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The Role of Habit Formation in General Equilibrium Macroeconomics

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Abstract

This Ph.D. thesis consists of two contributed papers. It builds on the recent dynamic macroeconomic literature on heterogeneous agents economies. Using the standard neoclassical growth model with infinitely lived agents facing incomplete insurance markets and idiosyncratic uncertainty, I study the consequences of their consumption-savings decisions for precautionary savings, wealth inequality and asset pricing when habit formation preferences are introduced.

In the first paper I examine the role of habit formation in shaping the wealth distribution in an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty. I compare the implications for precautionary savings and for wealth concentration between economies that only differ in the role played by habit formation. I find that habit formation brings a hefty increase in precautionary savings and very mild reductions in the statistics of wealth inequality.

The second paper extends the class of models that support the habits explanation for the equity premium puzzle in order to account for heterogeneity in earnings, wealth, habits and consumption. I find that, contrary to the earlier results in the literature, the presence of habits does not imply a price for risk much higher than the one implied by models with intertemporally separable preferences. The main reasons for this are general equilibrium ones. Firstly, with just two assets available, households can smooth out consumption fluctuations very well. Therefore, the higher utility losses of uncertainty imposed by habits will not command a high price of risk because households manage to avoid this risk. Secondly, the composition of the set of agents pricing the assets is sensitive to changes in the model. In an economy with habits, pricing agents turn out to be households facing very small consumption fluctuations. The model is extended to account for firms financing their capital through bonds. The qualitative results remain unchanged.

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Dedication

To my parents, who allowed me the privileged position and freedom to pursue my own path in life.

To Vània, who had to cope with my changing moods and lack of time for the last two years.

Als meus pares, que m'han donat totes les possibilitats i llibertat per poder seguir el meu propi camí a la vida.

A la Vània, que ha aguantat els meus canvis d'humor i falta de temps lliure durant els dos últims anys.

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[1]**This chapter is joint work with Antonia Díaz and Victor Ríos-Rull**

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Chapter 1

Introduction

My Ph.D. dissertation builds on the recent dynamic macroeconomic literature on heterogeneous agents economies. Using the standard neoclassical growth model with infinitely lived agents facing incomplete insurance markets and idiosyncratic uncertainty, I examine the consequences of their consumption-savings decisions for precautionary savings, wealth inequality and asset pricing when habit formation preferences are introduced. Habit formation has been recently used to improve the predictions of models based on time separable preferences in different fields where savings behaviour under uncertainty and the income fluctuation problem are the chief ingredients. However, few studies have done so outside the representative agent framework.

An important issue throughout the thesis is the notion of general equilibrium. This turns out to be very relevant when assessing the effect of the habit formation hypothesis on the macroeconomic variables. For given prices, the habit formation hypothesis produces big changes in households' behaviour. However, changes of behaviour mean changes of quantities in the aggregate variables and therefore new equilibrium prices. When this change in prices is added to the picture, most of the changes induced by a different preference hypothesis are undone and the overall equilibrium effect turns out to be modest. This is the main message of the thesis: habit formation does not substantially modify the predictions of our macroeconomic models. When the opposite has been said, it is mainly because the arguments have relied on partial equilibrium models. At this point of the thesis it is difficult to be more precise about this without entering into more detailed discussion. I would like to ask the reader to go through chapters 2 and 3 in order to get more precise arguments.

There are three short sections completing this introduction. Firstly, section 1.1 describes both intuitively and mathematically what I mean by habit formation. This overlaps a little with the contents of further chapters. When the overlap is too evident I refer the reader to the appropriate section where the issue is discussed in further detail. Secondly, section 1.2 describes the remainder of the thesis in a short and precise way. Thirdly, 1.3 makes a brief comment on the original solution methods used in this thesis and refers the reader to more detailed discussion within the text.

1.1 Habit formation

It is important to state clearly what I mean by habit formation. Some economists use the term habit formation to refer to a very specific type of goods or services such as smoking, going to the gym or having a husband. This usage of the habit term implies that there are some types of goods whose consumption display habit behaviour whereas there are some others that do not. What do we mean by habit behaviour? We mean that consumption of a certain good in different points of time is complementary. Observationally, this implies that high consumption of a certain good (say cigarettes) in one period of time is associated to high consumption in the following period. This has proved to be a very useful framework to study different issues and it has produced such interesting insights as the theory of rational addiction by Becker.[1]

However, in this thesis habit formation does not refer to a specific set of goods but to the whole bundle of goods and services that a person consumes. It should be seen as the idea of getting used to certain status, to a certain quality of life. People form habits over the whole level of consumption. The composition of the goods one consumes has nothing to do with the habitual behaviour. When enjoying a high level of consumption, one generates the need to keep consuming at that level in the future. In this context, As Campbell and Cochrane (1999) state it,

[1] See for example Becker (1996) which contains a collection of papers on this issue by the Nobel Prize laureate.

Habit formation captures a fundamental feature of psychology: repetition of a stimulus diminishes the perception of the stimulus and responses to it.

Mathematically, I define preferences displaying habit formation by the following utility function:

$$u(c,h) = \frac{\left(ch^{-\gamma}\right)^{1-\sigma}}{1-\sigma} \quad \text{with} \quad \gamma \in (0,1)$$

which is equivalent to writing:

$$u(c,h) = \frac{\left(c^{1-\gamma}\left(\frac{c}{h}\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma}. \tag{1.1}$$

where c is current consumption level and h is current stock of habit. The stock of habit is a weighted average of all past consumption. It can be represented with the following law of motion:

$$h' = \psi(c,h) = (1-\lambda)\,h + \lambda c \quad \text{with} \quad \lambda \in (0,1] \tag{1.2}$$

where h' denotes next period habit stock. This is the standard constant elasticity of substitution utility function over the composite good $c^{1-\gamma}\left(\frac{c}{h}\right)^{\gamma}$. Habit formation is defined by the following assumption:

$$\frac{\partial^2 u(c,h)}{\partial c \partial h} > 0$$

That is to say, marginal utility of consumption increases with past consumption. Having enjoyed high consumption in the past generates a need for high consumption in the present. Consumption in different points of time is complementary.

In other words, the habit formation hypothesis is the idea that individuals care not only about their level of consumption but also about how this level of consumption compares to the consumption they have enjoyed in the past. The utility function in expression 1.1 shows this very clearly: $(1-\gamma)$ is the weight of consumption level and γ is the weight of current consumption relative to past consumption.

The term habits refers to preferences where an increase in h lowers the utility derived from a given level of consumption. An individual enjoying certain level of consumption is better off if he was consuming less in the previous period than if he was consuming more in the previous period. If we accept interpersonal comparisons of utility, we could say that when comparing two middle class households, one that raised from poverty and another one that fell from past luxury, the former is happier. This intuition is captured by the assumption:

$$\frac{\partial u(c,h)}{\partial h} < 0$$

When this derivative is positive we talk about durability, and it has very different considerations.

Of course, one important aspect of habit formation is the length of the memory. That is to say, for how long past consumption affects current utility. The memory of the process is captured by the parameter $(1-\lambda)$ in equation 1.2. If memory is very short, say only last period consumption matters, then changes of status may not be so dramatic.[2] Imagine I face a sudden fall of income that sends me into poverty. If my preferences display habit formation this means that I care for my low level of consumption but also for the difference between my low level of consumption and my past higher consumption. However, if memory is short and I easily forget the distant past we can say that I get used to being poor quite easily. After few periods in poverty the difference between my current consumption and my stock of past consumption has vanished. If I care much more for how my current consumption compares to my past consumption than for the level of consumption itself (small γ) then this fall into poverty is not very bad in the long run.

This example tells us something important. Modelling habit formation is an issue not only of finding a functional form for preferences but also of finding the right values for the parameters. The functional forms in expressions 1.1 and 1.2 imply very different behaviors according to different values of the parameters γ, λ and σ. For example, somebody with big γ (big weight of relative consumption) and small $(1-\lambda)$ (short memory) will care much less about losing their job than somebody with big γ (big weight of relative consumption) and big $(1-\lambda)$ (long memory). It might well be that they cared less than somebody with small γ (big weight of consumption level). Caring more or less about such a contingency translates into different behaviour in trying to prevent it or insuring against it. A more detailed discussion on this issue can be found in section 2.3.2.

[2]Precisely, only previous period consumption matter when $(1-\lambda)=0$

1.1.1 More ways of modelling habit formation

Some economists have used functional forms for the habit formation hypothesis that differ from the one described in this section. There are mainly two variations.

The first variation, called *survival consumption* habit, refers to the utility function. Under this formulation the habit stock enters the utility function as a minimum consumption that updates itself following a law of motion in the line of equation 1.2. A more detailed discussion of the *survival consumption* habit and how it compares with the formulation used throughout this thesis can be found in section 2.3.

The second variation does not relate to the utility function but to the law of motion. It is called *external habit* or *catching up with the Joneses.* It is based on assuming that what matters to build the habit stock is not the history of the individual's own consumption but the history of society's consumption. Therefore, it is capturing quite a different idea: people care for how they do relative to their neighbours (and their neighbours' history). In a representative agent framework this variation can be seen as a particular case of the habit used in this thesis, which we can call *internal habit.* When an agent decides between consumption today and tomorrow, he takes into account that by consuming today he is increasing his habit stock and therefore bringing 'problems' for the future. He needs to predict which is going to be his consumption in all future periods in order to evaluate how much 'trouble' he is creating for himself by increasing the habit stock. Under the assumption of *external habit* this forward-looking effect does not exist because the effect of an individual's consumption on the aggregate consumption level is negligible. Therefore, *external habit* is technically as an *internal habit* for a household that does not recognize the fact that by increasing consumption today he generates the need for increasing consumption tomorrow.

However, the *external habit* in an heterogenous agents economy is different from what I have stated in the previous paragraph. In a representative agent economy individual consumption and aggregate consumption are equal to each other in equilibrium and therefore the dynamics of the individual and aggregate habit stock are the same. However, this is not true in a heterogeneous agents economy. Each individual's history of consumption is different from each other and different from the series of aggregate consumption. It is still true that economic agents do not internalize the effect of their own consumption into future habit stocks. However, there is an extra difference from an *internal habit* framework: it is the history of aggregate consumption (and hence the history of aggregate shocks) that matters and this is different from the history of individual consumption (and hence the history of idiosyncratic shocks). The implications of *external habit* in a heterogeneous agents economy is left for further research following my work in chapter 3.

1.2 Outline of the thesis

This thesis is made out of two independent, although closely related, research papers. The first paper, chapter 2 in this thesis, shows the importance of precautionary savings. In equilibrium, habit forming economic agents, facing uninsurable idiosyncratic risk to labour earnings, can successfully smooth out income fluctuations by raising their savings. This implies that, when compared to agents in an economy without habit formation, consumption fluctuates less. This result naturally brings the second paper, chapter 3, into the picture. The second paper looks at a similar economy extended with aggregate uncertainty. The differential return between the risky asset and the risk free bond of the economy, the equity premium, is analysed. Will habit formation households require a high differential rate of return to hold the risky asset in equilibrium or on the contrary will they reduce their idiosyncratic risk by raising precautionary savings? The answer is closer to this second option. A more detailed summary of these two papers follows in the next paragraphs.

In chapter 2 I study the role of habit formation in shaping the wealth distribution in an otherwise standard heterogeneous agents model economy with idiosyncratic uncertainty. I compare the implications for precautionary savings and for wealth concentration between economies that only differ in the role played by habit formation. Once preferences are properly adjusted so that the Intertemporal Elasticity of Substitution is the same in all model economies studied, I find that habit formation brings a hefty increase in precautionary savings and very mild reductions in the coefficient of variation and in the Gini index of wealth. I also find that the reductions in these measures of inequality also hold when I adjust the model economy so that aggregate savings are the same as in the economy without habit formation. These findings hold for both persistent and non persistent habits although for the former the quantitative size of the effects is much larger. I conclude that habit formation, while being a mechanism that increases the amount of precautionary savings generated in a model, does not change the implications for wealth inequality that arise from standard models.

Habit formation has been proposed as a possible solution for explaining the equity premium puzzle. Chapter 3 extends the class of models that support the habits explanation in order to account for heterogeneity in earnings, wealth, habits and consumption. I find that, contrary to the earlier results in the literature, the presence of habits does not imply a price for risk much higher than the one implied by models with intertemporally separable preferences. The main reasons for this are general equilibrium ones. First, with just two assets available, households can smooth out consumption fluctuations very well. Therefore, the higher utility losses of uncertainty imposed by habits will not command a high price of risk because households manage to avoid this risk. Second, the composition of the set of agents pricing the assets is sensitive to changes in the model. In an economy with habits, pricing agents turn out to be households facing very small consumption fluctuations. I also find these effects hold when I extend the standard model to allow for firms financing capital by bonds. In addition, I characterize three important properties of the model economy that relate to portfolio choice: willingness to hold risky assets (1) increases with wealth, (2) decreases with labour earnings and (3) decreases with habit stock.

1.3 Methodology

This is a quantitative work. Both chapters state clear quantitative questions and complex macroeconomic models are used in trying to answer them. As it is pervasive in this line of research, I look at US data and answer the questions for model economies set to mimic certain properties of the US economy.

The solution of the models, based on numerical methods, reaches high levels of complexity. In this thesis both models (chapter 2 and 3) have an extra state variable compared to similar models that have been solved before (like Aiyagari (1994) or Krusell and Smith (1997)). This extra state variable is the habit stock. The solution methods are of interest on their own since the extra state variable can be used to deal with many different economic questions. For example, one may want to consider a fertility model. The extra state variable would be in that case the number of children. Increasing the dimension of the state space increases the complexity of the algorithms as well as the computing power requirements. For a detailed explanation on the solution methods the reader may want to read the computational appendices of each chapter, sections 2.7 and 3.10.

Chapter 2

Habit Formation: Implications for the Wealth Distribution [1]

2.1 Introduction

Models with a large number of ex–ante identical agents with standard preferences subject to uninsurable, idiosyncratic shocks to income are the main tool used to answer two questions that many economists see as important: (1) what is the size of precautionary savings (savings held for the sole purpose of smoothing consumption across different contingencies)?, and (2) what accounts for the very large differences in assets holdings among American households? The accepted answer to the first question as posed, for example, by Aiyagari (1994) is that precautionary savings are small, no more than 3% of total savings. With respect to the second question, there is a relative consensus in the profession that a theory of wealth inequality based on standard and identical preferences and on uninsurable shocks to income can account for only a small part of the observed wealth inequality.[2]

[1]**This chapter is joint work with Antonia Díaz and Victor Ríos-Rull**

[2]For example Krusell and Smith (1998) postulate shocks to preferences to account for wealth inequality while Carroll (2000) argues that we should use models where consumers consider the accumulation of wealth as an end in itself or models where wealth yields a large unobservable flow of services. Quadrini and Ríos-Rull (1997) contains a review of the literature and its successes and failures in accounting for wealth inequality with uninsurable shocks to income. On the other hand, a recent paper, Castañeda, Díaz-Giménez, and Ríos-Rull (2000), argues that a suitably modified version of the basic model with identical and standard preferences and uninsurable shocks does account for the wealth inequality observed in the U.S. An important modification proposed by these authors is the use of a process for earnings with more volatility than those found in previous work.

In this paper we study, in the context described in the previous paragraph, the role played by habit formation in shaping the distribution of wealth, and hence, the answers to those two important questions.

Habit formation has been recently used to improve the predictions of time-separable models in different fields where savings behaviour under uncertainty and the income-fluctuation problem are the chief ingredients. For instance, some authors have pursued this path and studied various formulations of habit formation to improve our understanding of the equity premium puzzle.[3] Other authors have used this class of preferences to study the observed relationship between savings and growth.[4] Finally, Fuhrer (2000) shows how the presence of habits in consumption can generate slow and hump-shaped reactions of consumption to monetary and other shocks. Despite all this work with habits,[5] its implications for the determination of precautionary savings and for shaping the wealth distribution have not been explored. This is precisely the target of this paper.

We pursue this line because we think that the habit formation hypothesis can have a significant role in shaping the wealth distribution. Households with habits want, not only a smooth pattern of consumption, but also a smooth pattern of changes in consumption. This implies that households in habits economies dislike consumption fluctuations to a larger extent than their counterparts in a world of time-separable preferences do. This should increase the amount of precautionary savings. How much it will is one of the quantitative questions we address. Any effect on wealth concentration relies on an asymmetric impact, over different types

[3]Abel (1990) and Constantinides (1990) show that adding habit formation to an otherwise standard exchange model economy, the equity premium puzzle as stated by Mehra and Prescott (1985) disappears. The same result is obtained by Heaton (1995), Boldrin, Christiano, and Fisher (1997), Boldrin, Christiano, and Fisher (2001) and Campbell and Cochrane (1999). See also chapter 3 of this thesis for the opposite result

[4]The evidence shows that, across countries and across households, the growth rate of income has a positive and significant effect on the savings rate (see Edwards (1995), Carroll and Weil (1994), Deaton and Paxson (1994), for instance). To account for this observed pattern Carroll, Overland, and Weil (2000) modify the standard Ak model to display habit formation. They show that the model is successful to replicate the positive response of the savings rate to the growth rate of income.

[5]In this work, as in ours, households do not value leisure. The role of time non-separabilities in leisure is dormant since its early appearance in quantitative theory in Kydland and Prescott (1982). Also we do not look at the feature opposite to habit formation, that is durability of consumption, even though in the context of our model preferences could display durability of consumption by simply setting one parameter to a negative value. The reason is that the definition of wealth that we use already includes a large fraction of the stock of consumer durables.

of agents, of the habit formation hypothesis on the disutility of consumption fluctuations. On the one hand, when bad times strike, households will deplete their assets faster to ensure a mild decrease in consumption. But on the other hand, anticipating this problem, they will have accumulated some extra assets. In equilibrium, which force will dominate for each type of agent? This is the key for the second quantitative question addressed in the paper. As an anticipation of the results, we find that the behaviour of asset poor people turns out to be critical.

There is a variety of attempts trying to quantify the size of precautionary savings. The econometric literature offers diverse answers that range from the null importance of precautionary savings found by Dynan (1993) to the bigger size found by Carroll and Samwick (1998).[6] Within the macroeconomic literature, there are some attempts to measure the importance of precautionary savings using models with a large number of ex–ante identical agents subject to uninsurable, idiosyncratic risk. In the partial equilibrium context, it is found that precautionary motives rise substantially aggregate savings. For instance, Hubbard, Skinner, and Zeldes (1994) find that idiosyncratic uncertainty implies an increase in the aggregate capital-income ratio of 0.90 percent. Also Carroll and Samwick (1997) find that households facing higher uncertainty accumulate more wealth although such response is much lower than the one predicted in Hubbard, Skinner, and Zeldes (1994).[7] Finally, Cagetti (2000) finds that for individuals under 50 years old almost all savings respond to precautionary motives and that at the age of retirement wealth is twice as high as in a world without idiosyncratic uncertainty. However, in general equilibrium models the size of precautionary savings is substantially reduced, the reason being that, as aggregate savings increase, their rate of return fall. Aiyagari (1994) in a infinite horizon economy finds that precautionary savings are small, no more than 3 percent of total savings. Huggett (1996) finds similar result in a life-cycle economy.

Thus, the accepted answer seems to be that precautionary savings are small. As stated before, habit formation provides an extra reason for further expanding the

[6]They estimate that between 39 and 46 percent of wealth of individuals under 50 years is attributable to the extra uncertainty that some consumers face compared to the lowest uncertainty group. See Browning and Lusardi (1996) for a survey of the empirical literature on precautionary savings

[7]In particular, Carroll and Samwick (1997) show that to obtain a level of responsiveness of wealth similar to Hubbard, Skinner, and Zeldes (1994) estimates, the rate of time preference should be as high as 11 percent, as opposed to Hubbard, Skinner, and Zeldes (1994) who use a rate of 3 percent.

precautionary motive in spite of the fall in the return on savings.

To study the role of habits we compare a standard economy without habits (the benchmark economy) with various habits counterparts. We proceed by first looking at economies with the same parameterization except for the habits. The two economies do not have the same Intertemporal Elasticity of Substitution (henceforth IES). Hence, households in the habits and in the benchmark economy not only differ in their attitude towards risk, but also in their willingness to intertemporally substitute consumption. To better understand the implications of habits, we compare the benchmark economy with the habits economies recalibrated to have the same IES. To further study the role of habits in shaping wealth inequality we want to isolate the effect of habits on risk aversion from the induced effects brought by the changes in the interest rate. To this end, we compare the benchmark economy with the habits economies calibrated not only to have the same IES, but also the same aggregate savings. To analyze the effect of the persistence of habits on the level of precautionary savings and wealth inequality we study two habits economies: one in which habits respond very quickly to changes in consumption (non persistent) and other in which the response of habits is very slow (persistent).

