Sequential competitions with a middle-mover advantage

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Abstract

This paper investigates the incentives that drive advantageous positions in sequential competitions. Sequential competitions have been shown to have either a first- or last-mover advantage. In contrast, this paper illustrates a general sequential-move competition where the first- and last-moving agents are the least profitable while the middle-moving agent is guaranteed to earn the highest possible payoff. This result provide for a new intuition about the underlying incentives in a sequential decision structure which are tested using a multiple-round laboratory experiment. Experimental data aggregated across all rounds support the prediction of a first- and last-mover disadvantage along with a middle-mover advantage. Furthermore, the data suggest that subjects learn as they gain experience with this competition. In this manner, a sequential decision structure with inexperienced agents will benefit the first- and middle-moving agents, whereas the same decision structure with experienced agents will only benefit the middle-moving agents.

Keywords: middle-mover advantage; first-mover disadvantage; last-mover disadvantage; sequential competition; all-pay contest; voting; information cascade; Quantal Response Equilibrium; laboratory experiment

JEL Classification: C72; D71; D72; L25

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1 Introduction

This paper studies an environment where competitors make publicly observable choices according to an exogenously determined sequence. Previous research suggests that sequential competitions will endow an advantage to certain positions in the decision sequence. In most sequential settings, the advantageous position is either the first-mover or the last-mover.[1] The intuition behind these results is relatively straightforward. In some settings, first-movers can capitalize on a scarce resource, capture a desirable location, narrow down the competition, or develop a patented product, whereas in other scenarios, last-movers can learn about unknown market conditions or free-ride off of previous investments. However, a natural question is whether all sequential competitions can be classified as fair (where no inherent advantageous position exists), first-mover advantageous, or last-mover advantageous.

The main objective of this paper is to demonstrate a sequential competition which exhibits both a first- and last-mover *dis*advantage. In this manner, sections 3 and 4 produce theoretical and experimental results which suggest a "middle-mover" advantage. The model presents an exogenously determined sequential competition where agents choose perfectly observable actions with the motivation of maximizing their symmetric and publicly-known payoff function (section 3.1). The Limiting Logit Equilibrium (McKelvey & Palfrey 1995) of this highly symmetric model predicts that agents will earn different payoffs based solely on their position in the decision sequence (Results 1 and 2). In contrast with the previous literature, this paper's equilibrium predicts that first- and last-movers will earn the lowest expected payoffs (first- and last-mover disadvantage), while the middle-mover is predicted to earn the maximum possible payoff (middle-mover advantage). Intuitively, the equilibrium suggests that every agent's choice is influenced by two combating incentives. The "upstream" and "downstream" incentives encourage agents to make choices that adversely affect early-moving and late-moving agents, respectively (section 3.4).

Section 4.1 describes a laboratory experiment used to test the prediction of a middle-mover advantage. Subjects interact in a sequential voting competition for 20 rounds where, in each round, subjects are randomly assigned a new group and a new position in the decision sequence. Section 4.2 discusses the experiment's main findings which can be summarized with two points. First, the data support the model's prediction of a middle-mover advantage along with a first- and last-mover disadvantage. Second, the repeated context of the experiment

[1] First- or last-mover advantages are predicted in situations with scarce resources (Main 1955; Prescott & Visscher 1977), learning behavior (Spence 1981; Spence 1984), patents (Gilbert & Newbery 1982; Bresnahan 1985), asymmetric costs (Schmalensee 1982; Klemperer 1987), signaling incomplete information (Banerjee 1992; Bikhchandani et al. 1992), free-riding (Ghemawat & Spence 1985; Tellis & Golder 1996), and within evolutionary settings (Poulsen 2007).

unearths a relationship between a subject's experience with the competition and that subject's responsiveness to the two combating incentives. Inexperienced subjects respond primarily to the downstream incentive whereas experienced subjects are more equally responsive to both the upstream and downstream incentives. The relationship between experience and behavior refines the expected outcome in this type of sequential competition. More specifically, a sequential competition with inexperienced agents greatly disadvantages the last-movers whereas the same competition with experienced agents is disadvantageous to both the first- and last-movers. Section 4.4 summarizes all of the experimental results.

2 Literature Review and Motivation

Strategic behavior in sequential competitions has been studied in a variety of contexts. This paper contributes to four strands of literature focusing on firm-choices in a sequential-move market, sequential contests, herding behavior (or information cascades), and sequential voting.

A large literature focuses on identifying profitable positions in a marketplace with sequential firm decision-making. Markets with first-mover advantages are predicted when firms can learn through experience (Spence 1981; Spence 1984), impose R&D patents (Gilbert & Newbery 1982; Bresnahan 1985), accumulate scarce resources (Main 1955; Prescott & Visscher 1977), or enforce switching costs on consumers (Schmalensee 1982; Klemperer 1987). These works are summarized in Lieberman & Montgomery's 1988 award-winning paper entitled "First-mover advantages".[2] Last-mover advantages are less-commonly predicted but are expected in markets where rival firms can free-ride, in one way or another, off of initial investments made by early-moving firms (Ghemawat & Spence 1985; Tellis & Golder 1996). These works are summarized in Lieberman & Montgomery's follow-up 1998 seminal paper entitled "First-mover (dis)advantages". The focus on first-mover advantages *or* disadvantages has garnered thousands of publications. However, to the best of my knowledge, none of these papers have systematically explored an environment where *both* incentives exist, which is the goal of this current paper.

The closest previous work suggesting a middle-mover advantage is Lilien & Yoon's (1990) empirical investigation of the success rates of 112 markets with newly introduced industry-products. On average within each market they find that *"Success is lower for the first and second entrants; higher for third and fourth; and again lower for fifth and sixth, and subsequent entrants"* (pg 578, Result 2). While their finding aligns with *"a common premise of entry timing models in the economic literature"* (pg 570, Proposition 4), Lilien & Yoon's

[2] Bettis (1998) explains the award selection process and outcome.

result lacks a formal theory or substantiating explanation. This current paper aims to provide intuition for empirical works that find a middle-mover advantage.

In contest theory, agents typically make irreversible investments towards one objective in an effort to claim an available prize. Sequential all-pay contests (or auctions) are richly studied because there exists many real-world applications such as political lobbying (Becker 1983), patent provision (Wright 1983), sales (Varian 1980), R&D races (Dasgupta 1986), athletics (Frick 2003), job promotions (Rosen 1986), and wars of attrition (Bulow & Klemperer 1999). Settings have been studied where agents have complete information about each player's valuation of the prize (Hillman & Riley 1989; Baye et al. 1993) as well as settings where agents have private valuations and only know the distribution of agent valuations for a given population (Amann & Leininger 1996; Moldovanu & Sela 2001). Contests may be designed to allocate multiple prizes (Barut & Kovenock 1998; Moldovanu & Sela 2006).[3] The outcome of a sequential all-pay contest will be affected by many components such as the information available, the number of agents, heterogeneous abilities, the number or type of prizes available, participation constraints, and so on. This current paper holds constant these important concerns in order to solely focus on the relationship between an agent's expected payoff and that agent's position in the decision order of a sequential contest. Furthermore, instead of modeling agents that invest different levels of a resource (effort) toward one objective, this paper's model could be viewed as one where agents direct their (exogenously determined) level of a resource towards multiple objectives.[4] From this perspective, this paper's model can be described as a particular sequential all-pay contest with n homogeneous agents competing over $n-1$ identical prizes with complete information.

There also exists a rich literature investigating decision structures with observable and sequential actions that produce information cascades or "herd" behavior. Banerjee (1992) and Bikhchandani et al. (1992) famously model a sequential decision structure where each agent is privately endowed with a signal suggesting which action is optimal. After an agent chooses her action, this action is revealed to all of the agents who choose after her. The revelation of early actions influences later-moving agents to *herd* in the same direction as the earlier agents. Modeling agents with private information is commonly used to elicit herd behavior in sequential decision structures (Wit 1999; Dekel & Piccione 2000; Battaglini 2005). The model presented in this paper imposes a similar dynamic to these papers, in that

[3]This research analyzes how different prize allocations can influence the effort levels of the contestants. Typically, the analysis takes on the perspective of a contest designer who is motivated to maximize the total effort expended by the contestants. For example, if contestants have incomplete information, then allocating the entire prize to a "winner-take-all" structure will usually optimize the contestants' expected total effort (Moldovanu & Sela 2001; Moldovanu & Sela 2006).

[4]Using the model terminology introduced in section 3, each agent could be viewed as investing in four proposals where the proposal with the least amount of investment is selected by the group to be unfunded.

later-moving agents will observe and react to the choices of early-moving agents. However, this paper's model is different from previous work because it does not endow agents with private information, which is necessary to create the inefficiencies that stem from herding or cascading behavior. Results from this model suggest that, without private information, there exists two underlying incentives in an openly sequential procedure; one of which is related to the previous literature's herding incentive (this paper's downstream incentive).

This paper studies a setting where the action made by a group has a disproportionate consequence on a subset of agents. This type of voting or allocation structure is used in events ranging from informal committee selection problems to decisions made by courts of law. As an example of an informal selection problem, consider a committee of researchers allotting funding to research projects. Researchers have preferences over which projects they prefer and, in many cases, researchers within the committee even have their own proposal that they wish to have funded. With a limited budget, how should the committee decide which projects deserve funding? A natural way to proceed is to openly discuss and vote for the merits of each project until an allocation is agreed upon. A more formal example involves the U.S. Supreme Court's process of granting *certiorari*. Of the 10,000 cases submitted to the Supreme Court every year, approximately 80 cases are granted a writ of *certiorari*;[5] thus granting them a full review by the Court. While the exact method of granting *certiorari* is not explicitly defined, many judges have openly verified that the process is carried out in the following manner: *"Inside a closed conference room, the Chief Justice leads the meeting in which the Justices discuss the petitions and vote aloud on which cases they find more significant and deserving of deliberation. Voting begins with the Chief Justice and is followed by the Associate Justices according to seniority"*.[6] In addition, former-Justice William Rehnquist describes the decision as an openly sequential process: "*I review the memos and indicate on them the way I intend to vote at the conference. I don't necessarily always vote the way I had planned to vote, however; something said at the conference may persuade me either to shift from a 'deny' to a 'grant' or vice versa*" (Rehnquist 2002, p. 233). A Justice's relative seniority determines his or her sequential voting position which may significantly affect their influence on the decisions made by the Court. A group of people sequentially deciding which projects deserve to be funded or which cases deserve to be heard may not seem to be a strategic or competitive setting. However, these sequential settings become highly competitive if an agent prefers a certain set of outcomes over a different set of outcomes. Using the two examples above, strategic behavior should be expected in these stylized "citizen-candidate" models (Osborne & Slivinski 1996) where the people making the funding decisions are the same

[5]www.supremecourt.gov

[6]http://supreme.lp.findlaw.com/supreme_court/supcthist.html

people receiving the funding and when Supreme Court Justices have personal preferences over which cases they prefer to be heard by the court.[7] In these applications, the results from this paper predict that middle-moving committee members and middle-senior Justices are endowed with a considerable advantage.

3 Model

3.1 Set-up

The model consists of five agents $\mathcal{A} = \{A_1, A_2, A_3, A_4, A_5\}$ sequentially making choices over a set of five proposals: $\mathcal{P} = \{P_1, P_2, P_3, P_4, P_5\}$. This process is carried out openly and sequentially in order of the agent's subscript number. Without losing generality, each agent's sequential position is exogenously determined and publicly known by all agents at all times. Each agent chooses one proposal and whichever proposal is chosen by the most agents is determined as the group outcome. If P_i is the group outcome, then A_i earns a payoff of π, and the other four agents earn Π. The payoffs are related so that $\pi = \Pi - \varepsilon$, where ε represents the cost borne by the agent whose favored proposal is selected as the group outcome. The model generally defines $\varepsilon > 0$ in order to allow for a variety of cost magnitudes while uniquely determining Π to be greater than π. If one proposal is chosen by more agents than any other proposal, then the payoff vector consists of four agents earning Π and one agent earning π. In the case of a tie, agents involved in the tie earn the expected payoff of a randomized tie-breaking procedure while the agents who are not involved in the tie earn Π. For example, if two proposals, P_i and P_j, have each been chosen twice then A_i and A_j earn $\frac{\Pi+\pi}{2}$, while the other three agents earn Π. If all five proposals are each chosen once then all five agents earn $\frac{4\Pi+\pi}{5}$. This payoff structure is publicly known by all agents at all times.

