AN OPTIMISATION-BASED APPROACH FOR PROCESS PLANT LAYOUT

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Abstract

This paper presents an optimisation-based approach for the multi-floor process plant layout problem. The multi-floor process plant layout problem involves determining the most efficient - based on predefined criteria - spatial arrangement of a set of process plant equipment with associated connectivity. A number of cost and management/engineering drivers (e.g. connectivity, operations, land area, safety, construction, retrofit, maintenance, production organisation) have been considered over the last two decades in order to achieve potential savings in the overall plant design process.

This work constitutes an extension of the previous work by Patsiatzis and Papageorgiou [1] to address the multi-floor process plant layout problem. New features introduced modelled tall equipment with heights greater than the typical floor height in chemical process plants, with connection point at a design-specified height for each equipment.

The number of floors, land area, allocation of each equipment to a floor and the overall layout of each floor were determined by the optimisation model whilst preventing overlap of equipment. The connection costs, horizontal and vertical, as well the construction costs were accounted for with an overall objective to minimize the total cost. The overall problem was formulated as a mixed-integer linear programming (MILP) model based on a continuous domain representation and its applicability demonstrated by a number of illustrative examples. Results showed an increase in the number of equipment handled by the proposed models in modest computational times. Finally, symmetry breaking constraints were included to increase computational efficiency, and their performance tested with the last example.

Keywords

Multi-floor process plant layout; Mixed integer linear programming(MILP); Optimisation

1 Introduction

Over the past two decades, the layout of equipment in process plants have gained increasing relevance, from an economic, safety, operational and maintenance-related point of view.[2,3] Layout design generally involves obtaining a suitable arrangement of items and their connections within a predefined area based on pre-specified criteria. These items may be process vessels and equipment, work centres or departments within an organisation; with connections by pipes, conveyors, vehicular transport or any suitable material handling equipment.[3,4]
Layout designs have been found to have a tremendous impact on the costs and productivity of any system involved[2], as such researchers have constantly sought out methods to find the best solution to such design problems - generally referred to as the layout problem. When concerned with items that are needed for the production of goods or delivery of services, it has been referred to as the facility layout problem.[4] Amidst the classes of the facility layout problem (see Drira et al. for a detailed review) is the process plant layout problem, on which this work primarily focuses. Process plant layout problem addresses the layout problem for items in a typical chemical process plant.[5]
Previous approaches to process plant layout design were from a practical point of view, where decisions were made based on engineering judgement[6], but especially from the late 90s, more scientific approaches in terms of mathematical modelling have been adopted to solve the problem[1,7–12]. Most of the previous research have focused on a cost-reducing objective, with a varying number of scenarios. Georgiadis and Macchietto[9] proposed a mathematical programming approach with an objective to minimize cost - pumping, pipe connection and floor construction - and was formulated as a mixed integer linear programming (MILP) problem. Results showed the model was computationally efficient for a small number of units. Papageorgiou and Rotstein[12] further solved the process plant layout problem over a continuous spatial domain whilst introducing more realistic constraints for single-floor case studies. Barbosa-Povoa et al.[7,8] accounted for different inputs and outputs for each equipment, irregular shapes, a 3-dimensional representation, allowed for different production sections, and

safety and operability factors. These problems were all formulated as an MILP giving computationally efficient solutions for a small number of units.

Formulations to address scenarios with multiple floors have also been considered. Patsiatzis and Papageorgiou[10] developed an MILP formulation to allow for a multi-floor layout with floor construction costs (fixed and area-dependent) and vertical pumping costs. The model successfully handled up to 11 units within a modest CPU time. For larger number of units, Patsiatzis and Papageorgiou[1] proposed a rigorous decomposition approach and an iterative solution scheme, with the later producing a solution with a higher degree of optimality. An improvement-type algorithm was proposed by Xu and Papageorgiou[13] applicable to a single floor.

Apart from a cost minimisation objective, other authors have introduced other criteria to the objective function. Penteado and Ciric[5] presented a mixed integer non-linear programming (MINLP) problem not only to minimize piping cost, land allocation cost, but also financial risk and safety costs. Other factors have been the routing and layout of pipes[6], safety and risk assessment[11,14–17], long-length equipment[17–19], area minimisation[20]; with varying solution techniques[17,21–25].

This paper proposes a generic model to the multi-floor process plant layout problem, initially proposed by Patsiatzis and Papageorgiou[1] (an improvement on Patsiatzis and Papageorgiou[10]) but extended to account for long-length equipment, i.e. equipment typically spanning multiple floors, with connection points at design-specified heights for each equipment. Ku et al.[18] presented a MINLP model and accounted for tall equipment, minimizing total layout area with a hybrid optimisation method. This work seeks to propose a more efficient MILP model to handle a larger number of units than previously recorded. Symmetry breaking constraints are also introduced to reduce solution search over multiple optimal solutions.

The rest of this paper is structured as follows: the problem is described in section 2 with underlying assumptions; mathematical models are then proposed in section 3 to address the problem; case studies to show model performance and applicability are presented in section

4, where additional symmetry breaking constraints are also introduced and tested. Finally, concluding remarks are given in section 5.

2 Problem Description

The following assumptions are made in the model formulation. Equipment items are described by rectangular shapes. In all cases, rectilinear distances are assumed, with connections between equipment taken from their respective geometrical centres in the x-y plane. Vertical distances are measured from a design-specified height of each equipment. Each equipment is allowed to rotate in 90^o angles in the x-y plane, and must start from the base of each available floor. The floor height is fixed across all floors and if an equipment exceeds such height, it is allowed to extend through available floors.

The problem description for the process layout problem is as follows.

Given:

- a set of N equipment items and their dimensions (length, breadth and height);
- a set of K potential floors for layout;
- connectivity network amongst equipment;
- cost data (connection, pumping, land, and construction);
- floor height;
- space and equipment allocation limitations;
- minimum safety distances between equipment items;

determine:

- the total number of required floors for the layout;
- the base land area occupied;

- the area of floors;
- spatial equipment allocation to floors;

so as to: minimise the total plant layout cost.