We find that the presence of habits generates a volume of precautionary savings whose size goes from two to three times the volume of precautionary savings generated by the standard model, depending on the habits persistence. With respect to inequality we find that habits do decrease wealth inequality as measured by the coefficient of variation and the Gini Index by about 10 and 18 percent also depending on habits persistence (for instance, the Gini Index is 0.404 in the benchmark model without habits and it goes down to 0.339 in one of the habits economies). The reason of this decrease in wealth inequality is the different effect that habits has on households depending on their level of wealth. In short, wealth poor households increase their precautionary savings more than wealth rich ones do. First, other things equal, wealth rich households have a smaller proportion of their income in form of risky labour earnings. And second, wealth rich households have higher buffer stocks to smooth out consumption fluctuations (as a matter of fact, even without habit formation they are already very well self-insured, so there is no big need for extra cover in face of the higher disutility of consumption fluctuations).

Results, as we have stated, depend on the persistence of the habits process. When the habit stock is not persistent (as when it is given for instance by only previous period consumption) the effect on household's behaviour is much weaker than when the habit stock is persistent (as when it is given by the whole history of past consumption). The reason for this is as follows. Households care about fluctuations of consumption around their habit stock. Given a change in consumption, when the habit process is (not) persistent, it takes many (few) periods to catch up with the consumption level and therefore variations of consumption over the habit stock are big (small) as it is the lifetime utility loss.

These findings, although quantitatively smaller, also hold in model economies that generate Gini coefficients closer to those in the data.[8] In this case precautionary savings are between 1.5 and 2.7 its size in the non habits economy and the reduction in the inequality measures ranges between 3 and 8 percent. The larger changes correspond to economies with persistent habits. Thus, our assessment is that while the effect of habits in precautionary savings can be very big, the overall effect in wealth inequality is milder.

In this paper the vector of state variables includes both habits and assets, two variables directly controlled by the household. This feature complicates the numerical methods involved which leads us to use multidimensional splines to solve the problem of the household.

The rest of this paper is organized as follows. Section 2.2 describes the model, while Section 2.3 describes the calibration procedures. Section 2.4, describes the findings. Section 2.5 explores the robustness of the findings with respect to the process for earnings. Finally, section 2.6 concludes. Appendix 1 (section 2.7) describes how we solved the problem of the agent and how we computed equilibria. Appendix 2 (section 2.8) describes how the intertemporal elasticity of substitution is calculated in different economies.

2.2 The model economy

The economy is a growth economy with production populated by a measure one of households that live forever. We only look at steady states. Section 2.2.1 describes preferences with habits. Sections 2.2.2 and 2.2.3 describe the technology, including the production sector, the shock process that affect households and the market

[8]This is achieved by calibrating the process for labour earnings as proposed by Castañeda, Díaz-Giménez, and Ríos-Rull (2000).

arrangements. In Section 2.2.4 we write down the households problem while Section 2.2.5 presents a formal definition of steady state equilibrium.

2.2.1 Preferences

Households derive utility from current and past consumption. Current consumption is denoted by c. Past consumption affects the level of a stock of habits that we denote with $h \in [0, \infty)$. We write the evolution of habits as $h' = \psi(c, h)$, where we already use the recursive notation that is pervasive throughout the paper with primes denoting next period's values. We write the per period utility as $u(c, h)$, and total utility as $\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$. Notice that, since current consumption affects future per period utilities (by means of h_t), preferences over consumption are not time-separable.

2.2.2 Technology

Each period households receive a shock to their efficiency units of labour $e \in E = \{e_1, ..., e_{n_e}\}$. This shock is Markov with transition matrix, $\pi_{e,e'}$.

Aggregate output, Y, is produced according to an aggregate neoclassical production function that takes as inputs capital, K, and efficient units of labour, L, $Y = F(K, L)$. The aggregate labour input comes from aggregating all agents' efficiency units of labour. Aggregate capital results from aggregation of all assets. Capital depreciates at rate $\delta \in [0, 1]$.

2.2.3 Market arrangements

There are no state contingent markets for the household specific shock, e. Households hold assets $a \in A \equiv [\underline{a}, \infty)$ that pay interest at rate r. We assume that households are restricted by a lower bound on their assets holdings $\underline{a}$. This lower bound may arise endogenously as the quantity that ensures that the household is capable of repaying its debt in all states of the world or we can just set it exogenously as a borrowing constraint.[9] The absence of state-contingent markets and the presence of borrowing constraints are the ingredients needed to depart from

[9]See Huggett (1993) and Aiyagari (1994) for details. Quadrini and Ríos-Rull (1997) contains a review on this topic.

the representative agent framework which is silent about distributional issues in the cross-section.[10]

2.2.4 The household's problem

Since we only look at steady states, the individual household's state variables are its shock, its assets and its stock of habit, $\{e, a, h\}$. The problem that the household solves is

$$v(e,a,h) = \max_{c \geq 0, a' \geq \underline{a}} \; u(c,h) \; + \; \beta \sum_{e'} \pi_{e,e'} \; v(e', a', h') \tag{2.1}$$

$$\text{s.t.:} \qquad a' = e\,w + (1+r)\,a - c \tag{2.2}$$

$$h' = \psi(c,h) \tag{2.3}$$

where r and w are the return on assets and the rental rate for efficiency units of labour.

It is well known that under certain conditions problems of this type have a solution that we denote $a' = g^a(e,a,h)$, $c = g^c(e,a,h)$ with an upper bound on asset holdings, $\overline{a}$ and on the stock of habits $\overline{h}$, such that $\overline{a} \geq g^a(e,a,h) \geq \underline{a}$ and $\overline{h} > \psi(g^c(e,a,h),h) > 0$, for all $e \in E$, all $h \in \{h \mid 0 \leq h \leq \overline{h}\}$, and all $a \in \{a \mid \underline{a} \leq a \leq \overline{a}\}$. Sometimes we use the compact notation $s = \{e,a,h\}$ and $S = \{E \times [\underline{a}, \bar{a}] \times [0, \bar{h}]\}$. With respect to assets, the required conditions amount to have a low enough rate of return, $\beta < \frac{1}{1+r}$. Again, see Aiyagari (1994), Huggett (1993), or Quadrini and Ríos-Rull (1997) for details. With respect to habits, it suffices to have a bounded ψ.

It is possible to construct a Markov process for the individual state variables, from the Markov process on the shocks and from the decision rules of the agents (see Huggett (1993) or Hopenhayn and Prescott (1992) for details). Let $\mathcal{B}$ be the σ-algebra generated in S by, say, the open intervals. A probability measure x over $\mathcal{B}$ exhaustively describes the economy by stating how many households are of each type. Note that the first moment of x over e yields the aggregate labour input while the first moment over a yields aggregate capital.

[10]See Chatterjee (1994) or Alvarez and Díaz (2000).

Let $Q(s, B)$ denote the probability that a type $\{s\}$ has of becoming of a type in $B \subset \mathcal{B}$. Function Q naturally describes how the economy moves over time by generating a probability measure for tomorrow x' given a probability measure x today. The exact way in which this occurs is

$$x'(B) = \int_S Q(s, B) \; dx \tag{2.4}$$

If the process for the earnings shock is nice in the sense that it has a unique stationary distribution, then so has the economy.[11] Furthermore, this unique stationary distribution is the limit to which the economy converges under any initial distribution.[12]

2.2.5 Equilibrium

We have almost all the ingredients to define a steady state equilibrium. We only need to add the condition that marginal productivities yield factor prices as functions of x. Note that to obtain a steady state, we look for a measure of households x such that given the prices implied by that measure, households actions reproduce next period the same measure x. Formally, a steady state equilibrium for this economy is a set of functions for the household problem $\{v, g^a, g^c\}$, and a measure of households, x, such that: (i), Factor inputs are obtained aggregating over households: $A = \int_S \; a \; dx$, and $L = \int_S \; e \; dx$; (ii), factor prices are factor marginal productivities, $r = F_1(K, L) - \delta$, and $w = F_2(K, L)$; (iii), given x, K, and L, the functions $\{v, g^a, g^c\}$ solve the households' decision problem described in Subsection 2.2.4; (iv), the goods market clears: $\int_S [g^c(s) + g^a(s)] \, dx = F(K, L) + (1-\delta)K$, and (v), the measure of households is stationary: $x(B) = \int_S Q(s, B) \; dx$, for all $B \subset \mathcal{B}$.

2.3 Calibration

This paper explores the role of habits in quantitatively shaping the wealth distribution. To do this, we compare economies identical in every respect but the specification of preferences: while in a benchmark economy preferences display no habit

[11]For example if it satisfies the American–dream American–nightmare condition stated in Ríos-Rull (1998), then there is a unique stationary distribution of households over earning shocks, assets holdings and stock of habits.

[12]This does not mean that this will happen in equilibrium outside the steady state. The transition Q has been constructed under the assumption that the households think that prices are constant.

formation, in the other economies preferences display habits. We start by choosing the benchmark model economy to be essentially a version of Aiyagari (1994) that has become the standard in the literature that measures the size of precautionary savings. We describe the calibration of the benchmark model economy in Subsection 2.3.1 and that of the model economies with habits in Subsection 2.3.2.

Once we have a benchmark model economy without habits we have to calibrate the economies with habits. There is not a unique way to do this, since habits have been modelled in at least two ways. On the one side, there is a *survival consumption* branch. Past consumption piles up into a habit stock that determines a minimal consumption for today, below which utility is not defined.[13] On the other side, there is a *multiplicative habit* branch. Past consumption piles up into a habit stock that enters utility dividing today's consumption, capturing the notion that, under habit formation, it is not consumption level but relative consumption what matters.[14] Therefore, the two different approaches differ in two dimensions. First, the *survival consumption* household cares about the absolute difference between consumption and habit stock whereas the *multiplicative habit* consumer cares about the relative difference. And second, for the *survival consumption* household, consuming below the minimal level given by the habit stock is not defined (death) whereas it is well defined for the *multiplicative habit* consumer.

Regarding the first difference, the *survival consumption* representation has been preferred by authors working with the representative agent hypothesis in the field of asset pricing. As Campbell and Cochrane (1999) claim, one needs this formulation to get the equity premium negatively correlated with the cycle.[15] However, Krusell and Smith (1997) show that once we allow for heterogeneous agents an economy with no habits can deliver a negative correlation between the equity premium and the cycle. Furthermore, chapter 3 shows that this last result is preserved when adding a *multiplicative habit* in the heterogeneous agents economy.

Regarding the second difference, it is difficult to reconcile the *survival habit* approach with individual data. Even in the most conservative earnings process, any household can see its labour earnings halved between two consecutive periods. If

[13] Pioneered by Ryder and Heal (1973) and followed for instance by Constantinides (1990), Heaton (1995), Boldrin, Christiano, and Fisher (1997) or Dynan (2000).

[14] Used for instance by Abel (1990), Carroll, Overland, and Weil (2000) or Fuhrer (2000).

[15] Campbell and Cochrane (1999) put it this way: "As consumption declines toward the habit in a business cycle through, the curvature of the utility function rises".

one wants to replicate the U.S. data Gini coefficient for earnings, being unlucky may mean dividing earnings by a factor of 9 (see Section 2.5) in one period or even by 45 in two periods (in an extreme bad luck case). The *survival habit* utility function can hardly accommodate this variation in earnings if households do not accumulate huge precautionary savings. In contrast, variations in earnings in aggregate data are not so sharp and fit well in the *survival consumption* utility function.

We choose to work with the *multiplicative habit* utility function. The reason is twofold. First, the motives that brought the *survival consumption* representation into the picture are absent here. Namely, to have a representative agent economy display certain properties of data that an heterogeneous agents economy already does with the *multiplicative habit.* And second, since we calibrate and simulate our model economies to represent individual behaviour, the computational problem associated to solving the model when consumption falls below habit in the *survival consumption* case becomes very big.[16]

We choose to study two types of habits that differ in their persistence. Essentially persistent habits imply that current consumption enters negatively the per period utility function of all future periods, albeit in a decreasing manner. Non–persistent habits are those where the influence of current consumption ends next period since the per period utility function only depends on yesterday's and on today's consumption.

We have to choose not only the type of habits, but we also have to be very specific with respect to what is the habits counterpart to our benchmark model economy. We propose a sequence of economies in increasing order of appropriateness. First, we think of the habits economy as an economy like the benchmark with the addition of the term in habits but *keeping constant* all other parameters.

Habit formation breaks the link between risk aversion and the Intertemporal Elasticity of Substitution, IES. Thus, economies that keep their parameterization identical to that of the benchmark model economy except for the specification of habits differ not only in the degree of risk aversion but also in the IES. For this reason we also compare the benchmark economy with another model economy with habits but adjusted so that it has the same IES. This is achieved by changing one parameter of preferences.

[16]However, in section 2.4.2 we also provide some simulations for the *survival consumption* utility function to show that the qualitative results are the same.

This paper explores the role of habits in shaping the distribution of wealth. Part of our interest is in the size of precautionary savings, but another important part of our concerns is inequality. We want to separate the effects of habits on both characteristics and for this reason we also investigate an economy that has not only the same IES as the benchmark model economy, but also the same amount of precautionary savings, and hence of total savings.

2.3.1 The benchmark model economy (no habits)

In the benchmark model economy, preferences are of the CRRA form,

$$\sum_t \beta^t \frac{c_t^{1-\sigma} - 1}{1 - \sigma}$$

and we set a period to be one year. Parameter β is set at .96, which places the equilibrium interest rate around 4%. We set the Intertemporal Elasticity of Substitution, $\frac{1}{\sigma}$ to be equal to 0.5, a value that is around those most preferred by economists. This is our only departure from Aiyagari (1994) in the benchmark model economy since for all other choices we mimic his values. [17] Production occurs through a standard neoclassical production function $F(K_t, L_t) = K_t^{\theta} L_t^{1-\theta}$. Capital share is equal to 0.36 and the depreciation rate of capital δ is set equal to 0.08. Note that these are all standard values.

With respect to the process for earnings, Aiyagari (1994) sets an AR(1) in the logarithm of labour income. The process is fully described by two parameters: its persistence and its volatility. He chooses both values following estimates of Kydland (1984) that used PSID data and of Abowd and Card (1987) and Abowd and Card (1989) that used both PSID and NLS data. Then, he approximates the process by using a seven state Markov chain following the procedures described in Tauchen (1986). We follow the same directions although we reduce the Markov

[17] Aiyagari (1994) uses values of 1, 0.33 and, 0.2. Ghez and Becker (1975) and MaCurdy (1981), both using a life cycle model and explicitly accounting for leisure postulate a low value. Mehra and Prescott (1985) and Prescott (1986) discuss other estimates in the literature and conclude that a reasonable number is not too far from 1 (notice that the models they use have quarters as periods). Cooley and Prescott (1995) point out that this parameter is among the most difficult to pin down and settle for a value of 1. Hurd (1989) has a point estimate below one.

chain to three states.[18] We take our benchmark to be an autocorrelation of 0.6 and a coefficient of variation of 0.2.[19] We later provide results for an economy that has a lot more earnings dispersion, an economy capable of generating wealth dispersion more in accordance with the data.[20] The specific parameter values that we choose are summarized in Table 2.1.

Table 2.1: **Parameter values of the benchmark model economy**

	General Parameters		
β	σ	θ	δ
0.96	2	0.36	0.08
	Earnings Process		
$e \in \{e_1, e_2, e_3\} =$	$\{.78,$	$1.00,$	$1.27\}$
$\pi_{e,e'} =$	0.66	0.27	0.07
	0.28	0.44	0.28
	0.07	0.27	0.66
	Stationary Distribution		
$\pi^\star =$	0.337	0.326	0.337

2.3.2 Calibration of the economies with habits

As we have already stated, we use the specification of preferences used by Carroll, Overland, and Weil (2000) and Fuhrer (2000) based on that of Abel (1990) where the stock of habits h enters multiplying the level of consumption. The per period

[18] As we describe below, habits introduce an additional choice state variable which dramatically increases the computational costs of the project. By choosing three states we reduce computer time drastically in a margin that has never proved to be important.

[19] Aiyagari (1994) provides results for autocorrelations of 0.0, 0.3, 0.6 and 0.9 and for coefficient of variations of 0.2 and 0.4.

[20] Aiyagari (1994) fails to account for the amount of wealth inequality in the U.S. The highest value of the coefficient of variation of assets in any of his model economies is 1.13 compared to 6.09 in the data. As Castañeda, Díaz-Giménez, and Ríos-Rull (2000) points out, this is in part due to the process of earnings that he chooses. According to Díaz-Gimenez, Quadrini, and Ríos-Rull (1997) the coefficient of variation for U.S. earnings from the 1992 Survey of Consumer Finances is as big as 4.19 whereas the largest value Aiyagari uses is 0.4.

utility function is

$$u(c,h) = \frac{(c\, h^{-\gamma})^{1-\sigma} - 1}{1-\sigma} = \frac{\left[c^{1-\gamma} \cdot \left(\frac{c}{h}\right)^{\gamma}\right]^{1-\sigma} - 1}{1-\sigma}, \qquad 1 > \gamma > 0. \tag{2.5}$$

The second way of writing the specification highlights the fact that consumers care about a composite good which is a weighted average of the absolute value of consumption (being rich or being poor) and the relative level of consumption with respect to the past (being better or worse than usual). For $\gamma = 0$ we are in the no-habits case: only absolute consumption matters. For $\gamma = 1$ we are in the opposite case: only relative consumption matters. Notice, hence, that the same reason that makes households willing to smooth consumption levels is going to make them willing to smooth the ratio of consumption over the habit stock (so they are better off with several small changes than with a single big change).

The evolution of the stock of habits is given by the function

$$h' = \psi(c,h) = (1-\lambda)\; h \;+\; \lambda\; c, \qquad \lambda \in (0,1]\,. \tag{2.6}$$

Thus, the level of habit is a weighted average of the stream of past consumption. The parameter $(1-\lambda)$ measures the persistence of the habit stock. The higher the value of λ the lower the duration of the influence of current consumption in future per period utilities[21]. As λ decreases the effect of c in future utilities increases and the ability of current consumption to modify the habit stock is reduced.[22]

Calibrating the basic habits model economy requires choosing values for the parameters λ and γ. Notice that the benchmark model economy has a representation under this parameterization: the value of γ is zero (which makes irrelevant the value of λ).

There are several studies that try to estimate the parameters of habit formation in consumption. Some use individual level short panels, whereas some others use

[21]Notice that for $\lambda = 1$ we are in the particular case that today's habit stock is only yesterday's consumption or, in other words, today's consumption only affects tomorrow's utility.

[22]Notice also that setting $\lambda = 0$ is not equal to the limit case of $\lambda \to 0$. One can rewrite the law of motion for habits as $\widetilde{h}' = (1-\lambda)\widetilde{h} + c$ where $\widetilde{h} \equiv \frac{h}{\lambda}$ because λ is just a constant that does not affect the maximization. Under this representation we see that the role of λ is solely to control the persistence of the process and does not affect the strength of consumption in the habit stock. However, we cannot do this normalization when $\lambda = 0$ because it would imply dividing the utility function by zero, which does affect the maximization. Therefore, in this particular case λ is doing two things, namely, setting the persistence of habits equal to one and saying that consumption does not affect the habit stock.

aggregate time series. Some try to find out which parameterizations are consistent with certain asset pricing regularities whereas some others try to estimate consumption demand functions or first order conditions.[23] The heterogeneity of data sets and techniques rises to a very wide range of possible values for our γ and λ. Ideally, we would be looking for estimations consistent with our model in functional forms and length of period. Unfortunately, this is hard to find.

The closest model to ours is the one by Fuhrer (2000) who uses quarterly data on aggregate consumption data in non-durable goods and services (from NIPA) to estimate a log-linearized consumption function where habits enter multiplicatively in the utility function as in our model. He estimates $\gamma = 0.8$ and $\lambda = 0.9985$. An estimation of $\sigma = 6.11$ is consistent with the $IES = 0.5$ we use throughout the paper.[24]

Constantinides (1990) and Boldrin, Christiano, and Fisher (1997) try to find which pairs of parameters are consistent with the observed risk premium and with both the observed risk premium and risk free rate respectively. However, they use the *survival consumption* formulation to introduce habits in the utility function. Boldrin, Christiano, and Fisher (1997) find the best fit with the equivalent to our notation of $\gamma = 0.58$ and $\lambda = 0.70$.[25] Constantindes finds several pairs that fit the risk premium, with weight of habit γ ranging from 0.09 to 0.49 and corresponding persistence parameter λ ranging from 0.10 to 0.37.[26] Although these papers solve the equity premium puzzle as stated by Mehra and Prescott (1985) they still have some counterfactual implications as the excessive unconditional variance of the risk free rate. Heaton (1995) performs a similar experiment with monthly aggregate time series in non-durable consumption and services by NIPA getting $\gamma = 0.71$ and $\lambda = 0.58$. However, he also sets a more ambitious framework allowing for habit formation and consumption durability to interact (and not to offset each other as

[23]There are some others that just test for the presence of habit formation without estimating any closed form. A good example of these is Meghir and Weber (1996).

