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### Current Challenges and Future Trends in Analytical Fragility and Vulnerability Modelling

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The lack of empirical data regarding earthquake damage or losses has propelled the development of dozens of analytical methodologies for the derivation of fragility and vulnerability functions. Each method will naturally have its strengths and weaknesses, which will consequently affect the associated risk estimates. With the purpose of sharing knowledge on vulnerability modeling, identifying shortcomings in the existing methods, and recommending improvements to the current practice, a group of vulnerability experts met in Pavia (Italy) on April 2017. Critical topics related with the selection of ground motion records, modeling of complex real structures through simplified approaches, propagation of aleatory and epistemic uncertainties, and validation of vulnerability results were discussed, and suggestions were proposed to improve the reliability and accuracy in vulnerability modeling.

#### **INTRODUCTION**

The assessment of seismic risk of a portfolio of buildings represents the first step towards the development of risk reduction strategies, and generally requires three main components: a probabilistic seismic hazard model, an exposure dataset and a set of vulnerability functions. The

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latter component assumes special importance since a reduction of the seismic vulnerability (e.g., retrofitting interventions) can cause a direct decrease of the associated earthquake risk. In the last three decades, dozens of methodologies have been proposed for the analytical derivation of fragility and vulnerability functions, with varying levels of complexity and accuracy (e.g., D'Ayala et al. 2014, Pitilakis et al. 2014). Each methodology will inevitably lead to distinct vulnerability and fragility functions, even if an identical building class is considered (e.g., Silva et al. 2014a; Pitlakis 2015). It is thus fundamental to understand the limitations and potential bias in the existing practice, and explore solutions to increase the reliability of the resulting functions. As an example, Figure 1 illustrates yielding and collapse fragility functions for nonductile European reinforced concrete moment frames, collected from 24 studies (Crowley et al. 2014). These functions represent structures mostly built before the implementation of modern seismic regulations. Apart from variations due to site-specific conditions and/or different structural designs, these functions represent a similar building class, and thus a lower variability in the fragility functions would be expected. However, these results indicate coefficients of variation in the median yielding and collapse probability in the order of 60%, and a range of mean collapse probability for PGA equal to 0.5g between 5% and 100%.

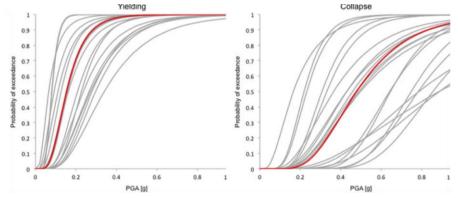


Figure 1 - Fragility functions for yielding and collapse damage states for non-ductile European reinforced concrete structures (adapted from Crowley *et al.* 2014).

This large dispersion is partially due to the consideration of different methods, formulae and assumptions during the fragility derivation process. The resulting estimates of damage or loss even from a selection of these fragility functions would certainly vary considerably, and potentially bias a decision-making process. Amongst the various decisions that a modeler has to undertake in implementing an analytical vulnerability derivation procedure, it is reasonable to identify the selection of the set of seismic inputs, the development of the numerical model, the estimation of the structural response and the statistical regression approach as the most influential steps. Currently, there is no consensus in the engineering community concerning the

ideal solution for each step. Moreover, the vast majority of the fragility models available in the literature are seldom compared with other existing models, and rarely tested against empirical data by applying ground motions from specific earthquake scenarios or within probabilistic risk applications that consider a myriad of possible future scenarios. This lack of testing often leads to biased results when these fragility curves are used in real-world applications. In order to discuss these limitations, and to improve the know how in the development of fragility and vulnerability functions, a panel of 12 experts from academia, the engineering sector and the catastrophe risk modeling community met on the 3-4 of April 2017 in Pavia (Italy). During the discussion of these topics, several recommendations were proposed to improve the current practice in vulnerability modeling, as outlined in the final section. The Authors acknowledge that the views presented herein reflect mostly the current European practice and research, and that other regions might not be adequately represented.

#### **DEFINITION OF SEISMIC INPUT**

#### SELECTION OF RECORDS FOR SITE VS REGIONAL VULNERABILITY ANALYSIS

Ground motions for the vulnerability assessment process can be viewed from one perspective as "dynamic loading protocols", a standardized loading to which a structure is subjected, in order to investigate its response. This approach makes the assessment procedure much simpler, as a standardized set of ground motions can be used in all analysis cases, saving the effort of selecting new ground motions and potentially facilitating "fair" comparisons of collapse capacity across building classes and locations, though such approach neglects sitespecific seismogenic characteristics, as further discussed below.

If this first perspective is adopted, generic suites of ground motions are appropriate for use in vulnerability assessment. There are a number of such sets available today (e.g., Somerville *et al.* 1997, Krawinkler *et al.* 2003, Baker *et al.* 2011). These sets of motions are popular because they allow users to obtain ground motions with a minimum amount of effort. They are also popular for cases where a number of building types are being studied, or where there is no specific site of interest. The conceptual problem with use of generic ground motions, however, is that they lead to a building being considered equally safe at all locations having the same hazard in terms of the intensity measure, *IM*, used to specify the collapse fragility. The question is whether the differences in the anticipated properties of strong ground shaking at those varying locations need to be further accounted for in a collapse risk assessment. In a second alternative perspective, the ground motions are viewed as our best estimate of what future ground motions at the site of interest, and with the *IM* level of interest, might look like. With this perspective, the ground motions for a given analysis should be representative of ground motions that a building at a specific site might actually experience. *Further, low-amplitude ground motions generally have different properties than high-amplitude motions, and so unique motions should be used at each intensity level of interest.* The use of such an approach will make the analysis results more predictive of building behavior at that site. The drawback is that it requires site-specific motions to be selected for each analysis case. This requires the knowledge of more information about the site, and consequently more analyst effort (Baker 2013). Depending upon the IM used for analysis, care to consider the unique spectral shapes, durations, and velocity pulses expected at the site may be needed. These requirements can be mitigated to some degree, however, by choosing a more advanced *IM* metric, as discussed in the following section.

