifically, they provide evidence that stress-tests conducted under the CCAR led to a decrease in affected banks' credit supply to small business. An overview of the recent history of stress-testing in the financial sector can be found in Dent et al. (2016).

In this paper, I propose an adverse selection credit market model with aggregate uncertainty where collateral plays a dual role. The banking sector features perfect competition and a bank earning zero expected profit in the presence of aggregate uncertainty makes a loss in bad aggregate states and a positive profit in the good aggregate state. This suggests a role for banking regulation to ensure that bank losses in bad aggregate states are not large enough to threaten the financial stability of the economy. To model this, I propose a reduced form version of a stress-test where firms face a regulatory constraint on the losses they are permitted to make on loans in the bad aggregate state.

Collateral in this model plays two roles. First, it can help mitigate the adverse selection problem by acting as a screening device. Second, it helps the bank satisfy the regulatory constraint by reducing the loss given default that the bank suffers in bad states of the world.

This paper is directly related to the literature on adverse selection in credit markets. Papers that focus on the use of collateral as a screening device in credit markets featuring adverse selection include papers such as Besanko and Thakor (1987) and Lacker (2001). The possibility of pooling and credit rationing equilibria were raised by Stiglitz and Weiss (1981) and Bester (1987) amongst others. This paper contributes to this literature by studying the interaction between the regulatory constraint and the adverse selection problem. This paper also complements the empirical literature on the impact of regulatory stress-testing on banks such as Acharya et al. (2018) and Cortés et al. (2018) by providing a theoretical mechanism through which more stringent stress-testing can impact lending outcomes.

2.2 Model

2.2.1 Environment

Firms are risk neutral and live for one period. They have access to a risky technology and are heterogeneous in the riskiness of their technology. There are two discrete firm types indexed by $i \in \{L, H\}$ with H-type firms featuring a higher probability of success than L-type firms. A firm that invests k_i in the initial period will produce output $y_i = (k_i)^{\nu}$ where $\nu \in (0,1)$ if successful and will produce zero output if unsuccessful. The probability of the risky technology producing successfully depends on both the firm type and the aggregate state of the economy $z_t \in \{z_R, z_B\}$ where z_t is iid over time. For a given aggregate state z_t, the probability of a risky project being successful takes two possible values $p_i(z_t) \in \{p_L(z_t), p_H(z_t)\}$. The H-type firms are assumed to be more successful in all states thus

$$p_H(z_t) > p_L(z_t) \quad \forall z_t \in \{z_R, z_B\} \tag{2.1}$$

It is also assumed that all firms are more successful in a boom (z_B) than a recession (z_R), thus

$$p_i(z_B) > p_i(z_R) \quad \forall i \in \{L, H\} \tag{2.2}$$

The economy is populated by a unit mass of firms. The mass of H-type firms is denoted by $\mu \in (0,1)$ while the mass of L-type firms is $1-\mu$. There is imperfect information as firms are aware of their type when making their decisions but the banks are not. Banks are aware of the distribution of firms in the economy. The expected probability of success for each agent before the realisation of the aggregate shock is defined as

$$\tilde{p}_i \equiv Prob(z_t = z_B)\, p_i(z_B) + [1 - Prob(z_t = z_B)]\, p_i(z_R)$$

It follows from (2.1) and (2.2) that $\tilde{p}_H \geq \tilde{p}_L$.

Firms enter the period with no assets and receive an endowment W at the end of the period. In order to invest in the project, firms must enter into a loan contract with a bank. It is assumed that the bank can observe the size of the firm's endowment. As the endowment is received at the end of the period, firms cannot use it to invest in a project but as banks observe W, firms are able to pledge this wealth as collateral.

Banks are risk neutral and competitive and are funded by insured depositors who earn a risk-free return denoted by $(1+r)$. Credit contracts between banks and firms of type i are denoted by the triple $\theta_i = \{R_i, k_i, \eta_i\}$ where R_i is the interest rate charged to the firm by the bank, k_i is the size of the loan and $\eta_i \in [0, W]$ is the amount of collateral sacrificed by the firm if it chooses to default on the payment $R_i k_i$. The end-of-period endowment forms the upper-bound on the amount of pledgeable collateral, while non-negative collateral (insurance) is ruled out.

Firms have limited liability and pay the payment $R_i k_i$ only if the project is successful. The bank is unable to stake a claim on the firm's end-of-period endowment W unless this is agreed upon between the two parties beforehand through posting collateral.

It is assumed that any contracts made between the bank and the firm cannot be contingent on the aggregate state. To motivate this assumption, consider the idea that the bank only learns of the aggregate state through defaults within its own loan book, if loans do not all default simultaneously, the bank is unable to enforce state-contingent payoffs to those served first. In order to treat all borrowers within a period equally, the bank must choose contracts that are not contingent on the aggregate state. This assumption and its motivation is similar to the sequential service assumption made in the bank run literature as discussed in Allen and Gale (2009).

The bank has a lower valuation of the posted collateral than the firm itself. It is assumed that firms value collateral at its face value of η_i while banks value it at a discounted rate $\gamma\eta_i$ with $\gamma \in (0,1)$. This assumption makes pledging collateral costly. The expected profit firm i receives from a loan $\theta_i = \{R_i, k_i, \eta_i\}$ is

$$u_i^F(\theta_i) = \tilde{p}_i[(k_i)^\nu - R_i k_i] - (1-\tilde{p}_i)\eta_i + W \tag{2.3}$$

The expected profit a bank makes on offering contract θ_i to a type i firm is

$$u_i^B(\theta_i) = \tilde{p}_i R_i k_i + (1-\tilde{p}_i)\gamma\eta_i - (1+r)k_i \tag{2.4}$$

2.2.2 A regulatory stress-test

It is assumed that the aggregate state is not known when contracts are set and the aggregate state is not contractable. Thus, if the loan is not fully collateralised, the loan is risky and the return is correlated with the aggregate state z_t. The bank may not have sufficient funds to repay the depositors should a recession occur, in which case the regulator will tax (lump sum) banks in the boom while making payments to depositors in a recession.

In order to reduce its exposure to the aggregate state, the regulator is able to perform a stress-test of the banking sector. As the model has only two aggregate states, the stress scenario will simply be the recession state z_R. The stress-test is parametrised by $\sigma \in [0,1]$, which is the hurdle rate associated with the test. To simplify the analysis, it is assumed that in order to pass the stress test, each contract[1]

[1]The assumption that the stress-test applies to each contract is more restrictive than if the stress-test applied to the portfolio of contracts offered. It can be motivated by appealing to the idea that banks are small and may only offer one contract in equilibrium.

must have an expected payoff conditional on z_R of at least $\sigma(1+r)k_i$ such that

$$p_i(z_R)R_ik_i + (1-p_i(z_R))\gamma\eta_i \geq \sigma(1+r)k_i \tag{2.5}$$

This is equivalent to stating that the bank must make a loss on each loan no greater than $(1-\sigma)(1+r)k_i$ when $z_t = z_R$. Additionally, it is assumed that the penalty for a firm failing the stress-test is infinite such that equation (2.5) is a constraint in the contracting problem.

The hurdle rate σ captures the severity of the stress-test. If $\sigma = 0$ then there is no restriction on loan risk. If $\sigma = 1$, in order to pass the stress-test, the bank is required to have sufficient funds to repay depositors in all states of the world and all loans must be fully collateralised.

2.2.3 Equilibrium Concept

The possible non-existence of a competitive Nash equilibrium in models with asymmetric information has been documented in Rothschild and Stiglitz (1976), Wilson (1977) and Riley (1979). To ensure that a competitive equilibrium exists, this paper adopts the concept of a Riley reactive equilibrium as described in Riley (1979).

A set of equilibrium contracts is a Riley reactive equilibrium if for any additional contract which generates an expected profit to the bank that makes the offer, there exists another contract which would yield a gain to the second bank and losses to the first.

The use of a Riley reactive equilibrium rules out a pooling equilibria in most situations and thus places a stronger requirement on the existence of pooling equilibria. An alternative equilibrium concept that could have been adopted is the Anticipatory equilibrium concept described by Wilson (1977). The Anticipatory equilibrium allows for the existence of pooling equilibria in a standard adverse selection model and as such is less restrictive on the existence of pooling equilibria.

Due to the presence of an upper bound on collateral, W, there remains the possibility that if this constraint binds then pooling equilibria can exist as a Riley reactive equilibrium. The idea that a binding upper bound on collateral can distort contracts was discussed in Bester (1987). This paper shows how regulatory constraints such as those imposed through stress-testing of banks makes it more likely that the upper bound on collateral binds and thus more likely that a pooling equilibria exists.

When the upper bound on collateral does not bind, pooling equilibria cannot exist as a Riley reactive equilibrium as discussed in Riley (1979). The pooling equilibria found in this paper as Riley reactive equilibria would also be pooling equilibria under the Wilson Anticipatory equilibrium.

2.3 Contracting Problem

If the upper-bound on collateral does not bind, the Riley reactive equilibrium will consist of two separating credit contracts: $\theta_i \equiv \{R_i, k_i, \eta_i\}$, $i \in \{L, H\}$ where the set of separating contracts solves the following maximisation problem

$$\max_{\theta_L,\theta_H} \quad \{\mu\left(\tilde{p}_H\left[(k_H)^{\nu} - R_H k_H\right] - (1-\tilde{p}_H)\,\eta_H + W\right) \qquad (2.6)$$

$$+ (1-\mu)\left(\tilde{p}_L\left[(k_L)^{\nu} - R_L k_L\right] - (1-\tilde{p}_L)\,\eta_L + W\right)\}$$

subject to

$$\tilde{p}_H\left[\left(k_H\right)^\nu - R_H k_H\right] - \left(1-\tilde{p}_H\right)\eta_H + W \geq \tilde{p}_H\left[\left(k_L\right)^\nu - R_L k_L\right] - \left(1-\tilde{p}_H\right)\eta_L + W \tag{2.7}$$

$$\tilde{p}_L\left[\left(k_L\right)^\nu - R_L k_L\right] - \left(1-\tilde{p}_L\right)\eta_L + W \geq \tilde{p}_L\left[\left(k_H\right)^\nu - R_H k_H\right] - \left(1-\tilde{p}_L\right)\eta_H + W \tag{2.8}$$

$$\tilde{p}_i R_i k_i + \left(1-\tilde{p}_i\right)\gamma\eta_i \geq \left(1+r\right)k_i \qquad i \in \{L,H\} \tag{2.9}$$

$$p_i\left(z_R\right)R_i k_i + \left(1-p_i\left(z_R\right)\right)\gamma\eta_i \geq \sigma\left(1+r\right)k_i \qquad i \in \{L,H\} \tag{2.10}$$

$$0 \leq \eta_i \leq W \qquad i \in \{L,H\} \tag{2.11}$$

Equations (2.7) and (2.8) are truth telling constraints that ensure both firm types reveal their true type to the bank. Equation (2.9) ensures that banks make non-negative profit in expectation. Equation (2.10) is the regulatory constraint discussed above and equation (2.11) requires that the collateral level of a contract be non-negative, thus ruling out the bank providing insurance to the firms, while also requiring that collateral be no greater than the firm's total endowment.

2.3.1 Firm Preferences

To express the contract space graphically, first note that the firm's expected payoff can be rewritten in terms of two dimensions (π_i, η_i), where $\pi_i \equiv (k_i)^\nu - R_i k_i$ is the payoff from a successful project given loan size k_i and the interest rate charged on the loan R_i.

Rewriting equation (2.3) in terms of π_i and η_i, the profit for firm i given a generic contract θ_i becomes

$$u_i^F = \tilde{p}_i \pi_i - (1 - \tilde{p}_i)\,\eta_i + W \tag{2.12}$$

The firm's iso-profit curves are linear in (π, η)-space with the following marginal rate of substitution

$$\left.\frac{d\pi}{d\eta}\right|_u = \frac{1 - \tilde{p}_i}{\tilde{p}_i} \tag{2.13}$$

The marginal rate of substitution between π and η depends on the firm type i. Due to the lower probability their project is successful, L-type firms have steeper iso-profit curves and are less willing to trade higher collateral requirements for a higher payoff if successful. As a result, in the presence of incomplete information, H-type firms are able to separate from L-type firms through a willingness to accept a contract with higher collateral requirement.

2.3.2 Optimal Contracts when the stress-test constraint is slack

The model presented in this paper is based on a standard collateral-screening model with the addition of a stress-test constraint, equation (2.10). This constraint may not bind in equilibrium, in which case the optimal contracting problem collapses to a standard adverse selection problem. Then, so long as the firm has sufficient wealth that the upper-bound on collateral does not bind, the optimal contracts under imperfect information consist of two separating contracts.

I now consider in more detail the solution to the simplified problem where the stress-test constraint, equation (2.10), does not bind. In this case, competition in the banking sector requires that banks earn zero expected profits and equation (2.9) holds with strict equality. Substituting equation (2.9) into the definition of π yields the following equation

$$\pi_i^{ZP}(k_i, \eta_i) = (k_i)^v - \left(\frac{1+r}{\tilde{p}_i}\right) k_i + \left(\frac{1-\tilde{p}_i}{\tilde{p}_i}\right) \gamma \eta_i \tag{2.14}$$

Equation (2.14) describes the largest feasible payoff π_i to a firm of type i for a given loan size k_i and collateral requirement η_i. As the firm's technology features decreasing returns to scale, there is an optimal loan size k_i^{ZP} which can be found from the derivative of equation (2.14) with respect to k_i and is given by

$$k_i^{ZP} = \left(\frac{v\tilde{p}_i}{1+r}\right)^{\frac{1}{1-v}} \tag{2.15}$$

An important feature of this simplified problem is that the optimal loan size is independent of the size of the collateral requirement. The interest rate charged to firms is

$$R_i^{ZP} = \left(\frac{1+r}{\tilde{p}_i}\right) - \left(\frac{1-\tilde{p}_i}{\tilde{p}_i}\right)\left(\frac{1+r}{v\tilde{p}_i}\right)^{\frac{1}{1-v}} \gamma \eta_i \tag{2.16}$$

Evaluating equation (2.14) at the optimal scale yields the following

$$\Pi_i^{ZP}(\eta_i) = (1-v)\left(\frac{v\tilde{p}_i}{1+r}\right)^{\frac{v}{1-v}} + \left(\frac{1-\tilde{p}_i}{\tilde{p}_i}\right) \gamma \eta_i \tag{2.17}$$

Equation (2.17) describes the frontier of payoffs to the firm, π_i, in $(\pi, \eta)^+$-space that earn banks zero expected profit. As $\gamma \in (0,1)$, the frontier for firm i

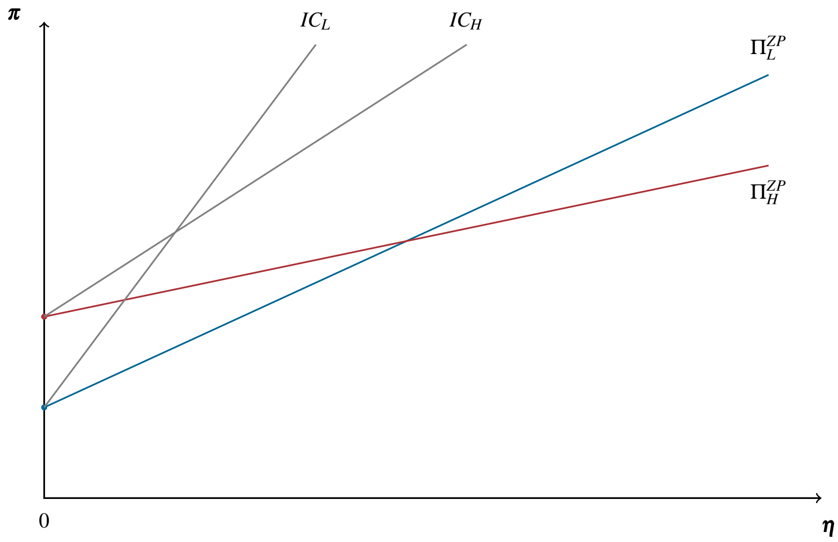

Figure 2.1: Optimal Contracts under full information when the stress-test constraint is slack

is increasing in η_i but has a slope that is lower than that of the firm's iso-profit curves. This is because, while banks value the collateral pledged ($\gamma > 0$), they place a lower value on collateral than firms do ($\gamma < 1$). Pledging collateral is inefficient and it follows that absent incomplete information, the optimal contracts are the corner solution where the frontier Π_i^{ZP} intersects the non-negativity constraint on collateral and thus $\eta_i = 0$. When $\eta_H = \eta_L = 0$, it follows from equation (2.17) that H-type firms receive a larger payoff than L-type firms due to their higher probability of success. This is shown graphically in figure (2.1).

With incomplete information, the L-type firms would wish to masquerade as H-type firms in order to obtain a larger payoff. Due to their higher probability of success, H-type firms choose a contract with a positive collateral requirement in order to separate from the L-type firms. As H-type firms have no incentive to masquerade as L-type firms the only truth-telling constraint that will bind in equilibrium will be that of the L-type firm, equation (2.8). The amount of collateral required for

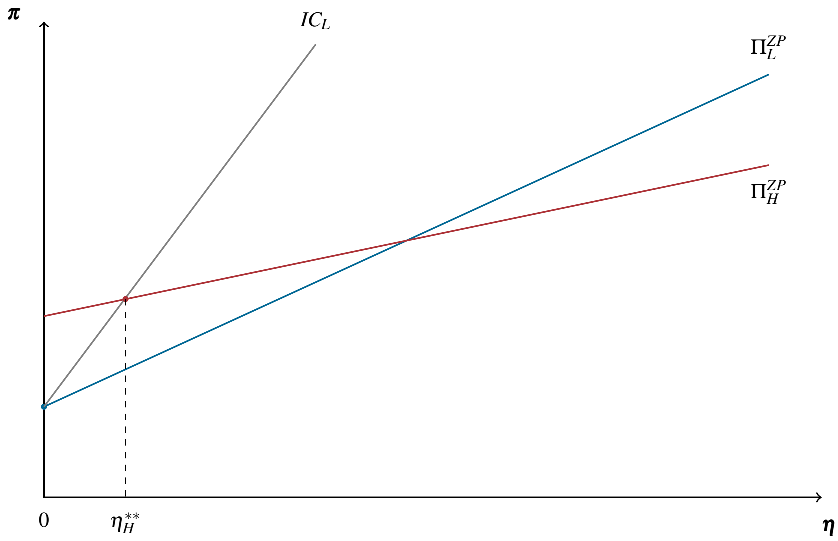

Figure 2.2: Optimal Contracts under incomplete information when the stress-test constraint is slack

separation is

$$\eta_H^{**} = (1-v)\left(\frac{v}{1+r}\right)^{\frac{v}{1-v}} \left[\frac{(\tilde{p}_H)^{\frac{1}{1-v}}\tilde{p}_L - \tilde{p}_H(\tilde{p}_L)^{\frac{1}{1-v}}}{\tilde{p}_H(1-\tilde{p}_L) - \tilde{p}_L(1-\tilde{p}_H)\gamma}\right] > 0$$

The collateral posted by H-type firms is strictly positive in the presence of incomplete information. Figure 2.2 illustrates these contracts graphically.

The solution to the simplified problem is also a solution to the full problem presented in section 2.3, when equation (2.10) does not bind. To establish when this is the case, first note that equation (2.10) can be rewritten as

$$\xi_i \tilde{p}_i R_i k_i + (1-\xi_i \tilde{p}_i)\gamma\eta_i \geq \sigma(1+r)k_i \tag{2.18}$$

where

$$\xi_i = \frac{p_i(z_R)}{\tilde{p}_i} \in (0,1)$$

is the ratio of the success probability conditional on a recession to the unconditional success probability of a firm of type i.

When equation (2.10) does not bind, competition in the banking sector ensures that banks earn zero profits in expectation and equation (2.9) holds with strict equality. Substituting equation (2.9) into equation (2.18) yields the following inequality

$$(1-\xi_i)\,\gamma\eta_i \geq (\sigma-\xi_i)\,(1+r)\,k_i \tag{2.19}$$

As both k_i and η_i are non-negative a sufficient condition for equation (2.10) to be slack is $\sigma \leq \xi_i$. The solution to the simplified problem also features $\eta_L = 0$ and thus $\sigma \leq \xi_L$ is also a necessary condition for this to be part of the solution to the full problem.

2.3.3 Full Information Contracts with the stress-test constraint

If the stress-test constraint becomes sufficiently tight, to the extent that $\sigma > \xi_i$, equation (2.19) shows that the stress-test constraint, equation (2.10), must bind for sufficiently low collateral and thus the frontier is no longer fully characterised by equation (2.17). I now characterise the contract frontier in the case where $\sigma > \xi_i$, which I denote by $\Pi_i(\eta_i)$.

From equation (2.19) it follows that when $\sigma > \xi_i$ equation (2.10) is slack if the contract features sufficiently high collateral. In this case, the optimal contract features a constant loan size, k_i, regardless of the collateral required. This loan size is defined by equation (2.15). Substituting this into equation (2.19) above and rearranging yields a cutoff level of collateral $\bar{\eta}_i^{ZP}$ defined by

$$\bar{\eta}_i^{ZP} = \frac{1}{\gamma}\left(\frac{\sigma - \xi_i}{1-\xi_i}\right)\left(\frac{1}{1+r}\right)^{\frac{v}{1-v}}(v\tilde{p}_i)^{\frac{1}{1-v}} \tag{2.20}$$

such that with $\eta_i \geq \bar{\eta}_i^{ZP}$, banks earn zero profit when offering the frontier contracts.

In this region of the frontier, equation (2.9) binds while the stress-test constraint, equation (2.10), is slack. When collateral is higher than the cutoff value $\bar{\eta}_i^{ZP}$, the frontier will be characterised by equation (2.17) and $\Pi_i(\eta) = \Pi_i^{ZP}(\eta)$.

Now consider the case where the stress-test constraint, equation (2.10), binds and the banks earn strictly positive profits in expectation so that equation (2.9) is slack. To derive the contract frontier in this situation, first substitute equation (2.10) into the definition of π to yield the following

$$\pi_i^{RC}(k_i,\eta_i) = (k_i)^v - \frac{\sigma}{\xi_i}\left(\frac{1+r}{\tilde{p}_i}\right)k_i + \left(\frac{\frac{1}{\xi_i} - \tilde{p}_i}{\tilde{p}_i}\right)\gamma\eta_i \tag{2.21}$$

Equation (2.21) describes the contract frontier in terms of loan size k_i and collateral requirement η_i. There is an optimal loan size k_i^{RC}, independent of collateral, which can be found from the derivative of equation (2.21) with respect to k_i and is given by

$$k_i^{RC} = \left[\left(\frac{\xi_i}{\sigma}\right)\left(\frac{v\tilde{p}_i}{1+r}\right)\right]^{\frac{1}{1-v}} \tag{2.22}$$

The interest rate charged to firms is

$$R_i^{RC} = \left(\frac{\sigma}{\xi_i}\right)\left(\frac{1+r}{\tilde{p}_i}\right) - \left(\frac{\frac{1}{\xi_i} - \tilde{p}_i}{\tilde{p}_i}\right)\left[\left(\frac{\sigma}{\xi_i}\right)\left(\frac{1+r}{v\tilde{p}_i}\right)\right]^{\frac{1}{1-v}}\gamma\eta_i \tag{2.23}$$

Evaluating equation (2.21) at the optimal loan size yields the following

$$\Pi_i^{RC}(\eta_i) = (1-v)\left[\left(\frac{\xi_i}{\sigma}\right)\frac{v\tilde{p}_i}{1+r}\right]^{\frac{v}{1-v}} + \left(\frac{\frac{1}{\xi_i}-\tilde{p}_i}{\tilde{p}_i}\right)\gamma\eta_i \tag{2.24}$$

The frontier defined above is a solution to the full problem whenever equation (2.10) binds and equation (2.9) is slack. This occurs when the following inequality is satisfied

$$(\sigma-\xi_i)(1+r)k_i > (1-\xi_i)\gamma\eta_i \tag{2.25}$$

The above inequality will hold when collateral is sufficiently low and σ is sufficiently high.

Substituting in the optimal loan size specified by equation (2.22) yields the following cutoff for collateral

$$\bar{\eta}_i^{RC} = \frac{1}{\gamma}\left(\frac{\sigma-\xi_i}{1-\xi_i}\right)\left(\frac{1}{1+r}\right)^{\frac{v}{1-v}}\left[\left(\frac{\xi_i}{\sigma}\right)v\tilde{p}_i\right]^{\frac{1}{1-v}} \tag{2.26}$$

such that the frontier of contracts with $\eta_i \leq \bar{\eta}_i^{RC}$ feature a binding stress-test constraint, equation (2.10), while banks earn positive profit and equation (2.9) will be slack. For collateral values lower than the cutoff value $\bar{\eta}_i^{RC}$, the frontier will be characterised by equation (2.24) and $\Pi_i(\eta) = \Pi_i^{RC}(\eta)$.

Comparison of the cutoffs, equation (2.20) and equation (2.26) yields the following strict inequality

$$\bar{\eta}_i^{RC} < \bar{\eta}_i^{ZP}$$

As $\sigma > \xi_i$ it follows that $k_i^{RC} < k_i^{ZP}$ and thus the frontier contracts in the region

$\eta < \bar{\eta}_i^{RC}$ features a lower loan size than the frontier contracts in the region $\eta \geq \bar{\eta}_i^{ZP}$. Furthermore, it follows from above that there is an interval $\eta_i \in \left(\bar{\eta}_i^{RC}, \bar{\eta}_i^{ZP}\right)$ where both the stress-test constraint, equation (2.10), and the bank profit constraint, equation (2.9), bind.