3 Mathematical Formulation

The mathematical formulation constitutes an extension to the work of Patsiatzis and Papageorgiou[1]. A full description of the model proposed by Patsiatzis and Papageorgiou[1] is available in the supporting information. Two formulations (A & B) are proposed, primarily differing in the manner tall equipment (which span multiple floors) are handled. The first handles multi-floor equipment as is, by a set of constraints as summarised in Table 1 for each of the three models proposed. The second formulation is a linearised form of the model proposed by Ku et al.[18] (a direct extension of Patsiatzis and Papageorgiou[1]), with multi-floor equipment divided into pseudo-single floor units equivalent to the number of consecutive floors they require.

Table 1: Summary of equations adopted in models A.1 - A.3

Constraints	Formulation A model		
	A.1	A.2	A.3
Floor	1 - 4, 5 - 6	1 - 4, 5 - 6	1 - 4, 5 - 6
Equipment orientation	9, 7 - 8	9, 7 - 8	9, 7 - 8
Multi-floor equipment	10-14	15	12, 13 and 16
Distance	24,21, 22, and 23	10, 12, 24 and 21, 22, 23,	24,21, 22, and 23
Non-overlapping	17 - 20	17 - 20	17 - 20
Area	31 - 36	31 - 36	31 - 36
Layout design	30, 25 - 29	30, 25 - 29	30, 25 - 29

Nomenclature

Indices

i,j	equipment item in models A.1 - A.3
i',j'	equipment item in model B
k, k'	floor number
p	pseudo units
s	rectangular area sizes

Sets

I	set of equipment item for models A.1- A.3
I'	set of equipment item for model B
MF	set of multi-floor equipments
P_i	sets of pseudo units for equipment i

Parameters

α_i, β_i,γ_i	dimensions of item i
M_i	number of floors required by item i
f_{ij}	1 if flow direction between i and j is positive; 0, otherwise
OP_{ij}	distance between the base and output point on equipment i
IP_{ij}	distance between the base and input point on equipment j
BM	a large number
FH	floor height
LC	land cost
$FC1$	fixed floor construction cost
$FC2$	area-dependent floor construction cost
C^c_{ij}	connection/piping costs

C_{ij}^{h}	horizontal pumping costs
C_{ij}^{v}	vertical pumping costs

Integer variables

NF	number of floors required for layout

Binary variables

O_i	1 if length of item i is equal to α_i; 0, otherwise
W_k	1 if floor k is occupied; 0, otherwise
n_{ijk}	1 if items i and j are assigned to floor k; 0, otherwise
N_{ij}	1 if items i and j are assigned to the same floor; 0, otherwise
V_{ik}	1 if item i is assigned to floor k
Q_s	1 if rectangular area s is selected for the layout; 0, otherwise
$E1_{ij}$, $E2_{ij}$	non-overlapping binary, a set of values of which prevents equipment overlap in one direction in the x-y plane
S_{ik}^{s}	1 if item i spans more than one floor and begins at floor k; 0, otherwise
S_{ik}^{f}	1 if item i spans more than one floor and terminates at floor k; 0, otherwise

Continuous variables

l_i	length of item i
d_i	breadth of item i
h_i	height of item i
x_i, y_i, z_i	coordinates of the geometrical centre of item i
R_{ij}	relative distance in x coordinates between items i and j,

	if i is to the right of j
L_{ij}	relative distance in x coordinates between items i and j,
	if i is to the left of j
A_{ij}	relative distance in y coordinates between items i and j,
	if i is above j
B_{ij}	relative distance in y coordinates between items i and j,
	if i is below j
U_{ij}	relative distance in z coordinates between items i and j,
	if i is higher than j
D_{ij}	relative distance in z coordinates between items i and j,
	if i is lower than j
TD_{ij}	total rectilinear distance between items i and j
NQ_s	linearisation variable expressing the product of NF and Q_s
AR_s	predefined rectangular floor area s
FA	floor area
X^{max}, Y^{max}	dimensions of floor area

3.1 Formulation A

In formulation A, multi-floor equipments are treated as a single unit starting on one floor k, spanning through consecutive floors and terminating at a higher floor k'. Floors are numbered from bottom to top. Three equivalent models - A.1, A.2 and A.3 are described which only differ by the sets of constraints used to model the multi-floor equipment.

3.1.1 Floor constraints

Every equipment i available is assigned to an equivalent number of floors, M_i.

$$\sum_k V_{ik} = M_i \qquad \forall\ i \tag{1}$$

where V_{ik} is a binary variable which determines if an equipment i is assigned to floor k. A variable, n_{ijk}, is introduced to determine if equipment i and j are assigned to floor k, given by:

$$n_{ijk} \geq V_{ik} + V_{jk} - 1 \qquad \forall\ i = 1, ..., N-1,\ j \neq i,\ k = 1, ..., K \tag{2}$$

$$N_{ij} \geq n_{ijk} \qquad \forall\ i = 1, ..., N-1,\ j \neq i,\ k = 1, ..., K \tag{3}$$

The variable N_{ij} takes the value of 1 if and only if items i and j are on any same floor. Furthermore, a floor will only exist if an equipment starts on it. For this, new variables S^s_{ik} and $S^f_{ik'}$ are introduced taking values of 1 if an equipment i starts at floor k and terminates at floor k' respectively. Thus:

$$S^s_{ik} \leq W_k \qquad \forall\ i, k \tag{4}$$

Also, a floor should exist only if an equipment is assigned to it, and not just pasing through it (for multi-floor equipment), and that the preceding floor is also occupied. This constraint is obtained from Patsiatzis and Papageorgiou[1]:

$$W_k \leq W_{k-1} \qquad \forall\ k = 2, ..., K \tag{5}$$

The number of floors required is then determined by equation 6[1].

$$NF \geq \sum_k W_k \tag{6}$$

3.1.2 Equipment orientation constraints

A 90° rotation of equipment orientation is allowed in the x-y plane. This is represented by equations 7 and 8[1].

$$l_i = \alpha_i O_i + \beta_i (1 - O_i) \qquad \forall\, i \tag{7}$$

$$d_i = \alpha_i + \beta_i - l_i \qquad \forall\, i \tag{8}$$

Rotation in the z-axis is deemed unrealistic as such equipment orientation is fixed for construction in virtually all cases.

$$h_i = \gamma_i \qquad \forall\, i \tag{9}$$

3.1.3 Multi-floor equipment constraints

In order for equipment items requiring more than one floor to span across successive floors, the constraints below are introduced. Three alternative models are shown (Models A.1, A.2 and A.3).

