Staircase patterns

Helen Thouless, Simon Lewis and Sue Gifford discuss young children's understanding of number patterns.

One grey day in late winter, Simon Lewis, an early years teacher, noticed that several boys in his class were in the playground and engrossed in putting the giant cable reels in a staircase growing pattern (see Figure 1). There was some consternation when they found that they could not find the fifteenth reel in order to complete the pattern. The boys knew that there should be another and hunted until they found it in the garden shed. This child-initiated activity gave Simon an ideal chance to initiate a discussion about number and patterns, during which the children discovered that, depending on where they stood, the pattern either went 1, 2, 3, 4, 5 or 5, 4, 3, 2, 1.

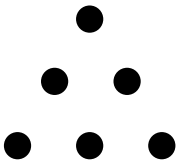
Figure 1: Growing pattern of reels.

Figure 2: Cuisenaire staircase.

This spontaneous mathematising was not an isolated incident; it was a product of the culture Simon had produced in his class. He expected children to play with numbers, to notice and produce patterns and to engage with learning that is huge and outdoors. This article is the story of how Simon set up this culture to focus on growing patterns.

The importance of patterns

Simon was part of our *Pattern Project* team, aiming to develop young children's pattern awareness following the work of Papic, Mulligan and Mitchelmore (2011) in Australia. The team consisted of Helen Thouless, Sue Gifford and a highly talented and committed group of early years teachers. We were interested in developing pattern awareness, because recent research has suggested that this improves children's mathematical achievement (Rittle-Johnson *et al*., 2017). Patterns can be defined as 'any predictable regularity, usually involving numerical, spatial or logical relationships' (Mulligan and Mitchelmore, 2009, p. 34). The most common patterns in the early years are repeating patterns and regular arrangements, such as on dice. Growing patterns, such as triangular or square number patterns, were seen as more advanced in Mulligan and Mitchelmore's programme. We had also found that a dotty triangle growing pattern was difficult for young children to copy, while few could say how it might grow by adding a row (Thouless and Gifford, 2019).



Papic *et al.* (2011) did not include growing patterns in their pre-school programme, but used them to assess pattern recognition at the end of Year 1. Similarly, we planned to introduce patterns, for instance with staircases going up in twos and fives, when we extended the project to work with 5- and 6-year-old children. However, simple staircase number images going up in ones are important to develop number understanding in the early years because they link cardinal and ordinal numbers. Young children do not readily recognise the 'one more than' relationship between counting numbers: connecting the cardinal value of each stack with the 'going up in ones' relationship helps children make this major leap in understanding that each consecutive number includes the previous number plus one (Gifford, 2014). This is the principle of hierarchical

Figure 3: Reflective staircases.

Figure 4: Numbers beyond ten.

inclusion, which is involved in all growing patterns. Understanding this requires part-whole recognition, or simultaneously seeing the whole number and its parts, which has been identified as a major milestone in young children's number knowledge. For these reasons, we considered growing patterns best left until the start of formal schooling and not appropriate for the pre-school setting.

How did Simon introduce staircase patterns?

In December Sue and Helen observed Simon working with a small group of children, three boys and a girl, at the horseshoe table. This is something that he did twice a week with all the children and was an opportunity for him to investigate what his children knew and to observe what they did when given a challenge. This vignette is an example of the tasks he gave children to do, although, of course, the types of tasks varied with the children he had in front of him and as the year progressed.

Simon put out five Cuisenaire rods assigning them the values one to five and asked, "How many do I have?" Andy responded, "This one, this one, this one, this one, this one" as he pointed to each rod, which showed good use of tagging but not such good use of the counting sequence. So, Simon tried again, "How many is that?" and Hassan immediately responded, "5" but could not explain how he knew it. These responses made Simon realise that this group could use some practice counting objects and so the whole group counted the blocks along with Simon.

Then Simon told the children that he was going to put the blocks in order from longest to shortest, but actually he put them in a staircase that went 5, 4, 3, 1, 2. This was very surprising to Hassan and Lydia, and Hassan said, "Three doesn't come after one." Simon asked, "Can you fix it?" and Hassan then repaired the error. Simon has found that making deliberate mistakes in patterns is a fun way to engage children and to assess their understanding of patterns. Once the children had seen Simon's correct sequence of Cuisenaire rods, they built their own (see Figure 2). Lydia originally ordered the rods 5, 4, 2, 1, 3 but corrected it only when she saw Simon's model, not when she looked at the other children's correct models.

Simon then gave each group member the numerals 1-5, and they matched the rods to the numeral and put the numbers in order. Kieran matched the numbers and rods in order very neatly. Hassan first put the rods in order and then matched them to the numerals and Lydia matched the rods to the numerals but did not put them in order, whereas Andy looked at the numerals 2 and 5 and initially thought they were the same. He then put the numerals in order, although he put the number four rod on the numeral 5 and vice versa. It was then that Kieran noticed that Simon had put down a 3 instead of a 4 and found the correct numeral. Again, Simon used an intentional error to engage his children's attention and to make sure that they looked at and compared their work with that of other participants.

This was as many activities as Simon could include when working with one small group on one day, and the children went off to participate in free play while Simon worked with another group. But Simon continued this practice of working with small groups of children throughout the following term and noticed some interesting developments that he and the children were able to capitalise on.