When equation (2.9) and equation (2.10) both bind, they can be combined to yield an equation for k_i as a function of the collateral requirement η_i. This is given by the following equation

$$k_i^{BC} = \left(\frac{1}{1+r}\right)\left(\frac{1-\xi_i}{\sigma-\xi_i}\right)\eta_i \tag{2.27}$$

Similarly, the interest rate can be written as

$$R_i^{BC} = (1+r)\left(\frac{1-\sigma+(\sigma-\xi_i)\,\tilde{p}_i}{(1-\xi_i)\,\tilde{p}_i}\right) \tag{2.28}$$

The frontier of feasible contracts in the case where equation (2.9) and equation (2.10) both bind, denoted as $\Pi_i^{BC}(\eta_i)$, can be found by substituting equation (2.27) and equation (2.28) into the definition of π which gives

$$\Pi_i^{BC}(\eta_i) = \left[\left(\frac{1}{1+r}\right)\left(\frac{1-\xi_i}{\sigma-\xi_i}\right)\gamma\right]^{\nu}\eta_i^{\nu} - \frac{1}{\tilde{p}_i}\left[\left(\frac{1-\xi_i}{\sigma-\xi_i}\right)-(1-\tilde{p}_i)\right]\gamma\eta_i \tag{2.29}$$

Over the interval $\eta_i \in \left(\bar{\eta}_i^{RC}, \bar{\eta}_i^{ZP}\right)$, $\Pi_i(\eta_i) = \Pi_i^{BC}(\eta_i)$.

The frontier of feasible contracts in the case of $\sigma > \xi_i$ can be summarised by the following

$$\Pi_i(\eta_i) = \begin{cases} (1-v)\left[\left(\frac{\xi_i}{\sigma}\right)\left(\frac{v\tilde{p}_i}{1+r}\right)\right]^{\frac{v}{1-v}} + \left(\frac{\frac{1}{\xi_i}-\tilde{p}_i}{\tilde{p}_i}\right)\gamma\eta_i & \eta \leq \bar{\eta}_i^{RC} \\ \left[\left(\frac{1}{1+r}\right)\left(\frac{1-\xi_i}{\sigma-\xi_i}\right)\gamma\right]^{v}\eta_i^{v} - \frac{1}{\tilde{p}_i}\left[\left(\frac{1-\xi_i}{\sigma-\xi_i}\right) - (1-\tilde{p}_i)\right]\gamma\eta_i & \eta \in \left(\bar{\eta}_i^{RC}, \bar{\eta}_i^{ZP}\right) \\ (1-v)\left(\frac{v\tilde{p}_i}{1+r}\right)^{\frac{v}{1-v}} + \left(\frac{1-\tilde{p}_i}{\tilde{p}_i}\right)\gamma\eta_i & \eta \geq \bar{\eta}_i^{ZP} \end{cases} \tag{2.30}$$

The derivative of $\Pi_i(\eta_i)$ with respect to η_i is:

$$\frac{\partial \Pi_i}{\partial \eta_i} = \begin{cases} \left(\frac{\frac{1}{\xi_i}-\tilde{p}_i}{\tilde{p}_i}\right)\gamma & \eta \leq \bar{\eta}_i^{RC} \\ v\left[\left(\frac{1}{1+r}\right)\left(\frac{1-\xi_i}{\sigma-\xi_i}\right)\gamma\right]^{v}\eta_i^{v-1} - \frac{1}{\tilde{p}_i}\left[\left(\frac{1-\xi_i}{\sigma-\xi_i}\right) - (1-\tilde{p}_i)\right]\gamma & \eta \in \left(\bar{\eta}_i^{RC}, \bar{\eta}_i^{ZP}\right) \\ \left(\frac{1-\tilde{p}_i}{\tilde{p}_i}\right)\gamma & \eta \geq \bar{\eta}_i^{ZP} \end{cases} \tag{2.31}$$

The gradient $\frac{\partial \Pi_i}{\partial \eta_i}$ always remains within the interval $\left[\left(\frac{1-\tilde{p}_i}{\tilde{p}_i}\right)\gamma, \left(\frac{\frac{1}{\xi_i}-\tilde{p}_i}{\tilde{p}_i}\right)\gamma\right]$ and is steepest when $\eta \leq \bar{\eta}_i^{RC}$. A requirement for the equilibrium contract to be an interior solution is that the gradient of the contract frontier at $\eta_i = 0$ is steeper than the iso-profit constraint. This holds if the following condition is satisfied

$$\frac{\gamma}{\xi_i} + (1-\gamma)\tilde{p}_i > 1 \tag{2.32}$$

When the above inequality is satisfied, the full-information contracts are an interior solution featuring $\eta_i > 0$ and can be illustrated graphically by the point where the frontier $\Pi_i(\eta_i)$ is tangent to firm i's iso-profit curve. This case is illustrated by figure 2.3 and the result is formally stated in the proposition below.

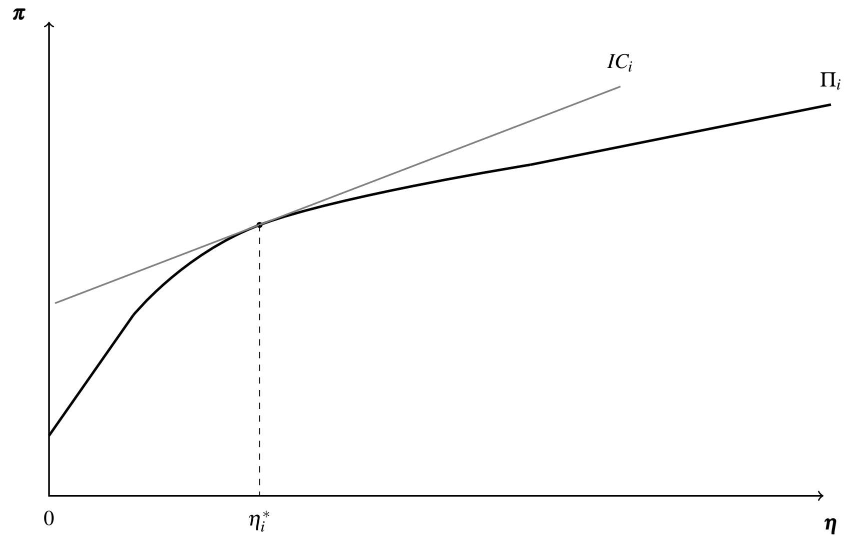

Figure 2.3: Frontier of feasible contracts when $\sigma > \xi$

Proposition 2.1. *If $\sigma > \xi_i$ and $\frac{\gamma}{\xi_i} + (1-\gamma)\,\tilde{p}_i > 1$ the full information competitive equilibrium contract for firm i will feature strictly positive collateral $\eta_i \in \left(\bar{\eta}_i^{RC}, \bar{\eta}_i^{ZP}\right)$ and equation* (2.9) *and equation* (2.10) *will bind.*

Proof. See Appendix 2.6.1 □

The optimal contract at the interior solution, when the conditions set out in proposition 2.6.1 are statisfied, is denoted by $\theta_i^* \equiv \{R_i^*, \eta_i^*, k_i^*\}$ with the individual terms given by

$$k_i^* = \left(\frac{v\tilde{p}_i\,(1-\xi_i)}{(1+r)\left(1-\xi_i+(\sigma-\xi_i)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)\right)}\right)^{\frac{1}{1-v}} \tag{2.33}$$

$$\eta_i^* = \frac{1}{\gamma}(\sigma-\xi_i)\left(\frac{1-\xi_i}{1+r}\right)^{\frac{v}{1-v}}\left(\frac{v\tilde{p}_i}{1-\xi_i+(\sigma-\xi_i)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}\right)^{\frac{1}{1-v}} \tag{2.34}$$

$$R_i^* = \left[1-(1-\tilde{p}_i)\left(\frac{\sigma-\xi_i}{1-\xi_i}\right)\right]\left(\frac{1+r}{\tilde{p}_i}\right) \tag{2.35}$$

The contract terms specified above highlight the distortion resulting from the stress-test constraint, equation (2.10). Both the loan size k_i^* and the interest rate charged are decreasing in σ and, for $\sigma > \xi_i$, they lie strictly below the values offered when the stress-test constraint does not bind. The collateral level η_i^* is now strictly positive, whether it is increasing or decreasing in σ depends on the parameter values of the model. The proposition below sets out more formally the requirements for η_i^* to be increasing in σ.

Proposition 2.2. *The collateral requirement of the full information optimal contract when $\sigma > \xi_i$ and $\frac{\gamma}{\xi_i} + (1-\gamma)\,\tilde{p}_i > 1$ is increasing in σ if $\left(\frac{1-v}{v}\right)\left(\frac{\gamma}{1-\gamma}\right) > (1-\tilde{p}_i)$ and decreasing in σ otherwise.*

Proof. See Appendix 2.6.2. □

To understand why required collateral may be decreasing in σ, first note that an increase in σ requires the bank to reduce the aggregate risk of the loan contract. This can be achieved either by increasing the collateral required or by reducing the size of the loan. As $\gamma < 1$, the use of collateral is costly and if the cost of collateral is sufficiently high (low γ) and the returns to scale of the project sufficiently large (high v), it may be optimal to reduce both the loan size and the collateral requirement in order to pass the stress-test.

The possibility of a corner solution when $\sigma > \xi_i$ is set out in the following proposition.

Proposition 2.3. *If $\sigma > \xi_i$ and $\frac{\gamma}{\xi_i} + (1-\gamma)\,\tilde{p}_i < 1$ the full-information competitive equilibrium contract for firm i features $\eta_i = 0$, $k_i = k_i^{RC}$ and $R_i = R_i^{RC}$. Equation*

(2.9) *holds as a strict inequality and thus banks make positive profit in expectation.*

Proof. See Appendix 2.6.1 □

If the condition for an interior solution is not satisfied, $\frac{\gamma}{\xi_i} + (1-\gamma)\,\tilde{p}_i < 1$, the slope of the frontier Π_i is always strictly less than the slope of the iso-profit curves of firm i for all η_i and the optimal collateral requirement will be at the lower-bound with $\eta_i = 0$.

2.3.4 Incomplete Information Contracts with the stress-test constraint

To simplify the analysis that follows, I make two additional assumptions. First I assume that $\frac{\gamma}{\xi_i} + (1-\gamma)\,\tilde{p}_i > 1$ for $i \in \{L, H\}$ so that the full information contracts feature positive collateral requirements. Second, I assume that the ratio of success probability in the bad aggregate state to the expected success probability is the same for both firm types and thus $\xi = \xi_H = \xi_L$. Given the second assumption, the feasible frontier of contracts will be concave for both firm types above the same threshold σ.

In the presence of incomplete information, the banks are unable to condition contracts on the firm type. As before, the relative slope of the firms' iso-profit lines means H-type firms are able to separate from L-type firms by accepting a contract with a larger collateral requirement. However, as shown in the previous section, the full-information contracts feature positive collateral and thus it is not immediately clear that either truth-telling constraint is violated by the full-information contracts.

As a first step, it is important to understand the shape of the contract frontiers for the two firm types. First note that $\Pi_L(0) < \Pi_H(0)$ and thus the frontier for L-type firms lies below that of H-type firms. The following proposition establishes that Π_L and Π_H cross exactly once.

Proposition 2.4. *The frontiers of feasible contracts for the two firm types Π_L and Π_H will cross exactly once in $(\pi,\eta)^+$-space for any $\sigma \in [\xi, 1]$ at a point $\tilde{\eta} > \eta_L^{ZP}$.*

Proof. See Appendix 2.6.3. □

It follows immediately from the above proposition that the frontier of contracts for the H-type firm lies above that of the L-type firm at the L-type firm's full-information contract, $\Pi_H(\eta_L^*) > \Pi_L(\eta_L^*)$. Thus the H-type firm would strictly prefer one its own full-information contract to that of the L-type firm and its truth-telling constraint, equation (2.7), will not bind. The relevant truth-telling constraint is equation (2.8).

To understand when equation (2.8) binds, I define the function ϕ_L as the profit L-type firms receives from their full information contract less the profit they would receive from from the H-type firm's contract

$$\phi_L \equiv \tilde{p}_L \pi_L^* - (1-\tilde{p}_L)\,\eta_L^* - \tilde{p}_L \pi_H^* + (1-\tilde{p}_L)\,\eta_H^* \tag{2.36}$$

If $\phi_L \geq 0$, the truth-telling constraint of L-type firms is satisfied at the full information contracts and there is no need for H-type firms to accept a higher collateral level in order to separate from L-type firms. If on the other hand $\phi_L < 0$, the truth-telling constraint for L-type firms is not satisfied at the full information contracts and equation (2.8) binds in the incomplete information case.

The lower-limit of ϕ_L as $\sigma \to \xi$ is

$$\lim_{\sigma\to\xi}\{\phi_L\} = \left[\left(\frac{\tilde{p}_L}{\tilde{p}_H}\right)^{\frac{v}{1-v}} - 1\right]\left[v^{\frac{v}{1-v}} - v^{\frac{1}{1-v}}\right] < 0 \tag{2.37}$$

For sufficiently low σ equation (2.8) will bind and H-type firms must accept higher collateral requirements in order to separate from L-type firms.

As the full-information contracts feature positive collateral requirements, unlike in the standard adverse selection model, even with imperfect information the truth-telling constraint may not bind as illustrated by the upper-limit of ϕ_L as $\sigma \to 1$

$$\lim_{\sigma\to 1}\{\phi_L\} = v^{\frac{v}{1-v}}\left[(1-v)\left(\frac{\tilde{p}_L + \frac{1}{\gamma}\left(\frac{\tilde{p}_L}{\tilde{p}_H} - \tilde{p}_L\right)}{\tilde{p}_L + \frac{1}{\gamma}(1-\tilde{p}_L)}\right)^{\frac{v}{1-v}} + v\left(\frac{\tilde{p}_H + \frac{1}{\gamma}\left(\frac{\tilde{p}_H}{\tilde{p}_L} - \tilde{p}_H\right)}{\tilde{p}_H + \frac{1}{\gamma}(1-\tilde{p}_H)}\right) - 1\right] > 0 \tag{2.38}$$

The value of ϕ_L is strictly positive as $\sigma \to 1$ and thus for sufficiently high values of σ, equation (2.8) does not bind. Appendix 2.6.4 shows that there is a threshold $\bar{\sigma}_L$ above which $\phi_L > 0$. For $\sigma \geq \bar{\sigma}_L$ both equations (2.7) and (2.8) will not bind at the full-information contracts and thus neither firm has an incentive to deviate from their full-information contracts, even in the imperfect information setting. This is stated formally in the proposition below.

Proposition 2.5. *There exists a threshold* $\bar{\sigma}_L \in (\xi, 1)$ *above which the truth-telling constraint for L-type firms, equation* (2.8)*, does not bind at the equilibrium separating contracts and below which it does bind at the equilibrium separating contracts.*

Proof. See Appendix 2.6.4. □

The separating contract for H-type firms when $\sigma \in (\xi, \bar{\sigma}_L)$ and assuming that

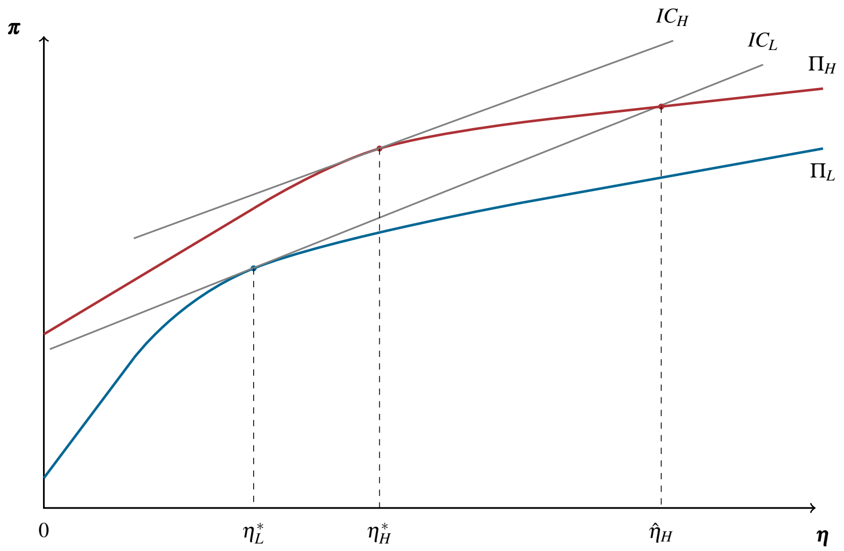

Figure 2.4: Asymmetric Information Contracts when $\sigma > \xi$

the wealth constraint does not bind, denoted by $\hat{\theta}_H$, can be found as the point on the frontier Π_H at which L-type firms are indifferent between this contract and their full-information contract θ_L^*. An example of this intersection is illustrated in figure 2.4.

For brevity, I omit the precise contract terms for H-type firms in the case where equation (2.8) binds as their precise formulation will depend on whether $\hat{\eta}_H$ is larger than $\bar{\eta}_H^{ZP}$ or not. From the properties of the contract frontier Π_H, in addition to a higher collateral requirement, it follows that the separating contract features a strictly larger loan size and a weakly lower interest rate relative to the full-information contract.

2.3.5 Contracts when the wealth constraint binds

I have assumed up to this point that firms have sufficient wealth to supply the collateral required by the bank and the upper-bound on collateral did not bind. I now relax this assumption and study the impact of a binding upper-bound on collateral

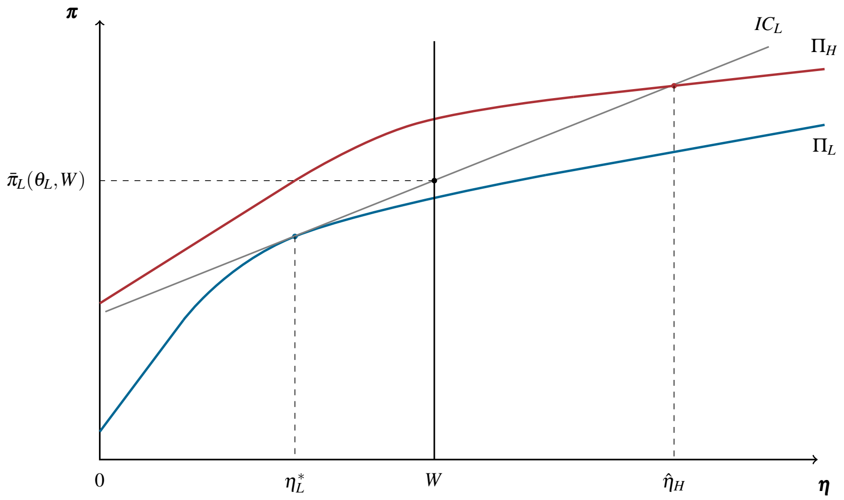

Figure 2.5: Contracts with binding wealth constraint when $\sigma > \xi$

on the optimal contracts.

In the absence of incomplete information, the binding wealth constraint distorts the contract terms when $\eta_i^* > W$. In this case the contract still lies on the frontier of feasible contracts at the point $\Pi_i(W)$.

If the incentive compatibility constraint and the wealth constraint both bind such that $\hat{\eta}_H > W$, the H-type firm's contract lies below the frontier Π_H. This case is illustrated in figure 2.5. The payoff π that the H-type firm receives when the wealth constraint binds $\pi_H^* = \bar{\pi}_L(\theta_L, W)$ can be found from the incentive compatibility constraint, equation (2.8), given the contract that the L-type firm receives θ_L and the upper bound on collateral W

$$\bar{\pi}_L(\theta_L, W) = (k_L)^{\nu} - k_L R_L + \left(\frac{1-\tilde{p}_L}{\tilde{p}_L}\right)(W - \eta_L)$$

When the wealth constraint binds, the H-type firm's contract lies off the frontier of contracts that satisfy equation (2.9) and equation (2.10) and the firm is indifferent between any pair of contract terms $\{k_H, R_H\}$ which yields a payoff of $\bar{\pi}_L(\theta_L, W)$. However, a bank can maximise its profit by offering the optimal loan size k_H^{ZP} , subject to the regulatory constraint (2.10), and increasing the interest rate charged to firms above the value R_H^{ZP}, earning strictly positive profit in expectation.

2.3.6 Pooling Contracts

When H-type firms pledge their entire wealth as collateral such that the upper-bound on collateral binds, there is the possibility of pooling contracts existing as a Riley Reactive Equilibrium. In a pooling contract, both H-type and L-type firms receive the same contract $\theta_P = \{R_P, k_P, \eta_P\}$. The expected probability of success conditional on aggregate state z_t is simply a weighted average of the success probabilities for L-type and H-type firms, weighted by the proportion of that firm type in the economy

$$p_P(z_t) = \mu p_H(z_t) + (1-\mu) p_L(z_t)$$

Again assuming that ξ is the same for both firm types, it follows that the unconditional expected probability of success is also a weighted average of the L-type and H-type probabilities

$$\tilde{p}_P = \mu \tilde{p}_H + (1-\mu) \tilde{p}_L$$

The frontier of feasible pooling contracts Π_P for $\sigma > \xi$ can be calculated as in the separating case but using the pooling probabilities such that

$$\Pi_P(\eta) = \begin{cases} (1-\nu)\left[\left(\frac{\xi}{\sigma}\right)\left(\frac{\nu\tilde{p}_P}{1+r}\right)\right]^{\frac{\nu}{1-\nu}} + \left(\frac{\frac{1}{\xi}-\tilde{p}_P}{\tilde{p}_P}\right)\gamma\eta_i & \text{if } \eta \leq \bar{\eta}_P^{RC} \\ \left[\left(\frac{1}{1+r}\right)\left(\frac{1-\xi}{\sigma-\xi}\right)\gamma\right]^{\nu}\eta_i^{\nu} - \frac{1}{\tilde{p}_P}\left[\left(\frac{1-\xi}{\sigma-\xi}\right)-(1-\tilde{p}_P)\right]\gamma\eta_i & \text{if } \eta \in \left(\bar{\eta}_P^{RC}, \bar{\eta}_P^{ZP}\right) \\ (1-\nu)\left(\frac{\nu\tilde{p}_P}{1+r}\right)^{\frac{\nu}{1-\nu}} + \left(\frac{1-\tilde{p}_P}{\tilde{p}_P}\right)\gamma\eta_P & \text{if } \eta \geq \bar{\eta}_P^{ZP} \end{cases} \tag{2.39}$$

Where the thresholds $\bar{\eta}_P^{RC}$ and $\bar{\eta}_P^{ZP}$ are found by substituting the pooling probabilities into equations (2.26) and (2.20). An important property of Π_P is that for any $\mu \in (0,1)$ and any $\sigma \in [0,1)$ it crosses Π_L and Π_P exactly once. This follows from the single-crossing condition of Π_L and Π_H and noting that for any $\mu \in (0,1)$ $\tilde{p}_P \in (\tilde{p}_L, \tilde{p}_H)$.

For a pooling contract θ_P to exist as a Riley Reactive Equilibrium, there must exist no deviating contract that would satisfy equations (2.9) and (2.10) while making the pooling contract either unprofitable or violate the stress-test constraint. That is, there cannot exist a cream-skimming contract.

It follows that a necessary condition for the existence of a pooling contract is for both L-type and H-type firms to prefer the pooling contract to the best separating contracts available, otherwise the firms would choose the separating contracts over the pooling contracts. Similarly, any pooling contract must lie on the frontier Γ_P of pooling contracts, otherwise a better pooling contract could be found that would be preferred by both types of firm.

Furthermore, as any pooling contract that attracts L-type firms occurs at a point where $\Pi_H(\eta_P) > \Pi_P(\eta_P)$, in most cases there will exist a profitable cream-skimming contract which will satisfy equations (2.9) and (2.10). Due to the relative slope of firm iso-profit lines, a cream-skimming contract will feature a higher payoff π in addition to a higher collateral requirement η than the pooling contract such that H-type firms will choose the cream-skimming contract to the pooling contract while L-type firms would prefer the pooling contract. Should such a cream-skimming contract exist, the pooling contract would be left with only L-type firms and the pooling contract would no longer be profitable.

However, the existence of a cream-skimming contract depends on the ability of H-type firms to accept a contract with higher collateral than the pooling contract. If the upper-bound on collateral binds at the pooling contract such that $\eta_P = W$, no contract with higher collateral can be offered and thus there exists no cream-skimming contract. In this case, the pooling contract survives as a Riley Reactive Equilibrium. This result is set out more formally in the proposition below.

