A DISCRIMINATION THEORY FOR THE MARKET POWER VERSUS THE EFFICIENCY HYPOTHESES UNDER REGIMES OF DISCRETE TECHNICAL SHOCKS

by

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ABSTRACT

A cooperative oligopoly model is developed where the production function of each firm is of a Cobb-Douglas type augmented for discrete and continuous, embodied and disembodied, exogenously determined technical change. Firms are divided into two groups depending on whether they have adopted the latest major process innovation or not. The econonomic rate of return to revenue is a function of collusion; the latter is parameterised and exogenous. The relation among relative market shares and relative technologies and rates of return for every two firms is established at the firm level while at the industry level the relation among the Herfindahl index, the ratio of the rates of return and the ratio of the technologies of the two groups is established. An analysis follows determining the circumstances under which the ambiguity between the market power and the differential efficiency hypotheses can be resolved by using either firm level or industry level conclusions. Two different versions of the model are considered: a) Capital adjustment is instantaneous and costless b) The cost of gross investment is a function of its own size.

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To the memory of my father,

Michael Dassios

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CHAPTER 5

CHAPTER 1

INTRODUCTION

1.1 A summary of the existing literature on the market power versus the differential efficiency hypotheses ambiguity.

The literature on the conflict between the market power versus the differential efficiency hypotheses may be classified into two categories. The first category tries to establish if there is any relationship between performance and market structure. It also seeks to describe the nature (causal or simultaneous) of this relationship. The empirical findings in the literature for the U.K. are quite divided as to whether in fact there does exist a statistically significant relationship between profitability and concentration. There is also contradictory evidence as to if this relationship is positive or negative. The second category focuses on identifying the nature of this relationship. Two different interpretations are offered for the existence of a positive and significant relationship. The so called collusion-profitability school (Bain, (1956), Collins and Preston, (1969)) argues that the higher concentration is, the higher tacit or explicit collusion will be. This will lead to an abuse of market power which will raise profitability above the competitive levels. On the other hand, the differential efficiency (Demsetz, (1973)) states that a positive correlation is reflecting the superior efficiency of some firms which are rewarded with high profits and high market shares.

The first category consists of articles which are concerned with the construction of static, dynamic or simultaneous equation models. Within such frameworks it is attempted to establish a relation between profitability and concentration, either of a theoretical or/and of an empirical nature.

Static models are usually one-period, profit maximising, Cournot or co-operative frameworks. Typical examples are the model by Cowling & Waterson (1976) and its later modification by Clarke & Davies (1982). In this second article the ambiguity as to whether collusion or efficiency is responsible for superior performance and high seller concentration is clearly demonstrated. In particular a theoretical relationship was established, which demonstrated that concentration (as measured by the Herfindahl index of concentration) is positively affected both by differentials in efficiency among firms (measured by the coefficient of variation of marginal costs) and by the degree of collusion within the industry (measured by the elasticity of conjectural variation). In a subsequent empirical paper by Clarke *et al* (1984) an attempt was made to first estimate this degree of collusion, and subsequently to test whether this was positively determined by concentration. More importantly, this article was one of the first steps towards intra-industry and time series analysis studies, regressing average margins against market shares employed from the census size distibutions for the period 1971-77.

Dynamic models tend to be profit maximising models over an infinite period horizon. The control variables in such models are the different types of expenditure that correspond to the price and non- price strategies the firm pursues in order to restrict potential entry and determine the stream of its profits. Using optimal control techniques, the optimal path for the firm's control variables is determined under the restriction of a function for the market structure (the state variable), which contains the variables that determine its evolution (Jacquemin (1971)). The sensitivity of the market structure to these variables determines the true degree of monopoly power the firm posesses because it describes the effect that different forms of firm conduct have on the long run market structure. The advantage of these models is that they investigate the feedback of the short run market power or differential efficiency to long run market structure. Therefore, they explicitly contribute to the question of endogeneity of the market concentration and its validity as a causal determinant of profits. More specifically, this refers to the market power abuse interpretation. The argument is that it is conduct that affects concentration rather than the opposite. As Jacquemin writes:

"...*Different forms of market conduct have the capacity to produce, either directly or as side effects, gradual changes in the structure of industries. According to such an approach, the more substantial is the market power of a firm, the more the market structure must be viwed as a strategic variable to the individual firm, not as an exogenously determined parameter. In the so called*

'market structure-conduct-performance' pattern, the causal relation can run from conduct to structure to performance."
The nature of the differential efficiency pattern is naturally not affected by the above argument since it is by definition that market structure and profitability are jointly determined through superior efficiency.

The need for models in which the concentration variable is endogenous triggered the development of simultaneous equations systems which incorporated an explicit concentration equation. For example, in Martin (1979) a dynamic concentration equation is included. This equation describes how concentration gradually approaches its long run equilibrium level following a partial adjustment rule. More generally, in simulaltaneous equations models (Geroski, (1982)), entry conditions, cost and demand conditions and industry concentration are considered to be mutually interacting and jointly determine industry profitability.

The second category mainly consists of articles which shift the focus of interest away from the industry level towards the firm level of analysis. The logic behind this shift is that the relationship between profitability and concentration (whether positive or not) contributes nothing to the resolvance of the differential efficiency- market power hypotheses ambiguity. The reason a positive relation reveals nothing has been already discussed in the opening paragraph and is in fact what triggered the ambiguity in the first place! On the other hand, a negative (or stricly speaking non-positive) relationship can either mean a more pro-competitive behaviour within the more concentrated industries or that the firms within such industries opt for a quiet life. Obviously, this second view has to be ruled out since the concept is based on managerial theories of the firm rather than on a profit maximising behaviour framework. However, some comfort can be drawn from the fact that according to an empirical study conducted by Clarke (1984) for the period 1970-76 in the U.K., profit margins were found to be more volatile in more concentrated industries. This argues against the possibility of a cosy arrangement among firms. Sticking with the first interpretation, which is compatible with the profit maximising condition, to the extent that concentration is the result of high collusion, a negative relationship points to a negative connection between profitability and collusion. This means that the latter has a pro-competitive impact, implying the rejection of the collusion-profitability hypothesis. Yet, the very same argument can apply for refuting the differential efficiency hypothesis. Therefore, whatever the sign of the relationship between market structure and profitability, the ambiguity between the two hypotheses can not be resolved since these have not been expressed in a way that makes them mutually exclusive at an industry level of analysis. Consequently, Demsetz offered an alternative way of approaching the subject. According to his theory, the effect of collusion is a public good while the effect of efficiency a private one. Therefore, the former benefits -in terms of profitability- small and large firms equally, while the latter guarantees high market share and rents for those who posess it. Firms who do not posess it suffer from a combination of low profitability and market share. Therefore, if the differential efficiency hypothesis is correct then the profit rates of small firms in highly concentrated industries should not differ from the profit rates achieved by small firms in less concentrated industries. If the collusion profitability hypothesis is at work, then the profitability of small firms in highly concentrated industries should exceed the profitability of small firms in less highly concentrated industries. Based on this Demsetzian perception, Harris (1988) argued that if firm profitability is positively related to the firm's market share and non positively related to the industry concentration, then this is sufficient for rejecting the collusion hypothesis when there is no product differentiation in the market. Note however that the Demsetz theory is actually **totally contradictory** to the seminal versions of the collusion-profitability school which explicitly rejected the argument that market concentration would increase the profitability of all firms; see more on this in the next section.

1.2 Re-stating the two hypotheses within a new model.

In the model cosidered below, innovation as well as collusion are treated largely as **exogenous** to the industry and the benefits of both are considered to be **private** goods.

Innovation is exogenous as in the Demsetz model. Therefore, as in Nelson and Winter (1978) **the regime of Schumpeterian competition is reversed by having the causal linkage running from the pace and pattern of innovation to the level of concentration.** This argues that successful innovators are rewarded with a relatively higher market share as compared to that of their rivals. Relatively higher profit rates **may** also be the reward of success if the adopters conjectures (implicit colllusion or/and tacit collusion) ensure that they produce at a level of output where their marginal costs are lower than the marginal costs of their competitors. However, the model also allows for the possibility that the conjectures of adopters will be such that they will determine an optimum level of output for these firms at which their marginal costs are higher that the marginal cost of the laggard firms. Obviously the probability of something like that happening is higher the smaller the gap in productivity between the innovators and their rivals is.

The exogeneity of collusion has already been discussed in the beginning of this chapter when refering to the work of Jacquemin and the endogeneity of the market structure.

The fact that the only form of innovation in the model is reduction in the production cost of a single, homogeneous product is not as restrictive as it may at first appear. Nelson and Winter (p. 530) argue that such a "..*model can equally well be interpreted in terms of product innovation. In this alternative interpretation, product improvement involves an equiproportional increase in the effectiveness of the product in every use; when such an improvement can be achieved with a smaller proportional increase in unit production cost, there has been an effective reduction in the cost at which 'efficiency units' are produced. To put it in another way, the firms in the model may be thought of as producing physically heterogeneous products that embody varying amounts of a single Lancasterian 'characteristic' per physical unit. The demand curve of the model is then to be interpreted as the demand curve for the characteristic, and prices of different products will vary in relation to the amount of the characteristic they contain. Thus, mere physical heterogeneity of the products of different firms does not necessarily render our model inapplicable. And, to the extent patterns of that buyer preference among products are dominated by the variation in a single quality dimension, the 'cost reduction' approach may, at a minimum, be a useful approximation*"

The model is based on the two distinct theories of innovation developed by Schumpeter. In his later work he maintained that the large existing enterprises within an industry are the engine of technical change. This suggests that innovation will be more intensive the larger the firms are. This contradicts his earlier work, in which he described the outsider, newcomer innovator enterpreneur as the main motive behind technical change. (For a distinction between these two see Winter (1984), pp.294-297) His later work is closer to the Demsetz theory although, as already mentioned, causality in Schumpeter is reversed since for Schumpeter it is the large enterprise which causes innovation and not vice versa. This obviously contrasts with his early work in which he argues that there is no reason why superior efficiency should be connected with a higher market share. In the model to be discussed each industry's firms are separated into two groups using as a criterion whether the firm in question has adopted the latest available production technique. Each innovation may originate either from an industry other that the one in which the innovation is applied (innovative entry and transfers) or from already existing firms within the industry in which the innovation is applied. According to empirical findings by Geroski and Steward, 1982, (more on this paper below) "*...entry innovations are somewhat more "technologically important" than the rest, and this feeling is strengthened by noting that* 36% *involve a new product* or *process...*".

Turning now to the assumption that not only innovation but also collusion is a private good, this is in direct contradiction to Demsetz. He considers collusion a public good and accordingly bases his resolving of the ambiguity on this supposedly public nature of collusion. **This model rejects the Demsetzian conviction of collusion benefiting small and large firms equally; instead it**

agrees with what Martin (1988) wrote:

"*...Failure to find a positive effect of market concentration on the profitability of smaller firms is taken as evidence against a " concentratiom-market power" relationship. But seminal versions of the concentration-market power hypothesis suggested that concentration would allow the exercise of market power primarly by large firms....*

A finding that concentration benefits primarily the profitability of larger firms would confirm these versions of the collusion-profitability hypothesis, which suggests that large firms are more profitable in concentrated industries because industries become concentrated when it is efficient to organise production in large units. The efficiency-profitability hypothesis has not been advanced in a way which distinguishes it from the collusion-profitability hypothesis....The "efficiency" and "market power" interpretations of a positive effect of market concentration on price-cost margins are complementary rather than mutually exclusive."

On this basis, the argument by Harris is incorrect since a negative (non positive) relation between concentration and profitability combined with a positive relationship between market share and profitability suggests only that it is large firms that reap the profits of monopoly power. The fact that this monopoly power is a private good no longer resolves the ambiguity because it can be either collusive practices or superior efficiency that are resposible for the existence of large dominant firms.

More interestingly, if the negative (non positive) relation between concentration and profitability is combined with an also negative relation between profit rate and market share then again the ambiguity is not resolved. If large firms own their bigness to superior efficiency then this **may or may not** lead to higher profitability depending on the conjectures of the firms. The definition of collusion in this model is broad since this may be either in an explicit and/or in a tacit form. Collusion is expressed in terms of what each firm conjectures that the degree of sensitivity of total output with respect to a change to its output is equal to. In other words, the collusion measure is constructed by **parameterizing the elasticity of the total industry output variation** for each firm (which is discussed in detail in a later chapter) and treating it as **exogenous** to the model. Consequently, when it is said that a firm is **more collusive** than another firm this translates as the former firm having a **higher elasticity of total industry output variation** (henceforth, ETIOV) than the latter. How does the above affect the interpretation of a negative relation between profitability and concentration combined with an also negative relation between market share and profitability? If collusion is in the form of conjectures expressed by ETIOV, then since conjectures are always positively related to profitability then the ETIOV will also be positively related to profitability. To the extent that this happens through an indirect route it means that collusion (ETIOV) affects market structure (creating dominant firms with monopoly power) and as a consequence profitability is affected. If this is true however, then it is not possible to have a negative relation between market share and profitability. Therefore when this relation is negative, this signals that ETIOV affects directly profitability and not through the route of increased monopoly power; a fact that negates the collusion hypothesis. To the extent that efficiency leads to growth but not to relatively higher profit rates, it implies that the large innovative firms have unfavourable conjectures that do not permit them to translate their superior efficiency into higher profitability. That however does not deny the differential efficiency hypothesis, it just means that efficiency is rewarded only in terms of growth and not in terms of profitability, since profitability is seperately determined through other direct or indirect sourses which are ultimately linked to the conjectures a firm bears.

Given the above arguments let us formalize the framework and define the hypotheses within it. Two distinct models are to be developed; the criterion for defining each one is the degree of flexibility of the capital. In the first model there is no capital adjustment cost and each firm adjusts its capital and labour inputs instantaneously. Therefore the long run profit maximisation model degenerates into a single period model. In the second model the cost of gross investment measured in efficiency units is a function of its size, making it necessary for the firm to explicitly look to the future. This implies a non trivial maximisation of the present value of its future stream of profits.

Although there is a stable long run equilibrium level of capital stock (towards which the firm moves either instantaneously or through a flexible accelerator model), this will be shifted by unexpected discontinuous jumps in the level of technology. In both models firms have constant expectations regarding the market conditions and the objective is to establish a relationship between the Herfindahl index and the ratio of economic profit rates of the two groups of firms.

It shall be demonstrated that **at the firm level** under certain conditions superior efficiency, even if combined with a higher degree of collusion, is not always linked to a higher market share. In such a case the DEH is rejected at the firm level. For example, several studies based on a data set for the U.K. compiled at the Science Policy Research Unit identifying more that 4000 major innovations over the period 1945-83 (Geroski and Steward, (1986), Pavitt *et al*, (1987), Geroski and Pomroy, (1988)) have noted a suprisingly large percentage of major innovations for small firms in very innovative, skill intensive industries. This challenges the Schumpeterian presumption that large size is necessary to faciliate innovativeness which had an effect on the European policy for supporting the creation of large firms in high technology industries. Note however, that in the model the case for superior efficiency does not require (as the Demsetz theory does) higher profitability and as a side-effect a higher market share. It merely requires that the more efficient firm will have a greater market share than the less efficient firm. Whether this superior efficiency will be rewarded by a relatively higher profitability depends on how the output decision of the firm will be affected by its conjectures. Therefore the differential efficiency hypothesis does not necessarily require greater profitability for the more efficient firm. Provided that certain conditions are satisfied, this will result in a negative relation at the industry level between profit rates and concentration. Consequently, **the differential efficiency hypothesis in the model (henceforth, DEH) is an augmented Demsetz theory capable of explaining both a positive as well as a negative relation between performance and market share. It should be stressed that this model DOES NOT interpret greater market share as evidence of lower cost; it merely states that given lower cost, if this is rewarded by a greater market share then the DEH is to be accepted (or more properly not rejected) at the firm level. On the other hand, if the relatively more efficient firm has a lower market share the differential efficiency hypothesis is to be rejected. Also, the model demonstrates that higher collusion is not always associated with a greater market share; if collusion is high, small firms may be attracted into the market and the market shares of the incumbents may actually decline. However, collusion in my model IS NOT a public good as Demsetz argued; it is more likely to benefit large firms in terms of profitability. But if small firms are more collusive in their behaviour than large firms are, then even if large firms are more profitable the market power hypothesis should be rejected; it is not via the abuse of market power that these**

large firms achieved this higher profitability. Consequently, in our model a greater market share -not a higher profit rate- is the crucial measure for rejecting or accepting the market power hypothesis (henceforth, MPH) at the firm level.

So at the firm level of analysis, the criterion for accepting or rejecting the differential efficiency hypothesis is whether a difference in efficiency between two firms has as a consequence a higher market share for the relatively more efficient firm (if it is not , then we shall reject the DEH). **At the industry level the criterion is whether an increase in the differential efficiency between two firms will lead to an increase or a decrease in concentration.** Geroski and Pomroy's empirical findings suggest that the introduction of innovations has a deconcentrating effect possibly because the small innovating firms grow at the expense of the large laggard firms. In terms of this model, such a finding implies the rejection of the DEH at the industry level since higher concentration can not be explained in terms of superior efficiency.

The market power hypothesis is to be accepted (rejected) at the firm level if the relatively more collusive firm (in terms of ETIOV) has a higher (lower) market share. **At the industry level the market power hypothesis is acceepted (rejected) if a further relative increase (decrease) in the ETIOV of the firm which already had the higher (lower) ETIOV between these two firms will lead to an increase in concentration.**

The decision not to use profitability as the crucial criterion in the model is based on the fact that "...*The "efficiency" and "market power" interpretations of a positive effect of market concentration on price cost margins are complementary rather than mutually exclusive*" and therefore incapable by definition of resolving the ambiguity between the two hypotheses.

CHAPTER 2

SETTING THE ENVIRONMENT OF THE MODEL

In the first section of this chapter the rules for seperating each industry into two groups are set out. Then the variables included in the production function are introduced and discussed in detail. In the second section alternative measures of capital are constructed by incorporating wholly or partly technology into the capital stock variable. In the third section the cost function for each firm is derived by using profit maximising model which assumes competitive behavior. In the final section a summary list of the variables defined in this chapter is given since these are going to be used repeatedly over the rest of the thesis.

2.1 The division of an industry in technological terms.

The firms within an industry will be divided into two groups, adopters (group A) and non adopters (group NA). The adopters'group consists of these firms that apply the best (in terms of output productivity) production method. The total number of these firms, N_t^A, is a function of imitation and innovation. On the other hand, $N_t - N_t^A$ denotes the number of firms that have not adopted the best available technique at time t.

A firm, or a group of technologically cooperating firms, introduces a major innovation at time t. Automatically, N_t^A will become equal to the number of firms that simultaneously introduced the innovation. If this latest innovation is fully protected by a patent, then N_t^A will not change until a new innovation occurs, when it will become equal to the number of firms in the group that introduced this new innovation. In this model technological lead changes hands over time as innovating firms can originate either from the former non-adopters group or from the former adopters group or from firms that have just entered the industry (entry innovations).

N_t^A does not refer to a single innovation but rather to a sequence of innovations. This implies that many imitation procedures, each corresponding to a different innovation, can go on simultaneously within the industry. Once a new innovation occurs the process of imitation for this specific innovation will commence either immediatelly, or after the patent protection period is over. This process will spread the new technology among the firms in the industry. This does not necessarily mean that the imitation process for the former technique is abandoned altogether; lack of information among different firms, the fact that the new innovation may involve only minor cost reductions, or more importantly complete patent protection for the latest innovation, might have as a result the phenomenon of some firms adopting earlier techniques.

Alternatively, the division of the industry may be viwed in terms of cost. AC_t^A denotes the **average** of the unit costs of the N_t^A firms which produce using the latest (lowest cost) production method at time t. The term "average" for AC_t^A is used to stress that the usage of the same production technique does not necessarily imply uniform unit costs for the group of firms that applies it. This will vary from one firm to another, e.g., depending on the level of output each firm produces (existence of scale economies), absolute capital advantages (advantages of a firm in raising money in imperfect capital markets) and differences in the prices of raw materials each firm faces (control over the inputs). In this model only scale economies are explicitly incorporated into the cost function. Equivalently, AC_t^{NA} denotes the **average** unit cost for the $N_t - N_t^A$ firms that do not use the best (in terms of cost) production method at time t. The difference between AC_t^{NA} and AC_t^A will be the source of excess profits for the adopters' group.

To illustrate the above arguments, suppose that the best available production method at time t applied to firm j's capital stock may be represented by the following function:

$$Q_{jt}^A = f_1(K_{jt}, L_{jt}; t)$$

where L_{jt} represents the quantity of labour and K_{jt} is the quantity of capital **measured net of both physical and technical (embodied and disembodied) obsolescence**. This means that we take as a unit of capital a unit of investment which embodies the latest technology available at time t. Thus the contribution to K_{jt}, as measured here, of one unit of investment at time $s < t$ is less than one full unit both because that investment has deteriorated physically and also because it is technically less efficient than one unit of investment at time t (Nelson, (1964) footnotes 13 and 18). This point will be further expanded in section 1.2. Q^A_{jt} is the output produced by a firm j (j= 1,2,...N^A_t) belonging to group A at time t. The time dimension is also seperately included in the production function to account for technical change. If we assume that the above function is of a Cobb-Douglas form then :

$$Q^A_j = A_t \upsilon_j (K_{jt})^a (L_{jt})^b = A_o \upsilon_j A'_t A''_t (K_{jt})^a (L_{jt})^b \quad j \in A \quad (2.1)$$

A'_t is an index for organizational technical change, in other words of all disembodied technical change concerning improvements either in the combination of existing inputs or in the quality (in terms of eduction, age and sex) of the labour force and other economy-wide contributions by government sponsored, academic and independent research. It encompasses all past and current innovations by former and current technologically leading firms. Let,

$$A'_t = \exp(yt) \quad (2.2)$$

where y is the rate of disembodied technical change which depends on the average managerial performance of the group. Managerial skills are considered as an additional input in the production function of the firm. The difference between this input and the other two is that while both the magnitude of capital and labour are determined by the firm the degree of managerial competence is not. One can not 'increase' or 'decrease' the latter as one pleases; therefore it may be reasonably argued that it enters the production function in an exogenous fashion. Although successful managerial practices can always be copied by rivals, the ability to cope with change is not. It is this ability, which an individual either has or has not, that determines to a large extent the rate of imitation. Differences among firms in managerial ability are accounted for by the firm specific effect υ which is described below.

A_t'' is the index of the 'quality-of-capital' technical change, the part of technical change which refers to all improvements in the quality of capital given its age distribution. Let:

$$A_t'' = \exp(a\text{L}t) \qquad (2.3)$$

where L is the average rate of quality improvement of new machinery in the industry being considered. Differences between firms in the rate of qualitative capital improvement are accounted for in the measure of capital K, which is net of obsolescence i.e. differences between firms as far as the synthesis (age distribution, since capital is of more than one vintage) of their capital is concerned have been incorporated into K.

A_o is an index for the productivity induced by that part of technology which is considered as constant by all the firms. Unexpected, discontinuous, discrete jumps in A_o, o = 1, 2, ... (i.e. from A_1 to A_2 say) constitute the source of the disequilibrium force Schumpeter described (creative distruction) and may be perceived as representating the major innovations within each industry.

υ_j reads as a time invariant variable that exhibits how efficient a firm belonging in the adopters' group is when it comes to applying major innovations. Jumps in υ's imply jumps in firm specific differences in managerial ability; these jumps are **independent** of jumps in A_0. Defining $\upsilon M_j = \ln(\upsilon_j)$ we have,

$$\upsilon_j = \exp(\upsilon M_j) \quad (2.4)$$

A_0 is not productivity actually realized by any firm in particular, it just denotes the potential maximum in productivity the latest major innovation offers. A_0 times υ_j gives A_{0j}, the **realized productivity** for firm j. This realized productivity is always below the currently available maximum potential productivity, i.e. υM_j's, $\forall j \in A$, are negative and consequently $A_{0j} < A_0$. Contrary to the neoclassical view, such differentials will persist in the long run. The disequilibrium Schumpeterian forces of technical change put firms into a situation where they have to face discrete, unexpected advancements in technical knowledge. Firms incorporate these advancements into their isoquant curves, each with a different degree of success depending on its managerial ability of adopting new methods of production. If the best available method of production did not change then these differentials in the adoption of this technique would gradually converge. Furthermore, through imitation all firms within the industry would adopt this technique. Subsequently, in the abscence of cost advantages profits in the long run would converge towards the normal competitive rate. So the classical model is just a special case of the Schumpeterian model. But as a general rule differentials persist and it is only the identity of the firms to whom these differentials correspond that changes. Managerial firm specific deviations from the productivity potential A_0 within group A may also reflect the fact that in a discrete time model two different firms both recorded as being in the adopters' group for a given period may well have introduced the latest technique at different points within that period. Therefore, that proportion of output recorded as having been produced in that period by a new technology differs from firm to firm. Since the model to be developed is in continous time, it is constrained by the simplification that during each very small time interval ∂t the whole of the firm's output has been produced by using the best technology available during this infinitesimal time interval. Differences among firms due to the fact that this is not entirely correct are subsumed in υ_j's.

To summarise the "growth of total output productivity" (Nelson, p.583) is going to be equal to:

$$\frac{\frac{\partial(A_t\upsilon_j)}{\partial t}}{A_t\upsilon_j} = \frac{A_0\upsilon_j(y+a_L)\exp[(y+a_L)t]}{A_0\upsilon_j\exp[(y+a_L)t]} = y+a_L \qquad (2.5)$$

This includes a_L which denotes advances embodied in new capital and y which does not. Note that $A_0\upsilon_j$ is not included in this term. This has happened because during an infinitesimal time interval A_0 is constant since it consists of discrete, distant in time innovations and therefore the probability of a jump occuring during ∂t is zero. These discrete innovations are unexpected both in their timing as well as in their magnitude. As a result they do not have any impact on the formations of expectations by the firms. Instead they are perceived as isolated, permanent, once-and-for-all jumps which may well be revolutionary, major and highly significant, but nevertheless not the sort of changes which firms can reasonably bet on their timing and magnitude. Consequently, the growth in total factor productivity is viwed as the result of a continuous chain of minor innovations. A permanent change in the values of y and/or L reflects a permanent change in the rate of continuous technical advancement characterising the industry. The production function for firm j, $j \in A$ (omitting the time subscripts from the output and factor variables) is:

$$Q_j^A = A_t \upsilon_j (K_j)^a (L_j)^b \tag{2.6}$$

The implicit assumption that makes possible the separation of an industry into two groups is that technical change (embodied or disembodied) does not alter the **shape** of the production function. If this did not hold it would not be possible to rule out the possibility of having two production techniques of which the one is superior to the other within certain ranges of output and inferior within others. This means that total cost curves should not have any points of intersection i.e. no 'switching' is permitted. Restricting all production functions to be of the same functional form ensures that.

In this model it is assumed that the series of major innovations given by o = 1,2.. m have all been granted patents for some specific period of time. (In U.K. when a patent is granted it is valid -as long as renewal fees are paid- for sixteen years.) This is based on the wide spread belief that patents are used to protect major innovations rather than mere improvements in production techniques (Stoneman, (1987)). However, there are objections as to how effective a measure of major innovations patents are. As Griliches *et al* (1988) argue, patents may be more of an intermediate output rather that a final output of superior products or production processes. Patents are applied for at an early stage of the inventive process. So there is ambiguity as to whether the invention will actually turn out to be a "winner" innovation. Obviously this uncertainty will be resolved during the first three or four years of the patent's life. Therefore renewal data on patents may prove very useful in constructing more accurate measures of patented output since they are an index of the quality of the patent which can supplement the quantity based patent count data. These more accurate measures may then be used to ensure that each subsequent innovation will be strictly better in terms of productivity as the restriction $A_1 < A_2 <\ldots< A_m$ requires. It is assumed that for as long as a new technology reigns it is going to be pefectly protected by a patent. The innovators have an interest to do that since as their innovation is the best within the industry, their returns from this patent justify payment of the annual renewal fee.

Each patent within group NA will become available to all of the firms within the group as soon as the time for which the patent was valid expires or even before that if the patentee judges that the returns from his patent no longer justify the payment of the annual renewal fee. The annual returns from patent protection decay very quickly over time, with rates up to 20 percent a year as Schankerman and Pakes (1986) write. Of course some non adopters may choose to adopt none of the available production techniques if they expect that the discounted future returns from adopting the production technique do not cover the costs of embodying it.

While for group A the index measuring the potential productivity is A_o, for group NA the index is B_o and it measures **the potential productivity accruing from the most efficient (in terms of productivity) innovation among the innovations that are available to all the firms in group NA, i.e. the innovations that are no longer protected by a patent**. Let τ^o_{o-1} denote the time gap between two consequtive jumps in technology, i.e. the time that elapses before A_{o-1} jumps to A_o. If technology say o = m is overcome by technology o = m+1, potential productivity in group A will jump from A_m to A_{m+1}. ($A_1 < A_2 < \ldots$). Then let t_{o-1} denote the time that elapses after innovation m+1 has occured (in this case o=m+1) and before innovation m (o-1=m) becomes available to **all** the firms in group NA because its patent either expired or was not renewed. Then it is assumed that a) τ^o_{o-1} is sufficiently large so that there is always time for innovation o-1 to become available before innovation o+1 occurs and therefore $t_{o-1}<\tau^o_{o-1}$ b) major innovations are protected by sufficiently lengthy patents to guarantee that the life of a patent will always be greater than the time gap between two innovations, which implies that $t_{0-1} > 0$ and A_0 is always larger that B_0. So when innovation m+1 occurs, A_0 jumps immediately from A_m to A_{m+1} while B_0 remains for a time equal to A_{m-1} (it had assumed that value before innovation m occured) and after a period equal to t_m it jumps to the value A_m. In other words a jump in A_0 will always be followed by a jump in B_0 after t_{0-1} time has elapsed.

Let V denote the gap in potential productivity between the two groups. When $t < t_{0-1}$ this will be equal to:

$$V = \frac{A_0}{B_0} = \frac{A_0}{A_{0-2}} > 1 \qquad (2.7)$$

When $t \geq t_{0-1}$, V becomes equal to:

$$V = \frac{A_0}{B_0} = \frac{A_0}{A_{0-1}} > 1 \qquad (2.7')$$

The justification for using such oversimplifying assumptions is that the model is not interested in the workings of innovation in itself, but rather on the the impact of innovation on productivity and how this is gradually 'passed on' to the market structure. More simply, what is pursued here is the original Schumpeterian version of creative destruction which postulates a long term impact of innovation on industrial structure. In this thesis this long term impact is isolated and investigated while taking the shorter term effects of structure on innovation as given. (See Baldwin and Scott, (1987), section 3.3 on the post 1960's reformulated version of the Schumpeterian hypothesis.)

The role of υ's in group NA is exactly the same as the role of υ's in group A and the former shares the same properties with the latter, i.e. they are time invariant and jumps in their values are independent of jumps in A_0. Defining $\upsilon M_l = \ln(\upsilon_l)$ we have,

$$\upsilon_l = \exp(\upsilon M_l) \tag{2.8}$$

where υM_l's are negative and therefore $B_{0l} = B_0 \upsilon_l < B_0$. Note that differences in productivity among firms in group NA owing to the fact that in this group, unlike group A, firms use different innovations, are accounted for in the capital stock measure K_{jt} and <u>not</u> in the υ's. The former includes the embodiment effect of all major discrete innovations and minor continuous technical improvements by accounting for the age distribution of the capital stock of firms.

To avoid overlapping between the two groups the following restrictions are imposed to hold for the gap in realized productivity for all $j \in A$ and $l \in NA$ at all times, irrespective of whether potential productivity in group NA has yet jumped into its new value or not.

$$V \frac{\upsilon_j}{\upsilon_l} > 1 \tag{2.9}$$

Another important issue is what exactly a and b stand for in our model. These are the <u>static</u> <u>output</u> <u>elasticities</u> with respect to capital and labour respectively. In other words they account for how changes in the magnitude of these inputs- but NOT on how changes in their productivity - will affect output. If in our model capital was not clear of technical obsolescence then in the "growth of total factor productivity" the part that needs to be embodied would not be equal to 'aL' but instead to $aL\left(1-\frac{\partial \bar{g}_{jt}}{\partial t}\right)$ where $\frac{\partial \bar{g}_{jt}}{\partial t}$ is the change in the average age of capital within the time interval ∂t (Nelson, p.586). In other words, $-L\frac{\partial \bar{g}_{jt}}{\partial t}$ accounts for the change in the gap between average quality and the quality of new equipment between time t and time $t+\partial t$. As a result a static model would overestimate 'a' and will also mislead us to believe that 'a' varies through time and among firms, while in reality it is the dynamic part $aL\frac{\partial \bar{g}_{jt}}{\partial t}$, which has not been accounted for, that is resposible for this variation. Therefore in our model 'a' is uniform between firms and stable through time since the differences in the age of capital have been already incorporated in K_{jt}. For this reason, it is henceforth assumed that the static output elasticities given by 'a' and 'b' are uniform among all firms and time invariant.

2.2 Definitions of the capital input.

Let us now develop in more detail the various definitions of capital to be used throughout this thesis. Commence by defining C_t, the capital in physical units:

$$C_t = \int_0^t I_s \exp[-\delta(t-s)]ds \qquad (2.10)$$

where I_s is gross investment in physical units at time s, where s < t and δ is the physical rate of depreciation . J_t^E is defined as capital in efficiency units:

$$J_t^E = \int_0^t I_s \exp(Ls)\exp[-\delta(t-s)]ds \qquad (2.11)$$

where L is the rate of capital augmenting technical progress and $I_s\exp(Ls)\exp[-\delta(t-s)]$ denotes how much the amount of investment made at time s is worth at time t using time's 0 investment units. J_t^F will be capital in full efficiency units including both embodied and disembodied technical change:

$$J_t^F = \int_0^t I_s \exp[(L+ \frac{y}{a})s]\exp[-\delta(t-s)]ds \qquad (2.12)$$

Finally, K_t denotes capital in reverse full efficiency units and is equal to:

$$K_t = \int_0^t I_s \exp[-(\delta+L+ \frac{y}{a})(t-s)]ds \qquad (2.13)$$

This explains clearly the argument that K_t incorporates the decay of capital due to continuous technological improvement. Consequently, (2.6) may be rewritten as follows:

$$Q^A_{jt} = A_0 \exp(\upsilon M_j)(J^F_{jt})^a (L_{jt})^b \quad \Leftrightarrow$$

$$Q^A_{jt} = A_0 \exp(\upsilon M_j)\left[\int_0^t I_{js}\exp[(L+\frac{y}{a})s]\exp[-\delta(t-s)]ds\right]^a (L_{jt})^b \quad (2.14)$$

where the term in the brackets is capital in full efficiency units, J^F_{jt}. Further modifying the above relation we get:[1]

$$Q^A_{jt} = A_{oj}\ \exp[(aL+\ y)t]\left[\int_0^t I_{js}\exp[-(\delta+L+\ \frac{y}{a})(t-s)]ds\right]^a (L_{jt})^b \quad (2.15)$$

Therefore the term in the brackets in the relation above is K_{jt}, capital in reverse full efficiency units. If we combine (2.14) with (2.15) we get:

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[1] As long as the production function is homogeneous, we can embody the disembodied rate of technical change. This means that:

$$Q^A_{jt} = A_0 \exp[yt + \upsilon M_j] f_2(J^E_{jt}, L_{jt})$$

If the above production function is homogeneous of degree u then this implies that:

$$Q^A_{jt} = A_{oj}\exp\left[\left(\frac{y}{u}u\right)t\right] f_2(J^E_{jt}, L_{jt}) =$$

$$= A_{oj} f_2\left[\exp\left[\left(\frac{y}{u}\right)t\right]J^E_{jt},\ \exp\left[\left(\frac{y}{u}\right)t\right]L_{jt}\right] = f_2(J^F_{jt},\ L^E_{jt})$$

where now labour is measured in efficiency units incorporating the disembodied technical progress. However, in the special case of a Cobb-Douglas production function because of its multiplicative form, this disembodied technical change has to be embodied in <u>either</u> of the two inputs or to be <u>split</u> between them. In no way does this special formulation implies a loss of generality.

$$J^F_{jt} = K_{jt}\exp\left[\left(L + \frac{y}{a}\right)\cdot t\right] \qquad \Leftrightarrow (2.16)$$

$$J^F_{jt} = \exp(-\delta t)\,{}_0\!\int^t I_{js}\cdot\exp\left[\left(L + \frac{y}{a} + \delta\right)s\right]ds \qquad (2.17)$$

Differentiating (2.17) with respect to time we obtain:

$$\dot{J}^F_{jt} = \exp\left[\left(\frac{y}{a} + L\right)t\right]\cdot I_{jt} - \delta J^F_{jt} \qquad \Leftrightarrow (2.18)$$

$$\dot{J}^F_{jt} = I^F_{jt} - \delta J^F_{jt} \qquad (2.19)$$

where I^F_{jt} is the gross investment at time t in full efficiency units. If we differentiate K_{jt} with respect to time we get:

$$\dot{K}_{jt} = I_{jt} - \left(L + \frac{y}{a} + \delta\right)K_{jt} \qquad (2.20)$$

Consequently, (2.19) and (2.20) give the capital accumulation equation for capital in full efficiency units and capital in reverse full efficiency units respectively. It will make no difference for the oligopoly model with no adjustment costs (developed in the next chapter) whether K_{jt} or J^F_{jt} is used as a measure of capital. But when developing the oligopoly model with adjustment costs, if these adjustment costs were expressed as a function of K_{jt} instead of J^F_{jt} then, as it is easily demonstrated in the appendix of the chapter developing this model, the adjustment costs would tend to zero as $t\to\infty$. Consequently, the adjustment costs model would degenerate into a no adjustment costs model.

2.3 Profit maximisation under perfect competition.

To derive the total, average and marginal cost function for each firm in each group we shall use a profit maximising model in which competitive behavior is assumed, i.e. each firm believes that the price will not change in respose to a change in its output. Then let the present value of the future flow of profits for firm j, j∈A, be equal to:

$$\int_0^{\infty} \exp(-ht)\pi_{jt}dt$$

Firm j wishes to maximise the above function subject to the two conditions given by the production function and the capital accumulation equation respectively:

$$Q_{jt}^{A} = A_t \upsilon_j (K_{jt})^a (L_{jt})^b$$

$$\dot{K}_{jt} = I_{jt} - \left(L + \frac{y}{a} + \delta\right) K_{jt}$$

where *h* is the market interest rate. The corresponding Lagrangean function is:

$$\mathfrak{L} = \int_0^{\infty}\left(\exp(-ht)\left(pQ^{A}_{jt} - wL_{jt} - q_t I_{jt}\right) + \right.$$

$$\left. + \lambda^*_t \cdot \left(\dot{K}_{jt} - I_{jt} + \left(L + \frac{y}{a} + \delta\right)K_{jt}\right)\right)dt$$

q_t is the price of new investment in physical units and w the price of a unit of labour. The first order conditions (with respect to L and I) are :

$$p\ b\left(\frac{Q^{*A}_{jt}}{L^{*}_{jt}}\right) = w \qquad (2.21)$$

$$-\exp(-ht)q_t - \lambda^*_t = 0 \qquad (2.22)$$

and using the Euler-Lagrange equation:

$$\dot{\lambda}^*_t = \exp(-ht)pa\left(\frac{Q^{*A}_{jt}}{K^{*}_{jt}}\right) + \left(L + \frac{y}{a} + \delta\right)\lambda^*_t \qquad (2.23)$$

Differentiating (2.22) with respect to time gives:

$$\dot{\lambda}^*_t = \exp(-ht)(-\dot{q}_t + hq_t)$$

Substituting for $\dot{\lambda}^*_t$ and λ^*_t in equation (2.23) gives:

$$p\ \frac{\partial Q^{*A}_{jt}}{\partial K^{*}_{jt}} = \left(h + L + \frac{y}{a} + \delta\right)q_t - \dot{q}_t$$

q_t has to be equal to $c_t \cdot \exp\left[\left(\frac{y}{a} + L\right)\cdot t\right]$ where c_t is the price of gross investment in full efficiency units. If q_t was more or less than $c_t \cdot \exp\left[\left(\frac{y}{a} + L\right)\cdot t\right]$ then as it is demonstrated in the appendix of the chapter on the oligopoly model with adjustment costs, the price of new investment would tend to zero as t ∞. Therefore,

$$p \frac{\partial Q^{*A}_{jt}}{\partial K^{*}_{jt}} = \exp\left[\left(L + \frac{y}{a}\right)t\right] \left((h + \delta) c_t - \dot{c}_t\right) \qquad (2.24)$$

Relation (2.24) gives the marginal productivity of capital. The R.H.S. of the above relation will be henceforth summarised by r_t which is the user cost of capital in reverse efficiency units.[2]

It is now possible to re-write the profit maximization problem as:

$$\int_0^{\infty} \exp(-ht) \left(pA_t \upsilon_j (K_{jt})^a (L_{jt})^b - wL_{jt} - r_t K_{jt}\right) dt$$

The first order conditions are:

$$p\ A_t\ \upsilon_j\ b\ (K^{*}_{jt})^a (L^{*}_{jt})^{b-1} = w \qquad (2.25)$$

$$p\ A_t\ \upsilon_j\ a\ (K^{*}_{jt})^{a-1} (L^{*}_{jt})^{b} = r_t \qquad (2.26)$$

2 Unless otherwise explicitly stated, it is assumed throughout the thesis that firms have constant expectations regarding h, y, δ, L, w and p.

