University College London
Department of Economics

Time Series Analyses of Consumption Grouped Data

Margherita Borella

A thesis submitted for the degree of
Doctor of Philosophy
University of London

September, 2001

Abstract

The general framework for this thesis is the analysis of the time series properties of grouped data. The first chapter focuses on the dynamic properties of durable expenditure. Consumers make durable purchases infrequently and usually in large amounts. Aggregating among agents can lead to very complicated dynamics in aggregate demand, as fluctuations will be driven both by fluctuations in the number of consumers making the purchase and by fluctuations in the size of the purchase. A dynamic index model is estimated using a long time series of cross section data drawn from the UK Family Expenditure Survey. The methodology used is well suited to analyse the dynamic response of durable expenditure to economy-wide shocks at different aggregation levels.

The second chapter aims at characterising the time series properties of individual consumption; data are drawn from the UK Family Expenditure Survey. The methodology consists in estimating multivariate moving average systems for grouped variables: this approach has the advantage of allowing to explicitly take into account the measurement error present in the individual measures of consumption and income.

A panel data technique is used in the third chapter to evaluate different models of the individual earnings process. The analysis is based on the Bank of Italy Survey of Household Income and Saving; the analysis is carried out exploiting the panel component of the Survey. This analysis permits to undertake a study of pension earnings distribution in Italy: the final chapter analyses how effective is redistribution of pensioners' income under different Social Security systems.

Acknowledgments

In doing my PhD and writing this thesis, I have run up a long list of acknowledgments. I am immensely grateful to my supervisor Orazio Attanasio for his help and patience; I also owe a great debt of gratitude to Agar Brugiavini, who encouraged me to take a PhD and supported me throughout it. I would particularly like to thank Richard Blundell and Elsa Fornero for very valuable discussion and advice. I am very greatful to Manuel Arellano, Fabio Bagliano, James Banks, Martin Browning, Onorato Castellino, Ian Crawford, Hidehiko Ichimura, Costas Meghir, Giovanna Nicodano, Sarah Tanner and Harald Uhlig for their helpful suggestions.

Participants to the seminars at University College London, CeRP, University of Turin, to the ESPE 1999 Conference, to the CEPR meeting on "New Approaches to the study of Business Cycles" (Madrid, January 31 1998), and to the IZA Conference "Pension Reform and Labor Markets" (Berlin, May 19-21, 2001) provided very useful comments.

Irreplaceable support and advice was provided by Charles Grant, Francesca Fabbri, Adriaan Kalwji, Astrid Kunze, Mario Padula, Lia Pacelli, Monica Paiella, Sonia Pereira and by my husband, Federico Revelli. I am also very greateful to my parents for their kind support throughout these years.

Finally, I greatly acknowledge the financial support from the University College London Graduate School and EEC (TMR, grant n. ERBFMBICT 961645), without which this research would not have been possible.

Declaration

No part of this thesis has previously been presented to any college or university for any degree.

Margherita Borella

London, 13 September 2001

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Introduction

This thesis is concerned with the analysis of the time series properties of consumption and its components on grouped data. The time series properties of aggregate consumption are well known in the time series literature; very little is known, however, about the stochastic properties of consumption at the individual level. The use of grouped data, built on the UK Family Expenditure Survey, permits to exploit the time dimension even in the absence of a genuine panel data set, and allows to focus on business cycle fluctuations. On the other hand, the lack of a longitudinal component in the data forces the analysis to ignore pure idiosyncratic variability and to focus instead on the dynamics of group averages.

Chapter 1 analyses the dynamic properties of cohort durable expenditure. The presence of adjustment costs and other sources of inertial behaviour can be very difficult to handle in a fully structural fashion. The pattern of observed variability, however, can be suggestive about the nature of the structural models generating the data. The empirical application makes use of a dynamic index model, which allows to identify aggregate shocks, common to all cohorts, and to study the group responses to those shocks. The technical framework for this analysis is given by the dynamic index model studied in Sargent and Sims (1977) and Geweke and Singleton (1981). This approach has been recently used in order to study the dynamic properties of disaggregated (regional or sectoral) time series data - e.g. wage rate data in different industries (Watson and Engle, 1983), sectoral employment (Quah and Sargent, 1994), output and hours of work in different sectors (Forni and Reichlin, 1996). The basic idea underlying this kind of models is to characterise the cross section dependence between the different series via the identification of possibly serially correlated common factors. Estimates show that the common shock that captures the comovements of all the variables provokes a cyclical reaction in all the variables considered. The movement in the relative price, which increases during the expansion phase, stresses the importance of this variable in the cycle, and it is consistent with the model in Caplin and Leahy (1999) where the price is endogenous and its increase in response to a demand shock smooths the response of car expenditure and the number of buyers.

In chapter 2 a different approach is taken to study the covariance pattern of consumption, income and of the interest rate. The methodology consists in estimating multivariate moving average systems for grouped variables: this approach has the advantage of allowing to explicitly take into account the measurement error present in the individual measures of consumption and income. Having characterized the dynamic properties of average group data, we map the pattern of correlations that emerges from the data to those implied by different theoretical models. A related study has been carried out by Altonji et al. (1987), who focus however on idiosyncratic variability by removing time effects. The focus of chapter 2 is, on the contrary, the modelling of business cycle frequency shocks; this approach might be particularly informative in characterizing smoothing mechanisms and to interpret the shocks identified in the analysis in terms of underlying structural models. Results show a prolonged dynamics of non-durable expenditure, which cannot be entirely explained by the influence of lagged shocks to income and to the interest rate. Some evidence on excess sensitivity of consumption to lagged income shocks has also been found, although results are not clear-cut. The long run response of consumption to a unit shock in the interest rate has been estimated to be about 1, both in a system in which income is included among the equations and in a system of consumption and interest rate alone.

With a change of focus, a panel data technique is used in chapter 3 in order to evaluate different models of the individual earnings process. The use of a longitudinal data set allows to study the purely idiosyncratic components of earnings, while aggregate shocks are removed in preliminary regressions. The analysis is based on the Bank of Italy Survey of Household Income and Saving and exploits the panel component of the Survey. The Bank of Italy Survey is drawn every two years: this feature raises identification problems as the first-order autocovariance is not observed. However, it is possible to use the panel dimension of the data set in order to discriminate between several specifications that imply different covariance patterns. In order to exploit the differences that may arise due to heterogeneous education attainments, estimates are performed by education group.

Results show that the AR(1) plus individual effect model provides the best characterisation of the unobserved component of the earnings process. The estimated autoregressive parameter however is well below unity, indicating stationarity.

Having characterised the dynamic properties of the Italian earnings process, in chapter 4 a study of pension earnings distribution in Italy is undertaken. The analysis explores how effective is redistribution of pensioners' income under different Social Security systems. In this final chapter, a traditionally redistributive earnings-related formula is compared with a contribution based pension formula. In particular, earnings related formulae may operate within cohort redistribution from poor to rich (this point has been raised, for example, by Castellino, 1995, and James, 1997). Contribution-based formulae that do not actively operate redistribution could then enhance equity by removing the inequities implicit in the earnings related systems. The work focuses in particular on the recent reforms undertaken in the Italian Social Security system: between 1992 and 1995 the Italian system was deeply reformed and is now moving from an earnings-related to a contribution-based scheme. Simulations have been calibrated on the Italian male dependent workers earnings process estimated in chapter 3 and on the Italian Social Security system, before and after the reforms undertaken in 1992 and 1995. Results show that the new contribution-based scheme (after the reform in 1995) reduces inequality among all groups considered, i.e. private or public dependent workers of different education groups.

Chapter One

A Dynamic Index Model for Durable Expenditure

1. Introduction

Since the work in Mankiw (1982), considerable effort in the literature has been devoted in order to model the consumers' decision to buy durable goods and to understand the dynamic behaviour of aggregate durable expenditure, and in particular how aggregate time series behaviour depends on individual behaviour.

In this chapter, the main task of the research is to analyse the dynamic properties of cohort durable expenditure. The empirical application makes use of a dynamics index model, which allows to identify aggregate shocks, common to all individuals, and to study the individual (cohort) responses to those shocks. The technical framework for this analysis is given by the dynamic index model studied in Sargent and Sims (1977) and Geweke and Singleton (1981). This approach has been recently used in order to study the dynamic properties of disaggregated (regional or sectoral) time series data - e.g. wage rate data in different industries (Watson and Engle, 1983), sectoral employment (Quah and Sargent, 1994), output and hours of work in different sectors (Forni and Reichlin, 1996). The basic idea underlying this kind of models is to characterise the cross section dependence between the different series via the identification of possibly serially correlated common factors.

As this approach is based on large T asymptotics, empirical implementation requires a data set with a long time series dimension. Unfortunately, most of the existing data sets containing information on consumption do not have a panel dimension, so the analysis is conducted on grouped data constructed using 25 years of quarterly cross sections from the UK Family Expenditure Survey (FES). The FES survey is not a panel; as every year the sample is drawn anew, individual households are not followed through time. However, it is possible to define groups of households (i.e., cohorts) according to a fixed membership rule, as the year of birth of the head of the household (Deaton, 1985). If the functional forms are linear in the parameters - but not necessarily in the data - mean cohort behaviour reproduces the form of individual behaviour and the cohort means can be treated as panel data. While purely idiosyncratic variability is not identified, this procedure makes it possible to identify dynamic effects at the group level.

Section 2 provides a description of a dynamic index structure, while section 3 reviews the main literature on the consumers' decision to buy a durable good. In section 4 the empirical specification of the dynamic index model is set up and section 5 discuss the estimation procedure. Section 6 provides a description of the data used in the estimation, and section 7 discusses results. Section 8 concludes the chapter.

2. A dynamic index structure

The dynamic index model has been used since the work of Geweke (1977) and Sargent and Sims (1977) to study the dynamic interrelations of a system in which the dependent variables are function of common and possibly unknown shocks. This approach has been more recently extended in order to estimate the dynamic properties of disaggregated time series data: Watson and Engle (1983) discuss an estimation procedure based on the time domain representation of the problem and apply their methodology to study wage rate data in different industries; Quah and Sargent (1994) use a similar approach to estimate a model of sectoral employment in which the number of sectors is greater than the number of time periods; Forni and Reichlin (1996) propose an estimation procedure in the case of individual data aggregated at group level and apply it to output and hours of work in different sectors.

Here I will show the basic structure of the model as applied to individual time series, in subsequent sections I will discuss identification problems and the estimation procedure.

Let x_t^i be a p-vector of individual variables observed for i= 1, ..., N, and u_t a q-vector of mutually independent and unobservable shocks common to all individuals and to all variables with q, the number of shocks, strictly less than the total number of variables, Np. A dynamic index structure is then defined as:

$$x_t^i = \mathbf{A}^i(L)u_t + \varepsilon_t^i \qquad (1)$$

where ε_t^i is an idiosyncratic shock, specific to a particular x_t^i variable for individual i at time t. Both the common macro shocks and the idiosyncratic shock can be serially autocorrelated, but they are pairwise orthogonal. The common macro shocks affect the x_t^i variables through the $(p \times q)$ matrix of lag polynomials $A^i(L)$; the coefficient on the lag polynomials are individual specific, so that different individuals will react differently to the same macro shock. In this model the covariance structure of the variables considered is restricted so that all the dynamic interrelations between the dependent variables are driven by a relatively small number of unobservable common factors.

As stressed in Quah and Sargent (1994), it is important to notice that in this model the common macro shocks and the idiosyncratic shocks are assumed to be orthogonal: this feature states the difference with the common trends model for cointegrated time series, in which the common trends are correlated with the equation-specific disturbances. In this context, where the equation-specific disturbances represent idiosyncratic shocks faced by the individuals, the orthogonality conditions imposed in the dynamic index model seem more appropriate.

In the present analysis, the variables of interest will be the expenditure on durable commodities (cars) for different individuals. The system is dynamic, so that the dynamic response of durable commodities to the unobserved common macro shocks will be captured.

Estimation of model (1) at the individual level would require a panel data set with a long time dimension. Not only most of the existing data sets containing information on consumption do not have a panel dimension, but the estimation of the system with a great number of individual observations would be in practice intractable. The analysis will be therefore conducted using grouped (or cohort) data built over a long time series of independent cross-sections. Cohorts are built defining a fixed membership rule (e.g. date of birth) so that groups of individuals belonging to the same cohort can be followed over time. This procedure, described in detail in Deaton (1985), consists in building time series of group averages of the variables of interest and in performing the analysis using the group averages. If the individual model presents some non-linearities the group averages can be constructed on the non-linear transformations of the variables at the individual level.

Model (1) can be rewritten in terms of cohort averages, defining, for cohort c=1, ..., C:

$$x_t^c \equiv \frac{1}{N_t^c}\sum_{j\in c} x_{j,t}\,, \qquad \varepsilon_t^c \equiv \frac{1}{N_t^c}\sum_{j\in c} \varepsilon_{j,t}\,, \qquad \mathbf{A}^c(L)u_t \equiv \frac{1}{N_t^c}\sum_{j\in c} \mathbf{A}_j(L)u_t$$

where N_t^c, the size of the cohort, varies with time and across cohorts. The model therefore takes the form:

$$x_t^c = \mathbf{A}^c(L)\,u_t + \varepsilon_t^c \tag{2}$$

In the present work, the dependent variables will be income, the average car expenditure, as well as the number of buyers for each cohort at each point in time. In section 4 I will show in more detail the empirical specification used in this work.

It is worth noticing that the analysis conducted at group level permits to have a time dimension in the data, and although idiosyncratic variability is not identified, different cohorts are allowed to respond in a different way to the common macro shocks.

3. Competing theories on durable expenditure

The modelling of the consumer's decision to buy a durable good is quite complex as the analysis cannot rely on the typical – and convenient – assumptions usually made to model non-durable consumption. The two major aspects of the problem can be identified in the intertemporal non-separability of preferences defined over durable goods and in the possibility of corner solutions, i.e. a consumer may decide not to buy a durable good at all in a certain period. I will analyse both these features in turn.

As a durable good lasts for many periods, any analytical representation of the problem faced by the consumer must take into account temporal dependencies. One way to model durable goods in a life-cycle context is to include the stock of durables in the utility function and to assume that the service flow provided by the good is proportional to the stock. The problem faced by the consumer can be written as:

$$\max E_t\left[\sum_{j=0}^{T-t}\beta^j u_{t+j}(c_{t+j},S_{t+j})\right]$$

s.t. $\quad A_{t+1}=(1+r_t)A_t+y_t-p_tc_t-v_td_t$

where β is the rate of time preference, c_t is non-durable consumption, S_t is the stock of durable held by the consumer and d_t is expenditure on the durable good at time *t*, A_{t+1} is wealth at the beginning of period *t+1*, p_t is the price of the non-durable good and v_t is the price of the durable good. The problem is inherently dynamic as the stock of durables is assumed to evolve as:

$$S_t=(1-\delta)S_{t-1}+d_t$$

where δ is the depreciation parameter. A rational consumer choosing the level of expenditure d_t will take into account the effect that this has on the stock also for subsequent periods.

Using a quadratic utility specification, Mankiw (1982) showed that durable expenditure should follow an ARMA(1,1) process with an MA coefficient equal to 1-δ:

$$e_t=\alpha_0\delta+\alpha_1e_{t-1}+\varepsilon_t-(1-\delta)\varepsilon_{t-1}$$

where ε_t captures the impact of all new information about the future that becomes available to the consumer at time *t*. The dynamic behaviour of this model is very simple. If there is a unit shock in the innovation to income in period *t*, then durable expenditure evolves as:

Time	ε	e
t	1	1
t+1	0	$\alpha_1 - (1-\delta)$
t+2	0	$\alpha_1(\alpha_1 - (1-\delta))$
t+3	0	$\alpha_1^2(\alpha_1 - (1-\delta))$
...	...	...

If $\alpha_1 < 1$ the model implies a smooth adjustment to the previous level of expenditure[1], while if $\alpha_1 = 1$ from time t+1 onwards expenditure settles at the new equilibruim value of δ.

Mankiw (1982) tested his model on US post-war data, but data did not support this model. In particular, data do not reject the hypothesis that durable expenditure is an AR(1) process, which implies that the estimated value for δ, the depreciation rate, is equal to one. This finding, known in the literature as "Mankiw's puzzle", motivated further research in order to characterize the time series properties of durable expenditure[2].

A possible explanation for the empirical failure of Mankiw's model is the presence of adjustment costs associated with changing the durable stock. Bernanke (1984, 1985) studied the problem faced by the consumer in the presence of convex (quadratic) adjustment costs. This simplifying assumption permits him to derive closed form solutions to the problem. Following Hall and Mishkin (1982), the exogenous income process is assumed to be composed of a deterministic part and a stochastic component. The latter is formed by a permanent component and by a transitory one, that is:

[1] Unless $1-\delta > \alpha$, in which case expenditure falls to a negative value and then goes gradually to zero.

[2] Caballero (1990) estimates a non parsimonious model for durable expenditure, and shows that changes in durable expenditure are actually consistent with the permanent income hypothesis, but the dynamic behaviour is more prolonged than predict by the simple Mankiw's model. An explanation for this "sluggish" behaviour is the presence of fixed costs, as analysed in Caballero (1993).

$$\tilde{y}_t = \bar{y}_t + y_t^R + y_t^W$$
$$y_t^R = y_{t-1}^R + u_t$$
$$y_t^W = \eta_t$$

where u_t and η_t are independent i.i.d. processes.

Revisions in permanent income are then equal to permanent shocks to income plus the annuity value of transitory shocks:

$$y_t^P - y_{t-1}^P = u_t + \beta\eta_t$$

where β is the annuization factor. The stock of durables is assumed to evolve as:

$$S_t = (1-\delta)S_{t-1} + d_t$$

The desired stock of capital S^* is proportional to permanent income, so that it evolves according to:

$$S_t^* - S_{t-1}^* = \alpha(u_t + \beta\eta_t)$$

Bernanke (1984, 1985) shows that given quadratic costs and a quadratic utility function the stock of capital evolves according to:

$$S_t - S_{t-1} = \lambda(S_{t-1}^* - S_{t-1})$$

and expenditure is:

$$e_t = \lambda(S_{t-1}^* - S_{t-1}) + \delta S_{t-1}$$

Assuming there is a unitary permanent shock to income, the response function is in this case:

Time	u	y	e
T	1	1	$\lambda\alpha$
t+1	0	1	$\lambda\alpha(1+\delta-\lambda)$
t+2	0	1	$\lambda\alpha(1+(\delta-\lambda)(2-\lambda))$
t+3	0	1	$\lambda\alpha(1+(1+(2-\lambda)(1-\lambda))(\delta-\lambda))$
...	0	1	...
t+n	0	1	$\alpha\delta$

If $\lambda > \delta$ the function in the first period is higher than the long run value, and it smoothly goes to the long run equilibrium afterwards. On the contrary, if $\delta > \lambda$ expenditure smoothly increases up to the long run equilibrium value.

This kind of models predicts that households smoothly adjust to the equilibrium, purchasing small quantities of durables until they reach the desired stock. This feature makes the convex adjustment cost model unattractive from a theoretical point of view and, more importantly, it is inconsistent with the data, as households adjust their stock infrequently.

Hence, non-convex costs have been introduced into the analysis: Grossman and Laroque (1990) proved in a simple setting that the optimal policy for a consumer who faces non-convex adjustment costs is an *(S,s)* policy of the kind studied in inventory policy. In order to describe the model, I follow Lam (1991), who sets the problem as in Bernanke (1984, 1985). However, instead of assuming quadratic adjustment costs, Lam assumes the desired stock of durables evolves according to a threshold adjument rule:

$$E_t = \begin{cases} S_t^* - S_t & \text{if } S_t^* - S_t > \gamma_U \\ S_t^* - S_t & \text{if } S_t^* - S_t < \gamma_L \\ 0 & \text{otherwise} \end{cases}$$

This implies that if the desired stock exceeds the existing stock by more than the upper threshold γ_U, the household will adjust its stock. Similarly, if the desired stock is less than the actual stock, so that the difference between the two is less then the lower threshold γ_L, the household will adjust.

A shock to income therefore has an effect only if the desired stock increase induced is big enough to make it convenient. Aggregating among agents can lead to very complicated dynamics in aggregate demand as fluctuations will be driven both by fluctuations in the number of consumers making the purchase and by fluctuations in the size of the purchase.

In particular, both macro and micro shocks affect both the target variable and the two trigger points. As Bertola and Caballero (1990) show, if all individuals are identical and there are only macro shocks in the economy, then the aggregate path of the target variable will be the same as for a single individual, with a single peak in correspondence to adjustment. On the opposite, if there are only idiosyncratic shocks, the aggregate will behave smoothly. However, heterogeneity is quantitatively important: Attanasio (2000) uses micro data in order to estimate an (S,s) rule and studies aggregation problems. He finds that, in response to a permanent shock to the desired target variable (defined as the stock of durables normalised by non-durables), the (S,s) model has a higher impact and is lower afterwards, as compared to the frictionless permanent income hypothesis and to the partial adjustment model.

Adda and Cooper (2000) develop a model where consumers are heterogeneous in vintages and study in particular dynamics due to changes in the cross-sectional distribution of cars (echo effects). If a large number of individuals, driven by a positive shock, purchase a car at a certain point in time, it can be expected that there will be a large number of consumers buying a new car *t* years later. Because of heteroscedasticity, however, the echo will be smaller than the original impact.

While in Adda and Cooper the equilibrium model is solved under the hypothesis of constant marginal costs, so that price is exogenous, Caplin and Leahy (1999) set up an equilibrium model and focus on the endogenous movements of the relative price. In order to solve the model, however, they rule out echoes effects in the demand of cars. They show that a linearization of the model leads to a VAR in the number and size and purchases. They find that after a permanent positive shock to income the increase in relative price creates an incentive for agents to delay purchases. This delay then smooths out the response to the shock, which explains the delayed response of durable expenditure to income innovations documented in Caballero (1990). If a shock is temporary, the threshold, after an initial increase, will eventually return to its long run value. This means that purchases, after an initial increase, will be lower than the average when the trigger point settles to its previous level. This kind of distributional dynamics, due to movements in the thresholds, is not ruled out in Caplin and Leahy's model.

4. Empirical specification

The dynamic behaviour of purchases is studied through a flexible dynamic index structure that allows to identify the effect of common shocks to variables present in the system. As in the recent literature that studies models with non-convex costs it has been stressed the importance of the behaviour of the number of consumers making the purchase and of the size of the purchase, the variables studied are: (cohort) average car expenditure, the fraction of buyers, (cohort) average income, and the relative price, defined as the ratio of the car price index and the non-durable price index. Sets of conditions for identification of the dynamic index model are discussed in Geweke and Singleton (1981). Here it is assumed that the common macro shocks are pairwise orthogonal and that they are orthogonal also to the idiosyncratic (cohort-specific) shocks. In addition, exclusion restrictions must be placed in order to identify more than one common shock. In what follows, two common shocks will be identified: first, a common component that identifies the comovement between all the variables present in the system; second, a component, orthogonal to the first one, that captures the commonalities between all the variables but income. Although this is not a causal analysis, the exclusion restrictions are obviously placed in order to distinguish between a shock that moves all the variables in the system and a shock that moves only prices and car purchases. The system can be written as:

$$\frac{1}{\tilde{N}_t^c}\sum_{j\in c}\log(dur)_t^j \equiv \log(dur)_t^c = A_d^c(L)u_t^1 + B_d^c(L)u_t^2 + \varepsilon_t^{dc}$$

$$\frac{\tilde{N}_t^c}{N_t^c} = A_{buy}^c(L)u_t^1 + B_{buy}^c(L)u_t^2 + \varepsilon_t^{buyc}$$

$$\frac{1}{N_t^c}\sum_{j\in c}\log(income)_t^j \equiv \log(income)_t^c = A_y^c(L)u_t^1 + \varepsilon_t^{yc}$$

$$\log(p_t^{dur}/p_t^{ndur}) = A_p(L)u_t^1 + B_p(L)u_t^2 + \varepsilon_t^p$$

for $c=1, \dots, C$,

where N_t^c is the size of cohort c in period t, and $\tilde{N}_t^c$ is the number of people in cohort c who buy a durable good at time t. Expenditure of the durable good is computed conditional on the individuals buying the good. $A(L)$, $B(L)$ and $C(L)$ are polynomials in the lag operator.

All variables are detrended (in a deterministic way), in the sense that the partial effect of demographic variables is taken out: the dependent variables in the analysis are then the fluctuations around the deterministic trend of durable expenditure and the other variables of interest. The possible presence of stochastic trends will be captured by the lag structure of the system, as it is shown in the next section.

5. Estimation

Since the works of Sargent and Sims (1977) and Geweke (1977), several approaches to estimate dynamic factors models have been devised in the literature. In particular, the issue of estimating this class of models in the presence of time series of cross-section data has been developed by Quah and Sargent (1994) who propose an estimation strategy based on the study by Watson and Engle (1983). This approach is based on the state space representation of the model, and therefore is carried out in the time domain. Frequency domain approaches to the estimation of dynamic index models for time series of cross-section, as in Forni and Reichlin (1996), are based on the assumption that the cross-section dimension is large enough for the idiosyncratic component to die out on average (Granger, 1987).

As the present analysis is carried out at the cohort level, and the number of groups considered is rather limited, this last approach is not suitable; on the contrary, maximum likelihood estimation of the state-space representation of the model has proved to be particularly profitable. It is this strategy that I now turn to describe.

The dynamic index model set up in the previous section can be written as:

$$x_t^c = A^c(L)u_t + v_t^c \tag{3}$$

where x_t^c is a $(p\times 1)$ vector of (possibly) cohort-specific variables, u_t is a $(q\times 1)$ vector of pairwise orthogonal shocks identical for all cohorts. $A_j(L)$ is a $(p\times q)$ matrix of polynomials in the lag operator L with maximum lag M_a and v_t^c is a $(p\times 1)$ vector of cohort-specific shocks. These components will incorporate genuine cohort-specific shocks as well as the average idiosyncratic shocks, including measurement error. The analysis is carried out assuming that all the components in v_t^c are pairwise orthogonal. In addition it is assumed that u_t and v_t^c are pairwise orthogonal.

The common shocks u_t are assumed to be generated by the autoregressive process:

$$\Gamma(L)u_t = \eta_t \tag{4}$$

where $\Gamma(L)$ is a diagonal matrix with kth entry given by:

$$1 - g_k(1)L - g_k(2)L^2 - \ldots - g_k(Mg)L^{Mg}$$

and the error term is white noise.