A helpful simplification of the model is to consider a group of agents tasked with selecting one agent to be eliminated from the group. In this simplification, an agent's payoff is maximized so long as they are not eliminated and, in line with this motivation, the model assumes that agents cannot "vote against themselves" by choosing their own proposals ($c(A_i) \neq P_i$). This model is closely related to a dynamic elimination contest (Konrad 2009; Stracke et al. 2014) with only one round. The timing of the model is as follows:

1. **Information**. Agents learn the position number and payoff function for all agents.
2. $\boldsymbol{c(A_1)}$. A_1 chooses a proposal from $\mathcal{S}_1 = \{P_2, P_3, P_4, P_5\}$. All agents observe this.

[7] Rehnquist suggests that Justices, indeed, have personal opinions over Court issues: "*Whether or not to vote to grant certiorari strikes me as a rather subjective decision, made up in part of intuition and part legal judgment*" (Rehnquist 2002, p. 234).

3. $\boldsymbol{c(A_2)}$. A_2 chooses a proposal from $\mathcal{S}_2 = \{P_1, P_3, P_4, P_5\}$. All agents observe this.
4. $\boldsymbol{c(A_3)}$. A_3 chooses a proposal from $\mathcal{S}_3 = \{P_1, P_2, P_4, P_5\}$. All agents observe this.
5. $\boldsymbol{c(A_4)}$. A_4 chooses a proposal from $\mathcal{S}_4 = \{P_1, P_2, P_3, P_5\}$. All agents observe this.
6. $\boldsymbol{c(A_5)}$. A_5 chooses a proposal from $\mathcal{S}_5 = \{P_1, P_2, P_3, P_4\}$. All agents observe this.
7. **Outcome**. The group outcome is determined and payoffs are realized.

3.2 Equilibrium refinement

As is commonly observed in sequential games with perfect information, this model has many Nash and Subgame Perfect Nash Equilibria (SPNE). For instance, if $c(A_1) = P_4$, $c(A_2) = P_4$, and $c(A_3) = P_4$, then any of the 16 combinations of possible choices by A_4 and A_5 are SPNE outcomes. There exists another 16 SPNE outcomes where $c(A_1) = P_5$, $c(A_2) = P_5$, and $c(A_3) = P_5$. In total, there are 320 SPNE outcomes of this form where three or more agents choose the same proposal.[8] There are additional SPNE outcomes with a tie in the group outcome.[9]

Because the SPNE concept offers hundreds of equilibria, this paper achieves a refined prediction by implementing a bounded rationality equilibrium concept. Intuitively, the refined equilibrium is solved for in two steps. First, agents are modeled as error-prone decision makers using the logistic specification of the Agent Quantal Response Equilibrium (McKelvey & Palfrey 1998). This model allows for different equilibrium predictions corresponding to different levels error-prone decision-makers in a sequential game. Second, the equilibrium strategies converge to one SPNE when the error signal is set to be arbitrarily small (McKelvey & Palfrey 1995; the Limiting Logit Equilibrium). This process is formally defined in what remains of this section.

The Quantal Response Equilibrium (McKelvey & Palfrey 1995; QRE hereafter) is a bounded rationality equilibrium concept that has been used to analyze behavior in groups (Guarnaschelli et al. 2000) and sequential settings (McKelvey & Palfrey 1998). The QRE concept is a statistical generalization of the Nash equilibrium (Goeree et al. 2016). Similar to Nash, the QRE is a fixed point where each agent is making choices according to a response function. Unlike the Nash concept, which assumes best-responding, the QRE's *quantal* response functions allows for the analyst to model agents that commit errors based on a specified distribution. By changing the model's free parameter, $\lambda \in [0, \infty)$, the analyst models different levels of error-prone decision-making. When $\lambda = 0$, agents are completely random and choose each available action with a uniform probability. With larger

[8] There are the suggested 32 SPNE outcomes with the group (A_1, A_2, A_3) choosing either P_4 or P_5. 32 more SPNE outcomes with the group (A_1, A_2, A_4) choosing either P_3 or P_5. 32 more SPNE outcomes with the group (A_1, A_2, A_5) choosing either P_3 or P_4. And so on.

[9] For example, $c(A_1) = P_4$, $c(A_2) = P_4$, $c(A_3) = P_1$, $c(A_4) = P_1$, and $c(A_5) \in \{P_2, P_3\}$.

λ parameterizations, agents are less affected by the error term and become more responsive to payoff differences in their actions. When $\lambda \to \infty$, agents have an arbitrarily small amount of noise and respond in a way that is consistent with the Nash Equilibrium.

While the QRE can adopt any distribution for the error terms, the literature typically specifies that each agent's errors as being drawn from an i.i.d. type-I extreme value distribution (Luce 1959). The logit-QRE is one specification within the family of Regular Quantal Response Equilibria (Goeree et al. 2005). This process generates a probabilistic response function for an agent in which actions that yield higher payoffs are more likely (but not necessarily) selected than actions that yield lower payoffs. With this structure, the probability that Agent i chooses action s is determined by the familiar Equation (1).

$$p_{i,s} = \frac{e^{\lambda E(\tilde{\pi}_{i,s}(p_{-i}))}}{\sum_{s'} e^{\lambda E(\tilde{\pi}_{i,s'}(p_{-i}))}} \qquad \forall s' \epsilon S \tag{1}$$

$E(\tilde{\pi}_{i,s}(p_{-i}))$ is the expected payoff earned by Agent i when choosing action s while the other agents within the model make choices according to their own probabilistic response functions p_{-i}. This λ-weighted expected payoff is divided by the sum of the λ-weighted expected payoffs earned by Agent i when choosing all other available actions, s'.

The Agent Quantal Response Equilibrium (McKelvey & Palfrey 1998; AQRE hereafter) extends the QRE concept to sequential games by using the agent-normal form representation of the game (Selten 1975). Each agent's strategy is represented by a set of different agents each who realize a unique sequence of choices made before them. The refined equilibrium prediction used in this paper is the logit-AQRE when agents are specified to have an arbitrarily small amount of noise ($\lambda \to \infty$). This "Limiting Logit Equilibrum" (McKelvey & Palfrey 1995; LLE hereafter)[10] can be found by starting at the center of the strategy space (where $\lambda = 0$ and agents are choosing randomly) and tracing the unique (or "principal") branch that converges to a Nash Equilibrium as $\lambda \to \infty$.

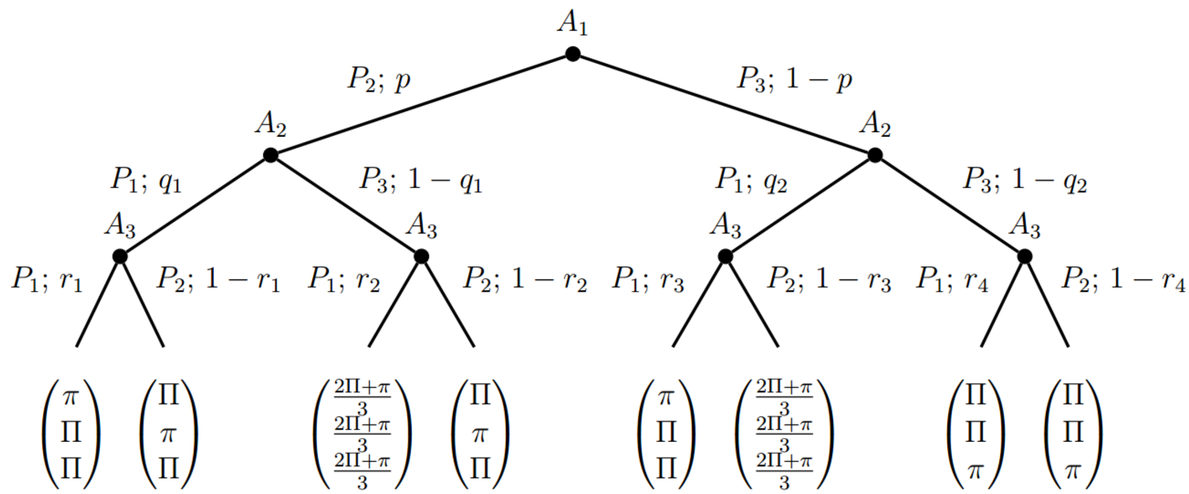
3.3 LLE predictions

To illustrate the intuition behind the refinement process, consider the logit-AQRE of this paper's model with 3 agents.[11] Figure 1 shows the extensive form representation along with the 7 equations required to characterize the logit-AQRE. Given a value for λ, this system has a unique solution which is solved recursively starting from the decision nodes associated with the choice of A_3 (r_1, r_2, r_3, and r_4) followed by the choice of A_2 (q_1 and q_2) and A_1 (p).

[10] The LLE has also been referred to as the "logit equilibrium" when applied to sequential games (McKelvey & Palfrey 1998).

[11] As discussed in section 3.4, the logit-AQRE and LLE become computationally difficult with many agents.

In the 3 agent model, the LLE is the $(r_1, r_2, r_3, r_4, q_1, q_2, p)$ that is solved for when λ is set to be sufficiently large.[12]



$$p = \frac{e^{\lambda\left\{q_1 r_1 \pi + q_1(1-r_1)\Pi + (1-q_1) r_2 (\frac{2\Pi+\pi}{3}) + (1-q_1)(1-r_2)\Pi\right\}}}{e^{\lambda\left\{q_1 r_1 \pi + q_1(1-r_1)\Pi + (1-q_1) r_2 (\frac{2\Pi+\pi}{3}) + (1-q_1)(1-r_2)\Pi\right\}} + e^{\lambda\left\{q_2 r_3 \pi + q_2(1-r_3)(\frac{2\Pi+\pi}{3}) + (1-q_2) r_4 \Pi + (1-q_2)(1-r_4)\Pi\right\}}}$$

$$q_1 = \frac{e^{\lambda\{r_1\Pi + (1-r_1)\pi\}}}{e^{\lambda\{r_1\Pi + (1-r_1)\pi\}} + e^{\lambda\left\{r_2(\frac{2\Pi+\pi}{3}) + (1-r_2)\pi\right\}}}; \qquad q_2 = \frac{e^{\lambda\left\{r_3\Pi + (1-r_3)(\frac{2\Pi+\pi}{3})\right\}}}{e^{\lambda\left\{r_3\Pi + (1-r_3)(\frac{2\Pi+\pi}{3})\right\}} + e^{\lambda\{r_4\Pi + (1-r_4)\Pi\}}}$$

$$r_1 = \frac{e^{\lambda\{\Pi\}}}{e^{\lambda\{\Pi\}} + e^{\lambda\{\Pi\}}} = \frac{1}{2}; \quad r_2 = \frac{e^{\lambda\left\{\frac{2\Pi+\pi}{3}\right\}}}{e^{\lambda\left\{\frac{2\Pi+\pi}{3}\right\}} + e^{\lambda\{\Pi\}}}; \quad r_3 = \frac{e^{\lambda\{\Pi\}}}{e^{\lambda\{\Pi\}} + e^{\lambda\left\{\frac{2\Pi+\pi}{3}\right\}}}; \quad r_4 = \frac{e^{\lambda\{\pi\}}}{e^{\lambda\{\pi\}} + e^{\lambda\{\pi\}}} = \frac{1}{2}$$

Figure 1: The extensive form and logit-AQRE for this paper's model with 3 agents.

The LLE for the 5 agent model is computed using the same approach. Every decision node in the extensive form representation has four possible choices and the number of decision nodes representing each agent is equal to the number of possible histories that an agent can realize. A_1 is the first mover and, therefore, has only one decision node. A_2 has 4 decision nodes corresponding to the 4 possible histories that A_2 could realize (either $c(A_1) = P_2$, $c(A_1) = P_3$, $c(A_1) = P_4$, or $c(A_1) = P_5$). A_3, A_4, and A_5 have 16, 64, and 256 decision nodes. In total, there are 341 unique agents each with a probabilistic choice that can be solved using 3 equations ($|\mathcal{S}_i| - 1$). This means that for each specified λ value, the logit-AQRE is characterized by 1,023 equations. As in Figure 1, this system is solved recursively starting

[12]Analyses of 3 agent and 4 agent models are in Appendix A. The LLE of these models also predict that the middle-moving agent is (weakly) expected to earn the highest payoff, which suggests that the theoretical results are not dependent on five agents or an odd number of agents.

with the 768 equations representing A_5. The Gambit program is used to calculate the logit-AQRE corresponding to different λ values (McKelvey et al. 2013). The unique mixed strategy LLE is found by computing these 1,023 equations when λ is set to be sufficiently large.