Proposition 2.6. *A pooling contract θ_P will be a Riley Reactive Equilibrium if i) the wealth constraint on collateral binds $\eta_P = W$, ii) the contract lies on the frontier of feasible pooling contracts $\pi_P = \Pi_P(W)$ and iii) L-type firms strictly prefer the pooling contract to their separating contract $\Pi_P(W) > \bar{\pi}_L(\theta_L, W)$*

Proof. First note that if $\Pi_P(W) > \bar{\pi}_L(\theta_L, W)$ then it follows that the separating contract for H-type firms features $\hat{\eta}_H = W$ and $\hat{\pi}_H = \bar{\pi}_L(\theta_L, W)$; thus both H-type and L-type firms will prefer the pooling contract to their separating contract. Next note that as $\pi_P = \Pi_P(W)$, the pooling contract lies on the feasible frontier and thus there exists no pooling contracts that are preferred by either agent to θ_P. Finally, as $\eta_P = W$ there exist no deviating contracts that only one agent type would prefer

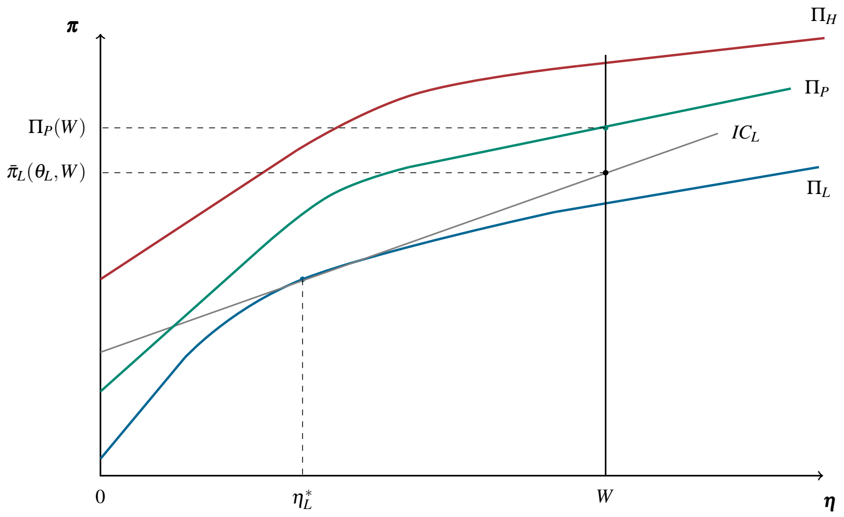

Figure 2.6: Pooling Contracts when $\sigma > \xi$

to the pooling contract and thus there does not exist any deviating contract that any agent type prefers and would satisfy equations (2.9) and (2.10). □

An example of a pooling contract existing as a Riley reactive equilibrium is illustrated in figure 2.6 in a case where $\sigma > \xi$. The stress-test constraint increases the amount of collateral required by contracts and hence makes it more likely that the upper-bound on collateral binds. However, pooling contracts may exist even when $\sigma \leq \xi$ if W is sufficiently low. For example, in the extreme case of $W = 0$, a pooling equilibrium exists for all values of σ because separation of types through collateral requirements is not possible.

The precise terms of the pooling contract, as in the separating case, depend where the contract is located on the frontier. More generally, the pooling contract results in a misallocation of capital relative to the separating contracts case. With separating contracts, H-type firms receive a larger loan size than L-type firms, reflecting the higher success rate of H-type firms. In a pooling equilibria, all contract terms, including capital are the same across firm times and the market no longer allocates more capital to firms with a higher probability of success.

2.4 Conclusion

This paper presented a one-dimensional adverse selection model of firm financing where lenders face a stress-test constraint that restricts the losses they can make in a recession. As the stress-test constraint tightens, banks value collateral not only as a screening device but also as a way of reducing the risk of loans. When the stress-test constraint on bank losses becomes sufficiently tight, for example following a financial crisis, the collateral requirements for loans increases and the ability of banks to screen firms may suffer.

The key result of this paper is that the tightening of the stress-test hurdle rate can result in a misallocation of capital as a result of a switch from separating contracts, where less risky firms get better credit terms to a pooling contract where firms of differing riskiness get the same contracting terms. This misallocation of capital is shown to lower aggregate productivity; something that is not normally associated with financial friction models where lower quality firms are usually offered relatively worse credit terms following a crisis. This result suggests policy makers should exercise caution when imposing regulatory constraint on banks. This also applies to stress-testing to the extent that announcing publicly the results of regulatory stress-tests will impose implicit capital requirements on banks.

This paper also emphasises the interaction between a firm's pledgeable collateral and the impact of banking regulation. The misallocation which occurs in a pooling equilibria can only occur if firms have insufficient pledgeable collateral. If on the other hand, firms have access to sufficient collateral, the tightening of the regulatory constraint may be beneficial in forcing an increased use in collateral and consequently a fall in lending risk.

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2.6 Appendix for Chapter 2

2.6.1 Proof of proposition 2.3

Consider the Lagrangian associated with maximising equation (2.6) subject to the bank's zero profit condition, (2.9) the liquidity restriction (2.10) and the non-negativity constraint (2.11). To simplify the FOCs I denote $T_i = R_i k_i$ Assume $\sigma > \xi$ and thus the stress-test constraint binds

$$\begin{aligned}\mathscr{L}_i &= \sum_i \{\mu_i (\tilde{p}_i [(k_i)^v - T_i] - (1-\tilde{p}_i)\eta_i + W) \\ &\quad + \lambda_B^i [\tilde{p}_i T_i + (1-\tilde{p}_i)\gamma\eta_i - (1+r)k_i] + \sum_i \lambda_\eta^{i-}\eta_i \\ &+ \lambda_\sigma^i [p_i(z_R)T_i + (1-p_i(z_R))\gamma\eta_i - \sigma(1+r)k_i]\}\end{aligned}$$

Where $\mu_H = \mu$ and $\mu_L = 1-\mu$.

The first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial k_i} = \mu_i v \tilde{p}_i (k_i)^{v-1} - (1+r)\left(\lambda_B^i + \sigma\lambda_\sigma^i\right)$$

$$\frac{\partial \mathscr{L}}{\partial \eta_i} = -\mu_i(1-\tilde{p}_i) + \lambda_B^i(1-\tilde{p}_i)\gamma + \lambda_\sigma^i(1-p_i(z_R))\gamma + \lambda_\eta^{i-}$$

such

$$\frac{\partial \mathscr{L}}{\partial T_i} = -\mu_i\tilde{p}_i + \lambda_B^i\tilde{p}_i + \lambda_\sigma^i p_i(z_R)$$

If $\eta_i > 0$ and thus $\lambda_\eta^{i-} = 0$ the other two multipliers are:

$$\lambda_\sigma^i = \mu_i \left(\frac{1-\tilde{p}_i}{1-\xi}\right)\left(\frac{1-\gamma}{\gamma}\right)$$

and:

$$\lambda_B^i = \mu_i \left[1 - \xi \left(\frac{1-\tilde{p}_i}{1-\xi}\right)\left(\frac{1-\gamma}{\gamma}\right)\right]$$

For this to be an equilibrium we require $\lambda_B^i > 0$ and thus:

$$1 > \xi \left(\frac{1-\tilde{p}_i}{1-\xi}\right)\left(\frac{1-\gamma}{\gamma}\right)$$

which implies:

$$\frac{1-p_i(z_R)}{p_i(z_R)}\gamma < \frac{1-\tilde{p}_i}{\tilde{p}_i} \tag{2.40}$$

Thus whenever equation (2.40) holds, the equilibrium will feature positive collateral $\eta_i > 0$ and the banks will make zero expected profit.

If instead equation (2.40) does not hold, then it follows from above that $\lambda_B^i = 0$ and thus the banks will make positive profits. Combining $\frac{\partial \mathscr{L}}{\partial \eta_i}$ and $\frac{\partial \mathscr{L}}{\partial T_i}$ when $\lambda_B^i = 0$ implies that $\lambda_\eta^{i-} > 0$ and thus $\eta_i = 0$. It follows from this that whenever (2.40) fails to hold, banks make positive profits in expectation and $\eta_i = 0$.

2.6.2 Proof of proposition 2.2

Collateral when $\sigma > \xi$ and the incentive compatibility constraint does not bind is:

$$\eta_i^* = \frac{1}{\gamma}(\sigma - \xi)\left(\frac{1-\xi}{1+r}\right)^{\frac{v}{1-v}} \left(\frac{v\tilde{p}_i}{1-\xi+(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}\right)^{\frac{1}{1-v}}$$

To see whether this is increasing in σ, simply consider the derivative of η_i^* with respect to σ which can be written as:

$$\frac{\partial \eta_i^*}{\partial \sigma} = \frac{1}{\gamma}\left(\frac{1-\xi}{1+r}\right)^{\frac{v}{1-v}}\left(\frac{v\tilde{p}_i}{1-\xi+(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}\right)^{\frac{1}{1-v}}$$
$$\times\left(1-\frac{\frac{1}{1-v}(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}{1-\xi+(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}\right)$$

For this derivative to be positive we require:

$$1 > \frac{\frac{1}{1-v}(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}{1-\xi+(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)}$$

which simplifies to:

$$1-\xi > \frac{v}{1-v}(\sigma-\xi)\left(\frac{1}{\gamma}-1\right)(1-\tilde{p}_i)$$

For this to hold for all $\sigma \in [0,1]$ we require the above to hold at $\sigma = 1$ which implies:

$$\left(\frac{1-v}{v}\right)\left(\frac{\gamma}{1-\gamma}\right) > (1-\tilde{p}_i)$$

Thus so long as the above condition holds, the derivative of η_i^* with respect to σ is always strictly positive and thus η_i^* increases at higher values of σ.

2.6.3 Proof of proposition 2.4

Note that for any σ and any η , the non-linear section of the frontiers are such that: $\Pi_H^*(\eta) > \Pi_L^*(\eta)\ \forall\sigma \in [0,1)$. Note also that $\Pi_i^B(\eta) \geq \Pi_i^*(\eta)\ \forall\eta \in \left[0,\bar{\eta}_i^A\right]$ and thus it follows that the frontiers cross exactly once if the crossing point of Π_L^B and Π_H^B occurs after η_L^A. For this to occur, first define the crossing point between Π_L^B and Π_H^B by $\tilde{\eta}$ which can be found from the following equation:

$$\left(\frac{p_H(z_L)}{\sigma(1+r)}\right)^{\frac{v}{1-v}}\left[v^{\frac{v}{1-v}}-v^{\frac{1}{1-v}}\right]+\frac{1-p_H(z_L)}{p_H(z_L)}\gamma\tilde{\eta}=$$
$$\left(\frac{p_L(z_L)}{\sigma(1+r)}\right)^{\frac{v}{1-v}}\left[v^{\frac{v}{1-v}}-v^{\frac{1}{1-v}}\right]+\frac{1-p_L(z_L)}{p_L(z_L)}\gamma\tilde{\eta}$$

Rearranging yields an equation for $\tilde{\eta}$:

$$\tilde{\eta}=\frac{1}{\gamma}\left(\frac{v}{\sigma(1+r)}\right)^{\frac{v}{1-v}}(1-v)\left([p_H(z_L)]^{\frac{v}{1-v}}-[p_L(z_L)]^{\frac{v}{1-v}}\right)\left(\frac{p_H(z_L)\,p_L(z_L)}{p_H(z_L)-p_L(z_L)}\right)$$

Now note that for single crossing we require:

$$\tilde{\eta}\geq\eta_L^A$$

Substituting the equations for $\tilde{\eta}$ and η_L^A and simplifying yields:

$$(1-v)\left([p_H(z_L)]^{\frac{v}{1-v}}-[p_L(z_L)]^{\frac{v}{1-v}}\right)\left(\frac{p_H(z_L)\,p_L(z_L)}{p_H(z_L)-p_L(z_L)}\right)\geq\frac{v}{\sigma}\left(\frac{\sigma-\xi}{1-\xi}\right)[p_L(z_L)]^{\frac{1}{1-v}}$$

Note that the RHS is maximised when $\sigma=1$ thus the above will hold for any $\sigma\in[0,1]$ if the following holds:

$$\left(\frac{1-v}{v}\right)\left([p_H(z_L)]^{\frac{v}{1-v}}-[p_L(z_L)]^{\frac{v}{1-v}}\right)\left(\frac{p_H(z_L)\,p_L(z_L)}{p_H(z_L)-p_L(z_L)}\right)\geq[p_L(z_L)]^{\frac{1}{1-v}}$$

This simplifies and rearranges to:

$$\left(\frac{\tilde{p}_H}{\tilde{p}_L}\right) \geq \left[\frac{1}{1-v}\left(\frac{\tilde{p}_H}{\tilde{p}_L} - v\right)\right]^{1-v}$$

Now note that at $\frac{\tilde{p}_H}{\tilde{p}_L} = 1$ the above holds with equality i.e.

$$1 = \left[\frac{1}{1-v}(1-v)\right]^{1-v}$$

Also note that:

$$\frac{\partial}{\partial\left(\frac{\tilde{p}_H}{\tilde{p}_L}\right)}\left\{\left(\frac{\tilde{p}_H}{\tilde{p}_L}\right) - \left[\frac{1}{1-v}\left(\frac{\tilde{p}_H}{\tilde{p}_L} - v\right)\right]^{1-v}\right\} = 1 - \left[\frac{1-v}{\frac{\tilde{p}_H}{\tilde{p}_L} - v}\right]^{v}$$

This derivative is greater than zero for all $\frac{\tilde{p}_H}{\tilde{p}_L} > 1$ and thus it follows that $\eta_L^A < \tilde{\eta}$ and thus Γ_L and Γ_H cross exactly once at a point $\gamma > \eta_L^B$.

2.6.4 Proof of proposition 2.5

Equation (2.36) can be written for the interval $\sigma \in (\xi, 1]$ as:

$$\begin{aligned}\phi_L &= \left(\frac{\frac{\tilde{p}_L}{\tilde{p}_H}\left(1-\sigma+(\sigma-\xi)\left(\tilde{p}_H+\frac{1}{\gamma}(1-\tilde{p}_H)\right)\right)}{1-\sigma+(\sigma-\xi)\left(\tilde{p}_L+\frac{1}{\gamma}(1-\tilde{p}_L)\right)}\right)^{\frac{v}{1-v}}\left[v^{\frac{v}{1-v}} - v^{\frac{1}{1-v}}\right] \\ &\quad - \left[v^{\frac{v}{1-v}} - v^{\frac{1}{1-v}}\left(\frac{1-\sigma+(\sigma-\xi)\left(\tilde{p}_H+\frac{1}{\gamma}\left(\frac{\tilde{p}_H}{\tilde{p}_L}-\tilde{p}_H\right)\right)}{1-\sigma+(\sigma-\xi)\left(\tilde{p}_H+\frac{1}{\gamma}(1-\tilde{p}_H)\right)}\right)\right]\end{aligned}$$

We can write the derivative of this with respect to σ as:

$$\begin{aligned}\frac{\partial \phi_L}{\partial \sigma} &= \frac{1}{\gamma} v^{\frac{1}{1-v}} \frac{1}{\left(1-\sigma+(\sigma-\xi)\left(\tilde{p}_H+\frac{1}{\gamma}(1-\tilde{p}_H)\right)\right)^2}\left((1-\xi)\left(\frac{\tilde{p}_H}{\tilde{p}_L}-1\right)\right) \\ &\quad -\frac{v}{1-v}\left(v^{\frac{v}{1-v}}-v^{\frac{1}{1-v}}\right)\left(\frac{\frac{\tilde{p}_L}{\tilde{p}_H}\left(1-\sigma+(\sigma-\xi)\left(\tilde{p}_H+\frac{1}{\gamma}(1-\tilde{p}_H)\right)\right)}{1-\sigma+(\sigma-\xi)\left(\tilde{p}_L+\frac{1}{\gamma}(1-\tilde{p}_L)\right)}\right)^{\frac{v}{1-v}-1} \\ &\quad \left(\frac{\frac{\tilde{p}_L}{\tilde{p}_H}(1-\xi)\left(\frac{1}{\gamma}-1\right)(\tilde{p}_H-\tilde{p}_L)}{\left(1-\sigma+(\sigma-\xi)\left(\tilde{p}_L+\frac{1}{\gamma}(1-\tilde{p}_L)\right)\right)^2}\right)\end{aligned}$$

For the derivative to be monotonically increasing we require:

$$\left(\frac{1-\sigma+(\sigma-\xi)\left(\tilde{p}_L+\frac{1}{\gamma}(1-\tilde{p}_L)\right)}{1-\sigma+(\sigma-\xi)\left(\tilde{p}_H+\frac{1}{\gamma}(1-\tilde{p}_H)\right)}\right)^{\frac{1}{1-v}}\left(\frac{\tilde{p}_H}{\tilde{p}_L}\right)^{\frac{v}{1-v}} > (1-\gamma)\,\tilde{p}_L$$

This will always hold as the left-hand side is strictly greater than 1 while the right-hand side is strictly less than one. Thus we can conclude that:

$$\frac{\partial \phi_L}{\partial \eta} > 0$$

As $\phi_L < 0$ when $\sigma \leq \xi$ and $\phi_L > 0$ when $\sigma = 1$, the above condition is sufficient to show that there exists a threshold $\bar{\sigma}_L \in (0,1)$ at which point $\phi_L = 0$.

Chapter 3

A Model of Credit Rationing in SME Loan Applications

3.1 Introduction

This paper creates a modelling framework for the study of SME loan applications that is consistent with existing survey data. Specifically, it aims to capture several observable features of the loan market. First, firms choose whether they apply for a loan, and if they do not apply for a loan it may be due to one of several reasons. For example, firms may not apply for a loan because they do not need a loan. Other firms may not apply even though they would want a loan because they believe they will not be able to obtain a loan. A firm's loan application may not be successful; a firm may receive only part of the funding it had requested, or it may have its loan application rejected outright. Firms may also choose to decline a loan offer they thought was not acceptable. All of these cases are observed situations in the data and suggest the existence of different types of credit rationing.

This paper presents a modelling framework that is consistent with outcomes of the SME loan market observed in the 'Survey on the access to finance of enterprises' (SAFE). Firms have access to some external funding with which they can attempt to fund a project of fixed investment size. The model presented in this paper is able to distinguish between two reasons for firms not to apply for a loan. Firms that do not require a bank loan are able to fund the project without a bank loan and so do not apply for a loan. Firms that do not think they will receive a bank loan choose not to apply but are unable to invest in the project without bank financing.

A key feature of the model is that there is uncertainty regarding the cost of the project at the point in time that the firm applies for a loan. Unlike other models of credit rationing which use adverse selection to generate credit rationing, such as the classic paper Stiglitz and Weiss (1981), firms are also unaware of the cost. In these adverse selection models, credit rationing occurs when banks are unable to distinguish between two firms of different types. However, these models are unable to explain the full range of loan outcomes and reasons why firms choose not to apply for loans. This paper explicitly models a loan application phase, where both banks and firms face uncertainty regarding the cost of the firm's project. Firm heterogeneity at the application phase results in firms endogenously choosing whether to apply for a loan or not. The uncertainty firms face at the application stage allows firms that apply for loans to experience the full range of loan outcomes observed in the data.

Analysis of the SAFE survey data suggests that competition in the banking sector may be a key determinant of SME loan outcomes. To address this possibility, I consider a differentiated banking sector using the spatial differentiation model of Hotelling (1929).

This paper is related to the extensive literature on credit rationing. An overview of the credit rationing literature can be found in Jaffee and Stiglitz (1990) who distinguish between several types of credit rationing. Examples of papers which feature what Jaffee and Stiglitz (1990) term 'redlining' where all firms of a given type are denied credit or 'pure credit rationing' where firms of a type are randomly denied credit can be found in Freimer and Gordon (1965), in a perfect information setting and Jaffee and Modigliani (1969) and Stiglitz and Weiss (1981) in an asymmetric information setting. In addition, Jaffee and Russell (1976) and Kjenstad et al. (2015) also discuss the case of 'price rationing' where borrowers obtain a loan but not of the desired size at the interest rate charged to them. Additionally, this paper is related to the literature on competition in the lending market. Villas-Boas and Schmidt-Mohr (1999) consider how spatial differentiation in the banking sector affects a credit market model with asymmetric model. As in their paper I introduce differentiation in the credit market using the Hotelling (1929) model of spatial differentiation.

This paper extends this literature by introducing an application phase, where firms choose whether to apply for a loan. This paper also provides a theoretical contribution in that there is the possibility of 'pure credit rationing' where the loan applications of observationally equivalent firms are rejected at random in a model without asymmetric information. This result is made possible because of the additional loan application phase. In the model, banks compete for loan applications rather than loans. Banks commit to offering a loan contract conditional on the realisation of the cost uncertainty with a given probability to firms. In some situations they grant loss-making loans with positive probability to encourage additional loan applications.

The introduction of a loan application phase allows for self-selection into the loan market and additional competition considerations. Parlour and Rajan (2001) offer an alternative model of competition in credit markets where many lenders simultaneously offer loan contracts to a borrower, with the borrower able to accept more than one contract and find that a competitive banking sector can still generate positive profit in equilibrium. Their simultaneous contracting framework however is not well suited to analysing survey data based on a single loan application as borrowers are able to accept multiple loan contracts. While this may be an appealing aspect of their model, the SAFE survey data only asks firms about the outcome of their last loan application.

3.1.1 Motivating Data

The 'Survey on the access to finance of enterprises' (SAFE) conducted jointly by the European Commission and the ECB is a bi-annual survey that has provided evidence of the financing conditions of Small and medium-sized enterprises (SMEs) since 2009. In order to provide summary statistics by country, I obtained the confidential dataset for all the waves between 2009H1 and 2013H1 inclusive. I focus on two survey questions and their responses for 11 Eurozone countries.[1] The first

[1] Data was available for other other EU countries but not in every wave. I restrict my sample to countries that appeared in every wave.

is Q7A part a) which asks firms whether they have applied for a bank loan in the last six months. The second is Q7B part a) which asks the firms that responded that they had applied for a loan what the outcome of their loan application was.

The SAFE questionnaire breaks the response of whether the firm had made a loan application into four possibilities. First, firms could respond that they had applied for a bank loan. Second, firms could state that they did not apply for a bank loan because of the possibility that their application would get rejected. Third, firms could state that they did not apply for a bank loan because they had sufficient internal funds. Finally, they could say they did not apply for a bank loan because of another reason. Ignoring the last response, which does not add any additional information, the survey still distinguishes between two possible reasons why a firm may not apply for a bank loan, either they do not require a loan or they do not think they will be able to obtain a loan.

Table 3.1 summarises the mean response to the loan application question by each country over the period 2009H1 to 2013H1 inclusive. The difference in the proportion of firms applying for loans across country is perhaps not surprising, but there are still interesting differences. The mean rate of firms applying for loans in the included countries is 24%. Italy, Spain, France and Greece all exhibit loan application rates higher than the mean while countries like Finland, Ireland and the Netherlands exhibit loan application rates lower than the mean. The dataset, includes the years following the financial crisis and it is likely that part of this variation may be due to how each country fared following the financial crisis. However, there are differences even amongst the so-called PIIGS, whose experience of the financial crisis was worse than that of the rest of Europe, with Ireland a particular outlier of this group. While Italy, Spain, Greece feature a higher than average loan application rate, the application rate in Portugal is slightly below the mean at 21% while the application rate in Ireland is one of the lowest of the sampled countries at 17%.

Turning now to the outcome of these loan applications, the SAFE questionnaire breaks the responses into one of four outcomes. First, firms could receive the full loan amount that they applied for. Second, the firm only received part of the loan that they had requested[2]. Third, the firm's loan application was rejected outright by the bank. Finally, the firm may deem the cost of the loan too high and reject the terms offered to them by the bank. In addition, not all firms apply for bank loans.

Table 3.2 summarises the response to the loan outcome question by each country. Again, there are differences in the rejection rates across countries. Of particular interest is the variation in outright rejections of loans where Greece, Ireland and the Netherlands all have a rejection rate far above the mean rejection rate of 13%.

The Netherlands appears to be an interesting outlier, having the highest loan rejection rate of 24% of the sampled countries and one of the lowest application rates of 13%. The rejection rate in the Netherlands is comparable to Greece and Ireland, while the application rate is comparable to Ireland, Portugal and Finland. Unlike Greece, Portugal and Ireland the Netherlands is one of the so-called PIIGS

[2]The wording of this question response has changed slightly between waves. In the first wave, firms were only able to respond 'Applied but only got part of it'. In later waves, this was broken down into two responses 'Applied and got most of it [BETWEEN 75% AND 99%] ' and 'Applied but only got a limited part of it [BETWEEN 1% AND 74%]'. To allow for comparison between waves, I group these two responses together and treat them as a response that the firm only receive part of the loan.

Table 3.1: Q7a: Applications for Bank Loans (Last 6 Months), by Country

	AT	BE	DE	ES	FI	FR	GR	IE	IT	NL	PT
Applied	0.23	0.26	0.24	0.34	0.16	0.31	0.29	0.17	0.34	0.14	0.21
Did not Apply: Feared Rejection	0.03	0.06	0.05	0.07	0.01	0.05	0.12	0.14	0.04	0.09	0.06
Did not Apply: Sufficient Funds	0.62	0.51	0.56	0.37	0.62	0.43	0.27	0.51	0.39	0.55	0.32
Did not Apply: Other	0.13	0.18	0.15	0.23	0.21	0.21	0.33	0.18	0.23	0.22	0.40

Source: ECB 'Survey on the access to finance of enterprises' (SAFE)

Notes: Table excludes respondents who reported Don't Know and the columns are re-weighted in order to sum to 1. Values are the average fraction of respondents for all waves between 2009H1 and 2013H1.

Table 3.2: Q7b: Result of Bank Loan applications (Last 6 Months), by Country

	AT	BE	DE	ES	FI	FR	GR	IE	IT	NL	PT
Fully Funded	0.83	0.76	0.81	0.51	0.85	0.80	0.40	0.48	0.63	0.50	0.59
Part Funded	0.13	0.14	0.11	0.31	0.08	0.09	0.34	0.25	0.24	0.18	0.25
Refused Terms	0.02	0.03	0.02	0.03	0.01	0.02	0.03	0.05	0.02	0.08	0.02
Rejected	0.03	0.07	0.06	0.15	0.06	0.10	0.23	0.23	0.12	0.24	0.13

Source: ECB 'Survey on the access to finance of enterprises' (SAFE
Notes: Table excludes respondents who reported Don't Know and the columns are re-weighted in order to sum to 1. Values are the average fraction of respondents for all waves between 2009H1 and 2013H1.