The cohort-specific shocks v_t^c are also assumed to have a finite autoregressive distribution:

$$B^c(L)v_t^c = \varepsilon_t \tag{5}$$

where $B^c(L) = diag[b_1^c(L)\ldots\ldots b_p^c(L)]$

Combining (5) and (4), equation (3) can be rewritten as:

$$\begin{aligned} B^c(L)x_t^c &= B^c(L)A^c(L)u_t + \varepsilon_t^c \\ &= \Phi(L)u_t + \varepsilon_t^c \end{aligned} \tag{6}$$

Equations (6) and (4) are easily written respectively as the measurement and the transition equations of the state-space representation of the model:

$$\begin{aligned} x_t &= \Phi\widetilde{u}_t + B\widetilde{x}_{t-1} + \varepsilon_t \\ \widetilde{u}_t &= G\widetilde{u}_{t-1} + \eta_t \end{aligned}$$

Defining $M_{a+b}=M_a+M_b$ and $M_h=Max(M_{a+b}, M_g)$, x_t is a (Cp) vector, $\widetilde{x}_{t-1}$ is a (CpM_b) vector containing x values for each variable p, each cohort C and each lag up to M_b; Φ is a ($Cp \times Q(M_h+1)$) matrix in which, if $M_{a+b}<M_h$, the last M_h-$Q(M_{a+b}+1)$ columns are zero, $\widetilde{u}_t$ is a $Q(M_h+1)$ vector containing the common shocks at time t and lagged, and the error term is a (Cp) vector containing the error terms of all the equations for all cohorts. Denoting an element of Φ as $f_{ij}^c(lag)$ where c represents cohort, i the equation, and j the factor, the measurement equation is:

$$\begin{bmatrix} x_{1t}^{1} \\ x_{1t}^{2} \\ \vdots \\ x_{1t}^{C} \\ \vdots \\ x_{pt}^{C} \end{bmatrix} = \left[\begin{array}{cccc:cc} f_{11}^{1}(0) & f_{12}^{1}(0) & \cdots & f_{1Q}^{1}(M_{a+b}) & \cdots & 0 \\ f_{11}^{2}(0) & \cdots & \cdots & f_{1Q}^{2}(M_{a+b}) & \cdots & 0 \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ f_{p1}^{C}(0) & \cdots & \cdots & f_{pQ}^{C}(M_{a+b}) & \cdots & 0 \end{array}\right] \begin{bmatrix} u_{t}^{1} \\ u_{t}^{2} \\ \vdots \\ u_{t}^{Q} \\ u_{t-1}^{1} \\ \vdots \\ u_{t-Mh}^{Q} \end{bmatrix} + \begin{bmatrix} b_{1}^{1}(1) & 0 & \cdots & b_{1}^{1}(2) & \cdots & 0 \\ 0 & b_{1}^{2}(1) & \cdots & 0 & \cdots & 0 \\ \cdots & & & & & \\ \cdots & & & & & \\ \cdots & & & & & \\ 0 & \cdots & b_{p}^{C}(1) & 0 & \cdots & b_{p}^{C}(M_{b}) \end{bmatrix} \begin{bmatrix} x_{1,t-1}^{1} \\ x_{1,t-1}^{2} \\ \vdots \\ x_{p,t-1}^{C} \\ \vdots \\ x_{p,t-Mb}^{C} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t}^{1} \\ \varepsilon_{1t}^{2} \\ \vdots \\ \varepsilon_{1t}^{C} \\ \vdots \\ \varepsilon_{pt}^{C} \end{bmatrix}$$

Similarly, denoting g_i(lag) an element of the matrix G (which is a square matrix of dimension $k(M_h+1)$), the transition equation can be written as:

$$\begin{bmatrix} u_{t}^{1} \\ \vdots \\ u_{t}^{Q} \\ u_{t-1}^{1} \\ \vdots \\ u_{t-Mh}^{Q} \end{bmatrix} = \left[\begin{array}{cccccccc:c} g_{1}(1) & 0 & g_{1}(2) & 0 & \cdots & g_{1}(M_{g}) & 0 & & \\ & \ddots & & & \cdots & & \ddots & & \mathbf{0} \\ 0 & g_{Q}(1) & 0 & g_{Q}(2) & \cdots & 0 & g_{Q}(M_{g}) & & \\ \hdashline & & & & & & & & \\ & & & \mathbf{I} & & & & & \mathbf{0} \\ & & & & & & & & \end{array}\right] \begin{bmatrix} u_{t-1}^{1} \\ \vdots \\ u_{t-1}^{Q} \\ u_{t-2}^{1} \\ \vdots \\ u_{t-Mh-1}^{Q} \end{bmatrix} + \begin{bmatrix} \eta_{t}^{1} \\ \vdots \\ \eta_{t}^{Q} \\ \hdashline \mathbf{0} \end{bmatrix}$$

Maximum likelihood estimation of this model is in principle straightforward. As shown for example in Harvey (1993) the log likelihood can be written as:

$$\log L(\theta) = \text{constant} - \frac{1}{2}\sum_{t=1}^{T} \log|F_t| - \frac{1}{2}\sum_{t=1}^{T} \zeta_t' F_t^{-1} \zeta_t$$

where both the covariance matrix, F_t and the vector of prediction errors, ζ_t, can be recursively computed using the standard Kalman filter equations (see Appendix for more details).

6. Data description

The data used in the estimation are drawn from the UK Family Expenditure Survey (FES) from 1974 to 2000. In that survey, about 8,000 families in the UK are interviewed each year and they are asked to fill diaries in which they record all the expenditures they make for two weeks. The survey records also information on demographic and labour supply variables for each member of the family. Every fortnight the sample is drawn anew, so that there is no panel dimension in the data.

From this sample I selected families in which the household head is married or cohabitating, and are living in England, Scotland or Wales. In addition, in order to have balanced sample in the estimation by cohort, I selected out families in which the household head was born before 1939 or after 1953. These exclusions result in a sample of 31,981 observations. Cohorts are then constructed defining three date-of-birth groups, as shown in table 1.

Table 1. Cohort definition

Cohort	Year of Birth	Age in 1974	Age in 2000	Mean Cell Size
1	1939-43	33	59	87.89
2	1944-48	28	54	114.44
3	1949-53	23	49	102.26

The variables used in the estimation are cohort averages of the logarithm of expenditure on cars, a variable that takes value 1 if the family bought a car and zero otherwise, the logarithm of family income and the relative price of cars, defined as the ratio between the car price index and the non durable price index. Expenditure on cars is conditional on buying, i.e. the cohort average is computed using only those families who report a positive expenditure on cars.
All the variables used in estimation have been detrended. In particular, all the variables considered have been regressed for each cohort on a polynomial in age[3] and on the logarithm of family size in order to capture the demographic trends, as well as on quarterly dummies in order to take seasonality into account[4]. Detrending has been performed one cohort at a time in order to

[3] Where "age" is the age of the household head.

[4] Unfortunately, respondents report whether they bought a car and how much they spent during the 3 months prior to the interview from 1988 (1) to 1996 (1). Before and after that period they are asked whether they purchased a car during the 12 months prior to the

remove also the cohort effects[5]. The relative price has also been detrended and it is expressed in logarithm.

7. Results

The two-shock model described in section 4 has been estimated by maximum likelihood as described in section 5. The dependent variables are the logarithm of car expenditure (conditional on buying), the proportion of buyers, the logarithm of income and the relative price for cars for each cohort and for each quarter. The averages for car expenditure have been computed only for the sub-sample of families who participate in the car market.

All variables have been deterministically detrended as described in the previous section. As in the model described in section 4, the first shock captures the comovement of all the variables present in the system, while the second shock captures the comovement of all variables but family income.

Results are shown in tables 2-5, where the estimated coefficients are reported. The idiosyncratic, cohort specific shocks did not appear to be serially correlated (i.e. M_b=0); in addition, lagged effects of the common shock did not appear after the second lag[6] (i.e. M_a=2). The factor loadings are shown in table 2, while the estimated (square roots of the) variances of the idiosyncratic components are displayed in table 3. The common shock has been estimated as an AR(8) process (M_g=8), with coefficients displayed in table 4. Table 5 reports the percentage of the variance of each dependent variable in the system captured by the common shock[7]. This percentage varies from 8.5 per cent for cohort 2 log expenditure, to 58 per cent for cohort 2 proportion of buyers. For the relative price the

interview. Therefore, log car expenditure and the dummy variable for buyers have been also regressed on a dummy that takes value 1 between 1988 (1) and 1996 (1), and zero elsewhere.

[5] This procedure allows different cohorts to have a different shape in the polynomials in age.

[6] Estimation with more lags, both in the transition and in the measurement equation, produced little difference in the results.

[7] This has been estimated equation by equation as 1 minus the ratio of the estimated cohort-specific shock variance to the sample variance of the corresponding variable. See Appendix for more details.

percentage of variance explained by the macro components is equal to 99.65 per cent.

A graphical representation of the results is presented in figure 1 and 2, where the response functions of the variables considered to a temporary unit shock to the first and second common component are shown. Starting with the first common component, which captures the comovement of all the variables considered, graphs show that a positive temporary shock in the common component results in an increase in the relative price and in income for all cohorts; in particular, both variables increase up to lag 6 or 7, then start declining reaching the baseline at lag 12-13, and remain below the baseline up to lag 24. The same cycle is then repeated, with smaller oscillations as time increases. For the three cohorts considered, the income response does not present main differences. In response to same shock, car expenditure and the fraction of buyers reaction is also shown. Both car expenditure and the fraction of buyers after an initial increase in response to the shock, start oscillating around the baseline, displaying a cycle very similar to income and price. However, they also present marked oscillations around this cycle, a feature that makes the picture, and especially the turn points in the cycle, less neat.

The reaction of the system to the second component, after a temporary unit shock, is a decrease in the relative price of cars, mirrored by a somewhat delayed increase in expenditure and in the fraction of buyers.

The same structure has been estimated on aggregate data, that is, using the same sample as before and aggregating it into a single cohort. Results for this model are shown in tables 6-9 and in figures 3 and 4. As in the previous model, the two common macro shocks have been estimated as independent AR(8) processes, with coefficients shown in table 8. As in the previous model, the first shock captures the comovement of all the variables present in the system, while the second shock captures the comovement of all variables but family income.

The estimated coefficients are shown in table 6 and 7. Table 9 reports the percentage of the variance of the dependent variable captured by the two common components. For the individual variables this percentage ranges between 24 and 56 per cent, while for the relative price is equal to 99 per cent.

Again, in order to discuss the implications of the estimated coefficients impulse response functions are reported for temporary unit shocks to both components. Figure 3 shows the response function to a shock in the first component of all the variables in the system. As the graphs show, all the variables considered react positively to the shock for the first periods, then they start declining and fall below the baseline around lag 13 (i.e. after 3 years). The oscillations are very similar to those estimated with the disaggregated model, and the smoothing due to aggregation is visible for car expenditure and the proportion of buyers. Summarizing, the reaction of all the variables in the system to a unit shock to the common component is positive and, although the structure does not support a causal analysis, it seems reasonable to interpret the increase in the relative price as driven by the increase in average expenditure in cars and in the number of buyers.

The reaction of the system to the second component, after a temporary unit shock, is an increase in the relative price of cars, mirrored by a decrease in expenditure and in the fraction of buyers. With the same caution as before, it is seems reasonable to interpret this second shock as a positive shock in the relative price of cars, which drives a reduction in the number of buyers and in the average expenditure on cars.

8. Conclusions

In this chapter the joint dynamics of cohort and aggregate car expenditure, fraction of buyers, income and price was analysed using a dynamic index model in order to identify common macro shocks to the variables of interest. Estimates show that the common shock that captures the comovements of all the variables provokes a cyclical reaction in all the variables considered. At the cohort-aggregate level, this cycle is quite neat for price and for income; it is present, although less neatly, also for car expenditure and for the fraction of buyers. The cycle is complete at about lag 24-26 (i.e. 6 – 6.5 years) for all variables, including average car expenditure and the fraction of buyers. The movement in the relative price, which increases during the expansion phase, stresses the importance of this variable in the cycle, and it is consistent with the model in Caplin and Leahy (1999) where the price is endogenous and its increase in response to a demand shock smooths the response of car expenditure and the number of buyers.

Table 2. Estimated coefficients (factor loadings)

	u_t^1	u_{t-1}^1	u_{t-2}^1	u_t^2	u_{t-1}^2	u_{t-2}^2
Log(car_1)	0.1356	0.0764	0.0013	-0.0350	0.2927	0.1306
(z)	(2.60)	(1.23)	(0.03)	(-0.25)	(2.06)	(0.89)
Log(car_2)	-0.0110	-0.0167	0.0707	-0.0165	0.2680	-0.0356
(z)	(-0.22)	(-0.31)	(1.54)	(-0.11)	(1.82)	(-0.25)
Log(car_3)	0.0332	0.0400	0.0476	-0.2315	0.0527	0.3514
(z)	(1.13)	(0.68)	(1.01)	(-1.66)	(0.43)	(2.47)
Buy_1	0.0008	-0.0076	0.0416	0.0588	0.0755	0.0100
(z)	(0.04)	(-0.37)	(3.01)	(0.92)	(1.31)	(0.18)
Buy_2	-0.0643	0.0716	0.0323	-0.0584	0.1198	0.0809
(z)	(-4.00)	(2.08)	(1.15)	(-1.07)	(3.15)	(1.35)
Buy_3	0.0210	0.0328	0.0091	-0.0939	0.1185	0.0632
(z)	(1.25)	(2.14)	(0.65)	(-2.35)	(2.71)	(1.55)
Log(income_1)	0.0345	-0.0014	0.0825	-	-	-
(z)	(1.84)	(-0.07)	(5.99)			
Log(income_2)	0.0338	0.0096	0.0667	-	-	-
(z)	(2.67)	(0.61)	(6.50)			
Log(income_3)	0.0336	0.0363	0.0499	-	-	-
(z)	(2.52)	(2.57)	(4.55)			
Relative price	0.0571	0.0378	0.0567	-0.0694	-0.1155	-0.0608
(z)	(13.94)	(5.29)	(8.59)	(-2.05)	(-26.16)	(-1.38)

Note: z is the ratio of the coefficient estimate to the estimated asymptotic standard error

Table 3. Residual variance estimates (square root)

	Cohort 1	Cohort 2	Cohort 3
Log(car)	1.4448	1.4601	1.3757
(z)	(12.78)	(12.79)	(12.76)
Buyers	0.5463	0.3272	0.3953
(z)	(12.27)	(9.67)	(11.20)
Log(income)	0.5056	0.3895	0.4008
(z)	(11.82)	(11.53)	(11.72)
Relative price	0.0249		
(z)	(2.42)		

Note: z is the ratio of the coefficient estimate to the estimated asymptotic standard error

Table 4. AR coefficients – shock equations

Lag	u_t^1	u_t^2
1	0.8958	0.1263
	(7.91)	(0.65)
2	-0.5986	0.1560
	(-5.23)	(1.00)
3	1.1759	0.3457
	(11.32)	(2.50)
4	0.1327	-0.3477
	(1.00)	(-2.31)
5	-0.8851	0.2278
	(-6.43)	(1.46)
6	0.4528	-0.1642
	(4.50)	(-1.11)
7	-1.0001	0.1682
	(-10.50)	(1.37)
8	0.6382	-0.1140
	(5.50)	(-0.82)

Note: in parenthesis the ratio of the coefficient estimate to the estimated asymptotic standard error

Table 5. Variance explained by macro components (percentage)

	Cohort 1	Cohort 2	Cohort 3
Log(car)	21.63	8.50	14.23
Buy	11.27	58.18	29.91
Log(income)	28.98	35.88	37.36
Relative Price	99.65		

Table 6. Estimated coefficients (factor loadings)

	u_t^1	u_{t-1}^1	u_{t-2}^1	u_t^2	u_{t-1}^2	u_{t-2}^2
Log(car_1)	-0.0235	-0.0242	-0.0333	-0.1308	0.0082	-0.1001
(z)	(-1.87)	(-0.75)	(-0.85)	(-1.10)	(0.34)	(-0.99)
Buy_1	-0.0059	-0.0049	-0.0161	-0.0207	-0.0223	-0.0150
(z)	(-1.85)	(-1.09)	(-0.96)	(-1.82)	(-2.12)	(-1.63)
Log(income_1)	-0.0190	-0.0324	-0.0237			
(z)	(-0.60)	(-3.44)	(-1.72)			
Relative price	-0.0250	-0.0384	-0.0250	0.0851	0.0132	0.0985
(z)	(-1.70)	(-1.70)	(-1.67)	(6.64)	(4.56)	(6.77)

Note: z is the ratio of the coefficient estimate to the estimated asymptotic standard error

Table 7. Residual variance estimates (square root)

	σ_ε^j	z
Log(car)	0.8196	(9.11)
Buyers	0.2871	(8.42)
Log(income)	0.2667	(8.55)
Relative price	0.0295	(4.54)

Note: z is the ratio of the coefficient estimate to the estimated asymptotic standard error

Table 8. AR coefficients – shock equations

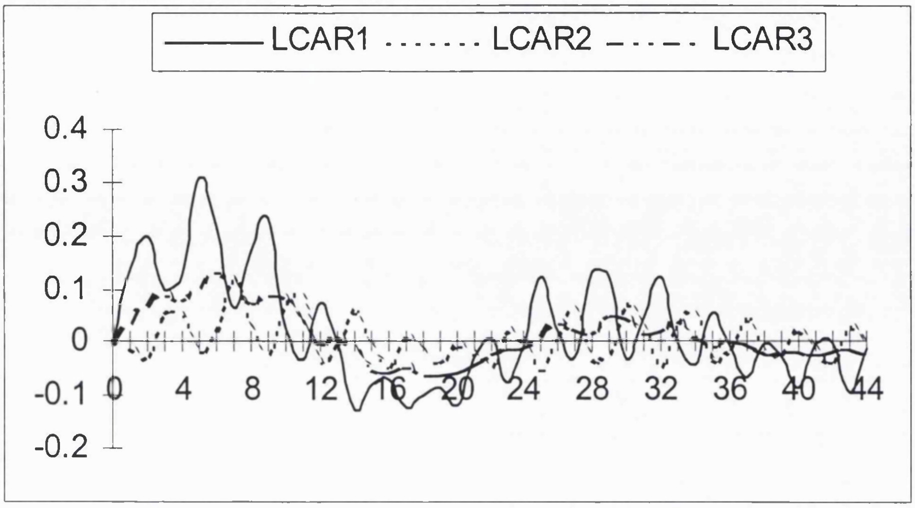
Lag	u_t^1	u_t^2
1	0.0315	1.2791
	(0.17)	(5.76)
2	2.1507	-1.7753
	(4.83)	(-4.19)
3	0.3793	1.8902
	(0.96)	(2.78)
4	-2.4155	-1.2458
	(-2.83)	(-1.55)
5	-0.6276	0.7824
	(-1.79)	(1.02)
6	1.6350	-0.4547
	(1.98)	(-0.79)
7	0.2328	0.1904
	(0.98)	(0.61)
8	-0.5225	-0.1073
	(-1.36)	(-0.54)

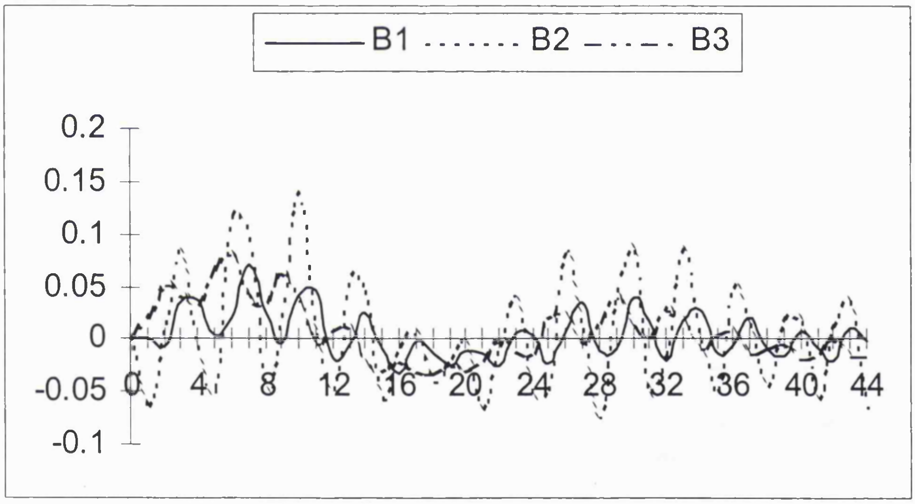
Note: in parenthesis the ratio of the coefficient estimate to the estimated asymptotic standard error

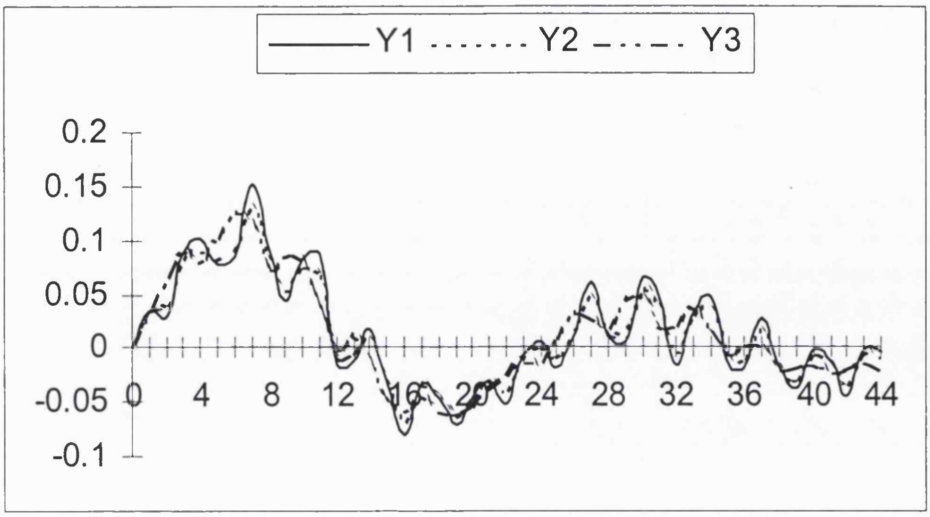
Table 9. Variance explained by macro components

	%
Log(car_1)	24.11
Buy_1	30.55
Log(income_1)	56.01
Relative Price	99.50

Figure 1. Cohort model – Response to the common shock



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0 4 8 12 16 20 24 28 32 36 40 44

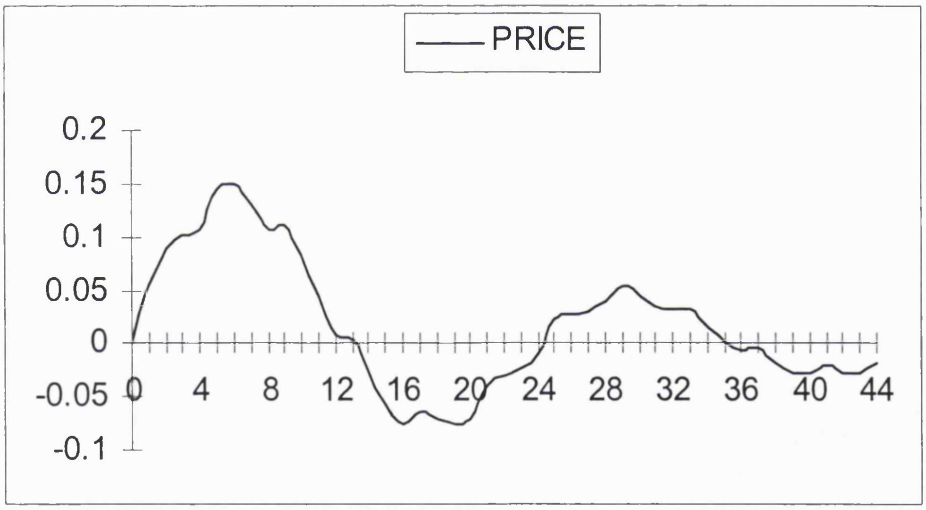
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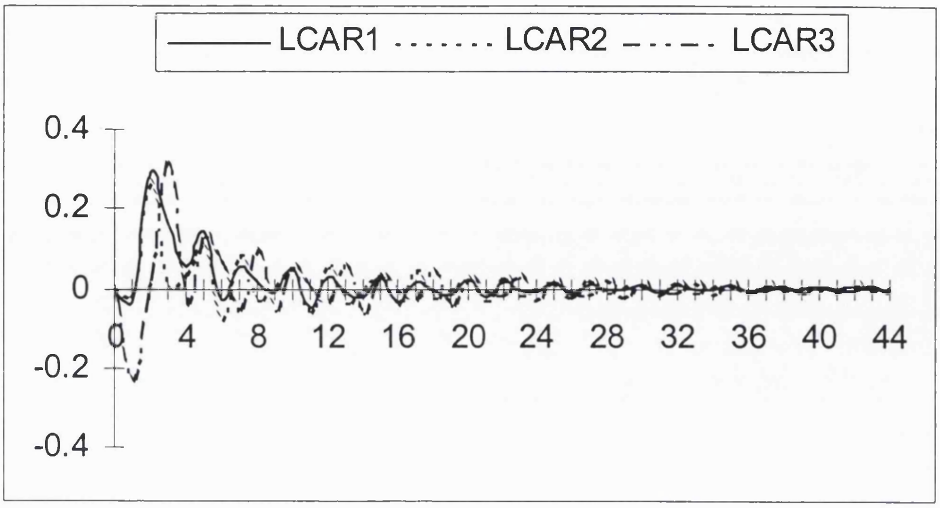
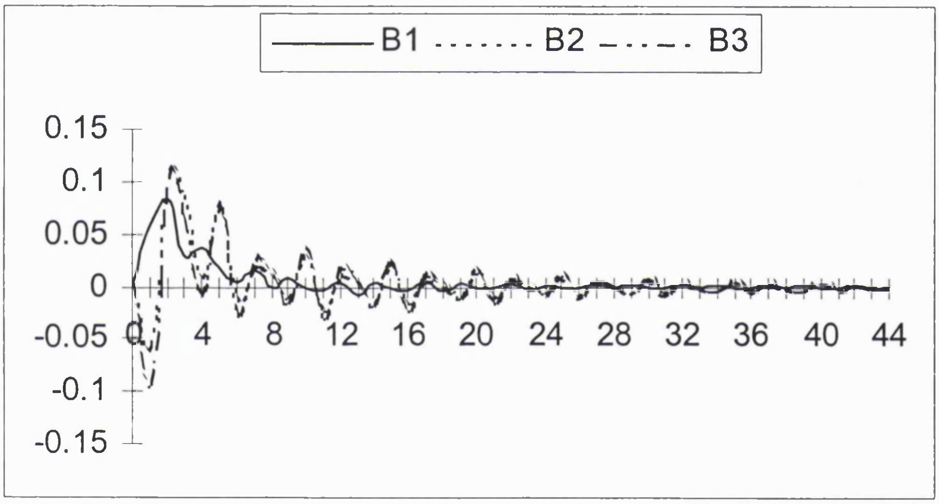
Figure 2. Cohort model – Response to second shock





