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THE RADIO AND INFRARED SPECTRUM OF EARLY-TYPE STARS UNDERGOING MASS LOSS

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SUMMARY

A unified model is presented for the radio and infrared spectrum of early-type stars surrounded by a gaseous ionized envelope resulting from mass loss. The cases of uniform mass loss (i) at constant velocity, and (ii) with accelerative effects taken into account, are treated. It is shown that the radio and infrared free–free spectra are predicted to be of the form $S_\nu \propto \nu^{0.6}$ except (typically) in the near infrared where the spectrum will become flatter. Various effects which may cause a deviation from a 0·6 spectral index are considered. Finally, it is shown how radio and infrared flux densities may be used to derive mass loss rates and to obtain lower limits to the number of ionizing photons emitted per second by the underlying stars.

1. INTRODUCTION

Recently there has been considerable interest in radio and infrared emission from early-type stars undergoing mass loss (e.g. in the radio region: Wade & Hjellming (1972); Braes, Habing & Schoenmaker (1972); Altenhoff & Wendker (1973); Hughes & Woodsworth (1973); Purton, Feldman & Marsh (1973); Seaquist & Gregory (1973); and Wright *et al.* (1974); and in the infrared: Woolf, Stein & Strittmatter (1970); D. A. Allen (1973); Gehrz, Hackwell & Jones (1974); Cohen, Barlow & Kuhi (1975) and Barlow & Cohen (1975, in preparation)). The radiation from many of these objects is believed to be due to thermal free–free emission from circumstellar ionized gas, since, for those objects detected in the radio region, the log flux density versus log frequency plot exhibits a positive slope, and because in many cases dust emission does not appear to be responsible for the excess emission detected in the infrared region. However, in both the radio and infrared ($\sim 10\ \mu$) regions, the flux density–frequency relation is consistently found to be intermediate between that expected for an optically thin homogeneous plasma ($S_\nu \tilde{\propto} \nu^{-0.1}$) and that for an optically thick plasma ($S_\nu \propto \nu^{+2}$).

For the object V1016 Cyg, Seaquist & Gregory (1973) have shown by an approximate argument that the assumption of a spherically symmetric but inhomogeneous plasma distribution leads to a spectral index intermediate between −0·1 and +2, and by model fitting have deduced a mass loss rate for this object. In this paper we show that a more exact treatment of the spectral flux distribution produced by an ionized, uniform, spherically symmetric mass loss flow leads to a spectrum essentially of the form $S_\nu \propto \nu^{0.6}$ in the radio and infrared regions, even if acceleration of the gas is taken into account. This intermediate slope results because at any frequency one is essentially seeing emission from gas down to a level at which the gas becomes optically thick. At higher frequencies one always sees down to deeper levels (higher densities) and consequently the total mass of gas contributing to the emission increases with increasing frequency.

We show that this simple $S_\nu \propto \nu^{0\cdot6}$ spectrum should apply to a variety of objects over a wide range of frequencies and recent infrared observations of some early-type stars are quoted to support the view that a simple linear spectrum connects the radio and infrared regions. It is argued that deviations from a 0·6 spectral index in the radio region may be caused either by variability due to non-uniform mass loss rates, or by an increasing fraction of neutral gas with distance (in the form of condensations) in the region of the flow responsible for the radio emission.

Finally, general expressions are derived in terms of observable quantities for the mass loss rate $\dot{M}$, the electron scattering optical depth, τ_e, and the number of ionizing photons required to be emitted per second by a star with the appropriate radio and infrared spectrum.

2. DERIVATION OF THE SPECTRUM

We consider first a 'uniform flow' model for mass loss from a star, in which ionized gas, originating at some radius R_c, is ejected at a uniform rate with constant velocity v_∞. The mass loss rate is then related to the gas number density n by

$$\dot{M} = 4\pi r^2 n \mu m_H v_\infty \tag{1}$$

where r is the radial distance from the centre of the star, μ is the mean atomic weight of the gas and m_H is the mass of a hydrogen atom. Therefore

$$n = \frac{A}{r^2} \tag{2}$$

where

$$A \equiv \frac{\dot{M}}{4\pi\mu m_H v_\infty}.$$

Now consider a thin cylindrical shell of the outflowing gas of projected radius q (see Fig. 1). The intensity of radiation emerging from this cylindrical shell is

$$\begin{aligned} I(\nu, T) &= \int_0^{\tau_{max}(q)} B(\nu, T)\, e^{-\tau}\, d\tau \\ &= B(\nu, T)\{1 - e^{-\tau_{max}(q)}\} \end{aligned} \tag{3}$$

