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How Fundamental is the Fundamental Assumption?

Nils Kürbis

RESUMEN

El supuesto fundamental de la justificación de la deducción basada en la teoría de la demostración al que se adhieren Dummett y Prawitz consiste en que "si tenemos un argumento válido para un enunciado complejo, podemos construir para él un argumento válido que termina con una aplicación de una de las reglas de introducción que gobieran su operador principal". Defiendo aquí que este supuesto, en su versión general, es defectuoso y que debería restringirse de manera que no se aplique a argumentos en general, sino sólo a demostraciones. Argumento también que el proyecto de Dummett y Prawitz de proporcionar una base lógica para la metafísica descansa sólo en el argumento restringido.

PALABRAS CLAVE: *semántica de teoría de la demostración, Michael Dummett, Dag Prawitz, teorías verificacionistas del significado, realismo* versus *antirrealismo*.

ABSTRACT

The fundamental assumption of Dummett's and Prawitz' proof-theoretic justification of deduction is that 'if we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator'. I argue that the assumption is flawed in this general version, but should be restricted, not to apply to arguments in general, but only to proofs. I also argue that Dummett's and Prawitz' project of providing a logical basis for metaphysics only relies on the restricted assumption.

KEYWORDS: *Proof-theoretic Semantics, Michael Dummett, Dag Prawitz, Verificationist Theories of Meaning, Realism vs. Anti-Realism*.

I. INTRODUCTION

The *fundamental assumption* of Dummett's and Prawitz' proof-theoretic justification of deduction has largely been ignored in the literature, even though Dummett and Prawitz assign great importance to it. It originates in Dummett's *The Logical Basis of Metaphysics* (henceforth *LBM*). As the name indicates, Dummett places it at the foundations of the proof-theoretic justification of deduction. Prawitz agrees that it is of central importance to their project, where the meanings of sentences are specified by what count as their direct verifications, so that the meanings of logical constants are specified in terms of their use in arguments: if the fundamental assumption fails, 'it probably means a failure also of verificationism' [Prawitz (1994), p. 375] and 'it is the whole verificationist project that is in danger when the fundamental assumption cannot be upheld' [Prawitz (2006), p. 523)]. Thus either a viable justification of the practice of ignoring the fundamental assumption is mandatory or a revision is called for. The aim of this paper is to justify the practice by arguing that the fundamental assumption should be dropped, as it is flawed. However, I shall argue that this is not a disaster for the proof-theoretic justification of deduction, as the assumption need not be made. As it is hardly ever discussed in the literature, workers in the field seem to agree implicitly with my result, so it is worth making it explicit.

I shall first give some background and introduce the fundamental assumption together with the notions of direct and indirect verifications, compositionality and the complexity condition. I'll then give four objections to the fundamental assumption. They are not all wholly original. Some of them can be found in Dummett's writings, whose own examination leaves the fundamental assumption 'very shaky' [*LBM* p. 277]. Next I'll observe that Dummett and Prawitz make very little use of the assumption in the development of their theory. In particular, it is not used in formulating the notion of harmony and an argument against classical logic can be given solely on the basis of compositionality and the complexity condition. I conclude that the assumption is unnecessary to establish the main conclusions of Dummett's and Prawitz' programme. Finally, I shall argue that this does not matter for Dummett's and Prawitz' larger project of providing a *logical basis for metaphysics*: for this, only purely logical reasoning needs to be taken into account. Restricted to this realm, something very similar in content to the fundamental assumption reappears as a theorem on the forms of proofs in systems of natural deduction, i.e. as a consequence, not a prerequisite, of the proof-theoretic justification of deduction.

II. Dummett's Fundamental Assumption

According to Dummett, sentences have *direct* or *canonical* as well as *indirect* verifications. To illustrate, suppose Brutus desires a fig and Porcia knows that he does and Brutus knows that Porcia knows this: then, if Porcia offers to Brutus what he desires, Brutus can either verify directly that Porcia offers him a fig by waiting and seeing what she has for him, or he can verify this indirectly by deducing from his knowledge what Porcia offers him.

