The Forced Safety Effect:

How Higher Capital Requirements Can

Increase Bank Lending

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Abstract

Government guarantees generate an implicit subsidy for banks. A capital require-
ment reduces this subsidy, through a simple liability composition effect. However, the
guarantees also make a bank undervalue loans that generates surplus in the states of the
world where it defaults. Raising the capital requirement makes the bank safer, which
alleviates this problem. We dub this mechanism, which we argue is empirically relevant,
the *forced safety effect.*

I.   Introduction

Since the global financial crisis, bank capital requirements have been substantially tightened.[1] The merits of these reforms have been fiercely debated. Higher capital requirements, the typical refrain goes, raise banks' costs of funds, thereby reducing credit provision and dampening economic activity.[2] However, increases in banks' private costs of funds are not necessarily relevant from a normative perspective (see, e.g., Hanson et al. (2011) and Admati et al. (2013)). Nonetheless, the idea that such increases result in less lending has seeped into conventional wisdom.

In this paper, we challenge such conventional wisdom. We develop a model in which capital is costly from a bank's perspective due to an implicit subsidy from a government guarantee. At a given level of lending, a higher capital requirement reduces the value of the subsidy, and hence, it increases the bank's weighted *average* cost of funds. But it also makes the bank safer; this can actually make the marginal loan more appealing and therefore induce an increase in lending.

How can the marginal loan become more appealing with a higher capital requirement?

To build intuition, it is useful to explain why the marginal loan may not have been financed in the first place. Despite implying a subsidy, a government guarantee can generate a mechanism analogous to the debt overhang problem in Myers (1977): the bank undervalues a loan (and potentially passes on it), if a portion of its surplus, in effect, accrues to the taxpayer, who is backing the guarantee. We dub this the *guarantee overhang* problem.

Making the bank safer means that loan surplus accrues to the bank shareholders in more

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[1]Specifically, minimum tier-one capital requirements were raised from 4 to 6% of risk-weighted assets, but additional extra "buffers" were created to adjust, inter alia, for the systemic importance of the institution, the economic cycle, and to prevent accidental breaches of the minimum. Effective requirements for large global banks are now in the double digits as a percentage of risk-weighted assets.

[2]See, for instance, Institute of International Finance (2011) on page 10.

states of the world. If, *for the marginal loan*, this surplus is positive *in these specific states*, forcing the bank to be safer makes the marginal loan more appealing as it alleviates the guarantee overhang.

Our model has a single period in which a representative bank faces a capital requirement and finances loans with a mix of liabilities that can be interpreted as deposits and capital. The bank starts with existing loans and can make new ones.

The bank maximises the expected payoff of initial shareholders. Deposits are insured by the government with no fee; hence, they are implicitly subsidised. This has two implications. First, the capital requirement is binding in equilibrium: the bank chooses lending and adjusts capital to meet the requirement. Second, the objective function can be written as the sum of the economic surplus from lending and a term that captures the value of the implicit subsidy (as in Merton (1977)).

We first assume that the payoffs to *new* loans are perfectly correlated and that new lending yields an increasing and strictly concave aggregate payoff. This allows us to study how the equilibrium level of lending responds to marginal changes in the capital requirement using a first order approach (we refer to the response to an increase as the *lending response*). The derivative of the subsidy with respect to lending, the marginal subsidy, is a wedge in the bank's first order condition. This wedge captures the underlying moral hazard problem arising from the guarantee. Economic surplus is independent of the capital requirement. Hence, if an increase in the requirement increases the marginal subsidy, the bank increases lending.

Increasing the capital requirement has two effects on the marginal subsidy. First, a smaller fraction of the marginal loan is financed by deposits. This generates a well understood composition effect: the bank substitutes subsidised deposits with capital, decreasing the marginal subsidy. This effect also captures exactly how the capital requirement raises the

3

bank's weighted average cost of funds.[3]

However, the change in the capital requirement also affects whether the bank defaults or not in any given state. To go further, we note that the appropriate measure of surplus from the marginal loan is its residual cash flow. This variable, which we denote Z, is the marginal loan's realised payoff minus the repayment on the deposits raised to finance it. In the states where the bank survives, Z comes as an addition to the shareholder's payoff. But if the bank defaults, Z accrues to the taxpayer.

We can now elaborate on the second effect, which we argue is overlooked by conventional wisdom. Consider the default boundary – that is, the set of states where the bank can just repay depositors. Increasing the requirement increases the buffer against losses and shifts this boundary. There are now more states where Z accrues to shareholders. In particular, increasing the capital requirement makes the shareholders internalise the expected value of Z along the default boundary.

This second effect captures that the requirement forces the bank towards safety. Because the bank could have chosen to be safer and to internalise these cashflows (by operating at a higher capital ratio than the requirement), but preferred not to, we dub the second effect the *forced safety effect* (FSE).

If, in expectation, the residual cashflows along the default boundary are positive, the bank is internalising cashflows that increase the shareholders' payoff. In this case, the FSE is positive and increases the value of the marginal subsidy. If the residual cashflows are negative, the FSE makes the bank internalise more losses, decreasing the value of the marginal subsidy and reinforcing the composition effect.

Our main theoretical contribution is to show that: (i) the FSE can be positive and (ii) the

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[3]The change in average funding costs is relevant in determining the impact on the bank's profit. See Kisin and Manela (2016) for a quantification.

FSE can dominate the composition effect, which is why lending can increase with the capital requirement.

In equilibrium, the bank may optimally choose to finance negative NPV loans and/or not to finance positive NPV loans. Pointing out that the guarantee overhang can lead to the latter is also a contribution of this paper. Reasoning in terms of residual cashflows also helps clarify the link between the guarantee and debt overhangs. At the the heart of both is that a portion of the residual cash flows from investment accrue to another stakeholder.[4] In Myers (1977), residual cash flows can only be positive.[5] In our context, residual cash flows can be positive or negative. This is why government guarantees can lead a bank to (i) under-value positive NPV loans and/or (ii) over-value negative NPV ones. The latter is typically interpreted in terms of risk shifting (Kareken and Wallace (1978)). We argue that the former has a similar interpretation: the bank under-values the loans precisely because some of the surplus they generate only reduces the risk shifted onto the taxpayer.

We calibrate the model and find an economically significant, positive FSE in plausible conditions: targeting the situation facing a global bank in 2017, we find that it fully offsets the negative forces (the lending response is slightly positive). However, at levels of capital requirements prevailing before the global financial crisis, lending responses are more likely negative.

Overall, our sensitivity analysis reveals that lending responses are likely to exhibit substantial variation. Hence, one should not expect a homogeneous relationship between the capital requirement and bank lending. We discuss the empirical predictions that arise from

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[4]Papers that link bank underlending to the debt overhang problem include Hanson et al. (2011), Admati et al. (2018), and Jakucionyte and van Wijnbergen (2018). Bank behaviour exhibiting symptoms of an overhang problem has been noted in different contexts in the recent literature. See for instance, Gropp et al. (2019) for evidence from stress tests, or the work in Duffie et al. (2019) showing that funding-value adjustments correspond to the transfer to existing debtholders associated with debt overhang.

[5]Investment is fully financed with equity, hence the residual cash flow is the cash flow itself.

our analysis in Section V and argue that our findings help reconcile results in the empirical literature.

The key ingredient for a positive FSE is a form of residual cashflow heterogeneity. It requires that, in equilibrium and in expectation along the default boundary, the residual cash flow of the marginal loan (i.e., Z) must be greater than the residual cash flow of the average asset on the bank's balance sheet (which is exactly 0 on the default boundary). For the marginal subsidy to be positive (so the bank passes on positive NPV loans), the same is required, except that the expectation is conditional on the whole default region, and not just the default boundary. This subtle difference implies that the FSE and the marginal subsidy can have different signs. Yet, they both rely on residual cash flow heterogeneity that, we argue, is missing from many models where a negative lending response always arises.

The relevant residual cashflow heterogeneity can come from imperfect correlation between legacy and new loans. It can also come from other sources such as heterogeneity among new loans, differences in capital requirements, or background risk. We consider examples that depart from the first order approach to show that a positive FSE (and/or a positive marginal subsidy) also can arise: (i) when *all* loans are perfectly correlated; or (ii) when the bank starts from scratch (i.e has no legacy loans or debt).

The idea that tighter capital requirements raise banks' average costs of funds and prompt a credit contraction has been formalised by Thakor (1996), among others. As Suarez (2010) discusses, a usual way to capture such an effect is to assume an exogenous cost of issuing outside equity. This effectively makes the capital requirement a tax on lending. In the extreme case where aggregate bank capital is in fixed supply, a higher capital requirement can only shrink banks' balance sheets. We do not restrict equity issuance in our model.

Many models linking capital requirements and bank lending assume that loans are in-

finitesimal, face the same capital requirement, and, conditional on an aggregate state, have independent and identically distributed payoffs. This payoff structure is convenient in that it enables aggregation to a representative bank. However, it implies that idiosyncratic risk is automatically diversified: the default region and the default boundary are pinned down by the aggregate state. Hence, residual cash flow on all loans are identically distributed over the default boundary. Since, by definition, the default boundary is the locus where the average residual cash flow is nil, it must be the case that it is nil in expectation for any given loan, including the marginal one. Therefore, the FSE can only be nil. Hence, such a specific structure assumes away the relevant heterogeneity.[6]

In Repullo and Suarez (2004) and Martinez-Miera and Suarez (2014), there are two types of conditionally *iid* loans. However, banks fully specialise in equilibrium. So, endogenously, all the loans on a given bank's balance sheet are identically distributed, meaning that the FSE is again nil. Full specialisation is also the key feature that implies a nil FSE in Rochet (1992) and Harris et al. (2017). In practice, however, it is impossible for banks to fully specialise, especially if they wish to be large.

To preserve tractability in our first order approach, we also impose some structure in the payoff of new loans (i.e., perfect correlation and diminishing returns). In isolation, a portfolio of such new loans can only exhibit a negative FSE. But imperfectly correlated legacy loans introduce sufficient heterogeneity to make a positive lending response possible.[7]

The main takeaway from our paper for the policy debate is that, whether the FSE domin-

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[6]Models following such an approach include Repullo and Suarez (2013), Corbae and D'Erasmo (2017), Elenev et al. (2017), Malherbe (2019), and Malherbe and McMahon (2019).

[7]Begenau (2018) potentially has the required heterogeneity for a positive FSE; however, in the spirit of Thakor (1996), she proxies the implicit subsidy with a reduced form function of which the capital requirement is not an argument. The cross-partial is nil. Nonetheless, she finds that a positive aggregate lending response can arise due to a general equilibrium effect: higher capital requirements can *decrease* banks' funding costs because they reduce the aggregate supply of deposits (which carry an endogenous convenience yield).

ates the composition effect or not, there are many situations in which it will make the lending response substantially less negative than otherwise. Overlooking this effect is tantamount to confusing how average, rather than marginal, costs of funds are affected by changes in capital requirements.

II. The environment

We present here the environment we will use for most of our analysis. Section IV will contain slight deviations from this, which we will specify in due course.

A. The baseline model

There are two dates, 1 and 2. There is a bank, a continuum of households who own the bank's liabilities, and a government. Households are risk neutral and do not discount the future; they supply funds perfectly elastically with an opportunity cost of funding of 1. We focus on the date 1 decision of the bank. The random variables A and B capture the realised state of the economy at date 2. They are distributed according to a joint function $f(A, B)$ with support $[a_L, a_H] \times [b_L, b_H]$, where $a_L, b_L \geq 0$ and with $\mathbb{E}[A] = \mathbb{E}[B] = 1$. Figure 1 summarises the bank's balance sheet.

Predetermined variables. As of date 1, there are legacy loans on the bank's balance sheet. Their total book value is $\lambda \geq 0$, and they generate a risky date 2 payoff $A\lambda$. Without loss of generality, the bank holds no cash. The bank has existing deposits that can be withdrawn at par at date 1. Given deposits are supplied perfectly elastically, it is only necessary to consider their end-of-date-1 level. Hence, we do not define notation for existing deposits. The book value of capital at the beginning of date 1 is denoted by $\kappa \geq 0$.

Figure 1: The bank's balance sheet

Assets	Liabilities
(new loans) x	$\kappa + c = \gamma_b x + \gamma_a \lambda$ (capital)
(legacy loans) λ	$d = (1 - \gamma_b)x + (1 - \gamma_a)\lambda$ (deposits)

Notes: The parameters γ_a and γ_b denote the capital requirement on legacy and new loans respectively, κ is existing capital and c is net issuance (which can be negative). The capital requirement is always binding in equilibrium; see Section B.

Decision variables. The bank decides how much to lend. We denote the total amount of new lending by $x \geq 0$. New loans also mature at date 2 and yield a stochastic payoff. Our baseline assumption is that new loans have a continuous payoff function $BX(x)$, which is increasing, strictly concave in x, twice differentiable in the strictly positive domain, with $X(0) = 0$, and $\lim_{x \to 0} X_x(x) = \infty$. In Section IV, we also consider linear and discrete lending opportunities.

At the same time, the bank adjusts its liabilities: capital and deposits. We denote the change in capital c. The change in capital can be negative (as long as $\kappa + c > 0$). In this case, the change can be interpreted as a dividend payment or the value of a share repurchase. If c is positive, it should be interpreted as the bank raising more capital. In this case, an amount c is raised in exchange for date 2 cashflow rights. The corresponding total repayment is denoted C. This repayment is determined in equilibrium and can be contingent on any realised variable. The bank's chosen level of deposits is denoted by d.

Even though they all refer to households, we use different terms for holders of different bank-issued liabilities. Initial shareholders own the initial (i.e. inside) equity, investors hold new capital, and depositors hold deposits.

Deposit insurance, taxes and the capital requirement. The government insures bank deposits with no premium: in the event the bank has insufficient cashflows to repay depositors in period 2, the government makes depositors whole, and breaks even via an ex-post lump-

9

sum tax on households. This is the source of moral hazard in the model. Given this, deposits pay no interest. If the bank defaults on deposits, no payment to any other liability is allowed.