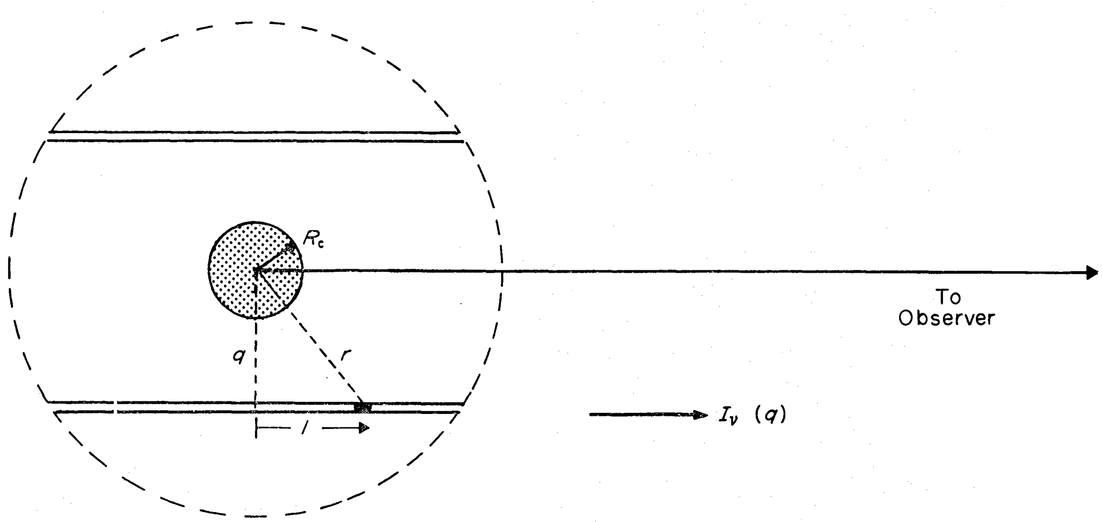

FIG. 1. *Geometry of the model.*

where $B(\nu, T)$ is the Planck function for frequency ν and temperature T, and $\tau_{\max}(q)$ is the total optical depth along the cylindrical shell. We have assumed a constant temperature T for the gas throughout.

The flux received by the observer from this infinitesimal shell is

$$dS_\nu = \frac{B(\nu, T)}{D^2}\{1 - e^{-\tau_{\max}(q)}\}\, 2\pi q\, dq \tag{4}$$

where D is the distance of the object.

Using the expression for the linear free–free absorption coefficient given by C. W. Allen (1973), we have,

$$\tau_{\max}(q) = K(\nu, T)\int_{-\infty}^{+\infty} n^2\, dl = \frac{K(\nu, T)\,\gamma A^2}{q^3}\cdot\frac{\pi}{2} \quad \text{for} \quad q \geqslant R_c \tag{5a}$$

and

$$\tau_{\max}(q) = K(\nu, T)\int_{\sqrt{R_c^2 - q^2}}^{\infty} n^2\, dl = \frac{K(\nu, T)\,\gamma A^2}{q^3}\,\frac{\pi}{2} \times\left\{\frac{1}{2} - \frac{1}{\pi}\arccos\left(\frac{q}{R_c}\right) - \frac{1}{\pi}\frac{q}{R_c}\left(1 - \frac{q^2}{R_c^2}\right)^{1/2}\right\} \quad \text{for } q < R_c \tag{5b}$$

where

$$K(\nu, T) = 3{\cdot}7 \times 10^8\{1 - \exp(-h\nu/kT)\}\, Z^2 g(\nu, T)\, T^{-1/2}\nu^{-3}$$

in cgs units; the ion number density is assumed equal to the total gas number density n; and the electron number density is equal to γ times the ion number density.