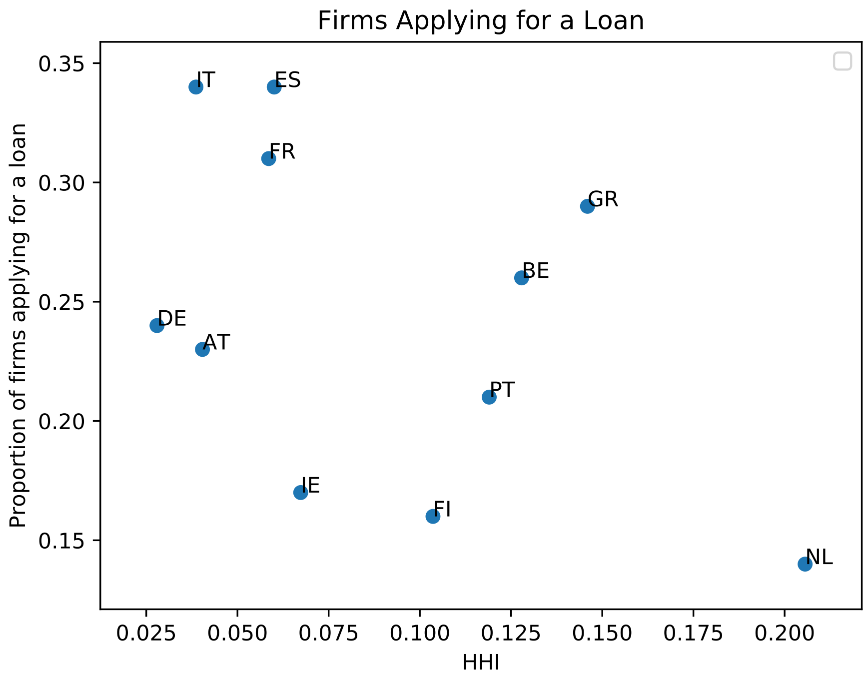

Figure 3.1: Proportion of Firms applying For a Loan vs HHI

Source: ECB Banking Structural Financial Indicators and SAFE survey data

that suffered most during the financial crisis. One possible explanation for the differences in the loan market observed in the Netherlands is that the Netherlands has the most concentrated banking sector of the countries analysed as measured by a mean Herfindahl–Hirschman Index (HHI) of total assets between 2009 and 2013 of 0.21.

To analyse the relationship between competition in the banking sector and the outcome of firm loans more formally, I use HHI data of total assets from the ECB. Figure 3.1 plots the HHI of the banking sector in each country with the proportion of firms applying for loans, which features weak negative correlation (-0.47), suggesting that fewer firms apply for loans in countries that feature less competition in the banking sector. Figure 3.2 plots the HHI of the banking sector in each country

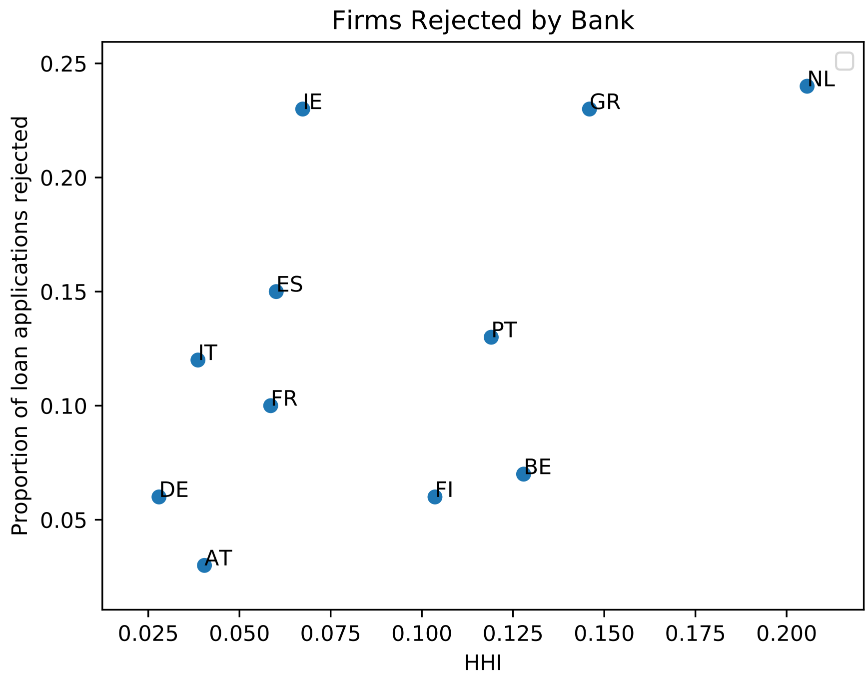

Figure 3.2: Proportion of Applications rejected by Bank and HHI

Source: ECB Banking Structural Financial Indicators and SAFE survey data

against the proportion of firms that apply for a loan and are rejected. This exhibits weak positive correlation (0.55), suggesting that firms are more likely to get their loan application rejected in countries that has less competition in the banking sector.

3.2 The Model

The model presented here is a partial equilibrium model of a credit market with firm-specific technology shocks and imperfect financial markets. The credit market has an explicit loan application phase, where firms choose whether or not to apply for a bank loan. The bank then chooses whether to reject the firm's loan application and, if the loan application is accepted, what terms to offer the firm. The firm then chooses whether to accept or reject the bank loan.

A risk-neutral firm has access to a project which requires a fixed level of investment K which is common to all firms. The project generates a stochastic rate of return $z \in \mathbb{R}^+$. The stochastic rate of return is realised at the end of the period and is not known at any point during the financing process. The shock $z \in \mathbb{R}^+$ is a random variable distributed according to a Pareto distribution with a cdf given by

$$G(z) = 1 - \left(1 + \frac{z}{\lambda}\right)^{-\eta} \tag{3.1}$$

where $\eta > 1$ and $\lambda > 0$ are scale parameters.

In addition to the investment level K, the firm must also pay a stochastic fixed cost in order to produce which takes one of two discrete values $\xi \in \{\xi_L, \xi_H\}$, with $\xi_L < \xi_H$. The fixed cost is realised after the source of the firm's funding has been arranged and thus the firm must choose whether to apply for a bank loan without full information of the cost of the project. It is assumed that both values of ξ have an equal chance of being realised.

The firm is able to finance the investment through a mixture of bank loans b and equity e. Issuing equity is subject to an exogenous cost function, which is increasing and convex in the amount of equity issued. In addition, there is a maximum amount of equity that each firm is able to issue $\bar{E} \in \mathbb{R}^+$. This upper-bound varies across each firm, is drawn at the beginning of the period, before the loan application is made and is observed by both the bank and firm. To simplify the analysis, the cost of equity is assumed to have the following functional form

$$\Psi(e) = e + \frac{\psi}{2}e^2 \quad e \in [0, \bar{E}] \tag{3.2}$$

If a firm wishes to obtain a bank loan, they must apply before the realisation of ξ. The loan terms, and whether or not the bank accepts the firm's loan application, depends on both $\bar{E}$ and ξ.

A bank loan consists of a loan quantity $b(\bar{E}, \xi)$, an interest rate $R(\bar{E}, \xi)$ charged on the loan and a probability that the bank provides a loan $\rho_B(\bar{E}, \xi) \in [0, 1]$. The probability, conditional on $(\bar{E}, \xi)$, that the bank rejects a loan application is $1 - \rho_B(\bar{E}, \xi)$.

Competition in the banking sector is modelled using the spatial competition model of Hotelling (1929). There are two banks located at each end of a line of unit length. Firms are continuously and uniformly distributed along the line. Differentiation in the credit market is captured by the distance between the firms and the banks as well as the associated travel cost. A firm that wishes to obtain a loan from a bank must pay a linear travel cost $t > 0$. The location of the firm on the line is denoted by $d \in [0, 1]$ which is defined as the distance from the left endpoint of the line. Banks are unable to observe the distance between bank and firm an thus cannot condition contract terms on d. Firms are only able to apply to one of the two banks for a loan and each bank offers a menu of contracts $\{b(\bar{E}, \xi_i), R(\bar{E}, \xi_i), \rho_B(\bar{E}, \xi_i)\}_{i \in \{L,H\}}$ conditional on each realisation of η. It is assumed that the bank commits to these contracts.

The loan is risky and firms are able to default on their repayment. If they do default the firm is liquidated and both the firm and the bank receive nothing. The firm is risk-neutral and looks to maximise its end of period payoff and thus will default only in the case where it has insufficient funds to repay its loan. There exists a threshold value of z, below which a firm with observable characteristics $(\bar{E},\xi)$ that obtains a loan featuring interest rate $R(\bar{E},\xi)$ and loan size $b(\bar{E},\xi)$ would default on its loan obligations. This threshold is denoted by the following equation

$$\hat{z}(\bar{E},\xi) = R(\bar{E},\xi)\, b(\bar{E},\xi) \tag{3.3}$$

While firms do not know the realisation of ξ when they apply for a bank loan, it is assumed they fully anticipate the loan contracts $\{b(\bar{E},\xi_i), R(\bar{E},\xi_i), \rho_B(\bar{E},\xi_i)\}_{i\in\{L,H\}}$ that the banks offer conditional on both $\bar{E}$ and ξ. To simplify the analysis I assume that ξ, d and $\bar{E}$ are all distributed independently of each other.

In order to produce, the firm must have secured sufficient funds in order to pay both the investment K and the fixed cost ξ and so the following budget constraint must be satisfied

$$K + \xi \leq b(\bar{E},\xi) + e(\bar{E},\xi) \tag{3.4}$$

The expected profit from producing for a firm $\bar{E}$ that obtains a loan $(b(\bar{E},\xi), R(\bar{E},\xi))$ following the realisation of the fixed cost ξ from the bank located at the left endpoint is

$$\Pr\{\text{Repay}\}\,\mathbb{E}[z|\text{Repay}] - \Pr\{\text{Repay}\}\,R(\bar{E},\xi)\, b(\bar{E},\xi) - \Psi(e(\bar{E},\xi))$$

where $e(\bar{E},\xi)$ is specified by equation (3.4).

Using equation (3.3), the Pareto distribution for z defined by equation (3.1) and the functional form for $\Psi(e)$ yields the following

$$\pi_F(\bar{E},\xi) = \left(\frac{\lambda}{\eta-1}\right)\left(1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)^{-\eta+1} - \left(e(\bar{E},\xi)+\frac{\psi}{2}\left[e(\bar{E},\xi)\right]^2\right) \quad (3.5)$$

The expected profit net of travel costs of the firm depends on the distance it is located from the left endpoint and is

$$u_F(d,\bar{E},\xi) = \pi_F(\bar{E},\xi) - td \quad (3.6)$$

Similarly, if the firm finances their project with a loan from the bank located at the right endpoint receives the following expected profit net of travel costs

$$u_F^C(d,\bar{E},\xi) = \pi_F^C(\bar{E},\xi) - t(1-d) \quad (3.7)$$

where $\pi_F^C(\bar{E},\xi)$ denotes the expected payoff the firm receives if it accepts the contract offered by the bank at the right endpoint.

Following the realisation of ξ, firms are able to reject the loan offer made to them by the bank. If they reject the bank loan, firms are only able to produce if $\bar{E} \geq K+\xi$ so that they have access to sufficient equity to fully finance the project. Otherwise, firms receive an outside option that is normalised to zero. The outside option of the firm therefore depends on the realisation of both $\bar{E}$ and ξ. I assume that the firm does not pay the transport cost if it does not receive a bank loan and so the firm's outside option is independent of d and can be written as follows

$$\bar{u}_F\left(\bar{E},\xi\right)=\begin{cases}\max\left\{\left(\frac{\lambda}{\eta-1}\right)-\left(K+\xi+\frac{\psi}{2}\left[K+\xi\right]^2\right),0\right\} & \bar{E}>K+\xi\\ 0 & \text{Otherwise}\end{cases} \tag{3.8}$$

Firms are more likely to accept a bank loan if they are located closer to a bank. The total expected payoff from applying for a bank loan from the left bank is

$$\Pi_F\left(d,\bar{E}\right)=\sum_i\frac{1}{2}\left[\rho_B\left(\xi_i\right)\max\left\{u_F\left(d,\bar{E},\xi_i\right),\bar{u}_F\left(\bar{E},\xi_i\right)\right\}+\left(1-\rho_B\left(\xi_i\right)\right)\bar{u}_F\left(\bar{E},\xi_i\right)\right] \tag{3.9}$$

Firm are distributed uniformly along the line and banks face symmetric problems, thus to simplify the exposition I will focus on the bank located at the left endpoint.

The expected payoff the firm receives should they choose not to apply for a bank loan depends on the value of $\bar{E}$ relative to ξ_H and ξ_L as this will determine after which realisations of ξ, if any, the firm is able to fund the project without a bank loan. The total expected payoff the firm receives when choosing not to apply for a bank loan is

$$\bar{U}_F\left(\bar{E}\right)=\begin{cases}\sum_i\frac{1}{2}\max\left\{\left(\frac{\lambda}{\eta-1}\right)-K-\xi_i+\frac{\psi}{2}\left[K+\xi_i\right]^2,0\right\} & \bar{E}>K+\xi_H\\ \max\left\{\frac{1}{2}\left[\left(\frac{\lambda}{\eta-1}\right)-\left(K+\xi_L+\frac{\psi}{2}\left[K+\xi_L\right]^2\right)\right],0\right\} & K+\xi_L<\bar{E}\leq K+\xi_H\\ 0 & \text{Otherwise}\end{cases} \tag{3.10}$$

Firms would prefer to apply for a loan from the left bank than not whenever

$\Pi_F(d,\bar{E}) > \bar{U}_F(\bar{E})$. The value of not applying for a loan is independent of the location of the firm d and thus *ceteris paribus* firms that are located further away from a bank are less likely to apply for a bank loan. There are two possible reasons why a firm may choose not to apply for a bank loan. First, if $\bar{E}$ is low, they require a large loan from the bank in order to produce, and the firm may anticipate that the bank would not be willing to supply them with financing. These firms would choose not to apply for a bank loan and would be unable to produce. This corresponds to firms whose survey response was 'Did not apply because of possible rejection'. The second possibility, is that $\bar{E}$ is sufficiently high that firms find it is preferable to finance the project solely through equity. This corresponds to the survey response 'Did not apply because of sufficient internal funds'.

Banks are risk neutral and have access to a perfectly elastic supply of deposits at a cost of $1+r>1$. The expected profit of a bank providing a loan $(b(\bar{E},\xi),R(\bar{E},\xi))$ to a firm $(d,\bar{E},\xi)$ is simply the expected value of the promised repayment less the cost of funds

$$\Pr\{\text{Repay}\}R(\bar{E},\xi)b(\bar{E},\xi)-(1+r)b(\bar{E},\xi)$$

Using equation (3.3) and the Pareto distribution for z defined by equation (3.1) yields the following

$$\pi_B(\bar{E},\xi)=\left(1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)^{-\eta}\hat{z}(\bar{E},\xi)-(1+r)b(\bar{E},\xi) \tag{3.11}$$

While the bank is unable to condition the terms of the bank loan on d, the bank takes into account the impact of d on the likelihood that the firm accepts the terms of their loan offer. As the mass of firms is uniformly distributed along d, from the point of view of the left-hand bank, the probability of a firm that applies for a loan with characteristics $(\bar{E},\xi)$ accepting a loan is

$$\rho_F(\bar{E},\xi)=\begin{cases}1 & \text{if } \pi_F(\bar{E},\xi)-d^*(\bar{E})t>\bar{u}_F(\bar{E},\xi)\\ 0 & \text{if } \pi_F(\bar{E},\xi)<\bar{u}_F(\bar{E},\xi)\\ \frac{1}{d^*(\bar{E})t}\left[\pi_F(\bar{E},\xi)-\bar{u}_F(\bar{E},\xi)\right] & \text{Otherwise}\end{cases} \tag{3.12}$$

where $d^*(\bar{E})$ is the furthest distance from the bank that a firm is willing to apply for a loan from the left-hand bank rather than the right-hand bank.[3] Consider a bank offering a set of contracts $\{b(\bar{E},\xi_i),R(\bar{E},\xi_i),\rho_B(\bar{E},\xi_i)\}_{i\in\{L,H\}}$ to a $\bar{E}$ firm, the total expected profit the bank would receive from this is

$$\Pi_B(\bar{E})=\sum_i\frac{1}{2}\left[\pi_B(\bar{E},\xi_i)\rho_B(\bar{E},\xi_i)\rho_F(\bar{E},\xi_i)\right] \tag{3.13}$$

The timing of the model is as follows. First, firms observe their upper-bound on equity issuance $\bar{E}$ and their location from the banks d, and decide whether to apply for a loan or not and if so, to which bank. Second, the value of fixed cost, ξ, is publicly revealed and the bank makes its loan decision, by offering contract terms $(b(\bar{E},\xi),R(\bar{E},\xi))$ to the firm or rejecting the loan application with probabil-

[3] The firm chooses between three options, applying for a loan from the left bank, applying for a loan from the right bank and not applying for a loan. As there is no cost to applying to a loan and in order to simplify the contracting problem, I model the case of firms not applying for a loan as if they had applied to the bank that offered them the best contracting terms and then rejected the contract with probability one.

ity $\rho_B(\bar{E},\xi)$. Third, the firm chooses whether to accept the loan terms, or to reject the bank's loan terms and take its outside option. Finally, the value of z is realised and the firm chooses to repay or default on any loan it may have received.

3.3 Contracting Problem

Consider the problem faced by the bank located at the left endpoint when faced with firms with equity cost parameter $\bar{E}$. The bank must choose a set of contracts $\{b(\bar{E},\xi_i), R(\bar{E},\xi_i), \rho_B(\bar{E},\xi_i)\}_{i\in\{L,H\}}$ that maximises the expected profit subject to the participation constraints of the firm. The bank's contracting problem can then be written as

$$\max_{\{b(\bar{E},\xi_i),R(\bar{E},\xi_i),\rho_B(\bar{E},\xi_i)\}_{i\in\{L,H\}}} \Pi_B(\bar{E})\, d^*(\bar{E}) \tag{3.14}$$

subject to

$$\sum_i \frac{1}{2}\left[u_F(d^*(\bar{E}),\bar{E},\xi_i)\,\rho_B(\xi_i)\,\mathbb{1}\{u_F(d^*(\bar{E}),\bar{E},\xi_i) > \bar{u}_F(\bar{E},\xi_i)\}\right] \geq$$
$$\sum_i \frac{1}{2}\left[u_F^C(d^*(\bar{E}),\bar{E},\xi_i)\,\rho_B^C(\xi_i)\,\mathbb{1}\left\{u_F^C(d^*(\bar{E}),\bar{E},\xi_i) > \bar{u}_F(\bar{E},\xi_i)\right\}\right] \tag{3.15}$$

$$K + \xi_i \leq b(\bar{E},\xi_i) + e(\bar{E},\xi_i) \quad \forall i \in \{L,H\} \tag{3.16}$$

$$e(\bar{E},\xi_i) \leq \bar{E} \quad \forall i \in \{L,H\} \tag{3.17}$$

$$0 \leq \rho_B(\bar{E},\xi_i) \leq 1 \quad \forall i \in \{L,H\} \tag{3.18}$$

There are two participation constraints that are relevant for the bank's problem. First, firms may choose to reject a contract offered to them and instead receive their outside option set out by equation (3.8). Second, firms could choose to apply to the competing bank. Firms closer to the left bank with $d \in [0, \frac{1}{2})$ will have a natural preference to apply to the left bank, but if the bank offered better contract terms than its competitor, it would be able to capture a larger share of the loan applications. This second participation constraint is set out by equation (3.15), which determines the share of loan applications made to the left bank, $d^* (\bar{E})$. Equation (3.16), equation (3.17) and equation (3.18) are feasibility constraints.

Conditional on the realisation of ξ, one of the two participation constraints will be relevant to the problem, either the firm will reject the loan offer with some positive probability as determined by equation (3.8), or the firm will accept the loan offer with probability one and competition from the other bank will be relevant. There are three possible cases to consider. The first, that of *full-scale competition*[4] occurs in which case $\rho_F (\bar{E}, \xi_L) = \rho_F (\bar{E}, \xi_H) = 1$ and firms will accept the loan offered regardless of their distance from the bank. The second case, is that of *local monopoly* where $\rho_F (\bar{E}, \xi_L) < 1$ and $\rho_F (\bar{E}, \xi_H) < 1$. In this case, the presence of the competing bank does not impact the contracts offered and both banks will act as a local monopolist to firms located sufficiently close to them, while firms located away from the banks, around $d = \frac{1}{2}$, will choose not apply for a loan. The third case, is the *mixed case* where neither of the above two cases holds. Which of these cases holds depends on $\bar{E}$ and t.

Finally, as both the left and the right bank face symmetric problems, in equilibrium both banks offer the same set of contracts to firms located at each end of the

[4]The terms that describe the cases here are commonly used in Spatial competition models such as Stole (1995) and in Villas-Boas and Schmidt-Mohr (1999).

line and thus in equilibrium $d^{*}(\bar{E}) = \frac{1}{2}$.

3.3.1 Equilibrium Loan Size

Before analysing the three cases set out above, I first analyse the part of the solution that is common to all three, namely the equilibrium loan size. From the first order conditions from the bank's constrained optimisation problem, if the upper-bound on equity does not bind, the equilibrium contract occurs at the point where the firm's marginal rate of substitution between the loan size $b(\bar{E},\xi)$ and the interest rate $R(\bar{E},\xi)$ equals that of the bank. It follows from the convexity of the equity cost function that the following inequality must hold in equilibrium

$$(1+\psi\cdot e(\bar{E},\xi))\left(\frac{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}\right)\leq 1+r \tag{3.19}$$

Equation (3.19) holds with equality whenever $e(\bar{E},\xi)<\bar{E}$ and can be rearranged to yield the following equation for $e(\bar{E},\xi)$ in terms of the default cutoff $\hat{z}(\bar{E},\xi)$

$$e(\bar{E},\xi)=\frac{1}{\psi}\left[(1+r)\left(\frac{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}\right)-1\right] \tag{3.20}$$

Using equation (3.4) yields an equation for $b(\bar{E},\xi)$ in terms of the default cutoff $\hat{z}(\bar{E},\xi)$

$$b(\bar{E},\xi)=K+\xi+\frac{1}{\psi}-\left(\frac{1+r}{\psi}\right)\left(\frac{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}\right) \tag{3.21}$$

A requirement for the firm with $e(\bar{E},\xi)<\bar{E}$ to want to borrow a positive quantity of debt is

$$1 + r < 1 + \psi(K + \xi) \tag{3.22}$$

This requires the marginal cost of financing the entire project through equity to be less than the risk free interest rate that the bank must pay on its deposits. I assume that equation (3.22) is satisfied at both ξ_L and ξ_H so that the optimal loan contract features a positive quantity of debt.

If on the other hand the upper-bound on equity binds such that $e(\bar{E}, \xi) = \bar{E}$, equation (3.19) holds as a strict inequality and the loan size can be found from the firm's budget constraint, equation (3.4) as

$$b(\bar{E}, \xi) = K + \xi - \bar{E} \tag{3.23}$$

Whether the upper-bound on equity holds in equilibrium or not depends not only on how tight the constraint on equity issuance is, $\bar{E}$, but also on the default probability. From the derivative of equation (3.20) with respect to $\hat{z}(\bar{E}, \xi)$ it follows that contracts with higher default probability are more likely to result in the equity constraint binding.

As both the equilibrium equity issuance $e(\bar{E}, \xi)$ and the loan size $b(\bar{E}, \xi)$ have been found in terms of the default probability, $\hat{z}(\bar{E}, \xi)$, the equilibrium loan contract can now be fully summarised by the default probability. The impact of a change in the default probability on the equity issuance and loan size is set out in the proposition below.

Proposition 3.1. *If $e(\bar{E}, \xi) < \bar{E}$, equity issued in the equilibrium contract is strictly* *increasing in the default probability of the contract* $\frac{\partial e(\bar{E},\xi)}{\partial \hat{z}(\bar{E},\xi)} > 0$ *and the debt issued in the equilibrium contract is strictly increasing in the default probability* $\frac{\partial b(\bar{E},\xi)}{\partial \hat{z}(\bar{E},\xi)} < 0$*. If* $e(\bar{E},\xi) = \bar{E}$, $\frac{\partial e(\bar{E},\xi)}{\partial \hat{z}(\bar{E},\xi)} = 0$ *and* $\frac{\partial b(\bar{E},\xi)}{\partial \hat{z}(\bar{E},\xi)} = 0$.

Proof. When the upper-bound on equity is slack, $e(\bar{E},\xi) < \bar{E}$, the first part of the proposition follows directly from differentiating equation (3.20) and equation (3.21) with respect to the default cutoff. When the upper-bound on equity binds, $e(\bar{E},\xi) = \bar{E}$, the second part of the proposition follows immediately from $e(\bar{E},\xi) = \bar{E}$ and equation (3.23). □

Proposition 3.1 shows that as the default probability of the contract increases, the firms would prefer to raise additional equity finance and reduce the size of the loan they obtain from the bank. When the upper-bound on equity binds, the firm is no longer able to reduce the loan size and the equity and debt levels do not respond to an increase in the default probability of the bank loan.