#### SELECTING INTENSITY MEASURE TYPES TO DEFINE FRAGILITY FUNCTIONS

The definition of fragility and vulnerability curves for single buildings has been traditionally done, with few exceptions, by anchoring the ground motion severity to a single *IM*. In fact, this approach is suggested in the Performance-Based Earthquake Engineering (PBEE) framework (Cornell 2000), which has become in the last two decades the main reference of risk assessment for specific structures. Limiting the pinpoint between ground motion characterizations and building response to a single *IM* rather than a vector of *IM*s was dictated simply by convenience. The use of more than one *IM* comes at a price since it requires carrying out vectorial rather than scalar seismic hazard (Bazzurro and Cornell 2002). This more complex path, however, can and in certain cases should be considered, especially when dealing with complex structures that require 3D modeling (Faggella *et al.* 2013).

The selection of the single "adequate" *IM* for building-specific vulnerability characterization has been the subject of a very fertile body of research for at least twenty years (e.g., Shome *et al.* 1998; Porter *et al.* 2007; Luco and Cornell 2007, Villar *et al.* 2017). The three criteria that have been used to guide the selection of the best *IM* for structural response or Engineering Demand Parameter, *EDP*, prediction were *practicality* (can we predict it at a site for a given earthquake scenario? Do reliable GMPEs exist for it?), *efficiency* (does it have a significant predictive power for the desired *EDP*?) and *sufficiency* (do different sets of ground motion records (GMRs) that have the same *IM* value induce statistically indistinguishable *EDP*?

distributions?) (Luco and Cornell 2007). For different types of buildings these criteria have led to many choices of *IM*s (Kostinakis *et al.* 2016). Peak ground acceleration, *PGA*, has been often used for very stiff buildings (and, less appropriately, for flexible ones as well). Spectral acceleration, *SA*, at the fundamental period of vibration ( $T_1$ ) has been used ubiquitously in the early days and then more recently only for first-mode dominated structures. Spectral accelerations at periods longer than  $T_1$  were used for better predicting the severely nonlinear part of the structural response with the goal of assessing the structural response and/or collapse probability. Various combinations of spectral accelerations at different periods were used in the attempt to capture the contribution of all vibration "modes" that are significant to the structural response in its linear and nonlinear range (Vamvatsikos and Cornell 2005).

None of these simple scalar *IM*s is, in general, a good predictor of the ensemble of *EDP*s that are crucial for the assessment of risk measured in terms of economic losses and downtime. Ideally, the predictor should also be good for levels of response both at the onset of damage and also for more severe damage levels and even collapse. Figure 2 shows the variability in IDR and PFA in a 7-story reinforced concrete building in Istanbul when tested against 20 ground motions corresponding to a 10% probability of exceedance in 30 years. These records were selected using the Conditional Spectrum (CS) method (Lin *et al.*, 2013) and considering four different *IM*s: *SA* at  $T_1$ , *SA* at  $T_2$  (second mode period), *SA* at  $T_H$  (1.5 times  $T_1$ ), and *AvgSA* (average of spectral acceleration values between  $0.2T_1$  and  $1.5T_1$ ).

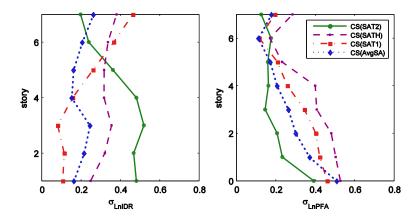


Figure 2. Variability of *IDR* and *PFA* of a post-1980 7-story reinforced concrete building at all stories induced by ground motions corresponding to 10% probability of exceedance in 30 years at the building site.

The three choices of *SA* are good predictors only for one of the two *EDP*s at some of the stories and only for some levels of shaking (see Kohrangi *et al* (2016) for additional results for other levels of shaking). The *AvgSA* instead, performs reasonably well for both *EDP*s for all

stories and, therefore, for loss estimation purposes it seems to be a superior choice of *IM*. The same study also employed *AvgSA* for the selection of ground motion records for three cities in Turkey, and concluded that this *IM* offers a superior sufficiency in comparison with other *IMs*. Moreover, unlike what happens with other complex *IMs* (e.g., Housner Intensity, vector-based intensities), this metric can be estimated using common GMPEs (e.g., Dávalos and Miranda 2018).

#### **EXPLORING PHYSICS-BASED GROUND MOTION SIMULATION**

The development of analytical vulnerability models requires the availability of "reliable" GMRs, as discussed above. Usually GMRs are selected and scaled from a database of existing records to represent target seismic characteristics. The inherent scarcity or total absence of suitable real GMRs for some specific scenarios (e.g., large-magnitude strike-slip events recorded at close source-to-site distances) makes the use of alternative options unavoidable (Brendon et al. 2017).

Recent advances in high-performance computing and understanding of complex seismic source features, path, and site effects, have led to an increasing interest in physics-based ground motion simulation. In fact, physics-based simulated (or "synthetic") ground motions capturing complex source features (such as spatially variable slip distributions, rise-time, and rupture velocities), path effects (geometric spreading and crustal damping), and site effects (wave propagation through basins and shallow site response) provide a valuable supplement to recorded ground motions. Specifically, simulated ground motions fulfill a variety of engineering needs (Bradley *et al.* 2017, Smerzini *et al.* 2016), such as seismic hazard assessment or assessment of seismic demand on structural and geotechnical systems through response history dynamic analysis, within the framework of PBEE. Other alternatives include stochastic-based artificial accelerograms (e.g., Vanmarcke et. al., 1997; Rezaeian and Der Kiureghian, 2011) and modified real records matching a given elastic target spectrum (e.g., Atkinson and Goda, 2010; Seifried and Baker, 2016).

Among engineers and risk modellers the general concern is that simulated ground motions may not be equivalent to real records in estimating seismic demand, and hence, in estimating the induced damage and loss to structures. Moreover, synthetic ground motions are not yet widely available in engineering practice, especially in regions where seismogenic faults and characteristics and the regional velocity structure are not well established. On the other hand, in California, the recently-released Southern California Earthquake Center (SCEC) Broadband Platform (BBP; Maechling *et al.* 2015) provides scientists and engineers with a suite of opensource tools to compute synthetic GMs using physics-based GM simulation models. A Technical Activity Group (TAG) focusing on Ground Motion Simulation Validation (GMSV) has been established by SCEC to develop and implement testing/rating methodologies via collaboration between GM modelers and engineering users. Similar efforts are also being made in Italy, through a web-repository (SYNTHESIS: SYNTHEtic SeISmograms database) containing synthetic waveforms for Italian scenario earthquakes coming from different simulation techniques (D'Amico *et al.* 2017) and in New Zealand via QuakeCoRE validation efforts (e.g., Bradley *et al.* 2017).