[24]Fuhrer (2000) also allows for a fraction of agents not to behave rationally but just to eat all their current income. He estimates this fraction to be 25% of the total population.

[25]This is an abuse of notation because their formulation is different from ours. We can somehow 'translate' parameters from one to the other seeing the *survival consumption* representation as $u(c_t, h_t) = \frac{(c_t - h_t)^{1-\sigma}}{1-\sigma}$ and $h_{t+1} = (1-\lambda)h_t + \gamma c_t$.

[26]Constantinides (1990) represents $u(c_t, h_t) = \frac{(c_t - h_t)^{1-\sigma}}{1-\sigma}$. The discrete time version of the law of motion for the stock of habits he uses is $h_t = \left(\frac{1}{1+a}\right)^t h_0 + b\sum_{i=1}^{t}\left(\frac{1}{1+a}\right)^{i-1} c_{t-i}$. Thus, we translate $\lambda = 1 - \frac{1}{1+a}$ and $\gamma = b$.

in Ferson and Constantinides (1991)) and targeting not only first moments of asset returns but also second moments. With this larger moments set he finds $\gamma = 0.00$ and $\lambda = 1.0$ if a pure habit model is used (which is evidence against habits) but $\gamma = 0.67$ and $\lambda = 0.18$ if interaction with durability is allowed for. In this last case we see a very high persistence (in monthly data) not found in other studies.[27]

Dynan (2000) also uses the survival consumption formulation with individual level data on food consumption from PSID. Because of data restrictions[28] she imposes $\lambda = 1.0$ (i.e., only yesterday consumption matters) to find that γ cannot be said to be different from zero.[29]

We find Fuhrer estimation as the closest one to our formulation. Since he uses quarterly data and we are calibrating for a model period of one-year length we see his $\gamma = 0.8$ as an upper bound and end up choosing $\gamma = 0.75$. As for the persistence parameter, we work with a pair of values at each side of the possible range: $\lambda = 1.0$ and $\lambda = 0.25$. The former is consistent with Fuhrer estimation whereas the latter, acknowledging the diversity of empirical results, will show us what happens at the other side of the persistence range.[30,31] Our choices are described in Table 2.2.

[27]Two other important papers in the asset pricing literature with habit formation are Campbell and Cochrane (1999) and Boldrin, Christiano, and Fisher (2001). However, they do not seem to us such a useful reference for calibrating our habit process. The former paper sets up an external process for the habit stock, that is to say, the habit stock does not depend on individual consumption but on aggregate consumption instead. In the steady state of our economies aggregate variables are constant and therefore an external habit stock would also be a constant. The latter paper is an effort to put together asset pricing and business cycles in a model with many ingredients beyond habit formation.

[28]She has the same problem as Meghir and Weber (1996). Time dimension in individual level data is very short and does not allow for estimations of persistent habits.

[29]She uses the survival consumption formulation with individual data. This also helps finding γ not different from zero since it is very difficult to accommodate the large individual variability of consumption with the endogenous survival consumption level unless γ is very small

[30]We have obviously tried different values of γ. In none of our experiments γ has changed the qualitative results. γ behaves just as an amplifier of the habit phenomenon.

[31]We find interesting to explore a low lambda in spite of Fuhrer's findings because of the following. Meghir and Weber (1996) and Dynan (2000) reject the hypothesis of habit formation using individual level data. However, the lack of long time series forces them to equalize habit stock to yesterday's consumption. If habits are very persistent, as Heaton (1995) suggests, it might well be the case that their rejection of the habit hypothesis is driven by the fact that, under very persistent habits, yesterday's consumption ability to modify the habit stock is small.

Table 2.2: **Basic Parameters of the Habit Economies**

	γ	λ
Low Persistence Economy	0.75	1.00
High Persistence Economy	0.75	0.25
Benchmark Model Economy	0.00	–

2.3.3 The value of the intertemporal elasticity of substitution

With CRRA preferences and no habit formation, preferences over different periods in time and over different contingencies are the same. Adding time non-separable preferences breaks this symmetry. As shown by Constantinides (1990) and Boldrin, Christiano, and Fisher (1997), habit formation breaks the link between the Intertemporal Elasticity of Substitution and the level of risk aversion. In order to disentangle the effect of habit formation in each dimension we look at model economies with habit formation exhibiting the same IES as our benchmark so that the results show the effect of habits on preferences over uncertain levels of consumption, not over intertemporally distributed consumption. In Appendix 2 (section 2.8) we show that, for a version of the representative agent model without uncertainty, $IES = \frac{1}{\gamma+(1-\gamma)\sigma}$.[32] This tells us two things. First, if individuals are not financially constrained, preferences towards consumption in different moments of time do not depend on the persistence of the habit stock, $1-\lambda$, but only on the magnitude of habits in the utility function, γ,. Second, with habit formation preferences towards consumption in different moments of time exhibit less curvature than without habit formation[33] (in other words, households desire less consumption smoothing). The reason for this being, as posed by Carroll, Overland, and Weil (2000), that

> *the gain or loss in utility associated with a given increase or decrease in consumption over a long horizon will be diminished by the associated movement in the habit stock.*

Therefore, when we want to keep IES as in the benchmark economy given γ, we will adjust σ.

[32]Carroll, Overland, and Weil (2000) already show this result in a continuous time Ak growth economy.

[33]For $\sigma > 1$ and $0 < \gamma < 1$

2.4 Results

In this section we report the findings from the various model economies. The results have been computed by solving the household's problem with a two–dimensional spline tensor product (that we ensure generates a concave function). We construct a sample of 5000 households.[34] Then, using the decision rule, the law of motion for the exogenous state and a random number generator, we simulate the decisions of these households to find a new distribution of households. We iterate until the main statistics of the samples converge. Then, we compare the statistics generated for all the economies studied by using in all of them the same realizations of the random numbers.[35]

In Subsection 2.4.1 we review the Aiyagari (1994)'s model economy, which we will refer to as the benchmark economy hereafter. We compare his findings to those of the representative agent deterministic version of his economy. Notice that the main statistics of the representative agent deterministic version of our model with habits are the same as those of the model without habits. Subsection 2.4.2 describes the properties of economies with habits, without any further adjustments. Sections 2.4.3 and 2.4.4 show the results for economies with habits where we recalibrate our model economy so that, respectively, IES and total savings match those of the economy without habits.

2.4.1 The benchmark model economy

The main characteristics of the benchmark model economy, that we refer to sometimes as economy B, are described in Table 2.3.

The first column shows the values of the key statistics of the deterministic representative agent counterpart of the benchmark economy.[36] We denote this economy as D. As we see the interest rate is 4.17% and the capital output ratio is 2.959. For comparison purposes, we have normalized output to one in this deterministic

[34]We have also tried larger samples. We see that sample sizes beyond our choice do not change aggregate results. However, for histograms and for reporting the shares of certain groups we use a larger sample size of 50000.

[35]For further details, see Ríos-Rull (1998).

[36]The deterministic model has been calibrated with the same parameters as those used in the benchmark economy and setting the labour endowment equal to the unconditional mean of the earning process.

Table 2.3: **Main statistics of the Benchmark Economy and its Deterministic Counterpart**

	D Deterministic Economy	B Benchmark Economy	Change $\frac{B-D}{D} * 100$
Aggregate Assets	2.959	3.015	1.9%
Output	1.000	1.007	0.7%
Capital Output ratio	2.959	2.994	1.2%
Interest Rate	4.17%	4.02%	-3.5%
Coeff. of Variation of Wealth	0.0	0.748	–
Gini Index of Wealth	0.0	0.404	–

economy. The second column includes the statistics of the benchmark model economy. We also report two measures of wealth dispersion, the coefficient of variation and the Gini Index. The last column reports the proportional variation in the main statistics of the benchmark model economy with respect to the deterministic economy. Note that both economies have different interest rates. This means that agents are responding to different prices.[37] We want to highlight two main things from this table. First, precautionary savings, that we define as the excess in total wealth that a given economy has over its deterministic counterpart, are small, less than 2%, confirming Aiyagari's findings. Second, under this parameterization, assets holdings are very evenly distributed. The Gini coefficient, for example, is 0.40 while is 0.78 in the U.S. data.

2.4.2 The unadjusted habits' economies

As stated, we compare two habits economies with the benchmark model economy. Except for the existence of habits, the two habits economies have the same parameterization as the benchmark model economy. They differ from each other in the persistence of the habits. The second column of Table 2.4 reports the main statistics of what we refer as the non–persistent habit economy, or economy N, while the fourth column refers to the persistent habit economy, or economy P. The third and fifth columns have the rates of change between the habits economies and the

[37]In fact if in the benchmark model economy the interest rate were set exogenously at the level of the deterministic economy, total assets will be unbounded. See Huggett (1993), Aiyagari (1994), Ríos-Rull (1998).

benchmark model economies.[38]

Table 2.4: **Main statistics of the Benchmark and the Unadjusted Habits Economies**

	B Benchmark Economy	N Non–Pers. Habits $\lambda = 1$	Change $\frac{N-B}{B}$	P Pers. Habits $\lambda = 0.25$	Change $\frac{P-B}{B}$
Aggregate Assets	3.015	3.012	-0.1%	3.035	0.7%
Output	1.007	1.006	-0.0%	1.009	0.2%
Capital Output ratio	2.994	2.993	-0.0%	3.007	0.4%
Interest Rate	4.02%	4.03%	0.2%	3.97%	-1.3%
Precautionary Savings	1.9%	1.8%	-4.6%	2.6%	34.4%
Coeff. of Var. Wealth	0.748	0.721	-2.7%	0.669	-10.6%
Gini Index Wealth	0.404	0.393	-2.7%	0.367	-9.2%

Regarding total assets, we see that both economies are quite similar to the non-habits case. Aggregate savings are nearly unchanged, moving in opposite directions. Whereas in the non persistent case they fall by just a 0.09%, they increase by 0.67% in the persistent case. Movements in output and capital-output ratio follow. Even in the persistent economy, precautionary savings stay quite low, at 2.56%. However, wealth dispersion changes more. With non persistent habits, our measures of dispersion fall by about 3 percent but they fall about 10 percent with persistent habits.

We have also carried out some simulations with the *survival consumption* form of habits for both non-persistent and persistent cases.[39] Precautionary savings are 1.9% and 2.0% respectively whereas gini indices are 0.402 and 0.399. Notice therefore the small increase in precautionary savings (even for the non-persistent case there

[38] As in the previous Subsection, we are looking at the general equilibrium version of the economies where the interest rates adjust to ensure that aggregate asset holdings equate aggregate capital. This means that interest rates are different than in economy B. If we keep fixed the interest rate (without letting it clear markets) we see what happens in absence of this price effect. Economy N has total assets of 2.87 and economy P of 4.11, which clearly shows larger changes than their general equilibrium counterparts.

[39] Writing the utility function as $u(c_t, h_t) = \frac{(c_t - \gamma h_t)^{1-\sigma} - 1}{1-\sigma}$ and the law of motion for the habit stock as given by equation 2.6 we set parameters as in economies N and P with one difference. We keep $\gamma = 0.1$. The reason for that is that with higher γ we start to need to solve for cases in which $c < \gamma h$. Getting around this problem implies a different strategy in the way to solve the household problem.

is a tiny increase hidden by the rounding) and the fall in the inequality measures.

Figure 2.1 shows the Lorenz curves for assets of the three model economies. We see how similar are the benchmark and the non–persistent unadjusted habit model economies while in the persistent unadjusted model economy the distribution is a little bit more even. We report the histograms of asset holdings in Figure 2.2. The histogram shows quite similar pictures for all economies.

Overall, we see that for economies that differ only in the specification of habits from the benchmark model economy, the implied differences for precautionary savings are not very big. They fall slightly for the non persistent economy and they increase for the persistent economy. This is a pattern we find throughout all experiments: precautionary savings are always larger for economies where habits are persistent than for economies where habits are non persistent. Inequality indicators fall in both economies, more in the persistent case.

However, the differences we have seen between the benchmark economy and the habits economies cannot be solely attributed to the effect of habits. In particular, the benchmark model economy has an intertemporal IES = 0.5 while that of the unadjusted economies with habits have a value of IES = 0.8. This means that households in the unadjusted habits economies have a smaller desire to smooth consumption intertemporally. In the next subsection we report the properties of economies with habits, both persistent and non persistent where the parameter σ has been adjusted to generate a $IES = 0.5$, the value of the benchmark model economy.

2.4.3 Adjusting habits to match the intertemporal elasticity of substitution

Habits break the link between the individual's willingness to choose a contingent consumption plan, measured by the Coefficient of Relative Risk Aversion and the individual's willingness to intertemporally substitute consumption, measured by the Intertemporal Elasticity of Substitution. Each measure is the inverse of the other in the representative agent version of the benchmark economy, the economy without habits. This is no longer true in an economy with habits. Thus, to investigate the effect of habits on the level of precautionary savings we need to isolate the effect of habits on the level of risk aversion from the effect on the IES. In this subsection

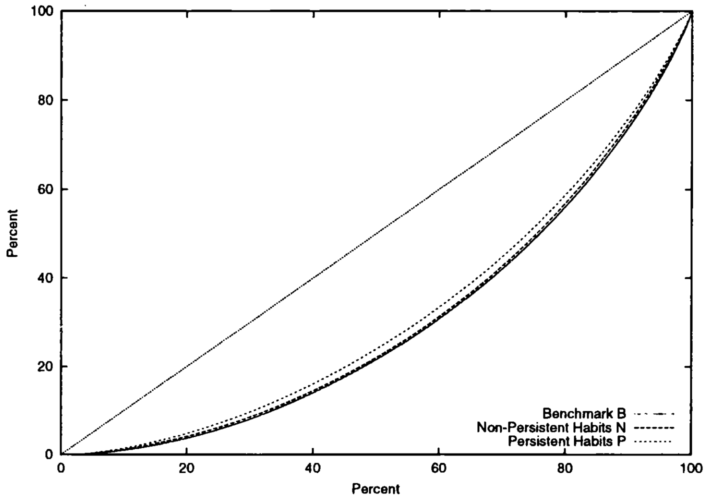

Figure 2.1: **Lorenz curve for assets: Unadjusted Economies, $\sigma = 2$, $\beta = 0.96$.**

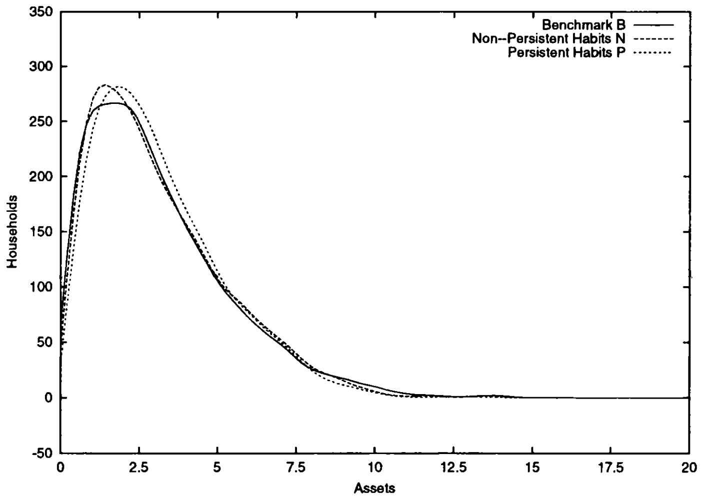

Figure 2.2: **Histogram for assets: Unadjusted Economies $\sigma = 2$, $\beta = 0.96$. General Equilibrium**

we recalibrate our habits economies so that the IES of the habits economies is the same as in the benchmark economy.[40]

Recall that in economies with habits $IES = \frac{1}{\gamma+(1-\gamma)\,\sigma}$, where γ is calibrated at 0.75 and IES targeted to 0.5. This implies that σ has to be increased to 5. Notice that this means that households in the unadjusted habits economies of the previous section have substantially less desire for smoothing consumption over time than they will in the economy where we adjust σ. We have then two new model economies, an economy with non-persistent habits adjusted so that its $IES = 0.5$, that we denote economy M, and an economy with persistent habits also adjusted to have its $IES = 0.5$, that we denote economy Q.

Table 2.5: **Main statistics of the Benchmark and the Adjusted Habits Economies** $IES = 0.5$

	B Benchmark Economy	M Non–Pers. Habits $\lambda = 1$	Change $\frac{M-B}{B}$	Q Pers. Habits $\lambda = 0.25$	Change $\frac{Q-B}{B}$
Aggregate Assets	3.015	3.066	1.7%	3.126	3.7%
Output	1.007	1.013	0.6%	1.020	1.3%
Capital Output ratio	2.994	3.027	1.1%	3.065	2.4%
Interest Rate	4.02%	3.89%	-3.2%	3.75%	-6.9%
Precautionary Savings	1.9%	3.6%	92.8%	5.7%	200.9%
Coeff. of Var. Wealth	0.748	0.676	-9.6%	0.611	-18.3%
Gini Index Wealth	0.404	0.371	-8.2%	0.339	-16.1%

The results are reported in Table 2.5. Notice that precautionary savings increase substantially in both economies with respect to the deterministic case. Now, they are 3.6% of total wealth in the deterministic case in the non-persistent habits economies and more than 5.6% in the persistent habits economies. So precautionary savings are between two and three times larger than in the benchmark model economy depending on the persistence. So habits indeed increase precautionary savings over the benchmark model economy, although perhaps their effect on aggregate capital is small.

With respect to wealth dispersion, we see an overall reduction of the inequal-

[40]We are referring to the IES of the deterministic representative agent version of each model throughout the paper.

ity indicators, which is more evident in the economy with persistent habits where the reduction in inequality is more dramatic. Figure 2.3 shows the Lorenz curves for assets of the benchmark model economies and of the habits economies with $IES = 0.5$, while Figure 2.4 reports the histograms of asset holdings. Here we start seeing a much clearer picture than in the economies with a larger Intertemporal Elasticity of Substitution. Inequality clearly goes down, especially for persistent habits. The Lorenz curve of the habits are much closer to the diagonal than those in the benchmark model economy and the histograms seem to be much tighter. It is also more evident that a big part of the distributional differences are due to poor people: households in economies with habits, not wanting to face fluctuations on consumption, make sure they do not hold too low asset levels. In any case, the reduction of inequality as measured by the statistics that we have chosen is always less than 20 percent.

2.4.4 Economies with the same savings as the benchmark model economy

To finish we want to isolate the effects of habits on inequality from those on precautionary savings. To this end we perform a second adjustment on the habit economies. We adjust the parameter β so that aggregate capital and hence the equilibrium interest rate is equal to that of the benchmark model economy.

Table 2.6: **Main statistics of the Benchmark and the Habits Economies Adjusted to have $IES = 0.5$ and identical precautionary savings.**

	B	M'		Q'	
	Benchmark	Non–Pers.	Change	Pers.	Change
	Economy	Habits		Habits	
		$\lambda = 1$	$\frac{M-B}{B}$	$\lambda = 0.25$	$\frac{Q-B}{B}$
β	0.960	0.959	-0.1%	0.957	-0.3%
Coeff. of Var. Wealth	0.748	0.678	-9.4%	0.616	-17.6%
Gini Index Wealth	0.404	0.372	-7.9%	0.341	-15.6%

We label M' and Q' the non-persistent and persistent economies respectively. Results are in Table 2.6. As we can see the statistics for inequality are essentially identical to those for the economies of the previous subsection with the same β as

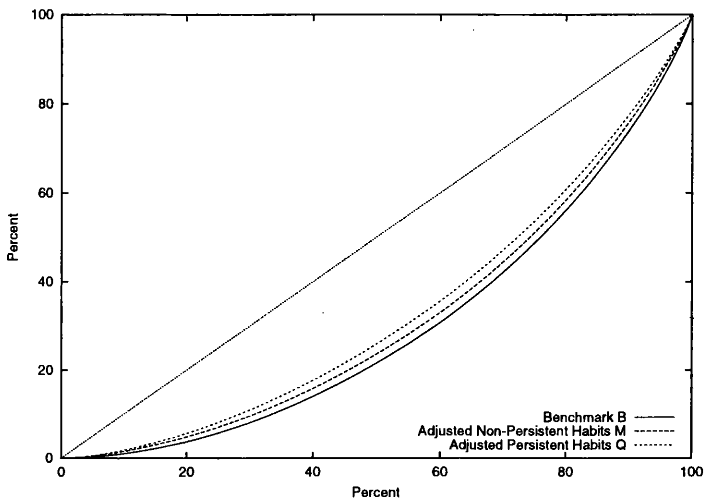

Figure 2.3: **Lorenz curve for assets: Economies Adjusted to the IES**

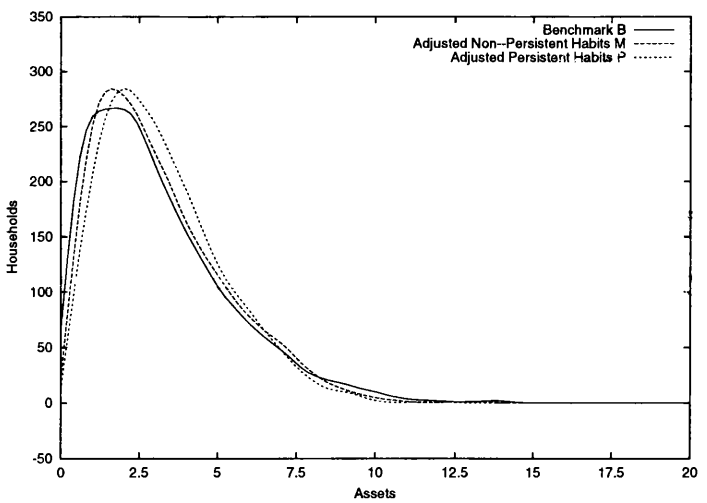

Figure 2.4: **Histogram for assets: Economies Adjusted to the IES**

the benchmark model economy. Figure 2.5 shows the Lorenz curves for assets of the benchmark model economy and of the habits economies adjusted to have the same IES and the same precautionary savings as the benchmark economy, while Figure 2.6 reports their histograms of asset holdings. Both the Lorenz curve and the histogram resemble the ones already seen in the previous section, but with the value added for the histogram that the means of the distribution for the three economies are set to be equal. This allows us to see where the differences in wealth dispersion lie. We basically see that, as already stated, the habit economies have much fewer people in low levels of assets and more people about the mean, with hardly no differences in the high values. Again, this effect is stronger for persistent habits.