The likelihood of firms applying for a loan and subsequently accepting a loan offer is determined by the profit they obtain from the loan contract. The link between the default probability and the firm profit of a loan contract is set out in the following proposition.

Proposition 3.2. *The profit a firm obtains from a given loan contract,* $\pi_F(\bar{E},\xi)$*, is strictly decreasing in the default probability of a loan contract,* $\hat{z}(\bar{E},\xi)$ *and hence both* $\rho_F(\bar{E},\xi)$ *and* $\Pi_F(d,\bar{E})$ *are weakly decreasing in* $\hat{z}(\bar{E},\xi)$*.*

Proof. By substituting equation (3.20) into the definition of $\pi_F(\bar{E},\xi)$ and differentiating yields $\frac{\partial \pi_F(\bar{E},\xi)}{\partial \hat{z}(\bar{E},\xi)} < 0$. The rest of the proposition follows from the definition of $\rho_F(\bar{E},\xi)$, equation (3.12), and $\Pi_F(d,\bar{E})$, equation (3.9). $\square$

Proposition 3.2 shows that in cases where the default probability $\hat{z}(\bar{E},\xi)$ is higher, firms are less likely to apply for loans and less likely to accept a loan offer should they apply.

Proposition 3.3. *If $e(\bar{E},\xi) < \bar{E}$ then the profit a bank receives from a contract, $\pi_B(\bar{E},\xi)$, is increasing for all equilibrium contracts with $b(\bar{E},\xi) > 0$. Otherwise if $e(\bar{E},\xi) = \bar{E}$, the profit a bank receives from a contract, $\pi_B(\bar{E},\xi)$, is increasing for $\hat{z}(\bar{E},\xi) \in \left[0, \frac{\lambda}{\eta-1}\right]$ and decreasing at $\hat{z}(\bar{E},\xi) > \frac{\lambda}{\eta-1}$.*

Proof. The second part of the proposition follows from substituting in equation (3.23) into the definition of bank profit, equation (3.11), and differentiating. The derivative is increasing for $\hat{z}(\bar{E},\xi) < \frac{\lambda}{\eta-1}$ and has a unique maximum at $\hat{z}(\bar{E},\xi) = \frac{\lambda}{\eta-1}$. The first part of the proposition follows from noting that equation (3.21) and $b(\bar{E},\xi) > 0$ implies $\hat{z}(\bar{E},\xi) < \frac{\lambda}{\eta-1}$. Then from equation (3.21) and equation (3.11), the derivative of $\pi_B(\bar{E},\xi)$ with respect to $\hat{z}(\bar{E},\xi)$ is positive for $\hat{z}(\bar{E},\xi) < \frac{\lambda}{\eta-1}$. $\square$

From the above proposition, there is a unique default probability that maximises the bank profit from the contract, $\pi_B(\bar{E},\xi)$, which occurs at $\hat{z}(\bar{E},\xi) = \frac{\lambda}{\eta-1}$. This default probability can only occur when the upper-bound of equity binds. Otherwise, firms will substitute away from bank debt and increase their equity issuance as the default probability approaches this level.

3.3.2 Full-Scale Competition

I now turn to the case of full-scale competition. When this case holds, all firms apply for loans and will accept the bank's loan offer so that $\rho_F(\bar{E},\xi_L) = \rho_F(\bar{E},\xi_H) =$

1. The first order conditions of the bank's problem yields, in addition to equation (3.19) discussed above, the following equation

$$\Pi_B(\bar{E}) = \frac{t}{2}\left(\frac{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}\right) \tag{3.24}$$

The above equation links the expected default probability of a contract $\hat{z}(\bar{E},\xi)$ to the total expected profit of the bank. This must hold for both ξ_L and ξ_H and thus the default probability remains constant regardless of the realisation of ξ, and $\hat{z}(\bar{E},\xi_L) = \hat{z}(\bar{E},\xi_H)$. Using the definition for $\Pi_B(\bar{E})$ set out in equation (3.13) yields a system of equations along with equation (3.19) which pins down the default probabilities.

While there does not exist a closed form solution for $\hat{z}(\bar{E},\xi)$, the properties of the equilibrium can be found using the implicit function theorem. I will focus on the impact of a change in the parameter t, which governs the market power of banks and hence the level of competition on the banking sector. An increase in t will decrease competition in the banking sector. The relationship between t and $\hat{z}(\bar{E},\xi)$ is set out in the proposition below.

Proposition 3.4. *When a full-scale equilibrium occurs and* $\rho_F(\bar{E},\xi_L) = \rho_F(\bar{E},\xi_H) = 1$*, an increase in t increases the default probability on loans* $\hat{z}(\bar{E},\xi)$ *for all* $\hat{z}(\bar{E},\xi) \in \left[0, \frac{\lambda}{\eta-1}\right]$*. Furthermore, if* $\hat{z}(\bar{E},\xi) > \frac{\lambda}{\eta-1}$ *the bank would make an expected loss by offering the loan contracts and instead rejects the loan application with probability one.*

Proof. In a full-scale equilibrium, the relationship between t and $\hat{z}(\bar{E},\xi)$ is given by equation (3.24). From equation (3.13), the derivative of $\Pi_B(\bar{E})$ is positive whenever $\hat{z}(\bar{E},\xi) \leq \frac{\lambda}{\eta-1}$ and by applying the implicit-function theorem to equation (3.24) the first part of the proposition immediately follows. From equation (3.24) it follows that $\hat{z}(\bar{E},\xi) \leq \frac{\lambda}{\eta-1}$ is a necessary condition for the bank to earn a positive expected profit on the set of contracts, $\Pi_B(\bar{E}) > 0$, and the second part of the proposition immediately follows. □

The last contract term to consider is $\rho_B(\bar{E},\xi)$, the probability that the bank rejects a firm's loan application. As discussed in proposition 3.4, if the bank makes an expected loss from offering the set of loan contracts, banks will always reject the loan applications, setting $\rho_B(\bar{E},\xi) = 0$. Furthermore, it follows from the bank's first order conditions that following the realisation of ξ, the bank will always accept the loan application, $\rho_B(\bar{E},\xi) = 1$, whenever $\pi_B(\bar{E},\xi) \geq 0$. Intuitively, a profit maximising bank would never reject a profitable loan application.

If $\pi_B(\bar{E},\xi_i) < 0$ for some $i \in \{L,H\}$ and $\Pi_B(\bar{E}) > 0$, the bank may choose to reject the loan application with some probability $\rho_B(\bar{E},\xi_i) \in (0,1)$. This is set out formally in the proposition below.

Proposition 3.5. *When a full-scale equilibrium occurs and i) $\pi_B(\bar{E},\xi_i) < 0$, ii) $\pi_B(\bar{E},\xi_j) > 0$ for some $i,j \in \{L,H\}$ and $j \neq i$, and iii) the following condition is met*

$$-\frac{t}{2}\pi_B(\bar{E},\xi_i) < \pi_B(\bar{E},\xi_j)\left(\pi_F(\bar{E},\xi_j) - \frac{t}{2}\right) \tag{3.25}$$

the bank will reject loan applications following the realisation of ξ_i with a probability $\rho_B(\bar{E},\xi_i) \in (0,1)$. Furthermore, the default probability will be given by the *following equation*

$$\rho_B\left(\bar{E},\xi_i\right) = -\frac{\pi_B\left(\bar{E},\xi_j\right)}{\pi_B\left(\bar{E},\xi_i\right)}\left(\frac{\pi_F\left(\bar{E},\xi_j\right)-\frac{t}{2}}{\pi_F\left(\bar{E},\xi_i\right)-\frac{t}{2}}\right) - \frac{\frac{t}{2}}{\left(\pi_F\left(\bar{E},\xi_i\right)-\frac{t}{2}\right)} \tag{3.26}$$

In the case where

$$-\frac{t}{2}\pi_B\left(\bar{E},\xi_i\right) \geq \pi_B\left(\bar{E},\xi_j\right)\left(\pi_F\left(\bar{E},\xi_j\right)-\frac{t}{2}\right) \tag{3.27}$$

the bank rejects the loan application following the realisation of ξ_i with probability one and $\rho_B\left(\bar{E},\xi_i\right) = 0$.

Proof. See Appendix 3.7.1. □

Proposition 3.5 raises the possibility that banks use cross-subsidisation across states, along with random loan rejections to incentivise firms to apply for loans, while limiting the losses they make following the realisation of ξ_i. This occurs because firms compete for loan applications rather than on individual contracts. Banks make a positive profit if the realised cost is ξ_j and are willing to make a loss in the case where the realised cost is ξ_i in order to satisfy the participation constraint of the firms and encourage additional loan applications.

The full-scale competition equilibrium occurs if the firms located at the mid-point of the line apply for a loan and accept the loan terms regardless of the realisation of ξ. This implies that the following must hold for both $i \in \{L,H\}$

$$\frac{t}{2} \leq \pi_F\left(\bar{E},\xi\right) - \bar{u}_F\left(\bar{E},\xi\right) \tag{3.28}$$

The right hand side of equation (3.28) is decreasing in $\hat{z}(\bar{E},\xi)$ and hence decreasing in t when the full-scale competition equilibrium occurs. It follows that an increase in t, and hence a decrease in competition in the banking sector, means it is less likely that the firms located at the mid-point will apply for and accept a bank loan and that the full-scale competition equilibrium will occur only if competition in the banking sector is high and t is sufficiently low. Another implication of equation (3.28) is that all firms will apply for a loan, either to the left bank if they are located at $d \in [0, \frac{1}{2})$ or to the right bank otherwise.

3.3.3 Local Monopoly

I now turn to the equilibrium case at the other end of the spectrum, which is the case of local monopoly. In this case, firms that are located sufficiently far from the banks choose not to apply for a bank and $\rho_F(\bar{E},\xi_i) \in (0,1)\ \forall i \in \{L,H\}$. The first order conditions of the bank's problem can then be written as

$$\pi_B(\bar{E},\xi) = \left(\frac{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)} \right) [\pi_F(\bar{E},\xi) - \bar{u}_F(\bar{E},\xi)] \tag{3.29}$$

This equation holds for both ξ_L and ξ_H. However, unlike in the full-scale competition case discussed above, the default probability is able to vary depending on the realisation of ξ.

As before, no closed form solution can be found but the properties of the equilibrium default probability $\hat{z}(\bar{E},\xi)$ can be found by applying the implicit function theorem to equation (3.29). Another important difference in the local monopoly case is that, so long as the local monopoly remains, the level of competition in the banking sector as governed by t, no longer impacts the equilibrium default probabilities offered to firms. This is evidenced by the fact that both equation (3.19) and equation (3.29) are independent of t.

Turning now to the probability of the probability that the bank reject a firm's loan application, the first order condition for the bank's problem yields the following condition

$$\rho_B(\bar{E},\xi) = \begin{cases} 1 & \text{if } \pi_B(\bar{E},\xi) \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.30}$$

In the local monopoly case, each bank has full monopoly power over the firms that choose to apply to them and there is no need for the bank to cross-subsidise across the two realisations of ξ. Instead banks only offer contracts to firms if they are profitable.

The local monopoly equilibrium occurs if firms located at the mid-point of the line do not apply for a loan. This implies that the following must hold for both $i \in \{L,H\}$

$$\frac{t}{2} > \pi_F(\bar{E},\xi) - \bar{u}_F(\bar{E},\xi) \tag{3.31}$$

This holds if t is sufficiently large. When the above equation is satisfied, firms that are located sufficiently far from the two banks, with d close to $\frac{1}{2}$, choose not to apply to either bank for a loan as they anticipate that the bank will not be able to offer them a loan contract which provides them with a higher payoff than their outside option. Whether or not these firms invest in the project depends on $\bar{E}$ relative to the cost of investment and whether they are able to profitably fund the project through equity alone.

3.3.4 Mixed case

In the mixed case equilibrium, all firms will apply to a bank regardless of their distance from them. Following one realisation of the fixed cost, ξ_i, all firms will accept the bank contract and $\rho_F(\bar{E}, \xi_i) = 1$ while following the other realisation ξ_j $j \neq i$, firms that are located furthest away from the banks will reject the loan offer and $\rho_F\left(\bar{E}, \xi_j\right) < 1$. The firm faces a local monopoly contract following the realisation of ξ_j and a full-competition contract following the realisation of ξ_i.

The contract offered to the firm following a realisation of ξ_j is identical to the local monopoly case and $\hat{z}\left(\bar{E}, \xi_j\right)$ is determined by equation (3.29) and as such is determined independently of t. In the case of $\rho_F(\bar{E}, \xi_i) = 1$, the first order conditions of the bank's problem yields the following equation which determines $\hat{z}(\bar{E}, \xi_i)$

$$\pi_B(\bar{E}, \xi_i) = \frac{t}{2}\left(\frac{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E}, \xi_i)}{1+\frac{1}{\lambda}\hat{z}(\bar{E}, \xi_i)}\right) \tag{3.32}$$

Applying the implicit function theorem applied to equation (3.32), it follows that the derivative of $\hat{z}(\bar{E}, \xi_i)$ with respect to t is positive and thus the default probability associated with the loan contract increases as the market power of the banks increase. The key difference between the mixed and full-scale competition cases is that equation (3.32) no longer contains the expected bank profit but rather the bank profit conditional on ξ_i. As a result, $\hat{z}(\bar{E}, \xi_i)$ is determined independently of the contract offered following the realisation of ξ_j and there is no cross-subsidisation between states. The bank rejects the firm's loan application with probability one if it were to make a loss on the loan, and accepts it with probability one if it were to make a profit, thus the policy for $\rho_B(\bar{E},\xi)$ is the same as in the local monopoly case, set out in equation (3.30).

The mixed case equilibrium occurs if all firms apply for a loan but the firms located at the mid-point of the line will reject the loan offer following the realisation of ξ_i while accepting the loan offer following the realisation of ξ_j. This implies that the following must hold

$$\frac{t}{2} \leq \pi_F(\bar{E},\xi_i) - \bar{u}_F(\bar{E},\xi_i) \tag{3.33}$$

$$\frac{t}{2} > \pi_F\left(\bar{E},\xi_j\right) - \bar{u}_F\left(\bar{E},\xi_j\right) \tag{3.34}$$

The mixed case will occur when t takes an intermediate value, higher than the value required by the full-competition case but lower than the value that would lead to the local monopoly case. In the mixed case, all firms apply for a loan irrespective of the distance from the bank as they all have a positive probability of ξ_i being realised and accepting the loan offer.

While it may be tempting to think j may only take one specific value, this is not the case. There are two reasons a firm may reject a bank loan offer. A firm may prefer not to produce rather than accept the loan terms, in which case it is more likely that $\xi_j = \xi_H$ as the higher fixed cost will lead to worse lending terms. However, it may be that the firm prefers to finance the project through equity only, rather than accept the loan terms and this is more likely when $\xi_j = \xi_L$.

Market power in the banking sector, t, affects the type of equilibrium in the contract market. From equation (3.28), the full-scale competition case occurs when the firms have low market power and t is low. Similarly, from equation (3.31), the local monopoly case occurs when firms have high market power and t is high and the mixed case occurs at intermediate values of t as can be seen from equation (3.33) and equation (3.34).

3.4 Credit Rationing

In their discussion of credit rationing, Jaffee and Stiglitz (1990) set out several definitions of credit rationing. In this section, I focus on three of their definitions and link them to the questionnaire responses in the ECB SAFE survey. I then show how they relate to the properties of the model and how a change in the market power in the banking sector impacts the possibility of credit rationing.

The first type of credit rationing that I focus on is that of 'redlining' which Jaffee and Stiglitz (1990) describe as the case where a lender refuses to grant credit to a borrower at any interest rate. The second type of credit rationing they refer to as 'pure credit rationing' which occurs when some individuals obtain loans while apparently identical individuals are unable to receive the same loan terms. Both of these types of credit rationing correspond to firms in the survey that applied for a loan and were rejected. The survey does not distinguish between these two types of credit rationing. In the model, the bank rejects a firm conditional on $\bar{E}$ and the realisation of ξ with probability $\rho_B(\bar{E},\xi)$. The 'redlining' definition of credit rationing corresponds to $\rho_B(\bar{E},\xi) = 0$ while the 'pure credit rationing' definition corresponds to $\rho_B(\bar{E},\xi) \in (0,1)$.

The possibility of 'redlining' in the full-scale competition case is covered by proposition 3.5. In the local monopoly or mixed equilibrium cases, the possibility of redlining is set out in the following proposition.

Proposition 3.6. *In a local monopoly or mixed equilibrium, a bank makes a loss on a potential loan contract,* $\pi_B(\bar{E},\xi) < 0$*, if and only if* $e(\bar{E},\xi) = \bar{E}$ *and* $\hat{z}(\bar{E},\xi) > \frac{\lambda}{\eta-1}$*. A necessary condition required for* $\pi_B(\bar{E},\xi) < 0$ *is*

$$\left(1+\frac{1}{\eta-1}\right)^{-\eta}\left(\frac{\lambda}{\eta-1}\right) < (1+r)(K+\xi-\bar{E}) \tag{3.35}$$

The bank will reject loan contracts with probability one conditional on the realisation of ξ *and* $\rho_B(\bar{E},\xi) = 0$.

Proof. Equation (3.29) in the local monopoly case and equation (3.32) in the mixed case, a necessary condition for $\pi_B(\bar{E},\xi) < 0$ is that $\hat{z}(\bar{E},\xi) > \frac{\lambda}{\eta-1}$. From proposition 3.3, if $e(\bar{E},\xi) < \bar{E}$ then $\hat{z}(\bar{E},\xi) < \frac{\lambda}{\eta-1}$ and $\pi_B(\bar{E},\xi) > 0$ and the first part of the proposition follows. In addition, proposition 3.3 states the bank maximises profit at $\hat{z}(\bar{E},\xi) = \frac{\lambda}{\eta-1}$. A necessary condition for $\pi_B(\bar{E},\xi) < 0$ is that $\pi_B(\bar{E},\xi) < 0$ at $\hat{z}(\bar{E},\xi) = \frac{\lambda}{\eta-1}$ and $e(\bar{E},\xi) = \bar{E}$. From equation (3.23) this implies $\left(1+\frac{1}{\eta-1}\right)^{-\eta}\left(\frac{\lambda}{\eta-1}\right) < (1+r)(K+\xi-\bar{E})$. The final part of the proposition follows from the bank's first order condition with respect to $\rho_B(\bar{E},\xi)$. □

The proposition above sets out when 'redlining' occurs. The bank profit equation is increasing in the default probability $\hat{z}(\bar{E},\xi)$ up to the point where $\hat{z}(\bar{E},\xi) = \frac{\lambda}{\eta-1}$. In both the local monopoly case and the mixed case, redlining only occurs when $\pi_B(\bar{E},\xi) < 0$ and $\hat{z}(\bar{E},\xi) > \frac{\lambda}{\eta-1}$. This can only occur when

$e(\bar{E},\xi) = \bar{E}$. Intuitively, banks that make negative profit on a contract are able to raise the default probability and hence the profit they make up to the maximum point $\hat{z}(\bar{E},\xi) = \frac{\lambda}{\eta-1}$ and will do so either until they are able to make a positive profit, or the firm is no longer willing to accept the loan offer. As the default probability and the interest rate charged to the firm increases, the firm would only be willing to accept the loan offer when the upper-bound on the amount of equity the firm is able to issue binds and the firm is unable to produce without a bank loan.

The requirements for $\pi_B(\bar{E},\xi) < 0$ set out in proposition 3.6 are independent of t and thus in both the local monopoly case and the mixed case, an increase in market power will not affect which loan applicants will be unable to obtain a bank loan. However, an increase in t will reduce the number of firms that apply for loans due to the fall in $\pi_F(\bar{E},\xi)$. The proportion of loan applicants being 'redlined' will only increase with t if the increase in the number of firms that don't apply and self-finance more than offsets the firms that switch between being redlined and not applying, fearing rejection.

The case of 'redlining' in the full-scale competition case is complicated by the possibility of 'pure credit rationing' occurring. The 'pure credit rationing' case leads to fewer firms being rejected than if they had simply been 'redlined' while the bank makes a negative profit on contracts in this situation. An increase in market power in the banking sector, t, makes it less likely that the full-scale competition equilibria occurs and hence less likely 'pure credit rationing' occurs.

The third form of credit rationing I consider is 'price rationing' which Jaffee and Stiglitz (1990) describe as the situation where a borrower receives a loan of a smaller size than desired at a given loan rate. This corresponds to firms in the survey that applied for a loan and were accepted but did not receive the full loan amount they had requested. To consider this form of credit rationing, note that the partial derivative of $\pi_F(\bar{E},\xi)$ with respect to the loan size is

$$\frac{\partial \pi_F(\bar{E},\xi)}{\partial b(\bar{E},\xi)} = 1+\psi \cdot e(\bar{E},\xi) - [1-G(\hat{z}(\bar{E},\xi))]R(\bar{E},\xi) \tag{3.36}$$

The above derivative captures the firm's loan demand at a fixed interest rate $R(\bar{E},\xi)$.Firms are price rationed if, at the equilibrium contract, equation (3.36) is positive as they would rather have a larger loan at the given interest rate. If equation (3.36) is negative, the firm would prefer to finance their project with less debt and are not price rationed.

The impact of t on the likelihood of 'price rationing' is detailed in the proposition below.

Proposition 3.7. *If $e(\bar{E},\xi)=\bar{E}$, then $\frac{d\{\partial \pi_F(\bar{E},\xi)/\partial b(\bar{E},\xi)\}}{dt} < 0$ and firms are less likely to be price-rationed when t increases. If $e(\bar{E},\xi)<\bar{E}$, a sufficient condition for $\frac{d\{\partial \pi_F(\bar{E},\xi)/\partial b(\bar{E},\xi)\}}{dt} < 0$ and thus for firms to be less likely to be price rationed following an increase in t is*

$$\left(\frac{1}{1+r}\right)(1+\psi(K+\xi)) < 2 \tag{3.37}$$

Proof. See Appendix 3.7.2. □

In the case where the equity constraint binds, an increase in t will decrease equation (3.36) and make it less likely that firms are credit constrained. Firms are least likely to be price rationed when $\bar{E}$ is close to, but less than the required investment level $K+\xi$. Intuitively, these firms are unable to self-finance the project but require only a small bank loan in order to finance the project, which they may be willing to pay a relatively large interest rate on. These firms would not be price rationed because, given the interest rate charged on their loan, they would prefer to increase the equity they issue and obtain a smaller loan but are unable to do so due to the binding constraint on equity.

In the case where the equity constraint does not bind, (3.36) is only decreasing in t if the cost of equity is not too high relative to the cost of deposits. However, even if this condition is satisfied, firms are more likely to be price rationed when $e(\bar{E},\xi) < \bar{E}$. This is because for a firm not to be price rationed, the loan size $b(\bar{E},\xi)$ must be sufficiently small and the interest rate charged $R(\bar{E},\xi)$ must be sufficiently high that equation (3.36) is negative. However, if this were the case, the firm would be better off attempting to fully self-finance the project and not having to pay the high interest rate charged by the bank.

3.5 Conclusion

This paper provides a framework to analyse SME loan applications. In particular it is able to capture key features of responses to the 'Survey on the access to finance of enterprises'. Firms that can self-finance a project may choose not to apply for a bank loan while still being able to produce. The model is able to distinguish this case from cases where a firm does not apply for a loan because they anticipate their application will be rejected with certainty.

The model also allows for different types of credit rationing. Firms can have their application accepted but be price rationed, where the firm would want a larger loan at the quoted interest rate, or they could see their application rejected outright by the bank. As in the data, firms may also reject a loan offer made by the bank.

The impact of competition also fits with some of the observed properties of the data. First, the proportion of firms applying for bank loans appears lower in countries that have less competition in the banking sector, as measured by the HHI of total assets in the banking sector. This feature also holds in the model, where a decrease in in the banking sector leads to more firms choosing to self-finance and fewer firms finding it profitable to apply for a bank loan regardless of whether they produce or not. Second, the proportion of firms that find their loan accepted appears lower in countries that have less competition in the banking sector. Here, the implications of the model are less clear cut, however, this result may hold in cases where, in response to an increase in the market power of the banking sector t, the increase in the proportion of firms that choose not to apply a bank loan and self-finance is larger than the proportion of firms that switch from being 'redlined' to not applying because they anticipate being rejected with probability one.

The framework presented here suggests that competition in the banking sector is a key determinant of both the applications and outcomes of SME loans, however it is not the only determining factor and further research should be undertaken to understand the relative importance of competition in the banking sector versus other factors such as aggregate shocks.

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3.7 Appendix for Chapter 3

3.7.1 Proof of proposition 3.5

First note that for a bank to be willing to loan to a firm, it must be the case that $\Pi(\bar{E}) > 0$ and thus $\pi_B\left(\bar{E},\xi_j\right) > 0$ for $j \in \{L,H\}$. Also note that $\pi_B\left(\bar{E},\xi_j\right) > 0$ implies $\rho_B\left(\bar{E},\xi_j\right) = 1$.

