Diagrammatic Reasoning and Propositional Logic

A Philosophical Study of Peirce's Alpha Existential Graphs

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Abstract

It has been the standard view of philosophers in this century that the use of diagrams is incompatible with rigour in logic; and that diagrams are dispensable from logical proof, in the sense that there is no proof of which a diagram is an essential part. This view has come under recent attack by logicians who have proved certain diagrammatic logics – for example, Venn diagrams and Peirce's Existential Graphs – to be sound and complete.

This thesis builds on these results, by arguing positively that a diagrammatic logic can have informational resources in virtue of its representational form which are not available to a logically equivalent sentential counterpart; that these resources can facilitate the process of logical inference; and that this implies a view of deductive inference generally which is not straightforwardly "deductivist", but incorporates observational and experimental processes.

The strategy followed is to examine the representational features of a particular diagrammatic logic – the alpha Existential Graphs, or EG – and to compare them to those of certain varieties of sentential logic (five-functor, two-functor and Polish notation SL), to which EG is extensionally equivalent. In the first half of the thesis, I introduce and situate the standard view; explore the distinctive features of diagrams, and diagrammatic representation; analyse the key idea of perspicuousness; and review current theories of information to determine a compatible background account. In the second half, I introduce EG, and show that it is more perspicuous in the required sense than SL; rebut a well-known claim that EG relies on some special notion of "Peirce provability", and describe how EG can motivate a non-deductivist account of deductive reasoning. Finally, I criticise the standard view of diagrams, and locate EG within Peirce's general theory of signs, underlining the strong motivations for Peirce's late move into diagrammatic logic.

A Note on Terminology

I use the phrase "sentential logic" hereafter simply to refer to logics with the representational form of sentences. The phrase is not intended to mark any distinction, or take any view as such, as to the content or subject matter of logic. In this usage, then, it is possible to say that a sentence of sentential logic *A* and a diagram of diagrammatic logic *B* are different representations of the *same* logical content. Among those who take the subject matter of logic to be (or to be like) propositions, *A* and *B* can then both be described as different forms of propositional logic, and representations of *A* and *B* can be described generally as propositional representations.

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1: Logical Reasoning and Diagrams

1.1 Context: The Dethronement of Diagrams

What is the logical status of diagrams? And what is the relation, if any, between diagrams and logical reasoning? Until recently the answers to both these questions have seemed self-evident to logicians. The standard view was given a well-known formulation by Tennant 1986 in connection with geometry:

> It is now commonplace to observe that the diagram [sc. triangle *ABC*] ... is only an heuristic to prompt certain trains of inference; that it is dispensable as a proof-theoretic device; indeed, that it has no proper place in the proof as such. For the proof is a syntactic object consisting only of sentences arranged in a finite and inspectable array. One is cautioned, and corrected, about ... the mistake of assuming as given information that is true only of the triangle that one has happened to draw, but which could well be false of other triangles that one might equally well have drawn in its stead.[1]

That is, diagrams have no logical status as such, indeed they are out of place in proof generally. Proofs are composed of sentences of logic; and a diagram is not a sentence, nor a sentence a diagram. Since proofs are composed of sentences, there can be no proof of which a diagram is an essential part. If a diagram has any role to play in proof, it is not syntactic but heuristic; to make the reasoner's cognitive processes faster or easier, perhaps. Where a diagram has played this role, however, the ultimate justification for the conclusion must come through a (sentential) proof. Especially so since diagrams may be misleading; the information derived from a piece of reasoning from diagrams may not, for example, count in the general case.

Similar expressions of this general view could be multiplied without difficulty.[2] Its origins appear to lie in mathematical and logical developments of the 19th century. Historically, the status of diagrams in mathematics had been all but unquestioned, secured by their role in Euclidean geometry. In Euclid's demonstrations, the diagrams are generally indispensable; if they are omitted, then what remains is no longer a demonstration of the theorem in question. Although Euclidean geometry was regarded for centuries as the paradigm of the axiomatic method, in fact it is not axiomatic in the modern sense. Its proofs are not directly derived from its axioms; rather, they are generated from the axioms by a concomitant process of construction, using ruler and compass according to the first three postulates (drawing a line segment, extending a line segment by another line segment, and drawing a circle about a given point with a certain line segment as radius). It is this process which, in the absence of an explicit theory of order, gives the requisite intuitive sense of continuity and (because it can be iterated) infinity of points in the line.[3] In Euclidean proofs, certain assertions are warranted only by the text of the proof (such as equality of lines or magnitudes), while other assertions (such as those relating to topological information about regions and intersections) are warranted solely by the diagrams themselves.[4] Diagrams not properly constructed cannot be utilised; the use of an improperly constructed diagram in Euclid is equivalent to the covert introduction of new axioms, an act which, in view of the cumulative proof-structure of the *Elements*, would serve to undermine the rigour and results of the whole project.[5] Within Euclidean geometry, thus, the constructed diagrams are indispensable both to the formal theory of proof in the system, and to the process of geometrical reasoning.

Historically, much of the influence of Euclidean geometry appears to have derived from the central role played in it by diagrams. In Euclid's system it seemed possible to intuit the process and results of geometrical reasoning, with a clarity and self-evidence not given in algebra. The diagrams themselves could be manipulated through the process of construction so as to yield new information and new results. And the use of diagrams seems to have played an important role in giving to Euclidean geometry an accessible, public and apparently indisputable character which for a long period ensured its paradigmatic status among the exact sciences. An awareness of this status is, as many have noted, a constant background theme in 17th and 18th century philosophy, and culminates in the project Kant sets himself in the first Critique: "This attempt to alter the procedure which has hitherto prevailed in metaphysics, by completely revolutionising it *in accordance with the example set by the geometers* and physicists, forms indeed the main purpose of this critique" (Kant 1781/7 Bxxii; italics added).

But it is well-known that this picture was superseded in several respects during the last century. In the first place, it was discovered that some proofs in Euclid were invalid or incomplete because of an over-reliance on features of the diagrams involved, a finding which may have reinforced a more general view that diagrams were often misleading. Secondly, geometry was given an axiomatisation by Hilbert which included the required theory of order, in which the specifically Euclidean act of construction therefore played no role, and which required no reference to diagrams.[6] New and more rigorous accounts were given of such key concepts as the denseness and continuity of the line. Thirdly, the development of non-

Euclidean geometries by Gauss, Riemann, Lobachevski and others undercut the apparently "given" nature of Euclidean geometry; and did so in terms which, far from requiring the use of diagrams, often appeared non-visualisable and only able to be formulated at all in equations. And these developments in geometry were to some extent paralleled by the discovery of counterintuitive results in analysis (such as continuous but nowhere-differentiable curves), which likewise fuelled a general scepticism about what could be considered intuitive, and indeed about the meaning of what it was to be intuitive at all.[7] It is this scepticism which motivates Frege's attempt to demonstrate the "autonomy" of arithmetic from intuition.

The case of Euclid illustrates both the changing status of diagrams and more general claims made against them: that diagrams are generally not compatible with rigour in mathematical proofs; that, as physical representations, they are insufficient to the generalised case; and that, because they can sometimes be visually misleading, they are not to be relied upon in warranting inferences or discovery, whatever their other merits may be.

Among logicians, the status of diagrammatic logics seems to have been undermined by their novelty, by real weaknesses in the principal systems employed, and by the comparative success of sentential logics. The notation of the *Begriffsschrift*, often considered diagrammatic, was widely criticised on and after publication as being cumbrous and unwieldy, and resistance to it may have contributed to the slow reception of Frege's ideas.[8] By contrast, the diagrammatic logics of Euler and Venn, though generally considered quite intuitive, were long known to have serious formal limitations. In Euler's original system it is difficult to represent contradictory information, for example, while Venn himself acknowledged that his diagrams could not generally represent hypothetical or disjunctive information.[9] Neither system can express multiple generality; until recently, neither system had been given a formalised syntax and semantics. Clearly, such basic weaknesses would, if unaddressed, gravely limit the expressiveness of these logics. The contrast with the achievements of sentential logic (truth-functional and predicate), though it would not have been so marked at the end of the 19th century, is quite evident today.

1.2 Diagrams Re-evaluated

Both within mathematics and logic, then, the standard view has been that diagrams have no place in rigorous logical reasoning; they are merely heuristic. It is interesting to compare this view with the working status of diagrams in the sciences and social sciences, where they retain a central place in both teaching and discovery. Three examples illustrate this point, and the tendency of practitioners to prefer diagrams to symbolic formulations in certain contexts of reasoning:

- Alfred Marshall, in his seminal *Principles of Economics*: "The use of [diagrams] requires no special knowledge, and they often express the conditions of economic life more accurately, as well as more easily, than do mathematical symbols ... [E]xperience seems to show that they give a firmer grasp of many important principles than can be got without their aid; and that there are many problems of pure theory, which no-one who has once learned to use diagrams will willingly handle in any other way."[10]

- Einstein, writing to Jacques Hadamard: "The words or the language, as they are written or spoken, do not seem to play any role in my mechanism of thought. The psychical entities which seem to serve as elements in thought are certain signs and more or less clear images, which can be voluntarily reproduced and combined ... The above mentioned elements are, in my case, of visual and some of a muscular type. Conventional words or other signs have to be sought for laboriously only in a secondary stage...".[11]

- Feynman diagrams. Schweber describes the emergence of two parallel but apparently different theories of quantum electrodynamics in the late 1940s: the first – mathematically very demanding – approach was developed by Schwinger and Tomonaga; the second, a diagrammatic approach which greatly facilitated the process of calculation, was developed by Feynman. Feynman diagrams were regarded as a useful curiosity until Freeman Dyson proved the two theories to give equivalent results in 1948/9, after which they were rapidly adopted by the majority of physicists working in the area.[12]

These examples underline the heuristic value of diagrams, which has already been noted: the extent to which they can help the reasoner to grasp concepts, illustrate ideas, avoid mistakes, formulate conjectures or develop proofs. To explain this phenomenon, philosophers have increasingly examined precisely what role diagrams can play in the processes of reasoning and discovery, both in the empirical sciences and in a priori disciplines such as mathematics and logic. Thus in mathematics, for example, it has been suggested that diagrams, or rather visual thinking, may generally be a means of discovery of theorems in geometry and arithmetic, but only in a very restricted range of cases in elementary analysis.[13] This is a stronger sense of "heuristic value" than the purely psychological; as well as any role they may have in cognition, the claim is that diagrams have epistemological value, in that they can be knowledge-yielding. By this means the heuristic status of diagrams has been analysed, and to some extent found to be justified, by philosophical inquiry.

What is it to be *merely* heuristic, however? As I have noted, on what has been called the standard view, proofs are syntactic objects composed of sentences, and diagrams are seen as merely supplements or addenda to proofs, and therefore eliminable from them. This very strong claim has the consequence that there cannot be a diagrammatic logic. It must, however, now be revised in two respects.

- As regards the claim that diagrammatic logics cannot be rigorously formalised, it has recently been demonstrated that Venn diagrams can be given their own syntax and semantics, with rules of transformation which make the system demonstrably sound and complete. The problem of representing disjunctive information can be overcome. Moreover, a modest extension of the system suffices to make it logically equivalent to a first-order monadic language.[14]

- As regards the claim that diagrammatic logics lack expressive power, it has been demonstrated that a less well-known but more sophisticated system, the existential graphs of C.S. Peirce, is logically equivalent to propositional logic in its Alpha version, and to first order predicate logic with identity in its more developed Beta version.[15] In its Gamma version, it can generate systems provably equivalent to the principal systems of modal logic.[16]

From a formal perspective, therefore, certain diagram systems can be considered logics in a full sense, and at least one diagrammatic logic is extensionally equivalent to the symbolic system most in current use. This suggests a response to the standard view: that one way in which diagrams go beyond the merely heuristic is in diagrammatic logics, for diagrams are not eliminable from diagrammatic proofs.

A holder of the standard view might respond to this claim in a number of different ways, which I will explore in Chapter 7. At this stage, the point is simply to notice the apparent possibility that a diagrammatic logic could combine the exactness and precision of syntax of a sentential logic, with the heuristic value of diagrams in general. Such a logic would be both secure and, in some sense still to be refined, intuitive. And, in particular, it would not be misleading, since the syntax of the system would define exactly what it was to be a well-formed diagram and what the effects of applying a given rule of inference were.

Various claims in this area have been made on behalf of diagrams, both within diagrammatic logics, and within diagrammatic presentations of non-diagrammatic logical systems. These contrast diagrams favourably with classical sentential logics. Such claims include: that diagrams prompt patterns of reasoning which are of a different character from those of classical logic, and which enable users to solve more complex problems; and that they are easier to learn, more intuitive in some sense, and more informative than their sentential counterparts.

Thus Roberts says of Peirce's existential graphs that "if a facile and perspicuous notation is one that can be quickly learned and easily manipulated, then years of experience with university students have convinced me that [Peirce's notation] is the most perspicuous, and *Principia* notation the least."[17] Brauner et al. say of the existential graphs: "our personal experiences in logic teaching within the humanities also suggest that one may benefit a lot from the use of graphs in the teaching of such logic courses".[18] And Barwise and Etchemendy claim that a diagrammatic presentation of Turing machines (*Turing's World*) allows students to solve significantly more complex computability problems than does its sentential equivalent, a 4-tuple notation; they note a discrepancy between what they consider traditional logical reasoning and the wider forms of valid reasoning actually engendered by diagrams, calling for a richer theory able to accommodate these cases too; and they describe how in the *Hyperproof* system diagrams prompt analytical approaches (e.g. proof by cases) in logic more intuitively than do sentences. The greater relative informativeness of the diagrams is ascribed to the ways in which they capture and differentiate between possibilities.[19]

1.3 Diagrammatic logic and information

Some of the issues raised above can best be addressed by cognitive psychology. But such claims for diagrammatic logics also raise a range of philosophical questions: How is the distinction between diagrams and sentences to be understood? How can diagrams in general be more informative or perspicuous than sentential representations? If diagrammatic logics are in some ways more informative or perspicuous than equivalent sentential logics, are they so in virtue of certain intrinsic features of all diagrams, or for some other reason (perhaps due to the specific rules of inference of the systems in question)? What are the advantages and disadvantages of a sentential logic as compared to its diagrammatic equivalent? (Even to pose the latter question may strike some as odd, given the overwhelming predominance of sentential logics among users.)

Perhaps the most fundamental question is this: what does it *mean* to say a diagram is more perspicuous than a sentence? In the case of extensionally equivalent logics *A* and *B*, what notion of information is able both to capture the fact of logical equivalence, and simultaneously to explain how *A* can be more informative than *B*? There would seem to be an air of paradox – if not of contradiction – about this claim, at three levels. First, some philosophers have questioned whether it makes sense to talk of logic as conveying information at all. Secondly, if logics *A* and *B* convey the same information, then it is not clear how one can also be more informative than the other. And finally, if two equivalent logics can be differently informative, it is not clear whether this distinction can be made in terms of a concept of information which is philosophically interesting and not purely psychological; that is, not simply reflective of human habits of thought.

These are deep issues, and I will not attempt to provide a general answer to them here. Rather, my strategy is a more limited one: to approach these questions by examining the representational features of a particular type of diagrammatic logic as against those of an expressively equivalent sentential counterpart. The logics to be used are Peirce's alpha existential graphs (hereafter EG) and various notational forms of sentential (truth-functional) logic (SL). This strategy has a number of advantages over a more general study of diagrammatic representation. First, since it is known in advance that these logics are both sound, complete and expressively equivalent, it will be possible to focus on their representational features without the concern that one may be more informative than the other simply for reasons of logical power. Second, the fact that the syntax and semantics can be precisely specified for both systems allows us to sidestep a number of problems which arise in more informal contexts from lack of clarity regarding the general or specific constraints on diagrammatic representation. These include problems of generality of content, of misrepresentation and of improper inference. In the cases to be discussed here, it will be clear what the content of a propositional representation is, whether such a representation is well-formed, and whether a given rule of inference is applicable.

Accordingly, this thesis falls naturally into two halves. In the first, I examine descriptions, diagrams and depictions as representational types, in order to isolate certain general features of diagrams in virtue of which they can convey information perspicuously (Chapter 2); and review current theories of information in order to determine a compatible background account of information (Chapter 3). This allows a more rigorous restatement to be given of the key idea of perspicuousness. In the second half, I introduce EG formally (Chapter 4); compare it to various types of SL (Chapter 5); rebut a recent claim about provability in EG which would severely undermine the utility of the system in proofs; and show how EG's relatively greater perspicuousness motivates a "non-deductivist" view of deductive reasoning, and indeed a specific distinction between two types of such reasoning (Chapter 6). The last chapter returns to some of the more general questions about diagrams raised above.

In adopting this approach I hope, not merely to provide limited answers to the questions posed above, but to shed light on three issues of broader interest. The first relates to the question whether, and in what sense, there may be a role for intuition in logic. For reasons already noted, the claim that geometry – let alone arithmetic – involves some act of intuition came to be questioned in the second half of the 19th century. In the case of logic, however, a plausible account of visual information and visual inference in a diagrammatic logic provides reason for thinking that some specific notion of intuition, close to the core sense of the word, is in play here.

The second question relates to the nature of deductive reasoning and proof. I have already mentioned claims that in some contexts the types of reasoning used in diagrams differ from those to be found in classical logic. While this may be true in other contexts, I want to suggest something different: that in this rather basic case, an analysis of EG motivates and supports the claim that deductive reasoning generally involves elements of observation and experimentation; and that this is also true of deductive reasoning about sentences, though its motivation is less evident here owing to their less perspicuous representational form. Such an account, unsurprisingly perhaps, has the potential to return us to a view of deduction more akin to that to be found in Euclid, in contrast to those who think of deduction as homogeneous.

The final question is not a philosophical question at all, but an historical one. Peirce invented the existential graphs in the early 1890s, and he gave up sentential logic as a result and never returned to it. This was not a trivial step, since Peirce had made very significant contributions to sentential logic during the previous decade: among his achievements were the invention of quantifiers around 1883, independently of Frege and in substantially the representational form which they take today; his demonstration that propositional logic can be developed using one operator c. 1880, a generation before the Sheffer stroke was invented; one of the earliest uses of the truth table method as a general test for validity; and the first use of what since become known as prenex normal form.[20] The question therefore arises as to why Peirce should have turned his back so completely on the new field, having made such fundamental discoveries and at precisely the time when it was undergoing so revolutionary a period of development. To the extent that the alpha graphs warrant, I hope to shed light during this discussion on the philosophical case for such a step.

2: Diagrams, Depictions and Descriptions

2.1 Introduction

Consider the following situation. Bill and Jane want to give their friends directions to their house. That is, they want to present their friends with some relevant information in a way which will (a) if followed correctly, guide them to their house, and (b) in case of error, will allow them to find their way back to the correct route. There is no obvious limit to the amount of collateral information required to meet goal (b). But some selection must clearly be made, both because their friends' ability to absorb information is finite, and because there is some information which would be actively misleading or distracting if included. And the choice of how the information is represented is important for similar reasons: what is needed is a method of representation which, as far as possible, compactly and explicitly represents the intended information, but which does not also convey irrelevant information.

There would seem to be three general methods to represent this information: via a description (say, a set of instructions), via a depiction (say, a photograph) and via a diagram (say, a sketch map).[21] And there will also be hybrid solutions from among the three.

What I want to do in this chapter is to compare these three representational types, in order to bring out some of the characteristic features of each; and to understand the idea of perspicuousness better. I will not attempt to give a general account of diagrammatic representation, though I will consider various conditions on a representation relation, and try to situate these comments within this wider debate. Two caveats are in order. First, I shall deal rather briefly, perhaps too briefly in places, with the substantial philosophical and psychological literature which has built up on the representational character of descriptions and depictions (less so, on diagrams), and the differences between them. Secondly, however, the discussion will unavoidably involve appeals to intuition as to what constitute tokens of each representational type. This is complicated by the very wide range of representations which can, with more or less justification, be classified as depictions or diagrams; intuitions here (especially about hard cases) can often diverge. Rather than try to offer any general definitions or get involved in specific sub-debates, I will assume – and try to operate within –

a common core of intuitions as to what constitutes a token of each representational type. That such types do have characteristic features – indeed, to what extent it is appropriate to differentiate three such types at all – should become clear from the discussion.

2.2 Descriptions, Depictions and Diagrams

The focus of this chapter is, then, mainly on the *representational* differences between descriptions, depictions and diagrams. There are also non-representational differences between these types (differences of use or function, for example; such as communication, problem-solving, conveying aesthetic or linguistic meaning, etc.) and some of these will assume importance later in the discussion. And within each type, for reasons I shall examine, sometimes non-representational characteristics (for example, the content of a description) can affect the perspicuousness of a given token.

Numerous attempts have been made to characterise and explain the representational differences between the three types. Representations have been classified as, to take a few examples, "graphical/linguistic", "analogical/Fregean", "analog/propositional", "graphical/sentential", "diagrammatic/linguistic".[22] Commentators have sought, not to give necessary and sufficient conditions on a representation's being of one type or another, but to identify a single property alleged to be characteristic of a given type. Such alleged properties include, for a given representation: the sequentiality or linearity of its sub-elements; its degree of compactness; whether or not its colour is relevant; whether or not it represents from a certain point of view; whether or not its target or range can be seen; its relative scope to misrepresent; its scope for labelling or annotation; whether it is "analogue" or "digital"; its degree of semantic and syntactic "density", and "relative repletion"; whether or not it includes certain characteristic types of shape or figure; whether it is or requires to be one-, two- or three-dimensional; whether or not it resembles, or is homomorphic or isomorphic to, its range or target; and whether or not it has certain properties (permitting free rides, restricted by over-specificity) imposed by the structural constraints on the relation between representations and their targets.[23]

There is little if any unanimity, at any level of detail, amid this plethora of claims, which reflect differing goals, methods, background disciplines and governing assumptions. As a basis for this discussion, then – and at the risk of travestying a much debated topic – I will simply set out a general characterisation of descriptions, depictions and diagrams which I believe captures some common intuitions and many of the insights above. I shall assume, here and later, that all such representations are well-formed, appropriately sized and visually salient. Such a general characterisation might go as follows:

- *Descriptions* are normally composed of sentences of natural language. Such sentences are linear sequences of words, which are symbols whose meaning is (very broadly) implicitly or explicitly established by convention among their users. When these words are scanned in a certain order by a suitably informed viewer, they can convey information. Such information may be general or particular.

