Output Feedback Robust Distributed Model Predictive Control for Parallel Systems in Process Networks with Competitive Characteristics

Shuzhan Zhang[1], Dongya Zhao[1*], Sarah K. Spurgeon[2]
1. College of New Energy, China University of Petroleum (East China), Qingdao, China, 266580;
2. Department of Electronic & Electrical Engineering, University College London, Torrington Place, London WC1E 7JE, UK.
$*$ Corresponding author's Email: dyzhao@upc.edu.cn

Abstract

The parallel structure is one of the basic system architectures found in process networks. This paper formulates control strategies for such parallel systems when the states are unmeasured. The competitive coupling and competitive constraints are addressed in the control design. A distributed buffer and pre-estimator are proposed to solve problems relating to coupling and timely communication whilst a distributed moving horizon estimator is employed to further improve the estimation accuracy in the presence of the constraints. An output feedback robust distributed model predictive control algorithm is then developed for such parallel systems. The Lyapunov method is used for the theoretical analysis which produces tractable linear matrix inequalities (LMI). Simulations and experimental results are provided to validate the effectiveness of the proposed approach.

Key Words: Parallel system, robust distributed model predictive control, estimation, output feedback

I. Introduction

In modern industrial chemical plants, a process network is comprised of many process units arranged in a complex structure [1]. Such a process network can be divided into elements with a series structure and elements with a parallel structure depending on the process interactions. A series structure is characterised by each subsystem being connected in series [2] so that the output of the former subsystem is the input of the latter subsystem [3], [4], [5]. The parallel structure covers the case where all subsystems are connected in parallel i.e. the inputs are obtained from the same bus while the corresponding outputs converge to another bus. In this case each subsystem is competing with the other subsystems because of resource limitations and there are couplings between information, mass and energy among all the subsystems. These couplings and constraints in the parallel system architecture are different from those in a series system. It is necessary to clearly describe the couplings and constraints within a parallel system before embarking on a control design [6], [7]. The authors have previously studied the parallel system and proposed a state-feedback based distributed model predictive control (DMPC) in which the competitive couplings and constraints have been initially defined to describe characteristics of the parallel system [8]. Assume that two subsystems are connected in parallel in the same bus. It is obvious that the sum of the inputs of the two subsystems must be less than or equal to the total input of the whole system. When the sum of the inputs of the two subsystems is equal to the total input of the whole system (this is the most common case in practice[9], [10]), it is the case that when the input of one subsystem increases, the input of the other subsystem must be correspondingly reduced. If the outputs of each subsystem are also connected in parallel, the outputs have the same characteristics. The system will not achieve the control objective and may even become unstable if the competitive couplings and constraints are not addressed appropriately.

The authors have solved the competitive coupling and competitive constraint problems by designing a state feedback DMPC for a parallel system in [8]. In this paper all of the states are assumed to be measurable. Note that it is almost impossible to measure all the states in practice. In this case a state observer may be a good choice to recover the unmeasured states. There are many output feedback MPC methods that have been developed [11], [12], [13]. Most of these approaches use observers in the control design, such as, Kalman filter [14], tube-based minimax observer [15], Luenberger observer [16], [17] and a moving horizon observer [18]. The Kalman filter approach uses the state equation and observer equation to obtain an optimal solution according to the linear unbiased minimum mean square error estimation criterion [19]. The tube-based minimax observer employs local feedback around a nominal or reference trajectory and keeps the trajectories resulting from the disturbance sequences in a possibly small neighbourhood of the nominal trajectory [20]. The closed loop poles of the error system when a Luenberger observer is used have negative real parts which ensures that the observation error converges to zero asymptotically [21]. However, these observers cannot address the constraints in a parallel system effectively and solutions may not be optimal. Note that moving horizon estimation (MHE) can cope with system constraints [22] and can make full use of the known information about the constraints to improve the accuracy of observation [23]. In essence, MHE is an online optimization method based on the most recent data [24] whereby the constraints can be expressed directly in the optimization to reformulate a quadratic program [25]. The approach has been successfully applied in practical process networks; for example, an MHE strategy has been proposed for detectable linear systems to solve the constraint problems in [26]. Note that most of the mentioned designs are for centralized control and the couplings among the subsystems have not been considered.

Output feedback distributed model predictive control (OFDMPC) is worthy of attention. There are two main problems to be solved when the output feedback control is distributed, one is the coupling problem, the other is the problem of timely communication. To date, very little literature has reported work in this field. An OFDMPC algorithm has been presented for a polytopic uncertain system subject to randomly occurring actuator saturation and packet loss [27]. In this work the authors assumed that the system parameters could be designed in advance so that the couplings can be known apriori and without any need for state estimation; the work has not considered the timely communication problem. A cooperative DMPC has been proposed for a class of large-scale systems composed of discrete-time linear subsystems which are coupled via the states [28]. Each subsystem was associated with a local MPC unit, a local predictor and a local observer. The coupling problem was solved by these local model predictive controllers which exchanged predicted input sequences via a delayed communication network. An OFDMPC algorithm has been proposed for a team of linear discrete-time subsystems which are coupled by the cost function [29]. Here the optimisation problem was reformulated despite the presence of couplings of cost function and dynamic state couplings.

Although the above work has addressed OFDMPC issues, it has not considered timely communication amongst the subsystems and the characteristics of the parallel system as defined in [8]. There will be considerable computation and even degraded control performance if the competitive couplings and competitive constraints of parallel systems are not dealt with appropriately.

