What is Wrong with Classical Negation?

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Abstract

The focus of this paper are Dummett's meaning-theoretical arguments against classical logic based on consideration about the meaning of negation. Using Dummettian principles, I shall outline three such arguments, of increasing strength, and show that they are unsuccessful by giving responses to each argument on behalf of the classical logician. What is crucial is that in responding to these arguments a classicist need not challenge any of the basic assumptions of Dummett's outlook on the theory of meaning. In particular, I shall grant Dummett his general bias towards verificationism, encapsulated in the slogan 'meaning is use'. The second general assumption I see no need to question is Dummett's particular breed of molecularism. *Some* of Dummett's assumptions will have to be given up, if classical logic is to be vindicated in his meaning-theoretical framework. A major result of this paper will be that *the meaning of negation cannot be defined by rules of inference in the Dummettian framework*.

> Für die *Negation* liegen die
> Verhältnisse nicht so einfach.[1]
> *Gentzen*

1 Introduction

Dummett's meaning-theoretical arguments against classical logic are divided into two kinds.[2] One kind comprises arguments based on the nature of knowing and understanding a language: here belong the manifestability and the acquisition arguments. These arguments aim to establish that the nature of speakers' understanding of a language does not warrant the assumption that every sentence is determinately either true or false. It is widely agreed that

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[1] 'The situation is not so easy for *negation*.' (Gentzen 1936, 511)

[2] This paper has been with me for a while. Many people have read or heard versions of it and contributed with their comments. Instead of trying to list them all, which would undoubtedly lead to unintended omissions, I'd like to single out two philosophers to whom I am particularly indebted. Bernhard Weiss, to whom everything I know about Dummett can be traced, and Keith Hossack, my *Doktorvater*, for his robust philosophical challenges. This paper would not have been written without their advice and encouragement. I would also like to thank the referees for *Grazer Philosophische Studien*, whose constructive criticism resulted in a substantial improvement of this paper.

they are either unsuccessful[3] or too underdeveloped to carry the force they are intended to carry—the latter point being attested to by Dummett himself, who admits that it is far from a settled issue what full manifestability amounts to.[4]

The other kind comprises arguments based on how the meanings of the logical constants are to be determined in the theory of meaning. They are the focus of the present paper. Using Dummettian principles, I shall outline three such arguments, of increasing strength, and show that they are unsuccessful by giving responses to each argument on behalf of the classicist[5].

It is crucial that in responding to these arguments a classicist need not challenge any of the basic assumptions of Dummett's outlook on the theory of meaning. In particular, I shall grant Dummett his general bias towards verificationism, encapsulated in the slogan 'meaning is use'. The second general assumption I see no need to question is Dummett's particular breed of molecularism. The point of the present paper is to investigate how, accepting these Dummettian assumptions, the classicist can counter Dummett's arguments.

Some of Dummett's assumptions will have to be given up, if classical logic is to be vindicated in his meaning-theoretical framework. I will argue that *the meaning of negation cannot be defined by rules of inference in the Dummettian framework*.

As Dummett's project is well known, the discussion of his views on the theory of meaning remains deliberately concise.

2   *tertium non datur*

2.1   Against *tertium non datur*

Dummett rejects holism, the view that the meaning of a word is determined by the whole language in which it occurs, as well as atomism, the view that the meaning of a word can be determined individually. In received terminology, the principle of compositionality states that the meaning of a syntactically complex expression depends on the meanings of its constituent expressions and the way they are assembled. Dummett argues for a more substantial principle, which he calls by the same name. 'The principle of compositionality is not the mere truism, which even a holist must acknowledge, that the meaning of a sentence is determined by its composition. Its bite comes from the thesis that the understanding of a word consists in the ability to understand characteristic members of a particular range of sentences containing that word.' (Dummett 1993, 225) The notion of complexity on which a molecular theory of meaning is built cannot be equated with syntactic complexity, but characterises semantic features of expressions. There are expressions an understanding of which requires an understanding of others first. For instance, whereas understanding the terminology of the theory of the colour sphere presupposes an understanding of colour words, the converse is not true: a speaker may be proficient in using colour words like 'red', 'green', 'yellow' and 'blue' without understanding the terms 'pure', 'mixed' and

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[3]Transposing Alexander Miller's arguments from the semantic realist to the adherent of classical logic (Miller 2002, 2003).

[4]*Cf.* the 'Preface' to (Dummett 1991).

[5]In defiance of the *OED*, where 'classicist' is reserved for persons who study Classics or followers of Classicism, I shall use this term to refer to adherents of classical logic.

'complementary colour' or what is meant by 'saturation', 'hue' and 'brightness'. The latter expressions are semantically more complex than the former. As Dummett puts it, a relation of dependence of meaning holds between them. 'What the principle of compositionality essentially requires is that the relation of dependence between [sets of] expressions and [sets of] sentence-forms be asymmetric.' (Dummett 1993, 223) The qualification 'sets of' is needed because there may be collections of expressions that, although they must form surveyable sets, can only be learned simultaneously; according to Dummett, this is true for simple colour words (*ibid.*). A theory of meaning employs the relation of dependence to impose on the expressions of the language 'a hierarchical structure deviating only slightly from being a partial ordering' (*ibid.*). It thereby exhibits how the language is learnable step by step. In learning a language, a speaker works his way up the hierarchy from semantically less complex to semantically more complex expressions. Mastering a stage in this process is to master everything a speaker needs to know about the meanings of the expressions constituting that stage, and it does not alter the speaker's understanding of the meanings of expressions constituting stages lower in the hierarchy. This is Dummett's molecularism in the theory of meaning. To avoid confusion with received terminology, I shall avoid using 'compositionality' where the semantic notion of complexity is concerned and instead use 'molecularity'.

Applying molecularity to proof-theory and combining it with the verificationism derived from the principle that meaning is use, according to Dummett a proof should never need to appeal to sentences more complex than that which is proved. It should be possible to transform any proof into one which satisfies this requirement. A speaker following a proof should always be able to work his way up from *less* complex assumptions to a *more* complex conclusion, where of course intermediate steps down through less complex sentences are allowed on the way up. Dummett puts forward the *fundamental assumption* of the proof-theoretic justification of deduction: 'if we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator.' (Dummett 1993, 254) Leaving out the technical details, the fundamental assumption ensures that we can always construct proofs in such a way that the sentences occurring in the proof can be ordered by the relation of dependence of meaning, as required by molecularity, in such a way that the conclusion occupies the highest point in the hierarchy.[6]

With this material, Dummett can give a compelling argument against classical logic on meaning-theoretical grounds. I shall follow traditional terminology and call $A \lor \neg A$ *tertium non datur*, which deviates from Dummett's terminology. A

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[6]According to Dummett, the fundamental assumption applies not only to arguments which are proofs, but also to the more general case of deductions with undischarged premises, which, as Dummett acknowledges, meets some formidable difficulties (Dummett 1993, Chapter 12). These difficulties are irrelevant to the arguments to be given here, as they only require that the fundamental assumption applies to theorems, in which case it is provable for intuitionist logic and some formulations of classical logic. In another paper I argue that, quite independently of the present considerations, it is best to restrict the fundamental assumption in this way (Kürbis 2012). Strictly speaking, we should also make a distinction between 'argument', 'canonical proof' and 'demonstration', but this introduces a complexity unnecessary in the present context. Arguments may contain 'boundary rules', which are rules allowing the deduction of atomic sentences from other atomic sentences, as well as arbitrary inferences (Dummett 1993, 254ff). Canonical proofs and demonstrations are essentially special cases thereof, formalised in a system of natural deduction satisfying Dummettian criteria.

proof of *tertium non datur* in the system of classical logic formalised in (Prawitz 1965) proceeds as follows:[7]

$$
\cfrac{
  \cfrac{
    \cfrac{
      \cfrac{\neg(A \vee \neg A) \quad \cfrac{\cfrac{\overline{A}^{\ 1}}{A \vee \neg A}}{}}{\bot}\ ^{2}
    }{\cfrac{\bot}{\neg A}^{\ 1}}
    \qquad
  }{A \vee \neg A}
}{}
$$

$$
\cfrac{\overline{\neg(A \vee \neg A)}^{\ 2} \qquad \cfrac{\cfrac{\cfrac{\overline{\neg(A \vee \neg A)}^{\ 2} \quad \cfrac{\cfrac{\overline{A}^{\ 1}}{A \vee \neg A}}{}}{\bot}}{\neg A}^{\ 1}}{A \vee \neg A}}{\cfrac{\bot}{A \vee \neg A}^{\ 2}}
$$