Equation (4) may now be integrated to obtain the total flux

$$S_\nu = \frac{B(\nu, T)}{D^2}.2\pi\left(\frac{\pi\gamma A^2 K(\nu, T)}{2}\right)^{2/3}.\left\{\int_{y_c}^{\infty} y(1 - \exp(-1/y^3))\, dy + f(y_c)\right\} \tag{6}$$

where

$$y \equiv q\left\{\frac{2}{\pi\gamma A^2 K(\nu, T)}\right\}^{1/3}, \quad y_c \equiv R_c\left\{\frac{2}{\pi\gamma A^2 K(\nu, T)}\right\}^{1/3}$$

and

$$f(y_c) \equiv \int_0^{y_c} y\left(1 - \exp\left[-\frac{1}{y^3}.\left(\frac{1}{2} - \frac{1}{\pi}\arccos\left(\frac{y}{y_c}\right) - \frac{1}{\pi}\frac{y}{y_c}\left(1 - \frac{y^2}{y_c^2}\right)^{1/2}\right)\right]\right) dy.$$

The term $f(y_c)$ corresponds to the contribution to the total flux by the cylinder of gas (of radius R_c) projected against the star. For large enough mass loss rates or low enough frequencies (see below), y_c tends to zero and thus $f(y_c)$ also tends to zero. We then obtain

$$S_\nu = 1{\cdot}33.2\pi.\left(\frac{\pi}{2}\right)^{2/3}\gamma^{2/3}\frac{A^{4/3}}{D^2}.B(\nu, T)\, K^{2/3}(\nu, T). \tag{7}$$

In the radio and infrared regions of the spectrum where $h\nu \ll kT$, equation (7) can be expressed in convenient units as

$$S_\nu = 23{\cdot}2\left(\frac{\dot{M}}{\mu v_\infty}\right)^{4/3}\frac{\nu^{2/3}}{D^2}\gamma^{2/3} g^{2/3} Z^{4/3}\ \text{Jy}\dagger \tag{8}$$

† 1 Jy = 10^{-26} W m^{-2} Hz^{-1}.

where $\dot{M}$ is in $M_\odot$/yr, D is in kpc, v_∞ is in km s^{-1}, ν is in Hz and g is the Gaunt factor. This flux, S_ν, depends only very weakly (through the Gaunt factor) on the actual value assumed for the constant temperature of the gas.

From equation (6) we have

$$S_\nu \propto \nu^{2/3} \tag{9}$$

neglecting the frequency dependence of the Gaunt factor. Inclusion of the variation of this factor with frequency in the radio region (C. W. Allen 1973) flattens the spectrum to

$$S_\nu \propto \nu^{0\cdot6} \tag{10}$$

while in the infrared the Gaunt factor varies much less than in the radio, so that relation (9) is essentially unaltered.

It can easily be shown that the result given by equation (7) is equivalent to receiving, at any frequency ν, the integrated flux emitted exterior to a radial distance, $r = R(\nu)$, corresponding to a radial optical depth from infinity of $\tau_\nu = 0\cdot244$. This characteristic radius of the emitting region is given by

$$R(\nu) = 2\cdot8\times10^{28}\gamma^{1/3}g^{1/3}Z^{2/3}T^{-1/2}\left(\frac{\dot{M}}{\mu v_\infty \nu}\right)^{2/3} \text{cm} \tag{11}$$

where T is in K, $\dot{M}$ is in $M_\odot$/yr, ν is in Hz and v_∞ is in km s^{-1}.

At a sufficiently high frequency ν_c, $R(\nu_c)$ equals the inner boundary radius of the expanding envelope R_c. In the uniform flow model considered here the density of emitting gas becomes zero inside R_c (ignoring the presence of the star). At frequencies higher than ν_c the whole shell is optically thin and there is no further increase in the flux with frequency. The spectrum will therefore begin to flatten with increasing ν as ν_c is approached and will remain flat until the Wien turnover. From (11) the value of ν_c is given by

$$\nu_c = 4\cdot7\times10^{42}.\frac{\gamma^{1/2}g^{1/2}Z}{T^{3/4}R_c^{3/2}}.\frac{\dot{M}}{\mu v_\infty} \text{ Hz.} \tag{12}$$

Additionally, close to the stellar surface one may expect the assumption of constant outflow velocity to break down since the gas will be undergoing the acceleration (presumably radiative) which takes it from rest to its final constant velocity, v_∞.