In order to design OFDMPC for parallel systems, the competitive couplings and competitive constraints are defined by using observed states and predicted control laws. Then, a data buffer is introduced which can store the most recent data. Before each iteration, this data can be used to generate a pre-estimate and to give an initial value for the MHE iteration at the current instant. In this way state information can be transferred to other subsystems in a timely way. The MHE approach is well-aligned with the proposed characteristics of the buffer. In this paper, an OFDMPC is derived for a parallel system which may be subject to uncertainty. The current states are estimated by distributed MHE (DMHE). By taking this route, a formulation of OFDMPC for parallel systems is established which possesses the following core features: competitive couplings and competitive constraints are described using the observed states of the parallel system; a pre-estimator is used for making a preliminary estimate of the states and the predictor is used for making an initial prediction of the control law. This information will be sent to the corresponding buffers which are proposed to store the most recent data about the competitive coupling and competitive constraints in the parallel system. Then the controller can use this information to calculate competitive couplings. The DMHE can use these pre-estimated states to further estimate the current states and improve the accuracy. The performance of the OFDMPC is guaranteed when the controller is applied to the parallel system. For this special class of system, application of the result and verification of the underlying assumptions are computationally tractable. The following is then achieved: A robust output feedback DMPC is proposed based on the predicted and estimated states. The robust stability of the closed loop parallel system is analyzed. A step by step control algorithm is given to realize its straightforward implementation. Finally, the effectiveness and performance is validated by extensive simulations and an experimental trial. When compared with the previous results of references [27] [28] [29], the main advantages are focused on (i) the competitive couplings can be calculated by using the pre-estimators and predictors. (ii) The information is communicated in a timely fashion among all the subsystems by using the buffers.

The paper is organized as follows. In Section II the parallel system with state estimation problem is formulated and the essential assumptions and definitions are given. In Section III the estimation errors are analysed. The ROFDMPC algorithm is proposed and its stability is addressed in Section IV. The results of simulations and an experimental trial are demonstrated in Section V to validate the proposed approach. Finally, some conclusions are drawn in Section VI.

II. Problem formulation

Consider the linear discrete time parallel system composed of N subsystems coupled via states and inputs in Figure 1. Subsystem i can receive information from all the other $(N-1)$ subsystems. The dynamic model with uncertainties in the ith subsystem is given by the following:

$$\begin{cases} x_i(k+1) = A_{ii}x_i(k) + B_{ii}u_i(k) + D_iw_i(k) \\ \qquad\qquad + \sum\limits_{j=1,j\neq i}^{N} [{\lambda_{ij}}^T(x_i(k) - \chi_{ij}x_j(k))A_{ij}x_j(k) + {\delta_{ij}}^T(u_i(k) - \sigma_{ij}u_j(k))B_{ij}u_j(k)] \\ y_i(k) = C_ix_i(k) + v_i(k) \end{cases} \tag{1}$$

where $x_i(k) \in \mathbb{X}_i \subseteq \mathbb{R}^{n_{x_i}}$ is the state vector, $u_i(k) \in \mathbb{U}_i \subseteq \mathbb{R}^{n_{u_i}}$ is the control input and $y_i(k) \in \mathbb{Y}_i \subseteq \mathbb{R}^{n_{y_i}}$ is the output vector, $w_i(k) \in \mathbb{W}_i \subseteq \mathbb{R}^{n_{x_i}}$ is an unknown disturbance and $v_i(k) \in \mathbb{V}_i \subseteq \mathbb{R}^{n_{y_i}}$ is measurement noise. $\mathbb{X}_i$, $\mathbb{U}_i$ and $\mathbb{Y}_i$ are polyhedral and polytopic constraint sets, respectively, $\mathbb{W}_i$ and $\mathbb{V}_i$ are $\mathbb{C}$-sets. $A_{ii} \in \mathbb{R}^{n_{x_i}\times n_{x_i}}$, $B_{ii} \in \mathbb{R}^{n_{x_i}\times n_{u_i}}$, $A_{ij} \in \mathbb{R}^{n_{x_i}\times n_{x_j}}$, $B_{ij} \in \mathbb{R}^{n_{x_i}\times n_{u_j}}$, $C_i \in \mathbb{R}^{n_{y_i}\times n_{x_i}}$, $D_i \in \mathbb{R}^{n_{x_i}\times n_{w_i}}$, the pairs (A_{ii}, B_{ii}) are assumed to be controllable and (A_{ii}, C_i) are assumed

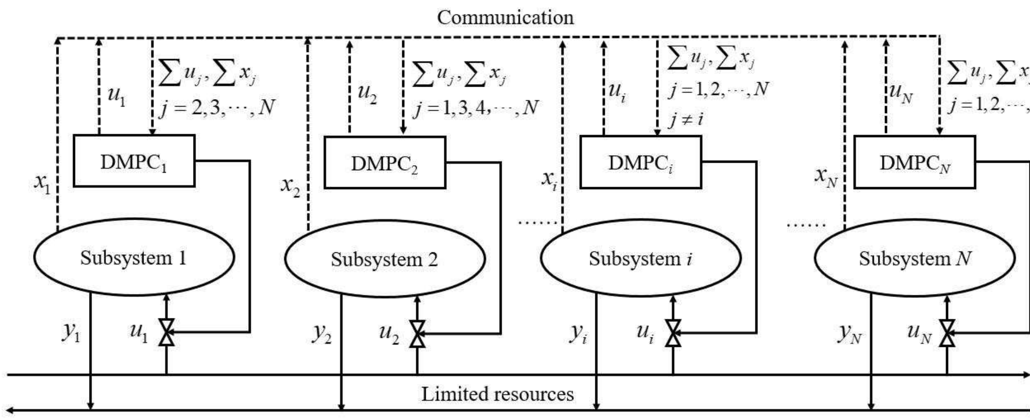

Fig. 1: Parallel structure in process networks.

to be observable. $\sigma_{ij} \in \mathbb{R}^{n_{u_i} \times n_{u_j}}$, $\chi_{ij} \in \mathbb{R}^{n_{x_i} \times n_{x_j}}$, $i, j = 1, \cdots, N, j \neq i$ are weighting matrices representing the competitive strength of the control input and system state respectively, $\delta_{ij} \in \mathbb{R}^{n_{u_i}}$ $\lambda_{ij} \in \mathbb{R}^{n_{x_i}}$, $i, j = 1, \cdots, N, j \neq i$ are weighting vectors of the competitive coupling of the control input and system state respectively where all elements in the vectors are positive.

Definition 1: For the ith subsystem of the parallel system, define

$$c_i^S = \sum_{j=1, j\neq i}^{N} {\lambda_{ij}}^T (x_i - \chi_{ij} x_j) A_{ij} x_j(k) \tag{2}$$

and

$$c_i^I = \sum_{j=1, j\neq i}^{N} {\delta_{ij}}^T (u_i - \sigma_{ij} u_j) B_{ij} u_j(k) \tag{3}$$

as the competitive couplings.