The proof violates molecularity: the *less* complex $A \vee \neg A$ is deduced from the *more* complex discharged assumption $\neg(A \vee \neg A)$. No proof of $A \vee \neg A$ which would satisfy Dummett's criteria can be given. For how should such a proof of $A \vee \neg A$ proceed? By molecularity and the fundamental assumption, $A \vee \neg A$ would have to be derived from A or from $\neg A$. Whichever it is, it must come from assumptions that are discharged in the process of the argument. It cannot be A, for this may be an atomic sentence and no atomic sentence follows from no premises at all.[8] It cannot be $\neg A$ either, for, if A is atomic, neither does $\neg A$ follow from no premises at all.[9] Hence it is not possible to meet Dummett's criteria on molecular theories of meaning *and* accept $A \vee \neg A$ as a theorem.[10]

   This argument against classical negation is remarkable. The main assumption it is based on is that a theory of meaning should be molecular, which is a very plausible assumption. It is not an argument that Dummett gives himself, but, being based purely upon Dummettian considerations, it is one that he could give, in particular as he thinks that double negation elimination or an equivalent classical negation rule like *consequentia mirabilis*, from $\Gamma, \neg A \vdash \bot$ to infer $\Gamma \vdash A$, violate constraints on molecularity. It is an argument that is very strong indeed.[11]

2.2   A classicist response

The appeal to molecularity in the argument against *tertium non datur* assumes that $A \vee \neg A$ and $\neg(A \vee \neg A)$ are of different semantic complexity. It is a fair question to ask—whether one is a classicist or not—what it is that a speaker needs to understand in order to understand $\neg(A \vee \neg A)$ that she does not need to understand in order to understand $A \vee \neg A$. On the face of it, there is nothing

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[7]I'll discuss various ways of formalising classical logic in Prawitz' system in due course and show what is wrong with them on the Dummettian plan. We can exclude ways of formalising logics that Dummett excludes, such as multiple conclusion logics.

[8]If A is not something like *verum*, but it is clear enough how the point is to be taken.

[9]If A is not something like *falsum*, *cf.* the previous footnote.

[10]For *special* areas of enquiry one may be able to show that either A or $\neg A$, as is the case in intuitionist arithmetic for atomic A. However, this is not a question of *logic*: it makes assumptions concerning the subject matter of the atomic sentences, and logic makes no such assumptions.

[11]It rules out even logics in which negation is conservative over the positive fragment, such as the relevant logic **R**. According to Belnap, responding to (Prior 1961), conservativeness is a requirement for the *existence* of a constant (Belnap 1962, 133f). This is not sufficient to ensure that the constant is a respectable one on Dummett's account, as other meaning-theoretical constraints have to be satisfied, too. Hence someone following Peter Milne's suggestion of viewing *consequentia mirabilis* as an introduction rule for A still needs to answer Dummett's molecularity constraint, as Milne himself notes (Milne 1994, 58f). The present paper can be seen as providing Milne with a solution to this problem.

in one that is not in the other. To understand $A \vee \neg A$ and $\neg(A \vee \neg A)$, one needs to understand $\neg$, $\vee$ and that A stands for a sentence. Dummett introduces two notions of complexity: syntactic complexity, related to what is normally called the principle of compositionality, and semantic complexity, his notion of molecularity. The two notions do not coincide. For the argument against *tertium non datur* to go through, it has to be assumed that the fact that $\neg(A \vee \neg A)$ contains $A \vee \neg A$ as a proper subformula, and is therefore syntactically more complex, carries over to their respective semantic complexities. I shall argue that this assumption is unmotivated.

Consider what Dummett says is involved in understanding 'or'. 'On a compositional [i.e. molecular] meaning-theory, to know the meaning of 'or', for example, is to be able to derive, from the meanings of any sentences A and B, the meaning of $\ulcorner A$ or $B \urcorner$ [...] To understand $\ulcorner A$ or $B \urcorner$, therefore, you must (i) observe the composition of the sentence, (ii) know what 'or' means, (iii) know what A and B mean.' (Dummett 1993, 222) Decompositing clauses (ii) and (iii) in the cases of $A \vee \neg A$ and $\neg(A \vee \neg A)$ end in the same final components: in each case you need to know what $\vee$, $\neg$ and A mean.

Arguably, clause (i) does not impart semantic complexity either. I cannot just observe the composition of a sentence in the abstract, as it were: understanding the composition is essentially tied to an understanding of the parts and how they are pieced together. To understand $\vee$, I need to understand that it takes two sentences and forms a sentence out of them. I also need to understand the principles of inference governing it. As there are two introduction rules for $\vee$, my understanding guarantees that I understand that the order of the sub-sentences plays a role in the composition, even though the two options are logically equivalent. As another example, take $\supset$: understanding this connective involves understanding that it forms a sentence out of two sentences and the rules of inference governing it. The latter guarantee that my understanding also involves an understanding that the meaning of the resulting sentence is different depending on which sentence I put to the left and which one to the right of $\supset$. How to compose sentences with these constants is an essential part of understanding them. It comes together with an understanding of what $\vee$ or $\supset$ mean that they put together sentences in a certain way, which results in the sentences having a certain composition.

In addition, the meanings of the logical constants are given in a completely general way. Concerning the understanding of logical constants, Dummett writes that 'the understanding of a logical constant *consists* in the ability to understand any sentence of which it is the principal operator: the understanding of a sentence in which it occurs otherwise than as the principal operator *depends on*, but does not go to constitute, an understanding of the constant.' (Dummett 1993, 224) The rules governing it tell us how to proceed when the constant applies to any sentences whatsoever. If I understand an operator and can apply it in one case (e.g. $\neg A$), I can also be expected to be able to apply it in any other case (e.g. $\neg(A \vee \neg A)$), given I understand the rest of the context, which *ex hypothesi* is so in the case of *tertium non datur*, as $\vee$ is understood. Of course we need to observe how the components are pieced together. But in piecing them together in one way or other, no new conceptual resources are required.

Following this line of reasoning, the classicist can point out that in fact the proof of *tertium non datur* does not violate molecularity. The difference in the syntactic complexity between $A \vee \neg A$ and $\neg(A \vee \neg A)$ does not carry over to the

semantic level. Exactly the same conceptual resources are needed to understand either of them.

Thus the classicist has a straightforward response to the Dummettian argument against *tertium non datur*. It proceeds entirely on Dummettian grounds, appealing only to principles that Dummett himself puts forward. It has the further implication of revealing the fundamental assumption to be an excessive requirement, if, as Dummett demands, it is applied strictly to the main operator of a theorem.

The phenomenon that syntactic complexity doesn't carry over to semantic complexity is more widespread than just the logical constants. Consider 'Fred paints the wall in complementary colours'. This is syntactically less complex than 'Fred paints the wall in red and green, or blue and orange, or purple and yellow'. However, it is semantically more complex, as I cannot understand the concept 'complementary colour' without understanding simple colour words. Similarly, Dummett suggests that 'child', 'boy' and 'girl' are expressions that occupy the same point in the partial ordering that dependence of meaning imposes on the expressions of the language. They can only be learnt together, where some logical relations between them need to be recognised as well (Dummett 1993, 267). If this is so, then, even though it is syntactically more complex, 'Hilary is a boy or a girl' is semantically as complex as 'Hilary is a child'.

2.3 Conclusion

Although unsuccessful, the Dummettian argument against *tertium non datur* is significant as it is an attempt to formulate an argument against classical logic purely on the basis of very general considerations about the form a theory of meaning has to take. It relies on the assumption that the difference in the syntactic complexity between $A \vee \neg A$ and $\neg(A \vee \neg A)$ carries over to the semantic level. The classicist can respond by denying that this is so.

The classicist response is not based on any specifically classical principles. In particular, it makes no reference to the fact that classical logic does not need $\vee$ as a primitive, which the Dummettian can counter by arguing that as ordinary language has an undefined 'or', logic should have $\vee$ undefined, too. The core and motivation of the classicist response can be accepted by philosophers of any logical bias. The argument against *tertium non datur* aims to establish that something is wrong with classical logic, if the framework of a Dummettian theory of meaning is assumed. The classicist response does not proceed by establishing that something is wrong with Dummett's favourite, intuitionist logic, but only that the argument fails to show that something is wrong with classical logic: we have not been given good reasons to believe that classical logic does not fit into the Dummettian framework.

Philosophy being what it is, straightforward arguments and simple responses won't settle the issue. In the next section, I shall give a second Dummettian argument against classical negation that aims to establish that negation in general does add to the semantic complexity of sentences, and I shall provide a corresponding classicist response.

