The role of mathematics in crime science

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1 Introduction

In 1940, towards the end of his career, the Cambridge mathematician GH Hardy wrote *A Mathematician's Apology*, which has come to be known as one of the most compelling accounts of what it is to study mathematics. One of the themes of the essay is the reverence afforded to pure mathematics - that which is not motivated by a real-world problem - to the extent that it has been characterised as a defence of mathematics as 'art for art's sake': that no truly worthy mathematics has any application in the real world. Although Hardy denies this, he does nevertheless state that 'nothing I have ever done is of the slightest practical use'; a statement somewhat at odds with the key principles of crime science. If this is indeed representative of the attitudes - and contributions - of mathematicians, the fact that a chapter on this topic should appear in this handbook is perhaps, therefore, surprising.

The views of Hardy, however, belong to a different age, and the role of mathematics in modern science has evolved to such an extent that applied research is now far more common. Indeed, the distinction between the two forms has been shown to be illusory, nowhere more starkly than in Hardy's own field of number theory, which is concerned with the intrinsic properties of numbers and the relationships between them. What was once thought to be the most 'inapplicable' field of all now forms the basis for the encryption systems by which all online transactions are protected: in many respects the most significant security measure in the modern world.

While it is relatively intuitive that mathematics should play a role in technological development and the study of physical systems, however, its relevance to social phenomena such as crime is much less obvious. Indeed social systems have traditionally been considered to be simply too complex to be suitable for mathematical treatment. The phenomena in question involve the actions of, and interactions between, human individuals, both of which are far more complex than the laws which govern physical systems. In recent years, however, a number of techniques and approaches have been developed in order to deal with the challenges posed by such complexity. These provide a foothold from which mathematical progress can be made, and have inspired a surge of interest in the application of mathematics to issues which were previously thought intractable. Among these is crime, the modelling and analysis of which has attracted substantial interest within the mathematical community in recent years.

Mathematics has a role to play across a number of aspects of crime science. By nature, it is a field that is concerned with formalisation and rigour, and it therefore provides a means by which concepts and relationships can be expressed in concrete terms. At its most basic, this may simply facilitate other work, for example by providing a framework for the analysis of social networks. The real power of the approach, however, lies in modelling: the encoding of behavioural mechanisms in simplified mathematical terms. By studying the properties and behaviour of such models, insight can be gained into the situations they represent: the range of possible outcomes, and the consequences of changes, for example. Given that many key concepts of crime science - intervention, evaluation and prediction - can be framed in such terms, the relevance of mathematical treatment begins to become much more apparent. Crucially, the approach also brings with it a uniquely logical and rigorous perspective, thereby complementing the strengths of other contributory disciplines.

The purpose of this chapter is to outline the aspects of mathematics which have greatest potential to be of value within a crime science approach. It will begin by giving an overview of the approaches and techniques which are likely to be of relevance, with particular emphasis on complexity science approaches. It will then give a number of examples of topics on which work has already been carried out, giving a brief review of past research and discussing its relevance to crime science. The chapter will conclude with a discussion of the prospects for the continued integration of mathematics within the crime science agenda.

2 What is mathematics?

So far in this chapter, the term 'mathematics' has been used in a general sense, and it seems sensible to begin by clarifying in what sense it will be used here. In truth, the term encompasses such a diverse range of approaches and techniques that it is difficult to define concisely, and attempts to do so - 'the study of quantity' or 'the study of patterns' - are so broad as to be uninformative. Maths is both a way of thinking and an array of techniques, many of which bear little relation to each other; the challenge here is to identify which of these can offer a meaningful contribution to crime science.

Some types of mathematics, of course, already arise frequently in the study of crime, and require no introduction: almost all research with a quantitative element will include some statistical analysis, for example. However, the role played by statistics in such work is primarily functional, in the sense that it provides tools which support other analytical approaches. Rarely are statistical methods the primary focus of criminological research, and their application to crime data is no different to that in many other fields. Although the techniques are undoubtedly sophisticated, the approach itself does not represent a singularly mathematical perspective.