From the first order condition for the bank's problem with respect to $\rho_B(\bar{E},\xi_i)$, the following must hold for an interior solution to $\rho_B(\bar{E},\xi_i)$

$$\frac{1}{2}\pi_B(\bar{E},\xi_i)+\frac{1}{t}\pi_B(\bar{E},\xi_i)\rho_B(\bar{E},\xi_i)\left(\pi_F(\bar{E},\xi_i)-\frac{t}{2}\right)+\frac{1}{t}\pi_B\left(\bar{E},\xi_j\right)\left(\pi_F\left(\bar{E},\xi_j\right)-\frac{t}{2}\right)=0 \tag{3.38}$$

For there to be a solution to the above equation, it must be the case that $\pi_B(\bar{E},\xi_i) < 0$ and

$$-\frac{t}{2}\pi_B(\bar{E},\xi_i) < \pi_B\left(\bar{E},\xi_j\right)\left(\pi_F\left(\bar{E},\xi_j\right)-\frac{t}{2}\right) \tag{3.39}$$

Then the rejection probability can be found as

$$\rho_B(\bar{E},\xi_i) = -\frac{\pi_B\left(\bar{E},\xi_j\right)}{\pi_B(\bar{E},\xi_i)}\left(\frac{\pi_F\left(\bar{E},\xi_j\right)-\frac{t}{2}}{\pi_F(\bar{E},\xi_i)-\frac{t}{2}}\right)-\frac{\frac{t}{2}}{\pi_F(\bar{E},\xi_i)-\frac{t}{2}} \tag{3.40}$$

In the case where

$$-\frac{t}{2}\pi_B(\bar{E},\xi_i) \geq \pi_B\left(\bar{E},\xi_j\right)\left(\pi_F\left(\bar{E},\xi_j\right)-\frac{t}{2}\right) \tag{3.41}$$

the bank rejects the loan application with probability one following the realisation of ξ_i and $\rho_B(\bar{E},\xi_i) = 0$.

3.7.2 Proof of proposition 3.7

If the equity constraint binds, equation (3.36) can be written as

$$\iota = \frac{(1+\psi\bar{E})(K+\xi-\bar{E}) - \left(1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)^{-\eta}\hat{z}(\bar{E},\xi)}{K+\xi-\bar{E}} \tag{3.42}$$

The first part of the proposition follows immediately from differentiation the above with respect to $\hat{z}(\bar{E},\xi)$ getting $\frac{\partial\iota}{\partial\hat{z}(\bar{E},\xi)} < 0$ and noting that $\frac{d\hat{z}(\bar{E},\xi)}{dt} \geq 0$.

For the second part of the proposition, first note that if the equity constraint does not bind, equation (3.36) can be written as

$$\begin{aligned}\iota = {} & \frac{1}{\psi}\left(\frac{(1+r)^2}{b(\bar{E},\xi)}\right)\left(\frac{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}\right)\psi\left(\frac{1}{1+r}\right)\left(K+\xi+\frac{1}{\psi}\right)\\ & -\frac{1}{\psi}\left(\frac{(1+r)^2}{b(\bar{E},\xi)}\right)\left(\frac{1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}{1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)}\right)\\ & -\frac{1}{\psi}\left(\frac{(1+r)^2}{b(\bar{E},\xi)}\right)\frac{\psi}{\lambda}\left(\frac{1}{1+r}\right)^2\left(1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)^{-\eta}\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\end{aligned} \tag{3.43}$$

The derivative of this with respect to $\hat{z}(\bar{E},\xi)$ is

$$\begin{aligned}\frac{\partial\iota}{\partial\hat{z}} = {} & -\frac{1}{\lambda}\left[\frac{\eta\left[2\left(1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)-\left(\frac{1}{1+r}\right)(1+\psi(K+\xi))\left(1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)\right]}{\left(1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)^3}\right]\\ & -\frac{1}{\lambda}\left[\frac{\psi}{\lambda}\left(\frac{1}{1+r}\right)^2\left(1-(\eta-1)\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)\left(1+\frac{1}{\lambda}\hat{z}(\bar{E},\xi)\right)^{-\eta-1}\right]\end{aligned} \tag{3.44}$$

A sufficient condition for the above derivative to be negative is

$$\left(\frac{1}{1+r}\right)(1+\psi(K+\xi)) < 2 \tag{3.45}$$

and the second part of the proposition immediately follows.

Chapter 4

Macroeconomic implications of insolvency regimes

4.1 Introduction

In the aftermath of the 2007/2008 financial crisis output in both the UK and the US fell considerably. Real GDP fell by 5.88 percent in the UK between 2007Q4-2009Q2 while real GDP fell by 4.24 percent in the US over the same period. Despite UK output falling further than in the US, the UK labour market remained surprisingly resilient, as employment fell by 1.65 percent between 2007Q4-2009Q2 compared to the US where employment decreased by 5.34 percent. The key driver of the fall in UK output was labour productivity, which fell 3.3 percent. In the US, labour productivity actually increased by 2.3 percent.[1]

This paper suggests a link between labour productivity and a country's insol-

[1] UK data is from the UK Office of National Statistics (ONS), US GDP data is from the US Bureau of Economic Analysis (BEA), US productivity and unemployment data from the US Bureau of Labor Statistics (BLS). Labour productivity for the UK is measured as output per hour worked for the whole economy. Labour productivity for the US is output per hour worked for the non-farm business sector.

vency regime. It is well documented, for example by Djankov, Hart, McLiesh and Schleifer (2008), that the UK insolvency regime is more creditor-friendly than in other countries, including the US. The UK insolvency regime features two main procedures, administration and liquidation. The stated aim of administration is to maintain the firm as a going concern and is similar in principle to the US Chapter 11 procedure. One key difference between the UK and US insolvency regimes is in the control firm ownership maintains once insolvency begins. In the US, Chapter 11 allows firm management to remain in place and a court arbitrates between debtor and creditor. In the UK, administration replaces management with a professional 'insolvency practitioner' or 'administrator'. The administrator has full control of the business during administration. Liquidation on the other hand is a simple winding-up process, similar to the US Chapter 7 procedure where the firm ceases trading and the assets of the firm are sold off in an attempt to satisfy creditors. The incentives of firm management to default on its debt will depend on the insolvency regime in place and may impact on the firm's production decisions through the interest rate on firm debt.

I model the UK insolvency regime using a firm dynamics model in the spirit of Hopenhayn (1992) with the addition of financial frictions. Firms have access to both equity and debt. Equity is subject to exogenous issuance costs as in Gomes (2001) while debt is modeled using the costly-state verification framework of Townsend (1979). I allow for the firm to endogenously choose between two insolvency procedures. The first, restructuring, like administration in the UK and Chapter 11 in the US, allows the firm to continue subject to agreement between the parties. The second, liquidation, as in the UK and the US through Chapter 7, involves firm exit. If the firm chooses to restructure its debt, the firm and lender must bargain over the proceeds from restructuring. I distinguish between a creditor-friendly regime such as the UK and a debtor-friendly regime such as the US through the firm's bargaining power during restructuring. Defaulting is costly and leads to a loss of efficiency. In particular, the cost of holding capital increases for high-risk firms. In the model, I find that more borrowing constrained firms have a lower capital-to-labour ratio and thus have lower labour productivity.

I calibrate this model to UK aggregate data and find that the creditor-friendly bankruptcy regime features better steady-state properties, with higher output and higher labour productivity. This result is driven by banks charging lower interest rates on debt in the creditor-friendly regime which in turn implies lower barriers to entry for firms and higher employment in the steady state. In order to explore the dynamics of the model, I analyse an unanticipated aggregate productivity shock. The model finds a response to shocks that are largely consistent with the UK and the US following the financial crisis. Specifically, employment falls most in the debtor-friendly regime while labour productivity falls more in the creditor-friendly regime. A debtor-friendly insolvency regime, while more costly in the steady-state allows firms to remain less borrowing constrained following an aggregate shock and as a consequence these firms hold more capital relative to their counterparts in a creditor-friendly regime.

In order to further establish the link between firm behaviour and labour productivity since the financial crisis, Figure 4.1 and Figure 4.2 show the change in the number of firms, employment by these firms and the ratio of employment to number of firms for the UK and US respectively. The key takeaway from these graphs is that following the financial crisis, the number of firms in the UK fell more, in percentage terms than employment. This results in a higher employment per firm, which is a crude measure of the average firm size. In the US this result is reversed, employment fell more than the number of firms and the average firm size fell. I consider two possible explanations for this behaviour. First, that the firms that exited the economy in the US tended to be larger compared to the UK. This would cause the average size of the firms remaining to fall. A second explanation is that continuing firms in the US reduced their employment to a much greater extent than in the UK, that is firms adjusted the intensive margin of employment more in the US than in the UK. In Figures 4.3 and 4.4 I examine whether these differences derive from selection effects, exit of larger firms in the US relative to the UK, or from adjustment of employment at the intensive margin. I find that the size of exiting firms increased marginally in both the UK and the US between 2008 and 2014. This suggests that the differences in employment are driven by incumbent firms adjusting their workforce. This paper presents a possible mechanism through which this can occur, driven by differences between the bankruptcy regimes in the UK and US.

Related Literature

This paper is related to the large literature that explores the interaction between financial frictions and firm dynamics. The firm dynamics build on Hopenhayn (1992), where firms are heterogeneous, face idiosyncratic productivity shocks and

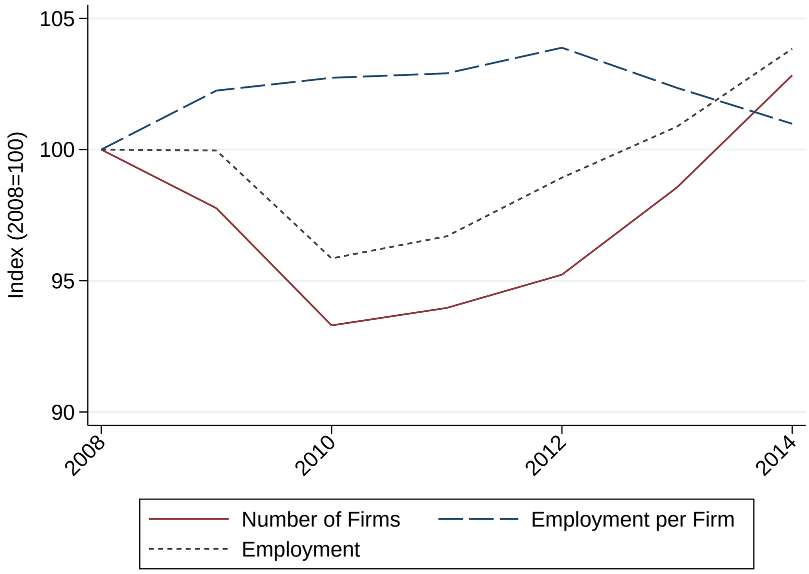


Figure 4.1: UK employment per firm since the financial crisis

Index of the total number of firms, total employment associated with those firms and the average employment per firm, 2008=100.

Source: Eurostat

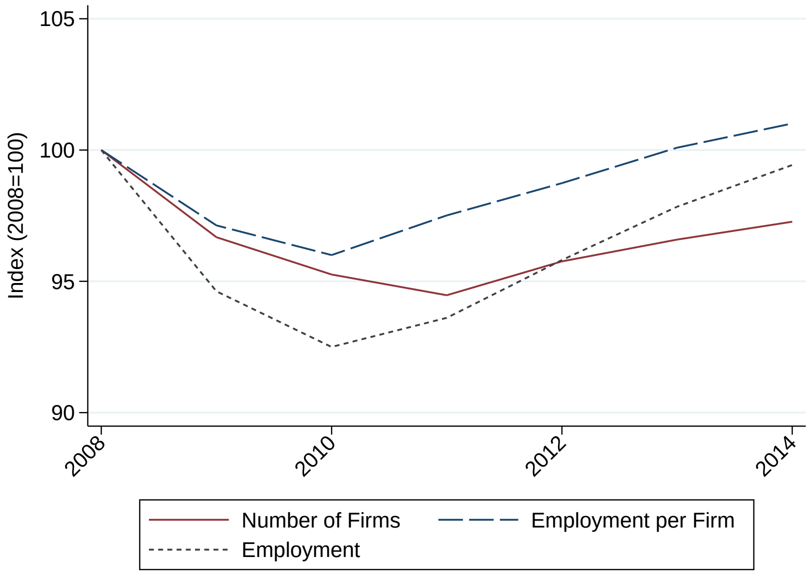


Figure 4.2: US employment per firm since the financial crisis

Index of the total number of firms, total employment associated with those firms and the average employment per firm, 2008=100.

Source: Business Dynamic Statistics.

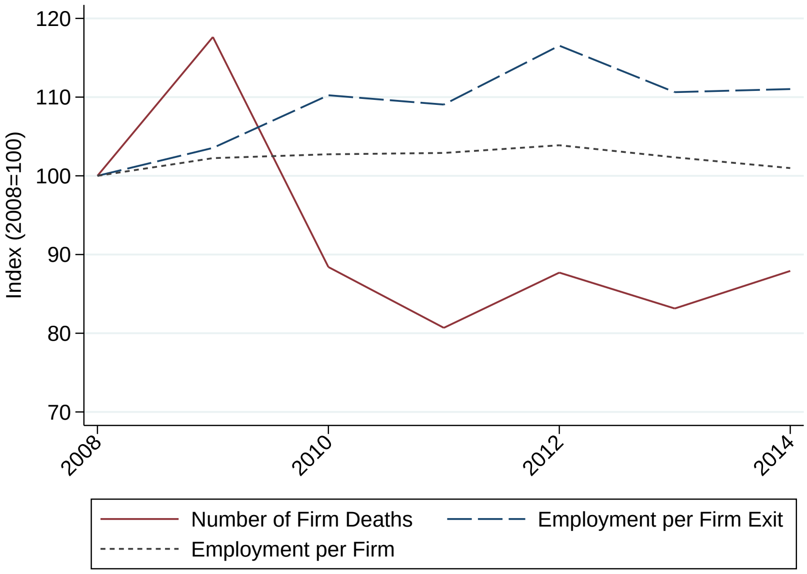


Figure 4.3: Size of UK firm exits since the financial crisis

Index of the total number of firm deaths, total employment associated with exiting firms and the average employment per firm, 2008=100.

Source: Eurostat

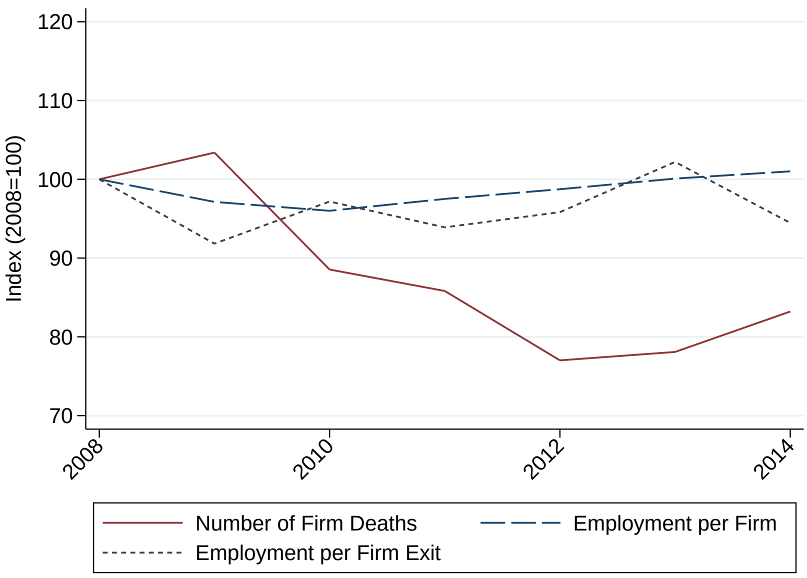


Figure 4.4: Size of US firm exits since the financial crisis

Index of the total number of firm deaths, total employment associated with exiting firms and the average employment per firm, 2008=100.

Source: Business Dynamic Statistics.

pay fixed costs to both enter and to continue production. Entry in this paper follows that of Clementi and Palazzo (2016); the mass of potential entrants is fixed and the free entry condition pins down the productivity of the marginal entrant. With the potential number of entrants fixed, the wage is sensitive to fluctuations in employment and allows for non-trivial transition dynamics in a model with no aggregate uncertainty. The addition of financial frictions to heterogeneous firm models has been explored by Cooley and Quadrini (2001), Covas and Den Haan (2012), and Clementi and Palazzo (2016).

This paper is related to the literature of credit markets. Specifically, it is related to models of debt such as the costly-state verification framework proposed by Townsend (1979) and featured in the work of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997), Bernanke Gertler and Gilchrist (1999) and others. Additionally, it is related to models of exogenous equity finance such as Gomes (2001), Cooley and Quadrini (2001) and Covas and Den Haan (2012). Papers on debt insolvency are less numerous, however Corbae and D'Erasmo (2017) also study the choice between restructuring and liquidation in the context of a heterogeneous firms model, focusing on the US system of Chapter 11 and Chapter 7. The main difference between this paper and theirs is the treatment of the labour market. They assume a fixed supply of inelastically supplied labour and model firm entry as in Hopenhayn (1992) where the free-entry condition pins down the equilibrium wage. This rules out the possibility of aggregate employment dynamics as without aggregate shocks, the wage remains constant and entry adjusts to clear the market. In my paper, I extend the existing literature on insolvency by assuming that households supply labour elastically and the mass of potential entrants is held fixed. This allows for fluctuations in both the wage and employment in response to unanticipated aggregate shocks. Another paper investigating the implications of creditor rights in insolvency on firm behaviour is Acharya, Amihud and Litov (2011) who study a model featuring two insolvency regimes, an 'equity-friendly system' as in the US and a 'debt-friendly-system' as in the UK and find that the insolvency regime impacts the leverage ratio. In related work, Acharya, Amihud and Litov (2011) study empirically the difference in insolvency regimes across countries and find having strong creditor rights in a country leads firms to reduce risk and become more reluctant to borrow.

This paper is also related to the literature on labour productivity, especially on the literature that focuses on the UK 'productivity puzzle'. Blundell, Crawford and Jin (2014) set out the empirical evidence underlying the 'productivity puzzle' and explore some of the possible causes behind it. This paper is not meant to provide a theory of the UK's low productivity, but rather to highlight the UK's insolvency regime as a possible contributing factor and to investigate the extent to which this is the case.

4.2 Model

Consider a discrete time general equilibrium model with a representative household and heterogeneous firms facing financial frictions. Firms are owned by households and produce a homogeneous good using two inputs; capital (k) and labour (n). Firms fund their costs of production through internal funds and two sources of external funding: equity (e) and debt (b). Issuing equity is subject to an external issuance cost while debt finance occurs through a one-period contract with competitive risk-neutral financial intermediaries. Debt is risky and firms can default on their debt. A firm that defaults on its debt faces an endogenous choice between two forms of insolvency; debt restructuring and liquidation. A firm that enters liquidation ceases trading and is forced to exit, firms receive nothing and financial intermediaries receive the revenue of the firm less a liquidation cost. A firm that restructures its debt remains in the market and is able to produce in the following period. The payoffs following a restructuring is the result of bargaining between the firm and the bank. There is a representative household that maximises lifetime utility. Household income is derived from labour income, asset holdings, and dividends from firms.

4.2.1 Firms

Firms enter the period with net worth (x). The inputs of capital (k) and labour (n) are decided one period in advance. At the beginning of the period the firm's revenue for the period is realised. Following the realisation of their revenue, firms decide whether or not to default on their debt and if a firm defaults, it chooses whether to enter insolvency or liquidation. Next, firms issue equity (e) or dividends and choose whether to produce in the following period. Finally, firms that choose to produce next period choose next period's capital (k'), labour (n') and the terms of debt financing (b', R') for the next period. With R' the interest rate charged by the bank. Figure 4.5 summarises the timing of a firm's problem. The timing of the firm's problem closely follows Cooley and Quadrini (2001).

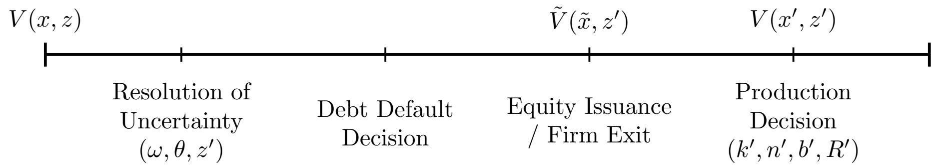


Figure 4.5: Timing of the model

In order to produce, firms must also pay a fixed cost of production $c_f > 0$. Firms that have positive net worth following the default decision are able to exit and issue a final dividend. Each firm produces a homogeneous output according to a decreasing returns to scale production function. The firm's production technology is given by

$$y = z\omega\left(k^{1-\alpha}n^{\alpha}\right)^{v} \qquad \alpha \in (0,1),\ v \in (0,1) \tag{4.1}$$

Firm-specific productivity consists of a persistent component z and a transitory component ω. There is no aggregate uncertainty in the model. The persistent productivity component $z \in \mathbb{R}_+$ follows an AR(1) process

$$\ln z' = \rho_z \ln z + \varepsilon_z' \tag{4.2}$$

where $\varepsilon_z' \sim N(\mu_{\varepsilon,z}, \sigma_{\varepsilon,z})$. The transitory productivity component is realised at the beginning of the period, after k, n and b have been chosen. It is assumed to be iid across firms and across time, orthogonal to ε_z, and with values $\omega \in \Omega \subset \mathbb{R}_+$ drawn from a distribution with cdf $G(\cdot)$.

The persistent component of productivity z' is observed at the end of the current period, before the firm decides if it will default on b and before the financing and production decisions of the next period are chosen.

At the beginning of each period, a firm is characterised by its net worth x and the realisation of its persistent productivity level z. Firms are risk-neutral and maximise the present discounted value of future dividends; firms discount dividends using the discount factor $(1+r)^{-1}$ where $1+r$ is the risk-free interest rate which is assumed to remain constant over time.

The present discounted value of future dividends for a firm with persistent productivity z and net worth x is denoted by $V(x,z)$. I denote the implicit opportunity cost of a firm with persistent productivity z' exiting as $\bar{x}(z')$ which is defined through the following equation

$$V\left(\bar{x}\left(z'\right),z'\right)=0 \tag{4.3}$$

As firms with zero net worth are able to exit the economy without incurring the fixed cost of production it follows that $V(0,z')\geq 0$ and the cost of exit will be weakly negative $\bar{x}(z')\leq 0$ for all values of z'.

In addition to using internal funding, firms are able to issue equity e and obtain debt financing b. Issuing equity is subject to an exogenously given cost function which is increasing in the amount of equity issued. The issuance cost function is given as

$$\psi(e)=\begin{cases}\frac{1}{2}\psi_0 e^2 & e\geq 0\\ 0 & e<0\end{cases} \tag{4.4}$$

The assumption of quadratic equity issuance is also made in Covas and Den Haan (2012).

An implication of the equity issuance cost is that a firm issuing a negative quantity of equity is equivalent to a dividend issuance. I economise on notation by allowing e to capture both equity issuance ($e > 0$) and dividend issuance ($e < 0$). The firm must purchase both capital and labour before production occurs. The firm's budget constraint is

$$b' + e + \tilde{x} = k' + \frac{1}{1+r}\left(wn' + c_f\right) \tag{4.5}$$

The term $\tilde{x}$ is the firm's end-of-period net worth after the realisation of revenues but before the firm issues equity, w is the aggregate wage. The last term in the firm's budget constraint reflects the requirement that firms have sufficient funds available to pay both workers and the fixed cost of production next period.

4.2.2 Financial Intermediaries

Firms borrow from competitive risk-neutral financial intermediaries. The opportunity cost for financial intermediaries of lending to firms is equal to the risk-free interest rate $(1+r)$. Financial intermediaries maximise their expected profits from lending. In equilibrium, free entry of financial intermediaries implies that they break even in expectation. A firm that repays its debt has the following end-of-period net worth

$$\tilde{x}_R\left(\omega, k, n, Rb; z\right) = z\omega\left(k^{1-\alpha}n^{\alpha}\right)^{\nu} + \left(1-\delta\right)k - Rb \tag{4.6}$$

where δ is the capital depreciation rate.

If a firm defaults on its debt it must choose to enter either liquidation or insolvency. A firm that enters liquidation is forced to exit and forfeit any current revenue and expected future earnings. The bank receives the firm's resources after production less a dead-weight loss equal to a fraction $(1-\underline{\theta}) \in (0,1)$ of the firm's resources after production. The total cost of liquidation is

$$(1-\underline{\theta})\left(z\omega\left(k^{1-\alpha}n^{\alpha}\right)^{v}+(1-\delta)k\right)-\bar{x}\left(z'\right) \tag{4.7}$$

Liquidation in this model is similar to the default costs in the costly state verification of Townsend (1979). Part of the firm's end of period net worth comes from selling its undepreciated capital at the end of the period; the liquidation cost includes a fire-sale cost on this transaction. A liquidated firm makes the following payment to financial intermediaries

$$T_L(\omega,k,n;z)=\underline{\theta}\left(z\omega\left(k^{1-\alpha}n^{\alpha}\right)^{v}+(1-\delta)k\right) \tag{4.8}$$

A firm that restructures its debt does not exit and the firm bargains with the bank over the resources after production less a dead-weight loss that results from restructuring. This dead weight loss is

$$(1-\theta)zE[\omega]\left(k^{1-\alpha}n^{\alpha}\right)^{v}+(1-\underline{\theta})(1-\delta)k \tag{4.9}$$

where $\theta\in\Theta\subset[0,1]$ is a firm specific recovery rate drawn from cdf $H(\theta)$ known before the firm decides on whether to default on its debt. The recovery rate θ is realised at the same time as ω and z', before the firm's default decision but after the debt contracts have been finalised.

