Essays on Credit Risk Modeling

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ABSTRACT

This thesis attempts to extend researches on two of the most important issues in credit risk modelling, the relationship between credit risk and interest rate risk, and the co-movement of credit spreads. The former plays an important role in credit risk pricing and risk management and has been actively studied over the past few years. Many extant theoretical models of credit risk pricing predict that there is a negative relation between credit spreads and interest rates. Recently there have been a few attempts to study this relationship in a dynamic way. It has been found that the relation is negative in the short run but in the long run it becomes positive. Recent empirical studies have also raised the issue of co-movement of credit spreads. There is evidence that credit and liquidity proxies can only explain a small part of the variations of the change of credit spreads. A large part of the unexplained residues can be attributed to systematic factors or unknown common factors. In this thesis we address the above two issues theoretically and empirically. In chapter 2, we develop a theoretical model, which can accommodate negative as well as positive relationship between credit spreads and interest rates. We empirically study the dynamic of this relationship in the framework of a Markov switching error-correction model in chapter 3. Chapter 4 is a comprehensive study of the of sovereign credit spreads. We use a reduced model to explore the relation between sovereign spreads and risk free interest rates, the co-movement of sovereign

credit spreads, the source of co-movement, the term structure of sovereign spreads and the influence of co-movement on sovereign term structure.

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CHAPTER 1

Introduction

This thesis aims to extend the current research on the two important issues of credit risk modelling, the correlation between credit spreads and interest rates, and the co-movement of credit spreads. The research on the former issue has been active over the past few years and recent studies look at it in a dynamic way. The latter has been raised by several recent studies and research on it is in its primitive state.

The relation between credit spreads and interest rates is one of the key elements of credit risk pricing. Even for the simplest discount defaultable bond pricing model,

$$P_t = E_t^* \left[e^{-\int_t^T r_s ds} 1 \left(\tau > T \right) \right] \tag{1.1}$$

where P_t is the bond price, r_t is the risk free interest rate, $1(\tau > T)$ is the indicator function which is one when no default happens before maturity T and zero otherwise, $E_t^*[\cdot]$ refers to expectation under risk neutral measure, one needs to make assumptions about the interest rate r_t and default risk $1(\tau > T)$ for the calculation of the expectation. Merton (1974) assumes the interest rate is constant. The static comparison shows that there exists a negative relation between credit spread and interest rate. In Longstaff and Schwartz (1995), the interest rate is assumed to be an O-U process. The model shows when the interest rate increases, the drift term of risk neutral default probability decreases and thus the credit spread decreases, resulting in a negative relation between the credit spread and the interest rate. Using a simple two period model, Annaert et al (1999) show

$$spread = \frac{(1+r)\,EDF}{1-EDF} \tag{1.2}$$

where r is the risk free interest rate and EDF is the expected default frequency. As long as the expected default frequency is a proper constant probability residing between 0 and 1, a positive relation exists between the risk free rate and the credit spread. They claim the analysis can be generalized to a multi-period coupon bond with a similar conclusion.

On the empirical side, Longstaff and Schwartz (1995) use a linear regression model to study the relation between Moody's bond index and Treasury rates and find that there is a negative relation between changes in interest rates and changes in credit spreads. To exclude the potential spurious negative relation induced by imbedded call options, Duffee (1998) constructs a non-callable index for the corporate bond spread. By using a vector regression analysis, he finds significant negative coefficient for the three month bill rate. Arak and Corcoran (1996) also find a negative relation between yields on private issues and riskfree rates when all variables are measured in levels.

Morris, Neal and Rolph (1998) extend the static analysis of the correlation into a dynamic framework. They argue that when corporate bond yields are cointegrated with Treasury rates, both the regressions in level and the usual regressions in first differences may lead to false inferences. Using 10 year constant maturity Treasury rates and Moody's Aaa and Baa seasoned indices over the period between 1960 to 1997, they find that each of the corporate yields is cointegrated with the Treasury rate. In the short run credit spreads are negatively related with Treasury rate but in the long run the relationship turns out to be positive.

Kiesel, Perraudin and Taylor (1999) analyze this relation in a non-parametric way. They find there are some cases in which the relation is positive and others in which it is negative even in the long run.

The inconsistency between empirical evidences and theoretical predictions leads to the first aim of this thesis:

A). Develop a model which can accommodate both positive and negative relationships between credit spreads and interest rates.

Morris et al (1998) report that credit spreads and interest rates are not only non-stationary but also are cointegrated, implying a dynamic movement of the relation. They find in the short run, credit spreads and interest rates are negatively related but in the long run, the relation becomes positive. In order to reduce the possible influence of imbedded option on the value of defaultable bond and thus the relation between credit spreads and interest rates, Barnhill et al (2000) carefully construct indices of corporate spreads of different ratings. They confirm the dynamics found by Morris et al (1999). However, both studies are based on linear error-correction models. It has been claimed interest rates and credit spreads contain information about business cycles and experience regime changes themselves (Ang et al (1998), Bernanke and Gertler (2000)). It is possible that apart from the conventional linear correlation, there might be other ways that the risk free interest rate interacts with the credit spread. Lekkos et al. (2002) use a non-linear multivariate smooth transition autoregression (STAR) model to study the common factors among US and UK swap markets. They find that the impact of risk free interest rate on the credit spreads on swap markets are regime dependent. When regime changes the correlation changes. Their results suggest that economic situation might be a source of co-movement. The second aim of this thesis, therefore, is to

B). Investigate the impact of economics situation on the correlation between risk free interest rates and credit spreads.

The analysis of credit spread determinants raises the issue of the co-movement of credit spreads. It has been shown that default risk and liquidity risk proxies might not be large enough in explaining the change of credit spreads. Collin-Dufresne, Goldstein and Martin (2001), Bakshi, Madan and Zhang (2001) and Elton Gruber, Agrawal and Mann (2001) conduct regression analysis of corporate bond markets and show that default related fundamentals, tax, various liquidity proxies such as the risk free interest rate, the leverage of the firm, the volatility of the return of the firm's total assets, trading volumes, and bid-ask spread can only explain a small part of the variations of the change of credit spreads. Eichengreen and Mody (1998) and Westphalen (2002) carried out similar studies on the sovereign bond market and show similar results. Eichengreen and Mody (1998) suggest that it is investors' sentiment, not fundamentals that determine variations of sovereign credit spreads. In Elton Gruber, Agrawal and Mann (2001), they show a large part of the residues of their regression can be attributed to Fama and French factors, supporting the conjecture of Eichengreen and Mody (1998). Collin-Dufresne, Goldstein and Martin (2001) conduct a principal component analysis of their regression residues and find a latent common factor can explain most of the residues. However, the latent common factor cannot be attributed to any set of variables. We are led to inquire

C). The co-movement of credit spreads, the source of the co-movement and the impact of the co-movement on credit term structure.

We proceed in three chapters.

In Chapter 2, we develop an equilibrium pricing model based on the work by Constantinides (1992) and Saa-Requejo et al (1999). We show that the quadratic model of Constantinides (1992) and the stochastic default boundaries of Saa-Requejo et al (1999) allows the sign of the relation between credit spreads and interest rates to depend on the state of economy. We also show the new pricing model keeps other nice properties of Longstaff and Schwartz (1995) and Saa-Requejo and Santa-Clara (1999).

Chapter 3 addresses the second aim of this thesis. We study the correlation between credit spreads and interest rate in a nonlinear framework. We estimate two bivariate processes: Treasury rates and AAA spreads, Treasury rates and BAA spreads using a Markov switching error correction model and Moody's corporate yield data ranging from 1960 to 2000. Our results show that apart from the conventional linear relationship, credit spreads and treasury rates are correlated through Markov regime switching variables. In one regime, there is an increasing credit spread, decreasing interest rate and high volatility, in the other one a decreasing credit spread, increasing interest rate and low volatility. In contrast to the linear analysis of Morris et al (1998) claiming that AAA spread and BAA spread contain no more information for each other, we find the regimes extracted from AAA spread and BAA spread are different, suggesting different factors might drive the movement of AAA spread and BAA spread.

We deal with the third aim in chapter 4. We study the co-movement and variance composition of sovereign term structure of the three largest Latin American countries via a term structure model. We decompose the term structure into common and entity specific components and undertake a joint estimation in the framework of two-step approach. We specify factors as Gaussian processes with different mean reversion rate and long run mean so that we can measure the variance composition of term structure and evaluate the impact of risk adjustment on the variance composition of term structures. We show that most of the variations of these sovereign spreads can be attributed to the latent common factor. However, this common factor can only account for a small part of the total variations under physical probability measure. Our results support the conjecture that market sentiment is the main reason for the co-movement of credit spreads. We also show that the term structure of yield spreads is upward sloping, and international interest rate is negatively related to the level and slope of yield spreads. The common factor parallel shifts the level of the term structures for all countries but leaves the slope of the term structure unchanged, resulting in a very stable slope across calm and crisis time.

The contributions of this thesis are:

A). We develop a corporate bond pricing model which allows the sign of the relationship between credit spreads and interest rates to vary in different states. Most of the extant models predict that the relation is negative.

B). We examine the dynamics of the relation between the risk free interest rate on the credit spread in a non-linear framework. Our results show that apart from the conventional linear relationship, credit spreads and treasury rates are correlated through Markov regime switching variables. In one regime, there is an increasing credit spread, decreasing interest rate and high volatility, in the other one a decreasing credit spread, increasing interest rate and low volatility. In contrast to the linear analysis of Morris et al (1998) claiming that AAA spread and BAA spread contain no more information for each other, we find the regimes extracted from AAA spread and BAA spread are different, suggesting different factors might drive the movement of AAA spread and BAA spread. And unlike the non-linear multivariate smooth transition autoregression (STAR) model, Markov switching error correction model allows us to estimate the date of regimes and the number of regimes endogenously. The cointegration between credit spreads and interest rates is also clearly taken care of.

C). We study the co-movement of sovereign spreads and the term structure of sovereign spreads comprehensively. Our results confirm the conjecture that risk premiums are the reason for the co-movement of credit spreads. We also examine the shape of the term structure, the impact of co-movement of credit spreads on the term structure and the influence of risk free interest rates on the sovereign spreads.

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CHAPTER 2

A Simple Model for Time-Varying Relationship Between Corporate Credit Spreads and Interest Rates

The relation between default risk and market risk is a key ingredient of modern risk management and defaultable bond pricing. Recent empirical studies suggest that the sign of this relation may change over time. This is inconsistent with the predictions of extant structural models that the sign of this relation is uniformly negative or positive for a firm. In this paper we develop an equilibrium pricing model based on the work by Constantinides (1992) and Saa-Requejo et al (1999). We show that the quadratic property of Constantinides (1992) and the stochastic default boundaries of Saa-Requejo et al (1999) allow the sign of the relation between credit spreads and interest rates to depend on the state of economy. We also show the new pricing model keeps other nice properties of Longstaff and Schwartz (1995) and Saa-Requejo and Santa-Clara (1999).

2.1 Introduction

The relation between credit spreads, the differences between corporate yields and government yields of the same maturity, and interest rates is an important part of corporate bond pricing and risk management. Many pricing models predict that the relation is negative: an increase in the riskfree rate will decrease a firm's credit spread. In his pioneering paper, Merton (1974) takes defaultable bond as an option on the diffusion process of the firm market value. Default occurs if the face value of the bond is less than the market value of the firm at maturity. The interest rate is assumed to be a constant. The relation between changes in interest rates and credit spreads is negative by static comparison. Black and Cox (1976) relax the assumption on the default time by allowing default to occur whenever the market value of the firm falls below certain exogenously given threshold level; Leland and Toft (1996) endogenise the default barrier. Static comparison shows that they generate the same relationship as Merton's.

Longstaff and Schwartz (1995) take into account of the dynamic of interest rate and allow it to be correlated with the dynamic of the firm's assets value. The drift term of their default process consists of two parts: the riskfree interest rate and the adjustment for correlation between interest rate and firm value. Since interest rate dominates the adjustment, no matter what the relation between interest rate and firm value is positive or negative, there is a clear negative relation between credit spreads and the interest rate: an increase in the interest rate will increase the drift term making default less likely thus decrease the credit spreads; A decrease in the interest rate will decrease the drift term making default more likely thus increase the corresponding credit spreads.

The negative relation is thought to be counter-intuitive. It seems more likely that a high nominal interest rate would increase the debt burden and should be associated with a high risk premium.

Using a simple two period model, Annaert et al (1999) show that

$$spread = \frac{(1+r)\,EDF}{1-EDF}$$

as long as the expected default frequency is a proper constant probability residing between 0 and 1, a positive relation exists between the riskfree rate and the credit spread. They claim the analysis can be generalized to a multi-period coupon bond with similar conclusion.

Leland and Toft (1996) suggest that the riskfree rate influences not only the discount rate but also directly influence the value of the underlying asset. Thus if riskfree rate increases, the value of the firm would decrease and the probability of default would correspondingly increase, implying a positive relation between interest rate and credit spreads.

The empirical evidences as to this relationship are mixed. In the linear regression framework, Longstaff and Schwartz (1995) study the relation between Moody's bond data and Treasury rates and find that there is a negative relation between changes in interest rates and changes in credit spreads. To exclude the potential spurious negative relation induced by imbedded call option, Duffee (1998) constructs a non-callable index for corporate bond spread. By using a vector regression analysis, he find significant negative coefficient for three month bill rate. Arak and Corcoran (1996) also find a negative relation between yields on privately issues and riskfree rates when all variables are measured in levels.

Morris, Neal and Rolph (1998) argue that when corporate bond yields are cointegrated with Treasury rates, both the regressions in level and the usual regressions in first differences may lead to false inferences. Using 10 year constant maturity Treasury rates and Moody's Aaa and Baa seasoned indices over the period between 1960 to 1997, they find that each of the corporate yields are cointegrated with Treasury rate. In the short run credit spreads are negatively related with Treasury rate but in the run the relationship turns to be positive.

Kiesel, Perraudin and Taylor (1999) analyze this relation in a non-parametric way. They find there are some cases in which the relation is positive and others in which it is negative even in the long run.

The inconsistency between empirical evidences and theoretical predictions leads some authors to build models able to accommodate the negative and positive relation between credit spreads and interest rates.

Saa-Requejo and Santa-Clara (1999) show that the relation can be either negative or positive depending on the relation between the firm's asset and the interest rate for a given bond. The innovation of their paper is that the boundary is the market value of the firm's total liability or the "bankruptcy " firm value, thus is stochastic and can covary with the value of the firm's asset. Since the market value of total lability or the bankruptcy firm values are actually the value of an asset, its risk-adjusted return should equal the riskfree interest rate. In this circumstance the level of interest rate is no longer the determinant of default probability. Whether the relation between interest rate and credit spreads is negative or positive depends on whether the correlation between the interest rate and firm value process is negative or positive. However, for a given firm the sign of the relation is fixed.

Chang at al (2000) present an equilibrium model in which the relation varies with the wealth of the economy. In their model, default-free term structure and default premia are determined simultaneously. The consumer's relative risk aversion in wealth increases with decreases in wealth. As the wealth drops, the default premium increases, the default-free interest rates go down.

In this paper, we introduce Saa-Requejo and Santa-Clara (1999)'s default mechanism into the default free Constantinides's economy. We show that the non-affine property of Constantinides' interest rate and the stochastic default boundary produce a similar result to Chang at al (2000) with regards to the relation between interest rate and credit spread: the sign of the relation depends on the state of economy. We also show the new pricing model keeps other nice properties of Longstaff and Schwartz (1995) and Saa-Requejo and Santa-Clara (1999).

2.2 Theoretical Framework

2.2.1 Constantinides's default free economy

In Constantinides' economy, a representative consumer economy represented by the augment probability space $(\Omega, \mathcal{F}, F, P)$, where $F = \{\mathcal{F}_t\}_{0 \leq t \leq T}$. the consumer maximizes the expected discounted sum of a strictly increasing concave Von Neumann-Morgensten utility function, U :

$$\max E_t^P \left[\int_t^{\infty} \beta^t U(C_t) dC_t \right] \tag{2.1}$$

where $E_t^P[\cdot]$ is the conditional expectation given all information up to time t , $\beta \in (0,1)$ the time discount factor, C_t is consumption at time t. The first order condition for the economy to be in equilibrium is:

$$U'(C_t)P_t^* = E_t^P\left\{\beta^{(T-t)}U'(C_T)P_T^*\right\} \tag{2.2}$$

where $U'(\cdot)$ is the derivative of $U(\cdot).P_t^*$ is the real price of an asset, $.P_{t,\omega}^* : [0,\infty) \times \Omega \to R^+$.

Suppose π_t is the price level at time t , $p_{t,\omega} : [0,\infty) \times \Omega \to R^+, P_t$ is the nominal price for the asset, the above condition is

$$U'(C_t)\frac{P_t}{\pi_t} = E_t^P\left\{\beta^{(T-t)}U'(C_T)\frac{P_T}{\pi_T}\right\} \tag{2.3}$$

Define

$$\xi_\tau = \beta^{\tau-t}\frac{U'(C_\tau)}{\pi_\tau}$$

then we have the first order condition for an asset's nominal price in equilibrium:

$$P_t = E_t^P\left(\frac{\xi_T}{\xi_t}P_T\right) \tag{2.4}$$

The riskless discount bond value at time t with maturity T is

$$B(t,T) = E_t^P\left(\frac{\xi_T}{\xi_t}\right) \tag{2.5}$$

If market are complete, then there is a unique ξ_T.

Constantinides's (1992) assumes a pricing kernel:

$$\xi_t = \exp\left[-(g+\frac{\sigma_0^2}{2})t + x_{0,t} + \sum_{i=1}^{N} x_{i,t}^2\right] \tag{2.6}$$

where

$$dx_{0,t} = \sigma_0 d\omega_{0,t},$$

$$E_0^P(x_{0,t}) = x_{0,0}$$

$$Var(x_{0,t}) = \sigma_0^2 t$$

$\{x_{i,t}, i = 1, ..., N\}$ are N state variables which follow Ornstein-Uhlenbeck process:

$$dx_{i,t} = k_i(\mu_i - x_{i,t})dt + \sigma_i d\omega_{i,t} \tag{2.7}$$

k_i is the adjustment speed of $x_{i,t}$ to its long run mean μ_i , μ_i and σ_i are constants, $\sigma_i > 0$. $\{\omega_{i,t}$, i=0,...N$\}$ are N independent standard Brownian motion. $x_{i,t}$ is normally distributed with

$$E_t^P(x_{i,t}) = \mu_i + (x_{i,t} - \mu_i)e^{-k_i(T-t)}$$

$$Var(x_{i,t}) = \frac{\sigma_i^2}{2k_i}\left(1 - e^{-2k_i(T-t)}\right)$$

Steady state distribution of $x_{i,t}$ is also normal with mean μ_i and variance $\frac{\sigma_i^2}{2k_i}$.

The price of riskless discount bond under this specification of state price is

$$B(t,T) = E_t^P\left(\frac{\xi_T}{\xi_t}\right)$$

$$= (\Pi_{i=1}^N H_{i,T})^{-\frac{1}{2}}$$

$$\cdot \exp \left\{ \begin{array}{c} (\sum_{i=1}^{N} k_i - g)(T-t) + \sum_{i=1}^{N} H_{i,T}^{-1} \left[x_{i,T} - \mu_i (1 - e^{k_i (T-t)}) \right]^2 \\ - \sum_{i=1}^{N} x_{i,t}^2 \end{array} \right\} \tag{2.8}$$

where

$$H_{i,T} = \frac{\sigma_i^2}{k_i} + \left(1 - \frac{\sigma_i^2}{k_i}\right) e^{2k_i (T-t)}$$

The instantaneous interest rate is

$$r_t = -E_t^P \left(\frac{d\xi_t}{\xi_t} \right) / dt$$

$$= g + \sum_{i=1}^{N} \left\{ -\sigma_i^2 - \frac{k_i \mu_i^2}{2\left(1 - \frac{\sigma_i^2}{k_i}\right)} + 2k_i \left(1 - \frac{\sigma_i^2}{k_i}\right) \left(x_{i,t} - \frac{\mu_i}{2\left(1 - \frac{\sigma_i^2}{k_i}\right)} \right)^2 \right\} \tag{2.9}$$

$$dr_t$$

$$= \sum_{i=1}^{N} \left\{ \begin{array}{c} 4k_i^2 \left(1 - \frac{\sigma_i^2}{k_i}\right) \left(x_{i,t} - \frac{\mu_i}{2(1 - \frac{\sigma_i^2}{k_i})} \right) ((\mu_i - x_{i,t}))\, dt \\ +4k_i \left(1 - \frac{\sigma_i^2}{k_i}\right) \left(x_{i,t} - \frac{\mu_i}{2\left(1 - \frac{\sigma_i^2}{k_i}\right)} \right) \sigma_i d\omega_{i,t} \end{array} \right\} \tag{2.10}$$

To guarantee that the interest rate r_t is positive, one must restrict parameters as follows:

$$\sigma_i^2 < k_i, i = 1, ..., N$$

$$g > \sigma_i^2 + \frac{k\mu_i^2}{2\left(1 - \frac{\sigma_i^2}{k_i}\right)}, i = 1, ..., N$$

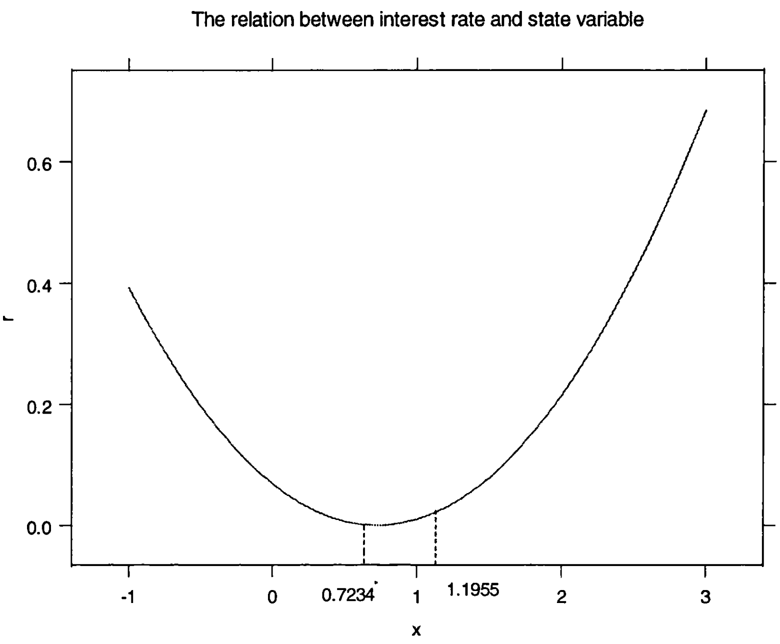

Figure 2.1: The relation between the interest rate r and the state variable x

There are several points to notice:

1) This interest rate process is guaranteed to be positive and it neither nests nor is nested by the interest rate process of CIR or translated CIR model.