Where mathematics does have the potential to make a distinctive contribution is through its ability to encode functional relationships. The crucial term here is 'functional': rather than simply expressing the numerical association between quantities, the purpose of such approaches is to describe the underlying mechanisms by which quantities are related. Rather than asserting that harsher sentences are associated with lower crime rates, for example, such a representation might describe the effect on the cost-benefit calculations of individual offenders, and express the result of this in the aggregated population. The duality between real-world processes and their representations means that analysis can be used to examine the true workings of the system, taking into account all instrumental relationships. This process can be summarised as mathematical modelling.

The basic premise of mathematical modelling is to produce a simplified representation of a system in quantitative terms. The process can be considered as a sequence of steps:

1) Observe some situation or phenomenon

2) Propose a hypothetical generative mechanism

3) Express this mechanism in mathematical terms

4) Verify that the behaviour of the model conforms to empirical observations

5) Investigate the properties of the model in order to gain more general insight

It is relatively easy to see how this framework can be applied in crime science. Certainly there is no shortage of interesting phenomena (many of which are quantified statistically) and behavioural hypotheses which seek to explain them. The main challenges therefore lie in 3), 4) and 5), and concern the translation between these real-world concepts and their mathematical representation. Each of these steps, however, has the potential to be of value in understanding criminal phenomena.

Stage 3) represents the key bridge between sociological and mathematical terminology. Because of the formality of mathematical language, model specification requires the researcher to express concepts (some of which may be qualitative in nature) in precise terms. This has a clarifying effect because it requires concepts to be concretely defined and relationships fully enumerated: any vagueness or inconsistency of theory must be eliminated. This is clearly well-aligned with the need for transparency and precision in crime science, and it is also notable that this aspect comes as a by-product of the modelling process: no mathematical analysis is required, and it is simply an exercise in specification.

The model built in stage 3) is a means of encoding a hypothesis so that its consequences can be explored quantitatively; in this sense, it represents a rigorous (and perhaps complex) thought experiment. The aim of stage 4) is to examine how well these consequences match with reality, in order to assess the validity of the model. This represents a test of the underlying hypotheses: if the model fails to reproduce real-world behaviour, the proposed mechanism cannot be adequate. If, on the other hand, the model conforms to empirical observation, the assumed process can be considered to be a feasible candidate explanation. The notions of 'necessary' and 'sufficient' are important here (and, indeed, throughout mathematics). 'Sufficient' indicates that a model (or feature of one) produces the correct behaviour, but does not rule out other (perhaps simpler) explanations. 'Necessary', on the other hand, denotes a feature which must be present in order to generate a behaviour. Establishing which aspects of theory are essential, and which are superfluous, is clearly of value in the context of crime science.

Stages 3) and 4) are not necessarily unique to mathematical modelling, and similar principles are present in other approaches, such as agent-based modelling. Where a mathematical approach is truly distinctive, however, is in the level of analysis which can be performed on models. While the workings of other models can be opaque and difficult to unpick, the use of generalised mathematical methods means that sophisticated analytical methods can be applied. This means that deep insight can be drawn into the behaviour of such models: their response to changes in parameters, their evolution over time, the range of possible outputs and the circumstances under which they arise. Despite the mathematical language, in many cases these issues correspond to real-world questions. Examples of these include:

- How would behaviour differ in an alternative setting (*e.g.* a different country)?
- Are there adverse phenomena (*e.g.* crime outbreaks) which have not been observed but could arise?
- How will activity change over time?
- What would be the effect of an intervention (*e.g.* a design change or prevention strategy)?

In all cases, the crucial point is that the findings are quantitative and based on rigorous representation of the underlying system.

Mathematical modelling is not, of course, without shortcomings. The approach is based on simplification and, while much of its analytical power is derived from this, it also represents a weakness. Models of this type will, by definition, omit some effects and fail to capture the nuance of others, and this renders them an easy target for criticism. Questions of the form 'why is effect X not included?' will be familiar to any modeller, and are frequently cited when attempting to discredit any particular model.