We therefore consider the case in which the outflowing gas is accelerated according to

$$\ddot{r} = (f-1)\frac{GM_*}{r^2}, \quad f>1 \tag{13}$$

where M_* is the mass of the central star; which leads to a density distribution of the form

$$n = \frac{A}{r^{3/2}(r-R_c)^{1/2}} \tag{14}$$

where R_c, the radius at which the outflowing gas would have zero velocity, is essentially the surface radius of the star, since the velocity at the surface is very much less than v_∞ for real cases. The new equation analogous to (5a) for the optical depth along an infinitesimally thin cylindrical shell now becomes

$$\tau_{\max}(q) = \frac{2K(\nu, T)\,\gamma A^2}{q^3}.\left\{\frac{R_c^2}{q^2}.\left(\frac{\pi/2+\arcsin(R_c/q)}{\sqrt{1-R_c^2/q^2}}-\frac{R_c}{q}-\frac{\pi}{2}\right)\right\} \quad (q\geqslant R_c) \tag{15}$$

which degenerates to (5a) for $q \gg R_c$ but becomes infinite irrespective of frequency as $q \rightarrow R_c$. The flux integral now becomes impossible to evaluate analytically but a physical understanding of the situation may be obtained as follows.

For sufficiently high mass loss rates (or, equivalently, sufficiently low observing frequencies) the quantity KA^2 appearing in (15) becomes large and τ_{max} becomes much greater than unity at values of q only moderately greater than R_c. In this case the effects of acceleration are unimportant and we recover the simple form given previously (equation (5a)). However, if the mass loss rate is low enough (or the frequency high enough) then τ_{max} does not become much greater than unity until q is very close to R_c. But, in this region, the density is increasing with decreasing radial distance so rapidly that we see essentially a blackbody of constant projected linear radius R_c at *all* frequencies higher than the critical value given effectively by relation (12). Additionally, the temperature of the gas will be changing from a value appropriate to the ionized gas ($\sim 10^4$ K) to that of the photosphere of the star. An exact calculation of the emission from regions *close* to the effective surface of a star undergoing mass loss (of particular importance in the near infrared, i.e. 0·6–5 μ) therefore requires a detailed solution of the equation of radiative transfer for an accelerating stellar atmosphere possessing a temperature gradient; a feat which will not be attempted here. However, in the next section evidence will be presented to show that in observed cases the spectrum undergoes a flattening in the near-infrared similar to that predicted by the simple uniform flow model.

3. COMPARISON WITH OBSERVATIONS

Fig. 2 shows spectra of a sample of stars for which radio observations at two or more frequencies have appeared in the literature.

For P Cygni, in addition to radio fluxes (Wendker, Baars & Altenhoff 1973), free–free fluxes at several wavelengths in the infrared are also plotted. These fluxes were derived from the infrared photometry of Gehrz *et al.* (1974), by correcting approximately for the presence of the underlying stellar blackbody continuum, corresponding to the dereddened Rayleigh–Jeans tail given by: $V = 4{\cdot}79$; $E(B-V) = +0{\cdot}61$; and a ratio of total to selective extinction of $R = 3{\cdot}0$ (neglecting the possible small perturbation of the V-band flux by free–free emission).

Inspection of the resultant spectrum of P Cygni reveals a flattening of the spectrum, at the highest infrared frequencies plotted, qualitatively similar to that predicted by the simple uniform flow model discussed in the previous section. At lower infrared frequencies it can be seen that the spectrum approaches a straight line relation, with a spectral index of +0·65 between 10 and 19·5 μ. Additionally, a spectral index of +0·65 is also found to connect the 5 and 10·7 GHz radio measurements. These indices are in very good agreement with the predictions of the uniform flow model.

If a simple connection over three decades of frequency is attempted between the radio and infrared fluxes for P Cygni, a spectral index of 0·75 is found between 10 GHz and 20 μ ($1{\cdot}5 \times 10^{13}$ Hz), where errors due to the correction for the underlying stellar blackbody continuum are small at this latter frequency. If an extrapolation (with a spectral index of 0·65) of the 10–20 μ spectrum of P Cygni is made to the radio region, predicted fluxes are found to be approximately twice those actually detected. Since the spectral indices in the infrared and radio alone are both equal to 0·65 and are in good agreement with model predictions, it is