Dividing (2.25) by (2.26) :

$$\frac{w}{r_t} = \frac{b}{a}\frac{K^*_{jt}}{L^*_{jt}} \Leftrightarrow K^*_{jt} = \frac{w}{r_t}\frac{a}{b} L^*_{jt} \qquad (2.27)$$

then :

$$Q^{*A}_{jt} = A_t \cdot \upsilon_j (L^*_{jt})^b (L^*_{jt})^a \left(\frac{w}{r_t}\frac{a}{b}\right)^a \Leftrightarrow$$

$$L^*_{jt} = A_t^{-\frac{1}{a+b}} \upsilon_j^{-\frac{1}{a+b}} (Q^{*A}_{jt})^{\frac{1}{a+b}} \left(\frac{r_t}{w}\frac{b}{a}\right)^{\frac{a}{a+b}} \qquad (2.28)$$

Substituting for L^*_{jt} into (2.27) gives

$$K^*_{jt} = A_t^{-\frac{1}{a+b}} \upsilon_j^{-\frac{1}{a+b}} (Q^{*A}_{jt})^{\frac{1}{a+b}} \left(\frac{r_t}{w}\frac{b}{a}\right)^{-\frac{b}{a+b}} \qquad (2.29)$$

Consequently, the total cost of producing the profit maximising output Q^{*A}_{jt} is equal to:

$$TC^A_{jt} = w\, A_t^{-\frac{1}{a+b}} \upsilon_j^{-\frac{1}{a+b}} (Q^{*A}_{jt})^{\frac{1}{a+b}} \left(\frac{r_t}{w}\frac{b}{a}\right)^{\frac{a}{a+b}} +$$

$$+ r_t\, A_t^{-\frac{1}{a+b}} \upsilon_j^{-\frac{1}{a+b}} (Q^{*A}_{jt})^{\frac{1}{a+b}} \left(\frac{r_t}{w}\frac{b}{a}\right)^{-\frac{b}{a+b}} \Leftrightarrow$$

$$TC^A_{jt} = k_{jt}\, (Q^{*A}_{jt})^{\frac{1}{a+b}} (A_t)^{-\frac{1}{a+b}} \qquad (2.30)$$

where, $D = (\frac{a}{b})^{-\frac{a}{a+b}}\left(\frac{a+b}{b}\right)$, $k_t = D\, r_t^{\frac{a}{a+b}} w^{\frac{b}{a+b}}$ and $k_{jt} = k_t\, \upsilon_j^{-\frac{1}{a+b}}$

Analogously, the profit maximising solution for the NA group gives the following total cost :

$$TC^{NA}_{jt} = k_{jt} \ (Q^{*NA}_{jt})^{\frac{1}{a+b}} \ (B_t)^{-\frac{1}{a+b}} \qquad (2.31)$$

the implicit assumption being that the adopters group, A, is facing the same cost conditions with the non adopters group, NA, in terms of capital and labour inputs. Rewriting (2.30) we have:

$$TC^{A}_{jt} = k_{jt} \ (A_t)^{-S} \ (Q^{*A}_{jt})^{S}$$

where $S = \frac{1}{a+b}$. The weighted average is:

$$TC^{A}_{t} = (A_t)^{-S} \ TQ^{A}_{t} \qquad (2.30a)$$

where $TQ^{A}_{t} = \frac{\sum^{A} k_{jt} (Q^{*A}_{jt})^{S+}}{Q^{*A}_{t}}$. Also,

$$AC^{A}_{jt} = k_{jt} \ (A_t)^{-S} \ (Q^{*A}_{jt})^{S-1} \qquad (2.32)$$

The weighted mean unit cost for the adopters' group is equal to:

$$AC^{A}_{t} = (A_t)^{-S} \ AQ^{A}_{t} \qquad (2.32a)$$

where $AQ^{A}_{t} = \frac{\sum k_{jt} (Q^{*A}_{jt})^{S}}{Q^{*A}_{t}}$ .Correspondingly,

$$MC^{A}_{jt} = S \ k_{jt} \ (A_t)^{-S} \ (Q^{*A}_{jt})^{S-1} \qquad (2.34)$$

and by summing and dividing we obtain the weighted mean marginal cost for the adopters :

$$MC_t^A = S\ AC_t^A \qquad (2.34a)$$

It is obvious that,

$$TC_{jt}^A = Q_{jt}^{*A}\ AC_{jt}^A = Q_{jt}^{*A}\ \frac{1}{S}\ MC_{jt}^A$$

Following exactly the same reasoning,

$$TC_{jt}^{NA} = k_{jt}\ (B_t)^{-S}\ (Q_{jt}^{*NA})^S$$

Weightening up we obtain the mean:

$$TC_t^{NA} = (B_t)^{-S}\ TQ_t^{NA} \qquad (2.31a)$$

where $TQ_t^{NA} = \dfrac{\sum^{NA} k_{jt} (Q_{jt}^{*NA})^{S+1}}{Q_t^{*NA}}$. Also,

$$AC_{jt}^{NA} = k_{jt} (B_t)^{-S} (Q_{jt}^{*NA})^{S-1} \qquad (2.33)$$

The weighted mean unit cost for group NA is:

$$AC_t^{NA} = (B_t)^{-S}\ AQ_t^{NA} \qquad (2.33a)$$

where $AQ^{NA} = \frac{\sum^{NA} k_{jt}(Q_{jt}^{*NA})^{S}}{Q_{t}^{*NA}}$. Similarly,

$$MC_{jt}^{NA} = S\ k_{jt}(B_{t})^{-S}(Q_{jt}^{*NA})^{S-1} \tag{2.35}$$

and the weighted mean marginal cost for group NA is:

$$MC_{t}^{NA}=S\ AC_{t}^{NA} \tag{2.36}$$

It should be stressed that since according to the first order profit maximising condition marginal cost should equal marginal revenue, then when competitive behavior is assumed marginal cost will have to be equal to the homogeneous price. Therefore, each firm will produce up to the point where its marginal cost is equal to the price, i.e. each firm will supply the amount of output for which his marginal cost becomes equal to the marginal cost of all other firms. Then we have the following cases (Varian (1984)):

a. If $S = 1$ then since marginal cost is no longer a function of the firm's output, by comparing relations (2.34) and (2.35) it is easily concluded that the price will in the long run equilibrium will be equal to the constant average cost of the adopter firm with the highest firm specific deviation. Only the firms that manage somehow to meet that average cost condition will be able to survive and profits for them will be equal to zero.

b. If $S < 1$ then because $p = MC < AC$ this is incompatible. Either some firm will hold out until other firms leave the industry and then start to behave as a monopolist or some sort of collusive outcome will result among firms with the highest possibility of surviving. In any case the competitive behaviour under such cost conditions is not sustainable.

c. If $S > 1$, or in other words when static returns to scale are less than one, then one may look at each firm as playing two roles. It is both the owner of some innovation and a producer. In his role as the owner of an innovation he considers selling or renting his innovation to a producer. Then, in his role as a producer he determines his optimal supply of output, and thus his optimal profits, counting the rent on the innovation as a fixed cost of production. Consequently, the total profit of firm j will be equal to the most anyone would be willing to pay to rent j's innovation which is equal to $p\,Q_j^* - TC_j$. Q_j^* is the optimal supply of output for firm j. As for TC_j, while it includes capital costs, it does not include the rent on the major innovation the firm uses. Moreover, since $p = MC$, $j = 1,2,\ldots,N$, the economic rate of return is equal to:

$$\frac{pQ_j^* - TC_j}{pQ_j^*} = \frac{p - AC}{p} = \frac{S\ AC - AC}{S\ AC} = 1-a-b \qquad (2.37)$$

Therefore, the economic rate of return each firm earns is uniform across all firms and depends on the static returns to scale. When $S = 1$ this is equal to zero, when $S < 1$ it is negative, and when $S > 1$ it is positive. The curiosity of having decreasing returns to scale even when innovative entry is possible can be explained using the same method with Varian, p.20. Define a new production function $F(A_0, J^F_{jt}, L_{jt})$ which is characterized by a constant returns to scale technology and where A_0 is considered to be a "pseudoinput". More specifically this production function is equal to:

$$F(A_0, J^F_{jt}, L_{jt}) = (A_0)^{1-a-b}(J^F_{jt})^a(L_{jt})^b \qquad (2.38)$$

The decreasing returns to scale production function is equal to:

$$f_3(J^F_{jt}, L_{jt}) = (J^F_{jt})^a(L_{jt})^b \qquad (2.39)$$

As a result,

$$F(A_0, J^F_{jt}, L_{jt}) = A_0\left(\frac{J^F_{jt}}{A_0}\right)^a\left(\frac{L_{jt}}{A_0}\right)^b = A_0 f_3\left(\frac{J^F_{jt}}{A_0}, \frac{L_{jt}}{A_0}\right) \qquad (2.40)$$

If A_0 is fixed (and it is mistakenly perceived as such by the firms) and the the units of measurement used are such that A_0 is fixed at 1, then we have exactly the same production function f_3 that we had before. Hence, the original decreasing returns to scale production function given by f_3 can be thought of as a restriction of the constant returns to scale production function $F(A_0, J^F_{jt}, L_{jt})$ for $A_0=1$.

To summarise, the model of competitive behaviour is over-restrictive for two reasons. First, there are limitations as to the values a+b can take. In particular a+b can not be larger than one. Second, for a+b less than one, even if one accepts that this is possible using the reformulation suggested above, the different level of profits earned by each firm solely depends on the differential efficiency each firm enjoys. As a result no ambiguity exists since by definition the MPH can be rejected in favour of the DEH.

2.4 Summary of variables.

A: group of firms applying the best available major innovation.

NA: group of firms applying any other innovation except the best major innovation.

N_t: total number of firms in the industry at time t.

N_t^A: total number of firms in group A at time t.

y: rate of continuous disembodied tecnical change.

L: rate of continuous embodied technical change.

A_0: index for the potential productivity of the best available innovation (group A).

B_0: index for the potential productivity available to all firms in group NA.

t_{0-1}: time elapsed after a new major innovation occurs before potential productivity in group NA jumps form A_{0-2} to A_{0-1}.

τ^0_{0-1}: time gap between two major innovations.

$V = \frac{A_0}{B_0}$: gap in potential productivity between the two groups.

υ_j: index of how efficiently a firm in either group applies a major innovation.

A_{oj}: realized productivity for a firm j, $j \in A$.

B_{ol}: realized productivity for a firm l, $l \in NA$.

$V\frac{\upsilon_j}{\upsilon_l}$: gap in realized productivity between two firms j and l, $j \in A$ and $l \in NA$.

I_t: investment in physical units at time t.

C_t: capital in physical units at time t.

J_t^E: capital in efficiency units at time t.

I_t^F: investment in full effciency units at time t.

J_t^F: capital in full efficiency units at time t.

K_t: capital in reverse full efficiency units at time t.

c_t: price of investment in full efficiency units at time t.

$q_t = c_t \exp\left[\left(\frac{y}{a} + L\right)t\right]$: price of investment in physical units at time t.

$r_t = [(h+\delta)c_t - \dot{c}_t]\exp\left[\left(\frac{y}{a} + L\right)t\right]$: user cost of capital in reverse full efficiency units at time t.

TC_t: total cost function at time t.

AC_t: average cost function at time t.

MC_t: marginal cost function at time t.

CHAPTER 3

THE GENERALISED CO-OPERATIVE, FREE OF CAPITAL ADJUSTMENT COSTS MODEL

The competitive behaviour model developed in the previous chapter is by definition inappropriate for discussing the ambiguity between the MPH and the DEH. The model assumes *a priori* a pattern of behaviour, i.e. the price taking one, as a result of which the MPH is ruled out while the DEH is the only possible alternative. Total profits are equal to the return on the major innovation each firm uses while the uniform economic profit rate solely depends on the static returns to scale characterising the production of the particular product. The model defined below is a general model incorporating different possible types of behaviour with the economic profit rate being a measure of this behaviour. Moreover, this model explicitly allows for average and marginal cost to be a non trivial function of output when $a+b \neq 1$. The latter is in contrast to Cowling and Waterson who ignored the dependance of average and marginal cost on output by simply restricting their model to $a+b=1$. In the first section the basic components of the model are set forth, and the assumptions about the firms behaviour are discussed in detail. In the second section the task of distinguishing between the MPH and the DEH at the firm level is undertaken. For the purpose of this the reader is reminded of the principles followed in this model for rejecting or accepting either of the hypotheses . The theoretical validity of these was discussed in the second section of Chapter 1. These principles are applied in a step-by-step procedure identifying in which cases the ambiguity is resolved at the firm level. In the subsequent section an analogous analysis at the industry level is undertaken. In the final section, a brief discussion of the possibilities for empirical application focuses mainly on the shortcomings in the availability of data for identifying the adopters and the non adopters group in each industry.

3.1 Setting up the model.

The key assumption in this model is that the installation of new capital is both *costless* and *instantaneous*. Consequently, the long run profit maximisation is identical to the short-run one. The profit function for each firm j within an industry i is of the form :

$$\int_0^{\infty} \exp(-ht)\Pi_j \, dt = \int^{\infty} \exp(-ht)\left(pQ_{jt} - TC_{jt}\right)dt =$$

$$= \int^{\infty} \exp(-ht)\left(pQ_{jt} - wL_{jt} - r_t K_{jt}\right)dt$$

where r_t is the user cost of capital as defined in Chapter 1. The first order conditions for the firm to maximise its profits, suppressing the time subscripts for simplicity's sake, are:

$$\left[p + \frac{\partial p}{\partial Q}\frac{\partial Q}{\partial Q_j}Q_j\right]\frac{\partial Q_j}{\partial L_j} = w \qquad \Leftrightarrow$$

$$\left[1 + \frac{\partial p}{\partial Q}\frac{\partial Q}{\partial Q_j}\frac{Q_j}{Q}\frac{Q}{p}\right]\frac{\partial Q_j}{\partial L_j} = \frac{w}{p}$$

Define $e=-\frac{p}{Q}\frac{\partial Q}{\partial p}$ as the absolute value of the elasticity of demand and $\psi_j=(1+\lambda_j)\frac{Q_j}{Q}$ as the **elasticity of total industry output variation** (ETIOV) measuring how elastic the firm believes that total industry output is with respect to a unit increase in its own output. Then the above relation becomes:

$$(1-\frac{1}{e}\psi_j)\frac{\partial Q_j}{\partial L_j} = \frac{w}{p} \qquad \Leftrightarrow$$

$$\sigma_j\frac{\partial Q_j}{\partial L_j} = \frac{w}{p} \qquad (3.1)$$

and similarly:

$$\sigma_j\frac{\partial Q_j}{\partial K_j} = \frac{r_t}{p} \qquad (3.2)$$

where Q_j is the profit maximising level of output and $\sigma_j=1-\frac{\psi_j}{e}$. Unless otherwise stated, in our work ψ_j will be treated as a parameter determined through the way firms form their conjectures as explained in more detail later. For the time being it is sufficient to say that our specification of conjectural beliefs by parameterizing ψ_j differs from the specifications used so far in the previous literature, which have parameterized instead the following quantities:

I) The conjectural derivatives of the form

$$\lambda_j = \frac{\partial \sum_{l \neq j}^{N} Q_l}{\partial Q_j} \qquad (3.12a)$$

λ_j being the expectation of firm j about its rivals output responses.

II) The elasticity of conjectural variation, equal to

$$\nu_j = \frac{\partial \sum_{l \neq j}^{N} Q_l}{\partial Q_j} \frac{Q_j}{\sum_{j \neq l}^{N} Q_l} \qquad (3.12b)$$

where, $\psi_j = \nu_j + \frac{Q_j}{Q}(1 - \nu_j)$.

III) The relative conjecture, (Martin, 1989) i.e. the percentage increase in j's rivals output as a response to a unit absolute change to j's output:

$$\omega_j = \frac{\partial \sum_{l \neq j}^{N} Q_l}{\partial Q_j} \frac{1}{\sum_{j \neq l}^{N} Q_l} \qquad (3.12c)$$

Since σ_j must always be positive[1] we have that:

[1]The firm will never produce for levels of output where the productivity of labour or capital is negative.

$$1+\lambda_j < e\frac{Q}{Q_j} \qquad (3.3)$$

From the Cobb-Douglas production function it is derived that:

$$\frac{\partial Q_j}{\partial L_j} = b\frac{Q_j}{L_j} \qquad (3.4)$$

$$\frac{\partial Q_j}{\partial K_j} = a\frac{Q_j}{K_j} \qquad (3.5)$$

By substituting (3.4) and (3.5) into (3.1) and (3.2) respectively the following are easily deduced:

$$\sigma_j bpQ_j = L_j w \qquad (3.6)$$

$$\sigma_j apQ_j = K_j r_t \qquad (3.7)$$

Also by dividing (3.6) by (3.7) relation (2.27) is deduced. As a proxy of the profit rate, the profits to sales ratio (π_j) is equal to:

$$\pi_j = \frac{pQ_j - TC_j}{pQ_j} = \frac{pQ_j - \sigma_j bpQ_j - \sigma_j apQ_j}{pQ_j} \qquad \Leftrightarrow$$

$$\pi_j = 1-\sigma_j(a+b) \qquad (3.8)$$

Schmalensee (1987, 1989) discriminated between accounting and economic rates of return by defining the first as the rate that includes in profits the return on capital while the latter does not. In this model both labour and capital renumeration is NOT included in profits. However, the 'rent' from the major innovation the firm uses is **included** in the profits. In this sense, π_j is a 'quasi' economic profit rate. What relation (3.8) states, is that this 'quasi' economic return is a mere proxy of the collusion parameter ψ_j. The higher ψ_j is, the higher π_j is going to be. In other words whether the firm will be in a position to extract the rent that corresponds to the major innovation it uses depends on how collusive (in terms of the ETIOV) this firm is. The profitability measure used varies considerably among the different empirical studies. However, the common characteristic in all of these studies is that accounting data on profitability are used to measure the economic profit rate. For U.S. what is normally used is the accounting rate of return on assets, while for the U.K. data limitations forced British studies to use either the accounting rate of return on revenue (as in Cowling and Waterson) or value added minus employee compensation on value added (as in Hart and Morgan, (1977), Clarke *et al*, e.t.c.).

Price cost margins (or in other words the Lerner index) is equal to:

$$\frac{p- MC_j}{p} = \frac{pQ_j - MC_j Q_j}{pQ_j} = \frac{pQ_j - S\ AC_j Q_j}{pQ_j} \Leftrightarrow$$

$$PMC_j = 1 - \sigma_j \qquad (3.9)$$

Therefore PCM_j is going to be equal to π_j only in the special case where the static returns to scale are equal to one. Using (3.8) the weighted average of profits to revenue for the industry as a whole can be calculated:

$$\pi = \sum^{N}\left(\frac{Q_j}{Q}\pi_j\right) = \sum^{N}\left(\frac{Q_j}{Q}\left[1- \sigma_j(a+b)\right]\right) \qquad (3.10)$$

Analogously, the weighted average of the profits, PCM, is equal to:

$$PCM = \sum^{N}\left[\frac{Q_j}{Q}\;\frac{p - MC_j}{p}\right] = 1- \sum^{N}\left(\frac{Q_j}{Q}\sigma_j\right) \qquad (3.11)$$

In the special case where $\sigma_j = \sigma$ for all firms we have that:

$$\pi = 1 - \sigma(a+b) \qquad (3.10)'$$

$$PCM = 1 - \sigma \qquad (3.11)'$$

Different assumptions about the way firms form their expectations will result into different values of ψ_j, the ETIOV of a firm j. The conjectural derivative for a firm j, j $\in$ NA is equal to:

$$\lambda_j = \frac{\partial \sum^{A} Q_l^{A}}{\partial Q_j^{NA}} + \frac{\partial \sum_{l \neq j}^{NA} Q_l^{NA}}{\partial Q_j^{NA}} = \alpha_{NA,A,j} + \alpha_{NA,NA,j} \qquad (3.12)$$

where $\alpha_{NA,A,j}$ represents the expectation of a non-adopter firm j as to the total output reaction of group A to a change in its output and $\alpha_{NA,NA,j}$ the expectation of a non-adopter firm j as to the total output response of the other non-adopters to a change in its output. In a similar fashion for a firm j∈A the conjectural variations are:

$$\lambda_j = \frac{\partial \sum^{NA} Q_l^{NA}}{\partial Q_j^A} + \frac{\partial \sum_{l \neq j}^{A} Q_l^A}{\partial Q_j^A} = \alpha_{A,NA,j} + \alpha_{A,A,j} \qquad (3.13)$$

It shall be assumed that **within each group** firms jointly maximimise their profits as a whole and then distribute them in such a way that they all earn equal price cost margins (and subsequently equal π's). The above assumption is described in the diagram below where for simplicitly it is assumed that there are just two firms in group A, j and *l*, and also $a+b < 1$. The joint profit maximising output is the point where the aggregate marginal cost curve for group A (derived by horizontally summing up the individual marginal cost curves) intersects the conjectured marginal revenue curve (depending on ψ^A) of group A. Each firm will then produce that quantity of output at which its individual marginal cost is equal to that of the group at the group's aggregate profit maximising level of output Q^A. Combining this with (3.9) implies that σ's (and consequently ψ's) have to be group uniform. Exactly equivalent things hold for group NA. When the industry is in equilibrium (which we shall determine later in this section as a consistent conjectures equilibrium) the price determined in this way for each group will be equal to p=f(Q) where f is the function of the **actual** demand curve and Q ($Q=Q^A+Q^{NA}$) is the sum of the group profit maximising output levels.

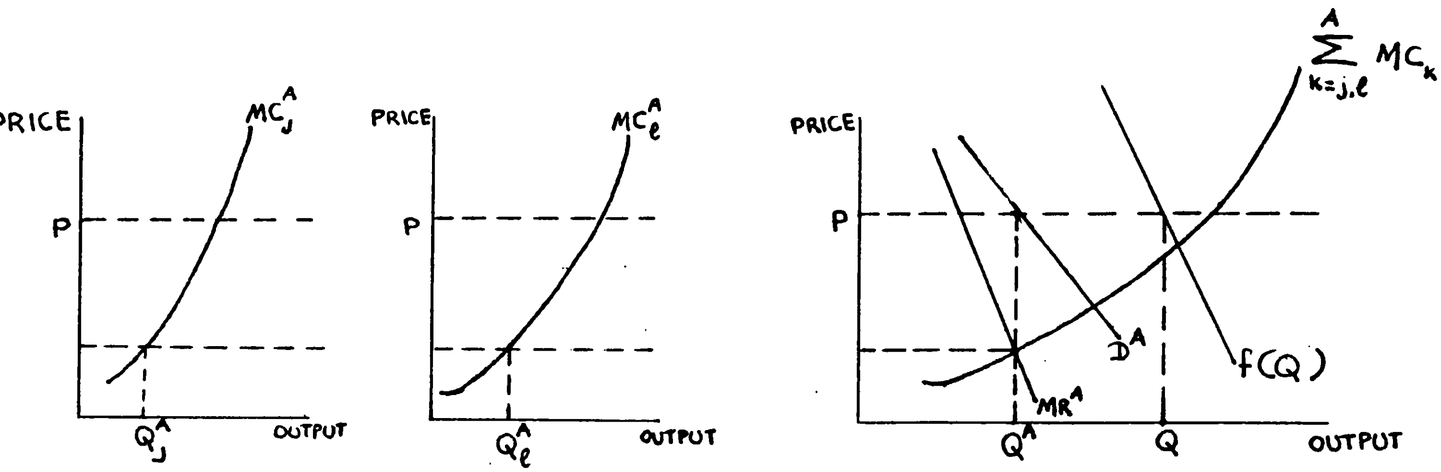


FIGURE 1

Since there is perfect collusion within each group this means that each firm believes that all the other firms within its group will react to output changes so as to maintain their market shares. Consequently:

$$\frac{\partial Q_l^A}{\partial Q_j^A} = \frac{Q_l^A}{Q_j^A} \qquad \forall j, l \in A \tag{3.14}$$

$$\frac{\partial Q_l^{NA}}{\partial Q_j^{NA}} = \frac{Q_l^{NA}}{Q_j^{NA}} \qquad \forall j, l \in NA \tag{3.15}$$

Summing (3.14) for all $l \in A$, $l \neq j$ and (3.15) for all $l \in NA$, $l \neq j$ we get respectively:

$$\alpha_{A,A,j} = \frac{\sum_{l \neq j}^{A} Q_l^A}{Q_j^A} = \frac{Q^A}{Q_j^A} - 1 \tag{3.16}$$

$$\alpha_{NA,NA,j} = \frac{\sum_{l \neq j}^{NA} Q_l^{NA}}{Q_j^{NA}} = \frac{Q^{NA}}{Q_j^{NA}} - 1 \tag{3.17}$$

Combining (3.16) with (3.13) and (3.17) with (3.12) we obtain respectively:

$$\psi^A = (1+\lambda_j)\frac{Q_j^A}{Q} = \frac{Q^A}{Q} + \alpha_{A,NA,j}\frac{Q_j^A}{Q} \tag{3.18}$$

$$\psi^{NA} = (1+\lambda_j)\frac{Q_j^{NA}}{Q} = \frac{Q^{NA}}{Q} + \alpha_{NA,A,j}\frac{Q_j^{NA}}{Q} \tag{3.19}$$

Since ψ's are group uniform parameters, the following will hold for group A and NA respectively:

$$\frac{1}{\frac{Q_j^A}{Q_l^A}} = \frac{\alpha_{A,NA,j}}{\alpha_{A,NA,l}} \quad \forall\ j,\ l \in A \tag{3.20}$$

$$\frac{1}{\frac{Q_j^{NA}}{Q_l^{NA}}} = \frac{\alpha_{NA,A,j}}{\alpha_{NA,A,l}} \quad \forall\ j,\ l \in NA \tag{3.21}$$

Consequently, the ratio of the market shares of two firms which belong in the same group is inversely related to the ratio of the corresponding conjectures of these two firms concerning the output behaviour of the other group.

The between group's conjectures are assumed to be exogenously determined, of an implicit nature and consistent. The consistency assumption means that after some short validation procedure there is convergence towards a locally rational equilibrium (as explicitly defined in Ulph, 1981) where each firm's conjectures about the other group's reactions are perfectly correct locally. Since each firm uses a reaction function which is a best reply to the reaction function of the rival group (in other words each firm is not just correct about the level of the other group's reaction but also about the function according to which it is acting, i.e. the possibility of "being right for the wrong reasons" is ruled out) in this equilibrium no firm has an incentive to change its output. **Therefore the equilibrium is a Cournot-Nash equilibrium in conjectures**. (Consistent conjectures equilibrium.) Neither group expects the output of the other group to remain literally constant, so it does not make sense to talk of each firm as taking the other group's output as given. However, it is part of each firm's belief that if he decides to make no change in its output, the other group will hold its output constant as well for as long as the equilibrium exists. This part of the firm's conjectures will by definition be confirmed in the equilibrium.

The non adopters group is informed of the existence of a new technology but unsure of the true characteristics of this technology unless it has already adopted it. The fact that the birth of a new technology becomes common knowledge is justified by the rationale that the adopting group firm has an interest in letting the rest of the firms know of its good luck since this will affect their output responses. In the case of a patent, things are even more simple since by law the characteristics of the patented innovation are fully known to the laggard firms but it is illegal to copy them. This common knowledge affects the reaction functions of both groups and - by definition of the consistent conjectures - their conjectures as well. For the output share of each group to remain constant after an increase in the technology gap between two groups the ratio of the ETIOV's (or the ratio of their respective σ's) of the two firms should also increase (decrease) by a sufficient amount so as to offset the impact of $\frac{A_0}{B_0}$ on the output shares. Instead of explicitly allowing for this positive relation between the technology gap and the gap in the ETIOV's for a given output distribution, it shall be left to the model itself to implicitly demonstrate this. For this to be possible it suffices to assume that **the ETIOV's of each group are given parameters for any firm joining this group. More simply, the identity of the firms adopting a certain form of behaviour changes as firms move from the one group to the other.** In other words, it does not matter who the king (peasant) is, but once a king (peasant) he is expected to behave like a king (peasant). Moreover since σ's in this model are the parameters used for measuring the behaviour of the firms (just as Cowling and Waterson used as a measure the conjectural derivatives, Clarke and Davies the elasticity of conjectural variation and Martin the relative conjecture) then relation (3.8) implies that π's are a mere proxy for the behaviour of each firm. In other words by definition the higher the σ of a firm is, the lower its profits to revenue ratio is going to be and vice versa. That explains why the market share rather than the profit rate is used in this model as a criterion for attempting to resolve the ambiguity between the two hypotheses.

It shall be demonstrated that by using the assumption of group-uniform σ's it is possible to derive a testable expression at the industry level revealing how the Herfindahl index is related to π^{A} and π^{NA}, the profits to revenue ratios of the two groups. But before doing that, a firm level analysis will be undertaken in order to determine under what circumstances it is possible to resolve the ambiguity between the MPH and the DEH at the firm level.

3.2 Firm level hypotheses discrimination analysis.

As it was argued in Chapter 1, at the firm level of analysis the criterion for not rejecting the DEH is whether a difference in the efficiency between two firms has as a consequence a larger market share for the relatively more efficient firm. Analogously, the criterion for not rejecting the MPH is whether the relatively more collusive firm (in terms of the ETIOV) has a relatively larger market share.

For applying these rules it is first necessary to rewrite the Cobb-Douglas function using relations (3.6) and (3.7). For simplicity, it is assumed that $r_t = \exp\left[\left(L+\frac{y}{a}\right)t\right]\left((h+\delta)c\right)$ i.e. the price of gross investment in full-efficiency units (c_t) is considered to be constant (c).

$$Q_j = G_j\left[\sigma_j a Q_j \frac{p}{r}\right]^a \left[\sigma_j b Q_j \frac{p}{w}\right]^b \qquad \Leftrightarrow$$

$$Q_j = \Psi_t G_j^{\frac{1}{1-a-b}} \sigma_j^{\frac{a+b}{1-a-b}} \tag{3.22}$$

where $\Psi_t = \left[a^a b^b \left(\frac{p}{r_t}\right)^a \left(\frac{p}{w}\right)^b\right]^{\frac{1}{1-a-b}}$ and $G_{jt} \begin{cases} = A_t \upsilon_j^A & \text{for } j \in A \\ = B_t \upsilon_j^{NA} & \text{for } j \in NA \end{cases}$

As it shall be demonstrated in the next chapter, relation (3.22) can also be derived by generalising the Cowling and Waterson model to allow for increasing and decreasing returns to scale. By dividing relation (3.22) for firm j with the same relation for firm *1* - where by definition j is always technologically superior to firm *1* - the following relation is obtained:

$$\frac{Q_j}{Q_1} = \left(\frac{G_j}{G_1}\right)^{\frac{1}{1-a-b}} \left(\frac{\sigma_1}{\sigma_j}\right)^{\frac{a+b}{a+b-1}} \tag{3.23}$$

The above relation will be analysed for the cases a+b>1 and a+b<1 in turn. (The case of a+b= 1 will be treated seperately in Chapter 4.) By definition the following hold:

(a) $V = \frac{A_0}{B_0} > 1$ for $j \in A$, $l \in NA$ where V is either equal to $\frac{A_0}{A_{0-2}}$ or equal to $\frac{A_0}{A_{0-1}}$ in accordance to relations (2.7) and (2.7′) respectively.

b) V = 1 for j, l belonging in the same group.

c) $\frac{G_j}{G_l} = V\,\upsilon_{jl} > 1$ for $j \in A$ and $l \in NA$ according to relation (2.9).

(d) $\upsilon_{jl} > 1$ for j, l belonging in the same group if j is the relatively more efficient firm in applying major innovations.

Begin with the case a+b>1. Returning to (3.23) this may be re-writen as:

$$\frac{Q_j}{Q_l}\left(\frac{G_j}{G_l}\right)^{\frac{1}{a+b-1}} = \left(\frac{\sigma_l}{\sigma_j}\right)^{\frac{a+b}{a+b-1}} \Leftrightarrow$$

$$\frac{Q_j}{Q_l}\,(M_{jl})^{-f} = (x_{lj})^{-f_1}$$

where $M_{jl} = \frac{G_j}{G_l} = V\upsilon_{jl} > 1$ according to (c), $x_{lj} = \frac{\sigma_l}{\sigma_j}$, $f = \frac{1}{1-a-b} < 0$ and $f_1 = \frac{a+b}{1-a-b} = f-1 < 0$. When $\sigma_j < \sigma_l$ ($\psi_j > \psi_l$, i.e the relatively more efficient firm has a relatively higher ETIOV as well) for Q_j to be larger than Q_l, or in other words for the DEH to be accepted it is required that:

$$(V)^{f}(\upsilon_{jl})^{f} > (x_{lj})^{f_1}$$

However, since the firm with the relatively higher ETIOV (firm j) also has the relatively greater market share the MPH can not be rejected (is accepted). As a result for this case neither of the two hypotheses can be rejected in favour of the other. Turning now to the case where the more efficient firm has a smaller market share which in terms of our theory translates as rejection of the DEH for this pair of firms, since $Q_j < Q_l$ we have:

$$(V)^{f}(\upsilon_{jl})^{f} < (x_{lj})^{f_1}$$

Since firm j which is the relatively more collusive (in terms of ETIOV) has a lower market share the MPH is also rejected. This implies that **the combination of both higher collusion and efficiency does not necessarily guarantee a higher market share.** The DEH is rejected (and so is the MPH) regardless of the fact that the more efficient firm will have a higher π than the less efficient firm since this is just a mere reflection of the ETIOV. The rejection of both hypotheses means that for this case the firm level of analysis is **not adequate** for explaining market shares either in terms of the DEH or in terms of the MPH. As it will be shown in the next section, such cases may be dealt with at the industry level. The criterions for accepting the DEH and the MPH at the industry level will prove sufficient for resolving the ambiguity. So for a+b > 1 and $x_{lj} > 1$ there are two possibilities:

either both hypotheses will be rejected or both hypotheses will be accepted.

Now turn to the case of a+b > 1 and $x_{lj} < 1$. Then the above inequality will always hold since by definition V $\upsilon_{jl} > 1$ and $0 > f > f_1$. Therefore, under these circumstances it is not possible to have $Q_j > Q_l$. **The less efficient firm will always have a larger market share and therefore the DEH will always be rejected.** Moreover, since the firm with the relatively higher collusion (as measured by ETIOV) has a higher market share the MPH is accepted. Therefore the ambiguity in this case is resolved with the DEH being rejcted in favour of the MPH.

For the case of a+b<1 ($f>f_1>0$), if $\sigma_j<\sigma_l$, for Q_j to be larger than Q_l it is required that:

$$(M_{jl})^{f} > (x_{lj})^{f_1}$$

Notice that for this case when $Q_j > Q_l$ (and therefore the DEH is accepted) for the same reasons as in the case a+b > 1 and $x_{lj} > 1$, the MPH will also be accepted. Also, as before, if $Q_j < Q_l$ then both hypotheses are rejected. Consequently for a+b < 1 and $x_{lj} > 1$ **either both hypotheses will be accepted or both hypotheses will be rejected.**

For a+b < 1 and $x_{lj} < 1$ the more efficient firm will **always** have a higher market share (since the above inequality will always hold) and therefore the DEH is to be accepted always. Moreover, the firm which is relatively more collusive is the firm with the relatively lower market share. Consequently, under these circumstances **the MPH is always rejected in favour of the DEH**. The DEH is accepted, this time regardless of the fact that the more efficient firm has a lower π than the less efficient firm (since $\psi_j < \psi_l$). **While in such a case the Demsetz hypothesis would have been rejected, our DEH will not be rejected because as already explained in the introduction, it is more broadly defined and capable of explaining a negative as well as a positive relation between market share and profitability**. Since neoclassical theory is unable to account for decreasing static returns in the long run it is also unable to incorporate this last case where the DEH clearly prevails over MPH hypothesis.

To summarize, at the firm level, when the more efficient firms are the more profitable ones ($x_{lj} > 1$), the analysis should be done at the industry level if one wishes to resolve the ambiguity. When the more efficient firms are the less profitable ones ($x_{lj} < 1$) then if a+b < 1 the MPH is rejected in favour of the DEH while when a+b > 1 then the DEH is rejected in favour of the MPH. For a+b < 1 , the firms that enjoy no cost advantage are more collusive (and thus more profitable) but do not wish to use their oligopolistic practices for obtaining a higher market share since average costs are an increasing function of size. On the other hand, the firms that enjoy genuine cost advantages can afford to sustain a higher market share since although a+b < 1 their productivity is relatively superior to that of the laggard firms, i.e. for this case innovativeness is the cause of large size as Demsetz argued. For $a+b > 1$, the laggard firms which are also relatively more collusive as reflected in their ETIOV make sure that they gain a greater market share since this will have a negative impact on their average costs. Moreover, while the more collusive firms are dominant in terms of size, the engine for innovativeness are the small firms. In other words for $a+b > 1$ and $x < 1$ the industry is characterised by small innovating firms, thus rejecting the Schumpeterian assumption of large innovative firms and justifying the need for anti-trust policy. In Chapter 4, after the derivation of relation (3.22) from the generalised Cowling and Waterson model, the four cases described in this section are diagramatically illustrated.

3.3 Industry level hypotheses discrimination analysis

It shall be now demonstrated that the cases for which the ambiguity between the MPH and the DEH was not resolved at the firm level (the MPH and the DEH were either both accepted or both rejected) can be resolved at the industry level. The DEH is to be accepted at the industry level if an increase in the technological gap between the two groups will lead to an increase in the level of concentration. The MPH is accepted if a further relative increase in the ETIOV of the group which already had the higher

ETIOV should lead to an increase in the level of concentration.

For applying these rules it is first necessary to establish a relationship between concentration on the one side and the gap in technology as well as the gap in ETIOV's between the two groups on the other side. By squaring (3.22) we derive:

$$(Q_j)^2 = (\Psi_t)^2 (G_j)^{2f} \ (\sigma_j)^{2f_1} \tag{3.24}$$

Combining (3.22) and (3.24) we can now obtain the Herfindahl index of concenration (where $\sum^N$ denotes the summation over all firms in the industry):

$$H = \frac{\sum^N (Q_j)^2}{\left(\sum^N Q_j\right)^2} = \frac{\sum\left((G_j)^{2f} \ (\sigma_j)^{2f_1}\right)}{\left[\sum^N\left((G_j)^f \ (\sigma_j)^{2f_1}\right)\right]^2} \tag{3.25}$$

If it is assumed that the σ's within each group are uniform then we may set that $\sigma^{NA} = x\sigma^A$. (where $\sigma_1^A = \sigma_2^A = \quad = \sigma_{N^A}^A$, $\sigma_1^{NA} = \sigma_2^{NA} = \ldots = \sigma_{N-N^A}^{NA}$). Such an assumption will imply equal profit rates within each group suggesting, as already discussed in section 3.1, the formation of a cartel in which each firm produces up to the level where its profit margins are equated to the profit margins of the other members of its group. This implies that for firms within the same group since $x_{lj} = 1$, when a+b > 1 then $Q_j < Q_l$ (DEH rejected) and when a+b < 1 $Q_j > Q_l$ (DEH accepted) while MPH is inconclusive at the firm level. Within each group for a+b > 1 the more efficient a firm is, the less it it will produce and exactly the opposite when a+b < 1, for it is in this way that the explicit collusive aggrement of equal π's within each group will be satisfied. Moreover, when there are no firm specific effects within a group, then each firm within that group will have an equal market share as (3.23) indicates.

The assumption of group-uniform σ's has the advantage of making our model considerably more simple for the task of establishing inter-industry relations while retaining freedom to account for fairly diverse paterns of behaviour since λ's and ν's are free to vary among firms.

We can now derive a new expression for π by incorporating the uniformity assumption into (3.10):

$$\pi = \frac{Q^A}{Q}\pi^A + \frac{Q^{NA}}{Q}\pi^{NA} \qquad \Leftrightarrow(3.26)$$

$$\pi = 1- \sigma^A (a+b)\left(\frac{Q^A}{Q} + \frac{Q^{NA}}{Q}x\right) \qquad \Leftrightarrow(3.26)'$$

$$\pi = 1-\sigma^A (a+b)\ x_1$$

where π^{NA} is the uniform profit rate for group NA, π^A is the uniform profit rate for group A, $x_1 = \frac{Q^A}{Q}(1-x) + x>0$ and $x= \frac{1-\pi^{NA}}{1-\pi^A}$.

Using the group uniformity assumption (3.25) may be rewritten as:

$$H = \frac{(\sigma^A)^{2f_1} \sum^{A} (G_j)^{2f} + (x\,\sigma^A)^{2f_1} \sum^{NA} (G_l)^{2f}}{\left[(\sigma^A)^{f_1} \sum^{A} (G_j)^{f} + (x\,\sigma^A)^{f_1} \sum^{NA} (G_l)^{f} \right]^2} =$$

$$\frac{1 + x^{-2f_1} V^{2f} \dfrac{\sum^{A} (\upsilon_j^A)^{2f}}{\sum^{NA} (\upsilon_l^B)^{2f}}}{\left[1 + x^{-f_1} V^{f} \dfrac{\sum^{A} (\upsilon_j^A)^{f}}{\sum^{NA} (\upsilon_l^B)^{f}} \right]^2} \; \frac{\sum^{NA} (\upsilon_l^B)^{2f}}{\left[\sum (\upsilon_l^B)^{f} \right]^2} =$$

$$\frac{1 + x^{-2f_1} V^{2f} C_1}{\left[1 + x^{-f_1} V^{f} C_2 \right]^2} C_3 \qquad (3.27)$$

where $C_1 = \dfrac{\sum^{A} (\upsilon_j^A)^{2f}}{\sum^{NA} (\upsilon_l^B)^{2f}}$, $C_2 = \dfrac{\sum^{A} (\upsilon_j^A)^{f}}{\sum^{NA} (\upsilon_l^B)^{f}}$, $C_3 = \dfrac{\sum^{NA} (\upsilon_l^B)^{2f}}{\left[\sum (\upsilon_l^B)^{f} \right]^2}$, with C_1, C_2, C_3 being always positive and additionally C_3 being smaller than one. Furthermore, C_4 is defined as:

$$C_4 = \frac{\sum^{A} (\upsilon_j^A)^{2f}}{\sum^{A} (\upsilon_j^A)^{f}} \; \frac{\sum^{NA} (\upsilon_l^B)^{f}}{\sum^{NA} (\upsilon_l^B)^{2f}}$$

It is now possible by partial differentiation to examine the effect on H from a change in the technological gap between the two groups and the effect of a change in the gap between σ's of the two groups the signs of which are the criteria in this model for accepting or rejecting the DEH and the MPH respectively.

$$\frac{\partial H}{\partial V} = \frac{(2\ f\ x^{-2f_1} V^{2f-1} C_1) C_3 (1+ x^{-f_1} V^{f} C_2)^2}{\left[1+x^{-f_1} V^{f} C_2\right]^4}$$

$$- \frac{2 \left[1+x^{-f_1} V^{f} C_2\right] f\ x^{-f_1} V^{f-1} C_2 \left[1+x^{-2f_1} V^{2f} C_1\right] C_3}{\left[1+x^{-f_1} V^{f} C_2\right]^4} .$$

Consequently, for $\frac{\partial H}{\partial V}$ to be positive when a+b<1 ($f, f_1 > 0$) it must be the case that:

$$2\ f\ C_1\ C_3\ x^{-2f_1} V^{2f-1} + 2\ f\ C_1\ C_2\ C_3\ x^{-3f_1} V^{3f-1}$$

$$- 2\ f\ C_2\ C_3\ x^{-f_1} V^{f-1} - 2\ f\ C_1\ C_2\ C_3\ x^{-3f_1} V^{3f-1} > 0 \quad \text{since } f, f_1 > 0 \quad \Leftrightarrow$$

$$C_1\ C_3\ x^{-2f_1} V^{2f-1} - C_2\ C_3\ x^{-f_1} V^{f-1} > 0 \quad \Leftrightarrow$$

$$C_4\ V^{f} > x^{f_1}$$

Therefore, for a+b < 1 when $C_4\ V^{f} < x^{f_1}$ $\frac{\partial H}{\partial V}$ is negative and when $C_4\ V^{f} > x^{f_1}$ $\frac{\partial H}{\partial V}$ is positive.

For $\frac{\partial H}{\partial x}$ to be positive when a+b < 1 it is required that:

$$-2 f_1 C_1 C_3 x^{-2f_1-1} V^{2f} - 2 f_1 C_1 C_2 C_3 x^{-3f_1-1} V^{3f} +$$

$$2 f_1 C_2 C_3 x^{-f_1-1} V^{f} + 2 f_1 C_1 C_2 C_3 x^{-3f_1-1} V^{3f} > 0 \quad \text{since } f_1>0 \qquad \Leftrightarrow$$

$$-C_1 x^{-2f_1-1} V^{2f} + C_2 x^{-f_1-1} V^{f} > 0 \qquad \Leftrightarrow$$

$$C_4 V^{f} < x^{f_1}$$

Consequently, for a+b < 1 when $C_4 V^f > x^{f_1}$ $\frac{\partial H}{\partial x}$ is negative and when $C_4 V^f < x^{f_1}$ $\frac{\partial H}{\partial x}$ is positive.