Such criticisms, however, miss the point of the exercise; models are not intended to provide comprehensive descriptions. The key principle is that, by sacrificing exact correspondence with reality, other advantages can be gained, and these concern both the theoretical and practical objectives of modelling. The theoretical aspect is summarised well by the dictum "the model is finished not when there is nothing left to add, but when there is nothing left to take away". In crime science, the ability to reduce a phenomenon to its essential components is part of the problem-solving approach, and mathematics can prove valuable in that context. In practical terms, the argument is simply that the value of a model is determined exclusively by its utility: the proof is in the pudding, and if a model is of use in preventing crime, any omissions or simplifications are immaterial.

3 Mathematical approaches to crime

As stated previously, crime is not an issue which has traditionally been considered suitable for mathematical modelling. This view is largely due to the number and intricacy of behaviours involved: crime is simply too complex an issue to be captured in a few equations. In recent years, however, there has been a rapid growth in research concerned with precisely this kind of large real-world system. While these may never be as amenable to analysis as classical topics, the application of bespoke techniques does allow progress to be made.

There is no universal definition of a 'complex system', primarily because the scope of the field is so broad, and it is most often defined either by example of by specifying the general properies which characterise it. Exactly which of these is emphasised depends on context, but three properties are common to all working definitions (see Newman, 2011):

1) The system is composed of a large number of components
2) The behaviour and interaction of these components is non-trivial
3) The collective behaviour of the system does not follow straight-forwardly from the micro-level mechanisms

It is the third of these properties - the presence of so-called 'emergent' behaviour - which truly distinguishes complex systems. Such phenomena are typically non-linear, and span a wide range, from pattern formation to chaotic behaviour. The common attribute is that they are 'irreducible', in the sense that they are only observed at the macro level.

That criminal phenomena can be considered in this way is relatively easy to see. Properties 1) and 2) are almost immediate since crime occurs in large societies and involves human activities. With regard to property 3), a number of criminological phenomena can be considered to be emergent. An appealing example is the classic 'broken windows' hypothesis (Wilson & Kelling, 1982): this specifies an individual-level effect (the change in an offender's perception as a result of environmental cues) which ultimately is manifested as an area-level problem (the presence of endemic, and perhaps more egregious, criminality). For crime more generally, the presence of unexpected effects and the difficulty of exercising control are characteristic of the non-linearities and feedback loops associated with complex social systems (Castellano *et al.*, 2009).

Complex systems arise in many domains, and examples can be found in ecology (Levin, 1998), finance (Mantegna & Stanley, 2000) and urban studies (Batty, 2007), to name only three. The focus here is on crime, however, and it therefore makes most sense to outline the various techniques that are most commonly applied. The field is inherently interdisciplinary and, whilst fundamentally mathematical, incorporates approaches from physics, economics and computer science. A selection of those which most readily apply to crime will now be described.

3.1 Dynamical systems

Dynamical systems theory is concerned with describing mathematically the evolution of quantities, or entities, over time. It is the aspect of complexity science which resembles classical applied mathematics most closely, and many of the models are closely related to those seen in other fields, such as fluid dynamics. The central objective is to describe the change over time of defined variables (f and g, say), which represent some properties of the system.

When the variables in question are continuous-valued (*i.e.*, can take a smooth range of values) the description most often comes in the form of differential equations. Such equations specify the rate of change of a variable (denoted $\frac{df}{dt}$, for example) as a function of itself and other variables, the form of which reflects the hypothesised mechanism. It is these rates of change which determine the behaviour over time, and typically one equation will be defined for each variable of interest. Taken together, these constitute a system of equations, and these encode the model.

When the rate of change of one variable is a function of the value of another variable (*e.g.* when $\frac{df}{dt}$ is a function of g), this represents an interaction, and the differential equations are said to be 'coupled'. It is these couplings which generate interesting behaviours, and it is the intricate patterns of coupling which generate much of the distinctive behaviour of complex systems. Equation-based models, however, are some of the most mathematically tractable, as the field is well-developed and includes a range of very powerful techniques (Arrowsmith & Place, 1992; Strogatz, 1994).