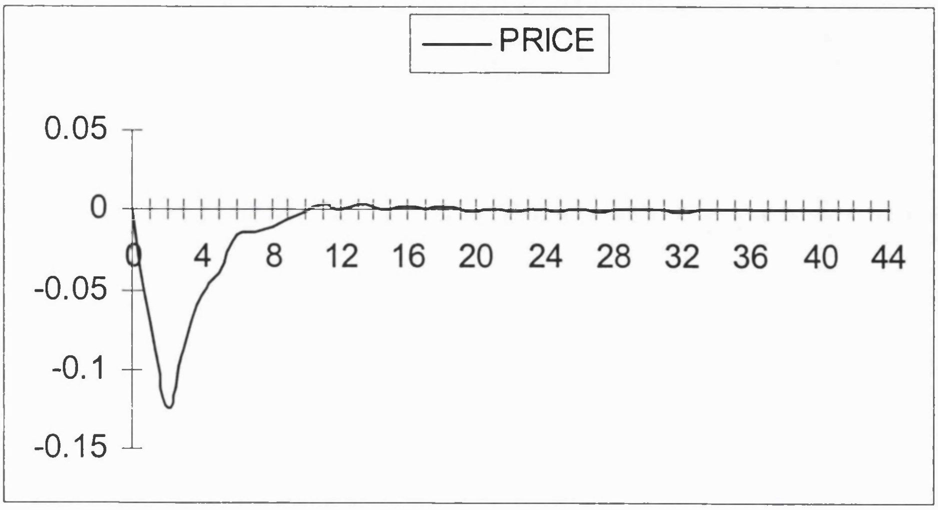
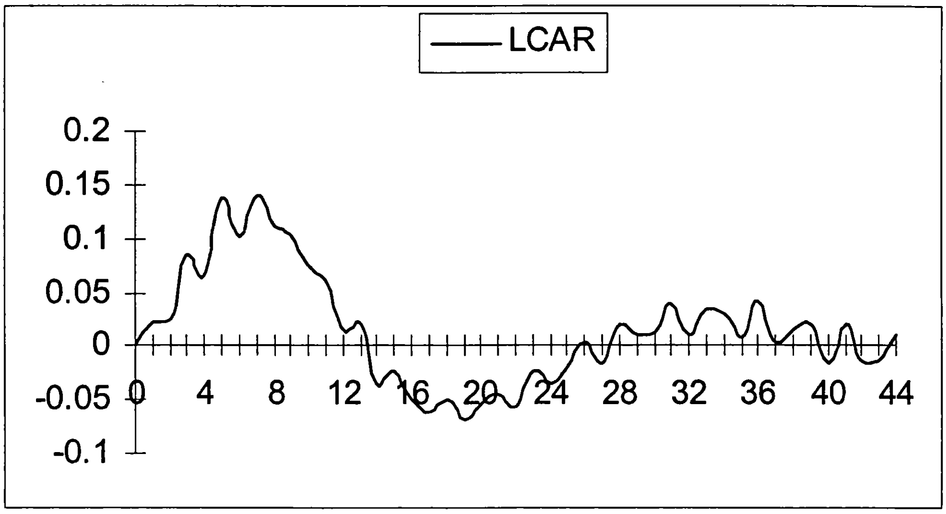
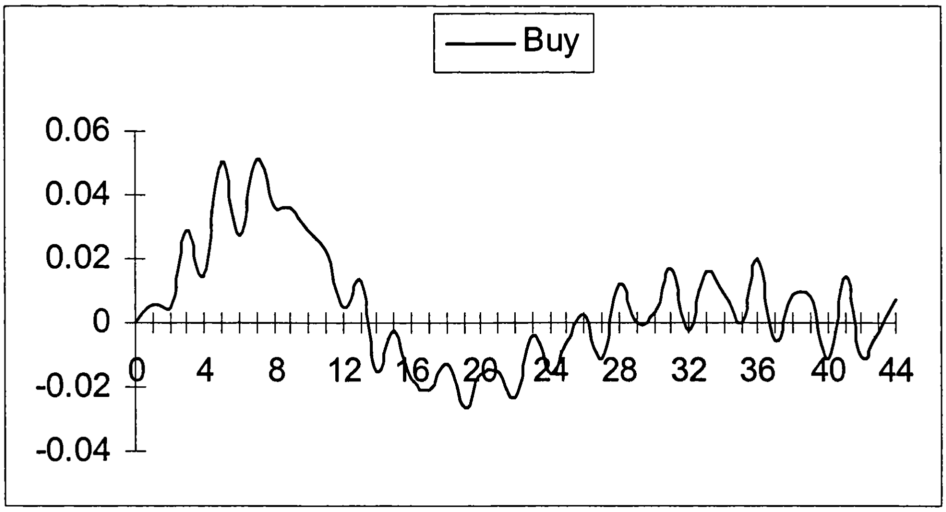
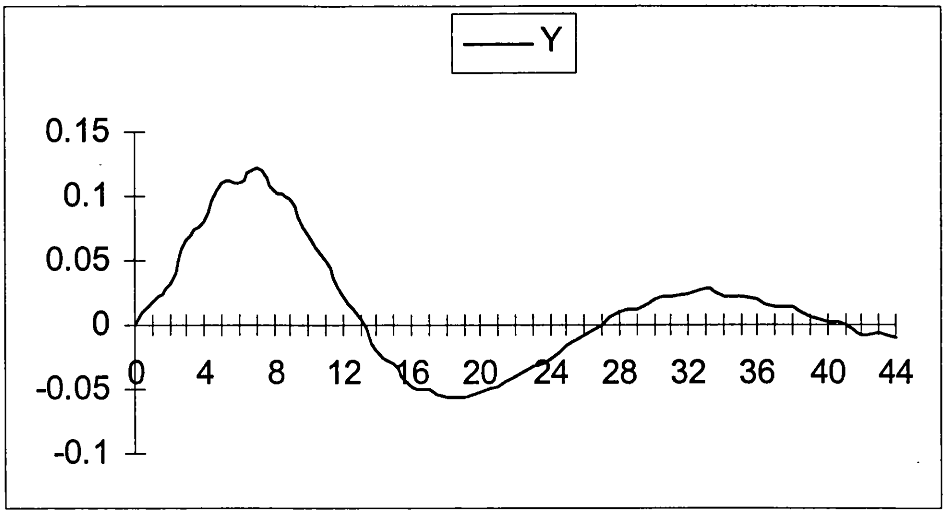


Figure 3. Aggregate model – Response to the common shock







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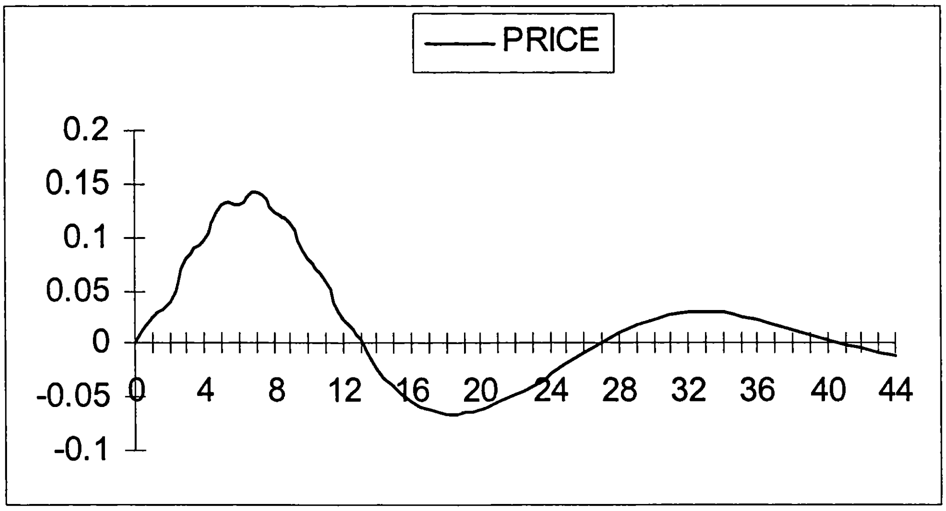

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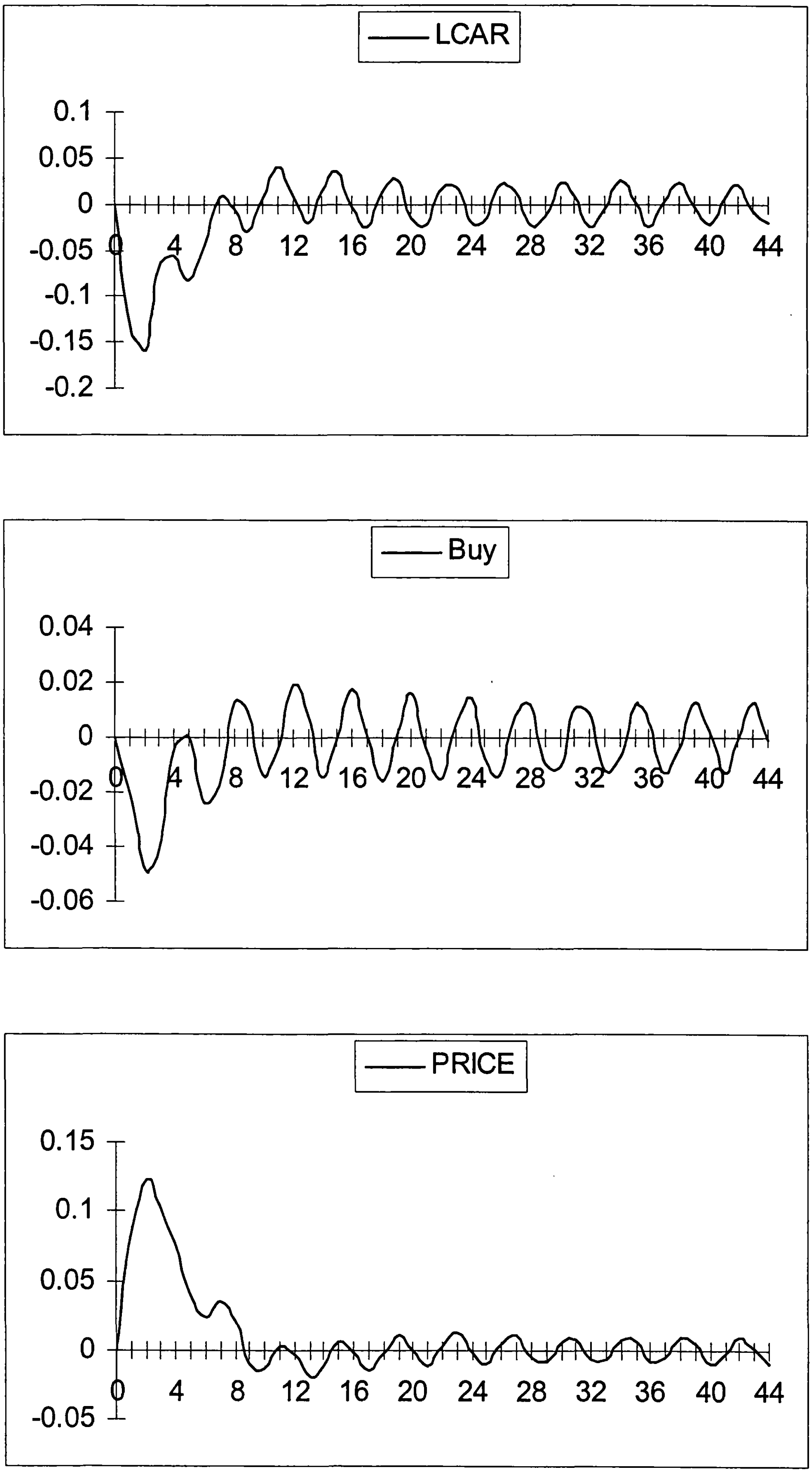
Figure 4. Aggregate model – Response to second shock



Appendix

The state space representation of the model is:

$$x_t = \Phi\widetilde{u}_t + \varepsilon_t$$
$$\widetilde{u}_t = G\widetilde{u}_{t-1} + \eta_t$$

with

$$E(\varepsilon_t \varepsilon'_\tau) = \begin{cases} \Sigma_\varepsilon & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$$

$$E(\eta_t \eta'_\tau) = \begin{cases} \Sigma_\eta & \text{for } t = \tau \\ 0 & \text{otherwise} \end{cases}$$

and both Σ_ε and Σ_η are diagonal matrices. The number of lags in the autoregressive representation of the cohort-specific shocks, M_b, has been set equal to zero for simplicity.

The Kalman filter algorithm is used to recursively the least squares forecast of the state vector conditional on information available at time (t-1):

$$\widetilde{u}_{t/t-1} = E(\widetilde{u}_t / X_{t-1}),$$

with mean squared error matrix:

$$P_{t/t-1} = E[(\widetilde{u}_t - \widetilde{u}_{t/t-1})(\widetilde{u}_t - \widetilde{u}_{t/t-1})']$$

The forecast for x_t conditional on information available at time (t-1) is:

$$x_{t/t-1} \equiv E(x_t / X_{t-1}) = \Phi\widetilde{u}_{t/t-1}$$

So, the prediction errors and the mean squared matrix needed to compute the likelihood are:

$$\zeta_t \equiv x_t - x_{t/t-1} = \Phi(\widetilde{u}_t - \widetilde{u}_{t-1}) + \varepsilon_t \quad \text{and}$$

$$F_t \equiv E(\zeta_t \zeta'_t) = \Phi P_{t/t-1}\Phi + \Sigma_\varepsilon$$

which can be computed recursively using the Kalman equations.

The percentage of the variance explained by the macro component computed in the result section is computed equation by equation as the complement to one of the ratio of the estimated variances of the cohort-specific components and the sample variance of each dependent variable. In other words, the variance of the dependent variables considered in this computation is conditional on the state vector:

$Var(x_t^j / \tilde{u}_t) = \Phi_j \Sigma_u \Phi'_j + \Sigma_\varepsilon^{jj}$ j=1, ..., Cp

where Φ_j is row j in the matrix of the factor loadings, and Σ_ε^{jj} is the j-th element on the diagonal of the cohort-specific shock variance matrix. The explained fraction of variance is then computed for each equation as:

$$\frac{\Phi_j \Sigma_u \Phi'}{Var(x_t^j / \tilde{u}_t)} = \frac{\hat{\Sigma}_\varepsilon^{jj}}{Var(x_t^j / \tilde{u}_t)} \qquad \text{j=1, ..., } Cp$$

Chapter Two

Stochastic Components of Individual Consumption:

A Time Series Analysis of Grouped Data[1]

1. Introduction

In the last 20 years, many empirical studies of consumption behaviour have focused on some version of an Euler equation for intertemporal optimization and have estimated structural parameters and tested the model by exploiting the overidentifying restrictions implied by such an equation. These studies, too numerous to be listed here, have used both aggregate and individual level data, reaching different conclusions about the validity of the model and about the magnitude of the structural parameters that can be identified within such a framework. It is important to stress that most of the implications of such theoretical structure are restrictions on the time series properties of consumption.

The time series properties of aggregate consumption (and some of its components) are well known in the time series literature. Indeed, some of these properties have stimulated the development of different theoretical models. Very little is known, however, about the stochastic properties of consumption at the individual level.

In this chapter, we propose a new methodology to analyse the time series properties of individual consumption expenditure. Our approach fills an important gap in the existing literature. Panel data have been used to study the

[1] This chapter is part of a research project joint with Orazio Attanasio.

time series properties of hours and earnings by several authors, including MaCurdy (1982), Abowd and Card (1989) and Moffitt and Gottshalk (1995). These studies, on the one hand model only labour market variables, on the other focus on the dynamic properties of purely idiosyncratic components and treat aggregate shocks as a nuisance parameter that is eliminated, together with deterministic life cycle effects, in preliminary regressions.

Our main goal is the characterization of the variance-covariance matrix of innovations to consumption and other variables of interest. The availability of a long time series of cross-sections allows us to focus on business cycle fluctuations rather than removing aggregate shocks by time dummies. On the other hand, the lack of a longitudinal component to our data forces us to ignore pure idiosyncratic variability and to focus instead on the dynamics of group averages.

Having characterized the dynamic properties of our average group data, we map the pattern of correlations that emerges from the data to those implied by different theoretical models.

Most of the studies in the labour economics literature are based on big N asymptotics, as they exploit the cross sectional variability to identify the parameters of interest. Because of this, some of them, such as Abowd and Card (1989), allow the coefficient of interest to vary over time. Our study, instead, focuses on the time series properties of individual consumption. The use of big T asymptotics is an important distinguishing feature of our approach. While we remove deterministic trends, we do not remove business cycle aggregate shocks. Indeed, our approach can be described as an attempt to model these aggregate shocks and is therefore based on big T asymptotics.[2]

[2] Another strand of the literature has focused on the time series properties of disaggregated business cycles. These studies, including Watson and Engle (1983), Quah and Sargent (1994), Forni and Reichlin (1996), focus on the time series properties and aim at characterizing the number of common factors and modeling their dynamic effects on the sectors considered.

The variables we consider are disposable income and consumption. We characterize the time series properties of these variables and we interpret the estimated variance covariance structure in terms of alternative models of individual behaviour.

The theoretical model on the background of our analysis is the life cycle model. It is therefore natural to study consumption in its relation to age and to divide the sample to form cohorts of individuals that are followed over time. Cohorts, however, are not the only interesting group that can be formed. One can consider education or occupation groups. Differences in the variability, persistence and covariance structures across these groups can be quite interesting.

The data we use are from the UK Family Expenditure Survey. The FES is, to the best of our knowledge, the longest time series of cross section containing exhaustive and detailed information on consumption, its components and several other variables of interest. We use surveys from 1974 to 2000 to form quarterly observations on several components of consumption at the cohort level.

The rest of this chapter is organized as follows. In section 2 we present our statistical model and discuss its identification and estimation. In section 3 we discuss the implications of different theoretical structures on the variance covariance structure we estimate in section 2. In section 4 we discuss the data sources and section 5 presents and discusses our empirical results. Section 6 concludes the chapter.

2. The Methodology

As stressed in the introduction, the main aims of this study are two. First, we would like to identify innovations to consumption and other variables and model their covariance structure. Second, we want to use this covariance structure to shed some light on the plausibility of alternative theoretical models.

As the data we have lack any longitudinal dimension, we are forced to use average grouped data to estimate any dynamic model. This means that we are not able to model idiosyncratic persistence, but only persistence at the group level.

As we focus on business cycle fluctuations, we remove from our data all age or cohort effects. Effectively, our methodology removes all deterministic trends from the data and interprets them as arising from a combination of cohort and age effects. This leaves us with group specific shocks whose covariance structure is the focus of this study.

In this section we sketch the main features of our approach. First we discuss the statistical model we want to identify. In particular, we write down the statistical model that we estimate and discuss the identifying assumptions we make.

As the nature of our data forces us to work with grouped data, effectively our model is a model of group shocks. However, as we construct group averages from our micro sample, the variables we observe are affected by two sources of variability. On the one hand, we have genuine group specific shocks. On the other, our sample averages are affected by measurement error arising from the limited size of our sample. The latter source of variability constitute a nuisance which can, however, be controlled for given the information on the sample structure and given the information on within cells variability. We discuss these issues in the second part of this section.

2.1. *The statistical model*

Let us consider a generic variable x_{ct}^{h}, where the index h denotes the individual household, the index t the time period and the index c the group to which household h belongs. Such a variable can be written as:

$$x_{ct}^{h} = \bar{x}_{ct} + \eta_{ct}^{h} \qquad (1)$$

where $\bar{x}_{ct}$ denotes the mean of the variable x for group c at time t. Given a sample in which group membership is observable, the mean in equation (1) can be easily estimated by the sample means. In the next subsection we discuss the problems that arise because we do not observe $\bar{x}_{ct}$ but are forced to estimate it on the basis of samples of limited size. In this subsection, we treat $\bar{x}_{ct}$ as observable.

Notice that the variables in (1) can be non-linear transformations of the variables of interest. As we work with micro data, we can control the aggregation directly. This turns up to be important for at least three reasons. First, the theory often implies relationships among non linear function of variables. Second, in the case of corners, the theoretical relationships one might want to consider at the aggregate level involve both means conditional on not being at a corner and overall means. Third, within cell heterogeneity, that is the variability of η_{ct}^{h}, is not only useful to correct for the measurement error in the estimation of $\bar{x}_{ct}$, but can also be informative to study the evolution of the inequality over time.

As stressed above, because we lack panel data, we can only study dynamic models by using grouped observations. That means that we can only study the dynamics of $\bar{x}_{ct}$. Any purely idiosyncratic persistence, embedded in η_{ct}^{h} cannot be recovered by our methodology. This might be a serious problem in evaluating the importance of precautionary saving or similar phenomena. Nothing much can be done about this except noticing that to get a handle on persistence at the individual level, a genuine panel dimension is necessary as it will be necessary to observe the covariance between individual variables in subsequent time periods. Having said that, however, it should be stressed that even a relatively short genuine panel might be sufficient to estimate a model which requires big T asymptotics if repeated panels are available: one can then group the cross moments and follow their dynamics over time.

The first step of our procedure consists in removing all deterministic trends from the variables of interest. Denoting as $\tilde{x}_{ct}^{h}$ the variables before detrending, these are likely to be affected by time, group and age effects. As within the framework of a life cycle model groups are often formed on the basis of the year of birth of the household head, group effects are essentially cohort effects. This implies the impossibility of disentangling age, time and cohort effects. In what follows we label all deterministic trends in the data as 'age and cohort' effects. While this label is arbitrary, disentangling the various effects is not the aim of the study which focuses, instead, on modeling the innovation to income and consumption at the business cycle frequency.

To remove deterministic changes, therefore, we regress the variables of interest on a cohort specific high order polynomial in age with cohort specific intercepts.

Because we use quarterly data, we also include seasonal effects in our first step regression.

$$\tilde{x}_{ct}^{h} = \delta_c + \sum_{i=1}^{3} \alpha_{ci} q_{ti} + f^{c}(t-c) + x_{ct}^{h} \tag{2}$$

where $f^{c}(t-c)$ is the cohort specific polynomial in age (obtained as time-year of birth), q's are quarterly seasonal dummies and δ_c the cohort specific intercepts. Equation (2) is estimated by OLS. The cohort average of the residual, x_{ct}, reflects both genuine time variation in group averages and measurement error arising from the limited sample sizes in computing the group averages.[3]

It should be stressed that equation (2) does *not* contain time dummies: as business cycle shocks (either common across groups or not) are the focus of the study, we do not want to remove them. Having estimated the parameters of such a regression, we interpret the cohort averages of the estimated residuals x_{ct} as deviations of the average cohort data from the deterministic trends present in the data. It is the cohort averages of these residuals that we model and study.

To take into account the possibility of stochastic trends (and because of the structural interpretation we give to the covariance structure that we estimate), we take the first differences of x_{ct} and study its time series properties. The model we propose to estimate is the following:

$$\begin{aligned} \Delta x_{ct} &= \sum_j \alpha_j^{xx} u_{c,t-j}^{x} + \sum_j \alpha_j^{xy} u_{c,t-j}^{y} + \sum_j \alpha_j^{xr} u_{t-j}^{r} \\ \Delta y_{ct} &= \qquad\qquad\quad \sum_j \alpha_j^{yy} u_{c,t-j}^{y} + \sum_j \alpha_j^{yr} u_{t-j}^{r} \\ r_t &= \qquad\qquad\qquad\qquad\qquad\quad \sum_j \alpha_j^{rr} u_{t-j}^{r} \end{aligned} \tag{3}$$

The vector x is non-durable consumption for each cohort. We can consider other choice variables such as components of consumption, hours of work and

[3] More efficient estimates could be obtained by controlling for the heteroscedasticity induced by different cell sizes and within cell variances.

participation rates by adding more equations to the system. These variables are affected by all the shocks present in the system. The vector y is disposable income; other variables of the same type would include variables that are assumed not to be determined by individual choice, at least at the frequency we are considering. These could include, for instance, wage rates for male and females, and they can be group specific. These variables are affected by all the shocks in the system with the exception of the shocks specific to the variables in the first group (the choice variables). r is the interest rate, and this kind of variable includes variables that are common across groups as well as not being determined by individual choice. This vector includes variables such as prices, both intertemporal (interest rates) and intratemporal (relative prices). Finally, the α are matrixes of the relevant dimension. A more detailed exposition of the type of model we estimate is given in the Appendix.

The MA structure in equation (3) is quite general, but imposes some important and strong restrictions that are used to (over-) identify the model. In particular, the triangular structure (at least for the contemporaneous shocks) is crucial for indentification. We assume that shocks to individually determined variables, such as non-durable consumption, do not affect the variables that are assumed to be given to the individual households.[4] In addition to these restrictions, we make the normalization assumption that the coefficient on the own residuals are equal to one. For instance, we assume that all the components of the diagonal of the matrix α^{xx}_0 are equal to one.

2.2. *Estimation*

There are several ways in which one can estimate the model (3). By making assumptions on the density of the shocks that enter the system (3) it is possible to compute the likelihood function associated with a given sample and estimate the parameters of interest by maximizing such a function. It would be also

[4] Identification requires that only contemporaneous shocks to 'choice' variables do not determine 'non-choice' variables. We make the stronger assumption partly for ease of notation.

possible to avoid making specific functional form assumptions, and use a method of moment estimator. In particular, one could compute the second moments of the time series of the variables of interest; these would include both the contemporaneous second moments and the autocorrelation. These moments depend on the parameters in model (3) and on the variance covariance matrix of the shocks. However, this line of estimation has lead to difficulties in the estimation of the variance covariance matrix of the parameters, and maximum likelihood estimation has been preferred and carried out (see the Appendix for a detailed description).

So far, for ease of exposition, we have assumed that the group means of the variables of interest are observed. As instead they are estimated using samples of limited size, we have to consider the measurement error problem that this induces. The fact that we use a time series of independent cross section and given that we are interested in the changes in the variable of interest induces an MA(1) structure to the residuals of our system. However, given that we know the cell size and we can estimate the within cell variance, we have a substantial amount of information on the measurement error that we can use to correct the estimated sample moments.

More specifically, the changes in the variables we model can be decomposed into two components,

$$\Delta x_{ct} = \Delta \bar{x}_{ct} + \Delta \eta_{ct} \qquad (4)$$

where η_{ct} is the measurement error, whose variance can be estimated consistently from the within cell variability and the cell size and which is assumed to be independent of $\Delta \bar{x}_{ct}$, the changes in the innovations we are interested in modeling. The time series variance of $\Delta \bar{x}_{ct}$ can then be consistently estimated by the sample equivalent of the following expression:

$$Var(\Delta \bar{x}_{ct}) = Var(\Delta x_{ct}) - E\left[\frac{2}{N_{ct}} Var_i(\eta_{ct}^h)\right] \qquad (5)$$

where the second variance has a subscript 'i' to stress that is a cross sectional covariance.[5]

The within cell variance can be estimated from the micro data. If we assume that the variance within cells is constant, one can estimate such a variance very efficiently. If one assumes that the cross sectional variances in equation (5) are known, one can considerably simplify the maximization of the likelihood function. We estimate such variances under the assumption that they are constant across cell and can be estimated using the entire sample. As we have around 23,000 observations, we treat such variances are known.

2.3. *Comparison with the existing literature*

One of the first papers to use panel data to infer the time series properties of individual level variables is the widely cited paper by MaCurdy (1982) in which the author models deviations of wages and earnings from a regression equation including several variables (such as age and demographics) and a set of period dummies. These deviations are found to be well represented by an IMA(2) model, where the second coefficient of the MA component, while significantly different from zero, is estimated to be quite small. MaCurdy (1982) uses data from the PSID. The same survey was later used by Abowd and Card (1989) who, generalizing MaCurdy's approach, allow for non stationarity of the processes for earnings and wages. While they also found that the autocovariances of order higher than two are not significantly different from zero (and small in magnitude), they reject the hypothesis of stationarity. Abowd and Card (1989) also related their findings about correlations and autocorrelations in terms of alternative structural models, performing a variance decomposition exercise.

Altonji et al. (1987) probably constitutes the study closest to the present one. They consider the covariances and autocovariances of wages, hours and food

[5] The latter variance will be different in each cell and at each point in time, if nothing else because cell sizes can be quite different. These variances can be averaged out to get a mean of the unconditional variance needed for the correction.

consumption in the PSID and use them to estimate several factor models which are then given a structural interpretation.[6]

The main conceptual difference between the existing studies in the literature and our approach is the fact that we do not remove time effects and, indeed, focus on the modelling of business cycle frequency shocks. For this reason we need to rely on T-asymptotics and for this reason our level of flexibility in analysing non stationarity is limited.[7] However, it is important to stress that the difference is not merely technical but of focus: we focus on the nature of business cycle shocks and how these are absorbed by consumption, its components and, possibly, other variables. In other words, ours is an attempt to model the time effects that are removed in other studies. [8] We think that our approach might be particularly informative in characterizing smoothing mechanisms and to interpret the shocks we identify in terms of underlying structural models.

The necessity of large T asymptotics is also consistent with the same requirement in the estimation of Euler equations in the absence of complete markets. In the next section we stress the interpretation of our estimated covariances in terms of the orthogonality restrictions implied by an Euler equation and alternative models. Notice that, at least in principle, we can allow for group specific fixed effects in all our equations without much difficulty. Situations in which this is appropriate are discussed in the next section.

[6] More recently, Moffitt and Gottshalk (1995) have used the evolution in the cross sectional variances in earnings to identify permanent and transitory components.