For $\frac{\partial H}{\partial V}$ to be positive when a+b >1 ($f, f_1<0$) it is required that:

$$2 f C_1 C_3 x^{-2f_1} V^{2f-1} - 2 f C_2 C_3 x^{-f_1} V^{f-1} > 0 \quad \text{since } f<0 \qquad \Leftrightarrow$$

$$C_1 C_3 x^{-2f_1} V^{2f-1} - C_2 C_3 x^{-f_1} V^{f-1} < 0 \qquad \Leftrightarrow$$

$$C_4 V^{f} < x^{f_1}$$

Therefore, for a+b > 1 when $C_4 V^f > x^{f_1}$ $\frac{\partial H}{\partial V}$ is negative and when $C_4 V^f < x^{f_1}$ $\frac{\partial H}{\partial V}$ is positive.

For $\frac{\partial H}{\partial x}$ to be positive when a+b >1 ($f, f_1<0$) it is required that:

$$-2 f_1 C_1 C_3 x^{-2f_1-1} V^{2f} + 2 f_1 C_2 C_3 x^{-f_1-1} V^{f} > 0 \quad \text{since } f_1>0 \qquad \Leftrightarrow$$

$$C_4 V^f > x^{f_1}$$

Consequently, for a+b > 1 when $C_4 V^f < x^{f_1}$ $\frac{\partial H}{\partial x}$ is negative and when $C_4 V^f > x^{f_1}$ $\frac{\partial H}{\partial x}$ is positive.

The above results are particularly interesting because they demonstrate that **at the industry level the effects of V and x on H are always of an opposite sign**. This suggests a positive relation between V and x confirming the prediction in section 3.1 which argued that an increase in $\frac{A_o}{B_o}$ will have to be met by a decrease in $\frac{\sigma^A}{\sigma^{NA}}$ if the distribution of output shares is to remain unchanged.

The signs of $\frac{\partial H}{\partial V}$ and $\frac{\partial H}{\partial x}$ serve as the criteria by which the DEH and MPH respectively should be rejected at the industry level. It is rather easy to interpret a positive or a negative $\frac{\partial H}{\partial V}$. A positive sign means that the differential efficiency hypothesis is correct since a higher concentration is the result of an increase of the gap in efficiency between the two groups. A negative sign suggests that an increase of the gap in efficiency has a negative impact on concentration and therefore high efficiency can not serve as an alternative interpretation of high concentration as Demsetz argued. How are we to translate the sign of $\frac{\partial H}{\partial x}$? This depends on whether x is larger or smaller than one. For $x \geq 1$ a positive sign implies that a relative increase in the ETIOV of group A (and consequently an increase in x) which already has an equal or relatively higher ETIOV (and, by definition, an equal or higher profit rate) than group NA increases concentration. In more detail, since $x = \dfrac{1 - \frac{1}{e}\psi^{NA}}{1 - \frac{1}{e}\psi^{A}} \geq 1$, $\psi^{A} \geq \psi^{NA}$ and since an increase in x translates as an increase in ψ^{A} and/or a decrease in ψ^{NA}, this implies that **an increase in x means an increase in the divergence between the ETIOV's of the two groups in favour of the group with the already higher ETIOV** (when $x > 1$). If an increase in the divergence of the ETIOV's brings an increase in H (in other words a positive $\frac{\partial H}{\partial x}$) then the MPH is to be accepted and if this increase brings a decrease in H (i.e. a negative $\frac{\partial H}{\partial x}$) then the MPH is rejected. On the other hand when $x < 1$ then a positive sign denotes that an increase in the ETIOV of group A, which is the group with the relatively smaller ETIOV (and, by definition, the one with the lower profit rate) will result to an increase in concentration which is in contradiction to the MPH. In more detail, since $x = \dfrac{1 - \frac{1}{e}\psi^{NA}}{1 - \frac{1}{e}\psi^{A}} < 1$, $\psi^{A} < \psi^{NA}$ and since an increase in x translates as an increase in ψ^{A} and/or a decrease in ψ^{NA} (for a constant elasticity of demand), **this implies that there is a convergence between the ETIOV's of the two groups since there is move in favour of the group with lower ETIOV.** If this decrease in the differentials in the ETIOV's between the two groups results to an increase in H then the MPH will be rejected. On the other hand if this decrease brings a decrease in H as well then the MPH will be accepted.

The above interpretations can now be applied to the industry level results. When $a+b > 1$ and $x \geq 1$ if $C_4^{f} V > x^{f_1}$ then $\frac{\partial H}{\partial V}$ is negative, which denotes that the DEH should be rejected while $\frac{\partial H}{\partial x}$

is positive, which means that the MPH is not contradicted and should not be rejected. On the other hand when a+b > 1, if x ≥ 1 and $C_4^f V^f < x^{f_1}$ then MPH is rejected in favour of the DEH. When a+b > 1 and x < 1 if $C_4^f V^f < x^{f_1}$ then neither MPH nor DEH can be rejected and a hybrid model with both forces at work seems appropriate, while if $C_4^f V^f > x^{f_1}$ both hypotheses are rejected. For a+b < 1 and x ≥ 1 if $C_4^f V^f > x^{f_1}$, the positively signed $\frac{\partial H}{\partial V}$ confirms the DEH while MPH is rejected on the grounds of a negative $\frac{\partial H}{\partial x}$ and if $C_4^f V^f < x^{f_1}$ then DEH is rejected in favour of the MPH. When a+b < 1 and x < 1 if $C_4^f V^f < x^{f_1}$ both the DEH and MPH are rejected while if $C_4^f V^f > x^{f_1}$ again neither the MPH nor the DEH can be rejected.

To summarise, at the industry level when x ≥ 1 the market power hypothesis and the differential efficiency hypothesis are mutually exclusive, while when x < 1 ambiguity exists since both or neither of the two hypotheses can be rejected against its alternative.

It is useful to contrast the above conclusions with the firm level conclusions of section 3.2. For a+b < 1 (where f, f_1 = f-1 are both positive) when ambiguity exists at the firm level because x ≥ 1 then this can be resolved at the industry level: if the gap in efficiency between the two groups is sufficiently large and/or the gap between the group σ's $\left(\frac{\sigma^{NA}}{\sigma^{A}}\right)$ sufficiently small (i.e. the gap in the conduct between the two groups, $\frac{\psi^{A}}{\psi^{NA}}$, is sufficiently small) for $C_4^f V^f$ to be greater than x^{f_1} then the MPH is rejected in favour of the DEH. On the other hand, if the gap between the group efficiencies is sufficiently small and/or the gap between the group σ's $\left(\frac{\sigma^{NA}}{\sigma^{A}}\right)$ sufficiently large for $C_4^f V$ to be smaller than x^{f_1} then the DEH is rejected in favour of the MPH. In the opposite case, i.e. if a+b < 1 and an ambiguity exists at the industry level because x < 1, then this can be resolved by using the results derived at the firm where MPH is always rejected in favour of the DEH. For a+b > 1 (where f, f_1= f-1 are both negative) when ambiguity exists at the firm level because x ≥ 1 this is resolved at the industry level: If the gap in efficiency between the two groups $\left(\frac{A_o}{B_o}\right)$ is sufficiently small and/or the gap between the group σ's sufficiently large for $C_4^f V$ to be larger that x^{f_1} then the DEH is rejected in favour of the MPH. On the other hand, if the gap in efficiency between the two groups is sufficiently large and/or the gap between the group σ's $\left(\frac{\sigma^{NA}}{\sigma^{A}}\right)$ sufficiently small for $C_4^f V$ to be smaller than x^{f_1} then the MPH is rejected in favour of the DEH. However, when the ambiguity is at the industry level because x < 1 then this can be resolved by simply refering to the firm level results which conclude that the DEH is rejected in favour of the MPH.

Consequently, it seems that when the adopters profit rate is equal to, or higher than, the profit rate of the non adopters (x ≥ 1) using the industry level conclusions is the correct tactic while when the adopters profit rate is lower than the non adopters profit rate (x < 1) using the conclusions derived at the firm level conclusions seems more appropriate. Table 1 provides a summary of the industry level conclusions when $x \geq 1$ and of the firm level conclusions when $x < 1$.

TABLE 1

Resolving the ambiguity between MPH and DEH

a+b > 1

$x \geq 1$	If $C_4V^f > x^{f_1}$ accept the MPH-reject the DEH. Else reject MPH and accept the DEH. **Ambiguity resolved** using the **industry** level results. †
$x < 1$	DEH rejected-MPH accepted. **Ambiguity resolved** using the **firm** level analysis conclusions. †

a+b < 1

$x \geq 1$	If $C_4V^f > x^{f_1}$ reject the MPH-accept the DEH. Else reject DEH and accept the MPH. **Ambiguity resolved** using the **industry** level results. †
$x < 1$	MPH rejected-DEH accepted. **Ambiguity resolved** using the **firm** level analysis conclusions. †

As a final point in this section it would be interesting to see whether the possibility of a negative relationship between market share and firm profitability at the firm level can also be a possibility at the industry level as a negative relation between π and the index of concentration. In order to do this $\frac{\partial \pi}{\partial x}$ shall be first calculated by utilising relation (3.26) while it is assumed for simplicity that π^{NA} (and therefore σ^{NA}) is constant.

$$\frac{\partial \pi}{\partial x} = \frac{\partial \pi}{\partial \pi^{A}} \frac{\partial \left(1 - \frac{1-\pi^{NA}}{x}\right)}{\partial x} =$$

$$= \frac{1}{x^{2}} (1- \pi^{NA}) \left(\frac{Q^{A}}{Q} + \pi^{A} \frac{\partial \left(\frac{Q^{A}}{Q}\right)}{\partial \pi^{A}} - \pi^{NA} \frac{\partial \left(\frac{Q^{A}}{Q}\right)}{\partial \pi^{A}} \right) \Leftrightarrow$$

$$\frac{\partial \pi}{\partial x} = \frac{1}{x^{2}} (1- \pi^{NA}) \left(\frac{Q^{A}}{Q} + (\pi^{A}-\pi^{NA}) \frac{\partial \left(\frac{Q^{A}}{Q}\right)}{\partial \pi^{A}} \right) \qquad (3.28)$$

As relation (3.28) reveals the sign of $\frac{\partial \pi}{\partial x}$ depends on x since when $x > 1$, $\pi^{A} > \pi^{NA}$, whereas when $x < 1$, $\pi^{A} < \pi^{NA}$. Moreover the sign depends on a+b since when $a+b < 1$, $\frac{\partial \left(\frac{Q^{A}}{Q}\right)}{\partial \pi^{A}}$ is negative since, *ceteris paribus*, an increase in the ETIOV of one group will decrease the output of all the firms in this group when a+b is less than one. On the other hand when a+b is larger than one then $\frac{\partial \left(\frac{Q^{A}}{Q}\right)}{\partial \pi^{A}}$ is going to be positive. Therefore it is going to be possible for $\frac{\partial \pi}{\partial x}$ to be negative either when $a+b > 1$ and $x < 1$ or when a+b < 1 and x > 1. If either of these two combinations holds and $\left| (\pi^{A}-\pi^{NA}) \dfrac{\partial\left(\dfrac{Q^{A}}{Q}\right)}{\partial \pi^{A}} \right| > \dfrac{Q^{A}}{Q}$ then $\dfrac{\partial \pi}{\partial x}$ will be negative.

If it is possible to have $\dfrac{\partial \pi}{\partial x}$ positive when $\dfrac{\partial H}{\partial x}$ is negative and vice versa, then obviously this means that there can be a negative relationship between H and π. Looking at the conditions which determine the sign of $\dfrac{\partial H}{\partial x}$, this is clearly possible. For example if both a+b and x are larger that one, then $\dfrac{\partial H}{\partial x}$ can be either positive (when $C_4(V)^{f} > x^{f_1}$) or negative (when $C_4(V)^{f} < (x)^{f_1}$), while $\dfrac{\partial \pi}{\partial x}$ can only be positive. More importantly, **the sign of the relationship between H and π reveals nothing about the rejection or not rejection of either of the two hypotheses. In other words, this sign adds nothing to the resolvance of the ambiguity between the market power hypothesis and the differential efficiency hypothesis**.

3.4 Estimation possibilities and data limitations.

If one wishes to determine which of the four possible conclusions apply for a particular industry, two conditions should be satisfied. First a model at the industry level is required which permits the estimation of C_4V^{f} and a+b as two separate parameters. Second, data should exist that make possible the calculation of x. Knowledge of x is essential for deciding whether the firm level or the industry level conclusions are going to be used, while the estimates of the two parameters are required so that when $x \geq 1$ it shall be possible to say whether C_4V^f is larger or smaller than x^{f_1}. Estimates for the two parameters will be derived by running a non linear regression based on relation (3.27). In particular, one may set as $\gamma_1=C_1(V)^{2f}$, $\gamma_2=C_2(V)^f$, $\gamma_3=C_3$ (therefore $\frac{\gamma_1}{\gamma_2}=C_4(V)^f$) and estimate from this regression γ_1, γ_2, γ_3 and f_1. All of the above mentioned parameters will vary between different industries and additionally γ_1 and γ_2 will vary from time to time as discrete jumps in innovations alter the magnitude of the efficiency gap, V. What is therefore required is a set of data that can account for these differences i.e. panel data at the industry level. The regression for industry i if a multiplicative disturbance term is added will be of the form:

$$H_{it} = \frac{1+ \gamma_{1it}(x_{it})^{-2f_{1i}}}{\left[1+ \gamma_{2it}(x_{it})^{-f_{1i}}\right]^2} \gamma_{3i}\eta_{it} \qquad (3.29)$$

Since panel data information on $x = \frac{1-\pi^{NA}}{1-\pi^{A}}$ is required, it is essential that for each industry data should not only account for innovative successes between the firms so as to successfully seperate the adopters from the non adopters in each cross section, but should also provide a continuous tracking of the major innovative activities of each firm through time. Patenting data (such as these available for the U.S. from the country's Patent Office, which provide data on the patenting behaviour of a large cross section of firms over significant time intervals) are an indicator of innovative activity. However, there are objections as to **how effective** a measure of major innovations patents are as already discussed in Chapter 2. According to a study by Pakes and Griliches (1980) patents are a fairly good indicator of differences in inventive activity across firms but short term fluctuations in their number between firms have a large noise component in them implying that they are not a good measure of inventive activity at the within level. However they derive this result based on something like a patent production function, focusing on the degree of correlation between patents and R&D expenditures. Consequently, their finding that *"...annual fluctuations in patenting at the individual firm level appear to be much less well related to current and past fluctuations in R&D expenditures..."* is not suprising; it confirms the commonly held presumption that patents measure only major innovations and do not account for fluctuations in productivity over small time intervals. Morover, it confirms the model's assumption that there are large time intervals elapsing between major innovations which can not be captured in Pakes and Griliches's empirical study since their sample accounts for eight years only.

No progress will be made in estimating γ_1, γ_2, γ_3 and f_1 unless some restrictions are imposed as to how parameters γ_1 and γ_2 vary between industries and time. Having said that, there are two ways of treating γ_1 and γ_2 (Judge *et al*, 1988). The first is to consider these two parameters as fixed and time invariant for each industry and proceed in estimating $\underline{\gamma}'=(\underline{\gamma}_1', \underline{\gamma}_2', \underline{\gamma}_3', \underline{f}_1')$, where $\underline{\gamma}'$ is a (1×4N) vector of unknown fixed parameters to be estimated within the framework of a seemingly unrelated regressions model. Alternatively, one may regard γ_{1it} and γ_{2it} as random parameters with means γ_{1i} and γ_{2i} respectively. If one defines $\varepsilon_{1it}=\gamma_{1it}-\gamma_{1i}$, $\varepsilon_{2it}=\gamma_{2it}-\gamma_{2i}$, then an alternative to (3.29) is the model

$$H_{it} = \frac{1+ \gamma_{1i}(x_{it})^{-2f_{1i}} + \varepsilon_{1it}(x_{it})^{-2f_{1i}}}{\left[1+ \gamma_{2i}(x_{it})^{-f_{1i}} + \varepsilon_{2it}(x_{it})^{-f_{1i}}\right]^2}\gamma_{3i} \qquad (3.30)$$

In (3.30) the disturbances ε_{1it} and ε_{2it} replace the *ad hoc* disturbance term η_{it} of the earlier model. The joint distribution function of H_{it} and ε_{2it} may be written as:

$$g(H_{it},\varepsilon_{2it}) = h(\varepsilon_{1it},\varepsilon_{2it})\,|J| \qquad (3.31)$$

where $h(\varepsilon_{1it},\varepsilon_{2it})$ is the joint density function of ε_{1it} and ε_{2it}, and $|J|$ is the absolute value of the Jacobian of the transformation, i.e.,

$$J = \det\begin{bmatrix} \frac{\partial\varepsilon_{1it}}{\partial H_{it}} & \frac{\partial\varepsilon_{1it}}{\partial\varepsilon_{2it}} \\ \frac{\partial\varepsilon_{2it}}{\partial H_{it}} & \frac{\partial\varepsilon_{2it}}{\partial\varepsilon_{2it}} \end{bmatrix} = \det\begin{bmatrix} \frac{\partial\varepsilon_{1it}}{\partial H_{it}} & \frac{\partial\varepsilon_{1it}}{\partial\varepsilon_{2it}} \\ 0 & 1 \end{bmatrix} = \frac{\partial\varepsilon_{1it}}{\partial H_{it}} \qquad (3.32)$$

Solving (3.30) for ε_{1it}:

$$\varepsilon_{1it} = \frac{\frac{H_{it}}{\gamma_{3i}}\left(1+\gamma_{2i}(x_{it})^{-f_{1i}} + \varepsilon_{2it}(x_{it})^{-f_{1i}}\right)^2 - \left(1+\gamma_{1i}(x_{it})^{-2f_{1i}}\right)}{(x_{it})^{-2f_{1i}}} \qquad (3.33)$$

Consequently,

$$J = \frac{\partial\varepsilon_{1it}}{\partial H_{it}} = \frac{\left(1+\gamma_{2i}(x_{it})^{-f_{1i}} + \varepsilon_{2it}(x_{it})^{-f_{1i}}\right)^2}{\gamma_{3i}(x_{it})^{-2f_{1i}}} \qquad (3.34)$$

Therefore by substituting (3.33) and (3.34) into (3.31) the random component ε_{1it} is eliminated. As a result, the log likelihood function is:

$$L = \ln \prod_{i,t} \int g(H_{it},\varepsilon_{2it})\,d\varepsilon_{2it} = \sum_{i,t} \ln\left[\int g(H_{it},\varepsilon_{it})\,d\varepsilon_{2it}\right]$$

As the above relation indicates, for deriving the log likelihood function one has to solve a very complicated numerical integration. This constitutes a major project in itself and hence is beyond the scope of this thesis.

Once γ_1,γ_2 and f_1 have been successfully estimated it is possible to determine for each industry whether a+b is larger or smaller than one and whether $C_4V x^{f f_1}-1$ is positive or negative and consequently conclude for each industry with $x \geq 1$, whether the MPH or the DEH is the prevailing hypothesis using the industry level conclusions. If $x < 1$ then the firm level conclusions should be used. **It is still not necessary to actually work at the firm level since one can identify in which case the industry belongs by simply looking at the estimates derived at the industry level.** If the estimates reveal that $a+b < 1$ and $x < 1$, the MPH is rejected in favour of the DEH and if they reveal that $a+b > 1$ and $x < 1$ then the DEH is rejected in favour of the MPH.

APPENDIX

In this appendix the different cases in which the ambiguity is resolved shall be illustrated in terms of reaction curves as these are given by relation (3.22). To keep things as simple as possible it is assumed that there are only two firms within the industry, one adopter which we call A and one non adopter which we call NA. As it was mentioned in chapter 3, section 2.1, the between group conjectures are locally consistent. Since we are dealing in this appendix with a duopoly then both $\lambda_A = \alpha_{A,NA}$ and $\lambda_{NA} = \alpha_{NA,A}$ are going to be locally consistent. The notion of consistency in its limited (local) version requires that '...*at the equilibrium quantities, the slopes of the actual reaction functions are equal to the conjectured slopes.*' (Bresnahan, 1981, p.938.) By the definition of consistency a change in the demand and cost conditions of any of the two firms would affect both λ_A and λ_{NA}, i.e. consistency makes conjectures endogenous. If these two conjectures were restricted to be linear ($\lambda_A(Q_A)$ and $\lambda_{NA}(Q_{NA})$ are both constants) then by definition ψ^A and ψ^{NA} would also have to be endogenous (since $\psi^A=(1+\lambda_A)\frac{Q_A}{Q}$, $\psi^{NA}=(1+\lambda_{NA})\frac{Q_{NA}}{Q}$), which is in contradiction with our model's assumption that ETIOV's are exogenously determined. However if we assume that conjectures are non linear functions of output then the parameters of the polynomials $\lambda_A(Q_A)$ and $\lambda_{NA}(Q_{NA})$ can be arbitrarily chosen so that these conjectures are both locally consistent (first derivative restriction) and satisfy the relations $\psi^A=(1+\lambda_A)\frac{Q_A}{Q}$,

$\psi^{NA}=(1+\lambda_{NA})\frac{Q_{NA}}{Q}$ for **exogenously determined** values of ψ^{A} and ψ^{NA}. However since one requirement of consistency is that conjectures are self-fulfiling in equilibrium (a property which it shares with any other type of model in which agents believe that if they do not change their output no one else will either) then **combinations of conjectures for which no equilibrium exists are ruled out.**

As it shall be deduced in detail in Chapter 4, relation (3.22) may be re-written for firm A as follows:

$$p\sigma_{A} = MC_{A} \quad \Leftrightarrow$$

$$(Q_{A})^{\frac{1-a-b}{a+b}} = E_{A}p \qquad (A.1)$$

where $E_{A} = (A_{OA})^{\frac{1}{a+b}}(\sigma_{A})(a+b)(k_{t})^{-1}$. If one sets $p = a_{D}-b_{D}(Q_{A}+Q_{NA})$, $(a_{D}, b_{D} > 0)$, then the reaction function for firm A is equal to:

$$(Q_{A})^{\frac{1-a-b}{a+b}} + E_{A}b_{D}Q_{A} = E_{A}(a_{D}-b_{D}Q_{NA}) \qquad (A.2)$$

Similarly the reaction function for firm NA is equal to:

$$(Q_{NA})^{\frac{1-a-b}{a+b}} + E_{NA}b_{D}Q_{NA} = E_{NA}(a_{D}-b_{D}Q_{A}) \qquad (A.3)$$

where $E_{NA} = (B_{ONA})^{\frac{1}{a+b}}(\sigma_{NA})(a+b)(k_{t})^{-1}$. The equilibrium point is the point of intersection of the two reaction curves. If the equilibrium price is equal to p^{*}, then:

$$p^* = \frac{(Q_A)^{\frac{1-a-b}{a+b}}}{E_A} = \frac{(Q_{NA})^{\frac{1-a-b}{a+b}}}{E_{NA}} \Leftrightarrow$$

$$Q_{NA} = Q_A \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \qquad \text{(A.4)}$$

where $\psi = \frac{1-a-b}{a+b}$. If the reaction function of firm A is solved with respect to the output of firm NA we have:

$$Q_{NA} = \frac{a_D}{b_D} - Q_A - \frac{(Q_A)^{\psi}}{E_A b_D} \qquad \text{(A.5)}$$

Respectively, if the reaction function of firm NA is solved with respect to the output of firm A we have:

$$Q_A = \frac{a_D}{b_D} - Q_{NA} - \frac{(Q_{NA})^{\psi}}{E_{NA} b_D} \qquad \text{(A.6)}$$

If one substitutes the equilibrium condition (A.4) into (A.5) we derive the following relation:

$$Q_A \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} = \frac{a_D}{b_D} - Q_A - \frac{(Q_A)^{\psi}}{E_A b_D} \Leftrightarrow$$

$$\frac{a_D}{b_D} - \left[1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right] Q_A - \frac{(Q_A)^{\psi}}{E_A b_D} = 0 \qquad \textbf{(A.7)}$$

Let us first examine the case where $\psi > 0$ $(a+b<1)$. Then in order to have the two reaction curves intersecting at positive outputs we require that there is a positive solution to relation (A.7), which is always true when ψ is positive. **In other words, under decreasing returns to scale the reaction curves always intersect at a positive output combination.** Moreover, in order to describe the exact form of the reaction curves it should be noted that the point where the reaction curve of firm A intersects the Q_{NA} axis is equal to the point where the reaction curve of firm NA intersects the Q_A axis which is equal to $\frac{a_D}{b_D}$. Furthermore, the slope of both reaction curves is negative as can be easily shown by taking the first derivative in relation (A.5) with respect to Q_A and the first derivative in relation (A.6) with respect to Q_{NA} (the latter gives the inverse of the slope of the reaction function), both of which are always negative when $\psi>0$.

$$\frac{dQ_{NA}}{dQ_A} = -1 - \frac{\psi (Q_A)^{\psi-1}}{E_A b_D} < 0 \qquad \textbf{(A.5')}$$

$$\frac{dQ_A}{dQ_{NA}} = -1 - \frac{\psi (Q_{NA})^{\psi-1}}{E_{NA} b_D} \Leftrightarrow$$

$$\frac{dQ_{NA}}{dQ_A} = \left(-1 - \frac{\psi(Q_{NA})^{\psi-1}}{E_{NA}b_D}\right)^{-1} \qquad (A.6')$$

As relationship (A.5') implies the absolute value of the slope of firm's A reaction function is always larger that one for $\psi>0$, while the absolute value of the slope of firm's NA reaction function is always smaller than one as (A.6') implies. In other words the tangent at any point on firm's A reaction curve is more steeply sloped that the tangent on firm's NA reaction curve at the same point. This condition guarantees the stability fo equilibrium when $\psi>0$. Additionally if one takes the second derivatives in these relations it is easily deduced that for $\psi>1$ the reaction curves are concave and for $0<\psi<1$ the reaction curves are convex. If in addition to $\psi>0$, $\sigma_A>\sigma_{NA}$ $(x<1)$, then E_A is larger that E_{NA} and consequently from relation (A.4) it is obvious that when $\psi>0$ and $E_{NA}<E_A$ then the equilibrium output of the adopter firm is larger that the equilibrium output of the non adopter firm. This corresponds to the case of $a+b<1$ and $x<1$ in chapter 3 where according to the firm level results the MPH is rejected in favour of the DEH. If $x>1$ and still $(A_{OA})^{\frac{1}{a+b}}\sigma_A > (B_{ONA})^{\frac{1}{a+b}}\sigma_{NA}$ then E_A would still be larger than E_{NA} and as a result the output of firm A would be larger that the output of firm NA meaning than in terms of the firm level criteria of chapter 3 both hypotheses would be accepted. However, in terms of the industry level criteria the MPH is rejected in favour of the DEH since this case corresponds to the case where $a+b<1$, $x>1$ and $C_4V^f>x^{f_1}$ (note that for the duopoly model $C_4V^f = \left(\frac{A_{OA}}{B_{ONA}}\right)^f$ and $f_1 = \frac{1}{\psi}$). If when x>1 we have that $(A_{OA})^{\frac{1}{a+b}} \sigma_A < (B_{ONA})^{\frac{1}{a+b}} \sigma_{NA}$ then because the equilibrium output of firm A is smaller than the equilibrium output of firm NA, in terms of the firm level criteria both hypotheses would be rejected while in terms of the industry level criteria the DEH is rejected in favour of the MPH since this case corresponds to the case where a+b<1, x>1 and $C_4V^f < x^{f_1}$.

If one wishes to move a step further and determine what the point where the reaction curve of firm A intersects the Q_A axis is equal to (and respectively what the point where the reaction curve of firm NA intersects the Q_{NA} axis is equal to), specific values for ψ>0 will have to be looked at, so that one can get an idea of how the equilibrium is determined under decreasing static returns to scale. If for example ψ = 1 (a+b = 0.5) then if we solve (A.2) for this value and for Q_{NA} = 0 we get that the point the reaction curve of firm A intersects the Q_A axis is equal to $\frac{E_A a_D}{(1+E_A b_D)}$, and respectively the point where the reaction curve of firm NA intersects the Q_{NA} axis is equal to $\frac{E_{NA} a_D}{(1+E_{NA} b_D)}$. It is obvious that both of these points are smaller than $\frac{a_D}{b_D}$. Moreover, as it was proven, for ψ>0 the absolute value of the slope of A's reaction curve has a steeper slope than NA's reaction curve.

Diagram A1 illustrates what happens when ψ = 1 and additionally $E_A > E_{NA}$. As a consequence of E_A being larger than E_{NA},

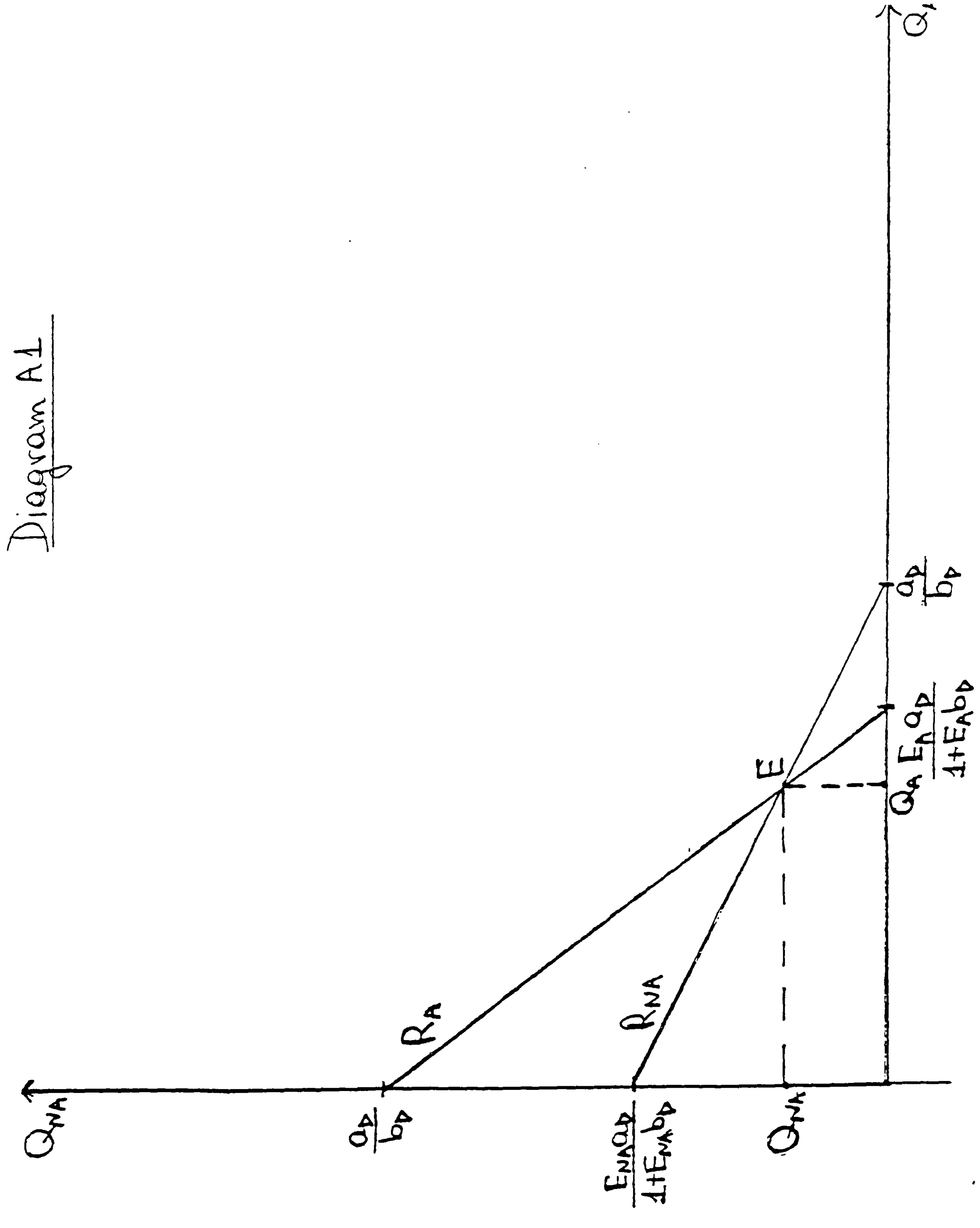


Diagram A1

$\frac{E_A a_D}{(1+E_A b_D)} > \frac{E_{NA} a_D}{(1+E_{NA} b_D)}$. At equilibrium (point E) the output of the adopter firm is larger than the equilibrium output of the non adopter firm. The case where $E_A<E_{NA}$ is illustrated in diagram A2. Here we have that $\frac{E_A a_D}{(1+E_A b_D)} < \frac{E_{NA} a_D}{(1+E_{NA} b_D)}$. At equilibrium, the output of the adopter firm is smaller than the output of the non adopter firm. If $\psi>1$ then diagram A3 illustrates the case where at equilibrium the output of firm A is larger than the output of firm NA. On the other hand, diagram A4 depicts the case where the equilibrium output of firm NA is larger than the equilibriun output of firm A. For the situation where $0<\psi<1$ the two respective cases are illustrated in diagrams A5 and A6 respectively.

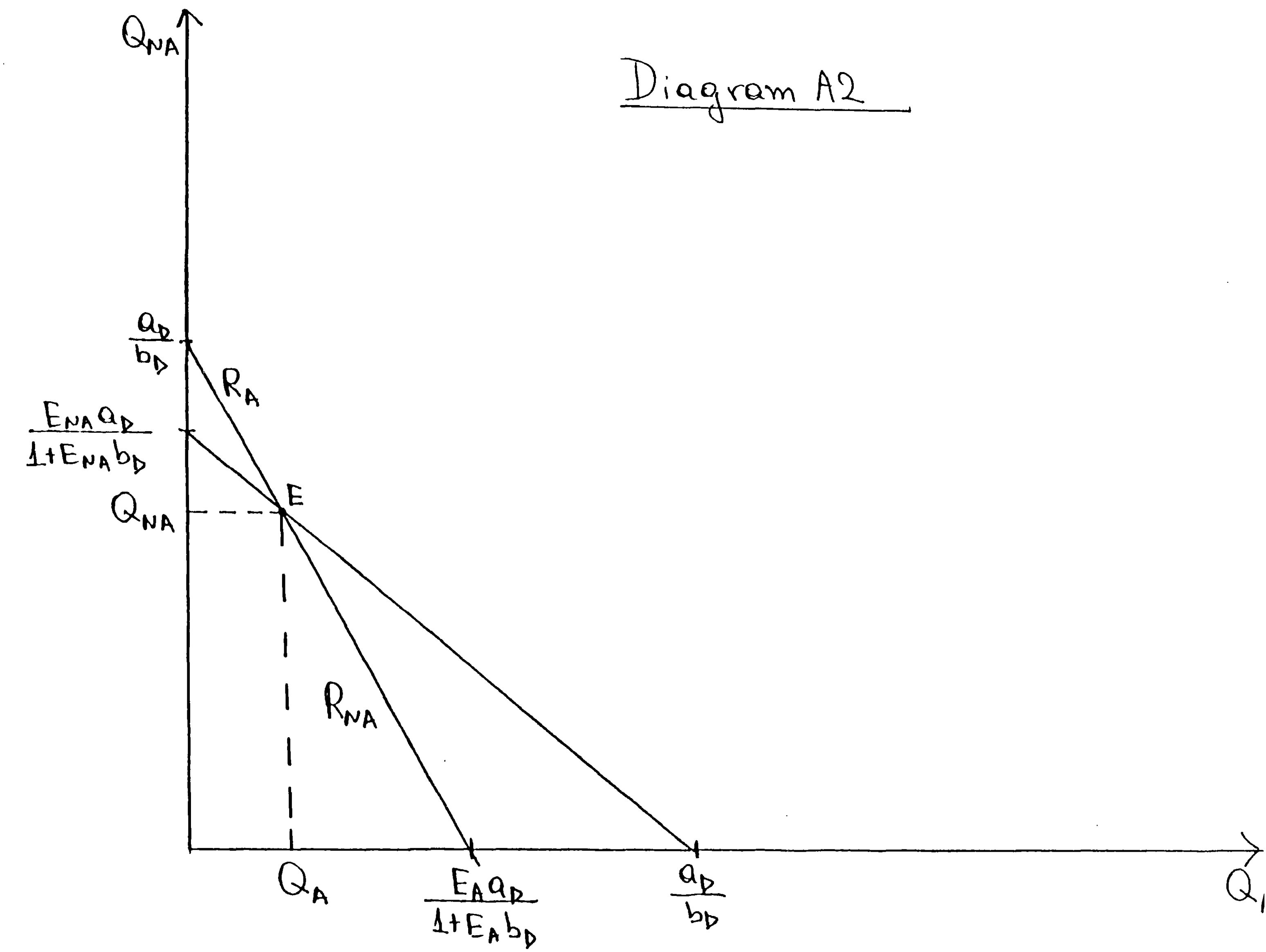
Let us now examine the case where $\psi<0$ (a+b>1). Set y to be equal to the L.H.S. of relation (A.7). If y is differentiated with respect to Q_A and the derivative is set equal to zero, then what is derived is the value of Q_A that corresponds to the maximum value of y. In particular this is equal to:

$$Q_{A\,max(y)} = \left[\left(1+\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}}\right) \frac{E_A b_D}{(-\psi)}\right]^{\frac{1}{\psi-1}} \qquad (A.8)$$

As a result the maximum value of y is equal to:

$$y_{max} = \frac{a_D}{b_D} - \left[1 + \left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}}\right] \left[\left(1+\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}}\right) \frac{E_A b_D}{(-\psi)}\right]^{\frac{1}{\psi-1}}$$

Diagram A2



Q_{NA}
$\frac{a_D}{b_D}$
R_A
$\frac{E_{NA} a_D}{1+E_{NA} b_D}$
E
Q_{NA}
R_{NA}
Q_A
$\frac{E_A a_D}{1+E_A b_D}$
$\frac{a_D}{b_D}$

Diagram A3

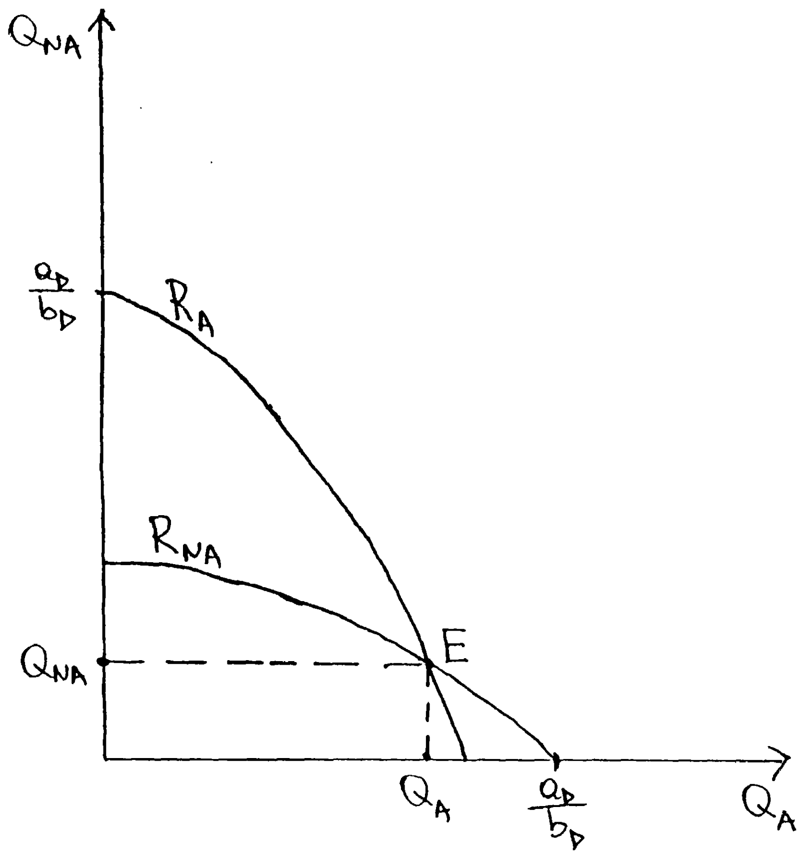


Diagram A4

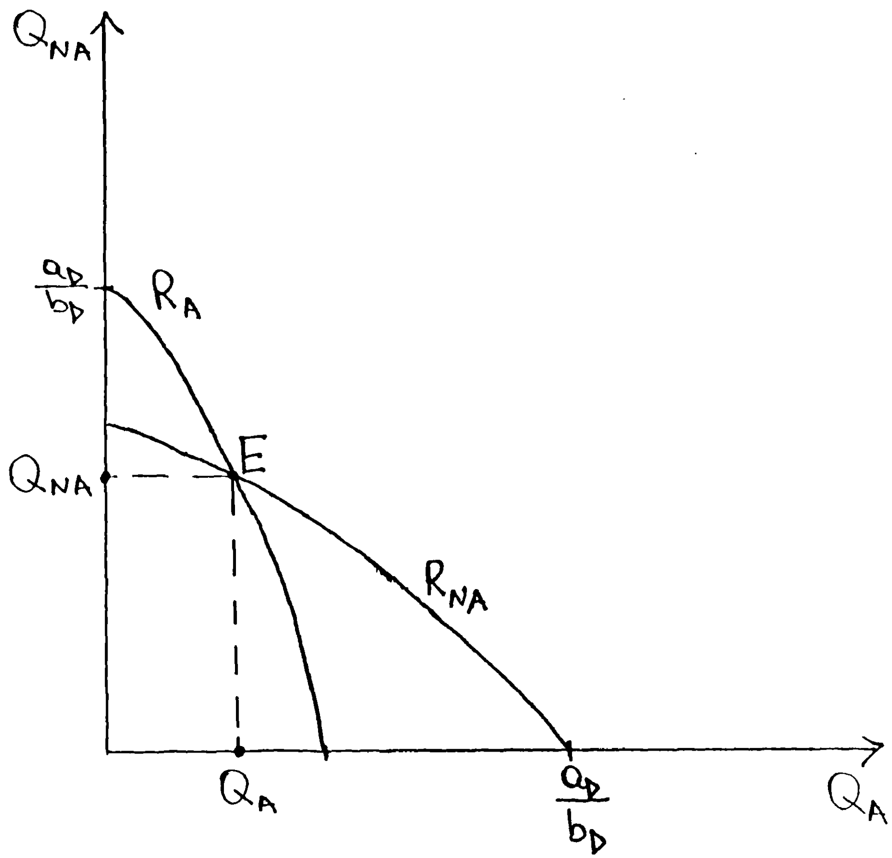


Diagram A5

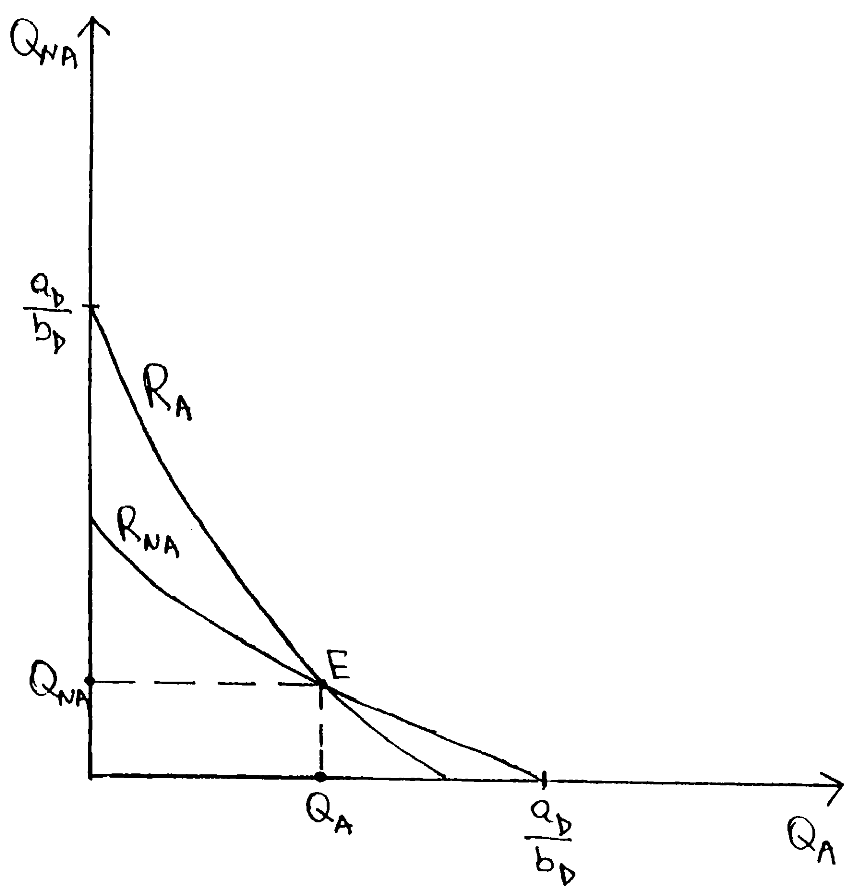
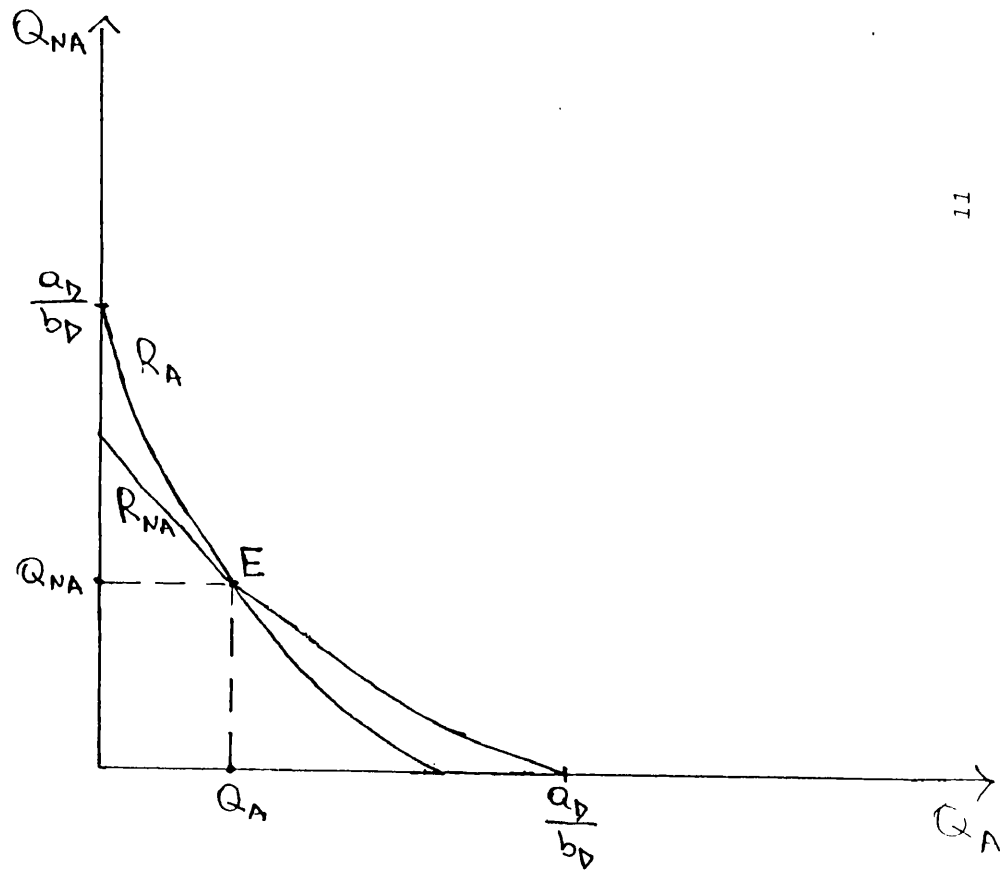