Mathematical analysis of dynamical systems can be used to characterise the evolution of a system over time and investigate issues such as pattern formation and stability. Stability refers to the effect on the system of small perturbations: a stable solution is one which is robust, whereas unstable solutions are liable to undergo dramatic qualitative changes. These are issues with important real-world interpretation: instability, in whatever form it takes, is generally an undesirable characteristic of a system, and is indicative of dramatic potential consequences.

Many criminal issues have a dynamic component - aggregate crime rates, levels of incarceration, spatial patterns, for example - and this type of modelling is a natural choice in such cases. The dynamic variables can be either macro-level (*e.g.* national crime rate) or micro- (*e.g.* the number of offenders at a certain location) and non-criminal quantities may also be included: a model for metal theft might incorporate commodity prices, for example. The challenge is to derive equations from hypothesised behaviours, but once this is done they can be used to investigate behaviour (or make forecasts) in great detail.

3.2 Networks

One of the topics which is most closely associated with complexity science is the study of networks. The term 'network' refers to a collection of discrete entities (called *nodes* or *vertices*) and connections between them (known as *edges* or *links*). Much of the terminology is derived from the mathematical field of 'graph theory', and networks can be thought of as graphs arising in the real world.

Social networks are one of the most well-known types of network, and provide a good illustrative example. In such networks, nodes represent actors (*i.e.* individuals) and links represent social relationships between them (such as friendship, communication or physical contact). Many other types of network are also studied, however: examples can be found in telecommunications (*e.g.* the internet), transport (*e.g.* air travel) and ecology (*e.g.* food webs). Network representations are versatile and can represent many different types of relationship: *directed* networks allow links to have directionality, and *weighted* networks are those in which some value can be associated with links (such as frequency of contact).

The study of networks involves both the analysis of real-world networks and their use in theoretical models. Research of the first type is concerned with characteristing the structure of networks, in particular by measuring the 'centrality' of nodes. Many different notions of centrality exist - a simple one is the number of links a node has - and these emphasise different aspects of structure. Research has shown that many real-world networks exhibit common statistical properties (Albert & Barabási, 2002), and a wide variety of examples has been explored (Costa *et al.*, 2011). This kind of analysis can be used to assess the importance of nodes, or the function they play in the network. A related topic is that of community detection, which seeks to identify meaningful groups of nodes.

Networks arise in a number of criminal contexts. Social relationships are a fundamental aspects of several criminological topics: while organised offending is perhaps the most common, network-based approaches can also be used to investigate relationships between victims, for example. As well as providing a convenient structure for data, analysis can offer insight into the roles played by various actors (*e.g.* command or brokerage) and reveal organisational signatures. Other criminal issues also involve networks. Transport networks are a key determinant of built environment and may therefore exert a significant influence on spatial phenomena. In a more general security context, network concerns are also crucial in understanding the resilience of infrastructure (*e.g.* power networks) and must be considered in the profiling of risk.

3.3 Game theory

Game theory is a field which has primarily been developed within economics and which concerns decision-making. The term 'game' is slightly misleading: it simply refers to a scenario in which two or more actors (the *players*) must decide between a number of possible actions, after which they will receive a payoff. The crucial point is that the payoff to a given player is dependent upon the actions of the other player(s): Player A's payoff after taking action X depends on which action was taken by Player B. Game theory is therefore the study of decision-making and strategy in situations where it is necessary to account for the behaviour of others. The object of the analysis is typically concerned with finding *equilibria*: strategies for each player which are mutually optimal and therefore represent rational behaviour.

Games can vary in a number of respects - the number of players, the number of stages, whether actions are taken simultaneously - and can be very complex. A number of simple games (2 players, 2 actions each), however, are very well-known, such as the Prisoner's Dilemma. Although games such as these may seem abstract or whimsical, in fact they capture the essence of much more general concepts (*e.g.* co-operation and defection) and can be applied to a range of real-world situations.

Game theory can be used to study adaptation and evolution. In this context, games are played repeatedly between actors in a population, and actors are able to update their strategies in response to the outcome of each round. Over time, actors may gravitate towards certain strategies, thereby suggesting that they are dominant in some sense. This can be applied to the study of social norms, and a famous example concerns the emergence of co-operation in a repeated Prisoner's Dilemma (Axelrod, 1984).