3 Classical negation rules

3.1 Against rules yielding classical negation

The Dummettian argument against *tertium non datur* focussed on a specific application of classical negation rules. The classicist response counters that this application cannot be objectionable on Dummettian grounds. The Dummettian should now focus more generally on the effect of rules that, when added to intuitionist logic, yield classical logic.

To illustrate the line of argument, assume classical logic is formalised by adding double negation elimination to intuitionist logic with the following rules for negation introduction and elimination and *ex falso quodlibet*:

$$\frac{\overline{A}^{\;i}}{\begin{array}{c}\Xi\\\bot\end{array}}\;i \qquad \frac{\neg A \quad A}{\bot} \qquad \frac{\bot}{B}$$

$$\frac{\bot}{\neg A}\;i$$

I'll discuss other ways of extending intuitionist to classical logic in due course.

To establish that these rules violate general constraints imposed on the theory of meaning, the Dummettian needs to point out that there are sentences B not containing negation which can be established as true *only* by using double negation elimination, such as Pierce's Law $((A \supset B) \supset A) \supset A$. Then the inference of B from $\neg\neg B$ would, on Dummettian principles, be constitutive of the meaning of B, because it licenses uses of B that are not possible independently of this move. Hence the meaning of B would depend on the meaning of $\neg\neg B$. But there is a component in $\neg\neg B$ the meaning of which has to be acquired independently of B, i.e. negation. To acquire an understanding of the meaning of negation, a speaker needs to acquire an understanding of the rules of inference for negation, which he doesn't have to know in order to know B. This is a case where syntactic and semantic complexity go hand in hand. For the Dummettian, $\neg\neg B$ counts not only as syntactically more complex than B, but also as more complex in the semantic sense. Thus by molecularity, the meaning of $\neg\neg B$ is dependent on the meaning of B and negation. This is a circular dependence of meaning: a speaker who wishes to command an understanding of B would first have to command an understanding of $\neg\neg B$, which, however, cannot be achieved independently of mastery of the meaning of B. A speaker could not break into the circle and learn the meaning of B. B could have no place in the partial ordering that the relation of dependence of meaning imposes on the language. Hence B cannot have a stable meaning at all. It follows that double negation elimination should be rejected, as it is incompatible with Dummett's molecularism and his interpretation of the principle that meaning is use.[12]

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[12]It is worth reflecting whether there are examples of non-logical sentences not containing negation that can only be verified by double negation elimination, if classical logic is used, i.e. whether the non-conservativeness of classical negation over the positive fragment of intuitionist logic applies also to non-logical sentences. Maybe the following is an example. Consider an embryo. Let's call it Hilary. An intuitionist would resist the temptation of asserting that Hilary is either a boy or a girl, as neither disjunct can yet be verified. But consider 'Hilary is neither a boy nor a girl'. Intuitionistically, this is equivalent to 'Hilary is not a boy and not a girl'. But an intuitionist might accept that if a child is not a boy, then it is a girl: arguably, verifying that a child is not a boy just is or must proceed via verifying that it is a girl. Hence if Hilary is neither a boy nor a girl, Hilary is a girl and not a girl, which is impossible. Hence, the intuitionist can conclude that it is not the case that Hilary is neither

In deriving a problematic sentence A of the kind we are interested in, double negation elimination need not be applied in the final step, so that the whole sentence to be derived is its conclusion. It may instead be applied to deduce a proper subsentence B of A. There will then still be a sentence that, in the process of the deduction, can only be derived by deriving its double negation first. What affects the part affects the whole: A cannot have a stable meaning if its subsentence B does not have one. Moreover, such a proof of A can always be transformed into one in which double negation elimination is the final step, and, for reasons to be explained later in this section, they both have to count equally as canonical verifications, and the problem that affects the one affects the other.

It is clear that what applies to double negation elimination equally applies to other rules for classical negation. The observation at the basis of the argument against double negation elimination—that there are sentences B not containing negation that can only be verified by applying double negation elimination—generalises. As classical negation is not conservative over the positive fragment of intuitionist logic, any rules for classical negation will enable us to derive sentences not containing negation using sentences containing negation. The Dummettian observes that, if classical negation rules are employed, there are sentences A not containing negation that can only be verified by a process that at some point appeals to the negation $\neg B$ of a subsentence B of A (not necessarily a proper subsentence). To understand $\neg B$ a speaker needs to understand something he does not need to understand in order to understand B: negation. Classical negation rules affect the use of B, as they affect the conditions under which it is assertible, and thus its meaning. This, once more, produces a circular dependence of meaning, just as in the case of double negation elimination.

There are other ways of extending intuitionist logic to classical logic than adding double negation elimination. We could add rules for implication, such as Pierce's Rule:

$$\overline{A \supset B}^{\ i}$$
$$\Pi$$
$$\frac{A}{A}_{\ i}$$

This rule violates molecularity. If A can only be verified by appeal to this rule, then the application of the rule would be constitutive of its meaning. But $A \supset B$ occurs in an undischarged assumption, so a speaker applying the rule needs to understand that sentence in order to be able to do so. However, the meaning of $A \supset B$ depends on the meaning of A. Again, there is a circular dependence of meaning between A and $A \supset B$.

If negation is defined in terms of $\bot$ and $\supset$, Peirce's Rule generalises a classical negation rule:

$$\overline{\neg A}^{\ i}$$
$$\Pi$$
$$\frac{A}{A}_{\ i}$$

a boy or a girl. The classicist would proceed to apply double negation elimination to conclude that Hilary is either a boy or a girl, even though there is no direct verification of the sentence. If this is plausible, then 'Hilary is either a boy or a girl' is an example of a sentence which, if classical logic is used, can only be verified by verifying its double negation first.

Even keeping ¬ primitive, the special case is no improvement on the general case: if A can only be verified by appeal to that rule, then the meaning of A is dependent on the meaning of ¬A, which appears in an undischarged premise, and conversely, ¬A is dependent on the meaning of A. Again there is a circular dependence of meaning. The same counts for *consequentia mirabilis*:

$$\frac{\overline{\qquad}}{\neg A}\ ^{i}$$
$$\Pi$$
$$\frac{\bot}{A}\ ^{i}$$

Another strategy is to add dilemma:

$$\frac{\overline{\quad}}{A}\ ^{i} \qquad \frac{\overline{\quad}}{\neg A}\ ^{i}$$
$$\Pi \qquad\quad \Sigma$$
$$\frac{B \qquad\qquad B}{B}\ ^{i}$$

Here the situation is slightly more complicated, but essentially the same. Any deduction that ends with an application of dilemma can be transformed into one that appeals to ¬B:

$$\frac{\overline{\quad}}{A}\ ^{1}$$
$$\Pi$$
$$\frac{\overline{\neg B}\ ^{2} \qquad B}{\frac{\bot}{\neg A}\ ^{1}}$$
$$\Sigma$$
$$\frac{\overline{B}\ ^{2} \qquad\qquad B}{B}\ ^{2}$$

The case we are interested in is where the final application of dilemma was part of a canonical verification of B. The transformed deduction contains a formula, ¬B, that the original one did not contain, and it contains additional applications of negation introduction and elimination. However, both deductions employ exactly the same conceptual resources. To follow the original proof, the speaker needs to understand negation. So he understands ¬B, as the understanding of negation, being a logical constant, is general. For the same reason, the additional applications of rules for negation only draw on resources the speaker who can follow the original deduction already needs to command. The transformed deduction may contain a maximal formula, if ¬A is the major premise of negation elimination in Σ. It can be removed: the conclusion of the rule will be $\bot$, and we can move the deduction leading to the minor premise on top of A in Π. If there is no such deduction, the case is trivial. The resulting deduction is still a deduction of B via ¬B. Thus there is no reason not to count the transformed deduction also as a canonical verification of B. Dummett does not require that every sentence has at most one canonical verification. Quite to the contrary. Understanding a sentence involves a grasp of the wealth of conditions under which it counts as conclusively verified. Each derivation must count as equally constitutive of the meaning of B, and once more we have a circular dependence of meaning, where B depends on ¬B, but ¬B depends on B.