Diagram A6



$$- \frac{1}{E_A b_D} \left[\left(1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right) \frac{E_A b_D}{(-\psi)} \right]^{\frac{\psi}{\psi-1}} =$$

$$= \frac{a_D}{b_D} - \left[1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \left(\frac{E_A b_D}{(-\psi)} \right)^{\frac{1}{\psi-1}}$$

$$- \left(\frac{E_A b_D}{(-\psi)} \right)^{\frac{\psi}{\psi-1}} \frac{1}{E_A b_D} \left(1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right)^{\frac{\psi}{\psi-1}} =$$

$$= \frac{a_D}{b_D} - \left[1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \left(\frac{E_A b_D}{(-\psi)} \right)^{\frac{1}{\psi-1}}$$

$$- \frac{(E_A b_D)^{\frac{1}{\psi-1}}}{(-\psi)^{\frac{\psi}{\psi-1}}} \left(1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right)^{\frac{\psi}{\psi-1}} =$$

$$= \frac{a_D}{b_D} - \left[1 + \left(\frac{E_{NA}}{E_A} \right)^{\frac{1}{\psi}} \right]^{\frac{\psi}{\psi-1}} (E_A b_D)^{\frac{1}{\psi-1}} \left((-\psi)^{\frac{1}{1-\psi}} + (-\psi)^{\frac{\psi}{1-\psi}} \right) \Leftrightarrow$$

$$y_{max} = \frac{a_D}{b_D} - \left[(E_A b_D)^{\frac{1}{\psi}} + (E_{NA} b_D)^{\frac{1}{\psi}} \right]^{\frac{\psi}{\psi-1}} \left((-\psi)^{\frac{1}{1-\psi}} + (-\psi)^{\frac{\psi}{1-\psi}} \right) \qquad \text{(A.8)}$$

Consequently, the condition for $y_{max} \geq 0$ is that:

$$a_D \geq \left[(E_A)^{\frac{1}{\psi}} + (E_{NA})^{\frac{1}{\psi}} \right]^{\frac{\psi}{\psi-1}} (b_D)^{\frac{\psi}{\psi-1}} \left((-\psi)^{\frac{1}{1-\psi}} + (-\psi)^{\frac{\psi}{1-\psi}} \right) \qquad \text{(A.9)}$$

When y_{max} is positive then as diagram A7 shows there are going to be two positive solutions (E_1 and E_2 respectively) and therefore the reaction curves of the two firms will intersect twice. If on the other hand $y_{max} = 0$ as illustrated in diagram A8 then the two reaction curves will touch at only one point while if $y_{max} < 0$ (diagram A9) then the two reaction curves will not intersect for any positive output combination. What we additionally can prove before we set out to draw the above three cases of the reaction curves is that if the two curves intersect then their first point of intersection will lie at points of positive slopes for both reaction curves. This is proved as follows. First we differentiate (A.5) with respect to Q_A in order to calculate the slope of the reaction curve of firm A:

$$\frac{dQ_{NA}}{dQ_A} = -1 + \frac{1}{E_A b_D}(-\psi)(Q_A)^{\psi-1} \qquad \text{(A.10)}$$

Moreover we differentiate (A.6) with respect to Q_{NA} in order to calculate the inverse of the slope of the reaction curve of firm NA:

Diagram A7

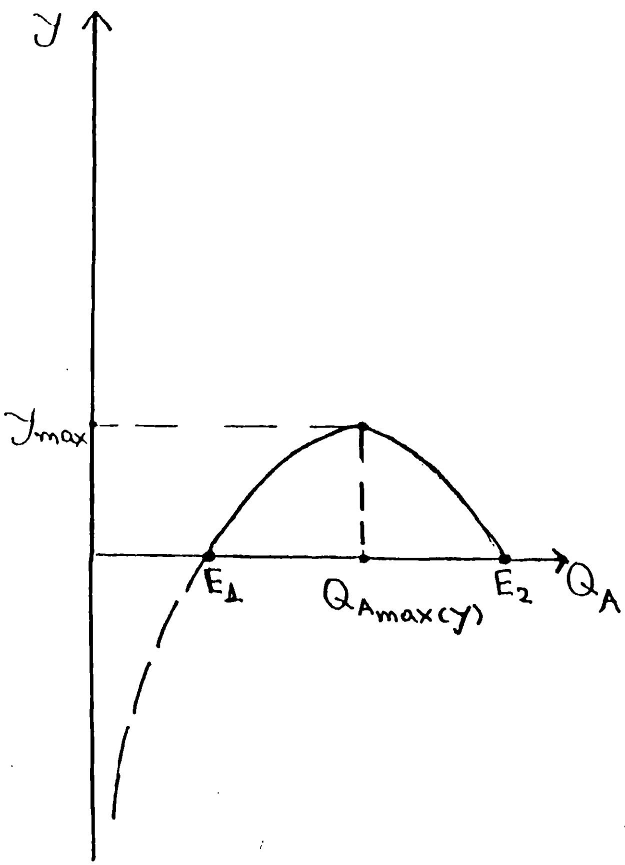


Diagram A8

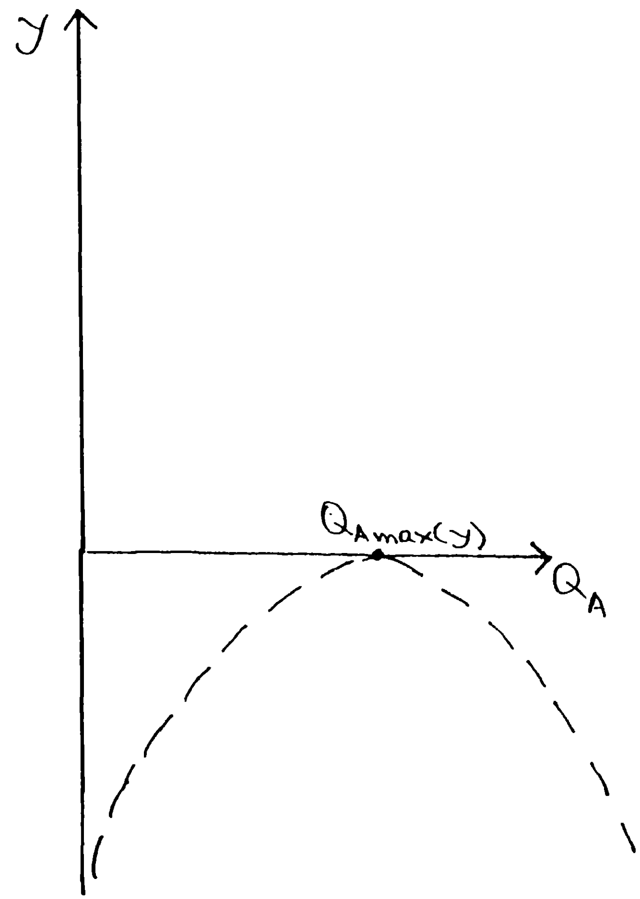
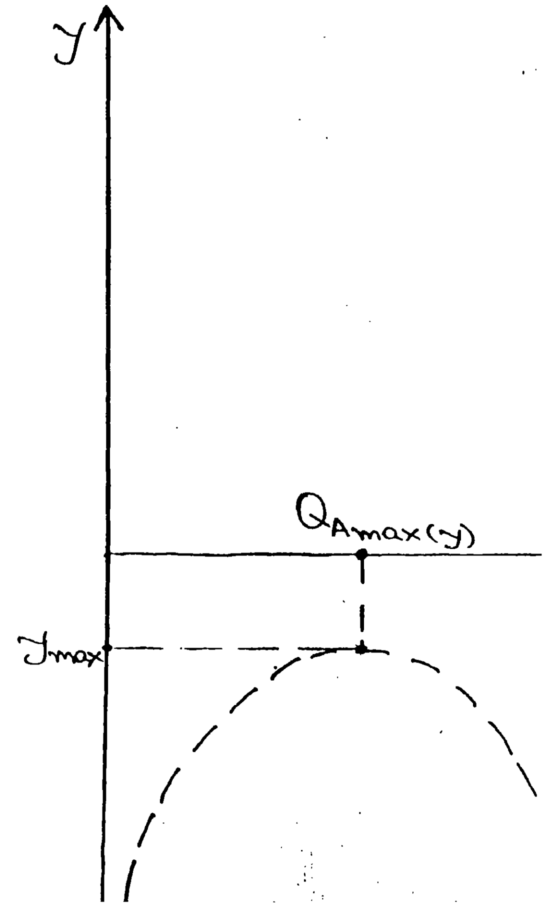


Diagram A9



$$\frac{dQ_A}{dQ_{NA}} = -1 + \frac{1}{E_{NA} b_D}(-\psi)(Q_{NA})^{\psi-1} \qquad \text{(A.11)}$$

If we substitute equilibrium condition (A.4) into (A.11) we get:

$$\frac{dQ_A}{dQ_{NA}} = -1 + \frac{1}{E_{NA} b_D}(-\psi)(Q_A)^{\psi-1}\left(\frac{E_{NA}}{E_A}\right)^{\frac{\psi-1}{\psi}} =$$

$$= -1 + \frac{1}{E_A b_D}(-\psi)(Q_A)^{\psi-1}\left(\frac{E_{NA}}{E_A}\right)^{\frac{-1}{\psi}} \qquad \Leftrightarrow$$

$$\frac{dQ_A}{dQ_{NA}}\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}} = -\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}} + \frac{1}{E_A b_D}(-\psi)(Q_A)^{\psi-1} \qquad \text{(A.12)}$$

Getting back to relation (A.10), if we set Q_A equal to $Q_{A_{max(y)}}$ we will get the following:

$$\frac{dQ_{NA}}{dQ_A} = -1 + \frac{1}{E_A b_D}(-\psi)\left(1+\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}}\right)\frac{E_A b_D}{(-\psi)} \qquad \Leftrightarrow$$

$$\frac{dQ_{NA}}{dQ_A} = \left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}} > 0 \qquad \text{(A.13)}$$

Moreover if we substitute for $Q_{A_{max(y)}}$ into (A.12) we derive:

$$\frac{dQ_A}{dQ_{NA}}\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}} = -\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}} + \frac{1}{E_A b_D}(-\psi)\left(1+\left(\frac{E_{NA}}{E_A}\right)^{\frac{1}{\psi}}\right)\frac{E_A b_D}{(-\psi)} \Leftrightarrow$$

$$\frac{dQ_A}{dQ_{NA}} = \left(\frac{E_{NA}}{E_A}\right)^{\frac{-1}{\psi}} > 0 \qquad (A.14)$$

What (A.13) and (A.14) imply is that at $Q_{A_{max(y)}}$ the slopes of both reaction curves are positive. This means that at the first intersection point, E_1 (diagram A7) which is smaller than $Q_{A_{max(y)}}$ the slopes of both reaction curves will also be positive. At the second intersection point, E_2 ($E_2 > Q_{A_{max(y)}}$), any combination of slopes for the two reaction curves may be valid. If there is only one intersection point, then since this is given by $Q_{A_{max(y)}}$ itself (diagram A8), again at this sole point of intersection both reaction curves will have positive slopes.

Having said the above, we are now ready to draw the diagrams of the reaction curves that correspond to the cases described in diagrams A7, A8 and A9 respectively. Diagram A10 corresponds to diagram A7. In this particular diagram the two equilibrium points have been drawn in such a way than in E_1 firm A produces more than firm NA while in E_2 firm A produces less than firm NA. If we follow a similar analysis as that for $\psi>0$, by taking into account equilibrium condition A.4 since ψ is now negative if Q_A is larger

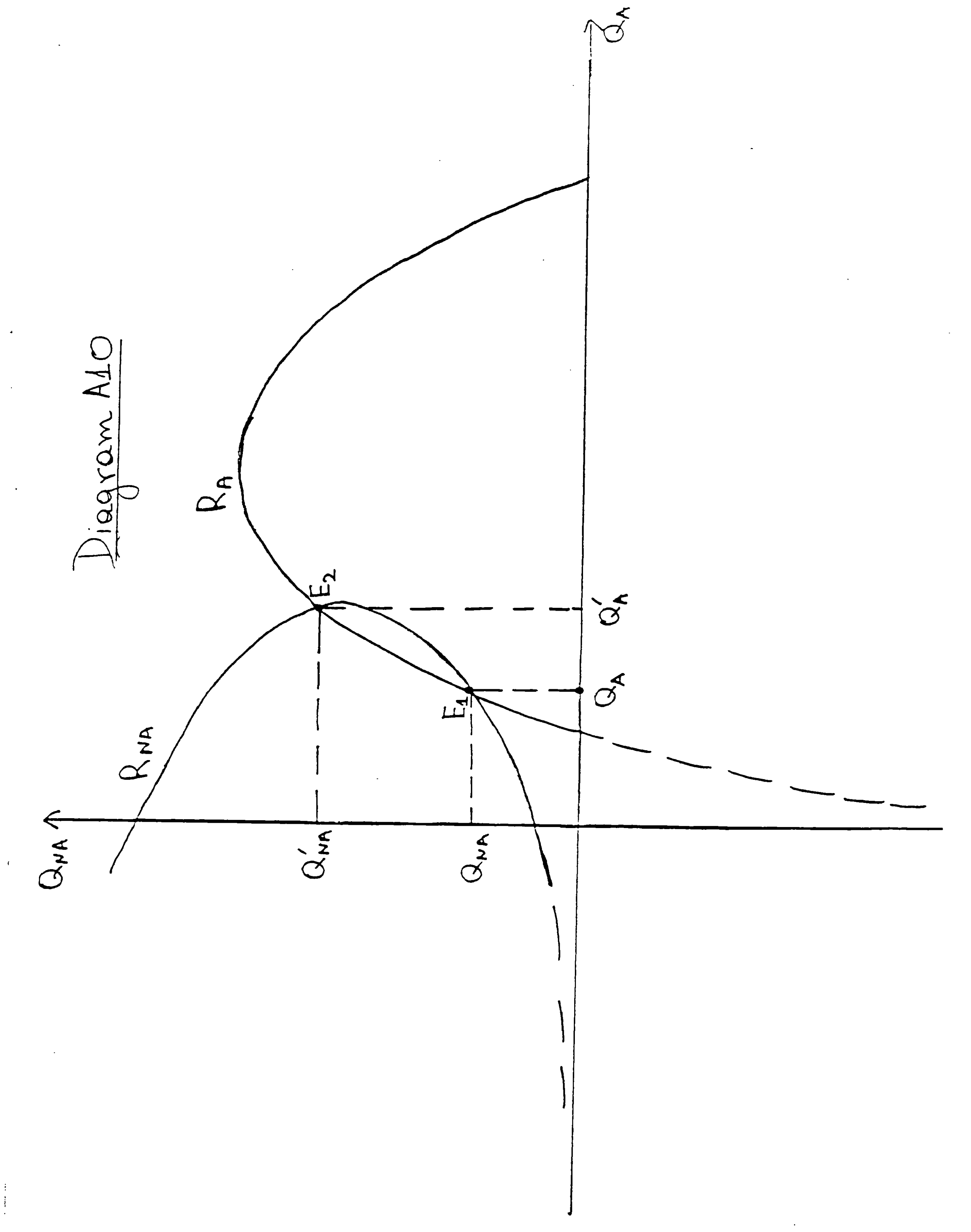


Diagram A10
R_A
R_{NA}
E_2
E_1
Q_A
Q'_A
Q_A
Q_{NA}
Q'_{NA}
Q_{NA}

than Q_{NA}, then this implies that E_{NA} is larger than E_A. For this to happen a necessary condition is that x>1. Therefore equilibrium E_1 corresponds to the case where a+b<1, x>1 and $C_4V^f > x^{f_1}$ (since $(A_{OA})^{\frac{1}{a+b}}\sigma_A < (B_{ONA})^{\frac{1}{a+b}}\sigma_{NA}$), in which according to the firm level criteria both hypotheses would be accepted while according to the industry level criteria the ambiguity is resolved by rejecting the DEH in favour of the MPH. On the other hand in E_2, E_{NA} is smaller than E_A (since Q_A is smaller than Q_{NA}), implying that this equilibrium point will either correspond to the case where a+b<1 and x<1, in which situation according to the firm level criteria the DEH is rejected in favour of the MPH, or in the case where a+b<1, x>1 and $C_4V^f < x^{f_1}$ where the ambiguity is resolved using the industry level criteria by rejecting the MPH in favour of the DEH. Diagram A11 corresponds to diagram A8. The way it is drawn, we have that firm A has a smaller equilibrium output than the equilibrium output of firm NA but the reverse situation is also a possible equilibrium outcome. Finally, diagram A12 corresponds to diagram A9. In this situation no equilibrium exists for the duopoly model with consistent conjectures. This means that '*...any duopoly equilibrium is dominated in cost terms by some single firm outcome. Hence it is the theory of entry, not of duopoly, which determines price* (output). *Monopoly equilibria do exist. Thus only equilibria with the cost minimizing number of firms in operation exist.*' (Bresnahan, p.940.) In other words the outcome will be a monopoly with firm A being the sole producer.

Diagram A11

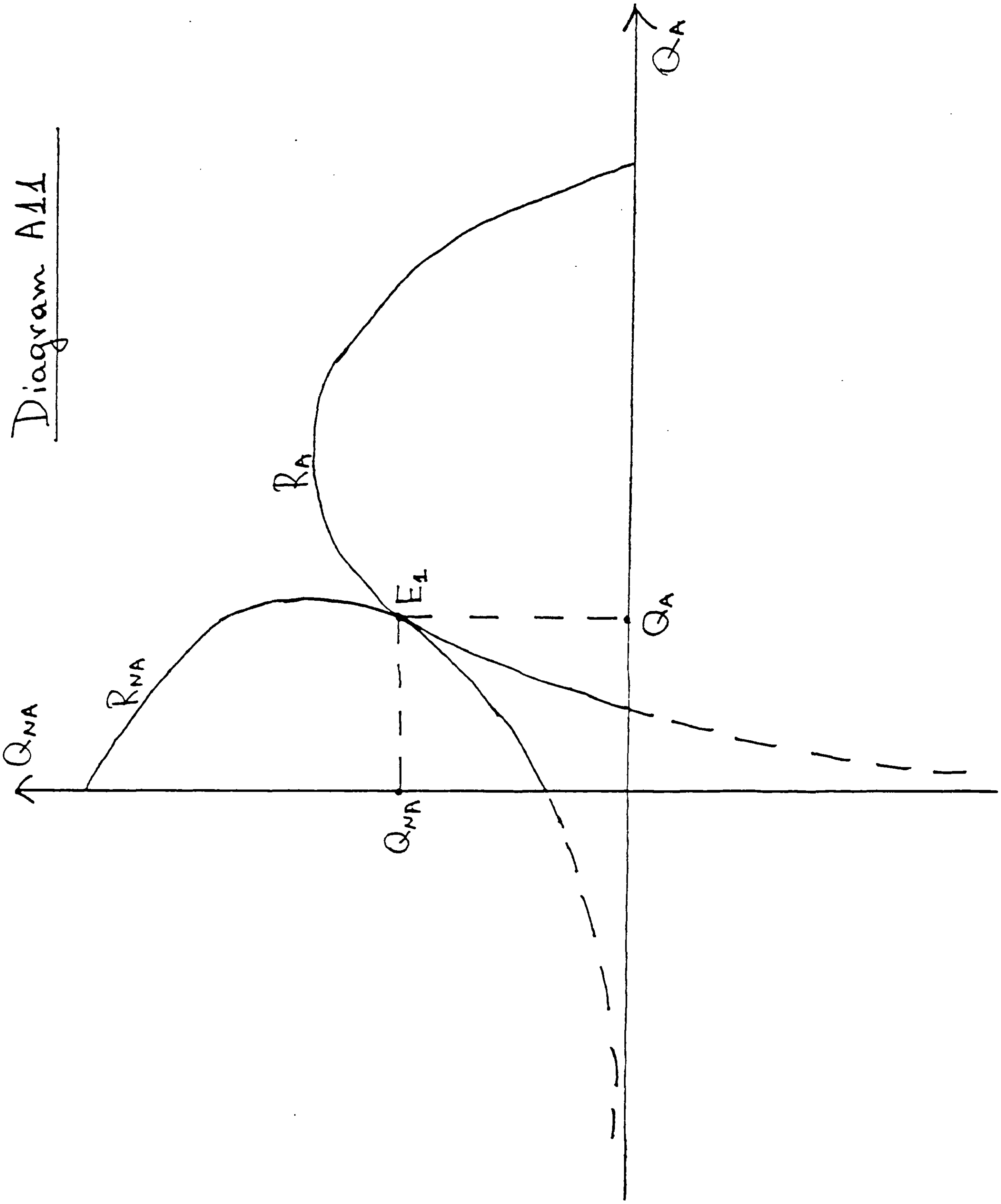
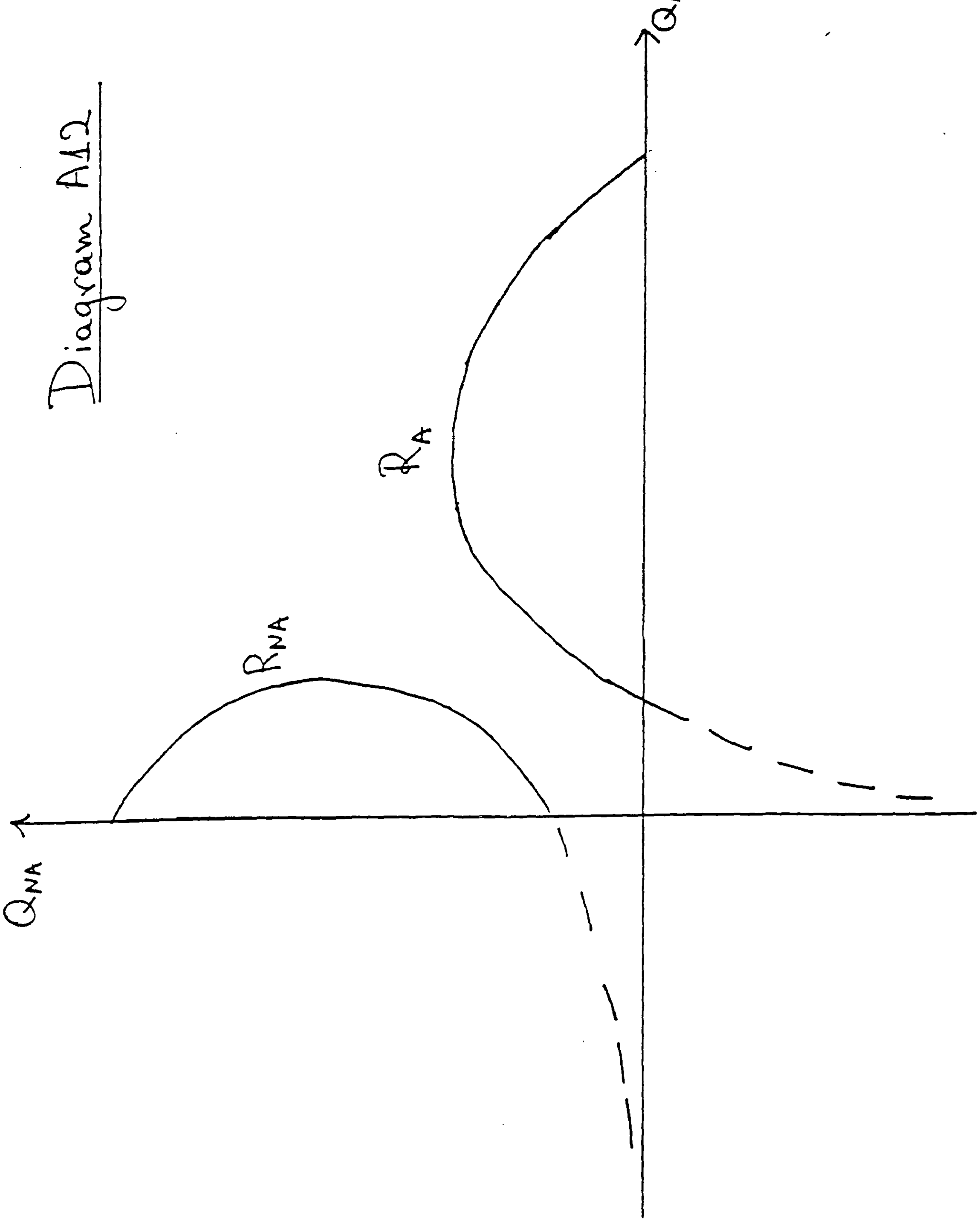


Diagram A12



CHAPTER 4

THE STANDING OF THE GENERALISED CO OPERATIVE MODEL WITHIN THE MARKET STRUCTURE-PERFORMANCE FRAMEWORK

The purpose of this chapter is to compare the model developed in chapter 3 to that of Cowling and Waterson. It shall be demonstrated that the former is a generalisation of the latter for all types of costs (increasing, constant and decreasing static returns to scale). As a proof for this, relation (3.22) is re-derived from the generalised C&W model. As a next step all the cases for resolving or not resolving the ambiguity at the firm level (as developed in section 3.2) are illustrated diagrammatically. Finally, the special case of a+b = 1 is discussed, demonstrating the procedure for resolving the ambiguity between the two hypotheses under constant static returns to scale.

4.1 Re-expressing the model in terms of the Cowling and Waterson model

Working in terms of the C&W model involves re-expressing everything in terms of cost rather than in terms of productivity. Consequently, superior efficiency is in terms of the cost curves each firm faces. Just as with efficiency in terms of productivity, it is assumed that there is no overlapping between any two firms in terms of cost, i.e. the marginal (average) cost curves of any two firms j and l will not intersect at any point (the V $\upsilon_{jl} > 1$

condition). The first order condition in the C&W model is:

$$Q_j = \frac{e\ Q}{1+\lambda_j} \left(1- \frac{MC_j}{p}\right) \tag{4.1}$$

For a homogeneous production function of degree ζ, the cost function will be homogeneous of degree $\frac{1}{\zeta}$ with repect to the output and consequently the marginal cost function will be homogeneous of degree $\frac{1}{\zeta}$ - 1. This means that MC_j will be a non-constant function of Q_j (among other variables) unless $\zeta = 1$. In other words by imposing that $\zeta = 1$, C&W do away with the problem of marginal cost being a function of Q_j. But if $\zeta \neq 1$ then the problem with (4.1) is that it is not a reduced form expression for Q_j. If both terms in (4.1) are multiplied by $\frac{1+\lambda_j}{Q}$ the following is derived:

$$(1+\lambda_j)\frac{Q_j}{Q} = e \left(1- \frac{MC_j}{p}\right) \tag{4.2}$$

Considering the L.H.S. as a parameter of the behaviour of the firm, Q_j may be expressed in its reduced form, provided that we find what MC_j is equal to. Assuming a Cobb-Douglas production function we may re-write the first order conditions (3.1) and (3.2) with respect to labour and capital as:

$$r_t = p\ \sigma_j\ G_{jt}\ (K_{jt})^{a-1}(L_{jt})^{b} \tag{4.3}$$

$$w = p\ \sigma_j\ G_{jt}\ (K_{jt})^{a}(L_{jt})^{b-1} \tag{4.4}$$

Dividing (4.4) by (4.3) relation (2.27) is derived. Then the Cobb Douglas is rewriten utilising relation (2.27) and, if the same procedure as in section 2.3 is followed, it can be easily deduced that MC_j is equal to:

$$MC_{jt} = \frac{1}{a+b} k_t (G_{jt})^{-\frac{1}{a+b}} (Q_{jt})^{\frac{1-a-b}{a+b}} \tag{4.5}$$

Relation (4.5) is identical to (2.34) for j∈A and identical to (2.35) for j∈NA. Combining (4.1) with (4.5) we get for jeA and *l*eNA respectively:

$$\sigma_j\ p = \frac{1}{a+b} k_t (A_t\ \upsilon_j)^{-\frac{1}{a+b}} (Q^A_{jt})^{\frac{1-a-b}{a+b}} \tag{4.6}$$

$$\sigma_l\ p = \frac{1}{a+b} k_t (B_t\ \upsilon_l)^{-\frac{1}{a+b}} (Q^{NA}_{lt})^{\frac{1-a-b}{a+b}} \tag{4.7}$$

From relations (4.6) and (4.7) it is easily deduced that, for a given level of output, ETIOV is inversely related to the efficiency index, a point already discussed in section 3.1. Solving this type of relation in terms of output the following is derived:

$$Q_{jt} = \left(\sigma_j\ p\ (a+b)\ (k_t)^{-1} (G_{jt})^{\frac{1}{a+b}} \right)^{\frac{a+b}{1-a-b}} \tag{4.8}$$

Substituting for k_t into (4.8):

$$Q_{jt} = \left(\sigma_j \; p \; (a+b) \left(\frac{b}{a+b}\right)\left(\frac{a}{b}\right)^{\frac{a}{a+b}} (r_t)^{-\frac{a}{a+b}} (w)^{-\frac{b}{a+b}} (G_{jt})^{\frac{1}{a+b}}\right)^{\frac{a+b}{1-a-b}} =$$

$$= \left(\sigma_j \; p \; (a)^{\frac{a}{a+b}} (b)^{\frac{b}{a+b}} (r_t)^{-\frac{a}{a+b}} (w)^{-\frac{b}{a+b}} (G_{jt})^{\frac{1}{a+b}}\right)^{\frac{a+b}{1-a-b}} \Leftrightarrow$$

$$Q_{jt} = (G_{jt})^{\frac{1}{1-a-b}} \sigma_j^{\frac{a+b}{1-a-b}} \left((p)^{a+b} (a)^{a} (b)^{b} (r_t)^{-a} (w)^{-b}\right)^{\frac{1}{1-a-b}} \qquad (4.8')$$

The above is identical to relation (3.22). Therefore the same expression for the optimal level of output is derived from the generalised C&W model.

4.2 A diagrammatic approach to the firm level discrimination analysis

Let the actual demand curve be of the form $p = f(Q)$. The marginal revenue as conjectured by a firm $j \in A$ is equal to:

$$MR_j^A = D_j^A \left(1 - \frac{1}{e(Q, \psi_j^A)}\right) \qquad (4.9)$$

where MR_j^A is the conjectured marginal curve for firm j, D_j^A is the conjectured demand curve and e the absolute value of elasticity of demand as a function of total output and the ETIOV of firm j. Equivalently, for a firm l, $l \in NA$, the relation is:

$$MR_l^{NA} = D_l^{NA}\left(1 - \frac{1}{e(Q,\psi_l^{NA})}\right) \qquad (4.10)$$

when in equilibrium the following conditions will hold:

$$D_j^A(Q_j^{*A}) = D_l^{NA}(Q_l^{*NA}) = p^* \qquad (4.11)$$

$$MR_j^A(Q_j^{*A}) = MC_j^A(Q_j^{*A}) \qquad (4.12)$$

$$MR_l^{NA}(Q_l^{*NA}) = MC_l^{NA}(Q_l^{*NA}) \qquad (4.13)$$

where Q_j^{*A} and Q_l^{*NA} are the profit maximising levels of output at equilibrium for firm j and l respectively, and $p^*=f(Q^*)$ where Q^* is the sum of all the profit maximising outputs in the industry at equilibrium. Henceforth, for simplicity we will drop the star superscript from the output expressions, but the reader is reminded that 'Q' expressions refer to profit maximising levels of output, just as it was implicitly assumed in the previous chapters.

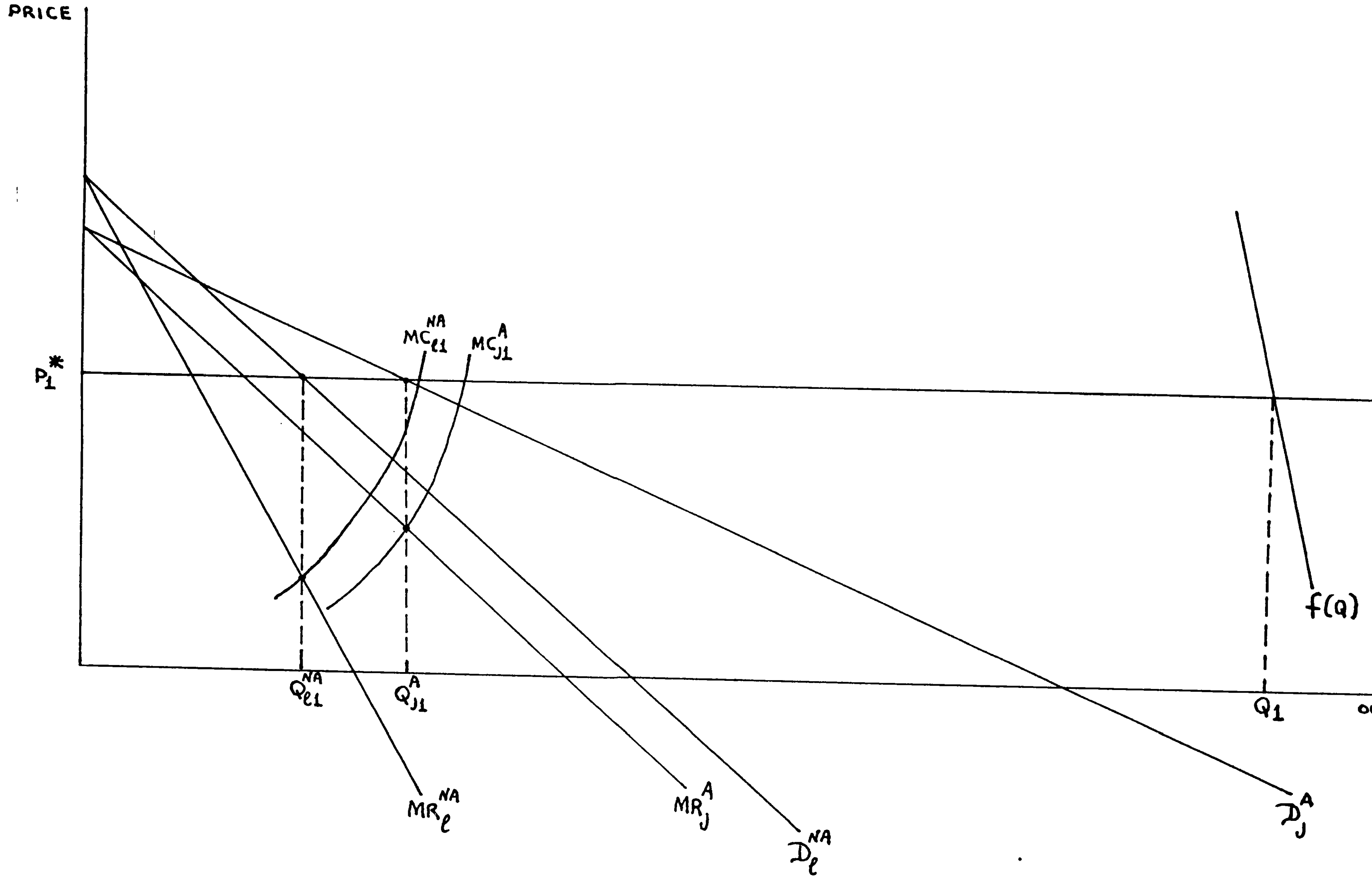
If the ETIOV of the adopter firm is smaller than the ETIOV of the non adopter firm (x<1) then utilising relations (4.6) and (4.7) the following will hold for the profit maximising firms when in equilibrium:

$$MR_j^A(Q_j^A) = MC_j^A(Q_j^A) > MR_l^{NA}(Q_l^{NA}) = MC_l^{NA}(Q_l^{NA}) \qquad \Leftrightarrow (4.14)$$

$$\frac{1}{e(Q,\psi_j^A)} < \frac{1}{e(Q,\psi_l^{NA})} \Leftrightarrow e(Q,\psi_j^A) > e(Q,\psi_l^{NA}) \qquad (4.15)$$

Therefore when $x<1$ then the demand curve as conjectured by the adopter firm is more elastic than the demand curve as conjectured by the non adopter firm. Using relations (4.11)-(4.15) the two cases of $x < 1$ for which the ambiguity was resolved at the firm level can be illustrated diagramatically. In particular, in Figure 2 the case of $a+b < 1$ for which the MPH was rejected in favour of the DEH is given by the curves with the subscript 1. The equilibrium price is p_1^* corresponding to an equilibrium aggregate output Q_1. The increasing marginal cost curves are MC_{j1}^A and MC_{l1}^{NA} respectively. Since in equilibrium, the two firms' conjectured marginal revenue curves MR_j^A and MR_l^{NA} intersect their respective marginal cost curves at levels of output Q_{j1}^A and Q_{l1}^{NA} such that condition (4.11) is satisfied, i.e. the price conjectured by each firm is equal to the equilibrium price p_1^*. The market share of the adopter firm is larger than the market share of the non adopter firm while the marginal cost of the adopter firm when producing output Q_{j1}^A is larger that the marginal cost of the non adopter firm which produces output Q_{l1}^{NA}, i.e. condition (4.14) is satisfied and as a consequence π_j^A is smaller than π_l^{NA}. Also as it can be

FIGURE 2



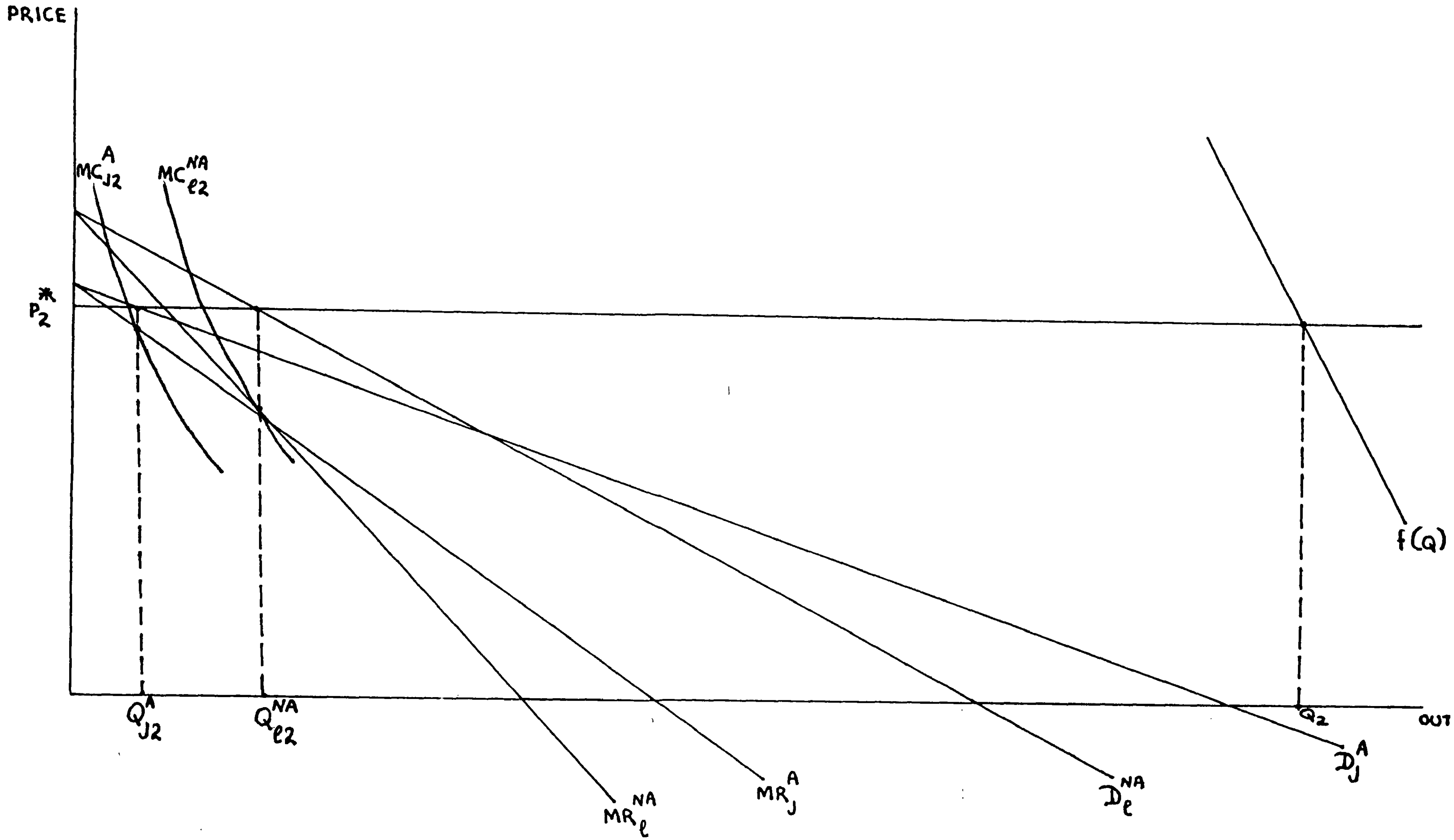
seen in Figure 2, the demand curve as conjectured by firm j, D_j^A, has been drawn in such a way that is more elastic that the demand curve as conjectured by firm *1*, D_1^{NA}, so that condition (4.15) is satisfied. The case of a+b > 1 for which the DEH was rejected in favour of the MPH is illustrated in figure 2'. The equilibrium price is p_2^* corresponding to an aggregate equilibrium output Q_2. The decreasing marginal cost curves are MC_{j2}^A and MC_{12}^{NA} respectively. The two firms' marginal revenue curves will intersect their respective marginal cost curves at points that correspond to optimal levels of output Q_{j2}^A and Q_{12}^{NA} such that the price as conjectured by each firm is equal to the equilibrium price p_2^*. The market share of the adopter firm will be smaller than the market share of the non adopter firm while, as before, π_j^A $<\pi_1^{NA}$ since the marginal cost of the adopter when producing output Q_{j2}^A is larger than the marginal cost of the non adopter when producing output Q_{12}^{NA}.