[7] Throughout the study we assume stationarity, that is that the first and second moments we will be considering do not vary over time. This might seem unfortunate as there is some evidence of non stationarity in micro studies such as Abowd and Card (1989) and Altonji et al (1987). However, it should be stressed that the moments we are considering are conceptually different from those identified by the papers cited. Given that our estimators are based on T-asymptotics, considering the possibility of time-varying moments would involve the parametrization of their changes. We have not pursued this line of research.

[8] This last statement is not completely accurate, as the studies do not remove *completely* time effects, but only those that are common across individuals.

The second main difference between our procedure and those in the studies cited above, is that we focus on grouped data, while the others use individual data. The main reason of our exclusive attention to grouped data is the lack of a longitudinal dimension in our data. This forces us to give up the possibility of identifying any purely idiosyncratic dynamics. However, the focus on (a fixed number of) groups and on the time series variability gives a greater flexibility in modelling cross sectional heterogeneity (i.e. across groups, in our case) and a better chance to identify genuine time series uncertainty (as distinct from cross sectional heterogeneity). Increasing the number of groups would allow one to use N-asymptotics and give greater flexibility in the analysis of non-stationary time series processes. In what follows we do not make any use of N-asymptotics arguments.

Third, this study can attempt at modeling the dynamics of several components of consumption and other relevant variables simultaneously. While other studies were limited by the availability of consumption measures (the PSID contains only information on food expenditure), the FES data set allows us to explore a much richer set up.

Finally, we only use the information on within group heterogeneity to correct the estimated sample moments for the small sample variability and measurement error. As pointed out by several authors, the evolution of within cell heterogeneity over time and age can be quite informative for a number of reasons. Deaton and Paxson (1994), for instance, relate the evolution of the within cell variance in consumption and income and check that the implications of the life cycle models for these cross sectional moments are satisfied. Blundell and Preston (1998), instead, use the differences between changes in the cross sectional variance of consumption and income to identify the variance of the permanent and transitory component of income.

A number of studies in the macro time series literature are slightly related to what we are doing. In particular, several studies papers have tried to identify the number of common factors in disaggregated business cycle models (see Watson and Engle (1983), Quah and Sargent (1994), Forni and Reichlin (1996)). A similar approach can be used to study dynamic behaviour of the components of consumption of several groups of individuals, as it has been pursued in chapter 1.

3. A structural interpretation of the results

Having estimated the parameters in model (3), one has to establish what are the implications of these estimates for alternative theoretical frameworks. Alternatively one can start from a theoretical framework and think of the implication that it has for the parameters of model (3).

The starting point for a structural interpretation of the parameters in system (3) is the life cycle model, interpreted as a flexible parametrization of a dynamic optimization problem in which the decision unit is the household.

We start with the simplest version of the life cycle model as an example of a way in which a theoretical framework can be used to impose restrictions on the parameters of model (3). We then complicate the model to introduce a number of realistic elements. We remove deterministic trends (including age effects) as well as family size effects from all the variables in our analysis: this implicitly assumes that the age and family size effects removed in the first step of our estimation procedure capture completely the effect of demographic variables and that these are considered as deterministic. As we focus on business cycle frequencies, we do not think that this assumption is particularly strong.[9]

3.1. *A simple version of the life cycle model*

A very simple version of the life cycle implies the following system of equations for a generic individual:

$$\log(\lambda_t) = E_t[\log(\lambda_{t+1})] + E_t r_{t+1} + k_t + \varepsilon_{t+1} \qquad (6)$$

$$\log(\lambda_t) = \log(U_x(x_t, z_t)) \qquad (7)$$

[9] If one thinks that the age polynomials used in the first step are not sufficient to remove the effect of demographic variables, and it is willing to retain the assumption that they are deterministic, these variables can be used in the first step regressions.

where the variable k is a function of the discount factor and of higher moments of the expectational error ε_{t+1}. U_x is the marginal utility of (non-durable) consumption, which is assumed to depend on consumption and a vector of observable and unobservable variables z. λ_t is the marginal utility of wealth and represents the effect of all present and future variables relevant for the optimization problem faced by the individual. r is the interest rate. The specification in equations (6) and (7) also assumes intertemporal separability, in that the marginal utility of consumption at t does not depend on variables from other time periods.

If we assume that the interest rate and k are constant over time and that the observable component of the vector z contains only deterministic variables that can be captured by the deterministic trends removed in our first step, equations (6) and (7) have very simple and strong implications for the model in (3).

First, one can simplify the model considerably eliminating the last equation (that refers to the interest rate). Furthermore, if the specification of the utility function is such that the marginal utility can be approximated by a linear function of log consumption (as it is the case, for instance, for a CRRA utility function), from equations (6) and (7) one can see that changes in log consumption can be related to the expectational error ε_{t+1} and therefore should not exhibit any serial correlation and should be uncorrelated with any information available at t.

It is potentially very important to consider the possibility of an unobservable component in the vector z. Such a component, which, for lack of a better term we label 'unobserved heterogeneity', captures those aspect of preferences that are not directly modeled and that are likely to be important for consumption. The time series properties of consumption innovations would then be clearly affected by the time series properties of such a term. For instance, if unobserved heterogeneity is constant over time, first differencing would remove it completely. If instead it evolves as a random walk, we would need to add a white noise term to the innovation of (log) consumption. Finally, if the level of such a variable is a white noise, there would be an MA(1) component in the Euler equation.

If one considers the fact that equations (6) and (7) refer to a single generic household, it is clear why, even in such a simple framework, aggregating such an equation across groups of households would generate group specific fixed effects. These could arise if, for instance, there are systematic differences across groups in the discount factors, higher moments of the expectational errors, or in the unobserved component of z.

The approach followed so far is quite similar to that of Altonji et al. (1987), with an important difference: the fact that we do not remove time effects.[10] A first and very simple generalization of the model which stresses the differences between our methodology and that of Altonji et al. is to consider time varying interest rates. This extention could be of particular interest as allows one to estimate the elasticity of intertemporal substitution.

Allowing for a time variable interest rate involves adding an additional equation to the model, so that one can measure the correlation between innovations to interest rates and consumption (and other variables). One can either assume that the interest rate is the same for all groups or allow for differences in intertemporal prices induced, for instance, by differences in marginal tax rates across groups. The latter approach, however, involves the necessity of measuring group specific interest rates.

If we consider an asset whose rate of return is roughly constant across groups and that is widely held, than equations (6) and (7) induce a set of additional restrictions on system (3). If we define with $A^{xx}(L)=\sum_{j=0}^{q}\alpha_j^{xx}L^j$, and analogously for $A^{xr}(L)$ and $A^{rr}(L)$, it is easy to show that an isoelastic utility function with a coefficient of relative risk aversion γ implies that: $\gamma(A^{xx}(L)u_{c,t+1}^{x}+A^{rx}(L)u_{c,t+1}^{r})=A^{rr}(L)u_{c,t+1}^{r}$. As this has to hold for every

[10] Altonji et al. (1987) follow two different strategies that could be, in principle be pursued here. The first consists in parametrizing the innovation to marginal utility of wealth as a function of the innovations of wages, non labour income and possibly their variables deemed to be relevant for the problem. The other is to use explicitly the Euler equation (8) to difference out λ_t. Here we follow only the latter approach.

possible realization of the residuals, the restrictions on the coefficients of system (3) are that $A^{xx}(L) = 0$ and $\gamma A^{xr}(L) = A^{rr}(L)$.

The second set of restrictions implies that, as long as interest rates are predictable, one can (over) identify the coefficient of relative risk aversion.

Should one encounter a rejection of such a restriction, several alternative specifications are possible depending on the nature of the rejection. If one violates the hypothesis that the coefficient on lagged consumption shocks are zero, these is probably an indication that there are some persistent taste shocks that should be incorporated into the model. Alternatively, the fact that the restriction about the proportionality of the coefficients on the interest rate lagged innovations is violated might be an indication of differences in interest rates and/or risk aversion across groups. If one more lag in the interest rate innovation enters the system even this possibility can be tested against more general misspecifications.

3.2. *Non separability with labour supply and other components of consumption*

Implicit in the formulation of the model above is the assumption that non durable consumption is separable from other components of consumption excluded from the analysis (such as durables and housing) as well as from leisure. The latter might be particularly important as deviations from this assumption could explain observed correlation between expected income and consumption. Indeed, in many empirical analysis of Euler equations based on micro data (such as those of Attanasio and Weber (1993), Blundell, Browning and Meghir (1994), Banks, Blundell and Preston (1994), Attanasio and Browning (1995), Attanasio and Weber (1995)) labour supply, and in particular, female labour force participation seems to play an important role.[11] This evidence is not entirely surprising, as many components of consumption

[11] Browning and Meghir (1991) test explicitely for the dependence of a demand system on labour supply behaviour.

expenditure are accounted by job related expenses or, in the case of female labour force participation, might substitute for home production services.

The generalization of the simple model proposed in the previous section is straightforward: equations (6) and (7) still hold, except that the marginal utility of consumption has to depend on the excluded commodities and on labour supply. It should be stressed that equations (6) and (7) are robust to the presence of various kinds of complications in the determination of durables and/or labour supply, such as fixed adjustment costs and the like. The marginal utility of non durable consumption is defined as a function of non durable consumption and the optimal level of the other relevant variables, regardless of how they are determined.

The difficulty, in the case of durable consumption, concerns the observability of the existing stock of durables at each point in time.[12] In the case of labour supply and in particular female labour supply, for which corner solutions are important, one has to allow the marginal utility depend explicitly on participation at the individual level. Given the nature of the data, this does not constitute an important problem. If the (log of) marginal utility of consumption depends additively on a participation indicator, aggregating equation (7) one has a model in which average changes in log non durable consumption depend, among other things, on the (changes) in participation rates at a point in time for a given cohort. It is therefore necessary to model female participation rates in a way analogous to the way in which we model non durable consumption, wages, or income. Such an equation can be easily added to the system of equations (7). It should be stressed that this procedure, while allowing the study of the properties of non durable consumption is silent about the determinants of labour force participation.

[12] One might try to construct group level estimates of the existing stock of durables by cumulating the observed expenditures. We have not yet attempted this procedure.

3.3. *Multiple commodities*

So far we have worked with the assumption of a single and homogeneous non durable consumption. More precisely, we have considered total non durable consumption and studied its allocation over time as a function of a single price index. Of course, this approach is only justified under stringent conditions on preferences (see Gorman, 1959).[13] It might therefore be important to model simultaneously the allocation of resources over time and, at an point in time, among several commodities. Furthermore, even when the Gorman aggregation conditions are satisfied, the study of a demand system can be of interest. Finally, the consideration of several Euler equations simultaneously might give more powerful tests of the model considered.

Let's then assume that **q** is a vector of m commodities, with prices **p.** Instead of equation (7) we will then have m equations relating the marginal utility of each commodity to its price and to the marginal utility of wealth λ :

$$\log(p_t^i) + \log(\lambda_t) = \log(U_{q^i}(q_t^i, z_t)); \quad i = 1,\ldots,m \qquad (7')$$

where the index i refers to the commodity. As we are writing (7') as an equality, we are implicitely ruling out the possibility of corners in any of the m commodities. From equation (7') it is also clear why it is important to have an unobserved component in preferences. Without it, one could consider equation (7') for two different commodities to eliminate the marginal utility of wealth and obtain an equation that has no error!

From our perspective, to map a system of equations such as (7') in anything like (3) involves considering the innovations in each commodity (and possibly in total consumption) and modelling the vector of relative prices. As far as the

[13] Blundell, Browning and Meghir (1994) and Attanasio and Weber (1995) address this issue. In the first paper, that uses UK data, the authors find that while the restrictions that would grant the use of a single price index are formally rejected, the use of a Stone price index constitute a good approximation of the 'true' price index that should be used. Attanasio and Weber (1995), using a slightly different parametrization of preferences and US data, find a more important role for a second price index.

latter are concerned, they can be treated in a fashion similar to the interest rate: they can be assumed to be constant across consumers. Three different possibilities are open for the treatment of commodities. On the one hand one can consider m-1 equations obtained by using a specific commodity as a benchmark and eliminate therefore the marginal utility of wealth from the system. This approach involves therefore to consider static relationships, possibly expressed in ratios of marginal utilities and relative prices. Second, one can use a function of total non durable expenditure as an approximation of the marginal utility of wealth and use it in each of the m equations. The last alternative is to use equation (6) in each of the expressions in (7') and therefore derive an Euler equation for each of the commodities considered.

Several considerations are in order. First, regardless of the approach used, the discussion above about the possibility that the vector z includes some choice variables is relevant here. The demand system that one obtains eliminating the marginal utility of wealth is effectively the *conditional* demand system discussed in detail by Browning and Meghir (1991).

Second, the first two approaches are essentially static and can be expressed in terms of the levels of the variables of interest. The residuals of the residuals of these equations arise from unobserved hetereogeneity across consumers (groups) and measurement error. Indeed, the first two approches give rise to equations that can in principle be estimated using cross sectional data (except that one has to have enough price variability, that can only be observed over time). On the other hand, these equations, within the framework of a life cycle model with intertemporally separable preferences, are uniformative about the way in which households react to shocks.[14] Third, the last approach, that of deriving m Euler equations, is intrinsically dynamic and is the natural extention of what we do considering a single commodity. This is the preferable line as it

[14] Meghir and Weber (1996) interpret any evidence of dynamics in a system of demand equations as an indication of intertemporal non-separability. Using US data they are unable to identify any dynamic effect, once they condition on durable and semi-durable consumption.

delivers some interesting restriction of the system of equation (7') (extended for changes in prices). In particular, one can see that the innovations to that system of equation, once one control for changes in prices, should be driven by a single factor: the innovations to the marginal utility of wealth.

4. Data

As in the previous chapter, the data set is drawn from the UK Family Expenditure Survey (FES) from 1974(1) to 2000(1). The sample has been built selecting all married or cohabitating couples, living in England, Scotland or Wales, whose head is an employee. In order to have a balanced sample in the estimation, families whose head was born before 1953 or after 1940 have been selected. These exclusions result in a sample of 23,379 families. Two seven-year-of-birth groups are then defined as described in table 1.

Table 1: Cohort definition

Cohort	Year of Birth	Age in 1974	Age in 2000	Mean Cell Size
1	1940-46	31	57	104
2	1947-53	24	50	118

The variables used in estimation are: non-durable consumption and disposable income, as well as prices for non-durable and consumption computed using the weights available from the FES. Non-durable consumption is defined as the sum of: food, alcohol and tobacco, fuel, clothing, transportation costs and services. Durable consumption (excluding cars) shown in table 2 is defined as the sum of household durable goods (including furniture, furnishings and the like) and audio-visual equipment.

In order to remove deterministic trends, (the logarithm of) the variables of interest have been regressed on a polynomial in age and on quarterly dummies, as well as on the logarithm of family size. Estimation has been carried out by OLS one cohort at a time, according to equation 2 in the text. The interest rate used in estimation is the 3-month treasury bill rate, which has been deflated using consumer price index constructed from the FES. The real interest rate has also been detrended.

5. Results

In table 2 a measure of the volatility of the variables considered in estimation is shown. This is defined as the standard deviation (times 100) of the changes in detrended log consumption and income, both total and corrected for the part that can be attributed to sampling error. This is computed as shown in equation 5. After taking into account the correction for measurement error, non durable consumption appears less volatile than disposable income, and according to table 2 sampling error in non-durable consumption accounts for a large part of its volatility. The volatility of expenditure on durables (excluding cars) conditional on buying, and of the number of families purchasing a durable each quarter is also shown.

Table 2. Volatility

	Total		After correction	
	Cohort 1	Cohort 2	Cohort 1	Cohort 2
Non-durable	6.19	5.48	2.31	2.14
Durable	23.34	20.76	13.17	11.99
Buyers	6.05	5.98	0.71	2.62
Income	5.22	5.41	2.56	3.67

Before considering estimation of systems of equations as in (3), some results on bivariate MA models of the relevant variables are shown in tables 3-7[15]. In particular, tables 3 and 4 show estimates of bivariate MA(1), MA(4), and MA(8)

[15] When considering one variable at a time, the number of equations is given by the number of cohorts, which is equal to 2 in the present case.

models for non durable consumption for both cohorts, both without and with correction for measurement error. The model estimated is therefore:

$$\Delta ND_{ct} = \sum_{j=0}^{q} \alpha_j^{ND} u_{c,t-j}^{ND} \qquad c=1, \ldots, C$$

where ND_{ct} is log non durable expenditure for cohort c at time t, and q is the number of lags. The measurement error term has been suppressed for ease of notation, and it is either disregarded (estimates without correction) or treated as shown in the Appendix (estimates with correction for measurement error). The coefficient on the lag zero shock (α_0^{ND}) is constrained to be one, and it is not shown in the tables.

Among the results the estimated covariance matrix of the shocks for the two cohorts is also reported[16]. Comparison between table 3 and table 4 reveals how measurement error influences estimates of the parameters. The MA(1) coefficient estimated without accounting for measurement error in table 3 is equal to -.93, while when removing the effect of measurement error the MA(1) coefficient is equal to -.15 (although the estimate is not statistically different from zero). Tables 3 and 4 also report estimates for MA(4) and MA(8) models. Comparison in the likelihood value for the MA(1) model and the MA(4) model reveals that the zero restrictions imposed by the one-lag specification are rejected. However, improvement in the likelihood given by the MA(8) model is rather small, and a likelihood ratio test that compares the MA(4) and the MA(8) in table 4 is equal to 3.9, distributed as a chi squared with 4 degrees of freedom, and the restrictions cannot be rejected. As shown in section 3.1, assuming a constant interest rate and a CRRA utility function, changes in log consumption should not exhibit any serial correlation. Before exploring the determinants of the rejection of such a restriction, arising from the results shown in table 4, it is informative to analyze the time properties of other variables of interest, namely disposable income and the interest rate.

[16] All variables have been multiplied by 10 in estimation.

Table 5 and 6 report results for bivariate MA models for (the logarithm of the first difference of) disposable income. In particular, table 5 shows estimates of MA(1), MA(4) and MA(8) models for the two cohorts considered without adjusting for measurement error, while table 6 reports the same estimates when the correction for measurement error is taken into account.

While the MA(1) coefficient is reduced in the model with correction for measurement error as compared to table 5 where no correction is made, this is still equal to -.5 and statistically different from zero in table 6. However, comparison in the values of the logarithm of the likelihood reveal that the restrictions imposed both by the MA(4) and by the MA(1) model should be rejected, suggesting a longer dynamic as compared to the non-durable consumption variable.

Finally, table 7 reports estimates for univariate MA models for the interest rate. Interest rate too shows a prolonged dynamics and the restrictions imposed by the MA(1) and MA(4) models are rejected.

Subsequent tables report results for multivariate models for different lag structures; all of them are corrected for the presence of measurement error. Tables 8 show results for an MA(1) system for non-durable consumption and the interest rate for both cohorts, while tables 9 and 10 show results for the same system with 4 and 8 lags. In the usual notation, this can be written as:

$$\begin{aligned} \Delta ND_{ct} &= \sum_{j=0}^{q} \alpha_j^{ND} u_{c,t-j}^{ND} + \sum_{j=0}^{q} \alpha_j^{ND-R} u_{c,t-j}^{R} \\ r_t &= \sum_{j=0}^{q} \alpha_j^{R} u_{c,t-j}^{R} \end{aligned} \qquad c=1, \dots, C$$

As reported in table 8, the one lag coefficient of the consumption shock is equal to +.3 (although it is not precisely estimated), while the interest rate shock affects consumption contemporaneously with a positive coefficient (+.56) and with one lag with a negative one (-.35). The coefficient of the lag zero interest rate shock in the consumption equation (i.e. α_0^{ND-R}) is the impact multiplier to a unit shock. If there is a permanent and unitary shock in the interest rate, however, the long run multiplier is roughly equal to 0.14 (i.e. $\sum_{j=0}^{q}\alpha_j^{ND\_R} \Big/ \sum_{j=0}^{q}\alpha_j^{R}$).

Results in table 9 report results for the same system with 4 lags. In the non-durable equation, coefficients relative to the shock of consumption are high but not precisely estimated. Their pattern, however, is difficult to reconcile with the theory exposed in section 3.1. The impact multiplier to a shock in the interest rate is equal to .42, while the long run multiplier of a unit shock in the interest rate is negative and equal to -1.4. This result may be due to the short lag specification, as the univariate model for the interest rate displayed a prolonged dynamics. Turning to the 8-lag model, inspection of table 10 reveals that the impact on consumption of an increase in the interest rate is equal to .84, while the multiplier of a permanent unit increase in the interest rate is equal to 1.02. Again, in table 10 coefficient on the lagged shock to consumption are big but very imprecisely estimated.

Tables 11, 12 and 13 show results of the system to which disposable income is added. In table 11, the MA(1) structure is estimated. Looking at the non-durable consumption equation, the lagged shock to consumption is equal to -.4 and it is precisely estimated. As it would be expected from the theory, a contemporaneous innovation to income has a positive impact on consumption. However, the lagged coefficient to an income innovation is negative and significantly different from zero, indicating the presence of excess sensitivity of consumption to past innovations to income.

The impact of the interest rate on consumption is positive and equal to .56, while the coefficient of a permanent unit increase in the interest rate is equal to .16.

Table 12 displays results for the same system with 4 lags. As before, coefficients are not precisely estimated. However, the likelihood ratio test rejects the restrictions imposed by the 1-lag model with respect to the 4-lag model. As in the case of a system with non-durable consumption and interest rate alone, the long run multiplier on the interest rate is equal to -1.8.

The 8-lag model is then presented in table 13. Again, parameter estimates display large standard errors, but the likelihood ratio test rejects the restrictions imposed by the MA(4) or MA(1) systems with respect to the 8-lag system. Similarly to the MA(8) model in consumption and interest rate, the long run impact of a unit permanent increase in the interest rate is equal to 1.04. As the long lag specification seems to be attributable mainly to long dynamics of the interest rate (and of disposable income too), a constrained version of the MA(8) model is also estimated and reported in table 14. Here, the consumption equation contains the u_t^{ND} and u_t^{Y} shocks lagged only to lag 4, while the interest rate shock is lagged to lag 8. According to the likelihood ratio test (with a value of 8.99 and 8 degrees of freedom) one cannot reject the restrictions imposed by the model in table 14. In the consumption equation, the excess sensitivity of consumption to lagged innovations to income is now less imprecisely estimated, with a coefficient on the first lag of -.6 (and a ratio coefficient – standard error equal to -1.02 and a p-value of about 16). However, constraining all the lagged coefficients of the income shock to be zero in the consumption equation would result in a rejection of the null hypothesis (results not shown). The impact of a shock in the interest rate to consumption is estimated to be .91, that is quite similar to the unconstrained MA(8) model. Similarly, the long run effect of a permanent and unitary shock to the interest rate is equal to 1.02.

6. Conclusions

In this chapter an analysis of the time series properties of individual consumption expenditure has been presented. The methodology consists in estimating multivariate moving average systems for individual (grouped) variables: this approach has the advantage of allowing to explicitly take into account the measurement error present in the individual measures of consumption and income. Data are drawn from the UK Family Expenditure Survey.

Results show a prolonged dynamics of non-durable expenditure, which cannot be entirely explained by the influence of lagged shocks to income and to the interest rate. Some evidence on excess sensitivity of consumption to lagged income shocks has also been found, although results are not clear-cut. The long run response of consumption to a unit shock in the interest rate has been estimated to be about 1, both in a system in which income is included among the equations and in a system of consumption and interest rate alone.

The (possible) effect of lagged income shocks to consumption, as well as the effect of lagged shocks to the unexplained component of consumption may be explained by a number of features that are not included in the version of the life cycle model considered. These include unobserved heterogeneity, habits, non-separability with other goods and/or non-separability with leisure. As sketched in section 3, the approach developed in this study may be profitably used in order to investigate in those directions, a task that is left for future research.

Table 3. Non-durable consumption, no correction for meas. error

	Coeff.	z	Coeff.	z	Coeff.	z
$u^{ND}_{c,t-1}$	-0.927	(-30.31)	-0.700	(-8.90)	-0.725	(-0.77)
$u^{ND}_{c,t-2}$			-0.088	(-0.97)	-0.083	(-0.30)
$u^{ND}_{c,t-3}$			-0.172	(-1.80)	-0.141	(-0.66)
$u^{ND}_{c,t-4}$			0.005	(0.06)	0.045	(0.48)
$u^{ND}_{c,t-5}$					0.010	(0.07)
$u^{ND}_{c,t-6}$					-0.008	(-0.06)
$u^{ND}_{c,t-7}$					-0.058	(-0.44)
$u^{ND}_{c,t-8}$					-0.040	(-0.42)
Covariance	0.2893	0.0425	0.2675	0.0275	0.2598	0.0228
Matrix	0.0425	0.1966	0.0275	0.1741	0.0228	0.1685
Log L	44.81		54.25		55.87	

Note: The covariance matrix is reported for $u^{ND}_{c,t} = \begin{bmatrix} u^{ND}_{1,t} \\ u^{ND}_{2,t} \end{bmatrix}$, which is (2×2) as estimation is carried on 2 cohorts. z is the ratio of the coefficient estimate to the estimated asymptotic standard error.

Table 4. Non-durable consumption, with correction for meas.error

	Coeff	z	Coeff.	z	Coeff.	z
$u^{ND}_{c,t-1}$	-0.150	(-0.82)	0.289	(1.08)	-0.347	(-0.32)
$u^{ND}_{c,t-2}$			-0.0003	(-0.31)	0.035	(0.07)
$u^{ND}_{c,t-3}$			-0.983	(-1.50)	0.237	(0.09)
$u^{ND}_{c,t-4}$			-0.152	(-0.27)	0.493	(0.26)
$u^{ND}_{c,t-5}$					-0.358	(-0.24)
$u^{ND}_{c,t-6}$					-0.958	(-0.65)
$u^{ND}_{c,t-7}$					0.070	(0.07)
$u^{ND}_{c,t-8}$					-0.075	(-0.10)
Covariance	0.0431	0.0274	0.0308	0.0118	0.0248	0.0120
Matrix	0.0274	0.0174	0.0118	0.0145	0.0120	0.0080
Log L	50.80		56.10		58.05	

Note: see note to table 3.

Table 5. Disposable income, no correction for meas. error

	Coeff	z	Coeff.	z	Coeff.	z
$u^{Y}_{c,t-1}$	-0.683	(-11.71)	-0.757	(-8.77)	-0.865	(-9.24)
$u^{Y}_{c,t-2}$			0.073	(0.68)	0.142	(1.34)
$u^{Y}_{c,t-3}$			-0.162	(-1.81)	-0.152	(-1.57)
$u^{Y}_{c,t-4}$			-0.066	(-0.75)	0.077	(0.59)
$u^{Y}_{c,t-5}$					0.007	(0.06)
$u^{Y}_{c,t-6}$					-0.115	(-1.01)
$u^{Y}_{c,t-7}$					0.008	(0.09)
$u^{Y}_{c,t-8}$					-0.162	(-1.78)
Covariance	0.1861	0.0003	0.1819	0.0072	0.1597	-0.0048
Matrix	0.0003	0.1791	0.0072	0.1646	-0.0048	0.1421
Log L	72.23		77.07		86.48	

Note: see note to table 3.

Table 6. Disposable income, with correction for meas. error

	Coeff	z	Coeff.	z	Coeff.	z
$u^{Y}_{c,t-1}$	-0.504	(-5.42)	-0.614	(-2.45)	-0.531	(-1.04)
$u^{Y}_{c,t-2}$			0.894	(3.36)	0.768	(0.95)
$u^{Y}_{c,t-3}$			-0.743	(-1.54)	-0.972	(-1.74)
$u^{Y}_{c,t-4}$			-0.365	(-0.96)	0.604	(0.55)
$u^{Y}_{c,t-5}$					-0.337	(-0.44)
$u^{Y}_{c,t-6}$					0.036	(0.71)
$u^{Y}_{c,t-7}$					0.347	(0.58)
$u^{Y}_{c,t-8}$					-0.754	(-1.51)
Covariance	0.0525	0.0196	0.0247	0.0109	0.0182	0.0066
Matrix	0.0196	0.0755	0.0109	0.0409	0.0066	0.0297
Log L	73.33		79.99		87.27	

Note: see note to table 3.