We can generalise dilemma with the following rule:[13]

$$\frac{\overline{A \supset B}^{\;i} \qquad \overline{B \supset C}^{\;i}}{\begin{array}{cc} \Pi & \Sigma \\ D & D \end{array}} \; i$$
$$\frac{}{D}$$

A deduction that ends in an application of this rule can be transformed into one in which D is a subformula of a discharged assumption:

$$\frac{\overline{B}^{\;1}}{A \supset B}$$
$$\Pi$$
$$\frac{\overline{D \supset C}^{\;2} \qquad D}{C}$$
$$\frac{}{B \supset C}^{\;1}$$
$$\frac{\overline{\top \supset D}^{\;2} \qquad \overline{\top}}{D} \qquad \frac{\Sigma}{D}$$
$$\frac{}{D}^{\;2}$$

Instead of $\top$, we could use an arbitrary, but suitably chosen tautology, say $D \supset D$. We could, indeed, also replace $\top$ with other suitably chosen sentences, in particular sentences E such that the meanings of sentences occurring in the original deduction depend on the meaning of E in the partial ordering that dependence of meaning imposes on the sentences of a language in a molecular theory of meaning. As in the case of dilemma, the transformed deduction uses exactly the same conceptual resources as the original deduction. Maximal formulas arising from the transformation can be removed. If the former was a canonical deduction of D, so is the latter. We get a circular dependence of meaning: D depends on $C \supset D$, which in turn depends on D, violating molecularity.

Adding corresponding axioms instead of rules cannot make a difference to the situation, as they are equivalent. Besides, axioms, according to Dummett, count as introduction rules. They introduce grounds for asserting sentences that are not matched by the consequences of asserting them, as laid down by the elimination rules for the main connective of the axiom. Axioms, then, immediately violate Dummett's verificationism.

In this section, I have only discussed specific cases of rules. It would be desirable to establish a general result to the effect that any rules that yield classical logic, when added to intuitionist logic, violate molecularity. The cases discussed are, however, the most prominent ones and are sufficiently varied to shift the burden of proof. Once more, we can describe Dummett as formulating a challenge: find rules that yield classical negation that won't violate general constraints on the theory of meaning. For the present purposes, we can leave matters here. I shall go on to discuss a classicist response to the concerns of the present section that will exonerate the classicist from answering this renewed challenge.

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[13]We get dilemma by replacing A with $\top$ and C with $\bot$. Deleting $A \supset$ gives yet another rule that yields classical negation. It is unacceptable to the Dummettian for similar reasons as the general version.

To finish, here is a conjecture for a formal result that I leave for another occasion: A deduction Π ending with an application of a rule that yields classical logic can be transformed into a deduction Π' of the same conclusion with the following properties: a) Π' finishes with an application of the same rule as Π; b) the conclusion of Π occurs as a proper subformula of a discharged premise; c) only rules applied in Π are applied in Π'; d) formulas in Π' not occurring in Π are composed of subformulas of formulas occuring in Π. The idea is the following. For introduction and elimination rule to be in harmony, they need to fulfil certain constraints. The resulting logic is intuitionist. To extend it to classical logic, rules need to be added that discharge assumptions containing logical constants, an option Dummett excludes (Dummett 1993, 297). Given further constraints on such rules, a general procedure can be specified that transforms deductions in the desired way. Thus any such rule violates molecularity.

3.2 Another classicist response

To counter the argument against rules yielding classical logic, it suffices to argue that classical negation rules do not, in fact, violate molecularity. What is needed is a further assumption, one that is very plausible from the classical perspective, but not inherently classicist: although $\neg A$ is *syntactically* more complex than A, this does not carry over to the crucial *semantic* notion of complexity at the foundation of Dummett's molecularism.

Peter Geach has proposed a view on negation which has the desired consequences. Geach holds that an understanding of negation and an understanding of affirmation[14] cannot be separated from each other. A speaker cannot understand Fa without understanding $\neg Fa$ and conversely: 'they go inseparately together—*eadem est scientia oppositorum*.' (Geach 1972, 79) Following Geach, I shall use 'predicate' to mean not a predicate letter, but a meaningful expression of a language, or alternatively 'concept'. Someone understanding a predicate needs to be able to distinguish between things to which it applies and things to which it does not apply. Understanding a predicate enables a speaker to draw this distinction. Thus understanding a predicate endows a speaker with a grasp of affirmation as well as negation. Consequently, 'the understanding of "not male" is no more complex than that of "male".' (*ibid.*) To grasp a concept is inseparable from grasping its negation, as 'knowing what *is* red and what *is not* are inseparable.' (Geach 1971, 25) A speaker cannot acquire a grasp of one without acquiring a grasp of the other: they are learnt together. Hence, according to Geach, a sentence and its negation are of the same semantic complexity.[15]

Geach's view on affirmation and negation is comparable to Dummett's view

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[14] Affirmation is not to be confused with assertion, which is a speech act. Historically, 'position' has also been used to denote the opposite of negation.

[15] Although Geach puts his point in terms of predicate negation, it carries over to sentential negation and was certainly intended to do so. This is particularly clear in the present context, as Dummett and, according to his interpretation, Frege would call $\neg F\xi$ the negation of the predicate $F\xi$ only in a derivative way. Strictly speaking, there is no predicate negation, according to Frege/Dummett. Negation is a function, and functions always have objects as values, whereas predicate negation would take functions as values (Dummett 1981, 40ff). $\neg F\xi$ is not constructed from $F\xi$ by applying negation, but in the same way as every predicate: from a sentence by omitting some occurrences of a name: from the sentence Fa we omit the name a to get the predicate $F\xi$, we apply negation to the sentence Fa to get $\neg Fa$ and drop a from it to get the predicate $\neg F\xi$. In the final analysis, any talk of predicate negation is explicable in terms of sentential negation.

on simple colour words. According to Dummett, in order to understand the meaning of 'red', e.g., I also need to understand other simple colour words, like 'brown', 'green' and 'yellow'. Geach makes the analogous point about a predicate and its negation: to understand 'red' requires understanding of 'not-red' and vice versa. Combining Dummett's and Geach's points, to understand what it means that something is red or green or blue etc., I also need to understand that what is green is not red etc. Saying what something is, is also saying what it is not, or as Spinoza says: *omnis determinatio est negatio*.

It is worth noting at this point that saying that Fa and $\neg Fa$ go inseparably together is one thing, rejecting the claim that $\neg Fa$ exhibits a composite structure quite another. Geach should not be understood as claiming the latter. Even if negation is as fundamental to understanding as affirmation, it makes a uniform contribution to sentences in which it occurs, and $\neg Fa$ may still be described as being composed of $\neg$ and Fa. Geach's point is only that in this case syntactic composition does not add to semantic complexity.

If the classicist adopts Geach's account of negation, there is an answer to the molecularity challenge posed by the argument against rules yielding classical negation. If negation and affirmation go inseparably together, then diagnosing a difference in the complexities of A and $\neg A$ relies on a misconception: it is wrong to measure their semantic complexity by observing that one contains a sentential operator in principal position that the other lacks. As a speaker acquires an understanding of both simultaneously, the same conceptual resources are required in understanding A and understanding $\neg A$. Transposing Geach's ideas to the Dummettian molecular theory of meaning, A and $\neg A$ occupy the same position in the partial ordering that dependence of meaning imposes on a language. Thus they have the same semantic complexity. If the sense of an expression is something a speaker has to know about the expression in order to be able to use it, then a theory of meaning along Geach's lines would specify simultaneously the senses of A and its negation $\neg A$. Correspondingly, establishing $\neg A$ as true is an operation of the same complexity as establishing A as true. Consequently, the argument against classical negation rules loses its force: a verification of B that proceeds via $\neg B$ does not result in a circular dependence of meaning, and hence unintelligibility, even if B does not contain negation and cannot be verified otherwise.

This completes the classicist response to the Dummettian argument against rules yielding classical negation. There are, however, no obvious reasons why Geach's view on negation should be restricted to the classicist. It is quite neutral. An intuitionist might accept it, too. The point that a sentence and its negation are of equal semantic complexity can be motivated independently of which rules negation is subject to. Initially at least, Geach makes no reference to classical logic.[16]

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[16]This is not affected by Geach's illustration of his view, an obvious reference to Frege's metaphor of concepts with sharp boundaries (Frege 1998, Vol. II: §56): 'A predicate may be represented by a closed line on a surface, and predicating it of an object be represented by placing the point representing the object on one or other side of this line. A predicate and its negation will then clearly be represented by one and the same line; and there can be no question of logical priority as between the inside and the outside of the line, which inseparably coexist.' (Geach 1972, 79) According to this picture, $\neg\neg A$ has the same content as A. This view is not needed to counter the molecularity challenge. Geach notices that the picture is problematic for vague predicates. It is only an illustration and not essential to Geach's philosophical point.

3.3 Conclusion

The argument against classical negation rested on the observation that, if classical logic is used, there are sentences not containing negation that can be verified only by a process that appeals to rules yielding classical negation, and that this leads to a violation of molecularity, due to the nature of those rules. The classicist response rested on the assumption that a sentence and its negation are of equal semantic complexity. This may be controversial. But as with the classicist response to the argument against *tertium non datur*, although this assumption is particularly attractive for classicists like Geach, it is not one that actually depends on any specifically classicist assumptions. An intuitionist could adopt it, too.