2) Even when N=1, knowledge of r_t does not unambiguously determine the value of state variable. The relation between interest rates r_t and state variable x is quadratic. We display this relation using parameters from empirical study by Biao Lu (1999) in Figure 1.

3) The state variable $x_{0,t}$ is irrelevant to determination of interest rate or riskless bond price. It is introduced to avoid unnecessary assumptions on the stationality of consumption and price level process. we can see later, it does not affect default risk process either. We can ignore it in our model.

2.2.2 Assumptions About Default RiskAssume that trading takes place continuously, markets are perfect, complete, and frictionless, without taxes, transaction costs or information asymmetries. The market value of a firm's total assets is given by the following geometric Brownian motion:

$$dV_t = (\mu_{vt} - \delta)V_t dt + \sigma_v V_t d\omega_{vt} \tag{2.11}$$

where μ_{vt} is percentage return of the assets, σ_v is a constant dividend rate. ω_{vt} is a standard Brownian motion.

The firm's total debt is assumed to be an asset with shocks from the underlying uncertainty. The specification of total debt as an asset held by some agents was introduced by Saa-Requejo and Santa-Clara (1999) .

$$dD_t = (\lambda_{dt} - \gamma)D_t dt + \sigma_{d1} D_t d\omega_{1t} \tag{2.12}$$

where λ_{dt} is the required return of total debt and γ is a constant payout rate to debt holders.

The firm continues to operate until its total asset value falls below its total debt value. By Ito's lemma, the time of default is equivalent to the first passage time of the following process hitting a constant boundary 0.

$$dZ_t = \mu_z dt + \sigma_z d\omega_{zt}$$

where

$$\mu_z = \mu_{vt} - \delta - (\lambda_{dt} - \gamma) - \frac{1}{2}(\sigma_v^2 - \sigma_{d1}^2) \tag{2.13}$$

$$\sigma_z d\omega_{zt} = \sigma_v d\omega_{vt} - \sigma_d d\omega_{1t}$$

.

The correlation between $d\omega_{zt}$ and $d\omega_{1t}$ is

$$\rho_{z1} = \frac{1}{\sigma_z}(\rho_{1v}\sigma_v - \sigma_{d1}) \tag{2.14}$$

2.3 Corporate Bond Pricing

Following Longstaff and Schwartz (1995), we assume that when default occurs, the firm defaults on all its obligations and the bondholder receives $1-w$ otherwise equivalent riskfree discount bonds. w is the loss ratio of the bond face value and depends on the seniority of the bond, the bargaining power of the bondholders etc. w is higher for a senior secured bond than a junior bond. For simplicity we assume w to be a constant , This is in line with many structure models and in this way, we avoid the complicate allocation of assets upon bankruptcy and can price the bond in a way that is independent of the capital structure of the firm. Within this framework, we can then write the time t value of a defaultable zero coupon bond with maturity T as

$$P_t = (1-w)B(t,T) + wE_t^P\left(\frac{\xi_T}{\xi_t}1(\tau > T)\right) \tag{2.15}$$

Where τ is the default time, $1(\tau > T)$ is the indicator function, $1(\tau > T) = 0$ when default occurs before maturity and $1(\tau > T) = 1$ when default does not happen within the life of the bond.

We now consider two cases which vary according to the assumptions made about correlations.

2.3.1 Case 1. Default risk is uncorrelated with the pricing kernel. Under this assumption the defaultable bond price is:

$$P_t = (1-w)B(t,T) + wE_t^P\left(\frac{\xi_T}{\xi_t}1(\tau > T)\right)$$

$$= (1-w)B(t,T) + wB(t,T)Q_t^P(\tau > T)$$

$$= B(t,T)[1 - wQ_t^P\{1(\tau < T)\}]$$

$Q_t^P\{1(\tau < T)\}$ is determined by a univariate arithmetic Brownian motion

$$dZ_t = \mu_z dt + \sigma_z d\omega_{zt}$$

with hitting boundary 0. we have an closed form solution (see Harrison(1990)):

$$Q_t^P\{1(\tau < T)\} = N\left(-\frac{Z_0 + \mu_z T}{\sigma_z\sqrt{T}}\right) + e^{-2\mu_z z_0/\sigma_z}N\left(-\frac{Z_0 - \mu_z T}{\sigma_z\sqrt{T}}\right) \qquad (2.16)$$

where $N(\cdot)$ is the distribution probability of standard normal variable.

2.3.2 Case 2 General situation In order to calculate the expectation

$$E_t^P\left(\frac{\xi_T}{\xi_t}1(\tau > T)\right)$$

we do the following things:

First unlike other pricing models which start from risk neutral probability measure, our framework is in the real world and we need to obtain risk price from the pricing kernel.

Under the assumption of complete market, there exits a unique probability measure Q under which discounted security prices follow a martingale(Harrison and Kreps (1979) and Harrison and Pliska (1981))

$$\frac{P_t}{B_t} = E_t^Q \left(\frac{P_T}{B_T} \right)$$

$$= E_t^P \left(\frac{P_T}{B_T} \frac{dQ}{dP} \right)$$

where

$$B_t = \exp \left(\int_0^t r_s ds \right)$$

$$\frac{dQ}{dP} = E \left(\int_t^T \gamma_s d\omega_s - \frac{1}{2} \int_t^T \gamma_s^2 ds \right)$$

γ_s is an F_t -adapted vector process.

$\frac{dQ}{dP}$ is the Radon-Nikodym derivative which take us from the real world measure P into the risk neutral world measure Q.

we can rewrite

$$\frac{\xi_T}{\xi_t} = \frac{B_t}{B_T} \frac{dQ}{dP}$$

$$= \exp \left(\int_t^T \gamma_s d\omega_s - \int_t^T (r_s + \frac{1}{2} \gamma_s^2) ds \right)$$

Taking derivative, we have

$$\frac{d\xi_t}{\xi_t} = -r_t dt + \gamma_t d\omega_t$$

$$= -r_t dt + \sigma_0 d\omega_0 + 2x_{1,t}\sigma_1 d\omega_1$$

By Girsanov's Theorem,

$$\omega_{0,t}^* = \omega_{0,t} - \int_t^T \sigma_0 ds$$

$$\omega_{1,t}^* = \omega_{1,t} - 2\int_t^T x_{1,s}\sigma_1 ds$$

we get bivariate processes under risk neutral measure:

$$dZ_t = \mu_z dt + \sigma_z d\omega_{zt}^*$$

here

$$\mu_z = -\delta + \gamma - \frac{1}{2}(\sigma_v^2 - \sigma_{d1}^2)$$

$$dx_{1,t} = (k_1(\mu_1 - x_{1,t}) + 2x_{1,t}\sigma_1^2)dt + \sigma_1 d\omega_{1,t}^*$$

Now we can write

$$P_t = (1-w)B(t,T) + wE_t^P\left(\frac{\xi_T}{\xi_t}1(\tau > T)\right)$$

$$= (1-w)B(t,T) + wE_t^Q\left(e^{-\int_t^T r(u)du}1(\tau > T)\right) \tag{2.17}$$

where expectation is taken under risk neutral measure Q.

We need to move $e^{-\int_t^T r(u)du}$ out of the expectation. This can be done by changing risk neutral measure to forward measure. The Radon-Nikodym derivative

$$\frac{dQ^T}{dQ} = \exp\left(\int_t^T b_{u,T} d\omega_s^T - \frac{1}{2}\int_t^T (b_{u,T}^2) ds\right) \tag{2.18}$$

where $b_{u,T}$ is the diffusion term of the riskless bond process

$$b_{u,T} = 2\sigma_1 \left\{H^{-1}[x_{1,t} - \mu_1(1 - e^{k_1(T-t)})] - x_{1,t}\right\}$$

Thus we have

$$P_t = (1-w)B(t,T) + wB(t,T)E_t^T(1(\tau > T))$$

$$= (1-w)B(t,T) + wB(t,T)Q_t^T(1(\tau > T))$$

$$= B(t,T)[1 - wQ_t^T\{1(\tau < T)\}] \tag{2.19}$$

where $Q_t^T(1(\tau > T))$ is determined by the bivariate processes

$$dZ_t = \left\{\mu_z - 2\rho_{1z}\sigma_1\sigma_z\left[\mu_1 H^{-1}\left(1 - e^{k_1(T-t)}\right) + \left(1 - H^{-1}\right)x_{1,t}\right]\right\} dt + \sigma_z d\omega_{zt}^T \tag{2.20}$$

$$dx_{1,t} = (k_1(\mu_1 - x_{1,t}) + 2\sigma_1^2 x_{1,t} - 2\rho_{1z}\sigma_1\sigma_z\left[\mu_1 H^{-1}\left(1 - e^{k_1(T-t)}\right) + \left(1 - H^{-1}\right)x_{1,t}\right] + \sigma_1 d\omega_{1,t}^T \tag{2.21}$$

with absorbing boundary $Z_\tau = 0$.

The yields for defaultable bond is

$$y_{t,T} = -\frac{\ln P_{t,T}}{T-t}$$

$$= -\frac{\ln B(t,T)}{T-t} - \frac{\ln(1 - wQ_t^T(1(\tau < T)))}{T-t}$$

$$\approx -\frac{\ln B(t,T)}{T-t} + \frac{wQ_t^T(1(\tau < T))}{T-t}$$

The credit spread is:

$$y_{t,T} - (-\frac{\ln B(t,T)}{T-t}) \approx \frac{wQ_t^T(1(\tau < T))}{T-t} \tag{2.22}$$

From (20) we can see both negative and positive relation can happen for a given bond:

If the parameters are such:

$$\rho_{z1} = \frac{1}{\sigma_z}(\rho_{1v}\sigma_v - \sigma_{d1}) < 0 \tag{2.23}$$

then in the region of positive relation between state variable and interest rate, there is a negative relation between interest rate and credit spread: when interest rate r_t increases, $x_{1,t}$ increases, the drift term of the default process increases. This means Z_t is expected to drift away from boundary at a faster rate, which reduces the forward measure probability of default. The required credit premium decreases accordingly. When there is a decrease in $x_{1,t}$ (r_t), the forward measure probability of a default and credit spread increase. This corresponds to the empirical regularity: the inverse relation between the default premia and the default-free rates which has been documented by Duffee (1998) and Longstaff and Schwartz (1995) and the phenomenon known as flight to quality: during period of crisis, default-free interest tend to fall dramatically and the default premium tend to increase.

In the region of negative relation between state variable and interest rate, there is a positive relation between interest rate and default risk: when interest rate r_t increases, the state variable $x_{1,t}$ decreases thus the drift term of the default process decreases. This makes Z_t drift towards the boundary and increases the forward probability of default.

If the parameters are such that:

$$\rho_{z1} = \frac{1}{\sigma_z}(\rho_{1v}\sigma_v - \sigma_{d1}) > 0 \tag{2.24}$$

the relation between interest rate and credit spread also depends on in which region the state variable is in. The analysis is the same as above but the relation is reversed.

2.4 Further explanation of the negative/positive relation of our Model

The state dependent correlation distinguishes our model from that of those in the current literature.

The influential Longstaff and Schwartz (1995)'s model is based on Vasicek interest model. We can see that the negative relation comes from their special modelling of the default risk: default occurs when assets reach a constant boundary. If the boundary is the value of an asset, then this relation disappears: correlation coefficient only affects the magnitude of credit risk but not the relation between the change of interest and credit spread.

Modelling the total liability of the firm as an asset, as in Saa-Requejo and Santa-Clara (1999) and our case, introduces the possibility of state dependent correlation. In this framework, the interest rate is no longer a dominant factor in determining the risk neutral default probability and the correlation of the interest rate and default risk depends on the correlation adjusted term in the risk neutral default process.

Saa-Requejo and Santa-Clara (1999) describe the situation when the interest rate is modelled as a CIR process. The correlation adjusted term in the risk neutral default process is a linear function of the square root of the positive interest rate $\sqrt{r_t}$. Default risk depends on state variable the interest rate. However, due to the linearity of the correlation adjusted term, change of interest rate affect change of credit spread monotonically. The relation between credit spread and the interest rate is uniformly signed, either negative or positive but not both.

The negative/positive relation between credit spreads and interest rates of our model derives from the quadratic form of interest rate of Constantinides (1992). Cheng and Scaillet (2002) explore the advantage of the quadratic model over the affine one in a more general framework: the linear-quadratic jump-diffusion

(LQJD) setting. They show that under some technical regularities the LQJD class can be embedded into affine class through the use of an augmented state vector. For the same number of factors, the LQJD model has the extra capacity to accommodate effects that cannot be handled in an affine model without introducing the extra pseudo-factors. In this framework, our one factor quadratic model is superior over the one factor affine interest rate model of Saa-Requejo and Santa-Clara (1999) in that while the relation between credit spreads and interest rates of theirs depends on only $\sqrt{r_t}$, it depends on two factors in our case, x_t and the augmented factor y_t (x_t^2). y_t (x_t^2) is restricted to be non-negative but the unrestricted x_t allows the relation to be positive as well as negative.

2.5 Simulation results and data

Simulation of the pricing model and prediction of the relation between credit spread and interest rate are displayed on the following charts. The parameters I choose for the default process are equal to those of Longstaff and Schwartz (1995) and other standard corporate bond pricing models wherever possible. The parameters for the state variables are the estimations by Biao Lu (1999) of Constantindes's one state version. $g = 0.083(19.8724), k_1 = 0.0799(11.5728), \mu_1 = 1.1955(6.8591), \sigma_1 = 0.1178(17.8528)$.

Interest rate reaches its minimum value

$$g - \sigma_1^2 - \frac{k\mu_1^2}{2\left(1 - \frac{\sigma_1^2}{k_1}\right)} = 2.50104E - 0.5$$

when

$$x_{1,t} = \frac{k_1 \mu_1}{2(k_1 - \sigma_1^2)} = 0.72339$$

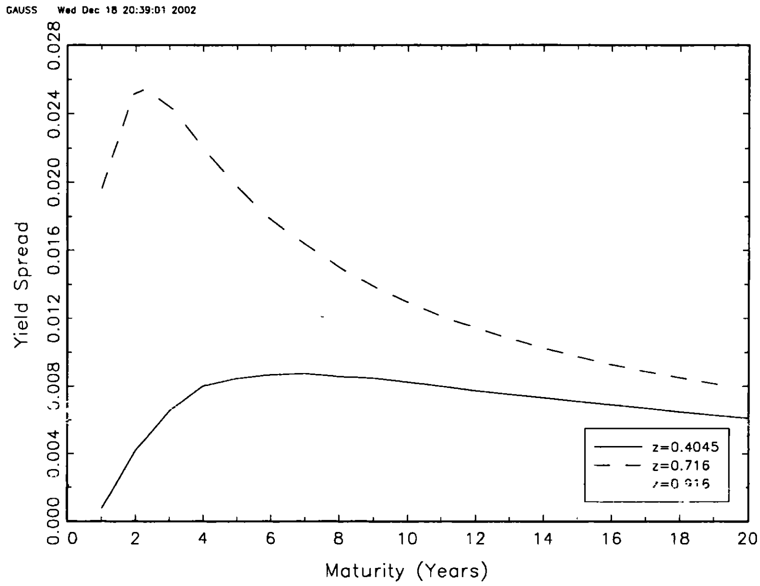

Figure 2.2: $r_0 = 0.04(x = 1.28), w = 0.2, \rho = -0.25, \sigma_v = 0.2, \sigma_{d1} = 0.1$.

Figure 2 shows how credit spread depends on the underlying firm quality. The shape and magnitudes of credit spreads are almost the same as Longstaff and Schwartz(1995)'s. For high rating ($z_0 = 0.916$) or medium rating ($z_0 = 0.716$) bonds, the term structure is monotone increasing. It means high rating firms are unlikely to default in the short term, but over long period of time the possibility of declining in quality increases. For low rating bonds ($z_0 = 0.4054$), the shape is hump-shaped which means low quality firm face immediate risk of default but over time their financial state will improve. $z_0 = 0.4054, 0.716, 0.916$ corresponds to $X = 1.5, 2.0, 2.5$ in Longstaff and Schwartz (1995).

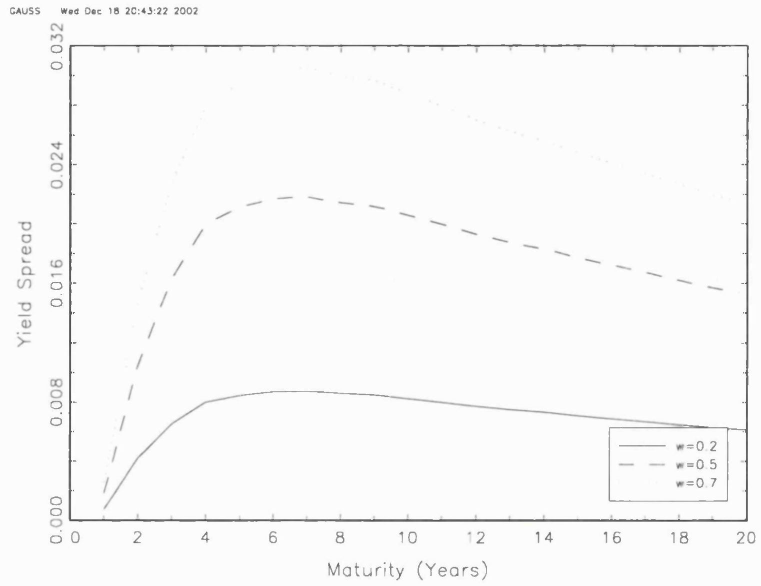

Figure 2.3: $r_0 = 0.04(x = 1.28), \rho = -0.25, \sigma_v = 0.2, \sigma_{d1} = 0.1, z_0 = 0.716$

Figure 3 shows credit spread is an increasing function of the written-down ratio w : a larger w requires a larger spread. This is the same as in Longstaff and Schwartz(1995). The shapes are also the same.

This chart shows the negative relationship between interest rate and credit spread, the same as in Longstaff and Schwartz(1995) and other standard pricing models. $r = 0.02, 0.04, 0.1$ corresponds to $x = 1.12, 1.28, 1.6$ respectively.

Figure 4 shows the particular result we get in this paper: when state variable $x < 0.72339$, the relation between interest rate and credit spread becomes positive. $r = 0.5, 0.4, 0.2$ corresponds to $x = 0.1, 0.18, 0.33$ respectively.