2.4.5 Final comments

We have seen two things: first, in habits economies precautionary savings are substantially higher and the level of wealth inequality is a little lower than in their non habits counterparts and, second, these effects are stronger in economies with persistent habits. We comment each result in detail.

Households in habits economies are more displeased with fluctuations in consumption than their counterparts in economies without habits. Households with habits want not only a stable pattern of consumption but also a stable pattern of variations in consumption. Consequently, households in habits economies hold more assets. We see this in both the size of precautionary savings and in the shape of the lower tail of the distribution. Regarding the size of precautionary savings, we have seen that once adjusting σ to keep IES unchanged, total precautionary savings doubles or triples depending on persistence, admittedly from quite a low value (1.9%). Regarding the shape of the lower tail, we observe that it is asset-poor households who increase more their asset holdings.[41] The reason for this is twofold. On the one hand, the share of uncertain labour earnings in their total income is higher for asset-poor households than for asset-rich households. On the other hand, asset-poor households are not well self-insured, which means that income fluctuations get easily translated into consumption fluctuations. On the contrary, asset-rich households

[41] Quantitatively, the bottom 5% of the assets distribution have an average stock of assets of 0.13 (0.2% share) in the benchmark economy whereas they have 0.27 (0.4%) and 0.31 (0.5%) in the economies labeled M' and Q'. On the other side, for the top 5% the average assets are 9.00 (15.0%) for the benchmark and 8.11 (13.5%) and 7.56 (12.6%) for the economies M' and Q'.

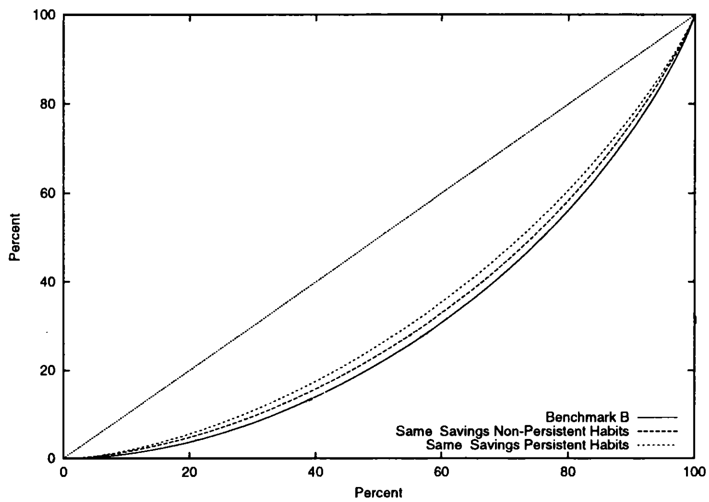

Figure 2.5: **Lorenz curve for assets: Economies adjusted to savings, and IES**

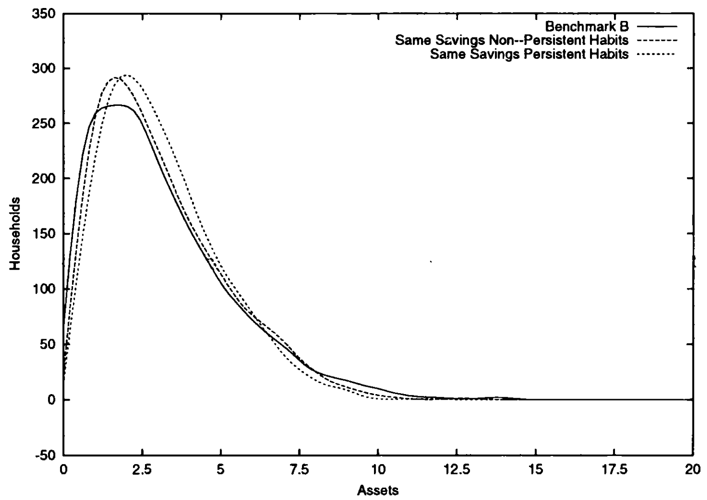

Figure 2.6: **Histogram for assets: Economies Adjusted to savings and IES**

have a stock of assets large enough to buffer fluctuations. So it is for the former households that an increase on the loss of utility due to consumption fluctuations is more likely to change savings behaviour. This asymmetric impact of habits on asset-poor and asset-rich households accounts for the reduction in inequality.

Both effects on precautionary savings and on inequality are stronger for persistent habits. A possible explanation for this is as follows. When habits are persistent, a fall in consumption today has a small impact in lowering the habit stock for tomorrow. In contrast, the opposite is true when habits are non persistent: a fall in consumption today is easily translated in a fall in the habit stock for tomorrow. Since not only the consumption level but also consumption relative to the habit stock matters, the fall in consumption is worse in utility terms if the habit stock stays stuck at its previous level than if it falls together with consumption. In a sense, non-persistent habits act as a safety net: being poor is not so bad because one gets easily used to it.

As we have stated, the overall effect on aggregate capital is not big. This is because the largest changes in savings behaviour are done by asset-poor people, whose share of total assets is very small.

We turn next to explore whether these findings are specific to our parameterization or also hold for a larger set of model economies.

2.5 Economies with high earnings variability

One of the problems that Aiyagari's economy has in trying to match the U.S. wealth distribution is that the earnings distribution itself, an exogenous element, is already lacking dispersion. In his benchmark economy, he sets the coefficient of variation for the earnings distribution to be equal to 0.2, which in our experiments gives a Gini index of 0.11. The values of the Coefficient of variation and the Gini index for the U.S. economy are respectively 4.19 and 0.63.[42] One interesting robustness analysis is, hence, to see how the conclusions change if we allow for an income process generating much more earnings inequality to a level similar to the U.S. data. Castañeda, Díaz-Giménez, and Ríos-Rull (2000) calibrate the earnings process (among other features of their model economy) so that a suitably modified version of Aiyagari's model

[42]See Díaz-Gimenez, Quadrini, and Ríos-Rull (1997).

accounts for the Lorenz curve of wealth observed in the U.S. We construct a 3 point Markov process that has some of the properties of the 4 point Markov process of Castañeda, Díaz-Giménez, and Ríos-Rull (2000)[43] that we report in Table 2.7. To get a high Gini coefficient with just three points in the Markov chain, one needs to make each state very different, in the process that we construct the endowment of the lucky households is almost 50 times the endowment of the unlucky ones. This process for earnings has a Gini index of 0.60.[44]

Table 2.7: **Earnings Process of the High Earnings Variability Economies**

$e \in \{e_1, e_2, e_3\} =$	{1.00,	5.29,	46.55}
$\pi_{e,e'} =$	$\begin{bmatrix} 0.992 & 0.008 & 0.000 \\ 0.009 & 0.980 & 0.011 \\ 0.000 & 0.083 & 0.917 \end{bmatrix}$		
$\pi^* =$	0.481	0.456	0.063

We run the same experiments that we run for the low earnings volatility process with this new earnings process, following the same calibration procedures. The only difference lies in the parameter β because we want the benchmark economy to have the same aggregate capital and interest rate as the benchmark economy with Aiyagari's earnings process. To do so, β must be lowered from 0.96 to 0.887. Higher variability calls for more precautionary savings.

What we find is that qualitative results remain unchanged. In the first panel of Table 2.8 we can see the no habits economy B^* against what we called economies M^* and Q^*: those with non persistent and persistent habits respectively with σ adjusted to have the same IES as in the no-habits economy. In the no-habits economy precautionary savings are 134.3%, a huge number compared to the 1.9% with

[43]There are many ways of implementing this reduction. Our choice should be seen as merely illustrative for the study of the properties that habit formation has on Economies with high Earnings variability.

[44]The process estimated by Castañeda, Díaz-Giménez, and Ríos-Rull (2000) includes retirees and it was designed for a model where households choose work effort. The process here is just intended to be in the ball park of that one. For instance, notice that even though the Gini Index is close to that in the data, its coefficient of variation is smaller than one half that in the data.

Table 2.8: **Main statistics of the High Earnings Variability Economies**

Habits Economies Adjusted to have $IES = 0.5$

	B^*	M^*		Q^*	
	No Habits	Non–Pers. Habits $\lambda = 1$	Change $\frac{N-B}{B}$	Pers. Habits $\lambda = 0.25$	Change $\frac{P-B}{B}$
Aggregate Assets	3.015	3.848	27.7%	5.948	97.3%
Output	1.007	1.099	9.2%	1.285	27.7%
Capital Output ratio	2.994	3.501	16.9%	4.626	54.5%
Interest Rate	4.02%	2.28%	-43.2%	-0.22%	-105.4%
Precautionary Savings	134.3%	199.1%	48.2%	362.3%	169.8%
Coeff. of Var. Wealth	2.491	2.405	-3.4%	2.274	-8.7%
Gini Index Wealth	0.857	0.831	-3.0%	0.805	-6.1%

Habits Economies Adjusted to have $IES = 0.5$ and identical precautionary savings.

	B^*	$M^{*'}$		$Q^{*'}$	
Precautionary Savings	134.3%	201.6%	50.2%	455.3%	239.1%
β	0.887	0.859	-3.1%	0.781	-11.9%
Coeff. of Var. Wealth	2.491	2.459	-1.3%	2.486	-0.2%
Gini Index Wealth	0.857	0.838	-2.2%	0.830	-3.2%

Aiyagari's earnings process. This is the consequence of having such a big variability in earnings. Remember that in Aiyagari's earnings process the endowment in good time is less than 50% higher than the endowment in bad times, whereas in the high earnings variability process, the endowment in good times is about 50 times larger than in bad times. As before, the habits economies exhibit higher precautionary savings, being the increase larger for the economy Q^* with persistent habits. In the non persistent habits economy, precautionary savings are 50% larger while in the persistent habits economy, they are more than 150% larger. This time the increase is over an already very large number,[45] making the role of habits very important in shaping this variable.

The model economies display a much higher coefficient of variation and Gini indices than the economies with Aiyagari's earnings process. Moreover, the values of the Gini index are even larger than the 0.78 of U.S. data.[46] This very high concentration of wealth can be seen both by means of the Lorenz curves plotted in Figure 2.7, that are much closer to the bottom right corner than the earlier ones, and by means of the shares of wealth held by selected groups of households reported in Table 2.9 where we can see that the share of wealth of the bottom 40% is zero and that of the top 10% is about 77%.

But what we really care about is the contribution of habits to shape inequality. We see that all our measures of inequality fall somewhat in the habits economies versus the no-habits economy, again more sharply for the economy with persistent habits. The main change in all economies occurs by having an increase in the share of the third and fourth quintiles at the expense of the fifth, especially of the households in the 80-95 percentiles. The fall of inequality in economies with habits adjusted to have the same precautionary savings is almost zero. However, the distribution of wealth has changed as it can be seen in Table 2.9, even if the total contribution of these changes to the coefficient of variation and the Gini Index is minimal.

To sum up, we just want to point out that the conclusions under this more

[45]Recall that these economies are parameterized so that the no-habits economy, B^*, has the same wealth as the benchmark model economy, B, and for this the discount rates have been reduced quite dramatically.

[46]As shown in Castañeda, Díaz-Giménez, and Ríos-Rull (2000) this type of process accounts for wealth inequality in an economy with a lot more detail built in. The actual number of the version that we use in this paper is not so important. Here, we are not after accounting for wealth inequality, but we are trying to measure the role of habit formation in changing our answers about wealth inequality and precautionary savings.

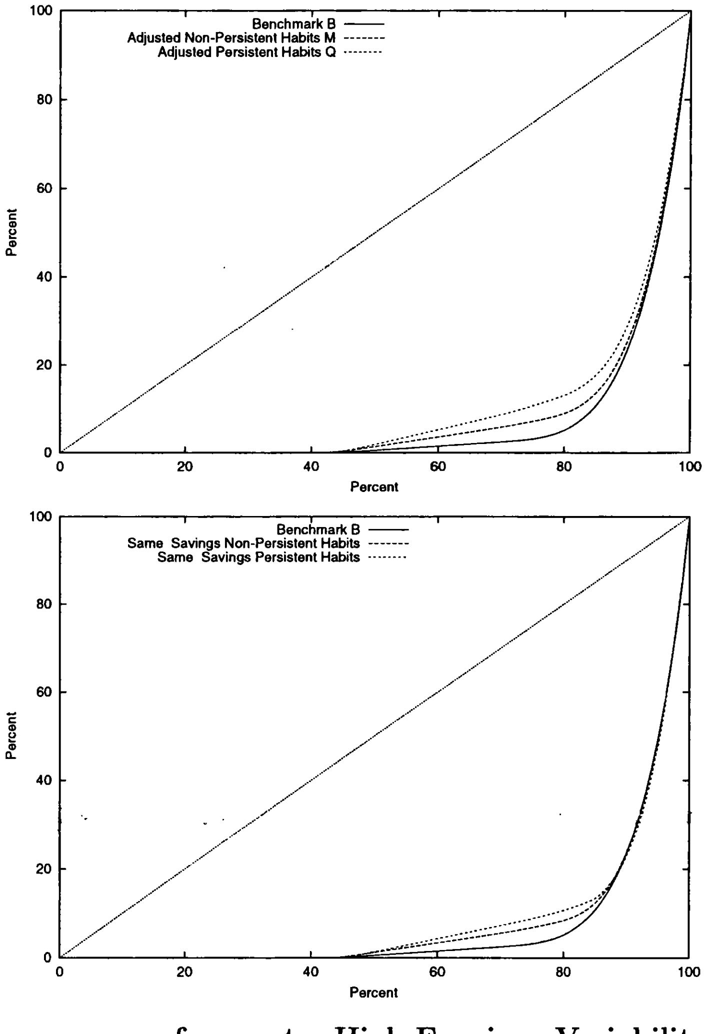

Figure 2.7: **Lorenz curve for assets: High Earnings Variability Economies.**

Table 2.9: **The Distributions of Wealth in the High Earnings Variability Economies**

	Quintiles					*Top Groups (%)*		
Economy	1st	2nd	3rd	4th	5th	Top 10%	Top 5%	Top 1%
B^* (No Habits)	0.00	0.00	1.51	3.60	94.89	76.62	50.62	13.65
M^* (IES–Adj Non-Pers)	0.04	0.04	3.52	5.35	91.04	74.75	49.47	12.91
Q^* (IES–Adj. Pers)	0.01	0.02	5.23	7.85	86.89	71.21	47.21	12.05
M'^* (Same Sav Non-Per)	0.01	0.01	3.35	5.04	91.60	76.54	50.89	12.89
Q'^* (Same Sav Per)	0.00	0.00	4.28	6.39	89.32	77.23	52.43	12.59

volatile earnings process strengthen those obtained with the less volatile Aiyagari's process. We find that precautionary savings (and aggregate capital) increase dramatically whereas our measures of inequality fall slightly, these effects being stronger for persistent habits. Also as before, we see that it is asset-poor households who proportionally raise more their asset holdings.

2.6 Conclusions

In this paper we have explored the role of habits in shaping the distribution of wealth. Our findings indicate that once we properly calibrate the economy to match the Intertemporal Elasticity of Substitution, the introduction of habits increases precautionary savings up to three times the (low) level of the benchmark model economy calibrated as in Aiyagari (1994). Its role in shaping inequality is that it reduces it: the inequality statistics go down in some cases 18%. These differences with the benchmark economy are more pronounced when habits are persistent than when they are not. When we calibrate our economies using an earnings process that better matches the observed earnings inequality in the U.S. we find the same qualitative results. Precautionary savings increase substantially from an already very large value and inequality indicators decrease a bit, with changes being bigger for persistent habits economies.

Habits affect the way households dislike consumption fluctuations. In this class of incomplete markets economies, where households are subject to idiosyncratic shocks, income fluctuations are only partially insurable through the accumulation of assets. Households with a long stream of good shocks hold big amounts of assets and reach a satisfactory degree of insurance. Households with a long stream of bad shocks are left with few assets and therefore have to bear consumption fluctuations. Not surprisingly, the comparison between an economy with habit formation and an economy without habit formation shows that it is asset-poor people behaviour that differs the most. Since the presence of habits makes consumption fluctuations more painful, those households with a small level of self-insurance will try to increase it by holding higher asset stocks. This makes the wealth distribution more even by reducing the number of people holding very low levels of assets.

2.7 Appendix 1. Computational procedures

To solve the consumer's problem described in section 2.2.4 we follow a successive approximations approach in the value function. Our individual state space contains two endogenous individual variables (assets and habits) as well as the exogenous idiosyncratic shock. This implies the need to create a two dimensional grid for the endogenous state and interpolate for solutions of assets and habits tomorrow different from the grid points. We will do this interpolation by two-dimensional splines. To our knowledge, there is no attempt done to solve a problem of this class through bidimensional splines. We explain below how we implement it. To solve for the steady state we proceed as follows. First, given a pair of prices $\{w, r\}$ we solve the household problem. Second, we compute the aggregate capital implied by this solution. This aggregate capital may or may not be consistent with the given prices $\{w, r\}$. If it is, they are the steady state prices. If it is not, we get a new pair $\{w', r'\}$ and repeat the process. Below we describe the procedure in more detail

2.7.1 Solving the household problem

The Contraction Mapping Theorem tells us that following a successive approximations strategy in the functional equation 2.1 will guarantee finding its fixed point. Moreover, any initial guess will do as long as it is concave in its endogenous arguments. More precisely, deriving the FOC we get the following system:

$$
\begin{aligned}
0 &= -u_c(c,h) + \beta \sum_{e'} \pi_{e,e'} \left[v_a\left(e', a', \psi(c,h)\right) - v_h\left(e', a', \psi(c,h)\right) \psi_c(c,h) \right] \qquad (2.7) \\
c &= e\,w + (1+r)\,a - a'
\end{aligned}
$$

which defines implicitly the policy functions $a' = g^a(e,a,h)$ and $c = g^c(e,a,h) = ew + (1+r)a - g^a(e,a,h)$. We substitute them back into equation 2.1 to get:

$$
v(e,a,h) = u[g^c(e,a,h), h] \; + \; \beta \sum_{e'} \pi_{e,e'} \quad v\left\{e', g^a(e,a,h), \psi[g^c(e,a,h), h]\right\} \quad (2.8)
$$

First, we choose a family of functions that the computer can understand. The problem we face here is one of two endogenous choice state variables. This means that we will need to compute the value function at any point in a bidimensional continuous support as well as at each point of the Markov process. We choose a bispline interpolation over a grid on a and h. Splines are very useful in this context because they guarantee continuous first and second derivatives. We need first derivatives to write the FOC and second derivatives to use Newton-based non-linear equation solvers. [47] Then, we guess an initial value function v^0, solve numerically the FOC 2.7 at each point (e_i, a_j, h_k) of the three-dimensional grid for the state space, get the policy functions $g^{0,a}(e_i, a_j, h_k)$ and $g^{0,c}(e_i, a_j, h_k)$, use the bispline interpolation to substitute them back into the functional equation 2.8 and get an updated v^1. If v^1 and v^0 are close enough we reached the fixed point. If they are not we iterate on, using v^1 instead of v^0 to get certain v^2.