The bank faces a capital requirement constraint that takes the form:

$$\kappa + c \geq (\gamma_a \lambda + \gamma_b x), \tag{1}$$

where $\gamma_a, \gamma_b \in (0, 1)$ are parameters that we refer to as the requirements on legacy and new loans respectively. Both parameters are set by the government. To be allowed to operate at date 1, the bank must have a book value of capital $(\kappa + c)$ sufficient to satisfy the capital requirement. Otherwise, the government shuts down the bank, initial shareholders walk away with 0, and we impose $x = c = 0$. The assumption $\mathbb{E}[A] = 1$ ensures that, in equilibrium, it is never profitable for the bank to shut down.[8]

In real-world regulation, there is typically no distinction between legacy and new loans (as new loans become a legacy immediately after being made). Hence, for most of what follows, we assume that $\gamma_a = \gamma_b = \gamma$, and refer to γ as *the requirement.* Still, the case of multiple requirements, which we study separately below, allows us to clarify some concepts. Our main result is that equilibrium lending may increase with γ; and this also holds for γ_a or γ_b separately.

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[8]We study date 1 bank closure in Appendix A.

B. Setting up the analysis

Date 2 default on deposits. If date 2 cashflows are too low to repay the depositors, the bank defaults on them. This happens when

$$d \;>\; BX + A\lambda$$

$$\underbrace{\text{promised repayment}} \qquad \underbrace{\text{total cash flow}}$$

We can then define two functions, either of which can be used to define the set of states where the bank is on the brink of default:
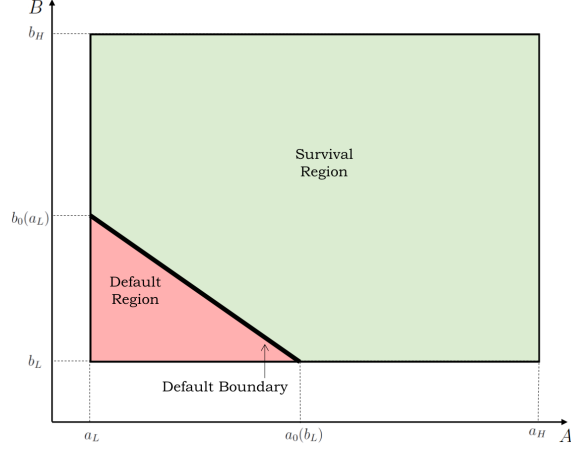
$$a_0(B) \equiv \frac{((1-\gamma_b)x + (1-\gamma_a)\lambda) - BX}{\lambda}; \; b_0(A) \equiv \frac{((1-\gamma_b)x + (1-\gamma_a)\lambda) - A\lambda}{X} \qquad (2)$$

We refer to these states as the *default boundary.* Figure 2 depicts the default boundary (the thick black line) and the whole default region (the red triangle) as subsets of the state space $\{[a_L, a_H] \times [b_L, b_H]\}$. Define the set Δ as all pairs $\{A, B\}$ in the (triangular) default region, the set $\backslash\!\backslash$ as all pairs along the default boundary, and the operators $\mathbb{E}\left[. \mid \backslash\!\backslash\right]$ and $\mathbb{E}\left[. \mid \Delta\right]$ as conditional expectations taken over these sets. The probability that the bank does not default is given by $p = \int \int_{\{A,B\} \notin \Delta} f(A, B) dA dB$.

Pricing of new capital. Investors act competitively, so that, in equilibrium, they just break even in expectation. For the moment, assume that the bank issues new capital (i.e., $c \geq 0$). Denoting $C(A, B)$ the contingent, date 2 repayment to new capital, we then have:

$$\int_{b_L}^{b_H} \int_{a_L}^{a_H} C(A, B) f(A, B) dA dB = c. \qquad (3)$$

Figure 2: Default region and boundary

To be able to interpret c as capital issuance, the underlying securities should be junior to deposits. Hence, we impose

$$C(A, B) \leq 0, \ \forall \{A, B\} \in \Delta. \tag{4}$$

We also impose limited liability for investors, which implies $C(A, B) \geq 0$, and for initial shareholders, which implies $C(A, B) \leq BX + A\lambda - d$. However, we do not restrict new capital to being a particular form of security. What matters is that capital will absorb losses before the guarantee is called. In practice, one can, for instance, think of it as seasoned equity or subordinated debt.

Initial shareholders' payoff. If c is positive, the expected final wealth of initial share-holders is:

$$w \equiv \int \int_{\{A,B\} \notin \Delta} [BX(x) + A\lambda - d - C(A,B)] f(A,B) dAdB$$

substituting break even condition (3) gives:

$$w = \int \int_{\{A,B\} \notin \Delta} [BX(x) + A\lambda - d] f(A,B) dAdB - c$$

Now, if c is negative, w is identical to the above, as initial shareholders will receive $-c$ with certainty at date 1. In the absence of frictions affecting the contracting between initial shareholders and investors in new capital, the shadow value of initial capital is equal to the price of new capital. Hence, it is unnecessary to treat positive and negative c as separate cases in what follows. And, accordingly, there is no need to distinguish between the owners of different classes of bank capital. For simplicity, we refer to them (both the initial shareholders and the investors in new capital) collectively as *the shareholders*.

The problem of the bank If the bank is safe (i.e., $p = 1$), shareholders are locally indifferent between any mix of capital and deposits that satisfies the requirement. If the bank defaults with strictly positive probability in equilibrium, the capital requirement always binds. From the bank's point of view, deposits are cheaper (depositors always break even, but sometimes at the expense of the taxpayer). Hence, the bank's problem boils down to finding a level of lending x^* that solves

$$\max_{x \geq 0} \int \int_{\{A,B\} \notin \Delta} [BX(x) + A\lambda - ((1-\gamma_b)x + (1-\gamma_a)\lambda)] f(A,B) dAdB - ((\gamma_b x + \gamma_a \lambda) - \kappa).$$

$$(5)$$

We refer to x^* as the equilibrium level of lending.

13

III.   The first order approach

We now impose $\gamma_a = \gamma_b = \gamma$. For simplicity, our propositions focus on the cases where the first order condition uniquely pins down an implicit function $x^*(\gamma)$, and where $p^*(x^*(\gamma), \gamma) < 1$, so that the capital requirement is relevant.[9] For better readability, we often omit function dependencies on x and γ. Additionally, we use subscripts for partial derivatives in these two variables and stars to indicate where functions are evaluated in equilibrium. For instance: $p_x^* \equiv p_x(x^*, \gamma)$.

A.   The sign of the lending response

The bank's objective function can be rewritten as

$$w(x) = \underbrace{X - x}_{\text{economic surplus}} + \underbrace{\int\int_{\{A,B\}\in\Delta} ((1-\gamma)(x+\lambda) - BX - A\lambda)\,f(A,B)dAdB}_{\equiv s(x,\gamma),\text{ i.e. the implicit subsidy}} + \kappa. \qquad (6)$$

The first term in equation (6) captures, intuitively, the economic surplus generated by new loans.[10] The second term integrates, over all the default states, the difference between the promised repayment to the depositors, $(1-\gamma)(x+\lambda)$, and the total cashflow available to the bank, $BX + A\lambda$. Under unlimited liability, this term would be the expectation of how much, ex-post, the shareholders would have to pay into the bank to make depositors whole. But instead, here, the taxpayer is footing the bill. This is why s should be interpreted as the

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[9]Because of the truncation, the objective function in 5 may have multiple peaks, or exhibit jumps or kinks. In these knife-edge cases, there may, for instance, be either no or several $x's$ that solve the first order condition. The FSE can not be isolated in these circumstances. Ignoring these cases enables a first order approach: focusing on small increases in γ helps build intuition and establish the possibility results in our propositions. However, as will become clear in Section IV, our main results do not hinge on this assumption.

[10]Since we assume $\mathbb{E}[A] = 1$, legacy loans are valued on the balance sheet at their expected value. Hence, $(\mathbb{E}[A] - 1)\lambda = 0$, and they do not appear in (6).

implicit subsidy to the bank's shareholders arising from the government guarantee.

Remark 1. The implicit subsidy corresponds to the expected net worth of the bank, when it is negative. As Merton (1977) has shown, deposit insurance can be interpreted as a (free, or at least mispriced) put option on the bank equity, with a strike price of 0. The implicit subsidy is therefore equal to the value of such an option.

The first order condition can be written as:

$$\underbrace{(X_x^* - 1)}_{\text{NPV}} + \underbrace{s_x^*}_{\text{marginal subsidy}} = 0. \tag{7}$$

The first term represents economic surplus maximisation. The second represents how the marginal loan affects the subsidy (we will derive s_x^* later).

At this stage, three points are in order. First, absent the implicit subsidy, only the opportunity cost of funds in the economy matters for investment decisions. The bank would then choose a level of lending consistent with Proposition 3 in **?**. Denoting this level x_{MM}, we have: $X_x(x_{\text{MM}}) = 1$.

Second, an increase in capital requirement unambiguously decreases shareholders' expected payoff: $\forall x, \ w_\gamma \leq 0$. Intuitively, for any x, the expected transfer from the taxpayer shrinks as the share of deposits in the bank's liabilities goes down. Formally:

$$w_\gamma = s_\gamma = -(1-p)(x + \lambda). \tag{8}$$

Third, the gap between x_{MM} and x^* is solely due to s_x^*. Intuitively, if an increase in γ increases the extent to which the marginal loan is subsidised, the *lending response* is positive (that is: $\frac{dx^*}{d\gamma} > 0$). Formally:

Lemma 1. (The sign of the lending response)

$$\frac{dx^*}{d\gamma} \lesseqqgtr 0 \Leftrightarrow s^*_{x\gamma} \lesseqqgtr 0$$

Proof. Formal proofs are in Appendix A. □

B. *The forced safety effect*

We have established that the sign of the lending response is that of $s^*_{x\gamma}$. Now, we turn to the underlying economic mechanism that can lead to $s^*_{x\gamma} > 0$.

Property rights and the marginal residual cash flow Issuing the marginal loan affects the bank's cashflows: the bank's date 2 revenue increases by BX_x, and the repayment due to depositors increases by $1 - \gamma$. Let us denote Z, the *residual* (i.e., net of deposit repayments) cash flow associated with the marginal loan:

$$Z(x, B) \equiv BX_x - (1 - \gamma).$$

Now, which stakeholder is entitled to Z depends on the realisation of the state variables. If the bank survives, the shareholders are the residual claimants. But if the bank defaults, shareholders walk away with zero, and the taxpayer becomes, in effect, the residual claimant.[11] What determines the bank's survival is the sign of the *total residual cash flow* $(A\lambda + BX - (1 - \gamma)(x + \lambda))$, which is different from Z. As we will see, Z plays a key role in our analysis.

The cross-partial derivative of the subsidy Our key result is:

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[11]Technically, the taxpayer is a claimant, in the sense that there is reduction in the transfer needed to make depositors whole.

Proposition 1.

$$s^*_{x\gamma} \quad = \quad \underbrace{-(1-p^*)}_{<0} + \underbrace{p^*_\gamma z^*_0}_{FSE \gtrless 0} \tag{9}$$

where

$$z^*_0 \equiv \mathbb{E}\left[Z^* \mid \backslash^*\right] = \frac{\int_{b_L}^{b^*_0(a_L)} Z^* f(a^*_0(B), B) dB}{\int_{b_L}^{b^*_0(a_L)} f(a^*_0(B), B) dB}$$

is the expected marginal residual cash flow conditional on being on the equilibrium default boundary. It is the case that (i) z^*_0 can be positive; (ii) this implies a positive forced safety effect; (iii) and can lead to a positive lending response: $s^*_{x\gamma} > 0$.

The cross-partial derivative is the sum of two components. The first term is negative. Raising γ reduces the portion of the marginal loan that is financed with (subsidised) deposits. Since the bank must substitute deposits (which it repays with probability p^*) with capital (which, in expectation, it repays in full), this change in the composition of liabilities reduces the marginal subsidy. We dub this effect *the composition effect.*

The second component captures that, keeping x^* constant, an increase in γ makes the bank safer: $p^*_\gamma > 0$. This corresponds to a shift in the default boundary: there are states of the world where the bank would have defaulted if not for the extra capital. In these states, the rights to the residual cashflow from the marginal loan (Z^*) switch from the taxpayer to the shareholders. In expectation, this raises the marginal subsidy by $p^*_\gamma z^*_0$. Since this term stems from the fact that the bank is forced to be safer (something it could have always chosen to do), we dub this effect the *forced safety effect.* To the best of our knowledge, this paper is the first to highlight such a mechanism.

Proposition 1 states that z^*_0 can be positive, which implies a positive FSE. The states where $Z^* > 0$ are defined by a threshold value $\hat{b}^* \equiv \frac{1-\gamma}{X^*_x}$ for B. Figure 3 depicts this in relation to the default boundary. It follows that z^*_0 will be positive if there is sufficient

17

Figure 3: Residual cashflows and the default boundary
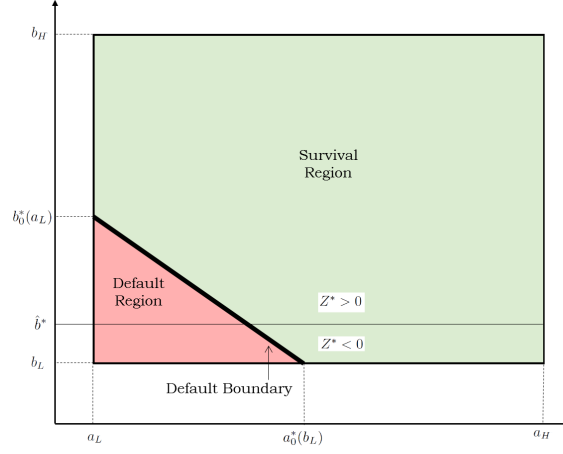
probability mass concentrated on the corresponding upper segment of the boundary. But how should one interpret $z_0^* > 0$? It means that, in expectation along the default boundary, the residual cashflows on the marginal loan are greater than those on the average loan.