Game theory applies naturally to criminal phenomena because rational behaviour is intrinsic to much criminological theory. Indeed the most fundamental decision of all - whether to commit a crime - can be framed in this way: crime offers a potential reward, but the actions of others determine whether it will instead result in capture and punishment. Many other issues could also be considered as games, such as willingness to intervene and severity of punishment. Resource allocation by law enforcement can also be framed as a game, and this can be used to explore preventative or defensive strategies.

4 Areas of application

The principles and techniques outlined in this chapter are versatile, in the sense that they can be applied to a wide range of topics; indeed, almost anything that can be quantified can, in principle, be modelled. Within crime science, a number of topics are particularly amenable to mathematical treatment, and research can be partitioned into a number of key areas. In this section, several of these topics will be reviewed: the motivation for, and value of, the mathematical approach will be described, and examples of research will be given.

4.1 Spatio-temporal patterns

The mathematical interest in spatio-temporal patterns of crime is an example of a field which has been motivated directly by empirical findings within geographical criminology. In particular, the observation of dynamic patterns within crime data - most notably the occurrence of hotspots and near-repeat victimisation - provides an ideal target for modelling: a stylised phenomenon for which analogies can be drawn with other physical processes. In this case, the apparent spreading of crime invites comparison with classical physical diffusion processes and related models of pattern formation.

In the case of urban crime, a number of behavioural hypotheses (including environmental theories) provide possible explanations for these patterns. The modelling task is therefore straightforward: to encode these hypotheses mathematically, and examine whether they are capable of generating patterns similar to those observed empirically. If a feasible model can be found, this is of potential value in a number of respects. At a basic level, it shows that the hypothesised behaviours do indeed provide a plausible explanation for real-world patterns. Furthermore, though, analysis of such a model can be used to investigate the conditions under which patterns form, and the way that they evolve. The latter of these is perhaps of greatest significance: if the dynamics of patterns can be understood, this raises the prospect that they can be extrapolated into the future. This amounts to spatial prediction, which is of clear potential value for prevention.

A prototypical example of this approach is the model of burglary introduced by Short *et al.* (2008).

In its most basic form, this model is specified in terms of the behaviour of individual actors: burglars move between houses arranged on a grid, and are attracted towards properties of higher 'attractiveness'. When attractiveness is sufficient, they carry out offences, each of which triggers a temporary elevation in the attractiveness of the victimised property (representing a 'boost' effect; see Pease, 1998). Crucially, the authors are then able to express this model (essentially an agent-based one) in the form of differential equations. By analysing these mathematically, it is possible to establish the conditions under which hotspots form, and describe explicitly their morphology and evolution. This goes beyond what can be deduced form agent-based simulations, in which such phenomena can only be observed qualitatively.

The Short model has been adapted and extended in a number of ways, most notably through the inclusion of police activity (Pitcher, 2010; Jones *et al.*, 2010). Motivated by the apparent role of street networks in crime patterning (see Davies & Johnson, 2014), a similar network-based model has also been proposed (Davies & Bishop, 2013). Other approaches have also been employed, with different theoretical emphasis: Nadal *et al.* (2010), for example, also include a notion of 'social tension' in their model, while the statistical approach of Mohler *et al.* (2011) models crimes by analogy with earthquakes. The real-world value of such modelling is exemplified by the fact that the latter of these forms the basis for a 'predictive policing' system in active use (Mohler *et al.*, 2015).

Although the majority of published models are concerned with high-volume urban crime, several other phenomena have also been considered. Recent high-profile instances of rioting have stimulated interest in that area, with models proposed for the cases of London (Davies *et al.*, 2013) and Paris (Berestycki *et al.*, 2015). In both cases, the models seek to combine behavioural principles (target selection, contagious behaviour) with the effect of spatially-varying demographic factors. Similar principles of target selection have also been employed in the context of maritime piracy (Marchione *et al.*, 2014), with a view to informing defensive strategies.