Direct verifications are linked to core uses of the expressions occurring in the sentence, whereas indirect verifications are further removed from them. '[Indirect verifications] will include deductive arguments involving sentences of an unbounded degree of complexity. It is this that requires us to distinguish between direct and indirect verifications of a statement, or, in mathematics, between canonical proofs and demonstrations of a more general kind. A direct verification of a statement is one which proceeds in accordance with the composition of the sentence by means of which it is expressed [...] When direct verification involves deductive reasoning, this reasoning must always proceed from less complex premises to a more complex conclusion' [*LBM* p. 229].

Indirect verifications of sentences have to be shown to be valid relative to direct ones: for any indirect verification, there must be a direct one. If this is not the case, it interferes with the partial ordering that *dependence of meaning* imposes on the language, as then the indirect verification licenses a use of the sentence not in accordance with its place in the ordering. Dependence of meaning is a relation Dummett takes to hold between sets of expressions based on the observation that 'the understanding of a word consists in the ability to understand characteristic members of a particular range of sentences containing that word' [*LBM* p. 225]. This leads to the *principle of compositionality*, which 'requires that the relation of dependence between [sets of] expressions and [sets of] sentence-forms be asymmetric' [*LBM* p. 223]. The qualification 'sets of' is needed because there may be collections of expressions that can only be learned simultaneously — Dummett mentions simple colour words like 'yellow', 'red', 'green', 'blue'. These however must form surveyable sets. Dependence of meaning has to have an end somewhere and can neither proceed *ad infinitum* nor in a circle. A compositional meaning-theory employs the relation of dependence to impose a partial ordering on the expressions of the language, which exhibits how the language is learnable step by step.

It is not easy to give a precise general characterisation of the distinction between direct and indirect verifications. But some such distinction is certainly motivated by compositionality. We may grant at least the following: even though there may not be obvious *positive* criteria for what counts as a direct verification of a sentence, there are reasonable and workable *negative* criteria for when something does not. We can agree with Dummett that a very elaborate argument for a very simple sentence, for instance, counts as an indirect verification, even though we may not be so sure what its direct verification is. 'A Case of Identity' in *The Adventures of Sherlock Holmes* contains a good example. I'm not sure what counts as a direct verification of the claim 'Mr Hosmer Angel is Mr James Windibank in disguise', but Holmes' later explanation to Watson surely does not: '"Well, of course it was obvious from the first that this Mr. Hosmer Angel must have some strong object for his curious conduct [of proposing to Windibank's step-daughter, exacting vows of fidelity from her, come what may, and disappearing the morning of the wedding after having alluded that something might happen to him], and it was equally clear that the only man who really profited by the incident, as far as we could see, was the stepfather. Then the fact that the two men were never together, but that the one always appeared when the other was away, was suggestive. So were the tinted spectacles and the curious voice, which both hinted at a disguise, as did the bushy whiskers. My suspicions were all confirmed by his peculiar action in typewriting his signature, which, of course, inferred that his handwriting was so familiar to her that she would recognise even the smallest sample of it. You see, all these isolated facts, together with many minor ones, all pointed in the same direction." – "And how did you verify them?" – "Having once spotted my man, it was easy to get corroboration. I knew the firm for which this man worked. Having taken the printed description, I eliminated everything from it which could be the result of a disguise — the whiskers, the glasses, the voice, and I sent it to the firm, with a request that they would inform me whether it answered to the description of any of their travellers. I had already noticed the peculiarities of the typewriter, and I wrote to the man himself at his business address asking him if he would come here. As I expected, his reply was typewritten and revealed the same trivial but characteristic defects. The same post brought me a letter from Westhouse & Marbank, of Fenchurch Street, to say that the description tallied in every respect with that of their employee, James Windibank. *Voilà tout*!"' The verification can only be indirect, as understanding the concept of being in disguise it is not necessary to know anything about family relations, typewriters or wine merchants of Fenchurch Street.

Be that as it may, the difficulties surrounding the general case need not deter Dummett in the proof-theoretic justification of deduction. Restricted to the special case of the logical constants, the distinction between direct and indirect verification seems reasonably clear: 'the introduction rules for a logical constant **c** represent the direct or canonical means of establishing the truth of a sentence with principal operator **c**' [*LBM* p. 252]; verifications proceeding otherwise are indirect. The *fundamental assumption* now is that 'if we have a valid argument for a complex statement, we can construct a valid argument for it which finishes with an application of one of the introduction rules governing its principal operator' [*LBM* p. 254]. Employing the distinction between direct and indirect verifications as applied to logical constants, it says that for every indirect verification of a sentence δAB we can find a direct one. Accordingly, Dummett presents the fundamental assumption as a requirement following from the thesis that the introduction rules for a logical constant define its meaning.