There are two possible problems associated with the approach just described. The first one is that the Contraction Mapping Theorem does not necessarily hold once we restrict the space of continuous and bounded functions to which v belongs to a some computer storable subspace. The second one is that the bispline approximation does not necessarily preserve concavity. Whereas we have not found any difficulty associated with the former problem, the latter deserves further comments. A spline is basically an interpolation mechanism that uses a third order polynomial in each interval between grid points. When a piece-wise linear approach is followed, what happens to a function to be approximated in one interval is totally

[47] We construct the bispline approximation by use of a tensor product of two unidimensional splines. An explanation on how to compute tensor products over two spaces of interpolating functions can be found in de Boor (1978), chapter XVII. In particular we use the algorithm implemented in the IMSL subroutine DBSINT.

independent of what happens to it in another one. However, this is not completely true for splines, since the requirement that first and second derivatives from the left and from the right at each grid point equal each other makes the polynomials in each interval not independent. A utility function is an object with sharp changes of slope, being first and second derivatives huge at low levels of c and h and much smaller at higher levels. This properties translate into the value function. Using few grid points means that, not only the approximation is worse than using many but also that certain properties of the function may be lost. Precisely, we observed that using a 15x15 grid[48] would lead, in the more extreme parameterizations[49], to a loss of concavity and, even worse, monotonicity. The reason of this is that the high first derivative of utility at low levels of c is translated into the adjacent intervals so that the splines approximation overshoots the function to be approximated. Only slowly the slope of the spline can go down and recover, creating a hump. To solve this, one needs to use many grid points close to zero, the area where this happens, to make sure the spline slope can fall gradually. We increased the grid to 75x20 points. Notice that this means solving the household's problem for 4500 points at each iteration

2.7.2 Solving for the steady state

Here we follow a standard procedure. We choose an initial guess r^1 and solve the household problem to get $g^a_{r^1}(e,a,h)$ and $g^c_{r^1}(e,a,h)$. Then, we guess an initial sample of individuals of size 5000 and apply to them $g^a_{r^1}(e,a,h)$, $g^c_{r^1}(e,a,h)$ and the law of motion for the Markov process 3000 times, which ensures in all experiments we have done that the main statistics of the sample are almost constant. This gives us the aggregate assets in the economy A_{r^1}. Then, we find r^2 such that the demand of capital by firms, $K(r)$ equals A_{r^1}. Say $r^1 < r^2$ (if not, relabel). Since capital demand is decreasing in r and aggregate assets are increasing in r, the steady state interest rate r^* belongs to the interval (r^1, r^2). From this point we start the iterative procedure. We take the middle point in the interval, call it r^3, and get the associated

[48]More dense close to the zeros than close to the upper bounds

[49]More extreme parameterizations mean high σ, high γ and high variability of the earnings process. All these three characteristics create higher differences in the marginal utility of consumption across grid points. The problems would first arise at those points with low consumption and high habits because the lower the consumption and the higher the habit, the higher the marginal utility of consumption

A_{r^3}. If $A_{r^3} < K(r^3)$ the interest rate we have tried is too small, so we set $r^1 = r^3$ and start again. If $A_{r^3} > K(r^3)$ the interest rate we have tried is too big, so we set $r^2 = r^3$ and start again. We stop when the distance between r^1 and r^2 is arbitrarily small. To ensure no sampling error is spoiling the convergence to r^* we use the same seed to initialize the random number generator in each iteration.

Some of our model economies (case of M and Q) are set to have the same interest rate and aggregate capital as a given one (B) by adjusting the time preference parameter β. The procedure used to get the steady state in these cases differs from the one just described in that r is fixed and in that we have to iterate in different values of β. The initialization of the procedure is not so clean because it is not possible to compute an interval (β^1, β^2) where our β^* belongs to. We proceed as follows. We guess an initial β^1. If $A_{\beta^1} < K^*$ we know we have a lower bound. If $A_{\beta^1} > K^*$ we know we have an upper bound. In the former (respectively latter) case we try higher (lower) betas until we find a β^2 for which $A_{\beta^2} > K^*$ ($A_{\beta^2} < K^*$). Then, since capital demand is invariant in beta and aggregate assets are increasing, we know that $\beta^* \in (\beta^1, \beta^2)$. From this point we can apply to β instead to r the iterative procedure described in the previous paragraph.

2.8 Appendix 2. Intertemporal elasticity of substitution

To correctly compare the habits economies with the non habits economy we want to make them equal in certain dimensions. One of these dimensions is the Intertemporal Elasticity of Substitution. The following theorem gives the explicit form for the IES in the habits economies. Notice that the IES is independent of the persistence of the habit stock.

Theorem 1. *For the certainty case with multiplicative habit, the Intertemporal Elasticity of Substitution in the steady state is independent of λ and equal to $\frac{1}{\gamma+(1-\gamma)\sigma}$*

Proof. We call multiplicative habit the case in which the instantaneous utility function is written as

$$u(c, h) = \frac{\left(c\, h^{-\gamma}\right)^{1-\sigma} - 1}{1-\sigma}, \qquad 1 > \gamma > 0.$$

Let's write down the Euler equation:

$$c_t^{-\sigma} h_t^{\gamma(\sigma-1)} - \beta\gamma\lambda \sum_{i=1}^{\infty} [\beta(1-\lambda)]^{i-1} x_{t+i} =$$
$$= \beta(1+r) \left\{ c_{t+1}^{-\sigma} h_{t+1}^{\gamma(\sigma-1)} - \beta\gamma\lambda \sum_{i=1}^{\infty} [\beta(1-\lambda)]^{i-1} x_{t+1+i} \right\}$$

where $x_t = c_t^{1-\sigma} h_t^{\gamma(\sigma-1)-1}$. Then, we can write the stock of habit as a function of all past consumption:

$$h_{t+1} = (1-\lambda)\, h_t + \lambda c_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i\, c_{t-i}$$

Steady state imposes $\frac{c_{t+1}}{c_t} = \eta$. Therefore, the habit stock in the steady state is:

$$h_t = A c_t$$

with $A \equiv \frac{\lambda}{\eta-(1-\lambda)}$, which means that consumption and habit stock grow at the same rate. Finally, in the steady state x_t becomes:

$$x_t = A^{\gamma(\sigma-1)-1} c_t^{-[\gamma+(1-\gamma)\sigma]}$$

With all this, we go to the Euler equation and replace the h and x by their steady state values:

$$A c_t^{-[\gamma+(1-\gamma)\sigma]} - \beta\gamma\lambda B \eta^{-[\gamma+(1-\gamma)\sigma]} c_t^{-[\gamma+(1-\gamma)\sigma]} =$$
$$= \beta(1+r) \left\{ A \eta^{-[\gamma+(1-\gamma)\sigma]} c_t^{-[\gamma+(1-\gamma)\sigma]} - \beta\gamma\lambda B \eta^{-2[\gamma+(1-\gamma)\sigma]} c_t^{-[\gamma+(1-\gamma)\sigma]} \right\}$$

where $B \equiv \sum_{i=0}^{\infty} [\beta(1-\lambda)]^i \left[\eta^{-[\gamma+(1-\gamma)\sigma]}\right]^i = \frac{1}{1-\beta(1-\lambda)\eta^{-[\gamma+(1-\gamma)\sigma]}}$

Getting $\eta^{-[\gamma+(1-\gamma)\sigma]}$ as common factor in the rhs lets us cancel out all the lhs to finally get:

$$1 = \beta(1+r)\eta^{-[\gamma+(1-\gamma)\sigma]}$$

or cleaner

$$\eta^{\gamma+(1-\gamma)\sigma} = \beta(1+r)$$

□

Theorem 2. *For the certainty case with survival consumption habit, the Intertemporal Elasticity of Substitution in the steady state is independent of λ and equal to $\frac{1}{\sigma}$*

Proof. We call survival consumption habit the case in which the instantaneous utility function is written as

$$u(c,h) = \frac{(c-\gamma h)^{1-\sigma} - 1}{1-\sigma}, \qquad 1 > \gamma > 0.$$

Let's write down the Euler equation:

$$(c_t - \gamma h_t)^{-\sigma} - \beta\gamma\lambda \sum_{i=1}^{\infty} [\beta(1-\lambda)]^{i-1} (c_{t+i} - \gamma h_{t+i})^{-\sigma} =$$

$$= \beta(1+r) \left\{ (c_{t+1} - \gamma h_{t+1})^{-\sigma} - \beta\gamma\lambda \sum_{i=1}^{\infty} [\beta(1-\lambda)]^{i-1} (c_{t+1+i} - \gamma h_{t+1+i})^{-\sigma} \right\}$$

Then, we can write the stock of habit as a function of all past consumption:

$$h_{t+1} = (1-\lambda) h_t + \lambda c_t = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i c_{t-i}$$

Steady state imposes $\frac{c_{t+1}}{c_t} = \eta$. Therefore, the habit stock in the steady state is:

$$h_t = A c_t$$

with $A \equiv \frac{\lambda}{\eta-(1-\lambda)}$, which means that consumption and habit stock grow at the same rate.

With all this, we go to the Euler equation and replace the h by its steady state value:

$$c_t^{-\sigma}(1-\gamma A)^{-\sigma} - \beta\gamma\lambda(1-\gamma A)^{-\sigma} B \eta^{-\sigma} c_t^{-\sigma} =$$
$$= \beta(1+r) \left\{ \eta^{-\sigma} c_t^{-\sigma}(1-\gamma A)^{-\sigma} - \beta\gamma\lambda(1-\gamma A)^{-\sigma} B \eta^{-2\sigma} c_t^{-\sigma} \right\}$$

where $B \equiv \sum_{i=0}^{\infty} [\beta(1-\lambda)\eta^{-\sigma}]^i = \frac{1}{1-\beta(1-\lambda)\eta^{-\sigma}}$

Getting $\eta^{-\sigma}$ as common factor in the rhs lets us cancel out all the lhs to finally get:

$$1 = \beta(1+r)\eta^{-\sigma}$$

or cleaner

$$\eta^{\sigma} = \beta(1+r)$$

□

Chapter 3

Pricing Risk in Economies with Heterogenous Agents and Incomplete Markets

3.1 Introduction

The equity premium puzzle, as stated by Mehra and Prescott (1985), uncovers the inability of the standard macroeconomic models to generate, for given parameter constraints, a differential return between risky and risk free assets as large as the one found in data. In other words, quantitative macroeconomic models produce a compensation for risk that is too small compared to its empirical counterpart.

One strand of the literature on asset pricing has proposed habit formation as an explanation for the equity premium. Habit formation increases the utility losses from consumption fluctuations and therefore increases the compensation for risk required to hold risky assets. Constantinides (1990), Abel (1990), Heaton (1995), Campbell and Cochrane (1999) and Boldrin, Christiano, and Fisher (1997) show how the extra parameters introduced by the habit formation hypothesis can be used to match the data on equity premium.

The original formulation of the puzzle, and the papers aforementioned, relied on the representative agent hypothesis, which implies that the consumption fluctuations faced by agents are equal to the fluctuations of aggregate consumption. Another branch of the asset pricing literature has attempted to allow for agents to differ in their earnings, wealth holdings and, consequently, in the consumption fluctuations they face. A higher variability of individual consumption should allow the equity premium to increase. However, it is found that without transaction costs or tight borrowing constraints, an asset structure of just risk free bonds and shares suffices to smooth out consumption fluctuations well enough so that the associated fluctuations in marginal utility do not command a high return on risky assets.[1]

This paper includes the habit formation hypothesis in the standard model with heterogenous agents and incomplete markets. In the model economy, agents differ in habit stocks as well as earnings and wealth holdings. The source of these differences is the absence of markets to insure against idiosyncratic shocks to labour income. This exercise extends the class of models used so far to show that habit formation can be a good explanation for the equity premium by allowing for heterogeneity in earnings and wealth and for consumption decisions of households. Instead of looking at the predictions for the equity premium once certain statistics on consumption data are given, it does so for given properties of earnings data. Consumption fluctuations are therefore derived as the result of optimal decisions of households.

As pointed out by Mankiw (1986) and Constantinides and Duffie (1996), the interaction between idiosyncratic uncertainty and aggregate uncertainty is crucial for the model to generate positive equity premia. In particular, it is required that the variance of the process for individual earnings is higher in downturns than in peaks. In this way, equity turns out to be an asset that pays well when less needed (in peaks agents face less volatility in earnings) and pays bad when things go wrong (in downturns earnings are more volatile). The model in this paper captures this feature by calibrating an employment process that generates higher and longer unemployment rates in downturns.[2]

The main result is that the habit formation hypothesis cannot reproduce the empirical consumption fluctuations needed to deliver high equity premia. Compared to the model without habits, what I find is that habit formation increases the equity premium by as much as 70%. However, the value found is still three orders of magnitude below its empirical counterpart. Also the Sharpe ratio increases by 70% but it still stays below its empirical estimates. The main reasons for this result

[1]See for example Heaton and Lucas (1996) and Krusell and Smith (1997)

[2]The interaction between earnings and aggregate shock turns out to be an empirical question. Is the variance of earnings negatively related to the cycle? Looking at PSID data, Storesletten, Telmer, and Yaron (2001) answer in the affirmative.

are general equilibrium ones. The first reason is that with just two assets, risk free bonds and shares, most households can insure themselves effectively against earnings shocks. Therefore, consumption fluctuations are small. Habit formation as an explanation for the equity premium relies on increasing the disutility of consumption fluctuations. The result is that the degree of consumption smoothness achieved is high enough to prevent the habit formation preferences from generating large fluctuations in marginal utilities. To be precise, the average over the model population of next period's expected consumption fluctuations falls by more than 40% when adding habits to the standard model. The second reason for this result is related to the composition of the set of agents pricing the assets. Typically, poorly insured agents, seeking for a hedge against risk, borrow as much as they can in risky assets to invest in bonds. On the contrary, well-insured agents go as short as they can with bonds and invest in risky assets. Only a small fraction in the middle have an interior solution to the portfolio choice problem. These agents form what I call the set of pricing agents. As one increases the disutility of risk (introduction of habit formation), the composition of the set of pricing agents changes. It will be better insured agents who will have interior solution. Therefore, not all the increase in the disutility of risk is translated into the equilibrium price of risk.

An accepted shortcoming for equilibrium models with production in trying to reproduce the equity premium is the lack of variability in the return of the risky asset. It is for this reason that Krusell and Smith (1997), following Hansen and Jagannathan (1991), decompose the equity premium into two parts: the price of risk and the amount of risk. In their main calibration, they do not get a sizeable price of risk. However, when they impose borrowing constraints, they find a price of risk similar to the one estimated by Lettau and Uhlig (1997). Consequently, they claim that for the equity premium to be matched, one only needs to increase the volatility of the return on physical capital.

To explore Krusell and Smith (1997) claim we need to expand somehow the volatility of the risky asset. To that purpose, I show how by increasing the volatility of the aggregate shock we get highly counterfactual implications for the volatility of the aggregate variables. Then, the paper also considers another departure from the standard model. It allows for firms to finance their capital by issuing risk bonds as well as shares. The introduction of leverage increases the volatility of the risky asset in a manner not inconsistent with the business cycles fluctuations of the macroeconomic variables. What I find is that the increase in the equity premium is quantitatively small. The interaction of habits and leverage produces a higher increase, but it still does not change the main results of the paper.

An interesting feature of the model is that it allows us to look at the portfolio choice of the agents. The results are as follows: willingness to hold risky assets (1) increases with wealth, (2) decreases with labour earnings and (3) decreases with habit stocks. The first result is a standard result already found in the literature. Wealthier households are further away from the borrowing constraints and therefore better insured against earnings uncertainty. The second result may be surprising. However, it should not be so. Once controlling for wealth, the role of the current earnings shock is solely to predict future earnings. Agents with high earnings also expect higher earnings in the future if, and only if, they remain employed. Since the source of earnings uncertainty related to the aggregate shock is the probability of being unemployed, they have more to lose by a downturn. The earnings variability conditional on aggregate shock is higher the higher the earnings level. Finally, the third result is also consistent with the literature. Households with higher habit stocks dislike fluctuations in consumption in a higher degree than households with lower habit stocks. Therefore, they are less willing to hold risky assets.

The rest of the paper continues as follows. Section 3.2 develops and details the model economy. Then, section 3.3 shows how I calibrate it to US data. Section 3.4 finds the optimality conditions and explains their implications. Then, section 3.5 looks at the benchmark economy and shows some results on portfolio choice. Section 3.6 shows two useful results. Firstly, by showing the effects of increasing the risk aversion parameter we can look at the composition effect. Secondly, it is showed how by increasing the amplitude of the business cycles fluctuations, the model equity premium increases while bringing some counterfactual implications at the same time. Next, section 3.7 shows how the habit formation hypothesis does not deliver big equity premia in a model where agents can save in order to avoid consumption fluctuations. Section 3.8 introduces leverage to the standard heterogeneous model in an attempt to increase the degree of riskiness of shares. Finally, section 3.9 concludes. The computational method is presented in the appendix (section 3.10.

3.2 The model economy

The basic structure of the economies in this paper is the standard growth model with aggregate uncertainty and heterogeneous agents with incomplete markets. The market incompleteness is the lack of insurance for the idiosyncratic shocks to labour earnings. We set a general framework to accommodate general consumer's preferences (allowing for habit formation) and leverage. We will then work out the non-habits and non-leverage cases as particular cases of the general structure.

3.2.1 Preferences

Preferences are stated generally to include habit formation. Non-habits preferences can be represented as a special parametric case. Households derive utility from both present and past consumption. Present consumption will be denoted by $c \in \mathcal{R}_+$ and past consumption will accumulate in a stock of habits denoted by $h \in \mathcal{R}_+$. The habit stock evolves according to the law of motion $h_{t+1} = \psi(c_t, h_t)$ with partial derivatives $\psi_c \in (0,1]$ and $\psi_h \in [0,1)$. Per period utility will be denoted by $u(c_t, h_t)$. Standard conditions on u_c apply. Habit formation hypothesis requires $u_h < 0$ and $u_{ch} > 0$. Agents are infinitely-lived and their total utility will be the infinite discounted sum of period utilities: $\sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$. Preferences are identical over households.

3.2.2 Technology

Output Y is produced using aggregate capital K and aggregate labour L. Output can be either consumed or invested to form productive capital. Capital is the only productive asset and depreciates at an exogenous rate $\delta \in [0,1]$. Aggregate labour is the sum of the economy's efficiency units of labour. We write the constant returns to scale production function as $F(z, K, L)$, where $z \in Z \equiv \{z_g, z_b\}$ is an exogenous stochastic technology level. It follows a Markov process represented by the transition probability matrix $\Gamma_z(z, z') \equiv \Pr(z_{t+1} = z' | z_t = z)$.

Households are subject to idiosyncratic shocks to labour earnings. I decompose the idiosyncratic risk into two parts. First, there is the employment shock. Employment possibilities $e \in E = \{0,1\}$ come stochastically and depend on the aggregate technology level z. At every period of time, households may $(e = 1)$ or may not

$(e = 0)$ be given an employment opportunity. Since agents do not value leisure the employment opportunity will be taken. Conditional on z and z' the process is iid and Markov with transition matrix $\Gamma_e(z, z', e, e') \equiv \Pr(e_{t+1} = e' | e_t = e, z_t = z, z_{t+1} = z')$. Second, when given an employment opportunity, agents also get an endowment of efficiency units of labour. Efficiency units of labour, represented by $\xi \in \Xi \equiv \{\xi_1, \xi_2, ..., \xi_{n_e}\}$, follow an iid process with Markov transition matrix $\Gamma_\xi(\xi, \xi') \equiv \Pr(\xi_{t+1} = \xi' | \xi_t = \xi)$. Notice that this process is independent of the aggregate shock z. When not given an employment opportunity households are assumed to operate a home technology that provides them with d units of consumption.

Since there is a continuum of households, a law of large number applies and the share of employed and unemployed people is only function of the aggregate shock. Total population is normalized to one and the unconditional expectation of efficiency units of labour $E[\xi]$ is also normalized to one. Unemployment rates of the economy in good and bad times are called u_g and u_b. Therefore, the amount of efficiency units L in the economy is a function $L(z)$ of the aggregate shock, with $L(z_g) = 1 - u_g$ and $L(z_b) = 1 - u_b$.

3.2.3 Market arrangements

Agents and firms trade two different assets: one-period risk free claims to consumption units $b \in B \equiv [\underline{b}, \infty)$ and shares for firm's ownership $s \in S \equiv [\underline{s}, \infty)$. One unit of the risk free bond b entitles a known payment of R^b units next period. One unit of shares s entitles a stochastic payment of R^s units next period, which will be a function of next period's aggregate shock. Notice that both assets are restricted by a lower bound. These lower bounds can be imposed as an exogenous borrowing constraint or can arise endogenously as the maximum borrowing that allows the agents to repay their debts in all states of the world. There are no state contingent markets for the idiosyncratic shocks. Therefore, neither employment nor efficiency units shocks can be insured against. The absence of state contingent markets plus the limitations in borrowing are the ingredients that allows the model to depart from the representative agent framework.[3] Firms rent labour in a competitive market (paying the marginal product w per efficiency unit of labour). Being a closed economy, households' aggregate savings will form the productive capital available in

[3]See Huggett (1993) and Aiyagari (1994) for details. Quadrini and Ríos-Rull (1997) contains a review on this topic.

the economy. Firms hire this capital to the households through two different markets. Part of it is obtained through the risk free one-period bonds. The remaining part comes in form of shares of firms' ownership, which take the residual marginal product of capital once bonds are paid.