A significant amount of research has been developed in recent years to validate ground motion simulation methods for engineering applications. As a recent example, Galasso *et al.* (2012; 2013) have investigated whether simulated ground motions are comparable to real records in terms of their nonlinear response in the domain of single degree of freedom (SDoF) oscillators and multiple degrees of freedom (MDoF) systems. The validation exercise using various *EDPs* indicated that, in most cases, the differences found in seismic demands produced by real and synthetic records are not significant, increasing to a certain degree the trust in the use of simulated motions for this type of analyses. Other validation exercises focused on the comparison of *IMs* from simulations and predictions from empirical models (GMPEs) (e.g., Rezaeian *et al.* 2015), or the comparison in terms of structural response of sets of simulations and recordings with similar elastic response spectra (e.g., Burks and Baker 2014). These types of validation exercises can highlight the similarities and differences between simulated and recorded GMs for a given method, which can support the improvement of the generation of synthetic records.

#### STRUCTURAL MODELLING FOR FRAGILITY ASSESSMENT

In this section, the current state of the art/practice in structural modeling for fragility assessment is outlined for several different structural systems, and the areas where future efforts should be focused are also summarized.

#### MASONRY

The fragility assessment of ordinary masonry buildings has traditionally been based on expert elicitation, empirical methods or simplified displacement-based models, rather than through structural modeling (i.e. nonlinear dynamic or static analysis). The macroseismic method (Lagomarsino and Giovinazzi 2006), which may be considered as an expert elicitation method, has proven to be very effective when validated with observed damage data. For example, data from 57,000 damaged structures after the L'Aquila earthquake (2009) allowed the verification of vulnerability modifier, which can be applied in portfolio risk analysis at urban or regional scales. Without considering vulnerability modifiers, the result of the application of the macroseismic method should be presented in an aggregated manner, as the outcome on a single building would be misleading. One of the main advantages of structural modeling approaches is that it is possible to assess the contribution of different uncertainties to the dispersion of the fragility function: record-to-record variability, different material and geometric parameters of the buildings in the class, or the definition of limit states thresholds (Pitilakis *et al.*, 2014). In a portfolio of masonry buildings, the dispersion related to the base shear or displacement capacity and the definition of limit states is usually comparable or even higher than those related to the seismic hazard.

Several methodologies with differing levels of accuracy have been proposed in the past for structural modeling of masonry buildings (e.g., Bernardini et al. 1990, D'Ayala et al. 1997, Calvi 1999, Glaister and Pinho 2003, Restrepo and Magenes 2004, D'Ayala 2005, Borzi et al. 2008, Molina et al. 2009, Oropeza et al. 2010, Rota et al. 2010, Pagnini et al. 2011, Lagomarsino and Cattari 2013). They are frequently based on a simplified capacity curve (force versus displacement, or spectral acceleration versus spectral displacement), evaluated after the a-priory definition of the collapse mode, and are usually sufficient for estimating building class vulnerability (again, not for a single building). However, it is worth noting that even if these methods are able to explicitly quantify the influence of relevant material, constructive and geometric parameters, they usually refer to a conventional simplified layout of the building. Therefore, it is necessary to ensure that the inter-building dispersion is lower than the intrabuilding counter part (see section on Vulnerability Modelling). To verify this issue, a set of representative prototype buildings for each building class should be defined and analyzed through detailed 3D MDOF nonlinear numerical models, in order to evaluate the inter-building dispersion, assuming other parameters as deterministic. The equivalent frame model, in which piers and spandrels elements are defined in terms of generalized forces (N, V, M) through specific failure criteria and deformation (drift capacity), can be very effective for this purpose (e.g., Lagomarsino et al. 2013). After this, simplified analytical models may be calibrated through these numerical simulations in terms of sensitivity to each single inter-building parameter. To this end, very accurate models (nonlinear continuous constitutive laws in finite element modeling, discrete element models) are not yet commonly applied in engineering practice, and future efforts will be needed to reduce their high computational effort and convergence problems close to collapse.

#### STEEL STRUCTURES

In steel structures, global collapse is often characterized by dynamic instability due to the development of second-order effects (Krawinkler, 2006). Local collapse is typically associated to *i*) excessive inelastic deformation demands imposed on structural members (e.g., buckling effects in beams, columns, bracing members); *ii*) large inelastic cyclic deformation demands that lead to low-cycle fatigue effects resulting in fracture of the members and/or connections; *iii*) failure of steel beam-to-column, brace or column base connections. The simulation of these failure modes in nonlinear analysis requires the adoption of robust numerical models able to simulate strength and stiffness deterioration effects. Additionally, the numerical models must allow controlling the potential for the development of low-cycle fatigue effects in structural members and connections.

Two modeling approaches are often adopted for the numerical modeling of framed structures: distributed (i.e. mechanical - Spacone et al., 1996) and concentrated (i.e. phenomenological – Ibarra et al., 2005) plasticity (e.g., Fragiadakis and Papadrakakis, 2008). Whilst the former approach allows explicit consideration of the material behavior, it is not suitable to simulate stiffness and strength degradation associated to the development of local buckling effects. The latter approach has the advantage of allowing the simulation of such effects. Typically, it requires the definition of a generalized force-displacement relationship, as well as hysteretic rules and corresponding stiffness and strength degradation parameters. Several experimental and numerical studies have been conducted on American and European steel profiles aiming at the characterization of moment-rotation relationships and the derivation of the strength and stiffness degradation parameters (e.g., Lignos and Krawinkler, 2011; Araújo et al. 2017). However, concentrated plasticity models also have important limitations, namely the fact that they do not explicitly account for the interaction between axial force and bending moment or bi-axial bending effects. Currently, there are no robust models available for this purpose, introducing therefore a limitation in the conduction of 3D structural analysis. Moreover, plasticity models are dependent on the stiffness and strength degradation parameters on the loading history, and the current models to simulate beam-column members do not properly account for the simulation of low-cycle fatigue effects, which are often the cause of structural collapse.

The representation of the connection behavior is of critical importance in response-history analysis, particularly when these components participate in the energy dissipation. Gusset plate connections can have an important influence on the response of steel bracing systems, particularly when out-of-plane bending of the braces occurs due to the development of flexural buckling. In such situations, nonlinear rotational springs are employed to represent the behavior of the gusset plate connections (Hsiao *et al.* 2012).