Finally, we check the time series of treasure rate and corporate bond rates from Bloomberg spread data. We choose daily data of corporate spreads of rating A2

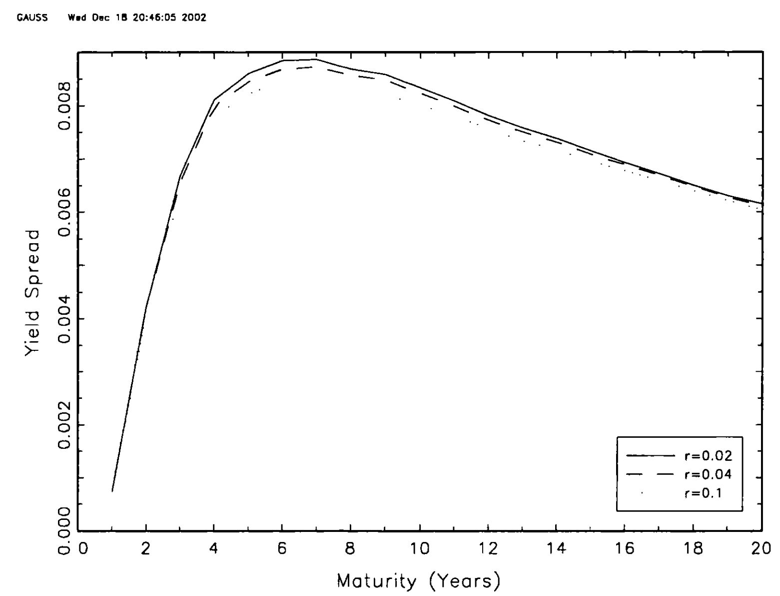

Figure 2.4: $w = 0.2, \rho = -0.25, \sigma_v = 0.2, \sigma_{d1} = 0.1.z_0 = 0.716$

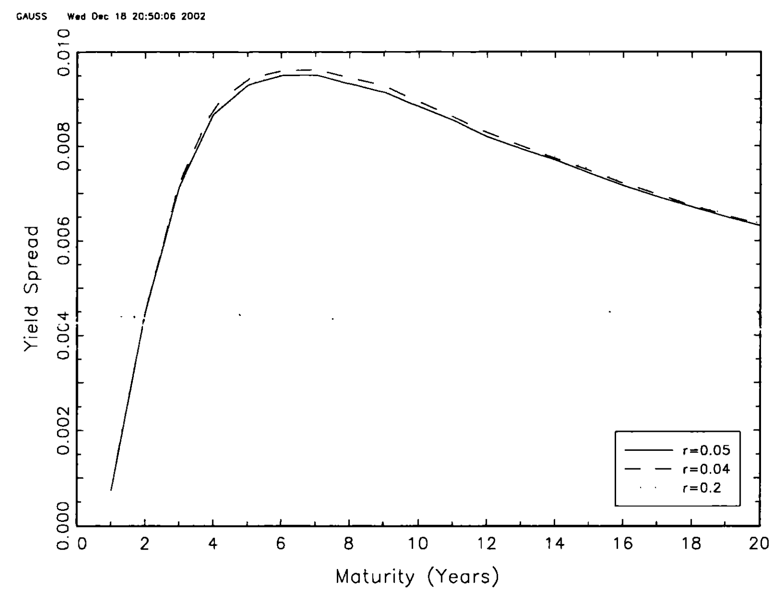

Figure 2.5: $w = 0.2, \rho = -0.25, \sigma_v = 0.2, \sigma_{d1} = 0.1.z_0 = 0.716$

Lags	Dickey-Fuller	Phillips-Perron
1	-4.31	-4.83
2	-3.86	-4.66
3	-3.44	-4.26
4	-3.51	-4.55
5	-3.50	-4.59

Table 2.1: The table shows the results of unit root tests using augmented Dickey-Fuller tests and Phillips-Perron tests for US treasure rate of maturity 0.25 year on a part of our sample.

with maturity 10 years and treasury rate with maturity 0.25 year between 1996 and 1998. As interest rates are supposed to be stationary in our model, we check the stationarity of the interest rate data. Table 1 reports the results of augmented Dickey-Fuller and Phillips-Perron unit root tests. All the test statistics show that we can reject the null hypothesis of a unit root in the time series of treasure rate over this range. We then partition this period with window length of 30 days. Both sample correlation and bootstrap estimation of the correlation have positive as well as negative values over this period. Even though more rigorous tests are needed, we suspect that the correlation is really state dependent, as predicted by our model.

		Dickey-Fuller	Phillips-Perron
1%	Critical Value	-3.4450	-3.4449
5%	Critical Value	-2.8673	-2.8672
10%	Critical Value	-2.5698	-2.5698

Table 2.2: MacKinnon critical values for rejection of hypothesis of a unit root

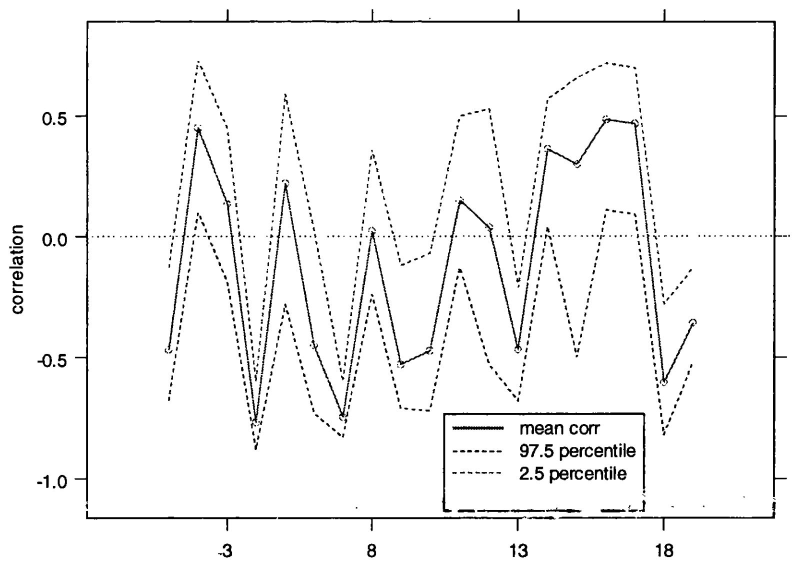

Figure 2.6: Sample correlation and bootstrap of the correlation. Windon length: 30 days. Resampling is performed independently. The number of replication is 1000. Data source: Bloomberg

2.6 Conclusion

The standard pricing model like Longstaff and Schwartz (1995) predicts that the relation between interest risk and credit risk is negative: when interest rates increase, the value of the firm's total assets increase and thus the likelihood of default decreases. This is in contrast with the recent empirical evidence that the relation in fact can be positive and negative. In this paper, we develop a model for the valuation of corporate bond, which can accommodate positive and negative relation. We find that there are two reasons for this uniform signed relation in standard pricing model: the affine interest rate model and the assumption of constant total debt values of a firm. We correct the first problem by using a general equilibrium algorithm. The interest rates are no longer affine function of state variables in this model. For the second problem, we model the value of the total debts of a firm as a stochastic process, which is correlated with the firm's asset value and risk free interest rates. In this context, the interest rates are no longer a dominant factor in determining default probability: it is dependent on the value of state variable as well as the correlation of the interest rates and the values of the firm's total assets. Our simulation results show that the model can match the empirical results while keeping other nice properties of the standard pricing models.

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CHAPTER 3

A Markov Switching Analysis of Credit Spreads and Interest Rates

In this paper we study the correlation between credit spreads and interest rate in a nonlinear framework. We estimate two bivariate processes: Treasury rates and AAA spreads, Treasury rates and BAA spreads using a Markov switching error correction model and Moody's corporate yield data ranging from 1960 to 2000. Our results show that apart from the conventional linear relationship, credit spreads and treasury rates are correlated through Markov regime switching variables. In one regime, there is an increasing credit spread, decreasing interest rate and high volatility; in the other one a decreasing credit spread, increasing interest rate and low volatility. In contrast to the linear analysis of Morris et al (1999) who claims that AAA spread and BAA spread contain no more information for each other, we find the regimes extracted from AAA spread and BAA spread are different, suggesting different factors might drive the movement of AAA spread and BAA spread.

3.1 Introduction

The relationship between credit spreads and interest rates is an important part of risk management and defaultable bond pricing. A proper risk management model needs to take good care of the correlation for the allocation of scarce capital. Assuming independence of credit risk and interest rate would underestimate the risk if the correlation is in fact positive and overestimate the risk if the correlation is negative. The interest rate is also taken as an essential determinant of credit spreads and assumptions about the correlation between the credit spread and the interest rate have to be made for the calculation of a defaultable instrument.

The relation predicted by extant theoretical models and displayed by empirical studies however is not unanimous. In the equilibrium models of Merton (1974), Longstaff and Schwartz (1995), an increase in interest rates will decrease a firm's credit spreads. Leland and Toft (1996) explore the possibility of direct influence and positive relation between interest rates and credit spreads and suggest that interest rates may not only influence the discount rates but also directly influence the value of the underlying asset. When interest rates increase, the value of the firm would decrease and the probability of default would increase, implying a positive relation between credit spreads and interest rates. In the empirical part, Longstaff and Schwartz (1995) and Duffee (1996) study the relation between the change of credit spreads and the change of interest rates and find a negative relation between the changes, which usually interpreted as a negative relation between the levels of interest rates and credit spreads.

Morris et al (1999) report that credit spreads and interest rates are not only non-stationary but also are cointegrated, implying a dynamic movement of the relation. They find in the short run, credit spreads and interest rates are negatively related but in the long run, the relation becomes positive. In order to reduce the possible influence of imbedded option on the value of defaultable bond and thus the relation between credit spreads and interest rates, Barnhill et al (2000) carefully construct indices of corporate spreads of different ratings. They confirm the dynamics found by Morris et al (1999).

All the above empirical studies assume a linear data generating process and the parameters in their model are constant during their sample period. It has been claimed interest rates and credit spreads contain information about business cycles and experience regime changes themselves (Ang et al (1998), Bernanke and Gertler (2000)). Recent financial crises such as the Russian defaults and the LTCM crisis suggest that during crises investments flight from default bond market to safe assets such as the Treasury bonds, inducing strong negative relation between credit spreads and interest rates, a phenomenon called "flight to quality". Constant correlation assumptions may not be appropriate in this situation and the negative correlation found in the above studies may actually be the result of the average effect of the constant parameter assumption if the sample period covers crises during which there exists a strong temporary negative relation. Taking into account the possible regime changes would help us to improve our knowledge of the relation and our techniques of risk management.

In this paper, we study the relation between credit spreads and interest rates in the framework of Markov regime switching error correction model. The regimes are governed by the Markov chain. The intercepts and variance-covariance can change in different regimes. In each regime, the processes are linear but the combination of processes is non-linear. A cointegrating vector characterizes the long run dynamics and the long run and short run dynamics are jointly estimated with the two regimes. In addition, we also include data during the recent severe crisis periods to detect if the positive long run relationship reported in recent studies is biased by the survival problem. The model identifies two regimes, one in which credit spreads are increasing, interest rates are decreasing and volatilities are high (recession) and one in which credit spreads are decreasing, interest rates are increasing and volatilities are low (normal time). The extra crisis data characterized by "flight to quality" just marginally reduces the magnitude of the positive long run relation found by Morris et al (1999) and Joutz et al (2000), reflecting the transitory feature of the negative relation. The contemporaneous correlation is negative in both regimes. The dynamics revealed by the regime dependent impulse response functions shows that the relation is negative at the beginning, then quickly becomes positive (in a about 2-3 months) across regimes. The pattern is almost the same in the two regimes with a slightly higher magnitude in normal regime. There is however extra negative short run relationship associated with regime transition, implying a higher magnitude of negative relationship when crisis happens or when economy begins to expand.

One related work is done by Lekkos al et (2002) who use a non-linear multivariate smooth transition autoregression (STAR) model to study the common factors among US and UK swap markets. They also identify two regimes and the impact of interest term structure on swap spreads depends on the regime. However, in order to apply STAR, they need to specify an observable (US interest slope) as the determinant of the state dynamics. Markov switching model is more flexible in the respect that there is no need to restrict the regime to be generated by some observables. The data will identify the dynamics. The regimes identified in our study matches the business cycles much better than Lekkos al et (2002). They find the regimes they identify based on the dynamics of the US slope do not correspond to the periods of economic expansion or recession.

3.2 Markov switching VAR model:

3.2.1 Definition:While the conventional time series models assume stable parameter representations over the entire sample period, the Markov-switching models allow the parameters to be estimated to switch between states. Hamilton (1988, 1989, 1990) popularizes Markov-switching models by offering convenient filtering algorithms and Krolzig (1997, 1998) integrate the Markov switching assumption into a vector-autoregressive model (VAR).

The MS-VAR can be described by a data degenerating process which is jointly determined by

- the observational equation:

$$y_t - \mu(s_t) = A_1(s_t)(y_{t-1} - \mu(s_{t-1})) + ... + A_p(s_t)(y_{t-p} - \mu(s_{t-p})) + u_t$$

if there is an immediate one time jump in the process mean $\mu(s_t)$, a change in the parameter matrices $A_j(s_t)$ and the variance -covariance matrix $\sum(s_t)$ after a change in the regime (MSM). In the above equation, $y_t = (y_{1t}......, y_{kt})'$ is a k-dimensional time-seires vector $(t = 1, ..., T)$, $\mu(s_t)$ is the mean of y_t and $A_j(s_t)$

are $(k * k)$ parameter matrices $(j = 1, ...p)$. The errors of this model are normally distributed

$$u_t|s_t \,\tilde{}\, N.I.D\left(0, \sum(s_t)\right)$$

The $\mu(s_t)$, $A_p(s_t)$, $\sum(s_t)$ are parameter shifting functions. Conditional on unobservable regime variable s_t, they are constant.

or

$$y_t = v(s_t) + A_1(s_t)\, y_{t-1} + ... + A_p(s_t)\, y_{t-p} + u_t$$

if the mean smoothly approaches a new level after the regime change (MSI-VAR). The $v(s_t)$ is again a parameter shifting function describing the regime-dependent intercept term.

- an unobservable discrete state Markov stochastic process defined by the transition probabilities

$$p_{ij} = \Pr(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i, j \in \{1, ..., M\}.$$

where s_t is the regime switching variable, M is the number of regimes and p_{ij} refers to the probability of transition from state i to state i.

The parameters of the underlying data generating process of the observed time series vectors y_t , $\mu(s_t), A_1(s_t) ... A_p(s_t)$, $\sum(s_t)$ may depend on the unobservable regime variable s_t, which represents the probability of be a different state of the world and is assumed that s_t is irreducible and ergodic.

Krolzig et al (1999) extend the MS-VAR to MS-VECM to accommodate non-stationary variables and cointegration between variables. They show if the reverse characteristic polynomial of the MSI-VAR $|A(\lambda)| = |I_k - A_1 - ..A_p\lambda^p|$ has one or more roots for $\lambda = 1$, $A(1) = 0$, and all other roots are outside the complex unit circle, the y_t variables are integrated and possibly cointegrated. If $y_t\tilde{}I(1)$ and there is a vector β such that $z_{t-p} = \beta' y_{t-p}$ is stationary, then y_t admits error correction representation. Subtracting y_{t-1} both sides of MSI-VAR and rearranging terms, we have

$$\triangle y_t = v(s_t) + \alpha\beta' y_{t-1} + \sum_{k=1}^{p-1} D_i \triangle y_{t-k} + u_t \tag{3.1}$$

where $D_i = -\left(I_k - \sum_{j=1}^{p-1} A_j\right)$ and the matrix $\alpha\beta' = I_k - \sum_{j=1}^{p} A_j = A(1)$ is singular.

The MS-VECM is closely related to the notion of multiple equilibria which can be characterized by an attractor of the system defined by the drift $\mu(s_t)$ and the long run equilibrium $\delta(s_t)$:

$$\triangle y_t - \mu(s_t) = \alpha\left[\beta' y_{t-1} - \delta(s_t)\right] + \sum_{j=1}^{p-1} D_i\left[\triangle y_{t-j} - \mu(s_t)\right] + u_t \tag{3.2}$$

where $v(s_t) = (I - D_1)\delta(s_t) - \alpha\mu(s_t)$ if $j = 1$.

MS-VECMs exhibit equilibrium as well as error correction mechanisms: in each regime disequilibrium are adjusted by the vector equilibrium correction mechanism. Since the regimes themselves are generated by the stationary irreducible Markov chain, the errors arising from regime shifts themselves are corrected towards the stationary distribution of the regimes.

MS-VECM offers us a useful tool to explore the bivariate process of credit spreads and interest rates. It has been documented that interest rates and credit spreads contain information of business cycles and they themselves experience regime changes. In addition, both credit spreads and interest rates exhibit non-stationary features and possible cointegrating relation. MS-VECM allows for regime switching as well as long run cointegrating relation.

3.2.2 Estimation:Estimation of MS-VAR models are based on the Expectation Maximization (EM) algorithm introduced by Dempster et al (1997).The EM algorithm an iterative ML estimation technique is designed for a general class of models in which the observed time series depends on some unobservable stochastic variables. Each iteration of the EM consists of two steps. The first step is to infer and filter the regime probabilities at each point of time. Agents are assumed to update their probability assessment using information entailed in each subsequent observation. This delivers an estimate of the smoothed probabilities of the unobserved state s_t. As a by-product of the filter-inferences, a likelihood function is derived. The next step is to maximize the likelihood function in order to obtain parameter estimates in the model. In the likelihood function, the conditional probabilities are replaced by the smoothed probabilities obtained from the first step. Equipped with the new parameter estimates, the filtered and smoothed probabilities are updated in the next filtering and inference stage and so on. The attractiveness of the Markov switching model is that the regimes are endogenously determined and the conditional regime probabilities are tracked down at each point of time. There is no a priori knowledge about the dates of the regime shifts.

When some or all variables in a VAR are non-stationary, standard asymptotic theory may not be applicable to the purpose of conducting statistical inference. In order to analyze the short run dynamics of the MS-VAR model while allowing for multiple equilibriums or Markov shifts in the equilibrium mean and/or the drift of the system at the same time, Krolzig (1997) proposes a two stage procedure for the Markov switching VAR equilibrium-correction model (MS-VECM). Since cointegrated systems with Markovian regime shifts can be characterized as a non-Gaussian cointegrated VAR of infinite order, Krolzig (1996) suggests a limited information approach to cointegration analysis using a pure finite-order VAR approximation of the underlying GDP without modelling the Markov switching at the first stage. Then conditional on the estimated cointegration matrix, we have

$$\triangle y_t = v\left(s_t\right) + \alpha\beta' y_{t-1} + \sum_{k=1}^{p-1} D_i \triangle y_{t-k} + u_t$$

the remaining parameters are estimated using the EM algorithm for ML estimation.

3.2.3 Specification testingThere are two kinds of tests in the framework of MS-VAR. Hamilton (1988, 1989, 1996), Hamilton and Perez-Quiros (1996), and Engel and Hamilton (1990) propose tests for autocorrelation, omitted ARCH, misspecification of the Markovian dynamics and omitted explanatory variables. These tests come as a by-product of the general estimation of the smoothed probabilities. The problem with these tests are that their small sample properties are very poor.

Another approach is to use graphical evaluation. The graphs include:

- Correlograms and distributions of the standardized residuals and prediction errors of the MS-VAR models estimated.

- Spectral densities: the smoothed functions of the model autocorrelations. Peaks indicate cyclical or seasonal behavior in the series.

- Density and QQ plots can be used to test the normality of the standardized residues and prediction errors of the models. Standard normal distribution can be used for comparison.

- Plots of the smoothed and prediction errors in the MS-VAR model.

3.2.4 Model Specification:Not all the parameters are necessarily dependent on the state of the world for a system. For the purpose to accommodate the short run "flight to quality" phenomenon, the tractability of the model and relative stable long run relationship, I specify the model as following:

Two states: $s_t = \{1, 2\}$. $s_t = 1$ is the boom state; $s_t = 2$ is the recession or crisis state. For credit spreads and interest rates, there is a shift in the drift across regimes. For example, the drift term of spreads $\nu_{spreads}(s_t = 1) > 0$ in the first regime (crisis) and negative (expansion) in the second regime. The same applies to interest level. The negative short run relation reported by the people above may actually because there is one opposite shift in the drift of interest rates and credit spreads caused by regime change, not because they really have a negative relation.

We can also accommodate state dependent equilibriums by: $z_{t-1} = \beta y_{t-1} + \delta(s_t)$. In different equilibriums economic variables may have different levels. This is the case of the so-called multiple equilibriums described by the sun spot theory in the analysis of sovereign credit risk. The same story applies to corporate bond market as well. In this framework we fix the long run correlation between variables to be the same across regime: β is a constant. It would be desirable if we allow β to change as regime switches but for practical reason, we proceed in this way, assuming the long run correlation is steady and changes in the long run is captured by the intercept.

We specify the innovations variance-covariance to be state-dependent:

$$u_t|\ s_t = NID\left(0, \sum(s_t)\right)$$

In this case, we allow contemporaneous correlations across variables to change across states: in boom, contemporaneous correlation might be high or positive while in crisis the correlation might be low or negative. If variance or/and correlations are state dependent, the impulse responses will depend on the history of the economics variables and the state where the shock occurs. The whole short run relationship will be different from that in the linear VAR-ECM analysis.

3.3 Data Description:

In order to analyze long run as well as short run relationship simultaneously, we need to use long time series of observations. There are two candidates to proxy credit spreads. One is difference between US swap rate data the constant maturity treasury rate data, and the other one is the difference between the corporate yield data and the constant maturity treasury rate data. Though swap spreads are widely used as a proxy for credit spreads by researchers such as Longstaff et al. (2000) and Yu (2000), we notice that the results by Duffie and Singleton (1997) and Grinblatt (1995) that a large part of the variations of swap spreads are attributed either to the market specialty or to the liquidity risk. The relation between swap spreads and interest rates may actually is not that of credit risk and interest rates. In this paper we choose to use Moody's seasoned corporate yield data and 10 year constant maturity Treasury rates on Federal Reserves H15 release. These data are month averages of daily rates. Credit spreads are constructed as the difference between corporate yields and the 10 year maturity treasury rates.

The Moody's indices are constructed from an equally weighted sample of yields on 75 to 100 bonds issued by large non-financial corporations. To be included in the indices, each bond issue must have a face value exceeding $100 million, a liquid secondary market and an initial maturity of greater that 20 years. The constant maturity Treasury rates are based on the yields of on-the -run Treasury bonds of various maturities and reflects the Federal Reserve's estimate of what the par or coupon rate would be for these maturities. The CMT rates are widely used in financial markets as indicators of Treasury rates for the most actively traded bond maturities.

Two potential biases might arise for the analysis of the relation between interest rates and credit spreads. One is AAA and BBB indices contain some callable bonds which induce a negative relation between spreads and non-callable Treasury rates. The second is The maturities of corporate indices are higher than those of Treasury rates. Morris et al (1999) discuss the problems and conclude that the overall bias is small.