Soon the children were no longer satisfied with building rows of Cuisenaire towers just to five. One girl was interested in finding rod that came after five. Once she found this rod, another girl said, “We will have to ask Simon to give us the number 6”. The first girl responded, “I know where there is a 6” and she went and got the numeral 6 off another display. She finished this session by matching the six rod with six one rods. In this episode, the children were interested in exploring the numbers beyond five, and achieved this by finding the six rod, ascertaining that six ones do fit on one six rod and matching this rod with a numeral. Simon had given the children the space to explore the mathematics of the Cuisenaire staircase and the children had the confidence to go beyond his expectations. The children continued to exceed Simon’s expectations and to build on each other’s ideas. When Adam put all the Cuisenaire rods in a staircase to ten, the other children were fascinated and copied his staircase to ten. When one child made their staircase backwards, they put it together with Adam’s so that the staircases ascended and descended. They then put all four staircases together to be symmetrical both vertically and horizontally (see Figure 3).

Later Adam was curious about what came after ten. He worked out that the next numbers would be ten and one, then ten and two, and then ten and three. For the following number he put together ten and six but adjusted the length of the six rod to continue the staircase shape, although that made bottom of the pattern no longer level (see Figure 4). Adam was beginning to figure out the structure of the teen numbers, although he did not yet have the vocabulary to name these numbers.

Soon the children were exploring staircase patterns with different resources. They were enthusiastic watchers of the *Numberblocks* series (https://www.bbc.co.uk/cbeebies/shows/numberblocks) and when they watched the episode for number 15, which included the *Step Squad* (see Figure 5), the class became very enthusiastic about making the *Step Squad* with a variety of materials.

Simon designed an activity where the children could stick blocks on a piece of paper to show the pattern. The children were very engaged by this activity, producing a variety of responses. Some children stuck the blocks onto the paper randomly, some could follow the growing pattern if given a template, and some could put the numbers in order but not match this to a visual growing pattern. Some children drew the *Numberblocks* characters, which showed that they realised that they could draw numbers in different ways, for example as a line or square. Marta made the staircase pattern as far as four and then made the dice pattern for five, and Adam made a pattern of number bonds to five. We realised that the range of responses showed that children were paying attention to different aspects of the pattern, such as numbers and shapes. These responses were similar to those we found when children were copying the triangle six pattern (Thouless and Gifford, 2019).

There were other resources that the children also explored. They made the *Step Squad* with blocks, bricks, *Numicon* and the aforementioned reels. At this stage of the year, not all children could accurately make the staircase to five (see Figure 6), but they readily engaged with the staircase shape in building a sequence of towers. Some children used the idea of the *Step Squad* to explore other areas of mathematics. Once Yves had built himself a *Step Squad* of bricks placed long ways up, he was curious about whether he was taller than the five bricks (see Figure 7). This deeper exploration of the *Step Squad*

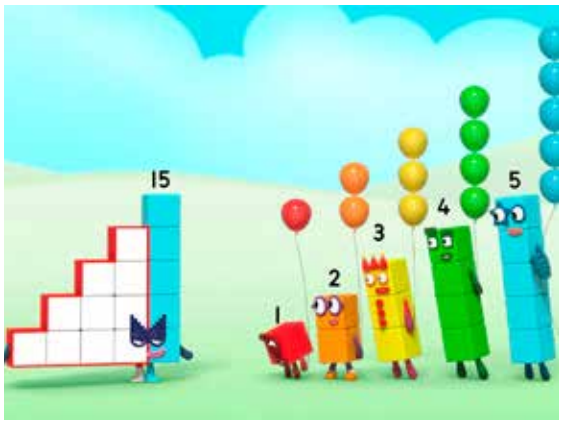


Figure 5: Number Blocks Step Squad.


Figure 6: Step squad with blocks.


Figure 7: How tall am I?

had occurred because one boy was enthusiastic about the image and the idea of investigating ways to make fifteen. Simon provided him with the materials and time to explore these ideas further and then the other children became enthusiastic to also explore these ideas and materials.

Implications

We have noted young children's enthusiasm for staircase arrangements in many settings and many adults share the same appreciation of ordered Cuisenaire rods. It seems that the intrinsic appeal of these regular arrangements can provide children with access to number patterns and relationships and encourage them to identify, recreate and even extend these. The giant staircase pattern with cable reels outdoors stimulated a mathematics talk session, with children standing on different sides of it noticing different things, such as the ascending and descending number sequences. Previously teachers in our team had led mathematics talks sessions by showing images on whiteboards. We realise that having a pattern big enough to walk round, apart from the 'wow' factor, provides more opportunities to notice different features. Other possibilities for outdoors maths talks could include vertical staircases with crates, triangular patterns with giant skittles, or arrays with tyres. Rich patterns like these afford both spatial and numerical regularities to remark upon.

The strategy that Simon used with the group was found to be effective by the project teachers. They would briefly present a problem to solve, such as a pattern with a deliberate mistake, with the challenge, "Can you fix it?" and then leave the materials out for children to copy and develop the pattern as they chose, or chose not to. The teachers were surprised by the number of children who carried on with these activities independently, often collaboratively. This allowed them to observe children's patterning and to listen to children explaining and correcting each other. And, of course, some children delighted in adopting the teacher role, providing challenges for others to extend a pattern or deliberate mistakes to spot. Teachers remarked on the way in which pattern work also fostered children's social and language development.

While the children learned from each other, they also paid close attention to the teacher's patterns and challenges. The role of the skilful teacher with the light touch is crucial here. In early years education, adult-led maths-focused activities are sometimes seen as pressurising children and depriving them of free play. What these examples show is how much a brief playful and challenging teaching episode can enhance children's pattern play.

We were struck by the pedagogical creativity of the teachers in the project group, who developed activities and stimulated children to create and investigate patterns way beyond expectations. In the process of teaching, the teachers would analyse children's difficulties and devise ways to overcome them, often using materials they had to hand. They devised accessible challenges, like copying a pattern, which could be extended to finding several ways to fix an error or generalise a pattern. In discussing this process with Simon, he added that sometimes he observed a child doing something and thought, "The others could do that too!" His planning was based, therefore, on his observations of children, in the best early years pedagogical tradition.

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