4.2 Gang territoriality and interaction

An interesting modelling challenge is posed in situations in which crime results from the interactions between two criminal populations, most naturally exemplified by inter-gang rivalry and violence. The prominent role of territoriality means that this has a spatial component, and significant parallels can be drawn with spatial ecology: gangs are analogous to species, competing for resources and territory. The situation can also be cast as a 'predator-prey' system, in which predation may correspond to efforts by law enforcement to reduce gang size. Understanding the dynamics of these populations can be of value in anticipating territorial changes, predicting outbreaks of violence, and suggesting measures by which antagonistic activity can be minimised.

An early attempt to employ an ecological approach to examine gang dynamics was the work of Crane *et al.* (2000), which framed the growth of gangs as a competitive process between existing gangs and social control mechanisms. The model displays behaviour consistent with real-world observations, including abrupt shifts in gang size, and finds conditions under which social control is most effective. More recent approaches have used a similar framework, but instead applied to the hostile relationships between gangs. Brantingham *et al.* (2012) examined the formation of territorial boundaries between pairs of gangs, taking into account the location of their central 'set spaces', and this was extended to the case of more than two gangs by Smith *et al.* (2012).

The ecological approach is not, however, the only one to have been applied to gang dynamics. Using methods from statistical physics, for example, Barbaro *et al.* (2013) model the marking of territory using graffiti, and establish conditions under which distinct divisions form. Such work is of potential practical value because it is at these interfaces where flashpoints of violence are likely to occur. On a related theme, the point process formulation has also been applied, particularly as a model of retaliation (Egesdal *et al.*, 2010; Short *et al.*, 2014). This approach has shown potential as a means of predicting missing data, particularly with respect to the identification of unknown participants in violent crimes (Stomakhin *et al.*, 2011).

4.3 Criminal networks

Many criminal phenomena involve interactions between actors, and networks provide a means by which these can be represented and studied. The interactions in question can take many forms - antipathetic relationships such as gang rivalries have been studied in this way (van Gennip *et al.*, 2013) - but most examples involve cooperative relationships, as in organised crime or terrorism. In such contexts, networks can be derived from various forms of data - communications, for example, or co-offending - and their structures examined. Various questions can then be addressed using social network analysis (Carrington, 2011): the extent to which a structure is hierarchical, the identification of roles and the measurement of influence. There are numerous examples of topics which have been investigated in this way, including organised crime (Ferrara *et al.*, 2014), terrorism (Krebs, 2002; Medina, 2014; Gill *et al.*, 2014) and trafficking (Mancuso, 2014).

The analysis of organised criminal networks is complicated significantly by their covert nature: by definition, such enterprises seek to conceal their association. How this can be addressed is a prominent theme of research in this area, with a number of approaches suggested for how probabilistic inferences about network structure can be made in clandestine settings (Gill & Freeman, 2013; Gerdes, 2014). The Dynamic Network Analysis approach introduced by (Carley, 2003) seeks to combine data from numerous sources with predictive network analysis techniques, and has been applied in the context of terrorism (Carley, 2006).

One of the most immediate motivations for studying criminal networks is the possibility of establishing how they might best be disrupted. The attack tolerance of networks has been well-studied (Albert *et al.*, 2000), and a number of general strategies exist, many of which focus on the elimination (via arrest, for example) of highly central or influential actors. However, recent research by Duijn *et al.* (2014) has suggests that such strategies are typically unsuccessful for criminal networks, which appear to display high resilience and the ability to re-organise after attack.

4.4 Resilience and inspection

While resilience of criminal networks is unwelcome, there are many other security contexts in which resilience is desirable. For potential targets of crime, for example, and especially those which are large inter-connected systems, the ability to withstand attack is something which a crime science approach would seek to foster. Since many such systems (*e.g.* infrastructure) can be represented as networks, mathematics can assist in this respect by examining their response to various types of attack. This can be used to identify weak points and to understand critical transitions: the points at which attacks are sufficient to cause catastrophic failure.