- *Depictions*, however, are not generally linear but two- (sometimes three-) dimensional; they are composed of elements which do not require to be scanned in any order by the viewer to convey information. These elements are not, at least in the first place, symbolic (though they can often be given symbolic interpretation). Let us say for purposes of discussion that depictions generally convey information by having a visual similarity of appearance to their objects.[24] However, depictions can also convey information through the relations of their elements to each other, where these represent relations between their objects. Such relations are what allow depictions to represent scenes. Such scenes are intrinsically particular, though it may be possible to give them a general interpretation: a depiction of an old man may convey some general attitude to or belief about the effects of old age, but in the first place it is a picture of a particular old man, actual or imagined. The resemblance generally characteristic of depiction thus places a constraint on interpretation.

- *Diagrams* are, in effect, an intermediate case. Like depictions, typically they are not linear but two- (sometimes three-) dimensional. Like sentences, they are often compositionally very flexible. Well-drawn diagrams are generally schematic; that is, they do not contain irrelevant detail. They can convey particular or general information, both through the conventionally established meaning of their symbol-elements, and through the relations of their elements to each other. The elements of a diagram need not be defined by convention, nor by any visual similarity to their objects. What constitute the visually relevant features of a diagram may vary with interpretation.

This comparison is of course, far from exhaustive; however, it captures two important intuitions: first, that in a spectrum of representations whose opposite poles are descriptions and depictions, diagrams in some sense occupy a middle place, closer to the latter than the former; second, that while all three types use symbols as marks (which must in some sense be interpreted to have meaning), they can all also employ symbolism at a higher level. Let me explain this last point with an example.[25] Take the case of a picture which represents the Holy Spirit as a dove. The picture bears a series of marks which we can recognise as a dove; but it also represents the Holy Spirit. It does not, one wants to say, *depict* the Holy Spirit; the fact that the Holy Spirit is connoted by the dove is a further piece of symbolic content, which is supplied by the observer's background information. This content is secondary; if the dove were not represented the Holy Spirit would not be. One wants to say, following Hopkins 1998, that there are two levels of representation here. The picture uses "bare" symbols; these are enough, when organised in a certain way, for it to convey information as a depiction. It also uses what I shall term "endowed" symbols, however; but these are not intrinsic to depiction, for a description of the same scene could represent the dove in a similar manner (the symbolic or allegorical content could be assigned to the word "dove").

I think this distinction, between what will later be called iconic and symbolic representation, is fundamental to understanding how descriptions, depictions and diagrams represent; and I will discuss it further below. For now, however, let us return to the broad characterisation of differences above. Clearly, this is far too abstract and vague as it stands. Part of the problem lies, as has been mentioned, with the sheer diversity of what can, without straining, be considered diagrams: many types of map, product instructions, Venn diagrams, Cartesian (X-Y) graphs, blueprints, Feynman diagrams, chemical valency diagrams, wiring diagrams and the wide variety of diagrams used in business (flowcharts, pie charts, scatter plots, gant charts, bar charts, organisational hierarchies etc.).[26] What do these have in common? And how can diagrams as a whole be more precisely differentiated from descriptions and depictions?

Before responding, let us first notice one background point. It seems generally to be a condition on *r*'s being a representation of X that it be subject to some underlying cause or intention. Thus, we consider a blueprint to represent a building in part because someone has caused or intended it to do so; we consider an ECG reading to represent heart activity because the machine has been (intentionally) set up to make it do so, and there is a causal relation between the heart's electrical outputs as measured and the shape of the line on screen. It is not enough that there be some purely accidental relation between the two; we do not consider that an ECG reading represents a graph of UK GNP even if the two are in fact identical. (Note that this underlying intention does not mean that a representation must convey the information it is intended to convey; it may in fact fail to do so, or the intender may not know in advance precisely what information he or she intends to convey. Nor does it mean that, even where it succeeds in conveying its intended information, it must convey only that information; there are cases in which the reasoner or creator of the representation has no such specific intention, indeed many diagrams are valuable because they give free rides to unintended or unexpected information.) Finally, there will clearly be information which is intended to be part of a representation, is relevant to the representation's conveying the intended information, but is not part of the intended information itself. Thus, in creating the representation that "there is a ball on a box", I intend that it contain seven words, it is relevant to its representing what it does that it contain seven words, but it is not part of the intended information that it do so.[27]

We can now say that one important property of descriptions is this: they need only express their intended information; they can leave unsaid and indeterminate some aspect of what is described; they need not convey more than is required. They are, one might say, *discreet.* A description of a ball on a box may be enormously detailed, or it may simply be the sentence: "there is a ball on a box". Depictions, by contrast, generally represent more information than is needed, in three ways. First, it may be in the nature of the specific type of depiction to display more information than is needed; thus, a photograph of a box will almost invariably contain additional information as to background, while a drawing could leave this information indeterminate while still remaining depictive. Second, though, there are some types of information which depictions must generally represent in order to depict other information. Thus, it is difficult simply to depict information about shape without also depicting information about size and orientation; and similarly, to depict information about texture, colour or intensity without (perhaps only vaguely) depicting information about shape. Thirdly, while the background may be indeterminate, it is difficult for a depiction to leave its intended information indeterminate, while remaining genuinely depictive: any depiction of a ball on a box is likely to show the ball in some particular location.[28] Depiction is, to this extent, intrinsically indiscreet.

The idea of discretion can be expressed as follows. For representation *r* and intended information P:

$$\text{Discretion} \quad = \quad \frac{\text{P}}{\text{total information which } r \text{ must convey to convey the information that P}}$$

That is, a representation is discreet to the extent that, in conveying information that P, it need only convey the information that P. (Of course, much will depend here on how "information" is defined, and on the role played by context and the observer's background knowledge; I discuss these issues in detail in Chapter 3.) The distinction between description and depiction can then be expressed thus: given a certain context, a well-formed description will tend to have discretion of close to 1. A well-formed depiction will typically never have discretion close to 1.

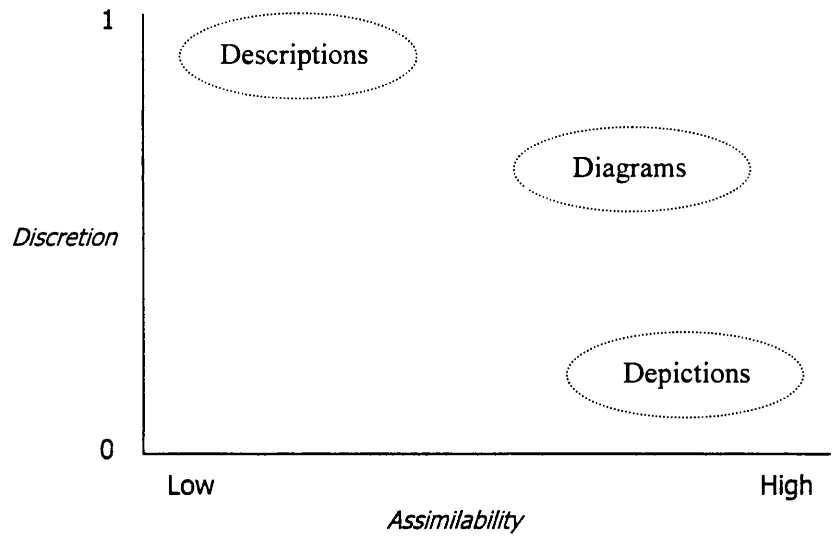
An important general property of depictions, by contrast, seems to be this: that the information they represent is typically easy for the observer to process or assimilate. In many cases, obtaining information from a depiction does not require a conscious process of inference; it is just seen, at a glance, in a way which is phenomenologically similar to how one sees the colour of one's shoes, for example. The content of a depiction can, moreover, be grasped in this way by a range of different viewers, speaking different languages and with different cultural backgrounds. Obtaining information thus does not seem, at this level at least, to be heavily dependent on background knowledge; or specifically, on knowledge of language. Descriptions, by contrast, are in natural language; for them to convey information requires an act of linguistic interpretation. Descriptions cannot simply be viewed, one might say; they must be *read.* The point here does not concern the degree of effort expended by the reasoner as such – one may obtain information from reading a description at a glance, without conscious inference – though it will, I think, in general be true that a (well-formed) depiction of (positive, non-disjunctive) given information will require less effort to process than a comparable description. Nor do I mean to suggest that one cannot also derive information from depictions more slowly, or through an explicit process of inference. Rather, the point concerns the cognitive resources which are brought into play in considering the representation. Descriptions seem to require symbolic interpretation, in the second "endowed symbol" sense described above; their content cannot be grasped at all except in terms of the (endowed) values assigned to their constituent symbols, and this seems to require the exercise of higher level resources of understanding. By contrast, depictions seem to be able to represent at a cognitively lower level, in terms of what I called "bare symbols" above; and this seems principally to require the exercise of perceptual resources. But of course there may also be a further level of representation in a depiction, in which symbolical or allegorical content is brought into play, and this will require a higher-level grasp of endowed symbols in the sense already mentioned.

Space does not permit a fuller justification of this claim about cognitive resources. A more fully-fledged account would discuss the psychological literature on levels of visual processing;[29] and, in epistemological terms, it would also explore the idea that the observer of a depiction has a general but defeasible entitlement (in the sense described by Burge 1993: that is, an epistemic warrant which may not be understood by, or indeed even accessible to, the observer) to derive certain information from it, and that this information is triggered by perception, at the "bare symbols" level, of resemblance relations between it and the depicted object. Much more needs to be said; but the effect of this move would be to connect the epistemology of depiction more closely to current work on *a priori* knowledge, and on testimony.[30]

Our goal here, however, is not to give a general constitutive account of diagrammatic representation as such, but to differentiate the three representational types more clearly. We can now be more precise about the status of diagrams. Diagrams score highly on the scale of discretion, it seems, though not in general as highly as sentences. A well-drawn diagram need not convey much if any extraneous information in order to convey the intended information that P. In this regard the great variety and multiplicity of types of diagram may come to the user's aid, allowing him or her to select a different type rather than allow indiscretion to occur; and the recognition that it is possible by such choices to avoid conveying extraneous information itself suggests that users generally expect diagrams as a type to be discreet. On the other hand, however, diagrams also score highly on the scale of ease of processing or assimilability. A diagram may serve as well as a picture, indeed in some contexts perhaps even better than a picture, at representing to an observer that there is a ball on the box.

This gives a fairly clear sense in which diagrams are indeed an intermediary form, between descriptions and depictions. Perhaps appropriately, this set of relationships can be viewed diagrammatically, as in fig. 2.2.1. (The circles are intended to indicate roughly where the central cases of each type fall; there will clearly be overlaps and borderline cases.)

Fig. 2.2.1



It is now possible to ask what the dynamic factors are which could affect the location of a particular type of representation along the two axes.

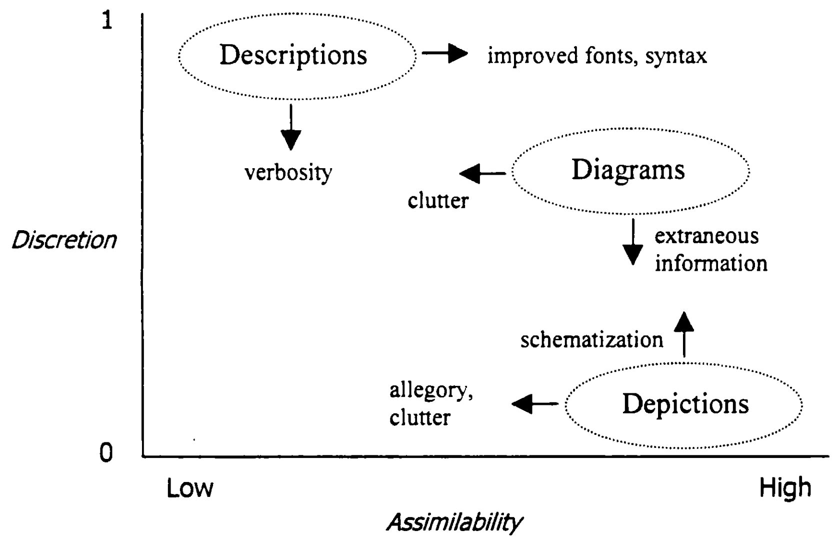
- Descriptions would lose discretion by conveying unnecessary collateral information; that is, through verbosity or periphrasis. They could increase their assimilability in two ways: at the level of form, perhaps through a change in font type, or through replicating the shape of their object, in a few rare cases;[31] at the level of content, perhaps through improved syntax, provided this did not affect the information conveyed.

- Depictions would lose assimilability by being overly symbolic or allegorical, as discussed, or by being cluttered (where "clutter" is defined as *uninformative* detail). Decoration could in principle either increase or decrease assimilability.[32] Depictions would gain discretion by becoming more schematic; that is, by omitting irrelevantly informative detail.

- Diagrams would lose assimilability by being cluttered; and would lose discretion by conveying irrelevant information.

These dynamic factors can also be represented, as follows:

Fig. 2.2.2



I want to suggest that this account can be used to analyse the concept of *perspicuousness*. What is generally meant by perspicuousness, or perspicuity? Something is perspicuous if it is "easily understood" or "clearly expressed", according to the OED. There is a visual metaphor in the Latin root of the word; that of "looking through" something to see something else, of easily discerning the information in a given representation. These senses are captured by the analysis above: perspicuousness can be understood in terms of the combination of discretion and assimilability. Both properties are needed: a representation that P which is discreet but hard to assimilate by the observer will not be a perspicuous representation at all; a representation which is assimilable but contains much irrelevant information will not be a perspicuous representation *that P*.

What is considered perspicuous will of course vary with context and between specific individuals. In general, this analysis suggests that a perspicuous representation – whether it is a description, a depiction or a diagram – does not convey more information than it requires to make its point, so to speak, and it does so in the way most assimilable by the observer. Conversely, a representation will be perspicuous if it avoids clutter, irrelevant information, verbosity, or unnecessary symbolic or allegorical content. Thus it is possible to compare two descriptions or two depictions on these criteria, and say of one token of a type that it is more perspicuous than the other. But it is also possible to compare tokens of different types, and to say that token *a* of type *A* is superior to token *b* of type *B* as a perspicuous representation of information that P. In both cases, some general trade-off will be required between the two criteria in order to make such judgements; but it does not seem problematic that people can and do make such trade-offs in precisely these terms (of clutter, irrelevance, verbosity etc.). Thus, this analysis seems to correspond both to our normal intuitions and to our normal linguistic practices.

2.3 Diagrams and Homomorphism

It would follow from this account that diagrams are, in general, a more perspicuous representational type than descriptions or depictions. It also, however, highlights the relative strengths of the other two types. A good diagram is, in a sense, *constrained* by its perspicuousness, which will be diminished by clutter or irrelevant information. On the other hand, descriptions are often not expected to be easy to process; so they can be made as lush, metaphorical, arcane or elliptical as the author wishes. Equally, depictions are not expected to be discreet; so their creator can add extraneous elements or detail. Where the goal is not the conveying of information as such, but the conveying of artistic or literary meaning, this flexibility may be very valuable. And of course the compositional nature of linguistic symbols gives descriptions unique scope to represent different types of information. Diagrams often have difficulty representing certain types of information, as I have mentioned; their flexibility comes in the scope they give the user to innovate and experiment, and to choose new formats to represent intended information without destroying the broader context in which information is to be conveyed.

The general value of diagrams lies, as we have noted, in how they assist the process of reasoning. But it will not do merely to say they do so in virtue of being perspicuous; something also needs to be said about the general relation between the diagram and its range or target (I use the latter words interchangeably henceforth), in virtue of which the reasoner is warranted in obtaining the intended information. I want to suggest here that differences in the representation relations involved seem to be what underlie the differing claims on our cognitive resources made by depictions and diagrams, as against descriptions.

Consider again the position of Bill and Jane, who have now decided to use a sketch-map of the local area to guide their friends. What is it about the sketch-map which entitles Bill and Jane to feel sure that it reliably conveys the requisite information? A first response might simply be: there is a resemblance between the diagram and the area. But the problem here is this: what are we to understand by resemblance? If resemblance is taken to mean "similarity of appearance", then this claim is false. There is little or no actual similarity of appearance between a sketch-map and the area which it represents (there is no danger, for example, of illusion; of mistaking one for the other). In the case of Feynman diagrams, which represent interactions between subatomic particles, we may not be able to see how the target "appears". In the case of a flow diagram showing phases of work, the target may have no visual appearance as such at all. If resemblance is not understood as similarity of appearance, however, it is hard to see what it could amount to, or what role it could play in a general account of diagrams. Many objects resemble each other in some respect, without conveying information about them. A resemblance account may be possible for depictions, despite such phenomena as caricatures, abstract pictures etc.; but it faces severe additional obstacles for diagrams.

A better account, which preserves the core intuition that there is something in common between the diagram and its range, would invoke the idea of similarity or identity of structure. On this view, elements of the diagram correspond (in some way to be specified) to elements of its range, and structural relations between the former correspond to structural relations between the latter. In the case of Bill and Jane's map, the symbol of the cross may correspond to (and so be used to identify, by visual metonymy) the church, and the beer glass the pub; and the distance between and orientation of the two symbols may correspond to the actual distance between and orientation of the two buildings on the ground. It would then be a *mistake* to locate the cross-symbol in a location on the map which does not correspond in the requisite way to the actual location of the church. Thus the structure of the diagram is, in this case, constrained by the structure of its range.[33] To the extent that the diagram possesses the requisite structural similarity or identity with its range, it can be used to convey information about it. Note that this structural correspondence need not – and often will not – extend to *all* the elements of the range; the diagram may simply represent the relevant elements. Thus a sketch map can be a diagram.

Can this overall account be extended to other type of diagram? It would seem that it can. Of course the conventions of the diagram in question will also play a role, as they do in the case of sketch-maps; a background grasp of these will be necessary for the viewer to derive the correct information. Thus, if George knows the relevant conventions for pie charts and is given a pie chart of car production by the major auto manufacturers in 1997, provided the requisite structural relationship exists between the range (the facts about car production) and the relevant markings on the diagram, he will be able to derive the information that, say, Ford sold more cars than Honda. Given certain background conventions, a Feynman diagram of two particles colliding will carry information that P provided the requisite structural relation exists between the marks in the diagram and the range of particles and their charges, interactions etc. And so on.

This structural correspondence account can be more precisely formulated mathematically, in terms of a homomorphic or isomorphic relation between diagram and range. A homomorphism is an order-preserving and injective (one-one) function or mapping between two structured sets of elements (in this case, the elements of the diagram and the relevant elements of the range); whereas an isomorphism is order-preserving but bijective (in a one-one correspondence). Within these constraints, there is clearly some latitude as to what are defined as elements of the diagram and of the range, and as to what properties or relations within or between the two are deemed to be preserved by the function. In the case of diagrams, adapting a definition advanced by Barwise and Hammer,[34] a homomorphic relation can be defined as follows:

1. Elements of a diagram Δ represent elements in a range D, with different types of range-element represented by different types of diagram-element.

2. Π is a function between Δ and D such that:

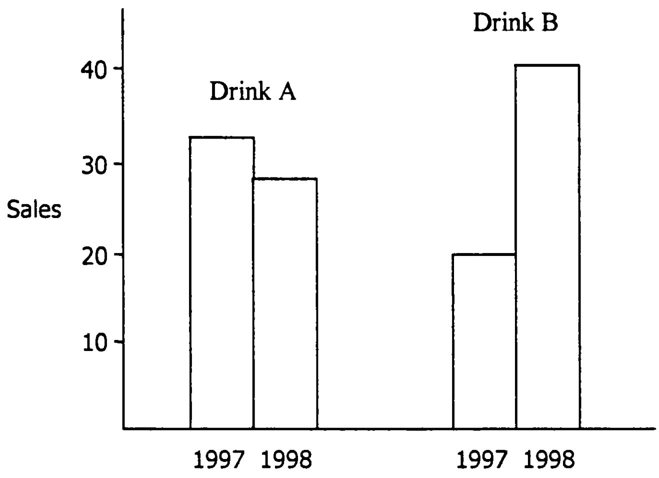
    a) if R is a relevant relation between elements of Δ, then there is a relevant relation S between the elements of D to which they are assigned by Π; and the converse is true;

    b) if a relevant relation between elements of Δ has some structural property (transitivity, reflexivity, symmetry etc.), then the same property holds between elements of D; and the converse is true;

    c) if an element d of Δ of some type T has a relevant property P, then Π(d) is an object of some type Π(T) with relevant property Π(P) on D; and the converse is true.

To see how this definition applies in practice, consider the diagram in fig. 2.3.1, a bar chart showing the sales of two brands of soft drink over two years.

Fig. 2.3.1



What is the range here? It can be thought of as comprising three types of element – the drink, the year, and the sales amount – in four ordered triples, of the form <D,Y,S>. In effect, these triples constitute four pieces of information to be conveyed: how much each of two products sold in each of two years. According to the definition above, then, for there to be the requisite structural correspondence between diagram and range requires (condition 1) that elements of the range be represented by elements of the graph, and that different elements of the range be represented by different elements of the graph. This condition is clearly satisfied: marks on the X-axis represent years, marks on the Y-axis represent sales, and the labelled bars represent the amount sold of each drink in each year.

It should be fairly clear that conditions 2(a) and (b) are also satisfied. For example, in the diagram the 1998 bar for drink A is lower than the 1997 bar; corresponding to this "lower than" relation in the graph is a "lower than" relation between the relevant numbers in the range. In the diagram, the relation between bar heights is asymmetric – if bar X is higher than bar Y, then bar Y is not higher than bar X – and so is the relation between the relevant numbers in the range.

Finally, imagine that the bar representing sales of drink B in 1998 has the following property: it is 20% above a line representing budgeted sales in the same year. Then, in virtue of the homomorphic relation the corresponding number in the range will *ipso facto* have the same property with respect to another number (the budgeted number) in the range. That is, condition 2(c) will be satisfied. And as the "relevance" restriction in the definition suggests, it may be the case that not all the visual properties of the diagram will be mapped by the relation to the range; some properties (e.g. the colour of the ink in the diagram) may be irrelevant.