Concerning moment-resisting frames, the component-based model is widely used to characterize the behavior of beam-to-column and column base connections. In simple terms, this approach consists of discretizing the connection into a number of components. The behavior of each component is characterized by a force-displacement relationship (e.g., bilinear). The assemblage of all the components allows deriving the global behavior of the connection (Faella *et al.* 1999). Whilst this approach is well established for static behavior, very limited guidance is available for the characterization of the cyclic behavior of steel connections. Recently, Augusto *et al.* (2017) proposed a procedure for the derivation of cyclic force-displacement relationships of beam-to-column connection components based on results obtained from detailed finite element analysis.

New structural systems and solutions represent additional challenges to the numerical modeling of steel structures. Light steel framing solutions consisting of the assemblage of cold-formed steel members are being built in high seismicity areas. These members are characterized by lower levels of ductility in comparison to traditional steel profiles. In spite of the significant number of research studies carried out during the last two decades, numerical models available to perform analytical characterization of the seismic vulnerability of this type of buildings are still limited. Another trend is the use of high strength steel. Despite the many advantages associated to the use of this steel, it is important that research studies are carried out targeting the characterization of the behavior of these members. The increase of yield stress turns a member more susceptible to buckling phenomena, which may impair its ductility and energy dissipation capacity.

#### **REINFORCED CONCRETE STRUCTURES**

Structural modelling for fragility assessment of reinforced concrete (RC) structures can be an extremely complex task. The investigation of the response of RC buildings started around the 1970s with the test of several specimens. Based on the results of cyclic tests of structural members of RC buildings (e.g. columns, beams, walls) many different semi-empirical models of deformation capacity and analytical models of hysteretic behavior of such structural elements were developed (e.g. Menegotto and Pinto 1973; Mander et al. 1988) and implemented in various tools. The hysteretic behavior of structural members of RC structures can be governed by a flexural behavior, shear behavior or other complex phenomena observed in seismic response of structures during earthquakes (e.g. bar slip and bond deterioration, beam-column joints, bucking of reinforcement). The process of development of nonlinear models of the components of RC structures is certainly not finished, and many challenges still exist. For seismic vulnerability analysis of RC buildings it is necessary to develop nonlinear models which can be applied to real complex structures with hundreds of structural components and to define the limitations of such models (e.g., Ramirez et al., 2012). The main issue is how to simultaneously account for different failure modes and interaction between them (axial, shear or flexural failure). By neglecting the axial and shear failure modes, the error in fragility analysis of existing buildings, which were not designed with consideration of capacity design principles, can be significant (e.g., Celarec and Dolsek 2013). Additional efforts are therefore needed to improve nonlinear models of existing reinforced concrete buildings. In addition, there is still a lack of models for walls and flat-slab reinforced concrete buildings, which represents a type of construction frequently adopted in regions with high seismic hazard (e.g. Chile). It is also necessary to develop guidelines for nonlinear modelling of different types of reinforced concrete structures in order to reduce the errors caused in prediction of their seismic response.

Many earthquakes in the last decades caused collapse of RC buildings which were not designed according to modern building codes. The main problem is that the seismic response of reinforced concrete buildings becomes extremely nonlinear in the vicinity of collapse (e.g., Zareian and Krawinkler 2007). Additional research is thus necessary in order to understand the ratio between the intensity measure causing collapse of RC structure and IM causing near collapse limit state, which can be, to some extent, estimated with nonlinear models already prescribed in existing building codes. It is known that this ratio can be structure-specific (e.g. Spence 2007), as well as dependent on the local seismic hazard. Additional research is also

required in order to improve the understanding of the relationship between the EDPs, which can be obtained from the results of structural analysis, and the physical damage of structural components of RC buildings (e.g. Martins *et al.*, 2016, FEMA P695 ). Such information is necessary to improve the definition of the limit states which are used in the fragility analysis.

#### NON-STRUCTURAL ELEMENTS

A significant part of the earthquake-related losses during recent earthquakes has been attributed to the damage to non-structural (NS) elements. These elements generally exhibit damage even at low seismic intensities and can affect the immediate functionality of buildings, in particular critical buildings. In the light of these considerations, the seismic performance of non-structural elements is nowadays recognized to be a key issue in PBEE. Within the loss estimation framework, the influence of NS components needs to be considered at different stages. First of all, it is important to understand whether the strength and stiffness of NS components should be considered in the numerical models, as they can affect the results of the structural analysis (e.g., Welch et al. 2014, CUREE 2003). In the last years, many researchers made efforts to develop rational methods in order to introduce the NS components in the seismic analysis. Heavy masonry infills are generally considered as NS elements, though it is well recognized that masonry infills modify the structural performance, and therefore should be considered in the structural analysis (Perrone et al. 2016, Dolsek et al. 2008). Mechanical and analytical models are nowadays available to simulate the influence of masonry infills on the seismic response, both from a local and global perspective (at least using simplified models). Except for masonry infills or other simple NS elements, the introduction of NS elements into vulnerability analysis of large numbers of buildings has not found application in practice. The seismic analysis considering the interaction between structural and NS components requires advanced numerical models with a large number of elements, the properties of which have not yet been well established (Filiatrault et al. 2014, FEMA P-58 2012).

The significant differences in the dynamic properties and natural frequencies of structural and NS elements could make numerical models ill conditioned for traditional modal analysis. At the same time, the direct analysis of structural and NS elements could introduce non-classical damping modes due to the large difference in damping characteristics. Based on these considerations, the vulnerability analysis of NS elements is generally conducted using the "cascading" approach, in which the performance of the structure is evaluated without considering the interaction with the NS elements. The floor acceleration/displacement time histories are then used as input for the analysis of the NS elements.

Regarding the loss estimation framework for building portfolios, the performance of the NS elements should be included in the damage calculations. This can be achieved through the identification of fragility functions for each NS component. However, there are only a few experimental studies available for NS elements, and consequently most of the fragility functions are based on expert judgment. The largest database of those functions is provided by FEMA P-58 (2012). Once the fragility functions have been defined, a statistical analysis is required to define which NS elements are installed in the building. Building Information Models (BIM) can be useful to identify performance targets and the quantities of both structural and NS elements. The detailing of all elements available in BIM is essential in the PBEE assessment framework in order to properly attribute damage characteristics (fragility functions), define the quantities (for the estimation of repair costs) and evaluate the repair time (e.g., Perrone *et al.* 2017). For this reason, the use of BIM can be a solution to introduce in a more refined manner the performance of NS elements in the vulnerability analysis of buildings.