The next Dummettian argument I shall consider aims at establishing that the negation of a sentence must be semantically more complex than the sentence itself. It differs from the argument against *tertium non datur* and rules yielding classical negation in that it not only attempts to show that something is wrong with classical logic, but also that intuitionist logic is the right logic.

4 *ex falso quodlibet*

4.1 Negation according to Dummett and Prawitz

The two Dummettian arguments against classical logic given so far fail to establish the desired conclusion that something is wrong with classical logic. Dummett needs a more forceful argument using more resources than just general constraints on the theory of meaning. The argument I shall turn to now is based on a very substantial additional theory, the *proof-theoretic justification of deduction*. Its core tenet is that the meanings of the logical constants, and thus negation, are to be defined by rules of inference governing them. It is an argument which not only is intended to point towards a deficiency in classical logic but also aims to establish that intuitionist logic is the correct logic.

Dummett argues that the meanings of logical constants should be given by self-justifying rules of inference governing them. To exclude connectives like Prior's *tonk*, these rules are required to be *in harmony*. For the present purposes, I do not need to go into the details of Dummett's account and can remain fairly informal about this notion.[17] Dummett demands that there be harmony between the canonical grounds of an assertion of a sentence with a main connective ∗ and the consequences of accepting it as true. Molecularity plays a role in motivating harmony: learning the meaning of logical connectives does not affect the meanings of expressions you have already learnt (nor, indeed, does what you have already learnt affect their meanings). Dummett claims that if the procedure of the proof-theoretic justification of deduction is followed, the meanings of the logical constants are given independently of a notion of truth that prejudges issues between classicists and intuitionists. The logic which turns out to be the justified one is the correct logic.[18]

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[17]I give formally precise definitions of harmony and stability in (Kürbis 2013), which work by specifying how to read off introduction from elimination rules and conversely.

[18]For details, *cf.* (Dummett 1993, chapters 11-13).

Dummett argues that the negation operator should be defined in terms of implication and *falsum*, $\neg A =_{def.} A \supset \bot$, as considerations of rules for an undefined negation operator show. There are two common options for the introduction rule. The first option is that $\neg A$ follows if A entails a contradiction:

$$
\cfrac{\cfrac{\overline{A}^{\ i}}{\substack{\Pi \\ B}} \qquad \cfrac{\overline{A}^{\ i}}{\substack{\Xi \\ \neg B}}}{\neg A}\ i
$$

It can hardly be claimed that the meaning of negation is defined by this rule: negation is already used in the premises.[19] Dummett himself employs a rule which suffers from the same inadequacy (Dummett 1993, 291ff):

$$
\cfrac{\cfrac{\overline{A}^{\ i}}{\substack{\Xi \\ \neg A}}}{\neg A}\ i
$$

A more promising option is to employ the introduction rule that $\neg A$ may be derived if A entails *falsum*:

$$
\cfrac{\cfrac{\overline{A}^{\ i}}{\substack{\Xi \\ \bot}}}{\neg A}\ i
$$

$\bot$ is governed by *ex falso quodlibet*, where B may be restricted to atomic formulas:

$$
\frac{\bot}{B}
$$

Negation introduction is harmonious with the rule *ex contradictione falsum*, needed for a complete account of negation:

$$
\frac{A \qquad \neg A}{\bot}
$$

An attempt at defining the meaning of negation in terms of the last three rules is unacceptable. The rules define the meaning of negation in terms of *falsum*, and the meaning of *falsum* in terms of negation: the rule for negation elimination is also a rule for *falsum* introduction, and the rule for negation introduction is also a rule for *falsum* elimination. Using these three rules leads to a circular dependence between the meanings of negation and *falsum*. Dummett argues that there should be no such circular dependence between the meanings of the logical constants (Dummett 1993, 257). Hence this is not a viable option for defining the meaning of negation by rules of inference in the Dummettian framework.

We are left with Dummett's option of defining $\neg A$ as $A \supset \bot$, where $\bot$ is governed solely by *ex falso quodlibet* and $\supset$ by its usual introduction and

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[19]Nonetheless, together with *ex contradictione quodlibet* as the elimination rule for negation, a system can be formulated in which deductions normalise.

elimination rules. Different arguments can be given why *ex falso quodlibet* satisfies the criterion of harmony. Prawitz argues that it is harmonious with the empty introduction rule (Prawitz 1979, 35). Dummett likens *falsum* to a universal quantifier over atomic formulas (Dummett 1993, 295). The details need not concern us here. What is important is that the negation so defined is intuitionist, not classical. Thus intuitionist logic is the correct logic according to Dummett's proof-theoretic justification of deduction.

Following this line of argument, classical negation can be excluded, as it requires the rule *consequentia mirabilis*:

$$\frac{\overline{\quad}^{\;i}}{\neg A}$$
$$\Xi$$
$$\frac{\bot}{A}\;i$$

As already discussed, this rule cannot be used to define the meaning of negation in terms of *falsum*, as it cannot count as defining the meaning of *falsum* independently of negation. It presupposes negation, which may occur in discharged premises. *Consequentia mirabilis* could only count as defining the meaning of *falsum* in terms of negation. But Dummett argues that the meaning of negation has to be defined in terms of *falsum*. Hence, once more, employing *consequentia mirabilis* produces a circular dependence of the meanings of *falsum* and negation. Dummett concludes that intuitionist negation does and classical negation does not satisfy the criteria of the proof-theoretic justification of deduction.[20]

It follows that the negation of a sentence is always semantically more complex than the sentence itself. $A \supset \bot$ is in general semantically more complex than A on anyone's account, as at least for some atomic propositions, a speaker can understand A without understanding $\supset$. Hence Dummett is in a position to claim that Geach's view that a sentences and its negation are of equal semantic complexity must be rejected in favour of a view on which A is less complex than $\neg A$.

4.2 The classical plan of attack

The rules governing classical negation do not fit the restrictions that Dummett's and Prawitz' proof-theoretic justification of deduction imposes on the form of self-justifying rules of inference. The classicist may, however, question whether this gives good reasons for rejecting classical logic. Dummett's and Prawitz' argument relies on the assumption that the meaning of negation can be defined by rules of inference. In the next section, I shall argue that this assumption is incorrect. *Ex falso quodlibet* fails to confer its intended meaning on $\bot$. Hence the meaning of intuitionist negation cannot be defined by rules of inference either. But then nothing can be amiss if the same is true for classical negation and *its* rules.

If rules of inference are not understood as completely determining the meaning of the constant they govern, then there is no rationale for requiring that they satisfy the demands of the proof-theoretic justification of deduction. For instance, as rules of inference alone are not sufficient to define the meanings

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[20]The discussion of the previous section contains the material necessary to exclude other ways of extending intuitionist logic to classical logic in a similar way.

of the connectives *F* and *P* with intended interpretation 'It will be the case that' and 'It has been the case that', tense logic is not subject to the proof-theoretic justification of deduction. The fact that the rules and axioms for *P* and *F* do not satisfy its requirements in no way shows that there is something wrong with them. The rules governing a connective are subject to the restrictions that the proof-theoretic justification of deduction imposes on the form of rules of inference if and only if the meaning of the connective is to be defined purely by the rules of inference governing it. Thus the fact that classical negation rules do not satisfy the criteria of the proof-theoretic justification of deduction is insignificant when it comes to reasons for rejecting classical logic.

4.2.1 The meaning of negation cannot be defined by rules of inference

Consider what $\perp$ is intended to be: a sentence that is false under any circumstances. Reading off its meaning from the rules governing it, the result should be that we cannot but say that $\perp$ is false. Although this characterisation of $\perp$ appeals to semantics, it does not violate the intended semantic neutrality of the proof-theoretic justification of deduction. It is legitimate to appeal to our semantic knowledge in order to see whether we have reconstructed it correctly in a given meaning-theory. Looking from the outside, as it were, at someone using $\perp$ according to the rule *ex falso quodlibet*, are we bound to say that he cannot mean anything but a false sentence with it? The requirement that no semantic assumptions enter the theory is fulfilled in this case, as no such assumptions enter the rule *ex falso quodlibet*. The question is: does it do the job it is supposed to do?