As this last point suggests, it may be that for some representations not all the above conditions are in fact satisfied. Where they are not – as with distance relations in, say, the London Underground map – the reasoner may go astray. To the extent they are, however, such representations will be homomorphic or isomorphic in some relevant respect to their ranges, and this structural similarity or identity can still make them valuable in reasoning. But this fact does mean that some diagrams can be *more* homomorphic – more structure-preserving – than others. Notice, moreover, that the range need not, often will not be, observable: in the case of a sketch map the homomorphism will include a correspondence between diagram elements and observable, concrete objects; with Feynman diagrams it will include one between diagram elements and partially or indirectly observable concrete particles; with Venn diagrams it will include one between diagram elements and abstract objects (sets).

It is generally true of diagrams that they are homomorphic or isomorphic to their ranges. This is not a distinguishing feature as such, however, for it is generally true of (figurative) depictions that they are isomorphic to their ranges. Indeed, in rare cases *descriptions* can have homomorphic properties; the temporal ordering of sentences may correspond to the succession of events described, as Barwise and Hammer point out.[35] Descriptions, however, are not generally homomorphic, whether the relevant elements be taken as either words or sentences.

Homomorphism gives to diagrams and depictions some features which are valuable for reasoning. One is this: it is possible to do part-whole reasoning quite easily using them, once the representation relation is understood. Imagine a (normal sized, well-drawn) diagram or depiction of a family tree, compared to a (concise, well-worded) description of the relation between the family-members. Viewing the diagram or depiction allows the reasoner first to identify a given relation – perhaps that of second cousin – and then see of any two members, wherever they are located, whether they satisfy the relation. This information is likely to be almost impossible for the reader of the description to obtain at all, unless it is used to compose a diagram, picture or mental image; or, unless a further set of rules is specified to capture the relation in question. Related to this is another feature: diagrams or depictions are useful for solving problems with limited information. In general, a portion of a diagram or depiction is a representation of a portion of its object or range. If the reasoner can construct a diagram of part of a range, this constitutes a secure basis for further reasoning. Part of a description, by contrast, may simply be a collection of apparently unrelated words or sentences.

2.4 Diagrams and Iconicity

In terms of being homo- or isomorphic, then, there is little to differentiate diagrams and depictions. Both have this feature in common: that knowing that there is a structural relation the reasoner can, simply by attending to the representation in hand, come to know information about a range, including information that was not originally part of the information needed to construct the representation. That is, the reasoner may notice some new visual feature of the diagram or depiction, and know – in virtue of the structural identity or similarity between the two – that this represents some further fact about or property of the range. Once the necessary relationship has been established between representation and range, the range becomes, in a sense, irrelevant to the process of reasoning (though not, of course, to its wider purpose). Since in many cases the range is too large (as often with maps), or too small (as with Feynman diagrams) to be observed, or is not observable as such at all (as with workflow diagrams), this makes reasoning from the diagram or depiction very advantageous. And it is notable that language often recognises this substitution of representation for range, as with arguments that begin "but the map says to go this way" etc.

This property is close to one which Peirce called *iconicity*; in the process of reasoning described above the representation acts as an *icon* of the relevant features of the range. The idea of iconicity is a complex one in Peirce's thought, with deep roots in his general theory of signs, and lies outside this discussion. Peirce often describes it in terms of a resemblance or similarity of character between a sign and its object. For reasons described above, the idea of resemblance is too vague to be of service; but the analysis given above can be extended to give a definition of iconicity which is broadly compatible with Peirce's view. Recall that by the definition 2 given above Π is a homo- or isomorphic relation between Δ and D such that for any relevant relation R between elements d_1, d_2 of Δ there is a relevant relation S between the elements s_1, s_2 of D to which they are assigned by Π; and the converse relation is also true. That is,

1. $d_1Rd_2 \leftrightarrow s_1Ss_2$.

If this is the case, and also:

2. it is possible to tell whether d_1Rd_2 just by observing Δ;

then Δ is an icon of D.

A representation will, then, be an icon of its range if and only if the reasoner can tell just by looking whether some relation or property holds of its elements; and the corresponding relation or property holds between elements of the range. Note that some representations can be more iconic than others; in some cases it will be easier to tell whether d_1Rd_2 just by looking. Thus, diagrams and depictions are almost always more iconic than descriptions; where there is resemblance (similarity of appearance) between representation and range a diagram may be more iconic than a depiction, or vice versa; where there is no resemblance, only diagrams will generally be able to serve as icons of the range for purposes of reasoning.

In using the phrases "just by observing" or "just by looking", here and in later chapters, I mean that the observer can grasp (or: is entitled to grasp) the information presented by the representation without a conscious process of inference. Often, this may simply require what I have termed relatively low-level cognitive resources. Of course, higher-level resources may also be involved; there may be, and often will be, endowed symbolism in an iconic representation. But this endowed symbolism involves, I would argue, a fundamentally different representation relation, one in which it is arbitrary – not given by any constraints of structural similarity or resemblance – as to what content is attributed to what symbol. This arbitrariness draws heavily on the observer's background knowledge, but it is what gives descriptions their scope and flexibility. By contrast, the flexibility of diagrams is not an intrinsic feature of any particular diagrammatic system, for these will always be structurally constrained (albeit in different ways); rather it comes in the scope to deploy different systems, with different advantages and disadvantages, with the relative assurance that because the representation relation preserves structure and draws less heavily on background knowledge, the observer will derive the intended information.

It is now possible to give a fuller explanation of why Bill and Jane should prefer the sketch map in the example above. In the first place, it is discreet; unlike the depiction, it does not contain irrelevant information. Secondly, however, it is iconic. The guest knows the relation which the map bears to the range; and can then tell just by observing that d_1Rd_2, and therefore that s_1Ss_2. Once such a general relation Π has been identified, observing a previously unnoticed relation or property xR^*y will warrant the inference that $\Pi xS^*\Pi y$ in the range; but this may be new information. The diagram can thus display new information over and above that with which it was constructed.

2.5 Conclusions

This chapter has explored the representational features of descriptions, depictions and diagrams respectively. It has been argued that these can be differentiated from each other by comparing their performance on two criteria, discretion and assimilability, and that these criteria constitute an attractive analysis of the idea of perspicuousness. On this view, diagrams are more perspicuous than either descriptions or depictions, though the latter have attributes which befit them for other purposes. Diagrams and depictions bear a representation relation to their ranges some of whose features can be made quite precise, and extended to capture the idea of iconicity. Descriptions, by contrast, seem to utilise a different type of representation relation altogether. Differentiating iconic from symbolic representation relations in the relevant respects seems to explain some intuitions as to how information is derived in each case, what cognitive resources are employed, and what are the sources of the differing utility and flexibility of depictions, diagrams and descriptions.

The position we have now reached bears directly on the comparison I want to make between the representational features of the existential graphs (EG) and those of various types of sentential logic (SL). In the first place, we need to distinguish between sentences of natural language and sentences of logic. While the two have in common the quality of discretion, sentences of logic – unlike sentences of natural language generally – are the results of a process of formalisation. In terms of the definition given earlier, they bear a homomorphic relation to their range; though, as we have seen, there can be degrees of homomorphism. In virtue of this, they can be iconic; it is possible to infer new information about their range just by looking at relations between their elements. Sentences of logic are thus similar to diagrams in this respect.

This analysis gives us the resources to differentiate, at the level of representational form, between diagrammatic and sentential logics, and to explore the sources of the perspicuousness (or lack of it) of each. But underlying this general account is, I have argued, a fundamental distinction between iconic and symbolic representation. What is needed now is a background account of information, in terms of which the idea of perspicuousness, and explicit or implicit information, can be understood. This forms the topic of the next chapter.

3: Diagrams and Information

In this chapter I want to examine the idea of information in more detail, and try to situate the discussion of diagrammatic representation and perspicuousness presented in Chapter 2, and the distinction drawn there between iconic and symbolic representation, within a broader theory of information. With this in hand, it will then be possible in Chapter 4 to introduce the system of Existential Graphs (EG); and in Chapter 5 to examine the informational characteristics of EG and sentential logic (SL).

What is sought, thus, is an account of information suitable both to diagrams and to logic. This chapter brings out some of the motivations for and constraints on such an account, by reviewing the most influential types of information theory: probability theories, possible worlds theories and inferential role theories. As the discussion will make clear, philosophers often mean different things by information, and it is important to make explicit from the outset some of the principal ways in which this concept can be analysed. At a bare, pre-semantic level, "information" can mean something like uninterpreted scientific or computer "data"; at the level of semantics, something like "the content of a proposition"; at the so-called pragmatic level, something like "implied (or implicated) meaning". "Information that P" can be factive (requiring that P be true), or not. Moreover, something can be regarded as "information" without thereby necessarily being regarded as "informative" – a word often connected with new information – though the converse does not seem to be true. And something can be regarded as "informative" in virtue of being new information, of being factive information, or of being (factive or non-factive) information which adds to or strengthens (or: ought to add to or strengthen) a person's existing beliefs (or: if factive, knowledge). And there will doubtless be other possible combinations.

Different intuitions can be primed by different uses of language; for example, to some ears "giving someone a piece of information" is a factive use, while "informing someone" is not. For my purposes, the core sense will be that of non-factive content, which may be informative in virtue of whether it adds or not to existing knowledge; and (contra Dretske)[36] I also believe this is the core sense in natural language, though nothing here hangs on this latter claim. But in discussing the views of others, I will try to make clear which sense of "information" or "informative" is or appears to be in play; and, substantively, I will argue in due course that some theories of information acknowledge but fail to accommodate the distinction drawn above between conveying information and being informative.

3.1 Theories of Information

Barwise and Seligman 1997 provides a convenient taxonomy of theories of information, which includes probability theories, possible worlds theories, and inferential role theories.[37]

A central theme across all these accounts has been the idea that information is related to the reduction of uncertainty. Take the case of a six-sided die: before it is thrown, Karen has no information about which number will come up; she is in a state of uncertainty between six possible alternatives. She is reliably informed before the throw, however, that the die is loaded to come up with either a two or a five; she is, as a result, better informed, since she knows that there are now only two possible alternatives. If she were reliably told that the die was loaded to come up with a five, she would be in a state of knowledge of the outcome; there would be just one possibility. Moreover, the more alternatives that are eliminated, the more information Karen has; the correlation between the reduction in the number of possibilities and the increase in information is a graduated one. And, in this example, the various degrees of information can be precisely quantified using probability theory. At the limit, where only one possibility exists, the probability of rolling a five would be 1.

3.2 Probability Theories

The relation between reduction in uncertainty and increase in information has been widely noted. It stands behind the equation Popper draws between the fertility and falsifiability of scientific conjecture, for example;[38] and it is a central plank in the branch of engineering known as communication theory. In the latter, probability theory is used to calculate the amount of information conveyed by a given event or representation between a source and a receiver.

Dretske's well-known approach to information, which is motivated by communication theory, is also probability-based. Given an objective probability distribution across a system or network, one can measure the amount of information flowing from a source to a receiver, and the amounts of equivocation (information generated but not received) and noise (information available to the receiver not generated by the source). Dretske suggests that this fact (*how much* information flows) can be used to place a quantitative constraint on a semantic theory of information (*what* information flows). Adapting Dretske's formalisation:

(PR) given background knowledge *k*, a representation *r* carries the information that P if and only if the conditional probability that P given *r* is 1 (and less than 1 given *k* alone).

For Dretske, information is factive, and – as the rider shows – intrinsically informative; without the rider, someone with background knowledge would *already* know that P, antecedently to any representation that P. On this view, *r* could not then carry the information that P.

Dretske emphasizes that for *r* to convey information, it must raise the conditional probability that P to 1, rather than some lower number which might merely make P very likely. His worry is this: take the case in which a message or signal that P is being sent from one person to another in series. If the probability that P were less than 1, then a sufficient number of resendings of the message would bring the conditional probability that P on receipt of the final message close to zero. It would then be hard to understand in what sense *r* could thought to convey *information* that P at all. Another case might be that in which a message that P is sent to a large number of different people at once. If the conditional probability that P is less than 1, then in some cases *r* will not convey the information that P at all. Whether *r* conveyed information would depend on whom it was sent to. In Dretske's phrase, information is supposed to be subject to the "xerox principle"; it is preserved no matter how often it is sent or resent.[39]

Like other probability theories, however, this account has difficulty in accounting for what has traditionally been considered *a priori* information. Consider the state of human knowledge before and after the proof of Fermat's theorem that there is no whole number solution of the equation $x^n + y^n = z^n$ for any $n > 2$. As a result of the proof, something was added to the state of our knowledge, and that something was the information that this equation is true. But probability theorists are forced to say, typically, *either* that probability theory does not apply to areas of *a priori* knowledge such as mathematics and logic; *or* that the probability of theorems in these areas is 1. In the former case, the probability theory has been given up, and some further theory is required to account for this information. In the latter case, the background knowledge *k* would be enough to give the conditional probability of 1 to each theorem; so that nothing could carry the information that it was true.

Given the plausibility of the claim that *a priori* knowledge of mathematics is possible, this objection is a particular problem for Dretske; both because of the commitments described above, and because his broader project is to connect information closely with knowledge.[40] Someone without these commitments might straightforwardly deny that it was possible to have information as such about logical or mathematical truths at all. The question is often asked: how can the process of eliciting what is already contained in the axioms of a system, or the premisses of an argument, yield information, if that information is to be considered as in any sense objective, and not simply as reflecting the user's ignorance or psychological limitations?

This argument is a variant of the much-debated "problem of deduction". I shall simply note two points. First, what is being asked here is not "how can the conclusion of a deductive argument carry information as such?", but "how can it carry *new* information"? To equate the two questions is to make the same assumption as Dretske, that information must be informative. Secondly, there is a false antithesis in this formulation between objectivity and psychology. Something may be contingent or a fact about human understanding and still be objective: it may be an objective fact that a person could come to know that P as a result of being informed that P. In that case the proposition that P would be information.

I will not pause further on this question, however. What is important is simply to note that, as it stands, a Dretske-type probability theory of information cannot be of service in analysing the informational characteristics of logics such as EG or SL.

3.3 Possible Worlds Theories

Probability measures as such are not in fact required for a theory to retain the insight that information is related to reduction of uncertainty. This point is illustrated by theories, by those such as Stalnaker, which identify information with sets of possible worlds. The possible worlds account extends the general project of possible worlds semantics into the area of information. Adapting the earlier formulation:

(PW) given background knowledge *k*, *r* carries the information that P if and only if in all the possible worlds compatible with *k* which contain *r*, P is true (and there is at least one possible world compatible with *k* in which not-P is true).

Here again, it is assumed that information is factive and informative.[41]

Possible worlds theories start from the suggestion that certain sets of possibilities are consistent with certain information, and that other sets are not; and that which are and are not provides a (more or less clear) constraint on what information is conveyed. However, they also encounter difficulty in explaining *a priori* information. Recall that the purpose of the bracketed rider, here as in the earlier account, is to ensure that the representation is informative; otherwise someone with background knowledge would *already* know that P, in advance of having observed *r*. But this creates a problem: mathematical and logical truths are defined in possible worlds semantics as true in all possible worlds. Thus, if P is a mathematical or logical truth, then nothing can convey the information that P, according to (PW) above, since to do so requires that there be at least one possible world in which not-P. But if the bracketed rider is dropped, then any representation would carry the information that P, and *r* would carry any mathematical or logical information. In either case, the possible worlds account fails: in the first case, because "5 + 7 = 12" carries the information that 5 + 7 = 12; and in the second because "5 + 7 = 12" does not carry the information that Fermat's Last Theorem is true.

This is a case of the so-called "granularity problem": the problem that possible worlds seem too coarse-grained to make the fine distinctions needed for accounts of information, knowledge or belief-attitudes. It has been addressed in a variety of ways: for example, Stalnaker proposes to treat mathematical and logical propositions as propositions about the semantic structure of particular linguistic expressions, to which we can have more fine-grained propositional attitudes than it seems we can to sets of possible worlds. Others have proposed to allow that different impossible propositions correspond, not all to the empty set of possible worlds, but each to different sets of impossible worlds.[42] These approaches both have significant drawbacks, however – of clarity, motivation or explanatory power – and I shall not discuss them here. The problem remains a real one.

3.4 Inferential role theories

In both natural language and logical contexts, the elimination of alternatives is generally correlated with increase in information, as has been noted. Thus, if you reliably say that either Bill or Harry drinks tea, and I already know Harry does not, I am entitled to conclude that Bill drinks tea. Similarly, if I am told that Fido is a dog, given normal background knowledge I am entitled to conclude that Fido has four legs, is an animal, furry, not spherical etc. These inferences generate new information from certain givens, together with any background knowledge; and indeed inference has sometimes been characterised as the derivation of implicit information from explicit information.

These considerations are what motivate *inferential role* theories of information. Following Barwise and Seligman 1997:

(IR) given background knowledge *k*, *r* carries the information that P if and only if someone with *k* could legitimately infer P from *r* (but could not from *k* alone).

In contrast to the previous accounts, inferential role theories try to capture the common intuition that information is somehow relative, not merely to the recipient's background knowledge, but to his or her processing capacity and other beliefs. Moreover, the core idea of information here is a wider one: it is not factive. Under (IR), it would be perfectly acceptable to say that a newspaper carried the information that the President was dead, without the President in fact being dead.

A theory such as (IR) above lacks the technical apparatus of probability or possible worlds theories behind it, and is less ramified and precise as a result. But, unlike the other two theories, it does not suffer the obvious objection associated with the need to account for *a priori* information; and the flexibility and variety of possible inferences seems to hold the promise that a suitably fine-grained account can be given of informational content.[43] Much more needs to be said, however, about the notions of inference and background knowledge in play here.

In particular, it is unclear what is meant by, and what are intended to be the constraints on, "legitimate" inference. Clearly, not just any inference will do, for otherwise any inferred conclusion would be counted as information. In the first place, it would seem, the inference must be a sound one. Secondly, however, it must bear some (still to be identified) relation to the background assumptions and knowledge, *k*. Thus, if I am told that Jim is at home, I am – given background knowledge *k* – entitled to infer the information that he can answer the door; I am *not*, one wants to say, entitled to infer the information that either he can answer the door or the moon is made of cheese, or 2 + 2 = 4 etc., although the latter inferences are logically valid.

The distinction to be drawn here is not that between logical implication and logical inference as such. Implication is a relation between propositions or sentences, while inference is a mental act. As Harman has stressed, these are distinct, and follow distinct rules: the reasoner confronted with "P" and "if P then Q" need not infer "Q", but may decide to reject one of the premisses instead.[44] Rather, the distinction here is between rational inference and deductively valid inference. Rational inference does not license adding inferred conclusions freely to one's stock of information, simply because these can validly be deductively inferred.

But even a restriction to rational inference may not be enough. Take the case in which Bill comes into the room and says "there's a fire outside". One wants to say that at the level of content, his utterance normally carries the information that there's a fire outside; and at the pragmatic level, that (for example) we should leave. Both levels could, it seems, be captured by (IR), if suitably constrained. But it might also be rational for one to infer that Bill is nervous. Would it be right to say that *his utterance carries* (still less: *contains*) *this information*? Bill may well not, after all, intend us to infer that he is nervous. We might prefer to say that this is derived additional information; but it remains information which we can rationally infer from the representation *r* plus background knowledge. But then it becomes unclear what is to be the dividing line between "carried information" and "derived additional information".

The idea of background knowledge, *k*, must also be better defined. For example, say the observer O's background knowledge *k* includes: if 2 + 2 = 5 then I'm a Dutchman. (IR) suggests that a representation that 2 + 2 = 5, given *k*, carries the information to O that he is a Dutchman; which seems implausible. On the other hand, as the tea example above illustrates, there is a class of inferences from a representation plus background knowledge which do carry information indirectly. These considerations suggest that there is some constraint of epistemic relevance governing what background knowledge can be appealed to, to warrant information-carrying inference.

The bracketed rider is also problematic. If *k* includes number theory, for example, then the force of the rider is to make it impossible for *r* to carry any information at all about number theory; if *r* were, (for example) "2 + 2 = 4", then it could not carry the information that 2 + 2 = 4, for this could be legitimately inferred by O from *k*. The rider thus seems to impose too strong a standard; one wants to allow that *r* can carry information (in the sense of content) that P to observer O even when O already knows that P. But to drop the rider would mean that *r* added nothing to the information contained in *k*; it would not be informative.

As the earlier examples illustrate, the general worry regarding (IR) is whether the desired ideas of inference and background knowledge can be made precise in a non-question-begging way. However, this last point highlights a tension between the general motivations behind all three theories. Each is, in effect, seeking to account simultaneously for two aspects of the idea of conveying information:

1) What it is for a representation (signal, message etc.) to *carry information*. This is broadly captured in each case by the idea of *r*'s raising the conditional probability that P to 1,[45] P's being true in the possible worlds which contain *r*, or P's being inferrable from *r* in the "right" way.

2) What it is for a representation (signal, message etc.) to *be informative*. Each theory requires that the information that P carried by *r* exceeds the background knowledge *k*; *r* will not be informative in the required sense unless it provides information over and above what the observer or reasoner already knows. This is expressed in each case by the bracketed rider.

Given suitable amendments, such theories may be valuable to account for *a posteriori* information. But as regards the *a priori* information of logic or mathematics, seeking to fulfil both goals together creates a dilemma. Either *k* includes the information that P or it does not:

(i) If *k does not* include P, then *k* cannot include the rules and conventions of logic or maths. This is because on each theory, respectively, for *r* to carry information the conditional probability given *k* alone must be less than 1, there must be some possible world compatible with *k* in which not-P is true, or P must not be inferrable in the right way from *k* alone. In each case, if P is a truth of logic or maths, the theory in question will not explain what it is for *r* to carry information that P, and goal 1 will not be met.

(ii) If, on the other hand, *k does* include the information that P, then *r* will not be informative; *k* will be enough for the reasoner to obtain the information that P. The bracketed rider will be irrelevant, and goal 2 will not be met.