Another motivation for making the fundamental assumption is a relation between it, compositionality and the *complexity condition*. 'The minimal demand we should make on an introduction rule intended to be self-justifying is that its form be such as to guarantee that, in any application of it, the conclusion will be of higher logical complexity than any of the premises and than any discharged hypothesis' [*LBM* p. 258]. The complexity condition ensures that the compositionality of the logic-free fragment of the language is kept intact after adding the logical constants. Thus, given an argument for A, by the fundamental assumption there always is an argument for A that proceeds according to its composition in a trivial, meaning-theoretically uninteresting sense—meaning no more than that it proceeds according to the way A is composed from its sub-sentential expressions. But this does not exclude that sentences are invoked in the argument that are of larger complexity than A — the fundamental assumption alone does not exclude introduction rules from having premises more complex than the conclusion. However, if the rules governing the logical constants satisfy the complexity condition, there is a verification that proceeds according to its composition in a substantial sense: this argument for A does not invoke sentences of higher complexity than A in any of its subarguments.

I need to dispose of a possible misunderstanding. It might be objected that the fundamental assumption is ridiculous, if it is an assumption about the form of deductions, as it would then entail that if $\Gamma \vdash A \vee B$, then $\Gamma \vdash A$ or $\Gamma \vdash B$. For if there is a deduction of $A \vee B$ from Γ, then, by the fundamental assumption, there is a deduction of $A \vee B$ from Γ which ends with a step by disjunction introduction. This rule has two forms: 'from A to infer $A \vee B$' and 'from B to infer $A \vee B$'. Hence by removing the last step from the deduction, either a deduction of A or a deduction of B from Γ could be constructed. But this cannot be correct. For there is a deduction of $A \vee B$ from $A \vee B$, and there should be neither a deduction of A from $A \vee B$ nor one of B from $A \vee B$. So the fundamental assumption is absurd. The misunderstanding is that the fundamental assumption is *not* applied to deductions, but to *supplementations* of arguments which are 'the result of replacing any complex initial premise by a canonical (sub)argument having that premise as its final conclusion' [*LBM* p. 255]. Here 'argument' is a notion slightly more general than 'deduction' as ordinarily used. The details need not concern us here, but arguments may contain steps by 'boundary rules' which allow the derivation of atomic sentences from other atomic sentences [*LBM* pp. 254f]. It is worth noting at this point, though, that two versions of the fundamental assumption can be extracted from Dummett's writings. Initially Dummett introduces the version given a few paragraphs earlier. Soon afterwards, however, he writes that 'the fundamental assumption is that, whenever we are entitled to assert a complex statement, we could have arrived at it by means of an argument terminating with at least one of the introduction rules governing its principal operator' [*LBM* p. 257], which seems to restrict it to arguments with premises we know to be true. Employing this second version of the fundamental assumption, what counts as an 'argument' has to be compared to $\vdash A$ rather than to $\Gamma \vdash A$. Although this may just be a slip of the pen, I shall suggest at the end of this paper that it is best to apply the fundamental assumption only to proofs of theorems, rather than deductions in general.

When correctly applied, the fundamental assumption does not lead to logical absurdities. There are, however, other, substantial reasons against making it, as I shall show in the next section.

III. Four Objections to the Fundamental Assumption

1. As the fundamental assumption favours *introduction* rules as giving the meanings of logical constants, applied to the case of disjunction this introduces a genuinely anti-realist thought: that a disjunction can only be asserted if we could also assert one of its disjuncts. This should be an illegitimate move if the aim is to establish which logic is the correct one *independently* of any prior postulations in favour of realism or anti-realism. Dummett concedes that for some constants it is more natural to view their meanings as determined by their elimination rules — arguably, disjunction is one of them. So the neutral approach is to reserve judgement as long as possible regarding the question whether it is the introduction or the elimination rules which define the meaning of a constant and to see whether neutral grounds can be given to decide this question. Dummett has not formulated a suitable converse of the fundamental assumption in case it is the elimination rules that determine the meanings of logical constants. Although he acknowledges this to be an option, Dummett does not consider it in detail, and what he has to say about such an approach is much less specific than the development of his favoured one. It is even less clear how the fundamental assumption and its converse would have to be applied in case we adopt an approach in which the meanings of some constants are determined by their introduction rules and of others by their elimination rules, but again, Dummett does not exclude this option.