Definition 2: The system with competitive coupling must satisfy $\sum \|u_i\| \leq \|u\|$, $\sum \|y_i\| \leq \|y\|$, ${\delta_{ij}}^T (u_i - \sigma_{ij} u_j) > 0$ and ${\lambda_{ij}}^T (x_i - \chi_{ij} x_j) > 0$, where u is the total input, y is the total output, $i, j = 1, \cdots, N$. These constraints are called competitive constraints.

Remark 1: In a parallel system under competitive constraints, when $u_j(k)$ increases , $\sum_{i=1, i\neq j}^{N-1} u_i$ may need to reduce because of the limited total input. The degree of reduction of $u_i(k)$ is affected by the weighting of the competitive strength σ_{ij} and the weighting of the competitive coupling δ_{ij}. The outputs have the same characteristics, that is, when $y_j(k)$ increases, $\sum_{i=1, i\neq j}^{N-1} y_i$ may need to reduce.

The subsystems are assumed to exchange information via a communication network. In the proposed method, each $DMPC_i$ packages its state information and predictive control sequence into one packet with a time-label and then sends it to the other subsystems over the network. Because the states are unmeasured, the competitive couplings c_i^S and c_i^I cannot know the state information corresponding to the other subsystems until the information has been estimated. This increases the communication time. In order to solve this communication problem, a pre-estimator and corresponding buffer strategy are proposed in this paper. The details are shown in Figure 2. In Figure 2, each local controller contains a pre-estimator, a predictor and a buffer. The buffer consists of two parts which correspond to the $(k-1)$th part and the (k)th part. For subsystem i, the buffer is named $buffer_i$. Following implementation of the control laws at sampling time $(k-1)$, all necessary information on the subsystems including $u_j(k-1)$, $u_i(k-1)$, $\hat{x}_i(k-1)$ and $\hat{x}_j(k-1)$ is sent and stored in the $(k-1)$th part of $buffer_i$, where $\hat{x}_i(k-1)$ and $\hat{x}_j(k-1)$ are the states estimated by the DMHE at sampling time $(k-1)$. At sampling time k and before calculating the control, the $predictor_i$ can use this information from the $buffer_i$ to preliminarily estimate the state $x_i^{ipre}(k)$ and control law $u_i^{ibuf}(k)$. Then the $Pre-estimator_i$ can use $x_i^{ipre}(k)$ and $u_i^{ibuf}(k)$ to estimate the states which are defined as $x_i^{ibuf}(k)$. After that, $x_i^{ibuf}(k)$ and $u_i^{ibuf}(k)$ are sent to the (k)th part of all the buffers. Meanwhile, the $buffer_i$ receives $x_j^{ibuf}(k)$ and $u_j^{ibuf}(k)$ from the jth subsystem ($j \neq i, j = 1, \cdots, N$). Then the competitive couplings can be calculated by using these packets. The $DMHE_i$ further estimates the current states on the basis of this pre-estimation. The following dynamic model

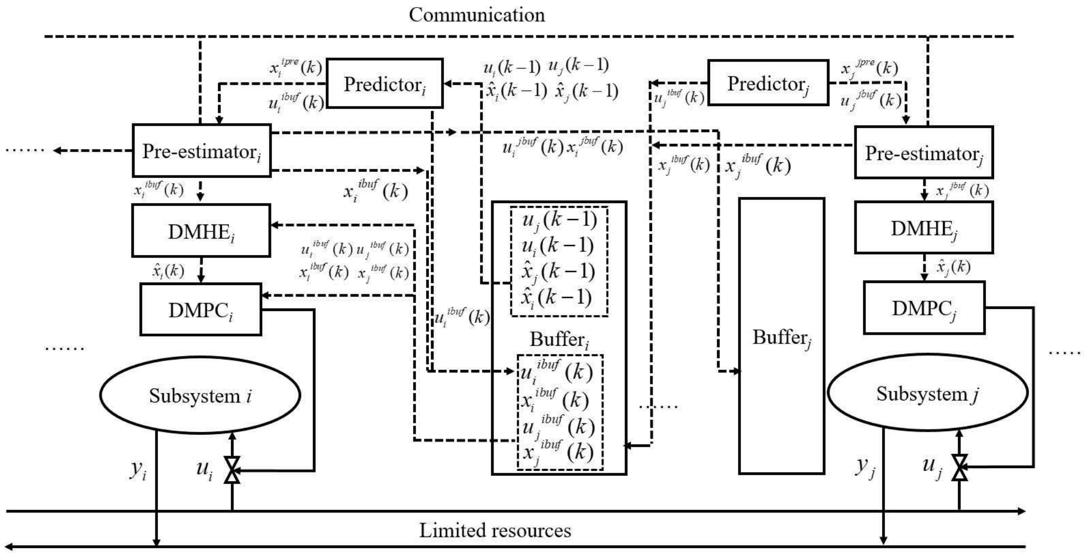

Fig. 2: The structure of the control system.

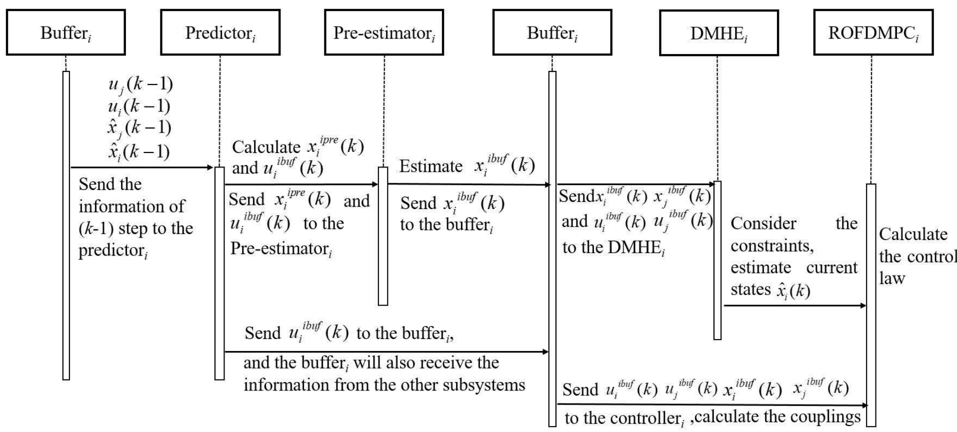

Fig. 3: The sequence diagram.