#### **TALL BUILDINGS**

The derivation of tall building fragility and vulnerability functions is a challenging task for several reasons. Most tall buildings are designed following the prescriptive conventional code regulations (e.g., ASCE 7 standards, International Building Code, Eurocode) that are tailored to provide a minimum safety for more regular and common building types. Tall buildings, on the other hand, shelter several hundreds of occupants and are sometimes constructed for multiple purposes (e.g. residential and office units in a compound complex) that require particular performance targets and relevant structural design measures. Currently, the PEER-Tall Building Initiative (PEER-TBI) and the Los Angeles Tall Building Seismic Design Council (LATBSDC) are two well-organized entities aiming to advance alternative performance-based nonprescriptive seismic design procedures for tall buildings. These organizations have been publishing performance-based design guidelines for tall buildings for about a decade (e.g., PEER 2017; LATBSDC 2018) that have been implemented in some new tall building constructions, particularly in California. Implementation of such nonprescriptive regulations as well as prescriptive provisions of standard codes would certainly inflate inter-building variability for tall building fragility and vulnerability assessment. The complex structural and nonstructural modeling assumptions in tall buildings would further complicate their fragility and vulnerability models because such components increase the intra-building uncertainty. The significance of inter- and intra-building variability in tall buildings have come into the engineering attention after their mixed performances during recent earthquakes in urban areas (e.g. Krishnan et al., 2006).

The inter- and intra-building variability as well as other sources of epistemic uncertainty in tall buildings that originate from structural, nonstructural and ground motion aspects require systematic approaches in handling their probabilistic loss estimation. The large number of random variables involved in tall building loss calculations and their intricate interdependency with respect to other structural systems are the compelling factors in the implementation of such systematic methodologies.

Given a performance level, the paramount distributions to convolve in the probabilistic loss integral constitute conditional distribution of EDP on IM (G(EDP|IM)), distribution of damage state (DS) conditioned on the EDP (G(DS|EDP)) and the distribution of loss decision variable (DV; such as repair/replacement cost) given DS (G(DV|DS)) (Jayaram et al., 2012). The nonuniform distribution of peak nonlinear structural response over the height of tall buildings require computation of losses at every story level that promotes the consideration of correlation between the EDPs at two different stories (Shome and Bazzuro, 2009). The correlation between EDPs help establishing bivariate distributions of G(EDP|IM) at each story, which improves the evaluation of the variation in loss. Besides, the consideration of epistemic uncertainties originating from modeling, quality of design and construction, analysis procedures and material properties would improve the predictive capacity of G(EDP|IM) (FEMA-355F, 2000). Since the contributions of structural and nonstructural subsystems to loss are critical in most tall buildings, multiple EDPs are used (e.g., peak story drift and peak floor acceleration) to define their conditional variations on the chosen IM. The multiple EDPs to account for the contribution of structural and nonstructural loss should be associated with corresponding fragility functions to establish the damage state distributions conditioned on EDP, G(DS|EDP). The considered fragilities should also account for model-to-model variability for a full probabilistic loss estimation that are currently addressed in several technical reports or publications (e.g. FEMA-355F (2000), FEMA-P58 (2012), Aslani and Miranda (2005)). Consideration of correlation between the DS of structural and nonstructural components is important while establishing reliable loss distributions conditioned on DS (i.e., G(DV|DS)). Based on the studies of Ramirez and Miranda (2009) G(DV|DS) should also account for the correlation between structural and nonstructural component costs.

Large numbers of variables, their uncertainty as well as their correlations with each other that are briefed in the above paragraph would make the numerical integration of probabilistic loss inconvenient. To this end, Shome et al. (2015) propose a Monte Carlo based simulation approach to compute the exceedance probability of loss for tall buildings by populating the conditional random variables described in the above distributions.

#### **VULNERABILITY MODELLING**

#### SAMPLING INDEX BUILDINGS: FEW MDOFS OR MANY SDOFS?

Whenever an analyst is trying to represent a population via a limited-size sample, be it election polling or vulnerability modeling, important questions always come up about the representativeness of the sample vis-à-vis the much larger population. The problem is always one of cost: the fewer voters/buildings one polls, the lower the cost of the analysis. Arguably, the issue is even more convoluted in vulnerability assessment compared to election polling, simply because the concept of a facsimile voter does not exist (at least not for a poll with any aspirations to legitimacy). On the other hand, a reduced-order model to approximate a building is often desirable, simply because it lowers the computational cost. In formal terms, the sampling issue arises when attempting to characterize an asset class according to a taxonomy (e.g., Brzev *et al.* 2013), typically defined by a number of attributes (e.g., material, lateral load resisting system, height, occupancy, age). To represent the class, a sample of so-called index buildings is selected, each being a single particular specimen building of the asset class, with given geometry and material properties (Porter *et al.* 2014). In terms of election polling, these are the sampled voters. If we fail to choose them right, we shall not estimate accurately the asset class vulnerability.

Assuming that sufficient data is available, it is possible to choose index buildings and appropriate weights using methods such as K-means clustering or classification trees. When sufficient knowledge is available on the building stock, expert opinion can substitute formal statistical methods. Still, this is not just a question of how, but also a question of how many. For example, Porter *et al.* (2014) suggests using one to seven index buildings to represent any single class, Vamvatsikos *et al.* (2017) have employed three archetypes for European steel concentrically-braced frames, while Silva *et al.* (2014b) and Babič and Dolšek (2016) used 100 models per archetype for Portuguese RC moment frames and archetypes of industrial precast building classes, respectively. Where do we draw the line when limited resources are available?

To be able to answer this question, we need to understand the two important sources of variability that need to be accounted for beyond just the well-known record-to-record variability: (a) the intra-building and (b) the inter-building variability.

Intra-building (or within-building) variability is mainly caused by uncertainty in the building properties, the model and the analysis method. In general, it suffices to say that the simpler the modeling approach (and the less data that is available), the more uncertain the results from assessing any archetype will be. Inter-building (or building-to-building) variability is instead due to the different buildings that comprise the asset class. In most realistic examples of asset classes, it is fair to assume that this is the dominant uncertainty. This is simply because one may expect to have a greater variability among different buildings of the same class than among different plausible models of a given structure.