2. There is a problem with the generality of the fundamental assumption. The argument for why it is considered necessary if the meanings of the logical constants are defined by their introduction rules is the following. Suppose a sentence δAB cannot be verified by the application of an introduction rule for δ, but only by the application of an elimination rule for another logical constant ε. In order to command a full understanding of δ – to be able to make all the uses of it one is entitled to make – the speaker has to understand ε. Hence the meaning of δAB is not determined completely by the meanings

of A, B and the introduction rule for δ, which accordingly does not determine the meaning of δ completely.

This argument is cogent if A and B are atomic sentences. It also generalises to complex A and B *if ε does not occur in them*. It is not plausible, however, that it generalises any further than that. If ε occurs in δAB, then a specification of the meaning of δAB appeals to a specification of the meaning of ε and understanding δAB requires an understanding of ε. But then a verification of δAB ending with an application of an elimination rule for ε would only employ the conceptual resources already required for understanding δAB. Understanding a logical constant is a general capacity. So if I understand a logical constant, I can be expected to apply the rules governing it in all cases where I also understand all the other expressions occurring in the sentences it connects. Thus the partial ordering dependence of meaning imposes on the language would not be endangered, if, by hypothesis, I understand all the subsentential expressions of δAB, even if its canonical verification ended with an application of an elimination rule for ε. If this correct, then the fundamental assumption is badly motivated because it is too general.

3. The fundamental assumption is only as clear as is the distinction between direct and indirect verifications. At first glance, in contrast to its application to other expressions of the language, the distinction seems admirably clear when applied to the logical constants. But Dummett concedes that we are sometimes justified in asserting $A \vee B$ without having any means of retrieving which of A or B is assertible. Dummett gives two examples. We may be able to infer 'That is either a boy or a girl over there' from 'That is a child over there', where 'the disjunctive conclusion was not arrived at by "or"-introduction, and may well not have been able to be on the basis of the observation actually made. […] Hardy may simply not have been able to hear whether Nelson said, "Kismet, Hardy" or "Kiss me, Hardy", though he heard him say one or the other: once we have the concept of disjunction, our perceptions themselves may assume an irremediably disjunctive form' [*LBM* p. 267]. Thus, not only does Dummett concede that we may derive disjunctive sentences without applying disjunction introduction, he goes as far as to acknowledge that it is possible to verify a disjunction *directly* without verifying either of its disjuncts. Dummett does not discuss any other logical constants in this context, but something analogous may plausibly be the case with negation: can't I infer 'This is not red' from 'This is green' or perceive directly that the wine is not in the fridge without having to derive an absurdity from contrary assumptions?

The upshot of this is that it is not the case that, for any logical constant, every direct verification of a sentence $A{*}B$ needs to end with the application of an introduction rule for $*$. This makes the distinction between direct and indirect verifications problematic even in the special case of the logical constants, and obviously this undermines the fundamental assumption: if it is not the case that only those verifications of a complex sentence count as direct which end with an application of an introduction rule for its principal connective, then the fundamental assumption is false.

Dummett's response to this problem is to claim that we are not concerned with *actual* verifications, but only with *potential* ones by a suitably placed observer. This response leads to my next objection to the fundamental assumption.

4. According to Dummett, the fundamental assumption needs to be interpreted in a suitable way. He concedes that 'what underpin the fundamental assumption are considerations that are not themselves proof-theoretic but are in a broad sense semantic: we are driven to invoke some notion of *truth*' [*LBM* p. 269]. Thus the fundamental assumption leads away from a purely proof-theoretic justification of deduction, which is not a bad thing, as it is implausible that no notion of truth enters it at all. Dummett needs to ensure that even if we are not *actually* in a position to proceed according to the fundamental assumption, an argument with a complex conclusion may nonetheless be valid. So the fundamental assumption needs to be interpreted in a suitable way: every complex sentence *could have* been verified by an application of an introduction rule for its principal connective by a suitably placed observer, who then *could have* produced a canonical argument for the sentence in question, even though *we* are not actually in a position to do so. In general, such an observer need not only be suitably placed, but also endowed with suitable powers, as such an observer would have to be able to, for instance, decide whether a very large number is either odd or even, so as to be able to produce a canonical argument for the statement that it is. Let's call such an observer 'ideal'. Then, if an argument is valid, an ideal observer could have produced a canonical argument for its conclusion.