In a sense, this dilemma is irrelevant for the probability theory and the possible worlds theory; for, as argued above, these theories fail to account for *a priori* information, whichever horn of the dilemma is chosen. For the inferential role theory, matters are less clear. I have already mentioned the need to place suitable constraints on the ideas of legitimate inference and background knowledge in (IR); and these include constraints of rationality and epistemic relevance. These suggest a possible strategy for the inferential role theorist: to retain the idea that *k* includes the information that P, and then give an account of what it is to be informative in terms not of logical, but of epistemic, possibility.[46] On this approach, *r* will be informative to the extent, not that O *could* (in principle, given the laws of logic) but *can* (as a matter of fact, given O's actual knowledge) in the right way infer, from *r* that P. This move allows *k* to include the laws of logic and mathematics, and it seems to pinpoint the area in which the other two theories go wrong; that is, through the attempt to capture the idea of being informative just in terms of mathematical or logical possibility. But, of course, in the absence of a sharp, non-question-begging account of the "right" kind of inference, or of background information, the worry will remain as to the explanatory power of (IR).

For my purposes, however, the point is this: to ground a comparison of the informational characteristics of two logics, an inferential role theory of some sort would seem the best alternative of the three. Such a theory captures an intuitively important aspect of information, by relativizing it to the observer's own capabilities. But it faces, and must resolve, a tension to be found in theories of information more generally; between the demands of what it is, respectively, to carry information and to be informative, and it must do so in a way, not yet properly defined, which is genuinely explanatory.

3.5 Displaying vs. Containing Information

The theories sketched above have all followed normal practice by talking in terms of a representation as "carrying" or "conveying" information. But these latter considerations suggest that there is an equivocation in this "carrying" or "conveying" terminology. Rather, a distinction is needed between a representation's *containing* information and its *displaying* information, or alternatively and equivalently, between a representation's *explicit* and its *implicit* information.

I want to focus on the containing/displaying distinction here. But I will suggest that this provides a context within which the carrying information/being informative distinction can be embedded. "Carrying information" can be understood in terms of what information a representation contains. "Being informative" can be understood in terms of what information a representation displays, and whether this in fact adds to the information previously possessed by the observer.

The containing/displaying, or implicit/explicit, distinction is one evident in natural language. Adapting the schema used above, it can be formally expressed thus:

(C) given background knowledge *k*, *r contains* the information that P if and only if someone could, in the right way, infer P from *r* given *k*.

(D) given background knowledge *k*, *r displays* the information that P if and only if *r* contains P and someone could obtain P from *r* given *k* just by looking.

(Again, "just by looking" is intended to include information acquired without a conscious process of inference.) The rider has been dropped; if the observer already knows P, then – on the definitions above – this does not prevent *r*'s containing or displaying the information that P. (As I discuss below, a representation can be *made to display* information it contains.) The idea of being informative can then be expressed as follows:

(I) *r* is informative that P to observer O if and only if *r* displays P and P is not already known (or, more weakly, believed) by O.

Thus, according to (I), for *r* to display (and *a fortiori* contain) information that P is a necessary but non-sufficient condition on *r*'s being informative.

Of course, the constraints on inference and background knowledge in inferential role theories will also apply here. But it is notable that this approach is able to capture many of the different meanings of information distinguished at the outset of this chapter. Contained information may or may not be factive, and it may be implicit or explicit in a representation. And (I) and (D) can between them capture the main senses of "informative": sometimes "*r1* is more informative than *r2*" is taken to mean simply "*r1* displays more information than *r2*", while sometimes it means "*r1* displays more *new* information than *r2*"; and suitably amended, (I) can capture the further sense in which it means "*r1* makes O more certain that P than *r2* does". So this approach does not face the general dilemma of informativeness described above.

3.6 Information Theory and Diagrams

How does this account work for each of the three types of representation discussed earlier: descriptions, depictions and diagrams? Take the example of a detailed, depictive map of the United States, showing cities, states and counties. It should be clear that this map contains a good deal of information. Given background knowledge of the symbolism, conventions, labelling etc., someone could infer in the right way all and only this information from the map. But only part of this information may be displayed – may be obtained just by looking. Thus, the map may display the information that LA is west of New York, but not the information that, for example, there are 3,056 counties in the continental US. If it is to be accurate, deriving this latter information will normally require a process of counting. Finally, the map may display information and not be informative. Someone specialising in geography or demography may come to it with a formidable array of existing relevant knowledge; the map will not be informative unless it adds some new information to this stock of knowledge.

This example can of course be readily extended to the case of a sketch map diagram. The London Underground map may, for example, contain but not display the information that there are 317 stations in the system; while it may display the information that there are fewer stations South than North of the Thames; and this information may or may not be informative.

Descriptions can also be treated in these terms, with the caveat that – for reasons sketched in Chapter 2 – they display relatively little information. There is only a thin sense in which the natural language sentences of a description *display* information, though given words or word-strings may do so. They do not, for example, bear any structural similarity or visual resemblance to the objects they represent. But they very evidently *contain* information; and it is this which allows the reader or user to make inferences. (In doing so, the user will doubtless employ rules or other background knowledge in *k*. But it is not necessary for him or her to be able to give an account of what process of inference is going on, or what rules are included in *k*, or why some combinations of words can convey information and others not; this does not affect his or her entitlement, in Burge's sense, to derive the given information.) The distinction between displaying and containing information thus remains quite intelligible.

This example is interesting for another reason. It illustrates how "conveying" (or "carrying") and "containing" information can come to be equated with each other if attention is focused solely on descriptions. In descriptions there is little displayed information available; so the displayed/contained distinction does not generally do much work. The opposite is true in the case of diagrams or depictions; and this motivates the need for a finer-grained terminology than that of "conveying" or "carrying" information.

It is, as has been seen, an attractive feature of inferential role theories that they relativise information to people's ability to absorb and process sensory inputs in a given context. Thus it is to be expected that what information is to be regarded as displayed and what as merely contained by a representation will differ from person to person, as well as being dependent on environment, lighting conditions etc. But, though this may mean the line must in some cases be drawn differently for different observers or reasoners, it does not threaten the displayed/contained distinction. At some level in any representation of even moderate complexity there will be a piece of information which is properly inferrable but indisputably beyond immediate observation (e.g. "the number of counties west of the Mississippi is 1,071", or "the number of consonants in sentence Y less the number of vowels is a prime number").

This account can now be extended to an explanation of how diagrams can misinform. Misinformation occurs when a diagram *d* displays information that P, but P is not in fact the case. As we saw earlier, diagrams are generally able to convey information at two levels: iconically, in virtue of a structural similarity or homomorphic relation between their own elements and elements of their ranges; and symbolically, in virtue of "endowed" symbols which carry information in virtue not of structural similarity but of conventionally established meanings. At the iconic level, misinformation will typically occur when the representation relation R between diagram *d* and range Δ does not hold for the features of *d* to which O is attending; the office floorplan indicates, say, that there is a store room in a given location where in fact there is not a store room. At the symbolic level, it will occur when the conventional relations in R between a symbol in *d* and the objects or properties it symbolises in Δ are carried over into cases where in fact they do not apply; as when a bar is given the wrong shading in a bar chart. In both cases the structural correspondence or conventional meaning is generally preserved elsewhere; this is why the representation can be regarded as a diagram at all, rather than simply as a confused collection of marks. For misrepresentation to occur thus requires a broader context of successful representation.

3.7 Perspicuousness Reconsidered

The display/contain, or implicit/explicit, distinction is quite intuitive. In particular, it is notable how many parallel distinctions have been made in the literatures of cognitive science, of epistemology, of logic and of diagrams.[47]

Contained information and displayed information are not, of course, mutually exclusive. Rather, displayed information is a sub-class of contained information: specifically, it is that portion of the total information content which can be derived from a given representation just by looking. A representation can contain information which is not displayed. But it cannot display information which it does not contain.

Moreover, contained information can be transformed into displayed information. The phrase "just by looking" includes both information derived at a glance, and information derived through more extensive observation (though not consciously inferred). However, the reasoner can *make the implicit explicit* by following certain procedures. For example, in order to make a given map display the information that there are 1,071 counties west of the Mississippi, the reasoner will need to count them. Once this has been done, this information can be memorised; if it is memorised then the reasoner may come to associate the information with that specific representation, or type of representation. When a similar example is given, it may be possible to derive the information once again just by looking;[48] the information may now be *displayed* by the representation. Another procedure for making the implicit explicit, this time in natural language, is to define the key words in a description.

Let us return now to the idea of perspicuousness. In Chapter 2 this was defined in terms of discretion and assimilability. On this view, *r* was a perspicuous representation that P to the extent that it did not convey more information than was needed to convey that P, and that it was easily assimilable by O. It is now possible to situate this view within a more general (inferential-role) theory of information: we can now say that *a representation is perspicuous to the extent that it contains only relevant information, and that it displays the information which it contains.*

This analysis can now be applied to the case under consideration: comparison of the informational characteristics of a sentential and a diagrammatic representation of a given range. In the examples to be considered, both representations will contain the same information, and there will be no extraneous information. The discretion of both representations will therefore be 1. The claim is this: while the two representations may *contain* the same information, they *display* different information, and different amounts of information. Thus one may be more perspicuous than the other. To the extent that one displays information which is new to the reasoner, it will be informative. Thus – to anticipate the answer to one of the questions raised at the end of Chapter 1 – two representations can in principle contain the same information and be differently perspicuous and differently informative.

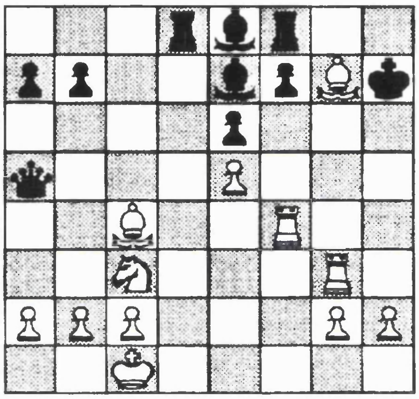
3.8 The Chess Example

Chapter 5 will apply these ideas to sentential and diagrammatic logics. An interesting preliminary case, however, is that of chess. There are both sentential and diagrammatic methods of representing all phases of a game of chess. Figure 3.8.1 sets out a typical point in a game, using both methods.

Figure 3.8.1(a) sentential notation

2WK5;WPWPWP3WPWP;2WKT3WR1;2WB2WR2;BQ3WP3;4BP3;BPBP2BBBPWBBK;
3BRBBBR2

Figure 3.8.1(b) diagram



The notation in Fig. 3.8.1(a) gives the positions of the pieces reading from left to right, going up the board rank by rank from White's perspective; empty squares are denoted by numbers, and occupied squares by the value of the piece which fills them (knights are denoted by KT to differentiate them from kings). Thus the first clause "2WK5" is interpreted as "two empty squares, then a white king, then five empty squares" and so on.

Since both representations convey information about the same position,[49] this is clearly a case in which they represent the same object or state of affairs. Both representations bear an isomorphic relation to the range. The rules of chess can be precisely formulated and apply to reasoning about the position represented in either type of representation: they constitute the background knowledge *k*. Both representations thus contain the same information; given *k*, the observer O could infer, in the right way, the same information from either representation.

However, it should be clear that they display quite different information, and/or amounts of information. The sentential version makes explicit which pieces are involved, but it does not display the array of black and white squares representing the board; this is something which is left implicit in the background knowledge *k*. Moreover, the position of the pieces is implicit; a process of inference would be required to place them on a board, for example. The diagrammatic version, however, bears an iconic relation to the board; one can infer from the isomorphism between diagram and domain that the White Knight can be taken by the Black Queen, for example, just by looking at the diagram. The relevant information – which moves are possible for each piece – is explicitly represented. Thus, certain moves can be observed to be impossible: for example, the move 1 … Q-K8, which would give check for Black if it were allowed under the rules.

Sentential and diagrammatic representations of chess therefore have different informational characteristics. These make them suitable for different functions. The sentential version can be useful as a means of representing a position or a game given limited space or notational resources; in a correspondence game, perhaps. But the diagrammatic version is significantly more perspicuous – it displays more of the information it contains – and so significantly better suited for reasoning.

In the next two chapters, I want to suggest that this example is analogous to the relation between various forms of sentential logic (SL) and the existential graphs (EG). Each system contains the same logical information as the other, but they display different information, and amounts of information. These characteristics make the two systems suitable for different functions; again, the diagrammatic version is more perspicuous, and so better suited for reasoning.

4: The Existential Graphs

I now want to apply the general analysis developed so far to the specific case of diagrammatic and sentential logics. This chapter, therefore, briefly sets out the syntax and semantics of the alpha existential graphs; chapters 5 and 6 analyse the graphs as a diagrammatic system, describe their informational resources, and compare these resources with those of different forms of sentential logic in the context of proofs.

The existential graphs are broadly divided into three: the alpha graphs (logically equivalent to propositional logic); the beta graphs (equivalent to first order predicate logic with identity); and the gamma graphs, which remained unfinished at Peirce's death but which broadly correspond to second (and higher) order logic and to modal logic.[50] Only the first part, concerning the alpha graphs, is described here. The discussion broadly follows Roberts 1973, with some amendments.[51]

4.1. Peirce and the Role of Logic

By way of background, it is important from the outset to bear in mind what the motivation is behind the graphs. The details of Peirce's view of logic are complex and much debated;[52] but a sketch will suffice for purposes of this discussion. Peirce construes logic very broadly, to include not merely deduction, but also what he considers the two forms of ampliative inference, induction and abduction (or hypothesis). Overall, logic is seen as intrinsically normative, in this sense: it holds certain patterns of reasoning up to a standard and examines them critically. Thus the logician seeks, not to describe existing patterns of reasoning (which Peirce considered to be psychologism), nor to prescribe such patterns (as might a preacher); but to describe the norms or standards implicit in valid reasoning, not merely as regards its validity, but also as regards its fertility, reliability and economy.[53] And these norms themselves reflect Peirce's underlying conception of the critical reasoner as a self-conscious and self-controlled shaper of inquiry towards truth.

Within this broad account, Peirce sometimes refers to deductive logic as "the science <u>of</u>

drawing necessary conclusions". This formulation is designed (rather confusingly to modern ears) to contrast with Peirce's view of mathematics, which he defined, following his father's usage, as "the science which draws necessary conclusions". But the underlying point is fairly clear. In Peirce's view, working mathematicians do not seriously doubt the vast majority of the proofs of mathematics, once these have been critically examined. The mathematician typically focuses, not on the process of reasoning, but on the results of adopting a given set of hypotheses. The position of the logician is quite different, however; he or she is self-consciously seeking to explicate and understand patterns of reasoning, and this requires him or her to go as slowly and carefully as is required to develop a precise grasp of these patterns, and to ensure the validity of their constituent inferences. It follows that for Peirce the task of formal logic, as a tool of logical analysis, is "to enable us to separate reasoning into its smallest steps so that each one may be examined by itself."[54]

In this respect, logic for Peirce is not what he termed a calculus: it is not a mechanism for drawing inferences or an aid to computation.[55] The purpose of a calculus, as Peirce saw it, was to abridge the process of inference in order to facilitate it, to reduce to the bare minimum the number of steps in a deduction between the premisses and the conclusion. A calculus is, for Peirce, a specific tool aimed at a specific problem or set of problems. A logic, by contrast, is an entirely general means by which the process of reasoning itself may be examined. It should be designed, in Peirce's view, to make the process of reasoning maximally explicit. As Peirce says (Ms 499): "But to say that the aim was to make the algebra as analytic of reasonings as possible is to say that the aim was to make every demonstration as long as it possibly could be made without being circuitous". He sums up the logic/calculus distinction nicely in a lecture of 1903: "The system of existential graphs is not at all intended for an aid in reasoning, but only for an aid in the *analysis* of reasonings".[56]

Peirce's conception of logic, thus, is as an aid in the analysis of reasoning. That is, he is *not* motivated by two of the leading preoccupations of 20th century philosophers of logic and language. He does not see logic as required to provide a foundation for mathematics, and he does not see logic as a means to analyse the deep structure of natural language. (This is not to say that his conception of logic cannot shed light on the logicist or natural language programmes, only that to do so is not part of its original motivation.)[57] Differently motivated, proponents of the logicist and natural language programmes seem (after Frege) to have had a clear preference for logics which were sentential in representational form. (And, of course, the clear inspiration for the symbolism of the quantifier was in fact the sentential notation developed by Peirce and his student Mitchell in the early 1880s.[58] This was adopted first by Schroder, later by Peano (with amendments, in the *Notations de Logique Mathematique* of

1894) and, later and under Peano's influence, by Whitehead and Russell (*Principia Mathematica*, 1910). The two-dimensional notation of the *Begriffsschrift*, by contrast, is to some modern eyes not sentential at all, but diagrammatic.[59] Frege regarded his notation highly, comparing it to a microscope in its ability to make logical structure precise and explicit, and some modern commentators have remarked on its (distinctively diagrammatic) virtues of immediacy and perspicuousness.[60] However, as mentioned in Chapter 1, it was ignored by others.) Thus, it appears that while the pioneers of quantification preferred diagrammatic notations for the sake of clarity, the new logic came to be assimilated to algebra, and perhaps to natural language, by mathematicians trained to be sceptical of diagrams as such.

This, however, is merely a conjecture. In the parallel case, it is clear that Peirce's view of logic led directly to his choice of representational form. Since he is seeking to render explicit what is implicit in logical reasoning, the goal of the existential graphs is above all to be perspicuous. In terms of syntax, they should be gap-free; in terms of representational form, they should be – as far as possible – iconic. A sentential form is, one might think, predestined to be inadequate to his purpose. As we shall see, there is a quite clear sense in which the system of existential graphs exploits its diagrammatic form to achieve this perspicuousness; what is important at this point, however, is to approach the graphs with the guiding recognition that they are differently motivated from logics within the dominant, sentential, tradition.

4.2 Syntax

The smallest steps into which a piece of logical reasoning may be separated are insertions and omissions; or, in modern terminology, introductions and eliminations. It is a primary function of the existential graphs (hereafter EG) as a logical system, therefore, to make such introductions and eliminations absolutely transparent and explicit.

There are three primitive types of symbol in the alpha graphs:

S1. The blank area on which the graphs are to be drawn.[61] This is known as the "sheet of assertion", or SA. A graph may be written anywhere on it.

S2. Propositional letters (e.g. "P", "Q", "R" …).

S3. A continuous, closed and non self-intersecting line, known as a "cut". This can be of any shape, but is normally drawn as a circle or oval.

Graphs can then be defined as follows:

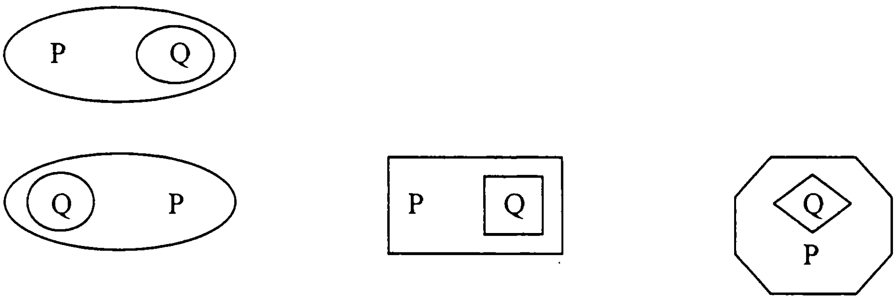
G1. Any part of the blank SA is a graph. A propositional letter is a graph.

G2. If Γ and Δ are graphs, then the juxtaposition of Γ and Δ is a graph.

G3. If Γ is a graph, then the enclosure consisting of a single cut with Γ alone inscribed within it is a graph.

Token-identity can be defined in terms of the construction history of the graphs.[62] Two graphs are tokens of the same type if and only if they can be constructed from sentence letters by applying the same operations of juxtaposition and enclosure in the same order. Thus, the four graphs below are all tokens of the same graph type.

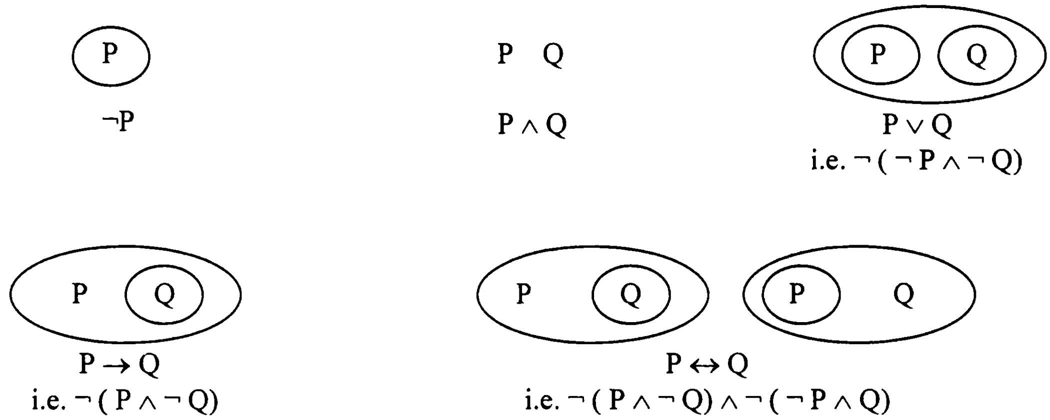
Fig. 4.2.1



4.3 Semantics

The normal semantics for the alpha graphs is as follows: to write a graph on SA is to assert it to be true; propositional letters stand for propositions; and to enclose a graph in a cut is to negate it. Thus there are, in effect, two logical operations in EG: negation (enclosing a graph in a cut), and conjunction (simultaneous juxtaposition of two or more graphs on SA). As the analogy with SL suggests, EG is expressively adequate; that is, it can express any truth-function whatever using just these two operators. Thus the principal truth functors in five-functor SL (hereafter 5SL) and two-functor (conjunction/negation) SL (2SL) can be expressed in EG as follows:

Fig. 4.3.1



As these examples indicate, graphs can be translated into formulas of SL by working from the outside inwards ("endoporeutically", in Peirce's terminology), representing cuts by the negation sign and the areas between two or more propositional letters, or between a propositional letter and a cut, by the conjunction sign. There is no need in EG, however, for symbolism or rules to govern grouping (that is, bracketing or rebracketing) of expressions, as in the 2SL formulas above. Rather, the cut symbol precisely defines the scope of the negation by what it encloses. Thus, in the case of the graph for material implication the subgraphs

"P" and ""

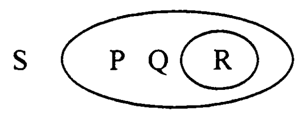
fall within the scope of the cut, as they fall within the brackets governed by the negation sign in the 2SL translation sentence. Peirce calls this feature, of one cut enclosing another, the "nesting" of cuts. There is no limit in principle to the amount of nesting which can take place in a graph.