for each subsystem i can be rewritten as:

$$
\begin{aligned}
x_i(k+1) &= A_{ii}x_i(k) + B_{ii}u_i(k) + D_i w_i(k) \\
&\quad + \sum_{j=1, j\neq i}^{N} [\lambda_{ij}{}^T(x_i^{ibuf}(k) - \chi_{ij}x_j^{ibuf}(k))A_{ij}x_j^{ibuf}(k) + \delta_{ij}{}^T(u_i^{ibuf}(k) - \sigma_{ij}u_j^{ibuf}(k))B_{ij}u_j^{ibuf}(k)] \\
y_i(k) &= C_i x_i(k) + v_i(k)
\end{aligned} \tag{4}
$$

where $u_i^{ibuf}(k)$ and $x_i^{ibuf}(k)$ are the predicted input and pre-estimated state of the ith subsystem, $u_j^{ibuf}(k)$ and $x_j^{ibuf}(k)$ are the predicted input and pre-estimated state of the jth subsystem, which are all stored in $buffer_i$.

The following assumptions are given:

Assumption 1: The controllers are synchronous.

Assumption 2: The controllers communicate only once within a sampling interval.

These assumptions are not restrictive. Assumption 1 is not unduly strong because in process control systems the sampling interval is long enough when compared to the computational time. Assumption 2 is appropriate because a single information exchange within a sampling interval is consistent with the requirement of minimizing the amount of data exchange via the network. Under these two assumptions, the logic of one control period can be shown in the sequence diagram Figure 3. It is clear that the coupling and timely communication problem can be solved by the buffer and the pre-estimator. Further the estimation accuracy can be improved by the $DMHE$.

Remark 2: The pre-estimated states $x_i^{ibuf} i = 1, 2, ..., N$ are calculated by the pre-estimator and the predictor can predict the control law $u_i^{ibuf} i = 1, 2, ..., N$. This information can be used to calculate the competitive coupling. Then, the DMHE is designed to accommodate the constraints and improve the estimation accuracy. Finally, a robust output feedback distributed model predictive controller is designed based on the estimated states.

III. State estimation

In this section, the pre-estimator is initially designed. Then, a DMHE is designed and the bounding sets for the DMHE errors are derived. Finally, the bounding sets for the pre-estimation errors and the overall estimation errors of the parallel system are derived.

A. Pre-estimator

To pre-estimate the states, it is assumed that there exists a Luenberger type pre-estimator for each subsystem. Consider the system defined by (1), the ith pre-estimator can be designed as:

$$\begin{aligned} x_i^{ibuf}(k+1) &= A_{ii}x_i^{ibuf}(k) + B_{ii}u_i(k) + L_i[y_i(k) - C_i x_i^{ibuf}(k)] \\ &+ \sum_{j=1,j\neq i}^{N} [\lambda_{ij}{}^T(x_i^{ibuf}(k) - \chi_{ij}x_j^{ibuf}(k))A_{ij}x_j^{ibuf}(k) + \delta_{ij}{}^T(u_i^{ibuf}(k) - \sigma_{ij}u_j^{ibuf}(k))B_{ij}u_j^{ibuf}(k)] \end{aligned} \tag{5}$$

where $x_i^{ibuf}(k)$ is the current estimate of $x_i(k)$ generated by the ith pre-estimator and L_i is the pre-estimator gain matrix. Note that the current estimate $x_i^{ibuf}(k)$ needs state information from all the other subsystems to calculate the coupling terms $\sum_{j=1,j\neq i}^{N} [\lambda_{ij}{}^T(x_i^{ibuf}(k) - \chi_{ij}x_j^{ibuf}(k))A_{ij}x_j^{ibuf}(k)]$. However, initially at the kth instant, $x_i^{ibuf}(k), i = 1, 2, \cdots, N$ is unknown. The $x_i^{ibuf}(k), i = 1, 2, \cdots, N$ in the coupling terms of (5) is replaced by $x_j^{ipre}(k), i = 1, 2, \cdots, N$, in which $x_i^{ipre}(k)$ is calculated by a local predictor. The predictor uses the explicit form of neighboring subsystems and is given by:

$$\begin{aligned} x_i^{ipre}(k) &= A_{ii}\hat{x}_i(k-1) + B_{ii}u_i(k-1) \\ &+ \sum_{j=1,j\neq i}^{N} \lambda_{ij}{}^T[\hat{x}_i(k-1) - \chi_{ij}\hat{x}_j(k-1)]A_{ij}\hat{x}_j(k-1) \\ &+ \sum_{j=1,j\neq i}^{N} \delta_{ij}{}^T[u_i(k-1) - \sigma_{ij}u_j(k-1)]B_{ij}u_j(k-1) \end{aligned} \tag{6}$$

where $\hat{x}_i(k-1)$ and $u_i(k-1)$ for all $i = 1, 2, \cdots, N$ are known at time step k and are stored in the $(k-1)$th part of the buffers. Then $x_i^{ipre}(k)$ can be calculated by (6) and these states are sent to the ith predictor to predict the input $u_i^{ibuf}(k)$. The predictor can be designed by using the method of [8] and the following dynamic model:

$$\begin{aligned} x_i^{ipre}(k+1) &= A_{ii}x_i^{ipre}(k) + B_{ii}u_i^{ibuf}(k) \\ &+ \sum_{j=1,j\neq i}^{N} [\lambda_{ij}{}^T(x_i^{ipre}(k) - \chi_{ij}x_j^{ipre}(k))A_{ij}x_j^{ipre}(k) + \delta_{ij}{}^T(u_i^{ibuf}(k) - \sigma_{ij}u_j^{ibuf}(k))B_{ij}u_j^{ibuf}(k)] \end{aligned} \tag{7}$$

where $u_j^{ibuf}(k)$ is the input of the jth subsystem. In (7), $u_i^{ibuf}(k) = K_i x_i^{ipre}(k)$, K_i can be calculated by using the method proposed in [8]. After $x_i^{ipre}(k), i = 1, 2, \cdots, N$ and $u_i^{ibuf}(k), i = 1, 2, \cdots, N$ are calculated by (6) and (7) respectively, $x_i^{ibuf}(k)$ can be estimated by the following estimator:

$$\begin{aligned} x_i^{ibuf}(k+1) &= A_{ii}x_i^{ibuf}(k) + B_{ii}u_i(k) + L_i[y_i(k) - C_i x_i^{ibuf}(k)] \\ &+ \sum_{j=1,j\neq i}^{N} [\lambda_{ij}{}^T(x_i^{ipre}(k) - \chi_{ij}x_j^{ipre}(k))A_{ij}x_j^{ipre}(k) + \delta_{ij}{}^T(u_i^{ibuf}(k) - \sigma_{ij}u_j^{ibuf}(k))B_{ij}u_j^{ibuf}(k)] \end{aligned} \tag{8}$$

Then, the pre-estimated state sequences $x_i^{ibuf}(k)$ and the predicted control sequences $u_i^{ibuf}(k)$ are obtained. They are stored in the (k)th part of $buffer_i$ and used to calculate the competitive couplings. The above process describes the selection mechanism of $buffer_i$ which can be summarized in the following algorithm.