A realist, who accepts classical logic, would have to hold that an ideal observer can verify every proposition or its negation. But according to Dummett, this cannot be justified merely by appeal to the powers of an ideal observer. The laws validated by the proof-theoretic justification of deduction 'remain invariant under considerable variation in the interpretation of the fundamental assumption, because it will still serve to validate them by the proof-theoretic justification procedure, without the need for further assumptions. When the strong realist interpretation is adopted, however, the situation changes: not all laws can any longer be validated by proof-theoretic means, because their validity depends not only on the fundamental assumption but on the further assumption of bivalence' [*LBM* p. 271]. Whether a proposition can be verified does not depend on anyone's capacities, but only on the proposition and what it is about. Take Dummett's example of Jones, who's dead now and has never been in a situation where he could have shown bravery. If we don't assume bivalence, we have no reason to assume that even the ideal observer will come up with an answer to the question whether Jones was brave. If Jones' actions leave that question open, we have no reason to believe that 'Jones is brave' is determinately either true or false, unless we assume the principle of bivalence. Similarly for Goldbach's Conjecture, for instance. Unless we accept bivalence, why should we exclude the option that mathematical reality leaves it undetermined whether every even number is the sum of two primes? Thus, even if we introduce the notion of an ideal observer to give a suitable interpretation of the fundamental assumption, according to Dummett the proof-theoretic justification of deduction still does not validate classical logic.

This line of argument, however, is dependent on what capacities we allow the ideal observer to have. Classicists and intuitionists agree that there are only two truth values, i.e. true or false. They disagree over whether every proposition determinately is one of the two. We can follow Dummett and exclude the option of there being more than two truth values as of relatively minor significance: 'A meaning-theory which substitutes, for the two-valued semantics, a finitely many-valued one represents a very trivial variation of this: we have merely been provided with a slightly more complicated mechanism for determining the truth or otherwise of a complex sentence in accordance with its composition from the subsentences. In such a semantic theory, truth, as we have been using this notion, corresponds to having a designated value' [*LBM* p. 305]. The difference between realists and anti-realists relies on there being propositions to which we are in no position to attach a truth value. For the anti-realist, the world is underdetermined: there are, in a sense, gaps in reality. But we are in no position to recognize any gaps: to assume that we can implies a contradiction, because we would then be in a position to recognise that it can never be verified and never be falsified, in which case it would be neither true nor false, which is impossible. All we can do is note that, as things stand, certain propositions have as yet neither been verified nor falsified. We cannot exclude the possibility that we will be in a position to decide these propositions. The question now arises what the ideal observer is able to do with a proposition to which we cannot attach a truth value. Assuming that no proposition can be neither true nor false, he wouldn't be able to recognise a gap in reality either. But why should we go for the option that the ideal observer is in the same position that we are in? In the case of Goldbach's Conjecture, why can't we allow the ideal observer to be able to complete the infinite task of checking each even number whether it is the sum of two primes? In the Jones case, why can't we allow the ideal observer to have *scientia media* or sufficient powers of counterfactual reasoning to be able to determine what Jones would have done had he been in a situation where he could have acted bravely? A realist may not have a problem with attributing such powers to the ideal observer; the anti-realist obviously would. But how are we to decide the question which position to take? It seems as if each position favours an account of the powers of the ideal observer suitable to its metaphysics. Put slightly differently, both the realist and the anti-realist agree that what the ideal observer could have verified extends our own recognitional capacities *ad infinitum*. But they disagree over what this means. The anti-realist insists we restrict ourselves to the potential infinite: the ideal observer's capacities extend our actual capacities in an arbitrarily large finite way, so that even the ideal observer cannot complete infinite tasks. The realist insists that we can allow the extension of our capacities by an actually infinite amount, so that the ideal observer can complete infinite tasks. The anti-realist will say that we cannot conceive of an actual infinite, the realist will say that we can. But the proof-theoretic justification of logical laws is not the place to decide that question, and so introducing the ideal observer doesn't help.