Model A.1 includes equations 10 - 14.

$$-V_{ik} + V_{i,k-1} + S^s_{ik} \geq 0 \qquad \forall\, i, k \tag{10}$$

$$-V_{ik} + V_{i,k+1} + S^f_{ik} \geq 0 \qquad \forall\, i, k \tag{11}$$

Equations 10 and 11 ensures that the binary variables S^s_{ik} and S^f_{ik} take a value of 1 if an equipment starts and ends on a particular floor, with each equipment being restricted to start and end on only one floor by:

$$\sum_k S^s_{ik} = 1 \qquad \forall\, i \tag{12}$$

$$\sum_k S^f_{ik} = 1 \qquad \forall\, i \tag{13}$$

The constraint to restrict equipment to occupy successive floors is then given as:

$$\sum_{k'=k}^{k+M_i} V_{ik'} \geq M_i.S^s_{ik} \qquad \forall\ i, k \tag{14}$$

Model A.2 models the multi-floor equipment constraint by the equation below:

$$\sum_{\theta=1}^{M_i-1} V_{i,k+\theta} \geq (M_i - 1).(V_{ik} - V_{i,k-1}) \qquad \forall\ i, k \tag{15}$$

Equation 15 ensures that multi-floor equipment occupy consecutive floors, the total number equalling the required number of floors M_i.

Model A.3 makes use of the binary variables S^s_{ik} and S^f_{ik} and is described using equations 12, 13 and 16 below.

$$V_{ik} - V_{i,k-1} = S^s_{ik} - S^f_{i,k-1} \qquad \forall\ i, k \tag{16}$$

In all cases (models A.1, A.2, and A.3) for the multi-floor equipment, every equipment item i spanning more than one floor starts on a single floor ($S^s_{ik} = 1$) and terminates on another floor ($S^f_{ik'} = 1$), occupying successive floors, the sum of which is equal to its required number of floors (M_i).

3.1.4 Non-overlapping constraints

To prevent two or more equipment to occupy the same space within a floor, constraints 17 - 20 are introduced[1]:

$$x_i - x_j + BM(1 - N_{i,j} + E1_{ij} + E2_{ij}) \geq \frac{l_i + l_j}{2} \qquad \forall\, i = 1, ..., N-1,\ j = 2, ..., N \tag{17}$$

$$x_j - x_i + BM(2 - N_{i,j} - E1_{ij} + E2_{ij}) \geq \frac{l_i + l_j}{2} \qquad \forall\, i = 1, ..., N-1,\ j = 2, ..., N \tag{18}$$

$$y_i - y_j + BM(2 - N_{i,j} + E1_{ij} - E2_{ij}) \geq \frac{d_i + d_j}{2} \qquad \forall\, i = 1, ..., N-1,\ j = 2, ..., N \tag{19}$$

$$y_j - y_i + BM(3 - N_{ij} - E1_{ij} - E2_{ij}) \geq \frac{d_i + d_j}{2} \qquad \forall\, i = 1, ..., N-1,\ j = 2, ..., N \tag{20}$$

3.1.5 Distance constraints

Distance constraints are described by equations 21, 22 and 23[1], to determine the relative distances in the x and y coordinates respectively.

$$R_{ij} - L_{ij} = x_i - x_j \qquad \forall (i,j):\ f_{ij} = 1 \tag{21}$$

$$A_{ij} - B_{ij} = y_i - y_j \qquad \forall (i,j):\ f_{ij} = 1 \tag{22}$$

$$TD_{ij} = R_{ij} + L_{ij} + A_{ij} + B_{ij} + U_{ij} + D_{ij} \qquad \forall (i,j):\ f_{ij} = 1 \tag{23}$$

Provision is made for connection between equipment i and j at design-specified heights of either equipment as described by equation 24;

$$U_{ij} - D_{ij} = FH \sum_k (k-1)(S^s_{ik} - S^s_{jk}) + OP_{ij} - IP_{ij} \qquad \forall (i,j):\ f_{ij} = 1 \tag{24}$$

where OP_{ij} represents the vertical distance from the base of equipment i to its output point, and IP_{ij} represents the vertical distance from the base of equipment j to its input point for connection i-j (Figure 1).

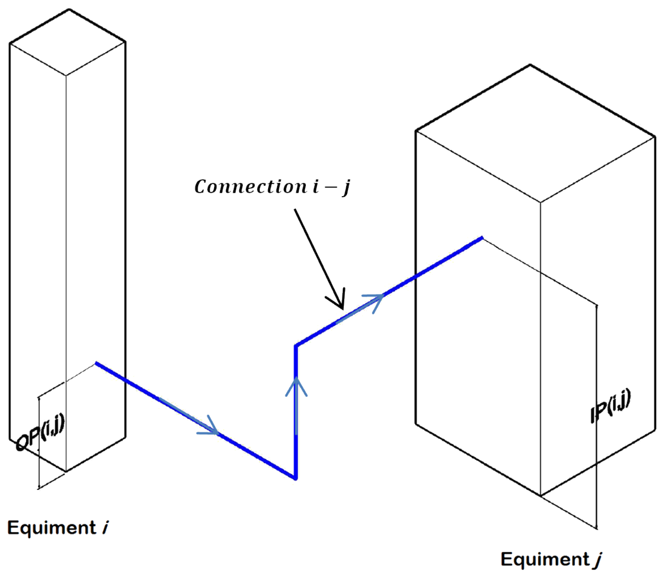

Figure 1: Multi-point connection illustration

3.1.6 Layout design constraints

Layout design constraints, 25 - 29[1], ensure that equipment are placed within the boundaries of a floor:

$$x_i \geq \frac{l_i}{2} \qquad \forall\, i \tag{25}$$

$$y_i \geq \frac{d_i}{2} \qquad \forall\, i \tag{26}$$

and the upper bound:

$$x_i + \frac{l_i}{2} \leq X^{max} \qquad \forall\, i \tag{27}$$

$$y_i + \frac{d_i}{2} \leq Y^{max} \qquad \forall\, i \tag{28}$$

The land area is then calculated as:

$$FA = X^{max} Y^{max} \tag{29}$$

An additional layout design constraint is included for the z-coordinates to define its geometrical centre, given by:

$$z_i = \frac{h_i}{2} \qquad \forall\, i \tag{30}$$

Equation 30 also ensures equipment i starts from the base of the floor k where it is placed.