Morris et al (1999) analyze this dataset sampled from 1960–1 to 1997-12. As their sample does not include the recent financial crises such as the Russian default 1998, the near collapse of LTCM in 1998 and the Brazilian crisis during which capitals flight to quality, inducing a strong negative relation between credit spreads and interest rates. The long run positive relation found by them may suffer from a survival biases. Hence we choose to use a bit longer sample starting from 1960-1 to 2000-8 which includes the above mentioned crises. Table 1 contains the summary statistics of the level of interest rates and credit spreads, and changes in them. During 1960-1 to 2000-8, the average of the 10 year Treasury rates is 7.37 percent, AAA spreads 0.72 percent and BBB spreads 1.72 percent. The mean monthly changes are close to zero. The standard deviation for 10 year Treasury rates is 2.53 percent, AAA spreads 0.406 percent and BBB spreads 0.64 percent. Compared with Morris et al (1999), we have a higher average of AAA spreads (0.729 vs. 0.684) and BBB spreads (1.72 vs. 1.69) and a lower average of 10 year maturity Treasury rates (7.34 vs. 7.46), reflecting the flight to quality effect during the added sample period. This extra negative movement of credit spreads and interest rates would give us an opportunity to check whether the positive relation reported by Morris et al (1999) is spurious or not.

The standard deviation for AAA spreads is higher but is almost the same for BAA spreads. The autocorrelation coefficients for all of the rates in level (0.93,

	AAA	BBB	CMT10	ΔAAA	ΔBBB	ΔCMT10
Mean	0.729	1.72	7.34	0.0036	0.0037	0.0023
Std	0.406	0.64	2.53	0.135	0.181	0.304
Max	1.84	3.82	15.32	0.61	1.04	1.61
Min	-0.17	0.29	3.71	-0.38	-0.53	-1.76
Auto	0.93	0.96	0.99	0.04	0.17	0.32

Table 3.1: Descriptive data of credit yields, credit spreads and treasury rates

0.96, 0.99 for AAA spreads, BAA spreads and CMT 10 year)show that the series are highly persistent.

We proceed using the two-step method proposed by Krolzig et at (1996). First we estimate the long run equilibrium equations. We use the extra data associated with recent crises to test the positivity of the long run relationship. We then take the long run equations as exogenous and estimate the system in the Markov framework.

3.4 Cointegration analysis:

3.4.1 Unit root test:We start with unit root test of individual time series using the Augmented Dickey-Fuller unit root test and Phillips-Perron unit root test. The null hypothesis is H_0: there is a unit root. We also check for possible $I\,(2)$ testing H_0 using first differences of the data. Table 3 presents the results. We cannot reject H_0 for the level of AAA spreads, BAA spreads and 10 year constant maturity treasury rates but we reject the null for their first differences. Thus the levels of AAA spreads, BAA spreads and 10 year maturity Treasury rate appear to be non-stationary I(1).

It is controversial that credit spreads and interest rates are non-stationary, as it implies an explosive structure over time. It is plausible however over an

	AAA	ΔAAA	BAA	ΔBAA	CMT10	ΔCMT10
ADF	-0.82**	-18.64	-0.64**	-9.23	-0.39**	-8.55
PP	-1.13**	-21.08	-0.74**	-18.23	-0.40**	-15.34
LAG	2	2	3	4	3	4

Table 3.2: Unit root test for credit spreads and Treasury rates

H_0	Eigenvalue	Max-Eigen Statistics		Trace Statistics	
No. of CE(s)		Statistic	5% Critical value	Statistic	Critical value
None	0.036	17.54*	14.07	20.34**	15.41
At most 1	0.006	2.81	2.81	2.80	3.76

Table 3.3: Cointegration test for AAA and Treasuey Rates

investment horizon. Recent studies by Duffee (1999) and Helwege (1996) show the term structure of corporate spreads is upward sloping. Duffee (1999) shows that because of risk adjustment from physical probability measure to risk neutral measure, credit spreads are explosive and the term structure is upward sloping. We therefore attribute the non-stationarity of Treasury rates to the risk premium and the investment horizon as well.

3.4.2 Cointegration Test:We are left with 3 non-stationary time series. We apply Johansen maximum likelihood procedure to test for the presence of a cointegration vector in the two bivariate processes: AAA spreads and 10 year interest rates, and BAA spreads and 10 year interest rate . The null hypothesis is that there is no cointegration vector (rank=0), the alternative is there is a single cointegration vector (rank=1). Table 4 and table 5 provide the results for the two system respectively.

For AAA spreads and 10 year Treasury rates, the Maximum eigenvalue statistics is significant at 5% level and the trace statistics is significant at 1% level,

H_0	Eigenvalue	Max-Eigen Statistics		Trace Statistics	
No. of CE(s)		Statistic	5% Critical value	Statistic	Critical value
None	0.038	18.76**	14.07	21.61**	14.41
At most 1	0.059	2.84	3.76	2.84	3.76

Table 3.4: Cointegration test for BAA and Treasury Rates

we reject the null that there is no integrating vector in favor of the alternative that there is one cointegrating vector. and BAA spreads, we reject the hypothesis that there is no cointegrating vector at the 1% level. The second eigenvalue statistics is not significant and we can not reject the null hypothesis that there is one cointegrating vector. For BAA spreads and the 10 year Treasury rates, both the Maximum eigenvalue statistics and the trace statistics are significant at 1% level and we reject the null that there is no cointegrating vector. We can not reject the null for the second cointegrating vector. Given the existence of cointegration between the AAA and the 10 year treasury rates, and between the BAA and the treasury rates, table 4 reports the corresponding cointegrating vectors. The long run relations can be written as:

$$Credit\ Spreads\ (AAA) = 0.0145\ [0.0362]\, Level\ of\ Treasury + 0.623 \tag{3.3}$$

$$Credit\ Spreads\ (BAA) = 0.161\, [0.042]\ \ Level\ of\ Treasury + 0.533 \tag{3.4}$$

In the long run, there is a positive relation between credit spreads and interest rate level for both AAA and BAA index. The positive relation is more pronounced for BAA (significant at 10% level) than AAA (not significant). A 1% increase in

Treasury rates will eventually increase AAA credit spreads for 0.0145% and BAA for 0.161%.

We also redo the cointegration estimation employing the data before the recent crises (from 1960,1 to 1997,12, same as Morris et al (2000)): the results are:

$$Credit\ Spreads\ (AAA) = 0.0278\ \ [0.0263]\, Level\ of\ Treasury + 4784$$

$$Credit\ Spreads\ (BAA) = 0.1775\ \ [0.037]\, Level\ of\ Treasury + 0.533$$

The negative relation caused by "flight to quality" during the Russian default, the LTCM crisis and the Brazilian crisis appears to be transitory and does not influence the long run relation very much: the coefficient for AAA increases to 0.0278 (still not significant), for BAA 0.1775 (significant at 10% level).

3.5 The MSIH-VECM

In the last section, we use a finite pure VAR of the VARMA representation of an MS-VAR process to get the cointegration vector. We use this cointegrating vector in the second stage of the estimation. The model we choose is MSIH(2)-VECM(2): two regimes, two lags of endogenous variables with possible shifts in the intercept and the variance-covariance matrix Σ. The lag length 2 is chosen according to AIC, HQ, SC model selection procedure. The unrestricted variance-covariance matrix Σ, especially the contemporaneous correlation, allows the data to detect different patterns of the short run dynamics in different regimes, and different patterns from those of linear VAR-VECM model.

$$\Delta spreads_t = v_1(s_t) + \sum_{p=1}^{2} A_{1p}\Delta spreads_{t-p} + \sum_{p=1}^{2} B_{1p}\Delta R_{t-p} + \alpha_1 z_{t-1} + u_{1t} \quad (3.5)$$

$$\Delta R_t = v_2(s_t) + \sum_{p=1}^{2} A_{2p}\Delta spreads_{t-p} + \sum_{p=1}^{2} B_{2p}\Delta R_{t-p} + \alpha_2 z_{t-1} + u_{2t} \quad (3.6)$$

where for AAA,

$$z_t = spreads_t - 0.0145R_t - 0.623$$

for BBB

$$z_t = spreads_t - 0.161R_t - 0.533$$

In this way we normalized $E_t[z_t] = 0$.

$$u_{it}|s_t \tilde{} NID(0, \Sigma_i(s_t))$$

$i = 1, 2$. The intercept $v(s_t)$ and the variance-covariance matrix of the innovations are state dependent. The autoregression coefficient A_{1p}, A_{2p} are kept as constant across regimes.

3.5.1 Regime existence test

Linearity Test Before we present the results from MSI-VECM, we need to test whether the non-linearity is necessary in describing the data. We test our MSI-VECM against a linear alternative.

Testing a Markov regime switching model is not straightforward because under the null hypothesis of constant coefficient model without regime switches, the probability associated with the additional regime are not identified. The presence of the nuisance parameters gives the likelihood surface sufficient freedom so that the scores associated with parameters of interest under the alternative may be identically zero under the null. Davis (1977, 1987) derived an upper bound for the significance level of the likelihood ratio test statistics under nuisance parameters. Some other procedures used for the derivation of the formal asymptotic distribution have been proposed by Hansen and Garcia and Perron, but they require the time consuming simulation of the data contained in a grid of values for the nuisance parameters. Ang and Behaert (1998) indicate that critical values of the $\chi^2(r+n)$ distribution can be used approximately where r is the number of restricted parameters and n is the number of nuisance parameters. In this paper we use the upper bound of Davis (1977, 1987), Ang and Behaert (1998) and AIC, Schwartz and HQ (Hannan and Quinn (1979)). They are the by-product of Krolzig' MS-VAR and reported in table 5 and table 6.

For both bivariate processes of AAA and 10 year Treasury rate, and BAA and 0 year Treasury rate, Ang and Behaert (1998) tests are significant at 1% level, LR-tests also reject the linearity significantly by invoking the upper bound of Davis (1977, 1987). Furthermore the AIC (with -2.24 vs. -1.6 for AAA and CMT10, -1.91 vs. -1.36 for BAA and CMT10), SC (with -2.05 vs. -1.47 for AAA and CMT10, -1.72 vs. -1.23 for BAA and CMT10), HQ (with -2.17 vs. -1.54 for AAA and CMT10, -1.84 vs. -1.31 for BAA and CMT10) are all in favor of the non-linear VECM.

Test of constant intercept and Homoscedasticity Our Markov switching regime model is characterized with either a shift in the intercepts, or a shift in the variance-covariance matrix Σ, or both. We've rejected the linearity hypothesis, ie, we reject the hypothesis that intercepts and variance-covariance matrix are both constant. In this section we try to find the regime switching characteristics by undertaking the following two tests:

Test 1: Null hypothesis: the variance-covariance matrix Σ is constant.

Alternative: there is a shift in the variance-covariance matrix Σ.

Test 2: Null hypothesis: the intercepts are constant.

Alternative: there are shifts in the intercepts.

Unlike the linearity test, under the null of constant intercepts, the unrestricted regime-dependent variance-covariance matrix Σ_i ensure the statistical identification of the model under the null. The same applies to the second test where the unrestricted intercepts ensure the statistical identification of the model under the null. The tests are nuisance parameter free and have standard distributions. We use likelihood ratio test which has a chi-square distribution with the number of degrees of freedom corresponding to the number of imposed restrictions. Table 6 shows the results of the likelihood ratio tests along with their AIC, Schwartz and HQ values. Test 1 is MSI-VECM (constant variance-covariance matrix Σ)against the unrestricted MSIH-VECM. The likelihood ratio statistics are $\chi^2\,(3)$ and they are significant at level of 1% for both AAA-CMT10 and BAA-CMT10. The values of AIC, HQ and SC are also smaller than their counterparts. Hence we reject the

Fitting (AAA)	MSIH-VECM	Linear VECM	MSI-VECM	MSH-VECM
logLik	565.65	401.87	401.87	560.38
AIC	-2.24	-1.60	-1.58	-2.23
HQ	-2.17	-1.54	-1.51	-2.16
SC	-2.05	-1.47	-1.41	-2.06
LR linearity test	327.59			
Chi (5)=[0.00]	Chi (7)=[0.00]	DAVIES=[0.00]		
Heteroskedasticity test	327.56^{**}			
Intercept test	10.54^{**}			

Table 3.5: linearity, heteroskedasticity and constant intecept tests for AAA and Treasury rate

Fitting (BAA)	MSIH-VECM	Linear VECM	MSI-VECM	MSH-VECM
logLik	485.72	344.74	344.93	482.5
AIC	-1.91	-1.36	-1.34	-1.90
HQ	-1.84	-1.31	-1.28	-1.84
SC	-1.72	-1.23	-1.18	-1.73
LR linearity test	281.97			
Chi (5) = [0.00]	Chi (7) = [0.00]	DAVIES= [0.00]		
Heteroskedasticity test	281.58^{**}			
Intercept test	6.44^{**}			

Table 3.6: linearity, heteroskedasticity and constant intecept tests for AAA and Treasury rate

constant variance-covariance matrix Σ hypothesis for both bivariate systems. Test 2 is MSH-VECM against the unrestricted MSIH-VECM. The statistics are $\chi^2(1)$ and are significant at 1% level. The values of AIC, HQ and SC are just slightly different from their counterparts. Overall we can reject there are no intercept shifts and no variance-covariance matrix Σ hypothesis.

3.5.2 Specification test:The analysis of the residuals are presented in figure 2 which shows there are no strong autocorrelation left in the errors. The prediction errors $\Delta y_t - E[\Delta y_t | Y_{t-1}]$ are based on the information set $Y_{t-1} = \{y_{t-1}, ...y_0\}$ and are assumed to be non-Gaussian. The smoothed standard errors are corrected for

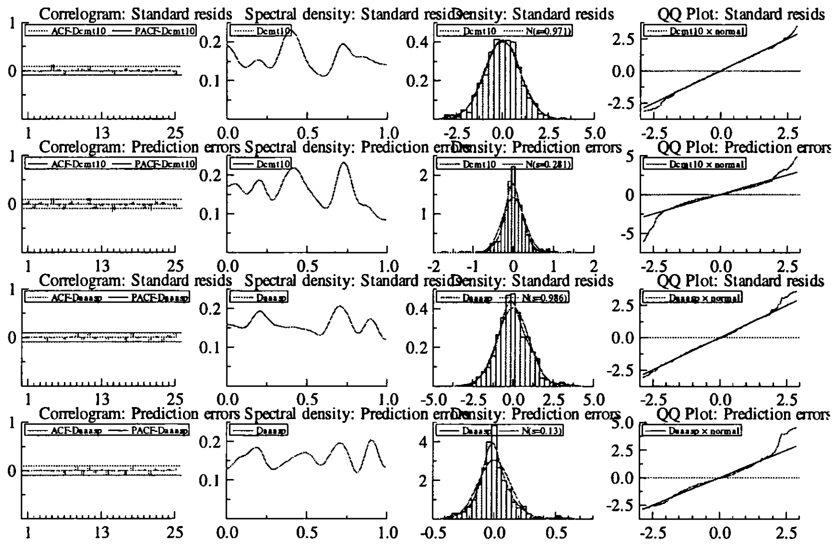

Figure 3.1: Specification test for AAA spread and Treasury rate

the effects of regime shifts

$$\sum_{M=1}^{2} \{\Delta y_t - E\left[\Delta y_t | s_t = m, Y_{t-1}\right]\} \Pr\left(s_t = m | Y_T\right)$$

and provide an inference of the Gaussian innovation process. The QQs show the smoothed standard errors appear to be normal.

3.5.3 Regime Identification:The estimated parameters using data from January of 1960 to August 2000 are presented in table 5 and 6.

For the bivariate system of AAA spreads and 10 year Treasury rate,

the transition matrix is

$$P = \begin{bmatrix} 0.9235 & 0.0765 \\ 0.0707 & 0.9293 \end{bmatrix}$$

where $p_{ij} = \Pr\left(s_t = i | s_{t-1} = j\right)$.

Both regimes are persistent with estimated duration a bit more than 1 year,

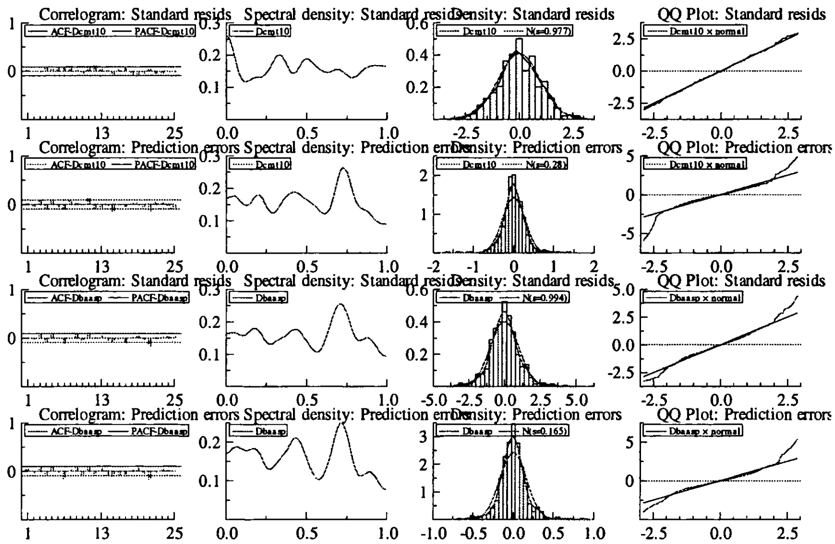

Figure 3.2: Specification test for BBB spread and Treasury rate

similar to that of Krolzig et al (2000). They find the average recession duration implied by the US output and employment is one and a half year. Regime 2 in our case is one year and two month. In regime 2, the intercept for credit spreads is a positive 0.020 and a negative -0.002 for Treasury rate. The standard deviation of credit spreads in regime 2 is 0.3713, higher that 0.1321 in regime 1. Endogenously regime 2 is associated with increasing credit spreads, decreasing interest rates and high volatility, features of recession. The contemporary correlation between interest rates and credit spreads is negative in both regimes but weaker in regime 2. The resulting regime probabilities are presented in figure 3. regime 2 clearly depicts the recent financial crises and the recession afterwards (from August of 1998 to the end of sample August of 2000), the economic trough in 1980-1982, March of 1975, November of 1970. There are some inconsistencies with the real economy recession. The recession in March of 1991 is classified as boom in this model.

AAA	Ergodic Probability	Duration	Observations
Regime 1	0.4803	13.08	237.7
Regime 2	0.5197	14.15	247.3

Table 3.7: Ergodic probability and duration for AAA spread and Treasury rate

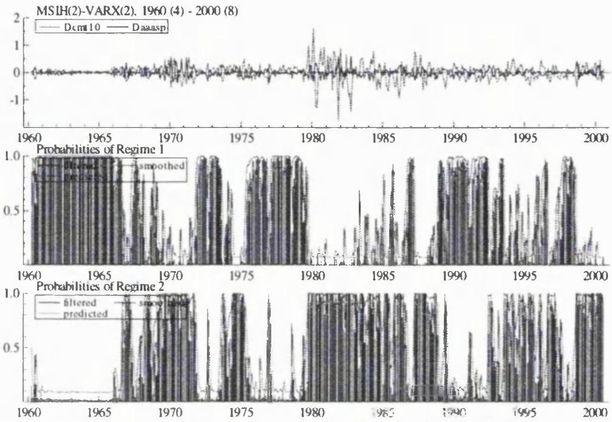

Figure 3.3: Regime Probability (AAA spread and Treasury rate)

This may reflect the difference between bond market from real economy. Overall, the classification of this model has much more economic meaning than those of Lekkos al et (2002) who exogenously specify interest slope as the determinant of the Markov switching process.

For the bivariate system of BAA and 10 year Treasury rates,

the transition matrix is:

$$P = \begin{bmatrix} 0.9604 & 0.0396 \\ 0.0363 & 0.9637 \end{bmatrix}$$

regime 2 again is associated with positive and high growth rate of credit spreads (0.019 vs. -0.015), negative and low growth rate of interest level (-0.0075 vs. 0.0118), and high volatility (0.20 vs. 0.11). The duration of regime 2 is 2.3 years, more persistent than AAA market and the US real economy. The recent crisis is

BAA	Ergodic Probability	Duration	Observations
Regime 1	0.4784	25.28	242.6
Regime 2	0.5216	27.57	242.4

Table 3.8: Ergodic probability and duration for BAA spread and Treasury rate

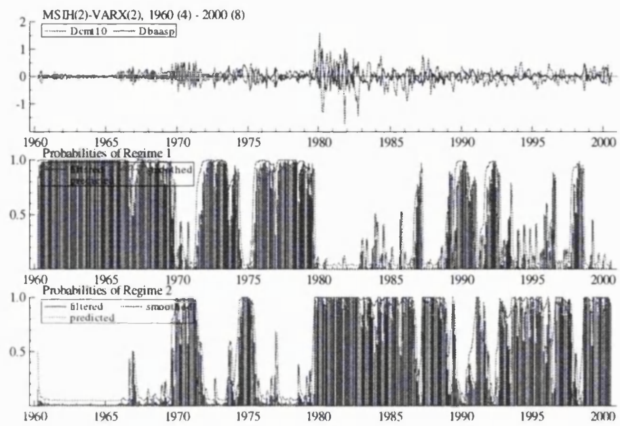

Figure 3.4: regime probability (BAA spread and Treasury rate)

also classified as in regime 2 (August 1998 to the end of sample August 2000). Other periods categorized as regime 2 are the periods from September 1969 to May 1971, April 1974 to May 1975, September 1979 to September 1986, April 1987 to December 1988, April 1989 to June 1989, October 1990 to August 1991, August 1992 to September 1997.Here the 1991 recession is identified as "recession" in BAA bond market. The system of BAA and 10 year Treasury seem to have more information about real economy in terms of regime identification. Table 5 displays the unconditional probabilities and figure 5 the resulting probability. Morris et al (1999) claims that AAA spread and BAA spread contain no more information for each other, here in our nonlinear framework we find the regimes extracted from AAA spread and BAA spread are different, suggesting different factors might drive the movement of AAA spread and BAA spread.