It is worth noting that in a later book, Dummett entertains the possibility that intuitionist logic is, as it were, the logic of our limitations: 'we cannot consistently envisage there being any such gap in a particular case; this would be to envisage a proposition's being neither true nor false, and this would be a contradictory supposition. […] God can know where a gap in reality occurs, by knowing neither the truth nor the falsity of some proposition; he has available to him a negation which is not available to us. It might therefore be urged that the logic of God's thought, a logic to whose application we cannot attain, is a three-valued, rather than a classical, one' [Dummett (2004), p.96]. Thus, if we allow the ideal observer to have the powers of verification Dummett here attributes to God, and we stick to the assumption that no proposition is neither true nor false, it follows that every proposition is either true or false. It is not so much the principle of bivalence that is at issue, rather, it is the principle both classicists and intuitionists agree on, that no sentence can be neither true nor false, plus which capacities of surveying what can be verified we can attribute to the ideal observer.

The appeal to an ideal observer in order to interpret the fundamental assumption, then, is problematic. Realists and anti-realists will assume the ideal observer to have capacities suitable to their respective notions of truth, i.e. metaphysics. We have not been given any criteria for deciding which notion of an ideal observer is the correct one. Evoking the ideal observer in the interpretation of the fundamental assumption results in placing metaphysical and epistemological issues at the foundations of the proof-theoretic justification of deduction, which, as the aim is to provide a *logical basis for metaphysics*, is detrimental to the project. The fact that the fundamental assumption stands in need of interpretation, and the kind of interpretation required according to Dummett, makes it counterproductive, given the larger aim of Dummett's project.

IV. Do we need the Fundamental Assumption?

Thus, as the fundamental assumption favours introduction rules, it introduces anti-realist prejudices; it is too general to be plausible; Dummett himself admits that it is possible to know that a disjunction is true without being in a position to verify either disjunct; and the appeal to an ideal observer to remedy the last point is not workable as classical and intuitionist logicians won't agree on the capacity of such an observer. On the one hand, Dummett considers the fundamental assumption to be a prerequisite of the proof-theoretic justification of deduction, and he gives an initial definition of the notion of valid canonical argument on the basis of it [*LBM* pp. 259ff]. On the other hand, Dummett admits that the fundamental assumption points away from a purely proof-theoretic justification of deduction, and he is forced to relax the requirements of his definition of validity, as the fundamental assumption is strictly speaking false when applied to some constants [*LBM* pp. 265ff]. Hence it is worth asking if the proof-theoretic justification of deduction can do without the fundamental assumption. In fact, the fundamental assumption has not attracted much attention in the literature, which suggests that the interest of the proof-theoretic justification of deduction is independent of it. I think this is so for the following reasons.

The proof-theoretic justification of deduction consists of a formal project with a philosophical motivation. The formal project is to establish the normalisation of deductions: it is a requirement on a proof-theoretically justified logic that its deductions normalise. But normalisation proofs are independent of the fundamental assumption, because it does not introduce any new formal concepts and does not impose any restrictions on the forms of rules of inference. Although Prawitz thinks that the fundamental assumption is important philosophically, there is no formal equivalent of it in his work on normalization of proofs [Prawitz (1965)] and the fundamental assumption is not appealed to there. The philosophical motivation of normalisability is the demand that the rules of inference governing the logical constants be in harmony, i.e. that the grounds for asserting a proposition are in harmony with the consequences of accepting it. This requirement has the effect of excluding connectives like Prior's *tonk*, but is of more general importance. The motivation for demanding that deductions normalise and that detours through maximal formulas may be removed is the thought that applying logic to premises to derive conclusions should not result in more information than we already had, and all information contained in the premises should remain extractable from the conclusions drawn. As Dummett puts it, 'the requirement that this criterion for harmony be satisfied conforms to our fundamental conception of what deductive inference accomplishes. An argument or proof convinces us because we construe it as showing that, given the premises hold good according to our ordinary criteria, the conclusion must also hold *ac-*

cording to the criteria we already have for its holding' [*LBM* p. 219]. This view of the nature of deduction is again independent of the fundamental assumption. Harmony is a feature of rules of inference. The requirement of harmony imposes restrictions on the form of rules of inference: the introduction and elimination rules for a logical constant must match in such a way that the grounds for asserting a proposition by application of an introduction rule match the consequences that can be drawn from the proposition by an application of an elimination rule. Harmony between introduction and elimination rules is the basis of normalization of proofs, as harmony between introduction and elimination rules is shown to hold by establishing that there are procedures for removing maximal formulas from deductions. Thus this aspect of the proof-theoretic justification of deduction is independent of the fundamental assumption.