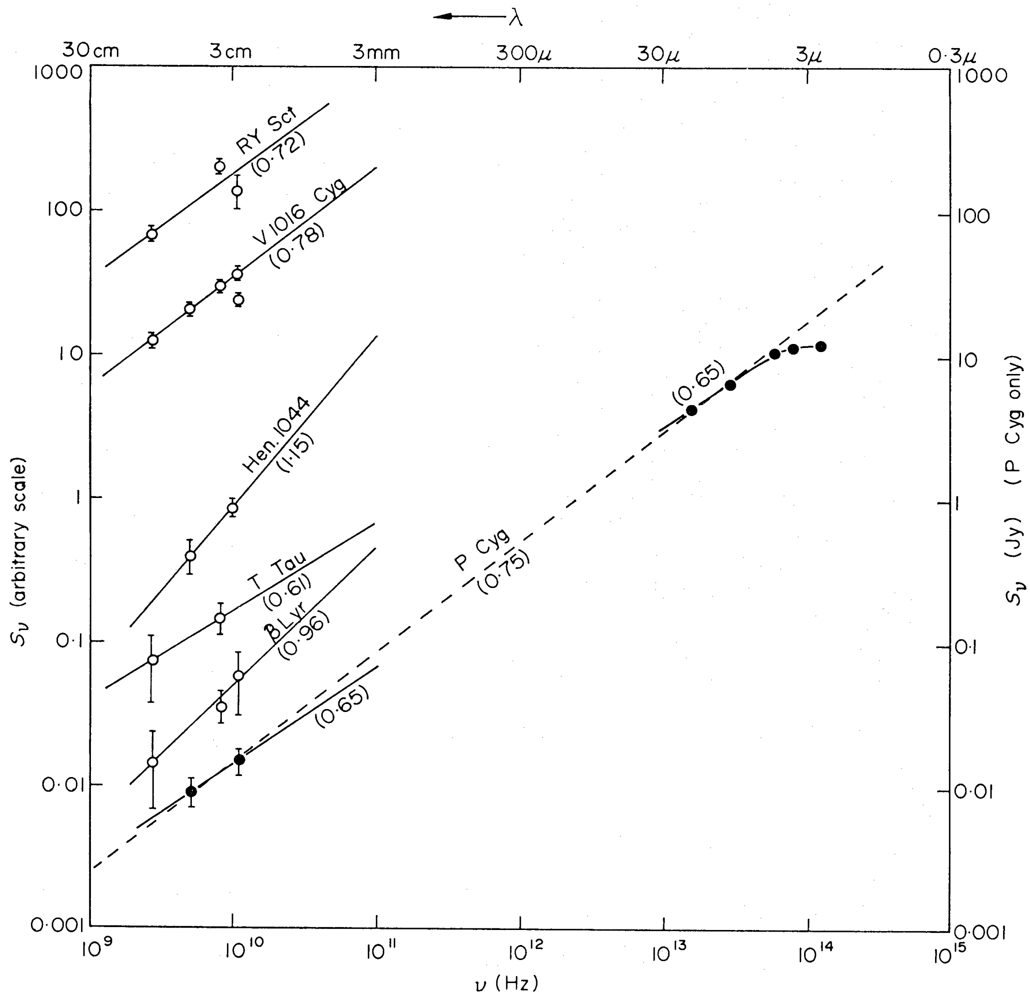

FIG. 2. *Flux density S_ν against frequency ν (in Hz) and wavelength λ for a sample of six radio stars. The left-hand ordinate is S_ν on an arbitrary scale and the right-hand ordinate is S_ν in Jy, applicable to P Cyg only. References used in constructing the radio spectra were Altenhoff et al.* (1973), *Hjellming, Blankenship & Balick* (1973), *Hughes & Woodsworth* (1973), *Purton et al.* (1973), *Seaquist & Gregory* (1973), *Spencer & Schwarz* (1974), *Wade & Hjellming* (1972), *Wendker et al.* (1973), *Wright et al.* (1974), *and the infrared spectrum of P Cyg was derived from Gehrz et al.* (1974), *as described in the text. The number in parentheses by each spectrum gives the spectral index.*

tempting to speculate that the discrepancy between the infrared and radio fluxes may be due to errors in the absolute flux calibration in one or both regions. However, an error of a factor of two is probably excluded. In any case, radio observations of other objects with intermediate free–free slopes in the infrared will either confirm or rule out this suggestion. For instance, recent infrared photometric observations of Wolf–Rayet stars (Cohen *et al.* 1975) show spectral indices of ~0·7 in the 3·6–10 μ region in several cases, from which 10 GHz radio flux densities at about the 10 mJy level may be predicted.