AAA	Δ**R**$_t$	Δ**Spreads**$_t$
regime dependent intercepts		
$v(s_{1t})$	0.006336 (0.01)	-0.01305 (0.0056)
$v(s_{2t})$	-0.002385 (0.0243)	0.01954 (0.0110)
Autoregression coefficients		
$\Delta spread_{t-1}$	-0.0479 (0.1347)	0.1050 (0.0671)
$\Delta spread_{t-2}$	0.2216 (0.0116)	-0.2270 (0.0598)
ΔR_{t-1}	0.3264 (0.0726)	0.0107 (0.0339)
ΔR_{t-2}	-0.1054 (0.0619)	-0.0313 (0.0303)
Adjustment coefficients		
α	-0.021 (0.025)	-0.044 (0.014)
Square root of variances		
$\sigma(s_{1t})$	0.1321	0.07437
$\sigma(s_{2t})$	0.3713	0.1665

Table 3.9: ML estimation results for AAA spread and Treasury. The model is MSI-VECM

BAA	ΔR_t	$\Delta Spreads_t$
regime dependent intercepts		
$v(s_{1t})$	0.0118(0.01)	-0.015 (0.0086)
$v(s_{2t})$	-0.0075 (0.0243)	0.019 (0.0135)
Autoregression coefficients		
$\Delta spread_{t-1}$	-0.0012 (0.1084)	0.2457 (0.0734)
$\Delta spread_{t-2}$	0.1011 (0.0931)	-0.0573 (0.0648)
ΔR_{t-1}	0.3894 (0.0759)	0.0107 (0.0479)
ΔR_{t-2}	-0.1762 (0.0746)	-0.0313 (0.0469)
Adjustment coefficients		
α	-0.001835 (0.02)	0.05190 (0.0145)
Square root of variances		
$\sigma(s_{1t})$	0.1440	0.1180
$\sigma(s_{2t})$	0.3687	0.2001

Table 3.10: ML estimation results for BAA spread and Treasury. The model is MSI-VECM

3.6 Short run correlation:

In order to have a meaningful interpretation of the estimates from the above structure models, some identification restrictions have to be imposed on the structure model. We impose such an order of the variables onto the system: ΔR_t, ΔAAA spreads for the first bivariate system and ΔR_t, ΔBAA spreads for the second bivariate system. Each variable has contemporaneous effects on itself and on variables below it. Such a triangular identification scheme corresponds to a Choleski decomposition of the $\Sigma(s_t)$ and makes $\Sigma(s_t)$ exactly identified. This implies that shocks to the treasury rates ΔR_t affect both the credit spreads and the Treasury rates themselves but shocks to credit spreads only affect themselves. This restriction is consistent with the standard two step estimation method in the reduced model for term structures. There interest rates affect credit spreads but no the reverse.

3.6.1 Contemporaneous correlation

Table 11 and 12 report the contemporaneous correlation estimates in linear VAR-VECM and our MSIH-VECM model. For linear model, the contemporaneous correlation estimates are -0.71 for AAA spreads and Treasury rates, and -0.78 for BAA spreads and Treasury rates. For MSIH-VECM model, AAA spreads and Treasury rates have a contemporaneous correlation of -0.84 in regime 1 and -0.70 in regime 2; BAA spreads and Treasury rates have a contemporaneous correlation of -0.87 in regime 1 and -0.78 in regime 2. In both cases the correlations are highly negative. BAA spreads have a higher magnitude of correlation with Treasury rates than that of AAA spreads. There

Contemporaneous correlation	AAA	BAA
Regime 1	-0.8395	-0.8734
Regime 2	-0.6992	-0.7747

Table 3.11: Contemporaneous correlation in MSI-VECM

Contemporaneous correlation	AAA	BAA
	-0.7147	-0.7821

Table 3.12: Contemporaneous correlation in linear VECM

are however not much difference across regimes.

3.6.2 Impulse response analysis

Credit spreads and Treasury rates are correlated not only through contemporaneous relation, but through lagged values as well. However the signs of the estimates for the coefficients of lagged values are mixed and the interpretation of these parameters is not straightforward. For this reason, the impulse response function is usually invoked to visualize the magnitude and the persistence of each variable's response to a shock to some specific variables. In the linear VAR model the impulse response function (IRF)s are calculated for one standard deviation impulse to the orthogonalized variables. For MSIH-VECM, the covariance matrix $\Sigma(s_t)$ is regime dependent, so we get different IRFs describing the response of the variables depended on the state of the system when the shock occurs. Furthermore, the system is also subjected to regime transition shocks.

- The response of shocks arising form the Gaussian innovations to each of the variables (corresponds to the IRF in linear Gaussian VARs).

$$E\left[y_{t+h}|u_t = \delta, \omega_{t-1}\right] - E\left[y_{t+h}|u_t = 0, \omega_{t-1}\right]$$

It is associated with the state-dependent variance-covariance matrix $\Sigma(s_t)$. To calculate this kind of IRFs, it is assumed that the given regime prevails throughout the duration of the response. For all the IRF, we choose the a time horizon of 35 months. Figure 5 and 6 show the Gaussian innovation IRFs for AAA spreads and BBB spreads in linear VAR systems, Figure 7 and 8 in MSIH-VECM. In both linear VAR and MSIH-VECM, the initial response of credit spreads to one standard deviation impulse to 10 year Treasury rates is negative. For the linear model, for one standard deviation increase of Treasury rates, AAA spread first decreases by 0.095 points then gradually the declines become less and reach its original level. For MSIH-VECM, AAA spreads first decrease by 0.05 points in regime 1 (growth), by 0.12 points in regime 2 (recession), then in both regimes quickly the spreads return to their original levels and continue to rise. It reaches 0.075 points above its original level in regime 1and 0.25 points in regime 2. The swing in regime 2 is larger than in regime 1 but the pattern are alike. For BAA spread, the linear model shows an increase in Treasury rate first decreases BAA spread by 0.14 points and then the spread increases until above its original level by 0.04 points. In MSIH-VECM, the patterns are similar as for AAA spread but with bigger magnitudes. For both AAA and BAA spreads, the patterns are similar across regimes but in recession regime, the down and high are larger.

- The response when there is a change in regime as from recession to normal.

$$E\left[y_{t+h}|s_t = j, \omega_{t-1}\right] - E\left[y_{t+h}|s_t = i, \omega_{t-1}\right]$$

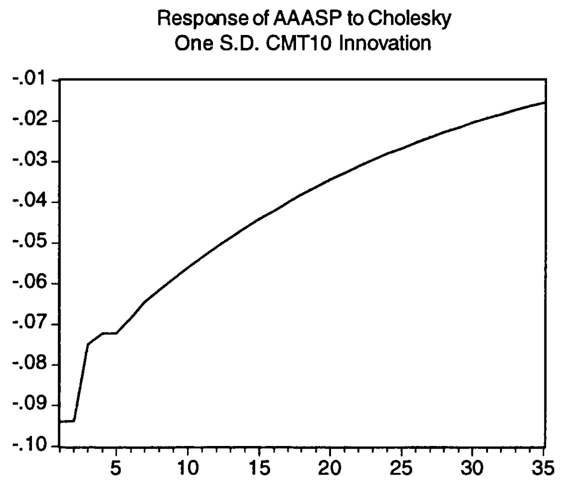

Figure 3.5: Reponse of AAA spread to shock of Treasury rate. The model is linear VECM

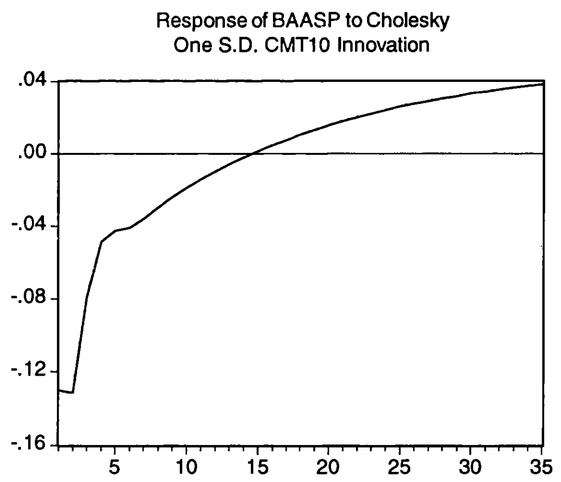

Figure 3.6: Reponse of BAA spread to shock of Treasury rate. The model is linear VECM

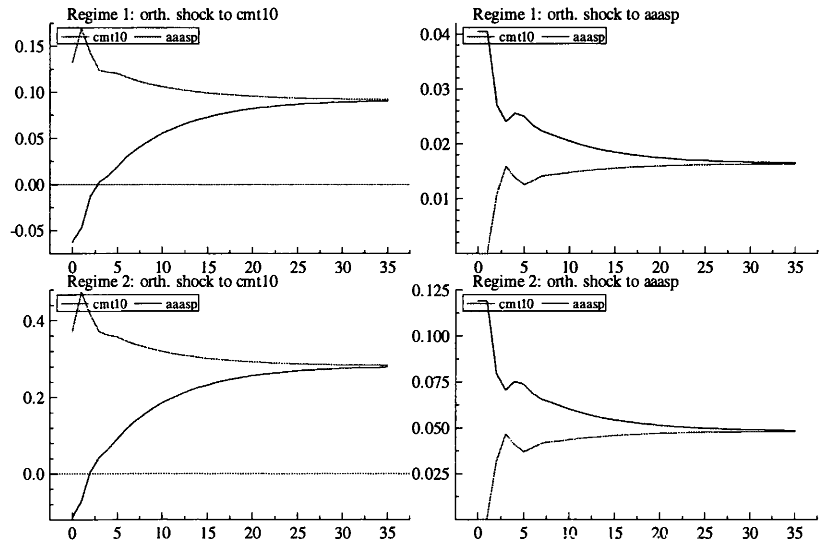

Figure 3.7: Impulse function of AAA spread and Treasury rate in MSI-VECM

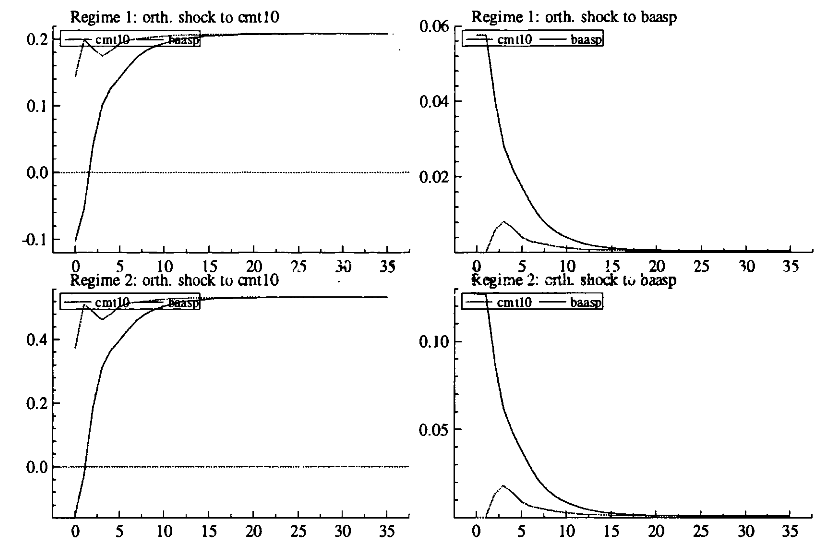

Figure 3.8: Impulse function of BAA spread and Treasury rate in MSI-VECM

It is determined by the changes of the current state and autoregressive transmission of the state-dependent intercepts.

- The dynamics when there is a move in the information structure from ergodic distribution to certainty regards the state.

$$E\left[y_{t+h}|s_t = j, \omega_{t-1}\right] - E\left[y_{t+h}|\omega_{t-1}\right]$$

The behaviors of credit spread and Treasury rate when we move from ergodic distribution to a sure state measure the responses to the changes in the phase of market cycle. Figure 9 on the diagonal show that there is a sudden decrease in AAA spread and an increase in Treasury rate when the market booms.

Previous studies on regime switching model only focus on the reponse of the system to Gaussian innovations within regimes, ignoring the effect of regime transition on the system. In our case, the dynamics of the relation between interest rates and credit spreads and magnitude of short run negative relation caused by the Gaussian innovations are the almost the same in both normal time and crisis time., implying the stability of the relation across regimes.

3.7 Conclusion

In this paper we study the relation using Markov switching error correction model developed by Krolzif (1997, 1998). This model allows multiple equilibria by allowing for regime shifts in intercepts and variance-covariance matrix, while taking into account the long run restriction of non-stationary processes. This methodology is suitable for the analysis of the non-stationary credit spread and

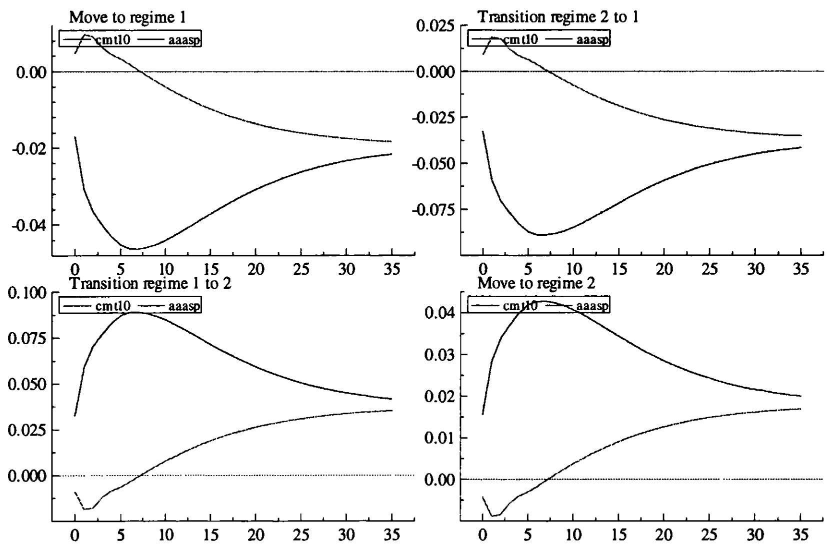

Figure 3.9: Response functions to regime transition shock for AAA spread and Treasury rate

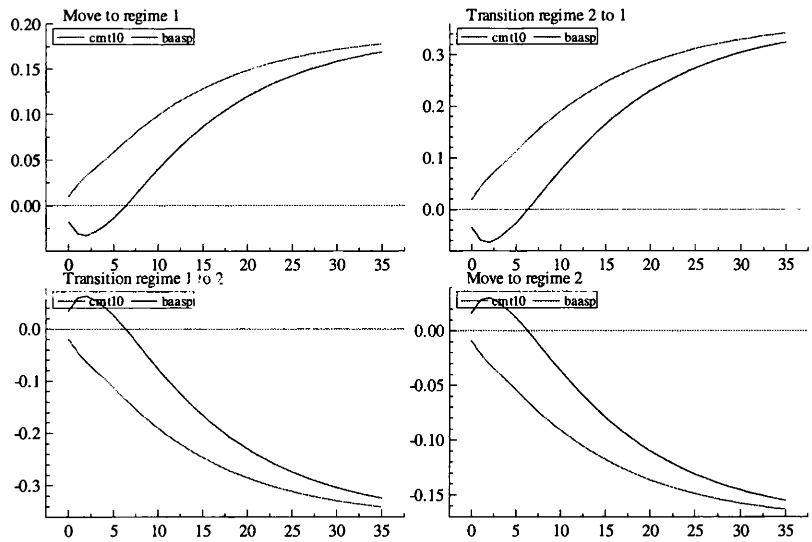

Figure 3.10: Response functions to regime transition shock for BAA spread and Treasury rate

Treasury rate processes and is in line with the findings by Ang and Berkaert (1998) who demonstrate that there is overwhelming evidence for multiple regimes in the data generating process of short interest rates and the regimes correspond well with business cycle expansions and contractions, and Gertler and Lown (2000) who show that high yield credit spread has had significant explanatory power for the business cycle and interpret their finding as possibly asymptomatic of financial factors at work in the business cycle, along the lines suggested by the financial accelerator. The model endogenously selects the dates of the regime shifts and distinguishes between two different regimes. One regime is associated with increasing credit spreads, decreasing interest rates and high volatility, the other one is characterized by decreasing credit spreads, increasing interest rates and low volatility. Our result shows that apart from the conventional linear correlation, credit spreads and Treasury rates are correlated through common regime switching variables. This result has significance for risk management and default related bond pricing in which the correlation usually is regard as a constant. The allocation of scarce capital should also take into consideration the economic situation. We also find that in contrast to the linear analysis of Morris et al (1999) claiming that AAA spread and BAA spread contain no more information for each other, the regimes extracted from AAA spread and BAA spread are different, suggesting different factors might drive the movement of AAA spread and BAA spread.

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CHAPTER 4

Sovereign Term Structure, Co-movement and Market Sentiment

In this paper we study the co-movement and variance composition of sovereign term structure of the three largest Latin American countries via a term structure model. The term structure approach of Driessen (2002) is not applicable, due to the relatively small number of issuers on sovereign bond markets. We therefore decompose the term structure into common and entity specific components and undertake a joint estimation in the framework of two-step approach. We specify the latent variables as Gaussian processes with different mean reversion rate and long run mean so that we can measure the variance composition of term structure and evaluate the impact of risk adjustment on the variance composition of term structures. We show that most of the variations of these sovereign spreads can be attributed to the latent common factor. However, this common factor can only account for a small part of the total variations under physical probability measure. Our results support the conjecture that market sentiment is the main reason for the co-movement of credit spreads. We also show that the term structure of yield spreads is upward sloping, and international interest rate is negatively related to the level and slope of yield spreads. The common factor parallel shifts the level of the term structures for all countries but leaves the slope of the term structure unchanged, resulting in a very stable slope across calm and crisis time.

4.1 Introduction

Credit spreads are the difference between defaultable bond yields and the otherwise equivalent risk free Treasury bond yields. There are mainly three reasons for the existence of the spreads. a) Default risk. Investors bear the risk of losing their investment due to the possible default event and the uncertainty of recovery value upon default, therefore requiring compensations. b) Liquidity risk and tax. Defaultable bond markets are usually illiquid, low volumes and involve high transaction costs. Liquidity premiums should be offered to investors to attract them to hold defaultable assets. c) Risk premiums. Investors are risk aversion and require risk premiums associated with the default and liquidity risks.

Most of the defaultable bond pricing models focus on default risk only. In the line of structural approach, Merton (1974) and Longstaff and Schwartz (1995) specify that credit spreads are determined by default event which happens when the values of the firms' total assets fall below a pre-specified lower boundary. The empirical performance of these models is not so satisfying (Jones, Mason, and Rosenfeld (1984), Anderson and Sundaresan (1996)). Credit spreads generated by these models are too low to match the data. There is not such a problem with the reduced approach proposed by Duffie and Singleton (1998). However as Pages (2001) shows, if the relation between interest rate and credit spread is negative, then the implied default probability can become negative. Liquidity premium should be include in the determinants of credit spreads.

Recent studies suggest that default risk and liquidity risk might not be large enough in explaining the change of credit spreads. Collin-Dufresne, Goldstein and Martin (2001), Bakshi, Madan and Zhang (2001) and Elton Gruber, Agrawal and Mann (2001) conduct regression analysis of corporate bond markets and show that default related fundamentals, tax, various liquidity proxies such as the risk free interest rate, the leverage of the firm, the volatility of the return of the firm's total assets, trading volumes, and bid-ask spread can only explain a small part of the variations of the change of credit spreads. Eichengreen and Mody (1998) and Westphalen (2002) carried out similar studies on the sovereign bond market and show similar results. Eichengreen and Mody (1998) suggest that it is investors' sentiment, not fundamentals that determine variations of sovereign credit spreads. In Elton Gruber, Agrawal and Mann (2001), they show a large part of the residues of their regression can be attributed to Fama and French factors, supporting the conjecture of Eichengreen and Mody (1998). Collin-Dufresne, Goldstein and Martin (2001) conduct a principal component analysis of their regression residues and find a latent common factor can explain most of the residues. However, the latent common factor cannot be attributed to any set of variables.

The recent studies raise the issues of both the source of credit spreads and the co-movement of credit spreads. Knowledge of the former helps us improve our portfolio risk management and of the latter has further significant implication for the pricing of credit derivatives such as CDOs and Credit basket derivatives as their payoffs depend on the correlation of default probability under risk neutral measure.

In this paper, we focus on the co-movement of sovereign credit spreads, the source of the co-movement and the impact of co-movement on credit spreads, in the line of term structure model.

There are several papers on the co-movement of corporate credit spreads in this framework.