3.2.4 The firm's problem

Firms will not choose the optimal composition of capital between bonds and shares. They take as given an exogenous share ρ of bonds over total capital. This share ρ will be called leverage. The exogeneity of ρ is quite a strong assumption. This paper does not intend to focus on the optimal leverage choice by firms in the context of heterogenous agents economy. Instead, it wants to assess the capability of certain mechanism, leverage, to provide us with the missing volatility of the economy's risky asset. We are trying to see if this increased volatility can cover some distance between the model equity premium and the empirical one.

Firms choose capital and labour solving the following static maximization problem:

$$\max_{K,L} \left\{ F(z,K,L) + (1-\delta)K - R^b \rho K - R^s (1-\rho) K - wL \right\}$$

R^b is the gross rate of return on bonds that clears the bonds market, and it is known by the firms when setting their capital demand. R^s and w, the gross return on shares and the wage rate, will be set equal to the marginal product of factors according to the FOC of the problem:

$$w = F_L(z,K,L) \tag{3.1}$$

$$R^s = \frac{1}{(1-\rho)} \left(F_K(z,K,L) + (1-\delta) \right) - \frac{\rho}{(1-\rho)} R^b \tag{3.2}$$

3.2.5 The household's problem

We formulate the problem recursively. Each individual state is given by the vector j formed by the agents's wealth ω, stock of habits h, employment opportunity e and efficiency units endowment ξ, plus the distribution of agents μ over this vector and the aggregate shock z.[4],[5] We define household wealth $\omega \in \Omega = [\underline{\omega}, \infty)$ as the sum

[4]Notice that when $e = 0$ the efficiency units endowment will be equal to zero.

[5]See section 3.2.6 for a clear definition of the model timing.

of bonds, shares, the income generated by them and the labour earnings[6]. μ is a probability measure over a $\sigma - algebra$ generated by the set $J \equiv \Omega \times R_{+} \times E \times \Xi$. The transition function for the measure μ is given by $\mu' = Q(\mu, z, z')$.[7]

Agents maximize the discounted sum of expected utilities by choosing consumption c, risk free bonds b and shares s subject to the feasibility constraints, the budget constraint, the law of motion for habits, the transition matrices for the exogenous shocks and the transition function for the aggregate state. The gross return on bonds depends on today's aggregate state (so it is known at the time of taking decisions) and the gross return on shares depends on today's aggregate state and also on next period's realization of the aggregate shock. The problem can be written as:

$$v(j, z, \mu) = \max_{c,b,s} \left\{ u(c, h) + \beta E_{e',\xi',z'|e,\xi,z} \left[v(j', z', \mu') \right] \right\} \tag{3.3}$$

subject to

$$c = \omega - b - s \tag{3.4}$$

$$h' = \psi(c, h) \tag{3.5}$$

$$\omega' = bR^b(\mu, z) + sR^s(\mu, z, z') + \Upsilon_{e'=1} w(\mu, z, z')\xi' + \Upsilon_{e'=0} d \tag{3.6}$$

$$(c, b, s) \geq (0, \underline{b}, \underline{s}) \tag{3.7}$$

where Υ_l is an indicator function that takes value 1 when the statement l is true and 0 otherwise. The expression $E_{l'|l}[var]$ is the operator for the mathematical expectation of var with respect the distribution of l' conditional on l. The laws of motion for e', ξ' and z' are implicit in the expectation operator. We are looking for the policy functions $c = g^c(j, z, \mu)$, $b = g^b(j, z, \mu)$ and $s = g^s(j, z, \mu)$.

3.2.6 Timing

Model period starts with households carrying certain amount of wealth ω, certain amount of habits h and knowing their idiosyncratic shocks e and ξ and the aggregate state formed by the distribution μ and the shock z. Households decide consumption c, bonds b and shares s. Firms decide the capital K' and labour L' demands.

[6]The lower bound $\underline{\omega}$ of Ω will be determined by the highest possible borrowing times the highest possible gross return plus the lowest possible labour income, ie, home production.

[7]Since the process for the employment shock depends on z and z' so will the transition function for the distribution of agents over the individual state vector j.

Then, the goods, bonds and shares markets clear. This sets productive capital K' and gross return on bonds R^b as functions of today's state z and μ. Then, nature provides the new shocks z', e' and ξ'. Labor market clears. Production $F(z', K', L')$ takes place. Factor inputs are rewarded their marginal products $w(\mu, z, z')$ and $R^s(\mu, z, z')$. Finally, the new wealth ω' is determined.

3.2.7 Equilibrium

An equilibrium for this economy is a set of functions $\{v, g^c, g^b, g^s, R^b, R^s, w\}$ and a law of motion Q such that:

1. Factor prices satisfy the firms' optimality conditions:

$$\begin{aligned} w(\mu, z, z') &= F_L(z', K', L(z')) \\ R^s(\mu, z, z') &= \frac{1}{(1-\rho)}(F_K(z', K', L(z')) + (1-\delta)) - \frac{\rho}{(1-\rho)} R^a(\mu, z) \end{aligned}$$

2. Given pricing functions $\{R^b, R^s, w\}$, a law of motion Q and the exogenous transition matrices $\{\Gamma_z, \Gamma_e, \Gamma_\xi\}$, functions $\{v, g^c, g^b, g^s\}$ solve the household problem

3. Labor market clears

$$L = L(z) = 1 - u_z$$

4. Shares market clears

$$\int g^s(j, z, \mu)\, d\mu = (1-\rho) K'$$

5. Bonds market clears

$$\int g^b(j, z, \mu)\, d\mu = \rho K'$$

6. Goods market clears

$$F(z, K, L) + (1-\delta) K = \int g^c(j, z, \mu)\, d\mu + \int g^b(j, z, \mu)\, d\mu + \int g^s(j, z, \mu)\, d\mu$$

7. The law of motion $Q(\mu, z, z')$ for the measure μ is generated by the optimal decisions $\{g^c, g^b, g^s\}$, the law of motion for habits ψ and the transition matrices $\{\Gamma_e, \Gamma_\xi\}$

Notice that conditions four and five imply

$$K' = \int g^b(j, z, \mu)\, d\mu + \int g^s(j, z, \mu)\, d\mu \tag{3.8}$$

which makes explicit the dependence of R^s and w on z and μ.

3.3 Calibration

Calibration will vary according to the differences in the economies we work with. However, some general principles can already be stated at this stage. To start with, we need to specify functional forms for our production function, instant utility function and law of motion for habits. Consistently with the lack of trend in US data for the factor shares, production function is assumed to be Cobb-Douglas,

$$F(z, K, L) = zK^{1-\theta}L^{\theta}$$

Utility function is assumed to be of the standard CES class. Habit formation is modelled as in Abel (1990), Fuhrer (2000) and in chapter 2 of this thesis. They characterize it by using the following utility function:

$$u(c, h) = \frac{(ch^{-\gamma})^{1-\sigma}}{1-\sigma} \quad \text{with} \quad \gamma \in (0, 1)$$

and the following law of motion for habits:

$$\psi(c, h) = (1 - \lambda)\, h + \lambda c \quad \text{with} \quad \lambda \in (0, 1]$$

Notice that the non-habits case has a representation under this formulation by setting γ equal to zero.[8]

[8]There is an alternative way of modelling habit formation in which habit stock enters utility function as a survival consumption level

$$u(c, h) = \frac{(c - \gamma h)^{1-\sigma}}{1-\sigma}$$

Implications are not necessarily the same. In representative agent frameworks this is the only way to have the Arrow-Pratt coefficient of risk aversion depending on the cycle and therefore to have a negative correlation between equity premium and the cycle. In a heterogeneous framework this is not necessary. See discussion in section 2.3.

The model period is imposed to be a quarter. Accordingly, depreciation of capital δ is set equal to 0.02 (see Cooley and Prescott (1995)) and intertemporal discount factor β is calibrated to produce an approximated capital output ratio of 10.75. Intertemporal elasticity of substitution is set to 0.5. Labor share θ to 0.64 (see Kydland and Prescott (1982)). Regarding the aggregate and employment shocks, z_g and z_b are chosen to be 1.01 and 0.99 whereas unemployment rates in good and bad times (u_g and u_b) are set to 0.04 and 0.1.[9] These standard values are consistent with the magnitude of fluctuations in postwar US macroeconomic series.

The two remaining conditions for the aggregate shock are that average duration of good and bad times equals 10 quarters each. The six remaining conditions for the employment shock are as follow. Firstly, average duration of unemployment spells in good and bad times are set to 1.5 and 2.5 quarters respectively. Secondly, to avoid aggregate labour being a state variable two extra conditions are imposed: (1) employment in good times must be the same regardless of last period being in good or bad times and (2) employment in bad times must be the same regardless of last period being in good or bad times:[10]

$$\begin{aligned}(1-u_g)\,\Gamma_e(z_g,z_g,1,1)+u_g\Gamma_e(z_g,z_g,0,1) &= \\ (1-u_b)\,\Gamma_e(z_b,z_g,1,1)+u_b\Gamma_e(z_b,z_g,0,1)\end{aligned}$$

$$\begin{aligned}(1-u_g)\,\Gamma_e(z_g,z_b,1,1)+u_g\Gamma_e(z_g,z_b,0,1) &= \\ (1-u_b)\,\Gamma_e(z_b,z_b,1,1)+u_b\Gamma_e(z_b,z_b,0,1)\end{aligned}$$

Third, probability for the unemployed finding a job when moving from good to bad times is set to zero and probability for the employed to enter unemployment when moving from bad to good times is also set to zero:[11]

$$\begin{aligned}\Gamma_e(z_g,z_b,0,1) &= 0 \\ \Gamma_e(z_b,z_g,1,0) &= 0\end{aligned}$$

[9]The aggregate shock introduces four parameters: the two levels of the shock and two independent parameters from the transition matrix. The employment shock introduces eight parameters: two for each of the four matrices.

[10]Aggregate labour becomes therefore just a function of the aggregate state.

[11]Notice that these 2 extra conditions satisfy:

$$\begin{aligned}\Gamma_e(z_g,z_b,0,1) &< \Gamma_e(z_g,z_g,0,1) \\ \Gamma_e(z_b,z_g,1,0) &< \Gamma_e(z_b,z_b,1,0)\end{aligned}$$

Regarding the efficiency units shock we establish three points and try to replicate some cross-section and time series statistics of US data. Table 3.1 shows these statistics (column 1) together with the ones produced by the chosen parameterization of the efficiency units shock (column 2). Table 3.2 presents the parameters for the efficiency units shock that generate the statistics in column 2 of table 3.1.[12]

Table 3.1: **Statistics of US data and simulated process**

	data	model
share of earnings of top 20%	60.2%	61.7%
share of earnings of bott 40%	3.8%	4.5%
gini index of earnings	0.61	0.55
persistence top 20%	68%	46%

Note: Cross section statistics in the first column are from SCF98. Persistence is the probability that those people on the top 20% in PSID 1989 are still there in PSID 1994

Table 3.2: **Stochastic process for efficiency units.**

	ξ_1	ξ_2	ξ_3
	30.0	8.0	1.0
	$\Gamma_\xi(\xi_1,.)$	$\Gamma_\xi(\xi_2,.)$	$\Gamma_\xi(\xi_3,.)$
$\Gamma_\xi(.,\xi_1)$	0.9850	0.0025	0.0050
$\Gamma_\xi(.,\xi_2)$	0.0100	0.9850	0.0100
$\Gamma_\xi(.,\xi_3)$	0.0050	0.0125	0.9850

Finally, the home production parameter b is set equal to 5% of the average quarterly earnings.[13]

[12] Cross section statistics are from SCF98. Persistence is the probability that those people in the top 20% in PSID 1989 are still there in PSID 1994. Notice that this corresponds to 20 model periods. See Budría, Díaz-Gimenez, Quadrini, and Ríos-Rull (2001) for details.

[13] The endowment when unemployed is important because it determines how much debt agents can hold while still being able to pay for interest with non-negative consumption. It needs to be set small enough so that unemployment is not a desirable state.

3.4 Solution of the model

Computation of this class of models is very demanding. In order to predict tomorrow's prices agents need to know the distribution of households μ over shocks, asset holdings and habit stocks and its law of motion $Q(\mu, z, z')$. Therefore, the state space contains an object (the probability measure μ) of infinite dimensionality which is far beyond current computer storing capabilities. To get around this problem, I follow the *partial information* approach used by Krusell and Smith (1997) which is itself an extension of the algorithm previously used by Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Krusell and Smith (1998) and explained in Ríos-Rull (1998). There are some novel issues in this implementation since consideration of habit formation increases the state space both at the aggregate and individual level. The approach is based on assuming that by only using part of the information contained in μ agents can predict tomorrow's aggregate state (and hence prices) almost as well as by using the whole distribution. Krusell and Smith (1998) show that typically the first moments of μ are enough. One finding of this paper is that the marginal distribution of agents over habits (or its first moment) does not bring any additional information in predicting tomorrow's state once we are considering the marginal distribution of assets (or its first moment).[14]

Technically, the idea is to replace the equilibrium equation (3.8) by

$$K' = f^K(z, K, H) \tag{3.9}$$

and introduce a new equation to predict aggregate habits

$$H' = f^H(z, K, H) \tag{3.10}$$

It will also be needed to approximate $R^a(z, \mu)$, which was a direct function of the distribution of agents, by:

$$R^b = f^{R^b}(z, K, H) \tag{3.11}$$

Under this approximation, the state space of the household problem is reduced. Instead of μ, consumers only need K and H to predict prices. Then, the first order conditions of the model will be:

$$\begin{aligned} &u_c(c, h) + \lambda\beta E_{e',\xi',z'|e,\xi,z}\left[v_h(j', z', K', H')\right] = \\ &\beta E_{e',\xi',z'|e,\xi,z}\left[v_\omega(j', z', K', H') R^s(z', K', R^b)\right] \end{aligned} \tag{3.12}$$

[14]See the computational appendix in section 3.10.

$$u_c(c,h) + \lambda\beta E_{e',\xi',z'|e,\xi,z}\left[v_h(j',z',K',H')\right] = \\ \beta E_{e',\xi',z'|e,\xi,z}\left[v_\omega(j',z',K',H')R^b\right] \tag{3.13}$$

These equations tell us that the utility loss of giving up one unit of consumption today (direct utility loss plus the discounted expected value of tomorrow's effect in the habit stock) must equal the mathematical expectation over different states of the discounted value of tomorrow's extra wealth units obtained investing on each type of asset. Notice that these equations imply:

$$E_{e',\xi',z'|e,\xi,z}\left[v_\omega(j',z',K',H')\left(R^s(z',K',R^b) - R^b\right)\right] = 0$$

and applying the law of iterated expectations:

$$E_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v_\omega(j',z',K',H')\right]\left(R^s(z',K',R^b) - R^b\right)\right] = 0 \tag{3.14}$$

which is what we will call the pricing equation. It tells us that the mathematical expectation of the difference between the returns of each asset weighted by the expected marginal value of wealth in each state, must equal zero.[15] This is the condition that non constrained optimizing agents will satisfy. Obviously, some agents will be in a corner solution by setting $b = \underline{b}$ or $s = \underline{s}$ and will not be able to satisfy equation 3.14.

The envelope conditions give us the value of one extra unit of wealth and one extra unit of habits:

$$v_\omega(j,z,K,H) = u_c(c,h) + \psi_c(c,h)\beta E_{e',\xi',z'|e,\xi,z}\left[v_h(j',z',K',H')\right] \tag{3.15}$$

$$v_h(j,z,K,H) = u_h(c,h) + \psi_h(c,h)\beta E_{e',\xi',z'|e,\xi,z}\left[v_h(j',z',K',H')\right] \tag{3.16}$$

where $\psi_c(c,h) = \lambda$ and $\psi_h(c,h) = 1-\lambda$.

For given forecasting laws 3.9, 3.10 and 3.11, solving the household problem will amount to solve the first order conditions 3.12 and 3.13 together with constraints 3.4, 3.5, 3.6 and 3.7. To do so I approximate the derivatives of the value function v_ω and v_h piece-wise linearly and use the envelope conditions 3.15 and 3.16 to update them. Then, I will also have to iterate in the forecasting laws space to find the forecasting laws that are consistent with the equilibrium. A detailed explanation of the procedure can be found in the computational appendix, section 3.10.

[15] Or more clearly, the value of investing in each assets has to be the same.

3.4.1 Equity premium

The equity premium is the difference between the expected return on the risky asset and the return in the risk free asset. We can write it formally in the context of our model as

$$E_{z'|z}\left[R^s\left(z', K', R^b\right)\right] - R^b$$

From the pricing equation 3.14, and using the definition of covariance, we can write

$$E_{z'|z}\left[R^{s\prime} - R^b\right] = -\frac{cov_{z'|e,\xi,z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right], R^{s\prime} - R^b\right]}{E_{z'|z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right]\right]}$$

where cov is the covariance operator. To simplify we use v'_ω as an abbreviation for $v_\omega\left(j', z', K', H'\right)$ and $R^{s\prime}$ as an abbreviation for $R^s\left(z', K', R^b\right)$. Further expanding we get:

$$\begin{aligned}&E_{z'|z}\left[R^{s\prime}\right] - R^b = \\ &-SD_{z'|z}\left[R^{s\prime}\right]\ CV_{z'|e,\xi,z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right]\right]\ Corr_{z'|e,\xi,z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right], R^{s\prime}\right]\end{aligned}$$

The first term, the conditional standard deviation of the return of the risky asset, is generally called the amount of risk. The product of the second and third terms form what is generally called as Sharpe ratio and measures the price of risk. The second term is the coefficient of variation on aggregate shock of the expectation of the marginal value of wealth. It measures the utility cost of aggregate fluctuations. The third term is the correlation on the aggregate shock between the expectation of the marginal value of wealth and the returns on the risky asset. The correlation term gives the sign of the equity premium. If high returns of the risky asset are associated with low marginal values of wealth, the risky asset entitles a positive risk premium. On the contrary, if the risky asset pays more when the marginal value of wealth is high it will then be seen as an insurance mechanism and it will entitle a negative premium. Recall that that aggregate shock has only two possible realizations. Conditional on e, ξ, z and z', the expectation over e' and ξ' of wealth ω' has only two possible realizations positively correlated with the aggregate shock and therefore the returns on shares.[16] Since v'_ω is a decreasing function of wealth it means that the correlation term will be exactly -1. We then rewrite:

$$E_{z'|z}\left[R^{s\prime}\right] - R^b = SD_{z'|z}\left[R^{s\prime}\right]\ CV_{z'|e,\xi,z}\left[E_{e',\xi'|e,\xi,z,z'}\left[v'_\omega\right]\right] \tag{3.17}$$

[16]Here, it is crucial the assumption that expected labour earnings are higher in good times than in bad times.

3.5 Benchmark economy

First of all I define the benchmark economy without habits and leverage. I calibrate it as stated in section 3.3. The absence of habits implies $\gamma = 0$ whereas the fact that firms are prevented from raising capital through bonds imposes $\rho = 0$. I allow for borrowing in shares and risk free bonds up to the level where agents cannot repay debt in all states of the world.[17] An intertemporal elasticity of substitution equal to 0.5 requires σ to equal 2. I call this economy MV (multivariate). The results are reported in the second column of table 3.3. Before looking at this economy I perform a preliminary exploration to assess the role of the efficiency units shock. So I also look at an economy equal to MV except for having all of its employed households with the same amount of efficiency units of labour.[18] I call this economy E (employment). Its results are in the first column of table 3.3. Notice that rates of return (and therefore equity premia) are in quarterly terms.

Table 3.3: **Statistics of simulated economies. Benchmark Economy**

	E	MV
k/y	10.74	10.75
$E_{z'\|z}[R^s - R^b]$	0.0004%	0.0017%
$SD_{z'\|z}[R^s]$	0.061	0.061
$Sharpe$	0.006	0.028
$\int CV_{z'\|z}\left[E[c']\right] d\mu$	0.28%	0.72%
$\int_{b>\underline{b},s>\underline{s}} CV_{z'\|z}\left[E[c']\right] d\mu$	0.28%	0.98%

Note: Rates of return are quarterly. Last two rows report the average (over all households and over pricing households respectively) of the conditional coefficient of variation of the expected consumption. The expectation operator on consumption refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

The equity premium for economy E is a very small 0.0004%, which is in the same order of magnitude as Krusell and Smith (1997) find, but much smaller than the empirical estimate for US data reported by Lettau and Uhlig (1997) of 1.99%. In this economy agents differ only on employment status and wealth. In figure 3.1 we can see the sample policy functions for agents in this economy. Unemployed

[17]This is approximate since this level depends on the realizations of R^b and R^s.

[18]All employed households are given a deterministic amount of efficiency units of labour equal to the unconditional mean of the process: $E[\xi]$

agents specialize in bonds. Among employed households, those with low levels of wealth specialize also in bonds. As wealth increases agents start to introduce capital to their portfolio. From a certain level of wealth on, the optimal investment is to borrow as much as possible in bonds and invest everything in capital. This implies creating a portfolio with higher expected returns and higher variability than the risky asset itself.