In order to solve for the LLE, the parameters (Π, π) are normalized to $(1, 0)$. This parameterization aligns with the experiment described in section 4. It is possible that the LLE prediction would be different with different payoffs (Tumennasan 2013; Zhang & Hofbauer 2016) or with the inclusion of (strictly dominated) strategies (Zhang 2016). However, there exists an arbitrarily large number of parameterizations that precisely align with the theoretical analysis carried out for the $(1, 0)$ normalization. Generally speaking, any parameterization that has the same "strategic" component as the $(1, 0)$ parameterization will have the exact same LLE (for more on this approach, refer to Jessie & Kendall 2015 and Jessie & Saari 2016). For example, adding a scalar value, x, to the payoff for A_i at all possible outcomes will yield the same LLE prediction.

Figure 2 shows the relationship between λ and A_1's logit-AQRE. When $\lambda = 0$, the logit-AQRE for A_1 is $prob(c(A_1) = P_2) = prob(c(A_1) = P_3) = prob(c(A_1) = P_4) = prob(c(A_1) = P_5) = 0.25$. As λ increases, these probabilities converge to A_1's LLE mixed strategy of $prob(c(A_1) = P_2) = prob(c(A_1) = P_5) = 0.5$ and $prob(c(A_1) = P_3) = prob(c(A_1) = P_4) = 0$.

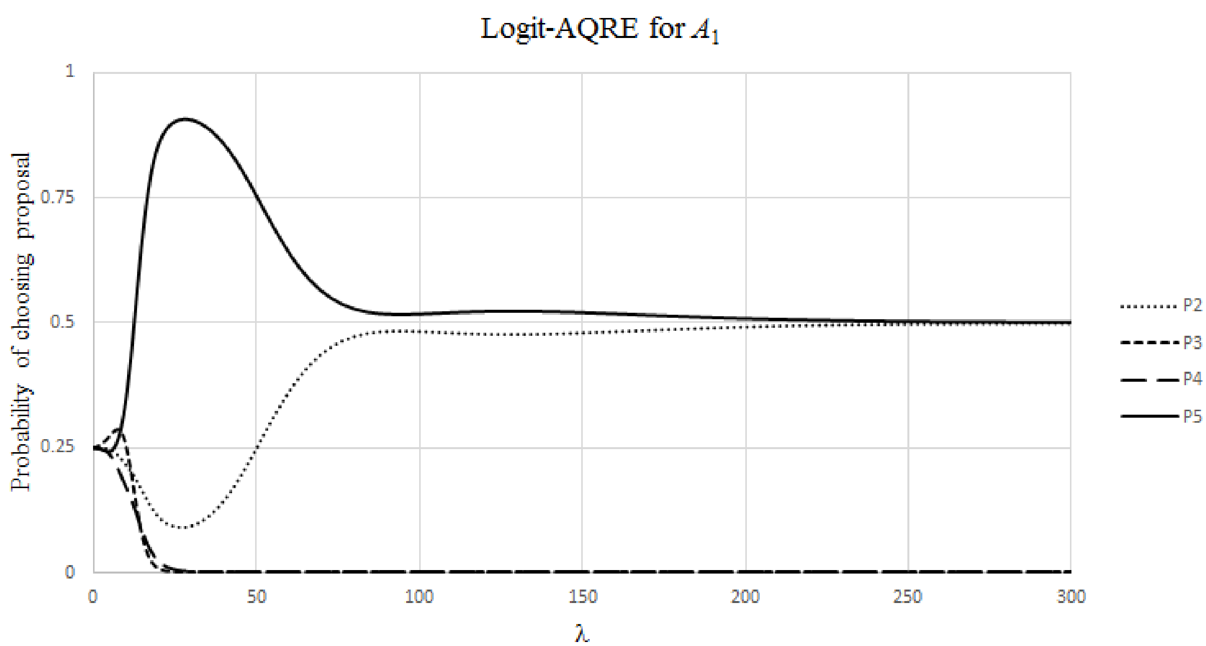


Figure 2: The logit-AQRE for A_1 by λ.

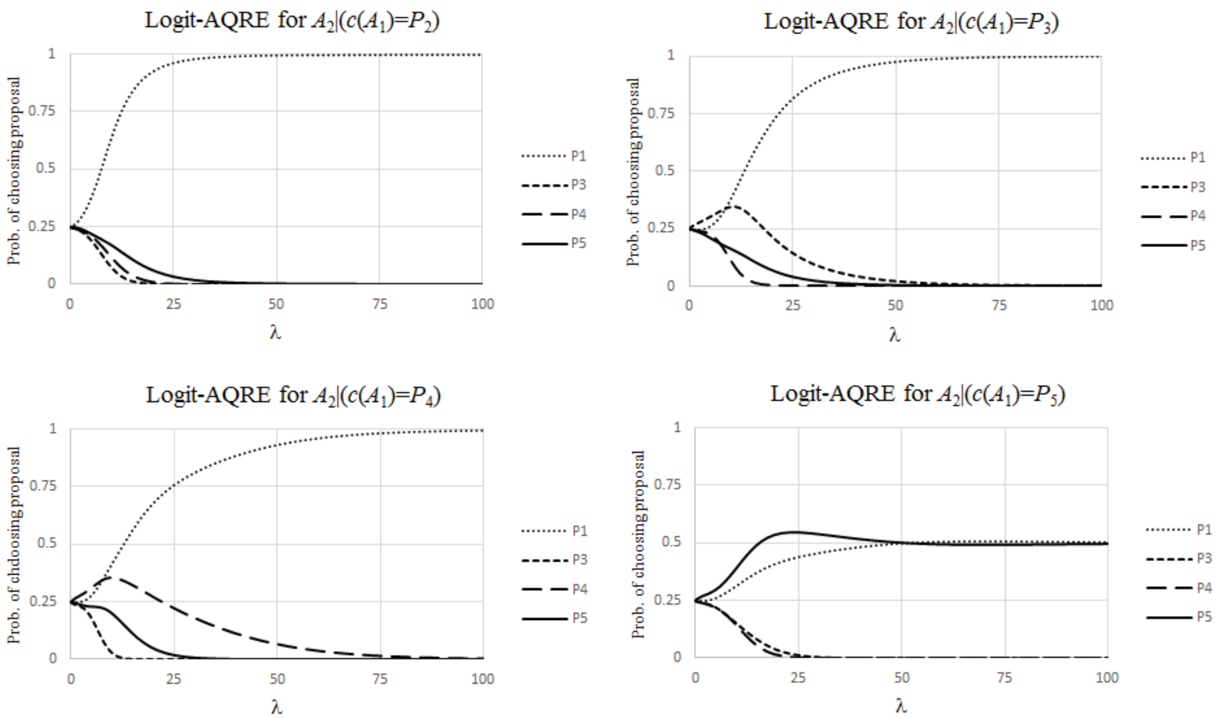


Figure 3: The logit-AQRE for A_2 by λ.

Figure 3 shows the relationship between λ and A_2's logit-AQRE across A_2's four decision nodes (corresponding to A_1's four possible choices). A_2's LLE mixed strategy is the strategy converged upon as λ increases. The LLE predicts that $prob(c(A_2) = P_1) = 1$ unless $c(A_1) = P_5$, in which case $prob(c(A_2) = P_1) = prob(c(A_2) = P_5) = 0.5$.

The LLE mixed strategy for A_3, A_4, and A_5 are spread across 16, 64, and 256 decision nodes, respectively. The output from this algorithm for all decision nodes is available in the supplemental materials.

In order to provide theoretical predictions in terms of expected actions and payoffs, attention is restricted to on-path equilibrium behavior. For instance, A_2's LLE behavior at $A_2|(c(A_1) = P_3)$ and $A_2|(c(A_1) = P_4)$ is ignored because $prob(c(A_1) = P_3) = prob(c(A_1) = P_4) = 0$. Ignoring behavior off of the equilibrium path allows for a full characterization of the LLE, shown in Figure 4.

Agent	History	LLE choice probabilities
A_1	–	Choose P_i for i= *2, 5* each with probability 0.5
A_2	P_2	Choose P_1 with probability 1
A_2	P_5	Choose P_i for i= *1, 5* each with probability 0.5
A_3	P_2, P_1	Choose P_4 with probability 1
A_3	P_5, P_1	Choose P_4 with probability 1
A_3	P_5, P_5	Choose P_i for i= *4, 5* each with probability 0.5
A_4	P_2, P_1, P_4	Choose P_i for i= *1, 2* each with probability 0.5
A_4	P_5, P_1, P_4	Choose P_1 with probability 1
A_4	P_5, P_5, P_4	Choose P_5 with probability 1
A_5	P_2, P_1, P_4, P_1	Choose P_i for i=*1, 2, 3, 4* each with probability 0.25
A_5	P_2, P_1, P_4, P_2	Choose P_i for i=*1, 2, 3, 4* each with probability 0.25
A_5	P_5, P_1, P_4, P_1	Choose P_i for i=*1, 2, 3, 4* each with probability 0.25

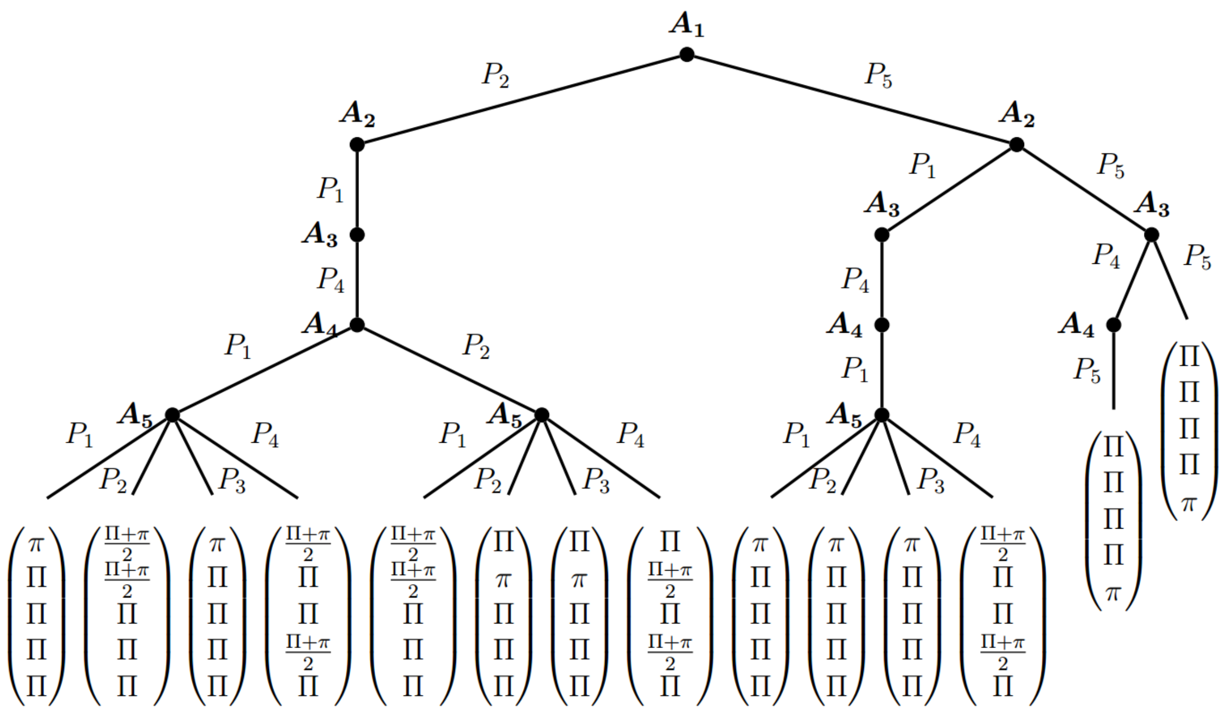


Figure 4: The on-path LLE choice probabilities.

The 14 equilibrium outcomes predicted by the LLE mixed-strategy is a drastic refinement from the hundreds of equilibria outcomes predicted by the SPNE. Surprisingly, P_3 is the only proposal that is never selected as the group outcome. No further analysis is needed to obtain this paper's main theoretical prediction of a middle-mover advantage.