Based on the reduced model of Duffee and Singleton (1997), Duffee (1999) specifies factors in his model as CIR processes and estimate the default processes for 161 firms with default intensity depending on a default-free interest rate and a firm-specific component. However, as Yu (2002) points out since in Duffee (1999) the only common factor in the intensity function is the interest rate, the model may not adequately capture the sources of common variation in yield spreads. Driessen (2002) extends Duffee (1999) by including common factors in the intensity function. He finds that the inclusion of common factors improves the precision of the model and after accounting for common factors, the firm specific factors become idiosyncratic and do not carry risk premiums.

Studies of the term structure of sovereign bonds have up to now focused mainly on the debts of a single country. Duffie et al. (2001) develop a comprehensive model to price dollar-denominated Russian bonds. Kesiwani (1999) and Pages(2001) apply Duffee's (1999) model to Brady bonds of some Latin American countries.

In this paper, we proceed in the framework of reduced approach to study the dynamics and co-movement of the term structures of a group of normal sovereign bonds of the three largest Latin American countries: Argentina, Brazil and Mexico. We cannot directly apply Driessen (2002), since for a small portfolio common factors cannot be taken as exogenous and are estimated separately from entity specific factors. We therefore decompose the term structure into an interest rate factor, a common factor and a country specific factor and perform a joint estimation using two-step estimation. As our focus is to measure the importance of different components of term structure in terms of variance contribution we specify factors as Gaussian processes rather than the widely used CIR processes. In order to accommodate stochastic risk premium which is supposed to be important in model specification for the Russian default period we allow the long run mean and mean reversion rate to change under physical and risk neutral measure. Longstaff, Liu and Mendell (2000) and Duffee (2000) show that this specification is able to accommodate more general forms of risk premium. Under this specification variance contributions of different components will be different under the two probability measures, allowing us to evaluate the impact of risk premium on the variance composition.

Our results show that most of the variations of the term structures can be attributed to the latent common factor. This is consistent with the regression analysis of Westphalen (2002) and Collin-Dufresne, Goldstein and Martin (2001). We also find that under physical probability measure the variance contribution of the common factor falls sharply and becomes quite insignificant, suggesting that the source of variations and co-movement of term structures is associated with risk premiums. Our result supports the conjecture of Eichengreen and Mody (1998) that investors' sentiment are responsible for the co-movement of sovereign spreads and is also consistent with the regression result of Elton Gruber, Agrawal and Mann (2001) that systematic risk premiums can explain a large part of the change of credit spreads. We also analyze the role of the common factor in determining the sovereign term structure and the shape of the term structure.

We organize the paper as follows. In section 2, we briefly review the literature on defaultable bond pricing. We set up the model in section 3. In section 4, we use the Kalman filter to estimate the risk free dollar interest rate process and extract interest rate factor, using US zero curve data from Bloomberg. In section 5, we undertake a joint estimation conditional on the extracted interest process, and analyzes the results and their implications. We conclude in section 6.

4.2 Literature Reviews

The modeling of term structures of defaultable bonds can be categorized into two groups:

- Structural models. In structure models, default happens when the entity's asset value fall below a specified default boundary. The defaultable bond is regarded as an option on the underlying asset's values. The models are pioneered by Black and Scholes (1973) and elaborated by Merton (1974), Longstaff and Schwartz (1995) and many others. Correlation between entities are introduced through equity correlation when they are extended to multi-issuer setting. This line of research is thus intuitively attractive and economically meaningful but suffers from weakness such as predictability of default probability at short end and the relative difficulty of implementing. Such models also miss out pricing factors such as liquidity and taxies.

- Reduced form models. In reduced form models the motivation of default is not specified and default process is simply described by an intensity function. The main examples are Jarrow and Turnbull (1995), Duffie and Singleton (1998), Lando (1998) and Madan and Unal (1998). Duffie and Singleton (1998) show that in this framework the pricing of a defaultable bond is reduced to that of an otherwise identical default free bond with a suitably adjusted risk free rate equal to the actual risk free rate plus the instantaneous probability of default. All of the familiar models and solutions of risk free term structure are applicable. In addition to their simplicity, reduced models treat default process as a surprise and sufficient short maturity spreads can be generated. Non-default pricing factors such as liquidity and taxes can be subsumed in the intensity function. Cross-section correlation can be introduced through interdependence of intensity functions.

Most of the term structure studies have been undertaken on corporate bond data. Though sovereign bond are also defaultable bonds, cares need to be taken when it comes to the analysis of sovereign spreads. They differ from corporate bonds in many respects including

- Enforceability. A distinctive characteristic of sovereign bond markets is the lack of legal mechanisms to enforce the contract. In the event of default, the bondholders of corporate bonds have the right to initiate proceedings in a bankruptcy court to seize the assets of the bond issuer. The nature of the jurisdiction however makes sovereign bond a legally unenforceable issue. A government default is largely a trade-off between reputation loss (the costs of future access to global credit markets), sanction costs (political, economical and seizure of exports and foreign assets) and the costs of payments, not its ability to repay.

- Cross-default clauses. Corporate bonds in US have cross-default clauses. Failure to honor one bond triggers default on all bonds and immediate repayment of principal on all bonds. A sovereign government may issue different bonds on different markets. Failure to repay some of its debts may not lead to the default of other bonds, as in the case of Russian default in August 1998.

- Rescheduling of payment and possible multiple default. A sovereign rarely makes an outright default. It can be in the bond holders' benefit to avoid bankruptcy costs by offering debt rescheduling or new lending.

- The number of issuers of sovereign debts is quite small compared with that of corporate bonds. In fact, in the widely used benchmark index constructed by Morgan for the emerging market there are only 15 countries and the highest weighting goes to these three countries.

More details for the specific features of sovereign debt can be found in a survey by Beatriz Armendariz de Aghion (1993).

There are some efforts trying to analyze sovereign term structure by modelling the complex sovereign default process in the framework of the structural model. Gibson and Sundaresan (1999) and Chang and Sundaresan (2000) explore the different enforceability of sovereign and corporate bonds and present theoretical models in which sovereign and corporate borrowers have different optimal strategies. To avoid the complexity of sovereign default Pages (2001) and Keswani (1999) apply Duffie and Singleton (1999)'s reduced model to analyze Latin American Brady bonds. Merrick (1999) uses a constant default intensity model and explores the recovery risk of sovereign bonds In the analysis of Russian dollar-denominated bonds, Duffie et al. (2001) show that liquidity risk, recovery risk, no cross-default clauses, and reschedule possibility can all be handled in their reduced model. All of the above papers focus on individual sovereign risk except for Keswani (1999) who gives a test of the existence of common factor among the Brady bonds he investigates.

The rapid growth of credit derivative market and portfolio risk management call for the analysis of the joint behavior of credit risks. Duffee (1999) estimate default processes of 161 firms with interest rate the only source of cross-section correlation. Recent empirical studies suggest that cross section correlation among default risks may not be fully explained by interest rate. Some unidentified common sources and systematic risk premiums might be responsible for much of the cross section correlations. For the corporate bond market, Collin-Dufresne, Goldstein and Martin (2001) and Pedrosa and Roll (1998) show that there is a dominant common source underlying credit spreads and observable financial and economics variables cannot explain this common source. Elton Gruber, Agrawal and Mann (2001) emphasize the role of systematic risk premium in determination of changes of credit spreads. They report that individual credit information can only explain a small part of changes of credit spreads and a large part are attributed to common factors which are used to explain systematic risk in stock markets. Motivated by these empirical results, Driessen (2002) extends Duffee (1999) by including latent common factors in the determination of credit spreads. He shows that the presence of common factors enhances model precision and that the common factors do carry risk premiums. Yu (2002) claims that it is important to include common factors in the intensity function in order to generate sufficient default correlation. He shows that the default correlation generated by Duffee (1999) is low but in Driessen (2002) the default correlation is sufficient large.

The results for sovereign bond market are similar to those of corporate bond market. Eichengreen and Mody (1998) find that only a fraction of the spread changes can be explained by interest rate, trade link and other observable economic factors. They claim investors' sentiments are responsible for co-movement of sovereign spreads. Westphalen (2001) conducts a similar study of sovereign debt to those of Collin-Dufresne, Goldstein and Martin (2001). He reports similar results that observable economic variables fail to explain a large part of the variations in sovereign spreads and one unidentified common factor is responsible for most of the variations in credit spreads.

Geyer et al (2001) find country specific factors help to explain the variation of sovereign spreads. They use a LISREL model to analyze of yield spreads from government bonds issued by member states of the European Monetary Union and find that a global factor that mainly represents the average level of the yield spreads and a country specific factor for each can sufficiently capture the main features of the data.

4.3 The Model

In this study we focus on the term structure and variance composition of sovereign spreads. Our main idea is

- We proceed in the reduced form approach. In reduced models, the value of a defaultable bond before default is defined as a promised stream of payment discounted at a rate that contains an adjustment for instant default probability and the value recovered when default takes place. In this way, we focus on risk modeling and avoid the complicated default mechanism of sovereign bonds.

- We specify the factors in the intensity process as latent variables in order to accommodate the empirical finding that there is an unidentified common factor in the residue of credit spreads after controlling for structural variables.

- In order to measure the variance contribution of different components, we specify all the factors as Gaussian processes.

- As empirical studies show, risk premiums can explain a large part of credit spreads and during crises investors' risk aversion can change dramatically, we allow the mean reversion rates and long run means of our Gaussian processes to change under physical and risk neutral probability measures. This specification not only can accommodate more general form of risk premium, but also allow us to evaluate the influence of risk premiums on the variance compositions.

- To account for the specific feature of sovereign bond market, we include a country specific factor for each intensity function.

- We estimate the model jointly with common and country specific factors. The three-step estimation by Driessen (2002) (conditional on the extracted risk free short rate, estimate the common factors then take the extracted common factors as exogenous to estimate the specific factors) is not be appropriate for sovereign bond markets and for small portfolios.

We start with building blocks of reduced form model:. Here we have a probability space and an increasing information sets $\{F_t : t \geq 0\}$ which define the resolution of information and uncertainty over time. Default free interest rate is denoted as r_t. Under conditions of absence of no arbitrage, there exists an equivalent risk neutral probability measure Q. Default free bond price is expressed as:

$$P_t(t,T) = E_t^Q \left[\exp\left(\int_t^T -r_u du \right) \right] \tag{4.1}$$

where E_t^Q denotes conditional expectation with respect to risk neutral measure Q

Default is an unpredictable jump with intensity h_t. That is, conditional on no default happening prior to time t, the probability of a country to default during the next instant $(t, t+dt)$ is $h_t dt$. Duffie and Singleton (1998) show that a defaultable bond price can be reduce to default free bond price formula with a default adjusted discount rate $r_t + h_t L_t$, risk free interest rate plus mean loss rate due to possible default and recovery rate when default occurs.

$$D_t\left(t,T\right) = E_t^Q\left[\exp\left(\int_t^T - \left(r_u + h_u L_u\right) du\right)\right] \tag{4.2}$$

where h_t is the risk neutral intensity of default and L_t is the risk neutral loss rate in market value in the event of default. To take into account liquidity risk and possible negative value of the model implied mean loss rate, in sample or out of sample, we follow Duffie and Singleton (1998) and Duffie, Pedersen and Singleton (2002) and add an additional term, l_t, which is interpreted as illiquidity premium, to the mean loss rate. We refer to $s_t = h_t L_t + l_t$ as the instantaneous spread.

Default bond price is now

$$D_t\left(t,T\right) = E_t^Q\left[\exp\left(\int_t^T - \left(r_u + s_u\right) du\right)\right] \tag{4.3}$$

Duffie, Pedersen and Singleton (2002) show that (3) can accommodate some the specific features of sovereign default risk. For instance, suppose liquidation events have intensity process h_1 and expected fractional loss process L_1, restructuring has intensity h_2 and expected fractional loss L_2, ... Assuming that no more than one of these events can happen at any one time, the total intensity process is

$h = h_1 + h_2 + ..h_n$, and the expected fractional loss is the intensity-weighted average, $L = \frac{h_1}{h}L_1 + \frac{h_2}{h}L_2 + ... + \frac{h_2}{h}L_2$. The instantaneous spread is then

$$s = L_t + l_t$$

The complexity of sovereign default is reduced to a single process s and except for the purpose of identification of the individual features, we only need to specify s and interest rate process r_t.

As our main concern is the variance composition of sovereign term structure, we choose to model factors as Gaussian processes instead of the widely used CIR processes. The standard Vasicek model suffers from constant risk premium and may not be an idea candidate to describe term structure during big crises such as Russian default when investors risk attitude changes dramatically. We therefore follow Longstaff, Liu and Mendell (2001), allowing mean reversion rates and long run means to be different under physical and risk neutral probability measure.

Under physical probability measures.

$$dr_t = k_r(\mu_r - r_t)dt + \sigma_r d\omega_{rt} \tag{4.4}$$

where μ_r is the long run mean, k_r is mean reversion rate, σ_r is the instantaneous volatility and $d\omega_{rt}$ is Brownian motion under physical measure.

Investors price bonds under risk neutral probability measure where risk adjustment is introduced. Under risk neutral measure:

$$dr_t = k_r^*(\mu_r^* - r_t)dt + \sigma_r d\omega_{rt}^* \tag{4.5}$$

Note here we make a risk adjustment: we have different long run mean and mean reversion rate under these two measures. The widely-used constant risk price Vasicek model changes only the long run mean while Cox-Ingersoll-Ross model only has a mean reversion rate adjustment. In both cases, risk prices are constant. In our preliminary experiments, we find they do not seem to match the data as well as this Vasicek model.

In this Vasicek model, the risk price associated with $d\omega_{rt}$ is

$$\left[\left(k_r - k_r^*\right) r_t + k_r^* \mu_r^* - k_r \mu_r\right] / \sigma_r \tag{4.6}$$

It is a linear function of state variable r_t. It is a increasing with r_t if $k_r < k_r^*$, that is, when the rate of reversion is smaller under risk neutral than under object measure.

This specification not only allows the model to fit the data better but also allows us to evaluate the role of risk adjustment in the sovereign term structure in terms of variance contribution.

Given the small size of the portfolio, we cannot hedge away idiosyncratic risks and need to take into account country-specific shocks. We specify the default-liquidity spread s_t (instantaneous sovereign spreads) for country i to be a function of the international dollar interest rates, a common shock and a country-specific shock.

$$s_{it} = \beta_i \left(r_t - \overline{r_t}\right) + \gamma_i x_t + z_{it} \tag{4.7}$$

where β_i, γ_i are constant which may be positive or negative. Also, $\overline{r_t}$ is the sample mean of r_t,; x_t is the common shock and z_{it}, $i = 1, 2, 3$, are the country-specific shocks that are uncorrelated across countries.

Since all shocks are latent variables, there should be at least as many series of observations as the number of factors to achieve identification. We give the proof in appendix A.

Our specification is similar to that of Janosi, Jarrow and Yildirm (2001)

$$s_{it} = \alpha_i + \beta_i r_t + \gamma_i x_t$$

where r_t is the three month T-bill rate and x_t is a standard Brownian Motion driving the S&P 500 index, thus their spread is a function of observables.

It is well known that the default of one entity can affect the default process of others. However, considering the complication of counter-party risk and the limited market data, we stick to the idea that default processes are independent conditional on economic wide shock r_t and x_t and assume that country-specific shocks are uncorrelated.

The effect of the interest rate on sovereign spreads has been documented by many empirical studies. The sign of the effect found by different studies have varies however. Calvo, Leiderman and Reinhart (1993), Fernandez Arias (1994), Kanminsky and Schmukler (2002) document a positive relation between US interest rates and emerging market sovereign spreads. It is interpreted that changes in US interest rates may affect borrowers' creditworthiness. Increases in US interest rates increase the debt burden borne by borrower countries and thereby reduce their ability to repay their debts. Higher premiums should be paid to compensate investors for the increased risks. International investors also try to seek high yields when US interest rates are low thus push down sovereign spreads of emerging market.

Eichengreen and Mody (1998), Kamin and Kleist (1997), Pages (2001), Keswani (1999), Duffie (2000) also provide evidence of the role of US interest rates on emerging market spreads. However, a negative relation was found in their studies. The explanations are that higher US interest rates may discourage emerging countries from issuing new debt and the reduced supplies bid up the prices of sovereign bonds and compress the yields spread. Flight to quality may also be the reason for this negative relation in that when sovereign default probability increases, investors become more risk averse and invest in safe assets such as US treasuries. The prices of sovereign bonds fall and the prices of US treasuries are bid up, resulting in a fall in treasury rate and a rise in sovereign spreads.

Structural models by Gibson and Sundaresan (1999) and Westphalen (2001) predict that the interest rates and credit spreads should be negatively related. A higher spot rate increases the risk neutral growth rate of the country wealth, leading to a decrease in the credit spread.

Common shock and country specific shocks are defined as follows:

Under the objective measure,

$$dx_t = k_x(\mu_x - x_t)dt + \sigma_x d\omega_{xt}$$

$$dz_{it} = k_{zi}(\mu_{zi} - z_{it})dt + \sigma_{zi} d\omega_{zit}$$

ω_{xt}, ω_{zit} are independent Brownian motions.

$$d\omega_{xt}d\omega_{zit} = 0$$

$$d\omega_{zit}d\omega_{zjt} = 0 \quad \text{when } i \neq j$$

Under risk neutral measure,

$$dx_t = k_x^*(\mu_x^* - x_t)dt + \sigma_x d\omega_{xt}$$

$$dz_{it} = k_{zi}^*(\mu_{zi}^* - z_{it})dt + \sigma_{zi} d\omega_{zit} \tag{4.8}$$

Risk prices for these $d\omega_{xt}$, $d\omega_{zit}$ are thus

$$\left[\left(k_x - k_x^*\right) x_t + k_x^* \mu_x^* - k_x \mu_x\right] / \sigma_x$$

$$\left[\left(k_{zi} - k_{zi}^*\right) z_{it} + k_{zi}^* \mu_{zi}^* - k_{zi} \mu_{zi}\right] / \sigma_{zi}$$

This specification allows us to estimate co-movement transition mechanism taking into account time-varying market sentiment. When $k_x > k_x^*$, $k_{zi} > k_{zi}^*$, the higher the values of x_t and z_{it}, the higher the market prices.

Under this specification, we have a closed form solution of risk free zero bond prices:

$$P_t\left(t,T\right) = A_r(t,T) \exp\left[-B_r\left(t,T\right) r_t\right]$$

where

$$A_r(t,T) = \exp\left[\gamma_r\left(B_r(t,T) - (T-t)\right) - \frac{\sigma_r^2 B_r(t,T)^2}{4k_r^*}\right]$$

$$B_r(t,T) = \frac{1}{k_r^*}\left[1 - \exp\left(-k_r^*(T-t)\right)\right]$$

$$\gamma_r = \mu_r^* - \frac{\sigma_r^2}{2k_r^{*2}}$$

Similarly, the price of the defaultable bond price with zero coupon is

$$\begin{aligned} D(T-t) &= A_{ri}^*(t,T)A_{xi}^*(t,T)A_{zi}(t,T) \\ &\quad \exp\left[-B_{ri}^*(t,T)\,r_t - B_{xi}^*(t,T)\,x_t - B_{zi}(t,T)\,z_t\right] \end{aligned} \tag{4.9}$$

where

$$A_{ri}^*(t,T) = \exp\left[\gamma_r^*\left(B_r(t,T) - (T-t)\right) - \frac{(1+\beta_i)^2\sigma_r^2 B_r(t,T)^2}{4k_r^*}\right]$$

$$A_{xi}^*(t,T)\exp\left[\gamma_{xi}^*\left(B_r(t,T) - (T-t)\right) - \frac{\gamma_i^2\sigma_r^2 B_x(t,T)^2}{4k_x^*}\right]$$

$$B_{ri}^*(t,T) = \frac{1+\beta_i}{k_r^*}\left[1 - \exp\left(-k_r^*(T-t)\right)\right]$$

$$B_{xi}^*(t,T) = \frac{\gamma_i}{k_x^*}\left[1 - \exp\left(-k_x^*(T-t)\right)\right]$$

$$\gamma_r^* = (1+\beta_i)\,\mu_r^* - \frac{(1+\beta_i)^2\sigma_r^2}{2k_r^{*2}}$$

$$\gamma_{xi}^* = \gamma_i\mu_x^* - \frac{\gamma_i^2\sigma_x^2}{2k_x^{*2}}$$

4.4 Estimation

4.4.1 Riskless interest rateIt would be desirable to estimate interest rate process and sovereign risk processes simultaneously. However, considering the large number of parameters and to fully utilize the long time series of Treasury rates, we use the standard two-step estimation method. In the first step we estimate and extract the interest rate process r_t. The conditional on the estimated parameters and the smoothed values of r_t, we estimate sovereign risks.

We use Bloomberg US stripped interest rates data on every Friday to estimate US interest term structure. The data starts from April 19 of 1991 to November 17 of 2000 with maturities ranging from 1, 2, 5 and 10 years. Table 1 show summary statistics for this data set. Interest rates over this period are quite stable and show mean reversion component from the small standard deviations and autocorrelations of level and first difference.