3.1.7 Area Constraints

In order to avoid bilinear terms in calculating the floor area, FA (eq. 29), equations 31 - 36[1] are introduced. The area of each floor is determined from a set of S predefined rectangular area sizes, AR_s, with dimensions $(\overline{X}_s, \overline{Y}_s)$.

$$FA = \sum_s AR_s Q_s \tag{31}$$

$$\sum_s Q_s = 1 \tag{32}$$

The floor length and breadth is selected from the chosen rectangular area size dimensions:

$$X^{max} = \sum_s \overline{X}_s Q_s \tag{33}$$

$$Y^{max} = \sum_s \overline{Y}_s Q_s \tag{34}$$

Also, a new term NQ_s is introduced in order to linearise the cost term associated with the number of floors:

$$NQ_s \leq K\,Q_s \qquad \forall\ s \tag{35}$$

$$NF = \sum_s NQ_s \tag{36}$$

3.1.8 Objective function

The objective function is the same for each model - to minimize the total cost associated with the connection cost, pumping cost, land area cost, floor construction cost and floor-area dependent cost. This is given as:

$$\begin{aligned} min \sum_i \sum_{j \neq i: f_{ij}=1} [C^c_{ij} TD_{ij} + C^v_{ij} D_{ij} + C^h_{ij}(R_{ij} + L_{ij} + A_{ij} + B_{ij})] \\ + FC1 \cdot NF + FC2 \sum_s AR_s \cdot NQ_s + LC \cdot FA \end{aligned} \tag{37}$$

3.2 Formulation B

Formulation B is a linearised model of Ku et al.[18] with adaptations from Patsiatzis and Papageorgiou[1] as described in the supporting information. The following additional constraints are included to account for the multi-floor equipment and design-specified connection height.

3.2.1 Multi-floor equipment constraint

Multi-floor equipment are split into pseudo units equivalent to the number of floors they occupy (Figure 2). This approach presents a more accurate representation for multi-floor equipment that have varying sizes along its height - due to a range (in type and size) of auxiliary units placed by such equipment. The drawback, however, is that an increase in the number of units in the MILP model is realised which can lead to larger computational times.

For all multi-floor equipment, $i \epsilon MF$, requiring M_i number of floors, each is split into M_i pseudo units. The first $M_i - 1$ units have heights equal to the floor height, and the last pseudo unit with the remainder. So, for each $i \epsilon MF$, there exists pseudo units, $p \epsilon P_i$; and the modified set of all equipments, I', becomes:

$$I' = (I \wedge (\neg MF)) \vee P_i$$

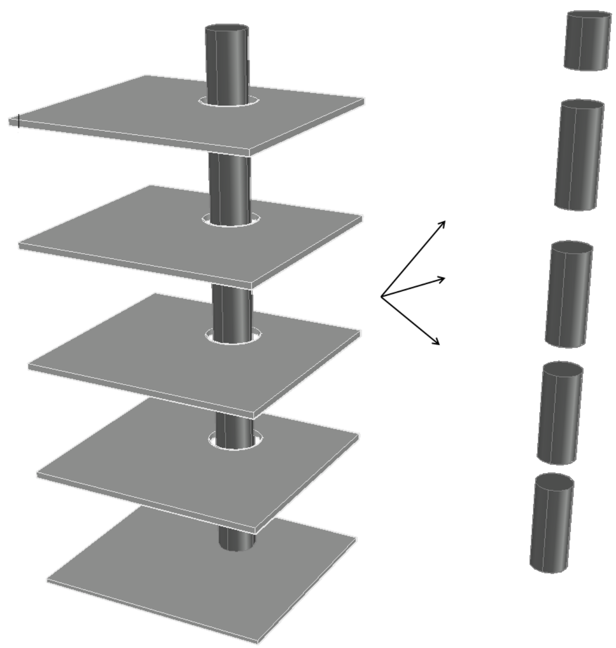

Figure 2: Pseudo-unit illustration

Subsequent pseudo units, $p + 1$, for each pseudo unit set, P_i, are then made to occupy successive floors by the constraint:

$$V_{p,k} = V_{p-1,k-1} \qquad \forall\ p \epsilon P_i, k \tag{38}$$

And for pseudo units to be placed directly above it's preceding counterpart:

$$x_p = x_{p+1} \qquad \forall\ p \epsilon P_i \tag{39}$$

$$y_p = y_{p+1} \qquad \forall\ p \epsilon P_i \tag{40}$$

and finally, consistency in 90º rotation is ensured for all pseudo units of a multi-floor equipment:

$$O_p = O_{p+1} \qquad \forall\ p \epsilon P_i \tag{41}$$

3.2.2 Distance constraints

For connection between equipment i' and j' at design-specified heights of either equipment, the constraint below is used;

$$U_{i'j'} - D_{i'j'} = FH \sum_k (k-1)(V_{i'k} - V_{j'k}) + OP_{i'j'} - IP_{i'j'} \qquad \forall (i',j') \epsilon I' : \ f_{i'j'} = 1 \tag{42}$$