Turning to the case for which the ETIOV of the adopter firm is larger than the ETIOV of the non adopter firm (x>1) then utilising relations (4.6) and (4.7) the following will hold for the two profit maximising firms at equilibrium:

$$MR_j^A(Q_j^A) = MC_j^A(Q_j^A) < MR_1^{NA}(Q_1^{NA}) = MC_1^{NA}(Q_1^{NA}) \quad \Leftrightarrow (4.16)$$

$$\frac{1}{e(Q,\psi_j^A)} > \frac{1}{e(Q,\psi_1^{NA})} \Leftrightarrow e(Q,\psi_j^A) < e(Q,\psi_1^{NA}) \qquad (4.17)$$

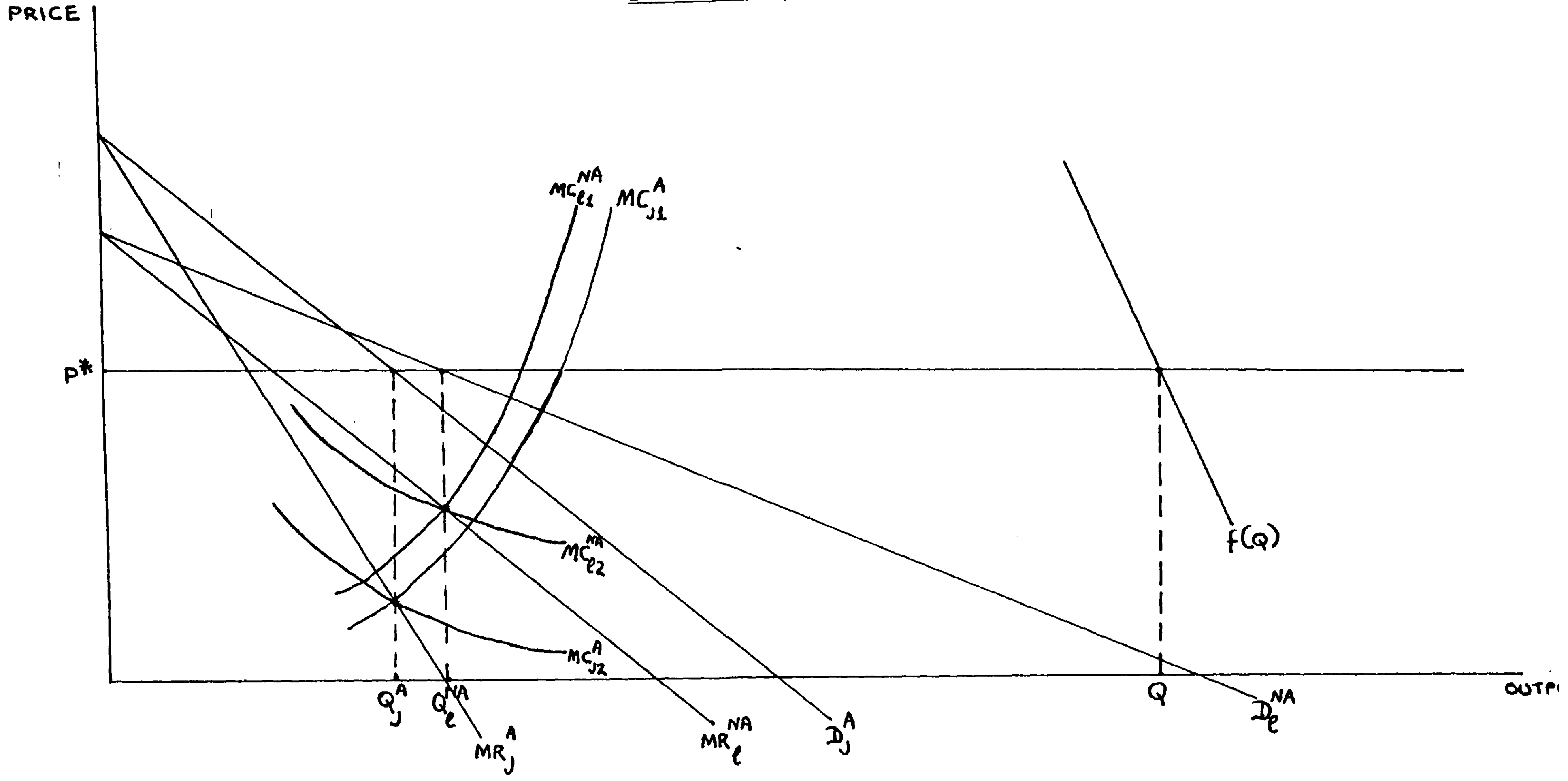
FIGURE 2'



PRICE
P_2^*
MC_{J2}^A
$MC_{\ell 2}^{NA}$
Q_{J2}^A
$Q_{\ell 2}^{NA}$
Q_2
OUT
$f(Q)$
MR_ℓ^{NA}
MR_J^A
D_ℓ^{NA}
D_J^A

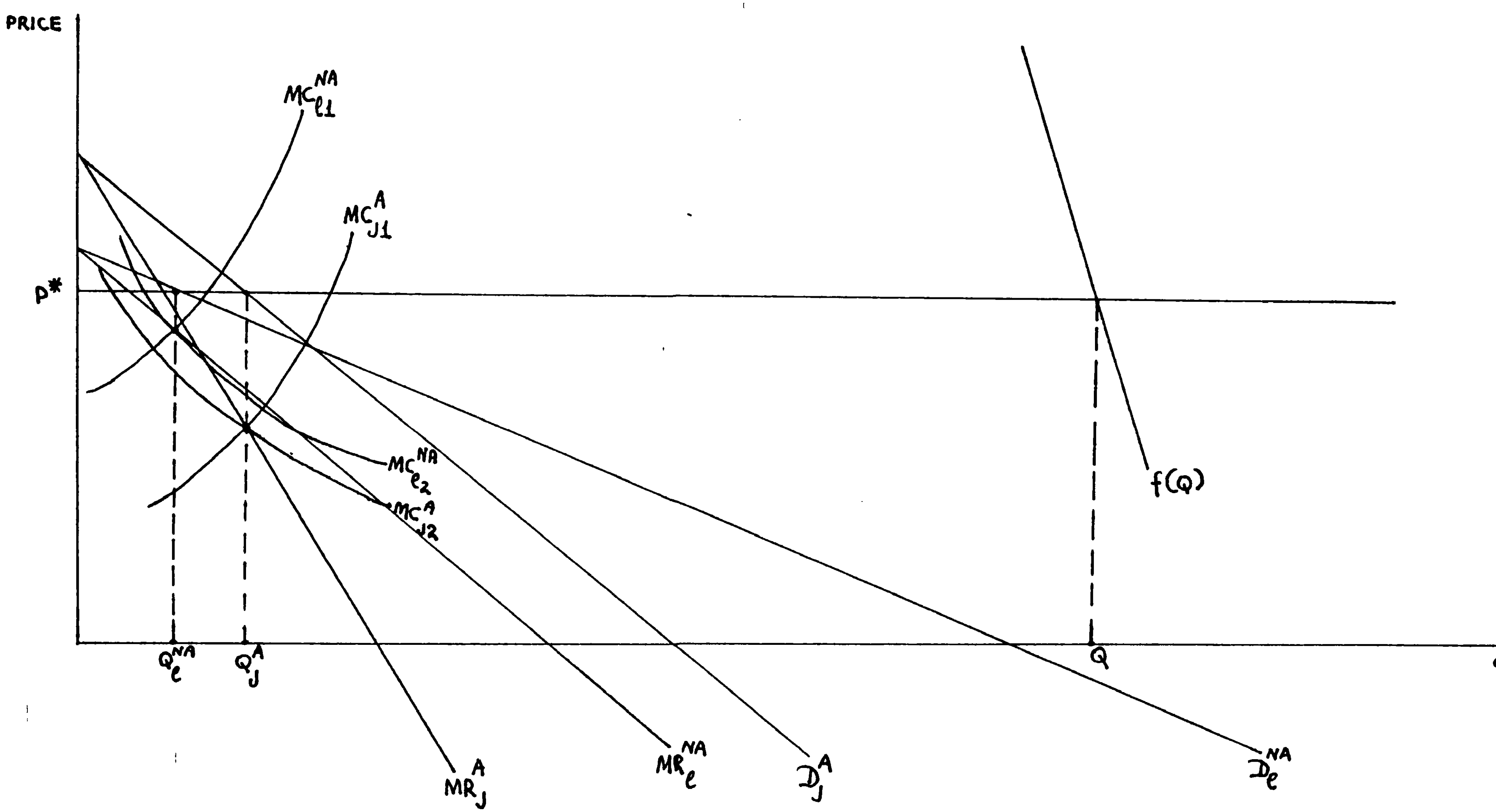
Consequently, when x>1 then the demand curve as conjectured by the adopter is less elastic than the demand curve as conjectured by the non adopter. Applying conditions (4.11)-(4.13), (4.16) and (4.17) the cases of a+b<1 and a+b>1 for which both the hypotheses are rejected are illustrated in Figure 3. The demand curve as conjectured by firm j, D_j^A is drawn so as to be less elastic than the demand curve as conjectured by firm l, D_l^{NA}, so that condition (4.17) is satisfied. For a+b<1 the marginal cost curves are MC_{j1}^A and MC_{l1}^{NA}. These intersect their respective marginal revenue curves at points that correspond to optimal levels of output Q_j^A and Q_l^{NA} such that the price as conjectured by each firm is equal to the equilibrium price p^*. The market share of the adopter firm is smaller than the market share of the non adopter firm while the marginal cost of the adopter firm at the optimum level of output, Q_j^A, is smaller than the marginal cost of the non adopter firm at the optimum level of output, Q_l^{NA}, in other words, π_j^A is larger than π_l^{NA} and condition (4.16) is satisfied. For a+b>1, the marginal cost curves are MC_{j2}^A and MC_{l2}^{NA} and the description is the same as for the case of a+b<1, i.e. the adopter firm has a lower market share and a higher π (because of a higher ETIOV) than the non adopter firm. But apart from the possibility of rejecting both hypotheses at the firm level when x>1, there is also the alternative possibility of accepting (not rejecting) both (either) of the hypotheses. This possibility is depicted in Figure 4 for both a+b<1 and a+b>1. The conjectured elasticities of demand are again drawn in such a way so as to ensure that D_j^A is less elastic

FIGURE 3



PRICE
P*
MC_{l1}^{NA}
MC_{J1}^{A}
MC_{l2}^{NA}
MC_{J2}^{A}
f(Q)
Q_J^A
Q_l^{NA}
Q
MR_J^A
MR_l^{NA}
D_J^A
D_l^{NA}
OUTP

FIGURE 4


PRICE
P*
MC_{l1}^{NA}
MC_{J1}^{A}
MC_{l2}^{NA}
MC_{J2}^{A}
f(Q)
Q_l^{NA}
Q_J^{A}
Q
MR_J^{A}
MR_l^{NA}
D_J^{A}
D_l^{NA}

than D_l^{NA}. For both a+b<1 and a+b>1, which are illustrated by the pairs of curves MC_{j1}^{A}, MC_{l1}^{NA} and MC_{j2}^{A}, MC_{l2}^{NA} respectively, it is demonstrated that at equilibrium the adopter firm has a larger optimal level of output than the non adopter while conditions (4.11)-(4.13), (4.16) and (4.17) are all satisfied.

Note that the equal π's whithin each group were the condition that guaranteed group uniform ETIOVs. The behaviour underlying the equal price cost margins explicit agreement within each group was illustrated in Figure 1 of section 3.1. If one goes a step further and assumes an industry-wide uniform level of ETIOV then the behaviour of every two firms irrespective in which group each one of them belongs will be described by Figure 1, i.e the behaviour of all firms in the industry is the behaviour of firms jointly maximising their profits. The reason for this is that **if within an industry all ETIOVs are uniform then an equilibrium can not exist for x = 1 unless the industry uniform ψ is equal to one**. This argument can be easily demonstrated using any of the diagrams of this chapter, for example, Figure 3. Since x=1 for any pair of firms (irrespective in which group each firm belongs) the conjectured demand curves for these two firms will coincide. Let the conjectured demand curve of firms j and *l* be D_j^A and their conjectured marginal revenue curve MR_j^A. Since the conjectured demand is the same for both firms, there is no way that condition (4.11) will be satisfied unless (a) their marginal costs intersect at the point of equilibrium (b) their marginal cost curves coincide (c) the idustry-wide uniform ETIOV is equal to one. Case

(a) is not possible since intersection of marginal (average) cost curves has been ruled out. Case (b) has been ruled out as well since $V\upsilon_{jl}$ has to be strictly larger than one irrespective of whether j and l belong in the same or different groups. Therefore the only possibility left is the case of ψ=1, which is nothing else but the joint profit maximisation case. The implication of $\psi_j=1$ for all $j \in N$ is that $\lambda_j = \frac{Q}{Q_j} - 1$ since then $\psi_j = (1+\lambda_j)\frac{Q_j}{Q} \simeq 1$. The equilibrium value of price and aggregate output is determined by the point where the curve deduced by the horizontal summation of all marginal cost curves, $\sum_j^N (MC_j)$, intersects the actual demand curve f(Q). Each firm will subsequently produce up to the point where its marginal cost becomes equal to the value of the marginal cost given by the joint profit maximisation, as described by Figure 1.

4.3 Hypotheses discrimination analysis under constant static returns to scale

When a+b = 1 the problem of (4.1) not being a reduced form expression for Q_j is no longer valid since as (4.5) implies marginal (and average) cost will no longer be a function of the level of output produced. Therefore there is no longer any need to parameterise the total industry output variation of firms in order to explicitly allow for the output variable included in the marginal cost function.

For a+b=1, the division of (4.7) by (4.6) gives that $x_{jl} = V\upsilon_{jl}$. In other words, the σ of each firm is an inverse measure of its productivity level which implies that it is no longer a suitable measure of the behaviour of each firm. As an alternative to σ, λ shall be used as a measure of the behaviour of the firms just as it was used in the Cowling and Waterson model originally.

Considering the ambiguity between the two hypotheses at the **firm level** let $\Lambda_{lj}=\frac{1+\lambda_l}{1+\lambda_j}$ where j is a more efficient firm than l is. When a+b=1 combining relation (4.6) with (4.7) gives:

$$1 - \frac{1}{e}(1+\lambda_j)\frac{Q_j}{Q} < 1 - \frac{1}{e}(1+\lambda_l)\frac{Q_l}{Q} \quad \Leftrightarrow$$

$$(1+\lambda_j)\frac{Q_j}{Q} > (1+\lambda_l)\frac{Q_l}{Q} \Leftrightarrow \Lambda_{lj} < \frac{Q_j}{Q_l} \qquad (4.18)$$

Consequently if $\Lambda_{lj} \geq 1$ $(\lambda_j \leq \lambda_l)$ then Q_j is always larger than Q_l and consequently DEH is accepted. However, since the more efficient firm is the less collusive, in terms of λ, the MPH is rejected. Therefore, for $\Lambda_{lj} \geq 1$ the ambiguity is resolved at the firm level. When Λ_{lj} is smaller than one $(\lambda_j > \lambda_l)$ then Q_j can be either smaller or larger than Q_l. If firm j has a larger market share than firm l has then both the DEH and the MPH are accepted, while if j has a smaller market share than firm l has then both hypotheses are rejected. Consequently when Λ_{lj} is smaller than one the ambiguity between the two hypotheses can not be resolved at the firm level.

Turning **at the industry level** of analysis, it is assumed that λ's are group uniform just as it was assumed for $a+b \neq 1$ that the σ's were group uniform. As a result, the difference in behaviour between the two groups is measured by $\Lambda = \frac{1+\lambda^{NA}}{1+\lambda^{A}}$. Moreover, since now efficiency is in terms of cost rather than in terms of productivity and since a+b=1, $MC_j=AC_j \ \forall \ j \in N$, the efficiency gap can be measured in terms of profitability. This is completely different from the situation developed in the model in chapter 3. The reasons that permit π to be a measure of efficiency rather than a measure of output behaviour are two. First, there are no scale economies which implies (via relation (4.5)) that marginal cost solely depends on the efficiency characterising a firm and is independent of the output decisions of the firm. Second, the assumptions concerning the firms'behaviour in our model no longer hold, i.e. **ETIOV is no longer an exogenously determined parameter** and therefore its dependance on market share can no longer be ignored. Market share will be jointly determined by marginal cost and conjectural variation, both of which are exogenously determined. Let π_P^A be the **potential profitability** for group A, that is, what the value of the profits to revenue ratio would be if a firm within that group had the managerial ability to realize the full potential in the productivity of major innovations (as given by index A_O). Analogously, let π_P^{NA} denote the **potential profitability** for the non adopters'group, the value of the profits to revenue if a firm in group NA had the managerial ability to realize the full potential in the productivity of major innovations (as denoted by B_0). $\upsilon^A_{\pi_P j}$ denotes the multiplicative deviation of a firm j, j∈A, from the potential maximum in the profits to revenue ratio for that group. As with the deviations in productivity all this firm specific deviations are time invariant and jumps in their value are **independent** of jumps in A_0. Also since no firm has a managerial ability such that it would be possible for it to realize the full potential profits to revenue implied by a major innovation we have that $\upsilon^A_{\pi_P j}<1$ for all j∈A. In other words, the realized (observed) profits to revenue for firm j as denoted by π^A_j is the product of the unobserved π^A_P times the equally unobserved $\upsilon^A_{\pi_P j}$. In exactly the same fashion we determine the firm specific deviation for the non adopters'group, which is denoted by $\upsilon^{NA}_{\pi_P l}$, l∈NA, and shares the same properties with $\upsilon^A_{\pi_P j}$. Consequently the gap in potential efficiency is now measured in terms of the gap in potential profitability and is given by $V' = \frac{\pi^A_P}{\pi^{NA}_P}>1$. The gap in realized profitability between firms j and l, j∈A and l∈NA, is equal to $M'_{jl}=V'\frac{\upsilon^A_{\pi_P j}}{\upsilon^{NA}_{\pi_P l}}>1$ which means that there is no overlapping in the profitability (efficiency) of firms belonging in different groups. For firms within the same group, if firm j's management is more efficient in applying major innovations than firm l's management is then $\frac{\upsilon_{\pi_P j}}{\upsilon_{\pi_P l}}>1$.

If one squares (4.1) and sums over all firms in the industry, the following relation is derived:

$$\sum^{N}(Q_j)^2 = \left(\frac{eQ}{p}\right)^2 \sum^{N}\left(\frac{p- MC_j}{(1+\lambda_j)}\right)^2 \tag{4.19}$$

Moreover if (4.1) is summed over all firms in the industry,it is derived that:

$$\left(\sum^{N} Q_j\right)^2 = \left(\frac{eQ}{p}\right)^2 \left[\sum^{N}\left(\frac{p- MC_j}{(1+\lambda_j)}\right)\right]^2 \tag{4.20}$$

Consequently,

$$H = \frac{\sum^{N}\left(\frac{p - MC_j}{1+\lambda_j}\right)^2}{\left[\sum^{N}\left(\frac{p - MC_j}{1+\lambda_j}\right)\right]^2} \tag{4.21}$$

Applying the group uniformity assumption on the λ's (4.21) becomes:

$$H = \frac{(1+\lambda^{A})^{-2} (\pi_p^{A})^2 \sum^{A}(\upsilon_{\pi_p j}^{A})^2 + (1+\lambda^{NA})^{-2} (\pi_p^{NA})^2 \sum^{NA}(\upsilon_{\pi_p l}^{NA})^2}{\left[(1+\lambda^{A})^{-1} (\pi_p^{A}) \sum^{A}(\upsilon_{\pi_p j}^{A}) + (1+\lambda^{NA})^{-1} (\pi_p^{NA}) \sum^{NA}(\upsilon_{\pi_p l}^{NA})\right]^2} =$$

$$= \frac{1 + \left(\frac{1+\lambda^{NA}}{1+\lambda^{A}}\right)^2 \left(\frac{\pi_P^A}{\pi_P^{NA}}\right)^2 \frac{\sum^{A}(\upsilon^{A}_{\pi_P j})^2}{\sum^{NA}(\upsilon^{NA}_{\pi_P l})^2}}{\left[1+\left(\frac{1+\lambda^{NA}}{1+\lambda^{A}}\right)\left(\frac{\pi_P^A}{\pi_P^{NA}}\right)\frac{\sum^{A}(\upsilon^{A}_{\pi_P j})}{\sum^{NA}(\upsilon^{NA}_{\pi_P l})}\right]^2} \frac{\sum^{NA}(\upsilon^{NA}_{\pi_P l})^2}{\left[\sum^{NA}(\upsilon^{NA}_{\pi_P l})\right]^2} \Leftrightarrow$$

$$H = \frac{1+ (\Lambda)^2 (V')^2 L_1}{(1+ \Lambda V' L_2)^2} L_3 \tag{4.22}$$

where $L_1 = \frac{\sum^{A}(\upsilon^{A}_{\pi_P j})^2}{\sum^{NA}(\upsilon^{NA}_{\pi_P l})^2}$, $L_2 = \frac{\sum^{A}(\upsilon^{A}_{\pi_P j})}{\sum^{NA}(\upsilon^{NA}_{\pi_P l})}$, $L_3 = \frac{\sum^{NA}(\upsilon^{NA}_{\pi_P l})^2}{\left[\sum^{NA}(\upsilon^{NA}_{\pi_P l})\right]^2}$. It is now possible to determine the sign of the effect from a change in the technological gap between the two groups on H as well as the sign of the effect on H from a change in the gap between the group λ's by calculating $\frac{\partial H}{\partial V'}$ and $\frac{\partial H}{\partial \Lambda}$ respectively.

$$\frac{\partial H}{\partial \Lambda} = \frac{2\ (V')^2 L_1 L_3\ \Lambda\left(1+ L_2 (V')\Lambda\right)^2 - 2\ L_2\ V' (1+ L_2 V'\Lambda) L_3 (1+ L_1 (V')^2 \Lambda^2)}{\left(1+ L_2\ V'\Lambda\right)^4}$$

So for $\frac{\partial H}{\partial \Lambda}$ to be positive it is required that:

$$L_1 (V')^2 \Lambda + L_1 L_2 (V')^3 (\Lambda)^2 - L_2\ (V') - L_1 L_2 (V')^3 (\Lambda)^2 > 0 \Leftrightarrow$$

$$V'\Lambda > \frac{L_2}{L_1}$$

If $V'\Lambda < \frac{L_2}{L_1}$ then $\frac{\partial H}{\partial \Lambda}$ will be negative.

For calculating $\frac{\partial H}{\partial V'}$ one should bear in mind that as in chapter 3 C_1, C_2, C_3 were independent of jumps in potential productivity (since the υ's were independent of jumps in potential productivity), here L_1, L_2, L_3 are independent of jumps in potential profitability (since υ_{π_P}'s are independent of jumps in potential profitability). Consequently,

$$\frac{\partial H}{\partial V'} = \frac{2\ (\Lambda)^2 L_1 L_3\ V' \left(1+\ L_2 (V')\Lambda\right)^2 - 2\ L_2\ \Lambda(1+\ L_2 V'\Lambda) L_3 (1+\ L_1 (V')^2 \Lambda^2)}{\left(1+\ L_2\ V'\Lambda\right)^4}$$

So for $\frac{\partial H}{\partial V'}$ to be positive it is required that:

$$L_1 (\Lambda)^2 V' + L_1 L_2 (V')^2 (\Lambda)^3 - L_2\ (\Lambda) - L_1 L_2 (V')^2 (\Lambda)^3 > 0 \quad \Leftrightarrow$$

$$V'\Lambda > \frac{L_2}{L_1}$$

If $V'\Lambda < \frac{L_2}{L_1}$ then $\frac{\partial H}{\partial V'}$ will be negative.

Consequently $\frac{\partial H}{\partial \Lambda}$ and $\frac{\partial H}{\partial V'}$ will be either both positive or both negative. A positively signed $\frac{\partial H}{\partial V'}$ means that an increase in the gap in potential efficiency (as measured by the ratio of profits to revenue) has caused an increase in the concentration ratio and therefore the differential efficiency hypothesis is accepted (not rejected). When, on the other hand, $\frac{\partial H}{\partial V'}$ is negative then the differential efficiency hypothesis can be rejected. The interpretation of the sign of $\frac{\partial H}{\partial \Lambda}$ is a bit more complex. When $\Lambda<1$ then an increase in Λ implies a convergence in the λ's of the two groups since the conjectural derivative of the group with the already higher conjectural derivative (λ^{A}) decreases and/or the conjectural derivative of the group with the comperatively lower conjectural derivative (λ^{NA}) increases. Consequently a positively signed $\frac{\partial H}{\partial \Lambda}$ means that the market power hypotheses can be rejected since a reduction in the divergence in output behaviour (as measured by the conjectural derivatives) between the two groups leads to an increase in the concentration ratio while a negative $\frac{\partial H}{\partial \Lambda}$ is translated as a non rejection (acceptance) of the market power hypothesis. For $\Lambda \geq 1$ a positive sign for $\frac{\partial H}{\partial \Lambda}$ implies that an increase in the conjectural derivative of the group with the already equal or relatively higher λ (λ^{NA}) and/or a decrease in the conjectural derivative of the group with the lower conjectural derivative (λ^{A}), or in other words an increase in the divergence between the λ's of the groups, increases concentration. Therefore the MPH can be accepted. On the other hand when $\Lambda \geq 1$ and $\frac{\partial H}{\partial \Lambda}$ is negative the market power hypothesis is rejected.

The above interpretations can now be applied to the industry level results. When $\Lambda<1$ if $V'\Lambda > \frac{L_2}{L_1}$ then $\frac{\partial H}{\partial V'}$ is positive which means that the DEH can be accepted and since $\frac{\partial H}{\partial \Lambda}$ is also positive the MPH is rejected. Analogously, when $\Lambda<1$ and $V'\Lambda < \frac{L_2}{L_1}$ then the DEH is going to be rejected in favour of the MPH. On the other hand, if $\Lambda \geq 1$ if $V'\Lambda > \frac{L_2}{L_1}$ then both the MPH and the DEH are accepted since both $\frac{\partial H}{\partial V'}$ and $\frac{\partial H}{\partial \Lambda}$ are positive. Moreover, when $\Lambda \geq 1$

and $V'\Lambda<\frac{L_2}{L_1}$ then both $\frac{\partial H}{\partial V'}$ and $\frac{\partial H}{\partial \Lambda}$ are negative and consequently both hypotheses are rejected.

To summarise, at the industry level when $\Lambda < 1$ the market power hypothesis and the differential efficiency hypothesis are mutually exclusive, while when $\Lambda \geq 1$ the ambiguity between the two hypotheses is not resolved since either both or neither of the two hypothesis is rejected against its alternative.

Therefore when $\Lambda<1$, if the gap in potential efficiency as measured by V', is sufficiently large and/or the gap in λ's as given by $(\Lambda)^{-1}=\frac{1+\lambda^{A}}{1+\lambda^{NA}}$ sufficiently small for $\frac{L_1}{L_2}V'$ to be larger than $(\Lambda)^{-1}$ then the MPH is rejected in favour of the DEH while if the gap in efficiency is sufficiently small and/or the gap in λ's, $\frac{1+\lambda^{A}}{1+\lambda^{NA}}$, sufficiently large for $\frac{L_1}{L_2}V'$ to be smaller than $(\Lambda)^{-1}$ then the DEH is rejected in favour of the MPH. When $\Lambda\geq1$ the the firm level conclusions are utilised, according to which the more efficient group will always have a larger market share and therefore the DEH is accepted while the market power hypothesis is rejected since $\lambda^{A}\leq\lambda^{NA}$.

Therefore for resolving the ambiguity between the two hypotheses estimates of Λ are required so that it can be determined whether Λ is larger or smaller than one. If in (4.22) we set as $\gamma_1'=\Lambda$ then the regression of industry i at time t, if a multiplicative disturbance term is added, is as follows:

$$H_{it} = \frac{1 + (\gamma'_{1it})^2 \dfrac{\sum^{A}(\pi^{A}_{ijt})^2}{\sum^{NA}(\pi^{NA}_{ilt})^2}}{\left[1 + \gamma'_{iit} \dfrac{\sum^{A}(\pi^{A}_{ijt})}{\sum^{NA}(\pi^{NA}_{ilt})}\right]^2} \frac{\sum^{NA}(\pi^{NA}_{ilt})^2}{\left[\sum^{NA}(\pi^{NA}_{ilt})\right]^2} \eta'_{it} \qquad (4.23)$$

since $(V')^2 L_1$ is equal to the ratio of the sum of the squares of realized profit rates of firms in group A over the same sum for group NA and $(V')L_2$ is the ratio of the sum of the realized profit rates of firms in group A over the same sum for group NA. Consequently, these two ratios can be calculated for each industry each time period provided that we have data on the profit rate of each individual firm and that it is possible to successfully seperate the adopters from the non adopters group. Once these two ratios are calculated then division of the one by the other gives what $\frac{L_1}{L_2}V'$ is equal to.

For estimating γ'_1 it is necessary to impose some restrictions as to how this parameter varies between industries and time. Analogously to section 3.4, γ'_1 can be regarded either as fixed and time invariant (seemingly unrelated regressions model) or as a random parameter with mean γ'_{1i} where $\varepsilon'_{1it}=\gamma'_{1it}-\gamma'_{1i}$. If the second method is pursued, then the alternative to (4.23) is the model

$$H_{it} = \frac{1 + (\gamma'_{1i} + \varepsilon'_{1it})^2 \dfrac{\sum^{A}(\pi^{A}_{ijt})^2}{\sum^{NA}(\pi^{NA}_{ilt})^2}}{\left[1 + (\gamma'_{1i}) \dfrac{\sum^{A}(\pi^{A}_{ijt})}{\sum^{NA}(\pi^{NA}_{ilt})} + (\varepsilon'_{1it}) \dfrac{\sum^{A}(\pi^{A}_{ijt})}{\sum^{NA}(\pi^{NA}_{ilt})}\right]^2} \frac{\sum^{NA}(\pi^{NA}_{ilt})^2}{\left[\sum^{NA}(\pi^{NA}_{ilt})\right]^2} \qquad (4.24)$$

If the joint distribution function of H_{it} and ε'_{1it} is $g(H_{it}, \varepsilon'_{1it})$ then the log likelihood function is:

$$L = \sum_{i,t} \ln\left[\int g(H_{it}, \varepsilon'_{1it})\, d\varepsilon'_{1it}\right]$$

Once γ'_1 has been successfully estimated either from (4.23) or from (4.24) it is possible to say whether Λ is smaller or larger than one. If $\Lambda<1$ then using the industry level conclusions we compare the value of $\frac{L_1}{L_2}V'$ (as it was calculated from the data) to the estimate of the value of $(\Lambda)^{-1}$. When $\frac{L_1}{L_2}V'$ is smaller than Λ then the DEH is rejected in favour of the MPH, while when $\frac{L_1}{L_2}V'$ is larger than Λ the MPH is rejected in favour of the DEH. On the other hand, if $\Lambda \geq 1$, utilising the firm level conclusions, the MPH is rejected in favour of the DEH since the more efficient group is less collusive than the laggards group is, while any firm belonging in the adopters'group has a greater market share than any firm belonging in the non adopters'group has.

CHAPTER 5

THE GENERALIZED-COOPERATIVE CAPITAL ADJUSTMENT COSTS MODEL

The model in chapter 3 was restricted by the imposition of two assumptions concerning capital: a) The installation of new capital is *costless* b) The installation of new capital is *instantaneous*. The first assumption rules out the possibility of the cost of gross investment being a function of its own size. If we drop the second assumption and introduce delivery lags into our model but still rule out adjustment costs and uncertainty, that is not sufficient to force the firms to look into the future and maximise the present value of their future stream of profits. However, the combination of delivery lags with uncertainty about the values of one or more of the exogenous parameters of the model and/or the existence of non linear adjustment costs will compel the firm to look into the future (if adjustment costs are linear they can be easily incorporated into the price of gross investment) and as a result dynamic and static maximisation will no longer be identical. The model developed below concentrates on the case of non linear adjustment costs, while delivery lags and uncertainty are both ruled out.

5.1 Setting up the model

The approach for developing a model which incorporates capital adjustment costs is similar to that by Nickell (1978).

Each firm maximises profits within an oligopolistic environment and tries to determine the dynamic optimum path for capital. It is assumed that each firm faces strictly convex adjustment costs.

Since there are no restrictions on the ability of the firm to adjust its labour force, the first order condition with respect to labour is given by the combination of relations (3.4) with (3.6):

$$p\,\sigma_j \frac{\partial Q_{jt}}{\partial L_{jt}} = p\,\sigma_j \frac{Q_{jt}}{L_{jt}} = A_{oj}(J_{jt})^{a}(L_{jt})^{b-1}b\sigma_j = \frac{w}{p} \qquad (5.1)$$

As already defined in chapter 2, c_t is the price of new investment in full efficiency units, I^F_{jt}, while the price of investment in physical units, I_{jt}, is equal to:

$$q_t = c_t \exp\left[\left(\frac{y}{a} + L\right)t\right] \qquad (5.2)$$

The reason for which q_t is a function of the rate of continuous technical change, as given by the exponent above, is that if q_t either understated or overstated the rate of continuous technical change, then as it shown in the appendix of this chapter the price of investment would become infinitely small or large as $t \to \infty$. Analogously, the adjustment costs function also has to include the rate of continuous technical change. If it did not, then as $t \to \infty$ adjustment costs would become infinitely small. Consequently, the model would degenerate into the no adjustment costs model of chapter 3. More generally, the appendix of this chapter examines what would happen if the adjustment costs function either understated or overstated the rate of continuous technical progress. Therefore the adjustment costs will depend on gross investment in full efficiency units rather than on gross investment in physical units. Moreover, the justification for assuming a convex form for the adjustment costs function is based on the assumption that the more highly technological an industry becomes the more specific to the firms that purchase them capital goods will become. As a result, the market for the capital goods purchased by these firms will become increasingly monopsonistic. In other words each firm's demand for each particular type (vintage) of capital will be a significant proportion of the total demand and therefore the supply price will increase with demand.

Given the above arguments, adjustment costs are given by a stricly convex function denoted by the expression $C(I_{jt})$ where when $I_{jt} = 0$ then $C(I_{jt}) = 0$ as well. Since these adjustment costs should account for technical change one may attribute to them the following quandratic form:

$$C(I_{jt}) = \beta_j \, (I^F_{jt})^2 \qquad (5.3)$$

If (5.3) is differentiated with respect to I_{jt} the following is deduced:

$$\frac{\partial C(I_{jt})}{\partial I_{jt}} = 2\beta_j \exp\left[\left(\frac{y}{a} + L\right)2t\right] I_{jt} = 2\beta_j \exp\left[\left(\frac{y}{a} + L\right)t\right] I^F_{jt} \qquad (5.4)$$

Writing from now on J_{jt} as J^F_{jt}, relation (5.1) may be re-written as:

$$L_{jt} = \left[\frac{w}{A_{oj}(J_{jt})^a \sigma_j p\ b}\right]^{\frac{1}{b-1}} \tag{5.5}$$

Using relations (2.16) and (3.5) the marginal revenue of capital, $p\ \sigma_j \frac{\partial Q_{jt}}{\partial K_{jt}}$, is equal to:

$$p\ \sigma_j \frac{\partial Q_{jt}}{\partial K_{jt}} = p\ \sigma_j \frac{\partial Q_{jt}}{\partial J_{jt}} \exp\left[\left(\frac{y}{a} + L\right)t\right]$$

Moreover,

$$p\ \sigma_j \frac{\partial Q_{jt}}{\partial J_{jt}} = p\ \sigma_j\ A_{oj}\ a\ (J_{jt})^{a-1}(L_{jt})^b = p\ \sigma_j\ a \frac{Q_{jt}}{J_{jt}} \tag{5.6}$$

Using relation (5.5) to substitute for L_{jt} in (5.6) we derive:

$$p\ \sigma_j \frac{\partial Q_{jt}}{\partial J_{jt}} = p\ \sigma_j\ A_{oj}\ a\ (J_{jt})^{a-1}\left[\frac{w}{A_{oj}(J_{jt})^a \sigma_j p\ b}\right]^{\frac{b}{b-1}} \Leftrightarrow$$

$$p\ \sigma_j \frac{\partial Q_{jt}}{\partial J_{jt}} = \left[p\ \sigma_j\ A_{oj}\right]^{-\frac{1}{b-1}} a \left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} \qquad (5.7)$$

If firm j is pursuing the optimal path of production then the total costs involved in installing one extra unit of capital stock should be equal to the gain resulting from the purchase of that extra unit of capital, the latter being equal to the present value of the marginal revenue of capital. Consequently,

$$\int_t^\infty \exp\left[-\left(h+\delta+\frac{y}{a}+L\right)(s-t)\right] p\sigma_j \frac{\partial Q_{js}}{\partial K_{js}}\, ds = \frac{\partial C(I_{jt})}{\partial I_{jt}} + q_t \Leftrightarrow$$

$$\int_t^\infty \exp\left[\left(\frac{y}{a}+L\right)t\right] \exp[-(h+\delta)(s-t)]\, p\sigma_j \frac{\partial Q_{js}}{\partial J_{js}}\, ds = \frac{\partial C(I_{jt})}{\partial I_{jt}} + q_t \Leftrightarrow$$

$$\exp\left[\left(\frac{y}{a}+L\right)t\right] \int_t^\infty \exp[-(h+\delta)(s-t)] \left[p\sigma_j A_{oj}\right]^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{js})^{\frac{1-a-b}{b-1}} ds =$$

$$= 2\beta_j \exp\left[\left(\frac{y}{a}+L\right)t\right] I^F_{jt} + c_t \exp\left[\left(\frac{y}{a}+L\right)t\right] \qquad \Leftrightarrow$$

$$2\beta_j I^F_{jt} + c_t = \int_t^\infty \exp[-(h+\delta)(s-t)] \left[p\sigma_j A_{oj}\right]^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{js})^{\frac{1-a-b}{b-1}} ds \qquad (5.8)$$

By taking the time derivative in (5.8) we obtain:

$$2\beta_j \dot{I}^F_{jt} + \dot{c}_t = \frac{d\exp\left[(h+\delta)t\right]}{dt} \frac{2\beta_j I^F_j + c_t}{\exp[(h+\delta)t]} +$$

$$+\exp[(h+\delta)t](p\sigma_j A_{0j})^{\frac{1}{1-b}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} \frac{d\int_t^{\infty}\exp[(h+\delta)(-s)]ds}{dt} \Leftrightarrow$$

$$2\beta_j \dot{I}^F_{jt} + \dot{c}_t = (h+\delta)\left(2\beta_j I^F_{jt} + c_t\right) - (p\sigma_j A_{0j})^{\frac{1}{1-b}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} \Leftrightarrow$$

$$2\beta_j \dot{I}^F_{jt} - 2\beta_j (h+\delta) I^F_{jt} + (p\sigma_j A_{0j})^{\frac{1}{1-b}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} = r_1 \quad (5.9)$$

where $r_1 = (h+\delta)c_t - \dot{c}_t$ is the user cost of capital in full efficiency units exclusive of adjustment costs. Note that for a firm belonging in the non-adopters group the same relation will hold, the only difference being that in the place of A_0 in (5.9) we have B_0 which is either equal to A_{0-1} or equal to A_{0-2} if the jump in A_0 has occured only recently and therefore there has not been sufficient time for the innovation 0-1 to become available to all the firms in the laggards'group. Note that as already discussed in section 2.1 both A_0 and B_0 are constant during an infinitesimal time interval dt since these two parameters are characterised by jumps of a discrete nature.

According to relation (2.19):

$$\dot{J}_{jt} = I^F_{jt} - \delta J_{jt} \quad (5.10)$$

The combination of relations (5.9) and (5.10) gives the trajectory of the optimal path in the $J\text{-}I^F$ plane. Substituting (5.10) into (5.9) we obtain:

$$2\beta_j \ddot{J}_{jt} + 2\beta_j \delta \dot{J}_{jt} - 2\beta_j (h+\delta) \dot{J}_{jt} - 2\beta_j (h+\delta) \delta J_{jt} +$$

$$+ (p\sigma_j A_{oj})^{\frac{1}{1-b}} a \left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} = r_1 \qquad \Leftrightarrow$$

$$\ddot{J}_{jt} - h\dot{J}_{jt} - (h+\delta)\delta J_{jt} +$$

$$(p\sigma_j A_{oj})^{\frac{1}{1-b}} \frac{a}{2\beta_j} \left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} = \frac{r_1}{2\beta_j} \qquad (5.11)$$

Let J_j^* be the value of capital stock in full efficiency units in a state of long run equilibrium. Since J_j^* is independent of time, $\dot{J}_j^* = 0$. If this is substituted into (5.10) it gives:

$$I_j^{F*} = \delta J_j^*$$

Combing the above with relation (2.16) gives:

$$I_{jt}^* = \delta K_{jt}^*$$

Substituting the above into (2.20) it is derived that:

$$\dot{K}^{*}_{jt} = -\left(L+\delta+ \frac{y}{a}\right) K^{*}_{jt} + \delta\, K^{*}_{jt} = -\left(L+ \frac{y}{a}\right) K^{*}_{jt}$$

In other words, if in the long run equilibrium net investment in efficiency units is equal to zero (i.e. if J^{*}_{j} is independent of time) then this implies that capital stock in reverse full efficiency units decreases by a rate equal to the rate of continuous technical change.