By construction, on the default boundary, the residual cashflow on the bank's average loan, over the the whole balance sheet, is nil $(BX^* + A\lambda - (1 - \gamma)(x^* + \lambda) = 0)$. If the marginal loan is exactly equivalent to the average loan on the balance sheet, then we would have $Z^* = 0$ at all points on the boundary, and the FSE would be nil. But if the marginal loan differs along some dimension (e.g. it may have a different return, or simply a different capital requirement), then Z^* can be positive along the boundary. In other words, for a non-zero FSE, heterogeneity in the residual cashflows generated by the bank's assets is required.

A common assumption in the macro-banking literature is that loans on a bank's portfolio are *iid* conditional on the aggregate state. Thus, the marginal loan is ex-post different from the average loan, but is identically distributed (and faces an identical capital requirement).

18

Hence, by assumption, $z_0^* = 0$ and the FSE is nil.

The marginal subsidy Heterogeneity in residual cashflows, or a lack thereof, is also key to the sign of the wedge in the first order condition (i.e. the marginal subsidy).

It is given by

$$s_x^* = -(1-p)z_\Delta^*, \tag{10}$$

where

$$z_\Delta^* = \mathbb{E}\left[Z^* \mid \Delta^*\right] = \frac{\int \int_{\{A,B\}\in\Delta^*} Z^* f(A,B) dA dB}{\int \int_{\{A,B\}\in\Delta^*} f(A,B) dA dB}.$$

The sign of the marginal subsidy is determined by the expected residual cashflows from the marginal loan in the default states, z_Δ^*. If $z_\Delta^* < 0$, the taxpayer subsidises the marginal loan by making good the losses the loan would otherwise impose on depositors. Here, $x^* > x_{\text{MM}}$: the bank finances negative NPV loans (as per Kareken and Wallace (1978), this is a well understood manifestation of risk-shifting). If $z_\Delta^* > 0$, then the positive residual cashflows instead reduce how much the taxpayer needs to chip in. So, at the margin, the subsidy is negative and so acts like a tax, reducing equilibrium lending below x_{MM}. This reflects the guarantee overhang we refered to in the Introduction, and will discuss in more detail in Section IV.

The default region is defined by the states in which the average loan has negative residual cashflows. Now, assuming that loans are conditionally *iid* (and face identical capital requirements) is akin to assuming that the $z_\Delta^* < 0$. Hence, under such an assumption, the marginal subsidy is always positive. As we now show, however, the contrary can be true as well.

A representative example The left panel in Figure 4 depicts a case where $x^*(\gamma)$ is U-shaped and the lending response is positive for intermediate values of γ. As we will discuss in

19

Figure 4: An example of $x^*(\gamma)$ and the bank's objective

$x^*(\gamma)$

$w(x)$
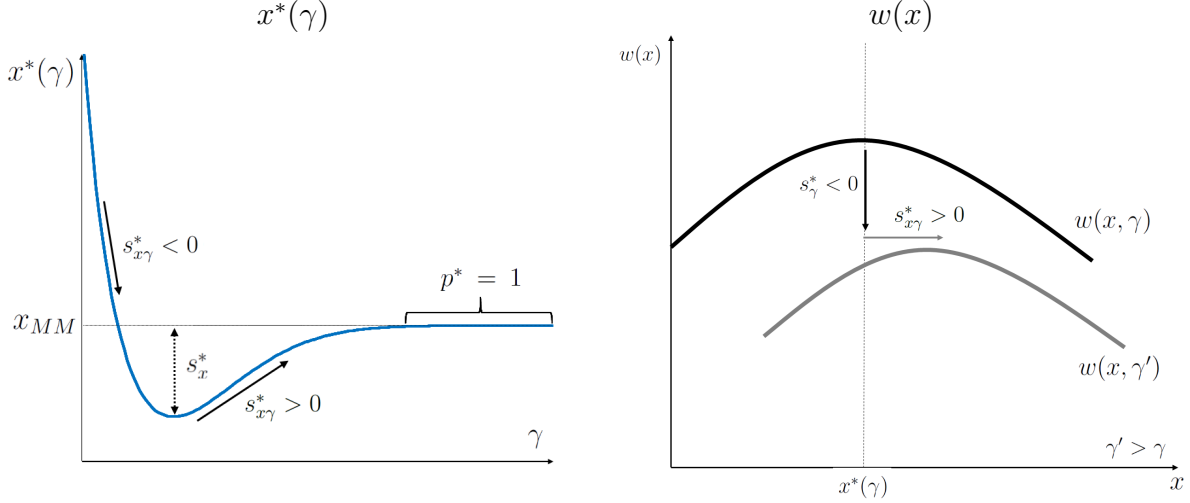
Notes: The left panel, provides a representative numerical example of $x^*(\gamma)$. The calibration is defined in Table I, except that the correlation between A and B and the tax rate are both set to zero. As explained in Section III, the sign of the slope of $x^*(\gamma)$ is given by the sign of $s^*_{x\gamma}$. The difference between the Modigliani-Miller level of lending, x_{MM}, and $x^*(\gamma)$ is a negative function of s^*_x. The right panel shows how the bank's objective, $w(x)$, is shifted down by an increase in the capital requirement when the lending response is positive at the initial equilibrium, $x^*(\gamma)$. If, instead, the lending response is negative, then $w(x, \gamma')$ peaks to the left of $x^*(\gamma)$.

Section V, this is a representative example of how the moral hazard generated by government guarantees can play out in the model. The right panel depicts how the positive lending response translates into movements in the bank's objective. That $w_\gamma < 0$ means that the payoff function associated with a higher capital requirement (denoted γ') is below the initial one. But it does not tell us whether it peaks to the left or to the right of the initial optimum. If $s^*_{x\gamma} > 0$, it peaks to its right, which means that the lending response is positive. As we discussed above and in the Introduction, the tractability assumptions commonly made in the literature, would restrict our model's equilibrium outcomes to $s^*_x < 0$ and $s^*_{x\gamma} < 0$. Relaxing such assumptions can lead to any sign combinations for these two objects. We develop insights for this in specific examples in Section IV.

20

C.   Multiple capital requirements

If the bank faces different capital requirements on legacy (γ_a) and new loans (γ_b), we obtain:

Proposition 2.

$$s^*_{x\gamma_a} = \underbrace{p^*_{\gamma_a} z^*_0}_{\gamma_a \ FSE} \tag{11}$$

$$s^*_{x\gamma_b} = -(1 - p^*) + \underbrace{p^*_{\gamma_b} z^*_0}_{\gamma_b \ FSE}. \tag{12}$$

Both $s^*_{x\gamma_a}$ and $s^*_{x\gamma_b}$ can be positive.

The first term in $s^*_{x\gamma_b}$ corresponds to the composition effect. That this effect is present in $s^*_{x\gamma_b}$ (and not in $s^*_{x\gamma_a}$) makes perfect sense; only γ_b alters how the marginal loan is financed. However, both $s^*_{x\gamma_b}$ and $s^*_{x\gamma_a}$ contain a term that corresponds to a forced safety effect. The intuition for the γ_a FSE is straightforward: a higher capital requirement on legacy loans makes the bank safer, and it then internalises the residual cashflows on the boundary, z^*_0. The intuition for the γ_b FSE is similar. Whether an extra capital buffer is associated with legacy or infra-marginal loans is irrelevant: in both cases it makes the bank safer.

So, if $z^*_0 > 0$, raising either requirement could boost lending. As a result, as we will show in Section C, a positive lending response can arise without legacy loans.

The relative strength of these two FSEs depends how much the boundary shifts when each requirement is raised (given by $p^*_{\gamma_a}$ and $p^*_{\gamma_b}$) and, therefore, depends on the shares of new and legacy loans on the balance sheet ($p^*_{\gamma_a}/p^*_{\gamma_b} = \lambda/x^*$). This allows us to decompose the effect of γ on the marginal subsidy. Starting from $\gamma_a = \gamma_b = \gamma$, we have: $s^*_{x\gamma} = -(1-p^*)+p^*_{\gamma_a} z^*_0+p^*_{\gamma_b} z^*_0$.

IV. Residual Cash Flow Heterogeneity

To illustrate how the FSE operates, why the implicit subsidy may act like a tax, causing banks to pass on positive NPV loans, and more generally to highlight the key role played by residual cash flow heterogeneity, we organise our discussion around four specific cases.

A. Perfectly homogeneous residual cashflows

A useful starting point is a special case where the FSE is always nil. This may occur if new loans are identical to the legacy loans. Here, the deviation from our baseline assumption is that the bank chooses $x \in (0, \bar{x})$, which yields a payoff:

$$BX(x) = Ax.$$

So the bank's choice is simply whether to scale up or not (with $\bar{x}$ as the maximum possible increase in scale). Considering a single capital requirement, we have:

$$s = \int_{a_L}^{a_0} (1 - \gamma - A)(x + \lambda)f(A)dA.$$

The default boundary is given by $a_0 = (1 - \gamma)$, and we assume that $a_L < (1 - \gamma)$ so that the bank defaults with positive probability (note that p is independent of x). We have:

$$s_x = -(1 - p)\underbrace{(\mathbb{E}[A \mid A < a_0] - (1 - \gamma))}_{<0} > 0.$$

Here, the marginal subsidy is positive and makes lending more appealing. Abusing notation somewhat, define x^* as the bank's optimum:

$$x^* = \begin{cases} \bar{x} & ; \mathbb{E}[A] + s_x > 1 \\ 0 & ; \mathbb{E}[A] + s_x < 1 \end{cases}$$

If $\mathbb{E}[A] < 1 < \mathbb{E}[A] + s_x$, we have a typical case of bank risk-shifting (Kareken and Wallace (1978)): a new loan has a negative NPV, but the bank finances it because an expected loss (equal to s_x) can be shifted to the taxpayer.

Here,

$$s_{x\gamma} = -(1-p) < 0. \tag{13}$$

The composition effect operates as usual, but the FSE is nil. Changes in γ still shift the default boundary (a_0 decreases and therefore p increases with γ), but $z_0^* = z_0 = a_0 - (1-\gamma) = 0$. In this scaling-up example, marginal and average residual cashflows are the same and the average residual cashflow is, by definition, nil along the boundary.

Let us define the *average subsidy* as $\frac{s}{x+\lambda}$. We have that:

$$\frac{\partial}{\partial \gamma} \left(\frac{s}{x + \lambda} \right) = -(1-p) < 0. \tag{14}$$

The expression is the same as in equation (13). This is intuitive: when residual cashflows on all loans are identical, the capital requirement has exactly the same effect on the marginal subsidy and the average subsidy. Equation (14) is not specific to the present example. It holds in general in our model and always corresponds to the composition effect.

In our interpretation, the composition effect is behind the claim (made repeatedly by bank

lobbies and sometimes by policymakers (see, for instance, Brooke et al. (2015), p5)) that increasing capital requirements would: (i) increase bank funding costs; and (ii) naturally lead to less lending. Equation (14) shows that the first part of the claim applies in our model: the average subsidy always falls in response to a higher capital requirement.

However, our analysis shows that the second part of the claim is a *non sequitur.* We believe it confounds the effect of the capital requirement on the *average* subsidy and the *marginal* subsidy. In the current example, there is no difference between these two objects. But, in general, they are different. And, as the comparison between equation (14) and equation (9) makes clear, this difference is precisely the FSE.

Indeed, as we will now show, $z_0^* = 0$ is a special case. Marginal and average residual cashflows can differ for several reasons, so that $z_0^* \neq 0$.

B.   Capital requirements as a source of heterogeneity

Even when new loans are identical to legacy ones, the FSE can be positive if the loans are subject to different requirements. Keeping, otherwise, the same structure as in the previous example, we have:

$$s = \int_{a_L}^{a_0} \left[(1 - \gamma_a - A)\lambda + (1 - \gamma_b - A)x \right] f(A)dA,$$

and

$$s_x = -(1 - p)\left(E[A \mid A < a_0] - (1 - \gamma_b) \right).$$

The default boundary is now given by $a_0(x) = (1 - \gamma_a)\frac{\lambda}{x+\lambda} + (1 - \gamma_b)\frac{x}{x+\lambda}$. The marginal residual cash flow on the boundary is $z_0(x) \equiv a_0(x) - (1 - \gamma_b) = \gamma_b - \frac{\lambda\gamma_a + x\gamma_b}{\lambda+x}$. So, if $\gamma_b > \gamma_a$,

then $z_0 > 0$, and the FSE will be positive:[12]

$$p_{\gamma_a} z_0 > 0, \; p_{\gamma_b} z_0 > 0.$$

This simple example is useful to illustrate how residual cashflows can be heterogeneous in spite of the loan cashflows being perfectly correlated. (We discuss the role of correlation in more detail in Appendix D).

C. *Heterogeneity among new loans*

Residual cash flow heterogeneity can, of course, emanate from heterogeneity in loan cashflows rather than capital requirements. As will soon become apparent, the U-shape in Figure 4 is due to heterogeneity between legacy and new loans. Here, we show that if the bank starts from scratch (i.e., $\lambda = \kappa = 0$), heterogeneity among new loans can also lead to a positive FSE.

A portfolio composition problem   In this example, we deviate from our baseline model and assume that there are n different unit lending opportunities indexed by i, and m equi-probable states indexed by j. Let B denote the $n \times m$ matrix of realised payoffs and $\boldsymbol{x} = \{x_1, x_2, ..., x_n\}$, where $x_i \in \{0, 1\}$, denote the bank's portfolio decision ($x_i = 1$ means that the loan is financed). In our general notation, the total realised payoff associated with a portfolio is given by:

$$BX(\boldsymbol{x}) \equiv \boldsymbol{x}B.$$

---

[12]Note that at $x = 0$, there are no infra-marginal loans, so $p_{\gamma_b} = 0$.

