Modelling motion from nature

Michael Rumbelow and Melissa Rodd explore how technology can exploit moving images for learning and teaching mathematics.

Introduction

A high-quality mathematics education … provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics, and a sense of enjoyment and curiosity about the subject. (Department for Education, 2014)

The ATM Science of Education (SoE) Working Group, *Using film in the teaching and learning of mathematics* was set up in 2018. Our SoE meetings inspire new ways of working in mathematics classrooms. In particular, we, Michael and Melissa, were keen to explore how contemporary technologies might afford using moving images or film in mathematics education contexts. Inspiration for investigation, in this case with Melissa's master's students came, like many ideas, from a technical hitch. Melissa, walking on a windy beach in north-west Scotland, entranced by the motion of the waves, filmed the scene on her phone. On playback, the phone had somehow rotated the frame by ninety degrees to a portrait orientation (see Figure 1). This change in orientation triggered, in Melissa's eye, the re-seeing of the waves as a set of parabolas which moved across the screen. And the movement of these parabolic waves was curious, provoking questions such as, "Aren't sines and cosines supposed to model waves?", as well as being aesthetically pleasing. This experience, which provoked mathematical curiosity and aesthetic pleasure as well as using filmic technologies seemed, to Melissa, to be a useful starting point for investigating film in mathematics education contexts. Subsequently, a session with mathematics education master's students was planned and addressed themes that included 'the appreciation of the beauty', mentioned in the national curriculum quotation that opened this article; mathematical modelling; digital technologies and drew attention to the links between mathematics and our sense of wonder at the natural world. Michael and Melissa were co-facilitators in the session, which was held during the summer term 2019. The rest of this article outlines some of the preparation for, and reports on the practice of, the teaching session.

Firstly, do the math

Being intrigued by the portrait movie of the waves on the beach, Melissa started to model this natural phenomenon, captured by the video on her mobile, using *Geogebra*. Starting with a single snapshot from the video copied and pasted into the background of *Geogebra,* a family of parabolas, defined by a parameter and controlled by a slider, was superimposed and adjusted until one member of the family of parabolas almost perfectly matched the shape of the wave (see Figure 1).

Figure 1: A parabola superimposed on a wave.

Secondly, tame the tools

The next challenge was to animate the curve to follow the wave as it moved in the film. The tool for this was to be the *iMovie* video-editor. However, you cannot import a video into *Geogebra* (version 5) like you can import a photo. This was a major technical issue and was addressed in two stages. Firstly, knowing that *Quicktime* enabled screen-recording, it was possible to create an *iMovie* of a *Geogebra* animation. This was done by sliding a parameter to animate the family of parabolas to match the waves' movement while simultaneously filming the screen using *Quicktime*. Secondly, this video of the moving *Geogebra* curve was mixed with the video of the waves within a video editor. This was done by mixing the *Quicktime* screen-recording of the animated parabola with the waves video taken on the beach in *iMovie* to effectively superimpose moving parabolas on top of the water waves. Now there was a film of a parabola, sweeping across the video clip, following, almost exactly, the curve of one of the travelling waves. Though fiddly, in principle this constituted an *ad hoc* method for modelling different waves or other natural movements using a parameterised family of curves.

Thirdly, discuss with colleagues

The SoE working group had already been viewing films made by Caleb Gattegno and Jean Nicolet in the 1960s for use in the classroom, of animated-by-hand moving conic sections and lines, similar to the graphs that can now be quickly created and animated using dynamic geometry software. The idea of learners creating their own films of movement in nature, then using dynamic geometry software to make animated models of these curves, which could then be edited together, seemed to combine several of the SoE group's interests. The modelling of nature video described above was presented at the working group and the subsequent group's discussion touched on issues such as:

- What are roles for aesthetics and 'appreciation of beauty' in learning mathematics? For example, are aesthetic choices in making films linked with aesthetic attraction to possible solutions to mathematical problems?
- How can film be used to focus and shift attention in classrooms? For instance, from a movement in a film of nature to a mathematical model of it.
- Can the practical and technical challenges of producing a film be beneficial, pedagogically, to learning mathematics? For example, how can technologies be managed in the classroom and how can we see the frustration of technical hitches as a stimulus rather than a blocker to creativity? Can making films heighten learners' sense of agency or creativity in learning mathematics?