In equilibrium (5.11) becomes:

$$(p\sigma_{j}A_{oj})^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J^{*}_{j})^{\frac{1-a-b}{b-1}} = r_{1} + r_{2j}J^{*}_{j} \qquad (5.12)$$

where $r_{2j} = 2\beta_{j}\delta(h+\delta)$ and $r_{1}+r_{2}J^{*}_{j}$ is the user cost of capital inclusive of adjustment costs in full efficiency units. In long run equilibrium the latter is equal to the marginal revenue of capital. Relation (5.12) implies that the equilibrium capital stock in efficiency units will shift with unanticipated jumps in A_{o} or in any other parameter. Of course if discontinuous jumps in A_{o} take a more "regular" form, it would be false to continue to assume that the producer will not try to incorporate these regularities into the production function by perceiving A_{o} as a random variable taking its value according to some distribution based on the observed past values of her individual A_{oj}. However, the values of A_{oj} may not be a correct indicator of the unobserved values of A_{o} since for some of the observed jumps in A_{oj} jumps in υ_{j} (which have been assumed to be independent of jumps in A_{o}) rather than jumps in A_{o} may be responsible. Moreover, the firm will need to assume a distribution according to which major innovations arrive, for example at some Poisson rate μ. In the unlikely case that firms are able to determine these two distributions then they will be able to form an expectation as to when the next jump is going to occur and what its impact on productivity is going to be. Then such jumps will no longer constitute an element of suprise in the way it was perceived by Schumpeter, but simply are sources of uncertainty for which the model has been accordingly formulated to account for. For jumps in B_0 note that the laggard firms still do not know the values of A_0 in the past, **since these were never realized (observed)**, and therefore they can not know with certainty what their individual B_{0j} is going to be equal to. Moreover, firms in the non adopters group will be able to determine the timing of the jump in B_0 only if they either know with certainty, or can form an expectation of, the magnitude of t_{0-1}. If both the magnitude of t_{0-1} and the size of the jump in B_{0j} was known, then during the time interval commencing with a jump in the adopters group from A_{0-1} to A_0 and ending when B_0 jumps from A_{0-2} to A_{0-1}, I^F_{jt} for a firm j, j∈NA, would satisfy the following differential equation (Nickell, pp 46-48):

$$2\beta_j \dot{I}^F_{jt} - 2\beta_j (h+\delta)[I^F_{jt} - \delta J^*_j(A_{0-2,j})] +$$

$$+(p\sigma_j A_{0-2,j})^{\frac{1}{1-b}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} [J_{jt} - J^*_j(A_{0-2,j})]^{\frac{1-a-b}{b-1}} = 0 \qquad (5.9')$$

which implies that investment in efficiency units would be above the level determined by the current value of B_0, A_{0-2}. After the jump in B_0 has taken place, i.e when $t \geq t_{0-1}$ and $B_0 = A_{0-1}$ then I^F_{jt} satisfies the following differential equation:

$$2\beta_j \dot{I}^F_{jt} - 2\beta_j (h+\delta)[I^F_{jt} - \delta J^*_j(A_{0-1,j})] +$$

$$+(p\sigma_j A_{0-1,j})^{\frac{1}{1-b}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} [J_{jt} - J^*_j(A_{0-1,j})]^{\frac{1-a-b}{b-1}} = 0 \qquad (5.9'')$$

The above relation has been applying for firms in group A since the time the unexpected jump from A_{0-1} to A_0 took place, i.e I^F_{jt}, $j \in A$, satisfies the following differential equation:

$$2\beta_j \dot{I}^F_{jt} - 2\beta_j (h+\delta)[I^F_{jt} - \delta J^*_j(A_{0j})] +$$

$$+(p\sigma_j A_{0j})^{\frac{1}{1-b}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} [J_{jt} - J^*_j(A_{0j})]^{\frac{1-a-b}{b-1}} = 0$$

Unless otherwise explicitly stated the model is restricted to refer **only to unexpected jumps** in A_0 or any other variable. The impact of such jumps on J_{jt} is determined below.

Let us determine the speed with which J_{jt} approaches its long run equilibrium value. Linearizing (5.11) in the neighbour of J^*_j, it is derived by using (5.12):

$$\ddot{J}_{jt} - h\dot{J}_{jt} -$$

$$\left[\delta(h+\delta)-(p\sigma_j A_{0j})^{-\frac{1}{b-1}}\frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_j^*)^{\frac{1-a-b}{b-1}-1}\left(\frac{1-a-b}{b-1}\right)\right]\left(J_{jt}-J_j^*\right)=0 \quad (5.13)$$

Set $\Gamma_j=\delta(h+\delta)-(p\sigma_j A_{0j})^{-\frac{1}{b-1}}\frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_j^*)^{\frac{1-a-b}{b-1}-1}\left(\frac{1-a-b}{b-1}\right)$. Provided that Γ_j is positive the equation will have two real roots of opposite sign equal to:

$$-\beta_{1j} = \frac{h-\sqrt{h^2+4\Gamma_j}}{2} \qquad (5.14)$$

$$\beta_{2j} = \frac{h+\sqrt{h^2+4\Gamma_j}}{2} \qquad (5.15)$$

Note that if Γ_j is negative then two possibilities exist:

(a) Equation (5.13) has no real roots and J_{jt} will oscillate with a solution of the form:

$$J_{jt} = \exp\left(h\,\frac{t}{2}\right)\left[C\cos(dt) + D\sin(dt)\right]$$

where C, D arbitrary constants, $d = \frac{\sqrt{h^2+4\Gamma_j}}{2}$ and $\beta_{3j} = \frac{h}{2} + id$, $\beta_{4j} = \frac{h}{2} - id$. Therefore, the motion will be oscillatory with an increasing amplitude.

(b) Equation (5.13) has two real roots both positive.

Consequently, convergence is only possible when Γ_j is positive. Then the solution of the homogeneous version of equation (5.13) is expressed by J_{jh} which is equal to:

$$J_{jh} = (J_{j0} - J_j^*) \exp(-\beta_{1j} t)$$

As a result, the general solution of equation (5.13) is the sum of the homogeneous solution plus the particular solution which is equal to J_j^*. Therefore the unique optimal path for J_{jt} will be the one which converges on J_j^* and has the form:

$$J_{jt} = J_j^* + (J_{j0} - J_j^*) \exp(-\beta_{1j} t) \tag{5.16}$$

where as (5.14) indicates $-\beta_{1j}$ is the negative root of (5.13). By taking the time derivative of (5.16) and combining it with (5.16) itself, the flexible accelerator formula for capital in full efficiency units is derived:

$$\dot{J}_{jt} = \beta_{1j}(J_j^* - J_{jt}) \tag{5.17}$$

It is obvious from (5.13) that Γ_j **is positive under decreasing static returns to scale** (a+b<1) since $\frac{1-a-b}{b-1} < 0$. Under increasing static returns to scale for Γ_j to be positive it is required that:

$$\delta(h+\delta) > (p\sigma_j A_{0j})^{\frac{1}{1-b}} \frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt}^*)^{\frac{1-a-b}{b-1} - 1} \frac{1-a-b}{b-1} \tag{5.18}$$

If we rewrite relation (5.12):

$$(p\sigma_j A_{0j})^{\frac{1}{1-b}} \frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_j^*)^{\frac{1-a-b}{b-1}-1} = \frac{r_1}{2\beta_j J_j^*} + \delta(h+\delta) \qquad (5.19)$$

Substitution of (5.19) into the right hand side of inequality (5.18) gives:

$$\delta(h+\delta) > \left[\frac{r_1}{2\beta_j J_j^*} + \delta(h+\delta)\right] \frac{1-a-b}{b-1} \qquad \Leftrightarrow$$

$$\frac{b-1}{1-a-b} > \frac{r_1}{2\beta_j J_j^* \delta(h+\delta)} + 1$$

If for simplicity it is assumed that $\dot{c} = 0$ then:

$$\frac{b-1}{1-a-b} > \frac{c}{2\beta_j J_j^* \delta} + 1 \qquad (5.20)$$

Consequently, **(5.20) is the condition for J_{jt} to converge in the long run under increasing returns to scale (a+b > 1).**

Corollary 1. Under increasing static returns to scale a necessary condition for J_{jt} to converge on J_j^* is that 1-a-b-(b-1)>0.

Relations (5.12) and (5.14), imply that both the speed of convergence towards the equilibrium and the long run equilibrium capital stock itself depend on the level of potential productivity, A_0. The sign of the impact of a jump in A_0 on J_j^*, *ceteris paribus*, can be determined using relation (5.12):

$$\frac{\partial\left[r_1 (J_j^*)^{-\frac{1-a-b}{b-1}} + r_{2j} (J_j^*)^{-\frac{1-a-b}{b-1}+1}\right]}{\partial J_j^*} \frac{\partial J_j^*}{\partial A_0} =$$

$$= \left(\frac{1}{1-b}\right) (A_0)^{-\frac{b}{b-1}} (p\sigma_j \upsilon_j)^{\frac{1}{1-b}} a \left(\frac{w}{b}\right)^{\frac{b}{b-1}} \quad \Leftrightarrow$$

$$\left[(h+\delta) c \frac{1-a-b}{1-b} (J_j^*)^{-\frac{1-a-b}{b-1}-1} + \left(\frac{1-a-b}{1-b}+1\right) 2\beta_j \delta (h+\delta) (J_j^*)^{-\frac{1-a-b}{b-1}}\right] \frac{\partial J_j^*}{\partial A_0} =$$

$$= \left(\frac{1}{1-b}\right) (A_0)^{-\frac{b}{b-1}} (p\sigma_j \upsilon_j)^{\frac{1}{1-b}} a \left(\frac{w}{b}\right)^{\frac{b}{b-1}} \quad \Leftrightarrow$$

$$\frac{\partial J_j^*}{\partial A_0} = \frac{a}{1-b} (A_0)^{-\frac{b}{b-1}} (p\sigma_j \upsilon_j)^{\frac{1}{1-b}} \left(\frac{w}{b}\right)^{\frac{b}{b-1}} \left[(h+\delta) c \frac{1-a-b}{1-b} (J_j^*)^{-\frac{1-a-b}{b-1}-1} + \right.$$

$$\left. + \left(\frac{1-a-b}{1-b} + 1\right) 2\beta_j \delta (h+\delta) (J_j^*)^{-\frac{1-a-b}{b-1}}\right]^{-1} \quad (5.21)$$

As it is obvious from relation (5.21) **under decreasing or constant static returns to scale** $\frac{\partial J_j^*}{\partial A_0}$ **is always positive**. A jump in A_0

will be followed by a jump in B_0 after time equal to t_{0-1} has elapsed. The impact of the jump in B_0 on J^*_l, $l\in NA$, is given by (5.21) if we substitute B_0 for A_0.

If relation (5.14) is differentiated with respect to A_0 then it is derived that $\frac{d\beta_{1j}}{dA_0}$ is equal to:

$$\frac{1}{4}\left(h^2+4\Gamma_j\right)^{-\frac{1}{2}}4\left[(p\sigma_j\upsilon_j)^{-\frac{1}{b-1}}\frac{a}{2\beta_j}\left(\frac{1}{b-1}\right)\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(A_0)^{-\frac{b}{b-1}}(J_j^*)^{\frac{1-a-b}{b-1}-1}\left(\frac{1-a-b}{b-1}\right)\right.$$

$$\left.-(p\sigma_j\upsilon_jA_0)^{-\frac{1}{b-1}}\frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}}\left(\frac{1-a-b}{b-1}\right)\left(\frac{1-a-b}{b-1}-1\right)(J_j^*)^{\frac{1-a-b}{b-1}-2}\frac{\partial J_j^*}{\partial A_0}\right]=$$

$$\left(h^2+4\Gamma_j\right)^{-\frac{1}{2}}\left[\frac{1-a-b}{(b-1)^2}(p\sigma_j\upsilon_jA_0)^{-\frac{1}{b-1}}(A_0)^{-1}\frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_j^*)^{\frac{1-a-b}{b-1}-1}\right.$$

$$\left.-(p\sigma_jA_{0j})^{-\frac{1}{b-1}}\frac{a}{2\beta_j}\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_j^*)^{\frac{1-a-b}{b-1}-1}(J_j^*)^{-1}\frac{\partial J_j^*}{\partial A_0}\left(\frac{1-a-b}{b-1}\right)\left(\frac{1-a-b}{b-1}-1\right)\right] \quad (5.22)$$

For $\frac{d\beta_{1j}}{dA_0}$ to be negative under decreasing or constant static returns to scale it is required that:

$$\frac{1-a-b}{(b-1)^2}(A_0)^{-1}-\left(\frac{1-a-b}{b-1}\right)\left(\frac{1-a-b}{b-1}-1\right)(J_j^*)^{-1}\frac{\partial J_j^*}{\partial A_0}<0 \quad \Leftrightarrow$$

$$(A_0)^{-1} - \left(1-a-b-(b-1)\right)(J_j^*)^{-1} \frac{\partial J_j^*}{\partial A_0} < 0 \qquad \Leftrightarrow$$

$$-1 > -\frac{\partial J_j^*}{\partial A_0} \frac{A_0}{J_j^*} \left(1-a-b-(b-1)\right) \qquad \Leftrightarrow$$

$$\frac{\partial J_j^*}{\partial A_0} \frac{A_0}{J_j^*} > \frac{1}{1-a-b-(b-1)} \qquad (5.23)$$

Relation (5.23) always holds as it is proved below.

Proof

If one multiplies both terms in relation (5.21) by $\frac{A_0}{J_j^*}$ then one can substitute for $\frac{\partial J_j^*}{\partial A_0} \frac{A_0}{J_j^*}$ in inequality (5.23), deriving the following:

$$\frac{a}{1-b} (p\sigma_j A_{0j})^{\frac{1}{1-b}} \left(\frac{w}{b}\right)^{\frac{b}{b-1}} \left[r_1 \frac{1-a-b}{1-b} (J_j^*)^{-\frac{1-a-b}{b-1}} + \right.$$

$$\left. + \left(\frac{1-a-b}{1-b} + 1\right) 2\beta_j \delta \frac{r_1}{c} (J_j^*)^{-\frac{1-a-b}{b-1}+1} \right]^{-1} > \frac{1}{1-a-b-(b-1)} \qquad \Leftrightarrow$$

$$\frac{a}{2\beta_j} (p\sigma_j A_{0j})^{-\frac{1}{1-b}} \left(\frac{w}{b}\right)^{\frac{b}{b-1}} > \frac{1}{1-a-b-(b-1)} \left[\frac{r_1}{2\beta_j} (1-a-b) (J_j^*)^{-\frac{1-a-b}{b-1}-1} + \right.$$

$$+[1-a-b-(b-1)]\frac{r_1}{c}\,\delta(J_j^*)^{-\frac{1-a-b}{b-1}+1}\Bigg]$$

If relation (5.19) is used, the the left hand side of the above inequality becomes:

$$\frac{r_1}{2\beta_j}(J_j^*)^{-\frac{1-a-b}{b-1}}+\delta\,\frac{r_1}{c}(J_j^*)^{-\frac{1-a-b}{b-1}+1}>$$

$$\frac{1-a-b}{1-a-b-(b-1)}\,\frac{r_1}{2\beta_j}(J_j^*)^{-\frac{1-a-b}{b-1}}+\frac{r_1}{c}\,\delta(J_j^*)^{-\frac{1-a-b}{b-1}+1} \Leftrightarrow$$

$$1-a-b-(b-1) > 1-a-b$$

which is true since $1-a-b\geq 0$. □

Therefore for $1-a-b\geq 0$ an unanticipated jump in potential productivity will have a negative effect on the equilibrium level of the capital stock and a positive effect on the speed of convergence.

Turning to the case of **increasing returns to scale,** $\frac{\partial J_j^*}{\partial A_0}$ is positive if:

$$r_1\frac{1-a-b}{b-1}(J_j^*)^{-\frac{1-a-b}{b-1}-1}+\left(\frac{1-a-b}{1-b}+1\right)2\beta_j\delta\frac{r_1}{c}(J_j^*)^{-\frac{1-a-b}{b-1}}>0 \quad \Leftrightarrow$$

$$\frac{c}{2\beta_j\delta J_j^*}+1<\frac{b-1}{1-a-b}$$

The above is the same as condition (5.20) which should be satisfied if J_{jt} is to converge in the long run when a+b>1.

Consequently, if there is to be convergence, $\frac{\partial J_j^*}{\partial A_0}$ **has to be positive irrespective of whether the firm operates under increasing, constant or decreasing returns to scale. Obviously the same applies for jumps in p or in** σ**'s or in** υ**'s.**

From (5.22) $\frac{d\beta_{1j}}{dA_0}$ will be positive when 1-a-b<0 if:

$$\frac{1-a-b}{(b-1)^2} (A_0)^{-1} - \left(\frac{1-a-b}{b-1}\right)\left(\frac{1-a-b}{b-1} -1\right) (J_j^*)^{-1} \frac{\partial J_j^*}{\partial A_0} > 0 \quad \Leftrightarrow$$

$$(A_0)^{-1} - \left(1-a-b-(b-1)\right) (J_j^*)^{-1} \frac{\partial J_j^*}{\partial A_0} < 0 \quad \Leftrightarrow$$

$$-1 > - \frac{\partial J_j^*}{\partial A_0} \frac{A_0}{J_j^*} \left(1-a-b-(b-1)\right) \quad \Leftrightarrow$$

$$\frac{\partial J_j^*}{\partial A_0} \frac{A_0}{J_j^*} > \frac{1}{1-a-b-(b-1)}$$

It can be easily demonstrated that the above always holds. The proof follows the same procedure as in the case of 1-a-b ≥ 0 which leads to the requirement that 1-a-b-(b-1)>1-a-b which is always true since as corrolary 1 states 1-a-b-(b-1) has to be positive even when a+b>1. □

Consequently, if the firm is producing under increasing

static returns to scale then, when a jump in technology occurs, the speed of adjustment towards the new and higher level of equilibrium capital stock will be quicker than before. More explicitly, the impact of a jump in A_0 on β_{1j} can be split into a direct and an indirect component. The indirect effect of A_0 on β_{1j} through J_j^* $(\frac{\partial\beta_{1j}}{\partial J_j^*} \frac{\partial J_j^*}{\partial A_0})$ always dominates over, and always has an opposite sign to the direct effect of A_0 on β_{1j} $(\frac{\partial\beta_{1j}}{\partial A_0})$. So when a+b>1 the positive indirect effect dominates over the negative direct effect and when a+b<1 the negative indirect effect dominates over the positive direct effect.

To summarise, the first thing that has been derived in this section is the first order condition of capital stock under non-zero adjustment costs. Subsequently, the long run equilibrium level of capital stock as well as the speed of convergence towards that level were determined. The rest of the section examined how these two variables are affected by unexpected jumps either in technology, or in demand (as reflected by p), or in behaviour (as reflected by the σ's).

5.2 Firm level hypotheses discrimination analysis

The analysis below will follow the same criteria with those that were used in chapter 3 for resolving the ambiguity at the firm level of analysis.

In order to determine whether a relatively more efficient (and/or more collusive) firm has a higher market share as compared to the market share of a less efficient firm, it is essential to determine first what the market share of a firm is equal to both on the path towards the long run equilibrium as well as when in long run equilibrium.

The first order condition for labour as given by relation (3.6) is:

$$wL_{jt} = \sigma_j bpQ_{jt} \tag{5.24}$$

$$wL_j^* = \sigma_j bpQ_j^* \tag{5.25}$$

The above conditions are satisfied when the firm is on the optimum path towards the long run equilibrium (where output is given by expression Q_{jt}) or in the long run equilibrium (where output is given by expression Q_j^*) respectively.

Combining condition (5.6) with (5.7) and (5.12) it is deduced that:

$$r_1 + r_2 J_j^* = \sigma_j ap\left[\frac{Q_j^*}{J_j^*}\right] \tag{5.26}$$

Rewriting now (5.12) gives:

$$J_j^* = \left[\sigma_j p A_{oj} (a)^{1-b} \left(r_1 + r_{2j} J_j^*\right)^{b-1} (b)^b (w)^{-b}\right]^{\frac{1}{1-a-b}} \qquad (5.27)$$

By substituting (5.27) into (5.5) it is derived that the long run equilibrium labour input is given by the following expression:

$$L_j^* = \left[\frac{w}{A_{oj} p b \sigma_j}\right]^{\frac{1}{b-1}} (J_j^*)^{-\frac{a}{b-1}} \qquad \Leftrightarrow$$

$$L_j^* = \left[\sigma_j p A_{oj} (a)^a \left(r_1 + r_{2j} J_j^*\right)^{-a} (b)^{1-a} (w)^{a-1}\right]^{\frac{1}{1-a-b}} \qquad (5.28)$$

Consequently if we substitute into the Cobb Douglas production function for capital and labour using (5.27) and (5.28) respectively, the following expression is derived:

$$Q_j^* = A_{oj} (p\sigma_j A_{oj})^{\frac{a+b}{1-a-b}} \left[a^{1-b} \left(r_1 + r_{2j} J_j^*\right)^{b-1} b^b w^{-b}\right]^{\frac{a}{1-a-b}} \left[a^a b^{1-a} \left(r_1 + r_{2j} J_j^*\right)^{-a} w^{a-1}\right]^{\frac{b}{1-a-b}}$$

$$= \Psi_1 (A_{oj})^{\frac{1}{1-a-b}} (\sigma_j)^{\frac{a+b}{1-a-b}} \left(r_1 + r_{2j} J_j^*\right)^{-\frac{a}{1-a-b}} \qquad (5.29)$$

where $\Psi_1 = \left[a^a b^b \left(\frac{p}{w}\right)^b p^a\right]^{\frac{1}{1-a-b}}$.

When the firm is on the optimum path towards the long run equilibrium then if relation (5.5) is combined with relation (5.16) the following expression is derived for the labour input:

$$L_{jt} = \left[\frac{w}{A_{0j}pb\sigma_j}\left[J_j^* \left(1-\exp(-\beta_{1j}t)+\left(\frac{J_{j0}}{J_j^*}\right)\exp(-\beta_{1j}t)\right)\right]^{-a}\right]^{\frac{1}{b-1}} \Leftrightarrow$$

$$L_{jt} = L_j^*(\theta_{1jt})^{-\frac{a}{b-1}} \qquad (5.30)$$

where $\theta_{1jt}=1-\exp(-\beta_{1j}t)+\left(\frac{J_{j0}}{J_j^*}\right)\exp(-\beta_{1j}t)$. Therefore, the optimum path of output which a profit maximising firm pursues is:

$$Q_{jt} = A_{0j}(L_{jt})^b(J_{jt})^a = A_{0j}(L_j^*)^b(\theta_{1jt})^{-\frac{ab}{b-1}}(J_j^*)^a(\theta_{1jt})^a \Leftrightarrow$$

$$Q_{jt} = Q_j^* \ (\theta_{1jt})^{-\frac{a}{b-1}} \qquad (5.31)$$

The profits to revenue across the optimal trajectory π_j if we use relations (5.24) and (5.26) is:

$$\pi_{jt} = \frac{pQ_{jt} - wL_{jt} - (r_1+r_{2j}J_j^*)J_{jt}}{pQ_{jt}} =$$

$$= \frac{pQ_{jt} - \sigma_j pbQ_{jt} - \sigma_j ap\left(\frac{Q_j^*}{J_j^*}\right)J_{jt}}{pQ_{jt}}$$

By substituting relation (5.31) into the above the following relation is derived:

$$\pi_{jt} = \frac{pQ_{jt} - \sigma_j pbQ_{jt} - \sigma_j ap\left(\dfrac{(\theta_{1jt})^{\frac{a}{b-1}} Q_{jt}}{(\theta_{1jt})^{-1} J_{jt}}\right) J_{jt}}{pQ_{jt}} \Leftrightarrow$$

$$\pi_{jt} = 1 - \sigma_j \left[a(\theta_{1jt})^{\frac{1-a-b}{1-b}} + b \right] \qquad (5.32)$$

The long run equilibrium level of profits to revenue is equal to:

$$\pi_j^* = \frac{pQ_j^* - wL_j^* - (r_1 + r_{2j} J_j^*) J_j^*}{pQ_j^*} = \frac{pQ_j^* - \sigma_j pbQ_j^* - \sigma_j ap\left(\dfrac{Q_j^*}{J_j^*}\right) J_j^*}{pQ_j^*} \Leftrightarrow$$

$$\pi_j^* = 1 - \sigma_j (a+b) \qquad (5.33)$$

Note that as (5.32) implies, when the firm is on its optimal path towards the equilibrium, profit rates do not solely depend on their respective σ but also on the speed of convergence (β_{1j}) and on the relative jump in the target (long run equilibrium) capital stock $\left(\dfrac{J_{j0}}{J_j^*}\right)$. Given the conclusions of section 5.1 this means that π_{jt} is indirectly affected by the potential efficiency, the σ and its respective firm specific effect, υ_j. Imposing the group uniformity assumption on σ's **does not imply equal profit rates for the firms within each group when on the optimal path**. However, as (5.33) demonstrates, the group uniformity assumption implies a common long run equilibrium level of profit rate for each group, towards which all the firms within that group converge through their corresponding optimal paths.

By dividing (5.31) for firm j by the same relation for firm l - where by definition j is more efficient than l in terms of realized productivity- the following relation is obtained:

$$\frac{Q_{jt}}{Q_{lt}} = \frac{Q_j^*}{Q_l^*} \frac{(\theta_{1j}t)^{\frac{a}{1-b}}}{(\theta_{1l}t)^{\frac{a}{1-b}}} \tag{5.34}$$

If (5.29) is substituted into (5.34):

$$\frac{Q_{jt}}{Q_{lt}} = \left(\frac{A_o}{B_o}\frac{\upsilon_j}{\upsilon_l}\right)^{\frac{1}{1-a-b}} \left(\frac{\sigma_l}{\sigma_j}\right)^{-\frac{a+b}{1-a-b}} \left(\frac{r_1+r_{2j}J_j^*}{r_1+r_{2l}J_l^*}\right)^{-\frac{a}{1-a-b}} \frac{(\theta_{1j}t)^{\frac{a}{1-b}}}{(\theta_{1l}t)^{\frac{a}{1-b}}}$$

$$\Leftrightarrow \frac{Q_{jt}}{Q_{lt}} = (V)^f (\upsilon_{jl})^f (x_{lj})^{-f_1} (r_{lj})^{af} (\theta_{jlt})^{\frac{a}{1-b}} \tag{5.35}$$

where $r_{lj} = \dfrac{r_1+r_{2j}J_l^*}{r_1+r_{2l}J_j^*}$ and $\theta_{jlt} = \dfrac{(\theta_{1j}t)^{\frac{a}{1-b}}}{(\theta_{1l}t)^{\frac{a}{1-b}}}$. Similarly, when the firms are in equilibrium:

$$\frac{Q_j^*}{Q_l^*} = (V)^f (\upsilon_{jl})^f (x_{lj})^{-f_1} (r_{lj})^{af} \tag{5.36}$$

By definition the following always hold:

(a) $V = \frac{A_0}{B_0} > 1$ for $j \in A$ and $l \varepsilon NA$, where V is either equal to $\frac{A_0}{A_{0-2}}$ or equal to $\frac{A_0}{A_{0-1}}$ in accordance to relations (2.7) and (2.7′) respectively.

(b) V = 1 for j, l belonging in the same group.

(c) $V \upsilon_{jl} > 1$ for $j \in A$ and $l \in NA$ according to relation (2.9).

(d) $\upsilon_{jl} > 1$ for firms belonging in the same group since by definition j is more advanced than l in terms of realized productivity.

Additionally, it is assumed that β_j in (5.3) is equal to β, say, which is uniform for all firms. The implication of this is that r_2's are uniform across all firms in the industry like r_1's are. Then when $J_j^* > J_l^*$, $r_{lj} < 1$ and when $J_j^* < J_l^*$, $r_{lj} > 1$. Using this assumption two theorems are proved below.

THEOREM 1

A necessary condition for r_{lj} to be smaller than one is that the equilibrium market share of the more efficient firm will be larger than the market share of the less efficient firm. $(Q_j^* > Q_l^*)$

Proof

As a first step both sides in relation (5.29) are raised to the power of $\frac{1}{a+b}$:

$$(Q_j^*)^{\frac{1}{a+b}} = \left[a^a b^b \left(r_1 + r_2 J_j^*\right)^{-a} w^{-b}\right]^{\frac{1}{(1-a-b)(a+b)}} (p\sigma_j)^{\frac{1}{1-a-b}} A_{0j}^{\frac{1}{(1-a-b)(a+b)}} \quad (5.37)$$

J_j^* can be expressed in terms of Q_j^* if (5.37) is substituted into (5.27):

$$J_j^* = \left(\frac{a}{b}\right)^{\frac{b}{a+b}} \left(\frac{r_1 + r_2 J_j^*}{w}\right)^{-\frac{b}{a+b}} (A_{0j})^{-\frac{1}{a+b}} (Q_j^*)^{\frac{1}{a+b}} \quad (5.38)$$

The above relation implies that if $J_j^* > J_l^*$, ($r_{lj} < 1$) then:

$$(M_{jl})^{-\frac{1}{a+b}} \left(\frac{Q_j^*}{Q_l^*}\right)^{\frac{1}{a+b}} (r_{lj})^{\frac{b}{a+b}} > 1 \Rightarrow \left(\frac{Q_j^*}{Q_l^*}\right) > 1$$

where $M_{jl} = V\, \upsilon_{jl}$. □

<u>THEOREM 2</u>

A sufficient (though not necessary) set of conditions for the equilibrium user cost of a more efficient firm j **to be higher than the equilibrium user cost of a firm** l **inferior to** j **in terms of efficiency (i.e.** $r_{lj}<1$**) is that** $a+b<1$ **(**f, $f_1>0$**) and** $x_{lj} \leq 1$.

<u>Proof</u>

Dividing relation (5.27) for firm j by the same relation for firm l we get:

$$\left(\frac{J_j^*}{J_l^*}\right)^{1-a-b}\left(\frac{r_1+r_2J_j^*}{r_1+r_2J_l^*}\right)^{1-b} = M_{jl}\,\frac{1}{x_{lj}}$$

Since the right hand side of the above relation is larger than one when $x_{lj} \leq 1$, the left hand side also has to be greater than one. When a+b < 1 this implies that:

$$J_j^* > J_l^* \Leftrightarrow r_1 + r_2J_j^* > r_1 + r_2J_l^* \Leftrightarrow r_{lj} < 1 \qquad \square$$

If one wishes to express market share in terms of equilibrium capital stock in efficiency units (J_j^*) rather that in terms of user cost inclusive of adjustment costs $(r_1+r_2J_j^*)$, the latter can be substituted for by using (5.12) into (5.29).

$$(Q_j^*) = (\Psi_1)(A_{oj})^f\sigma_j^{f_1}\left[(p)^{-\frac{1}{b-1}}\,a\left(\frac{w}{b}\right)^{\frac{b}{b-1}}\right]^{-af}(A_{oj})^{\frac{af}{b-1}}(\sigma_j)^{\frac{af}{b-1}}(J_j^*)^{af\frac{1-a-b}{1-b}} =$$

$$=(a)^{af-af}\left(\frac{b}{w}\right)^{bf+\frac{baf}{b-1}}(p)^{f_1+\frac{af}{b-1}}(A_{oj})^{f\left(\frac{b-1+a}{b-1}\right)}(\sigma_j)^{f_1+\frac{af}{b-1}}(J_j^*)^{\frac{a}{1-b}} =$$

$$=\left(\frac{b}{w}\,p\right)^{\frac{b}{1-b}}(A_{oj})^{\frac{1}{1-b}}(\sigma_j)^{\frac{b}{1-b}}(J_j^*)^{\frac{a}{1-b}} \qquad \Leftrightarrow$$

$$Q_j^* = (\Psi_2)(A_{oj})^{\frac{1}{1-b}}(\sigma_j)^{\frac{b}{1-b}}(J_j^*)^{\frac{a}{1-b}} \qquad (5.39)$$

where $\Psi_2=\left(\frac{b}{w}p\right)^{\frac{b}{1-b}}$. Consequently relations (5.35) and (5.36) become respectively:

$$\frac{Q_{jt}}{Q_{lt}} = (V)^{\frac{1}{1-b}}(\upsilon_{jl})^{\frac{1}{1-b}}(x_{lj})^{-\frac{b}{1-b}}(J^*_{jl}\theta_{jlt})^{\frac{a}{1-b}} \qquad (5.40)$$

$$\frac{Q^*_j}{Q^*_l} = (V)^{\frac{1}{1-b}}(\upsilon_{jl})^{\frac{1}{1-b}}(x_{lj})^{-\frac{b}{1-b}}(J^*_{jl})^{\frac{a}{1-b}} \qquad (5.41)$$

We are now ready to commence the firm level analysis, with the purpose of identifying cases for which the ambiguity between the DEH and the MPH is resolved.

5.2a The long run equilibrium

This subsection examines what happens when firms are in long run equilibrium. For this purpose relation (5.36) is used. The analysis commences with the case a+b>1 (f, $f_1<0$). Let $\sigma_j<\sigma_l$ ($\psi_j>\psi_l$, $x_{lj}>1$), i.e the relatively more efficient firm j has a larger ETIOV than that of firm l, and $J^*_j\leq J^*_l$ ($r_{lj}\geq 1$), i.e. the equilibrium total user cost of firm j is less than, or equal to, the equilibrium total user cost of firm l. Under the above circumstances for Q^*_j to be larger than Q^*_l, or in other words for the DEH to be accepted, it is required that:

$$(M_{jl})^{f}(r_{lj})^{af} > (x_{lj})^{f_1}$$

Since the relatively more efficient firm is also the relatively more collusive one ($\psi_j > \psi_l$) the MPH is accepted as well. As a result neither of the two hypotheses is rejected in favour of the other. Turning now to the case where the relatively more efficient firm has a relatively smaller equilibrium market share ($Q_j^* < Q_l^*$), this implies that:

$$(M_{jl})^{f}(r_{lj})^{af} < (x_{lj})^{f_1}$$

Since $\frac{Q_j^*}{Q_l^*} < 1$ and $\frac{\psi_j}{\psi_l} > 1$, this means that even the combination of higher collusion and higher efficiency does not guarantee a higher market share. In this last case both the DEH and the MPH are rejected since the firm which is both less collusive and less efficient enjoys a higher market share. Note that via relation (5.33), $x_{lj} > 1$ implies that $\pi_j^* > \pi_l^*$; i.e. the rejection of both hypotheses happens regardless of the fact that the more efficient firm is the more profitable one as well. This means that in this case the firm level of analysis is **not adequate** for explaining differences in market shares either in terms of the differential efficiency hypothesis or in terms of the market power hypothesis. Therefore, as in chapter 3, it shall be attempted to resolve the ambiguity using the industry level criteria instead. However, notice that if $Q^*_j>Q^*_l$, then both the DEH and the MPH are compatible with the case of a positive relation between market share and firm profitability since the relatively more efficient and more collusive (in terms of ETIOV) firm has a relatively higher market share and a relatively higher profit rate. In this case the analysis is also transfered to the industry level, this time not because the firm level criteria are not adequate for explaining differences in market share in terms of the two hypotheses, but because the **ambiguity between the two hypotheses can not be resolved.**

If a+b>1, $x_{lj}<1$ and $r_{lj}\geq 1$ then the inequality:

$$(M_{jl})^{f}(r_{lj})^{af} < (x_{lj})^{f_1}$$

will always hold since $(M_{jl})^{f}(r_{lj})^{af}<1$ and $(x_{lj})^{f_1}>1$. Therefore it is not possible to have $Q^*_j>Q^*_l$, i.e the DEH is always rejected. Because $x_{lj}<1$ then $\psi_j<\psi_l$ and since $\frac{Q^*_j}{Q^*_l}<1$ the MPH is accepted. **So in this case the ambiguity is resolved: the DEH is rejected and the MPH is accepted (not rejected).**

When $r_{lj}<1$ then one can not determine whether $(M_{jl})^{f}(r_{lj})^{af}$ is larger or smaller than $(x_{lj})^{f_1}$. However, according to Theorem 1 when $r_{lj}<1$ then Q^*_j must be larger than Q^*_l. In other words when $r_{lj}<1$,

$$(M_{jl})^{f}(r_{lj})^{af} > (x_{lj})^{f_1}$$

Therefore, irrespective of whether a+b is larger or smaller than one and of whether x_{lj} is smaller, equal or larger than one the DEH is always accepted when $r_{lj}<1$. As before, if $x_{lj}>1$ then the MPH is also accepted while if $x_{lj}<1$ then the MPH is rejected since the more collusive firm has a lower market share. **So when** $r_{lj}<1$ **and** $x_{lj}<1$, **the ambiguity is resolved in favour of the DEH at the firm level.**

For a+b<1 ($f,f_1>0$) and $x_{lj}>1$, $r_{lj}\geq 1$ - for the same reasons as in the case of a+b>1, $x_{lj}>1$ and $r_{lj}\geq 1$ - when $Q_j^*>Q_l^*$ both hypotheses are accepted while when $Q_j^*<Q_l^*$ both are rejected. On the other hand, as it is implied by Theorem 2, the combination a+b<1, $r_{lj}\geq 1$ and $x_{lj}\leq 1$ is impossible. Moreover, given Theorem 1 when $r_{lj}<1$ then if $x_{lj}>1$ both the DEH and the MPH are accepted, while if $x_{lj}<1$ MPH is rejected and the DEH is accepted.

To summarize, the firm level conclusions are as in the table on the following page.

LONG RUN EQUILIBRIUM.

Resolving the ambiguity between MPH and DEH at the firm level

a+b > 1

$x_{1j}>1$ $r_{1j}\geq 1$	Either both the DEH and the MPH are accepted or both are rejected.
$x_{1j}<1$ $r_{1j}\geq 1$	DEH rejected, MPH accepted. Ambiguity resolved at the firm level.
$x_{1j}>1$ $r_{1j}<1$	Both the MPH and the DEH are accepted.
$x_{1j}<1$ $r_{1j}<1$	DEH accepted, MPH rejected. Ambiguity resolved at the firm level.

a+b < 1

$x_{1j}>1$ $r_{1j}\geq 1$	Either both the DEH and the MPH are accepted or both are rejected.
$x_{1j}>1$ $r_{1j}<1$	Both the MPH and the DEH are accepted.
$x_{1j}<1$ $r_{1j}<1$	DEH accepted, MPH rejected. Ambiguity resolved at the firm level.

To summarize, at the firm level and when in long run equilibrium, when the more efficient firms are the more profitable ones, it is better to focus the analysis at the industry level. When the **more efficient firms are the less profitable ones** ($x_{lj}<1$) then the ambiguity is resolved and the intuition behind the results goes as follows: If a+b<1, the firms that enjoy no cost advantage are relatively more collusive (as it is reflected in their profitability) but have no interest to use oligopolistic practices to obtain a higher market share since average costs are an increasing function of size. Consequently, the optimum long run level of both the investment in full efficiency units and the output is relatively smaller for the laggard firms. On the other hand the more efficient firms can afford to have a higher long run profit maximising level of investment and output since although a+b<1 they have the relative advantage of superior productivity. For a+b>1, let us start with the laggard firm making sure that it gains via its oligopolistic practices a greater long run equilibrium level of investment and market share and therefore the DEH is rejected while the MPH is accepted. If an innovation takes place, then as it was proved the jump in potential productivity always has a positive impact on the long run equilibrium level of investment. Then, as relation (5.41) implies, an increase in V will affect positively the ratio $\frac{Q_j^*}{Q_l^*}$, both directly and indirectly (through the increase in J_{jl}^*). If the gap between A_0 and A_{0-1} is sufficiently enlarged so as to make $J_j^*(A_0)$ larger than $J_l^*(A_{0-1})$ and Q_j^* larger that Q_l^* then the model will come to reject the MPH in favour of the DEH. So, as long as $r_{lj} \leq 1$ and $Q_j^* < Q_l^*$, the larger the long run equilibrium gap in efficiency is, the more J_j^* and Q_j^* will approach J_l^* and Q_l^* respectively. If finally the efficient firm overcomes the laggard in terms of their respective J^*'s then by Theorem 1 since $r_{lj} < 1$, Q_j^* is larger that Q_l^* and therefore the DEH is accepted while the MPH is rejected.

5.2b The optimal path towards the long run equilibrium.

It shall be assumed that t_{o-1}, the time that elapses after innovation m+1 (o=m+1) has occured in group A and before innovation m (o-1=m) becomes available to all the firms in group NA (that is before a jump in B_o takes place), is of a sufficient magnitude so that by the time it is over all firms in group A will have approximately reached their equilibrium values, i.e. for all $j \in A$, $\exp(-\beta_{1j} t_{o-1}) \cong 0$ and if this is combined with (5.16) it means that

$$J_{jt} \cong J_j^*(A_o) \qquad \forall j \in A \quad \text{for} \quad t = t_{o-1}$$

Moreover it is assumed that τ_{o-1}^o $(\tau_{o-1}^o > t_{o-1})$, the time gap between two innovations, is sufficiently lengthy so that when the time interval $\tau_{o-1}^o - t_{o-1}$ is over, all firms in group NA will have approximatelly reached their equilibrium values, i.e. for $l \in NA$ $\exp[-\beta_{1l}(\tau_{o-1}^o - t_{o-1})] \cong 0$. Combining this with (5.16) means that

$$J_{lt} \cong J^*_l(A_{0-1}) \quad \forall l \in NA \quad \text{for } t = \tau^0_{0-1} - t_{0-1}$$

The above two relations imply that either the firms in group A or the firms in group NA will be on the optimal path towards the equilibrium but **never both** groups simultaneously. More simply, when a jump in A_0 takes place (from A_0 to A_{0+1} say) the firms in group NA have already approximately reached their equilibrium values (the convergence process in group NA is over and has just only started in group A) and when a jump in B_0 takes place (from A_{0-2} to A_0) the firms in group A have already approximately reached their equilibrium values (the convergence process for the firms in group A is over).

Taking into account the above assumptions, it shall be examined whether the conclusions of the cases where the ambiguity was resolved in the long run equilibrium still hold when the firm is on the optimal path towards the long run equilibrium.

Starting with the cases where a+b>1 and $x_{lj}<1$, let us assume that two firms j and l ($j \in A$ and $l \in NA$) start from a former equilibrium position for which say $J^*_j(A_0) \leq J^*_l(A_{0-1})$ ($r_{lj} \geq 1$, $Q^*_j \leq Q^*_l$) and then a jump occurs in the potential productivity of group A from A_0 to A_{0+1} such that $J^*_j(A_{0+1}) > J^*_l(A_{0-1})$ ($r_{lj} < 1$, $Q^*_j > Q^*_l$). Then as firm j starts to converge towards its new equilibrium positions there will be a particular point in time, say t^*, after which its optimal capital stock (J_{jt}) and output (Q_{jt}) will become larger that the firm's l corresponding equilibrium capital stock ($J^*_l(A_{0-1})$)and output (($Q^*_l(A_{0-1})$). In other words,

$$Q_{jt} \le Q_l^* \qquad \text{when} \quad t < t^*$$

and

$$Q_{jt} > Q_l^* \qquad \text{when} \quad t \ge t^*$$

Therefore while the new equilibrium conditions entail that the DEH should be accepted and the MPH rejected (since $a+b>1$, $x_{lj}<1$ and $r_{lj}<1$) we see that for as long as $t<t^*$ the correct decision is to reject the DEH and accept the MPH instead.