Consider now the radio spectra of the other objects presented in Fig. 2. Although the spectral indices range between 0·65 and 1·15, it is clear that the relation predicted by equation (10) is in substantial agreement with the observations, while the homogeneous density spectral distribution can definitely be excluded for objects with more than two spectral points. This conclusion was also reached by Seaquist

& Gregory (1973) for the particular case of V1016 Cygni. The present model also removes the necessity for the arbitrary assumption of an optical thickness turnover between the two observing frequencies for those objects having only two spectral points.

A moderate dispersion of spectral indices about a value somewhat greater than 0·6 is evident in Fig. 2, although part of this dispersion is probably caused by the low signal to noise ratios of some detections. Additionally, there is evidence that the radio emission from many of these objects may be variable (see e.g. Fig. 2 for V1016 Cyg and Gregory & Seaquist (1973) for R Aqr). Our model predicts that a varying mass loss rate from the star will have a greater effect on the spectrum at higher frequencies, since at lower frequencies the emission originates from a radially more extensive region of the ionized gas, implying a greater temporal smoothing of any irregularities in the mass loss rate. The flux density enhancement should be greater at higher frequencies and thus variability more pronounced. Radio measurements made at this higher frequency would then systematically overestimate the true flux if the signal to noise ratio of the detections was low: a spurious apparent steepening of the spectrum would result.

We now consider several other mechanisms which might produce variations of the spectral index from the 0·60 value. This value results essentially from assuming an $n \propto r^{-2}$ density relation, where n is the ionized gas number density. In the general case having $n \propto r^{-\beta}$, the flux density–frequency relation found using the same method as in Section 2 gives $S_\nu \propto \nu^{(4\beta-6\cdot2)/(2\beta-1)}$, valid for $\beta \geq 1\cdot5$. Now, for any physically reasonable case of non-spherical geometry, the total gas number density will fall off less rapidly than $n \propto r^{-2}$ (e.g. as r^{-1} for a disc geometry and independent of r for a cylindrical geometry) producing a more flattened radio spectrum. Since several objects have spectral indices greater than 0·6, it is obvious that a non-spherical geometry cannot explain their flux distribution.

Alternatively, we consider whether a decreasing ionized gas fraction can substantially change the spectral index in the radio region. For example an $S_\nu \propto \nu^{0\cdot8}$ spectrum will be produced by an $n \propto r^{-2\cdot25}$ law for the *ionized* gas, which, assuming the *total* gas density follows an r^{-2} law, requires the ionized fraction $x(r) = (n_+/_{\text{total}})$ to decrease as $r^{-0\cdot25}$. One mechanism which might be suggested for producing a radial decrease in x is the absorption by the gas flow of all the available ionizing photons from the exciting star. The limiting radius R_s of the region to which Lyman continuum photons can penetrate (analogous to the Strömgren radius) can be found by the usual procedure (Spitzer 1968), but with $n = Ar^{-2}$. For a pure hydrogen gas this leads to

$$\frac{R_c}{R_s} = 1 - \frac{N_{uv}.R_c}{2\pi A^2 \alpha} \tag{16}$$

where α is the hydrogen recombination coefficient to levels with $n \geq 2$ and N_{uv} is the total number of ionizing photons passing through radius R_c per second.

This relation implies that the radius of the ionized region will normally either be close to R_c or at infinity, depending upon whether the value of $A \equiv (\dot{M}/4\pi\mu m_H v_\infty)$ is greater than, or less than the critical value:

$$\left(\frac{\dot{M}}{4\pi\mu m_H v_\infty}\right)_{\text{crit}} = \left(\frac{N_{uv}.R_c}{2\pi\alpha}\right)^{1/2}. \tag{17}$$

If the value of A is below the critical value then all of the outflowing gas will be completely ionized. If, however, A is greater than the critical value, then $R_s \simeq R_c$

and the outflowing gas exterior to R_s will begin to recombine. Integration of the recombination rate as a function of radial distance leads to:

$$\left(\frac{1}{x(r)}-1\right) = \left(\frac{\alpha A}{R_s \mu v_\infty}\right)\left(1-\frac{R_s}{r}\right). \tag{18}$$

In the radio region $R_s/r \ll 1$ and thus $x(r)$ is essentially independent of r and has a constant (small) value of

$$x_{\min} = \left(1+\frac{\alpha A}{R_s \mu v_\infty}\right)^{-1} \tag{19}$$

and we therefore conclude that this process cannot affect the spectral index in the radio region (and would in any case make it very unlikely that the radio flux would be large enough to be detected, since $x_{\min}$ is typically $\sim 10^{-4}$).