The resulting model (B) thus consists of constraints 38 - 42; and equations S1-S15; S17-S28 in the original model of Patsiatzis and Papageorgiou described in the supporting information $\forall\ i', j' \ \epsilon\ I'$; and objective function:

$$\begin{aligned} min \sum_{i'} \sum_{j' \neq i' : f_{i'j'}=1} [C^c_{i'j'} TD_{i'j'} + C^v_{i'j'} D_{i'j'} + C^h_{i'j'} (R_{i'j'} + L_{i'j'} + A_{i'j'} + B_{i'j'})] \\ + FC1 \cdot NF + FC2 \sum_s AR_s \cdot NQ_s + LC \cdot FA \end{aligned} \tag{43}$$

4 Case Studies

In this section, a description of the case studies applied to each formulation is shown. Each example was modelled using GAMS[26] modelling system v24.7.1 with the CPLEX v12.6 solver on an Intel® Xeon® E5-1650 CPU with 32GB RAM, and the layout plotted using Autodesk AutoCAD® 2017. Each run was solved to global optimality. For the floor area, five alternative sizes (10m, 20m, 30m, 40m and 50m) were used in examples 1 and 2, seven alternative sizes (10m, 20m, 30m, 40m, 50m, 60m and 70m) in example 3, giving a total of 25 and 49 possible area sizes respectively, except otherwise stated. Data on connectivity costs can be obtained from process simulation results of a case study. Pipe sizing values are obtained from design equations, matched with commercially available pipe sizes and the costs estimated per unit length for a selected material of construction based on the components in each stream. Pumping costs are estimated based on current electricity prices and material flow rate in each stream, and construction costs are extrapolated from past data on similar plant design based on structural engineering calculations. For all case studies, data on equipment dimensions, connectivity and construction costs, and vertical connection points are included in the supporting information. For all three examples investigated in this work, both formulations A and B yield equivalent optimal plant layouts and the same objective value, with main differences in layout orientation and reflection. Thus, only the optimal layout result for model A.1 is presented for each example. Layout results for the other models are available in the supporting information.

4.1 Example 1

Example 1 is an Ethylene oxide plant used in the work of Patsiatzis and Papageorgiou[10], originally presented by Penteado and Ciric[5]. The plant consists of 7 units with process flow diagram shown in Figure 3. The summary of the model statistics and computational performance is shown in Table 2 and the optimal layout in Figure 4.

The results obtained gave a total cost of 66,262.0 rmu - 22% connection costs, 44% pumping costs and 34% construction costs. Each floor had an area measuring 20mx20m, totalling two floors selected for construction out of four provided to the model. Table 2 shows that the models in formulation A are more computationally efficient. Formulation B was inherently larger in size for each case study, as a greater number of units needed to be handled than in formulations A.

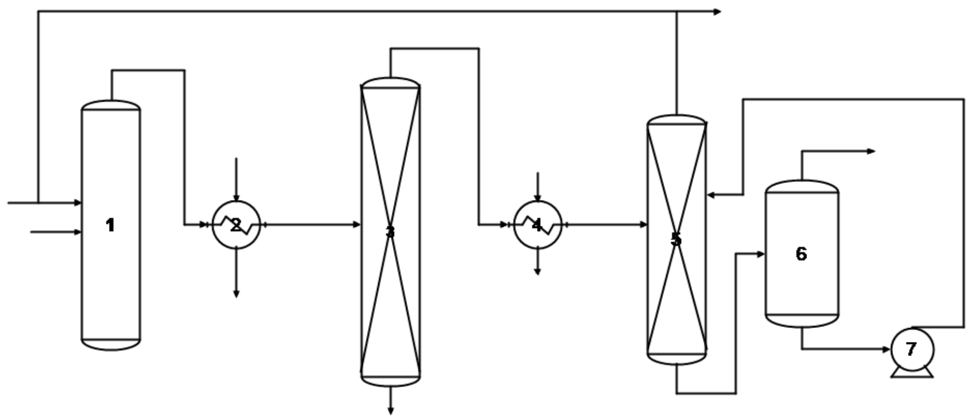

Figure 3: Flow diagram of Ethylene Oxide plant

The layout of equipment in Figure 4 showed that the two multi-floor equipment (3 and 5) started on the first floor and terminated on the second floor for all models, as they require two floors based on their height. The total cost - 66,262.0 rmu - is also greater than the value obtained by Patsiatzis and Papageorgiou[1] (50,817 rmu). The 30% increase in cost is attributed to the additional consideration of the connection points being measured from the design-specified heights, and the consequent change in equipment layout. Previous considerations assumed a connection from the mid-point of an equipment height, but current results show that such assumption is not realistic as reflected in the cost difference.

Table 2: Summary of model statistics and computational performance for Example 1

	EO plant (7 units)			
	A.1	A.2	A.3	B
Total Cost (rmu)	66,262.0			
CPU (s)	1.5	1.4	1.7	3.0
Number of binary variables	112	112	112	167
Number of continuous variables	225	204	225	130
Number of equations	424	381	382	633

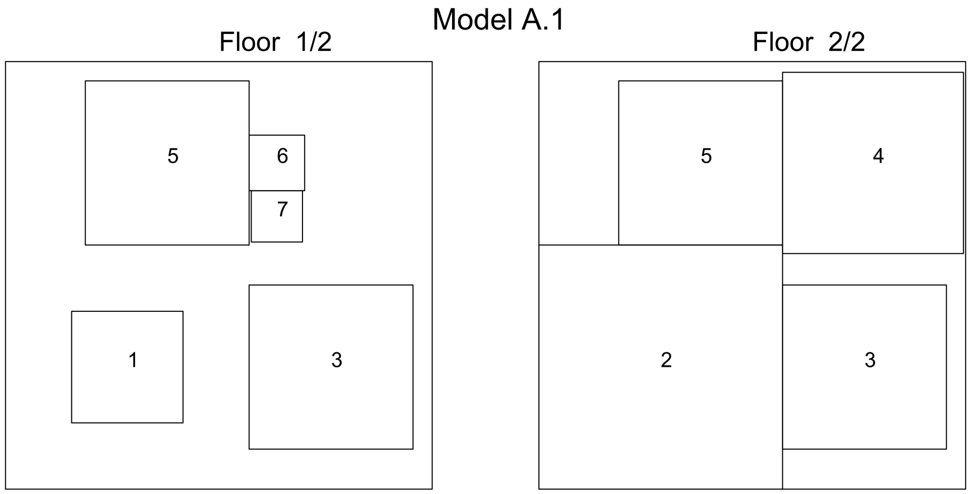

Figure 4: Example 1 layout results

4.2 Example 2

Example 2 is an Urea production plant, simulated with Aspen PLUS® v8.0. It consists of 8 units, 2 of which exceed the floor height of 8m. The process flow diagram of the plant is shown in Figure 5. A total of 25 possible area sizes were used ranging from 5m - 45m. Each unit had to be placed a minimum of 4m from another in either direction. The summary of the model statistics and computational performance is shown in Table 3.