The above statement essentially promotes quantity, hoping that sampling more archetypes captures the dominant inter-building variability. Figure 3 shows three conceptual strategies for capturing the dispersion within a building class. The first reflects the actual population distribution, the second captures the modes, and the third aims at modeling the modes and the distribution around them. If we also consider additional properties, plus the correlation among them, it becomes obvious that a small sample of archetypes will likely be inadequate. Still, can we really ignore the quality of each sampled model? If we could only guarantee that a reduced-order model and an approximate method of assessment would only introduce some additional intra-building variability around each sampled model of Figure 3 (b or c) without changing its central value (i.e. without bias), then our problem would be solved. Unfortunately, reduced-order models and simplified analysis approaches may consistently bias the results of assessment, thus not only adding some additional uncertainty, but also shifting the estimated vulnerability to higher or lower values. For example, employing 2D or 1D models that ignore plan asymmetry is bound to produce unconservative estimates of response.

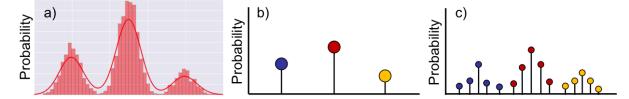


Figure 3 - Alternative representations of dispersion within a building class. a) actual population distribution, b) sampling to capture the modes, c) sampling to capture both the modes and the distribution around each.

The two obvious extremes of this quantity versus quality question would be, on one hand using few archetypes that are designed to exact requirements and analyzed via 3D MDOF models and nonlinear dynamic approaches with carefully selected GMs, versus employing a large number of SDOF models with a backbone curve derived via rudimentary calculations and/or expert opinion (i.e., no finite element analysis). At present, it is not easy to say which one is closest to the "truth", but it is not too difficult to state that both are suboptimal and the ideal approach may be to use a combination of both, whereby the detailed 3D MODF models are used to calibrate the simpler SDOF models.

#### STATIC VERSUS DYNAMIC ANALYSIS

Beyond just the sampling of archetypes, the analysis approach to be employed is a significant consideration. Typically, the important distinction is whether nonlinear static or dynamic analysis will be employed. The former can be one of many static approaches (e.g., FEMA 440), following either the equivalent linearization or the displacement modification approach to translate the static results into dynamic responses. Instead, dynamic approaches employ rigorous, computationally expensive time history analysis under multiple GM records selected according to one of the paradigms discussed in earlier sections.

Again, there are two important considerations to keep in mind, one of bias and one of variance. As an example, Figure 4 shows the resulting error in estimating the response of a 4-story regular RC frame used in FEMA P695. Employing static pushover based approaches introduces non-negligible errors in the assessment that will increase for force/moment quantities or quantities of any kind as more local results (e.g., story drifts or plastic hinge rotations rather than roof drift) are sought. Actually, story shears and overturning moments tend to be consistently underestimated even for this simple mid-rise structure, introducing an unconservative bias when assessing brittle modes of failure. They get even worse for the taller 8-story. In other words, there is a price to pay for the simplicity of the static pushover, especially if one does not respect (and account for) its many limitations.

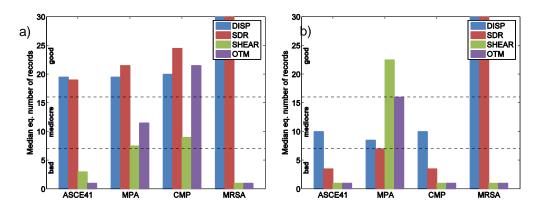


Figure 4 - Errors in estimating roof displacement (DISP), story drift ratio (SDR), story shear (SHEAR) and story overturning moment (OTM) for a) 4-story and a b) 8-story RC frames. Three different pushover-based approaches (ASCE-41, MPA, CMP) and the modal response spectrum analysis (MRSA) are compared in terms of the number of GM records that would cause the same error in dynamic analysis (Fragiadakis *et al.* 2014).

In terms of variance, most code-like pushover approaches tend to disregard record-to-record variability, offering a single "central" (mean or median, typically unspecified) response estimate for any quantity of interest. Obviously, this is not acceptable for vulnerability purposes. While there is little we can do to fix bias (other than using a different analysis approach), we can at least inject the proper variance into the static pushover. For displacement modification approaches, this can be achieved through any of the newest generation of R-µ-T (strength ratio ductility - period) relationships that have appeared in the literature. Specifically, Vamvatsikos and Cornell (2005) and Ruiz-Garcia and Miranda (2007) offer detailed median and dispersion information, while Peruš et al. (2013) proposed a web-based methodology for the prediction of approximate IDA curves. Such results are useful in upgrading existing  $R-\mu$ -T methods that only offer a central value estimate. By putting such existing approaches within a consistent fragility assessment framework, Silva et al. (2017) have offered an open-source toolkit whereby one can undertake building fragility assessment with considerable consistency among different approaches. Such an example is presented by Casotto *et al.* (2018) considering the approaches of Dolsek and Fajfar (2004), Ruiz-Garcia and Miranda (2007) and Vamvatsikos and Cornell (2005). Targeting a more casual user, Baltzopoulos et al. (2017) has offered a re-implementation of the Vamvatsikos and Cornell (2005) approach, adding further sources of dispersion due to the MDOF and a graphical user interface, to offer results of similar quality.

The advantage of pushover-based methods is that these methods are intuitive and easy to use by engineering practitioners. With some additional efforts, engineering practitioners can estimate structure-specific fragility function for a given facility (Dolsek, 2016). Although the approach is an approximation, it can still improve loss assessment of a building stock since the uncertainty in loss estimation is reduced by the use of structure-specific fragility functions. In addition, several attempts have been made recently in order to extend the use of pushover-based methods to buildings with significant impact of higher-modes effect (e.g. Kreslin and Fajfar, 2012, Brozovič and Dolšek 2014). The envelope-based pushover analysis procedure (Brozovič and Dolšek, 2014) assumes that the seismic demand for each response parameter is controlled by a predominant system failure mode that may vary according to the ground motion. It was shown that the accuracy of the approximate percentile response expressed in terms of IDA curves does not decrease with the height of the building or with the intensity of GM if the seismic response is predicted by the envelope-based pushover analysis procedure.