The resilience of networks to attack has been studied for some time (Latora & Marchiori, 2005), with critical phenomena investigated for energy networks in particular. More recent work has considered inter-dependencies between networks, such as those between the internet and energy distribution networks, which are believed to be responsible for previous blackouts (Buldyrev *et al.*, 2010). The potential for catastrophic failure in such systems is striking (Gao *et al.*, 2011), suggesting that such inter-dependencies should be taken into account when assessing the risks to such facilities. Unfortunately, little work has explicitly considered the security implications of such vulnerability; the work of Carvalho *et al.* (2014), considering the response of energy networks to real-world security risks, such as conflicts, is a notable exception.

One respect in which vulnerability and protection have been studied in depth is in the assignment of security resources to vulnerable locations. Such situations have been modelled as adversarial games between attackers (*e.g.* terrorists) and defenders (*e.g.* security forces). In these games, the attackers have a choice of targets, each of which has different reward and risk, and the task of the defender is to assign resources in a way which minimises this threat (Tambe, 2012). Crucially, however, both sides can take into account the decisions of the other (*e.g.* terrorists can observe how well-defended each target is) when deciding on a course of action, and this makes the formulation as a game particularly apt. By solving these models, the optimal allocation of defensive assets can be calculated, which will typically involve a mixture of spatial assignments. Such strategies have been employed in real-world settings, including inspection at airports (Pita *et al.*, 2008) and on transportation systems (Zhang *et al.*, 2013).

4.5 Economic models

Inspection, in the wider sense of identifying wrongdoing, is one of the topics examined in the economic modelling of crime, which is typically concerned with studying the level of criminality - as a social phenomenon - in the population as a whole. Although this is perhaps not as well-aligned with the crime science approach as the other topics discussed here, such is the volume of work that it would be remiss not to discuss it at all. A more comprehensive review, however, can be found in the article by Gordon (2010).

The general approach is well-illustrated by the seminal work of Becker (1968) which initiated the field, and which is discussed at greater length in the chapter by Manning elsewhere in this handbook. Crime is postulated to be a rational choice, in which the economic outcomes of crime are captured in a 'social loss function'. This incorporates the volume of, and damage accruing from, offending, which is then balanced against the probability, severity and cost of punishment. Minimising this function suggests how many resources should be devoted to detection and punishment in order to achieve a situation which is economically optimal; in essence, how much crime should be 'tolerated' in society. This exemplifies the macro-level viewpoint of such models: effects are considered at a societal level. Various effects have been studied in this way: the propensity to offend (Nadal *et al.*, 2010), the effect of punishment (Demougin & Schwager, 2003; Gordon *et al.*, 2009a), and the role of inequality (Ehrlich, 1973; Deutsch *et al.*, 1992; Bourguignon *et al.*, 2003).

Later models have sought to account for the effect of social interactions on crime: Glaeser *et al.* (1996) suggested that this may explain variations in crime rates between US cities, and Gordon *et al.* (2009b) presented a general model for choices under social influence. The effect of punishment on criminal organisation has also been studied from an economic perspective (Garoupa, 2007). The role of interaction naturally leads to analogies with epidemiology, and in particular the notion that criminality is a contagious phenomenon (Crane, 1991). Compartmental models, in which the population can be partitioned into a number of states (*e.g.* 'susceptible' and 'criminal') and individuals transition between these, are common in the context and have also been proposed for crime (Campbell & Ormerod, 1997).

A more recent trend also considers compartmental models, but from a game-theoretic perspective. The model proposed by (Short *et al.*, 2010), for example, includes four types of individual - 'informants', 'villains', 'paladins' and 'apathetics' - each of which have different propensities to commit crime and to serve as witnesses in criminal investigations. The game proceeds iteratively, with individual interactions (potential crimes) resulting in payoffs to each individual involved. After each round, individuals can transition between states, imitating the strategies of more successful counterparts, and ultimately the purpose of the model is to track the number of individuals in each state. The primary finding of the research is that informants are essential to the formation of a crime-free society, but the model is versatile: by manipulating the payoff structure, it is possible to investigate many strategies and interventions (Short *et al.*, 2013). Indeed, this is indicative of the value of economic models: they facilitate the analysis of macro-scale policy changes on criminality as a whole.