Two features of the restructuring cost are worth emphasising. First, the restructuring cost depends on the expected value of the revenue shock rather than the realisation of ω which adds a fixed cost element to the restructuring cost. As the liquidation cost is decreasing in the realisation of ω this ensures that, everything else equal, a lower realisation of ω makes it more likely that a firm chooses liquidation over restructuring. Second, the cost of restructuring features the same fire-sale cost on undepreciated capital as in the liquidation case, this is a simplifying assumption.

A firm that begins the restructuring process can be forced into liquidation by either the firm or the bank and therefore both parties take their payoffs from firm liquidation as their outside option and any remaining surplus is then bargained between the firm and the bank. A defaulting firm will restructure only if there is a positive surplus obtained over firm liquidation. The surplus from restructuring over liquidation is

$$S_B\left(\theta,\omega,z',k,n;z\right)=\left(\left(1-\underline{\theta}\right)\omega-\left(1-\theta\right)E\left[\omega\right]\right)z\left(k^{1-\alpha}n^{\alpha}\right)^{\nu}-\bar{x}\left(z'\right) \tag{4.10}$$

The surplus from restructuring over liquidation is simply the cost of liquidation, equation (4.7), less the cost of restructuring, equation (4.9). It follows that this surplus will be positive in situations where restructuring is less costly than liquidation.

The firm's bargaining weight is denoted by $\phi\in[0,1]$ and is a key parameter in the modelling of insolvency regimes. Higher values of ϕ imply that the bargaining power lies mostly with the firm and the insolvency regime is a more debtor-friendly regime such as the US while low values of ϕ means the bargaining power lies with the bank and the insolvency regime is a creditor-friendly regime such as the UK.

A firm that restructures will begin the next period with the following cash-in-hand

$$\tilde{x}_B\left(\theta,\omega,z',k,n;z\right)=\phi S_B\left(\theta,\omega,z',k,n;z\right)+\bar{x}\left(z'\right) \tag{4.11}$$

A restructured firm makes the following payment to financial intermediaries

$$T_B\left(\theta,\omega,z',k,n;z\right)=\left(1-\phi\right)S_B\left(\theta,\omega,z',k,n;z\right)+\underline{\theta}\left(z\omega\left(k^{1-\alpha}n^{\alpha}\right)^{\nu}+\left(1-\delta\right)k\right) \tag{4.12}$$

where the first term is the share of the surplus from restructuring which goes to the financial intermediary, while the second term is the payoff to the financial intermediary from liquidation which is also their outside option as they are able to refuse to restructure a firm's loan and instead force the firm into liquidation.

4.2.3 Households

There is a risk-neutral representative household that discounts the future at rate $\beta\in(0,1)$ and maximises the following utility function

$$\sum_{t=0}^{\infty}\beta^t\left[C_t+\gamma\left(\frac{N_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right)\right],\quad \gamma>0,\ \eta>0 \tag{4.13}$$

where C_t is aggregate consumption, N_t is the aggregate labour supplied by the household and the parameter η is the Frisch elasticity. Households own both firms and financial intermediaries and buy risk-free bonds B_t from the financial intermediaries which are used to lend to firms. They maximise the discounted present value of utility subject to the following budget constraint

$$C_t + B_{t+1} + \int s_{jt+1} p_{jt} dj = w_t N_t + (1 + r_t) B_t + \Pi_t^B + \int s_{jt} \left(d_{jt} + p_{jt}\right) dj \quad (4.14)$$

where p_{jt}, d_{jt} and s_{jt} denote the price, dividends and fraction of shares in firm j owned by the household and Π_t^B denotes the profits of financial intermediaries.

As households are risk-neutral, the risk-free interest rate will be constant across time and firms and households discount the future at the same rate. The first order conditions for the household labour supply is given by

$$\gamma N_t^{\frac{1}{\eta}} = w_t \quad (4.15)$$

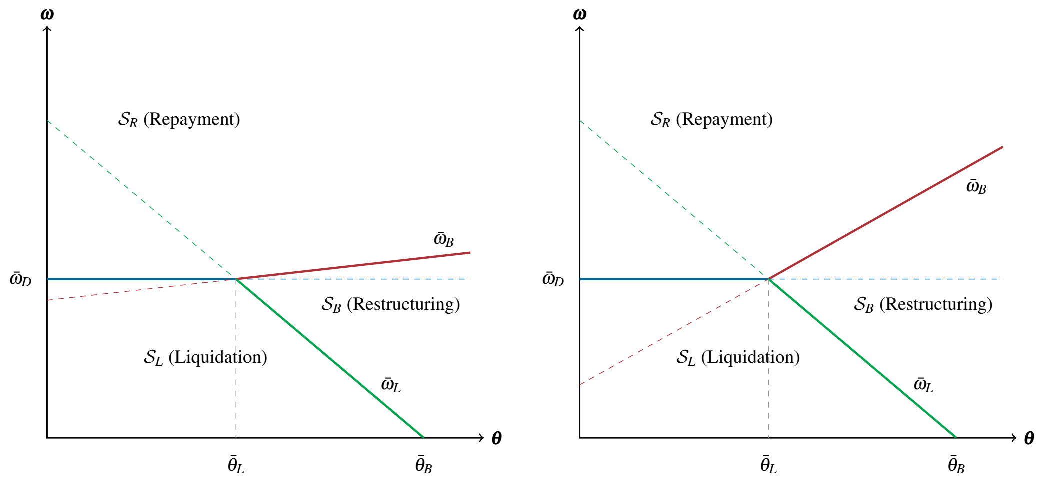
This equation determines the aggregate labour supply in the economy.

4.3 Equilibrium

4.3.1 Debt Resolution

At the beginning of every period, a firm that borrowed in the previous period must make a decision between repayment and default. If the firm defaults it must decide between restructuring it debt or liquidating the firm. The default decision the firm makes at the beginning of the next period will impact the interest rate the firm is charged on debt in the current period.

Figure 4.6 is an illustration of the firm's debt resolution decision in (θ, ω)-space for a hypothetical (z', k, n, Rb, z). The space can be partitioned into three



(a) Low ϕ **(b)** High ϕ

Figure 4.6: Firm's debt resolution decision

areas.

When the realisation of the revenue shock ω is high and the value of restructuring (θ) isn't too high, the firm will repay its loan. If the revenue shock ω is low and the value of restructuring θ is also low, the firm will choose liquidation. Finally, if ω isn't too high but the value of restructuring θ is high, the firm will choose restructuring over both liquidation and repaying its loan. The firm share of restructuring, ϕ is also key to the firm's debt resolution decision. Panel a) shows the case where ϕ is low and hence the restructuring area is small while panel b) shows the case where ϕ is high and hence the restructuring area is larger. Also note that firms will only choose liquidation over restructuring if the surplus from restructuring is negative, as a result, a change in ϕ doesn't impact the boundary between the restructuring and liquidation regions.

More formally, we can separate the space into three subsets. The first subset $\mathscr{S}_R(z', k, n, Rb; z)$ is the region where the firm will repay its debt. The second subset

$\mathscr{S}_B(z',k,n,Rb;z)$ is the region where the firm will default on its debt and restructure its debt. The final subset $\mathscr{S}_L(z',k,n,Rb;z)$ is the region where the firm will default on its debt and will liquidate the firm. Formally these regions are defined as follows

$$\mathscr{S}_R\left(z',k,n,Rb;z\right)=\left\{(\theta,\omega)\in\Theta\times\Omega : x_R\left(\theta,\omega,z',k,n,Rb;z\right)\geq x_B\left(\theta,\omega,z',k,n;z\right),\right.$$
$$\left. x_R\left(\omega,z',k,n,Rb;z\right)\geq\bar{x}\left(z'\right)\right\} \tag{4.16}$$

$$\mathscr{S}_B\left(z',k,n,Rb;z\right)=\left\{(\theta,\omega)\in\Theta\times\Omega : x_R\left(\theta,\omega,z',k,n,Rb;z\right)< x_B\left(\theta,\omega,z',k,n;z\right),\right.$$
$$\left. x_B\left(\theta,\omega,z',k,n;z\right)\geq\bar{x}\left(z'\right)\right\} \tag{4.17}$$

$$\mathscr{S}_L\left(z',k,n,Rb;z\right)=\left\{(\theta,\omega)\in\Theta\times\Omega : x_R\left(\theta,\omega,z',k,n,Rb;z\right)< x_B\left(\theta,\omega,z',k,n;z\right),\right.$$
$$\left. x_B\left(\theta,\omega,z',k,n;z\right)<\bar{x}\left(z'\right)\right\} \tag{4.18}$$

The boundaries of these sets can be characterised by cutoffs of θ and $\bar{\omega}$. First consider the case where a firm is indifferent between repayment and liquidation, then the following must hold

$$\bar{\omega}_D\left(z',k,n,Rb;z\right)=\max\left\{\frac{Rb+\bar{x}(z')-(1-\delta)k}{f(k,n)},0\right\} \tag{4.19}$$

For ω less than this cutoff, the firm will always default on its outstanding debt. Next, for $\omega<\bar{\omega}_D(z',k,n,Rb;z)$ the firm will be indifferent between restructuring and liquidation if the following equation holds

$$\theta = [1 - (1 - \underline{\theta})\,\omega] - \frac{-\bar{x}(z')}{f(k,n)} \tag{4.20}$$

This equation is decreasing in ω. Using this allows us to define cutoffs $\bar{\theta}_B(z',k,n;z) \geq \bar{\theta}_L(z',k,n,Rb;z)$ such that a defaulting firm will always prefer restructuring if $\theta > \bar{\theta}_B(z',k,n;z)$ and a defaulting firm will always be liquidated whenever $\theta < \bar{\theta}_L(z',k,n,Rb;z)$ where the cutoffs are defined by the following equations

$$\bar{\theta}_L\left(z',k,n,Rb;z\right) = \max\left\{\left[1 - (1 - \underline{\theta})\,\bar{\omega}_D\left(z',k,n,Rb;z\right)\right] - \frac{-\bar{x}(z')}{f(k,n)}, 0\right\} \tag{4.21}$$

$$\bar{\theta}_B\left(z',k,n;z\right) = \max\left\{1 - \frac{-\bar{x}(z')}{f(k,n)}, 0\right\} \tag{4.22}$$

For values of $\theta \in \left[\bar{\theta}_L(z',k,n,Rb;z), \bar{\theta}_B(z',k,n;z)\right]$ whether a defaulting firm will restructure or liquidate depends on the realisation of ω. Specifically, there will be a cutoff $\bar{\omega}_L(z',k,n;z)$ such that if $\omega \geq \bar{\omega}_L(z',k,n;z)$ a defaulting firm will restructure while if $\omega < \bar{\omega}_L(z',k,n;z)$ a defaulting firm will be liquidated. The cutoff is defined by the following equation

$$\bar{\omega}_L\left(z',k,n;z\right) = \max\left\{\frac{\bar{x}(z') + (1-\theta)\,f(k,n)}{(1-\underline{\theta})\,f(k,n)}, 0\right\} \tag{4.23}$$

In cases where the firm has some bargaining power during restructuring ($\phi > 0$) the firm may choose to restructure when $\omega > \bar{\omega}_D(z',k,n,Rb;z)$. For this to occur, the recovery rate from restructuring must be sufficiently high, that is $\theta > \bar{\theta}_L(z', k, n, Rb; z)$. The firm will prefer restructuring over repayment of its debt whenever $\omega < \bar{\omega}_B(\theta, z', k, n; z)$ where the cutoff is defined by the following equation

$$\begin{aligned}\bar{\omega}_B\left(\theta, z', k, n, Rb; z\right) = \max\Bigg\{ & \bar{\omega}_D\left(z', k, n, Rb; z\right) \\ & +\phi\left(\frac{\left(\theta - \bar{\theta}_L(z', k, n, Rb; z)\right) f(k,n) + \bar{x}(z')}{\left[1 - \phi\left(1 - \underline{\theta}\right)\right] f(k,n)}\right), 0\Bigg\}\end{aligned} \tag{4.24}$$

I refer to this case as strategic default as the firm has sufficient funds to be able to repay its loan but chooses to restructure its debt as they receive a higher net-worth by doing so. This is the only cutoff which depends on ϕ, specifically, $\bar{\omega}_B(\theta, z', k, n, Rb; z)$ is increasing in ϕ and thus for a given (z', k, n, Rb, z) firms have a greater incentive to restructure in a creditor-friendly (high ϕ) regime than in debtor-friendly (low ϕ) regime.

4.3.2 Bank's Problem

The expected profit of a bank for a given debt contract $(k, n, Rb, b; z)$ is written as

$$\begin{aligned}\Pi_B(k, n, Rb, b; z) = & E_{z'|z}\left[Rb \int_{\mathscr{S}_R(z', k, n, Rb; z)} d\left[G(\omega) \times H(\theta)\right]\right] \\ & + E_{z'|z}\left[\int_{\mathscr{S}_B(z', k, n, Rb; z)} T_B\left(\theta, \omega, z', k, n; z\right) d\left[G(\omega) \times H(\theta)\right]\right] \\ & + E_{z'|z}\left[\int_{\mathscr{S}_L(z', k, n, Rb; z)} T_L(\omega, k, n; z)\, d\left[G(\omega) \times H(\theta)\right]\right] \\ & - (1+r)\, b\end{aligned} \tag{4.25}$$

For a given contract $(k, n, Rb, b; z)$ the profit of the bank is strictly decreasing in the firm's bargaining power ϕ. There are two reasons for this. First as discussed in the previous section, a firm with high bargaining power is more likely to default on its debt and enter the restructuring process and the bank's profit from restructured debt is strictly less than if the debt was repaid. Second, as the bank has less bargaining power, it will receive a lower payment when the debt is restructured.

4.3.3 Firm's problem

Following the realisation of its revenue and its default decision, a firm that is not liquidated has cash-in-hand $\tilde{x}$ and knows the persistent component of its productivity for the next period z'. The firm can now choose to produce in the next period or it can issue a final dividend and exit. The equity issuance problem is written as follows

$$\tilde{V}\left(\tilde{x}, z'\right) = \max_{e} \left\{ -\left(e + \psi(e)\right) + \frac{1}{1+r} V\left(\tilde{x} + e, z'\right), \tilde{x} - \psi(-\tilde{x}) \right\} \tag{4.26}$$

The value function $\tilde{V}(\cdot, \cdot)$ is not everywhere differentiable. Specifically, there will be a point of non-differentiability at the point where the firm is indifferent between default and repayment as well as at points of indifference between exit (without default) and production. Nevertheless, by applying Theorem 1 from Clausen and Strub (2016) it follows that at the optimal solution to the equity issuance problem the following first order condition is satisfied[2]

$$\frac{1}{1+r}\frac{\partial V\left(\tilde{x}+e,z'\right)}{\partial e}=1+\frac{\partial\psi\left(e\right)}{\partial e} \tag{4.27}$$

A firm with $\partial V/\partial e > 1+r$ will issue equity until they are no longer borrowing constrained. A firm with $\partial V/\partial e = 1+r$ is no longer borrowing constrained and will be indifferent between issuing dividends and accumulating additional assets. To ensure that firms do not accumulate too many bonds and the asset market clears, I assume that in this situation shareholders demand that firms issue dividends rather than accumulate assets. This ensures that there is a maximum net-worth for a firm conditional on z.[3]

The firm's problem can be written recursively as

$$\begin{aligned} V\left(x,z\right)=\max_{\{k,n,b,R\}}\Bigg\{ & E_{z'|z}\left[\int_{\mathscr{S}_R(z',k,n,Rb;z)}\tilde{V}\left(\tilde{x}_R\left(\omega,k,n,Rb;z\right),z'\right)d\left[G\left(\omega\right)\times H\left(\theta\right)\right]\right] \\ & +E_{z'|z}\left[\int_{\mathscr{S}_B(z',k,n,Rb;z)}\tilde{V}\left(\tilde{x}_B\left(\theta,\omega,z',k,n;z\right),z'\right)d\left[G\left(\omega\right)\times H\left(\theta\right)\right]\right]\Bigg\} \end{aligned} \tag{4.28}$$

subject to

$$\tilde{V}\left(\tilde{x},z\right)=\max\left\{\max_{e}\left\{-\left(e+\psi\left(e\right)\right)+\frac{1}{1+r}V\left(\tilde{x}+e,z\right)\right\},\tilde{x}-\psi\left(-\tilde{x}\right)\right\} \tag{4.29}$$

[2] To apply Theorem 1 from Clausen and Strub (2016) and obtain the first order condition presented in this section, it is necessary to construct a 'differentiable lower support function', this is made possible by the differentiability of the function $\psi(e)$. If this function was not differentiable, as in Gomes (2001) then this would not be possible and we would not be able to use Clausen and Strub's theorem here.

[3] A common assumption made here is that firms discount the future at a rate smaller than $1/1+r$. I avoid making this assumption here so that there exist unconstrained firms in equilibrium. I will exploit the existence of these firms in my calibration strategy.

$$b + x = k + \frac{1}{1+r}\left(wn + c_f\right) \tag{4.30}$$

$$\Pi_B\left(k, n, Rb, b; z\right) = 0 \tag{4.31}$$

The firm maximises expected utility by choosing a contract $\{k, n, b, R\}$. The expectation in equation (4.28) is over future technology z' and the transition of net wealth x depends on the realisation of ω, z' and the firm's choice of whether it repays or restructures its loan. Equation (4.29) combines the firm's equity issuance and exit decision, equation (4.30) specifies the firm's budget constraint and equation (4.31) specifies that due to perfect competition, banks makes zero profit in expectation.

4.3.4 Firm Entry

Every period there is a constant mass $M > 0$ of prospective firms. Each firm draws an initial productivity level z_0 from a distribution $G_E(\cdot)$. Firms observe their initial productivity level and then decide whether to enter the market or not. In order to enter, a firm must pay a fixed entry cost $c_e > 0$. Entrants fund the cost of entry through an initial equity issuance and enter the economy with zero net-worth $x = 0$. The value of a prospective entrant which receives an initial productivity level z_0 is

$$V_E\left(0, z\right) = \max_{e}\left\{-\left(e + \psi\left(e\right)\right) + \frac{1}{1+r}V\left(e, z\right)\right\} \tag{4.32}$$

Firms will only enter if their initial productivity level is sufficiently high and there is a cutoff value $\bar{z}$ such that firms enter when $z_0 > \bar{z}$ with the cutoff defined by the following free-entry condition

$$V_E(0, \bar{z}) = c_e \tag{4.33}$$

Firms that enter the market decide on their production inputs and financing for the following period. Entrants do not produce until the period following their entry.

4.3.5 Recursive Competitive Equilibrium

A recursive competitive equilibrium consists of (i) value functions V, $\tilde{V}$, and V_E, (ii) policy functions $n(x,z,\Gamma)$, $k(x,z,\Gamma)$, $b(x,z,\Gamma)$, $R(x,z,\Gamma)$, $e(\tilde{x},z',\Gamma)$ and $\bar{z}(\Gamma)$ (iii) wage function $w(\Gamma)$ and (iv) distribution of firms Γ such that

1. The value functions $V(x,z)$, $\tilde{V}(\tilde{x},z')$ and policy functions $n(x,z,\Gamma)$, $k(x,z,\Gamma)$, $b(x,z,\Gamma)$, $R(x,z,\Gamma)$, $e(\tilde{x},z',\Gamma)$ solve the incumbent firm's problem

2. The value function $V_E(0,z)$ and the policy functions solve the prospective firm's problem free entry condition hold for entrants and firms enter the market only if $z \geq \bar{z}$ where $V_E(0,\bar{z}) = c_e$

3. Given the wage function $w(\Gamma)$ and the interest, the labour, equity and bond markets clear

4. The distribution of firms is consistent with firm decision rules and evolves

according to the following law of motion

$$\begin{aligned}\Gamma_{t+1}\left(x',z'\right) = &\int_{\mathscr{S}_R} \left(1-\tilde{\chi}_R\left(\omega,x;z\right)\right) \mathbb{1}_{\{x',z'|x,z\}} \Gamma_t\left(x,z\right) d\left[G\left(\omega\right)\times H\left(\theta\right)\right] \\ &+ \int_{\mathscr{S}_B} \left(1-\tilde{\chi}_B\left(\theta,\omega,z',x;z\right)\right) \mathbb{1}_{\{x',z'|x,z\}} \Gamma_t\left(x,z\right) d\left[G\left(\omega\right)\times H\left(\theta\right)\right] \\ &+ M\int_{z>\bar{z}} \mathbb{1}_{\{x',z'|x=0,z\}} dG\left(z\right) \end{aligned} \tag{4.34}$$

where $\mathbb{1}_{\{x',z'|x,z\}}$ is the indicator function given the firm's policy function. $\tilde{\chi}_R\left(\omega,x;z\right)$ and $\tilde{\chi}_B\left(\omega,x;z\right)$ are the exit rules for firms following repayment and restructuring of debt respectively. These equations are given by

$$\begin{aligned}\tilde{\chi}_R\left(\omega,x;z\right) =& \mathbb{1}\left\{\tilde{V}\left(\tilde{x}_R\left(\omega,k\left(x,z\right),n\left(x,z\right),R\left(x,z\right)b\left(x,z\right);z\right),z\right) < \right. \\ &\left. \tilde{x}_R\left(\omega,k\left(x,z\right),n\left(x,z\right),R\left(x,z\right)b\left(x,z\right);z\right)\right\}\end{aligned} \tag{4.35}$$

$$\begin{aligned}\tilde{\chi}_B\left(\theta,\omega,z',x;z\right) =& \mathbb{1}\left\{\tilde{V}\left(\tilde{x}_B\left(\theta,\omega,z',k\left(x,z\right),n\left(x,z\right);z\right),z\right) < \right. \\ &\tilde{V}\left(\tilde{x}_B\left(\theta,\omega,z',k\left(x,z\right),n\left(x,z\right);z\right),z\right) < \\ &\left. \tilde{x}_B\left(\theta,\omega,z',k\left(x,z\right),n\left(x,z\right);z\right)\right\}\end{aligned} \tag{4.36}$$

The definition of a recursive equilibrium here allows me to analyse the dynamic impact of shocks to the model. I will also consider the numerical solution to a stationary equilibrium which occurs at the point where $\Gamma_{t+1}\left(x',z'\right) = \Gamma_t\left(x,z\right)$. For a stationary distribution to exist, we require that firms have a sufficiently large probability of exiting. This will occur so long as there exists some combination of $\left(\omega,z\right)$

for all x at which point firms choose to exit.

4.3.6 Impact of the Insolvency Regime

Before turning to the calibration of the model, I first discuss some of the channels through which the insolvency regime can impact the real economy and specifically, labour productivity. The insolvency regime is determined by the firm's bargaining power in restructuring ϕ. In this section, I focus on the impact of ϕ on the capital-to-labour ratio, which is a key component, along with aggregate TFP, of labour productivity. The mechanism through which the capital-to-labour ratio is made clear in the following equation for the capital-to-labour ratio which I obtained from the first-order conditions of the firm's problem

$$\frac{k}{n} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{w}{r+\delta+(1-\delta)\Lambda_k(k,n,Rb;z)}\right) \tag{4.37}$$

where $\Lambda_k(k,n,Rb;z)$ is a distortion to the capital-to-labour ratio given by the following equation

$$\begin{aligned}\Lambda_k(k,n,Rb;z) = (1-\underline{\theta})\,E_{z'|z}&\left[\int_{\bar{\theta}_L(z',k,n,Rb;z)}^{1} G\left(\bar{\omega}_B\left(\theta,z',k,n,Rb;z\right)\right)dH\theta\right] \\ &+(1-\underline{\theta})\,E_{z'|z}\left[H\left(\bar{\theta}_L\left(z',k,n,Rb;z\right)\right)G\left(\bar{\omega}_D\left(z',k,n,Rb;z\right)\right)\right]\end{aligned} \tag{4.38}$$

which is simply the expected default probability multiplied by $(1-\underline{\theta})$. Thus equation (4.37) provides a direct mechanism through which an increase in the default probability leads to a decrease in the capital-to-labour ratio and hence a fall in labour productivity.

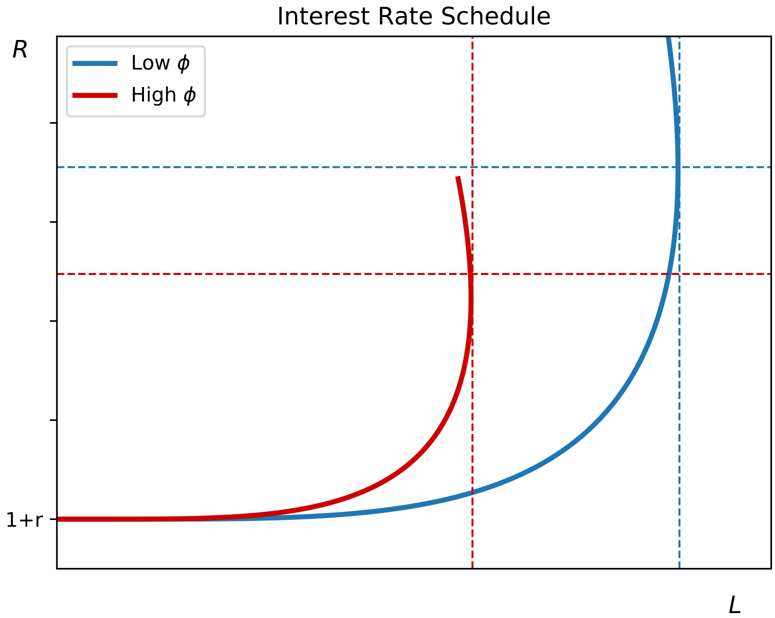


Figure 4.7: Impact of ϕ on firm leverage

As illustrated by Figure 4.6, an increase in ϕ will mean the firm has a greater incentive to restructure loans that it could otherwise repay. However, banks will anticipate the lower repayment probability and will adjust loan terms accordingly, which would result in higher interest rates being charged to the firm. As is common to models based on the costly state verification model of debt, the interest rate schedule offered to the firms is backward-bending and there is an upper limit to firm leverage where $L = \frac{b}{x+b}$ is firm leverage. An illustration of this is set out in Figure 4.7 where a higher ϕ results in higher interest rate and a lower debt capacity.