The portfolio choice results can be restated in the following form. First, among employed people, wealth-rich households buy aggregate risk in exchange of higher expected payoffs from unemployed ones. And second, holding wealth constant, employed households buy aggregate risk from unemployed ones. The former is a standard result. Higher wealth means (1) having a lower proportion of labour earnings in next period's expected income (2) and being further away from the borrowing constraints. Therefore, (1) the variability of expected income is lower and (2) it translates to a lesser degree into consumption variability. The latter is a result also already found by Krusell and Smith (1997). It derives from the assumptions embedded in the transition matrix for the employment opportunity shock $\Gamma_e(z, z', e, e')$. Namely, the fact that $\Gamma(z, z_g, 1, 1) - \Gamma(z, z_b, 1, 1) < \Gamma(z, z_g, 0, 1) - \Gamma(z, z_b, 0, 1)\ \forall z$. This implies that the conditional variance of labour earnings is higher for unemployed households. That is to say, the difference between good and bad times is higher for unemployed households.

Let's now turn to MV, the benchmark economy. Results are in the second column of table 3.3. We see that equity premium raises to 0.0017%, more than 4 times its value in the economy without efficiency shocks. However, the equity premium delivered by economy MV is still very far from data estimates. The higher riskiness introduced by the efficiency units shocks increases the market price of risk.[19] One way to see this is by looking at the individual consumption fluctuations. We need a measure of the difference between households' expected consumption next period when aggregate shock is good and households' expected consumption next period when aggregate shock is bad. I compute the coefficient of variation of the expected consumption for every individual and then take the average. This statistic

[19]Notice, however, that the efficiency units shock is not correlated with the aggregate shock. What the efficiency units shock does is change the conditional expectation of earnings when employed, increasing the gap between employment and unemployment for those who expect to have high ξ in next period and decreasing it for those who expect to have low ξ. In terms of equation 3.17 there is an increase in the covariance term.

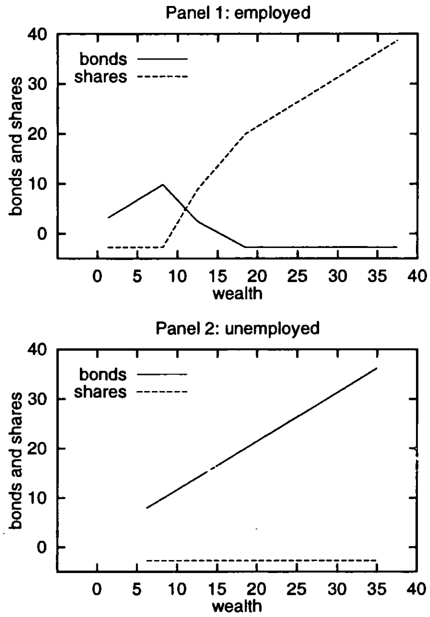

Figure 3.1: **Simulated policy functions. Economy** E

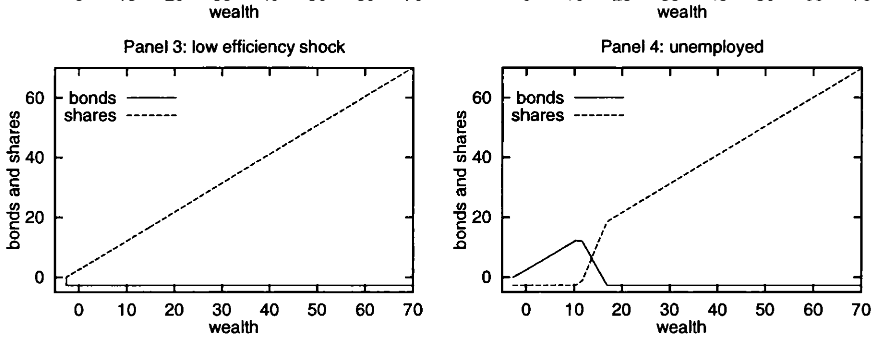

Figure 3.2: **Simulated policy functions. Economy** MV

is reported in row 5 of table 3.3. We see a big increase in individual consumption fluctuations: from 0.28% in economy E to 0.98% in economy MV. Row 6 computes the same measure but only for unconstrained agents.

Policy functions are in figure 3.2. Now the picture is a bit different. Among employed people, those with high efficiency units specialize in bonds if they have low levels of wealth and in shares if they have high levels. A small fraction in the middle buy both. The same behaviour appears for households with mid-efficiency units of labour although the changing points happen for much lower levels of wealth. Those with low efficiency units specialize in capital (and sell as much of bonds as they can). Among the unemployed, for low levels of wealth agents specialize in bonds and for high levels in capital.

Two results should be emphasized. Firstly, holding idiosyncratic risk constant, wealth-rich households buy aggregate risk from wealth-poor ones. Secondly, low-efficiency households are insuring high-efficiency ones. The first results is the same as in economy E. The second one is at first surprising. Notice, however, that it should not be so. Once we control for wealth, the sole role of the efficiency units shock is to predict next period's efficiency units endowment.[20] The process for ξ is such that $E\left[\xi'/\xi, e'=1\right]$ is increasing in ξ. I.e., conditional on being employed next period, the expected amount of efficiency units next period is increasing in the amount of efficiency units in the current period. Therefore, the higher ξ, the larger the difference between being employed and unemployed and hence the larger the conditional variability of expected labour earnings. This result relies on unemployment risk being unrelated to the earnings position.[21] An important note is that in economy MV unemployed households are not the least willing to hold risk as was the case in economy E. It turns out to be that unemployed households face lower earnings volatility than high-efficiency employed households because, even if the process for employment is more volatile for them, the difference of expected earnings between employment and unemployment is much lower.

Empirical results by Bertaut and Starr-McCluer (2000) are not inconsistent with these findings. Using SCF data, Bertaut and Starr-McCluer (2000) estimate house-

[20]The amount of resources brought by the shock are already accounted for in the wealth holdings.

[21]The result does not need to hold when unemployment probability is related to the skill level. High efficiency workers may face a higher differential in earnings between employed and unemployed status but they might also have a lower probability of unemployment. Overall, conditional variance of earnings could be lower for high earners than for low earners

holds' portfolio choices as function of wealth, education, income and other household observable variables. They find that (1) wealth always increases the share of risky assets in the portfolio and (2) income decreases the share of risky assets in the portfolio. The control for education means that we have to read the result on income as net of fixed heterogeneity, which make proxy the model's lack of correlation between earnings and aggregate uncertainty.

As a final comment, notice that there are two types of constrained agents. Firstly, agents wanting a higher hedge against risk who try to insure themselves by going as short as possible in shares and investing in risk free bonds. And secondly, agents wanting higher expected returns who open their positions by going as short as possible in bonds and investing everything in shares thus creating portfolios with higher expected returns and higher variability than the risky asset itself. It is not necessarily true, then, that what matters for the price of risk are the dynamics of consumption of wealthy households. What matters are the dynamics of consumption for the unconstrained agents. What we find is that for the benchmark economy these agents turn out to face quite low consumption fluctuations.

3.6 Some extreme experiments

Before looking at the results for habits, this section focuses on some calibration experiments of the model in order to understand it better. I take as reference the two components of the equity premium that we see in equation 3.17, namely, the price of risk and the amount of risk. Firstly, in section 3.6.1 the risk aversion is increased to show that, even for implausibly high values, the model allows for neither big equity premia nor big price of risk. This experiment is useful in order to present the composition effect. Secondly, section 3.6.2 focuses on the volatility of the risky asset. It measures the volatility of the macroeconomic fluctuations generated by an aggregate shock whose amplitude is targeted to close the gap between the model's equity premium and that of the actual data.

3.6.1 Risk aversion

One key finding of the representative agent models on asset pricing is that by increasing risk aversion (and for this class of utility functions this means decreasing the intertemporal elasticity of substitution) one can increase the predicted equity premium up to its empirical estimate. This model teaches us two important mechanisms that counteract this force. The first one is precautionary savings. Households suffering higher disutility from consumption fluctuations may increase their savings in order to avoid them. In this way, they have a higher buffer stock to prevent consumption from fluctuating too much. The second one is the composition effect. Households suffering higher disutility from consumption fluctuations may change their portfolio choice in order to avoid them. Buying a lower fraction of the asset whose returns are negatively correlated with the variance of the earnings process helps face lower consumption fluctuations. Therefore, the set of agents unconstrained in their portfolio problem will see its members changed.

To see this second mechanism at work, we compare a model with $\sigma = 2$ (our benchmark economy MV) with two other models with $\sigma = 5$ and $\sigma = 10$. Some statistics of these three economies are reported in table 3.4.

Table 3.4: **Statistics of simulated economies. High σ**

	MV	MV	MV
	$\sigma = 2$	$\sigma = 5$	$\sigma = 10$
k/y	10.75	10.77	10.70
$E_{z'\|z}[R^s - R^b]$	0.0017%	0.0057%	0.0123%
$SD_{z'\|z}[R^s]$	0.061	0.061	0.061
$Sharpe$	0.028	0.093	0.201
$b > \underline{b}$ & $s > \underline{s}$	1.31 (12.79%)	1.37 (12.88%)	2.21 (10.99%)
$b = \underline{b}$	1.00 (79.10%)	0.97 (80.13%)	0.96 (74.52%)
$s = \underline{s}$	0.53 (8.05%)	0.60 (6.98%)	0.71 (6.68%)

Note: Rates of return are quarterly. Last three rows indicate average wealth of selected group over economy's average wealth (in parenthesis the proportion of households in each group)

We see that the price of risk increases from 0.0268 for $\sigma = 2$ to 0.2010 for $\sigma = 10$ (this is 7.5 times bigger). However, it is still below 0.27, the lower bound estimated by Lettau and Uhlig (1997). Equity premium increases in the same proportion (volatility of risky asset being unchanged). However, even for $\sigma = 10$ the economy is still far away from the empirical estimate of 1.99 of Lettau and Uhlig (1997).

As one increases risk aversion, disutility of risk grows. However, there is a composition effect partly offsetting the rise in the economy's price of risk. Only a small fraction of agents (12.8% in the economy with $\sigma = 2$) have interior solution in the portfolio problem. As shown in section 3.5, for a given idiosyncratic shock, there is a monotonic relationship between wealth and portfolio risk, with wealth-poor households buying less risk than wealth-rich ones. It turns out to be that very poor agents and very rich ones do not price risk because they are in a corner solution. As disutility of risk increases the poorest among the households with interior solution stop buying shares (going to corner solution) and the poorest among those specialized in shares start introducing some bonds in the portfolio (becoming pricing agents). The pool of households pricing aggregate risk becomes wealthier. This can be seen in table 3.4. Relative wealth of pricing agents is 1.31 in the economy with $\sigma = 2$, 1.37 in the economy with $\sigma = 5$ and 2.21 in the economy with $\sigma = 10$. This means that the average wealth of pricing agents moves from 31% higher than the economy average to 121% higher. The claim is that if the same households that price risk in the economy with $\sigma = 2$ were pricing the risk in the economy with $\sigma = 10$, then equity premium in the economy with $\sigma = 10$ would be higher.

3.6.2 Volatility of aggregate shocks

As table 3.3 shows, volatility of returns on shares is too low compared to the values estimated from data. An argument presented by Krusell and Smith (1997) is that this class of models cannot reproduce the equity premium because of the lack of variability of the risky asset. They find a Sharpe ratio in the same order of magnitude as in real data, although in order to get it they need to restrict borrowing of both assets to zero. In this section I reproduce their finding on the Sharpe ratio and show what can be obtained by increasing the volatility of the aggregate shock. I define a new economy, labelled NB (no borrowing), with the only difference with respect to MV that borrowing is not allowed ($\underline{b} = 0$ and $\underline{s} = 0$). I compare it to another economy, labelled NB', similar to NB but with a different calibration for the aggregate shock z. I change the values of z_g and z_b from 1.01 and 0.99 to 1.20 and 0.80. That is, good (bad) technology shock improves (worsens) productivity by as much as 20% respect average instead of by just 1%.

Some statistics from these economies are reported in table 3.5. Economy NB gets, as stated by Krusell and Smith (1997), a Sharpe ratio close to 0.27, the value estimated by Lettau and Uhlig (1997). The problem is that standard deviation of the

Table 3.5: **Statistics of simulated economies. High volatility of aggregate shock**

	NB	NB'
k/y	10.75	10.76
$E_{z'\|z}[R^s - R^b]$	0.0142%	0.1388%
$SD_{z'\|z}[R^s]$	0.0614	0.4576
$Sharpe$	0.232	0.308
$sd\,[Y_{cyc}]$	2.27	16.80
$sd\,[C_{cyc}]$	1.02	3.48

Note: Rates of return are quarterly. Y_{cyc} and C_{cyc} are the hp-detrended series for aggregate output and consumption

risky asset is too small compared to their estimated value of 7.5. Consequently, the equity premium is still two orders of magnitude below empirical estimates. Increasing volatility of the aggregate shock works as expected. Economy NB' multiplies the volatility of capital returns by a factor larger than 7. With it, the equity premium increases a lot, up to 0.1388%, which is just one order of magnitude below the empirical estimates. One may want to expand the amplitude of the aggregate shock up to the level that the volatility of the return on capital matches its empirical counterpart. However, expanding the amplitude of the aggregate shock not only rises the volatility of the returns on capital but also of all the macroeconomic variables of the model. For example, business cycles volatility of output in economy NB' rockets to 16.80, about ten times what we find in US postwar data, whereas consumption volatility more than triples.[22] Therefore, we should think of some other mechanism to increase the volatility of the returns on shares. This is what we do in section 3.8.

A final result to look at is the role of borrowing constraints. Notice that economy NB is directly comparable to economy MV in section 3.5, with the only difference lying in the fact that economy NB does not allow for borrowing. As already shown by Krusell and Smith (1997), the equity premium and Sharpe ratio increase by an order of magnitude.

[22]Business cycles volatility refers to the standard deviation of the hp-detrended series.

3.7 Habits

Habit formation preferences have been proposed as an explanation for the equity premium. A common feature of the literature is to calibrate a process for consumption from data. Then, given the process for consumption, the FOC of the model generate the price of risk consistent with it. This paper takes a different approach. It does not calibrate the process for consumption but the process for earnings. Then, forward-looking consumers decide weather to let earnings fluctuations translate into consumption fluctuations or to use the assets available in order to try to smooth them out. When moving from a setting without habits to one with habits, not only the marginal utility of consumption changes but also the consumption fluctuations do.

In this section I introduce habit formation into the general model. I need to choose values for the parameters γ and λ. However, it is not clear which values to pick. There are some empirical papers estimating habit parameter values. However, results are very different among them, depending on the data and model specification.[23] The strategy followed in this paper is to allow for a 'strong' habit process and interpret the results as an upper bound. Firstly, I set $\gamma = 0.75$. γ determines the weight of habits on the utility function. The value chosen implies that households care much more about relative consumption than about consumption level.[24] Secondly, I set $\lambda = 0.25$. This generates a highly persistent habits process. A value of 1 would mean that only previous period consumption matters whereas a value smaller than 1 means that the whole history of past consumption enters the habit stock. Small values of λ imply that consumption in the distant past still has a lot of weight on current period's habit stock (or in other words, that current period's consumption hardly modifies next period's habit stock). A persistent habit process is used by Constantinides (1990) and Heaton (1995) to obtain sizeable equity premia. In addition, chapter 2 shows in a similar model without aggregate risk that the more

[23]Some empirical studies on habit formation are Fuhrer (2000), Dynan (2000) and Heaton (1995). See section 2.3.2 for details.

[24]Notice that utility function can be rewritten as

$$u(c,h) = \frac{\left(c^{1-\gamma}\left(\frac{c}{h}\right)^{\gamma}\right)^{1-\sigma}}{1-\sigma}$$

which shows the role of γ. Consumers care for the level of present consumption and for present consumption relative to past consumption (or habits). γ gives the weight of the latter.

persistent the process, the higher the effect of habits in the consumption/savings decision. Finally, I set $\sigma = 5$ in order to keep the intertemporal elasticity of substitution equal to 0.5.[25]

Table 3.6: **Statistics of simulated economies. Habits**

	MV	MVH
k/y	10.75	10.74
$E_{z'\|z}[R^s - R^b]$	0.0017%	0.0029%
$SD_{z'\|z}[R^s]$	0.061	0.061
$Sharpe$	0.028	0.047
$\int CV_{z'\|z}[E[c']]\,d\mu$	0.72%	0.40%
$\int_{b>\underline{b},s>\underline{s}} CV_{z'\|z}[E[c']]\,d\mu$	0.98%	0.50%

Note: Rates of return are quarterly. The last two rows report the average (over all households and over pricing households respectively) of the conditional coefficient of variation of the expected consumption. The expectation operator on consumption refers to the joint distribution of e' and ξ' conditional on e, ξ, z and z'.

Some statistics are reported in the second column of table 3.6 under the name of MVH (multivariate with habits). Sharpe ratio increases to 0.047, that is, 80% higher than in the benchmark economy MV (with the same increase in the equity premium because of the unchanged amount of risk). Thus we see that with the habit formation hypothesis the price of risk increases. However, the value delivered by the simulated model is still small relative to the 0.27 empirical value we are targeting. Given the earnings fluctuations, with habit formation preferences agents save more in order to avoid consumption fluctuation. This can be observed in the calibrated value of β (remember that β is calibrated to produce a capital output ratio of 10.75). In economy MV β is required to be 0.945 whereas in economy MVH β is required to be 0.882. As a consequence, household's expected consumption fluctuations fall. The 5^{th} row in table 3.6 quantifies this. The average over all households of the coefficient of variation on next period's expected consumption falls from 0.72% to

[25]as shown in section 2.8, intertemporal elasticity of substitution becomes

$$\frac{1}{\gamma + (1-\gamma)\,\sigma}$$

Economies without habits ($\gamma = 0$) keep the property of intertemporal elasticity of substitution being the inverse of risk aversion. However, when habits are introduced, the link between preferences over different states and over different time periods is broken.

0.40%.

Besides, the set of agents with non-corner solutions in the portfolio problem changes. It will be better-insured agents who will be pricing risk in economy MVH. This means that the agents pricing the assets in the economy with habits face lower consumption fluctuations than the agents pricing the assets in the economy without habits. Quantitatively, the average over pricing households of the coefficient of variation on next period's expected consumption falls from 0.98% to 0.50%. This is a higher fall than when averaging over the whole economy, in line with the intuition of a shift in the composition of the set of pricing agents.

Policy functions show a similar pattern to economy MV. In figure 3.3 there are the policy functions as a function of wealth. In this case the relation with wealth is not one to one since for every wealth level there are different values of the habit stock. Figure 3.4 holds wealth ω constant and shows the theoretical policies for bonds and shares as function of the habit stock. Clearly, the willingness to hold risky assets decreases with the habit stock. Higher habit stocks make consumption fluctuations less desirable.

3.7.1 Final comment

In short, what we observe is that households have changed their behaviour. Once we introduce habits they do not sit and ask for a high compensation to hold shares in their portfolios. The higher utility losses due to habit formation are mitigated by smoothing out consumption fluctuations to a higher degree. Overall, the price of risk is higher but not as much as it would be if we kept the consumption fluctuations constant.

This is the essence of the critique to previous papers that found that habit formation solves the equity premium puzzle. By taking the consumption process as given and then changing the preferences of households, one forces the agents in the model to bring all the adjustment on prices (equity premium) without giving a chance to change quantities (consumption fluctuations). Of course, this does not mean that the consumption process from data is wrong. It just asks the question of where the models get it from (and implicit in this, its relationship with the earnings process).