The layout result is shown in Figure 6. All 4 floors were assigned units, with each floor having an area measuring 15mx5m, and a total cost of 117,431.0 rmu - 6% connection, 26% pumping and 68% construction costs. The two multi-floor units (units 2 and 4) occupied successive floors proportional to their height (floors 1-4 and 2-3 respectively). Computational results showed global optimality was achieved by all models in a relatively short time (under 5s). Overall results show that the model is capable of handling process-specific conditions whilst deciding floor placement even for multi-floor units.

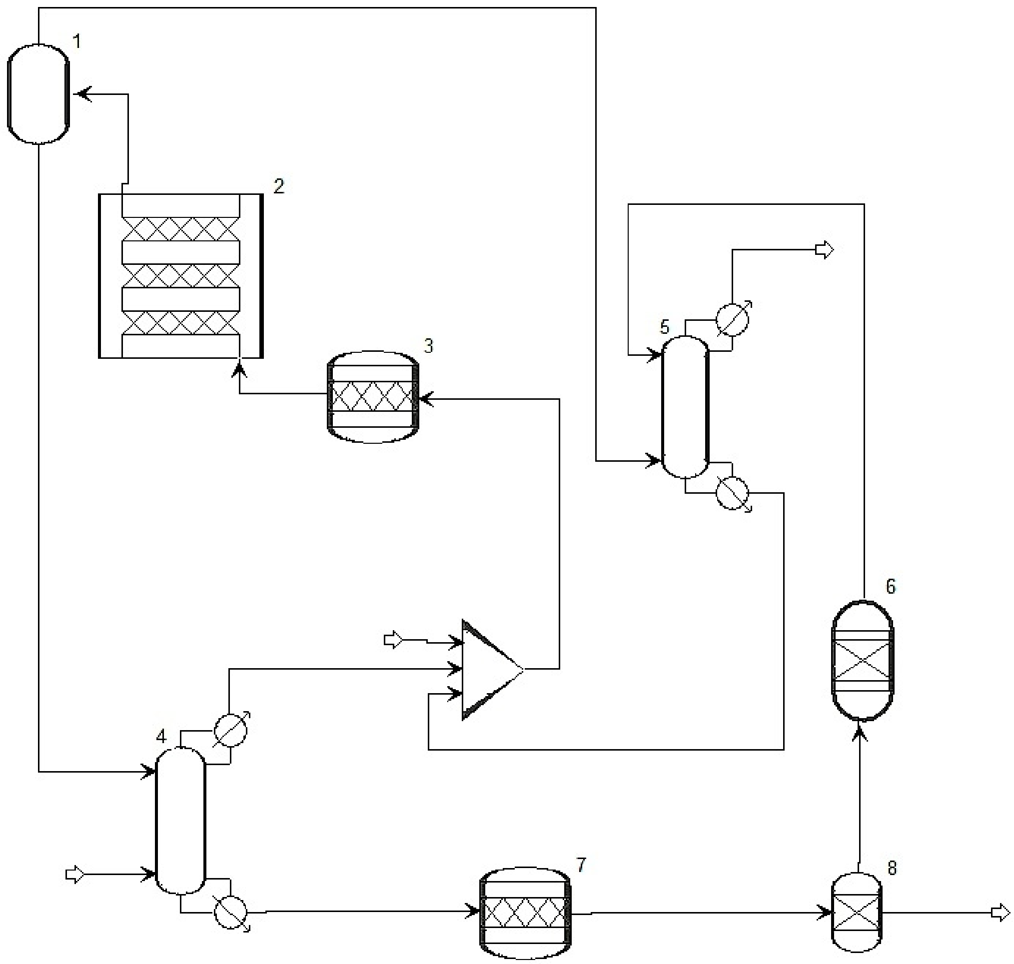

Figure 5: Flow diagram of Urea Production

Table 3: Summary of model statistics and computational performance for Example 2

	Urea Production (8 units)			
	A.1	A.2	A.3	B
Total Cost (rmu)		117,431.0		
CPU (s)	4.8	4.9	2.9	4.0
Number of binary variables	145	145	1453	281
Number of continuous variables	315	283	315	159
Number of equations	611	547	547	1,279