I think not. The intuitive content of *ex falso quodlibet* may be explained as follows: it says about $\perp$ something like 'If you say this, you might as well say anything'. $\perp$ is intended to be the ultimate unacceptable sentence, because everything follows from it. But what is it that makes a sentence from which everything follows unacceptable? It is that we assume that there are some sentences which are false.[21] If 'anything' covered only *true* sentences, there is nothing absurd in a sentence that entails that you may as well say anything. But it is a contingent feature of language that some sentences are false. Nothing prevents the atomic sentences of the language of intuitionist logic from all being true, and in that case every sentence, atomic and complex, would be true. Under these conditions, $\perp$ could be true. So *ex falso quodlibet* does not give the intended meaning to $\perp$, as it is not the case that we cannot but say that it is false.[22] More precisely, if every atomic sentence of the language was true, then far from $\perp$ having to be false, it might be true. If all we know about $\perp$ is what *ex falso quodlibet* tells us, then for all we know $\perp$ might be equivalent to the conjunction of all atomic sentences, and if they are all true, $\perp$ would be true.[23] So there are circumstances under which $\perp$ may be true, namely if all atomic sentences are

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[21]Some philosophers might prefer the view that what is unacceptable about a sentence from which everything follows is that there is no such thing. As they won't accept Dummett's and Prawitz' views on how negation should be defined, we may exclude them from consideration.

[22]In section 4.3.2 I argue that the lack of an introduction rule for $\perp$ does not remedy this.

[23]Dummett acknowledges the possibility of all atomic sentences of a language being true (Dummett 1993, 295). He also appears to countenance that complex sentences not containing negation can be logically true (Dummett 1993, 266ff). This suggests that maybe he envisages a solution along the lines of section 4.3.1 below, which, however, I shall show not to be workable.

true. So we are under no necessity to say that ⊥ is always false.[24]

Ironically, the reason why the definition of the meaning of *falsum* via *ex falso quodlibet* is appealing is that implicitly it appeals to *different models* for the language. This smuggles in semantic assumptions. It assumes that ⊥ is interpreted as having the same truth-value under every interpretation. This is not something that could be got from the rule. It is an assumption about how the semantics of ⊥ is to be given, which is external to the rule and thus illegitimate in the present context: it would not be the rule alone that determines the meaning of ⊥.

Dummett faces a predicament. He argues that from the proof-theoretic perspective, the meaning of negation needs to be given in terms of ⊥. But for *ex falso quodlibet* to confer on ⊥ the meaning of a constantly false sentence, the 'anything' it stands for would need to cover some formulas containing negation, it being understood that *A* and ¬*A* are never true together. So the meaning of ⊥ can only be given with reference to negation. This is circular.[25]

The classicist and the intuitionist are consequently in exactly the same situation with respect to their attempts at defining the meaning of negation proof-theoretically. Dummett claims that the use of *consequentia mirabilis*, the rule specifying the use of both *falsum* and negation in classical logic, engenders a circular dependence of meaning between negation and *falsum*, and it now has been established that the same can be said about intuitionist negation.[26]

I conclude that the meaning of negation cannot be defined purely proof-theoretically by rules of inference in the Dummettian framework. Consequently, if Dummett's proposal is that the meaning of a logical constant can be defined purely in terms of its use in deductive arguments *if and only if* this use can be characterised by harmonious introduction and elimination rules, then he is wrong. Even though in intuitionist logic *falsum* is governed by harmonious rules, its meaning cannot be defined by these rules. Only the *only if* part holds. There are logical constants the meaning of which cannot be determined by the harmonious rules governing them.[27]

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[24]This argument occurred to me several years ago. I had to discover that other people found it as well, in particular (Hand 1999). Milne makes the related point that any deduction of a negated sentence relies on negated premises or discharged hypotheses. He concludes that 'it is quite impossible for ¬-introduction to determine the meaning of ¬' (Milne 1994, 61). The argument has its full force, however, only if it is placed in the larger context in which it is produced here, because of the multi-layered nature of Dummett's argument against classical logic: even if the meaning of negation cannot be defined proof-theoretically, some response is needed to the molecularity challenge. Incidentally, an analogous argument purporting to show that the intended meaning of ⊤ cannot be given by rules of inference has a rather less clear status. ⊤ has only an introduction rule, but no elimination rule, which specifies that it follows from every sentence. In a language which contains just ⊤ and atomic sentences, where all atomic sentences are false, ⊤ could be false. But any language can be extended to contain logical constants defined by rules of inference, in particular ⊃. Then there will always be true sentences in a language.

[25]A designated absurdity like 0 = 1 instead of ⊥ makes no difference. It is hard to see how *ex absurdo quodlibet* might then be justified, if not because one already accepts *ex contradictione quodlibet* and uses 0 = 1 as inducing a contradiction, which is again circular. This works at best in special contexts like arithmetic where 0 = 1 does the job it is supposed to do due to the axioms of arithmetic, hence not purely due to rules of inference governing it. In section 4.3.1, I argue that a more mundane absurdity like '*a* is red and green all over' does not do the trick either.

[26]In **R** there is even less of a chance of defining the meaning of negation in terms of rules of inference: the relevant *falsum* constant **f** is not governed by *any* rules which are not also negation rules. At the very outset it must be assumed that we either understand relevant *falsum* or negation.

[27]According to Gentzen, *ex falso quodlibet* has a *Sonderstellung* amongst the rules of inference: 'it does not belong to one of the logical symbols, but to the propositional symbol [⊥]' (Gentzen 1934,

4.2.2 Consequences for the theory of meaning

The ingenious idea of Dummett's proof-theoretic justification of deduction can be characterised as follows. On the basis of the assumption that speakers can follow rules of inference and a concept of truth, which is neutral in the sense that none of its logical properties are specified prior to an investigation into which logic is the correct one, the proof-theoretic justification of deduction defines the meanings of the logical constants, amongst them negation. The resulting rules for negation then settle the question which properties truth has. As these rules are intuitionist, the principle of bivalence is not fulfilled. Only *positive* notions are appealed to as the primitive notions of the theory of meaning, *viz.* truth, assertion, affirmation, but not *negative* ones, like falsity, denial and negation. Assuming both notions of truth and falsity as basic would prejudge issues between classicists and intuitionists, because each will assume these notions to stand in their favourite logical relations to each other. The classicist will assume notions of truth and falsity that satisfy the principle of bivalence, whereas the intuitionist will assume notions which don't. The proof-theoretic justification of deduction was designed to settle the debate between classicists and intuitionists on neutral grounds. The choice of primitives, truth and rules of inference, rather than truth and falsity, was supposed to ensure this neutrality.[28]

The definition of the meaning of negation in terms of rules of inference fails. The attempt turns out to be circular. In proof-theory, just as we assume that the meanings of the atomic sentences of the language are given, we need to assume that the meanings of their negations are given, too. The main insight to be drawn from the present discussion is that *positive* as well as *negative* primitive notions are needed in the theory of meaning. The argument of the last section once more suggests Geach's view on negation, so that speakers' understanding of the meaning of negation is an additional primitive of the proof-theoretic justification of deduction.

If the meaning of negation cannot be given purely by rules of inference, its rules are of a different nature from the rules of those connectives where this is possible. In the latter case, we can give the rules governing a constant from scratch, so to speak: a speaker can be taught the concept by being taught the rules. Just as we must assume prior understanding of 'It will be the case that' and 'It used to be the case that' in formalising tense logic, as learning the rules and axioms of tense logic are not sufficient to impart this understanding on a speaker, we must assume that we possess the concept of negation prior to formalisation. Laying down rules of inference for negation builds on this understanding. Although the rules tell us something about the intended interpretation of the symbol, they cannot impart understanding of the concept formalised.[29]

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189). Adopting this view cannot help Dummett and Prawitz, as the question remains where our understanding of ⊥ comes from.

[28]The point can also be made by noting that, if truth and falsity are chosen as primitives, intuitionists and classicists need to say something about the relation between the two notions, e.g. that nothing can be both true and false. This relies on using negation in the metalanguage, as in 'If *A* is true, *A* is not false'. Arguably, the negation of the object language will then mirror the properties of negation in the metalanguage, and hence, because classicists and intuitionists will each use their favourite logic in the metalanguage, neither has given a neutral justification of logical laws.

[29]Milne may have something similar in mind, when he says that its rules 'characterise' negation (Milne 1994, 85). Restricted to negation, it is in line with the views of Arthur Prior, who argued that inferential relations and truth-tables are devices of 'putting people on the track of the meaning of a word' and 'can help us in this way to fix the meaning of a word': they are a piece of 'informal

4.3 Three counter-arguments refuted

A Dummettian who'd rather not assume an understanding of the meaning of negation as a primitive might attempt to modify the proof-theoretic justification of deduction as a response to the argument that the meaning of negation cannot be defined by rules of inference. In the following, I shall discuss three accounts that attempt to do so. I shall show that each of them, though possibly interesting in their own rights, fails to satisfy Dummettian strictures imposed on the proof-theoretic justification of deduction.