Let us now assume that potential productivity jumps from A_{0-1} to A_0 in group NA. By the time this innovation becomes available in group NA the convergence process is over in group A. If $J_j^*(A_{0+1}) \le J_l^*(A_0)$ ($r_{lj} \ge 1$, $Q_j^* \le Q_l^*$) then there will be some time t^* that will have to elapse before the firm's *l* optimal capital stock (J_{lt}) and optimal output (Q_{lt}) become equal to, or larger than, the equilibrium capital stock ($J_j^*(A_{0+1})$) and equilibrium output ($Q_j^*(A_{0+1})$) of firm j. In other words,

$$Q_j^* > Q_{lt} \qquad \text{when} \quad t < t^*$$

and

$$Q_j^* \le Q_{lt} \qquad \text{when} \quad t \ge t^*$$

Therefore while the new equilibrium conditions entail that the DEH should be rejected and the MPH accepted we see that for as long as $t<t^*$ the correct decision is to accept the DEH and reject the MPH instead.

To conclude for a+b>1 and $x_{lj}<1$ the ambiguity between the two hypotheses is still resolved but it is possible that at the early stages of the convergence process the criteria for resolving the ambiguity may have to be reverse of what the criteria for $t \geq t^*$ are. Note that t^* is an inverse function of $\frac{A_0}{B_0}$ and the β_1's.

Obviously, such a situation can **not** arise for the combination of a+b<1 and $x_{lj}<1$ since as it is implied from Theorem 2 r_{lj} is in this case always smaller than one. In other words when a+b<1 and $x_{lj}<1$ there is no equilibrium state for which J_j^* will be smaller than (or equal to) J_l^*.

As a conclusion to the firm level analysis, the main result is that, as in the case of no adjustment costs, **a necessary and sufficient condition for the ambiguity between the two hypotheses to be resolved at the firm level is that** $x_{lj}<1$. **However caution in interpreting the results should be exercised since in the case where** a+b>1, $x_{lj}<1$ **one may at the initial stages of the convergence process reject the correct hypothesis and accept the false hypothesis**. A summary of all possible combinations and their respective conclusions is given by the table on the following pages.

THE OPTIMAL PATH TOWARDS THE LONG RUN EQUILIBRIUM

Resolving the ambiguity between MPH and DEH at the firm level

a+b > 1

$x_{1j}>1$ $r_{1j}\geq 1, t \lesseqgtr t^*$	Either both the MPH and the DEH are accepted or both are rejected.
$x_{1j}<1$ $r_{1j}\geq 1, t\geq t^*$	DEH rejected, MPH accepted. Ambiguity resolved at the firm level. †
$x_{1j}<1$ $r_{1j}\geq 1, t<t^*$	Either the DEH rejected and the MPH accepted or the MPH rejected and the DEH accepted. †
$x_{1j}>1$ $r_{1j}<1, t\geq t^*$	Both the MPH and the DEH are accepted.
$x_{1j}>1$ $r_{1j}<1, t<t^*$	Either both the DEH and the MPH are accepted or both are rejected.
$x_{1j}<1$ $r_{1j}<1, t\geq t^*$	DEH accepted, MPH rejected. Ambiguity resolved at the firm level. †
$x_{1j}<1$ $r_{1j}<1, t<t^*$	Either the DEH accepted and the MPH rejected or the MPH accepted and the DEH rejected. †

$a+b < 1$

$x_{1j}>1$ $r_{1j}\geq 1, t \lesseqgtr t^*$	Either both the MPH and the DEH are accepted or both are rejected.
$x_{1j}>1$ $r_{1j}<1, t\geq t^*$	Both the MPH and the DEH are accepted.
$x_{1j}>1$ $r_{1j}<1, t<t^*$	Either both the MPH and the DEH are accepted or both are rejected.
$x_{1j}<1$ $r_{1j}<1, t \lesseqgtr t^*$	DEH accepted, MPH rejected. Ambiguity resolved at the firm level.

5.3 Industry level hypotheses discrimination analysis in a state of long run equilibrium

In this section it shall be attempted to resolve the equilibrium state cases for which the ambiguity was not resolved at the firm level of analysis. The reader is reminded that the criteria for accepting the DEH at the industry level of analysis is whether an increase in the gap in potential productivity will have a positive impact on the level of concentration. As for the MPH, this is to be accepted if a further relative increase in the ETIOV of the group which already had a higher ETIOV results in an increase in the level of concentration.

For applying these rules it is first necessary to establish a relationship between concentration and the firm specific variables included on the right hand side of relation (5.29). Squaring (5.29) it is derived that:

$$(Q_j^*)^2 = (\Psi_1)^2 (A_{0j})^{2f} (\sigma_j)^{2f_1} \left(r_1 + r_2 J_j^*\right)^{-2af} \tag{5.42}$$

Equivalently, squaring (5.39) gives:

$$(Q_j^*)^2 = (\Psi_2)^2 (A_{0j})^{\frac{2}{1-b}} (\sigma_j)^{\frac{2b}{1-b}} (J_j^*)^{\frac{2a}{1-b}} \tag{5.43}$$

Moreover, summing (5.29) over all firms and then squaring:

$$\left[\sum^{N} Q_j^*\right]^2 = (\Psi_1)^2\left[\sum (A_{oj})^{f}(\sigma_j)^{f_1}\left(r_1+r_2 J_j^*\right)^{-af}\right]^2 \tag{5.44}$$

Similarly, the summation of (5.39) over all firms and the subsequent squaring of the deduced relation gives:

$$\left[\sum^{N} Q_j^*\right]^2 = (\Psi_2)^2\left[\sum (A_{oj})^{\frac{1}{1-b}}(\sigma_j)^{\frac{b}{1-b}}(J_j^*)^{\frac{a}{1-b}}\right]^2 \tag{5.45}$$

Summing (5.42) over all the firms in the industry and then dividing the result with (5.44) yields the following expression for the Herfindahl index of concentration:

$$H^* = \frac{\sum^{N}(Q_j^*)^2}{\left[\sum(Q_j^*)\right]^2} = \frac{\sum^{N}(G_{oj})^{2f}\ (\sigma_j)^{2f_1}(r_1+r_2 J_j^*)^{-2af}}{\left(\sum^{N}(G_{oj})^{f}\ (\sigma_j)^{f_1}\ (r_1+r_2 J_j^*)^{-af}\right)^2} \tag{5.46}$$

Alternatively, summing (5.43) over all the firms in the industry and then dividing the result by (5.45) yields the following Herfindahl index of concentration:

$$H^* = \frac{\sum^{N}(Q_j^*)^2}{\left[\sum(Q_j^*)\right]^2} = \frac{\sum^{N}(G_{oj})^{\frac{2}{1-b}}(\sigma_j)^{\frac{2b}{1-b}}(J_j^*)^{\frac{2a}{1-b}}}{\left(\sum^{N}(G_{oj})^{\frac{1}{1-b}}(\sigma_j)^{\frac{b}{1-b}}(J_j^*)^{\frac{a}{1-b}}\right)^2} \tag{5.47}$$

Imposing the same general assumption as in the no adjustment costs model of group uniform σ's ($\sigma^{NA}=x\sigma^{A}$) then such an assumption will still imply equal profits within each group if firms are in a stationary state. By incorporating the group uniformity of profitability into relation (5.33) the industry weighted average of profitability is equal to:

$$\pi^{*}= \frac{\pi^{*A}}{Q^{*}} \sum_{j}^{A}(Q_{j}^{*}) + \frac{\pi^{*NA}}{Q^{*}} \sum_{l}^{NA}(Q_{l}^{*}) \qquad \Leftrightarrow(5.48)$$

$$\pi^{*}= 1- \sigma^{A}(a+b)\left(\frac{Q^{*A}}{Q^{*}} + \frac{Q^{*NA}}{Q^{*}}\, x\right) \qquad \Leftrightarrow(5.48')$$

$$\pi^{*}= 1- \sigma^{A}(a+b)x_{1}^{*}$$

where π^{*NA} is the uniform profit rate for group NA, π^{*A} the uniform profit rate of group A, $x_{1}= \frac{Q^{*A}}{Q^{*}}(1-x)+x$ and $x= \frac{1- \pi^{*NA}}{1- \pi^{*A}}$.

Using the group uniformity assumption, (5.46) may be rewritten as:

$$H^{*}=\frac{(\sigma^{A})^{2f_{1}}(A_{0})^{2f}\sum^{A}\left[(\upsilon_{j})^{2f}(r_{1}+r_{2}J_{j}^{*})^{-2af}\right]+(x\sigma^{A})^{2f}(B_{0})^{2f}\sum^{NA}\left[(\upsilon_{l})^{2f}(r_{1}+r_{2}J_{l}^{*})^{-2af}\right]}{\left[(\sigma^{A})^{f_{1}}(A_{0})^{f}\sum^{A}\left[(\upsilon_{j})^{f}(r_{1}+r_{2}J_{j}^{*})^{-af}\right] + (x\sigma^{A})^{f_{1}}(B_{0})^{f}\sum^{NA}\left[(\upsilon_{l})^{f}(r_{1}+r_{2}J_{l}^{*})^{-af}\right]\right]^{2}}$$

$$\Leftrightarrow H^{*} = \frac{1 + (x)^{-2f_1}(V)^{2f}C_1}{\left[1 + (x)^{-f_1}(V)^{f}C_2\right]^2} C_3 \tag{5.49}$$

where:

$$C_1 = \frac{\sum^{A}\left((\upsilon_j)^{2f}(r_1 + r_2 J_j^{*})^{-2af}\right)}{\sum^{NA}\left((\upsilon_l)^{2f}(r_1 + r_2 J_l^{*})^{-2af}\right)}$$

$$C_2 = \frac{\sum^{A}\left((\upsilon_j)^{f}(r_1 + r_2 J_j^{*})^{-af}\right)}{\sum^{NA}\left((\upsilon_l)^{f}(r_1 + r_2 J_l^{*})^{-af}\right)}$$

$$C_3 = \frac{\sum^{NA}\left((\upsilon_l)^{2f}(r_1 + r_2 J_l^{*})^{-2af}\right)}{\left[\sum^{NA}\left((\upsilon_l)^{f}(r_1 + r_2 J_l^{*})^{-af}\right)\right]^2}$$

and

$$C_4 = \frac{\sum^{A}\left((\upsilon_j)^{2f}(r_1 + r_2 J_j^{*})^{-2af}\right)}{\sum^{A}\left((\upsilon_j)^{f}(r_1 + r_2 J_j^{*})^{-af}\right)} \frac{\sum^{NA}\left((\upsilon_l)^{f}(r_1 + r_2 J_l^{*})^{-af}\right)}{\sum^{NA}\left((\upsilon_l)^{2f}(r_1 + r_2 J_l^{*})^{-2af}\right)}$$

Equivalently, (5.47) becomes:

$$H^{*} = \frac{1 + (x)^{-\frac{2b}{1-b}} (V)^{\frac{2}{1-b}} C_1'}{\left[1 + (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^2} C_3' \qquad (5.50)$$

where:

$$C_1' = \frac{\sum^{A}\left((\upsilon_j)^{\frac{2}{1-b}} (J_j^{*})^{\frac{2a}{1-b}} \right)}{\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}} (J_l^{*})^{\frac{2a}{1-b}} \right)}$$

$$C_2' = \frac{\sum^{A}\left((\upsilon_j)^{\frac{1}{1-b}} (J_j^{*})^{\frac{a}{1-b}} \right)}{\sum^{NA}\left((\upsilon_l)^{\frac{1}{1-b}} (J_l^{*})^{\frac{a}{1-b}} \right)}$$

$$C_3' = \frac{\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}} (J_l^{*})^{\frac{2a}{1-b}} \right)}{\left[\sum^{NA}\left((\upsilon_l)^{\frac{1}{1-b}} (J_l^{*})^{\frac{a}{1-b}} \right)\right]^2}$$

and

$$C_4' = \frac{\sum^{A}\left((\upsilon_j)^{\frac{2}{1-b}} (J_j^{*})^{\frac{2a}{1-b}} \right)}{\sum^{A}\left((\upsilon_j)^{\frac{1}{1-b}} (J_j^{*})^{\frac{a}{1-b}} \right)} \; \frac{\sum^{NA}\left((\upsilon_l)^{\frac{1}{1-b}} (J_l^{*})^{\frac{a}{1-b}} \right)}{\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}} (J_l^{*})^{\frac{2a}{1-b}} \right)}$$

Both the parameters of relation (5.50) as well as the parameters of relation (5.49), are in a 'non reduced' form since neither J_j^{*} nor $r_1 + r_2 J_j^{*}$ are exogenous as (5.12) clearly indicates. Since σ's have been assumed uniform within each group, differences in capital in full efficiency units within each group are attributable solely to the different values of υ's as an inspection of (5.12) clearly demonstrates. Differences in the J^*'s between firms belonging in different groups are attributable not only to the firm specific variable υ but also to the group specific variables, namely potential productivity and the ETIOV of each group. Therefore, any attempt to calculate the change in H^* induced by a change in either V or x should account for the fact that, unlike the model of adjustment costs, the parameters are a function of these two variables.

Assume for simplicity that B_0 and σ^{NA} are constant. Using (5.50) the partial derivative of H^* with respect to the gap in potential productivity is:

$$\frac{\partial H^*}{\partial V} = \frac{\frac{2}{1-b}(V)^{\frac{2}{1-b}-1} x^{-\frac{2b}{1-b}} C_1' C_3' \left[1+ x^{-\frac{b}{1-b}} V^{\frac{1}{1-b}} C_2'\right]^2}{\left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^4} +$$

$$+ \frac{\frac{\partial C_1'}{\partial V} C_3' (x)^{-\frac{2b}{1-b}} (V)^{\frac{2}{1-b}} \left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^2}{\left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^4}$$

$$- \frac{\left[1+ (x)^{-\frac{2b}{1-b}} (V)^{\frac{2}{1-b}} C_1'\right] C_3' 2 \left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right] \frac{1}{1-b} (V)^{\frac{1}{1-b}-1} (x)^{-\frac{b}{1-b}} C_2'}{\left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^4}$$

$$- \frac{\left[1+ (x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}}C_1'\right]C_3' 2\left[1+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}C_2'\right]\frac{\partial C_2'}{\partial V}(V)^{\frac{1}{1-b}}(x)^{-\frac{b}{1-b}}}{\left[1+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}C_2'\right]^4}$$

For $\frac{\partial H^*}{\partial V}$ to be positive it is required that:

$$2\frac{1}{1-b}C_1'C_3'(x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}-1} + 2\frac{1}{1-b}C_1'C_2'C_3'(x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}-1} +$$

$$+ \frac{\partial C_1'}{\partial V}C_3'(x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}} + \frac{\partial C_1'}{\partial V}C_2'C_3'(x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}}$$

$$- 2\frac{1}{1-b}C_2'C_3'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}-1} - 2\frac{1}{1-b}C_1'C_2'C_3'(x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}-1} +$$

$$- 2\frac{\partial C_2'}{\partial V}C_3'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial V}C_1'C_3'(x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}} =$$

$$= 2\frac{1}{1-b}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}-1}\left[C_1'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - C_2'\right] +$$

$$+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}\left[\frac{\partial C_1'}{\partial V}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial V}\right] +$$

$$+ (x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}}\left[\frac{\partial C_1'}{\partial V}C_2' - 2\frac{\partial C_2'}{\partial V}C_1'\right] > 0 \quad \Leftrightarrow$$

$$2\frac{1}{1-b}\left[C_1'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - C_2'\right] + V\left[\frac{\partial C_1'}{\partial V}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial V}\right]$$

$$+ (x)^{-\frac{2b}{1-b}} (V)^{\frac{2}{1-b}+1} \left[\frac{\partial C_1'}{\partial V} C_2' - 2\frac{\partial C_2'}{\partial V} C_1' \right] > 0 \qquad (5.51)$$

Moreover, we have that:

$$\frac{\partial C_1'}{\partial V} = \frac{\partial C_1'}{\partial A_0} B_0 = \frac{\frac{2a}{1-b}\sum^{A}\left[(\upsilon_j)^{\frac{2}{1-b}} (J_j^*)^{\frac{2a}{1-b}} (J_j^*)^{-1} \frac{\partial J_j^*}{\partial A_0}\right]}{\sum^{NA}\left[(\upsilon_l)^{\frac{2}{1-b}} (J_l^*)^{\frac{2a}{1-b}}\right]} B_0 \qquad (5.52)$$

Since $\frac{\partial J_j^*}{\partial A_0} > 0$ (section 5.1) and $b<1$ (since $1-a-b-(b-1) = 2(1-b)-a > 0$), $\frac{\partial C_1'}{\partial V}$ is positive. It is easily demonstrated that the same applies for $\frac{\partial C_2'}{\partial V}$. The partial derivative of H^* with respect to x is:

$$\frac{\partial H^*}{\partial x} = \frac{-\frac{2b}{1-b}(x)^{\frac{2b}{1-b}-1} (V)^{\frac{2}{1-b}} C_1' C_3' \left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^2}{\left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^4} +$$

$$\frac{\frac{\partial C_1'}{\partial x} C_3' (x)^{-\frac{2b}{1-b}} (V)^{\frac{2}{1-b}} \left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^2}{\left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^4} +$$

$$\frac{\left[1+ (x)^{-\frac{2b}{1-b}} (V)^{\frac{2}{1-b}} C_1'\right] C_3' 2 \left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right] \frac{b}{1-b}(x)^{-\frac{b}{1-b}-1} (V)^{\frac{1}{1-b}} C_2'}{\left[1+ (x)^{-\frac{b}{1-b}} (V)^{\frac{1}{1-b}} C_2'\right]^4}$$

$$- \frac{\left[1+ (x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}}C_1'\right]C_3' 2\left[1+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}C_2'\right]\frac{\partial C_2'}{\partial x}(V)^{\frac{1}{1-b}}(x)^{-\frac{b}{1-b}}}{\left[1+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}C_2'\right]^4}$$

For $\frac{\partial H^*}{\partial x}$ to be positive it is required that:

$$- 2\frac{b}{1-b}C_1'C_3'(V)^{\frac{2}{1-b}}(x)^{-\frac{2b}{1-b}-1} - 2\frac{b}{1-b}C_1'C_2'C_3'(x)^{-\frac{3b}{1-b}-1}(V)^{\frac{3}{1-b}} +$$

$$+ \frac{\partial C_1'}{\partial x}C_3'(x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}} + \frac{\partial C_1'}{\partial x}C_2'C_3'(x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}}$$

$$+ 2\frac{b}{1-b}C_2'C_3'(x)^{-\frac{b}{1-b}-1}(V)^{\frac{1}{1-b}} + 2\frac{b}{1-b}C_1'C_2'C_3'(x)^{-\frac{3b}{1-b}-1}(V)^{\frac{3}{1-b}} +$$

$$- 2\frac{\partial C_2'}{\partial x}C_3'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial x}C_1'C_3'(x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}} =$$

$$= -\frac{2b}{1-b}(x)^{-\frac{b}{1-b}-1}(V)^{\frac{1}{1-b}}\left[C_1'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - C_2'\right] +$$

$$+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}\left[\frac{\partial C_1'}{\partial x}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial x}\right] +$$

$$+ (x)^{-\frac{3b}{1-b}}(V)^{\frac{3}{1-b}}\left[\frac{\partial C_1'}{\partial x}C_2' - 2\frac{\partial C_2'}{\partial x}C_1'\right] > 0 \quad \Leftrightarrow$$

$$- 2\frac{b}{1-b}\left[C_1'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - C_2'\right] + x\left[\frac{\partial C_1'}{\partial x}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial x}\right]$$

$$+ (x)^{-\frac{2b}{1-b}+1}(V)^{\frac{2}{1-b}}\left[\frac{\partial C_1'}{\partial x}C_2' - 2\frac{\partial C_2'}{\partial x}C_1'\right] > 0 \qquad (5.53)$$

Moreover,

$$\frac{\partial c_1'}{\partial x} = \frac{\partial c_1'}{\partial \sigma^A} \frac{\partial\left(\frac{\sigma^{NA}}{x}\right)}{\partial x} = - \frac{1}{x^2} \sigma^{NA} \frac{\partial c_1'}{\partial \sigma^A} =$$

$$\frac{\frac{2a}{1-b}\sum^{A}\left[(\upsilon_j)^{\frac{2}{1-b}}(J_j^*)^{\frac{2a}{1-b}}(J_j^*)^{-1}\frac{\partial J_j^*}{\partial \sigma^A}\right]}{\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J_l^*)^{\frac{a}{1-b}}\right]}\left(-\frac{1}{x^2}\sigma^{NA}\right) \qquad (5.54)$$

The above relation implies that $\frac{\partial c_1'}{\partial x}$ is negative since $\frac{\partial J_j^*}{\partial \sigma^A}$ is positive. In the same way, it can be shown that $\frac{\partial c_2'}{\partial x}$ is negative. Using (5.21) it can be easily demonstrated that:

$$\frac{\partial J_j^*}{\partial A_0} A_0 = \frac{\partial J_j^*}{\partial \sigma^A} \sigma^A \qquad (5.55)$$

The above combined with (5.52) and (5.54) means that:

$$\frac{\partial c_1'}{\partial A_0} = \frac{1}{A_0} \frac{\partial c_1'}{\partial \sigma^A} \sigma^A \qquad \Leftrightarrow$$

$$\frac{\partial c_1'}{\partial A_0} = \frac{\partial c_1'}{\partial V} \frac{1}{B_0} = \frac{1}{A_0} \frac{\partial c_1'}{\partial x}(-x^2)\frac{\sigma^A}{\sigma^{NA}} \qquad \Leftrightarrow$$

$$\frac{\partial c_1'}{\partial V} V = - \frac{\partial c_1'}{\partial x} x$$

Equivalently:

$$\frac{\partial C_2'}{\partial V} V = - \frac{\partial C_2'}{\partial x} x$$

Substituting the above relations into inequality (5.53) gives:

$$- 2\frac{b}{1-b}\left[C_1'(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - C_2'\right] - V\left[\frac{\partial C_1'}{\partial V}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial C_2'}{\partial V}\right]$$

$$- (x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}+1}\left[\frac{\partial C_1'}{\partial V} C_2' - 2\frac{\partial C_2'}{\partial V} C_1'\right] > 0 \qquad (5.56)$$

Comparing (5.56) to (5.51), the following conclusions are easily derived: if $C_4'(V)^{\frac{1}{1-b}} > (x)^{\frac{b}{1-b}}$ then given that $\frac{\partial H^*}{\partial x}$ is negative, $\frac{\partial H^*}{\partial V}$ is positive and given that $\frac{\partial H^*}{\partial V}$ is negative, $\frac{\partial H^*}{\partial x}$ is positive. If $C_4'(V)^{\frac{1}{1-b}} < (x)^{\frac{b}{1-b}}$ then given that $\frac{\partial H^*}{\partial V}$ is positive, $\frac{\partial H^*}{\partial x}$ is negative and given that $\frac{\partial H^*}{\partial x}$ is positive, $\frac{\partial H^*}{\partial V}$ is negative.

As before, a positive $\frac{\partial H}{\partial V}$ is interpreted as a non-rejection of the DEH and a negative $\frac{\partial H}{\partial V}$ as a rejection of the DEH. For $\frac{\partial H}{\partial x}$ and $x \geq 1$ a positive sign implies that a relative increase in the ETIOV of group A (and consequently an increase in x) which already has an equal or relatively higher ETIOV (and, by definition, an equal or higher profit rate) than group NA, increases concentration. Since an increase in x translates as an increase in the divergence of the ETIOV's, then if this increase causes an increase in H^*, the MPH is to be accepted. On the other hand, if this increase in the divergence of the ETIOV's causes a decrease in H^* then the MPH is to be rejected. When $x<1$, then a positive sign denotes that an increase in the ETIOV of group A, which is the group with the relatively smaller ETIOV (and by definition the group with the relatively lower profitability) will result to an increase in concentration which is a contradiction to the MPH. More simply, since an increase in x translates as a convergence between the ETIOV's of the two groups, then if a decrease in the difference in the magnitudes of the two ETIOV's results to an increase in H, the MPH is to be rejected. On the other hand, if this decrease causes a decrease in H^* the MPH is to be accepted.

The above interpretations can now be applied to the industry level results. Consequently, if one sets $\Delta = C_4' \, (V)^{\frac{1}{1-b}} - (x)^{\frac{b}{1-b}}$, then the following results hold:

$$1)\ x > 1,\ \Delta > 0: \left.\begin{array}{l} \text{a) reject DEH, accept MPH} \\ \text{b) accept DEH, reject MPH} \\ \text{c) accept DEH, accept MPH} \end{array}\right\} MPH \cup DEH$$

In other words, in this case at least one hypothesis is accepted, i.e. the unshaded area ($MPH^c \cap DEH^c$) is not possible as diagram 1a depicts.

$$2)\ x > 1,\ \Delta < 0: \left.\begin{array}{l} \text{a) reject DEH, accept MPH} \\ \text{b) accept DEH, reject MPH} \\ \text{c) reject DEH, reject MPH} \end{array}\right\} MPH^c \cup DEH^c$$

In this case at least one hypothesis is rejected, i.e. the unshaded area ($MPH \cap DEH$) is not possible as diagram 1b demonstrates.

$$3)\ x \le 1,\ \Delta > 0: \left.\begin{array}{l}\text{a) accept DEH, accept MPH}\\ \text{b) accept DEH, reject MPH}\\ \text{c) reject DEH, reject MPH}\end{array}\right\} MPH^c \cup DEH$$

In this case if DEH is rejected (MPH accepted) ⇒ MPH is rejected (DEH is accepted). MPH $\cap$ DEH^c, the unshaded area in diagram 1c, is not possible.

$$4)\ x \le 1,\ \Delta < 0: \left.\begin{array}{l}\text{a) accept DEH, accept MPH}\\ \text{b) accept MPH, reject DEH}\\ \text{c) reject DEH, reject MPH}\end{array}\right\} MPH \cup DEH^c$$

In this case when MPH is rejected (DEH is accepted) ⇒ DEH is rejected (MPH is accepted). What is not possible is MPH^c $\cap$ DEH, which is the unshaded area in diagram 1d.

Note that the impossible areas of each case add up to all the possibilities, i.e. $(MPH^c \cap DEH^c)+(MPH \cap DEH)+(MPH \cap DEH^c)+(MPH^c \cap DEH)=\Omega$. If the above industry conclusions are compared to the industry conclusions of the no adjustment costs model it is seen that the range of possibilities of the former are unfortunately wider. In particular, for the no adjustment costs model in cases 1 and 2 only choices a and b are possible, while for cases 3 and 4 only choices a and c are possible. The next section examines whether the possibilities can be narrowed down by deriving a reduced form expression for H^*. In order to do this, one should try to approximate the solution of (5.12) for J^*.

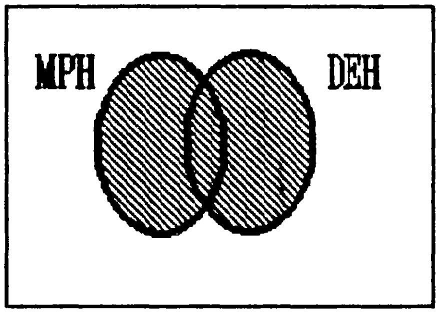


Figure 1a

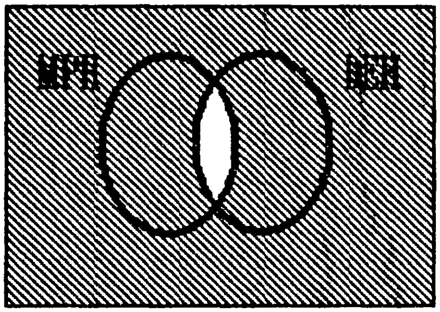


Figure 1b

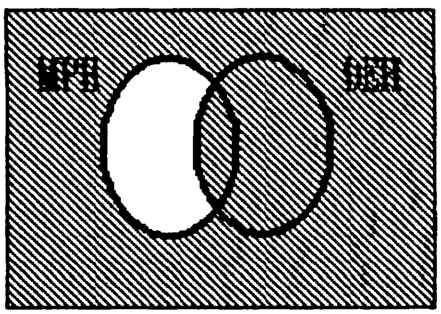


Figure 1c

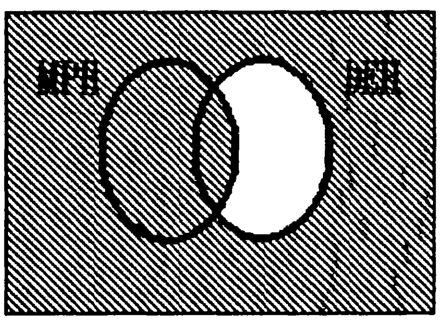


Figure 1d

5.4 Estimation possibilities for the equilibrium state; an approximation.

As it was proved in section 5.1, each J_j^*, $j \in A$, is an increasing function of A_o, σ^A and υ_j and each J_l^*, $l \in NA$, an increasing function of B_o, σ^{NA} and υ_l. Let us re-write r_{lj} in the following form:

$$r_{lj} = \frac{r_1 + r_2 J_l^*}{r_1 + r_2 J_j^*} = \frac{\frac{c}{2\beta\delta} + J_l^*}{\frac{c}{2\beta\delta} + J_j^*} \tag{5.57}$$

If it is assumed that c is small as compared to the parameter in the adjustment costs function times twice the depreciation rate, then as an approximation $\frac{c}{2\beta\delta} \cong 0$. As a result,

$$\frac{J_l^*}{J_j^*} \cong r_{lj} \tag{5.58}$$

The approximation $\frac{c}{2\beta\delta} \cong 0$ implies that (5.12) can be explicitly solved for J_j^*. In particular, (5.12) may be rewritten as:

$$J_j^* = \left[(p\sigma_j A_{oj})^{\frac{1}{1-b}} \left(\frac{a}{r_2}\right) \left(\frac{w}{b}\right)^{-\frac{b}{1-b}} \right]^{\frac{1-b}{1-a-b-(b-1)}} \Leftrightarrow$$

$$J_j^* = \left[(p\sigma_j A_{oj}) \left(\frac{a}{r_2}\right)^{1-b} \left(\frac{w}{b}\right)^{-b} \right]^{\frac{1}{1-a-b-(b-1)}} \tag{5.59}$$

Relation (5.59) implies that parameters C_1' and C_2' can be expressed as functions of V, x and the sum of the firm specific deviations from the potential maximum efficiency of each group. (Parameter C_3' only depends on the firm specific deviations since B_0 and σ^{NA} cancel out from the nominator and the denominator.) More specifically, these parameters can be rewritten as:

$$C_1' = (V)^{2g_1}(x)^{-2g_1} \frac{\sum^{A} \exp[(\upsilon M_j) 2\gamma_4]}{\sum^{NA} \exp[(\upsilon M_l) 2\gamma_4]} = g_3 (V)^{2g_1}(x)^{-2g_1} \qquad (5.60a)$$

$$C_2' = (V)^{g_1}(x)^{-g_1} \frac{\sum^{A} \exp[(\upsilon M_j) \gamma_4]}{\sum^{NA} \exp[(\upsilon M_l) \gamma_4]} = g_5 (V)^{g_1}(x)^{-g_1} \qquad (5.60b)$$

$$C_3' = \frac{\sum^{NA} \exp[(\upsilon M_l) 2\gamma_4]}{\left[\sum^{NA} \exp[(\upsilon M_l) \gamma_4]\right]^2} \qquad (5.60c)$$

where $g_2 = [1-a-b-(b-1)]^{-1} > 0$, $g_1 = \frac{ag_2}{1-b}$, $\gamma_4 = g_1 + \frac{1}{1-b} = \frac{1+ag_2}{1-b}$. Obviously since g_2 is always positive (corrolary 1) then this implies that b has to be always less than one. Consequently, g_1 and γ_4 are also always positive. Substituting (5.60a)-(5.60c) into (5.50) and adding a multiplicative disturbance term, the regression for industry i is of the form:

$$H^{*}_{it} = \frac{1 + \gamma_{1it} x_{it}^{-2\gamma_{5i}}}{\left[1 + \gamma_{2it} x_{it}^{-\gamma_{5i}}\right]^2} \gamma_{3i} \eta_{it} \qquad (5.61)$$

where $\gamma_1 = g_3 V^{2\gamma_4}$, $\gamma_2 = g_5 V^{\gamma_4}$, $\gamma_3 = C'_3$, $\gamma_5 = \gamma_4 - 1 = \frac{b}{1-b} + g_1 > 0$. Similarly to regression (3.29) in section 3.4, all these parameters vary between different industries and additionally γ_1 and γ_2 will vary from time to time as discrete jumps in innovations alter the magnitude of the gap in potential productivity, V. Therefore data that are capable of accounting for these occasional jumps are required. Also panel data information on major innovations are required for calculating the ratio $x = \frac{1-\pi^{*NA}}{1-\pi^{*A}}$. (For the availability and effectiveness of such data refer to section 3.4.)

Estimates of the parameters γ_1, γ_2, γ_3 and γ_5 can be derived either by running regression (5.61) assuming that γ_1 and γ_2 are fixed and time invariant parameters or by regarding these two as random parameters with means γ_{1i} and γ_{2i} respectively. In this second case if one defines $\varepsilon_{1it} = \gamma_{1it} - \gamma_{1i}$ and $\varepsilon_{2it} = \gamma_{1it} - \gamma_{2i}$ then an alternative to (5.61) is the model

$$H^{*}_{it} = \frac{1 + \gamma_{1i} (x_{it})^{-2\gamma_{5i}} + \varepsilon_{1it} (x_{it})^{-2\gamma_{5i}}}{\left[1 + \gamma_{2i} (x_{it})^{-\gamma_{5i}} + \varepsilon_{2it} (x_{it})^{-\gamma_{5i}}\right]^2} \gamma_{3i} \qquad (5.62)$$

From then on the procedure for estimating the parameters in (5.62) is identical to the procedure followed for estimating the parameters in (3.30) as described in section 3.4.

Since g_3, g_5 and γ_3 are all independent of V and x and

$\gamma_5=\gamma_4-1$, when differentiating H^* with respect to V and x one can use the conclusions for the no adjustment costs model (table 1, section 3.3). In particular, γ_5 takes the place of f_1 and γ_4 the place of f. Therefore since b<1 ($\gamma_4,\gamma_5>0$), this corresponds to the case of a+b<1 ($f,f_1>0$). As a result, when $x\geq 1$ one can determine whether the MPH is to be rejected and the DEH accepted or vice versa by using the industry level conclusions of section 3.3. More specifically, when $\frac{\partial H^*}{\partial V}$ is positive and $\frac{\partial H^*}{\partial x}$ is negative ($\frac{\gamma_1}{\gamma_2}>(x)^{\gamma_5} \Leftrightarrow \frac{g_3}{g_5}(V)^{\gamma_4}>(x)^{\gamma_5}$) the MPH is rejected and the DEH is accepted, while when $\frac{\partial H^*}{\partial V}$ is negative and $\frac{\partial H^*}{\partial x}$ positive ($\frac{g_3}{g_5}(V)^{\gamma_4}<(x)^{\gamma_5}$) the DEH is rejected and the MPH is accepted. This significantly narrows down the number of possible cases since the only possibilities for $x\geq 1$ is the intersection of cases 1 and 2 as derived from the model in its structural form, i.e. (MPH$\cup$DEH) $\cap$ ($MPH^c \cup DEH^c$), which means that the two hypotheses are mutually exclusive when $x\geq 1$ at the industry level of analysis. On the other hand, when x<1, if one takes into account relation (5.58) and divides relation (5.59) for a firm j, $j\in A$, by relation (5.59) for a firm *l*, $l\in NA$, it is deduced that:

$$r_{lj} = \left((M_{jl})^{-1}(x_{lj})\right)^{g_2} \qquad (5.63)$$

Relation (5.63) implies that when $x_{lj}<1 \Rightarrow r_{lj}<1$. This means that when $x_{lj}<1$, the case from the table in section 5.2a that corresponds to it is the combination $a+b<1$, $x_{lj}<1$ and $r_{lj}<1$. For this case the DEH is accepted and the MPH rejected. As a result, when x is calculated by the data to be less than one, **it is not necessary to run the regression since it can be concluded before hand (from the firm level conclusions) that the DEH is accepted and the MPH rejected.** The fact that unlike the no adjustment costs model, in the long run equilibrium of this model the rejection of the MPH and acceptance of the DEH is the only possibility when $\pi^{*A}<\pi^{*NA}$, is due to the convergence restriction as expressed in corollary 1 which implies that g_2 is always positive. This makes J_j^* an increasing function of A_0 (or B_0), σ^A (or σ^{NA}) and υ_j irrespective of whether $a+b<1$ or $a+b>1$. It also means that γ_4 and γ_5 are always positive. This contrasts with the fact that in regressions (3.29) and (3.30) f_1 could be either positive or negative. In relations (5.61) and (5.62) γ_5 takes the place of f_1 and therefore the case of rejecting the DEH and accepting the MPH when $a+b>1$, $x_{lj}<1$ and $r_{lj}\geq1$ is ruled out both because γ_5 is always positive and also because the combination of $x_{lj}<1$ and $r_{lj}\geq1$ is no longer possible. Threfore the possibility of rejecting the DEH in favour of the MPH when $x<1$ has to be ruled out in the long run equilibrium state of the capital adjustment costs model.

Let us summarise the conclusions as far as the ambiguity between the two hypotheses is concerned, when the approximation (5.58) is accepted. When the ETIOV of group A is smaller than the ETIOV of group NA (and consequently the equilibrium profits of the former are smaller than those of the latter) then the firms that enjoy no cost advantage have no interest to use their oligopolistic practices for obtaining a higher market share. On the contrary, their behaviour is output restrictive since they know that this is the only way to keep their average and marginal costs low. On the other hand, although the static returns to scale are adverse, the firms in group A can afford a relatively higher market share since their superior productivity compensates for this adversity. As a result, the differential efficiency hypothesis is accepted and the market power hypothesis is rejected. When $x \geq 1$, then if the gap in efficiency is sufficiently large and/or the gap in σ's (as given by x) sufficiently small for $\frac{g_3}{g_5}(V)^{\gamma_4}$ to be larger than $(x)^{\gamma_5}$ then the MPH is rejected in favour of the DEH. Else, if the gap in efficiency is sufficiently small and the gap in σ's sufficiently large for $\frac{g_3}{g_5}(V)^{\gamma_4}$ to be smaller than $(x)^{\gamma_5}$ then the DEH is rejected in favour of the MPH.

As a final point it is interesting to examine whether the possibility of a negative relationship between equilibrium market share and equilibrium firm profitability can also be a possibility at the industry level, reflected by a negative relation between π^* and H^*. In order to do this, one has to first calculate relation $\frac{\partial \pi^*}{\partial x}$ by utilising relation (5.48) while it is assumed for simplicity that π^{*NA} (and therefore σ^{NA}) is constant:

$$\frac{\partial \pi^*}{\partial x} = \frac{\partial \pi^*}{\partial \pi^{*A}} \frac{\partial \left(1 - \frac{1-\pi^{*NA}}{x}\right)}{\partial x} =$$

$$= \frac{1}{x^2}\ (1-\pi^{*NA}) \left(\frac{Q^{*A}}{Q^{*}} + \pi^{*A} \frac{\partial\left(\frac{Q^{*A}}{Q^{*}}\right)}{\partial \pi^{*A}} - \pi^{*NA} \frac{\partial\left(\frac{Q^{*A}}{Q^{*}}\right)}{\partial \pi^{*A}} \right) \Leftrightarrow$$

$$\frac{\partial \pi^{*}}{\partial x} = \frac{1}{x^2}\ (1-\pi^{*NA}) \left(\frac{Q^{*A}}{Q^{*}} + (\pi^{*A}-\pi^{*NA})\ \frac{\partial\left(\frac{Q^{*A}}{Q^{*}}\right)}{\partial \pi^{*A}} \right) \qquad (5.64)$$

As relation (3.28) reveals the sign of $\frac{\partial \pi^{*}}{\partial x}$ depends on x since when $x \geq 1$, $\pi^{*A} \geq \pi^{*NA}$, whereas when $x<1$, $\pi^{*A} < \pi^{*NA}$. Combining (5.59) with (5.39) gives:

$$Q_j^{*A} = \left(\left(\frac{b}{w}\right)^{2b} (A_{oj})^{2} (p\sigma^{A})^{a+2b} \right)^{\frac{1}{1-a-b-(b-1)}} \qquad (5.65)$$

Since, *ceteris paribus*, an increase in x means a relative increase in the ETIOV of group A, which in turn means an increase in π^{*A}, (5.65) implies that $\frac{\partial\left(\frac{Q^{*A}}{Q^{*}}\right)}{\partial \pi^{*A}}$ is always negative irrespective of whether $a+b<1$ or $a+b>1$. As a result, $\frac{\partial \pi^{*}}{\partial x}$ is going to be negative only when $x \not> 1$ and $\left| (\pi^{*A}-\pi^{*NA})\ \frac{\partial\left(\frac{Q^{*A}}{Q^{*}}\right)}{\partial \pi^{*A}} \right| > \frac{Q^{*A}}{Q^{*}}$.

If it is possible to have $\frac{\partial \pi^{*}}{\partial x}$ positive when $\frac{\partial H^{*}}{\partial x}$ is negative and vice versa, then obviously this implies a negative relationship between H^{*} and π^{*}. Looking at the conditions which determine $\frac{\partial H^{*}}{\partial x}$ (as derived from the reduced form relation (5.61)), this is clearly possible. For example, when x<1 then $\frac{g_3}{g_5}(V)^{\gamma_4}$ is always going to be larger that $(x)^{\gamma_5}$ and therefore $\frac{\partial H^*}{\partial x}$ is negative while $\frac{\partial \pi^*}{\partial x}$ can only be positive. More generally, the sign of the relationship between H^* and π^* adds nothing to the task of resolving the ambiguity between the market power hypothesis and the differential efficiency hypothesis. This quality carries over from the firm level of analysis, where the sign of the relationship between equilibrium market share and equilibrium profitability at the firm level revealed nothing about which hypothesis (if any) is at work.