Algorithm 1:

Step 1 ($(k-1)$th part-update): At time step $k-1$, after implementation of the control actions, $buffer_i$ receives the state sequences $\hat{x}_j(k-1)$ and the predicted control sequences $u_j(k-1)$ from the other subsystems $j=1,\cdots,N, j\neq i$, and stores $\hat{x}_i(k-1)$ and $u_i(k-1)$ from the ith subsystem. These valid packets are written in the $(k-1)$th part of $buffer_i$.

Step 2 (Prediction): At time step k, initially the information in the $(k-1)$th part of $buffer_i$ is sent to the $predictor_i$, and $x_i^{ipre}(k)$ can be calculated by (6) with the information. Then, $u_i^{ibuf}(k)$ can be predicted by using the method of [8] and the dynamic model (7).

Step 3 (Pre-estimation): The Luenberger type pre-estimator (8) is solved to obtain $x_i^{ibuf}(k)$ with the information of $x_i^{ipre}(k)$ and $u_i^{ibuf}(k)$.

Step 4 ((k)th part-update): The pre-estimated state sequences $x_i^{ibuf}(k)$ and the predicted control sequences $u_i^{ibuf}(k)$ are stored in the (k)th part of $buffer_i$. Meanwhile, $buffer_i$ receives $x_j^{ibuf}(k)$ and $u_j^{ibuf}(k)$ from the other subsystems, $j=1,\cdots,N, j\neq i$. The information in the (k)th part of all buffers is used to calculate the competitive couplings.

Remark 3: In this paper, the predictor uses the DMPC which is proposed in [8] to predict the input. This method can be used since following the calculation from (6), all the state information ($x_i^{ipre}(k), i=1,2,\cdots,N$) can be obtained. The corresponding state estimation errors will be analyzed in subsection 3.3.

B. Distributed moving horizon estimator

To improve the estimation of the states, a distributed moving horizon estimator is designed for each subsystem by solving the following constrained optimization problem at each time step k:

$$\min_{\hat{x}_i(k-Ne_i)} \frac{1}{2}\left(\sum_{l=k-Ne_i}^{k} \|y_i(l)-C_i\hat{x}_i(l)\|^2 + \|\hat{x}_i(k-Ne_i)-x_i(k-Ne_i)\|^2_{\vartheta_i}\right) \tag{9}$$

s.t.

$$\begin{aligned} \hat{x}_i(l+1\,|k) &= A_{ii}\hat{x}_i(l\,|k) + B_{ii}u_i(l\,|k) + D_i w_i(l\,|k) \\ &\quad + \sum_{j=1,j\neq i}^{N} R_{ij}^{ibuf}(l\,|k)A_{ij}x_j^{ibuf}(l\,|k) + \sum_{j=1,j\neq i}^{N} P_{ij}^{ibuf}(l\,|k)B_{ij}u_j^{ibuf}(l\,|k) + L_i[y_i(l\,|k)-\hat{y}_i(l\,|k)] \\ l &= k-Ne_i,...,k-1 \end{aligned} \tag{10}$$

$$\begin{aligned} \hat{y}_i(l\,|k) &= C_i\hat{x}_i(l\,|k) \\ l &= k-Ne_i,...,k \end{aligned} \tag{11}$$

$$\begin{aligned} &\sum \|u_i\| \le \|u\| \\ &\sum \|\hat{x}_i\| \le \|\hat{x}\| \\ &\sum \|\Psi_i y_i\| \le \|\psi\| \\ &{\delta_{ij}}^T(u_i-\sigma_{ij}u_j) > 0 \\ &{\lambda_{ij}}^T(\hat{x}_i-\rho_{ij}\hat{x}_j) > 0 \end{aligned} \tag{12}$$

where $R_{ij}^{ibuf}(l\,|k) = {\lambda_{ij}}^T[x_i^{ibuf}(l\,|k)-\chi_{ij}x_j^{ibuf}(l\,|k)]$ and $P_{ij}^{ibuf}(l\,|k) = {\delta_{ij}}^T[u_i^{ibuf}(l\,|k)-\sigma_{ij}u_j^{ibuf}(l\,|k)]$ are scalars, (Ne_i+1) is the estimation horizon, l is an positive integer, $\hat{x}_i(k)$ is the current estimate of $x_i(k)$ by the $DMHE_i$, ϑ_i is a nonnegative weight for the $DMHE_i$, $\Psi_i \in \mathbb{R}^{n_{y_i}\times n_{y_i}}$ is a matrix which is related to competitive coupling, $\psi_{is}>0$, $i\in\{1,2,...,N\}$, $s\in\{1,2,...,n_{y_i}\}$. $\hat{x}_i(l\,|k)$ denotes the predicted value of $\hat{x}_i$ at time step l calculated at time step k. The optimal solution of (9)-(12) is shown by $\hat{x}_i(k-Ne_i)$ and the initial value of $\hat{x}_i(k-Ne_i)$ is provided by the pre-estimator. An optimal sequence of the states is obtained from (9) in which the current optimal state estimate of subsystem i is denoted by $\hat{x}_i(k)$. In (9), $\|\hat{x}_i(k-Ne_i)-x_i(k-Ne_i)\|^2_{\vartheta_i}$ is the arrival cost.

