

## HOLMMHD: A VERSATILE MAGNETOHYDRODYNAMICS CODE

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Under certain conditions, electrically conducting fluids such as astrophysical plasmas can be described with the equations of magnetohydrodynamics (MHD; Verscharen et al. 2019). In conservative form, the ideal MHD equations include the continuity equation,

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0, \quad (1)$$

the momentum equation,

$$\frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot \left[ \rho \mathbf{U} \mathbf{U} + \left( p + \frac{B^2}{8\pi} \right) \mathbf{I}_3 - \frac{\mathbf{B} \mathbf{B}}{4\pi} \right] = 0, \quad (2)$$

the energy equation,

$$\frac{\partial W}{\partial t} + \nabla \cdot \left[ (W + p) \mathbf{U} - \mathbf{U} \cdot \frac{\mathbf{B} \mathbf{B}}{4\pi} \right] = 0, \quad (3)$$

Faraday’s law,

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{U} \times \mathbf{B}), \quad (4)$$

and the source-free condition of the magnetic field,

$$\nabla \cdot \mathbf{B} = 0, \quad (5)$$

where  $\rho$  is the mass density,  $\mathbf{U}$  is the velocity,  $p$  is the scalar pressure,  $\mathbf{B}$  is the magnetic field,

$$W \equiv \frac{p}{\gamma - 1} + \frac{\rho U^2}{2} + \frac{B^2}{8\pi} \quad (6)$$

is the energy density, and  $\gamma$  is the polytropic index. This closed set of coupled equations describes the time evolution of an isotropic and polytropic magnetofluid without resistivity and viscosity. In general, the nonlinear nature of the MHD equations makes a numerical solution necessary.

We present the HolmMHD code, written in Fortran 90, which solves Equations (1) through (6) on a three-dimensional, Cartesian grid. HolmMHD closely follows the excellent tutorial by Raeder (2001), in which further details can be found. The code performs time steps through a second-order, explicit predictor-corrector scheme and reduces the time step autonomously whenever this adjustment becomes necessary according to the Courant–Friedrichs–Levy criterion. For its spatial discretization, HolmMHD uses a staggered grid (Evans & Hawley 1988), in which the electric-field vector components are located on the edges of the grid cells, the magnetic-field and flux vectors are located on the faces of the grid cells, and all other variables are located in the centers of the cells. This geometric choice guarantees that solutions fulfill Equation (5) to within numerical accuracy during the entire simulation, provided that the initial condition fulfills Equation (5) throughout the simulation domain.

For the evaluation of the divergence terms  $\nabla \cdot \Phi$  in Equations (1) through (3), where  $\Phi$  represents the mass flux, momentum flux, or energy flux, HolmMHD uses a hybrid scheme. For simplicity, we describe this scheme for a one-dimensional setup only. The flux component  $\Phi_{i+1/2}$  (and likewise  $\Phi_{i-1/2}$ ) associated with a cell variable  $X_i$  on the face of cell  $i$  is calculated as

$$\Phi_{i+1/2} = \lambda \Phi_{i+1/2}^H + (1 - \lambda) \Phi_{i+1/2}^L, \quad (7)$$

where

$$\lambda \equiv \max \left[ 0, \min \left( 1, \frac{X_i - X_{i-1}}{X_{i+1} - X_i} \right) \right] \quad (8)$$

is the min-mod flux limiter (Harten et al. 1976),

$$\Phi_{i+1/2}^H \equiv \frac{7}{12} (\Phi_{i+1/2} + \Phi_{i+3/2}) - \frac{1}{12} (\Phi_{i-1/2} + \Phi_{i+5/2}) \quad (9)$$

is the fourth-order central flux,

$$\Phi_{i+1/2}^L \equiv \frac{1}{2} (\Phi_{i+1/2} + \Phi_{i+3/2}) - \frac{1}{2} (X_{i+1} - X_i) \max(w_i, w_{i+1}) \quad (10)$$

is the Rusanov flux,

$$w_i \equiv \left\{ \frac{1}{2} \left[ c_s^2 + v_A^2 + \sqrt{(c_s^2 + v_A^2)^2 - 4c_s^2 \frac{(\mathbf{B}_i \cdot \hat{\mathbf{e}}_x)^2}{4\pi\rho_i}} \right] \right\}^{1/2} + |(\mathbf{U}_i \cdot \hat{\mathbf{e}}_x)|, \quad (11)$$