4.3.1 The nature of atomic sentences

One retort is to claim that Dummett's atomic sentences cannot be atomic in the sense of Wittgenstein's *Tractatus* (Wittgenstein 2003, 6.3751) and of formal logic, where they are independent of each other and no conjunction of atomic formulas is always false and no disjunction of them is always true. If ⊥ is to do its job, amongst Dummett's atomic sentences there must be some that *exclude* each other and cannot be true together. Surely this is supported by ordinary language, where there are such mutually exclusive atomic sentences, say '*a* is red' and '*a* is green'. Then *falsum* could not but be false, as it entails mutually exclusive atomic sentences.[30]

At a first glance, this looks like a natural way out. However, it defeats its purpose. To adopt this approach is in fact to admit that the proof-theoretic definition of the meanings of the logical constants fails in the case of negation, as it is obviously not a purely proof-theoretic definition. Proof-theory is not concerned with what the atomic sentences of a language are like; any collection will do. That the amendment is spurious is also seen if we consider that if it was adopted it would be a matter of luck that we have a language with a decent negation. Couldn't it be that a language is as the *Tractatus* claims it to be and lacks mutually exclusive sentences? Thus even if it is granted that some languages may contain mutually exclusive sentences, there are circumstances under which ⊥ need not be false, namely if a language fails to have this property. Far from solving any problems for Dummett and Prawitz, it should evoke Frege's comments on Mill's gingerbread arithmetic: 'wie gut doch, dass nicht Alles in der Welt niet- und nagelfest ist' (Frege 1990, 9); how convenient indeed that our language is such that it contains the sentences it does in fact contain, as otherwise we couldn't do logic properly.

Rhetoric aside, one might of course try to advance arguments that for some reason or other there must always be true as well as false sentences in a language, or that a language could not be as Wittgenstein would have it in the *Tractatus*, or at least that any language could always be extended in such a way as to contain mutually exclusive sentences, or that the meanings of sentences are propositions and there are true ones and false ones amongst them. I have already mentioned that according to Dummett, using truth and falsity both as primitive fails to meet his requirements. Quite generally, the amendments just suggested cannot ensure that the meaning of negation can be defined by rules of inference. They all leave proof-theory and rely on assumptions external to it. One might object

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pedagogy' (Prior 1964, 160 & 164).

[30]This was my initial reaction when I found the argument of section 4.2.1. (Tennant 1999) also proposes it in reply to (Hand 1999).

that if molecularity as a principle motivated by the philosophy of language may enter the proof-theoretic justification of deduction, then why not also let other theses shape the theory, like the ones just mentioned, which maybe could also be argued for in the philosophy of language? This question misses the point that there is a crucial difference between molecularity and these further theses. Molecularity is a principle that enters the *form* of the rules. Contrary to that, these further theses affect their *content*. But the content was precisely what was to be determined *exclusively* by the rules. Hence no matter how well these theses might be established in the philosophy of language, making them an essential part of the proof-theoretic justification of deduction has the effect of letting the theory collapse.

Although the 'amendments' to Dummett's theory mentioned in this section may very well be interesting *new* approaches to defining the meaning of negation, they are in fact not amendments at all, but *incompatible* with Dummett's approach.

4.3.2 Falsity and assertibility

Another attempt is to argue that the intended meaning of $\perp$ is captured by the rules governing it, as $\perp$ is governed by an elimination rule only and no introduction rule. So it has no grounds for its assertion. Hence there are no conditions under which it may be correctly asserted, hence under which it is true. So it can only be false.[31]

First, this a *non sequitur* and still does not guarantee that $\perp$ is indeed always false. Although being always false is a *sufficient* condition for something not to have grounds for its assertion, this is not *necessary*. That something has no grounds that warrant its assertion does not entail that it is false. It could be that we cannot assert it because we cannot put ourselves in a position to assert all the premises it relies on. No one would claim that the conclusion of the ω-rule is always false.

Secondly, the attempt is of no use in the present context. An intuitionist could be perfectly happy with the claim that *ex falso quodlibet* determines the meaning of $\perp$ completely. On an intuitionist understanding of falsity, if it can be proved that something has no warrant, then it is false, and nothing is easier than showing that this holds for $\perp$, as it has no introduction rule. The problem is that this reasoning presupposes the anti-realist's notion of truth, explained in terms of assertibility. That something is unassertible entails that it is false only given the anti-realist notion of truth. Hence if this line of thought were used in the explanation of the meaning of *falsum*, it would certainly not be true of intuitionist logic that 'its logical constants can be understood, and its logical laws acknowledged, without appeal to any semantic theory and with only a very general meaning-theoretical background.' (Dummett 1993, 300) An analogous way would obviously be open to the classicist, using *his* preferred notion of truth. No explanation of the meaning of $\perp$ that satisfies the requirements of the proof-theoretic justification of deduction in being semantically neutral is forthcoming.[32]

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[31]*Cf.* (Prawitz 1979) and (Read 2000, 139).

[32]Appeal to warrants is not in itself biased towards intuitionism. Read describes himself as giving an account of the meanings of the logical constants in terms of what warrants an assertion of a complex formula with the constant as main connective (Read 2000, 130). He proposes infinitary rules for the quantifiers (*ibid.*, 136ff). If Read's notion of a warrant was an anti-realist one, it would follow

4.3.3 Empty succedents

Maybe the argument put forward to show that the meaning of negation is not definable in Dummett's way asks for the impossible, given the framework he chose for formalising logic: if an arbitrary B is said to follow from $\bot$, fair enough, $\bot$ might be true. But isn't this shortcoming easily rectified if, instead of B, we allow an empty space to occur?[33] To explain validity in the modified natural deduction framework, we adopt a suitable modification of an explanation of the validity in sequent calculi, where multiple and empty conclusions are allowed: a sequent $\Gamma : \Delta$ is valid if, whenever all of Γ are true, some of Δ are true. Surely then, if from $\bot$ only emptiness follows, it must be false.

No doubt, this reasoning towards an always false $\bot$ is unassailable. The only problem with it is that it has the cart before the horse in the context of the proof-theoretic justification of deduction. The explanation of the validity of sequents is a *semantic* one: an inference is valid if it is truth-preserving. On Dummett's view of the matter, the proof-theoretic justification of deduction must forswear the use of semantic notions in defining validity and instead define it in proof-theoretic terms: harmonious rules are self-justifying and valid purely by virtue of their form. That these rules are truth-preserving is a consequence of harmoniousness. The explanation of the validity of sequents does not fit with Dummett's outlook and, indeed, makes the proof-theoretic justification of deduction a rather idle pursuit. Without it, there is again no guarantee that interpretations of the language on which *falsum* is true are excluded, even if empty spaces are employed.

5 Conclusion

To sum up the dialectics of this paper, the argument against *tertium non datur* was intended as an argument that appeals only to very general considerations about the form a Dummettian theory of meaning has to take. It assumes that there is a difference in semantic complexity between $A \lor \neg A$ and $\neg(A \lor \neg A)$. The classicist can respond by pointing out that this assumption is unwarranted, as the same conceptual resources are required to understand each of them. The argument against rules yielding classical negation is an attempt to improve upon the situation by making a further assumption: that there are negation-free sentences B the double negation of which is true. Then the rules for classical negation licence uses of B not otherwise licensed, which results in a circular dependence of meaning, contradicting Dummett's requirement of molecularity. This argument assumes that $\neg B$ is semantically more complex than B. A classicist can counter by arguing that a sentence and its negation should count as being of the same semantic complexity, as their understanding requires the same

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that we can assert the negation of every universally quantified sentence. The assumption that $\forall x F x$ is assertible would entail a sentence which is never assertible, namely that we have checked an infinite number of sentences F_{t_i}, understood as an *actual, completed* infinity, for otherwise we could not proceed to draw the conclusion. Analogously for existentially quantified formulas. Read's notion of a warrant needs to be understood in a realist sense: for some warrantedly assertible sentences it is not within our powers to obtain those warrants. Read does not give a *neutral* justification of classical logic, but a rather unsurprising one on the basis of a realist notion of warrant, which is hard to distinguish from a realist notion of truth. Similar remarks apply to Hacking, who also recommends infinitary quantifier rules (Hacking 1979, 313).

[33] As suggested in (Tennant 1999).

conceptual resources on the part of the speaker. Adopting Geach's view of negation, A and $\neg A$ occupy the same position in the partial ordering dependence of meaning imposes on the expressions of a language in a molecular theory of meaning. The Dummettian response is an attempt to establish that the negation of a sentence is indeed semantically more complex than the sentence itself. The argument is based on the proof-theoretic justification of deduction and aims to achieve two things: first, $\neg A$ needs to be defined as $A \supset \bot$, which is undeniably more complex than A, and secondly, only intuitionist but not classical negation is governed by rules of inference satisfying the requirements imposed. The classicist response is to point out that the meaning of $\bot$ cannot be defined by rules of inference in the Dummettian framework, and hence the meaning of negation cannot so be defined either. Thus the fact that the rules for classical negation do not fulfil the requirements of the proof-theoretic justification of deduction does not warrant its rejection. I conclude that Dummett has not formulated a fair objection to classical logic on the basis of considerations about the form a theory of meaning has to take.