The optimization problem (9) can be rewritten as the following convex Quadratic Program (QP):

$$\begin{aligned} &\min_{z_i} \tfrac{1}{2}(\hat{x}_i(k-Ne_i))^T H_{i1}\hat{x}_i(k-Ne_i) + {H_{i2}}^T\hat{x}_i(k-Ne_i) + r_i \\ &s.t. \qquad G_i\hat{x}_i(k-Ne_i) \le \Xi_i \end{aligned} \tag{13}$$

where G_i and Ξ_i are constant matrices with appropriate dimensions representing the constraints of (10)-(12). Here, $\hat{x}_i(k-Ne_i)$ is an unknown vector of optimization (9) and r_i is a constant term. The corresponding matrices H_{i1} and H_{i2} in (13) are:

$$H_{i1} = (A_i(Ne_i))^T A_i(Ne_i) + \vartheta_i \tag{14}$$

$$H_{i2}{}^T = -(y_i)^T(S_i(Ne_i))^T A_i(Ne_i) + (u_i)^T(B_i(Ne_i))^T A_i(Ne_i) + \sum_{j=1}^{N} (x_j^{ibuf})^T (A_{ij}(Ne_i))^T A_i(Ne_i) - (\hat{x}_i(k-Ne_i))^T \vartheta_i \tag{15}$$

where

$$A_i(Ne_i) \triangleq \begin{bmatrix} C_i \\ C_i A_{iL} \\ \vdots \\ C_i(A_{iL})^{Ne_i} \end{bmatrix}, A_{ij}(Ne_i) \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_i A_{ij} & 0 & \cdots & 0 \\ C_i A_{iL} A_{ij} & C_i A_{ij} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_i(A_{iL})^{Ne_i-1} A_{ij} & C_i(A_{iL})^{Ne_i-2} A_{ij} & \cdots & C_i A_{ij} \end{bmatrix}$$

$$B_i(Ne_i) \triangleq \begin{bmatrix} C_i \\ C_i B_{iL} \\ \vdots \\ C_i(B_{iL})^{Ne_i} \end{bmatrix}, B_{ij}(Ne_i) \triangleq \begin{bmatrix} 0 & 0 & \cdots & 0 \\ C_i B_{ij} & 0 & \cdots & 0 \\ C_i A_{iL} B_{ij} & C_i B_{ij} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_i(A_{iL})^{Ne_i-1} B_{ij} & C_i(A_{iL})^{Ne_i-2} B_{ij} & \cdots & C_i B_{ij} \end{bmatrix}$$

$$L_i(Ne_i) \triangleq \begin{bmatrix} 0 & 0 & \cdots & \cdots & 0 \\ C_i L_i & 0 & \cdots & \cdots & 0 \\ C_i A_{iL} L_i & C_i B_{ij} & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ C_i(A_{iL})^{Ne_i-1} L_i & C_i(A_{iL})^{Ne_i-2} L_i & \cdots & C_i L_i & 0 \end{bmatrix}$$

$$S_i(Ne_i) \triangleq I - L_i(Ne_i)$$

$$A_{iL} = A_{ii} - L_i C_i$$

In (15), for updating matrix H_{i2}, the current and past measured outputs, past inputs, past predicted state and a prior estimate of the calculated state $\hat{x}_i(k-Ne_i)$ are needed.

Now defining the error between the actual and estimated states as $\tilde{x}_i(k-Ne_i) := x_i(k-Ne_i) - \hat{x}_i(k-Ne_i)$, the following proposition is given.

Proposition 1: For each subsystem i, $\tilde{x}_i(k-Ne_i)$ is bounded and there exists a $\mathbb{C}$-set $\tilde{\mathbb{X}}_i$ that if $\tilde{x}_i(k=0) \in \tilde{\mathbb{X}}_i$ then $\tilde{x}_i(k) \in \tilde{\mathbb{X}}_i$ for all $k > 0$.

Proof. Firstly, a dynamic equation based on the QP Active Set Strategy for $\tilde{x}_i(k)$ is pursued. Applying the Karush-Kuhn-Tucker (KKT) conditions [30] to the optimization problem (9) yields

$$\begin{aligned} &\hat{x}_i(k-Ne_i) = -H_{i1}{}^{-1}(H_{i2} + G_{iA}^T \lambda_{iA}) \\ &\lambda_{iA} = -(G_{iA} H_{i1}{}^{-1} G_{iA}^T)^{-1}(G_{iA} H_{i1}{}^{-1} H_{i2} + \Xi_{iA}) \\ &\lambda_{iA} > 0 \end{aligned} \tag{16}$$

where λ_{iA}, G_{iA} and Ξ_{iA} are active Lagrange multipliers and the corresponding matrices, respectively. Notice that from (14), if $\vartheta_i \geq 0$, then $H_{i1} > 0$ and $H_{i1}{}^{-1}$ exists. Substitute (14)-(15) into (16)

$$\begin{aligned} \tilde{x}_i(k-Ne_i) &= \bar{A}_{ie}\tilde{x}_i(k-Ne_i-1) + \bar{D}_{iw} w_i^{k-1}(k-Ne_i-1) + \bar{D}_{iv} v_i^{k-1}(k-Ne_i-1) + H_{i1}{}^{-1} G_{iA}^T \lambda_{iA} \\ \lambda_{iA} &= -(G_{iA} H_{i1}{}^{-1} G_{iA}^T)^{-1} G_{iA}(\bar{A}_{ie}\tilde{x}_i(k-Ne_i-1) \\ &\quad + \bar{D}_{iw} w_i^{k-1}(k-Ne_i-1) + \bar{D}_{iv} v_i^{k-1}(k-Ne_i-1) - \tilde{x}_i(k-Ne_i)) \end{aligned} \tag{17}$$

where $\bar{A}_{ie} \triangleq H_{i1}{}^{-1}\vartheta_i A_{iL}$, $\bar{D}_{iw} \triangleq H_{i1}{}^{-1}[\vartheta_i D_i, -(A_i(Ne_i))^T D_i(Ne_i)]$, $\bar{D}_{iv} \triangleq H_{i1}{}^{-1}[\vartheta_i L_i, -(A_i(Ne_i))^T S_i(Ne_i)]$, $w_i{}^{k-1}(k-Ne_i-1) \triangleq col(w_i(k-Ne_i-1), ..., w_i(k-1))$, $v_i{}^{k-1}(k-Ne_i-1) \triangleq col(v_i(k-Ne_i-1), ..., v_i(k-1))$ and $D_i(Ne_i)$ is defined as

$$D_i(Ne_i) \triangleq \begin{bmatrix} 0 & 0 & \cdots & \cdots \\ C_i D_i & 0 & \cdots & \cdots \\ C_i A_{iL} D_i & C_i D_i & 0 & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ C_i(A_{iL})^{Ne_i-1} D_i & C_i(A_{iL})^{Ne_i-2} D_i & \cdots & C_i D_i \end{bmatrix}$$