Table 7. Interest rate

	Coeff	z	Coeff.	z	Coeff.	z
$u^R_{c,t-1}$	0.503	(9.34)	0.020	(0.28)	0.617	(8.35)
$u^R_{c,t-2}$			0.094	(1.35)	0.353	(4.00)
$u^R_{c,t-3}$			-0.322	(-4.82)	-0.213	(-2.89)
$u^R_{c,t-4}$			-0.698	(-11.64)	-0.447	(-4.05)
$u^R_{c,t-5}$					-0.555	(-5.55)
$u^R_{c,t-6}$					-0.178	(-1.44)
$u^R_{c,t-7}$					0.034	(0.26)
$u^R_{c,t-8}$					-0.056	(-0.58)
Variance	0.0321		0.0229		0.0171	
Log L	126.64		143.78		159.35	

Note: see note to table 3.

Table 8. Consumption and interest rate, 1 lag

	Non-durable	z	Interest Rate	z
$u^{ND}_{c,t}$	1			
$u^{ND}_{c,t-1}$	0.312	(1.01)		
$u^R_{c,t}$	0.561	(2.57)	1	
$u^R_{c,t-1}$	-0.352	(-1.49)	0.513	(9.47)
Covariance	0.0171	0.0108	0.0321	
Matrix	0.0108	0.0068		
Log L	181.81			

Note: the covariance matrix of $u^{ND}_{c,t}$ is reported under the "non -durable" columns, the variance of u^R_t is reported under the "interest rate" column.

Table 9. Consumption and interest rate, 4 lags

	Non-durable	z	Interest Rate	z
$u^{ND}_{c,t}$	1			
$u^{ND}_{c,t-1}$	0.831	(0.72)		
$u^{ND}_{c,t-2}$	-0.110	(-0.35)		
$u^{ND}_{c,t-3}$	-1.136	(-0.75)		
$u^{ND}_{c,t-4}$	-0.597	(-0.56)		
$u^{R}_{c,t}$	0.424	(1.49)	1	
$u^{R}_{c,t-1}$	-0.430	(-1.19)	0.319	(3.70)
$u^{R}_{c,t-2}$	0.247	(0.70)	-0.239	(-3.03)
$u^{R}_{c,t-3}$	-0.461	(-1.40)	-0.638	(-10.23)
$u^{R}_{c,t-4}$	0.119	(0.41)	-0.370	(-4.71)
Covariance	0.0144	0.0049	0.0228	
Matrix	0.0049	0.0072		
Log L	204.32			

Note: see note to table 8.

Table 10. Consumption and interest rate, 8 lags

	Non-durable	z	Interest Rate	z
$u_{c,t}^{ND}$	1			
$u_{c,t-1}^{ND}$	0.444	(0.12)		
$u_{c,t-2}^{ND}$	-0.420	(-0.09)		
$u_{c,t-3}^{ND}$	-0.073	(-0.03)		
$u_{c,t-4}^{ND}$	0.806	(0.15)		
$u_{c,t-5}^{ND}$	-0.221	(-0.03)		
$u_{c,t-6}^{ND}$	-1.129	(-0.12)		
$u_{c,t-7}^{ND}$	-0.514	(-0.09)		
$u_{c,t-8}^{ND}$	0.091	(0.04)		
$u_{c,t}^{R}$	0.839	(2.01)	1	
$u_{c,t-1}^{R}$	-0.212	(-0.42)	0.623	(6.96)
$u_{c,t-2}^{R}$	0.240	(0.57)	0.360	(3.49)
$u_{c,t-3}^{R}$	-0.646	(-1.37)	-0.190	(-2.32)
$u_{c,t-4}^{R}$	0.331	(0.66)	-0.399	(-3.36)
$u_{c,t-5}^{R}$	0.285	(0.63)	-0.533	(-4.93)
$u_{c,t-6}^{R}$	0.137	(0.30)	-0.175	(-1.48)
$u_{c,t-7}^{R}$	-0.381	(-0.91)	0.030	(0.22)
$u_{c,t-8}^{R}$	0.084	(0.24)	-0.051	(-0.49)
Covariance	0.0100	0.0037	0.0171	
Matrix	0.0037	0.0020		
Log L	226.99			

Note: see note to table 8.

Table 11. Consumption, income and interest rate, 1 lag

	Non durable	z	Income	z	Interest rate	z
$u_{c,t}^{ND}$	1					
$u_{c,t-1}^{ND}$	-0.396	(-2.66)				
$u_{c,t}^{Y}$	0.670	(3.56)	1			
$u_{c,t-1}^{Y}$	-0.357	(-1.71)	-0.498	(-4.48)		
$u_{c,t}^{R}$	0.568	(2.65)	0.561	(3.01)	1	
$u_{c,t-1}^{R}$	-0.319	(-1.31)	-0.340	(-1.72)	0.506	(9.12)
Covariance	0.0200	0.0131	0.0395	0.0130	0.0324	
Matrix	0.0131	0.0090	0.0130	0.0716		
Log L	303.40					

Note: see note to table 8.

Table 12. Consumption, income and interest rate, 4 lags

	Non durable	z	Income	z	Interest rate	z
$u_{c,t}^{ND}$	1					
$u_{c,t-1}^{ND}$	0.153	(0.06)				
$u_{c,t-2}^{ND}$	0.0002	(0.03)				
$u_{c,t-3}^{ND}$	-0.483	(-0.15)				
$u_{c,t-4}^{ND}$	-0.668	(-0.41)				
$u_{c,t}^{Y}$	0.803	(3.01)	1			
$u_{c,t-1}^{Y}$	-0.331	(-0.76)	-0.732	(-2.77)		
$u_{c,t-2}^{Y}$	-0.239	(-0.65)	0.384	(1.05)		
$u_{c,t-3}^{Y}$	-0.189	(-0.59)	-0.302	(-1.13)		
$u_{c,t-4}^{Y}$	-0.040	(-0.13)	-0.361	(-1.41)		
$u_{c,t}^{R}$	0.458	(1.57)	0.522	(1.68)	1	
$u_{c,t-1}^{R}$	-0.424	(-1.06)	-0.453	(-1.20)	0.292	(3.17)
$u_{c,t-2}^{R}$	0.209	(0.52)	0.041	(0.10)	-0.239	(-2.96)
$u_{c,t-3}^{R}$	-0.488	(-1.26)	-0.239	(-0.78)	-0.624	(-9.39)
$u_{c,t-4}^{R}$	0.159	(0.49)	0.009	(0.03)	-0.382	(-4.50)
Covariance	0.0100	0.0031	0.0389	0.0132	0.0232	
Matrix	0.0031	0.0066	0.0132	0.0620		
Log L	334.01					

Note: see note to table 8.

Table 13. Consumption, income and interest rate, 8 lags

	Non durable	z	Income	z	Interest rate	z
$u_{c,t}^{ND}$	1					
$u_{c,t-1}^{ND}$	0.147	(0.01)				
$u_{c,t-2}^{ND}$	0.793	(0.03)				
$u_{c,t-3}^{ND}$	-0.247	(-0.03)				
$u_{c,t-4}^{ND}$	0.423	(0.02)				
$u_{c,t-5}^{ND}$	-0.550	(-0.02)				
$u_{c,t-6}^{ND}$	0.115	(0.01)				
$u_{c,t-7}^{ND}$	-0.961	(-0.08)				
$u_{c,t-8}^{ND}$	-0.751	(-0.09)				
$u_{c,t}^{Y}$	0.861	(1.50)	1			
$u_{c,t-1}^{Y}$	-0.485	(-0.64)	-0.672	(-1.24)		
$u_{c,t-2}^{Y}$	-0.072	(-0.10)	0.551	(0.95)		
$u_{c,t-3}^{Y}$	-0.797	(-0.84)	-0.840	(-1.66)		
$u_{c,t-4}^{Y}$	0.535	(0.58)	0.752	(0.86)		
$u_{c,t-5}^{Y}$	0.320	(0.41)	-0.367	(-0.50)		
$u_{c,t-6}^{Y}$	-0.250	(-0.35)	-0.004	(-0.01)		
$u_{c,t-7}^{Y}$	0.160	(0.20)	0.327	(0.62)		
$u_{c,t-8}^{Y}$	-0.248	(-0.39)	-0.766	(-1.65)		
$u_{c,t}^{R}$	0.926	(1.60)	0.910	(1.67)	1	
$u_{c,t-1}^{R}$	-0.192	(-0.32)	-0.468	(-0.78)	0.641	(5.35)
$u_{c,t-2}^{R}$	0.309	(0.61)	0.135	(0.23)	0.449	(2.53)
$u_{c,t-3}^{R}$	-0.703	(-1.26)	-0.161	(-0.34)	-0.101	(-0.85)
$u_{c,t-4}^{R}$	0.334	(0.52)	0.098	(0.15)	-0.311	(-2.05)
$u_{c,t-5}^{R}$	0.302	(0.56)	0.053	(0.11)	-0.533	(-3.92)
$u_{c,t-6}^{R}$	0.113	(0.19)	0.115	(0.24)	-0.221	(-1.77)
$u_{c,t-7}^{R}$	-0.230	(-0.42)	0.085	(0.17)	-0.005	(-0.03)
$u_{c,t-8}^{R}$	-0.015	(-0.03)	0.074	(0.15)	-0.106	(-0.88)
Covariance	0.0034	0.0011	0.01777	-0.00004	0.01757	
Matrix	0.0011	0.00057	-0.00004	0.02940		
Log L	367.78					

Note: see note to table 8.

Table 14. Consumption, income and interest rate, 8 lags, constrained

	Non durable	z	Income	z	Interest rate	z
$u^{ND}_{c,t}$	1					
$u^{ND}_{c,t-1}$	-0.273	(-0.09)				
$u^{ND}_{c,t-2}$	0.114	(0.03)				
$u^{ND}_{c,t-3}$	-0.177	(-0.07)				
$u^{ND}_{c,t-4}$	-0.662	(-0.73)				
$u^{ND}_{c,t-5}$	-					
$u^{ND}_{c,t-6}$	-					
$u^{ND}_{c,t-7}$	-					
$u^{ND}_{c,t-8}$	-					
$u^{Y}_{c,t}$	0.896	(2.22)	1			
$u^{Y}_{c,t-1}$	-0.603	(-1.02)	-0.851	(-2.02)		
$u^{Y}_{c,t-2}$	-0.097	(-0.19)	0.625	(1.28)		
$u^{Y}_{c,t-3}$	-0.384	(-0.61)	-0.544	(-1.47)		
$u^{Y}_{c,t-4}$	0.227	(0.44)	0.246	(0.37)		
$u^{Y}_{c,t-5}$	-		-0.174	(-0.27)		
$u^{Y}_{c,t-6}$	-		0.105	(0.25)		
$u^{Y}_{c,t-7}$	-		0.072	(0.15)		
$u^{Y}_{c,t-8}$	-		-0.484	(-1.40)		
$u^{R}_{c,t}$	0.911	(1.88)	0.942	(1.87)	1	
$u^{R}_{c,t-1}$	-0.186	(-0.33)	-0.493	(-0.87)	0.640	(5.98)
$u^{R}_{c,t-2}$	0.328	(0.70)	0.164	(0.28)	0.445	(2.96)
$u^{R}_{c,t-3}$	-0.738	(-1.37)	-0.177	(-0.38)	-0.103	(-1.01)
$u^{R}_{c,t-4}$	0.317	(0.54)	0.095	(0.15)	-0.309	(-2.42)
$u^{R}_{c,t-5}$	0.310	(0.64)	0.055	(0.10)	-0.531	(-3.97)
$u^{R}_{c,t-6}$	0.122	(0.26)	0.114	(0.28)	-0.216	(-1.91)
$u^{R}_{c,t-7}$	-0.238	(-0.44)	0.093	(0.21)	0.002	(0.02)
$u^{R}_{c,t-8}$	0.020	(0.05)	0.076	(0.17)	-0.102	(-0.89)
Covariance	0.0225	0.0059	0.0228	0.0007	0.0176	
Matrix	0.0059	0.0043	0.0007	0.0404		
Log L	363.29					

Note: see note to table 8

Appendix

The state space representation of the model

For ease of notation, a multivariate MA(1) model is considered in which there are three variables (i.e. first difference of log consumption, of income, and the interest rate) and two cohorts. These summarize the three types of variables included in the model:

$$
\begin{aligned}
\Delta x_t^1 &= u_t^{x1} + \alpha_1^{xx} u_{t-1}^{x1} + \alpha_0^{xy} u_t^{y1} + \alpha_1^{xy} u_{t-1}^{y1} + \alpha_0^{xr} u_t^r + \alpha_1^{xr} u_{t-1}^r + \eta_t^{x1} - \eta_{t-1}^{x1} \\
\Delta x_t^2 &= u_t^{x2} + \alpha_1^{xx} u_{t-1}^{x2} + \alpha_0^{xy} u_t^{y2} + \alpha_1^{xy} u_{t-1}^{y2} + \alpha_0^{xr} u_t^r + \alpha_1^{xr} u_{t-1}^r + \eta_t^{x2} - \eta_{t-1}^{x2} \\
\Delta y_t^1 &= u_t^{y1} + \alpha_1^{yy} u_{t-1}^{y1} + \alpha_0^{yr} u_t^r + \alpha_1^{yr} u_{t-1}^r + \eta_t^{y1} - \eta_{t-1}^{y1} \\
\Delta y_t^2 &= u_t^{y2} + \alpha_1^{yy} u_{t-1}^{y2} + \alpha_0^{yr} u_t^r + \alpha_1^{yr} u_{t-1}^r + \eta_t^{y2} - \eta_{t-1}^{y2} \\
r_t &= u_t^r + \alpha_1^{rr} u_{t-1}^r
\end{aligned}
$$

The model may be easily written in state space representation, where the state vector is:

$$
\xi_{t+1} = \begin{bmatrix} u_{t+1}^{x1} & u_{t+1}^{x2} & u_{t+1}^{y1} & u_{t+1}^{y2} & u_{t+1}^{r} & \eta_{t+1}^{x1} & \eta_{t+1}^{x2} & \eta_{t+1}^{y1} & \eta_{t+1}^{y2} & u_t^{x1} & \dots & \eta_t^{y2} \end{bmatrix}'
$$

of dimension equal to $(n^*c+z)\times(q+1)+n^*c^*2=k$, where q is the number of lags, n is the number of cohort specific variables, c is the number of cohorts, z is the number of fixed-across-cohort variables, and the second term in the sum is the measurement error terms (which does not depend on the number of lags in the model).

Define:

$M=(n^*c+z)$ i.e. the number of dependent variables in the model (5 in the example);

k the dimension of the space vector;

k_1=M+n*c i.e. the number of variables at time $t+1$ in the space vector plus the measurement error component at time $t+1$.

The state and measurement equations are:

$$
\begin{aligned}
\xi_{t+1} &= F\xi_t + v_t \\
X_t &= H\xi_t
\end{aligned}
$$

where X_t is the vector of M dependent variables, H is a $(M\times k)$ matrix containing the α parameters as well as block of zeros, F is a $(k\times k)$ matrix of zero's and one's

and ν is the state equation disturbance vector, where the first k_1 entries are given by the variables at time t+1 in the state vector, and all the other entries are always zero. The variance-covariance matrix Q of ν is a diagonal matrix apart from the entries in which there is the correlation among error terms for the same cohort. The measurement equation has no noise, so its variance-covariance matrix, R, is equal to zero. All these matrices are described in the last section.
The log likelihood function of the model is given by:

$$\log L = -\frac{NT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log\left|G_t\right| - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t{}' G_t^{-1}\varepsilon_t$$

which is the prediction error decomposition form of the likelihood. The prediction errors are given by:

$$\varepsilon_t = X_t - H\hat{\xi}_{t/t-1}$$

with associated MSE:

$$G_t = HP_{t/t-1}H' + R$$

where the matrix R is the variance-covariance matrix of the disturbance term in the measurement equation, and in this case is equal to zero.
The prediction errors and their MSE's can be calculated using the Kalman filter recursions:

$$\hat{\xi}_{t/t-1} = F\hat{\xi}_t$$
$$P_{t/t-1} = FP_{t-1}F' + Q$$
$$\hat{\xi}_t = \hat{\xi}_{t/t-1} + P_{t/t-1}H'(HP_{t/t-1}H' + R)^{-1}(X_t - H\hat{\xi}_{t/t-1})$$
$$P_t = P_{t/t-1} - P_{t/t-1}H'(HP_{t/t-1}H' + R)^{-1}HP_{t/t-1}$$

Definition of the matrices

Matrix H, in the measurement equation, is given by:

$$\begin{bmatrix} 1 & 0 & \alpha_0^{xy} & 0 & \alpha_0^{xr} & 1 & 0 & 0 & 0 & \alpha_1^{xx} & 0 & \alpha_1^{xy} & 0 & \alpha_1^{xr} & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \alpha_0^{xy} & \alpha_0^{xr} & 0 & 1 & 0 & 0 & 0 & \alpha_1^{xx} & 0 & \alpha_1^{xy} & \alpha_1^{xr} & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & \alpha_0^{yr} & 0 & 0 & 1 & 0 & 0 & 0 & \alpha_1^{yy} & 0 & \alpha_1^{yr} & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & \alpha_0^{yr} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & \alpha_1^{yy} & \alpha_1^{yr} & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_1^{rr} & 0 & 0 & 0 & 0 \end{bmatrix}$$

The matrix F, in the transition equation, is given by:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}$$

where the three zero blocks are of dimension ($k_1 \times k_1$), and I is an identity matrix of dimension k_1. If there are two lags matrix F takes the form:

$$\begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix}$$

and so on.

The error term in the transition equation is:

$$\upsilon_{t+1} = \begin{bmatrix} u_{t+1}^{x1} & u_{t+1}^{x2} & u_{t+1}^{y1} & u_{t+1}^{y2} & u_{t+1}^{r} & \eta_{t+1}^{x1} & \eta_{t+1}^{x2} & \eta_{t+1}^{y1} & \eta_{t+1}^{y2} & \mathbf{0} \end{bmatrix}'$$

where the zero block is a vector of dimension k-k_1.

The ($k_1 \times k_1$) upper-left block of the variance-covariance matrix of v, Q, is:

$$\left[\begin{array}{ccccc|cccc}
\sigma^2_{ux1} & \sigma^2_{ux1x2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sigma^2_{ux1x2} & \sigma^2_{ux2} & 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\
0 & 0 & \sigma^2_{uy1} & \sigma^2_{uy1y2} & & & & & \\
\vdots & 0 & \sigma^2_{uy1y2} & \sigma^2_{uy2} & 0 & & & & \\
 & \vdots & 0 & 0 & \sigma^2_{ur} & 0 & 0 & 0 & 0 \\
\hline
 & & \vdots & 0 & 0 & \sigma^2_{\eta x1} & 0 & \sigma^2_{\eta x1y1} & 0 \\
 & & & \vdots & 0 & 0 & \sigma^2_{\eta x2} & 0 & \sigma^2_{\eta x2y2} \\
 & & & & 0 & \sigma^2_{\eta x1y1} & 0 & \sigma^2_{\eta y1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \sigma^2_{\eta x2y2} & 0 & \sigma^2_{\eta y2}
\end{array}\right]$$

while the other three (k-$k_1 \times k$-k_1) blocks are zero.

The variances of the measurement error terms are treated as known in the estimation.

Chapter Three

The Error Structure of Earnings: an Analysis on Italian Longitudinal Data

1. Introduction

The availability of longitudinal surveys has allowed researchers to model the individuals' covariance pattern of earnings over time. Several authors using US panel data have performed this kind of study[1]. In particular, MaCurdy (1982) develops a set of statistical procedures in order to choose among different specifications of the error structure. In his application to the Michigan Panel of Income Dynamics, his preferred specification is given by an MA(2) model applied to the change in (the logarithm of) earnings, which implies an ARMA(1,2) model with a unit root for the same variable expressed in levels.

Using the Panel Study of Income Dynamics (PSID), Moffitt and Gottschalk (1995) model the unobserved component of (the logarithm of) earnings as the sum of a transitory component and a permanent component; in their preferred specification the permanent component is modelled as a random walk process.

In this chapter, the same line of research is followed in order to characterise the time series properties of earnings in Italy, using the panel data set drawn from the Bank of Italy Survey of Households' Income and Wealth (SHIW). The Bank of Italy Survey is drawn every two years: this feature raises identification problems as the first-order autocovariance is not observed. It is therefore not

[1] Among others, Lillard and Willis (1978), Abowd and Card (1989), and Gottschalk and Moffitt (1994) use the PSID data set in order to characterise the earnings process.

possible to distinguish among stationary models that imply one-lag covariances in the structure, as would be the case of an MA(1) component. However, it is possible to use the panel dimension of the data set in order to discriminate between several specifications that imply different covariance patterns. In particular, it is possible to characterize both the standard permanent-transitory model and models that contain AR(1) components. In addition, in order to exploit the differences that may arise due to heterogeneuos education attainments, estimates are performed by education group.

Results show that the AR(1) plus individual effect model provides the best characterisation of the unobserved component of the earnings process. The estimated autoregressive parameter however is well below unity, indicating stationarity.

Section 2 develops the theoretical models that will be tested in the empirical analysis, section 3 gives a brief description of the data set, and section 4 presents the results. Section 5 concludes.

2. Models for the Earning Process

The empirical formulation for the earning process typically used in the literature[2] is:

$$y_{it}^{a} = X_{it}^{a}{}'\beta + u_{it}^{a}$$

y_{it}^{a} is the natural logarithm of real earnings of the *i*-th individual at time *t*, where the index *a* (age) has been added to stress the fact that the variables in the model may as well depend on the position of the individual over the life-cycle. X_{it}^{a} is a $(k\times1)$ vector of observable variables, β is a $(k\times1)$ vector of unknown parameters, and u_{it}^{a} is an error term which represents unobserved characteristics determining earnings. The variables included in X are a polynomial in age, which captures the life-cycle profile of earnings, measures of education and other information available about the labour supply behaviour of the individuals

[2] Among others, see Lillard and Willis (1978), MaCurdy (1982), Abowd and Card (1989) and Moffitt and Gottschalk (1995).

in the sample. In addition, time dummies for each period are included in order to capture the common period effects. Consequently, the disturbances u_{it}^a are assumed to be independently distributed across individuals but not over time. Modelling their covariance structure is the main concern of this study. Several specifications have been proposed and tested in the literature: here the attention is concentrated on those specifications that can be identified using the Bank of Italy panel, which collects data every two years.

Permanent-Transitory Model

The simplest model for the earnings structure that has been studied in the literature is the permanent-transitory model, where the unobserved component of earnings for an individual i of age a is decomposed into a permanent component which is time invariant (μ_i) and a transitory idiosyncratic shock (ε_{it}^a).

$$u_{it}^a = \mu_i + \varepsilon_{it}^a \qquad (1)$$

where both μ_i and ε_{it}^a are i.i.d. with zero mean and variance equal to σ_μ^2 and σ_ε^2, respectively.

Estimation of this model is feasible if one observes the variance of earnings and its covariance. The theoretical moments are:

$$\text{var}(u_{it}^a) = \sigma_\mu^2 + \sigma_\varepsilon^2 \qquad \text{and}$$

$$\text{cov}(u_{it}^a, u_{i,t-s}^{a-s}) = \sigma_\mu^2 \qquad s=1, 2, \ldots$$

The major implications of this model are that the variance and the covariances of the unobserved component of earnings are constant over time. In addition, the theoretical covariances are identical at different lags. From those conditions it is clear that the model can be identified observing, in addition to the variance, the covariances at lag 2, 4, and so on. Therefore the parameters in the model can be identified using the Bank of Italy panel data set.

More realistic models

A model that has proved to be a good characterization of the earning process in the US is a model where the transitory component exihibits some autocorrelation. Assuming the transitory component follows an AR(1) process, the unobserved component of earnings can be written as:

$$\begin{aligned} u_{it}^{a} &= \mu_i + z_{it}^{a} \\ z_{it}^{a} &= \alpha\, z_{i,t-1}^{a-1} + \omega_{it}^{a} \end{aligned} \qquad (2)$$

where ω_{it}^{a} is an i.i.d. stochastic process with zero mean and variance σ_{ω}^{2} and μ_i is defined as before. This structure can be estimated if one observes the variance of earnings for a given individual and its covariance at different ages and points in time. The variances of this process can be summarised by the following recursions:

$$Var(u_{it}^{a}) = \sigma_{\mu}^{2} + Var(z_{it}^{a})$$

where:

$$Var(z_{it}^{\underline{a}}) = \sigma_{\underline{a}}^{2}$$

and:

$$Var(z_{it}^{a}) = \alpha^{2}\, Var(z_{it}^{a-1}) + \sigma_{\omega}^{2} \qquad \text{with } a > \underline{a}$$

Similarly, the covariances are defined as:

$$Cov\left(u_{it}^{a}, u_{it}^{a-s}\right) = \sigma_{\mu}^{2} + \alpha^{s}\, Var(z_{it}^{a-s}) \qquad s=1, 2, \ldots$$

where the formulas reflect the fact that the AR(1) component arises from a finite process starting at age $\underline{a}$, the age at which individuals enter the labour market. Estimation of a finite process allows to overcome the problems associated with unit roots, as the recursion formulas are well defined even if the autoregressive parameter is equal to (or greater than) one.

Contrary to standard time series analysis, the initial values of the autoregressive component should not be treated as known constants in models for longitudinal data where the time dimension is tipically quite small[3]. Here the autoregressive process is assumed to start at age $\underline{a}$, and the variance of the zero mean initial distribution of the process z_{it}^{a} ($\sigma_{\underline{a}}^{2}$) is estimated.

A generalization of the autoregressive model just discussed is a model in which the transitory component ω_{it}^{a} is not i.i.d. but displays some autocorrelation. To take a concrete example, consider the case in which ω_{it}^{a} is an MA(1) process:

$$\omega_{it}^{a} = \xi_{it}^{a} + \theta\xi_{i,t-1}^{a-1} \qquad (3)$$

The theoretical moments implied by this structure are shown in the Appendix. It should be noticed that the autocovariance function of an ARMA(1,1) model depends on the MA parameter at lags greater than one. However, failure to observe the first order autocovariance may render the empirical identification of such a parameter more problematic[4].

Estimation is carried out using the minimum distance method, which compares the sample moments to the theoretical ones (Chamberlain, 1984). Denoting the ($m\times1$) vector of sample moments as π and the vector of theoretical moments as $\pi(\alpha)$, which depends on ($n\times1$) unknown parameters (with $n<m$), the minimum distance method minimizes the function:

$$\min_{\alpha}(\pi - \pi(\alpha))'V(\pi - \pi(\alpha))$$

where V is a weighting matrix. When V is taken to be the inverse of the matrix of fourth moments the estimator is the well-known optimal minimum distance (OMD). However, Altonji and Segal (1996) warn about the bias that arises when estimating covariance structures of this type, and suggest the use of the equally

[3] See Anderson and Hsiao (1982) and MaCurdy (1982).

[4] In order to ease identification, the initial values of the process in this case have been set equal to zero.

weighted minimum distance (EWMD) which replaces V with the identity matrix. The latter strategy will be used in the estimation.