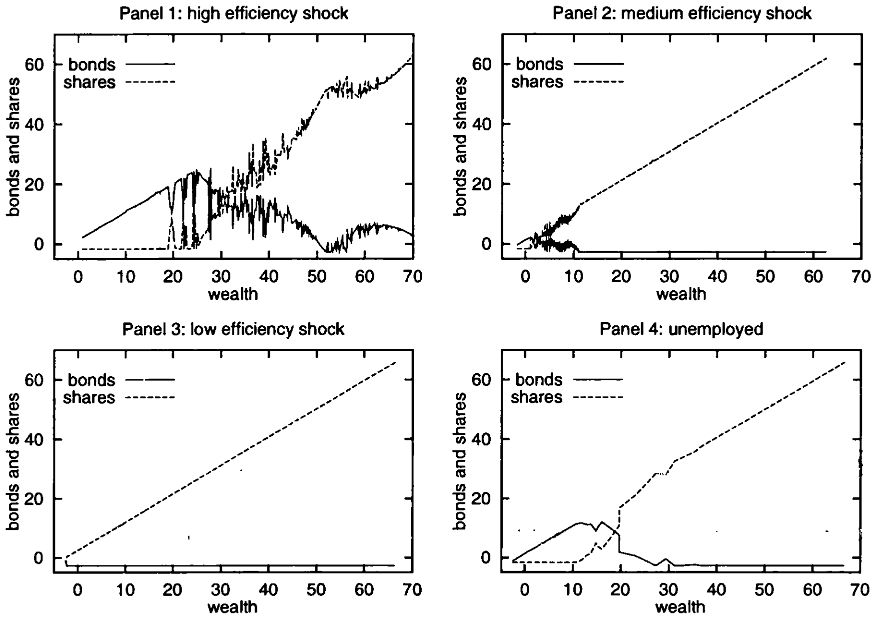

Figure 3.3: **Simulated policy functions. Economy** MVH

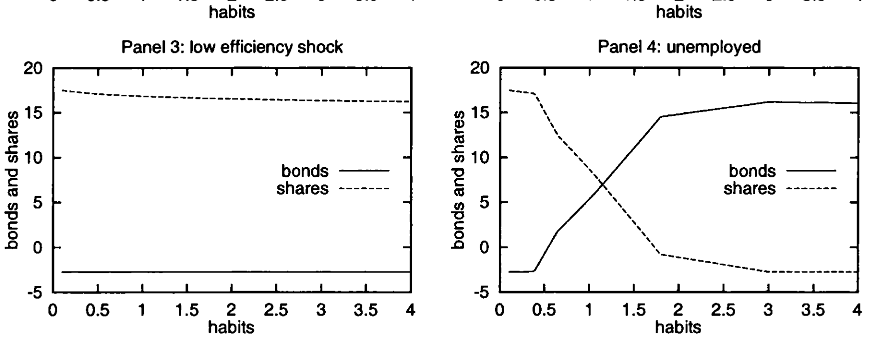

Figure 3.4: **Theoretical policy functions. Economy** MVH

3.8 Leverage

As seen in section 3.6.2, increasing volatility of the aggregate shock to try to match actual equity premium brings strong counterfactual implications about the cyclical properties of the aggregate series. We would need to increase volatility of the returns on the risky asset without changing the amplitude of business cycles fluctuations. A possible solution is leverage. In allowing firms to finance capital through debt, a smaller share of equity has to support the same variability in the marginal product of capital. Therefore, returns on equity become more volatile.[26] I follow a very parsimonious approach. Instead of developing a theory of leverage I just set its level exogenously to explore its implications. A second step would be to get the share of bond financing as the optimal choice of profit maximizing firms. I call the share of capital financed by bonds ρ.[27] Applying the standard deviation operator to equation 3.2, the role of ρ in relating the standard deviation of the return on shares to the standard deviation of the marginal product of capital becomes clear:

$$SD_{z'|z}\left[R^s\left(z', K', R^a\right)\right] = \frac{1}{(1-\rho)} SD_{z'|z}\left[F_K\left(z', K', L(z')\right)\right]$$

The standard deviation of the marginal return on capital in the model is roughly 0.06. Lettau and Uhlig (1997) estimate a value for the standard deviation of the S&P500 stock exchange index of about 7.5. If we wanted to match the empirical standard deviation of the return on shares we would need ρ to be about 0.992 (more than 99% of capital being financed by bonds). However, empirical estimates of ρ are much lower. This leaves us with two incompatible moments to match and just one parameter. I choose a value of ρ equal to 0.5 for illustrative purposes.[28] Notice that this choice implies just doubling the expected volatility of the marginal product of capital (or return on capital in a non-levered economy). We call the levered economy MVL (multivariate with leverage).

Column 2 of table 3.7 presents some statistics for the MVL economy. The result is that the equity premium remains almost unchanged due to a drop in the

[26]Storesletten, Telmer, and Yaron (2001) propose a different solution. They set the depreciation rate of capital to be stochastic. In this way they have an extra parameter, the standard deviation of the depreciation rate, and are able to match the standard deviation of capital. However, as they acknowledge, this also generates processes for aggregate consumption and investment that are far too volatile

[27]Therefore, the debt-to-equity ratio would be $\frac{\rho}{1-\rho}$

[28]The estimates for US reported by Benninga and Protopapadakis (1990) are not far from $\rho = 0.5$

Table 3.7: **Statistics of simulated economies. Leverage**

	MV	MVL	$MVHL$
k/y	10.75	10.75	10.73
$E_{z'\|z}[R^s - R^b]$	0.0017%	0.0018%	0.0040%
$SD_{z'\|z}[R^s]$	0.061	0.123	0.123
$Sharpe$	0.028	0.015	0.032
$b > \underline{b}$ & $s > \underline{s}$	1.31 (12.79%)	0.71 (17.85%)	0.95 (34.32%)
$b = \underline{b}$	1.00 (79.10%)	0.93 (46.24%)	0.85 (41.32%)
$s = \underline{s}$	0.53 (8.05%)	1.25 (35.84%)	1.23 (24.35%)

Note: Rates of return are quarterly. The last three rows indicate average wealth of selected group over economy's average wealth (in parenthesis the proportion of households in each group)

economy's price for risk. The last three rows of 3.7 tells us that the structure of constrained and unconstrained people has changed a lot. This can be seen more clearly in the policy functions (figure 3.5). They look very different from the ones generated by economy MV. Now employed households with high-efficiency shock specialize completely in bonds. Mid-efficiency shock households buy bonds for low levels of wealth and start to introduce shares as they get richer, specializing in them for high levels of assets. Low-efficiency consumers specialize in shares. Unemployed specialize in bonds. Consistently with previous results, we see that (1) keeping the idiosyncratic shock fixed, wealthier people tend to buy risk from the rest of the economy and that (2) keeping wealth constant, low-efficiency households also buy risk from the rest of the population.

The result is at first puzzling. We should expect the price of risk to remain unchanged and the amount of risk to raise the equity premium. In a complete markets economy agents should be able to reproduce their portfolios by changing the proportions of each asset. However, with borrowing constraints agents cannot necessarily reproduce their previous portfolio returns structure by selling and buying in the asset markets.

Finally, column 3 of table 3.7 shows some statistics of a simulated economy with habits and leverage. I call this economy $MVHL$ (multivariate with habits and leverage). The equity premium raises to 0.0041%, this is 140% higher than in economy MV and 40% higher than in economy MVH. The interaction between habits and risk looks important. When adding leverage to the economy with habits,

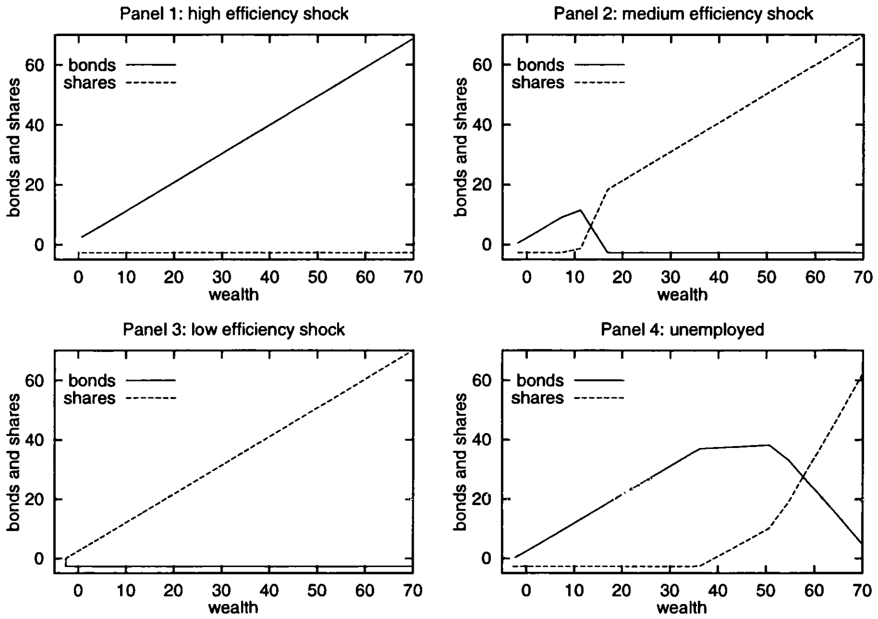

Figure 3.5: **Simulated policy functions. Economy** MVL

the price of risk falls but not as much as to compensate for the increased volatility. Therefore, the equity premium increases.

3.9 Conclusions

This paper shows how the habit formation hypothesis, which has been proposed as a possible explanation for the equity premium, is perhaps not such a good candidate. In a model that allows for consumers to use just two assets to insure themselves against idiosyncratic shocks to labour income, the addition of habit formation increase neither the Sharpe ratio nor the equity premium much. There are two general equilibrium features that prevent the Sharpe ratio from growing too much. Firstly, precautionary savings. With habit formation households trying to avoid higher utility losses from consumption fluctuations will use asset markets to avoid them. Secondly, what I call composition effect. When the disutility of risk increases (as it does with habit formation) the pool of agents pricing this risk changes. It is better insured agents who will have unconstrained solutions to their portfolio choice problem. This prevents the increase in disutility of risk to fully translate into an increase in the equilibrium price of risk.

The literature on habits and asset pricing has tended to take consumption fluctuations as given without worrying about how forward-looking agents generated them. The argument is that, for given statistics of consumption data, increasing the disutility from consumption fluctuations should increase the premium attached to the risky asset. However, in a general equilibrium model with heterogeneous agents anticipation of higher disutility from fluctuations makes households save in advance in order to smooth their consumption profiles. The result is that the consumption fluctuations generated by the model with habits are smaller than the ones generated by a model without habits. Another way to see this result is that, for our calibrated process on earnings, the habit formation hypothesis is inconsistent with the empirical consumption fluctuations.

In an additional extension, the paper also tries to increase the volatility of the returns on the risky asset. It does so by using leverage. An economy with habits and leverage produces a higher equity premium than the economy with only habit formation. The increase in the amount of risk of shares comes together with a fall in the Sharpe ratio. Although the former effect is bigger than the latter the interaction of higher risk with the habit formation hypothesis does not change the main results of the paper.

One interesting feature of this setup is the portfolio decision of agents according to their state. Higher wealth implies higher willingness to hold risky assets, higher earnings imply lower willingness to hold risky assets and higher habit stocks imply lower willingness to hold risky assets. These are clear testable implications for the heterogeneous agents model. The failure of the model in generating large equity premia should not disregard these portfolio choice implications.. What the model says is that it is difficult to see the equity premium as a risk premium given the earnings shock households face. Even if one considers habit formation. Recent work by Ellen McGrattan and Edward Prescott claims that it might well be that the equity premium is not a risk premium at all once intangible assets, foreign assets and different taxation issues are taken into account[29].

Admittedly, the structure of earnings uncertainty is quite simple. It is assumed that efficiency units of labour are unrelated to aggregate risk. Further research on the relation of the aggregate shock with the distribution of efficiency units of labour

[29] See McGrattan and Prescott (2000a) and McGrattan and Prescott (2000b).

should allow us to specify a more detailed earnings processes.

3.10 Appendix 1. Computational Procedures

This appendix explains the computer algorithm used to solve the model. The algorithm is based in the *partial information* approach used by Krusell and Smith (1997). They were extending previous work by Castañeda, Díaz-Giménez, and Ríos-Rull (1998) and Krusell and Smith (1998). Ríos-Rull (1998) explains it in good detail. Basically, as stated in section 3.4, solving the household problem implies, maximizing equation 3.3 subject to the constrains 3.4, 3.5, 3.6 and 3.7 and to the forecasting rules 3.9, 3.10 and 3.11. The problem is that the forecasting rules f^K, f^H and f^{R^a} are not known. We start explaining how to solve the household problem for given forecasting rules and then we discuss how to find the equilibrium ones.

3.10.1 Solving the household's problem

For the household problem the state space is given by the vector $j = \{\omega, h, e, \xi\}$ plus z, K and H. We collapse e, ξ and z into one variable ϵ that can take $n_\epsilon = n_z (n_\xi + 1) = 8$ different values. We are therefore left with the two endogenous individual state variables ω and h, the exogenous stochastic shock ϵ and the exogenous (at the household level) aggregate variables K and H. Define labour earnings in terms of the newly defined exogenous stochastic process ϵ as $labear(\epsilon, K)$. Households have to solve the following system formed by the FOC, the constraints and the forecasting rules:

$$
\begin{aligned}
0 &= u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] \\
&\quad -\beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega',h',\epsilon',K',H')\,R^s(\epsilon',K',R^b)\right] \\
0 &= u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] \\
&\qquad\qquad -\beta E_{\epsilon'|\epsilon}\left[v_\omega(\omega',h',\epsilon',K',H')\,R^b\right] \\
c &= \omega - b - s \\
h' &= \lambda c + (1-\lambda)\,h \\
\omega' &= bR^b + sR^s(\epsilon',K',R^b) + labear(\epsilon',K') \\
(b,s,c) &\geq (\underline{b},\underline{s},0) \\
K' &= f^K(\epsilon,K,H) \\
H' &= f^H(\epsilon,K,H) \\
R^b &= f^{R^b}(\epsilon,K,H)
\end{aligned}
\tag{3.18}
$$

which for a given pair $\{v_\omega^0, v_h^0\}$ delivers the policy functions $\{g^{0,c}, g^{0,b}, g^{0,s}\}$. Then, substituting both of them into the right hand side of the EC

$$
\begin{aligned}
v_\omega(\omega,h,\epsilon,K,H) &= u_c(c,h) + \lambda\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right] \\
v_h(\omega,h,\epsilon,K,H) &= u_h(c,h) + (1-\lambda)\,\beta E_{\epsilon'|\epsilon}\left[v_h(\omega',h',\epsilon',K',H')\right]
\end{aligned}
$$

we get a new pair $\{v_\omega^1, v_h^1\}$. These two systems define a mapping T from certain space into itself. Solving the household problem amounts to finding a fixed point of this mapping, i.e., a pair such that $\{v_\omega^1, v_h^1\} = \{v_\omega^0, v_h^0\}$. It remains to be defined a class of functions, which the computer can understand, for $\{v_\omega, v_h\}$. For every value of ϵ, we approximate $\{v_\omega, v_h\}$ piece-wise linearly in a four-dimensional grid[30]. Given an initial guess $\{v_\omega^0, v_h^0\}$, we solve the system 3.18 to get the policy functions $\{g^{0,c}, g^{0,b}, g^{0,s}\}$. Then, using the envelope conditions, we obtain a new pair $\{v_\omega^1, v_h^1\}$. If the new pair $\{v_\omega^1, v_h^1\}$ is close to $\{v_\omega^0, v_h^0\}$ we have find a fixed point of the mapping T and we take $\{g^{0,c}, g^{0,b}, g^{0,s}\}$ as the solution of the model. If not, we update $\{v_\omega^0, v_h^0\} = \{v_\omega^1, v_h^1\}$ and start again. Notice that there is no contraction theorem for this mapping, which means that there is no guarantee to succeed by using this successive approximations approach. For the iterations to make good progress, it turns out to be very important to select proper initial conditions $\{v_\omega^0, v_h^0\}$.

[30]In the K and H dimension there is not much curvature, so we use much fewer points than in the ω and h dimensions. We typically use 6 points for the aggregate variables, 35 for wealth ω and 10 for habit stock h. For these two variables the grid is much thicker at its beginning than at its end since it is for small values that there is more curvature.

3.10.2 Finding the equilibrium forecasting rules

The nature of the stationary stochastic equilibrium implies finding a distribution μ. The *partial information* approach is based in finding a vector of forecasting functions $f \equiv \{f^K, f^H, f^{R^a}\} \in \mathcal{F} \equiv \mathcal{F}^K \times \mathcal{F}^H \times \mathcal{F}^{R^a}$ consistent with rational expectations. I.e., given that agents forecast K, H and R^b with certain f, the simulated economy should display this same behaviour. Or in other words, the simulated series for K, H and R^b should be 'well' predicted by f. The idea is to start with an initial f^0, solve the household's problem defined in section 3.10.1, simulate the economy for a long series of periods and estimate a new f^1 within the same parametric class $\mathcal{F}$. Krusell and Smith (1997) show that one needs to make one correction to this procedure. To be precise, market for bonds does not clear in every period and state. In order to achieve the bond market clearing in every period and state, we define the following problem:

$$v\left(\omega, h, \epsilon, K, H, R^b\right) = \max_{c,a,s}\left\{u(c,h) + \beta E_{\epsilon'|\epsilon}\left[v\left(\omega', h', \epsilon', K', H'\right)\right]\right\} \tag{3.19}$$

subject to

$$\begin{aligned}
c &= \omega - a - s \\
h' &= \lambda c + (1-\lambda)\, h \\
\omega' &= bR^b + sR^s\left(\epsilon', K', R^b\right) + labear\left(\epsilon', K'\right) \\
K' &= f^K\left(\epsilon, K, H\right) \\
H' &= f^H\left(\epsilon, K, H\right) \\
R^b &= f^{R^b}\left(\epsilon, K, H\right) \\
(c, b, s) &\geq (0, \underline{b}, \underline{s})
\end{aligned}$$

This problem differs from the original one in the fact that R^b is an state variable for today, although tomorrow's R^b is perceived to follow the forecasting rule f^{R^b}. I.e., tomorrow's value function is given by problem 3.3. In this manner one can find an R^b that exactly clears the bond market. Solution of this problem delivers $g^c\left(\omega, h, \epsilon, K, H, R^b\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^b\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^b\right)$. At this stage we can state the algorithm as follows

1. Take an initial f^0

2. Solve the household' problem given by 3.18

3. Simulate the economy.

   (a) Set an initial distribution of agents over ω, h and ϵ.

   (b) Iterate to find the R^b that clears the bond market. Give an initial $R^{b,0}$ and solve the problem 3.19 to find $g^c\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,0}\right)$. If there is an excess of lending in the bond market try $R^{b,1} < R^{b,0}$, if there is an excess of borrowing try $R^{b,1} > R^{b,0}$. Go on until finding an $R^{b,*}$ the clears the market[31,32]

   (c) Get next period distribution over ω, h and ϵ by using $g^c\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$, $g^b\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and $g^s\left(\omega, h, \epsilon, K, H, R^{b,*}\right)$ and drawing a new value for the shock ϵ.

   (d) Come back to step (b). Do it for a large number of periods.

4. Use the simulated series for K, H and $R^{b,*}$ to estimate f^1

5. Compare f^1 and f^0. If they are similar we are finished, if not start again by setting $f^0 = f^1$ and going back to point 2.

There is just one last issue to be clarified. Which is the proper class $\mathcal{F}$ where to define our forecasting rules? In a problem without habit formation Krusell and Smith (1997) show that a linear function on the first moment of the wealth distribution is enough. We set the following rules

$$\begin{aligned}
\log K' &= \left\{ \begin{array}{ll} cf_{kg0} + cf_{kgk}\log K + cf_{kgh}\log H & \text{if } z = z_g \\ cf_{kb0} + cf_{kbk}\log K + cf_{kbh}\log H & \text{if } z = z_b \end{array} \right\} \\
\log H' &= \left\{ \begin{array}{ll} cf_{hg0} + cf_{hgk}\log K + cf_{hgh}\log H & \text{if } z = z_g \\ cf_{hb0} + cf_{hbk}\log K + cf_{hbh}\log H & \text{if } z = z_b \end{array} \right\} \\
\log R^b &= \left\{ \begin{array}{ll} \begin{array}{c} cf_{Rg0} + cf_{Rgk}\log K + cf_{Rgh}\log H + cf_{Rgkk}(\log K)^2 \\ +cf_{Rghh}(\log H)^2 + cf_{Rgkh}\log K \log H \end{array} & \text{if } z = z_g \\ \begin{array}{c} cf_{Rb0} + cf_{Rbk}\log K + cf_{Rbh}\log H + cf_{Rbkk}(\log K)^2 \\ +cf_{Rbhh}(\log H)^2 + cf_{Rbkh}\log K \log H \end{array} & \text{if } z = z_b \end{array} \right\}
\end{aligned}$$

[31] Or until $R^{b,1} \simeq R^{b,0}$

[32] An alternative (and computationally cheaper) approach would be to solve the problem generally for R^b and then interpolate different values $R^{b,0}$, $R^{b,1}$, ... until market clears. The problem with this is inexactitude. We would need an extremely fine grid on R^b to make the results along different periods of the simulation consistent among them.

Our findings are that we do not need as much information. Aggregate habits do not improve the forecasting. This actually means that aggregate habits turns out not to be a state variable of the system. Forecasting rules end up being:

$$\log K' = \left\{ \begin{array}{ll} cf_{kg0} + cf_{kgk} \log K & \text{if } z = z_g \\ cf_{kb0} + cf_{kbk} \log K & \text{if } z = z_b \end{array} \right\}$$

$$\log R^b = \left\{ \begin{array}{ll} cf_{Rg0} + cf_{Rgk} \log K + cf_{Rgkk} (\log K)^2 & \text{if } z = z_g \\ cf_{Rb0} + cf_{Rbk} \log K + cf_{Rbkk} (\log K)^2 & \text{if } z = z_b \end{array} \right\}$$

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