As will be seen below, application of a rule of inference can result in an empty cut; that is, a cut enclosing no propositional letter. An empty cut is interpreted, not as the negation of a proposition (since it encloses no propositional letter), but as the negation of the possibility of a proposition. It is known as the pseudograph, and is treated by Peirce as a proposition which is always false. It is thus equivalent in SL to the zero-place truth-functor "$\bot$", the constant false proposition; and its negation, the empty double cut, is equivalent to the truth-functor "$\top$", the constant true proposition.

4.4 Rules of Inference

A graph of anything other than a simple assertion or negation will feature more than one cut, and therefore a degree of nesting. In EG this degree is formalised by the idea of levels within a graph. The SA is taken to be an even level, and, moving inwards, every cut signifies a change of level, alternating between even and odd. An area is said to be "oddly enclosed" if it is surrounded by an odd number of cuts, and "evenly enclosed" if it is surrounded by an even number of cuts, or by no cuts at all.

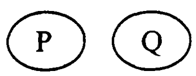
Fig. 4.4.1



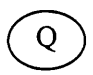
Thus in fig. 4.4.1 – which could be translated in 5SL as $S \land \neg((P \land Q) \land \neg R)$ or $S \land ((P \land Q) \rightarrow R)$ – "S" is on an even level, "P" and "Q" are on an odd level, and "R" is on an even level.

The inference rules for EG can now be stated. As with all logics the purpose of EG is to enable the reasoner to move invariably from true premisses to true conclusions merely by application of the permitted inferences. The inference rules consist of five permitted operations by which one graph may be manipulated into another in the certainty that the relevant operation is truth-preserving. These are stated below, together with examples and analogous logical principles in SL. (Since we will be dealing with interpreted graphs henceforth, I give the SL translations in semantic terms; but analogous syntactic versions could easily be given.)

R1. Erasure (ER): Any evenly enclosed graph may be erased. Thus from

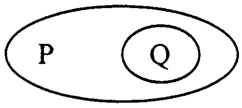


it is permissible to infer, given that SA is an even level

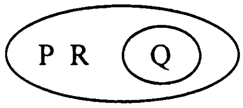


This is equivalent to the principle in SL that (using Greek letters to denote schematic form) $\varphi \land \psi \models \varphi$.

R2. Insertion (IN): Any graph may be written on any oddly-enclosed level. Thus from

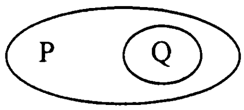


it is permissible to infer

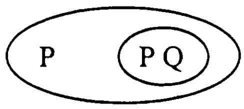


This is equivalent to the principle in SL that $\varphi \rightarrow \psi \models (\varphi \wedge \chi) \rightarrow \psi$. Further propositional letters are merely being added to the antecedent of a conditional proposition. It is clear that this cannot affect the truth value of the whole proposition.

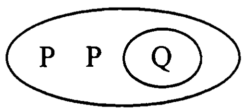
R3. Iteration (IT): Any graph which occurs in a given area A may be inserted in the same area, or in an area nested in A. Thus from



it is permissible to infer, for example,



or



This is analogous to the principles in SL that $\varphi \rightarrow \psi \models \varphi \rightarrow (\varphi \wedge \psi)$ and $\varphi \rightarrow \psi \models (\varphi \wedge \varphi) \rightarrow \psi$. Iterating the antecedent of a conditional, or adding the antecedent to the consequent, cannot affect the truth value of the whole proposition.

R4. Deiteration (DE): Any graph which might have been derived by iteration may be erased. If graph G is identical to another graph within area A (but not within any cut in A), or within an area in which A is nested, G may be erased.[63] Thus the examples in R3 above may be reversed.

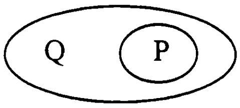
R5. Double Cut (DC): Two cuts together with nothing between them may be removed from or added to any graph at any level. This is similar to the rules of double negation in SL.

4.5 Proofs

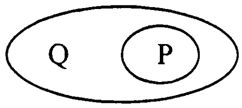
Two relatively straightforward examples should make clear at this stage broadly how proofs can be established in EG. The entailments are given in SL first for ease of checking.[64]

Proof 4.5.1: $P \models Q \rightarrow P$

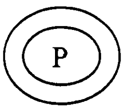
This translates into EG as follows:

$P \models$ 

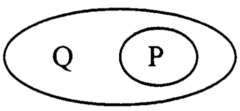
The proof is accomplished when the premiss (on the left) has been transformed into the conclusion (on the right) by applying the rules of inference. The rule invoked at each stage is stated in the right-hand column, together with the line of the graph being transformed.

Proof 4.5.1: $P \models$ 

1. P — Premiss

2. 

1, DC insertion

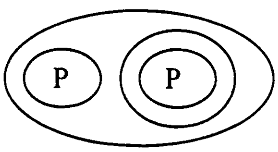
3. 

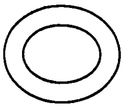
2, IN of Q.

As a second example, I prove a tautology. Since this is true on any possible interpretation of the propositional letters (in SL, there is nothing on the left of the turnstile), the proof must start from an empty SA.

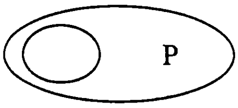
Proof 4.5.2: $\models P \vee \neg P$

This translates into EG as follows:

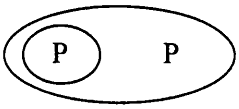
Proof 4.5.2: $\models$ 

1. 

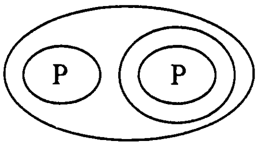
DC insertion

2. 

1, IN of P

3. 

2, IT of P

4. 

3, DC insertion.

This proof exemplifies two further points. First, since the SA is an even level, it is not permissible in EG to write any graph on it except a double cut; one or more propositional letters can then be written on the odd level of the graph, allowing the proof to start in earnest. This is the only permissible way to begin a proof starting from a blank SA. Secondly, however, it may not be clear at first sight how the graph in line two is to be read, and therefore whether the proof at that stage is truth-preserving. Recall that an empty cut is called the pseudograph, and always takes the value false. Then the graph in line two may be translated into SL as $\neg(\perp \wedge P)$. And this is true regardless of the value of P.

This chapter has briefly surveyed the syntax and semantics of the alpha existential graphs, and located them within Peirce's overall conception of logic. We now turn to examine more closely the informational resources of EG versus those of SL.

5: EG and Diagrammatic Information

The last chapter briefly set out the syntax and semantics of EG. I turn now to examine the informational characteristics of EG as a diagrammatic logic. For this purpose, it will be convenient to compare it to both five-functor ($\neg$, $\rightarrow$, $\wedge$, $\vee$, $\leftrightarrow$) and two-functor (in this case: $\neg$, $\wedge$) sentential logic, or 5SL and 2SL respectively.

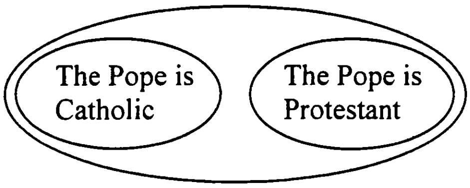
5.1 EG as a Diagrammatic Logic

Are graphs in EG diagrams? It certainly seems that the representational form of EG is quite different to that of sentential logics. Conjunction apart, there is little of the visual similarity to sentences of natural language which motivates 5SL, for example. Consider:

1. The Pope is Catholic or the Pope is Protestant
2. If it rains, then I will get wet
3. Fido is a cordate if and only if he has a heart

To varying degrees, these sentences bear a certain visual similarity to their translation sentences in 5SL. (This can be brought out by translating them into "loglish" or "logicese"; replacing the key phrase by a logical symbol leaves a sentence-like representation.) And in both 5SL and natural language the sentences must be scanned in a certain order in order to derive the requisite logical content.[65] But compare (1) above with its EG translation:

Fig. 5.1.1



As the token-identity definition for EG makes clear, there is nothing intrinsically linear about this graph; what counts are, broadly speaking, the "topological" relations amongst its elements (cf. fig. 4.2.1 above). Moreover, it is not necessary to read (or to be able to read) the graph from left to right: it may be scanned (outside-in; though there are other ways of seeing the graphs, to be discussed below) from any direction; or it may be seen whole, at a glance. Indeed, graphs in EG are generally non-sequential and (except in the simplest case) two dimensional,[66] containing cuts – enclosing lines (oval or circular) of a type normally associated with diagrams.

So, on a first view, the graphs appear to be diagrammatic. Can this claim be made more precise? One might try to do so by noting that the interpreted graphs generally bear a homomorphic relation to their ranges:

1. Basic elements in the range D – atomic propositions – are represented by basic elements of a diagram Δ – propositional letters – with different types of range-element represented by different types of diagram-element. The negation of a proposition in D is represented by the cut in Δ.

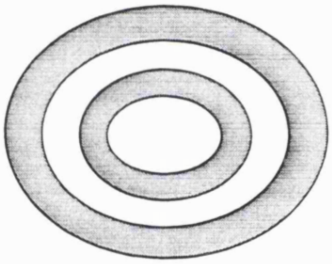
2. There are two basic relations in EG between elements of Δ: the one-place relation R of "lying inside a cut", which corresponds to the one-place relation S in D of "being negated"; and the two-place relation R* of co-existing in a given area with no intervening cut, which corresponds to the relation S* in D of conjunction.

3. Complex elements in D – complex propositions – are represented by complex elements of Δ – propositional letters and cuts – with different types of range-element represented by different types of diagram-element. Thus, the relation of "φ or ψ" is given in EG for any φ, ψ by the binocular-like pattern illustrated above. It is straightforward to see how a similar analysis can be given for implication and the biconditional in D.

But of course homomorphism alone is not enough: it is a non-sufficient condition on a representation's being a diagram; depictions are also generally homomorphic, as are sentences of SL. Instead, I suggested in Chapter 2 that we could better differentiate diagrams in terms of two ideas: discretion and assimilability (or ease of processing). It should be clear that graphs in EG are discreet: unnecessary iteration apart, as logical representations they do not convey more information than is necessary to convey the desired information. And they are also highly assimilable. Their representational form is exceptionally simple. There is *but one* basic visually salient relation: that of whether a given graph or set of graphs lies inside or outside a cut. It is typically (under normal conditions, for well-formed graphs of appropriate size) immediately evident on looking at a graph G whether it has any cuts, and whether any propositional letters lie inside or outside those cuts. (Of course, much of this immediacy will be lost for very complex propositional representations of either EG or SL.) The analysis of Chapter 2 thus supports the intuition that the graphs are diagrams.

The visually simple relation described above also, interestingly, defines the scope of the rules of inference of EG.[67] First, the scope of the rule of iteration (IT) is defined for a graph G as the area within which G lies, and any cuts nested in that area; the scope of deiteration (DE) is defined by extension. Secondly, the odd and even levels of a graph constitute, in effect, a concentric set of rings. This is brought out by the shading in Fig. 5.1.2 below, where the shaded areas are odd levels and the light areas are even. (And, as Peirce suggests, there is nothing to prevent the user generally writing graphs in EG in this shaded form, or indeed otherwise annotating or labelling them, if desired.)

Fig. 5.1.2



The rule of insertion (IN) permits any graph to be inscribed, in effect, on any shaded area; and the rule of erasure (ER) permits any graph to be erased from any unshaded area. Thus the scope of all four rules is given by the basic visually salient relation of lying inside or outside a cut.[68]

5.2 Linear EG and 2SL

It is an interesting feature of EG that it has an alternative, linear, representational form in which cuts are represented by pairs of brackets. In linear EG the translation of the 5SL truth-functors is as follows:

<u>5SL</u>	<u>Linear EG</u>
$\neg P$	(P)
$P \wedge Q$	PQ
$P \vee Q$	((P)(Q))
$P \rightarrow Q$	(P(Q))
$P \leftrightarrow Q$	(P(Q))((P)Q)

Peirce invented linear EG as a convenience for typesetting purposes.[69] However, he believed – and it has been noted by others[70] – that it is less intuitive than the two-dimensional version. Why should this be? And are graphs in linear EG sentences or diagrams? We can now say the following: since EG and linear EG are evidently isomorphic to each other, both will be homomorphic to their range, as described above. The fact of homomorphism, though it makes both valuable in reasoning, does not in itself distinguish between them. So too for discretion. But the two can be clearly distinguished in terms of assimilability. The linear notation does not contain the cut as such, but replicates it using brackets. It is much less evident in linear EG how far a graph is nested within another, and on what level a given graph stands; the brackets do not literally enclose areas within which insertions or eliminations may be made, but are packed together like letters in a word, as though the intervening area had no logical value. Moreover, they must be paired off and counted to ensure a graph of any complexity is well-formed. Finally, the linear format makes it less clear whether or not two graphs are tokens of the same type. Given that discretion is equal, "normal" EG's greater assimilability makes it more perspicuous than linear EG. And this fits with a pre-theoretical intuition that the latter is sentential and the former diagrammatic.

I have already noted the syntactic similarity between EG and 2SL. As in the case of linear EG above, it should be clear that the information displayed by formulas in 2SL is generally less assimilable than that displayed in the corresponding graphs in EG. In the first place, 2SL requires extensive use of bracketing in order to capture the scope of the negation operator. Again, such brackets do not merely add to the length of formulas; they also impede their intelligibility by forcing the reasoner to pair them off in order to check well-formedness, or that the correct scope restrictions have been observed. Bracketing is, then, a significant impediment to perspicuousness of representational form.[71] Secondly, there are a number of relevant basic visual relations for propositional letters in formulas of 2SL: lying inside or outside a pair of brackets, being preceded by a negation sign, and being preceded by the conjunction sign, for example. By contrast, as I have emphasized, there is but one basic visually salient relation in EG, that of lying inside or outside a cut. Thirdly, it seems plausible to suggest that, because they are simple and visually salient groupings, the graphs are easier for the mind to assimilate. There is evidence in the psychological literature that the mind can assimilate no more than four "chunks" of visual information at a time. The groupings formed by the graphs are identifiable units which can be seen as chunks, while it is not clear how formulas of 2SL, composed as they are of strings of symbols no one of which is more visually salient than another, can be grouped into chunks for purposes of reasoning.[72]

5.3 Displayed Information and Transparency Properties

We can now make use of the distinction developed in Chapter 3 between displayed and contained information. An interpreted propositional representation in a given logical system *contains* the information that P if and only if given certain background knowledge *k*, someone could infer, in the right way, the information that P from it; and it *displays* the information that P if and only if it contains P, and someone could obtain P from *r* given *k* just by looking. Thus two representations will contain the same information just in case someone could infer in the right way all the same information from each representation; they will be logically equivalent. But this is not to say that they display the same information, for in fact it may be possible to obtain the information that P from one just by looking, but not from the other. What information can be obtained just by looking, without a conscious process of inference, is a matter of the capabilities of the observer, of the specific marks on the representation, and of how perspicuous the representational system is overall.

For example, take the following two formulas in 5SL:

5.1. $\neg P \rightarrow ((P \vee Q) \leftrightarrow Q)$
5.2. $P \rightarrow (Q \rightarrow (P \wedge Q))$

Both are tautologies; they are therefore logically equivalent. So they contain the same information. But they do not display the same information; one cannot (normally) obtain the same information from them just by looking. It would be possible, for example, for someone to come to know (1) without thereby knowing (2). Indeed, if 5.1 and 5.2 did not display different information, then there could be nothing informative about the statement that the two were logically equivalent. By extension, such well-established truths of logic as de Morgan's laws would, on this view, have to be regarded as uninformative, instead of being the useful shortcuts they are.

As has been noted, the basic logical operators of EG are those of conjunction and negation.

That is, EG formalises compound propositions by analysing them into atomic components which are conjunctively asserted or denied subject to certain scope restrictions. This gives EG a syntactic similarity to 2SL; and graphs in EG can be translated into 2SL as a result, as has been noted.

Now EG and 2SL have an interesting property, which 5SL largely lacks. Call it *transparency of equivalence*.

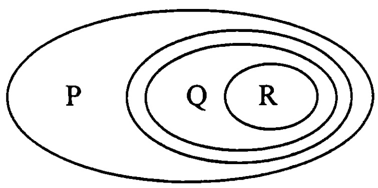
(TEQ) Two propositional representations in logic L are transparently equivalent if and only if they are logically equivalent and can be seen to be so by someone aware of the semantics of L without the conscious application of any rules of inference; otherwise they are opaquely equivalent, if equivalent at all.[73]

Some pairs of propositional representations (e.g., P and $\neg\neg P$) are transparently equivalent in all of 5SL, 2SL and EG, since the reasoner can obtain each from the other without consciously applying any rule of inference (and these are classical logics, with the requisite rules of double negation). But there is a large class of propositions whose representations in 5SL are opaquely equivalent, but can be transparently equivalent in EG and 2SL. For example, take the formulas in 5SL below:

S1. $P \rightarrow (Q \rightarrow R)$
S2. $P \rightarrow \neg(Q \wedge \neg R)$
S3. $\neg(P \wedge \neg(Q \rightarrow R))$
S4. $\neg(P \wedge \neg(\neg(Q \wedge \neg R)))$

S1-S4 are all logically equivalent to each other. But they can all be represented by just one graph in EG:

Fig. 5.3.1



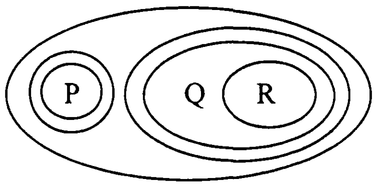
and one formula in 2SL: $\neg(P \wedge \neg(\neg(Q \wedge \neg R)))$ – i.e. S4 above. In the case of any pair of S1-S4 in 5SL, one or more rules of inference will normally have to be consciously applied for the two formulas to be seen to be equivalent. In the case of 2SL and EG, however, the inference is immediate: each side will be a token of the same representation type.

Moreover, take the following sentences in 5SL:

S5. $\neg P \vee (Q \rightarrow R)$
S6. $\neg P \vee (\neg Q \vee R)$
S7. $\neg P \vee \neg(Q \wedge \neg R)$

S5-S7 are all logically equivalent to each other. Again, they can all be represented by just one graph in EG:

Fig. 5.3.2



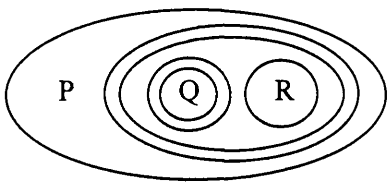
and one formula in 2SL: $\neg(\neg\neg P \wedge \neg(\neg(Q \wedge \neg R)))$.

Finally, take:

S8. $P \rightarrow (\neg Q \vee R)$
S9. $\neg(P \wedge \neg(\neg Q \vee R))$

S8-S9 are logically equivalent to each other. Again, they can be represented by just one graph in EG:

Fig. 5.3.3



and one formula in 2SL: $\neg(P \wedge \neg(\neg(\neg\neg Q \wedge \neg R)))$.

But of course, the three graphs above are transparently equivalent; one can see that they are identical, double cuts notwithstanding, at a glance. (And it might be argued – though with more difficulty, I think – that the same is true of the respective 2SL formulas.) But this is a piece of *new information*: even if a user of 5SL knew that any two formulas of S1-S4, or S5-

S7, or S8-S9 were logically equivalent, he or she might not know that S2 and S7, S5 and S9, or any other two formulas of S1-S9, were logically equivalent to each other.

TEQ is not a universal feature of any logic; if it were, then all tautologies in the given logic would be transparently equivalent. As the double negation example shows, many (perhaps all) logics have some transparency of equivalence; for those where the laws of double negation are not permitted, there are likely to be other cases of transparent equivalence.

Moreover, one can note a more fundamental property which might be called *transparency of entailment*.

(TEN) For any two propositional representations φ, ψ in logic L: φ transparently entails ψ if and only if φ entails ψ and can be seen to do so by someone aware of the semantics of L without the conscious application of any rules of inference; otherwise φ opaquely entails ψ, if it entails ψ at all.

Clearly, transparency of equivalence can be defined in terms of transparency of entailment.

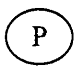
TEQ and TEN are valuable properties in a given logic, for they greatly assist the process of proof. Take the following formula, formed from S3 and S8 above.

5.3 $\neg(P \wedge \neg(Q \rightarrow R)) \models P \rightarrow (\neg Q \vee R)$

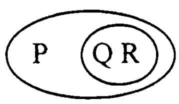
Someone could be unaware that this is a valid entailment, and come to know it as a result of a proof, thereby learning something new about logic. The proof would likely involve the application of a series of rules of inference. For the inexperienced reasoner, this will involve some conscious effort. For the experienced logician, however, the proof may well be almost automatic. Indeed, he or she may just "see" that the left hand formula entails the right without a conscious awareness of applying any rules. Experience with 5SL may enable such a reasoner to infer directly, without an application of any rules of inference of which the reasoner is then aware, that this entailment holds. However, it seems unlikely that such a reasoner is following no rule at all; rather, he or she appears to be unconsciously following a meta-rule to the effect that formulas of representational type A logically entail formulas of representational type B.

This question does not arise in the case of the equivalent proofs in EG and 2SL. These simply require the reasoner to write down the translation representations of the two formulas in 5.3, and then observe that both sides of the turnstile are in fact identical under the token identity criteria of each. No rules are applied; there is no question of *proving* the EG entailment. Thus 5.3 is a member of a (very large, it would appear) class of formulas whose proofs are motivated simply by their opacity of entailment in 5SL. Such proofs are unnecessary in EG and 2SL.