The effect of γ_b on portfolio decision   Unless n and m are small, the bank maximisation problem is intractable analytically. But it can be solved numerically to show that 3 key features typically emerge (since there are only new loans, only γ_b is relevant):

1. Unless γ_b is high enough, the bank generally optimally chooses to both *finance negative NPV loans* and *not finance positive NPV loans.*

2. A loan that is not financed under a capital requirement γ_b may end up being financed under a capital requirement $\gamma_b' > \gamma_b$, and vice versa.

3. Total lending for the bank, as a function of γ_b, follows a sort of *U shape.*
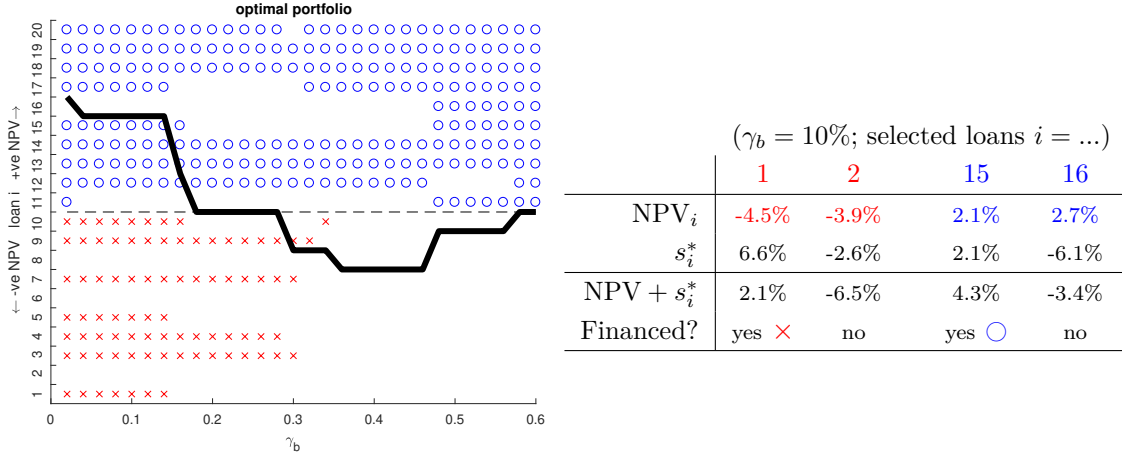
Figure 5 provides an example of optimal portfolio choices across values of γ_b (the bank chooses among $n = 20$ loans whose payoffs in the $m = 100$ states are independently drawn; Appendix C provides exact details). We draw payoffs so that loans 11 to 20 have a positive NPV and would form the optimal portfolio in the absence of the guarantee. As we can see, γ_b dramatically affects how the bank deviates from this benchmark. Understanding why requires looking at residual cashflows in the default region.

Residual cashflows and risk-shifting   Any loan is potentially marginal. Consider, for instance, a loan k that is not included in the equilibrium portfolio. In equilibrium, the marginal subsidy associated with this loan can be expressed as:

$$s_k^* \equiv - \underbrace{\frac{1}{m} \sum_{j \in \Delta^*} Z_{kj}}_{\text{expected Z over } \Delta^*} + \underbrace{\frac{1}{m} \sum_{i \in \{\Pi^* \cup k\}} \sum_j H_{jk}^* Z_{ij}}_{\text{"boundary shift" effect}} . \tag{15}$$

The first term is simply minus loan k's expected residual cash flow over the equilibrium default region; equivalent to equation (10). The additional second term arises as we are in a

Figure 5: Discrete New Loans with Cashflow Heterogeneity

	($\gamma_b = 10\%$; selected loans $i = ...$)			
	1	2	15	16
NPV_i	-4.5%	-3.9%	2.1%	2.7%
s_i^*	6.6%	-2.6%	2.1%	-6.1%
$NPV + s_i^*$	2.1%	-6.5%	4.3%	-3.4%
Financed?	yes ✕	no	yes ◯	no

Notes: The left panel provides an example of optimal portfolio choices across values of γ_b, with $n = 20$ assets, and $m = 100$ states (see Appendix C for details). On the x-axis is the capital requirement at 2-percentage-point increments. On the y-axis is the corresponding portfolio choice. Loans are sorted accorded to their NPV, in increasing order. Loans 1 to 10 have negative NPV and loan 11 to 20 have positive NPV. For a given capital requirement, a circle (for positive NPV loans) or a cross (for negative ones) means that the corresponding loan is in the optimal portfolio of the bank. The solid line indicates how many loans are in the portfolio. For instance, at $\gamma_b = 10\%$, there are 15 loans that are financed, including loan 1 and six others which all have negative NPV. In contrast, loans 11 and 16, which have positive NPV, are not financed. The right panel compares, for this case, the NPV and marginal subsidy of loans 1 and 16 to two loans of comparable NPVs, but for which the bank's decision is in line with NPV.

discrete case. Financing loan k shifts default the boundary, which also affects the marginal subsidy: The second term captures the resulting change in the expected residual cash flow of the equilibrium portfolio (where Π^* denotes the set of loans by number included in the optimal portfolio and H_{jk}^* is a function that takes the value -1, 1, or 0 depending on whether the inclusion of loan k in the bank portfolio adds state j to the default region, excludes it from the region, or does not affect its presence in it).[13]

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[13]To see that the first term corresponds to the marginal subsidy in the continuous case, denote $z_\Delta^*(k)$ the conditional expectation of Z_{kj} over the default region, and note that: $\frac{1}{m}\sum_{j\in\Delta^*}(-Z_{kj}) = -(1 - p^*)z_\Delta^*(k)$. The equivalent of the second term is nil under the first order approach. Indeed, by definition, the *total* residual cash flow is nil on the default boundary.

Loans are included in the optimal portfolio if[14]

$$\text{NPV}_i + s_i^* > 0.$$

So, the bank finances negative NPV loans if their marginal subsidy is large enough. As above, there is a classic risk-shifting interpretation as the bank shifts losses on these loans onto the taxpayer.

Loans are excluded from the optimal portfolio if:

$$\text{NPV}_i + s_i^* < 0.$$

So, the bank passes on positive NPV loans if their marginal subsidy is sufficiently negative. If the bank were to add one of these loans to its portfolio, it would increase the portfolio NPV, but this would be more than offset by the decline in the expected transfer from the taxpayer. In a sense, financing these loans would provide hedging, but this hedging would only benefit the taxpayer. Deciding not to hedge is comparable to deciding to increase risk. Hence, we argue that not financing positive NPV loans can also be interpreted as risk-shifting. (Below we also provide an interpretation related to guarantees generating an overhang problem.)

Composition and forced safety effects Holding the portfolio and default region fixed, an increase in γ_b reduces the marginal subsidy. The partial derivative of the first term in equation (15) is $-(1 - p^*)$; i.e., the composition effect.

However, a forced safety effect is present. The intuition follows Section III. More capital shifts the default region. This alters s_i^* for all loans. Moreover, in this discrete case, this

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[14]For such loans, the maginal subsidy can be written as $s_k^* \equiv -\frac{1}{m} \sum_{j \in \Delta^*} Z_{kj} + \frac{1}{m} \sum_{i \in \{\Pi^* \setminus k\}} \sum_j \left(-H_{jk}^* \right) Z_{ij}.$

causes the bank to reshuffle its portfolio, which alters the default region again, and so on. The net result is that any loan (with positive or negative NPV) that is not financed at some γ_b may be financed at some $\gamma_b' > \gamma_b$, and vice versa.

The U-shape   Total lending exhibits a U-shape in Figure 5 (comparable to the example in Figure 4). To see how this shape emerges, note first that at low levels of γ_b, many negative NPV loans are made, bringing the total number of loans substantially above x_{MM}. Second, as γ_b increases, the number of negative NPV loans in the portfolio tends to decrease.   Third, at low or high levels of γ_b most positive NPV loans are financed, but at intermediate values, many of them are passed on, leading to a U-shape in positive NPV lending.

This example relies on heterogeneity in the payoffs associated with new loans. Building on Section B, we could instead consider discrete, identical (positive NPV) loans that have different capital requirements (or risk-weights). As we show in Appendix D, this also delivers a U-shape.

D.   *Legacy loans as a source of heterogeneity*

Our final example provides analytic intuition for why U-shapes emerge in many cases.

Let us return to our baseline assumption where $BX(x)$ is strictly increasing and strictly concave, which allows us to use the first order approach. However, let us assume that new loans are safe, normalise $B = 1$, and that $\lambda > 0$ and $a_L < 1 - \gamma$, so that the bank may fail in equilibrium. The first-order condition, with a single capital requirement γ, reads

$$X_x^* - 1 + (1 - p^*) \underbrace{\left((1 - \gamma) - X_x^*\right)}_{=-Z^*} = 0.$$

Rearranging it gives

$$X_x^* = (1 - \gamma) + \frac{\gamma}{p^*}. \tag{16}$$
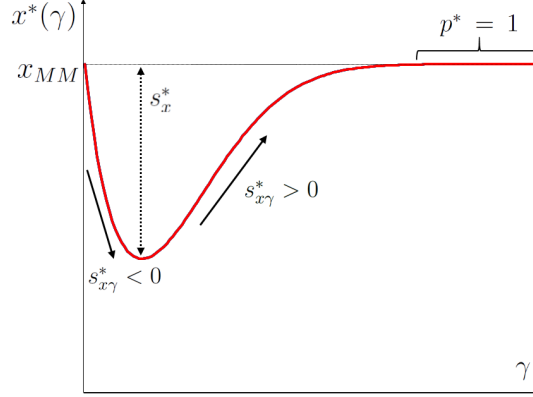
This equilibrium condition makes it clear that, as soon as $p^* < 1$ (and $\gamma > 0$), we have $X_x^* > 1$: the bank passes on positive NPV new loans. The intuition is similar to the example above, but now with more structure on the cashflows. Legacy loans can make the bank default. New loans are safe, with $X_x^* > 1$. Hence, they produce positive residual cashflows in all states: $Z^* > 0$. Intuitively, the marginal subsidy is negative. It acts as a tax, and the bank's optimal lending cut-off is below x_{MM}.

From first-order condition (16), when $\gamma = 0$, or γ is high enough (so that $p^* = 1$), $x^* = x_{MM}$. In between, as described, $p^* < 1$ and x^* is strictly below x_{MM}. This provides analytic intuition for a U-shape relationship and, therefore, the existence of a positive lending response. Figure 6 provides an example where $x^*(\gamma)$ is a well behaved U-shape. Adding risk to new loans typically makes the lending response intersect the y-axis above x_{MM} (like in Figure 4), which links back to our observation in the previous example that, at very low values of γ, many negative NPV loans are financed.

E. Overhang problems

Positive residual cashflows in the default region reduce the expected transfer from the tax-payer. So, as we established in Section B, if $z_\Delta^* > 0$, the marginal subsidy is negative and acts like a tax, reducing equilibrium lending below x_{MM}. This is very similar to what happens in the classic debt overhang problem of Myers (1977). Furthermore, a positive FSE implies the bank now internalises some of the positive residual cash flows in the default region. In this sense the FSE can be interpreted as alleviating an overhang problem. However, this interpretation only works for a subset of the different meanings "overhang problem" can have

Figure 6: Example of $x^*(\gamma)$ in the case with safe new loans: a well behaved U-shape

in the literature. In particular, the overhang problem may refer (explictly or not) to: (i) the presence of risky (long term) debt; (ii) an implicit transfer of positive cash flows to creditors; (iii) the resulting under-invesment problem; or the combination of the three. In the context of our model, only (ii) is fully consistent with the proposed interpretation.

In what follows, we formalise this statement and explore further the links with Myers (1977).

Definition. An overhang problem occurs in equilibrium if $\exists A \in \Delta^*, Z(A) > 0$.

This means that there are *some* states in the equilibrium default region where the marginal loan has positive residual cash-flows. As described in Section B, these positive cash-flows are, in effect, transferred to the taxpayer. Hence, our definition corresponds to notion (ii) above.

Ceteris paribus, an overhang problem makes the bank undervalue the marginal loan. However, nothing prevents Z being negative in *other* default states. These negative residual cash-flows are *losses* that are *shifted* onto the taxpayer. Ceteris paribus, they make the

bank overvalue the marginal loan. Whether the overhang problem or loss shifting problem dominates overall depends on the sign of z_Δ^*, which is the expectation of Z over *all* default states. As discussed, this expectation pins down the sign of the marginal subsidy and whether the bank lends more or less than x_{MM}.

Now, importantly, the sign of the FSE does not hinge upon the sign of z_Δ^*. It instead depends on the sign of $z_{\backslash}^*$. That is, it only depends on whether or not the overhang problem dominates along the default *boundary*. If it does, then the FSE is positive and, indeed, encourages more lending *because* it alleviates the overhang problem. But this can also happen when $z_\Delta^* > 0$. That is, a positive FSE can exacerbate an over-lending problem. This is why the interpretation that a positive FSE alleviates an overhang problem does not work well with notion (iii) above.

Moreover, a higher capital requirement can also worsen under-lending. To see this, it is useful to draw further links with the classic debt overhang problem.

In the model of Myers (1977), a firm has existing risky assets and existing debt, on which it will default in some states of the world. The firm can choose to raise equity to finance a positive NPV investment but may pass on it because the cash flow it generates accrues to existing debtholders in the default states. This can be mapped into a special case of our model. To see this, just impose $\gamma_b = 1$ (i.e., new investment must be fully financed with capital). Then, the residual cash flow on the investment is the cash flow itself. Hence, $Z > 0$. As a result: i) the marginal subsidy can only be negative; ii) the FSE can only be positive; and iii) increasing γ_a can only increase lending. Reinterpreting $1/\gamma_a$ as the initial leverage of the bank (where a fraction $1 - \gamma_a$ of legacy loans are funded with uninsured legacy debt), we obtain the classic interpretation: less initial leverage generates more investment.