Result 1. *In an openly sequential competition with five agents, A_3 is the only agent to earn the maximal payoff in all LLE outcomes.*

When agents make choices according to their LLE mixed-strategy, it is possible to assign a probability of realizing any of the 14 outcomes in Figure 4. For example, the far left outcome in the extensive form representation in Figure 4 is realized when $c(A_1) = P_2$, $c(A_2) = P_1$, $c(A_3) = P_4$, $c(A_4) = P_1$, and $c(A_5) = P_1$. These events are realized with respective probabilities of .5, 1, 1, .5, and .25 which means that the probability of realizing this outcome is $0.5 \times 1 \times 01 \times 0.5 \times 0.25 = 0.05 = \frac{1}{16}$. With a $\frac{1}{16}$ chance, A_1 earns π and the other four agents earn Π. Using this approach for all 14 possible outcomes, it is possible to calculate the expected payoff for each agent.[13]

Result 2. *In an openly sequential competition with five agents, the LLE predicts the following expected payoff for each agent:*

$$E(payoff(A_1)) = \Pi - \tfrac{14}{32}\varepsilon = \Pi - 0.4375\varepsilon$$
$$E(payoff(A_2)) = \Pi - \tfrac{7}{32}\varepsilon = \Pi - 0.21875\varepsilon$$
$$E(payoff(A_3)) = \Pi - \tfrac{0}{32}\varepsilon = \Pi$$
$$E(payoff(A_4)) = \Pi - \tfrac{3}{32}\varepsilon = \Pi - 0.09375\varepsilon$$
$$E(payoff(A_5)) = \Pi - \tfrac{8}{32}\varepsilon = \Pi - 0.25\varepsilon$$

The payoffs predicted by Results 1 and 2 serve as theoretical predictions pertaining to advantageous positions in this type of sequential competition. This experiment described in the next section will test these hypotheses:

Hypothesis 1. The middle-moving agent (A_3) will earn the highest payoff. Therefore, there exists a middle-mover advantage.

Hypothesis 2. The first- and last-moving agents (A_1 and A_5) will earn the lowest payoffs. Therefore, there exists a first- and last-mover disadvantage.

The LLE prediction is derived assuming that agents are error-less payoff-optimizers. In addition, this paper's computational equilibrium technique and uncommon prediction (of a middle-mover advantage) makes it especially unlikely that laboratory subjects will perfectly mimic the model's prediction. However, given that learning is commonly observed in multi-period laboratory experiments, I anticipate that the behavior of subjects who are experienced with the competition will be more likely to follow the model's predictions that the behavior of inexperienced subjects. Thus, the experiment will test a third hypothesis.

Hypothesis 3. The payoffs of subjects in late rounds will more closely resemble the LLE prediction than will the payoffs of subjects in early rounds.

[13] See Appendix B for more details on this calculation.

3.4 Summary of section 3

Before testing these hypotheses, it is beneficial to summarize the intuition from this section.

Why not use the SPNE as the basis for the paper's hypotheses? The model presented in section 3.1 has hundreds of pure-strategy SPNE. One approach would state that, in this context, equilibrium selection is an empirical question. This approach would run the experiment described in section 4.1 and state that the best-fitting SPNE are the SPNE-outcomes that are most often observed in the data. This alternative approach would not change the experimental results shown in section 4.2. However, the explanation of these results would, by definition, be ad-hoc and would almost certainly rely on unobservable psychological features such as revenge or retaliation. In contrast, this paper refines the set of SPNE ex-ante with the use of the LLE. This provides a unique mixed-strategy equilibrium prediction across 14 possible outcomes which does not rely on unobservable features in the utility function.

Why use the LLE? The LLE is used because it is theoretically and intuitively well-suited for this sequential environment. Theoretically, the LLE corresponds to a subset of the sequential equilibria, which classifies it as a true refinement of the Nash concept (McKelvey & Palfrey 1998; Turocy 2010). For large enough λ, tracing the "principal branch" of the logit-AQRE to find the LLE is guaranteed to converge to a unique mixed-strategy prediction (Turocy 2005, Lemma 2). Also, using extreme-value i.i.d. errors avoids the known empirical issues associated with the QRE concept (Goeree et al. 2005; Haile et al. 2008). Using the LLE aligns with a long history of using stochastic best response functions with an arbitrary level of noise (Blume 2003; Sandholm 2010). Intuitively, this refinement process aligns with the story of error-prone decision-makers who converge to one SPNE as their errors become arbitrarily small (section 3.2). An alternative intuition is that agents incur payoff shocks associated with each action which are not observed by other agents (or by the theorist). This may capture agents who have an unobserved preferences over which proposal to choose. For instance, A_2 might choose P_3 because she doesn't like the number 3. This intuition may be particularly appealing to some readers given that the model includes agents that are indifferent to many different outcomes (so long as their favored proposal is not selected). However, it is important to note that a benefit of using the LLE is that it produces an equilibrium prediction of agents who solely act in their selfish interest. Therefore, a huge advantage of this approach is that the model's prediction does not rely on any specific other-regarding or social-preference utility formulations.

An important limitation of using the LLE is the practical difficulty involved with extending this paper's results to include any finite number of agents (and proposals).[14] As was shown in section 3.3, calculating the logit-AQRE and LLE in this sequential setting is difficult even with a modest number of agents. In general, for a model with X agents choosing one of the $X-1$ possible proposals ($c(A_i) \neq P_i$), the agent-normal form represents the ith agent in the decision sequence with $(X-1)^{i-1}$ unique agents. Each unique agent has a probabilistic choice function similar to Equation (1) which is characterized by $X-2$ equations.[15] Taken together, this means that the total number of equations for a model with X agents is $\sum_{i=1}^{X}(X-2)\cdot(X-1)^{i-1}$. This formula illustrates the exponential relationship between a model's number of agents and the required number of equations. For example, a model with 10 agents would require the logit-AQRE to solve a system of 3,486,784,400 equations.

In spite of the computational difficulty, the LLE is more attractive than other refinement methods because it retains the equilibrium assumption and it avoids having to make assumptions about off-path equilibrium play. For example, a different refinement approach could model agents that, at all information sets, choose their Nash Equilibrium strategy with probability γ, and with probability $(1-\gamma)$ they would mix uniformly over their entire strategy set. However, these agents are not incorporating into their strategy the fact that *other* agents are also mixing according to γ. In this way, the "noisy Nash Model" deviates from the LLE because it does not impose equilibrium play (McKelvey & Palfrey 1998). A different refinement approach could "prune" the tree using backwards induction based on assumptions made (by the theorist) about off-path equilibrium play. There are many possible assumptions available and any such assumption would require an extensive explanation about its legitimacy. The LLE requires no additional assumptions. The LLE is essentially a tracing procedure that ends at the limiting case of the logit-AQRE. This tracing procedure does not require any assumptions about off-path equilibrium play as all actions are played with a positive probability. In the limit, as λ approaches infinity, the probability that many of these actions are played becomes arbitrarily small. In this way, the "pruning" is done within the model itself.

What did we learn? Section 3 presents the first theoretical model of a sequential competition where the middle-moving agents have an advantage. This novel prediction is particularly surprising because the set-up and assumptions are entirely standard. A middle-mover advantage is predicted from a model with complete information, symmetric payoffs, equilibrium play, and completely selfish agents. The only factor that differentiates agents' choices and

[14]Indeed, demonstrating a middle-mover advantage in sequential competitions with a general number of agents would be a valuable extension.

[15]There are only $X-2$ equations because $1-(p_1+p_2+...+p_{X-2}) = p_{X-1}$.

payoffs is their location in the decision sequence. While agents in different positions have different equilibrium strategies, the strategies of all players can be classified as a balancing across what I refer to as the "upstream" and "downstream" incentives.

Any agent has an incentive to choose proposals favored by agents who are near the end of the decision sequence. Doing so encourages other agents to choose the same proposal with the goal of coordinating on a proposal favored by a late-moving agent before that agent has a chance to choose (thus making their choice trivial). Hence, agents have an incentive to choose proposals that are *downstream* of the decision sequence.[16] In addition, any agent has an incentive to choose proposals favored by agents who are near the beginning of the decision sequence. Doing so encourages other agents to choose the same proposal with the goal of coordinating on a proposal favored by an early-moving agent that has already publicly made their choice. Hence, agents have an incentive to choose proposals that are *upstream* of the decision sequence.

These incentives can also be interpreted as endowing agents with different levels of impact and information. For example, early-moving agents benefit from their position because they make their choice before a target proposal has been selected or before a majority is reached. However, these agents have to make their choice without information about what other agents have chosen. In this way, early-moving agents are rich in impact, but poor in information. Conversely, late-moving agents benefit from their position because they make their choice after observing the choices of many agents. However, it is more likely that their turn comes after their proposal has been targeted or settled upon by a majority. In this way, late-moving agents are rich in information, but poor in impact. With this intuition, middle-moving agents have a balance of information and impact.[17] In fact, one interpretation of section 3 is that it formalizes a natural conclusion about the best position in this sequential competition. The best position is one that allows an agent to observe some information but which also ensures that the agent's choice has an impact on the outcome.

[16]The downstream incentive somewhat resembles the mechanism behind herding and information cascade models. However, the driving force behind the downstream incentive and these earlier phenomena are fundamentally different. In herding models, the incentive-to-herd is driven by an agent's continual process of updating of their imperfect belief based on revealed choices. However, this paper's downstream incentive stems from a model that assumes perfect information. Rather than a belief-updating process, this paper's downstream incentive is a strategic avenue leveraged by agents in order to increase their expected payoff.

[17]Previous research on sequential competitions could be viewed as relying on *one* of these incentives in order to drive either a first- or last-mover advantage (see Appendix C for examples). The intuition behind this paper's combating incentives is highly related to work explaining the disadvantage of voting early in an endogenously determined vote (Dekel & Piccione 2014). The novelty of this current paper is to illustrate that the co-existence of these combating incentives will benefit the agents moving in the middle of the decision sequence.

4 Experiment

4.1 Design

A laboratory experiment is used to test the prediction of a middle-mover advantage (Fischbacher 2007; Appendix D contains screen shots). In order to provide a clear environment, subjects are labeled as “voters” deciding the allocation of “prizes”. In alignment with the theoretical parameters, the subjects’ task is to take part in an open and sequential vote to determine which of the four voters will receive a prize (of $1) for that round. In each round, within each group, the voter with the most number of votes will not receive a dollar while the other four voters are awarded a dollar. If there is a two (or five) voter tie, the computer program randomly selects one of the two (or five) voters to earn $0. Each session consists of an instructions phase, a test of comprehension, 20 decision rounds, and a questionnaire. Subjects were paid for all rounds which yielded an average earnings of $16. In addition to this, all subjects received a $7 show-up payment. Each session was completed within 90 minutes.

Subjects make choices on computer terminals which are separated by physical dividers. Subjects are not permitted to communicate at any time during the experiment. Each subject is labeled within their group by their position in the vote. For instance, the fourth voter in each group is labeled “Voter 4”. Before each round, subjects are told that every subject is randomly assigned to a new group and a new voting position. After each round, subjects are perfectly informed of each voter’s choice and the subsequent outcome. The decision round ends when all five subjects have made a choice or when one voter has received three votes against them.

Four sessions were conducted using a total of 100 undergraduate students at a large public university. These 100 subjects made 400 collective decisions (group outcomes). The following results use data that are aggregated across all four sessions. The aggregated data consist of 370 group decisions where one agent receives more votes than any other agent and 30 group decisions with two-agent ties that are broken by a random number generator. Five-agent ties are never observed.

4.2 Main results

Figure 5(a) compares the LLE distribution of expected payoffs (black bars) with the observed average payoffs from each position (grey bars) for all 20 rounds (400 outcome data set). Figure 5(b) compares the LLE with the same data divided into early rounds (first ten rounds; dark grey bars) and late rounds (last ten rounds; light grey bars). In both figures, the theory and data show that subjects in the middle of the voting order earn higher average payoffs. This is further explored in what remains of this subsection.