Substituting λ_{iA} into $\tilde{x}_i(k-Ne_i)$, it follows that $\tilde{x}_i(k-Ne_i) = \bar{A}_{ie}\tilde{x}_i(k-Ne_i-1) + \bar{D}_{iw} w_i{}^{k-1}(k-Ne_i-1) + \bar{D}_{iv} v_i{}^{k-1}(k-Ne_i-1)$ and for time step $k+1$, $\tilde{x}_i(k-Ne_i+1) = \bar{A}_{ie}\tilde{x}_i(k-Ne_i) + \bar{D}_{iw} w_i{}^k(k-Ne_i) + \bar{D}_{iv} v_i{}^k(k-Ne_i)$.

Rewrite (10) for $l = k - Ne_i$:

$$\hat{x}_i(k - Ne_i + 1\,|k) = A_{ii}\hat{x}_i(k - Ne_i\,|k) + B_{ii}u_i(k - Ne_i\,|k) + \sum_{j=1,j\neq i}^{N}[R_{ij}^{ibuf}(k - Ne_i)A_{ij}x_i^{ibuf}(k - Ne_i) + P_{ij}^{ibuf}(k - Ne_i)B_{ij}u_i^{ibuf}(k - Ne_i)] + L_i(y_i(k - Ne_i) - \hat{y}_i(k - Ne_i\,|k)) \tag{18}$$

Further, (4) can be rewritten in the following form:

$$\begin{aligned} x_i(k - Ne_i + 1) &= A_{ii}x_i(k - Ne_i) + B_{ii}u_i(k - Ne_i) \\ &+ \sum_{j=1,j\neq i}^{N}[R_{ij}^{ibuf}(k - Ne_i)A_{ij}x_i^{ibuf}(k - Ne_i) + P_{ij}^{ibuf}(k - Ne_i)B_{ij}u_i^{ibuf}(k - Ne_i)] + D_iw_i(k - Ne_i) \end{aligned} \tag{19}$$

Subtracting (18) from (19): $\tilde{x}_i(k - Ne_i + 1) = A_{iL}\tilde{x}_i(k - Ne_i) + D_iw_i(k - Ne_i) - L_iv_i(k - Ne_i)$. By iteration, it is obtained that:

$$\tilde{x}_i(k+1) = (A_{iL})^{Ne_i}\tilde{x}_i(k - Ne_i + 1) + D_{iD}{w_i}^k(k - Ne_i + 1) - D_{iL}{v_i}^k(k - Ne_i + 1) \tag{20}$$

where $D_{iD} \triangleq [D_i(A_{iL})^{Ne_i-1}, \cdots, D_i]$ and $D_{iL} \triangleq [L_i(A_{iL})^{Ne_i-1}, \cdots, L_i]$. Multiplying both sides of (20) by $(A_{iL})^{-Ne_i}$:

$$\tilde{x}_i(k - Ne_i + 1) = (A_{iL})^{-Ne_i}\tilde{x}_i(k+1) - (A_{iL})^{-Ne_i}D_{iD}{w_i}^k(k - Ne_i + 1) + (A_{iL})^{-Ne_i}D_{iL}{v_i}^k(k - Ne_i + 1) \tag{21}$$

Rearranging the terms of (21):

$$\tilde{x}_i(k+1) = (\tilde{A}_i)\tilde{x}_i(k) + \tilde{w}_i \tag{22}$$

where $\tilde{A}_i \triangleq {A_{iL}}^{Ne_i}A_{ie}{A_{iL}}^{-Ne_i}$, $A_{ie} \triangleq {H_{i1}}^{-1}\vartheta_iA_{iL}$, $\tilde{w}_i \triangleq ([0, D_{iD}]+[\tilde{A}_iD_{iD}, 0]+{A_{iL}}^{Ne_i}D_{iw})w_i(k-Ne_i)^k+{A_{iL}}^{Ne_i}D_{iv}\times v_i(k - Ne_i)^{k+1}$ and $\tilde{w}_i$ is considered as a disturbance which lies in the $\mathbb{C}$-set $\tilde{\mathbb{W}}_i$ defined by $\tilde{\mathbb{W}}_i = S_{iw}\bar{\mathbb{W}}_i(Ne_i) \oplus S_{iv}\bar{\mathbb{V}}_i(Ne_i)$, where $S_{iw} \triangleq [0, D_{iD}] + [\tilde{A}_iD_{iD}, 0] + (A_{iL})^{Ne_i}D_{iw}$, $S_{iv} \triangleq [0, D_{iL}] + [\tilde{A}_iD_{iL}, 0] + (A_{iL})^{Ne_i}D_{iv}$, $\bar{\mathbb{W}}_i(Ne_i) \triangleq \left(\underbrace{\mathbb{W}_i \times \cdots \times \mathbb{W}_i}_{Ne_i+1}\right)$ and $\bar{\mathbb{V}}_i(Ne_i) \triangleq \left(\underbrace{\mathbb{W}_i \times \cdots \times \mathbb{W}_i}_{Ne_i+2}\right)$, the symbol $\oplus$ denotes the Minkowski sum. Note that A_{iL}, A_{ie} and $\tilde{A}_i$ are Schur matrices. Therefore, there exists a $\mathbb{C}$-set $\tilde{\mathbb{X}}_i$ that is robust positively invariant for (22)[31]. It follows that $\tilde{A}_i\tilde{\mathbb{X}}_i \oplus \tilde{\omega}_i \subset \tilde{\mathbb{W}}_i$ and if $\tilde{x}_i(k = 0) \in \tilde{\mathbb{X}}_i$ then $\tilde{x}_i(k) \in \tilde{\mathbb{X}}_i, \forall k \geq 0$. Q.E.D.