There are two further points to note. First, a graph in EG may transparently entail another graph, without this simply being a matter of the observer's seeing that a double cut could be inserted or removed. Rather, it may be because the observer can see that a more complex graph can be inserted or removed from a given area. Thus, to take a simple example, in EG



transparently entails, by IN



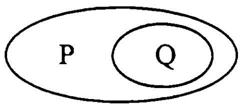
But it should be clear that in 5SL, for some users at least, $\neg P$ may not transparently entail $P \rightarrow (Q \wedge R)$. And indeed in 2SL, for some users at least, $\neg P$ may not transparently entail $\neg(P \wedge \neg(Q \wedge R))$. (If this seems doubtful, the point can be made by considering the same insertion in a rather more complex graph.) Again, proofs of (again, the large class of) such entailments are motivated simply by their opacity of entailment in both languages.

Secondly, though one may loosely say that EG has more TEQ and TEN than 5SL or 2SL, this is only a manner of speaking. TEQ and TEN are properties of pairs or sets of propositional representations in each logic. Though comparisons between logics can be used (as I have done) to bring the properties out, TEQ and TEN are not, I think, strictly quantitatively comparable across logics: first, because there are an indefinite number of pairs of propositional representations in each logic to be assessed, and it is implausible to claim that there are more pairs with TEQ and TEN in one logic than another; and secondly, because it seems to be true that there are some sentence-pairs (e.g. $P \leftrightarrow Q$ and $\neg P \leftrightarrow \neg Q$) which have TEQ in 5SL but not, when translated, in EG.

It is, perhaps, better to say that these transparency properties are intrinsic to a given logic, constraining how some proposition can be expressed in it; that is, constraining the translation of the proposition into the logic from its expression in natural language or thought. In the case of EG, these constraints remove much of the opacity which pairs of propositional representations have in other logics, avoiding the need for a large class of proofs as a result; and, from the perspective of reasoning, this is an important gain in efficiency and analyticity. But of course, someone who valued, not perspicuousness, but the connection to natural language, the flexibility and the ability to abbreviate formulas offered by the additional symbolism of 5SL, might reasonably prefer it on these grounds.[74]

5.4 Truth Conditions in EG and 2SL

It seems that the properties of TEQ and TEN are the result of the syntactic parsimony of EG and 2SL. For in both systems the operations of conjunction and negation are basic, and the other truth functors are defined in terms of them, and not as symbols in their own right. Thus in EG and 2SL there is only one basic way to represent material implication, for example; as



and $\neg(P \wedge \neg Q)$ respectively. In 5SL, however, there are of course three ways to represent this relation: $P \rightarrow Q$, $\neg(P \wedge \neg Q)$, and $\neg P \vee Q$, of which the first has a certain similarity to the "if-then" relation in natural language. To many, these three formulas are transparently equivalent as they stand; but, as we have seen, when combined in various ways they can form compound formulas which are not transparently equivalent. (Similar considerations apply in the case of TEN.) This is not to say that the symbolism of 5SL is not well-motivated; as well as the virtues mentioned above, it allows the user to avoid the heavy bracketing of 2SL, and can facilitate proof relative to 2SL. But it does appear that 5SL's relative lack of transparency of equivalence and entailment can be traced to its additional symbolism.

The representations of the derived truth functors also exhibit a further interesting feature of EG and 2SL, which is not shared by their equivalents in 5SL. This is that *they iconically represent the relevant lines in their respective truth tables.* Thus in 2SL, the compound representation for material implication is $\neg(P \wedge \neg Q)$. Reading this atomically, in terms of conjunction and negation, this represents the claim that the whole proposition is true, when it is not the case that P is true and Q is false. But this is just the relevant line of the truth table for material implication. Thus 2SL allows the user to read off the relevant line of the truth table for this derived functor just by looking at the formula for it. In 5SL, the user must remember the relevant information contained in the endowed symbol "→". Information about the truth conditions of $P \rightarrow Q$ is not available just by looking at it, for the representation relation here is not iconic but symbolic.

Exactly the same is true of the other derived functors. In each case, the formula in question is simply given by the denial of the false line(s) in the truth table (to which the formula given by conjoining the remaining lines is equivalent). Disjunction is represented in 2SL as $\neg(\neg P \wedge \neg Q)$; this iconically represents that the relation is true when it is false that both disjuncts are false. The biconditional is represented in 2SL as $\neg(P \wedge \neg Q) \wedge \neg(\neg P \wedge Q)$; this iconically represents that the relation is true when it is false that P is true and Q is false, or P is false and Q is true. (The biconditional is slightly more complex than the other functors since it has two relevant lines in the truth table, both of which are represented.) Again, in these cases the desired information cannot simply be read off the 5SL versions.

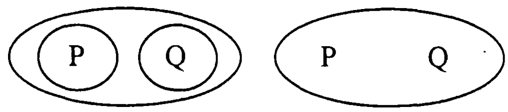
As the isomorphism between 2SL and EG would imply, EG has the same property; and this can be checked by examining the representations of the derived functors below (that of material implication has already been given above):

Fig. 5.4.1

$P \vee Q$ $\qquad$ $P \leftrightarrow Q$

The graphs are thus iconic representations of the truth conditions of the derived truth functors; one can tell just by looking at them what the truth conditions of the given functor are.[75] And of course it is trivial to observe that the same is true for conjunction and negation in all three logics under review.

It has often been noted that the relation between 2SL and truth tables allows a simple proof that 2SL, and some other two-functor logics, are expressively adequate; one simply conjoins the denial of the false lines in the truth table for the functor in question.[76] Similarly, the iconic representation of truth conditions makes it very straightforward in EG to define other functors; the procedure is, again, simply to note the false lines in the relevant truth table (substituting letters, enclosed or unenclosed by a cut, for truth values), deny them by encircling them in a cut, and write down the resulting graph. Thus, for example, an exclusive disjunction is false just when both the disjuncts are false or both are true; denying these yields the following:



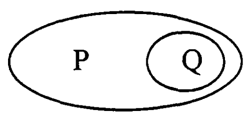
Contrast the position with 5SL. As we have seen, in 5SL the relations of material implication, disjunction and the biconditional are each represented in a single symbol; that is, through a conventional association between a given mark and some defining content (the truth table for the sign in question). This makes the notation compact. But there is no iconic representation here; one cannot simply read off the truth condition(s) for the relation in question just by observing a representation of it.

Iconic representation of truth conditions is, I think, more than a curiosity. There is some psychological evidence that a logic which iconically represents the truth conditions of its operators is more perspicuous to users in certain reasoning tasks. Such a logic, and some supportive early results, is outlined by Johnson-Laird and Byrne 1991.[77]

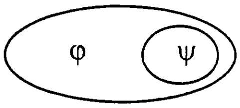
5.5 "Seeing As"

So far, this chapter has discussed some properties which are shared by EG and 2SL in virtue of the syntactic similarity between them. It has become clear that certain symbols defined as primitives in 5SL – $\vee$, $\rightarrow$, $\leftrightarrow$ – can be derived as compound symbols in EG and 2SL, compounds which can then be reasoned about as primitives in their own right. I now want to explore this idea in more detail.

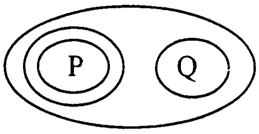
As noted earlier, in EG there is only one basic way to represent the relation of material implication, in a graph known as the "scroll" by Peirce:



Using the endoporeutic approach described in Chapter 4 this graph is read "atomically", moving from outside in and interpreting each of its symbols in turn; and this gives an SL translation of $\neg(P \wedge \neg Q)$. But this is not the only way the graph can be seen. Alternatively, it could be seen "molecularly", as a single compound representation of the following schematic form:



This would give a 5SL translation of $P \rightarrow Q$. Finally, it could be seen as the result of erasing a double cut enclosing P. Restoring the double cut would give the graph below. This has a 5SL translation of $\neg P \vee Q$, among others.

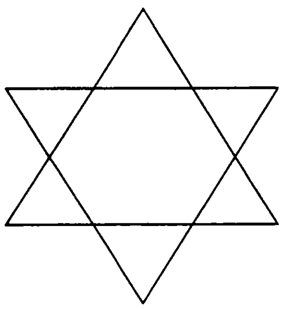


And this graph is transparently equivalent to the first graph above.

By allowing a single graph to be seen in a variety of different ways, therefore, EG achieves what 5SL achieves by differences in symbolism. The ability of the graphs to be seen in many different ways by the user allows them to display information which is not displayed by any one translation formula in 5SL; compare S1-S9 above. And if this displayed information is new to the reasoner, then it will be informative in the sense defined in Chapter 3. To use a term developed by Shimojima, the ability to see the graphs in different ways thus gives the reasoner a *free ride* to some new information.[78] This is a characteristic feature of diagrams in general.

The idea of "seeing as" in EG – or as she refers to it, multiple readings – has been given a precise formalisation by Sun-Joo Shin.[79] Like free rides, it is a distinctive property of diagrammatic representations generally. Take for example the Star of David:

Fig. 5.5.1



This can be seen as composed of a variety of different regular polygons; as a hexagon surrounded by triangles, as two triangles superimposed on each other, or as three overlapping parallelograms. Standard tests on visualisation in cognitive psychology explore this capacity, as do tests of the so-called Gestalt switch.[80] Other well-known examples of the phenomenon of "seeing as" would appear to include: seeing a diamond as a square, the duck-rabbit, two vases vs. a face, the old woman-young woman, the Necker cube and other visual illusions. Nor is the phenomenon of "seeing as", though characteristic of diagrams, restricted to them; for example, certain derived formulas in 2SL can, as we have noted, be seen and reasoned about as truth-functional primitives for disjunction, implication and the biconditional.[81]

5.6 Conclusions

To sum up: EG is a diagrammatic system, and has an exceptionally simple representational form, with one salient visually basic relation. Syntactically, it is similar to 2SL, with two logical operations, from which the standard zero, one and two-place truth-functors can be derived. Both EG and 2SL have a high degree of transparency of equivalence and entailment, where 5SL seems to have a low degree; both iconically represent the truth conditions of their truth functors, where 5SL does not; and both allow the reasoner to see certain compound graphs or formulas as primitive symbols for their derived truth functors. This gives them both much of the efficiency of syntactically richer languages, without the need for further rules of inference. The additional symbolism of 5SL has advantages; but the mutual interdefinability of its symbols has the effect of making opaque certain relations of equivalence and entailment which become transparent in logics with a more austere syntax.

Unlike 2SL, however, EG is not burdened with the need for bracketing to capture scope restrictions; and it seems to allow the reasoner to process information from the graphs more easily than is possible in the case of 2SL. Thus EG is more assimilable than either 2SL or 5SL; and more so than Polish notation, for similar reasons.

Utilising the distinction between displayed and contained information, one can now say that *graphs in EG generally display more information than sentences in 5SL*; recall that each of S1-S9 contains the same information (they are logically equivalent), but each displays a different portion of the information it contains. By contrast the translation graph in EG displays *all* the information in S1-S9, for any of these may be derived from the graph by the reasoner, simply by seeing it in different ways. And *graphs in EG generally display more information than sentences in 2SL*, because their extra visual simplicity and diagrammatic form allow the information to be obtained more easily. Recall that perspicuousness was defined in Chapter 3 in terms of the degree to which a representation displayed information which it contained; and in Chapter 2 in terms of the combination of discretion and assimilability. Adopting these definitions respectively, it therefore seems fair to conclude that EG is more perspicuous than either 5SL or 2SL.

This provides a plausible response to the apparent paradox, described in Chapter 1. There it was asked "if logics A and B convey the same information, how can one be more informative than the other?" The answer appears to be, roughly: "it is true that in one sense logics A and B can convey the same information. But there is an ambiguity in the word "convey" when applied to logics as representational systems, as between *containing* information and *displaying* information. The degree to which two representations contain the same information is a matter of *whether or not* they are logically equivalent given the rules of the logic in question. The degree to which they display the same information is generally a matter of how far they have the properties of transparency of equivalence and entailment, the visual simplicity of their representational form, and the degree to which that form allows their information to be assimilated by the reasoner."

Finally, the contained/displayed distinction underlines how opacity of equivalence and entailment motivates the need for proof. For the function of a constructive proof (in one sense of "construction") in propositional logic is to make explicit that a set of propositional representations Γ can be manipulated by the laws of logic until it displays all and only the same information as a target propositional representation A. The function of a non-constructive proof is to make a given propositional representation B exhibit an inconsistency in the information it contains. Generally, it is the fact that the two sides of a possible entailment do not display the same information which motivates the proof in the first place. In the next chapter, I examine proof procedure in EG in more detail.

6: Diagrams and Proof

It was seen in Chapter 4 that Peirce believed the task of any logic was to make the process of reasoning maximally explicit. Chapter 5 looked at some of the sources of the perspicuousness of EG. This chapter examines proof procedure in EG more closely. With a more developed account of proof procedure in EG, it then becomes possible to see how EG exemplifies a further claim of Peirce's – that there are different varieties of deductive reasoning – and to explore and evaluate these varieties of reasoning in more detail.

6.1 Hammer on "Peirce Provability"

Eric Hammer has recently made an important claim regarding proofs in EG, which deserves detailed discussion. In both Hammer 1995 and Hammer 1996, Hammer ascribes to Peirce a view of logical provability which is more restricted than that standardly held by logicians. Hammer says of EG as a system:

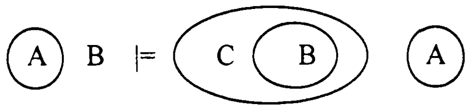
> In contrast to a more standard notion of provability[, in EG there] is a notion of "provability" of a graph from a set of graphs. *This notion of provability does not allow one to refer back to the previous stages of a proof, say to use a version of conjunction introduction.* Rather, one is restricted to making incremental changes to a single diagram. Thus, as is often done in actual reasoning with diagrams, one successively modifies a single diagram. (Hammer 1996, p. 130; italics added).

This claim is set out in more detail later in the same article, following Hammer 1995:

> Peirce has a somewhat different notion of provability than that defined above [i.e. the standard notion]. In particular, he does not have the rule of juxtaposition, instead compensating in other ways. For Peirce, a graph G is provable from a set Γ of graphs if and only if one can juxtapose a number of graphs in Γ, then apply the above rules of inference, excluding juxtaposition, successively to this graph, arriving eventually at G. *One is not ever allowed to recopy a graph.* A proof starts with juxtaposition of members of Γ. This graph is modified with a rule of inference to get a new graph, this new graph is modified to get another new graph, and so on until G is reached. *One never has the opportunity to go back to earlier graphs and collect together information about them. Any needed information must be carried along as one proceeds.* (Hammer 1996, p. 139; italics added).

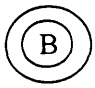
Hammer illustrates this claim by giving a "standard" proof, for which he specifically adds a rule of conjunction introduction to the normal rules of inference of EG. This he contrasts with a "Peircean proof" of the same entailment in EG, using the normal rules without conjunction introduction. The "standard proof" which he gives is as follows:

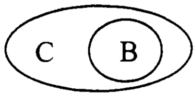
For a given set of graphs, Γ = {  , B}:

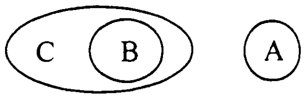
Proof 6.1.1: 

1.  A member of Γ

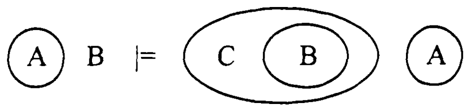
2. B A member of Γ

3.  2, DC insertion

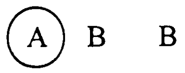
4.  3, IN of C

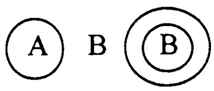
5.  1, Juxtaposition

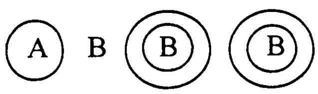
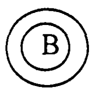
The "Peircean proof" given by Hammer is as follows:

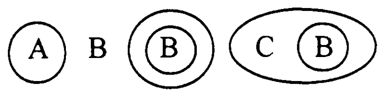
Proof 6.1.2: 

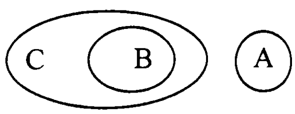
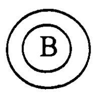
1.  Premiss

2.  1, IT of B

3.  2, DC insertion

4.  3, IT of 

5.  4, IN of C

6.  5, ER of B, 

As with the quotation above, this proof is given in both Hammer 1995 and Hammer 1996.

The suggestion seems to be, then, that Peirce uses a specific notion of provability in EG; and that this is a more restricted notion than that now standardly held by logicians. (There may, however, also be an implication that proofs in EG are longer or more elaborate than they should be, as a result of adopting this more restricted view of provability; certainly, the comparison between the proofs given above bears this interpretation, since the second proof is noticeably longer and more elaborate than the first.)

Is this claim damaging to an account of EG? It would in general seem to be a serious limitation on a logic if its inference rules did not permit it to incorporate results already proved from a given set of premisses Γ, without carrying them forward. For then every proof of a given result R would have to contain all the proofs of the prior results on which R was based. A logic which imposed this restriction would be "memoryless"; and it would quickly become impossibly cumbersome and unwieldy in use.

Now Peirce would of course have had no objection to the general claim that proofs in EG could be long and elaborate. As was seen in Chapter 4, at one point Peirce even goes so far as to say (Ms 499): "But to say that the aim was to make the algebra as analytic of reasonings as possible is to say that the aim was to make every demonstration as long as it possibly could be made without being circuitous". Hammer's claim usefully pinpoints a tension in this view. In the case of long proofs, the explicitness desired by Peirce cuts against the reasoner's being able to grasp, or to keep track of, the proof at all. To understand the reasoning in a long proof may require short cuts, or it may require prior results to be brought forward without having to be re-proven. Indeed, to achieve the desired proof, the reasoner may need to invoke results proven by others, but which he or she regards as reliable; the skill of the reasoner may come, that is, from recognising that such results can be combined in a way which constitutes a proof of the desired conclusion.[82]

So the question arises, how Peirce could have believed that EG was genuinely analytic of reasonings: first, if its proof-procedures were so much at odds with the established practices of mathematicians and logicians, practices which enormously simplified the process of proof; secondly, if its own core notion of provability made it intrinsically cumbersome and unwieldy in use? For any but the shortest proof, EG would be, not analytic of reasonings, but an impediment to them.

The problem becomes still more evident in view of two further facts.

- The first is simply that Peirce was of course a highly accomplished logician and mathematician. He was also very well-versed both in the history of logic and mathematics, and in the work of his contemporaries. Although proof theory continued to develop during his lifetime, and did not start to achieve modern levels of rigour until the work of Hilbert, it is certain that Peirce was aware of – indeed constantly seeking to improve – the best practices of the time. His father, Benjamin Peirce, was one of the most eminent mathematicians in America; and Peirce himself travelled widely in Europe. We know from his book reviews and letters, as well as more formal writings, that he was familiar with the work of most of the leading European mathematicians, including Cauchy, Dedekind, Riemann, Bolyai, Klein and Cantor. (And the same is true for that of mathematical logicians such as Schroeder, Peano and Russell.)

- Secondly, Peirce had made a close critical study of Euclid; and Euclidean geometry was the paradigm case of a cumulative proof structure, one in which the initial results were proved from the axioms and postulates, these in turn providing the premisses for later results. As described in Chapter 1, it is this structure which makes Euclidean geometry possible as a system at all.[83]

Clearly then, if Peirce did actually hold the view of "Peirce provability" which Hammer attributes to him, this would greatly undermine the value of EG as a system; and it would do so both in Peirce's own terms (given his aims for EG), and by comparison with the best contemporary and historical practice in mathematics and logic.

But did he hold such a view? It should first be noted that Hammer offers no textual support – from Peirce's own writings or from any collateral source – for his claim. At times, Peirce does talk in ways compatible with the memoryless view; for example, he likens SA to a blackboard and operations with EG to progressive amendments of graphs using chalk and eraser.[84] However, he never endorses it. Based on a survey and computerised text search of the Collected Papers (Peirce 1931/58; hereafter CP), a review of the existing volumes of the Chronological Edition and a review of Peirce's writings on mathematics (Peirce 1976), I would venture to doubt that there is any substantial textual evidence in Peirce's published writings for Hammer's claim; and I have located none in the unpublished writings.

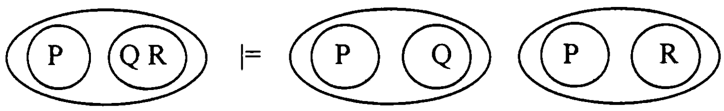
In fact, Hammer seems to rely on a general argument, as follows: the inference rules for EG do not permit any graph to be inserted on an even level except the empty double cut. But a juxtaposition of an earlier result would (unless that result were itself, improbably, a double cut) seem to constitute an insertion in even. So the juxtaposition of an earlier result is not possible.

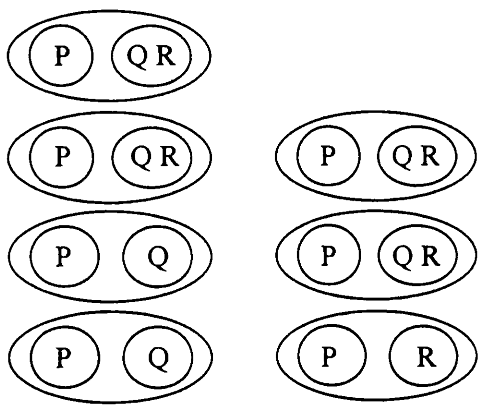
As far as I am aware, Peirce nowhere considers this view, for the reasons mentioned. But in fact it is clear, simply in virtue of the resources of EG itself, that Hammer's argument does not succeed. It is true that the inference rules for EG do not allow insertion-in-even. But they do allow the *iteration* on an even level of an existing graph. Take a proof in EG of one of the laws of distribution:

(LD) $P \vee (Q \wedge R) \models (P \vee Q) \wedge (P \vee R)$.

In EG this becomes

Proof 6.1.3:





1. Premiss
2. 1, IT of whole graph
3. 2, ER of first R
4. 3, ER of second Q

In step 2 of this proof it is quite permissible to inscribe the graph a second time on SA (an even level), because this is an act of iteration. Thus it is not in general true that graphs cannot be juxtaposed, only that this cannot be the result of an act of insertion-in-even, which is impermissible.[85]

To this it might well be objected that it leaves untouched the main claim Hammer is making, which is precisely that an *earlier* result cannot be juxtaposed. In this case, an existing graph has been iterated. But, the suggestion might go, for an earlier result to be iterable requires that it be carried forward to the current step in the proof. Thus proofs in EG are memoryless in the way Hammer's claim requires.