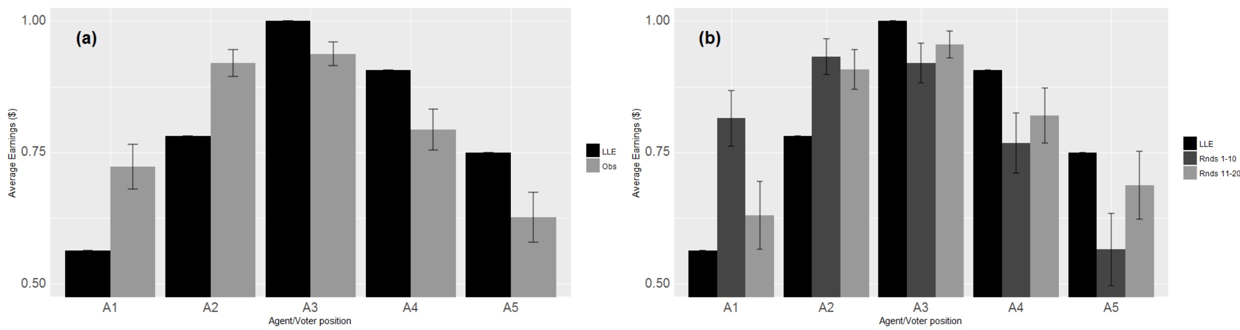


Figure 5: The black bar is the LLE prediction. In 5(a), the grey bar is the average payoff for each position in all 20 rounds (400 data points). In 5(b), the darker grey bar and the lighter grey bar represent the average payoff for each position in the first 10 rounds (200 data points) and the last 10 rounds (200 data points), respectively. 95% confidence intervals are displayed. Both graphs represent 370 outcomes where one unique agent is selected by the group and 30 two-agent ties.

An OLS regression controlling for subject-level fixed effects regresses a subject's earnings in a round on dummy variables identifying the subject's position in the voting sequence in that round. Table 1 analyzes data divided into choices made in all 20 rounds, in the first 10 rounds, and in the last 10 rounds. Within each range of rounds, the five regressions vary which dummy is omitted, which allows for a relative comparison controlling for subject-level fixed effects. Also, note that the constant in each regression is the average earnings for each round for the position dummy variable that is omitted.

Position	Rnds 1-20 (1)	Rnds 1-20 (2)	Rnds 1-20 (3)	Rnds 1-20 (4)	Rnds 1-20 (5)	Rnds 1-10 (1E)	Rnds 1-10 (2E)	Rnds 1-10 (3E)	Rnds 1-10 (4E)	Rnds 1-10 (5E)	Rnds 11-20 (1L)	Rnds 11-20 (2L)	Rnds 11-20 (3L)	Rnds 11-20 (4L)	Rnds 11-20 (5L)
A_1	–	-0.195	-0.214	-0.071	0.098	–	-0.117	-0.105	0.048	0.250	–	-0.278	-0.325	-0.189	-0.058
		(0.026)	(0.026)	(0.026)	(0.026)		(0.037)	(0.037)	(0.037)	(0.037)		(0.037)	(0.037)	(0.037)	(0.037)
A_2	0.195	–	-0.019	0.124	0.293	0.117	–	0.012	0.165	0.367	0.278	–	-0.047	0.088	0.220
	(0.026)		(0.026)	(0.026)	(0.026)	(0.037)		(0.037)	(0.037)	(0.037)	(0.037)		(0.037)	(0.037)	(0.037)
A_3	0.214	0.019	–	0.143	0.312	0.105	-0.012	–	0.153	0.355	0.325	0.047	–	0.135	0.267
	(0.026)	(0.026)		(0.026)	(0.026)	(0.037)	(0.037)		(0.037)	(0.037)	(0.037)	(0.037)		(0.037)	(0.037)
A_4	0.071	-0.124	-0.143	–	0.169	-0.048	-0.165	-0.153	–	0.203	0.189	-0.088	-0.135	–	0.132
	(0.026)	(0.026)	(0.026)		(0.026)	(0.037)	(0.037)	(0.037)		(0.037)	(0.037)	(0.037)	(0.037)		(0.037)
A_5	-0.098	-0.293	-0.312	-0.169	–	-0.250	-0.367	-0.355	-0.203	–	0.058	-0.220	-0.267	-0.132	–
	(0.026)	(0.026)	(0.026)	(0.026)		(0.037)	(0.037)	(0.037)	(0.037)		(0.037)	(0.037)	(0.037)	(0.037)	
Const.	0.723	0.919	0.937	0.795	0.626	0.815	0.932	0.920	0.768	0.565	0.630	0.908	0.955	0.820	0.688
	(0.019)	(0.019)	(0.019)	(0.019)	(0.019)	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)	(0.026)
Obs.	2,000	2,000	2,000	2,000	2,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000	1,000
F.E. #	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
df	7	7	7	7	7	7	7	7	7	7	7	7	7	7	7
Log-like	-873	-873	-873	-873	-873	-440	-440	-440	-440	-440	-427	-427	-427	-427	-427

Table 1: OLS of earnings by position including fixed effects. Standard errors in parentheses.

Using the data from all 20 rounds, Table 1 shows that A_1's average payoff of 0.723 is higher than A_5's average payoff of 0.626 by a significance level less than 0.001. Because of this, the two positions can be ranked as $A_1 \succ A_5$. In addition, Table 1 states that A_3's average payoff of 0.937 is higher than A_2's average payoff of 0.919 in all 20 rounds, but that this difference is not significant. Therefore, the two positions are ranked as $A_3 \sim A_2$. The average earnings at each position can be ranked in this manner for rounds 1-20, rounds 1-10, and rounds 11-20.

Rounds 1-20	$A_3 \sim A_2 \succ A_4 \succ A_1 \succ A_5$
Rounds 1-10	$A_2 \sim A_3 \succ A_1 \sim A_4 \succ A_5$
Rounds 11-20	$A_3 \sim A_2 \succ A_4 \succ A_5 \sim A_1$

Table 2: Ranking the average payoffs earned by each position. These rankings are based on statistically significant differences between the average earnings of each position shown in the regression results in Table 1.

Table 2 supports this paper's experimental result of a middle-mover advantage. The earnings of the first-, fourth-, and last-moving subjects are always significantly less than the earnings of the middle-moving agent. These results are highly significant when analyzing all 20 rounds or only the early or late rounds. The middle-mover earns slightly more than the subject in the second position when analyzing all 20 rounds or the late rounds. However, the difference in earnings between A_3 and A_2 are never significant.

Result 3. (Hyp. 1) *A_2 and A_3 earn the most in early, late, and all rounds.*

Table 2 illustrates a last-mover disadvantage when compared to the other four positions, which is particularly prevalent in the early rounds. Table 2 also illustrates a first-mover disadvantage that is most prevalent in late rounds.

Result 4. (Hyp. 2) *The last-mover (A_5) earns the least in early rounds. The first- and last-movers (A_1 and A_5) earn the least in late rounds.*

Figure 5(b) shows that each position's late-round average payoff is closer to the LLE prediction than the same position's early-round average payoff. Table 3 shows that these differences are highly significant for the first- and last-mover and marginally significant for the middle-mover.

Result 5. (Hyp. 3) *The LLE payoffs are better reflected in data from the late rounds, compared to the early rounds.*

	A_1	A_2	A_3	A_4	A_5
Rounds 1-10	0.82 (.03)	0.93 (.02)	0.92 (.02)	0.77 (.03)	0.56 (.03)
Rounds 11-20	0.63 (.03)	0.91 (.02)	0.96 (.01)	0.82 (.03)	0.69 (.03)
p-value	<0.001	0.416	0.072	0.215	0.007

Table 3: Average earnings by early and late rounds. Standard errors in parentheses. p-values from Wilcoxon sign-ranked test using paired data. These tests compare the heights of the darker grey and lighter grey bars in Figure 5(b).

4.3 Additional results

Table 4 shows the choices of A_1, A_2, and A_3 in all 20 rounds as well as the same data divided into early and late rounds.

	A_1				A_2				A_3			
	P_2	P_3	P_4	P_5	P_1	P_3	P_4	P_5	P_1	P_2	P_4	P_5
Rounds 1-20	.08 (33)	.13 (50)	.31 (122)	.49 (195)	.27 (108)	.09 (36)	.25 (99)	.39 (157)	.30 (120)	.13 (52)	.21 (85)	.36 (143)
Rounds 1-10	.07 (13)	.14 (27)	.30 (60)	.50 (100)	.17 (33)	.13 (25)	.27 (54)	.44 (88)	.22 (44)	.15 (29)	.23 (45)	.41 (82)
Rounds 11-20	.10 (20)	.12 (23)	.31 (62)	.48 (95)	.38 (75)	.06 (11)	.23 (45)	.35 (69)	.38 (76)	.12 (23)	.20 (40)	.31 (61)

Table 4: Proportion of choices by A_1, A_2, and A_3 (number of observations in parentheses).

As shown in Table 4, subjects in the first-moving position rarely choose P_2. This is in contrast with the LLE prediction which predicts an equilibrium where first-moving subject chooses P_2 half of the time. Furthermore, the choices made from the first-moving position do not change over the course of the experiment.

Result 6. *Contrary to the LLE prediction, first-moving subjects are least likely to choose* P_2.

Unlike the first-moving agent, A_2 and A_3 change their behavior over the course of the experiment. As shown in Table 4, subjects in these positions increase the frequency that they chose the upstream target (P_1) in late rounds. While A_2 and A_3 decrease the frequency that they choose any of the other three proposals, the largest decrease is in their likelihood to choose the downstream target (P_5).

Table 5 shows the proportion of choices by A_1, A_2 and A_3 conditional on the previous choices made in that round. This table shows A_2's two most-chosen proposals conditional on the 4 possible choices from A_1 and it shows A_3's two most-chosen proposals conditional on realizing each of these 8 choice histories.

(a) **Rounds 1-20**

A_1	P_2 .08 (33)				P_3 .13 (50)				P_4 .31 (122)				P_5 .49 (195)			
A_2	P_1 .79 (26)		P_5 .09 (3)		P_1 .38 (19)		P_3 .58 (29)		P_1 .17 (21)		P_4 .75 (92)		P_1 .22 (42)		P_5 .75 (146)	
A_3	P_1 .54 (14)	P_2 .42 (11)	P_2 .67 (2)	P_5 .33 (1)	P_1 .95 (18)	P_5 .05 (1)	P_1 .69 (20)	P_2 .31 (9)	P_1 .57 (12)	P_4 .24 (5)	P_1 .12 (11)	P_4 .77 (71)	P_1 .62 (26)	P_5 .26 (11)	P_1 .11 (16)	P_5 .79 (116)

(b) **Rounds 1-10**

A_1	P_2 .07 (13)				P_3 .14 (27)				P_4 .30 (60)				P_5 .50 (100)			
A_2	P_1 .62 (8)		P_3 .23 (3)		P_1 .22 (6)		P_3 .74 (20)		P_1 .08 (5)		P_4 .85 (51)		P_1 .14 (14)		P_5 .83 (83)	
A_3	P_1 .38 (3)	P_2 .63 (5)	P_1 .33 (1)	P_2 .67 (2)	P_1 .83 (5)	P_5 .17 (1)	P_1 .65 (13)	P_2 .35 (7)	P_1 .40 (2)	P_5 .40 (2)	P_2 .12 (6)	P_4 .78 (40)	P_1 .43 (6)	P_5 .43 (6)	P_1 .11 (9)	P_5 .82 (68)

(c) **Rounds 11-20**

A_1	P_2 .10 (20)				P_3 .12 (23)				P_4 .31 (62)				P_5 .48 (95)			
A_2	P_1 .90 (18)		P_5 .10 (2)		P_1 .57 (13)		P_3 .39 (9)		P_1 .26 (16)		P_4 .66 (41)		P_1 .29 (28)		P_5 .66 (63)	
A_3	P_1 .61 (11)	P_2 .33 (6)	P_2 .50 (1)	P_5 .50 (1)	P_1 1.00 (13)	–	P_1 .22 (2)	P_2 .78 (7)	P_1 .63 (10)	P_4 .25 (4)	P_1 .15 (6)	P_4 .76 (31)	P_1 .71 (20)	P_5 .18 (5)	P_2 .13 (8)	P_5 .76 (48)

Table 5: Proportion of choices by A_1, A_2, and A_3 conditioning on previous choices (number of observations in parentheses).

For A_2, P_1 is one of the two most-chosen proposals conditional on any choice by A_1. Compared to early rounds, the probability that A_2 chooses P_1 is higher in later rounds, regardless of A_1's choice. A similar pattern is observed for A_3 (with one exception). A_2 and A_3's evolved behavior in late rounds explains the large differences in payoffs between the early and late rounds for the first- and last-movers shown in Figure 5(b).

Result 7. *A_2 and A_3 are more likely to choose P_1 in late rounds than in early rounds.*

Figure 6 shows the average payoffs for each position conditional on the choice made by A_1. This figure illustrates two insights. First, although A_1 earns more in the early rounds, the relationship between A_1's proposal-choice and A_1's average earnings are the same in early and late rounds. More specifically, A_1's average payoff is higher when choosing downstream targets (either P_4 or P_5) rather than choosing upstream targets (either P_2 or P_3). This is in contrast with the LLE prediction which predicts an equilibrium where first-moving subjects would earn the same by following either incentive (choosing either P_2 or P_5). A Wilcoxon sign-ranked test on all 20 rounds states that A_1 earns significantly less by choosing P_2 instead of P_5 (p-values < 0.001).