However, instabilities may occur in the flow leading to neutral condensations of much higher than ambient density. Since circumstellar dust emission is known to originate from several of these objects (D. A. Allen 1973), implying dust condensation in the gas flow, this process is not unlikely. In order to produce a spectral index typically of 0·8, the postulated instabilities would have to act in such a manner as to produce the required ionized gas fraction distribution $x(r) \propto r^{-0 \cdot 25}$ in the region of the flow responsible for the radio emission. If the formation of neutral condensations does affect the radio spectra (leaving aside the possibility of variable mass loss rates) then the 0·6 value of the spectral index would represent a lower limit to the expected values of the indices, in agreement with the data presented in Fig. 2.

4. DERIVATION OF MASS LOSS RATES

We now consider how measured values of the radio and infrared fluxes may be used to derive estimates of mass loss rates. Manipulation of equation (8) leads to a relation for the mass loss rate $\dot{M}$ of the form

$$\dot{M} = 0 \cdot 095 \frac{\mu v_\infty S_\nu^{3/4} D^{3/2}}{Z \gamma^{1/2} g^{1/2} \nu^{1/2}} M_\odot/\text{yr} \tag{20}$$

where v_∞, the terminal velocity of the mass loss flow, is in km s^{-1}; S_ν, the flux observed at a frequency ν, is in Jy; D is in kpc; and ν is in Hz.

Equation (20) for the mass loss rate $\dot{M}$ is valid provided that the flux is measured in a region of the spectrum obeying the $S_\nu \propto \nu^{2/3}$ relation, i.e. provided $\nu < \nu_c$ (see equation (12)). This condition will be satisfied in the radio region, but the highest frequency in the infrared for which it is satisfied depends on the combination of v_∞, $\dot{M}$ and R_c for the particular star. If the infrared free–free emission from the gas of a star undergoing uniform mass loss can be separated out from the underlying stellar continuum, an estimate of the mass loss rate may be made, although if the measurement is at a frequency $\nu > \nu_c$ then an underestimate of this rate will result.

Having derived a mass loss rate for a particular star one should check that this rate, in combination with the other parameters of the flow, does not imply an excessive value for τ_e, the total electron scattering optical depth in the flow from R_c to the observer. For the uniform flow model described by (1),

$$\tau_e = \frac{\sigma_e \gamma A}{R_c} \tag{21a}$$

while for the accelerating flow case given by (14)

$$\tau_e = \frac{2\sigma_e \gamma A}{R_c} \tag{21b}$$

where σ_e is the Thomson scattering cross-section of free electrons. In order to get an upper limit to τ_e we adopt the latter value, corresponding to

$$\tau_e \leqslant \frac{4 \times 10^{19} \gamma \dot{M}}{R_c \mu v_\infty} \tag{22}$$

where $\dot{M}$ is in $M_\odot$/yr, R_c is in cm and v_∞ is in km s^{-1}. This value of the electron scattering optical depth will be appropriate to the optical region of the spectrum but not to the radio, since in the latter region the emission originates from a characteristic radius $R(\nu) \gg R_c$.

Finally, we may now use equation (17) to derive a lower limit to the number of ionizing photons emitted per second by a star with a mass loss rate $\dot{M}$, this lower limit being the number of photons per second required to maintain ionization of the observed mass loss flow. We find for a pure hydrogen gas, taking the value of the recombination coefficient α from Spitzer (1968), that

$$N_{uv} \geqslant \frac{1.5 \times 10^{77}}{T^{1/2} R_c} \left(\frac{\dot{M}}{\mu v_\infty}\right)^2 \text{ s}^{-1} \tag{23}$$

where $\dot{M}$ is in $M_\odot$/yr, v_∞ is in km s^{-1}, T, the temperature of the gas, is in K and R_c is in cm. Substitution of equation (20) for $\dot{M}$ into equations (22) and (23) then gives expression for τ_e and N_{uv} in terms of observable quantities (τ_e and N_{uv} will then be independent of knowledge of v_∞, since the quotient ($\dot{M}/\mu v_\infty$) is determined from S_ν by (20)).