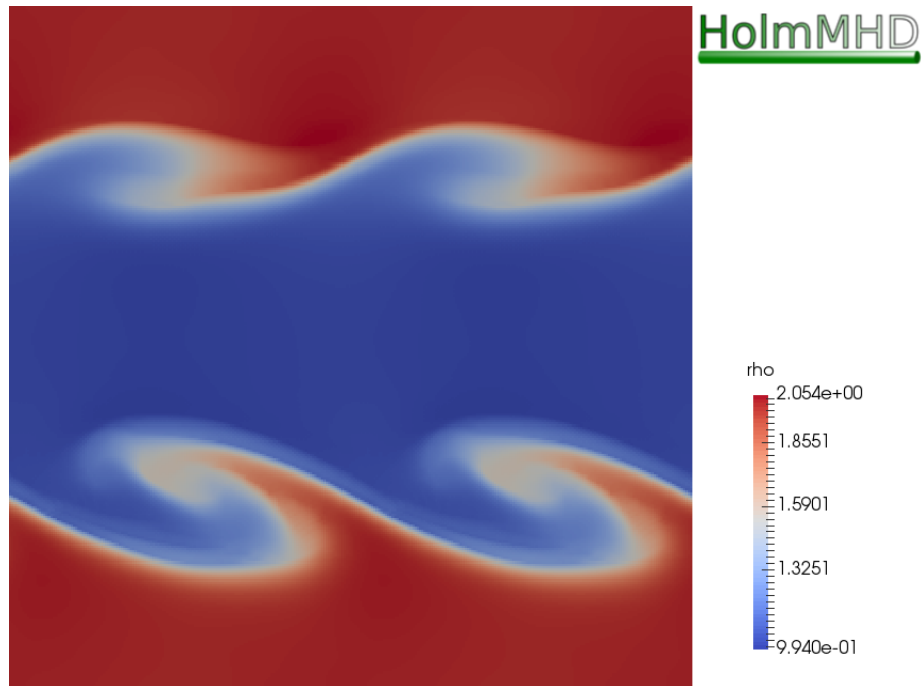
$c_s \equiv \sqrt{\gamma p_i / \rho_i}$  is the sound speed,  $v_A \equiv |\mathbf{B}_i| / \sqrt{4\pi\rho_i}$  is the Alfvén speed, and  $\hat{\mathbf{e}}_x$  is the unit vector in the direction parallel to the  $\Phi$ -component at hand (in a one-dimensional setup, this corresponds to the only direction on which all variables depend upon). We note that  $w_i$  is the maximum speed with which a fast magnetosonic wave crosses the cell  $i$  in the  $\hat{\mathbf{e}}_x$ -direction when it propagates in the  $\hat{\mathbf{e}}_x$ -direction and is convected by  $\mathbf{U}_i$ . By mixing the fourth-order central scheme with the first-order Rusanov scheme depending on the magnitude of the gradients in the solution, this numerical scheme is more diffusive in regions with large gradients while it preserves a higher accuracy in regions with smaller gradients. The divergence of the flux in cell  $i$  is then given by  $\nabla \cdot \Phi = (\Phi_{i+1/2} - \Phi_{i-1/2})/dx$ , where  $dx$  is the spatial step width. The extension of this scheme to two or three dimensions is straight-forward through the additivity of the divergence operator. The code employs periodic boundary conditions in all three dimensions.

HolmMHD applies an efficient MPI parallelization scheme that only communicates the relevant faces, edges, and corners of the simulation domains between the relevant processes. The code writes  $\rho$ ,  $\mathbf{U}$ ,  $\mathbf{B}$ ,  $p$ , and  $\nabla \cdot \mathbf{B}$  into binary HDF5 output files in parallel and creates an associated XDMF file to read the data into visualization software like ParaView. The code performs well on a set of standard benchmark tests such as a spherical hydro-blast wave, the Orszag–Tang vortex, Alfvén waves in one and three dimensions, the MHD aligned rotator, an isotropic three-dimensional decaying-turbulence setup, and the Kelvin–Helmholtz instability (see Figure 1).

The main design principles of the HolmMHD code are versatility and efficiency to make the code highly adaptable to a broad range of MHD applications. Consequently, the source code is publicly available under the permissive BSD 2-clause license for download (Verscharen 2019, Codebase: <https://github.com/danielver02/HolmMHD>).

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*Software:* HolmMHD (Verscharen 2019); doi:10.5281/zenodo.3270613; <https://github.com/danielver02/HolmMHD>



**Figure 1.** Two-dimensional Kelvin–Helmholtz instability after 500 time steps. The color-coding shows the fluid density. We resolve the simulation domain in  $300 \times 300$  spatial steps. Initially, the flow is directed toward the left-hand side in the inner, lower-density domain, and towards the right-hand side in the outer, higher-density domain. We use  $\gamma = 5/3$  and apply a weak, constant magnetic field in the direction of the flow in the outer domain.

#### REFERENCES

- Evans, C. R., & Hawley, J. F. 1988, *ApJ*, 332, 659
- Harten, A., Hyman, J. M., & Lax, P. D. 1976, *Communications in Pure Applied Mathematics*, 29, 297
- Raeder, J. 2001, *Space Plasma Simulation*, 84
- Verscharen, D. 2018, *HolmMHD*, v1.0, Zenodo, doi:10.5281/zenodo.3270613, as developed on GitHub
- Verscharen, D., Klein, K. G., & Maruca, B. A. 2019, arXiv e-prints, arXiv:1902.03448