Result 8. *A_1's average earnings are higher when choosing P_5 compared to choosing P_2.*

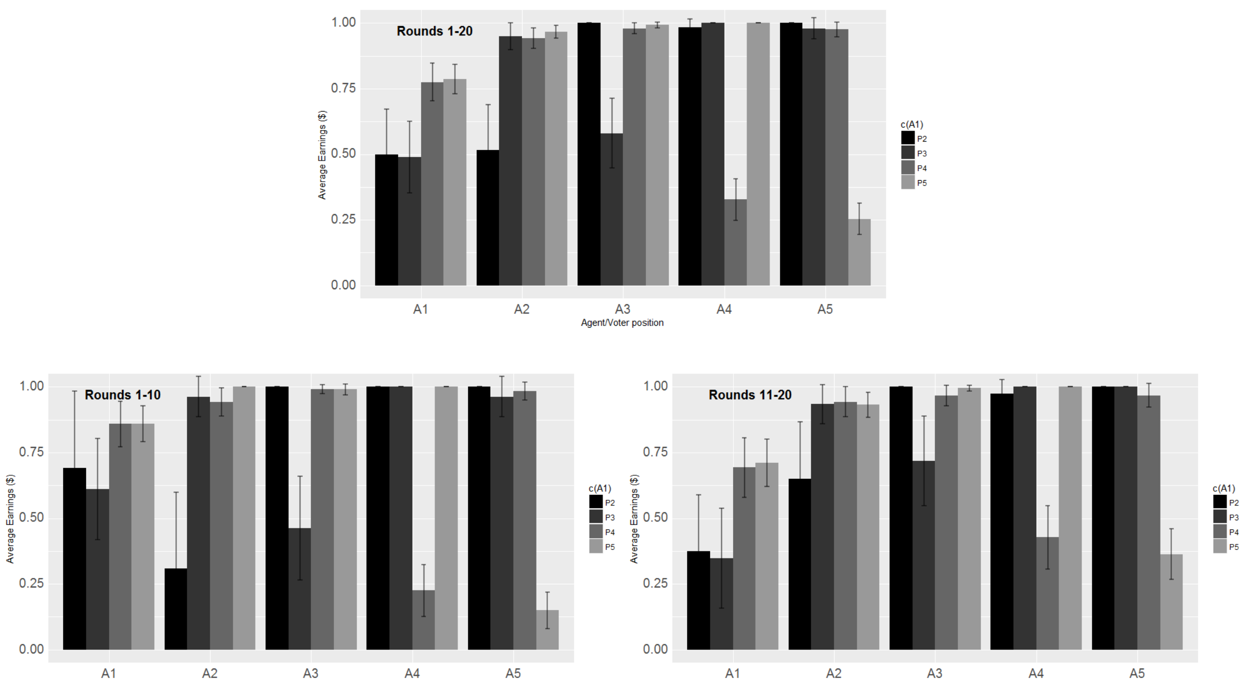


Figure 6: Each graph shows the average payoffs that each position earns conditional on A_1's choice. 95% confidence intervals are displayed. Rounds 1-20 use 400 data points. Rounds 1-10 and Rounds 11-20 use 200 data points each.

Figure 6 shows that the earnings of A_2, A_3, A_4, and A_5 are always lowest when the A_1 chose their favored proposal. However, the insight from this figure is to show that the decisive power of A_1's choice is significantly lower in the late rounds. Table 6 shows that each agent's average earnings conditional on being chosen by A_1 is lower in rounds 1-10 compared to rounds 11-20. As shown in Table 5, the cause of this change in payoffs is connected to change of behavior by A_2 and A_3. These agents were more likely to mimic the choice of A_1 in early rounds when compared to late rounds.

	A_2	A_3	A_4	A_5
Rounds 1-10	\$0.31 (13)	\$0.46 (27)	\$0.22 (60)	\$0.15 (100)
Rounds 11-20	\$0.65 (20)	\$0.72 (23)	\$0.43 (62)	\$0.36 (95)
p-value	0.054	0.071	0.017	<0.001

Table 6: Average earnings by early and late rounds conditional on A_1 choosing an agent's preferred proposal. Total number of observations in parentheses. p-value from Wilcoxon sign-ranked test using unpaired data. These tests compare the heights of the same colored bars across Figure 6's "Rounds 1-10" and "Rounds 11-20".

Result 9. *In late rounds, $c(A_1)$ is less decisive in determining payoffs.*

4.4 Summary of section 4

Results 3 and 4 provide strong support for a middle-mover advantage along with a first- and last-mover disadvantage. One possible cause of these advantageous positions is the existence of the upstream and downstream incentives. Results 5, 7, and 9 suggest a consistent interpretation of the relationship between responsiveness to these two incentives and experience with the competition. Inexperienced subjects are more likely to mimic the choice made by A_1 and are more likely to choose downstream proposals (P_5) which drives a significant last-mover disadvantage. However, as subjects gain experience with the competition, they are increasingly likely to choose upstream proposals (P_1) which drives a first- and last-mover disadvantage. Table 3 shows that this learning effect decreases the earnings of upstream agents (A_1 and A_2) in late rounds while it increases the earnings of downstream agents (A_4 and A_5). In addition, since equilibrium refinement plays a key theoretical role, an added benefit of the experiment is the support that it provides for using LLE in this context, particularly with experienced subjects.

While the theory and experiment agree on the advantageous positions, the choices observed in the experiment do not quantitatively fit the theoretical prediction. This is particularly true for A_1 who rarely chooses the upstream proposals (Result 6) and who earns less by choosing upstream proposals (Result 8). Clearly these results are related. If, empirically, A_1 is earning less by choosing upstream proposals then it follows that A_1 will be less likely to do so. Consider two post-hoc explanations for this behavior that could be tested in future work. First, subjects may be motivated by preferences other than strict payoff-maximization.

Subjects in the A_1 position might avoid choosing P_2 in order to reduce the probability that A_2 chooses P_1.[18] As Table 5 shows, A_1's belief about A_2's reciprocation is supported by the experiment data, particularly in the late rounds. When $c(A_1) = P_2$, $c(A_2) = P_2$ 79% of the time (26 out of 33), and 90% of the time in the last 10 rounds (18 out of 20). By contrast, when $c(A_1) \in \{P_3, P_4, P_5\}$, $c(A_2) = P_1$ 22% of the time (82 out of 367), and only 13% of the time in the first 10 rounds (25 out of 187). Second, subjects may require more rounds in order to learn the optimal strategy. As the experiment progresses, A_2 and A_3 adapt their strategy to focus more on upstream proposals, suggesting an effect of learning. This shift in behavior negatively affects A_1, who experiences the largest decrease in payoffs over the course of the experiment. It is possible that, with more rounds to learn, A_1 might alter their strategy towards choosing upstream proposals more often.

5 Conclusion

This paper models a sequential competition where agents have symmetric payoff functions and perfect information. The sequential-move model captures aspects from settings such as markets, all-pay contests, herding behavior, and voting. The model suffers from multiple equilibria, which is a common issue in the previous literature. Instead of relying upon other-regarding preferences, learning, differentiated costs, or imperfect information, this paper utilizes a well-known equilibrium refinement strategy which endows each agent with an arbitrarily small amount of noise. This Limiting Logit Equilibrium prediction challenges the standard prediction of either a first- or a last-mover advantage within a sequential-move framework. This paper's model provides a general, albeit somewhat unusual, sequential-move competition where the opposite result is observed; the first- and last-movers are the most disadvantaged. Moreover, this paper champions a new position in a sequential competition: the middle-mover (Result 1). The extent of a middle-mover advantage is theoretically quantified by calculating each agent's expected payoff based solely on their position in the decision sequence (Result 2). This paper finds that middle-movers are at such an advantage that they are theoretically predicted to earn the maximum possible payoff. This prediction is the result of agents responding to both an upstream incentive and a downstream incentive. In an openly sequential competition, agents have an incentive to marginalize agents who have either already acted (upstream) or who are many moves away from acting (downstream). The combination of these two incentives also drives an early- and late-mover disadvantage.

Data from a laboratory experiment support the prediction of a middle-mover advantage as well as a first- and last-mover disadvantage (Results 3 and 4, respectively). Additionally, the

[18] Theoretically, A_1 should anticipate this choice by A_2, as $prob(c(A_2) = P_1 | c(A_1) = P_2) = 1$.

experiment illustrates a relationship between the advantageous positions and experience with the sequential competition. Inexperienced subjects primarily respond to the downstream incentive, whereas experienced subjects are more responsive to both upstream and downstream incentives (Results 5, 7, and 9). This implies that a sequential competition with inexperienced agents will produce a last-mover disadvantage whereas the same competition with experienced agents will produce a first- and last-mover disadvantage.

This paper's main contribution is to demonstrate a sequential decision structure that defies the conventional prediction of either a first- or last-mover advantage. These results are especially enlightening when they are applied to a market or voting context. In a market setting that mimics this paper's model, firms will be most profitable if they choose to enter *after* the first competitors, but *before* the final competitors (as was empirically found in Lilien & Yoon 1990). Within a sequential voting context, the model and experiment assume that agents use their vote strictly as a way to maximize their own payoff. This approach ignores sincere policy related preferences that agents may have about which proposals deserve to be approved. The real-world analogue almost certainly contains a mix of these policy related concerns as well as strategic concerns. However, if agents are strategically motivated in a similar way as described in this paper, at least to some extent, the results of this paper predict a competition where the middle-moving voters have a significant advantage over the first- and last-movers.

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Appendix A: 3 and 4 Agent Models

Figure 7(a) shows the on-path LLE mixed-strategy prediction of the three-agent model described in Figure 1. The expected payoff for each position is represented in Figure 7(b). The LLE of the three-agent model is intuitive. A_1 will be in a 50-50 coin flip with either A_2 or A_3, and A_1's task can be viewed as a choice over who "flips the coin". Even though A_2 is (weakly) expected to earn the most, this analysis suggests that more than three agents are required in order to capture an upstream and downstream incentive.

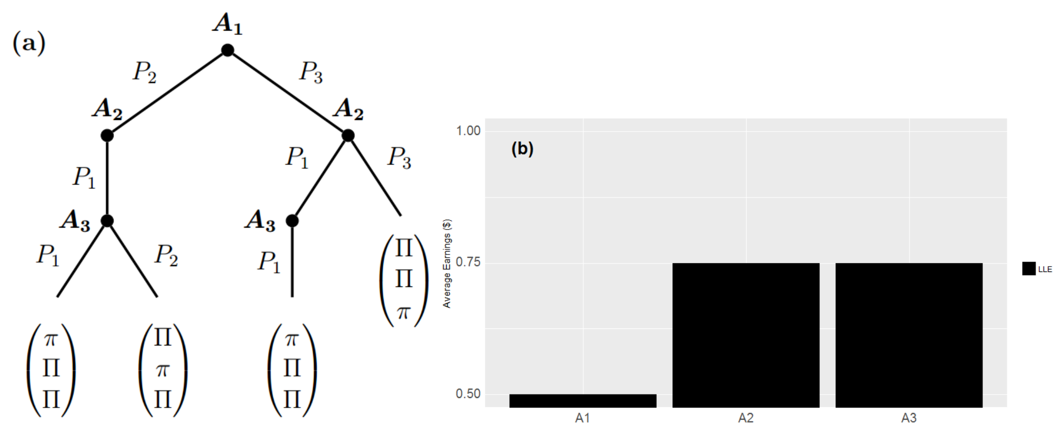

Figure 7: **(a)** On-path LLE choice probabilities for the model with 3 agents. Similar to Figure 4 in the main text. **(b)** The black bars are the theoretical expected payoff for each position with the experimental setting of $\Pi = 1$ and $\pi = 0$. The black bars in (b) are comparable to Figure 5 in the main text.

In an openly sequential competition with three agents, the LLE predicts the following expected payoff for each agent:

$$E(payoff(A_1)) = \Pi - \tfrac{1}{2}\varepsilon = \Pi - 0.5\varepsilon$$
$$E(payoff(A_2)) = \Pi - \tfrac{1}{4}\varepsilon = \Pi - 0.25\varepsilon$$
$$E(payoff(A_3)) = \Pi - \tfrac{1}{4}\varepsilon = \Pi - 0.25\varepsilon$$

Figures 8(a) and 8(b) show the on-path LLE mixed-strategy prediction and the expected payoff for each position, respectively, in this model with four agents.