Overall, in a question of static versus dynamic approaches, the dynamic ones will eventually win. We know from many GM studies that spectral shape, duration, etc. have considerable impact on response and vulnerability. There is no way to account for these impacts when considering static analysis. It is only a matter of computational cost that drives us to use the pushover. As our computational power increases, nonlinear dynamic analysis will become our main workhorse.

#### ACCOUNTING FOR TIME-DEPENDENCE AND SOIL-STRUCTURE INTERACTION

At present, the vast majority of the analytical vulnerability studies (and consequently risk assessments) assume that the mechanical properties of the buildings remain the same throughout time, and the analyses are performed considering fixed base conditions. However, it has been demonstrated that the so-called ageing effects can cause changes in the mechanical properties of the building stock, depending on the type of construction. On reinforced concrete structures, corrosion may cause degradation of concrete cover, loss of steel-concrete bond strength and loss of steel cross-sectional area, which in turn significantly reduces the ductility of the structure. Recent studies have demonstrated an increase in the probability of damage on the order of 25% for concrete buildings at least 50 years old (Pitilakis et al. 2013). On masonry buildings, the chemical deterioration of the mortar or bond can reduce their thickness and mechanical properties, which directly affects the displacement capacity of the structures (Benedetti et al. 2010). The available studies on this topic usually focus on the development of various numerical models reflecting different stages of deterioration, which are then tested against sets of ground motion records in order to assess their likelihood of damage against increasing levels of shaking (e.g. Yalciner et al. 2012, Pitilakis et al. 2013). Another source of increasing vulnerability throughout time is the potential damage due to seismic events that might occur within the lifecycle of a structure (Iervolino *et al.* 2015). For example, the 2015 M7.8 Gorkha earthquake affected one third of the Nepali population and caused complete or extensive damage in more than half a million buildings. The seismic vulnerability of these structures has been significantly altered, which means that recent earthquake risk models for Nepal are now obsolete (e.g., Chaulagain *et al.* 2015). These aspects in vulnerability modeling, which are frequently neglected, can affect significantly the structural vulnerability of the building stock.

In addition to ageing effects and the potential damage from past earthquakes, soil-structure interaction (SSI) can also affect significantly the seismic performance of structures, depending on the foundation type. The consideration of SSI in an analytical model can be performed by simulating kinematic interaction schemes, which can result in an elongation of the natural period of the soil-structure system and an increase of the damping due to the energy dissipation (Veletsos and Meek 1974). Neglecting SSI is reasonable when the structure is founded on rock or very stiff soil. However, in softer soil formations, SSI can modify the structural performance leading to either beneficial or unfavorable effects, depending on the dynamic properties of the soil, structure, and input motion (frequency content, amplitude, significant duration - e.g., Dutta *et al.* 2004; Rajeev and Tesfamariam 2012). Recent work on this subject proved that the introduction of SSI might modify considerably the computed fragility curves in case of compliant systems (Karapetrou *et al.* 2015).

Despite the recognized influence of the aforementioned phenomena, their consideration can be a challenging and time-consuming process for large-scale risk analyses. An alternative could be the consideration of secondary factors, which aggravate or attenuate the seismic vulnerability depending on the characteristics of the system (e.g., age, state of conservation, local site conditions, foundation system), similarly to what is often practiced in the catastrophe modeling industry.

#### **CONSEQUENCE MODELLING**

The output metrics of a risk assessment might be the number of damaged or collapsed buildings, which can be directly obtained from analytical fragility functions, but in the majority of applications, it is instead the consequences of this damage/collapse that needs to be estimated. Within an analytical risk framework, consequence models that relate the distribution of damage to the probability of injury, death, damage repair costs, or even downtime, need to be developed. This area of research has not received as much attention as analytical fragility modeling, and often the risk analyst will resort to the use of empirical consequence models. In this case, data such as the number of fatalities within a collapsed building (e.g., So and Pomonis, 2014, Coburn and Spence, 2002) or the cost of repairing damaged buildings from past earthquakes (e.g., Bal *et al.* 2008, Di Pasquale and Goretti 2010) is collected and normalized by exposure data, such as the number of people in the building at the time of the severe damage or collapse, or the replacement cost of the damaged buildings. The collection of such data is fundamental for a better understanding of the impacts of earthquakes and for better calibrating and validating loss models, but the direct use of these models within an analytical risk model is still questionable and ambiguous.

There needs to be compatibility between the damage and collapse states used in the analytical fragility functions and those employed in the consequence model, and this might not always be the case when these two models are developed separately, within different contexts. Furthermore, the repair costs collected after earthquakes might account for repair techniques that would not be used in other countries, and there might be legal frameworks that determine when buildings need to be demolished and replaced that differ from country to country (e.g., Bal *et al.* 2008).

There are a number of recent studies that have developed country-specific repair cost models for use with analytical fragility functions (e.g., Martins *et al.* 2016), but less developments have been made towards robustly connecting analytical fragility modeling with fatality modeling. One drawback in using existing data on fatalities from past earthquakes within an analytical risk models is that a clear description of the collapse mechanism and extent of collapse of each building where fatalities have been reported in past earthquakes is not available, and at best is case and country dependent. Another difficulty in developing fatality models that are compatible with analytical fragility functions is that often the latter do not explicitly model collapse, but instead focus on the near collapse limit state which is appropriate for estimating repair costs and for code compliance, but is not adequate for fatality modeling. Some developments in this direction are discussed in Crowley *et al.* (2017), whereby software to explicitly model different extents of collapse is used together with a fatality model that is dependent on the amount of collapsed debris, but there is still a need for more insight into the consequences of collapse modeling for different structural systems, together with data on the number of fatalities in buildings with different levels of collapse.

#### **EPISTEMIC UNCERTAINTIES**

Despite the improvements in structural modeling (as discussed in the previous sections), there is still uncertainty in the response estimates of buildings to GM, which arises due to inaccuracies in the numerical models used to represent the real inventory of buildings. These inaccuracies might arise due to lack of data regarding the buildings (e.g., no detailed drawings, poor knowledge of material properties) or due to simplifications made in the modeling (e.g., non-structural elements not modeled, stiffness and strength degradation of the structural components not explicitly modeled). Quantification of this modeling uncertainty is not straightforward, but a number of default values are available in the literature (e.g., FEMA P-58, 2012) and as more experimental tests of components and full scale buildings become available, there is scope to quantify the bias or lack of precision of the structural modeling methodology used in the development of analytical fragility functions (Bradley, 2013).