3. The Data

In order to model the earnings structure and its time series properties, the panel sample from the Bank of Italy Survey has been used. This is the most comprehensive survey of individual data in Italy and it contains detailed information on household members' demographic characteristics and labour supply variables. The Survey has been run since 1977, but it has a panel dimension only since 1989. Data are available until 1998 so that there are 5 consecutive waves of the sample that can be used in estimation[5].

Each wave about 8,000 families representative of the Italian population are interviewed; approximately 40% of them are interviewed in subsequent waves. However, only 10% of households interviewed in 1989 have been interviewed up to 1995. Therefore, the sample used in the analysis has been built using all individuals who have been interviewed for at least two consecutive waves of the survey. The use of an unbalanced sample in estimation considerably reduces the sample attrition bias present in panel data sets.

The dependent variable used in the analysis is built upon the logarithm of real annual gross earnings of each individual in the sample who reported positive earnings and classified himself as dependent worker (either in the private or in the public sector)[6]. Annual gross earnings have been deflated using the ISTAT consumer price index, and they are expressed in 1998 prices. The analysis is carried out using only male workers aged between 22 and 60, as for male workers the participation issue is less stringent than for female workers. After

[5] The available years are: 1989, 1991, 1993, 1995 and 1998. This implies that it is possible to compute the sample covariances of order two, three, four, five and so on.

[6] Earnings are gross of income tax but net of Social Security contributions. The variable actually reported in the Survey is "normal annual net earnings". However, as detailed demographic information is available in the data set, gross earnings have been computed for each individual in the sample. I am in debt to Agar Brugiavini who kindly provided me with the algorithm used to build gross earnings.

applying the selection criteria, the overall sample consists of 5,231 observations, of which 3,329 employed in the private sector.

The variable actually used in the analysis is built as the residuals from regressions of the logarithm of gross earnings on a polynomial in age and cohort and time dummies, controlling for education. In particular, the sample has been divided into 6 year-of-birth groups, in order to remove cohort effects in the variable of interest[7]. The youngest cohort is formed by individuals born between 1963 and 1967 included, and the eldest by individuals born between 1938 and 1942 included. In the analysis, individuals in the youngest cohort are considered as aged 24 in 1989, 26 in 1991 and so on. The other cohorts are treated similarly. Regressions are then performed by education group using as regressors a polynomial in age and cohort and time dummies, both for private and for public employees[8].

Estimates of the different specifications for the unobserved component of earnings are computed splitting the residuals into four groups, arising from two education groups for each sector, public and private. The two education groups are: high school dropouts (2864 observations, of which 2106 employed in the private sector) and high school and college graduates (2267 observations, of which 1223 employed in the private sector). College graduates on their own would form a sample of 222 and 425 observations in the private and in the public sector respectively, which has been considered too small to be treated separately in the analysis.

Figures 1 and 2 plot the sample variance and the second- and fourth-order covariance of the (residuals of) gross earnings both for private and for public sector dependent workers against age. Both figures show that variances and

[7] The quantitative importance of the cohort effects in the cross-sectional variance of earnings has been documented for example by Deaton and Paxon (1994) and Storesletten et al. (2000).

[8] It is not possible to separately identify age, cohort and time effects without any further assumption, as they are linear combinations of one another. In the analysis it is therefore assumed that the time effects are orthogonal to a time trend and add up to zero.

covariances do not appear to increase over time, a feature that is captured by stationary models. In addition, covariances of increasing order appear to decrease slowly, indicating some persistence in unobservable earnings.

4. Results

Minimum distance estimation of the models described above has been performed separately for private and public sector workers. In addition, estimates for different education groups are reported.

Tables 1 and 2 report estimates for the permanent-transitory model described by equation (1) respectively for private sector and public sector employees. Each table reports estimates both for the whole sample and for the two education groups: 1) high school dropouts, and 2) high school and college graduates. Similarly, tables 3 and 4 show estimated coefficients for the AR(1) model with fixed effect, and tables 5 and 6 present estimates of the parameters for the ARMA(1,1) model. In addition, for each table a Wald test is reported, built on the null hypothesis that the parameters are not statistically different in the two sub-samples considered[9].

For private sector dependent workers, the parameter estimates of the permanent-transitory model in table 1 imply that the overall variance of the unobserved component for the entire sample is 0.077, with a permanent variance of 0.036. Columns 2 and 3 in table 1 show the estimated coefficients for the two education groups considered: high school dropouts and high school and college graduates. The Wald statistics, however, indicates that the differences in the estimates are not statistically significant.

In table 2 estimates for the public sector dependent workers of permanent-transitory model are reported. The overall variance estimated for the whole sample is 0.058, expectedly lower than the overall variance for private sector employees. In particular, the overall variance for high school dropouts is 0.044, while for high school and college graduates is 0.066. The Wald statistics

[9] See Appendix for more details.

indicates that the parameter estimates for the two groups are in this case statistically different from each other.

Turning to the AR(1) estimates, table 3 shows that in the private sector the autoregressive parameter is statistically different from zero. The value of α for the whole sample is equal to 0.54, a value that indicates stationarity of the estimated process. The fixed effect variance is also estimated to be different from zero. Differences in the estimates of the two groups are not statistically significant.

The AR(1) model for the public sector employees is presented in table 4. The autoregressive parameter is precisely estimated and is higher then the parameter estimated for private sector dependent workers. The variance of the permanent component is not statistically different from zero. The Wald statistics suggests that differences in the parameters of the two groups are statistically significant.

Estimates of the ARMA(1,1) model plus a fixed effect are shown in tables 5 and 6 for private and public sector workers respectively. The moving average parameter is not statistically different from zero, while the other parameter estimates are close to those obtained for the AR(1) representation. In addition, the residual sum of squares are very close for the two models.

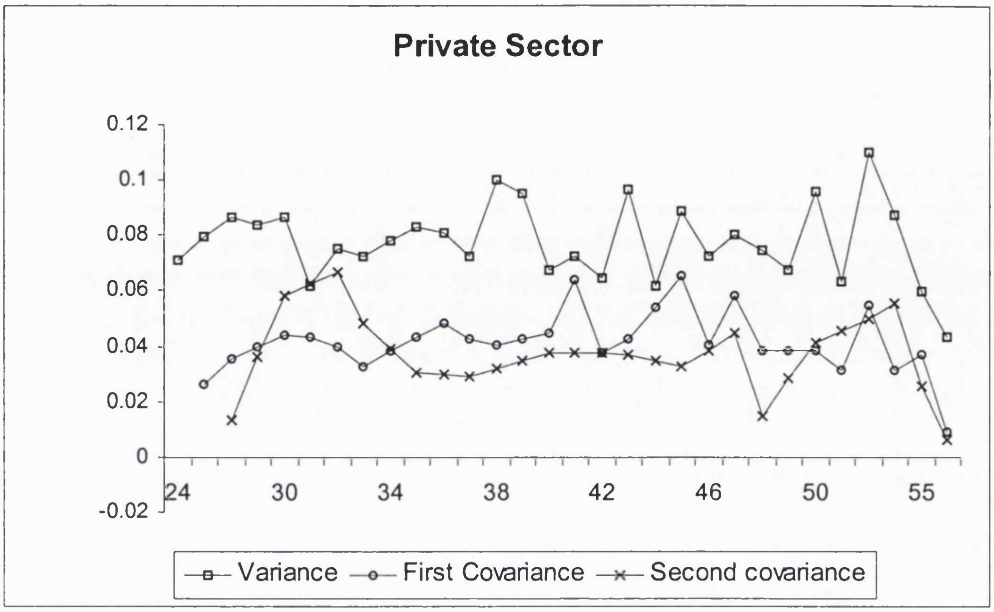
This evidence suggests that, given the data set used, the best characterization for the unobserved component of earnings in Italy seems to be represented by the sum of a stationary AR(1) model and a fixed effect.

5. Conclusions

In this study the panel drawn from the Bank of Italy Survey of Households' Income and Wealth has been used in order to characterise the covariance structure of the unobserved component of earnings.

Various models have been estimated for different sectors (private and public) and for different education groups, in order to exploit the differences that may arise due to heterogeneuos education attainments. The specification that better captures the features of the data is a model given by the sum of an AR(1) component and an individual fixed effect. The autoregressive coefficient has been estimated to be around 0.55 in the private sector and 0.8 in the public sector. In the latter group, parameter differences among education groups are found statistically significant, while in the former differences in the estimated parameters for the two education groups do not appear to be statistically significant.

Figure 1: Variance and covariance



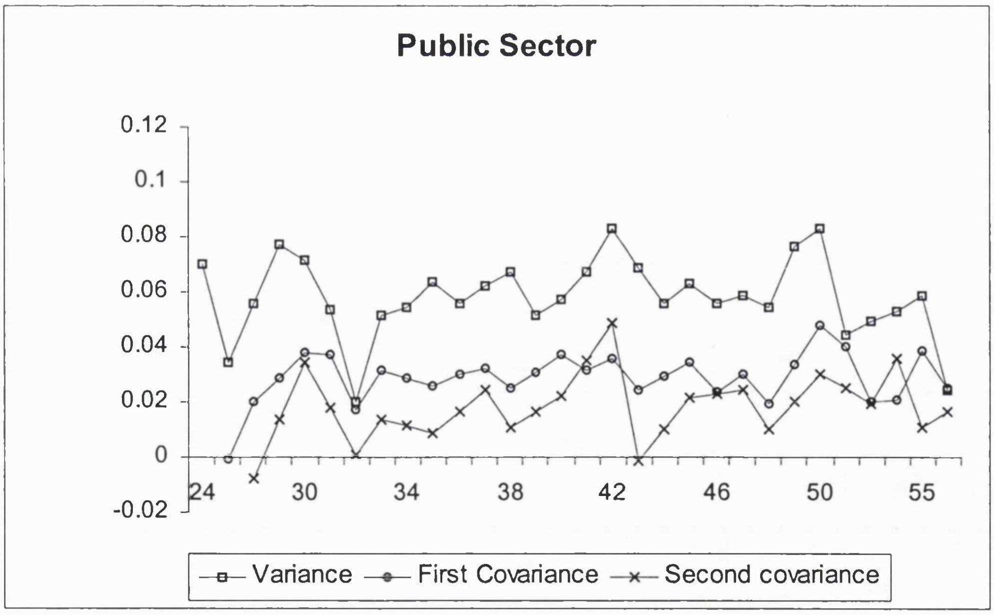


Table 1. Private sector dependent workers

	Whole sample	Education: no high school	Education: high school and college
σ^2_μ	0.036	0.034	0.039
t-stat	(21.22)	(16.57)	(12.46)
σ^2_ε	0.041	0.041	0.041
t-stat	(15.24)	(12.01)	(8.97)
Sum of Squared Residuals	*0.0214*	*0.0240*	*0.0572*
N. of Obs.	*3329*	*2106*	*1223*

Wald statistic: $\chi^2(2)=3.16$

Table 2. Public sector dependent workers

	Whole sample	Education: no high school	Education: high school and college
σ^2_μ	0.020	0.016	0.023
t-stat	(10.08)	(9.43)	(8.57)
σ^2_ε	0.038	0.028	0.043
t-stat	(13.00)	(9.57)	(10.52)
Sum of Squared Residuals	*0.0186*	*0.0180*	*0.0321*
N. of Obs.	*1902*	*758*	*1144*

Wald statistic: $\chi^2(2)=36.76$

AR(1) Model

Table 3. Private sector dependent workers

	Whole sample	Education: no high school	Education: high school and college
σ_{η}^{2}	0.028	0.027	0.032
t-stat	(6.25)	(5.55)	(3.88)
α	0.573	0.545	0.533
t-stat	(7.26)	(5.41)	(3.23)
σ_{ω}^{2}	0.033	0.034	0.034
t-stat	(11.12)	(8.91)	(6.87)
$\sigma_{\underline{a}}^{2}$	0.037	0.032	0.045
t-stat	(2.79)	(1.94)	(1.94)
Sum of Squared Residuals	*0.0199*	*0.0229*	*0.0558*
N. of Obs.	*3329*	*2106*	*1223*

Wald statistic: $\chi^2(4)=3.74$

Table 4. Public sector dependent workers

	Whole sample	Education: no high school	Education: high school and college
σ_{η}^{2}	-0.010	0.004	-0.060
t-stat	(-0.60)	(0.52)	(-1.07)
α	0.809	0.712	0.908
t-stat	(11.65)	(7.70)	(17.40)
σ_{ω}^{2}	0.024	0.021	0.022
t-stat	(8.84)	(6.59)	(7.24)
$\sigma_{\underline{a}}^{2}$	0.046	0.018	0.098
t-stat	(1.64)	(1.71)	(1.42)
Sum of Squared Residuals	*0.0147*	*0.0147*	*0.0259*
N. of Obs.	*1902*	*758*	*1144*

Wald statistic: $\chi^2(4)=9.89$

ARMA(1,1) Model

Table 5. Private sector dependent workers

	Whole sample	Education: no high school	Education: high school and college
σ_{η}^{2}	0.028	0.027	0.033
t-stat	(6.68)	(5.15)	(4.16)
α	0.619	0.645	0.473
t-stat	(6.30)	(6.92)	(0.93)
θ	-0.200	-0.360	0.097
t-stat	(-0.59)	(-0.79)	(0.08)
σ_{ω}^{2}	0.029	0.025	0.036
t-stat	(3.53)	(2.26)	(3.50)
Sum of Squared Residuals	*0.0199*	*0.0227*	*0.0559*
N. of Obs.	*3329*	*2106*	*1223*

Wald statistic: $\chi^2(4)=1.98$

Table 6. Public sector dependent workers

	Whole sample	Education: no high school	Education: high school and college
σ_{η}^{2}	0.001	0.003	-0.002
t-stat	(0.18)	(0.51)	(-0.15)
α	0.765	0.709	0.795
t-stat	(11.64)	(7.31)	(15.31)
θ	-0.204	0.119	-0.247
t-stat	(-0.65)	(0.31)	(-0.64)
σ_{ω}^{2}	0.023	0.021	0.024
t-stat	(3.65)	(6.77)	(2.92)
Sum of Squared Residuals	*0.0149*	*0.0147*	*0.0271*
N. of Obs.	*1902*	*758*	*1144*

Wald statistic: $\chi^2(4)=2.59$

Appendix

Estimation

Estimation is carried out using the minimum distance method, which compares the sample moments to the theoretical ones (Chamberlain, 1984).

The sample moments are built using the residuals of log earnings as described in section 3 in the text. Denoting the $(m\times 1)$ vector of sample moments as π and the vector of theoretical moments as $\pi(\alpha)$, which depends on $(n\times 1)$ unknown parameters (with $n<m$), the minimum distance method minimizes the function:

$$\min_{\alpha}(\pi-\pi(\alpha))'V(\pi-\pi(\alpha))$$

where V is a weighting matrix. Following the findings in the study by Altonji and Segal (1996) on the bias that arises when estimating covariance structures of this type, the identity matrix has been used in estimation, i.e. $V=I$. The equally weighted minimum distance estimator obtained has the following distribution[10]:

$$\sqrt{N}(\alpha_{EWMD}-\alpha)\xrightarrow{d}N(0,W)$$

The variance-covariance matrix is defined as:

$$W=(G'G)^{-1}G'VG(G'G)^{-1}$$

where G is a $(m\times n)$ matrix of first derivatives, and V is the $(m\times m)$ variance-covariance matrix of the moments considered. Each element in V is computed using the residuals for each observation i:

$$v_{m,m'}=\frac{1}{N}\left(\frac{1}{N}\sum_{i=1}^{N}(\pi_{im}-\pi_{m}(\hat{\alpha}_{EWMD}))(\pi_{im'}-\pi_{m'}(\hat{\alpha}_{EWMD}))\right)$$

As the panel is unbalanced, a different number of individuals will contribute to different elements in W. To ease notation, this is left implicit in the above formula.

[10] Under some regularity conditions. See Hansen (1982) for a detailed exposition.

It has been tested whether the parameters are different for the two education groups considered in the estimation. Given the asymptotic normal distribution of the EWMD estimator and the fact that the two samples are independent, the Wald statistic has been computed to test the joint hypothesis that all the parameters are equal in the two groups (group 1 and group 2):

$$X = \left(\hat{\alpha}^1 - \hat{\alpha}^2\right)' \left(W^1 + W^2\right)\left(\hat{\alpha}^1 - \hat{\alpha}^2\right)$$

which is distributed as a chi-square with n (the dimension of the parameter vector) degrees of freedom.

Mapping

1) permanent-transitory model

$$u_{it}^a = \mu_i + \varepsilon_{it}^a$$

where i.i.d. measurement error is captured by the transitory component.
Estimation of this model is feasible if one observes the variance of earnings and its covariance. The theoretical moments are:

$$\mathrm{var}(u_{it}^a) = \sigma_\mu^2 + \sigma_\varepsilon^2 \qquad \text{and}$$

$$\mathrm{cov}(u_{it}^a, u_{i,t-s}^{a-s}) = \sigma_\mu^2 \qquad s=2, 3, 4,...$$

2) AR(1) model

$$u_{it}^a = \mu_i + z_{it}^a$$
$$z_{it}^a = \alpha z_{i,t-1}^{a-1} + \omega_{it}^a$$

Individuals start working at age $\underline{a}$ ($\underline{a} = 24$)
Moments are built as:

$$Var(u_{it}^a) = Var(\mu_i) + Var(z_{it}^a)$$

$$Var(\mu_i) = \sigma_\mu^2$$

$$Var(z_{it}^{\underline{a}}) = \sigma_{\underline{a}}^2$$

$$Var(z_{it}^{a}) = \alpha^2 Var(z_{it-1}^{a-1}) + \sigma_{\omega}^2 \qquad a > \underline{a}$$

$$Cov\left(u_{it}^{a}, u_{it-s}^{a-s}\right) = \sigma_{\mu}^2 + Cov\left(z_{it}^{a}, z_{it-s}^{a-s}\right) \qquad s=2,3,4...$$

$$Cov\left(z_{it}^{a}, z_{it-s}^{a-s}\right) = \alpha^s \, Var(z_{it-s}^{a-s}) \qquad s=2,3,4...$$

3) ARMA(1,1) model

$$u_{it}^{a} = \mu_i + z_{it}^{a}$$
$$z_{it}^{a} = \alpha \, z_{i,t-1}^{a-1} + \xi_{it}^{a} + \theta \xi_{it-1}^{a-1}$$

Moments are built as:

$$Var(u_{it}^{a}) = Var(\mu_i) + Var(z_{it}^{a})$$

$$Var(\mu_i) = \sigma_{\mu}^2$$

$$Var(z_{it}^{\underline{a}}) = \sigma_{\xi}^2$$

$$Var(z_{it}^{a}) = \alpha^2 Var(z_{it-1}^{a-1}) + \left(1 + \theta^2\right)\sigma_{\xi}^2 \qquad a > \underline{a}$$

$$Cov\left(u_{it}^{a}, u_{it-s}^{a-s}\right) = \sigma_{\mu}^2 + Cov\left(z_{it}^{a}, z_{it-s}^{a-s}\right)$$

$$Cov\left(z_{it}^{a}, z_{it-s}^{a-s}\right) = \alpha^s \, Var(z_{it-s}^{a-s}) + \alpha^{s-1}\theta \, \sigma_{\xi}^2 \qquad s=2,3,4...$$

Sample Moments

Sample moments have been built on the residuals of regressions of the logarithm of gross yearly earnings on an age polynomial and cohort and time dummies. Age, cohort and year effects cannot be separately identified without making some further assumptions, as they are linear combinations of one another. In the analysis it is therefore assumed that the time effects are orthogonal to a time trend and add up to zero.

In order to control for education, regressions are estimated separately for each education group (up to 5 years, 8 years, 13 years or 17+ years of education). To compute the sample moments, only two education groups have been considered (high school dropouts and high school and college graduates).

Five-year date-of-birth cohorts have been built, the younger cohort being born in 1963-1967, and the oldest in 1938-1942. The resulting cohorts are six and they are observed for 5 time intervals.
In the panel data set, individuals belonging to the younger cohort are observed at ages 24, 26, 28, 30 and 33. For these individuals it is therefore possible to compute 5 variances, 3 second-order covariances, one third covariance and so on. Other cohorts are treated similarly. In total there are 30 variances, 18 second order covariances, 6 lag three covariances, and so on.

Chapter four

Social Security Systems and the Distribution of Income:

An Application to the Italian Case

1. Introduction

In principle, Social Security systems also aim at redistributing resources towards low-income groups. Earnings based (EB) schemes traditionally include mechanisms in order to actively redistribute income within the insured workers. Floors, ceilings, and survivor benefits are among the tools through which redistribution occurs.

In practice, EB systems may operate redistribution in the opposite direction, i.e. from poor to rich[1]. This kind of redistribution arises from various features of the EB systems, notably from the benefit computation formula, which takes into account only the last (or the last few) wage, and therefore guarantees an overgenerous pension to individuals with steeper earnings profiles, typically high earners.

In this framework, it has been argued (James, 1997) that contribution based (CB) formulae could enhance equity by removing the inequities implicit in the earnings related systems. In particular, the contribution-based scheme removes unequal treatments like early retirement benefits and advantages to workers with steep earnings profiles. However, other inequities may be introduced by the new system: the treatment of low income groups, and especially the

[1] Among others, the point has been raised by Castellino (1995) and James (1997).

treatment of workers who have non-continuous working careers, play a major role in assessing how much a Social Security system is able to redistribute income to low income groups.

This work focuses in particular on the recent reforms undertaken in the Italian Social Security system: between 1992 and 1995 the Italian system was deeply reformed and is now moving from an earnings-related to a contribution-based scheme[2]. The pre-1992 system was highly generous and redistributive, characterised by high replacement rates and various forms of redistribution of income from rich to poor. However, often redistribution operated in a perverse way, as highlighted in Castellino (1995).

From a redistributive earnings-related pension formula, the system is gradually moving to a contribution-based one with no direct redistributive aim[3]. According to the 1995 reform, after the (long) transition towards the new regime, the benefit will be based on the payroll taxes paid during the entire working period, (virtually) capitalised at the GDP nominal growth rate and converted into an annuity according to actuarial fairness.

The objective of this chapter is to study a particular aspect of the problem: the distributional implications deriving by the determination of the benefit on the basis of the entire working history of the individuals as opposed to the earnings-related scheme in which the benefit is computed on the basis of the last few years' wages.

The study is conducted through a simulation procedure which allows to construct an earnings profile for each individual. In order to make the comparison among the two benefit formulae, individuals are assumed to have continuous and long careers. In the Italian earnings-related system, however, it was possible to receive a "seniority" pension after 35 years of contributions

[2] The transition period, however, will be very long, as only workers who entered the labour market in or after 1996 will receive a pension completely computed according to the contribution-based scheme.

[3] However, there will still be redistribution between married and unmarried males and between men and women.

(reduced to 20 years for public sector dependent workers) without any actuarial correction. This feature was clearly an additional benefit offered to workers with continuous careers who could retire at a relatively young age. After the 1995 reform will be fully phased in, however, seniority pensions will disappear and uniform rules will apply to all workers.

It should also be noted that the effect of the reforms for individuals with discontinuous careers, who are likely to experience low lifetime earnings, are not analysed in this framework. The distributional impact of the reform on the lowest percentiles of the population requires a different kind of analysis and is left for future research.

The parameters needed are obtained from the estimation of the income process based on Italian panel data, drawn from the Bank of Italy Survey on Households' Income and Wealth, and presented in chapter 3. Having simulated the earnings history for individuals of a particular cohort, the pre-1992 and the post-1955 pensions are computed for each individual and the resulting distribution is analysed.

Results show that the new contribution-based scheme (after the reform in 1995) reduces inequality among all groups considered, i.e. private or public dependent workers of different education groups.

In section 2 an overview of the Italian Social Security system and of its recent reforms are reported, while in section 3 the formulae used to compute the benefits arising from different social security formulae are described. Section 4 reviews the data used and the methodology used for simulating the earnings profiles; and section 5 describes the results. Section 6 concludes.

2. The Social Security System in Italy

The reforms that took place in Italy in 1992 and in 1995 have deeply changed the pension system. The main features of the traditional pre-1992 system and of the new system resulting after the last reform in 1995 (promulgated during the Dini government) can be summarised as follows.

The traditional system was characterised by an earnings-related pension formula; it was highly generous and redistributive, characterised by high replacement rates and various forms of redistribution of income from rich to

poor. However, different schemes were (and still are) in place with different rules. In the main scheme, the Pension Fund for Private Employees (FPLD), the pension was based on the 2% of the average of the last five years multiplied by the number of working years. In the State scheme, for public employees, the pension was computed with the same mechanism but on the final wage. The system aimed to be redistributive: floors and ceilings were in place in order to enhance equity, as well as generous survivors benefits, the computation of virtual contributions for workers temporarily out of the labour force and so on.
However, as highlighted in Castellino (1995), the old system was often redistributive in a perverse way. In particular, if floors and ceilings were operating in the sense of redistributing from rich to poor, other features of the old pension system were operating in the opposite direction. Specifically, as the earning based benefits were computed on the basis of the last 5 years wages (or even the last wage for public-sector workers) employees with increasing wage profiles (typically high earners) ended up with overgenerous pensions.
In addition, "seniority" pensions were in place with different rules for different categories of workers: for private sector employees it was possible to claim a seniority pension after 35 years of work, while public sector employees could retire after 20 years of work (15 years for married women). In both cases, seniority pensions were computed with the same mechanism as the old-age pensions, without any actuarial correction for age difference at retirement.
The effect of within-cohort redistribution in the old system is the result of all those features, and it is not clear *a priori* in which direction it works.
From a redistributive earnings-related pension formula the system is gradually moving to a contribution-based one with no direct redistributive feature. After the 1995 reform[4] the pension is based on the payroll taxes paid during the entire

[4] That is, after the 1995 reform will be fully in place. As previously described, individuals who started working in or after 1996 will receive a benefit computed according to the 1995 reform. Individuals who were already active in the labour force in 1995 will receive a pension computed with the *pro-rata* mechanism. For a detailed exposition of the Italian Social Security system and its recent reforms, see for example Brugiavini and Fornero (2001) and

working period (virtually) capitalised at the GDP nominal growth rate and converted in annuity according to actuarial fairness. Ceilings still apply in the sense that contributions are not paid on the fraction of earnings above a certain threshold. As the benefit is computed on the basis of the contributions paid, however, ceilings do not have any redistributive feature[5]. Different schemes and seniority pensions will gradually disappear, and flexibility of retirement age is introduced. In particular, workers can retire before reaching age 65, either if they paid contributions for not less than 40 years, or if they are aged 57 or more and the benefit they are entitled to is greater than 1.2 times the yearly income support provided to the elderly in needs[6]. That limit does not apply when workers reach age 65: at that age any worker can claim his pension and, if eligible, means-tested old-age income support.

3. The earnings- and contribution-based formulae in Italy

In order to study the distributional impact of earnings- and contribution-based formulae, two scenarios have been built: the earnings based (EB) and the contribution based (CB) scheme. As previously described, the EB scheme, i.e. the method that was in place in Italy before the 1992 reform, is an earnings related pension formula: the amount of the benefit is computed on the basis of the last 5 years average earnings, multiplied by a coefficient equal to 0.02 and by the number of years during which the worker has paid the contribution to the Social Security system:

$$P_{EB} = a * 0.02 * \sum_{i=1}^{5} w_{age-i+1} / 5 \qquad \text{(EB)}$$

where *age* is the individual's age in his final working year, *w* is his gross yearly earnings indexed for inflation, and *a* is the number of years the individual has been active in the labour market. This formula is modified for public sector

Brugiavini (1999).

[5] In fact, this is an advantage offered to high-income earners if the composition of the pension portfolio is inefficiently unbalanced in favour of the pay-as-you-go component.

[6] Means-tested income support is provided in Italy to every person aged 65 or more.

dependent workers so that the average of the last five wages is replaced by the last wage (w_{age}).