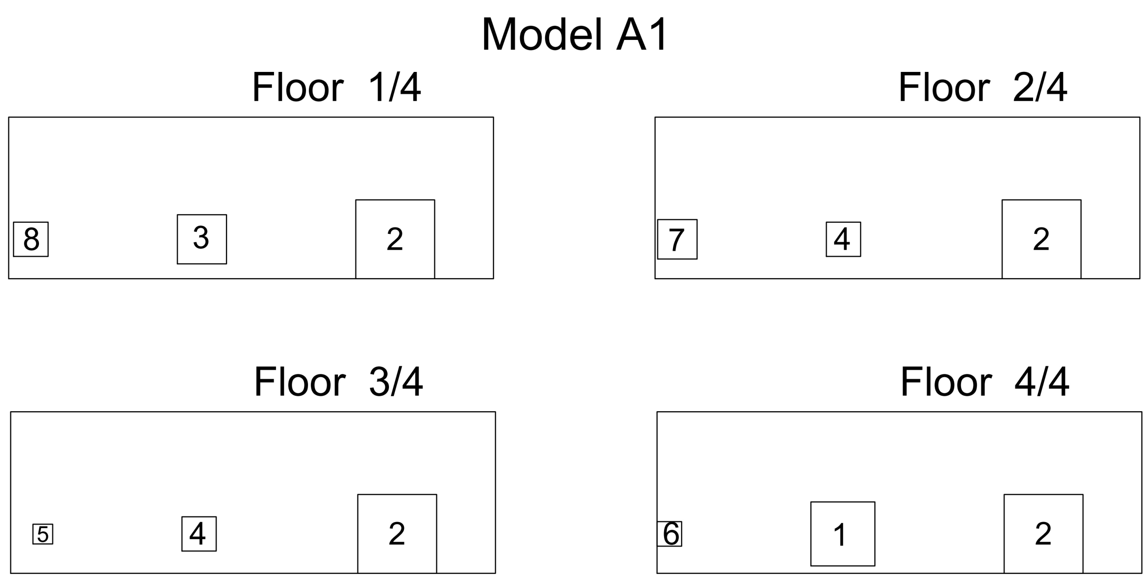

Figure 6: Example 2 layout results

4.3 Example 3

Example 3 is a Crude Distillation plant with preheating train, simulated with Aspen HYSYS® v8.0. It consists of 17 units, with 5 (pre-flash drum (unit 5), atmospheric distillation tower (unit 7), fired heaters 1 and 2 (unit 6 and 12), and debutaniser(unit 15)) exceeding the floor height of 5m. The process flow diagram of the plant is shown in Figure 7.

An additional condition that each of the multi-floor equipment starts from the ground floor

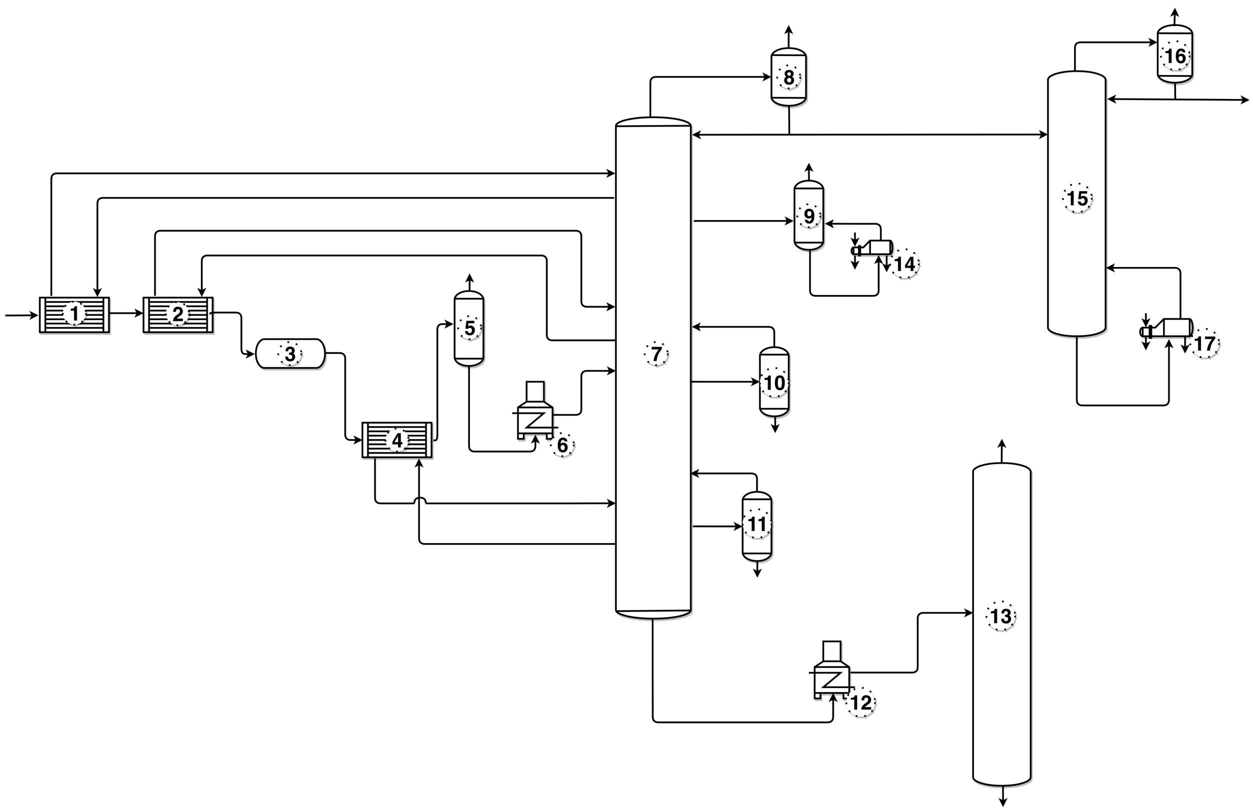

Figure 7: Flow diagram of Crude Distillation Plant with Preheating train

was imposed, as such represents a realistic representation from a construction point of view. The summary of the model statistics and computational performance is shown in Table 4 and the optimal layout in Figure 8.

The results obtained gave a total of four(4) possible floors, out of an available seven (7) which each floor measuring 20mx20m. A total cost of 749,691.4 rmu was obtained across all formulations as seen in Table 4. The higher relative pumping costs (50%) when compared with previous examples is attributed to the greater number of units considered. Connection and construction costs constituted 6% and 44% of the total cost respectively.

Table 4: Summary of model statistics and computational performance for Example 3

	CDU Plant (17 units)			
	A.1	A.2	A.3	B
Total Cost (rmu)	749,691.4			
CPU (s)	292.3	105.0	44.1	131.6
Number of binary variables	568	568	568	1,569
Number of continuous variables	1,507	1,388	1,507	387
Number of equations	3,213	2,993	2,975	11,483

The layout of equipment is shown in Figure 8. As all the multi-floor equipment (5, 6, 7, 12 and 15) starting floors were predefined, they all started from the first floor through the number of floors required based on their height. It is worthy of noting that although equipment 14 required 5 floors based on its height, a total of 4 floors was decided by all formulations. This is because the construction of a fifth floor was deemed unnecessary as no other equipment other than equipment 14 was to be placed on such floor in order to obtain an optimal solution, and the additional construction cost for a fifth floor was eliminated. So, although a multi-floor equipment can span a specified number of floors based on its height, not every one of those floors need to be constructed. Practical examples include multi-floor layouts about fired heaters with long stacks, distillation columns and flare stacks.