However, banks can finance new investment with insured deposits (i.e., $\gamma_b < 1$). Hence,

for banks, Z^* can have either sign. Our main focus is on a positive FSE, which occurs when $Z^* > 0$. But $Z^* < 0$ yields a the implication, at odds with what happens in the classic model: an increase in γ_a can make the bank pass on a positive NPV loan.[15]

Another difference with Myers (1977), is that the guarantee overhang cannot be fixed by shortening debt maturity. In the classic debt overhang problem, if existing debt were to mature before investment took place (or if its interest rate was renegotiable), the problem would not occur, as the price of debt would reflect its market value (taking into account new investment). In our model, debt takes the form of demand deposits which can be raised at the time investment take place. So, debt maturity is not part of the problem and notion (i) above is not relevant here. Indeed, Section C establishes that an overhang problem occurs (and may lead to under-lending) even in the absence of initial debt. This is why we find it more appropriate to refer to the problem as *a guarantee overhang* rather than a debt overhang.

V. Empirical relevance

In real-world regulation, there is generally a single capital requirements that applies to risk-weighted assets. Accordingly, we now interpret the potential difference in requirements as a difference in risk weights. Formally, we just rewrite the constraint as:

$$\kappa + c \geq \gamma(\alpha\lambda + \beta x),$$

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[15]Here's a simple numerical example: there are 3 equiprobable states {I, II, III}. There are 10 legacy loans, with a vector of total payoffs $\{7, 9, 14\}$. There is a single new lending opportunity with payoff vector $\{1.4, 0.5, 1.25\}$. All loans are of size one, so the new loan has positive NPV. Initially, $\gamma_a = \gamma_b = 0.1$. Facing these, the bank fails in states I and II, and optimally decides to finance the new loan. Raising γ_a to 0.2 for instance makes the bank survive in state II. This means that the bank internalises the negative marginal residual cash flow in this state ($Z_{II} = 0.5 - 0.9 = -0.4$). This more than offsets the positive Z_{III} and the bank prefers now to pass on the new loan.

where $\alpha \equiv \frac{\gamma_a}{\gamma}$ and $\beta \equiv \frac{\gamma_b}{\gamma}$ capture risk weight parameters.

A. *Extending the model*

To provide a meaningful calibration, we extend the baseline model with two additional features.

Taxes and tax shields First we introduce the tax advantage of debt – another reason why banks may find capital relatively costly. To capture this, we assume that the bank faces a tax rate τ on positive profits, net of interest expenses on deposits. In order to introduce a meaningful tax shield, we also assume that households have an opportunity cost of funds $1 + \rho > 1$, so that the interest rate on deposits is ρ. To maintain our normalisation, we now set $\mathbb{E}[A] = \mathbb{E}[B] = 1 + \rho$.

We formally describe how the tax interacts with $x^*(\gamma)$ in Online Appendix I. Here, we only illustrate, with Figure 7, the main qualitative effect of the tax shield on the lending response. The solid red line depicts an example of $x^*(\gamma)$ without the tax ($\tau = 0$). This is the U-shape relationship of Section 4.4 (where new loans are safe). The blue dashed curve is the case with the tax. As we can see, the tax shield tilts the relationship clockwise.

This is intuitive, as with tax deductibility, an increase in γ generates a composition effect that is similar to the one above and is stronger the higher the tax rate and the interest rate paid on deposits. (To make the effect visually obvious, we set both the tax and the interest rates at very high levels in Figure 7.)

Competition and aggregate demand for loans Second, we model competition between banks. Our baseline assumption has been that the bank faces a downward sloping demand for loans, independent of the the capital requirement. In practice, however, the loan demand for a

Figure 7: The shape of $x^*(\gamma)$ with corporate income tax
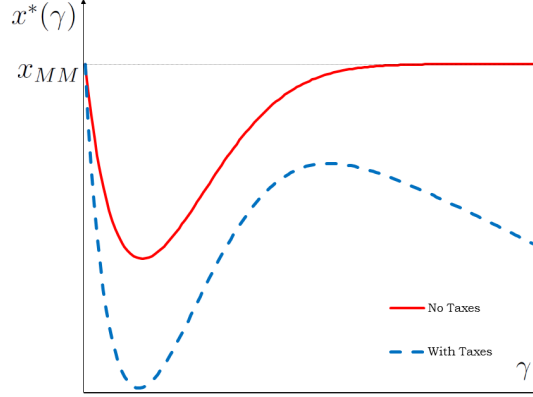
Notes: This figure illustrates the effect of taxes on $x^*(\gamma)$. The red line is equivalent to that in Figure 6. The blue dashed line corresponds to the same calibration with a corporate income tax $\tau = 50\%$ and an interest rate $\rho = 8\%$. The horizontal dotted line is the MM level of lending.

bank is affected by the loan supply of other banks and, therefore, by the capital requirements they face.

We capture imperfect competition in a Cournot fashion: There is a given number ν of identical banks, all face the same capital requirement γ, and all pick their optimal level of lending taking other banks' decisions as given.[16] Up to a normalisation that we will introduce later, the payoff function of the representative bank takes the form:

$$BX(x) = Bx\left((x + x')^{-\eta}\right),$$

where x' captures the *total* lending by other banks, and η is a parameter that captures the elasticity of aggregate loan demand.

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[16]A Cournot approach is analytically convenient, but we also believe that it is particularly meaningful if one conisders that banks first choose their level of capital (which, given the capital requirement, creates a capacity constraint) and then compete in price (i.e., in interest rate) in the market for loans. Schliephake and Kirstein (2013) have shown that this results in an elegant application of Kreps and Scheinkman (1983): The equilibrium outcome corresponds to that under Cournot competition. Other papers using Cournot competition for banks include Corbae and D'Erasmo (2017) and Jakucionyte and van Wijnbergen (2018).

Our equilibrium concept is a symmetric Nash equilibrium. Assuming it is unique, it corresponds to the fixed point (i.e. $x^* = x'/(\nu - 1)$) that solves the representative bank's first-order condition.[17]

B. *Calibration*

Our benchmark calibration aims at capturing a plausible situation facing a major international bank in 2017. Table I summarises this calibration.

The capital requirement is not a straightforward object to calibrate. In the model, what matters is loss-absorbing liabilities as a percentage of the bank's assets. Even after accounting for risk weights, this is not necessarily the same object as the headline regulatory capital requirement that the policy debate focuses on. Relevant considerations include: (i) Banks have hybrid liabilities that both may or may not count towards the requirement and may or may not have implicit guarantees attached to them; (ii) there are different requirements for different types of capital; (iii) banks hold voluntary buffers above the requirements (for instance, to prevent small shocks from leading to violations); and (iv) requirements vary across jurisdictions, types of banks (for example, banks deemed to be globally systemic now face higher requirements), and with macroeconomic conditions (this is the role of counter-cyclical capital buffers).

To circumvent the issue, we present our results for a wide range of values of γ; that is, we display the $x^*(\gamma)$ functions. Still, we need a reference value to centre the calibration. For ease of interpretation, we use a headline number of 13% of risk-weighted assets for the requirement. Under Basel III, this roughly corresponds to the Tier 1 capital requirement (including systemic, conservation, and pillar 2 buffers) that globally systemically important

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[17]In our numerical explorations, we have not encountered multiple fixed points.

Table I: Benchmark Calibration

Parameter	Value	Definition	Calculation	Source(s)
γ	0.13	capital requirement	Tier 1 Risk Based Minimum Capital Requirement of Globally Systemically Important Banks.	BCBS (2017) - Table B.4
α	0.5	risk weight on new loans	Average risk weights.	Mariathasan and Merrouche (2014)
β	0.5	risk weight on legacy loans		
x_{MM}	1	MM level of lending	Normalisation.	
ρ	0.012	interest rate	Average 1 year constant maturity US treasury yield (2017).	Federal Reserve Board - Release H.15
σ_A	0.041	standard deviation of $log(A)$	Target $p = 0.97$: annual frequency of banking crises in OECD countries 1970-2012.	Valencia and Laeven (2012)
σ_B	0.041	standard deviation of $log(B)$		
μ_A	$log(1+\rho)-0.5\sigma_A^2$	expectation of $log(A)$	existing loans fairly valued.	
μ_B	$log(1+\rho)-0.5\sigma_B^2$	expectation of $log(B)$	$E[B] = 1 + \rho$, implies $x_{MM} = 1$.	
λ	4	book value of legacy loans	x_{MM} normalised, and $x_{MM}/\lambda \implies 20\%$ of loans maturing per year.	van den Heuvel (2009)
ν	12	number of banks	Loan spread over deposit rate = 2%	Bernanke et al. (1999)
η	0.2	interest elasticity of demand	interest elasticity of demand on mortgage debt estimated from UK loan-to-value notches.	Best et al. (2015)
τ	0.24	corporate tax rate	OECD average corporate tax rate 2005-2017.	OECD tax database

banks face.[18]

Average risk weights are typically around 50% (see Mariathasan and Merrouche (2014)), so we use this number for α and β. Again, in practice, there is variation across banks and over time.

We calibrate τ to match the average statutory corporate tax rate among OECD countries; this corresponds to 24% in 2017. We interpret the period in our model as one year. Hence, we calibrate the interest rate ρ to match the average 1-year constant maturity US treasury bond yield: 1.2% in 2017.

We select parameters so that $x_{\mathrm{MM}} = 1$ in the benchmark calibration and any alternatives presented. To this effect, we rescale the representative bank's gross return function:

$$BX(x) = Bkx\left((x + x')^{-\eta}\right), \tag{17}$$

where $k = \nu^{\eta}/(1 - \frac{\eta}{\nu})$, and $\mathbb{E}[B] = 1 + \rho$.

We calibrate η to match the interest elasticity of demand on residential mortgage debt estimated from UK loan-to-value notches (Best et al. (2015)). We choose ν to target the average spread on new loans in the model $\left(\mathbb{E}[B]\frac{X^*}{x^*} - 1 - \rho\right)$, which we calibrate at 2%, consistent with Bernanke et al. (1999). This gives $\nu = 12$.

We calibrate the book value of legacy assets (λ) to 4 such that, if the bank chooses $x = x_{\mathrm{MM}}$, 20% of loans on the balance sheet were made in the current period. This is in line with values in the literature (see, for example, van den Heuvel (2009)). To abstract from bank closure (see Appendix A), we assume that $\kappa > \gamma\lambda$.

We model the joint distribution of $f(A, B)$ as a log-normal. We assume that legacy loans

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[18]See BCBS (2017) and EBA (2017) for recent assessments; Table A in BoE (2015) describes a breakdown of different requirements.

are held at fair value on the bank's balance sheet; that is, $\mathbb{E}[A] = 1 + \rho$. We further assume that A and B have identical standard deviations, which we calibrate by targeting the bank's equilibrium default probability ($1 - p^*$ in the model).

The appropriate calibration for $1 - p^*$ is the probability that the implicit subsidy is in the money and creditors benefit from a taxpayer transfer. Determining the value of p^* from the price of bank securities is challenging, due specifically to the need to strip out the value of any expected transfer. Instead, we use realised frequencies. Valencia and Laeven (2012) find that there have been 40 banking crises among the 34 OECD members over the period 1970-2012, which suggests a target value of $p^* = 0.97$ or a 3% annual probability of default (Martinez-Miera and Suarez (2014) use a similar value in their calibration).

Last, we set the correlation parameter in the joint distribution of A and B equal to 0.5. This choice is arbitrary, and we run sensitivity analysis on it below.

C. Results

Benchmark example Figure 8 displays the $x^*(\gamma)$ curve (the left panel) and the associated probability of survival $p^*(\gamma)$ (right panel) for our benchmark calibration. As we can see, $x^*(\gamma)$ is slightly upward-sloping region when γ is within the range of 11% to 21%. At lower values of γ, the slope is negative (and steeper at very low values). From values 21% onward, the curve is downward sloping again: the bank is in fact very safe, and the effect of the tax shield dominates.

At the reference value ($\gamma = 13\%$, indicated by the vertical dotted line), the slope is positive. However, the lending response has little economic significance. For instance, a capital requirement increase from 13% to 14%, would generate an increase in lending of 0.02%. However, this is still very different from conventional wisdom and the typical concern

Figure 8: Equilibrium Lending and Survival Probability Under the Benchmark Calibration

(a) $x^*(\gamma)$ (b) $p^*(\gamma)$
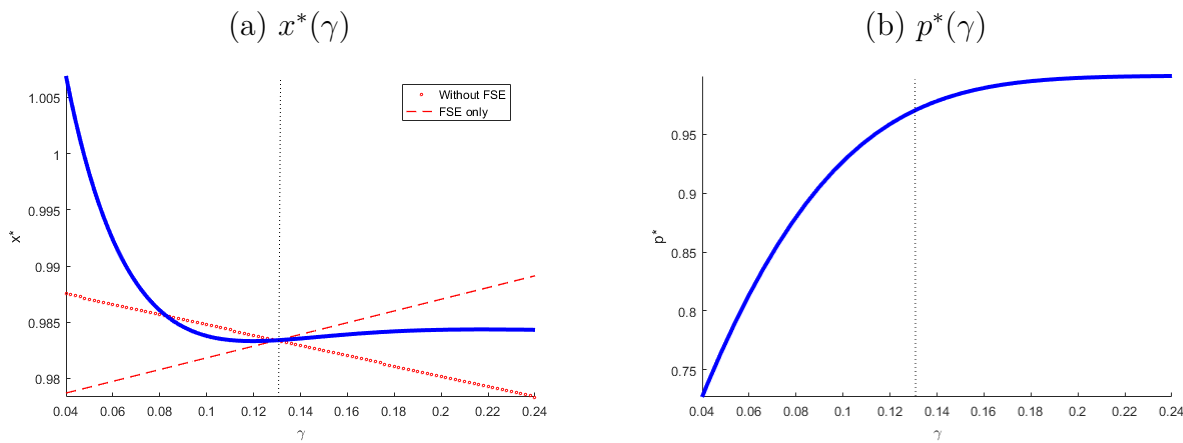
Notes: This figure depicts outcomes for the calibration defined in Table I. Panel a: equilibrium lending for the representative bank under the alternative levels of γ; the downward sloping dotted red line depicts the slope $x^*(\gamma)$ would have, at $\gamma = 13\%$, absent the FSE; the upward sloping dashed red line depicts, instead, the slope of $x^*(\gamma)$ at that point if only the FSE was operating. Panel b: corresponding equilibrium survival probability for the representative bank under the alternative levels of γ. In both panels, the vertical dashed line denotes the reference level of the capital requirement.

that such a policy change would provoke a cut in lending. And the reason for the absence of a cut is the forced safety effect. To illustrate this, we have added two counterfactual slopes to the left panel of 8. The dotted line shows what the slope of $x^*(\gamma)$ would be (at $\gamma = 13\%$) absent the FSE (i.e. if the slope was exclusively driven by the composition effect plus the tax shield). The dashed line shows what the slope would be in the case where only the FSE is active. The true tangent to $x^*(\gamma)$ at this point is essentially flat. This illustrates that the FSE can have the same magnitude as the forces that pull towards a negative lending response and can even overcome them. On this basis, we argue that the effect is quantitatively relevant.