The CB scheme is a contribution-based formula according to which the contributions paid by the worker throughout his life are virtually capitalised at a rate that reflects GNP growth. Actuarial fairness is achieved by multiplying the present value of the contributions by a coefficient that reflects the age of the person retiring from the job market, as well as demographic and GNP growth. The CB pension, for all categories of workers, is then computed as:

$$P_{CB} = \left(\sum_{i=\underline{a}}^{age} c_i \ * (1+g)^{age-i} \right) \cdot \delta_{age} \qquad \text{(CB)}$$

where c_i is the contribution paid by the worker at age i, g is GNP growth (assumed to be constant and equal to 1.5% in the simulations), δ is a coefficient of actuarial fairness and $\underline{a}$ is the age at which the worker entered the labour market.

It is clear that in order to compute the benefit deriving from scheme EB and scheme CB it is necessary to know the entire earnings history of each individual. The simulation technique used to build such a population is described in the next section.

Floors and ceilings are in place in the earning based system, and lower and upper limits on pensionable earnings have been also introduced in the new contribution based system. Computation of the benefits for individuals in the simulated population should take into account this feature. However, in what follows the simulated population will include only dependent workers with continuous careers, and in this setting floors and ceilings never become binding.

Using the simulated earnings histories it is therefore possible to compute the benefits for each individual according to the two different schemes, and to study the distributional implications of the pension formulae considered. Simulated earnings profiles represent gross earnings net of Social Security contributions. In order to simplify the comparison between the two regimes, a constant payroll tax rate equal to 32.7% has been applied, as this was the payroll tax rate effective in 1998. Of the Social Security contributions, 8.89% is paid by the worker, while the remaining 23.81% is paid by the employer.

4. Earnings Simulation

In order to build the age-earnings profiles needed to implement the simulations, the panel data set in the Bank of Italy Survey of Households' Income and Wealth (SHIW) has been used. As described in chapter 3, the panel data are available for the years 1989, 1991, 1993, 1995, and 1998. The earnings process for each individual is assumed to be the sum of a deterministic observable component and a stochastic unobservable component:

$$y_{it}^{j} = X_{it}^{j}{}'\beta + u_{it}^{j}$$

where y_{it}^{j} is the natural logarithm of real gross earnings of the *i*-th individual aged *j* at time *t*. X_{it}^{a} is a (k×1) vector of observable variables, β is a (k×1) vector of unknown parameters, and u_{it}^{a} is an error term which represents unobserved characteristics determining earnings. The dependent variable is gross earnings (net of Social Security contributions) of male dependent workers working full time and observed at least for two consecutive waves. Annual gross earnings have been deflated using the ISTAT consumer price index, and they are expressed in 1998 prices.

The sample obtained has been divided into groups according to the sector of activity (private/public) and to the education level (high school dropout, high school graduate, college graduate). In addition, in order to take into account cohort effects, six year-of-birth groups have been created. The youngest cohort is formed by individuals born between 1963 and 1967 included, and the eldest by individuals born between 1938 and 1942 included. In the analysis, individuals in the youngest cohort are considered as aged 24 in 1989, 26 in 1991 and so on. The other cohorts are treated similarly. Regressions are then performed by education group using as regressors a polynomial in age and cohort dummies, both for private and for public employees. Estimated coefficients for the age polynomial for each education/sector group are then used to build earnings profiles.

Gross earnings-age profiles for the different groups of interest are shown in Figures 1 and 2. The profiles shown are in levels, in 1998 prices, and are the ones relative to the youngest cohort. Public sector workers display, on average, flatter earnings profiles. In particular, for college and high school graduate workers the first wage is quite close in the two sectors considered; however, the average yearly wage rate of growth in the private sector is 2% per year for college graduates and 1.5% for high school graduates, while in the public sector the average rates of growth are 0.9% and 0.5% respectively. High school dropout workers exhibit a similar profile in both sectors, with an average yearly wage rate of growth of about 0.5%[7].

In order to simulate a different age-earnings pattern for each individual, an estimate of the parameters underlying the structure of the unobservable component of earnings is needed. Using the results obtained in chapter 3, the unobserved component of earnings for individual *i* of age *j* is decomposed into a permanent component which is time invariant (μ_i) and an AR(1) component (z_{it}^j).

$$u_{it}^j = \mu_i + z_{it}^j$$
$$z_{it}^j = \alpha z_{i,t-1}^{j-1} + \omega_{it}^j$$

where ω_{it}^j is an i.i.d. stochastic process with zero mean and variance σ_ω^2, and μ_i is i.i.d. with zero mean and variance equal to σ_μ^2. The AR(1) component arises from a finite process starting at age $\underline{a}$, the age at which individuals enter the labour market.

The parameters used to calibrate the simulation are based on the estimates in the previous chapter (chapter 3, tables 3 and 4). For convenience, parameter estimates are shown in tables 1 and 2 at the end of this chapter. In the simulation, all the unobservable components are assumed to be drawn from normal distributions, with zero mean and variance given by the variance estimates.

The simulated population has been built according to the structure of the 1998 sample of male dependent workers, employed both in the private and in the

[7] The average rate of growth of real GNP between 1989 and 1998 has been in Italy equal to 1.5%.

public sector. Of 15,000 observations, 30% are public sector and 70% are private sector employees. In the public sector, 18% of workers have a college degree, 42% are high school graduates and the rest has with a primary school degree. In the private sector, 63% of workers have only a primary school degree, 32% are high school graduates and only 5% are college graduates.
In the artificial population, individuals are assumed to start working at age 22 (college graduates at 25) and to retire at age 60.

5. Results

Having simulated the earnings profiles for a number of individuals as described in the preceding section, it is possible to compute the benefits resulting from the two scenarios considered: the earnings-related and the contribution based formulae.
Table 3 reports the mean of the final wage, and of the pension computed both with the earning based and with the contribution based formula. Gross yearly earnings (gross of Social Security contributions and of the income tax) are reported, as well as gross yearly pensions. For the whole sample (15,000 observations) the final year gross wage is roughly 41.5 million lira; the yearly gross pension computed with the earnings-related formula is 32.7 million lira while the benefit computed with the contribution based formula is 32 million lira.
Those results also depend on the assumption of a capitalisation of contributions at a rate equal to 1.5%. With the economy growing at a faster rate, the contributions paid by each worker would be virtually capitalised at a higher rate and the resulting CB benefit would be more generous. However, as the distribution of the benefits is the same for different assumptions on GNP growth, results are shown only for this base case.
Figures in table 3 imply an average gross replacement ratio of 78% for the EB pension, and of 77% for the CB benefit, where the figures are obtained dividing the average gross pension benefit by the average final year gross wage. This procedure amounts to compute the weighted average of the individual replacement ratios, weighted by the final wage.

As individual data are available, it is also possible to compute the individual replacement ratio, defined as the ratio of the individual pension benefit to the individual final wage, and to study its distribution. Unweighted averages of individual replacement rates are reported in table 4. The average of the individual replacement ratio under the EB regime is about 80%, while under the CB regime this is about 83%. Turning to the sub-groups considered, table 4 shows a tendency of a higher replacement rate when the benefit is computed according to the contribution-based method. The reverse is true for high school and college graduates employed in the private sector. For those two groups the pension computed with the CB formula is lower than the benefit that would have been received under the EB regime; the resulting gross replacement ratio falls from 80% to 72% for high school graduates and from 77% to 62% for college graduates.

In tables 5 and 6 the deciles of the distribution of the gross replacement rates are shown for the different groups considered. For the whole population the median replacement rates in the EB and the CB schemes are quite close. In both regimes, however, the replacement rates vary quite considerably: under the EB scheme, for example, the value of the replacement rate at the last decile is equal to 98%, while at the first decile it falls to 67%. The deciles under the CB regime range from 58% to 112%.

The distribution of the EB benefit for the public sector follows from the fact the only the last wage is used to compute the benefit (as opposed to the last five years used in the private sector). This implies a gross replacement rate of 78% for high school dropouts and high school graduates, and of 72% for college graduates, who have a shorter working career.

In table 7 the average gross replacement ratios by wage decile are reported: in both schemes, the lower the wage decile, the higher the replacement ratio the system ensures to the individual. Both for the whole population and for the two sectors separately, the CB compared to the EB scheme provides higher replacement rates to individuals in the bottom wage deciles, and conversely, the lower replacement rates to individuals who are in the higher wage deciles.

Turning to the level of the computed benefits, in table 8 a few measures of inequality are computed for the two pensions and for the different groups considered, in order to summarise the departure of the distribution from equity. The coefficient of variation is the ratio of the standard deviation of the variable of interest to its mean: the higher the coefficient of variation, the higher the inequality. The standard deviation of the variables in logarithm is also shown, and higher values also in this case represent higher inequality. Finally, the Gini coefficient is computed: it is defined as the ratio to the mean of half the average over all pairs of the absolute deviations between people. If the distribution of the variable considered is perfectly egalitarian, the Gini coefficient is 0, while if one individual owns the total amount available, while the others own nothing, the Gini coefficient is equal to 1.

The various indices give the same picture: the pension computed with the CB formula appears to have a more equal distribution then the EB pension.

Turning to the different categories of workers considered, in the private sector the reduction in inequality induced by the CB pension is lower compared to its effect in the public sector[8]. This is because the EB pension in the public sector was computed on the basis of the last wage only, thus reflecting its variability, and not on an average wage.

Graphical analysis based on Lorenz curves is a useful tool in comparing the inequality exhibited by different distributions. As the Lorenz curves for the two distributions are very close to each other, the transformed Lorenz curve has been built for the whole population as well as for the different sectors and education groups considered (Figures 3-6). The x-axis is the cumulative fraction of population – starting from the poorest – as in the standard Lorenz curve, while on the y-axis the difference between the cumulative fraction of the variable of interest and the line of complete equality (the 45 degree line) is plotted. The lower the curve, the less unequal is the distribution of the variable considered. The graphs show that the two curves do not cross each other, so

[8] It should be noticed that the inequality measures considered here do not exhibit the property of decomposition, so that overall inequality cannot be decomposed in an additive way into inequality within and between groups. It is nonetheless possible, however, to assess the effect of the different pension formulae within the groups considered in the analysis.

that, in terms of inequality, the distribution of the CB pension always Lorenz dominates the distribution of the EB pension[9].

It should be noticed that, as no individual in the simulated population is hitting the floor or the ceiling, the two pensions computed in this case contain no direct redistributive feature. The reduction in inequality therefore follows uniquely from the fact that in one case (the EB situation) the pension is computed on the basis of the last five years of earnings, while in the other the benefit is computed using the entire earnings history of the individual.

As the variables of interest differ in their means, the generalised Lorenz curves (Shorrocks, 1983) are also plotted (Figures 7-10). This is a plot of the cumulative fraction of the population against the cumulative fraction of the variable of interest multiplied by its mean. In this setting, the distribution with the higher mean cannot be dominated, as the end point of the curve is the overall mean, but if the distribution with the higher mean is also more unequal than the other, it is possible that the curves cross each other and none of the distributions dominates the other. Generalised Lorenz dominance of one variable to another implies that the social welfare associated with the former is greater than social welfare associated with the latter[10]. In this setting the population considered is only a sub-group of the whole population (namely, one generation of retiring dependent workers); this implies that the implications drawn are valid only for the sub-group considered and do not take into account the welfare of the society as a whole.

For the whole population considered (Figure 7) the two curves cross each other only in the last part, while for all the other percentiles the pension computed with the CB formula dominates the pension computed with the EB formula.

The analysis for each of the sub-groups considered (Figures 8-10) reveals that the EB pension dominates the CB pension only for the sub-groups of high

[9] As the curves do not cross each other, the Lorenz curve gives the ordering according to inequality.

[10] Where the social welfare function is non-decreasing in each of its arguments and s-concave. For an overview of these concepts, see Atkinson (1983) and Deaton (1997).

school and college graduates employed in the private sector. For those groups, the reduction in average pension is not compensated by the reduction in inequality. On the contrary, graduates in the public sector suffer a (small) drop in average pension which is compensated by the drop in inequality. For the other groups the CB pension dominates.

6. Conclusions

This study analysed the distribution of pensioners' income under different Social Security systems. In particular, the distributional impact of a pension deriving from an earnings-related formula and a pension deriving from a contribution-based formula has been studied. Simulations have been calibrated on Italian male dependent workers earnings histories and on the Italian Social Security system, pre- and post-reform.

Results show that the new contribution-based scheme (after the reform in 1995) reduces inequality among all groups considered, i.e. private or public dependent workers of different education groups. The generalised Lorenz curve shows that for the overall population considered (one generation of retiring dependent workers) the (small) reduction in average benefit is compensated by the reduction in inequality, with the exception of the highest percentiles. However, within groups with a steeper age-earnings profile (high school and college graduates employed in the private sector) the generalised Lorenz curve associated with the contribution-based scheme is dominated by the distribution associated with the old earnings-related scheme.

Figure 1

Figure 2

□: College Graduates
Δ: High School Graduates
O: High School Dropouts

Gross Earnings are net of Social Security contributions. Y-axis labels are expressed in million liras at 1998 prices, for the cohort born in 1965.

Parameter estimates for the earnings process

Table 1. Private sector dependent workers

	Education: high school dropout	Education: high school and college
σ_η^2	0.027	0.032
t-stat	(5.55)	(3.88)
α	0.545	0.533
t-stat	(5.41)	(3.23)
σ_ω^2	0.034	0.034
t-stat	(8.91)	(6.87)
$\sigma_{\underline{a}}^2$	0.032	0.045
t-stat	(1.94)	(1.94)
Sum of Squared Residuals	*0.0229*	*0.0558*
N. of Obs.	*2106*	*1223*

Table 2. Public sector dependent workers

	Education: high school dropout	Education: high school and college
σ_η^2	0.004	-0.060
t-stat	(0.52)	(-1.07)
α	0.712	0.908
t-stat	(7.70)	(17.40)
σ_ω^2	0.021	0.022
t-stat	(6.59)	(7.24)
$\sigma_{\underline{a}}^2$	0.018	0.098
t-stat	(1.71)	(1.42)
Sum of Squared Residuals	*0.0147*	*0.0259*
N. of Obs.	*758*	*1144*

Table 3. Simulation Results (Means)

	Number of Individuals	Final Wage	Earnings Based (EB) Benefit	Contribution Based (CB) Benefit
Whole sample	15000	41.667	32.679	32.092
Private	10600	42.101	33.313	31.933
High School Dropouts	6678	33.611	27.176	28.371
High School Graduates	3392	52.694	41.091	36.382
College Graduates	530	81.289	60.857	48.328
Public	4400	40.619	31.153	32.475
High School Dropouts	1760	36.293	28.309	30.367
High School Graduates	1848	41.107	32.064	33.656
College Graduates	792	49.091	35.346	34.406

Notes: -Values are expressed in million liras;

- Final wage is gross of income tax and of Social Security contributions;
- EB and CB benefits are gross of income tax.

Table 4. Individual Gross Replacement Ratio

	EB	CB
Whole sample	0.805	0.833
Private	0.819	0.820
High School Dropouts	0.832	0.885
High School Graduates	0.801	0.722
College Graduates	0.771	0.618
Public	0.769	0.866
High School Dropouts	0.780	0.871
High School Graduates	0.780	0.901
College Graduates	0.720	0.773

Note: The replacement ratio is computed as the average of the ratio of the individual's yearly gross pension to the yearly gross earnings (gross of income tax and of social security contributions paid by the worker).

Table 5. Individual Gross Replacement Ratio: deciles

	Whole Population		Private Sector		Public Sector	
Percentage	EB	CB	EB	CB	EB	CB
10	0.67	0.58	0.65	0.58	0.72	0.57
20	0.72	0.65	0.69	0.65	0.78	0.65
30	0.75	0.71	0.73	0.70	0.78	0.72
40	0.78	0.76	0.77	0.75	0.78	0.77
50	0.78	0.81	0.80	0.80	0.78	0.83
60	0.78	0.86	0.84	0.85	0.78	0.89
70	0.83	0.92	0.88	0.91	0.78	0.96
80	0.89	1.00	0.93	0.98	0.78	1.05
90	0.98	1.12	1.01	1.08	0.78	1.19

Table 6. Individual Gross Replacement Ratio: deciles by education group

Earnings Based

	Private Sector			Public Sector		
Percentage	HS dropouts	H.S.	College	H.S. dropouts	H.S.	College
10	0.66	0.64	0.60	0.78	0.78	0.72
20	0.71	0.68	0.65	0.78	0.78	0.72
30	0.75	0.72	0.69	0.78	0.78	0.72
40	0.78	0.75	0.72	0.78	0.78	0.72
50	0.82	0.79	0.76	0.78	0.78	0.72
60	0.85	0.82	0.80	0.78	0.78	0.72
70	0.89	0.86	0.83	0.78	0.78	0.72
80	0.95	0.92	0.89	0.78	0.78	0.72
90	1.03	0.99	0.96	0.78	0.78	0.72

Contribution Based

	Private Sector			Public Sector		
Percentage	HS dropouts	H.S.	College	H.S. dropouts	H.S.	College
10	0.65	0.54	0.47	0.66	0.55	0.48
20	0.72	0.59	0.51	0.72	0.63	0.55
30	0.77	0.63	0.54	0.76	0.71	0.60
40	0.82	0.67	0.57	0.80	0.78	0.65
50	0.87	0.71	0.60	0.85	0.85	0.73
60	0.91	0.75	0.63	0.90	0.93	0.80
70	0.97	0.79	0.67	0.95	1.02	0.88
80	1.04	0.85	0.72	1.02	1.13	0.97
90	1.14	0.93	0.79	1.13	1.31	1.13

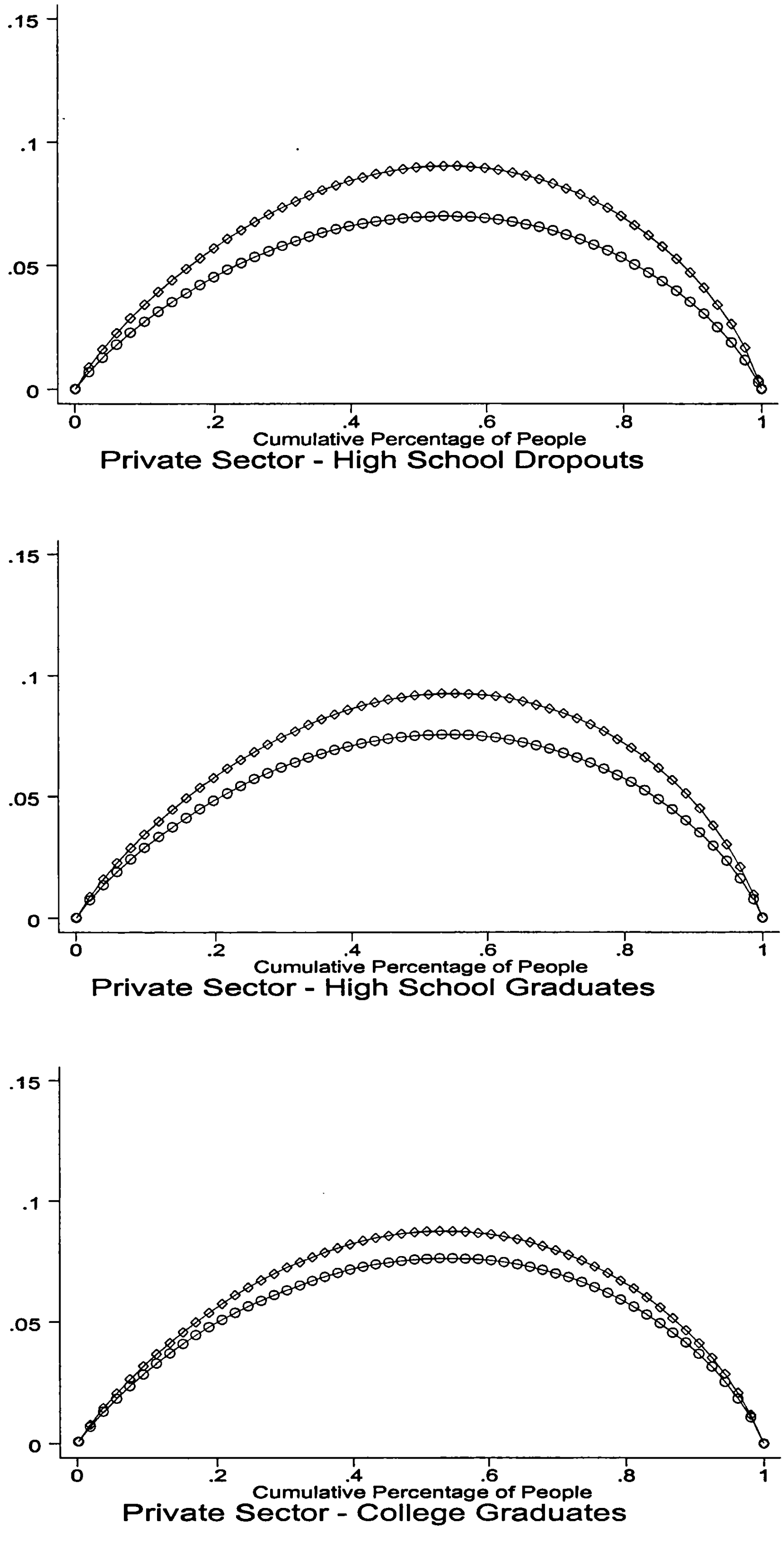
Table 7. Individual Gross Replacement Ratio: average by wage deciles

	Whole Population		Private Sector		Public Sector	
Deciles	EB	CB	EB	CB	EB	CB
1	0.922	1.168	0.968	1.128	0.774	1.297
2	0.860	1.023	0.898	0.995	0.773	1.078
3	0.838	0.947	0.867	0.930	0.774	0.982
4	0.816	0.891	0.841	0.873	0.774	0.928
5	0.801	0.838	0.815	0.820	0.773	0.873
6	0.791	0.797	0.801	0.786	0.770	0.816
7	0.780	0.748	0.782	0.740	0.771	0.769
8	0.768	0.704	0.766	0.703	0.768	0.712
9	0.753	0.651	0.748	0.652	0.762	0.662
10	0.719	0.566	0.706	0.570	0.752	0.546

Table 8. Inequality Measures

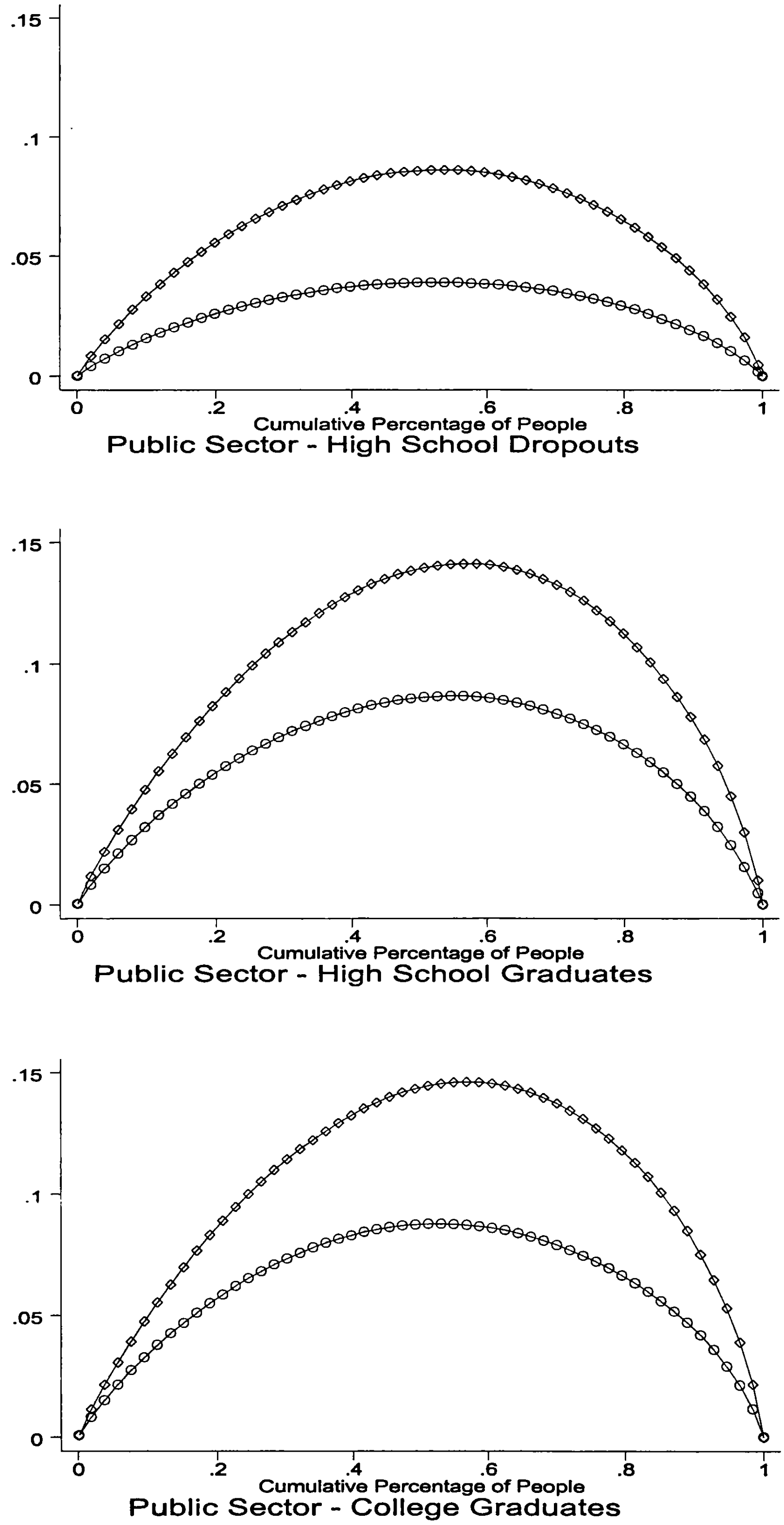
	Earnings Based Benefit			Contribution Based Benefit		
	Coeff. Variation	S.D. of logs	Gini coefficient	Coeff. Variation	S.D. of logs	Gini coefficient
Whole population	0.37	0.33	0.19	0.23	0.22	0.12
Private	0.36	0.33	0.19	0.25	0.24	0.13
High School Dropouts	0.23	0.23	0.13	0.18	0.18	0.10
High School Grad.	0.24	0.23	0.13	0.19	0.19	0.11
College Graduates	0.22	0.22	0.12	0.20	0.19	0.11
Public	0.34	0.31	0.18	0.19	0.19	0.10
High School Dropouts	0.22	0.22	0.12	0.10	0.10	0.06
High School Grad.	0.36	0.35	0.20	0.22	0.22	0.12
College Graduates	0.38	0.37	0.21	0.23	0.22	0.12