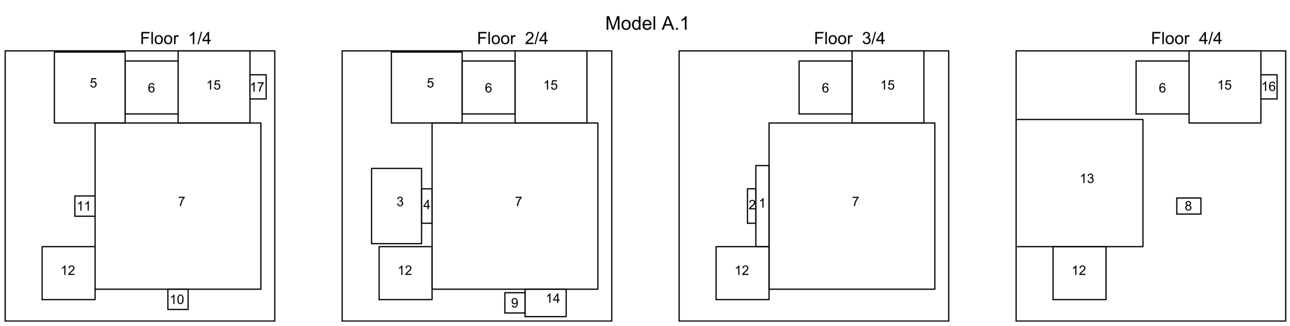

Figure 8: Example 3 layout results

4.3.1 Symmetry breaking constraints

For each of the case studies solved, it becomes evident from the layout results (presented above and in supporting information) that there can exist multiple optimal solutions for each problem. These multiple optimal solutions differ only in layout orientation/reflection and/or translation of equipment on the layout area, resulting in inefficient CPU usage[27], especially for larger problem instances. In order to increase computational efficiency and reduce symmetry, additional constraints, adapted from Westerlund and Papageorgiou[27] are introduced:

$$x_i + y_i - x_j - y_j \geq \alpha \cdot N_{ij} \quad (44)$$

$$E1_{ij} = 0 \quad (45)$$

where $\alpha = min\left(\frac{l_i}{2}, \frac{d_i}{2}\right) + min\left(\frac{l_j}{2}, \frac{d_j}{2}\right)$. These fix the relative position of i and j as well as one of the non-overlapping binary variables, $E1_{ij} = 0$. The latter enforces either of $x_i - x_j \geq \frac{l_i + l_j}{2}$ or $y_i - y_j \geq \frac{d_i + d_j}{2}$ according to equations 17 - 20, i.e., unit i is relatively locked in a compass point North East of unit j hence breaking the symmetry.

The above symmetry breaking constraints were applied to example 3 - Crude distillation plant with pre-heating train - by incorporating equations 44 and 45 in Model A.1 where the choice of equipment i and j are among multi-floor units only, based on the following three criteria[27]:

a - Equipment i and j having the highest connection cost: units 5 and 6;

b - Two equipment with the largest area: units 7 and 15;

c - Two equipment with the smallest area: units 6 and 12.

The resulting models were solved to global optimality and a solution of 749,691.4 rmu was obtained in all alternative choices of equipment, the same as model A.1 without these constraints. Their computational CPU times are shown in Table 5 - where for each unit selection alternative, the resulting model is named appropriately - model A.1a for model A.1 with symmetry breaking constraints with equipment i and j chosen by criterion a, having the highest connection cost, and so on. The results show that the new symmetry breaking constraints has improved computational efficiency by one order of magnitude. Computational improvements were also found for the other models developed and examples presented in this work.

Table 5: Summary of computational performance for Example 3; Model A.1 with symmetry breaking contraints

Model	CPU (s)
A.1	292.3
A.1a	31.2
A.1b	33.7
A.1c	71.2

5 Concluding remarks

An extension of the MILP model by Patsiatzis and Papageorgiou [1] was proposed to address new concerns in the multi-floor process plant layout problem. These concerns included the presence of multi-floor equipment in certain process plants with design-specified input and output connection points along the height of an equipment.

A total of four (4) models were proposed, grouped in two broad classes. The first class handled multi-floor equipment as they were (single tall units) having constraints to ensure multi-floor equipment were assigned to consecutive floors, with three (3) alternative models resulting. For the second class, multi-floor equipment were broken up to single-floor-pseudo-units and the layout solved based on a linearised model of Ku et al. [18].

Each of the four (4) models were validated with case studies and model performance were ascertained and compared. All models solved problems of up to 17 units well under 7 minutes, as compared to previous work [1] which only handled a maximum of 11 units simultaneously. Total cost distribution were consistent with expected values - construction and pumping costs taken up the larger portion, with connection costs following. All models were able to handle multi-floor equipment, and decide whether a floor be constructed and used even if a multi-floor equipment were assigned to it. In most cases though, formulations A was found to be more computationally efficient than B.

Finally, symmetry breaking constraints were introduced reducing the availability of multiple optimal solutions that lead to greater CPU usage. These constraints fixed the relative positions of two units i and j. The choice of units i and j were based on three criteria: largest connection, largest areas or smallest area amongst multi-floor equipment. A reduction in computational time up to 31.2s was obtained for example 3, as compared to 292.3s without symmetry breaking.

Further work will entail model validation with larger case studies and the development of efficient computational methods, e.g. decomposition techniques.

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Supporting Information Available

A full description of the model proposed by Patsiatzis and Papageorgiou [1], data for the case studies presented, and layout results for models A.2, A.3 and B is available as supporting information.

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Notes

The authors declare no competing financial interest.

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