Now let us consider instead an 8% capital requirement. The curve is downward sloping and steeper, which is more in line with conventional wisdom. Given that this percentage is the one mandated by Basel I, the regulation in place in most countries throughout the 1990's and most of the 2000's, this constitutes a plausible situation facing the banks before the global financial crisis. In this case, going from a capital requirement of 7% to 8% causes a lending

cut of 0.2% and an increase in lending spreads of 5bps. Note also the higher probability of a bailout in the right panel: a stronger composition effect is the root cause of the negative lending response.[19]

Discussion and sensitivity analysis We believe that our benchmark numbers for the calibration are plausible. However, they only constitute one example; we do not wish readers to take our results as a prediction that raising capital requirements today would necessarily cause most banks to slightly increase lending. Nor would we want to claim, on the basis of the paragraph above, that all banks would have shown steep negative lending responses in the run-up to the global financial crisis.

Rather we argue that lending responses are likely to show a lot of variation in both the time series and in the cross section, and in both sign and magnitude. To illustrate this, Figure 9 shows how the lending response of the representative bank in the benchmark example changes when we alter parameter values one at a time. It is possible to generate much steeper positive lending responses. One can, for instance, as in Panel a, make the legacy loans overvalued (think of a large stock of non-performing loans), or decrease the correlation between A and B (Panel b). And vice versa: undervalued legacy loans and higher correlation make the lending response more negative, and relatively steep in some cases. Combining the two, we find that even with a high correlation, there is positive response when legacy assets are overvalued (see Figure B2 in Online Appendix D).

Finally, in the same online appendix, we consider heterogeneity among banks and show that small differences in initial conditions, or bank-specific capital requirements, can lead to

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[19]Empirical evidence from pre-crisis sample periods generally points to a negative lending response (see for instance, Hancock and Wilcox (1994); Francis and Osborne (2012); Aiyar et al. (2014); **?**). Our model predicts that a negative lending response is more likely at low *levels* of the capital requirement. It is, therefore, conceivable that future empirical research, with sample periods under the stricter requirements of the new Basel III regime, will have different findings.

Figure 9: Sensitivity analysis on equilibrium lending
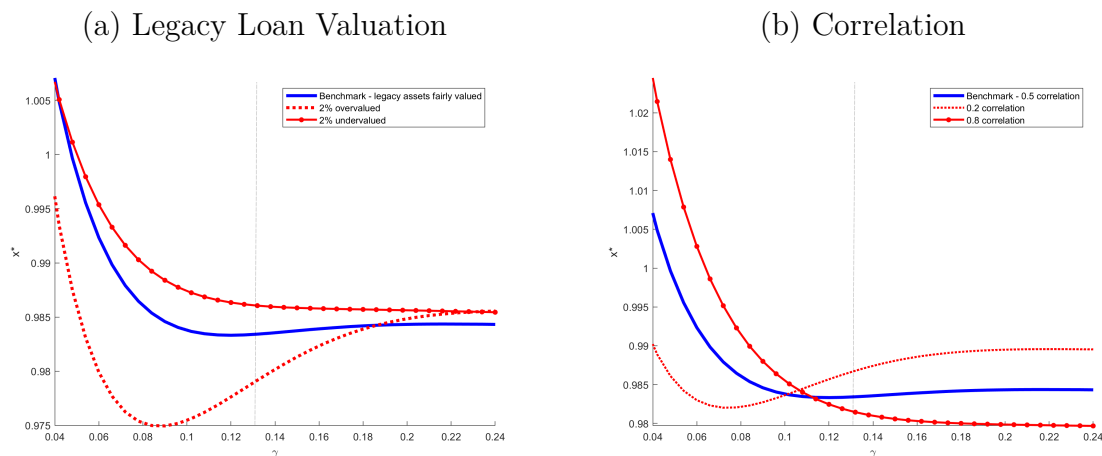
(a) Legacy Loan Valuation

(b) Correlation

Notes: The panels show equilibrium lending for the representative bank under the alternative levels of γ when the benchmark calibration defined in Table I is modified in a single dimension. All panels: the blue line denotes benchmark calibration; the vertical dashed line denotes the reference level of the capital requirement. Panel a: the red line with markers corresponds to legacy loans being undervalued by 2% ($E(A) = 1.02(1 + \rho)$), the red dashed line is legacy loans overvalued by 2% ($E(A) = 0.98(1 + \rho)$). Panel b: the red line with markers denotes when $log(A)$ and $log(B)$ have 0.8 correlation; the red dashed line is when $log(A)$ and $log(B)$ have 0.2 correlation.

very different responses, as competition provides a feedback effect.[20]

D. Empirical predictions and links to the literature

Our results show that one should not necessarily expect a stable relationship between capital requirements and lending. This may explain why empirical exercises looking at similar regulatory interventions but differing samples have come to different conclusions (compare for instance Gropp et al. (2019) and Bassett and Berrospide (2018) evidence on stress tests).[21] Extrapolating evidence from specific settings or time periods should be done with caution.

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[20]In addition, Online Appendix I discusses sensitivity over parameters governing the strength of the tax shield.

[21]They study stress-test-induced increases in the capital requirements in Europe in 2011 and the United States in 2013-2016, respectively. Both use types of difference-in-differences estimators where size based stress test eligibility criteria determines treatment. Despite the similar settings, the conclusions are very different: Gropp et al. (2019) find a reduction in lending, and Bassett and Berrospide (2018) find that, if anything, lending goes up.

In spite of the unstable relationship, we can use our model and calibration exercise to formulate empirical predictions:

1. **Banks that have a high risk of failure are likely to have a negative lending response because of the strength of the composition effect.** One implication of this is that if bankers wish to argue that higher capital requirements would lead to a substantial decrease in lending, they must also believe that the composition effect, and hence default probabilities, are large. This also means banks are receiving substantial subsidies in the first place.

2. **Banks for whom marginal loans are likely to deliver good cashflows when the rest of their assets perform badly are more likely to have a positive lending response due to the FSE.** This could occur because legacy loans and new loans are different in some respects; for instance, in the correlation of their returns or because legacy loans are overvalued in some way (in the spirit of panels A and B in Figure 9 above).[22] One particular case when such a situation may occur is after major economic crises, when new opportunities may open up even as banks are struggling with their legacy loans. Since new capital regulation typically arises after crises, this may be a fortuitous coincidence from a policy perspective. However, as we have argued, heterogeneity in cashflows may not just be an issue of legacy versus new loans. The bank may have sectoral heterogeneity in its lending opportunities that create differences in the cashflows among potential new loans.[23] Alternatively, a securities or investment

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[22]Our model also predicts that banks will overvalue new loans that are similar to legacy loans. Landier et al. (2015) provide evidence of this behaviour from a precrisis US subprime lender.

[23]One specific case is geographical variation in lending. For example, Puri et al. (2011) show that state banks in Germany that were heavily invested in US subprime loans cut back on loans to German retail borrowers during the 2007-2009 financial crisis. Given the relative safety of German retail borrowers we conjecture that the residual cashflows on these loans was very different from US subprime lending and a higher capital requirement may have prevented this effect.

banking division may generate cashflows orthogonal to the bank's loans.

3. **Within a portfolio, a difference-in-differences approach can identify the composition effect:** Looking at our model, imagine that the bank could make two types of new loans, and that the risk weight was raised on just one of them. The bank would do less of that type of lending relative to the other type. If this makes the bank safe enough, it could still expand its whole balance sheet, which would reflect a strongly positive FSE. Interestingly, if these loans were otherwise identical, the relative lending response would also be proportional to the composition effect.[24] This aligns with the empirical study of Behn et al. (2016), who show that banks cut back on loans in portfolios that face an increase in capital requirements *relative* to other similar loans in untreated portfolios.

4. **Targeted regulatory interventions can generate forced safety effects that affect all bank lending (and investment) decisions:** Consider a bank that has two main lines of business. Imagine that the regulator substantially restricts risk taking in one of them. This will make the bank safer and make it internalise the residual cashflows of the other line of business in more states. If those are positive (perhaps because this line of business generates relatively safe returns), the bank will expand in that dimension. This prediction is consistent with Acharya et al. (2018), which finds that when the Central Bank of Ireland imposed restrictions on the issuance of risky loans to urban borrowers, banks that were initially heavily exposed aggressively

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[24]To see this, imagine that new loans x_1 and x_2 have identical payoff functions and residual cashflows in all states of the world, and x_1 must be financed with γ_1 of capital. We have:

$$\frac{dx_1^*}{d\gamma_1} = \left(-(1-p^*) + p_{\gamma_1} z_0^*\right) / \left(-w_{x_1 x_1}^*\right)^{-1}, \frac{dx_2^*}{d\gamma_1} = \left(p_{\gamma_1} z_0^*\right) / \left(-w_{x_2 x_2}^*\right)^{-1},$$

where z_0^* is the expected residual cashflow along the default boundary for both loans. The assumptions mean that $w_{x_2 x_2}^* = w_{x_1 x_1}^*$ so the differential effect of γ_1 on x_1^* and x_2^* is proportional to the composition effect.

expanded their issuance of loans to safer borrowers, in rural counties.

VI.   Conclusion

This paper's contribution to the policy debate is to point out that capital being costly for banks does not imply a negative lending response to higher capital requirements. The forced safety effect can counteract the liability composition effect, which overturns conventional wisdom. Even if the FSE is not strong enough to dominate, a more conservative interpretation of our results is that FSE can be, in some circumstances, an important countervailing force that can mitigate the contractionary effect of tighter regulation.

The necessary ingredient to generate this effect in our model is heterogeneity in the bank's residual cashflows. Our calibration introduces this through differences between legacy loans and new loans. We view this as a form of heterogeneity that is relevant for banks that, after all, are going concerns with long-term loans. However, as we have argued, other sources of heterogeneity can make the FSE positive.

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Appendix

A.   Participation constraint and bank closure

In our model, the shadow value of initial capital is equal to the price of new capital. It is therefore irrelevant how much capital comes from new shareholders versus how much was already on the bank's books. This is why the value of κ does not affect the value of x^*.

Still, initial shareholders have the option to close the bank at date 1 and walk away with zero. This means that κ alters their participation constraint. To see this, note that the participation constraint is:

$$w^* = (\mathbb{E}\,[B]\,X^* - x^*) + (\mathbb{E}\,[A]\,\lambda - \lambda) + s^* + \kappa \geq 0. \tag{18}$$

Under our assumption that $\mathbb{E}[A] = 1$, the second term disappears, and the constraint is always satisfied ($X(0) = 0$, $s(0) \geq 0$ so at worst the constraint boils down to $\kappa \geq 0$). However, if $\mathbb{E}[A] < 1$, the participation constraint can be violated and κ becomes relevant. First, consider $\kappa \geq \gamma\lambda$. Here, closing the bank is never the best option for shareholders. This is because, under limited liability, even operating at $x = 0$ gives the bank's shareholders a positive payoff in expectation. However, if $\kappa < \gamma\lambda$, shareholders must first raise new capital if the bank is to operate. When the option value of operating the bank is low (i.e., when $\mathbb{E}[A]$ is low and new loans do not generate much surplus), operating may not be worth the cost of recapitalisation.

The participation constraint is then violated.[25]

Now, the key point we want to make is that when $\mathbb{E}[A] < 1$, increasing γ may make the bank close at date 1. Formally:

Proposition 3. *Assume $a_L, b_L > 0$. If $\kappa + (X(x_{MM}) - x_{MM}) < (1 - \mathbb{E}[A]) \lambda$, there exists a $\overline{\gamma} < 1$ such that for all $\gamma \geq \overline{\gamma}$, the bank closes at date 1.*

Proof. If $a_L, b_L > 0$, then $\forall x$, there exists a $\overline{\gamma} < 1$ such that $s(x) = 0$. Then, $x^* = x_{\mathrm{MM}}$ and the participation constraint (18) simplifies to $(X(x_{\mathrm{MM}}) - x_{\mathrm{MM}}) + (E[A] - 1) \lambda + \kappa \geq 0$. The condition in the Proposition ensures that it is violated. $\qquad \square$

To understand Proposition 3, first note that a high γ makes it more likely that $\kappa < \gamma\lambda$. Second, γ reduces the subsidy (see Equation 8), so that $s^* = 0$ for some sufficiently large γ. Given $s^* = 0$, if initial equity plus the surplus on new loans is insufficient to cover the expected losses on legacy assets, then the bank will shut down. The implication is that, for distressed banks, it is possible that $x^*(\gamma)$ is only upward sloping when $\gamma > \overline{\gamma}$ and the bank would always choose to close rather than increase lending in response to a requirement increase.