In practice

Melissa teaches a course designed to develop practising mathematics teachers' mathematical problem-solving skills and to provide a safe space for course participants to take risks in their choice of mathematical challenges they take. The course includes two Saturdays where experimental attitudes are encouraged. On one of the Saturdays, a one and a half hour session was earmarked for playing with digital tools, motion and images from nature. As well as legitimising risk in teaching, the session offered opportunities for participants to engage in genuine mathematical modelling with digital tools. Michael and Melissa planned and taught this session collaboratively.

The students and the plan

The group was made up of around 25 adult students, in a university classroom. Many of the participants were full-time international students with limited teaching experience and most of the rest were teachers who worked in the London area and were studying for a master's degree part-time. The session was structured around five key phases:

1. Teacher-led whole-group discussion of the pre-task of overlaying a graph on a static image in *Geogebra*, linking it with the plan to integrate animated graphs onto moving video, and demonstration of an animated parabola superimposed onto a moving wave.
2. Small-group sharing of students' videoclips and discussion of movement noticed in the clips; individual work creating an animated graph in *Geogebra* to overlay on the videoclip; a demo and individual experimentation with creating 3D models in the *Geogebra 3D* app to overlay as augmented reality 'live' on phone-shot video.
3. Individual work integrating *Geogebra* animated graphs with the videoclip.
4. A demonstration and individual experimentation with creating 3D models in the *Geogebra* 3D app to overlay as augmented reality live on phone-shot video.

5. Individual work integrating *Geogebra* animated graphs with the video-clip.

In order to familiarise the students with working with images within dynamic geometry software, Melissa worked on an activity with them before the Saturday session, which involved sliders in *Geogebra*. This involved pasting an image into *Geogebra* (one of the leaning tower of Pisa was used) then modelling the 'leaning' by sliding a and b in $y = ax + b$ so that the straight line matches the leaning tower's sloping side. As preparation for the video session, the students were asked to film or source a short clip of movement in nature in which they could discern some movement that they would like to model mathematically.

Discussion

We base our discussion around the three bullet points above.

Aesthetics

> Mathematics have a triple aim. They must furnish an instrument for the study of nature. But that is not all: they have a philosophic aim and, I dare maintain, an aesthetic aim. (Poincaré, 1905)

Making or finding a good clip involves expressing your own aesthetic taste. Superimposing graphical models onto it similarly involves aesthetic choices, of line colour and thickness, but also of standards of accuracy and how smoothly it fits the movement. Realising the vision one has in one's head of how it should look requires mathematical modelling and problem-solving activity as well as the use of tools, tamed or otherwise. Hence aesthetics as motivator stimulated mathematical modelling with digital tools. The feedback loops in this modelling were governed by both the participants' mathematical and their aesthetic considerations.

What was observable in the room was strong engagement with the personal project of getting the mathematical model of the movement in nature both accurate and pleasing to view. Satisfying both these criteria was important. The freedom to pursue individual tinkering or small group sharing worked for the vast majority. A few participants, who were less mathematically trained, would have probably found more satisfaction working on linear static models, like the leaning tower, as they seemed rather over-faced by trying to model the dynamic-with-curves-on clips they brought in. In this classroom experiment the interface between the beauty of nature and the beauty of mathematics was brought together and one informed the other in multiple ways that we will need to work on further to adequately describe the process.

Film

Film is a medium that enables the capturing and sustaining of attention because of the entrancement of motion and the surprise afforded by change of movement. We are interested in students' attention being shifted from watching to producing a mathematical model of the motion discerned. It seemed important that the participating students brought their own videos both as personal investment and to share what they liked.

From an educator's point of view, one can ask, "Can making films heighten learners' sense of agency or creativity in learning mathematics?" The result of a short film appears very different from traditional mathematics solutions; symbols and text may be