Contrast the fundamental assumption with the complexity condition. The complexity condition is the formal equivalent of compositionality and imposes restrictions on the forms of rules of inference. Although the rules for classical negation seem to exhibit a somewhat different shape from the intuitionist ones, that does not change the fact that classical logic normalises, if suitably formalised. To exclude classical logic from being a justified logic, some other considerations are needed. This can be achieved by appealing to compositionality. By compositionality, the meaning of $\neg\neg A$ is dependent on the meaning of A and negation. It may happen that a sentence of a language where double negation elimination is employed can be verified *only* via its double negation. In such a case the move from $\neg\neg A$ to A would contribute to the meaning of A, because it licenses uses of A not otherwise possible, as *ex hypothesi* no other verification is available. Hence the meaning of A would depend on the meaning of $\neg\neg A$. This is a circular dependence of meaning and hence, Dummett would claim, A cannot have a stable meaning at all. A speaker could not break into the circle and learn the meaning of A, which could have no place in the partial ordering that dependence of meaning imposes on the language. Of course, classical logic can be formulated with rules other than double negation elimination. But any such formulation will violate either the complexity condition or the restriction motivated by compositionality that there should be no circular dependence of meaning amongst the logical constants. Besides, it will remain the case that the verification of A will somehow make use of $\neg A$, hence the meaning of A depends on the meaning of $\neg A$, which in turn depends on the meaning of $\neg$ and A, so that the circular dependence of meaning has not been avoided. For the argument to go through it is sufficient that double negation elimination is assumed to be valid, no matter whether it is a derived or primitive rule of inference. Again, the fundamental assumption has not been appealed to. Compositionality suffices. So the major result of the proof-theoretic justification of deduction, that only intuitionist, but not classical logic, turns out to be justified can be established without appeal to the fundamental assumption.[1]

V. Towards a Logical Basis of Metaphysics without the Fundamental Assumption

According to Dummett, objections 3 and 4 arise when the fundamental assumption is applied to arguments in ordinary discourse for empirical propositions. The problems do not arise if applied to purely logical reasoning, i.e. proofs of theorems, because in this case it can be proved that something that looks very much like the fundamental assumption holds for suitably formulated systems: $\vdash A{*}B$ if and only if there is a proof of $A{*}B$ that ends with an application of an introduction rule for $*$. This holds, for instance, for the systems of intuitionist and classical logic in [Prawitz (1965)]. Furthermore, this proof will only use rules of inference for connectives contained in $A{*}B$, so that objection 2 cannot arise either. Of course, this is not the fundamental assumption, as it is not an assumption at all. Rather, it is a consequence of a suitably formulated logic.

It is also possible to formulate a converse of the fundamental assumption in case it is the elimination rules that specify the meanings of the logical constants. In that case, we should be interested in showing whether something is a contradiction, i.e. not in the case where something follows from the empty set, $\vdash A$, but rather in the case where the empty set follows from something, $A \vdash$, or rather $A \vdash \bot$, as this is what corresponds in a calculus of natural deduction to the $A \vdash$ of a sequent calculus. We can call such a construction a refutation of a contradiction. In a suitably formulated system of classical or intuitionist logic, it can be proved that $A \vdash \bot$ if and only if there is a proof in which begins with an application of an elimination rule to A.

Dummett thinks that it is necessary to make the fundamental assumption and apply it to arguments in general, not just proofs, as it is arguments for empirical propositions that we are normally engaged in in everyday reasoning. Dummett is concerned about the actual use speakers make of expressions, including the logical ones. Intuitionist implication and disjunction, for instance, do not accord well with the use of 'or' and 'if then' in ordinary discourse. We don't assert 'If *A* then *B*' just in case we are in possession of an argument that shows how an argument for *B* may be constructed from an argument for *A*, as reflection on, e.g., Dummett's own example 'If you enter this room, you'll die before nightfall' shows. Similarly we don't just assert '*A* or *B*' if we are in possession of a way of finding either an argument for *A* or one for *B*. To match the use of English 'or' and 'if then' with the use of intuitionist $\vee$ and $\supset$, the fundamental assumption needs to be interpreted in the way that gives rise to objections 3 and 4. It is, however, plausible that this concern with the use of 'or' and 'if then' in ordinary discourse is not well motivated. It may of course be an interesting question in how far the meanings of the logical constants of a system of natural deduction reflect the meanings of 'or', 'all', 'if-then' in natural language. But it is plausible that we do not need to take these complications into account: if the thesis is that the meanings of the logical constants are completely specified by the rules of inference governing them, then they may be given from scratch by those rules of inference. The logical constants of formal systems do not need to match up with any expressions of natural language at all. If the meanings of the logical symbols are given by their rules of inference, we do not need to look at ordinary discourse for a specification of their meaning and use: the rules are enough.