B.   Proofs

Lemma. *1.* (The sign of the lending response)

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[25]To see this, rewrite the participation constraint as:

$$\underbrace{\int\int_{\{A,B\}\notin\Delta^*} (BX^* + A\lambda - (1 - \gamma)(x^* + \lambda)) f(A, B) dA dB}_{\text{option value of operating}} \geq \underbrace{(x^* + \lambda)\gamma - \kappa}_{\text{new capital needed}} .$$

50

$$\frac{dx^*}{d\gamma} \lesseqqgtr 0 \Leftrightarrow s^*_{x\gamma} \lesseqqgtr 0.$$

Proof. This result follows directly from the implicit function theorem applied to the first order condition. □

Proposition. *1*

$$s^*_{x\gamma} = \underbrace{-(1 - p^*)}_{<0} + \underbrace{p^*_\gamma z^*_0}_{FSE \gtrless 0}$$

where

$$z^*_0 \equiv \mathbb{E}\left[Z^* \mid \mathbb{N}^*\right] = \frac{\int_{b_L}^{b^*_0(a_L)} Z^* f(a^*_0(B), B) dB}{\int_{b_L}^{b^*_0(a_L)} f(a^*_0(B), B) dB}$$

is the expected marginal residual cash flow conditional on being on the equilibrium default boundary. It is the case that (i) z^*_0 can be positive; (ii) this implies a positive forced safety effect; (iii) and can lead to a positive lending response: $s^*_{x\gamma} > 0$.

Proof. The subsidy is

$$s = \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} \left((1 - \gamma)(x + \lambda) - BX - A\lambda\right) f(A, B) dA dB.$$

Define $\Omega(x, B) \equiv \int_{a_L}^{a_0(B)} \left((1 - \gamma)(x + \lambda) - BX - A\lambda\right) f(A|B) dA$. So

$$s = \int_{b_L}^{b_0(a_L)} \Omega(x, B) f(B) dB$$

The marginal subsidy is given by

$$s_x = \frac{\partial b_0(a_L)}{\partial x} \underbrace{\Omega(x, b_0(a_L))}_{0} + \int_{b_L}^{b_0(A_L)} \Omega_x(x, B) f(B) dB,$$

with

$$\Omega_x(x, B) = \frac{\partial a_0(B)}{\partial x} \underbrace{((1-\gamma)(x+\lambda) - BX - a_0(B)\lambda)}_{0} - \int_{a_L}^{a_0(A_L)} \underbrace{BX_x - (1-\gamma)}_{Z(x,B)} f(A)dA.$$

So

$$s_x = -\int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} Z(x, B)f(A, B)dAdB = -(1-p)z_\Delta,$$

where

$$z_\Delta = \mathbb{E}\left[Z \mid \Delta\right] = \frac{\int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} Z(x, B)f(A, B)dAdB}{\int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} f(A, B)dAdB}.$$

Alternatively we can write: $s_x = (1-p)(1-\gamma) + \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A, B)dAdB$. The cross partial derivative is therefore

$$s_{x\gamma} = -p_\gamma(1-\gamma) - (1-p) - \frac{\partial}{\partial\gamma} \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A, B)dAdB.$$

Since $a_0(b_0(a_L)) = a_L$,

$$\frac{\partial}{\partial\gamma} \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A, B)dAdB = -\int_{b_L}^{b_0(a_L)} \frac{(x+z)}{z} BX_x f(a_0(B), B)dB.$$

Also,

$$p_\gamma = \frac{(x+z)}{z} \int_{b_L}^{b_0(a_L)} f(a_0(B), B)dB,$$

so we have

$$s_{x\gamma} = -(1-p) + p_\gamma z_0 \; ,$$

where

$$z_0 \equiv \mathbb{E}\left[Z \mid \backslash\right] = \frac{\int_{b_L}^{b_0(a_L)} (BX_x - (1-\gamma)) f(a_0(B), B) dB}{\int_{b_L}^{b_0(a_L)} f(a_0(B), B) dB}.$$

The remainder of the proof is by example. We provide examples of positive lending responses in Section V. Now, having proved (iii) by example, (ii) must be true, and, therefore, (i), as well, since the composition effect is always negative. □

Proposition. 2.

$$s_{x\gamma_a}^* = \underbrace{p_{\gamma_a}^* z_0^*}_{FSE} \tag{19}$$

$$s_{x\gamma_b}^* = -(1-p^*) + \underbrace{p_{\gamma_b}^* z_0^*}_{FSE}. \tag{20}$$

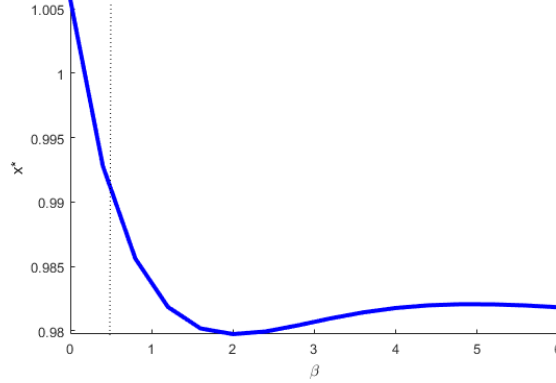Both $s_{x\gamma_a}^*$ and $s_{x\gamma_b}^*$ can be positive.

Proof. Starting from

$$s = \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} \left((1-\gamma_b)x + (1-\gamma_a)\lambda - BX - A\lambda\right) f(A, B) dA dB,$$

we get:

$$s_x = (1-p)(1-\gamma_b) + \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A, B) dA dB;$$

$$s_{x\gamma_b} = -(1-p) - p_{\gamma_b}(1-\gamma_b) - \frac{\partial}{\partial\gamma_b} \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A, B) dA dB.$$

$$s_{x\gamma_a} = -p_{\gamma_a}(1-\gamma_b) - \frac{\partial}{\partial\gamma_b} \int_{b_L}^{b_0(a_L)} \int_{a_L}^{a_0(B)} BX_x f(A, B) dA dB.$$

53

Figure A1: The lending and the capital requirement on new loans

Notes: This figure plots x^* as a function of β, the risk weight on new loans (which is set to 0.5 in the baseline calibration). All other parameters adhere to the calibration in Section V, in particular $\gamma = 13\%$.

Noting that $a_0(B) \equiv \frac{((1-\gamma_b)x+(1-\gamma_a)\lambda)-BX}{\lambda}$ and rearranging in a similar way as in the proof of Proposition 1 gives Equations (19) and (20). That $s^*_{x\gamma_a}$ can be positive follows from Proposition 1 as $s^*_{x\gamma} > 0 \implies s^*_{x\gamma_a} > 0$. We prove that $s^*_{x\gamma_b}$ can be positive by example. Recall that in our calibration in Section V, we have $\gamma_b = \beta\gamma$. Figure A1 presents x^* as a function of β holding γ fixed at its calibration of 13% in Table I. All other parameters also adhere to the Table I calibration. As can be seen, at the baseline level of $\beta = 0.5$, lending is decreasing in the capital requirement on new loans. But for the range of β between 2 and 5 (or γ_a between 26% and 65%), lending is increasing in the capital required against new loans. □

C.   Solving the problem with discrete loans

The numerical example in Figure 5 is set up and solved as follows. We set $n = 20$ and $m = 100$, each state being equiprobable. Let B be the $n \times m$ matrix of returns. We first independently draw each element of B from a *Lognormal*$(0, 1)$. Then, we rescale payoffs to

assign loans different NPV: We draw 10 numbers from a $U(0.95, 1)$ and 10 from a $U(1, 1.05)$. Using these numbers (in ascending order), we rescale the columns in B such that the averages of these columns match these numbers. We then solve for the optimal portfolio for different γ by grid search over all possible portfolio combinations.

D.   Additional Numerical Results

Discrete Loans and Heterogeneity in Capital Requirements We show here an example of a positive lending response when the bank can only choose to make identical new loans that face heterogeneous capital requirements. We reconsider the model in Section C but impose that the matrix B is composed of n identical $m \times 1$ columns such that the loans always deliver identical payoffs. We then rescale B such that the loans all have a small positive NPV of 0.2%. Last, loan i is attached a specific risk weight such that its effective capital requirement γ_i, is given by $\gamma_i \equiv \frac{2i}{20} \times \gamma$. This corresponds to risk weights from 10% to 200% in 10% increments. Again we solve for the optimal portfolio through grid search.

Figure A2 presents the relationship between total lending and γ for a randomly drawn B. The black line plots the total level of lending and a black circle indicates whether a particular loan is financed.

First, note that since the payoffs across loans are identical, for any given state j, loan 1 has the smallest realised residual cashflow. Using our definition of the marginal subsidy in a discrete state space (Equation 15), this means that $s_1^* \geq s_2^* \geq \ldots \geq s_{20}^*$. Given that all loans are equal NPV, this generates a clear pecking order: loans with the lowest risk weight are always financed first.

Second, a clear U-shape emerges. To understand where the familiar U-shape emerges from first consider the two extremes. If the capital requirement, γ, is very high, the bank

Figure A2: Numerical Example: Discrete New Loans with Risk Weight Heterogeneity
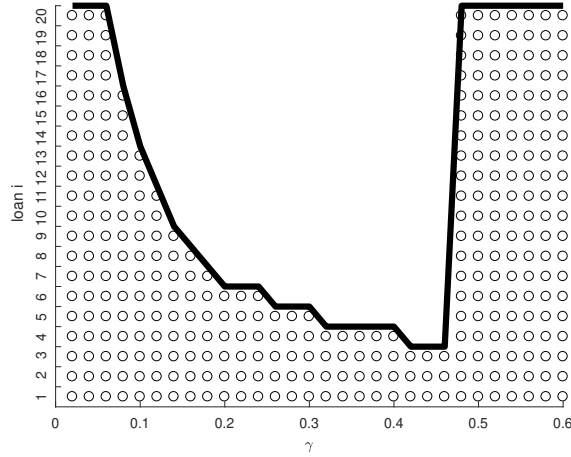
Notes: The figure provides an example of optimal portfolio choices for different values of γ, with $n = 20$ assets, and $m = 100$ states. On the x-axis is the capital requirement at 2 percentage point increments. On the y-axis is the corresponding portfolio choice. Loans are sorted in increasing order, according to their risk weight (which is given by $\frac{2i}{20}$). That is, loan #20 has the highest risk-weight. For a given capital requirement, a circle means that the corresponding loan is in the optimal portfolio of the bank. The solid line indicates how many loans are in the portfolio.

never defaults, this means that $s_i^* = 0, \forall i$ and since all loans are positive NPV they are all financed. Now consider the other extreme when the capital requirement is very low: now the bank defaults with positive probability and even the 20th loan, with the highest risk weight, generates negative residual cashflows in the default states. This means that $s_i^* > 0, \forall i$ and so all loans are financed. Now consider an intermediate level of γ. The bank will still default, however the capital requirement will be sufficiently high such that s_i^* is negative, and greater than the NPV in absolute terms, for loans with a high risk weight. These loans are therefore not financed.

The Role of Correlation   To illustrate the link between the correlation of assets' payoffs and the distortion arising from government guarantees, Figure A3 revisits the numerical example in Figure 5. It presents for each loan $i \in \{1, 2, \ldots, 20\}$ and two different levels of the capital requirement ($\gamma_b = 6\%$ and $\gamma_b = 40\%$): (i) the correlation between loan $i's$ payoff

56

Figure A3: Numerical Example: The link between correlation and the marginal subsidy
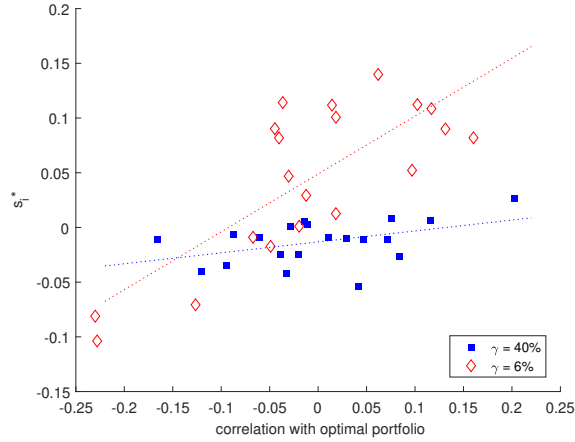
Notes: The graph presents for loans $i = \{1, 2, \ldots, 20\}$ in Figure 5 the relationship between (i) the correlation between loan $i's$ payoff and the aggregate payoff on all other loans in the bank's portfolio (x-axis); and (ii) the marginal subsidy, s_i^* (y-axis). Red diamonds indicate observations when $\gamma_b = 6\%$ the blue squares when $\gamma_b = 40\%$.

and the aggregate payoff on all other loans in the bank's portfolio; versus (ii) the marginal subsidy, s_i^*.

Three points stand out. First, the correlation is positively related to the marginal subsidy. If the loan is highly correlated with the rest of the bank's portfolio it is more likely to have low residual cashflows when the bank is in default resulting in a higher s_i^*.

Second, however, the correlation is not a sufficient statistic for s_i^*. There are occasions when a loan has a high s_i^* even though the loan's correlation with the overall portfolio is relatively low. This is because s_i^* depends only on the residual cashflows in the default region. So this outcome is perfectly plausible if the loan is only weakly related to the overall portfolio most of the time but generates low residual cashflows in expectation when the bank fails.

Third, the relationship between the correlation and s_i^* and is weakening in the level of the capital requirement. The intuition for this is straightforward: the higher the capital requirement, the safer the bank and thus default is more of a tail event. This means that the

57

overall correlation, across all states of the world, between cashflows is less informative about whether loan i will perform badly in the default region.

Internet Appendix for The Forced Safety Effect: How Higher Capital Requirements Can Increase Bank Lending [26]

I. The Model with Taxes

For simplicity, we consider the example in Section D of the main text (with $B = 1$). Adding the tax and assuming that households have an opportunity cost of funds $1 + \rho > 1$ means the bank's objective function now reads:

$$w = \underbrace{X - (1 + \rho)x}_{\text{economic surplus}} + \underbrace{\int_{a_L}^{a_0} \left((1 - \gamma)(1 + \rho)(x + \lambda) - X - A\lambda \right) f(A)dA}_{\equiv s(x,\gamma), \text{ i.e. the implicit subsidy}}$$

$$- \underbrace{\tau \int_{a_1}^{a_H} \left(X + A\lambda - (1 + (1 - \gamma)\rho)(x + \lambda) \right) f(A)dA}_{\equiv t(x,\gamma), \text{ i.e. the expected tax bill}} + \kappa,$$

where $a_1 = \frac{1}{\lambda}\left((1 + (1 - \gamma)\rho)(x + \lambda) - X \right)$ denotes the threshold such that the bank has positive taxable income if $A > a_1$. The expected tax bill is then given by $t(x, \gamma)$, all other terms derive directly from the main text, except that they now account for ρ.