5 Summary and outlook

One of the central objectives of crime science is to bring to the study of crime the kind of rigour and formalism that more traditionally characterises the physical sciences. It is natural, therefore, that mathematics, which provides much of the language and apparatus for these fields, should play a role in crime science. This chapter has sought to outline the particular ways in which mathematical approaches can contribute to crime science, by identifying those techniques which are most likely to be of use and by reviewing promising applications.

What is clear from the chapter as a whole is that the range of techniques relevant to crime science extends far beyond the classical disciplines of mathematics. It is no coincidence that interest in the modelling of crime has grown in line with a broadening of mathematics: several of the techniques described, such as network science, are relatively recent developments and are closely linked with increases in computational power and data availability. The prospect of further advances in these areas, and computational social science more generally, suggests that the field has a bright future: the scope and sophistication of mathematical approaches to crime science are both likely to increase.

Research to date has addressed a diverse array of criminal problems, ranging from the spatio-temporal patterning of crime to the size and structure of criminal groups. In each case, it is clear that mathematics offers a distinctive perspective: it is hard to imagine how predictive policing, for example, could properly be done without recourse to quantitative modelling. In most of the other examples cited, the absence of mathematics would not necessarily nullify the underlying principles, but would render them far more difficult to formalise and test.

Of course, offering formalisation is not an end in itself, and it makes sense to return now to the core issue of this chapter: the extent to which these approaches contribute to the crime science agenda. Although the examples cited here demonstrate the clear potential of the field, there is undoubtedly still work to be done to align it more fully with the aims of crime science.

Certainly mathematical modelling provides an ideal framework for the application of the scientific method to criminal phenomena: as outlined, it is the natural language for the encoding, analysis and possible falsification of hypotheses. Nevertheless, the extent to which this is realised in practice - particularly with respect to the comparison of models with real world data - remains open to question. Modelling has shown great promise in replicating various general phenomena from criminology (*e.g.* hotspot formation), and it is tempting to view these as having strengthened the case for the underlying behavioural hypotheses that they encode. However, the fact that many of these approaches are formulated in extremely simplified settings - idealised 2-dimensional domains, for example - limits the extent to which they can be believed to represent the real world.

Of perhaps even greater concern is the difficulty in differentiating between candidate models. In several contexts, a number of distinct approaches have been shown to successfully replicate a desired behaviour - hotspot formation is again a prominent example - and the question naturally arises of which is correct. This is, in many respects, an impossible judgement to make: when the target behaviour is as generic as 'areas of elevated crime risk', any model which produces such areas cannot be viewed as anything other than successful. In seeking to replicate such generic behaviours, therefore, the bar is perhaps being set too low, and models can too easily be viewed as viable.

This issue, however, is as much a challenge for crime science more generally as it is for mathematical modelling. To continue the previous analogy, the bar for such mathematical models can only be set higher if such a bar exists; in many cases, however, the extent to which phenomena have been characterised in empirical work remains relatively vague, to the extent that they are not sufficiently well-defined to discriminate between model outputs. Only when notions such as 'hotspot' can be defined at the level of precision that would be expected in the study of physical systems can techniques from those fields reasonably be expected to contribute to an equivalent extent. Stylised facts can only go so far in providing a target for modelling, and the increasing sophistication of empirical methods will, in time, address this issue.

Of course, a further concern for mathematical approaches relates to another fundamental pillar of crime science: their practical applicability. Success in replicating phenomena of interest will only be meaningful outside the academic context if it can be translated into real-world interventions - and in this respect there is much research still to do. The field of predictive policing is a clear example of the use of models to guide real-world activity, and has shown promising results with respect to crime prevention outcomes. Nevertheless, research concerning such real-world implementations is at a very early stage, and questions remain about the viability of such activity under many current policing models. For several of the other approaches outlined in this chapter, there is even more work to be done to explore the real-world applicability of mathematical approaches.

Looking to the immediate future, it seems clear that real-world applicability should be brought to the forefront of efforts to apply mathematics in the context of crime science. The approaches detailed in this brief overview are well-suited to the field, and have shown clear promise, but will only be of value to crime science if the gap to real-world applicability can be bridged. Although this may require a shift in focus away from deeper mathematical analysis, it will ensure that the relevance of mathematics to the study of crime continues to grow.

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