Furthermore, firms that are more highly leveraged will require higher output in order to repay the loan and hence the default probability is increasing in leverage. This allows Figure 4.8 to be rewritten in terms of Λ_k and L as illustrated by Figure 4.8. In cases where the bank response to high ϕ is sufficiently strong, a higher ϕ

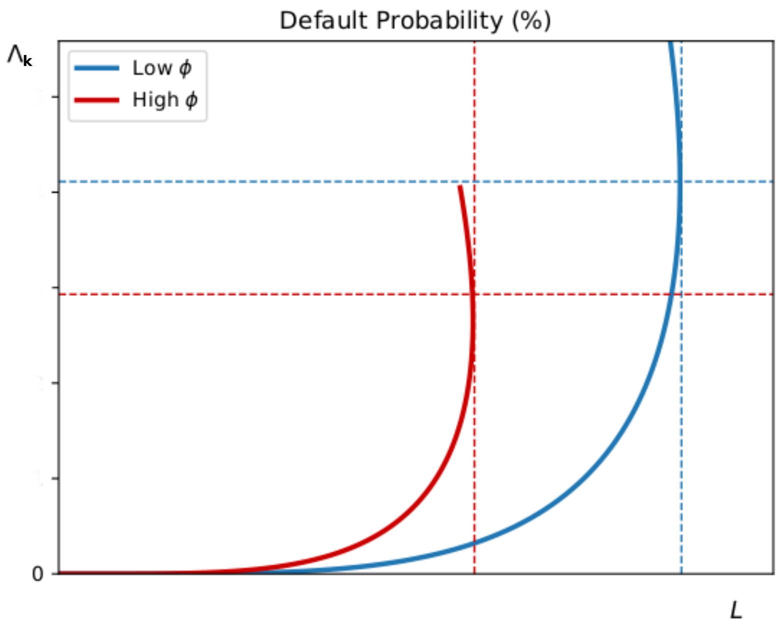


Figure 4.8: Impact of regime on default probability

results in lower debt capacity and lower maximum default probability.

4.4 Calibration

I solve the model numerically using a baseline calibration of the model to UK data since the financial crisis. One period in the model is a year. The distribution of entrants is assumed to be the stationary distribution implied by the AR(1) process for z. I approximate the process for z using the method described in Tauchen (1986). The distribution for $\theta \in [0,1]$ is assumed to be a standard uniform distribution. The distribution of the revenue shocks ω are log-normal with $\mu_\omega = -\frac{1}{2}\sigma_\omega^2$ so that $E[\omega] = 1$.

The model parameters are split into two categories, those calibrated using the model through indirect inference and those that are calibrated outside of the model or taken from standard values found in the literature. The first set of parameters are set out in Table 4.1 and are not inferred using the model. The second set of parameters are jointly calibrated using moments from the model. Table 4.2 sets out the benchmark values and provides a summary of the moments targeted.

Parameter		Value	Source
Interest rate	r	0.04	annual interest rate
Depreciation rate	δ	0.1	standard parameter
Discount factor	β	1/1.04	inverse of $1+r$
Labour production elasticity	α	0.65	standard parameter
Decreasing returns parameter	v	0.85	standard parameter
Mass of potential entrants	M	0.405	set so that $N = 0.72$
Labour utility parameter	γ	1.55	set so that $w = 1$
Frisch elasticity	η	0.75	Chetty et al. (2011)
Persistence of z productivity	ρ_z	0.597	estimated from UK firm data

Table 4.1: Calibrated parameters

The risk-free interest rate r is set to 4% which is a commonly used value in annual models. The discount factor β is set to be the inverse of $1+r$ so that firms discount the future at the same rate as firms. The depreciation rate of capital δ is set to 0.1. The Frisch elasticity η is set to 0.75 which is a value suggested by Chetty et al. (2011) for representative agent macro models. The utility function parameter γ is chosen so that the household's labour supply equation is consistent with the wage w which is normalised to 1. This, coupled with the mass of potential entrants M pins down the employment level in the model and M is calibrated to ensure that steady state employment equals 0.72 which is approximately the employment rate of the UK.

The parameters of the production function are similar to those used in the literature, with the labour production elasticity α set to 0.65 which is approximately the labour share of income in the UK and the decreasing returns to scale parameter

Parameter		Value	Target
Firm Bargaining Power	ϕ	0.149	Ratio of administrations to liquidations
Fixed cost of production	c_f	0.279	Entry rate of UK firms
Cost of entry	c_e	0.866	One year survival rate of UK firms
s.d of technology process	σ_z	0.183	Debt-to-asset ratio
s.d. of revenue shocks	σ_ω	0.487	Liquidation rate of UK firms
Liquidation recovery rate	$\underline{\theta}$	0.210	Default recovery rate
Equity issuance cost	ψ	1.48	Equity-to-asset ratio

Table 4.2: Model-estimated parameters

v set to 0.85 which is used in other similar models such as in Corbae and D'Erasmo (2017).

The parameters for the persistence of the firm technology shock ρ_z is estimated independently from the other parameters using Compustat data for UK firms. To estimate the parameters we follow a method similar to that described in Blundell and Bond (2000) and Cooper and Haltiwanger (2006), in particular, I estimate the following equation

$$\ln y_{it} = \rho_z \ln y_{it-1} + a_1 \ln n_{it} + a_2 \ln n_{it-1} + a_3 \ln k_{it} + a_4 \ln k_{it-1} + A_t + \tilde{\eta}_{it} \tag{4.39}$$

where y_{it} is the firm's revenue, k_{it} is capital, n_{it} is employment and the parameters are estimated using a dynamic panel data model.

The regression estimated in equation (4.39) could be used to estimate the variance of productivity shocks, however, in the model, this would correspond to a combination of σ_z and σ_ω. Instead, I calibrate the standard deviation of the technology parameter σ_z to the average debt-to-asset ratio of the Compustat firms. The standard deviation of the revenue shock σ_ω is calibrated to the proportion of firms in the UK declaring bankruptcy. Data from the UK Insolvency Service suggests

Targeted Moments (%)	Model	Data
Firm entry rate	10.7	12.3
Debt-to-asset ratio	55.4	47.9
Equity-to-asset ratio	19.2	18.0
Proportion of defaulting firms	0.651	0.660
Ratio of restructures to liquidations	10.6	10.8
Debt recovery-rate in default	22.4	20.2
One year survival rate	90.4	90.5
Untargeted Moment (%)	**Model**	**Data**
Average spread on borrowing	3.13	2.05

Table 4.3: Calibrated Moments

that 0.66% of the total number of firms in the UK declare bankruptcy every year. The fixed cost of production c_f is calibrated to the average startup rate of UK firms between 2008 and 2015, while the cost of entry (c_e) is calibrated to the one year survival rate of UK firms. The data for both of these targets is obtained from the Eurostat Structural business statistics database.

Armour et al. (2012) study the impact of a 2003 change in UK Bankruptcy law. They find that post-2003, in the UK the debtor recovers through Administrations 20.2% of their claim on average. I choose the liquidation recovery rate $\underline{\theta}$ to match this recovery rate. In the UK insolvency regime, firms that default and attempt to restructure enter administration rather than another form of insolvency. I use data from the UK Insolvency Service on the average proportion of total insolvencies that are administrations between 2008 and 2017 to calibrate the firm's bargaining power ϕ which in the model governs the likelihood that a firm restructures rather than enters bankruptcy. The equity cost parameter ψ is chosen to match the equity-to-asset ratio of the UK firms sampled in Compustat.

Table 4.3 sets out the moments used to calibrate the model and the model fit.

The model fits the data reasonably well although it struggles somewhat to capture the entry rate of firms and the debt-to-asset ratio. The model also does a reasonable job at fitting the average spread on borrowing, which was an untargeted moment. The data for the UK spread on borrowing comes from the Bank of England.

4.5 Results

4.5.1 Steady State

In this section I explore the steady state properties of the benchmark model and compare it to the steady state of a model that features a more debtor-friendly insolvency regime. The debtor-friendly regime uses the same parameters as the benchmark model, with the exception of the firm's bargaining power during debt restructure (ϕ) which is increased. The increase in ϕ is calibrated to the ratio of firm restructurings to liquidations in the US economy. The data comes from the American Bankruptcy Institute and is calculated as the ratio of Chapter 11 bankruptcies to the total number of Chapter 11 and Chapter 7 bankruptcies. This ratio is 17.6% which is larger than the target of the benchmark model of 10.8% which was calibrated to UK data. The resulting parameter increase in the debtor-friendly is set to 0.273 which results in a ratio of restructurings to liquidations of 17.9% which is close to the targeted moment. As the mass of entrants is fixed at the same quantity for the two models, the wage adjusts to ensure the labour market clears.

I refer to the benchmark model as the creditor-friendly model with $\phi_L = 0.149$ and compare it to the debtor-friendly model which features $\phi_H = 0.273$. Table 4.4 compares the aggregate values of the steady states for the two regimes. While the

	Creditor-friendly (ϕ_L)	Debtor-friendly (ϕ_H)
Bargaining Power (ϕ)	0.148	0.273
Wage (w)	1.0	0.995
Employment (N)	0.720	0.717
Aggregate Capital (K)	2.65	2.65
Output (Y)	3.84	3.83
Y/N	5.34	5.36
K/N	3.62	3.68

Table 4.4: Steady State Aggregates of Insolvency Regimes

Moments (%)	Creditor-friendly (ϕ_L)	Debtor-friendly (ϕ_H)
Firm entry rate	10.7	10.5
Debt-to-asset ratio	55.4	54.3
Equity-to-asset ratio	19.2	18.7
Proportion of defaulting firms	0.651	0.535
Ratio of restructures to liquidations	10.6	17.9
Debt recovery-rate in default	22.4	24.3
One year survival rate	90.4	90.5
Average spread on borrowing	3.13	3.09

Table 4.5: Moments of Insolvency Regimes

change in the aggregate values is minimal, the ϕ_H economy has lower output, lower aggregate capital and a lower equilibrium wage. This is because the increase in firm bargaining power leads to firms getting charged higher interest rates. This is shown in table 4.5 which compares the moments of the two models. The ϕ_H economy features a slightly higher spread on firm borrowing despite a lower default probability. This is due to the impact of the firm's bargaining power on the average recovery rate of loans in default.

The fall in the equilibrium wage means that for this calibration, the ϕ_H economy features a slightly lower capital-to-labour ratio and thus lower labour productivity as measured by the output to employment ratio as found in table 4.4 Both regimes feature lower than optimal capital investment. This is due to the financial frictions distorting the relative price of capital and labour.

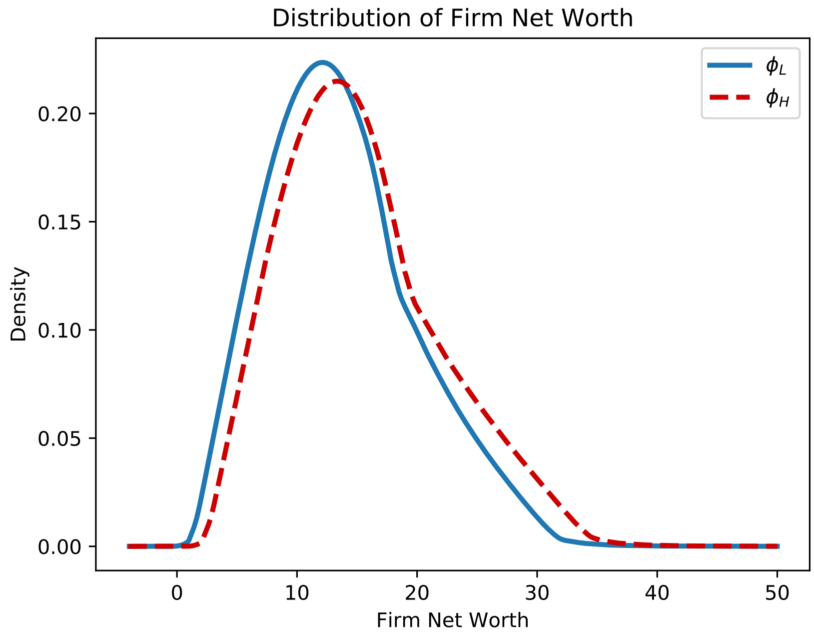


Figure 4.9: Distribution of Firm Net Worth (x) in the Steady State

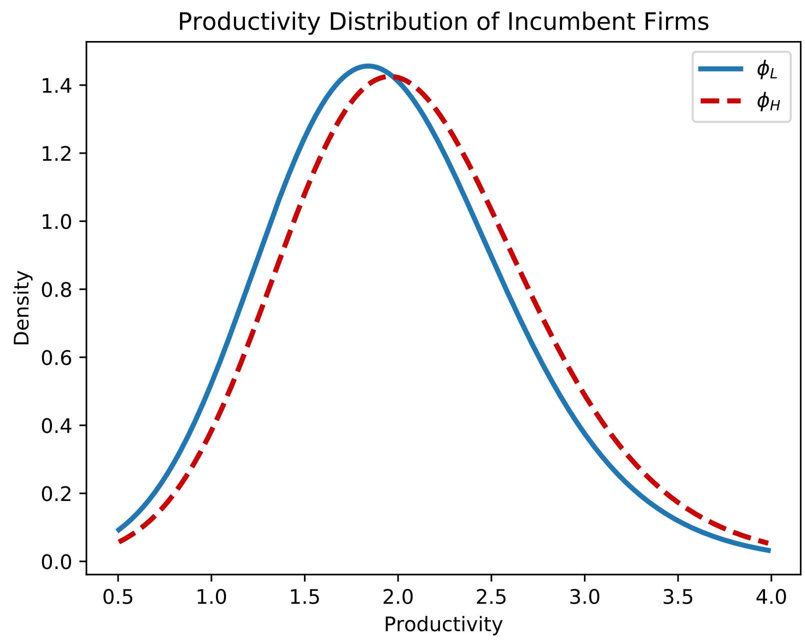


Figure 4.10: Distribution of Persistent Firm Productivity (z) in the Steady State

As discussed earlier, equation (4.38) specifies a distortion on the capital-to-

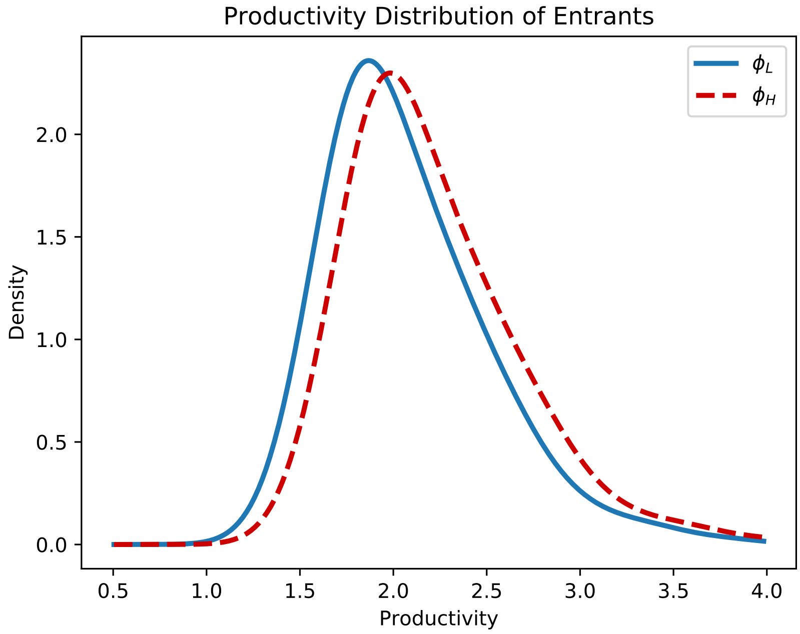


Figure 4.11: Distribution of Persistent Firm Productivity (z) of Entrants

labour ratio which increases with the firm's probability of default. This is due to the fire-sale cost of selling depreciated capital if the firm defaults which makes holding capital less efficient for firms with a higher default probability. The ϕ_H economy features a lower default probability which lowers the value of Λ_k and this is the key driver of the increase in the capital-to-labour ratio from the ϕ_L economy to the ϕ_H economy..

The unconditional distributions of firm net worth and persistent productivity z are shown in Figure 4.9 and Figure 4.10 respectively. The ϕ_H economy features more selection into the economy and thus the distribution of firm net worth x and productivity z shift to the right.

The increase in firm bargaining power, if everything else were constant would result in the bank charging higher interest spreads. However, in the ϕ_H economy, firms are unable to borrow as much and are less highly leveraged, hence why the spread on borrowing actually decreases. Both of these factors make production more costly in the ϕ_H economy and thus less productive firms exit sooner and there is more selection in firm entry. Figure 4.11 shows the productivity distribution of entrants in the steady state. As the mass of potential firms M is held constant between the two economies, the greater selection in the ϕ_H economy results in fewer entrants, a smaller mass of firms in the stationary distribution and a higher survival rate of newly entering firms. The fall in the equilibrium wage again dampens the variation of the distributions across regimes.

4.5.2 Dynamic Response

In this section I present the dynamic response of the model to an unexpected aggregate shock to the distribution of firms. The shock is a negative shock to the mean of the revenue shock distribution such that $\mathbb{E}[\omega]$ falls by 0.1 standard deviations. The shock lasts for only one period and following its impact, reduces the revenue and hence the net wealth of firms in the economy. Firms with sufficiently low net wealth will exit the economy, reducing the mass of incumbent firms below the steady state value. As the mass of entrants is constrained by the value of M, the wage will adjust to ensure the labour market clears. I assume that firms fully anticipate the path of wages, which after the initial probability zero shock follow a deterministic path. As the wage changes, the firm's policy functions also change. From a technical point of view, this requires the firm's problem to be solved in order to find the wage that clears the market.

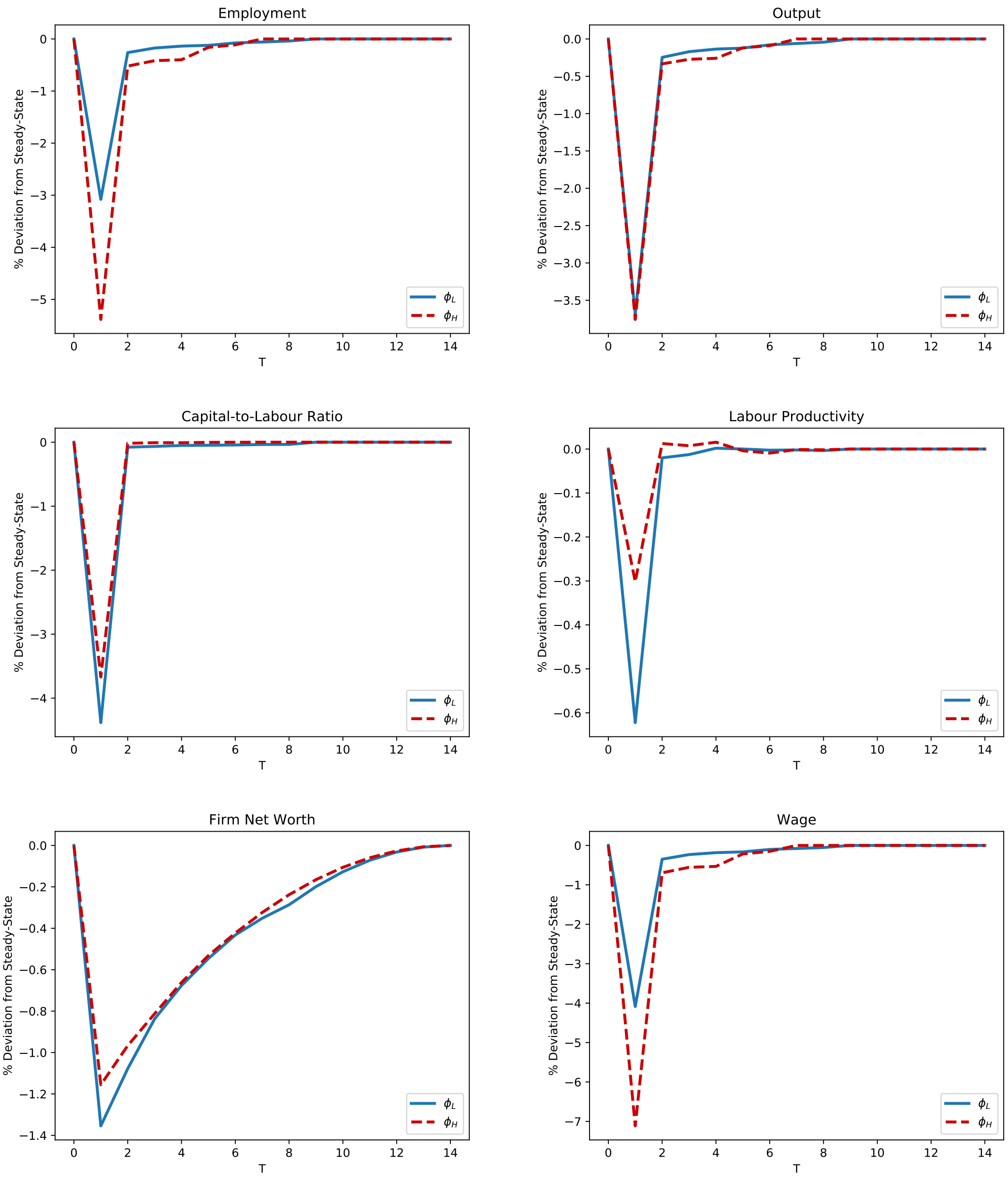


Figure 4.12: Dynamic Response to a negative shock to Firm productivity z

Figure 4.12 sets out the response of the two regimes to the negative shock to z. Employment falls less in the creditor-friendly (ϕ_L) than the debtor-friendly (ϕ_H) regime, employment falling 3 percent on impact in the ϕ_L case compared to 5.4 percent with ϕ_H. Labour productivity, measured as the ratio of output to employment, falls in both models, but falls most in the creditor-friendly regime (ϕ_L). In this experiment, the ϕ_H economy features leads to a 75 percent larger peak drop in employment and a 20 percent smaller fall in labour productivity following the negative productivity shock.

Labour productivity falls as firms are more borrowing constrained when their bargaining power is low and this in turn leads to firms lowering their capital holdings, with the capital-to-labour ratio falling more in the creditor-friendly regime. The response of output to the shock is similar in both regimes, suggesting that the employment and productivity responses roughly offset. The firm's bargaining power during restructuring may be costly in the steady-state, but provides a degree of insurance when the firm, or the economy, is hit by a negative shock. Firms that default as a result of the shock are more likely to restructure their debt when their bargaining power is high and firms that restructure their debt will begin the next period with higher net worth and thus are less borrowing constrained than if their bargaining power was low.

While the wage falls further in the debtor-friendly insolvency regime, this occurs for the same reasons as the fall in employment as w and N move together as can be shown from equation (4.15). As firms hit by the negative productivity shock more firms exit and employment falls. As employment falls, the household labour supply condition results in a falling wage. In the creditor-friendly bankruptcy regime, borrowing constrained firms substitute from capital to labour and the resulting fall in employment is less, hence the equilibrium wage falls less. The debtor-friendly regime does not fully capture the US experience, labour productivity still falls in the model while it rose slightly following the crisis. Fully capturing the response of the US economy was not the purpose of this exercise and I chose to calibrate the model to UK data only. However, the current formulation of the model would struggle to reverse the direction of the labour productivity response. This is because the capital-to-labour ratio will fall as the firm's default risk increases and a negative shock will increase the firm's probability of default. Nevertheless, the model presented here highlights how the difference in insolvency regimes between the UK and the US can partly explain the fall in labour productivity the UK witnessed since the financial crisis.

4.6 Conclusion

This paper presents a heterogeneous firm model of the UK's creditor-friendly insolvency regime and investigates a hypothetical change in the insolvency regime to a US-style debtor-friendly regime. The insolvency regime is modeled as a costly-state verification model where firms have the option to restructure their debt. Default costs create a wedge on the capital-to-labour wedge, effectively raising the price of holding capital for high-risk firms. As a consequence of this, firms that are more borrowing constrained have a lower capital-to-labour ratio and thus lower labour productivity. In the steady-state, less firm bargaining power results in lower firm borrowing rates and higher output. The dynamic response of the benchmark model to a negative shock matches the response of the UK to the financial crisis. In particular, following a negative shock to firm productivity firms in the creditor-friendly regime substitute from capital to labour. This dampens the fall in employment while causing labour productivity to fall. Firms in a debtor-friendly insolvency regime, while more costly in the steady-state allows firms to remain less borrowing constrained following a large aggregate shock and thus these firms hold relatively more capital relative to their counterparts in a creditor-friendly regime. In addition to the static benefits of the UK regime the results highlight the trade off in the dynamic response to shocks. The counterfactual debtor-friendly insolvency regime led to a 75 percent larger peak drop in employment following the negative productivity shock but a 20 percent smaller fall in labour productivity.

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