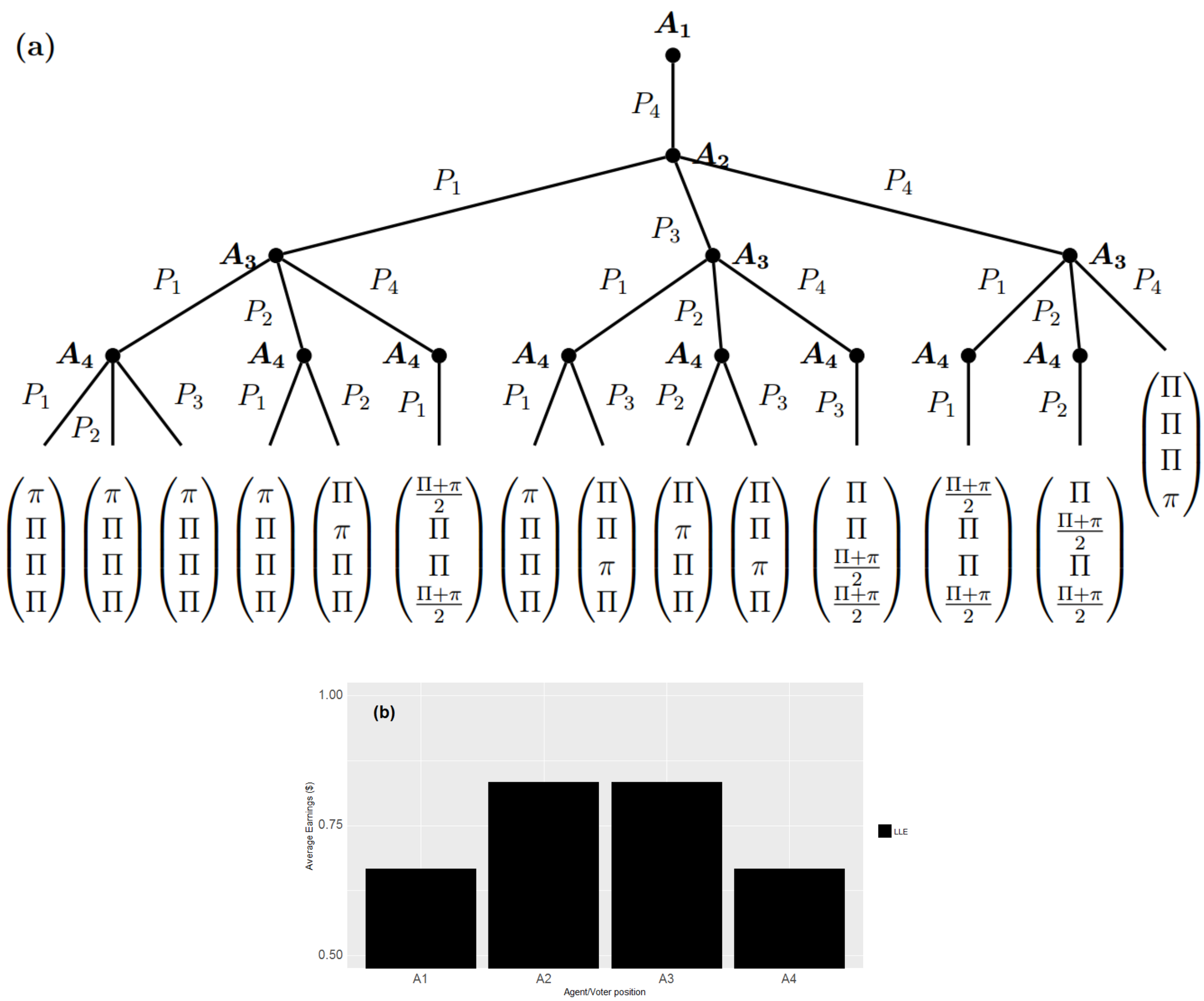

Figure 8: **(a)** On-path LLE choice probabilities for the model with 4 agents. Similar to Figure 4 in the main text. **(b)** The black bars are the theoretical expected payoff for each position with the experimental setting of $\Pi = 1$ and $\pi = 0$. The black bars in (b) are comparable to Figure 5 in the main text.

In an openly sequential competition with four agents, the LLE predicts the following expected payoff for each agent:

$$E(payoff(A_1)) = \Pi - \tfrac{1}{3}\varepsilon \approx \Pi - 0.333\varepsilon$$
$$E(payoff(A_2)) = \Pi - \tfrac{1}{6}\varepsilon \approx \Pi - 0.167\varepsilon$$
$$E(payoff(A_3)) = \Pi - \tfrac{1}{6}\varepsilon \approx \Pi - 0.167\varepsilon$$
$$E(payoff(A_4)) = \Pi - \tfrac{1}{3}\varepsilon \approx \Pi - 0.333\varepsilon$$

The LLE of the model with four agents reflects the main results of this paper - a middle-mover advantage with a first- and last-mover disadvantage. This suggests two ways in which the paper's main results are robust. First, it is less-likely that a middle-mover advantage would diminish with an increase in agents. Second, a middle-mover advantage is shown in competitions with either an odd or an even numbers of agents. The main difference between the models is in A_1's LLE choice. In the four-agent model, A_1 earns a higher expected payoff when choosing P_4 than by choosing P_2: $E(payoff(A_1|c(P_4))) = \Pi - \frac{1}{3}\varepsilon > \Pi - \frac{1}{2}\varepsilon = E(payoff(A_1|c(P_2)))$. Because of this, A_1's LLE strategy is to always follow the downstream incentive and choose P_4. One interpretation is that the first-moving agent requires more than three (other) competitors in order to credibly designate an upstream target-proposal other than P_1. As is seen in Figure 4, when the first-moving agent has four competitors, the LLE predicts that A_1 will respond to the upstream incentive by choosing P_2.

Appendix B: Expected Payoff Calculations

Model with 5 agents (see Result 2 in main text)

Agent 1:

$$E(payoff(A_1)) = \frac{\pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\pi}{16} + \frac{\pi}{16} + \frac{\pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{8} + \frac{\Pi}{8}$$
$$E(payoff(A_1)) = \frac{18\Pi+14\pi}{32}$$
$$E(payoff(A_1)) = \frac{18\Pi+14(\Pi-\varepsilon)}{32}$$
$$E(payoff(A_1)) = \Pi - \frac{14}{32}\varepsilon = \Pi - 0.4375\varepsilon$$

Agent 2:

$$E(payoff(A_2)) = \frac{\Pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\pi}{16} + \frac{\pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{8} + \frac{\Pi}{8}$$
$$E(payoff(A_2)) = \frac{25\Pi+7\pi}{32}$$
$$E(payoff(A_2)) = \frac{25\Pi+7(\Pi-\varepsilon)}{32}$$
$$E(payoff(A_2)) = \Pi - \frac{7}{32}\varepsilon = \Pi - 0.21875\varepsilon$$

Agent 3:

$$E(payoff(A_3)) = \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{8} + \frac{\Pi}{8}$$
$$E(payoff(A_3)) = \Pi$$

Agent 4:

$$E(payoff(A_4)) = \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{(\Pi+\pi)}{2\cdot 16} + \frac{\Pi}{8} + \frac{\Pi}{8}$$
$$E(payoff(A_4)) = \frac{29\Pi+3\pi}{32}$$
$$E(payoff(A_4)) = \frac{29\Pi+3(\Pi-\varepsilon)}{32}$$
$$E(payoff(A_4)) = \Pi - \frac{3}{32}\varepsilon = \Pi - 0.09375\varepsilon$$

Agent 5:

$$E(payoff(A_5)) = \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\Pi}{16} + \frac{\pi}{8} + \frac{\pi}{8}$$
$$E(payoff(A_5)) = \frac{24\Pi+8\pi}{32}$$
$$E(payoff(A_5)) = \frac{24\Pi+8(\Pi-\varepsilon)}{32}$$
$$E(payoff(A_5)) = \Pi - \frac{8}{32}\varepsilon = \Pi - 0.25\varepsilon$$

Model with 4 agents (see Appendix A)

Agent 1:

$$E(payoff(A_1)) = \frac{\pi}{27} + \frac{\pi}{27} + \frac{\pi}{27} + \frac{\pi}{18} + \frac{\Pi}{18} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{\pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{9} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{\Pi}{9} + \frac{\Pi}{9}$$
$$E(payoff(A_1)) = \frac{36\Pi+18\pi}{54}$$
$$E(payoff(A_1)) = \frac{36\Pi+18(\Pi-\varepsilon)}{54}$$
$$E(payoff(A_1)) = \Pi - \frac{18}{54}\varepsilon \approx \Pi - 0.3333\varepsilon$$

Agent 2:

$$E(payoff(A_2)) = \frac{\Pi}{27} + \frac{\Pi}{27} + \frac{\Pi}{27} + \frac{\Pi}{18} + \frac{\pi}{18} + \frac{\Pi}{9} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{\pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{9} + \frac{\Pi}{9} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{\Pi}{9}$$
$$E(payoff(A_2)) = \frac{45\Pi+9\pi}{54}$$
$$E(payoff(A_2)) = \frac{45\Pi+9(\Pi-\varepsilon)}{54}$$
$$E(payoff(A_2)) = \Pi - \frac{9}{54}\varepsilon \approx \Pi - 0.1667\varepsilon$$

Agent 3:

$$E(payoff(A_3)) = \frac{\Pi}{27} + \frac{\Pi}{27} + \frac{\Pi}{27} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{9} + \frac{\Pi}{18} + \frac{\pi}{18} + \frac{\Pi}{18} + \frac{\pi}{18} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{\Pi}{9} + \frac{\Pi}{9} + \frac{\Pi}{9}$$
$$E(payoff(A_3)) = \frac{45\Pi+9\pi}{54}$$
$$E(payoff(A_3)) = \frac{45\Pi+9(\Pi-\varepsilon)}{54}$$
$$E(payoff(A_3)) = \Pi - \frac{9}{54}\varepsilon \approx \Pi - 0.1667\varepsilon$$

Agent 4:

$$E(payoff(A_4)) = \frac{\Pi}{27} + \frac{\Pi}{27} + \frac{\Pi}{27} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{\Pi}{18} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{(\Pi+\pi)}{2\cdot 9} + \frac{\pi}{9}$$
$$E(payoff(A_4)) = \frac{36\Pi+18\pi}{54}$$
$$E(payoff(A_4)) = \frac{36\Pi+18(\Pi-\varepsilon)}{54}$$
$$E(payoff(A_4)) = \Pi - \frac{18}{54}\varepsilon \approx \Pi - 0.3333\varepsilon$$

Appendix C: Constructing Examples

Settings with either a first- or last-mover advantage can be generated by applying this paper's concept of upstream and downstream incentives. Such an approach could apply this paper's model including an additional feature that mutes or eliminates one of the two incentives; thus leaving only one disadvantaged position. Two examples are provided here.

One possible feature prohibits agents from knowing the decision sequence of agents choosing after them. As in this paper's model, agents perfectly observe the choices made before them and have perfect knowledge about the set of agents who will move after them. However, in an alternative model, agents have incomplete information about the decision sequence of the agents moving after them. This natural addition reflects settings with endogenous entry. The uncertainty about the late-movers' identity virtually eliminates the downstream incentive while having no effect on the upstream incentive. Because of this, the model will predict outcomes where late-movers prosper at the expense of early-movers (late-mover advantage).

An alternative feature could impose a small cost on the agent who is first to choose a proposal while subsequent choices on the same proposal are costless. In this model, the first-mover is guaranteed to incur this "initial investment" cost. However, the first-mover is guaranteed to receive the high payoff of Π (minus the investment cost) by choosing a proposal that is far enough down the decision sequence because early-moving agents are incentivized to free-ride and choose whichever proposal has already been chosen by the first-mover (which is now costless). Early-moving agents who were previously indifferent between responding to the upstream or downstream incentive with costless choices (in the original model), now strictly prefer following the downstream incentive. In such settings, we should expect that early-movers earn a higher average payoff than do late-movers (early-mover advantage).

Appendix D: Experiment Screen Shots

Welcome

Page 1 of 6

Welcome Message

Welcome to this experiment at UC Irvine. Thank you for participating.