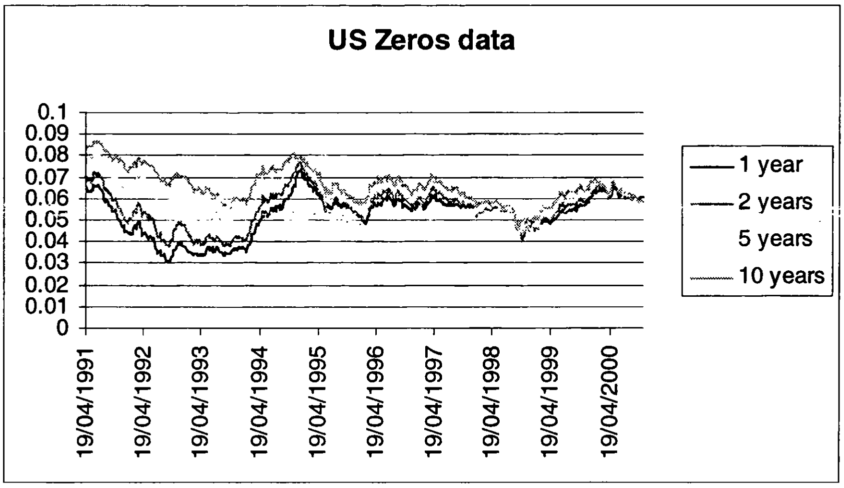


Figure 4.1: US zero coupon rates over the time period from 19/04/1991 to19/04/2000. Source: Bloomberg.

	Mean	Std	Minium	Median	Maximum	Autocorr
1 yr	0.052399	0.009631	0.0301	0.0553	0.0737	0.986
2 yr	0.055746	0.008745	0.0373	0.0574	0.0773	0.983
5 yr	0.061174	0.007848	0.0424	0.061	0.0811	0.979
10yr	0.065983	0.008046	0.0461	0.0652	0.0868	0.980

Table 4.1: Description statistics (Level) this table reports the summary statistics for the level of US zero coupon bond rates

	Mean	Std	Minium	Median	Maximum	Autocorr
1 yr	-8.47E-06	0.001280	-0.0041	0	0.0046	-0.092
2 yr	-2.36E-05	0.001395	-0.0042	-0.0001	0.0049	-0.045
5 yr	-4.25E-05	0.001390	-0.0042	-0.0001	0.0048	-0.079
10yr	-5.06E-05	0.001296	0.0461	-0.0001	0.0052	-0.117

Table 4.2: Description statistics (first difference) this table reports the summary statistics for the level of US zero coupon bond rates

We estimate interest rate process by standard Kalman filter popular in term structure estimation. Since we have four series of observations and one underlying state variable, we assume observations are contaminated by independent noises with mean zero and variances σ_1^2, σ_2^2, σ_5^2, σ_{10}^2.

Estimation results:

Almost all of the estimators for risk free process are significant at conventional significance level. The long run mean and mean reversion rate are different under

	estimates	std
μ_r	0.05295075	0.0059
k_r	0.62099281	0.3493
σ_r	0.01277396	0.0004
μ_r^*	0.07727961	0.0010
k_r^*	0.24919833	0.0154
σ_1	0.00292961	0.0001
σ_2	0.00000000	0.0002
σ_5	0.00446177	0.0001
σ_{10}	0.00684913	0.0002

Table 4.3: parameter estimates of riskfree interest rate process

physical probability measure from risk neutral probability measure, reflecting time varying compensation and market sentiment towards risk free assets. The model matches 2-year maturity zero curve very well but becomes worse for 10-year curve. As we estimate default processes taking the estimated risk free process as the real process, our results might be biased by the two-step approach.

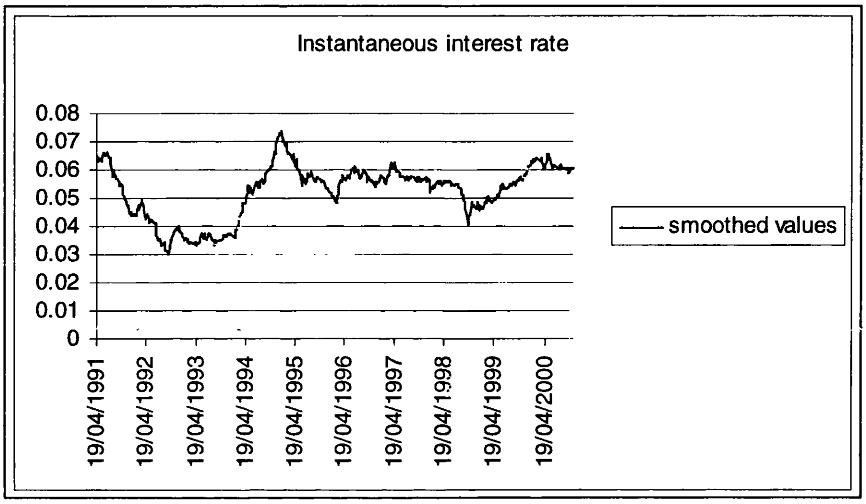


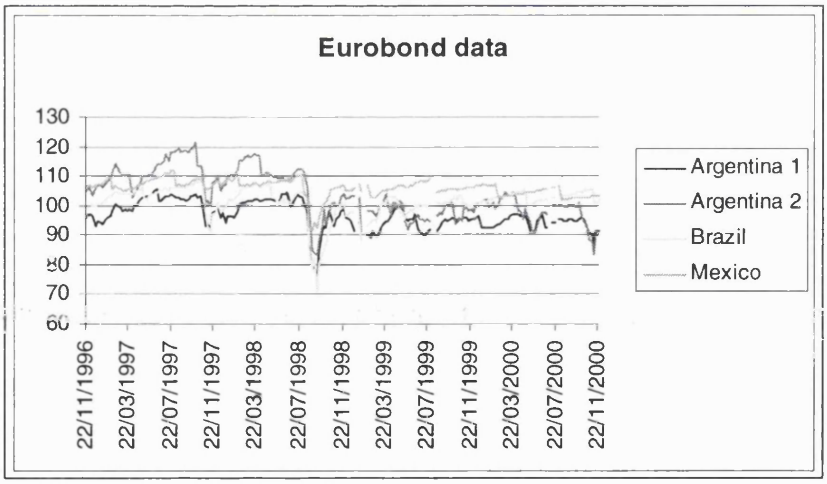
Figure 4.2: Extracted smoothed values of instantaneous interest rate

4.4.2 Estimation of Sovereign bondBecause of their higher trading volume and liquidity, Brady bonds have been the focus of term structure modeling and comovement and contagion research. However, since Bray bonds are based on special debt plans with institutional structures like rolling guarantees and oil recapture clauses, they differ from other sovereign bonds from the same countries and even stripped Brady bond spreads may not reflect the real financial costs and sovereign risks. In this paper, we focus on Eurobonds and analyze the corresponding sovereign risks. The data are mid-prices of four normal sovereign bonds of the three big countries:

Country	Issue date	Coupon type	Coupon date	Maturity	Coupon
Argentina	1993	fixed, Semi-annual	20/06, 20/12	20/12/03	8 3/8
Argentina	1996	fixed, Semi-annual	09/04, 09/10	09/10/06	11
Brazilian	1996	fixed, Semi-annual	05/05, 05/11	05/11/01	8.875
Mexico	1996	fixed, Semi-annual	06/02,06/08	06/02/01	9.75

Table 4.4: Data Description. All bonds are semi-annual fixed coupon bond without callability, collaterals and other non-standard features. Data Sources: Datastream, Bloomberg.

Argentine, Brazil and Mexico. All the bonds are denominated in US dollar so that we can avoid the complication of exchange risk. The sources are DataStream and Bloomberg. Even though daily price data are available, we choose to use weekly observations (every Friday) to reduce stale price and liquidity problems. The data starts from 02/11/96 to 24/11/00, covering the East Asian crisis, the Russian default, LTCM and the Brazilian currency crisis. The following table and figure present summary statistics for the gross data.



Sovereign bond prices over the time of 22/11/1996 to 22/11/2000. Data Source: Datastream, Bloomberg.

The changes of prices during this period are not as much as their corresponding

	Mean	Std	Minium	Median	Maximum
Argentina 03	96.518	4.78	74.96	96.00	105.80
Argentina 06	103.68	8.20	83.10	102.09	121.22
Brazilian	101.23	5.30	71.02	102.17	108.27
Mexico	105.89	3.02	91.08	106.40	111.80

Table 4.5: Descriptive statistics

Corr	Argentina 03	Argentina 06	Brazilian	Mexico
Argentina 03	1			
Argentina 06	0.84	1		
Brazilian	0.71	0.58	1	
Mexico	0.64	0.64	0.63	1

Table 4.6: Bond price correlation matrix

Brady bonds, due to their relatively shorter maturities and some other institutional reasons. The correlations among them are very high ranging from 0.83 to 0.63, suggesting common factors may be very important. The prices reach the lowest values during the Russian crisis.

We estimate the model using the extended Kalman filter. Coupon bond prices are sums of future cash flows discounted by the sovereign rates from zero coupon bond prices.

$$P_{c,t} = \sum_{i=1}^{N} C_i D\left(T_i - t\right) + FD\left(T_i - t\right) \tag{4.10}$$

where C_i is the coupon, N is the number of remaining number of coupon, T_i is the coupon date, F is the principal. We assume the data are observed with independent noises. We suppose the noises are normally distributed $N\left(0, \sigma_i^2\right), i = 1, 2, 3, 4,$. We set $\sigma_1^2 = \sigma_2^2$, the two Argentina bonds are observed with equal noises. Since the coupon price is not a linear function of state variables, we need to linearize it around the last prediction values. We leave the detailed procedure in appendix c.

Not all the parameters in the model can be identified from the data. The first identification problem comes from the long run means of the process x_t and z_{it}, μ_x^* and μ_i^*. As Dai and Singleton (2000) and Longstaff, Liu and Mandell (2000) point out, for Gaussian process, only linear combination of the long run mean under risk neutral measure can be identified from bond price data. Like Longstaff, Liu and Mandell (2000), we set $\mu_i^* = 0$ to avoid this identification problem. The second problem is the identification of the common factor. Under risk neutral measure, the common process can be expressed as

$$d(\gamma_i x_t) = k_x^*((\gamma_i \mu_x^*) - (\gamma_i x_t))dt + (\gamma_i \sigma_x)\, d\omega_{xt}$$

Only $\gamma_i \mu_x^*$, $\gamma_i \sigma_x$ can be identified. We set the sensitivity of Argentine as the benchmark $\gamma_A = 1$.

4.5 Results

4.5.1 Model fit

Table 8 reports the model fit for the four sovereign bonds. The model fits the data very well. The root mean square errors for Argentina with maturity at 2003 and maturity at 2006 are 2.38 and 2.75 dollars, for Brazilian bond 2.25 dollars and for Mexican bond only approximately 1 dollar. The mean errors and mean absolute errors are also very small compared with the face value of 100 dollars.

4.5.2 Sovereign spreads and interest rate

The results show that sovereign spreads are negatively related with US interest rate: The sensitivities to interest changes are -1.0848, -2.4286 and -1.3184 and all of them are significant under conventional

	Estimate	Std
μ_A	0.1043	0.0674
k_A	0.4849	0.3584
σ_A	0.0109	0.0025
k_A^*	-0.0892	0.0332
μ_B	0.1800	0.1458
k_B	1.6814	1.5500
σ_B	0.0181	0.0055
k_B^*	-0.0957	0.0537
μ_M	0.0956	0.0885
k_M	2.0334	2.9328
σ_M	0.0100	0.0039
k_M^*	-0.1424	0.0965
μ_x	-0.0736	0.0664
k_x	4.3069	4.1150
σx	0.0262	0.0045
μ_x^*	0.0138	16.8309
k_x^*	-0.0003	0.0000
γ_B	2.0746	0.3058
γ_M	1.1683	0.2880
β_A	-1.0848	0.4161
β_B	-2.4286	0.7473
β_M	-1.3184	0.2945
σ_1	1.4220	0.0828
σ_2	0.6200	0.1265
σ_3	0.1245	0.0437

Table 4.7: Parameter estimates for common factor and country specific factor in the sovereign short spreads

	Argentina 03	Argentina 06	Brazil	Mexico
RMSE(US$)	2.38	2.75	2.25	0.995
ME(US$)	-0.059	-0.023	0.066	0.034
MAE(US$)	1.57	1.79	1.12	0.48

Table 4.8: This table reports Root Mean Square Error (RMSE), Mean Error (ME) and Mean Absolute Error (MAE). All the units are US dollar.

significance level. The negative relation is consistent with results reported by Duffee (2002), Eichengreen and Mody (1998), Kamin and Kleist (1997), Pages (2001), Keswani (1999). However, the magnitude is much larger than their estimates: all of them are larger than unity in absolute value, which means not only spreads are negatively related to interest rate but sovereign yields are so as well.

To double-check the results, we perform a linear regression on the 5 year constant maturity Treasury rate, using the yield inverted from bond prices. The yield spread is proxied by the difference between bond yield and the 5 year constant maturity Treasury rate. We have a similar result. It appears that sovereign spreads are much more sensitive to the international interest rates than those of corporate bonds. We suspect that probably for large trading volumes, the US Treasury bonds and the Latin American sovereign bonds are two important substitutes. The price change of US Treasury may have strong substitution effect on the Latin American sovereign bonds. The large negative correlation may also arise from the different sample period. Our sample starts from the end of 1996 and ends in late 2000, covering Russian default and currency devaluation, the severest global crisis of 1990s, the Asian crisis and the failure of Brazilian currency peg as well as LTCM. During these crises, there is a well-known phenomenon: Flight to Quality: fund managers shed their investment in assets in crisis countries and seek safety in US treasury bonds, pushing down US treasury rates.

4.5.3 Sovereign Spread Term Structure

The Shape of Term Structure Our model specifies that the shape of the sovereign term structure is determined by the term structures of the three component processes: interest rate, common shock and country specific shocks.

$$-\beta_{ir}\overline{r}+\left(-\frac{\ln\left[A_{zi}\left(\tau\right)\right]}{\tau}+\frac{B_{zi}\left(\tau\right)}{\tau}z_{1}\right)+\left(-\frac{\ln\left[A_{xi}\left(\tau\right)\right]}{\tau}+\frac{B_{x}\left(\tau\right)}{\tau}\beta_{ix}x\right)$$
$$+\left(-\frac{\ln\left[A_{ri}\left(\tau\right)\right]}{\tau}\frac{B_{r}\left(\tau\right)}{\tau}\beta_{ir}r\right) \tag{4.11}$$

Under risk neutral measure, interest rate is mean reverting with reversion rate at 0.25. Its half-life time is 2.77 years and the implied term structure of this factor (the term in the third bracket) can be upward sloping, downward sloping as well as hump-shaped, depending on different initial value of this factor. The common factor exhibits essentially no mean reversion (mean reversion rate is 0.00) under risk neutral measure and its term structure (the term in the second bracket) is a flat line. Country-specific shocks in our study are all explosive under the risk neutral measure. For Argentina, Brazil and Mexico, the mean reversion rates under physical measure are respectively 0.4849, 1.6814, 2.0334 but -0.089, -0.096, -0.1424 under risk neutral measure. The term structure associated with country specific factors (the term in the first bracket) is upward sloping. The overall effect of these three factors at their fitted mean values is an upward sloping term structure. Figure 3 shows the term structures of these three countries at factors' sample mean fitted values.

Spread term structure describes the financial costs associated with a debt scheduling. The shape of term structure reflects expectation about future pos-

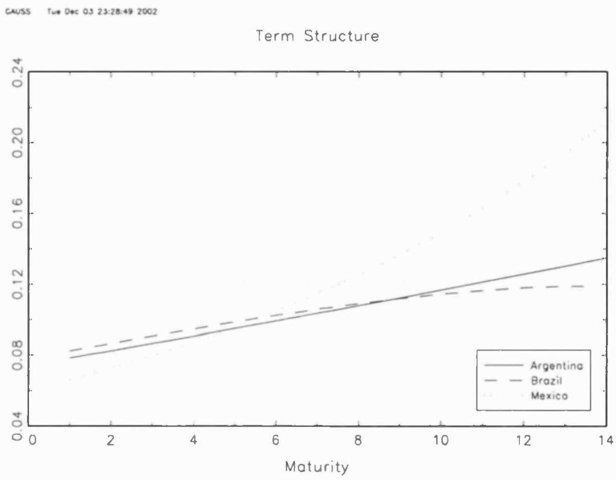


Figure 4.3: Term structure at sample mean fitted values.

sible default risk and risk compensation. An upward-sloping term structure is usually interpreted as that for a good quality entity immediate default is less likely to happen but over time credit quality may deteriorate and compensation for default risk should increase. For low quality entity, immediate default risk is higher but with the time passing by its financial state might improve and risk compensation decreases in the end. Figure 1 displays the increasing term structure for all the three countries over the sample period. Studies on Brady bond however show a different pattern of term structure. Pages (2001) estimates Brazilian sovereign term structure using Brazilian Brady bond prices over period of 1995-2000 and concludes that the term structure is hump-shaped. The fitted mean hazard rate is 10.3%. The fitted value in our study is only 2.7% for Brazil. The shape of the term structure and mean fitted hazard rate indicate not only Brady bond spread is not an appropriate reference of finance costs but also they may imply different credit quality.

<u>The impact of each component on term structure</u> To analyze the impact of each component on term structure in detail, we rewrite (11) into

$$A_i(\tau) + \frac{B_{zi}(\tau)}{\tau} z_1 + \frac{B_x(\tau)}{\tau} \beta_{ix} x + \frac{B_r(\tau)}{\tau} \beta_{ir} r \tag{4.12}$$

where

$$A_i(\tau) = -\beta_{ir}\bar{r} - \frac{1}{\tau}\left(\ln\left[A_{zi}(\tau) A_{ri}(\tau) A_{xi}(\tau)\right]\right)$$

The term structure is the sum of a constant and the three factors weighted by their factor loadings. Figure 2, 3, 4 display the four components for the three countries at the mean fitted values. For all the three countries, the common factor components are nearly constant across maturities, representing a parallel shift in the entire spread curve. The country specific factor component increase with maturity. An increase in the country specific factor will increase yield spread at long end more than at short end. The interest rate factor itself is mean reversion and its factor loading $\frac{B_r(\tau)}{\tau}$ is downward sloping. But because of the negative β_{ir}, the interest factor component $\frac{B_r(\tau)}{\tau}\beta_{ir} r$ is increasing. A decrease in the interest rate will increase the component, which will increase the term structure more at long end than at short end, resulting in a negative relation between interest rate and the slope of the term structure. Both interest rate factor and country specific factors contribute to the slope of the term structure.

The result that the country-specific factors carry risk premiums is in contrast to results of Driessen (2002) that in the corporate market issuers' idiosyncratic shocks are diversifiable and carries no risk premium. There are several reasons for this. Firstly, we use a small group of sovereign bonds. The country specific

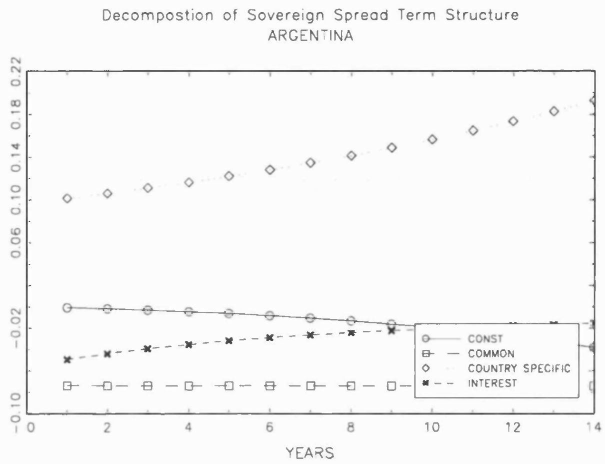


Figure 4.4: Decomposition of Argentina term struture. The figure displays the four components of term structure at the fitted mean value of factors.

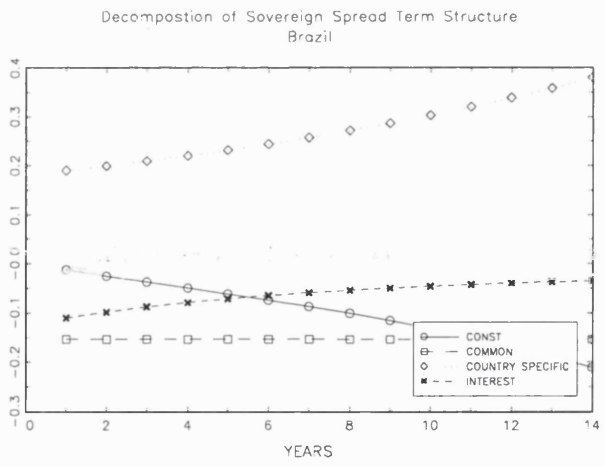


Figure 4.5: Decomposition of Brazilian term struture. The figure displays the four components of term structure at the fitted mean value of factors.