5.5 Industry level hypotheses discrimination analysis; the optimal path towards the long run equilibrium

Using (5.31) in combination with (5.39), if the same procedure for obtaining (5.49) and (5.50) is followed, the following expression for H_t, the Herfindahl index on the optimal path, is deduced:

$$H_t = \frac{1+ (x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}}D_{1t}'}{\left[1+ (x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}D_{2t}'\right]^2} D_{3t}' \qquad (5.66)$$

where:

$$D'_{1t} = \frac{\sum^{A}\left[(\upsilon_j)^{\frac{2}{1-b}}(J_j^* \ \theta_{1jt})^{\frac{2a}{1-b}}\right]}{\sum^{NA}\left[(\upsilon_l)^{\frac{2}{1-b}}(J_l^* \ \theta_{1lt})^{\frac{2a}{1-b}}\right]} = \frac{\sum^{A}\exp\left[\left(\frac{\upsilon M_j}{a} + \ln(J_{jt})\right)\frac{2a}{1-b}\right]}{\sum^{NA}\exp\left[\left(\frac{\upsilon M_l}{a} + \ln(J_{lt})\right)\frac{2a}{1-b}\right]} \quad (5.67a)$$

$$D'_{2t} = \frac{\sum^{A}\left[(\upsilon_j)^{\frac{1}{1-b}}(J_j^* \ \theta_{1jt})^{\frac{a}{1-b}}\right]}{\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J_l^* \ \theta_{1lt})^{\frac{a}{1-b}}\right]} = \frac{\sum^{A}\exp\left[\left(\frac{\upsilon M_j}{a} + \ln(J_{jt})\right)\frac{a}{1-b}\right]}{\sum^{NA}\exp\left[\left(\frac{\upsilon M_l}{a} + \ln(J_{lt})\right)\frac{a}{1-b}\right]} \quad (5.67b)$$

$$D'_{3t} = \frac{\sum^{NA}\left[(\upsilon_l)^{\frac{2}{1-b}}(J_l^* \ \theta_{1lt})^{\frac{2a}{1-b}}\right]}{\left[\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J_l^* \ \theta_{1lt})^{\frac{a}{1-b}}\right]\right]^2} = \frac{\sum^{NA}\exp\left[\left(\frac{\upsilon M_l}{a} + \ln(J_{lt})\right)\frac{2a}{1-b}\right]}{\left[\sum^{NA}\exp\left[\left(\frac{\upsilon M_l}{a} + \ln(J_{lt})\right)\frac{a}{1-b}\right]\right]^2} \quad (5.67c)$$

Notice that the parameters are a continuous function of time through the inclusion of the capital (in full efficiency units) on the optimal path, $J_{jt}=J_j^*\left(1- \exp(-\beta_{1j}t)+ \frac{J_{j0}}{J_j^*} \exp(-\beta_{1j}t)\right)$. In this section the sign of the impact of an unexpected jump in V or x is going to be determined. For variety, this time A_0 and σ^A are going to be assumed constant. The following two theorems hold for a firm on the unique convergent optimal path:

THEOREM 3

For all $l \in NA$,

$$\frac{\partial J_{lt}}{\partial B_0} B_0 = \frac{\partial J_{lt}}{\partial \sigma^{NA}} \sigma^{NA}$$

Proof

Taking the partial derivative with respect to B_o on both sides of relation (5.16) we have:

$$\frac{\partial J_{1t}}{\partial B_o} = \frac{\partial J_1^*}{\partial B_o}(1-\exp(-\beta_{11}t)) + \frac{\partial \beta_{11}}{\partial B_o} t\exp(-\beta_{11}t)(J_1^* - J_{10}) \qquad (5.68)$$

From (5.21) and (5.22) it is derived respectively that:

$$\frac{\partial J_1^*}{\partial B_o} B_o = \frac{\partial J_1^*}{\partial \sigma^{NA}} \sigma^{NA} \qquad (5.69)$$

$$\frac{\partial \beta_{11}}{\partial B_o} B_o = \frac{\partial \beta_{11}}{\partial \sigma^{NA}} \sigma^{NA} \qquad (5.70)$$

Substituting (5.69) and (5.70) into (5.68) we get that:

$$\frac{\partial J_{1t}}{\partial B_o} B_o = \frac{\partial J_{1t}}{\partial \sigma^{NA}} \sigma^{NA} \qquad (5.71)\square$$

THEOREM 4

$$\frac{\partial D'_{3t}}{\partial V} V = - \frac{\partial D'_{3t}}{\partial x} x$$

Proof

Taking into account Theorem 3 and differentiating both sides of (5.67c) with respect to V the following is derived:

$$\frac{\partial D'_{3t}}{\partial V} = \left(-\frac{A_o}{(V)^2}\right)\left[\frac{\frac{2a}{1-b}\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}}(J_{lt})^{\frac{2a}{1-b}}(J_{lt})^{-1}\frac{\partial J_{lt}}{\partial B_o}\right)}{\left[\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J^*_l\ \theta_{1lt})^{\frac{a}{1-b}}\right]\right]^2}\right.$$

$$\left.-\frac{\frac{2a}{1-b}\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}}(J_{lt})^{\frac{2a}{1-b}}\right)\sum^{NA}\left((\upsilon_l)^{\frac{1}{1-b}}(J_{lt})^{\frac{a}{1-b}}(J_{lt})^{-1}\frac{\partial J_{lt}}{\partial B_o}\right)}{\left[\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J^*_l\ \theta_{1lt})^{\frac{a}{1-b}}\right]\right]^3}\right] =$$

$$= \left(-\frac{A_o}{(V)^2}\right)\frac{\sigma^{NA}}{B_o}\left[\frac{\frac{2a}{1-b}\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}}(J_{lt})^{\frac{2a}{1-b}}(J_{lt})^{-1}\frac{\partial J_{lt}}{\partial \sigma^{NA}}\right)}{\left[\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J^*_l\ \theta_{1lt})^{\frac{a}{1-b}}\right]\right]^2}\right.$$

$$\left.-\frac{\frac{2a}{1-b}\sum^{NA}\left((\upsilon_l)^{\frac{2}{1-b}}(J_{lt})^{\frac{2a}{1-b}}\right)\sum^{NA}\left((\upsilon_l)^{\frac{1}{1-b}}(J_{lt})^{\frac{a}{1-b}}(J_{lt})^{-1}\frac{\partial J_{lt}}{\partial \sigma^{NA}}\right)}{\left[\sum^{NA}\left[(\upsilon_l)^{\frac{1}{1-b}}(J^*_l\ \theta_{1lt})^{\frac{a}{1-b}}\right]\right]^3}\right] =$$

$$= \left(-\frac{A_o}{(V)^2}\right)\frac{\sigma^{NA}}{B_o}\frac{\partial D'_{3t}}{\partial \sigma^{NA}} = -\frac{1}{V}\frac{\sigma^{NA}}{\sigma^{A}}\frac{\partial D'_{3t}}{\partial \sigma^{NA}}\sigma^{NA} = -\frac{x_t}{V_t}\frac{\partial D'_{3t}}{\partial x_t} \Leftrightarrow$$

$$\frac{\partial D'_{3t}}{\partial V}V = -\frac{\partial D'_{3t}}{\partial x}x \qquad (5.72)\square$$

Analogously, it can be proven that:

$$\frac{\partial D'_{1t}}{\partial V} V = - \frac{\partial D'_{1t}}{\partial x} x \qquad (5.73)$$

$$\frac{\partial D'_{2t}}{\partial V} V = - \frac{\partial D'_{2t}}{\partial x} x \qquad (5.74)$$

Let us make clear at this point that when we look at the impact of x and V on H_t, **we are only interested in the direct impact of V or x on H_t** and not in the indirect (side) effects which accrue from the inter-relation between V and x, as implied in relation (5.32). These side effects determine the magnitude of the deviation of π_{jt} from its target rate π_j^* and also how quickly the former approaches the latter. As soon as the convergence process is over these indirect effects dissappear. In other words we consider not only in equilibrium but also on the optimal path towards the equilibrium both jumps in V as well as jumps in x as **exogenously determined**. As a result the partial derivatives $\frac{\partial H_t}{\partial x}$ and $\frac{\partial H_t}{\partial V}$ coincide with their respective 'aggregate' derivatives $\frac{dH_t}{dx}$ and $\frac{dH_t}{dV}$. **This means that when a jump in V (or x) occurs, x (or V) and all the other parameters are not affected by this jump.** As a result, it can be easily demonstrated that the long run equilibrium conclusions as to when $\frac{\partial H^*}{\partial x}$ and $\frac{\partial H^*}{\partial V}$ are positive or negative hold exactly the same for $\frac{\partial H_t}{\partial x}$ and $\frac{\partial H_t}{\partial V}$ in the optimal path case. More specifically, for $\frac{\partial H_t}{\partial V}$ to be positive it is required that:

$$2\frac{1}{1-b}\left[D'_{1t}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - D'_{2t}\right] + V\left[\frac{\partial D'_{1t}}{\partial V}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial D'_{2t}}{\partial V}\right] +$$

$$+(x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}+1}\left[\frac{\partial D'_{1t}}{\partial V}D'_{2t} - 2\frac{\partial D'_{2t}}{\partial V}D'_{1t}\right] +$$

$$+\frac{\partial D'_{3t}}{\partial V_t}\left(1+(x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}}D'_{1t}\right)\left(1+(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}D'_{2t}\right)(x)^{\frac{b}{1-b}}(V)^{-\frac{1}{1-b}}V>0 \quad (5.75)$$

Similarly, for $\frac{\partial H_t}{\partial x}$ to be positive it is required:

$$-\frac{2b}{1-b}\left[D'_{1t}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - D'_{2t}\right] + x\left[\frac{\partial D'_{1t}}{\partial x}(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}} - 2\frac{\partial D'_{2t}}{\partial x}\right] +$$

$$+(x)^{-\frac{2b}{1-b}+1}(V)^{\frac{2}{1-b}}\left[\frac{\partial D'_{1t}}{\partial x}D'_{2t} - 2\frac{\partial D'_{2t}}{\partial x}D'_{1t}\right] +$$

$$+\frac{\partial D'_{3t}}{\partial x}\left(1+(x)^{-\frac{2b}{1-b}}(V)^{\frac{2}{1-b}}D'_{1t}\right)\left(1+(x)^{-\frac{b}{1-b}}(V)^{\frac{1}{1-b}}D'_{2t}\right)(x)^{\frac{b}{1-b}}(V)^{-\frac{1}{1-b}}x>0 \quad (5.76)$$

Substituting (5.72)-(5.74) into (5.76) it is easily proven that, as in the long run equilibrium state, the following hold: if $D'_{4t}(V)^{\frac{1}{1-b}} > (x)^{\frac{b}{1-b}}$ then given that $\frac{\partial H_t}{\partial x}$ is negative, $\frac{\partial H_t}{\partial V}$ is positive and given that $\frac{\partial H_t}{\partial V}$ is negative, $\frac{\partial H_t}{\partial x}$ will be positive. If $D'_{4t}(V)^{\frac{1}{1-b}} < (x)^{\frac{b}{1-b}}$ then given that $\frac{\partial H_t}{\partial V}$ is positive, $\frac{\partial H_t}{\partial x}$ is negative and given that $\frac{\partial H_t}{\partial x}$ is positive, $\frac{\partial H_t}{\partial V}$ is negative. Consequently the same range of industry level conclusions as those described in figures 1a-1d of section 5.3 carry over to the structural model along the optimal path. Therefore, as before, some approximation is required which by making possible the derivation of a reduced form expression for H_t will narrow down the set of possible conclusions.

5.6 Estimation possibilities for the optimal path towards the long run equilibrium state; the approximation approach

In this section relation (5.66) will be rewritten in a reduced form by using the same approximation as in section 5.3; namely $\frac{c}{2\beta\delta} \cong 0$ and as a result relations (5.58) and (5.59) hold. Start by writing (5.66) in a more explicit form; then H_t is equal to the following expression:

$$\frac{(\sigma^A)^{\frac{2b}{1-b}}(A_o)^{\frac{2}{1-b}}\sum^{A}\left[(\upsilon_j)^{\frac{2}{1-b}}\left(J_j^* \theta_{1jt}\right)^{\frac{2a}{1-b}}\right]+(\sigma^{NA})^{\frac{2b}{1-b}}(B_o)^{\frac{2}{1-b}}\sum^{NA}\left[(\upsilon_l)^{\frac{2}{1-b}}\left(J_l^* \theta_{1lt}\right)^{\frac{2a}{1-b}}\right]}{\left[(\sigma^A)^{\frac{b}{1-b}}(A_o)^{\frac{1}{1-b}}\sum^{A}\left((\upsilon_j)^{\frac{1}{1-b}}\left(J_j^*\theta_{1jt}\right)^{\frac{a}{1-b}}\right)+(\sigma^{NA})^{\frac{b}{1-b}}(B_o)^{\frac{1}{1-b}}\sum^{NA}\left((\upsilon_l)^{\frac{1}{1-b}}\left(J_l^*\theta_{1jt}\right)^{\frac{a}{1-b}}\right)\right]^2}$$

If relation (5.59) is substituted into (5.13) the following is derived:

$$\ddot{J}_{jt} - h\dot{J} - \left[\delta(\delta+h) + \frac{1-a-b}{1-b}\,\delta(h+\delta)\right]\left(J_{jt} - J_j^*\right) = 0 \Leftrightarrow$$

$$-\beta_1 = \frac{h - \sqrt{h^2 + 4\delta(\delta+h)\left(\frac{1-a-b-(b-1)}{1-b}\right)}}{2} \tag{5.77}$$

Therefore this approximation makes β_1 independent of the A_0's, σ's, υ's and J^*'s. As a result it forces β_1 to be uniform across all the firms in the industry; the velocity of convergence is identical for all firms. To determine what the θ_1's are equal to, we first have to determine what J_{jo} is equal to. The initial endowment of capital stock at the beginning of each convergence process is approximately equal to the long run equilibrium capital stock of the previous convergence process. As a result we have that for all $j \in A$

$$J_{j0} = \left((p\sigma^{A}_{-1} A_{0-1} \upsilon_j) \left(\frac{a}{r_2}\right)^{1-b} \left(\frac{w}{b}\right)^{-b} \right)^{\frac{1}{1-a-b-(b-1)}}$$

and for all $l \in NA$:

$$J_{l0} = \left((p\sigma^{NA}_{-1} B_{0-1} \upsilon_l) \left(\frac{a}{r_2}\right)^{1-b} \left(\frac{w}{b}\right)^{-b} \right)^{\frac{1}{1-a-b-(b-1)}}$$

If we assume that υ's are time invariant then the θ_1's are uniform within each group and equal to:

$$\theta^{A}_{1t} = 1- \exp(-\beta_1 t) + \left(\frac{\sigma^{A}_{-1} A_{0-1}}{\sigma^{A} A_{0}} \right)^{g_2} \exp(-\beta_1 t) \qquad (5.78)$$

$$\theta^{NA}_{1t} = 1- \exp(-\beta_1 t) + \left(\frac{\sigma^{NA}_{-1} B_{0-1}}{\sigma^{NA} B_{0}} \right)^{g_2} \exp(-\beta_1 t) \qquad (5.79)$$

Consequently, the following expression for H_t is derived:

$$H_t = \frac{1+ (x)^{-2\gamma_5} (V)^{2\gamma_4} g'_{3t}}{\left(1+ (x)^{-\gamma_5} (V)^{-\gamma_4} g'_{5t}\right)^2} \gamma_3 \qquad (5.80)$$

where

$$g'_{3t} = \left(\frac{(\theta^{A}_{1t})}{(\theta^{NA}_{1t})} \right)^{\frac{2a}{1-b}} \frac{\sum^{A} \exp[(\upsilon M_j) 2\gamma_4]}{\sum^{NA} \exp[(\upsilon M_l) 2\gamma_4]} = \left(\frac{(\theta^{A}_{1t})}{(\theta^{NA}_{1t})} \right)^{\frac{2a}{1-b}} g_3 \qquad (5.81)$$

$$g'_{5t} = \left(\frac{(\theta^{A}_{1t})}{(\theta^{NA}_{1t})} \right)^{\frac{a}{1-b}} \frac{\sum^{A} \exp[(\upsilon M_j) \gamma_4]}{\sum^{NA} \exp[(\upsilon M_l) \gamma_4]} = \left(\frac{(\theta^{A}_{1t})}{(\theta^{NA}_{1t})} \right)^{\frac{a}{1-b}} g_5 \qquad (5.82)$$

If we differentiate θ^{A}_{1t} with respect to V in relation

(5.78), while assuming that B_0 remains constant, we have:

$$\frac{\partial \theta^A_{1t}}{\partial V} = B_0 \frac{\partial \theta^A_{1t}}{\partial A_0} = -g_2\ B_0 \exp(-\beta_1 t) \left(\frac{A_{0-1}}{A_0}\right)^{g_2} (A_0)^{-1} \quad \Leftrightarrow$$

$$\frac{\partial \theta^A_{1t}}{\partial V}\ V \left(\frac{A_0}{A_{0-1}}\right)^{g_2} = -g_2 \exp(-\beta_1 t) \tag{5.83}$$

Similarly, if we differentiate θ^A_{1t} with respect to x in relation (5.78), while assuming that σ^{NA} remains constant, we have:

$$-\frac{\partial \theta^A_{1t}}{\partial x}\ x \left(\frac{\sigma^A}{\sigma^A_{-1}}\right)^{g_2} = -g_2 \exp(-\beta_1 t) \tag{5.84}$$

As a result we have

$$\frac{\partial \theta^A_{1t}}{\partial V}\ V \left(\frac{A_0}{A_{0-1}}\right)^{g_2} = -\frac{\partial \theta^A_{1t}}{\partial x}\ x \left(\frac{\sigma^A}{\sigma^A_{-1}}\right)^{g_2} \tag{5.85}$$

Differentiating now g'_{3t} in (5.81) with respect to V we get:

$$\frac{\partial g'_{3t}}{\partial V} = \frac{2a}{1-b} \left(\frac{(\theta^A_{1t})}{(\theta^{NA}_{1t})}\right)^{\frac{2a}{1-b}} g_3 (\theta^A_{1t})^{-1} \frac{\partial \theta^A_{1t}}{\partial V} \quad \Leftrightarrow$$

$$\frac{\partial g'_{3t}}{\partial V} = \frac{2a}{1-b}\, g'_{3t}\, (\theta^A_{1t})^{-1}\, \frac{\partial \theta^A_{1t}}{\partial V} \quad \Leftrightarrow$$

$$\frac{\partial g'_{3t}}{\partial V}\, V \left(\frac{A_0}{A_{0-1}}\right)^{g_2} = \frac{2a}{1-b}\, g'_{3t}\, (\theta^A_{1t})^{-1}\, \frac{\partial \theta^A_{1t}}{\partial V}\, V \left(\frac{A_0}{A_{0-1}}\right)^{g_2} \quad \Leftrightarrow$$

$$\frac{\partial g'_{3t}}{\partial V}\, V \left(\frac{A_0}{A_{0-1}}\right)^{g_2} = - \frac{\partial g'_{3t}}{\partial x}\, x \left(\frac{\sigma^A}{\sigma^A_{-1}}\right)^{g_2} \tag{5.86}$$

Equivalently, it can be proved that

$$\frac{\partial g'_{5t}}{\partial V}\, V \left(\frac{A_0}{A_{0-1}}\right)^{g_2} = - \frac{\partial g'_{5t}}{\partial x}\, x \left(\frac{\sigma^A}{\sigma^A_{-1}}\right)^{g_2} \tag{5.87}$$

Moreover, we have that:

$$\frac{\partial g'_{3t}}{\partial V} = \frac{2a}{1-b}\, g'_{3t}\, (\theta^A_{1t})^{-1}\, \frac{\partial \theta^A_{1t}}{\partial V} \quad \Leftrightarrow$$

$$\frac{\partial g'_{3t}}{\partial V} = \frac{2a}{1-b}\, \frac{g'_{3t}}{g'_{5t}}\, g'_{5t}\, (\theta^A_{1t})^{-1}\, \frac{\partial \theta^A_{1t}}{\partial V} \quad \Leftrightarrow$$

$$\frac{\partial g'_{3t}}{\partial V} g'_{5t} = 2\ g'_{3t}\ \frac{\partial g'_{5t}}{\partial V} \tag{5.88}$$

Equivalently it can be proved that:

$$\frac{\partial g'_{3t}}{\partial x} g'_{5t} = 2\ g'_{3t}\ \frac{\partial g'_{5t}}{\partial x} \tag{5.89}$$

Differentiating both sides of (5.80) with respect to V while assuming that B_o remains constant, it is easily proved that for $\frac{\partial H_t}{\partial V}$ to be positive it is required that

$$2\gamma_4\left(g'_{3t}(x)^{-\gamma_5}(V)^{\gamma_4} - g'_{5t}\right) + V\left(\frac{\partial g'_{3t}}{\partial V}(x)^{-\gamma_5}(V)^{\gamma_4} - 2\frac{\partial g'_{5t}}{\partial V}\right) +$$

$$+\ (x)^{-2\gamma_5}(V)^{2\gamma_4+1}\left(\frac{\partial g'_{3t}}{\partial V}g'_{5t} - 2\frac{\partial g'_{5t}}{\partial V}\ g'_{3t}\right) > 0 \tag{5.90}$$

Equivalently for $\frac{\partial H_t}{\partial x}$ to be positive (assuming σ^{NA} remains constant) it is required that

$$-2\gamma_5\left(g'_{3t}(x)^{-\gamma_5}(V)^{\gamma_4} - g'_{5t}\right) + x\left(\frac{\partial g'_{3t}}{\partial x}(x)^{-\gamma_5}(V)^{\gamma_4} - 2\frac{\partial g'_{5t}}{\partial x}\right) +$$

$$+\ (x)^{-2\gamma_5+1}(V)^{2\gamma_4}\left(\frac{\partial g'_{3t}}{\partial x}g'_{5t} - 2\frac{\partial g'_{5t}}{\partial x}g'_{3t}\right) > 0 \tag{5.91}$$

Substituting (5.88) into inequality (5.90) gives:

$$2\gamma_4\left[g'_{3t}(x)^{-\gamma_5}(V)^{\gamma_4} - g'_{5t}\right] + 2V\frac{\partial g'_{5t}}{\partial V}\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \qquad (5.92)$$

Substituting (5.89) into inequality (5.91) gives the following condition for $\frac{\partial H_t}{\partial x}$ to be positive:

$$-2\gamma_5\left[g'_{3t}(x)^{-\gamma_5}(V)^{\gamma_4} - g'_{5t}\right] + 2x\frac{\partial g'_{5t}}{\partial x}\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \quad \Leftrightarrow$$

$$\left(-\gamma_5 g'_{5t} + x\frac{\partial g'_{5t}}{\partial x}\right)\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \quad \Leftrightarrow$$

$$-\left[\left(\frac{b}{1-b} + \frac{a}{1-b}g_2\right)g'_{5t} - x\frac{\partial g'_{5t}}{\partial x}\right]\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \qquad (5.93)$$

Substituting (5.87) into (5.92) gives the following condition for $\frac{\partial H_t}{\partial V}$ to be positive:

$$2\gamma_4\left[g'_{3t}(x)^{-\gamma_5}(V)^{\gamma_4} - g'_{5t}\right] - 2x\frac{\partial g'_{5t}}{\partial x}\left(\frac{\sigma^A A_{0-1}}{\sigma^A_{-1} A_0}\right)^{g_2}\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0$$

$$\left[\gamma_4 g'_{5t} - x\frac{\partial g'_{5t}}{\partial x}\left(\frac{\sigma^A A_{0-1}}{\sigma^A_{-1} A_0}\right)^{g_2}\right]\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \quad \Leftrightarrow$$

$$\left[\left(\frac{1}{1-b} + \frac{a}{1-b}g_2\right)g'_{5t} - x\frac{\partial g'_{5t}}{\partial x}\left(\frac{\sigma^A A_{0-1}}{\sigma^A_{-1} A_0}\right)^{g_2}\right]\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \qquad (5.94)$$

Since we have that:

$$\frac{\partial g'_{5t}}{\partial x} = \frac{a}{1-b}\left(\frac{(\theta^{A}_{1t})}{(\theta^{NA}_{1t})}\right)^{\frac{a}{1-b}} g_5 (\theta^{A}_{1t})^{-1} \frac{\partial \theta^{A}_{1t}}{\partial x} =$$

$$\frac{a}{1-b}\, g_2\, g'_{5t}\, (x)^{-1} \exp(-\beta_1 t) \left(\frac{\sigma^{A}_{-1}}{\sigma^{A}}\right)^{g_2} \qquad (5.95)$$

Substituting (5.95) into inequality (5.94) gives :

$$g'_{5t}\left(\frac{1}{1-b} + \frac{a}{1-b}g_2 - \frac{a}{1-b}g_2 \exp(-\beta_1 t)\left(\frac{A_{0-1}}{A_0}\right)^{g_2}\right)\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0 \Leftrightarrow$$

$$g'_{5t}\left(\frac{1}{1-b} + \frac{a}{1-b}g_2\left(1-\exp(-\beta_1 t)\left(\frac{A_{0-1}}{A_0}\right)^{g_2}\right)\right)\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1\right) > 0$$

Since $\left(1-\exp(-\beta_1 t)\left(\frac{A_{0-1}}{A_0}\right)^{g_2}\right)$ is always positive then this means that $g'_{5t}\left(\frac{1}{1-b} + \frac{a}{1-b}g_2\left(1-\exp(-\beta_1 t)\left(\frac{A_{0-1}}{A_0}\right)^{g_2}\right)\right)$ is always positive.

Consequently, the condition for $\frac{\partial H_t}{\partial V}$ to be positive becomes:

$$\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 > 0 \qquad (5.96)$$

Analogously, if we substitute (5.95) into inequality (5.93) the following is derived:

$$-g'_{5t}\left(\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)\left(\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4}-1\right) > 0$$

If $\left[1-\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right]$ is positive then $g'_{5t}\left(\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is also positive. As a result the condition for $\frac{\partial H_t}{\partial x}$ to be positive is:

$$\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 < 0 \qquad (5.97a)$$

On the other hand if $g'_{5t}\left(\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is negative (a somewhat remote case since for this to hold $\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)$ would have to be considerably larger than one and the convergence process at its intial stages so that t is rather small), then the condition for $\frac{\partial H_t}{\partial x}$ to be positive is:

$$\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 > 0 \qquad (5.97b)$$

Consequently if $\left(\frac{b}{1-b} + \frac{a}{1-b}g_2 - \frac{a}{1-b}g_2 \exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is positive then for $\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 > 0$ $\frac{\partial H_t}{\partial V}$ is positive and $\frac{\partial H_t}{\partial x}$ is negative and for $\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 < 0$ $\frac{\partial H_t}{\partial V}$ is negative and $\frac{\partial H_t}{\partial x}$ is positive. If $\left(\frac{b}{1-b} + \frac{a}{1-b}g_2 - \frac{a}{1-b}g_2 \exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is negative then for $\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 > 0$ both $\frac{\partial H_t}{\partial V}$ and $\frac{\partial H_t}{\partial x}$ are positive and for $\frac{g'_{3t}}{g'_{5t}}(x)^{-\gamma_5}(V)^{\gamma_4} - 1 < 0$ both $\frac{\partial H_t}{\partial V}$ and $\frac{\partial H_t}{\partial x}$ are negative.

Since θ's are group uniform, then π's are also group uniform and equal to:

$$1-\pi^A_t = \sigma^A\left[a\ (\theta^A_{1t})^{\frac{(1-a-b)}{1-b}} + b\right] \qquad (5.98)$$

$$1-\pi^{NA}_t = \sigma^{NA}\left[a\ (\theta^{NA}_{1t})^{\frac{(1-a-b)}{1-b}} + b\right] \qquad (5.99)$$

If (5.99) is divided by (5.98), the following expression for x is derived:

$$x = \frac{1 - \pi_t^{NA}}{1 - \pi_t^{A}} \frac{a(\theta_{1t}^{A})^{\frac{1-a-b}{1-b}} + b}{a(\theta_{1t}^{NA})^{\frac{1-a-b}{1-b}} + b} \tag{5.100}$$

Consequently, the following expression for H_{it} is derived:

$$H_{it} = \frac{1 + \left(\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{-2\gamma_{5i}} (V)^{2\gamma_{4i}} g''_{3it}}{\left[1 + \left(\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{-\gamma_{5i}} (V)^{\gamma_{4i}} g''_{5it}\right]^2} \gamma_{3i} \tag{5.101}$$

where

$$g''_{3it} = \left(\frac{a(\theta_{1t}^{A})^{\frac{1-a-b}{1-b}} + b}{a(\theta_{1t}^{NA})^{\frac{1-a-b}{1-b}} + b}\right)^{-2\gamma_{5i}} g'_{3it}$$

$$g''_{5it} = \left(\frac{a(\theta_{1t}^{A})^{\frac{1-a-b}{1-b}} + b}{a(\theta_{1t}^{NA})^{\frac{1-a-b}{1-b}} + b}\right)^{-\gamma_{5i}} g'_{5it}$$

$$\gamma_{3i} = \frac{\sum^{NA} \exp[(\upsilon M_l) 2\gamma_{4i}]}{\left[\sum^{NA} \exp[(\upsilon M_l)\gamma_{4i}]\right]^2} \tag{5.102}$$

Let us set the following:

$$\gamma''_{1it} = (V)^{2\gamma_{4i}} g''_{3it} = (V)^{2\gamma_{4i}} g'_{3it} \left(\frac{a(\theta^{A}_{1t})^{\frac{1-a-b}{1-b}} + b}{a(\theta^{NA}_{1t})^{\frac{1-a-b}{1-b}} + b} \right)^{-2\gamma_{5i}} \quad (5.103)$$

$$\gamma''_{2it} = (V)^{\gamma_{4i}} g''_{5it} = (V)^{\gamma_{4i}} g'_{5it} \left(\frac{a(\theta^{A}_{1t})^{\frac{1-a-b}{1-b}} + b}{a(\theta^{NA}_{1t})^{\frac{1-a-b}{1-b}} + b} \right)^{-\gamma_{5i}} \quad (5.104)$$

Substituting (5.103) and (5.104) into (5.101) gives the following expression for the Herfindahl index of concentration for industry i:

$$H_{it} = \frac{1+ \left(\dfrac{1-\pi^{NA}_{t}}{1-\pi^{A}_{t}} \right)^{-2\gamma_{5i}} \gamma''_{1it}}{\left[1+ \left(\dfrac{1-\pi^{NA}_{t}}{1-\pi^{A}_{t}} \right)^{-\gamma_{5i}} \gamma''_{2it} \right]^{2}} \gamma_{3i} \quad (5.105)$$

Note that γ''_{1it} and γ''_{2it} do not just jump from time to time as discrete unexpected jumps in innovations increase or decrease V but are continuous functions of time through their dependance on θ^{A}_{1t} and θ^{NA}_{1t}. Therefore in order to derive a regression based on relation (5.105), one should regard these parameters as random with means γ''_{1i} and γ''_{2i} respectively. If one defines $\varepsilon''_{1it}=\gamma''_{1it}-\gamma''_{1i}$, $\varepsilon''_{2it}=\gamma''_{2it}-\gamma''_{2i}$ then the following regression is derived:

$$H_{it} = \frac{1+\left(\dfrac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{-2\gamma_{5i}}\gamma''_{1i}+\left(\dfrac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{-2\gamma_{5i}}\varepsilon''_{1it}}{\left[1+\left(\dfrac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{-\gamma_{5i}}\gamma''_{2i}+\left(\dfrac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{-\gamma_{5i}}\varepsilon''_{2it}\right]^2}\gamma_{3i} \qquad (5.106)$$

In order to obtain estimates for the parameters γ_1'', γ_2'', γ_3 and γ_5 in (5.106) one has to follow the same procedure that was used for obtaining estimates for the parameters of regression (3.30) in section (3.4).

Once these parameters have been estimated then we devide γ''_{1it} by γ''_{2it} and compare this ratio to $\left(\dfrac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{\gamma_{5i}}$. If we get that:

$$\frac{\gamma''_{1it}}{\gamma''_{2it}} = (V)^{\gamma_{4i}}\frac{g'_{3it}}{g'_{5it}}\left(\frac{a(\theta_{1t}^{A})^{\frac{1-a-b}{1-b}}+b}{a(\theta_{1t}^{NA})^{\frac{1-a-b}{1-b}}+b}\right)^{-\gamma_{5i}} > \left(\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{\gamma_{5i}}$$

Then if we use relation (5.100) the above inequality means that

$$(V)^{\gamma_{4i}}\frac{g'_{3it}}{g'_{5it}} > (x)^{\gamma_{5i}}$$

which means that if $\left(\frac{b}{1-b} + \frac{a}{1-b}g_2 - \frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma_{-1}^{A}}{\sigma^{A}}\right)^{g_2}\right)$ is positive then $\frac{\partial H_t}{\partial V}$ is positive and $\frac{\partial H_t}{\partial x}$ is negative, while if

$\left(\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is negative then both $\frac{\partial H_t}{\partial V}$ $\frac{\partial H_t}{\partial x}$ are positive. In the same way if

$$\text{(V)}\quad \gamma_{4i}\frac{g'_{3it}}{g'_{5it}}\left(\frac{a(\theta^A_{1t})^{\frac{1-a-b}{1-b}}+b}{a(\theta^{NA}_{1t})^{\frac{1-a-b}{1-b}}+b}\right)^{-2\gamma_{5i}} < \left(\frac{1-\pi^{NA}_t}{1-\pi^A_t}\right)^{\gamma_{5i}}$$

Then this means that if $\left(\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is positive then $\frac{\partial H_t}{\partial x}$ is positive and $\frac{\partial H_t}{\partial V}$ is negative while if $\left(\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma^A_{-1}}{\sigma^A}\right)^{g_2}\right)$ is negative both $\frac{\partial H_t}{\partial x}$ and $\frac{\partial H_t}{\partial V}$ are negative.

However for being able to interpret the sign of the derivatives $\frac{\partial H_t}{\partial V}$ and $\frac{\partial H_t}{\partial x}$ one has to know whether x is smaller or larger than one. Although the value of $\frac{1-\pi^{NA}_t}{1-\pi^A_t}$ can be calculated (provided that there are sufficient data to do so), the fact that this is found to be equal or larger (smaller) than one at time t does not mean that x will also be equal or larger (smaller) than one. Therefore given a sample of industries for a rather small number of years, one will have to ensure than none of the industries included in this sample has experienced during any of these years a violent jump either in terms of innovative activity or in terms of behaviour. This implies a carefull selection of industries so that the case of an industry where the convergence process in any of the groups of this industry is at its initial stages has been ruled out.

Once this has been done then one can guarantee that for each industry the ratio $\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}$ is a good proxy for x and also that

$\frac{b}{1-b}+\frac{a}{1-b}g_2-\frac{a}{1-b}g_2\exp(-\beta_1 t)\left(\frac{\sigma_{-1}^{A}}{\sigma^{A}}\right)^{g_2}$ is always positive even if $\frac{\sigma_{-1}^{A}}{\sigma^{A}}$ is considerably larger that one since t is going to be rather large and therefore $\exp(-\beta_1 t)$ rather small (closer to zero rather than to one). As a result one can base its conclusions concerning the ambiguity between the two hypotheses on the criteria derived for the later stages of of convergence. Then if $x \geq 1$ the industry level conclusions can be used. If $(V)^{\gamma_{4i}}\frac{g'_{3it}}{g'_{5it}} > \left(\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{\gamma_{5i}}$ then the MPH is rejected $\left(\text{since } \frac{\partial H_t}{\partial x}<0\right)$ and the DEH accepted $\left(\text{since } \frac{\partial H_t}{\partial V}>0\right)$, while when $(V)^{\gamma_{4i}}\frac{g'_{3it}}{g'_{5it}} < \left(\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}\right)^{\gamma_{5i}}$ the DEH is rejected and the MPH accepted. On the other hand, when $\frac{1-\pi_t^{NA}}{1-\pi_t^{A}}<1$, since according to the relation (5.63) when x<1 then r_{1j} has to be less than one as well and since both γ_4 and γ_5 are always positive, this case corresponds to the optimal path firm level conclusions in section 5.2b for the combination a+b<1, x<1 and $r_{1j}<1$. As a result for x<1 and $r_{1j}<1$ the MPH is rejected and the DEH is accepted according to the firm level conclusions.

APPENDIX 1

If the sellers of capital goods are unable to incorporate into the price of gross investment the fact that both its quality as well as how efficiently this is used by a firm is improving, then they will falsely charge a price equal to c (the price of one unit of capital stock in full effciency units) instead of q_t for a unit of investment in physical units. More generally, if because of a lack of precise engineering information, the agents providing the capital input fail to account for the full extent of technical progress or alternatively overestimate the magnitude of the continuous innovation process, then total cost involved in installing one extra unit of capital stock is equal to:

$$\frac{\partial C(I_{jt})}{\partial I} + q'_t = 2\beta_j \exp\left[\left(\frac{y}{a} + L\right)t\right] I^F_{jt} + c\ \exp(\varphi t) \qquad \text{(A1.1)}$$

Note that $\frac{y}{a}+L-\varphi$ is the magnitude of the underestimation (overestimation if the expression is negative) by the capital market of the rate of technical change. Consequently, (5.8) can now be rewritten as:

$$2\beta_j I^F_{jt} + c\ \exp\left[\left(\varphi - \frac{y}{a} + L\right)t\right] =$$

$$= \int_t^{\infty} \exp[-(h+\delta)(s-t)](p\sigma_j A_0)^{-\frac{1}{b-1}} a \left(\frac{w}{b}\right)^{\frac{1}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} ds \qquad \text{(A1.2)}$$

In the case where the sellers of capital goods do not fully account for the rate of technical progress, then as $t \to \infty$, the price of investment in physical units tends to zero, thus implying that in the long run the only cost the firms have to face for using capital is its cost of adjustment. On the other hand, if they overestimated the power of technical progress, in the long run the price of investment would become infinitely large and so the firms would have no incentive to produce since there is no finite price they could charge for their output which would allow them to cover their costs. However, it is rather difficult to envisage a situation where technical information is included in a precise manner in the adjustment costs and imperfectly in the price of capital, especially if adjustment costs are the result of monopsonistic elements in the market for capital goods. On the other hand, this seems more plausible when adjustment costs are the direct result of the installation of the new technology. But if this is true, then as Nickell correctly argues, there is no good reason as to why these costs should be of a convex form.

If there is an underestimation or an overestimation for technical change both in the price of gross investment as well as in the adjustment costs, then the total cost of gross investment in physical units is equal to:

$$\frac{\partial C(I_{jt})}{\partial I} + q'_t = 2\beta_j \exp(\varphi t) I'_{jt} + c \exp(\varphi t) \qquad \text{(A1.3)}$$

As a result, (5.8) becomes:

$$2\beta_j I'_{jt}\ \exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right] + c\ \exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right] =$$

$$= \int_t^{\infty} \exp[-(h+\delta)(s-t)]\,(p\sigma_j A_o)^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}}\, ds \qquad \text{(A1.4)}$$

Taking the time derivative in both sides yields:

$$\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right] 2\beta_j \dot{I}'_{jt} +$$

$$+ \left(\varphi - \frac{y}{a} - L\right)\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right](2\beta_j I'_{jt} + c) =$$

$$\frac{\partial \exp[(h+\delta)t]}{\partial t}\ \frac{\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right](2\beta_j I'_{jt} + c)}{\exp[(h+\delta)t]} +$$

$$\exp[(h+\delta)t]\,(p\sigma_j A_o)^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}}\ \frac{\int_t^{\infty}\exp[(h+\delta)t]\,ds}{\partial s} \Leftrightarrow$$

$$\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right] 2\beta_j \dot{I}'_{jt} = \left(h+\delta+L+\frac{y}{a} - \varphi\right)\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right](2\beta_j I'_{jt} + c)$$

$$- (p\sigma_j A_o)^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}} (J_{jt})^{\frac{1-a-b}{b-1}} \Leftrightarrow$$

$$\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right] 2\beta_j \dot{I}'_{jt} - 2\beta_j I_{jt}'\left(h+\delta+L+\frac{y}{a} - \varphi\right)\exp\left[\left(\varphi - \frac{y}{a} - L\right)t\right]$$

$$-(p\sigma_j A_o)^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_{jt})^{\frac{1-a-b}{b-1}} = \left(h+\delta+L+\frac{y}{a}-\varphi\right)\exp\left[\left(\varphi-\frac{y}{a}-L\right)t\right]c \qquad \text{(A1.5)}$$

In the case of underestimation, as $t\rightarrow\infty$ only the non linear part on the left hand side remains and the rest goes to zero. In other words in the $\lim_{t\to\infty}$ the differential equation degenerates into:

$$(p\sigma_j A_o)^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_{jt})^{\frac{1-a-b}{b-1}} \rightarrow 0 \Leftrightarrow (J_{jt})^{\frac{1-a-b}{b-1}} \rightarrow 0$$

The above means that when 1-a-b<0 the capital stock will be tend to zero, i.e the long run equilibrium level of investment in efficiency units is equal to zero. On the other hand, when 1-a-b>0 then, since the exponent of capital stock is negative, J_j will tend to infinity in the long run. So both posible solutions are trivial.

In the case of overestimation, if one divides both sides of relation (A1.5) with $\exp\left[\left(\varphi-\frac{y}{a}-L\right)t\right]$ then this becomes:

$$2\beta_j \dot{I}'_{jt}-2\beta_j I_{jt}'\left(h+\delta+L+\frac{y}{a}-\varphi\right)$$

$$-\exp\left[\left(\frac{y}{a}+L-\varphi\right)t\right](p\sigma_j A_o)^{-\frac{1}{b-1}} a\left(\frac{w}{b}\right)^{\frac{b}{b-1}}(J_{jt})^{\frac{1-a-b}{b-1}} = \left(h+\delta+L+\frac{y}{a}-\varphi\right)c$$

Since $\frac{y}{a}+L-\varphi$ is now negative, the non linear term in the relation above tends to zero as $t\to\infty$ and one gets the following differential equation:

$$2\beta_j \dot{I}'_{jt} - 2\beta_j I'_{jt}\left(h+\delta+L+\frac{y}{a}-\varphi\right) = \left(h+\delta+L+\frac{y}{a}-\varphi\right)c \qquad \text{(A1.6)}$$

Then since $I^F_{jt}=I'_{jt}\exp\left[\left(\frac{y}{a}+L-\varphi\right)t\right]$ if this is combined with (2.19) it implies that:

$$\dot{J}_{jt}=I'_{jt}\exp\left[\left(\frac{y}{a}+L-\varphi\right)t\right]-\delta J_{jt} \Leftrightarrow I'_{jt}=\exp\left[\left(\varphi-\frac{y}{a}-L\right)t\right](\dot{J}_{jt}+\delta J_{jt})$$

Substituting the above into (A1.6),

$$2\beta_j\exp\left[\left(\varphi-\frac{y}{a}-L\right)t\right]\left[\ddot{J}_{jt}+\left(\delta+\varphi-L-\frac{y}{a}\right)\dot{J}_{jt}+\left(\varphi-\frac{y}{a}-L\right)\delta J_{jt}\right]$$

$$-2\beta_j\left(h+\delta+L+\frac{y}{a}-\varphi\right)\exp\left[\left(\varphi-\frac{y}{a}-L\right)t\right](\dot{J}_{jt}+\delta J_{jt}) = \left(h+\delta+L+\frac{y}{a}-\varphi\right)c$$

Dividing both sides of the above equation with the expression $2\beta_j\exp\left[\left(\varphi-\frac{y}{a}-L\right)t\right]$ it becomes:

$$\exp\ddot{J}_{jt}-h\dot{J}_{jt}-\delta(h+\delta)J_{jt}=\left(h+\delta+L+\frac{y}{a}-\varphi\right)\exp\left[\left(\frac{y}{a}+L-\varphi\right)t\right]c \qquad \text{(A1.7)}$$

When $t\to\infty$, relation (A1.7) degenerates into a homogeneous second order differential equation and its characteristic equation has two roots, a positive and a negative. For convergence we keep only the negative root which is $-\beta_1'$. Then the solution of (A1.7) is equal to:

$$J_{jt} = J_{j0}\exp(-\beta_1' t)$$

and obviously in the long run J_{jt} will tend to zero; in other words the overestimation from the sellers side of the rate of technical progress makes the price of investment in the long run infinetely large and as a consequence firms have no profit to operate for. Consequently, in the case of overestimation we again have a trivial solution.

To summarise, it has been demonstrated that the underestimation or overestimation of the price of investment in physical units and/or of the adjustment costs accruing from this investment leads to trivial solutions only for the long run equilibrium level of capital stock.

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