You are about to participate in a study of decision-making, and you will be paid for your participation in cash, privately at the end of this session. What you earn depends partly on your decisions and partly on chance.

Please turn off your cell phone.

The entire session consists of multiple rounds. During each round, you will amass U.S. currency that will be paid to you at the end of the session. All rounds will take place through the computer terminals.

It is important that you do not communicate with any other participants during the session.

When you are ready, please click "Continue" to go to the instructions.

Continue

Basic Instructions

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Instructions Message Part 1

During each round you will be randomly placed into a group with 4 other participants. Each 5-participant group must decide how to distribute four "prizes". Each prize is worth one dollar. No participant can receive more than one prize in the same round, and the one-dollar prize cannot be split between more than one participant. This means that, in each group, and in every round, four participants will be awarded a one-dollar prize, and one participant will not.

Each 5-participant group will vote to determine which participant will NOT receive a prize. Instead of every participant voting at the same time, votes will be decided upon one at a time. It is important to note that voters are casting votes *against* other voters.

To do this, each participant in every group will be randomly assigned a voting role (Voter 1, Voter 2, Voter 3, Voter 4, or Voter 5). These voting roles represent the position in which each participant will vote. Voter 1 is the first voter, Voter 2 is the second voter, Voter 3 is the third voter, Voter 4 is the fourth voter, and Voter 5 is the fifth (and last) voter for that round.

Each round will start with Voter 1 voting against a voter (other than himself). While Voter 1 is making his/her decision, Voters 2, 3, 4, and 5 will click the "OK" button and wait until it is their turn to vote. Voter 1's decision will be shown to all five voters and then it will be Voter 2's turn to vote. While Voter 2 is making his/her decision, Voters 1, 3, 4, and 5 will click the "OK" button to wait. This process will continue for Voter 3, Voter 4, and Voter 5.

Who each voter has voted against will be displayed all five voters in the center of their screen. The number of votes that each voter has against him/her is also displayed on the bottom of the screen.

OK

Payoffs

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Instructions Message Part 2

Whichever voter has the most votes against him/her will not receive a prize in that round. Since there are only 5 people voting, if a voter has 3 votes against him/her, then he/she is sure to be chosen as the voter who will not receive a prize in that round. Because of this, if a voter has 3 votes against him/her then the round is over even if there are voters who have not yet voted in that round.

If there is a tie for the highest number of votes, then the computer will randomly choose which of the tied voters will not receive the prize. The randomization that determines who does not receive a prize is only over the voters who are tied for the highest vote.

For example, consider a scenario where Voter 1 and Voter 2 both have 2 votes against them at the end of the voting round. The computer will randomly determine which of these two voters does not receive a prize. There is a 50% chance (1 out of 2) that Voter 1 is selected to not receive a prize and there is a 50% chance (1 out of 2) that Voter 2 is selected to not receive a prize. Voters 3, 4, and 5 are guaranteed to receive a prize in that round.

Similarly, if a vote ends with every voter having one vote against him/her, then the computer randomly determines which of the five voters does not receive a prize. In this case, there is a 20% chance (1 out of 5) that any voter is selected to not receive a prize in that round.

Each round will consist of 1 voting outcome. A round ends when the votes have been cast and prizes have been awarded. At the beginning of each round you will be randomly re-grouped with participants in the room. You will also be randomly assigned a new voting number. After 20 rounds, the session ends.

OK

Comprehension Test

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Questions

To test your understanding, please answer the following questions about a possible round outcome.

Suppose that we are at the end of a round where all five votes have been cast. The following voting record represents the number of votes that each voter has against them.

Votes against Voter 1	Votes against Voter 2	Votes against Voter 3	Votes against Voter 4	Votes against Voter 5
1	2	2	0	0

The voting record will be displayed at all times during each round. In this case, the voting record tells us that Voter 1 has been voted against once, Voter 2 has been voted against twice, Voter 3 has been voted against twice, and Voters 4 and 5 have been voted against zero times.

What is the percent chance that Voter 1 will be selected to NOT receive a prize? ○ 0% ○ 20% ○ 50% ○ 100%

What is the percent chance that Voter 2 will be selected to NOT receive a prize? ○ 0% ○ 20% ○ 50% ○ 100%

What is the percent chance that Voter 5 will be selected to NOT receive a prize? ○ 0% ○ 20% ○ 50% ○ 100%

OK

Correct Comprehension Test

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You answered all of the three questions CORRECTLY! For review, please look at the explanations below.

Votes against Voter 1	Votes against Voter 2	Votes against Voter 3	Votes against Voter 4	Votes against Voter 5
1	2	2	0	0

Question 1: What is the percent chance that Voter 1 will be selected to NOT receive a prize?
Answer 1: Voter 1 is not tied for having the most votes. Because of this, Voter 1 is guaranteed to receive a prize. The percent chance that Voter 1 does not get a prize is 0%.

Question 2: What is the percent chance that Voter 2 will be selected to NOT receive a prize?
Answer 2: Voter 2 is tied for having the most votes with Voter 3. Because of this, Voter 2 has a 50% chance to be chosen to not receive a prize.

Question 3: What is the percent chance that Voter 5 will be selected to NOT receive a prize?
Answer 3: Voter 5 is not tied for having the most votes. Because of this, Voter 5 is guaranteed to receive a prize. The percent chance that Voter 5 does not get a prize is 0%.

General Explanation

The computer would randomly decide whether Voter 2 or Voter 3 will not receive a prize. Both voters have a 50% chance of not receiving a prize in this round. Voters 1, 4, and 5 are guaranteed to receive a prize in this round.

OK

Incorrect Comprehension Test

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Uh-Oh! You answered one or more of the questions incorrectly. Please read the explanations below.

Votes against Voter 1	Votes against Voter 2	Votes against Voter 3	Votes against Voter 4	Votes against Voter 5
1	2	2	0	0

Question 1: What is the percent chance that Voter 1 will be selected to NOT receive a prize?
Answer 1: Voter 1 is not tied for having the most votes. Because of this, Voter 1 is guaranteed to receive a prize. The percent chance that Voter 1 does not get a prize is 0%.

Question 2: What is the percent chance that Voter 2 will be selected to NOT receive a prize?
Answer 2: Voter 2 is tied for having the most votes with Voter 3. Because of this, Voter 2 has a 50% chance to be chosen to not receive a prize.

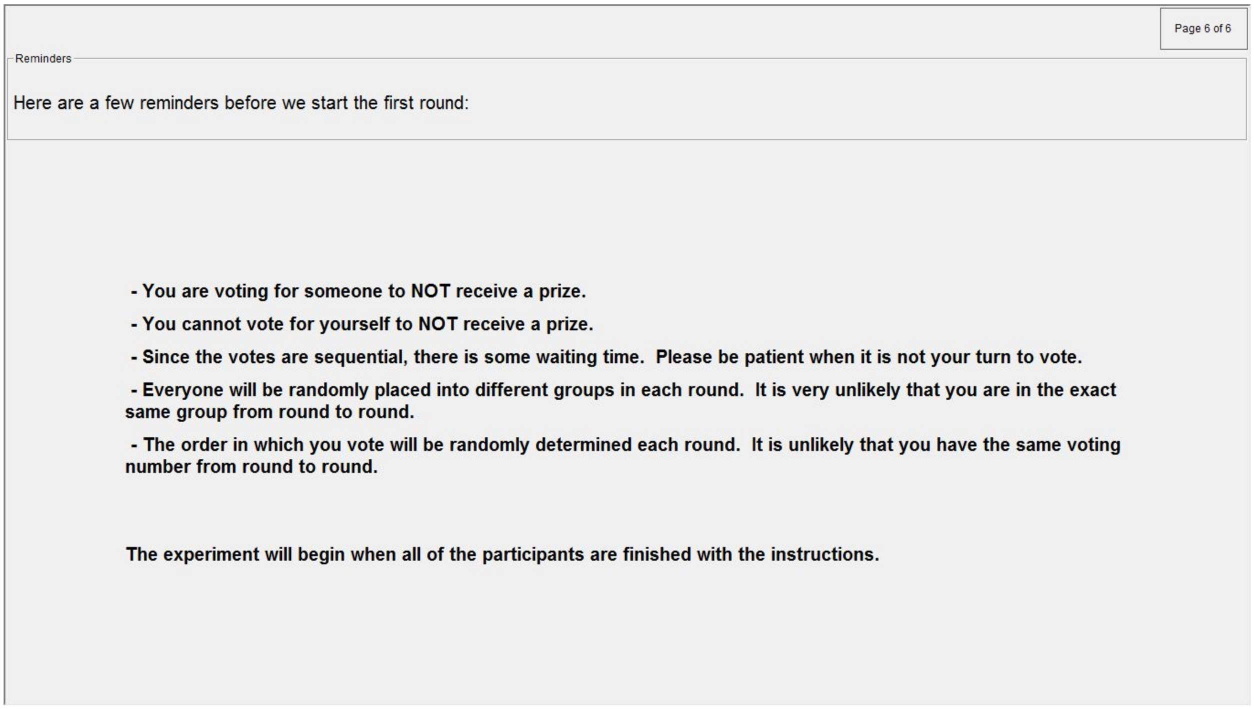
Question 3: What is the percent chance that Voter 5 will be selected to NOT receive a prize?
Answer 3: Voter 5 is not tied for having the most votes. Because of this, Voter 5 is guaranteed to receive a prize. The percent chance that Voter 5 does not get a prize is 0%.

General Explanation

The computer would randomly decide whether Voter 2 or Voter 3 will not receive a prize. Both voters have a 50% chance of not receiving a prize in this round. Voters 1, 4, and 5 are guaranteed to receive a prize in this round.

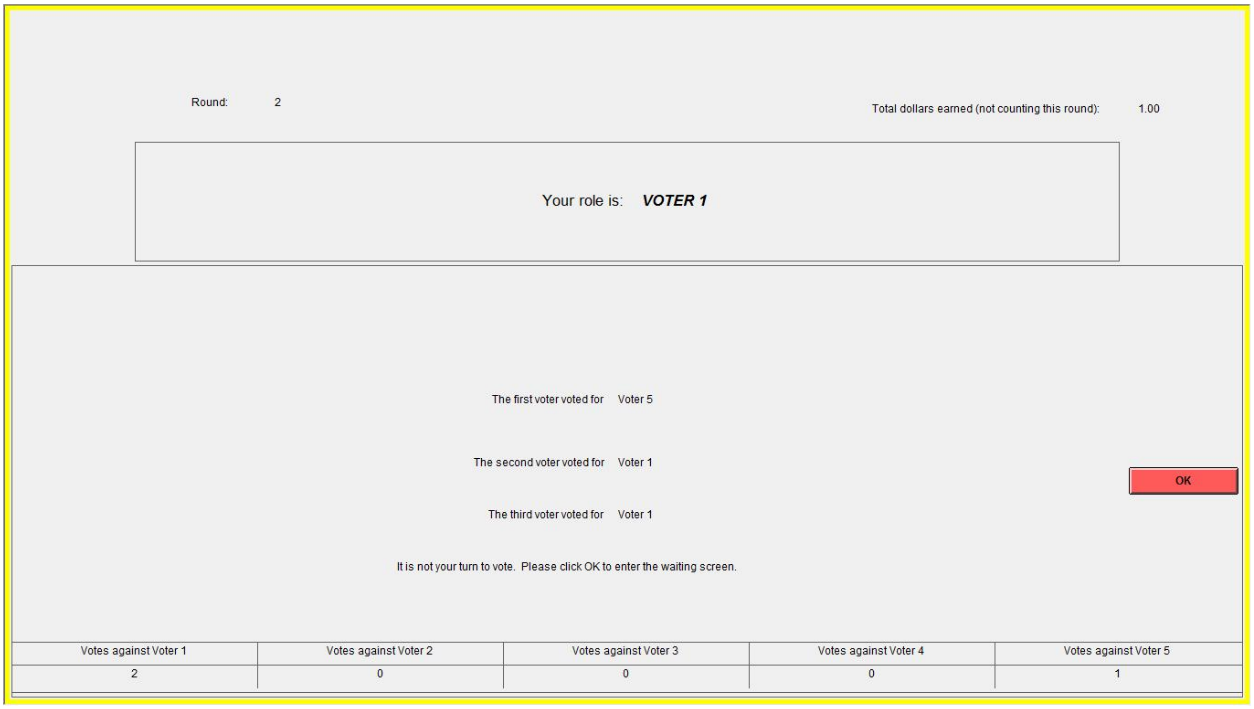
OK

Reminders



Decision

Waiting



Results