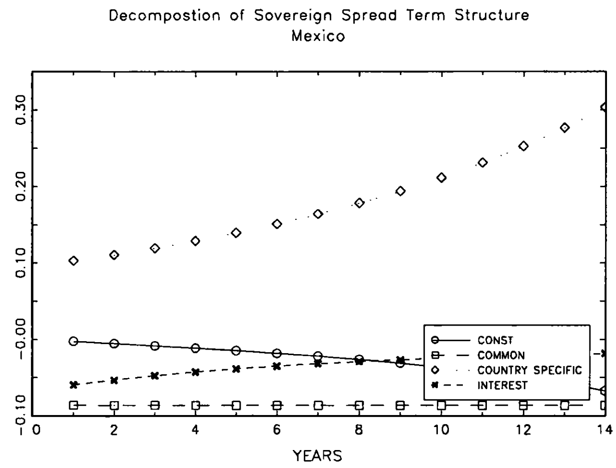


Figure 4.6: Decomposition of Mexico term struture. The figure displays the four components of term structure at the fitted mean value of factors.

factor actually may not be specific but rather common with some other sovereign bonds. Secondly, it may be because of the specific features of the sovereign bond market. The number of sovereign bond issuers is very limited compared with that of corporate bond market. In the widely used benchmark index constructed by Morgan for emerging market there are only 15 countries and the highest weighting go to these three countries. In contrast, in the corporate bond market study by Driessen (2002), there are 104 issuers and it is plausible that with such a large portfolio issuer's specific risk will be hedged away and will not carry risk premiums. Thirdly, as documented in the literature review part, the incentive for a sovereign government to default its debt is very different from that of a corporate entity. Whether and when to declare a default depend not on its ability to pay, but largely on the government's willingness to pay, on the country's political conditions and default history, and on the government's trade off between default costs and benefits. This is a rather individual decision. Fourthly, the world or regional economies are not fully integrated. Empirical work in international CAPM shows that country specific factors play a role in determining the return of an international asset. We note that in an empirical analysis of European government yield spreads by Geyer et al (2001), they find strong empirical evidence for a global factor that mainly represents the average level of the yield spreads and for a country specific factor for each issuer. The country specific factor for all the five European zone issuers are compensated for systematic risk and in their CIR specification for the factors have mean reversion rate under risk neutral measure $(k + \lambda) < 0$, implying an explosive process and an upward sloping term structure for this factor. This is consistent with our results for the three Latin American countries. In the study of the pricing of Russian sovereign bond, Duffie et al (2001) find that even for bonds from the same sovereign issuer, bond specific risks carry risk premiums and the mean reversion rates for the specific factors are all negative, implying an upward sloping term structure for these factors.

Nevertheless, the country specific factor appears to be rather stable during the sample period. We show in the next subsection that it is the latent common factor and the international interest rate that explains most of the variations of the sovereign yield spreads.

4.5.4 Volatility AnalysisThe sensitivities of the three countries to common shock are 1 (set as benchmark), 2.07, 1.17 for Argentina, Brazil and Mexico and all are significant. Their respective sensitivities to the interest rate, -1.08, -2.42, -1.31.

We cannot however say that Brazil is most vulnerable to common shocks, since the variation of the country specific factor of Brazil is also the largest. We compute the impact of international factors on the sovereign spread term structures of these three countries.

First, we compute the impact of the latent common factor on the dynamics of yield spread.

By Ito's lemma, the dynamics of sovereign spread with maturity $(T-t)$ in our Vasicek model is

$$dR(\tau) = A_i^{'}(\tau)\,dt + \frac{B_{zi}(\tau)}{\tau}\sigma_{zi}d\omega_{zi} + \frac{B_x(\tau)}{\tau}\beta_{ix}\sigma_x d\omega_x + \frac{B_r(\tau)}{\tau}\beta_{ir}\sigma_r d\omega_r \quad (4.13)$$

where

$$A_i^{'}(\tau) = \frac{B_{zi}(\tau)}{\tau}k_{zi}^{*}\left(\mu_{zi}^{*} - z_i\right) + \frac{B_x(\tau)}{\tau}k_x^{*}\beta_{ix}\left(\mu_x^{*} - x\right) + \frac{B_r(\tau)}{\tau}\beta_{ir}\left(\mu_r^{*} - r\right)$$

The variance of yield spread is

$$\begin{aligned} Var_{total} &= \left(\frac{B_{ri}(t,T)\,\beta_{ir}\sigma_r}{T-t}\right)^2 + \left(\frac{B_{xi}(t,T)\,\beta_{ix}\sigma_x}{T-t}\right)^2 \\ &\quad + \left(\frac{B_{zi}(t,T)\,\sigma_{zi}}{T-t}\right)^2 \end{aligned}$$

Variance contributed by the latent factor is

$$Var_{common} = \left(\frac{B_{xi}(t,T)\,\beta_{ix}\sigma_x}{T-t}\right)^2 \quad (4.14)$$

We measure the relative contribution of the latent factor by the ratio of variances:

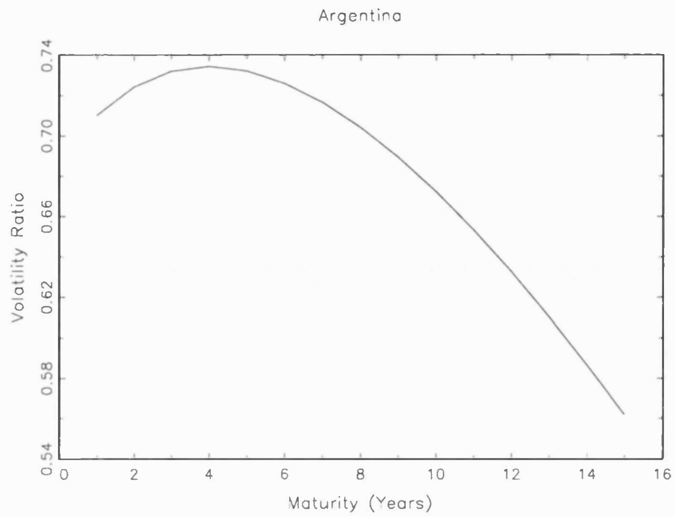


Figure 4.7: The ratio of the variance of the common factor to total variations.

$$R = \frac{Var_{latent}}{Var_{total}} \tag{4.15}$$

i =Argentina, Brazil, Mexico. Figure 7, 8, 9 display the term structure of the variance contribution of the latent factor to the total variance.

The variations of the level of term structure explained by the latent common factor for all the three countries are at least 55 percent across maturities from 1 year to 15 years. If we consider that the longest maturity from the beginning of our sample period is about 10 years and restrict our attention to maturity from 1 year to 10 year, the explained variations increase to at least about 70 percent. We claim the latent common factor is the main course of the variation of these sovereign spreads.

The slope of the term structure reflects investors' expectation of future sovereign default risk and the associated risk premiums. We define the slope to be

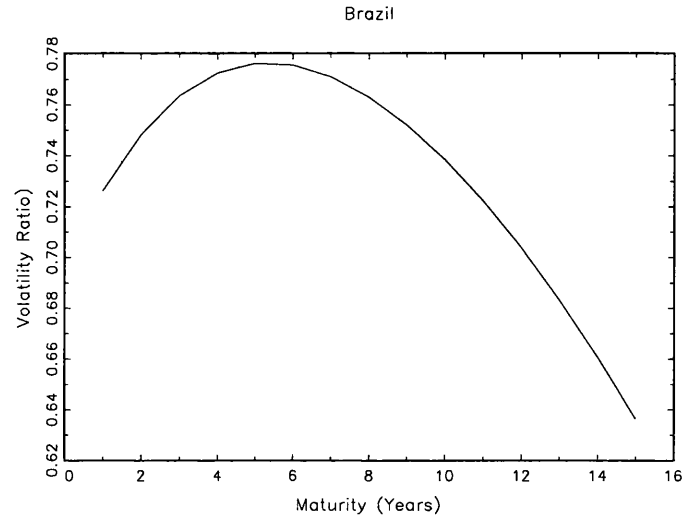


Figure 4.8: The ratio of the variance of the common factor to total variations.

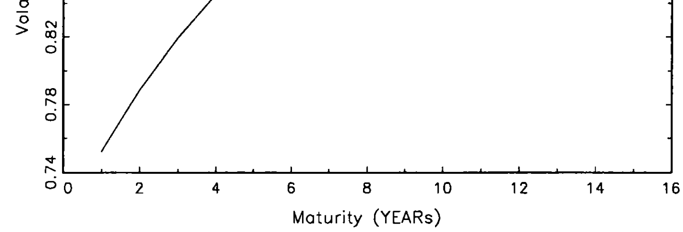


Figure 4.9: The ratio of the variance of the common factor to total variations.

the difference between the spreads of 10 year maturity and 1 year maturity. The variance of the slope is

$$\begin{aligned} Var\left(Slope_{total}\right) &= \left(\frac{B_{ri}\left(t,T_2\right)\beta_{ir}\sigma_r}{T_2-t} - \frac{B_{ri}\left(t,T_1\right)\beta_{ir}\sigma_r}{T_1-t}\right)^2 \\ &+ \left(\frac{B_{zi}\left(t,T_2\right)\sigma_{zi}}{T_2-t} - \frac{B_{zi}\left(t,T_1\right)\sigma_{zi}}{T_1-t}\right)^2 \end{aligned} \tag{4.16}$$

where $T_2 - t = 10$, $T_1 - t = 1$. The component associated with the latent common factor does not appear in (16) since it only parallel shifts the term structure and disappears in the difference of the spread of the short end and the long end. We measure the magnitude of the variance of the slope relative to that of the variance of the level.

$$R_{slope/level} = \frac{Var\left(Slope_{total}\right)}{Var\left(Level_{total}\right)}$$

We also measure the comovement of the slope by

$$R_{common/slope} = \frac{Var\left(Slope_{common}\right)}{Var\left(Slope_{total}\right)}$$

where

$$Var\left(Slope_{common}\right) = \left(\frac{B_{ri}\left(t,T_2\right)\beta_{ir}\sigma_r}{T_2-t} - \frac{B_{ri}\left(t,T_1\right)\beta_{ir}\sigma_r}{T_1-t}\right)^2$$

The results are reported in table 9.

Since the latent common factor, the main source of variation of term structure, does not affect the slope, the variations of the slope are very small. The largest is Argentina with 15 percent of the variation the level while Mexico and Argentina with only 9 percent of the variation of the level. The stable slope of the spread

	Argentina	Brazil	Mexico
$R_{slope/level}$	0.09	0.15	0.09
$R_{common/slope}$	0.57	0.41	0.82

Table 4.9: R(slope/level) is the ratio of the varince of the slope to that of the level. The slope is defined as the difference between the spread of 10 years maturity to that of 1 year maturity. R(common/slope) is the variance contribution of the international interest rate to the total variance of the slope.

term structure implies that investors' expectation of future sovereign default risk does not change much across calm and crisis time.

The impact of common shock is also very large. For Mexico, the variance contribution of the common factor (the international interest rate) is 82 percent while for Argentina 57 percent and Brazil 41 percent.

However, if under physical probability measure, we have a different picture of variance composition. The variation explained by the common factor fall dramatically. The variations explained by the common factor are less than 10 percent for bonds with maturities longer than 2 years, implying that it is the risk premium, rather than the real default probability that cause the sovereign bond market to move together

Under the specification of our Gaussian process, the change of mean reversion rate and long run mean from physical probability measure to risk neutral probability measure reflects the required risk premiums by the investors. Even though instantaneous volatility does not change under different probability measures, the volatility of spreads will change, due to the change of mean reversion rates. Under physical probability measure, the mean reversion rate for the common factor is 4.3. An instantaneous shock will quickly die out and influence bond spreads at

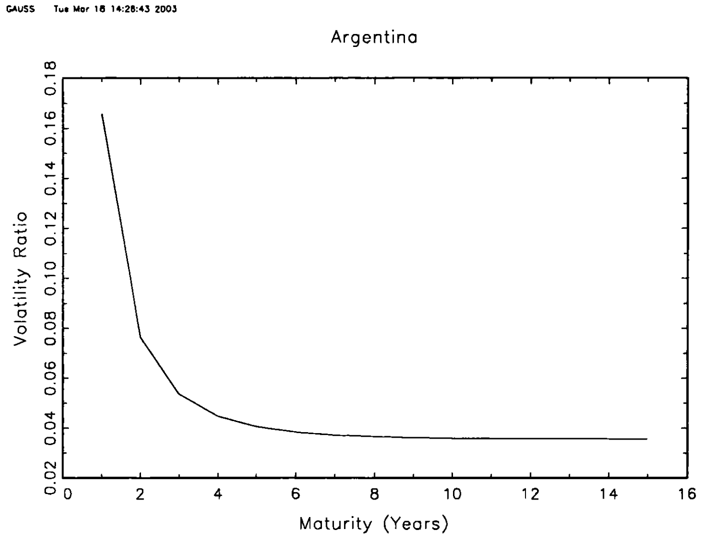


Figure 4.10: The ratio of the variance of the common factor to total variations, after controlling for the risk premium.

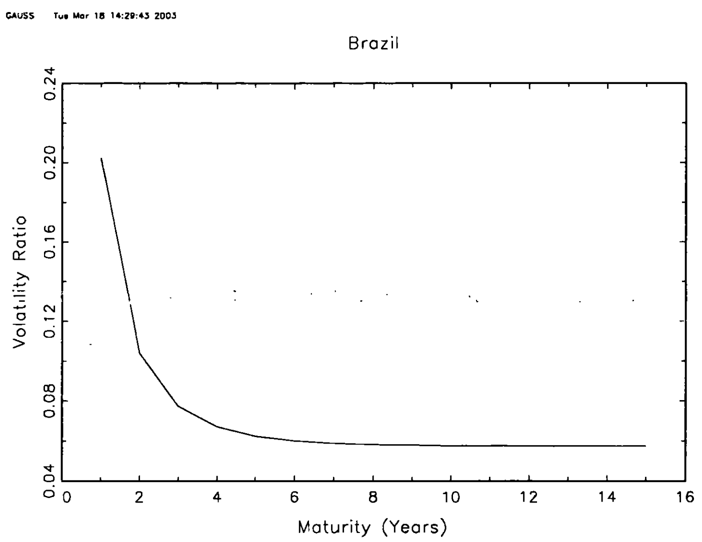


Figure 4.11: The ratio of the variance of the common factor to total variations, after controlling for the risk premium.

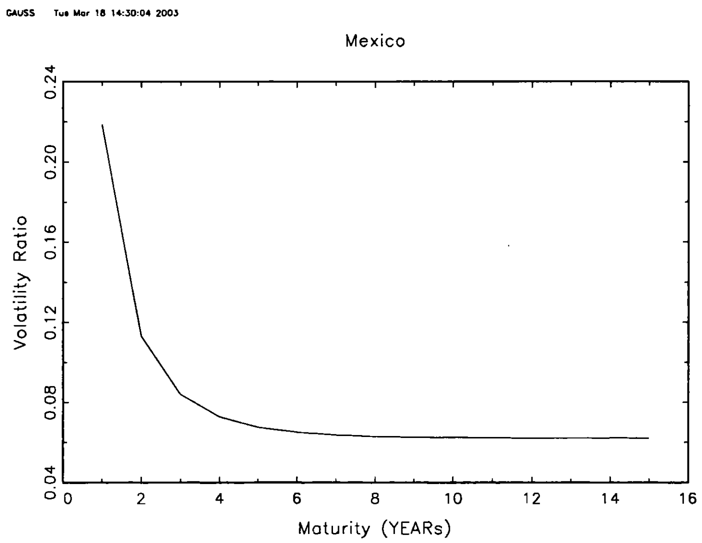


Figure 4.12: The ratio of the variance of the common factor to total variations, after controlling for the risk premium.

normal maturities very little. Under risk neutral probability measure the common factor is a random walk. An instantaneous shock will be persistent and influence bond spreads at all maturities. The sharp increase of variance contribution from physical probability measure to risk neutral measure implies that most of the variation of sovereign spreads and the comovement of sovereign spreads come from risk premiums.

In summary, we show that

- The sovereign spread term structures of these three countries are upward sloping.

- The latent common factor is responsible for most of the variations of the sovereign yield spreads.

- Market sentiment is responsible for the variation and comovement of sovereign spreads.

- The level of these sovereign term structures is rather volatile, due to the common and the international interest rate factors..

- The slopes of term structure are very stable, implying investors' foresight of future default risk does not change very much.

- The interest rate is negatively related to the level and slope of sovereign spreads.

4.6 Conclusion

The study of the joint behavior is useful for the pricing of basket credit derivatives and credit portfolio risk management. Most of the relevant literature has focused on corporate bond market. In this paper, we analyze the cross-section correlation of a group of sovereign bonds from three different countries. In order to accommodate the complexity of sovereign default process and the possible segmented country economies, we include a country-specific factor in the discount rates for different countries and allow correlation between sovereign spreads and international interest rates. We estimate the model by the standard Kalman and the extended Kalman filter.

We find that the latent common factor is responsible for most of the variations of the yield spreads, a result consistent with Westphalen (2001)'s principal component analysis of a larger group of sovereign bonds. We show that the different role of the latent common factor, the international interest rate and the country specific factor in determining the term structure of sovereign spreads. In all cases the latent common factor unanimously shifts the level of yield spreads of these countries in the same direction. The international interest rate is negatively correlated with the level and slope of sovereign spreads of these three countries, a result resembling those of Duffee (1999), Driessen (2002) and many others on corporate bond market. Apart from interest rates, there is a latent common shock that explains most of the variations of sovereign spread. The variations explained by the common factor fall dramatically if we control the risk premium associated with this common factor. The interest rate and the common factor explain most of the volatility of yield spread of these countries, reflecting the contagion phenomenon in the sovereign bond market and the important effect of international shocks on sovereign spreads. The country-specific factors carry risk premiums and contribute to the slope of credit term structure, but contribute little to the total variations of yield spreads. Our results highlight the comovement of sovereign term structure and the importance of risk premiums in determining the variation and comovement of sovereign spreads.

Appendix A: Observability of extended Kalman filter

The short sovereign spread is

$$s_{it} = \beta_i \left(r_t - \overline{r_t}\right) + \gamma_i x_t + z_{it}$$

$i = 1, 2, 3.$

Measurement equations:

$$\begin{aligned} Y_{1t} &= P_{1c,t}(s_{1t}) + \varepsilon_{1t} \\ Y_{2t} &= P_{2c,t}(s_{1t}) + \varepsilon_{1t} \\ Y_{3t} &= P_{3c,t}(s_{2t}) + \varepsilon_{3t} \\ Y_{4t} &= P_{4c,t}(s_{3t}) + \varepsilon_{4t} \\ E_{t-1}(\varepsilon_{jt}\varepsilon'_{jt}) &= H \end{aligned}$$

$j = 1, 2, 3, 4.$ $H{=}diag\ (\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2)\,.P_{c,jt}(s_{it})$ is a nonlinear function of s_{it}.

Transition equations

$$z_{it} = d_z + T_z z_{i,t-1} + \eta_{zt} \qquad E_{t-1}(\eta_{zt}\ \eta'_{zt}) = R(z_{i,t-1})$$

$$x_t = d_x + T_x x_{t-1} + \eta_{xt} \qquad E_{t-1}(\eta_{xt}\ \eta'_{xt}) = R(x_{t-1})$$

η_{zt} , η_{xt} are independent of each other. $R(z_{i,t-1})$ is diagonal,.$R(x_{t-1})$ is constant.

Observability Condition for Nonlinear Kalman filter:

$$\left[Z\ ZT.......ZT^{i-1}......ZT^{n-1}\right]$$

has rank n =dimension of state vector. Here $n = 4$, Z is Jacobian of observation matrix $P_{c,t}(s_{it})$ at $t|t-1$ for every t.

Thus all the four factors can be identified.

Appendix B: Kalman filter estimation of interest rate process

Observation equations:

$$
\begin{aligned}
y\left(T-t\right) &= -\frac{1}{T-t}\ln\left(P\left(T-t\right)\right) \\
&= -\frac{1}{T-t}\ln\left(A_r\left(T-t\right)\right)+\frac{1}{T-t}B_r\left(T-t\right)r_t
\end{aligned}
$$

$T-t = 1$ year, 2 years, 5 years, 10 years. Since we have 4 series of observations and one underlying state variable, we assume observations are contaminated by independently noises with mean zero and variances σ_1^2, σ_2^2, σ_5^2, σ_{10}^2.

Transition Equation:

$$r_t = d_r + T_r r_{t-1} + \eta_{rt} \qquad E_{t-1}(\eta_{rt}\ \eta'_{rt}) = R(r_{t-1})$$

$$d_r = \mu_r\left(1 - e^{-k_r\Delta t}\right)$$

$$T_r = e^{-k_r\Delta t}$$

$$R(r_{t-1}) = \frac{\sigma_r^2}{2k_r}(1 - e^{-2k_{ri}\Delta t})$$

Appendix C: Extended Kalman filter estimation of sovereign bond processes

Measurement equations:

$$Y_t = P_{c,t}(z_{it}, x_t) + \varepsilon_t \qquad E_{t-1}(\varepsilon_t \varepsilon_t') = H$$

H=$diag$ $(\sigma_1^2, \sigma_2^2, \sigma_3^2, \sigma_4^2)$.

and transition equations

$$z_{it} = d_z + T_z z_{i,t-1} + \eta_{zt} \qquad E_{t-1}(\eta_{zt}\ \eta_{zt}') = R(z_{i,t-1})$$

$$x_t = d_x + T_x x_{t-1} + \eta_{xt} \qquad E_{t-1}(\eta_{xt}\ \eta_{xt}') = R(x_{t-1})$$

$$E_{t-1}(\eta_{zt}\eta_{xt}'\ ) = 0$$

$$d_z = \mu_{zi}\left(1 - e^{-k_{zi}\Delta t}\right)$$

$$d_x = \mu_x\left(1 - e^{-k_x\Delta t}\right)$$

$$T_z = e^{-k_{zi}\Delta t}$$

$$T_x = e^{-k_{xi}\Delta t}$$

where the $R(z_{t-1})$ is a diagonal matrix with elements on the diagonal

$$\frac{\sigma_{zi}^2}{2k_{zi}}(1 - e^{-2k_{zzii}\Delta t})$$

$i = 1, 2, 3.$

$$R(x_{t-1}) = \frac{\sigma_x^2}{2k_x}(1 - e^{-2k_{xi}\Delta t})$$

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