Within seismic hazard modeling, significant attention has been given to the modeling of epistemic uncertainty, and today any state-of-the-art seismic hazard model will include a logic tree to account for epistemic uncertainties in both the source and ground-motion models. The SSHAC guidelines used in the nuclear power industry (USNRC, 2012) propose a structured process to capture the "centre, body and range" of uncertainty in hazard modeling, such that the next generation of hazard models (which should be based on increased data) should have results that fall within the confidence limits of a PSHA undertaken today. However, apart from a few exceptions (e.g. Schotanus *et al.*, 2004), the majority of probabilistic seismic risk assessments carried out to date have not accounted for the epistemic uncertainty within the fragility and consequence models, and thus the overall uncertainty in the risk assessment is being underestimated. The inclusion of fragility and consequence models within logic trees requires further attention and should become standard practice in future probabilistic risk assessments, both to provide appropriate uncertainty bounds to decision makers and to allow the impact of improved data collection and modeling on these components of the risk model to be explicitly tracked in the future.

#### VALIDATION AND VERIFICATION OF FRAGILITY MODELS

The dissemination of fragility or vulnerability models is usually performed through technical reports, conference proceedings or peer-reviewed publications. Whilst the scientific validity of the results is usually evaluated through a peer-review process, the verification of the

performance of the fragility or vulnerability functions is usually disregarded. This trend might lead to unrealistic results when the functions are finally used in real applications, such as the rapid estimation of damage, development of earthquake scenarios, or assessment of annualized earthquake losses for insurance purposes. Several reasons can be identified for this lack of verification:

Insufficient damage data: Damage data is usually only available in seismically active regions, with either sufficient resources to organize field surveys, or where external support is provided (e.g., EEFIT or EERI campaigns). Then, in regions where such data is available, most of the field campaigns focus on identifying failure mechanisms that led to the collapse of structures (forensic surveys, which can improve the development of numerical models), and not on statistically complete data collection and processing, which are critical for the validation of fragility and risk models. Moreover, even when data from several missions are available, the protocols to collect and classify damage data (i.e. damage scales and classification of each building into a building class) are often distinct, which prevents merging the data into a harmonized dataset.

<u>Unavailability of exposure data</u>: In order to verify the performance of the vulnerability functions throughout earthquake scenarios or probabilistic seismic risk, it is necessary to use an exposure model defining the spatial distribution, economic value, occupants and vulnerability classifications of the assets exposed to seismic hazard. However, such datasets are still rare and usually only available within the private sector. Furthermore, even when some exposure data exists, it might represent the building portfolio after a few years since the occurrence of the seismic event (e.g., at the time of the latest housing census). Finally, most of the large-scale exposure modeling methods rely on proxies (e.g., housing census, satellite imagery) to estimate building count, and on expert judgment to assign building classes, which are obviously subjective methodologies introducing important and uncontrollable epistemic uncertainty.

<u>Difficulties in characterizing the level of ground motion</u>: Even if adequate damage and exposure databases are available, it is still necessary to define the ground shaking across the affected region in order to verify the performance of the vulnerability functions. However, this component of the verification process is usually affected by a large aleatory variability (if intensity or GMPEs are used), even when a dense network of recording stations is available (which is only the case in a few nations).

Lack of expertise in loss modeling: the verification, validation and calibration of fragility and vulnerability functions might simply be out of the scope of the study, and beyond the expertise of the vulnerability modelers.

Some of the aforementioned issues can now be mitigated due to recent developments in earthquake engineering and seismology. For example, the ShakeMap system (Worden *et al.* 2016) of the USGS provides GM data, which takes into consideration information from recording stations and observed damage. For single structures, health-monitoring techniques, using ambient noise or small earthquake records can be used to update the numerical model, which can lead to an improvement of the fragility functions (Karapetrou et al., 2017). Exposure data can be accessed through large-scale initiatives, such as HAZUS (also covering Canada), H2020 European projects SERA and EPOS, and the Global Earthquake Model. There is also a multitude of open tools with intuitive graphical user interfaces capable of performing earthquake scenarios or probabilistic seismic risk assessment (e.g., ELER, SELENA, CAPRA, OpenQuake - Silva *et al.* 2014c). An application of the OpenQuake-engine and the USGS ShakeMap system to assess earthquake damage and losses in Italy and New Zealand is described in Silva and Horspoll (2018), while a simulation of past events for the purposes of evaluating earthquake models is described in Villar and Silva (2017).

#### CONCLUSIONS

We presented our opinions regarding the current practice in vulnerability modeling, along with the existing limitations, and possible future trends. If one would try to predict how seismic vulnerability should be modeled in twenty years, the experts concluded that most likely the advances in physics-based ground motion simulation and characterization of faults and geology around the world will allow the generation of sufficient ground motion records to overcome limitations in current strong motion databases, or to perform structural analysis solely with simulated ground motion. Additional numerical elements and material constitutive laws will be developed or improved, which will allow a better simulation of the seismic performance for single (MDOF) building analysis, and will enable a better calibration of sets of SDOF oscillators for building portfolio assessment. In this process, well-established modifying factors will be used to account for the possible SSI and ageing effects. Moreover, while nonlinear static procedures will still be valuable for practitioners (who might be less comfortable with complex numerical simulations), improved nonlinear time history analysis will be the most frequently

adopted approach to assess the response of structures. This shift will be allowed due to the greater capacity of computational resources, the improvement of the user experience of structural analysis tools, and the availability of additional numerical models that will increase the reliability and accuracy of the results.

A departure from the usual scalar intensity measures (e.g., *PGA*, *PGV*, *SA* at T<sub>1</sub>) to define fragility and vulnerability functions is also expected, which should propel the development of new GMPEs (Kohrangi *et al.* 2017), or the addition of modules in existing risk analysis tool to make use of more *sufficient* and *efficient* IMs (e.g., average *SA*, *Housner Intensity*). With the greater emphasis in the control of direct and indirect consequences proposed by modern seismic regulations, a better modeling of fatalities and loss of functionality (e.g., damage in NS components and contents) is expected, as opposed to the usual fragility curves in terms of a limited number of structural damage states. Due to a greater availability of damage and loss data, a stronger focus will be dedicated to the verification and validation of the resulting functions, which will consequently improve the quality of earthquake risk analysis, and resilience of the society.

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