The tax adds a new wedge in the first order condition:

$$X_x^* - (1 + \rho) + s_x^* - t_x^* = 0, \tag{B1}$$

where $t_x = q\tau \left(X_x - (1 + (1 - \gamma)\rho) \right)$ with $q = \int_{a_1}^{a_H} f(A)dA$ is the probability that the bank pays tax. Intuitively, $t_x^* > 0$. Ceteris paribus, the tax reduces equilibrium lending.

Now, the question we are interested in is how the tax, and the tax shield, affect the shape of the lending response. The reasoning follows the same logic as before: what happens depends on the cross-partial derivatives. But now, we have:

$$w_{x\gamma}^* = s_{x\gamma}^* - t_{x\gamma}^*.$$

The introduction of a tax has two effects on $w^*_{x\gamma}$:[27] a direct effect that materialises through the appearance of a term $t^*_{x\gamma}$, and an indirect effect that works through a change in the value of $s^*_{x\gamma}$.

The direct effect: a tilt We have argued that the main effect of tax shield on the lending response is a tilt (see Figure 7 of the main text). It emerges because $t^*_{x\gamma} > 0$.[28] Intuitively, the wedge driven by taxation is increasing in the capital requirement due to the deductibility of interest on deposits. As γ gets sufficiently high and the bank becomes safe, $s_x = 0$, and lending converges to the downward sloping, near diagonal line that makes up the right most region of the blue dotted curve in Figure 7. The slope of $x^*(\gamma)$ is proportional to $t^*_{x\gamma}$ in this region and comparing $x^*(\gamma)$ to x_{MM} makes the tilt more evident.

The indirect effect The tax, and the tax shield also affect $s^*_{x\gamma}$ because they affect x^*. Consistent with the tilt described above, the tax decreases x^*, but the effect is attenuated by the shield. There are three effects on $s^*_{x\gamma}$. First, less (positive NPV) lending reduces profitability, which increases the probability of default $(1 - p^*)$. Second, it affects p^*_γ. Third, a lower x^* raises the equilibrium marginal return to lending X^*_x, which therefore raises Z^* and makes the forced safety effect more positive.

The tax and tax shield It is important to differentiate between taxes and tax shields. Absent the deductibility of interest payments, $t_{x\gamma} = 0$, and the tax only matters for the lending response through its impact on $s^*_{x\gamma}$. Now, introducing a tax shield on deposits means both that $t^*_{x\gamma} > 0$ and that the marginal tax rate, t^*_x, is lower, which in turn potentially

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[27]The tax also affects w^*_{xx}, and therefore, the magnitude of the lending response. As before, the lending response's sign is solely determined by $w^*_{x\gamma}$.

[28]$t^*_{x\gamma} = q^*\tau\rho + q^*_\gamma\tau\left(X^*_x - (1 + \rho(1 - \gamma))\right) > 0$

reduces the impact of the tax on $s^*_{x\gamma}$.

Risky new loans Introducing the corporate income tax (and the tax shield) when new loans are risky (as in Section III of the main text) creates additional effects. Given that the expression for $t(x, \gamma)$ is very similar to that for $s(x, \gamma)$, the derivations are very similar, and it should not be too surprising that counter-intuitive cases can arise for some sets of parameter values. In particular, it is possible that $t^*_{x\gamma} < 0$ (which would then contribute to a positive lending response) and $t^*_x < 0$.[29] However, these are specific cases that are arguably unlikely to be relevant in practice.

Taxes in our calibration exercise. Figure B1 illustrates the role of taxes in our calibration exercise. The left panel shows the effect of a cut in the tax rate from 35% to 21% mimicking the 2017 US tax reform. The main effect is to shift the level of lending downwards; however, the difference in the slope of $x^*(\gamma)$ is quantitatively small. This serves to illustrate that in our benchmark calibration the presence of the tax is does not have a meaningful impact on $dx^*/d\gamma$. In panel (b), we show the effect of interest rates. Given our scaling assumption on returns, interest rates are only relevant through the tax shield and as the figure illustrates they have a more meaningful impact on the slope of $x^*(\gamma)$ tilting the function in line with the logic described above.

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[29]A negative marginal tax requires that the marginal loan transfers cashflows from states where the rest of the bank's assets are generating a tax profit to states where there is a tax loss. Hence, the marginal loan then reduces the expected tax bill and, absent other frictions, the tax can lead to the bank making negative NPV loans. In turn, a change in the capital requirement shifts the boundary between a taxable losses and profits. If the marginal loan generates a large tax loss on this boundary, then it is possible that $t^*_{x\gamma} < 0$.

Figure B1: Taxes, interest rates and equilibrium lending
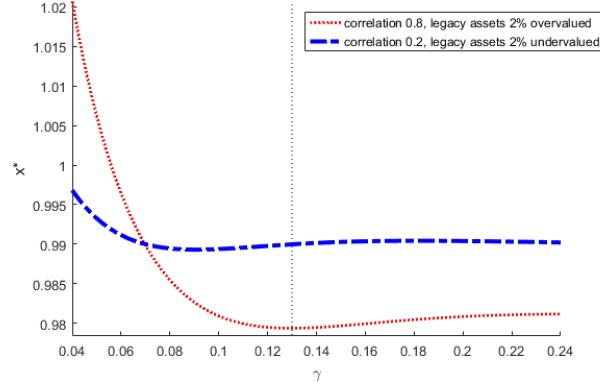
(a) Taxes

(b) Interest Rates
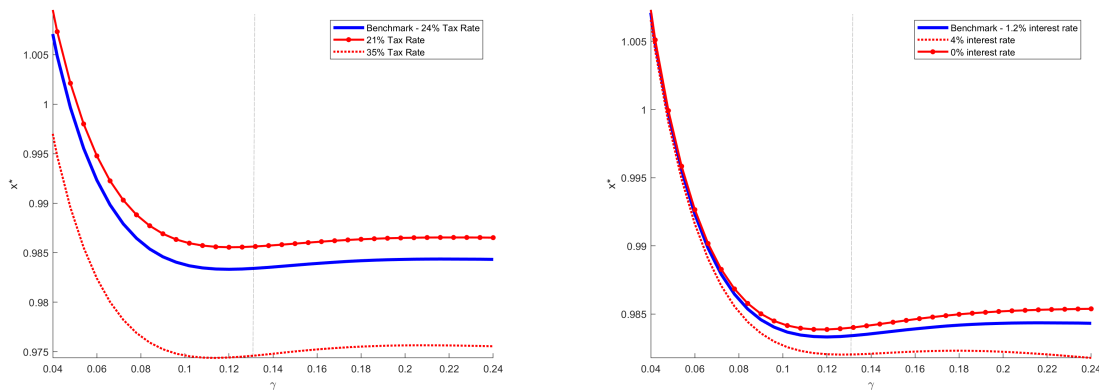
Notes: The panels show equilibrium lending for the representative bank under the alternative levels of γ when the benchmark calibration defined in Table I is modified in a single dimension. All panels: the blue line denotes benchmark calibration; the vertical dashed line denotes the reference level of the capital requirement. Panel a: the red line with markers is $\tau = 0.21$, the red dashed line is $\tau = 0.35$. Panel d: the red dotted line is $\rho = 0.0$, the red dashed line is $\rho = 0.04$.

II. Further Numerical Results from the Calibrated Model

Correlation and overvaluation In Section V of the main text, we claim that even with a large correlation between A and B, the lending response can be positive when legacy assets are overvalued. Figure B2 illustrates this. In addition, it shows that if legacy assets are undervalued but the correlation between A and B is low, it is also possible for the lending response to be negative.

Capital requirements at the bank and banking system level Our numerical exercise Section V of the main text considered ν identical banks all facing an identical capital requirement. In general, the lending of an individual bank is more sensitive to an idiosyncratic change in its individual capital requirement. This is simply because an individual bank faces a much shallower residual demand curve for loans than the total banking system (Kisin and

5

Figure B2: Sensitivity analysis: combinations of correlation and overvaluation

Notes: The figure show equilibrium lending for the representative bank under the alternative levels of γ when the benchmark calibration defined in Table I of the main text is modified on two dimensions. The blue dashed line denotes line is when $log(A)$ and $log(B)$ have 0.2 correlation and legacy loans are undervalued by 2% ($E(A) = 1.02(1 + \rho)$). The red dotted line is when $log(A)$ and $log(B)$ have 0.8 correlation and legacy loans are overvalued by 2% ($E(A) = 0.98(1 + \rho)$). The vertical dotted line denotes the reference level of the capital requirement.

Manela (2016) make a similar point). To show this, Figure B3 presents (in the red dotted line) the optimal level of lending for an individual bank when its requirement is changed, assuming all other banks lend at a rate given by the initial equilibrium when $\gamma = 0.13$. The blue solid line shows the symmetric equilibrium for comparison. As can be seen, the dotted curve is much steeper.

Bank heterogeneity Heterogeneity among banks can also substantially alter how banks respond to a change in aggregate capital requirements. To illustrate this, take one potential dimension in which banks could be heterogeneous: legacy loan valuation. Imagine that, instead of all banks having fairly valued legacy loans, as in the baseline calibration, there are two equally sized groups of banks: strong banks, whose assets are in fact better than their valuation (i.e. they are undervalued) and weak banks that have overvalued legacy loans (or unrecognised losses). Still, on average in the economy, legacy loans are fairly valued.

The two groups of banks will typically have different levels of lending. Broadly speaking,

Figure B3: The lending for an individual bank versus the banking system
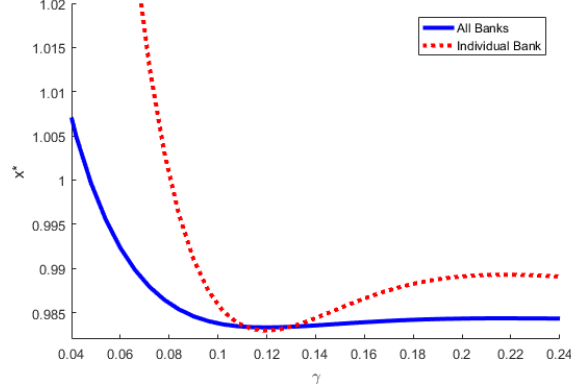
Notes: We use the benchmark calibration as in Table I of the main text. The blue line shows results for $x^*(\gamma)$ assuming all banks face the same capital requirement under the benchmark calibration. The red dotted line shows the optimal lending decision for a single bank facing a change in its own capital requirement, and we modify equation (17) such that x' (lending at other banks) is held fixed at $(\nu - 1)x^*$ evaluated at $\gamma = 0.13$.

strong banks will be safer and so the marginal subsidy is likely to be smaller in absolute magnitude. However, since the marginal subsidy can take both signs, either group could lend more than the other.

The banks will also respond differently to a change in their capital requirement. Put differently, each group has a different "U-shape" and, at a given γ, these shapes have different slopes with potentially different signs. As we describe below, competition further complicates the situation.

Such environment is highly non-linear. So aggregate lending, and the aggregate lending response to a change in the capital requirement, is affected by heterogeneity. In turns out that, compared to our representative bank benchmark case, introducing heterogeneity can either magnify, mitigate, or even flip the sign of the aggregate lending response.

Table Bi provides an example of how heterogeneity can play out. It shows the change in lending response following a capital requirement increases of 1 and 2 percentage points respectively, with weak (strong) banks having legacy loans that are 1% overvalued (underval-

7

ued). All other parameters are in line with Table I of the main text, and the initial capital requirement is 13%.

The responses for the representative bank are both small (they correspond to our benchmark result, see Figure 8 of the main text). It is immediately obvious that, with heterogeneity, the two groups of banks are each more sensitive to the requirement. One interpretation of what happens is the following. Weak banks are on the upward sloping portion of their U-shape. Therefore, their individual response is to increase lending. For the sake of simplicity, assume that stronger bank's individual response is initially nil. Now we can consider the effect of competition. The lending increase by weak banks decreases the residual demand facing strong banks. As a result strong banks cut lending. But this increases the residual demand facing weak banks, so they lend more (which also makes them safer and feeds the FSE), and so on and so forth. In equilibrium, we end up with polarised responses.

While this qualitative description is accurate for both columns in Bi, between the two cases the magnitudes and aggregate response differ markedly. When capital requirements are increased by 2 percentage points, this has a dramatic effect on weak banks: they become much safer, and end up lending much more. Competition amplifies this further and we see a 2.9% increase in lending. While this crowds out strong bank lending, the net effect is to boost aggregate lending by 0.06%, double the increase in the representative case. In contrast, a 1 percentage point increase gives a smaller boost to weak banks. Since strong banks seem to be initially on a downward sloping portion of their U-shape their response actually dominates, and overall aggregate lending falls.

Table Bi: Heterogeneous lending responses to a capital requirement Increase

	(1) 1% increase	(2) 2% increase
Representative Bank Response (%)	**0.021**	**0.034**
Aggregate Lending (%)	**-0.014**	**0.062**
Weak Banks: 1% Overvalued (%)	0.084	2.873
Strong Banks: 1% Undervalued (%)	-0.046	-0.642

Notes: The table shows the lending response to a 1 percentage point (column 1) and 2 percentage point (column 2) capital requirement increase starting at $\gamma = 13\%$. The first row shows how the representative bank responds under the benchmark calibration. The final three rows considers how the responses are altered by heterogeneity among banks. Specifically, we assume that half of banks have legacy loans that are 1% overvalued ($E(A) = 0.99(1 + \rho)$), and half have legacy assets that are 1% undervalued ($E(A) = 1.01(1 + \rho)$). We denote these weak and strong banks respectively; and the second and third columns present the lending response in each group. Note that at the initial value of $\gamma = 13\%$, strong banks lend more, hence the aggregate lending response places more weight on the reaction of strong banks. All other parameters are calibrated as in Table I of the main text.