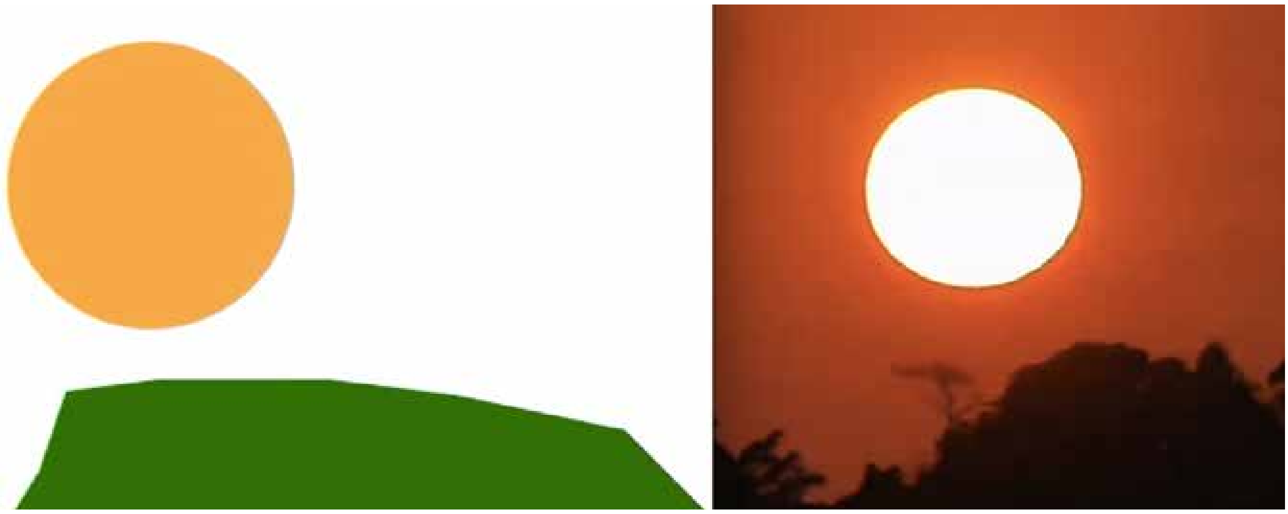

Figure 3: Screen shot from a student's video of the sun rising (left) compared with the internet-sourced video the student used as a model; in the student's video, the 'sun' rises at the same speed as the 'real sun'.

absent, for example, and the only judgement of correctness is a subjective, aesthetic one. However, as Caleb Gattegno (2011, p 3) pointed out, the urge to express oneself can be even stronger than the urge to communicate. And the mathematics learned in order to do so is potentially just as sophisticated as that required for an exam.

Participants in the master's session were invited to send in their results and several did; an example is shown in Figure 3.

Technologies

> Technology is stuff that doesn't work yet. (Adams, 1999)
>
> In both primary and secondary schools, teachers should use their judgement about when ICT tools should be used. (Department for Education, 2014)

What we observed in the session was that some technologies had become invisible, like using video on one's phone or sourcing clips from the internet, but editing and mixing video was indeed still technological for the majority. So, technology is analogous to learning. We only notice it and call it learning when we are struggling with it. And, when new technology frustrates us, as video-mixing tools did for the majority in our group, unless there is a motivation to persevere we are likely to discard it. In the case of this teaching session, there was sufficient energy to stimulate conversations and creative thinking so that the frustration and challenge of technical hitches was, in many instances, a stimulus rather than a blocker to creativity. Indeed, the technical hitches were part of the problem solving activity.

What advice would we give to someone considering teaching with 'stuff that doesn't work yet?' In this case, we offered the students high levels of agency in developing mathematical modelling, using digital tools and representations of families of curves sourced from nature. When working with school children or college students, a teacher has to assess how to cope with the sheer plethora of devices and versions of software their students might use. And, while technical problems will occur and can be a source for creativity, too many such problems overwhelm teacher and students. From Melissa's experience, there are some classes of 15-16-year-old students where a similar lesson could have worked, but not all such classes. Some of the teacher participants said that they intended to use moving images in their subsequent classroom teaching.

On reflection

The experience of the session contrasted the simplicity and power of the aesthetic pleasure of creative film-making, particularly of the natural world, as a motivation for learning mathematics, with the complexity of mastering new technology. And while technology that 'doesn't work yet' can be frustrating, there is also an aesthetic pleasure, a delight, in discovering a tool that makes it possible to create pleasing things. The activity did suggest that creating aesthetically pleasing films of movement in nature, integrated with dynamic graphical mathematical models, may offer a way of expressing one's 'appreciation of the beauty' in both nature and mathematics.

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For more information about the Science of Education working group go to https://www.atm.org.uk/News/atm-science-of-education-working-group-using-film-in-the-teaching-and-learning-of-mathematics/201458.

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