Figure 3: Transformed Lorenz curves, Private Sector



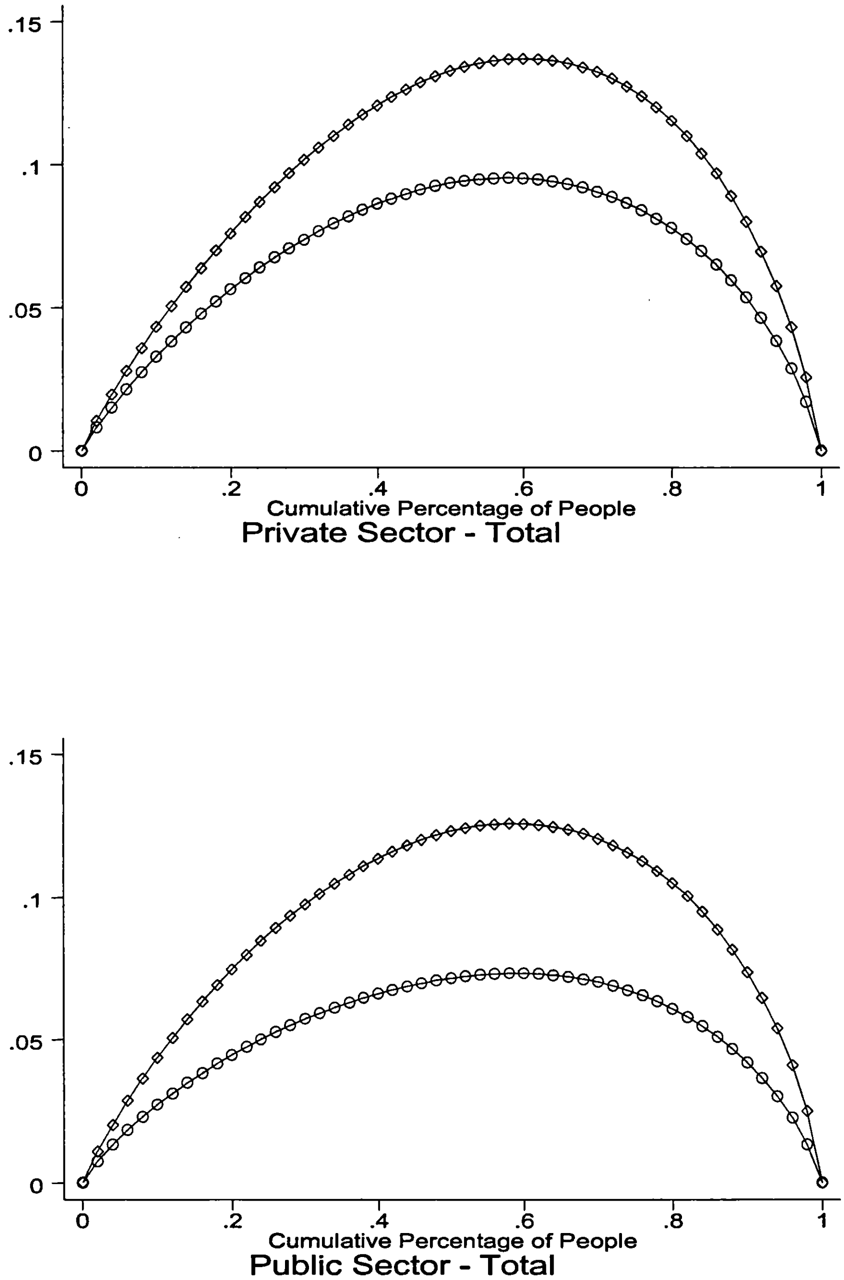
□: Earnings Based
O: Contribution Based

Figure 4: Transformed Lorenz curves, Public Sector



□: Earnings Based
O: Contribution Based

Figure 5: Transformed Lorenz Curve, Private and Public Sector



□: Earnings Based
O: Contribution Based

Figure 6: Transformed Lorenz curve, whole population

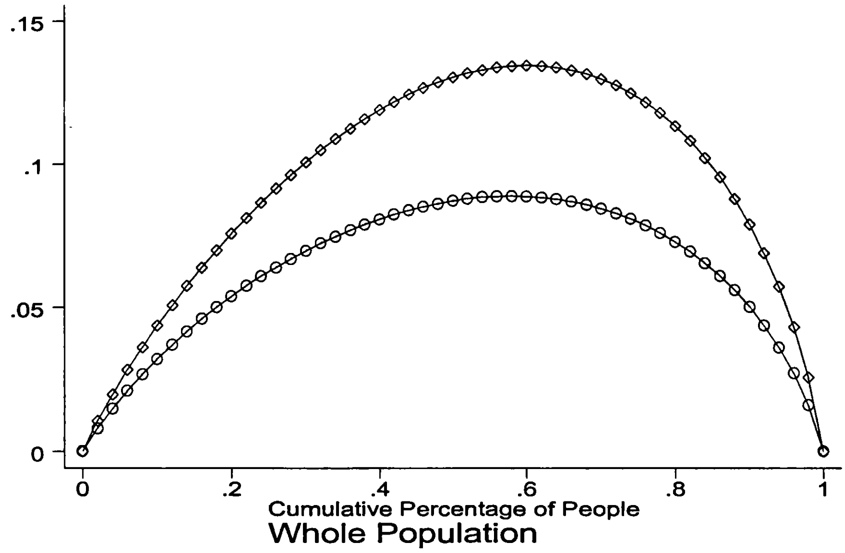
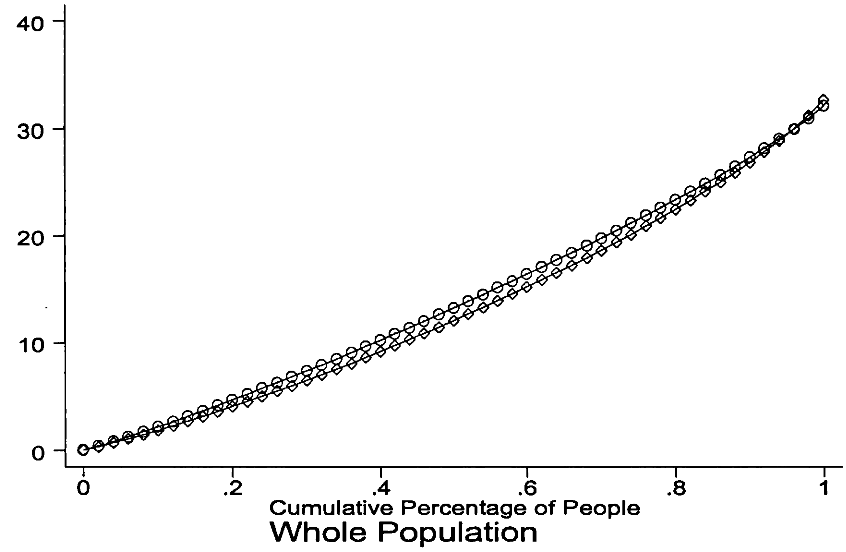
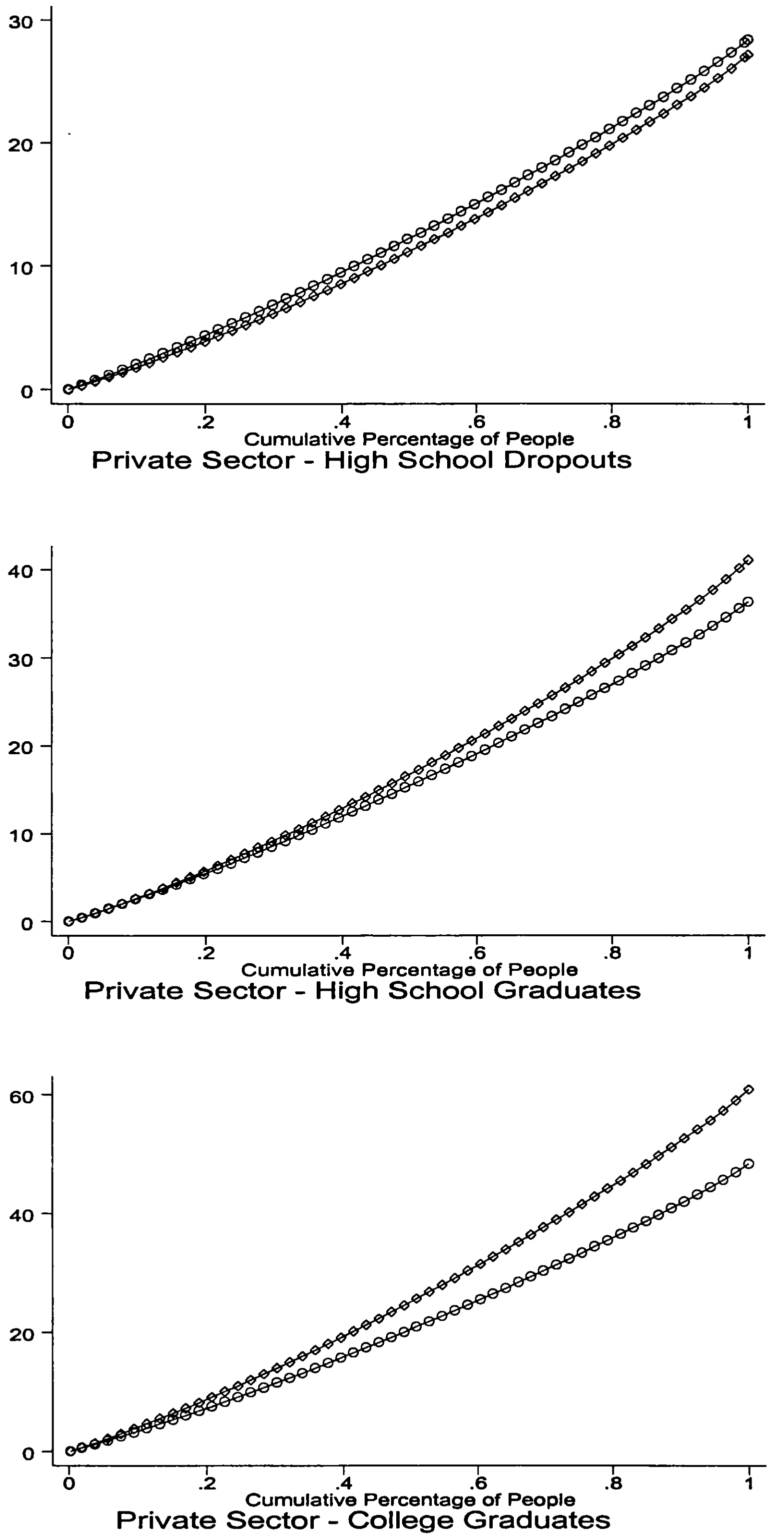


Figure 7: Generalised Lorenz curve, whole population



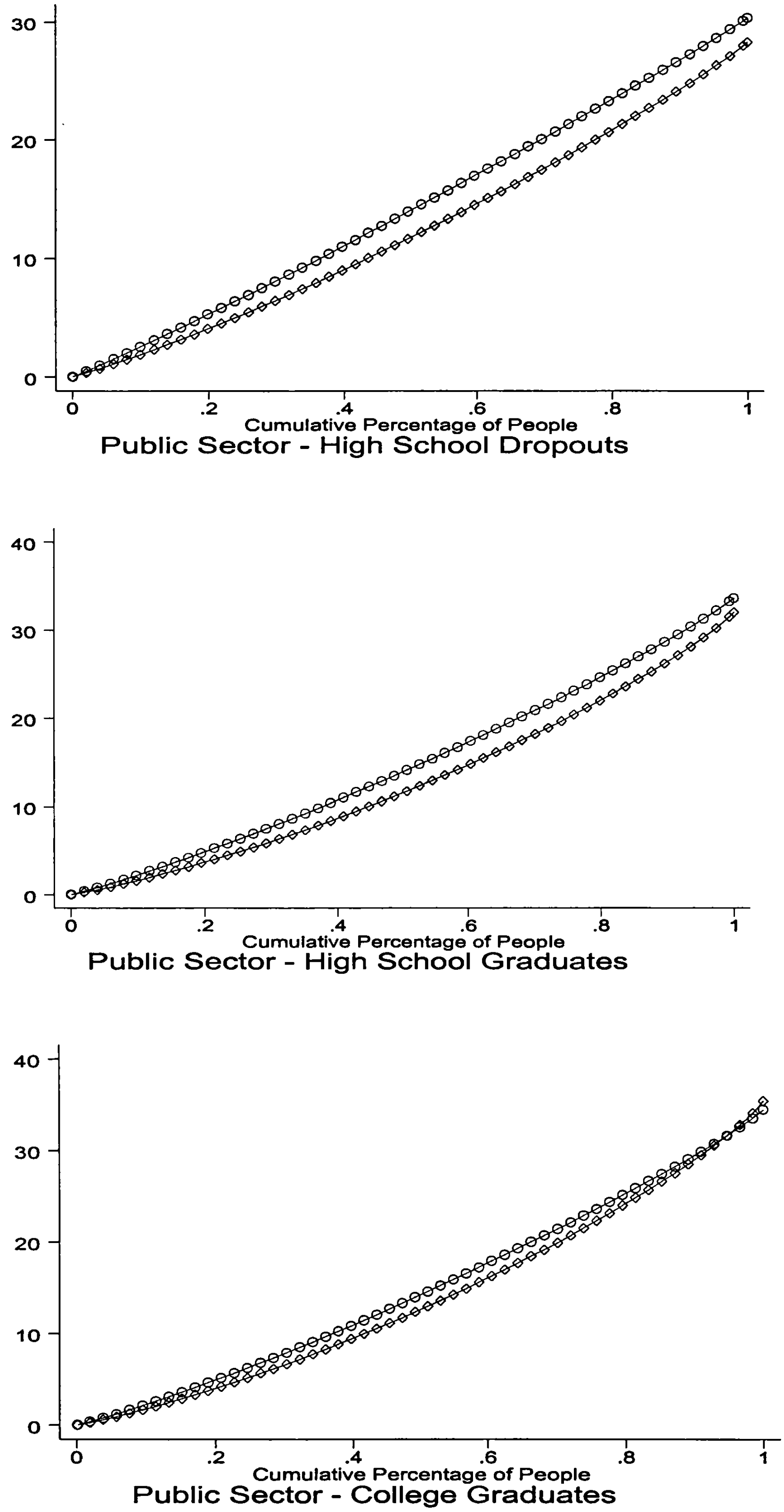
□: Earnings Based
O: Contribution Based

Figure 8: Generalised Lorenz curve: Private Sector



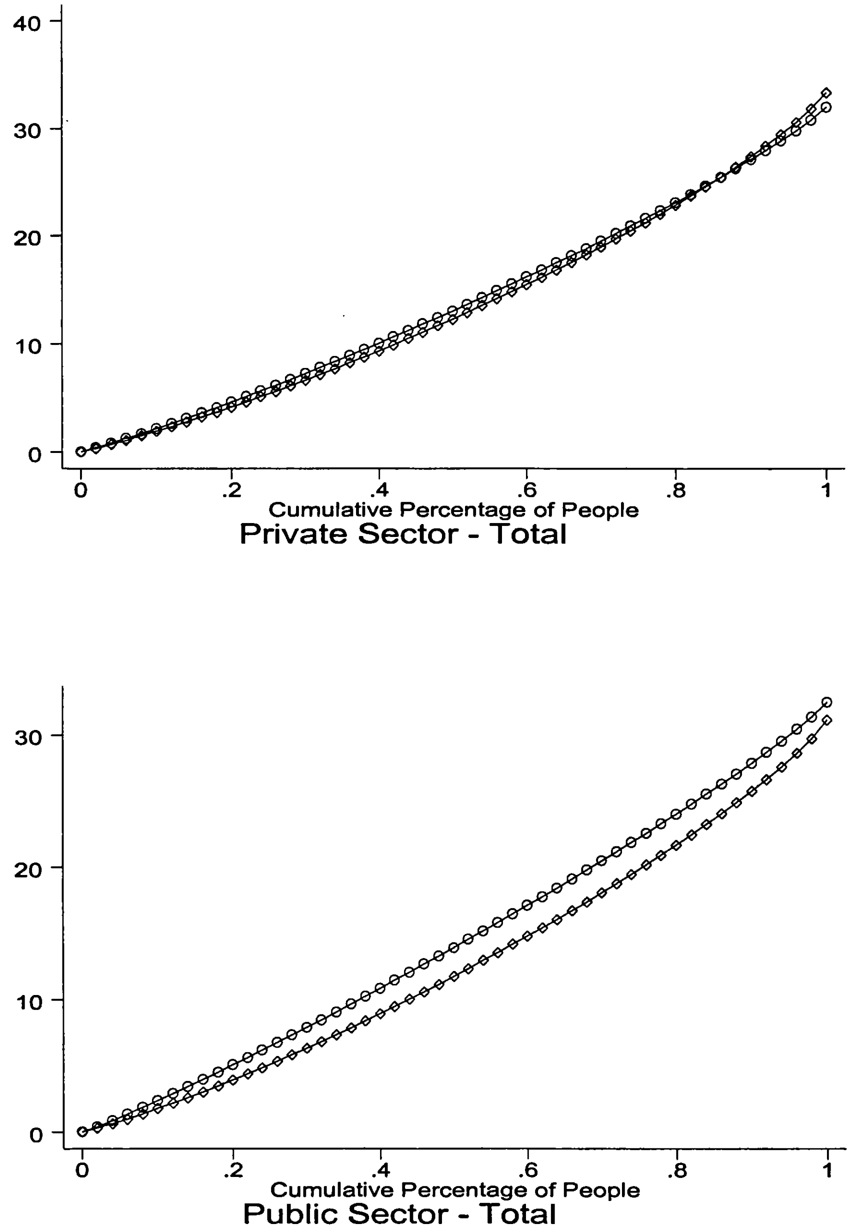
□: Earnings Based
O: Contribution Based

Figure 9: Generalised Lorenz curve: Public Sector



□: Earnings Based
O: Contribution Based

Figure 10: Generalised Lorenz Curve, Private and Public Sector



□: Earnings Based
O: Contribution Based

Conclusions

In this thesis an analysis of the time series properties of consumption on grouped data has been carried on.

In particular, I first have analysed the dynamic properties of cohort durable expenditure. The empirical application used a dynamic index model, which allows to identify aggregate shocks, common to all cohorts, and to study the group responses to those shocks. Estimates show that the common shock that captures the comovements of all the variables provokes a cyclical reaction in all the variables considered. The movement in the relative price, which increases during the expansion phase, stresses the importance of this variable in the cycle, and it is consistent with the model in Caplin and Leahy (1999) where the price is endogenous and its increase in response to a demand shock smooths the response of car expenditure and the number of buyers.

A different methodology has then been used in an analysis of the time series properties of consumption, income and of the interest rate. Such methodology consists in estimating multivariate moving average systems for grouped variables: this approach has the advantage of allowing to explicitly take into account the measurement error present in the individual measures of consumption and income. Results show a prolonged dynamics of non-durable expenditure, which cannot be entirely explained by the influence of lagged shocks to income and to the interest rate. Some evidence on excess sensitivity of consumption to lagged income shocks has also been found, although results are not clear-cut. The long run response of consumption to a unit shock in the interest rate has been estimated to be about 1, both in a system in which income is included among the equations and in a system of consumption and interest rate alone.

With a change of focus, a panel data technique has been used in order to evaluate different models of the individual earnings process. The analysis is based on the Bank of Italy Survey of Household Income and Saving and exploits the panel component of the Survey. Various models have been estimated for different sectors (private and public) and for different education groups, in order to exploit the differences that may arise due to heterogeneous education attainments. The specification that better captures the features of the data is a model given by the sum of an AR(1) component and an individual fixed effect. The autoregressive coefficient has been estimated to be around 0.55 in the private sector and 0.8 in the public sector. In the latter group, parameter differences among education groups are found statistically significant, while in the former differences in the estimated parameters for the two education groups do not appear to be statistically significant.

Having characterised the dynamic properties of the Italian earnings process, a study of pension earnings distribution in Italy has been undertaken. The analysis explores how effective is redistribution of pensioners' income under different Social Security systems. Simulations have been calibrated on the Italian male dependent workers earnings process estimated in chapter 3 and on the Italian Social Security system, before and after the reforms undertaken in 1992 and 1995.

Results show that the new contribution-based scheme (after the reform in 1995) reduces inequality among all groups considered, i.e. private or public dependent workers of different education groups. The generalised Lorenz curve shows that for the overall population considered (one generation of retiring dependent workers) the (small) reduction in average benefit is compensated by the reduction in inequality, with the exception of the highest percentiles. However, within groups with a steeper age-earnings profile (high school and college graduates employed in the private sector) the generalised Lorenz curve associated with the contribution-based scheme is dominated by the distribution associated with the old earnings-related scheme.

References

Abowd, J. and D. Card (1989), "On the Covariance Structure of Earnings and Hours Changes", *Econometrica*, 57, 411-445.

Adda, J. and R. Cooper (2000), "The Dynamics of Car Sales: A Discrete Choice Approach", NBER Working Paper n. 7785.

Altonji, J.G., A.P. Martins and A. Siow (1987), "Dynamic Factors Models of Consumption, Hours and Income", NBER Working Paper n. 2155.

Altonji, J.G. and L.M. Segal (1996), "Small Sample Bias in GMM Estimation of Covariance Structures", *Journal of Business and Economic Statistics*, 14(3), 353-366.

Anderson, T.W. and C. Hsiao (1981), "Estimation of Dynamic Models with error components", *Journal of the American Statistical Association*, 76, 598-606.

Anderson, T.W. and C. Hsiao (1982), "Formulation and Estimation of Dynamic Models Using Panel Data", *Journal of Econometrics*, 18.

Atkinson, A.B. (1983), "The Economics of Inequality", second edition, Oxford University Press, Oxford.

Attanasio, O.P. (2000), "Consumer Durables and Inertial Behaviour: Estimation and Aggregation of (S,s) Rules for Automobile Purchases", *Review of Economic Studies*, 67(4).

Attanasio, O.P. and M. Browning (1995) "Consumption over the Life-Cycle and over the Business Cycle", *American Economic Review*, 85, 1118-1137.

Attanasio, O.P. and G. Weber (1993), "Consumption Growth, the Interest Rate and Aggregation", *Review of Economic Studies*, 60,631-49.

Attanasio, O.P. and G. Weber (1995), "Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey", *Journal of Political Economy*, 103, 1121-1157.

Banks, J., R. Blundell and I. Preston (1994), "Life-Cycle Expenditure Allocations and the Consumption Cost of Children", *European Economic Review*, 38, 1234-78.

Bernanke, B. (1984), "Permanent Income, Liquidity, and Expenditure on Automobiles: Evidence from Panel Data", *Quarterly Journal of Economics*, 99, 587-614.

Bernanke, B. (1985), "Adjustment Costs, Durables, and Aggregate Consumption", *Journal of Monetary Economics*, 15, 41-68.

Bertola, G., and R.J. Caballero (1990), "Kinked Adjusted Costs and Aggregate Dynamics", in O. J. Blanchard and S. Fisher (eds), *NBER Macroeconomics Annual*, Vol. 5.

Blundell, R., M. Browning and C. Meghir (1994), "Consumer Demand and the Lifetime Allocation of Consumption", *Review of Economic Studies*, 61, 57-80.

Blundell, R. and I. Preston (1998), "Consumption Inequality and Income Uncertainty", *Quarterly Journal of Economics*, Vol. CXIII, Issue 2, May, 603-640.

Browning, M. and C. Meghir (1991), "The Effect of Male and Female Labour Supply on Commodity Demands", *Econometrica*, 53, 503-44.

Brugiavini, A. (1999) "Social Security and Retirement in Italy", in J. Gruber and D. Wise. eds., *Social Security and Retirement around the World*, The University of Chicago Press.

Brugiavini, A. and E. Fornero (2001) "Pension Provision in Italy", forthcoming in Richard Disney and Paul Johnson Eds., *Pension Systems and Retirement Incomes across OECD Countries*, Edward Elgar, London 2001.

Caballero, R.J. (1990), "Expenditure on Durable Goods: A Case for Slow Adjustment", *Quarterly Journal of Economics*, 105, 727-43.

Caballero, R.J. (1993), "Durable Goods: An Explanation for Their Slow Adjustment", *Journal of Political Economy*, 101, 351-384.

Caplin, A. and J. Leahy (1999), "Durable Goods Cycles", NBER Working Paper n. 6987.

Castellino, O. (1995) "Redistribution Between and Within Generations in the Italian Social Security System", *Ricerche Economiche*, 49, 317-327.

Chamberlain, G. (1984), "Panel Data", in *Handbook of Econometrics*, eds. Z. Griliches and M. Intriligator. Amsterdam: North Holland.

Deaton, A. (1985), "Panel Data from Time Series of Cross-Sections", *Journal of Econometrics*, 30, 109-26.

Deaton, A. (1997) "The Analysis of Household Surveys: a Microeconometric Approach to Development Policy", Published for the World Bank, The Johns Hopkins University Press.

Deaton, A. and C. Paxon (1994), "Intertemporal choice and inequality", *Journal of Political Economy*, 102, 437-467

Forni, M., and L. Reichlin (1996) " Dynamic Common Factors in Large Cross-Sections" *Empirical Economics*, 21, 27-42.

Geweke, J.F. (1977), "The Dynamic Factor Analysis of Economic Time Series Models", in: D. J. Aigner and A. S. Goldberger (eds), Latent Variables in Socio-Economic Models, Noth-Holland, Amsterdam.

Geweke, J.F., and K.J. Singleton (1981), "Maximum Likelihood 'Confirmatory' Factor Analysis of Economic Time Series", *International Economic Review*, 22, 37-54.

Gottschalk, P. and R.A. Moffitt (1994), "The Growth of Earnings Instability in the U.S. Labor Market", *Brooking Papers on Economic Activity*, 2:1994, 217-272.

Gorman, W.M. (1959), "Separable Utility and Aggregation", *Econometrica*, 27, 469-481.

Granger, C.W.J., (1987), "Implications of Aggregation With Common Factors", *Econometric Theory*, 3, 208-22.

Grossmann S.J. and G. Laroque (1990), "Asset Pricing and Optimal Portfolio Choice in the Presence of Illiquid Durable Consumption Goods", *Econometrica*, 58, 25-51.

Hall, R.E and F. Mishkin, (1982), "The Sensitivity of Consumption to Transitory Income: Estimate from Panel Data on Households", *Econometrica*, 50,461-81.

Hansen, L.P. (1982), "Large Sample Properties of Generalized Method of Moments Estimators", *Econometrica*, 50(4), 1029-54.

Harvey, A.C., (1993), "Time Series Models", The London School of Economics (eds), Second Edition.

James, E. (1997) "Pension Reform: Is There an Efficiency-Equity Trade-Off?", World Bank Working Paper n. 1767.

Lam, P.S. (1991), "Permanent Income, Liquidity and Adjustment of Automobile Stocks: Evidence from Panel Data", *Quarterly Journal of Economics*, 106, 203-230.

Lillard, L.A. and R.J. Willis (1978), "Dynamic Aspects of Earning Mobility", *Econometrica*, 46, 985-1012.

MaCurdy, T.E. (1982), "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis", *Journal of Econometrics*, 18, 83-114.

Mankiw, N.G. (1982), "Hall's Consumption Hypothesis and Durable Goods", *Journal of Monetary Economics*, 10, 417-25.

Meghir, C. and G. Weber (1996), "Intertemporal Non-Separability or Liquidity Constraints? A Disaggregate Analysis on US Panel Data", *Econometrica*, 64(5), 1151-81 .

Moffitt, R.A. and P. Gottschalk (1995), "Trends in the Covariance Structure of Earnings in the U.S.: 1969-1987", mimeo.

Quah, D. and T. Sargent (1994), "A Dynamic Index Model for Large Cross Sections". In: Stock J., Watson M. (eds) Business Cycles, indicators and forecasting. NBER and University of Chicago.

Sargent, T.J., and C.A. Sims (1977) "Business Cycle Modelling without Pretending to Have Too Much *A Priori* Economic Theory", in: C. A. Sims (eds), New Methods in Business Cycle Research. Minneapolis, Federal Reserve Bank of Minneapolis.

Shorrocks, A.F. (1983) "Ranking Income Distributions", *Economica*, 50, 3-17.

Storesletten, K., C.I. Telmer, and A. Yaron (2000), "Consumption and Risk Sharing Over the Life Cycle", mimeo.

Watson M.W. and R.F. Engle, (1983) "Alternative Algorithms for the Estimation of Dynamic Factor, MIMIC, and Time Varying Coefficient Regression Models", *Journal of Econometrics*, 23, 385-400.