However, this objection also fails. *For the whole proof may be regarded as a single graph.* On this view, the lines of a proof do not require to be read as "replacing" or "succeeding" one another. On the contrary, they can be read as cumulatively constructing a much larger graph, which is the graph not of the *result* of the proof, but of the *whole proof itself.* Each line then displays both the iteration of the previous line – by IT, a permissible transformation – *and* the result of applying the rule of inference in question. All the lines are written on SA, hence it is legitimate to read them as conjunctively asserted under the rules of EG. Since all the rules of inference are truth-preserving, considering the whole proof as a graph will not affect its truth. Moreover, two or more proofs may be regarded as a single graph, provided that they proceed only from the same premisses.

But this may seem simply to compound the problem. After all, if the whole proof is to be regarded as a single graph, then what is the relation between this graph and the subgraph of the conclusion? And how does this provide EG with the needed ability to incorporate prior results? The answer is as follows: recall that it is a rule of inference of EG that any graph or subgraph lying on an even level may be erased (the rule denoted by ER). The sheet of assertion, SA, is an even level. So, in order to obtain a graph of the conclusion of a proof from the graph of the proof, one need only apply ER once to erase all the subgraphs on prior lines, leaving the subgraph on the final line. This will then be the graph of the conclusion.

But this point can be generalised. On the view we are describing, talk of lines of a proof is simply a manner of speaking. Existential graphs do not have "lines" as such; they can be read from any direction, as has already been noted in Chapters 4 and 5. It is not, therefore, necessary to restrict the scope of an act of erasure to prior whole lines of the proof, once the proof itself is considered as a single graph. Rather, any subgraph on an even level, *whether it constitutes or occurs within what was previously called a line*, can be erased. Thus if from a given set of graphs Γ a specific result R1 is proved and a proof is sought on the basis of it of a new result R2, it is perfectly legitimate in EG to use R1 as a result in a proof of R2, without having to prove R1 again *within* the proof of R2; for this is simply the equivalent of considering the proofs of R1 and R2 as a single graph deriving from the same set of premisses, and erasing all the steps in R1 except the conclusion, R1 itself. These will remain part of the graph of the proof of R2.

This can be expressed diagrammatically. For any set of graphs $\Gamma = \{G_1, G_2, \ldots G_n\}$, a set of proofs $\{P_1, P_2, \ldots P_n\}$ may be represented schematically as follows, where "$G_{tnnn} \ldots$" represents a group of one or more graphs reached by applying the rules of inference of EG:

Fig. 6.1.1

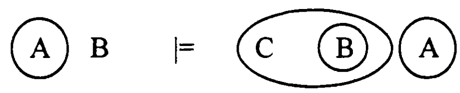
P_1	**P_2**	**P_n**
Γ	Γ	Γ
G_{t1}	G_{t2}	G_{tn}
G_{t11}	G_{t21}	G_{tn1}
G_{t111}	G_{t211}	G_{tn11}
…	…	…
C_1	C_2	C_n

Provided the premisses Γ remain the same, it is legitimate in EG to regard any or all of proofs $\{P_1, P_2, \ldots P_n\}$ as a single graph; and this graph could if necessary be constructed by iterating Γ on SA and applying the rules of inference as required in each case.

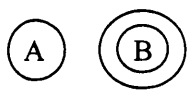
Say, however, that the reasoner now wishes to bring forward some prior results of reasoning – they need not be the *conclusions* of prior proofs – into a new proof. That is, say for example he or she wishes to use subgraphs G_{t11}, G_{t2} and G_{tn11} as part of a new proof P_{n+1}. When proofs $\{P_1, P_2, P_n\}$ are regarded as a single graph, this is perfectly acceptable. For it merely requires an act of erasing all but the required graphs from SA, using ER. The proof can then proceed using the set of graphs $\{\Gamma, G_{t11}, G_{t2}, G_{tn11}\}$ to the desired new conclusion. And in general, it will always be possible to inscribe a tautology on SA, since it remains true under any interpretation.

How does the "whole graph" approach work in practice? Take a revised version of Hammer's proof (proof 6.1.2 above), revised since the original is longer than it needs to be.[86]

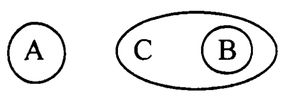
Proof 6.1.2 revised:



Premiss



[IT] DC insertion



[IT] insertion of C

Seen sequentially, this is a standard proof in EG, of the type seen in previous chapters. Seen as a whole graph, the reasoner has simply iterated each line (square-bracketed in the right-hand column) and then applied the relevant rules of inference. Thus any combination of the parts of the proof are conjunctively assertible. For example it would be equally possible to infer

But, of course, when a proof is seen as a single whole graph, then the act of iterating more than one instance of a given subgraph becomes superfluous. There is thus no need to repeat the cut-A or B subgraphs above; indeed to do so simply clutters up the proof, since no new information is added. It is in *this* sense, then, that – as Hammer correctly identifies as a characteristic of diagrammatic reasoning generally – "one successively modifies a single diagram" in proofs in EG, in sharp contrast to the redundancy of the "memoryless view" of proof in EG.

There are, then, two ways of seeing proofs in EG. In the first place, a proof can be seen sequentially, with the conclusion taking shape as one line of graphs follows another; this is similar to the standard reading of proofs in SL. However, a proof may also be seen as a single graph; indeed, two or more graphs derived from Γ may be viewed as a single graph. On this reading there is no sequence as such to the *proof*, only a sequence to *the construction of* the proof; any subgraph or set of subgraphs may be erased from SA, and what remains will be an entailment of Γ. And on this latter approach it is perfectly valid to bring forward or appeal to the prior results of reasoning in deriving new results.

Hammer's general mistake, then, is to assume that the flexibility of EG is exhausted by the first, sequential, approach. But note that distinguishing the two approaches only re-establishes parity between proofs in EG and proofs in sentence logics which already have conjunction introduction or a similar rule. However, EG has a feature which is not shared by sentential logics, even 2SL, to which it is syntactically similar, as we have seen. There is no symbol in EG for the conjunctive assertion of two graphs; they are asserted simply by being written on SA. In SL, a rule of detachment must be applied in order to isolate a sentence φ from an earlier sentence $\psi \wedge \varphi$, and the detached sentence must be written on a new line. In EG, since the proof can be seen as a whole graph, this is not necessary; the reasoner can immediately observe, even within "lines", what is being conjunctively asserted at all stages of the proof.

Differentiating the "sequential" and "whole graph" approaches helps to make explicit the different uses to which logic can be put in proof. Where the goal is to *produce* a proof, to do some constructive reasoning using EG, then the "whole graph" approach will be adopted; there is simply no need to iterate the additional subgraphs, which add clutter without information. Indeed, the reasoner may find it convenient not to write the graphs down sequentially at all, but to cluster them on the page or categorise them in other ways which experience suggests will aid the process of inference (just as one way to extract words from a string of letters in solving a crossword is often to write the letters in a circle).[87] When the goal is to *communicate* a proof, however, it will be appropriate to set each line out formally in the normal manner, so that the proof can be publicly checked by others. It is, in my view, important not to assimilate these two functions; that is, not to assume that the reasoning involved in communicating and following a proof is the same as the reasoning involved in constructing one.[88]

6.2 The Corollarial/Theorematic Distinction

This analysis can, I believe, shed some light onto the relation between EG and a central part of Peirce's account of deductive reasoning. Peirce's views on deduction are quite scattered and not always clear, and I shall simply sketch them for purposes of this discussion.[89]

Peirce believed deductive reasoning could be classified in various different ways. In particular, he says of deduction:

> "It is either Corollarial or Theorematic. A Corollarial Deduction is one which represents the conditions of a deduction in a diagram, and finds from the observation of this diagram, as it is, the truth of the conclusion. A Theorematic Deduction is one which, having represented the conditions of the conclusion in a diagram, performs an ingenious experiment upon the diagram, and by observation of the diagram, so modified, ascertains the truth of the conclusion." (CP 2.267)

Moreover, the modification introduced by the theorematic step is eliminated, according to Peirce, before the conclusion. As he put it in a draft of the Carnegie Institution application of 1903, "what I call the theorematic reasoning of mathematics [sc. the general science of deductive reasoning] consists in so introducing a foreign idea, using it, and finally deducing a conclusion from which it is eliminated" (Peirce 1976, 4.42).

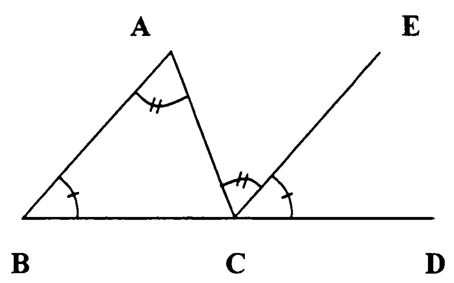
Elsewhere, Peirce describes deductive procedure as follows:

"The procedure of the mathematician is, first, to state his hypothesis in general terms; second to construct a diagram, whether an array of letters or symbols with which conventional "rules", or permissions to transform, are associated, or a geometrical figure, which not only secures him against any confusion of all and some, but *puts before him an icon by the observation of which he detects relations between the parts of the diagram other than those which were used in its construction.* This observation is the third step. The fourth step is to assure himself that the relation observed would be found in every iconic representation of the hypothesis. The fifth, and final, step is to state the matter in general terms." (Peirce 1976, 3.749; italics added)

In a corollarial deduction, thus, the reasoner inspects a diagram and directly observes the desired conclusion in it; it is then a trivial matter to apply the relevant rules in order to isolate the conclusion. In a theorematic deduction, the reasoner is not able to observe the desired conclusion directly in a given diagram. How is he or she to make progress? The claim is: only by a process of hypothesis and experimentation. The reasoner experiments by transforming the diagram in some way, by applying a rule of inference. He or she then inspects the resulting diagram. At this stage, either it is possible to make a corollarial inference to the conclusion, or it is not. If not – and if the reasoner wishes to continue the proof – the process of theorematic inference must be repeated. Finally, a diagram is generated which permits a corollarial inference, and the proof can be concluded; or it must be given up. Since the final diagram is reached by a corollarial inference, the diagram produced by the theorematic step has been eliminated by the time the conclusion is reached.

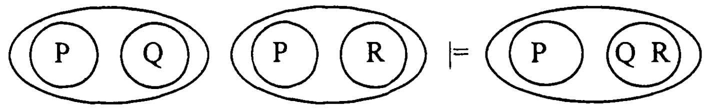
The detailed list of steps given by Peirce above bears a strong resemblance to the six formal divisions of a proposition, the formal procedure used in Euclidean geometry.[90] This impression is reinforced by the "corollarial/theorematic" terminology; and Peirce finds examples of corollarial and theorematic reasoning in Euclid. An example of the former is given by Shin 1997; I want to focus on the theorematic case, of which one example is the well-known proof (Proposition 32 of Book 1 of the Elements) that the internal angles of a triangle ABC sum to two right angles.

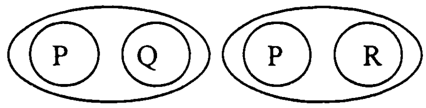
Fig. 6.2.1



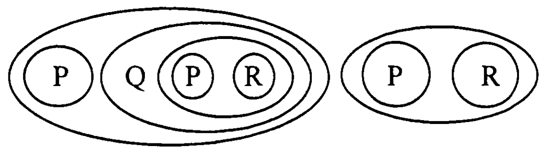
The key step in this proof is the drawing of a auxiliary line CE, parallel to line AB of the triangle; dividing the external angle at C such that ∠ACE = ∠BAC, the reasoner can then note the new information that ∠ABC and ∠ECD are equal. This step is not a corollarial one; it does not follow trivially by inspecting the triangle and applying an axiom or postulate of the system, some general rule of reasoning or some already established proposition. Rather, drawing such a line is a constructive act; it is permitted by the postulates of the system, *but it is not logically implied by those postulates.* It requires some act of choice by the reasoner. The reasoner has, in fact, many choices: he or she can draw a straight line anywhere on or near the figure, extend an existing line segment, or draw a circle around an existing point. Thus there are many possible blind alleys, some of which the reasoner will doubtless explore in constructing the proof before making the right choice. Once the right selection has been made, the necessary corollarial inferences can go through; and the auxiliary line then becomes irrelevant to the conclusion, which concerns only the internal angles of the triangle.

With this in mind, it is easy to see how Peirce's distinction finds application in EG. Take a sequential proof in EG of one of the laws of distribution – the opposite entailment of that in Proof 6.1.3, i.e. $(P \vee Q) \wedge (P \vee R) \models P \vee (Q \wedge R)$.

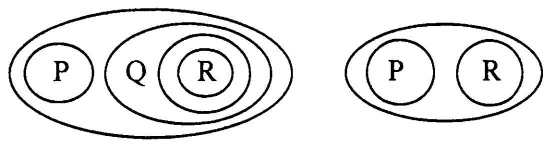
Proof 6.2.1:


1.


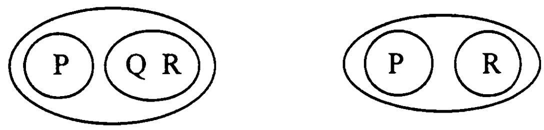
Premiss

2.


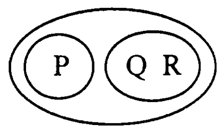
1, IT

3.


2, DE of


4.


3, DCN

5.


4, ER

Here the theorematic step is the inference from line 1 to line 2. Once this has been made, the graph of the conclusion can be directly observed by competent reasoners in EG in the left-hand graph, and it is a straightforward matter to isolate it and complete the proof, via a series of corollarial inferences; for less experienced reasoners, the following inference may also be theorematic. In both cases, the diagram introduced by the theorematic step has been eliminated from the conclusion.

I will not give further examples of theorematic reasoning in EG; these can very easily be identified in other proofs in this and earlier chapters. But some further explanatory comments are in order.

- The corollarial/theorematic distinction is one which Peirce came to – almost certainly out of dissatisfaction with Kant's distinction between analytic and synthetic truths, to which it bears some resemblance[91] – well before he developed EG. Thus Peirce's experience with EG was not in fact a historical reason for his reaching the corollarial/theorematic distinction. However, the representational form of EG is one which allows the distinction to be articulated very clearly, much more clearly than do sentential logics. For this reason EG can be used, philosophically, to motivate the distinction in the first instance.

- In line with his general practice, Peirce uses the word "diagram" very widely; as described in Chapter 2, it includes the sentences of sentential logics. Thus, in applying the corollarial/theorematic distinction to "diagrams" as such, Peirce is intending to underline its generality. In particular, it is not merely restricted to quantified languages, as Hintikka (1980) suggests. Ketner (1985) gives (non-quantificational) examples illustrating the distinction using EG, Euler circles and sentential logic.

- There is some apparent ambiguity in the idea of one diagram's being directly observable in another. If it is taken literally, then any act of insertion or iteration in EG – no matter how trivial – would not be corollarial, since it would give rise to a graph which could not be directly observed in the prior graph. It seems preferable to follow Peirce's own words: corollarial inference is that in which the reasoner "finds from the observation of [a] diagram, as it is, the truth of the conclusion", where this observation includes the reasoner's becoming aware, without a conscious process of inference, of a possible conclusion-diagram featuring an insertion or iteration, where this cannot literally be observed *in* the premiss-diagram.

Indeed, as I have noted, a very experienced reasoner might obtain the conclusion of the whole proof above simply from observing the premiss. This inference might involve no process of reasoning or application of rules of inference of which the reasoner was aware. Of course, this need not mean no rule at all is being applied; the inference might, rather, involve some form of meta-rule of the kind already discussed. If this is right, then the boundary-line between the corollarial and the theorematic will not be an absolutely fixed one. What counts as a theorematic step will depend in part on the experience and skill of the user; an experienced reasoner with EG is likely to be able to make the direct observations characteristic of corollarial reasoning at a much earlier stage than the novice, and to perceive short cuts in the way already described. At the limit, he or she will find certain patterns of logical reasoning straightforward, which might appear unmotivated or even counterintuitive to the novice.

Clearly this short discussion falls well short of a detailed argument to the effect that the corollarial/theorematic distinction should be generally recognised; for this the reader should turn to the literature mentioned, perhaps also noting the often influential parallel views of other philosophers such as Poincaré, Popper and Lakatos. My goal, rather, is to situate the corollarial/theorematic distinction within the broader context of EG set out above and in earlier chapters.

In Chapter 5 I suggested a means of seeing proofs in terms of the distinction between contained and displayed information. On this account, to establish a conclusion from some premisses is to make public and explicit that one logical representation (or set of representations) can be manipulated by the laws of logic until it displays the same information as a given target representation. But how exactly does a proof perform this function? The corollarial/theorematic distinction provides an answer. Corollarial inference is able to proceed only because the desired information is *already displayed* by the graph – that is, it can be obtained by the reasoner just by looking; this is what makes it trivial. It is theorematic reasoning, then, which bears the burden in a proof of transforming the representation in question so that it displays the desired information; that is, of isolating and bringing forward for display the desired information from the larger amount of information which the representation contains. In the Euclidean proof, similarly, it is the theorematic step of drawing an auxiliary line which causes the diagram to display the desired information.

How, then, do the two types of reasoning operate? Both are initiated by an act of observation, whereby the reasoner notices the visually salient features of a representation, and compares them to those of the desired conclusion. Of course, if the reasoner can directly observe the conclusion, then the inference is merely corollarial. *It is here that the perspicuousness of a particular notation shows its value; for some notations more than for others, it will be easier for the reasoner to make corollarial inferences.* In particular, notations which have the properties of transparency of equivalence and entailment, and which allow the phenomenon of "seeing as" – of symbol-clusters which can be taken as complex primitives, as discussed in Chapter 5 – will be at an advantage; for it is precisely in these cases that the reasoner will see new ways of decomposing or developing a representation so as to reach the desired conclusion. Observation is then, as Peirce says, a matter of detecting "relations between parts of the diagram other than those which were used in its construction."

Turning to EG in particular: it is, we have now seen, a mistake to assume that proofs in EG must be sequential and memoryless. This would limit the scope of possible observation by a reasoner to graphs in the line of proof immediately prior to the inference to be made. But I have suggested above that it is equally permissible in EG to see a proof as a single whole graph. When this is done, it greatly expands the representations to which the reasoner may potentially attend. (These can in principle include both representations produced in other proofs from the same premisses, and tautologies. Of course, when the proof is *communicated* to others, adequate separate justification will be required for those representations which are not produced as part of the current proof.) That is, in constructing a proof, at any given stage the act of observation may include not only all the prior graphs in the proof, but also other graphs which can be inscribed since they are known to be true in the system.

Considering the act of observation in this way creates a parallelism between proofs in EG and proofs in a standard logic such as 5SL; for in these logics it is typically permissible to bring forward earlier lines of a proof by conjunction introduction, as we have seen. It also suggests that the act of observation is, in a sense, dynamic; it is an act of interrogation, whereby the reasoner scans, assesses and selects from existing graphs. In some cases, the reasoner will simply see that the desired conclusion follows from a given graph or subset of available graphs. In others, since it is no longer a matter of inspecting simply the latest line of the proof, it may require time, effort and experimentation on the part of the reasoner to identify the graph or graphs in which the conclusion can be observed.

The second stage – the stage characteristic of theorematic reasoning – is that of experimentation, according to Peirce. Here again, seeing the proof as a whole graph encourages a slightly different interpretation from the standard accounts. On the standard accounts, in order to make progress the reasoner must find some way to transform the diagram in the latest line of the proof so as to display desirable new information. The diagram to which the reasoner must attend is settled and identifiable, and this immediately constrains which rules of inference can be applied. On the "whole graph" view the reasoner's task is slightly different. On the one hand, any subgraph or combination of subgraphs on SA is available as the object of a possible theorematic transformation, which might seem greatly to complicate the task of reasoning; on the other, for precisely the same reason the user has greater displayed information to hand from which to make an inference. The "whole graph" view both liberates the amount of displayed information with which the reasoner has to work, so to speak, and complicates the process of experimentation.

In what sense, then, is such reasoning in logic experimental? Indeed, it may be tempting to ask the prior question: how could any reasoning in logic be experimental? After all, it is usual to think of experiment in connection with the sciences of nature, as a matter of testing and weighing evidence; and it is not plausible, it might be said, to see logical transformations as motivated by evidence. Such a general topic lies well outside the scope of this discussion. But there are two features of Peirce's account which deserve mention.

First, in the process of constructing a proof, there will be stages where progress can obviously be made by making a particular transformation. Equally, however, there will clearly be stages at which the reasoner may have no obvious way to proceed. That is, as a result of a process of observation there may be a number of apparently plausible candidate transformations, any one of which could – given the reasoner's then-existing knowledge – lead to the desired conclusion. All of these transformations are permitted by the rules of inference of the system; but there is clearly a cost in time and effort to a mistaken selection. How, then, is the reasoner to make progress? Possible strategies might include one or some of: random selection, deliberately re-evaluating the experience gained in similar situations for possible clues, and visualising – for as many steps as possible – the outcome of a possible transformation. Whichever option is chosen, however, it will be a hypothesis; it will be permissible by the rules of inference of the system, but it will not be directly motivated by them or by any actual knowledge that it will lead to the desired conclusion. It will, in this sense, be experimental; it will, in Peirce's terminology, be an abductive rather than a deductive inference. If the transformation causes the formula to display the information needed for a process of corollarial inference to take place, it will have been successful. If not, its success will hinge on the outcome of a further piece of theorematic reasoning.