We now apply equations (20), (22) and (23) to two stars observed at radio frequencies: P Cygni (Wendker *et al.* 1973), and T Tauri (Spencer & Schwarz, 1974). Both stars are found to have radio spectral indices close to 0·7. For P Cygni the following parameters were used: a 10·68 GHz flux of 15 mJy (Wendker *et al.* 1973); a distance D of 1·8 kpc, corresponding to its absolute visual magnitude of −8·4 (Hutchings 1970); a core radius R_c of 35 $R_\odot$, corresponding to that of a B1 Supergiant (Panagia 1973); and a terminal flow velocity, v_∞, of 240 km s^{-1} (de Groot 1969). The formula given by C. W. Allen (1973) for the Gaunt factor in the radio region was used for a temperature of 10^4 K. $\mu = 1.26$, $\gamma = 1$ and $Z = 1$ were assumed. We find $\dot{M} = 1.2 \times 10^{-5}\ M_\odot$/yr; $\tau_e \leq 0.7$; $N_{uv} > 1.0 \times 10^{48}$ s^{-1}. It should be noted that for many early-type emission line stars velocities of expansion derived from ultraviolet spectra are greater than those deduced from optical spectra, so that the use of the latter for P Cygni may lead to an underestimate of v_∞ and therefore of $\dot{M}$. The lower limit to N_{uv} deduced for P Cygni corresponds to that expected for a B0 Supergiant according to the tables of Panagia (1973).

For T Tauri the following values for the parameters have been used; an 8·1 GHz flux of 8 mJy (Spencer & Schwarz 1974); and a distance of 0·16 kpc, a terminal velocity of expansion v_∞ of 225 km s^{-1}, and a core radius R_c of 4·6 $R_\odot$ (Kuhi 1964). A temperature T of 10^4 K was assumed for the ionized gas and $\mu = 1.26$, $\gamma = 1$ and $Z = 1$. We obtain $\dot{M} = 2 \times 10^{-7}\ M_\odot$/yr; $\tau_e \leq 0.1$; $N_{uv} > 2.7 \times 10^{45}$ s^{-1}. The lower limit to N_{uv} deduced for T Tauri corresponds to that expected for a main sequence B1 star (Panagia 1973), whereas T Tauri has a spectral type of K1e (Kuhi 1964). Suffice it to say that the problem of the maintenance of excitation in T Tauri has been recognized before and collisional processes have been suggested (Kuhi 1964).

Although mass loss from T Tauri stars is well established, these objects have also been considered to be related to protostars. It is interesting to note that the density distribution in the gaseous shell of a collapsing protostar will also be approximately of the form $\rho \propto r^{-2}$ (Larson 1969; Penston 1969). If the star formed is sufficiently massive to ionize the surrounding gas, a spectrum of the form $S_\nu \propto \nu^{0.6}$ would again be expected.

5. DISCUSSION

The method described here for the determination of stellar mass loss rates from infrared or radio flux measurements of stars with the appropriate spectrum possesses certain advantages over the classical spectroscopic curve of growth techniques used in the optical and ultraviolet regions. Using the free–free spectrum technique, mass loss rates are derived from the integrated emission of *all* ion species, including hydrogen, whereas mass loss estimates by the spectroscopic method depend on the difficult problem of deriving heavy element column densities, with the concomitant ionization equilibrium and abundance assumptions (the free–free spectrum method requires a distance estimate and the spectroscopic method requires a characteristic radius estimate (e.g. Morton 1967). Both methods require a velocity estimate).

However, in the infrared the free–free spectrum method encounters the problem of disentangling the stellar blackbody continuum contribution (and the contribution, if any, due to dust emission), whilst in the radio generally low flux levels are expected from most stars, due to the declining form of the spectrum. For these reasons, given reasonable future improvements in detector efficiencies in the 30–100 μ region, air- or space-borne observations in this wavelength range may hold promise for the determination of mass loss rates, since the rapidly declining stellar Rayleigh–Jeans continuum should be negligible in this region, whilst the free–free flux will typically be a factor of a hundred higher than in the radio.

The results described in this paper will be applied to infrared observations of Wolf–Rayet stars made by Cohen *et al.* (1975) and to infrared observations of OBA Supergiants and Of stars by Barlow & Cohen (1975, in preparation).

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