The complications with the fundamental assumption arise from taking into account arguments for empirical propositions. But metaphysics is not an empirical subject and it is not one that needs to be tied to everyday, ordinary reasoning. Of course, it would be desirable if metaphysics accords with untutored intuitions. But Dummett is a revisionist. He countenances the possibility that metaphysics will go against initial intuitions. Despite his revisionism, metaphysics should accord with untutored intuition, where there is no other basis for explaining what the terms of the metaphysician mean. For instance, if the metaphysician uses primitive terms in his theory that cannot be defined and have no analogue in ordinary language, the charge that we cannot understand him seems justified. But this problem cannot possibly arise in the case of a logical basis of metaphysics: the primitives, i.e. the logical constants, are defined entirely in terms of rules of inference. To provide a logical basis for metaphysics, it should suffice to account for purely logical reasoning: the complications arising from non-empirical reasoning are irrelevant to the project. The aim is achieved by the demand for harmony between introduction and elimination rules, normalisation of proofs and the complexity condition on rules of inference. Indeed, the metaphysical conclusion Dummett derives from his account of the justification of deduction is based on, not that certain inferences derived from some premises are invalid, but that $A \vee \neg A$ is not a theorem of intuitionist logic: anti-realism corresponds to the failure of that proposition being a theorem.

This is not to suggest that all there is to metaphysics is what we can read off the justified logic. If a system of metaphysics were formalised, it would presumably contain axioms of an extra-logical character. Whether the formal results that hold for the logic still hold for the whole system cannot be decided without actually having that system, except that any metaphysical axioms would have to be conservative over the logic. But as the proof-theoretic justification of deduction only gives the logical basis of metaphysics, the broad outline within which metaphysics is to be pursued, the latter requirement suffices to ensure the metaphysics is still anti-realist.

If we restrict consideration to purely logical reasoning, the content of the fundamental assumption reappears as something which is neither fundamental nor an assumption, but a theorem about the form of normalised proofs in systems of natural deduction. It is provable that in the case of both, classical and intuitionist logic as formulated by Prawitz, for any *theorem*, there is a *proof* of it which ends with an application of an introduction rule for its main connective. So instead of demanding that the fundamental assumption hold for any kind of argument, restricting consideration to purely logical reasoning we can observe that every theorem has a direct verification proceeding in accordance with its composition. This is a result that is formally tractable, as opposed to what Dummett originally had in mind. It is a consequence of the proof-theoretic justification of deduction rather than a precondition for it. But it is plausible that it is all we need if we are interested in a logical basis of metaphysics: for then we only need to take into account purely logical reasoning, and so all we need to take into account are *proofs* not Dummett's wider notion of an argument and its supplementation.[2]

Department of Philosophy
University of Sheffield
45 Victoria Street, Sheffield, S3 7QB, United Kingdom
E-mail: n.kurbis@sheffield.ac.uk

NOTES

[1] In [Kürbis (n.d.)] I consider possible responses to this argument on behalf of the classicist and the costs at which they come to the proof-theoretic justification of deduction.

[2] I would like to thank the two referees for **teorema**, whose constructive criticisms helped to improve this paper.

REFERENCES

DUMMETT, M. (1993), *The Logical Basis of Metaphysics*, Cambridge, MA, Harvard University Press.

— (2004), *Truth and the Past*, New York, Columbia University Press.

KÜRBIS, N. (n.d.), 'What is Wrong with Classical Negation?', unpublished manuscript.

PRAWITZ, D. (1965), *Natural Deduction*, Stockholm, Göteborg, Uppsala, Almquist and Wiksell.

— (1994), 'Review of Michael Dummett: *The Logical Basis of Metaphysics*', *Mind*, vol. 103, pp. 373-376.

— (2006), 'Meaning Approached via Proofs', *Synthese*, vol. 148, pp. 507-524.