Secondly, the techniques of evaluating parallel experience or visualisation are, in effect, attempts by the reasoner to find some specific epistemic warrant for selecting one option over another. Thus there is, in effect, an ambiguity in the word "evidence". Without seeking to beg too many questions, one might say the following: in the natural sciences, experimentation is a matter of investigating the world; the general question at issue is whether a hypothesis is true or false. It can be rational to see a hypothesis as progressively entrenched, if not confirmed, by repeated successful experimentation and, if initial experiments fail, to seek new evidence for the hypothesis by attempting a different type of experiment. In logic, however, the principle question to be addressed does not concern the truth or falsehood of the hypothesis as such, but rather its fertility or explanatory power: whether and how it constitutes a step towards the proof of the conclusion.[92] The reasoner does not generally derive greater warrant for a hypothesis from repeated successful experimentation. And indeed it is hard to understand what could constitute new evidence for choosing one hypothesis over another in a theorematic inference, over and above the results of testing it and observing the outcome.

But if "evidence" is taken to mean something like "external mark which can reliably be taken as a reason for doing P rather than Q", then theorematic reasoning involves an assessment of evidence. This assessment of evidence may, or may not, warrant a decision to adopt one hypothesis over another. Is such a warrant empirical or *a priori*? I can only tentatively sketch here some of the considerations which might play a role in a response to this question; much will depend on exactly how one understands the key terms. In general, one wants to say that the warrant for inferences in logic is *a priori*, in at least the following sense: it does not derive its justificational force from any appeal to experience. Matters are, however, complicated by considering what could warrant the selection of one hypothesis in a diagrammatic logic over and above another. Here, it seems, the effect of the diagram may be that of triggering some specific perception of resemblance, a memory or some habitual disposition; it could then be rational to adopt some hypothesis, consciously or unconsciously, because of the experience of the success of similar moves elsewhere. We might then want to say that the specific inference was (in part) empirically warranted. However, in the *a priori* context of logic, the warrant for the *knowledge obtained* by successful inference – the basis of justification or entitlement for that knowledge – could, it seems, remain *a priori*. But clearly a great deal more needs to be said.[93]

To return: someone who is predisposed to believe that there is no scope for observation or experiment in deductive reasoning is likely to remain unpersuaded by the corollarial/theorematic distinction. And opposition to it seems to receive support from what one might term a clear "deductivist" strand in the philosophy of logic, which sees deductive reasoning as a homogeneous, graduated process in which some information is extracted from other information. On this view, the process of proof is a matter of analysis, in which reason alone is used. Since deduction (on this view) employs reason alone, it is not observational, beyond the observation required to see a propositional representation; and since deduction is not observational, there is no reason to prefer one logic to another on the basis of perspicuousness. The process of analysis may be either shorter or longer, but it is a homogeneous and unified type of reasoning, and can as such be contrasted with induction (and, as some would have it, hypothesis or abduction), which is characteristic of the sciences of nature. From the deductivist perspective, talk of experimentation is to be equated with an empirical view of logic as a whole, in the style of Mill.

A Peircean approach suggests that this view is far too sweeping. For the analysis of logical reasoning, it is rational to prefer a logic which is perspicuous in representational form to one which is not. EG is perspicuous: at the level of representation, it has a high degree of discretion and assimilability; at the level of information, it displays more of the information it contains than do its equivalent sentential counterparts. So EG is more analytic of logical reasoning than these sentential logics. And this is not an empty claim: something hangs on it. For EG's perspicuousness can be used to motivate a distinction within what is normally considered deductive reasoning, between corollarial and theorematic inference. This distinction is obscured by the representational form of sentential logics, which gives rise to catch-all descriptions of deductive reasoning in terms of "information extraction" and the like. And it is also obscured, it would seem, by a mistaken assimilation of the two types of reasoning involved respectively in the communication and construction of proofs. At the very least, then, EG can motivate a non-deductivist, experimental view of deductive reasoning – in EG itself, or in diagrammatic logics more broadly – which is worthy of examination. But, if the corollarial/theorematic distinction is regarded as a plausible analysis of deductive reasoning in logic or mathematics generally, then these considerations may seem to provide further reason to question traditional distinctions between the types of reasoning to be found in these *a priori* disciplines, and those to be found in the natural sciences.

7: Conclusions

Let us return again to the standard view of diagrams outlined in Chapter 1:

> It is now commonplace to observe that the diagram [sc. triangle *ABC*] ... is only an heuristic to prompt certain trains of inference; that it is dispensable as a proof-theoretic device; indeed, that it has no proper place in the proof as such. For the proof is a syntactic object consisting only of sentences arranged in a finite and inspectable array. One is cautioned, and corrected, about ... the mistake of assuming as given information that is true only of the triangle that one has happened to draw, but which could well be false of other triangles that one might equally well have drawn in its stead.

It is a consequence of this view that, since proofs are composed of sentences, there can be no proof of which a diagram is an essential part. But what does this mean? In what sense are diagrams supposed to be generally dispensable from proofs?

One interpretation of this "dispensability thesis" might be:

1. Diagrams are always formally out of place in proofs, which are (finite, inspectable) syntactic objects consisting only of sentences.

This appears to be what is intended in the quotation (note the restriction to sentential proofs). But it is false, as we have seen; there are formal proofs in diagrammatic logics which are demonstrably sound and complete.

But the dispensability thesis might also be interpreted thus:

2. Diagrams are always dispensable from proofs: one can always present the proof in a non-diagrammatic way while remaining faithful to the reasoning involved in its construction.

It is not clear here what precisely is meant by "faithful to the reasoning involved in its construction". But assuming that (2) is intended broadly – to challenge the claim that there are distinctive patterns of reasoning with diagrams – then it too appears false. As we have seen, diagrammatic representation is generally iconic, not symbolic; and diagrammatic reasoning seems to involve different cognitive resources from those in sentential reasoning. And specifically, the patterns of visual reasoning to be found in EG are different from those in sentential logics (even 2SL, which has some degree of iconicity).

Alternatively, the dispensability thesis might be taken to mean:

3. Diagrams are always dispensable from proofs: one can always substitute sentences for diagrams within a proof while delivering the same conclusions.

But this is also false for EG, unless "sentences" are taken to include linear EG (clearly not what is intended); one cannot substitute a sentence of some other (sentential) logic in an EG proof and still have a (sentential or EG) proof of the required conclusion.[94] [95]

But even if (3) were true, the advocate of diagrams may reasonably ask: so what? In terms of formal logic, the syntactic admissibility in principle of diagrams is now a recognised fact; and, more generally, the diagram theorist has always acknowledged the greater overall flexibility of sentences (among their other virtues), and the fact that diagrammatic representation operates under specific and distinctive sets of constraints. What matters is what these constraints are, how they operate, what other factors are in play in diagrammatic representation, and how such representation allows diagrams to play the central role they do in reasoning. An *ab initio* dismissal of diagrams from logic or mathematics is, on this view, not merely dogmatic; it breaches what Peirce calls the first rule of reason, by blocking the way of future inquiry.

What light, then, has this inquiry in particular shed on those further matters of interest described in Chapter 1? First, I think it supports the views of philosophers who have claimed that there can be a role in logical reasoning for "intuition" in some sense; and that diagrammatic logics can, in virtue of their form, be more intuitive than sentential logics. EG is more intuitive than SL, on this view, because it is more perspicuous: in virtue of its transparency properties, EG allows the reasoner to obtain more information from a graph than from an equivalent sentential representation, just by looking. (And this quasi-perceptual idea of intuition *of* a concrete representation – rather than the more general supposed propositional attitude or faculty of intuition *that*[96] – comes close to the visual notion which lies at the etymological heart both of the English word "intuition" and its German equivalent, *Anschauung*.) The wider point is aptly summarised by Hilary Putnam, comparing Frege and Peirce:

> "Present-day readers of Frege's famous texts often suppose that the view that something like geometrical intuition has a role in arithmetic (let alone in logic itself!) was, in fact, overthrown by Frege once and for all. (Present-day readers are likely to think we don't even need "intuition" in *geometry* itself, since we can arithmetize geometry.) But Peirce's view highlights the fact that this is, to put it mildly, not obviously right. Peirce argued that *the recognition that a structure is a deduction* is itself akin to geometrical intuition ... Doing logic diagrammatically highlights the way such a proposal might be right; however, the view can also be held if one uses "proof trees" or linear proofs or some other system. We believe that this issue is far from sufficiently explored at the present time. But it is fascinating that the two inventors of predicate calculus (which we today regard as the heart of symbolic logic) disagreed on so fundamental a metaphysical issue, Frege seeing logic as totally nonempirical and Peirce seeing logic itself as involving something like mental experimentation with diagrams." (Peirce 1992, italics in original).

Secondly, this inquiry suggests that, at least as regards alpha EG, Peirce's decisive move into diagrammatic logic in the 1890s was not ill-motivated. To the contrary: not only is alpha EG more "analytic of reasonings", in Peirce's sense, than the most well-known propositional logics at present, it seems probable that it was so compared to the principal alternatives of the time, though I have not surveyed them here. And, as I argued in Chapter 6, this perspicuousness has value, for it makes explicit – and can be used to motivate – the corollarial/theorematic distinction in deductive reasoning; and this is an achievement whether or not that distinction is ultimately accepted.

In fact, Peirce's diagrammatic turn is directly embedded in his pragmatism and late theory of signs, or "semeiotic". The distinction between displayed and contained information can be recast in semeiotic terms: at the level of the sign, in relation to its dynamic object, in terms of the distinction between the iconic and the symbolic aspects of the sign; and, at the level of the interpretant – or the effect of the sign on some mind or quasi-mind – in terms of the distinction between the immediate and the logical interpretants of the sign.[97] Reinterpreting perspicuousness in these terms shifts the focus away from a preoccupation with diagrams vs. sentences, and towards the iconic and the symbolic as such; and it provides a formal context within which these modes of representation, and the different claims they make on us in thought and action, can be located. And the significance of EG as a notation, as of any other type of representation, is pragmatically captured for Peirce in terms very similar to what I have called perspicuousness. As he says in an article on the "Logic of Relatives" (1897):

> The third grade of clearness consists in such a representation of the [designated] idea that fruitful reasoning can be made to turn upon it, and that it can be applied to the resolution of difficult practical problems.[98]

Endnotes

[1] Tennant 1986, p. 304.
[2] Dedekind (1872) gives a canonical formulation: "even now such resort to geometric intuition in a first presentation of the differential calculus, I regard as exceedingly useful, from a didactic standpoint, and indeed indispensable, if one does not wish to waste too much time. But that this form of introduction ... can make no claim to being scientific, no-one will deny". Cf. Gardner 1958 ch. 2, Shin 1994 ch. 1, Brown 1997, Friedman 1992 p. 58.
[3] For a fuller account see Friedman 1992, ch. 1.
[4] Manders 1996 develops this point in detail.
[5] In practice Euclid does not always obey this rule. Cf. Kneale and Kneale 1962 p. 380 (though this account does not bring out the centrality of the process of construction in Euclid).
[6] Though of course Hilbert was well aware of the heuristic value of diagrams; compare the many diagrams in Hilbert 1899. Hilbert and Cohn-Vossen 1952 is an explicit attempt to bring a range of relatively difficult results to a wider audience by making them more intuitive (*anschaulich*) via diagrams.
[7] Cf. Hahn 1933/1980; and one might also note Landau's famous diagram-free textbook on calculus (Landau 1950).
[8] For detailed discussion see Gillies 1992.
[9] Venn 1886, pp. 521-4.
[10] Marshall 1890, quoted in Larkin and Simon 1987.
[11] Hadamard 1945, pp. 142-3.
[12] Schweber 1994, Miller 1986/1998.
[13] Giaquinto 1992, 1993, 1994.
[14] Shin 1991, 1994.
[15] Zeman 1964, Roberts 1973.
[16] Van den Berg, Brauner 1999.
[17] Roberts 1973, p. 126.
[18] Brauner at al. 1999.
[19] Barwise and Etchemendy 1998.
[20] Peirce 1935 (hereafter CP) 3.328-58; 4.12-20; 3.359-403; ibid. Cf. Roberts 1973, Appendix 1 for a detailed summary of Peirce's achievements in logic; and Putnam 1990, Chapter 18 for additional comment.
[21] Classifying a sketch map as a diagram is not intended to be controversial (compare: a map of the London Underground, which is widely regarded as a diagram). Some maps are better classified as depictions, however.
[22] By, respectively, Shimojima; Sloman; Palmer and Lindsay; Stenning, Oberlander and Inder; Larkin and Simon, Barwise and Etchemendy, and Hammer. See Shimojima 1996b for further discussion.
[23] Some of these alleged properties are advanced by more than one person; cf. Larkin and Simon 1987, Hopkins 1998, Eberle 1995, Goodman 1976, Barwise and Hammer 1996, Shimojima 1996b.
[24] An "experienced resemblance" account of depiction is defended, especially against Goodman-type worries, in Hopkins 1998.
[25] Taken from Hopkins 1998.

26 For types of diagram generally, see Harris 1996; for specific types of graphs, see Kosslyn 1994.
27 I do not discuss the idea of relevance here, but cf. Sperber and Wilson 1986.
28 Shimojima 1996 analyses this overspecificity in the case of diagrams persuasively in terms of structural constraints between representations and their targets.
29 For psychological/neurological evidence on symbol-processing, see Deacon 1998.
30 On testimony, see Coady 1992.
31 A similar phenomenon is occasionally seen in works of devotion (such as "the Altar" and other pattern poems of George Herbert); and also in the Mouse's tale in *Alice in Wonderland*.
32 Contra Tufte 1983/97, whose data-ink ratio assumes decoration must decrease assimilability.
33 And, of course, this relationship may hold in reverse; as in the case where a builder follows an architect's drawing in locating a house.
34 Barwise and Hammer 1996, p. 71f.
35 Barwise and Hammer 1996, p. 74.
36 For Dretske 1981, the nuclear sense is factive (p. 45).
37 In fact, Barwise and Seligman distinguish four types of information theory, also identifying a family of "state space" theories. There is a family resemblance between "state space" theories and the better-known "possible worlds" theory, however, and this discussion focuses on the latter. Barwise 1997 suggests that state space theories have resources (not shared by possible worlds theories) which allow them to account for *a priori* information; but I cannot explore this idea here.
38 Popper 1959.
39 Cf. Dretske 1981, ch. 2.
40 Dretske recognises that his account cannot include *a priori* information, though only in an endnote to the final chapter of Dretske 1981. But he claims, remarkably, that this is "not a serious omission, since [sic] ... every analysis of knowledge seems compelled to make a similar exception" (p. 265). But this hardly constitutes an *argument* for the omission's not being serious.
41 E.g. by Stalnaker 1984, p. 13 (factive) and, implicitly, p. 25 (informative).
42 E.g. Yagisawa 1988.
43 As, in the case of conceptual content, by Peacocke 1992.
44 Harman 1986, ch. 2.
45 This may *also* be informative, of course, in the sense of increasing the recipient's certainty that P.
46 Where, roughly speaking, something is epistemically possible for O if it is true given the state of O's actual knowledge. This requires much further qualification, however; cf. DeRose 1991.
47 Such as "computation/information" (Larkin and Simon 1987); "immediate/non-immediate implication" (Harman 1986); "surface/depth information" (Hintikka 1970); and "what a graphic shows/what lies behind it" (Tufte 1983, ch. 7).
48 And there is significant evidence for this phenomenon in cognitive psychology, notably in the arguments over descriptionalism (the view that mental images are structural descriptions). See Tye 1991, ch. 5.
49 The position is taken from Shirov-Kramnik, Groningen 1993; Black played 1 ... R-KKt1 and the game was later drawn by perpetual check. However, 1 ... QxKt!! wins for Black.
50 Zeman 1964, Roberts 1973.
51 For more information, cf. Roberts 1973, Zeman 1964, Hammer 1995 or (an introductory text) Ketner 1996; the first three contain metalogical proofs of the soundness and completeness of the alpha system. A decision procedure for the alpha graphs is given by Roberts 1997. For the beta graphs and gamma graphs, see especially Roberts 1973, Zeman 1997 and also the work of Van den Berg and Brauner.
52 On logic see, for example, Peirce 1992/1997, Kent 1987 and Fisch 1986.
53 CP 7.220. Cf. Dipert 1995, p. 47, though he does not fully distinguish the two senses of

"normative" as here.
[54] Lowell Institute Lectures 1903, Ms 455, p. 2 (quoted in Roberts 1973).
[55] Cf. CP 4.373, 4.533.
[56] Ms 450, emphasis added. See also Ms 454.
[57] On Peirce and logicism, see Haack 1993 and Houser 1993.
[58] Cf. Putnam 1990, pp. 256-7.
[59] E.g. it is explicitly treated as diagrammatic in Hughes 1993b.
[60] E.g. "the notation undoubtedly allows one to perceive the structure of a formula *at a glance*, and to perform substitutions with ease" (van Heijenoort 1967, p. 2); "the diagrams ... offer *immediate and graphic representations* of the logical relations and procedures that Frege was concerned with" (Hughes 1993b, p. 265; italics added).
[61] Peirce often talks of "scribing" a graph. But it is more natural to talk of "drawing" or even "writing" a graph.
[62] See Hammer 1995, p. 99.
[63] R3 and R4 are difficult to state concisely. Here I follow a formulation of John Sowa.
[64] Note that "entailment" here and later is not intended to have any modal force, but to have its standard metalogical meaning of logical consequence; or, when applied to formulas, of a formula $\varphi \models \psi$ in which ψ is a logical consequence of φ.
[65] Interestingly, Russell (1919, p. 149) notes this sequentiality as something of a drawback, though a necessary one: " "p or q" implies "q or p". This [principle] would not be required if we had a theoretically more perfect notation, since in the concept of disjunction there is no order involved ... but since our symbols, *in any convenient form*, inevitably introduce an order, we need suitable assumptions for showing that the order is irrelevant" (italics added).
[66] Of course, strictly speaking, sentences, letters and words are two dimensional; and, moreover, there are alphabets (such as Hebrew and Korean) which utilise non-linear spatial relations between letters in forming words. But what is meant should be clear.
[67] Except of course for DC, which is the simplest of all; it can be inserted or erased from anywhere on SA.
[68] Here one notes what seems to be the guiding motivation of the rules of inference in EG. As Peirce puts it in an unpublished draft letter to Lady Welby (L463): "our objects of knowledge are limited. We cannot insert one without some evidence. We can erase any without incurring responsibility. Our objects of ignorance are an infinite multitude and we cannot exclude any without evidence, while we can insert any we like, since it [sc. the insertion] is merely to represent ourselves as ignorant."
[69] Contrast the more robust attitude taken by Frege regarding his notation, in an article of 1896: "the comfort of the typesetter is certainly not the *summum bonum*" (quoted in van Heijenoort 1967, p. 2).
[70] E.g. Hammer 1995, p. 96.
[71] As emphasized by Kneale and Kneale 1962, p. 521f. And of course a system such as Lukasiewicz's Polish notation, which avoids brackets, cannot generally be regarded as high in perspicuousness, since it has low transparency properties (see below). Unlike 5SL, moreover, it bears little or no visual similarity to sentences of natural language.
[72] For chunking cf. Kosslyn 1994, ch. 1 and Appendix 3.
[73] I am grateful to Marcus Giaquinto for suggesting and helping to formalise this terminology.
[74] Peirce notes carefully (L237): "it is rather a blemish upon [a logical] system if it needlessly distinguishes equivalent propositions. But there may be a good reason for doing so."
[75] Was Peirce aware of the relationship between EG and truth tables? Probably from the date of their inception, almost certainly from the early 1900s. Peirce uses what Shosky 1997 terms the truth table technique as early as 1885; Clark 1997 and Zellweger 1997 show that as early as 1902 Peirce had written out a truth table for the 16 binary truth functional connectives; and by 1909 Peirce had developed truth tables for a three-valued logic (see Fisch 1986, p. 171).
[76] E.g. Bostock 1997, pp. 43-4.
[77] Chapter 3 passim.
[78] Shimojima 1996 p. 31f.

[79] Shin 1998. For "seeing as" in general, cf. Giaquinto 1993b.
[80] See, for example, Kosslyn 1994, p. 7; Tye 1991 pp. 49-50 and references.
[81] One notes the parallelism with the *Begriffsschrift* notation, which also permits the phenomenon of "seeing as" (Frege 1879, paras. 5-7).
[82] Something similar to this process appears to have occurred in the case of Wiles's proof of Fermat's Last Theorem. Of course this does not detract at all from the achievement.
[83] That Peirce was well aware of the value of the cumulative proof structure of the *Elements* is confirmed by CP 4.427, where he specifically comments on the fertility of Euclid's method.
[84] E.g. CP 4.396, 6.203. I owe this point to Nathan Houser.
[85] As Ken Ketner has suggested, juxtaposition would also be permissible in the case of tautologies as the result of a *derived tautology rule*; that is, a derived rule to the effect that it is always permissible to write a tautology on SA, since it can never affect the truth value of whatever is already inscribed.
[86] The corrected proof is three lines long, and is therefore in fact *shorter* than the "standard-style" proof which Hammer gives for comparison purposes.
[87] Again, this flexibility to reorder, prompt and track information is characteristic of reasoning with diagrams; see Cox 1999.
[88] On the communication/construction distinction, see also Barwise and Etchemendy 1998.
[89] The interested reader should consult Ketner 1985, Zeman 1986, Shin 1997 and Levy 1997. See also Hintikka 1980 for Hintikka's (in my view, unsuccessful) attempt to relate the corollarial/theorematic distinction to his own views on quantification and surface form; this is discussed in the Ketner and Shin articles cited above.
[90] These are enunciation (*protasis*), setting-out (*ekthesis*), definition or specification (*diorismos*), construction (*kataskeue*), proof (*apodeixis*) and conclusion (*symperasma*). See Heath 1956, p. 129 ff.
[91] For discussion of the relationship between the two distinctions, see Shin 1997.
[92] As well as explanatory power, Peirce also suggests (e.g. CP 7.220) that hypothesis-selection be guided by considerations of testability and economy.
[93] On obtaining *a priori* knowledge from diagrams, cf. Giaquinto 1993b and 1998.
[94] But note that it may be true that for any proof in which a diagram occurs, there is another proof of the "same" conclusion with no diagrams.
[95] I owe much of this discussion to conversations with Marcus Giaquinto.
[96] Cf. Parsons 1980/1994.
[97] Of course, much more needs to be said here to justify these summary remarks, especially about the role of the indexical (in Peirce's sense) aspect of the sign. For a general treatment of this topic, see Liszka 1996, ch. 2.
[98] CP 3.457.

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