The classicist responses to the Dummettian arguments do not challenge the meaning-theoretical assumptions of Dummett's programme, molecularity and the principle that meaning is use. They do not appeal to any assumptions which are specifically classical, such as a realist notion of truth. The responses prejudge no issues between classicists and intuitionists. No charge of circularity can be put against them.

The strength of the classicist line of defence is also its weakness. Nothing in the proposed answers to the Dummettian challenges suggests that classical logic has to be preferred over intuitionist logic. An intuitionist can accept all the assumptions made in the classicist responses. The Dummettian programme, modified in the light of the fact that the meaning of negation cannot be defined proof-theoretically by adopting negation as a primitive notion along Geach's lines, is logically rather more neutral than Dummett had thought his original project to be: it is compatible with both classical and intuitionist logic. I shall leave the question what conclusions to draw from this for another occasion.

6 Appendix

I argued that Geach's view on negation suggests itself as a supplement to the proof-theoretic justification of deduction, so that negation is an additional primitive on the same par as affirmation. There is a promising alternative approach that shares the insight that positive as well as negative primitives are needed. Huw Price has suggested that sense should be specified in terms of two primitive speech acts, assertion and denial, where negation can be defined in terms of them.[34] The difference is important enough: Price suggests to double pragmatic primitives, I suggest to double semantic ones.

I omitted bilateralism in the main part of this paper, as it is reasonably far removed from Dummett's original framework and deserves consideration on its own rights. At the request of several readers, I add this appendix to say a few words about Price's and Rumfitt's approach. I discuss them in detail

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[34]See (Price 1983), (Price 1990), (Price 2015). (Smiley 1996) and (Humberstone 2000) follow up some of Price's ideas. (Rumfitt 2000) calls the position *bilateralism* and provides a formal development of a logic for assertion and denial.

in two separate papers. To avoid giving away too much of their content, I'll restrict myself to summarising results established there. There is, however, an independent point to this appendix, namely to indicate that it is preferable to leave the 'unilateral' framework of proof-theoretic semantics as it is and adopt the two primitives affirmation and negation rather than change the framework to one in which the primitives are assertion and denial. Even if we accept that negation is a primitive, that doesn't mean we can't say anything interesting about it, so towards the end I also say a few words about how I envisage an account of negation to proceed.

Most philosophers accept Frege's view that there is no need to posit a primitive force of denial (Frege 1918, 153). We cannot understand certain inferences, such as those where the minor premise is rejected and the major premise is a conditional with a negated antecedent, in terms of a force of denial, as a speech act cannot be embedded into a conditional. We need negation as a sentential operator. But then denial is redundant, as we can define it in terms of negation and assertion. Price's and Rumfitt's accounts are more complicated than the unilateral account. It seems as if bilateralism only succeeds in introducing needless complexities.

Bilateralism and unilateralism aren't, however, equivalent theories, according to Price and Rumfitt. They aim to meet a well-known Dummettian challenge: to provide a framework for a theory of meaning that justifies classical logic but does not suffer from the shortcomings Dummett claims such an approach must face, by ensuring that it provides for a notion of sense intelligible to the kind of speakers that we are. Even Dummett and his most ardent followers, I think, agree that it would be preferable if classical logic were the justified one. Price and Rumfitt claim that bilateralism succeeds in justifying classical logic, whereas unilateralism does not. If that is correct, then the complexities of bilateralism are justified, as they result in establishing a theoretical *desideratum*, namely the justification of classical logic.

In two papers on bilateralism, one on Price and one on Rumfitt, I argue that each approach fails to justify classical logic as the unique logic. Price's account, ironically, works better for intuitionist logic. In a similar vein, it is possible to formulate an intuitionist bilateral logic in Rumfitt's framework in which the rules are harmonious, just as they are for classical logic. Thus the complexities bilateralism introduces into the debate fail to serve their purpose of justifying classical logic as the unique correct logic. This means that the unilateral approach of the current paper is to be preferred over their bilateral approach on methodological grounds.

In the paper on Price, I regiment Price's account by formulating axioms that capture the concepts Price employs in his argument that bilateralism justifies classical logic. Price proposes a pragmatic account of belief in terms of the differences they make to speakers' actions. My formalisation shows a certain amount of redundancy in the concepts Price employs. It turns out that the axioms entail consequences about the notion of making a difference that Price can't accept: if classical logic is correct, the notion is either vacuous or highly problematic. As my axiomatisation follows Price's wording very closely, it cannot be argued that the result merely shows my axiomatisation to be wrong. I show how a very small modification—adding a 'not' at a place in an axiom characterising disbelief where one would expect one anyway—insures that the notion of making a difference regains its interest. The theory is then, however,

best seen as intuitionist, and classical logic cannot be established on the basis of it. My axiomatisation uses all the resources Price provides, so to get classical logic, Price needs to extend his account. This may of course be possible, and I consider Price's options, but all this establishes is that both alternatives are possible, not what Price had intended to show, namely that only the classical version is justified.

Rumfitt poses the intuitionist a challenge: to provide a bilateral account of intuitionist logic in which the rules of the system are in harmony. Rumfitt demands of the intuitionist a specification of what in general follows from the denied negation of a formula that is harmonious with the introduction rule for denied negations. Classicists and intuitionists agree that the denied negation of a formula follows from its assertion. The harmonious elimination rule, according to Rumfitt, is that the asserted formula follows from its denied negation. This is only acceptable to the classicist, not the intuitionist. I show how to formulate different rules that are also harmonious, but result in an intuitionist bilateral logic. Thus Rumfitt's challenge is met. This is not the place to go into the formal details, but harmonious rules for an intuitionist bilateral logic can be formulated by making a fuller use than Rumfitt himself does of the possibilities offered by the formal framework of bilateral logics.

As neither Price's nor Rumfitt's approach lends itself exclusively to the classicist, but in each case an intuitionist alternative can be formulated, for methodological reasons—that a simpler theory is to be preferred over an equivalent more complex one—it follows that the unilateral approach proposed in this paper comes out as superior to its bilateral rivals.

Rumfitt's formalism also faces an independent problem of how to interpret deductions carried out in it. In Rumfitt's bilateral logic, the premises, discharged assumptions and conclusions are supposed to be understood as asserted or denied formulas. Rumfitt accepts that speech acts cannot be embedded in other speech acts. Thus, the formulas in Rumfitt's system cannot be understood as being prefixed by 'It is assertible that' and 'It is deniable that', as these are sentential operators and can be embedded. Rumfitt's bilateral formalism faces a fundamental conceptual problem: what does it mean to assume an assertion or a denial in a deduction? Arguably, this makes no sense, as it is plausible that making an assumption is a speech act.

Even if the meaning of negation cannot be defined by rules of inference within proof-theoretic semantics, we can still give an account of it. This is the aim of another paper of mine. For the purposes of this appendix, an indication of the general idea should suffice. Just as the meaning of a predicate, say 'is red', cannot be given purely by rules of inference, but the colour red has to figure in how its meaning is determined, the meaning of 'not' has to be given by reference to something other than rules of inference. Inferential relations may play an important role in determining the meaning of an expression even if that meaning cannot be completely determined by rules of inference. The predicate 'is red' gets its meaning from the inferential relations is stands in with other colour terms and what it refers to, the colour red. The structure colours exhibit together validates inferences such as that what is red is not green. Negation enters the understanding of concepts that exhibit complex inferential structures, like the colour words, and thus cannot be understood without a grasp of that structure. Certain metaphysical consideration may enter the Geachean account, but that is unsurprising: the relation between affirmation and negation is connected

to facts about the world. It is the point where metaphysics enters logic. If the meaning of negation cannot be defined within proof-theoretic semantics, this means that it loses the purity that Dummett envisaged it to have. It is important, however, to stay as neutral as possible when it comes to the question of whether classical or intuitionist negation is the correct one. Another question to be addressed in my paper is whether, on the basis of my Geachean account of negation, Dummett's complaints about multiple conclusion logics can be shown to be unfounded: this gives a smooth and elegant route to justifying classical logic. The paper aims to show how, building on Geach's ideas, a viable account of negation can be given that fills the gap in proof-theoretic semantics identified in the present paper, but nonetheless stays true to its spirit.

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