C. Stability of state estimation

The estimation error for the DMHE is defined as before: $\tilde{x}_i(k) \triangleq x_i(k) - \hat{x}_i(k)$. The estimation errors for the predictor and pre-estimator are defined as: $e_i^{ipre}(k) \triangleq x_i(k) - x_i^{ipre}(k)$, $e_i^{ibuf}(k) \triangleq x_i(k) - x_i^{ibuf}(k)$, respectively.

Proposition 2: For each subsystem i, if $e_i^{ibuf}(0)$, $\tilde{x}_i(0)$, $\hat{x}_i(0)$ and $x_i^{ipre}(0)$ are bounded, the pre-estimator gain matrix L_i satisfies: $\|L_i\| > \frac{1-\|A_{ii}\|}{\|C_i\|}$, the state feedback gain matrix K satisfies: $\|K\| < \min\{-\frac{1-\left\|\breve{A}(l)\right\|}{\left\|\breve{B}(l)\right\|}, -\frac{1-\left\|\widehat{A}(l)\right\|}{\left\|\widehat{B}(l)\right\|}\}$, then $e_i^{ibuf}(k)$ is bounded and there exists a $\mathbb{C}$-set $\mathbb{E}_i^{ibuf}$ that if $e_i^{ibuf}(k = 0) \in \mathbb{E}_i^{ibuf}$ then $e_i^{ibuf}(k) \in \mathbb{E}_i^{ibuf}$ for all $k > 0$, where

$$\breve{A}(l) = \begin{bmatrix} A_{11} & \cdots & \hat{R}_{1i}(l)A_{1i} & \cdots & \hat{R}_{1N}(l)A_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{R}_{i1}(l)A_{i1} & \cdots & A_{ii} & \cdots & \hat{R}_{iN}(l)A_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{R}_{N1}(l)A_{N1} & \cdots & \hat{R}_{Ni}(l)A_{Ni} & \cdots & A_{NN} \end{bmatrix}, l = 0, 1, \cdots, k-1$$

$$\breve{B}(l) = \begin{bmatrix} B_{11} & \cdots & \hat{P}_{1i}(l)B_{1i} & \cdots & \hat{P}_{1N}(l)B_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{P}_{i1}(l)B_{i1} & \cdots & B_{ii} & \cdots & \hat{P}_{iN}(l)B_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{P}_{N1}(l)B_{N1} & \cdots & \hat{P}_{Ni}(l)B_{Ni} & \cdots & B_{NN} \end{bmatrix}, l = 0, 1, \cdots, k-1$$

$$\widehat{A}(l)=\begin{bmatrix} A_{11}-L_1C_1 & \cdots & R_{1i}(l)A_{1i} & \cdots & R_{1N}(l)A_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ R_{i1}(l)A_{i1} & \cdots & A_{ii}-L_iC_i & \cdots & R_{iN}(l)A_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ R_{N1}(l)A_{N1} & \cdots & R_{Ni}(l)A_{Ni} & \cdots & A_{NN}-L_NC_N \end{bmatrix}, l=0,1,\cdots,k-2$$

$$\widehat{B}(l)=\begin{bmatrix} B_{11} & \cdots & P_{1i}(l)B_{1i} & \cdots & P_{1N}(l)B_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{i1}(l)B_{i1} & \cdots & B_{ii} & \cdots & P_{iN}(l)B_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{N1}(l)B_{N1} & \cdots & P_{Ni}(l)B_{Ni} & \cdots & B_{NN} \end{bmatrix}, l=0,1,\cdots,k-2$$

$$K=\begin{bmatrix} K_1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & K_i & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & K_N \end{bmatrix}$$

Proof: Subtracting (8) from (1):

$$\begin{aligned} & {e_i}^{ibuf}(k+1) \\ &= x_i(k+1)-x_i^{ibuf}(k+1) = A_{ii}e_i^{ibuf}(k) - L_iC_ie_i^{ibuf}(k) \\ &+ \sum_{j=1,j\neq i}^{N} \{ {\lambda_{ij}}^T[x_i(k)-\chi_{ij}x_j(k)]A_{ij}x_j(k) - {\lambda_{ij}}^T[x_i^{ipre}(k)-\chi_{ij}x_j^{ipre}(k)]A_{ij}x_j^{ipre}(k)\} \\ &+ \sum_{j=1,j\neq i}^{N} \{\delta_{ij}^T[u_i(k)-\sigma_{ij}u_j(k)]B_{ij}u_j(k) - \delta_{ij}^T[{u_i}^{ibuf}(k)-\sigma_{ij}{u_j}^{ibuf}(k)]B_{ij}{u_j}^{ibuf}(k)\} \\ &= \sum_{j=1,j\neq i}^{N} \{[R_{ij}(k)A_{ij} - {R_{ij}}^{ibuf}(k)A_{ij}]x_j^{ipre}(k) + P_{ij}(k)B_{ij}F_jx_j(k) - {P_{ij}}^{ibuf}(k)B_{ij}K_jx_j^{ipre}(k) + R_{ij}(k)A_{ij}e_j^{ibuf}(k)\} \end{aligned} \tag{23}$$

where $R_{ij}(k)={\lambda_{ij}}^T[x_i(k)-\chi_{ij}x_j(k)]$ and $P_{ij}(k)={\delta_{ij}}^T[u_i(k)-\sigma_{ij}u_j(k)]$ are scalars. For the whole system, e^{buf} has the following form:

$$\underbrace{\begin{bmatrix} e_1^{1buf}(k+1) \\ \vdots \\ e_i^{ibuf}(k+1) \\ \vdots \\ e_N^{Nbuf}(k+1) \end{bmatrix}}_{e^{buf}(k+1)} = \underbrace{\begin{bmatrix} A_{11}-L_1C_1 & \cdots & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & A_{ii}-L_iC_i & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & \cdots & A_{NN}-L_NC_N \end{bmatrix}}_{A^{buf}} \underbrace{\begin{bmatrix} e_1^{1buf}(k) \\ \vdots \\ e_i^{ibuf}(k) \\ \vdots \\ e_N^{Nbuf}(k) \end{bmatrix}}_{e^{buf}(k)} \tag{24}$$

$$+\underbrace{\begin{bmatrix} 0 & \cdots & P_{1i}(k)B_{1i}F_i+R_{1i}(k)A_{1i} & \cdots & P_{1N}(k)B_{1N}F_N+R_{1N}(k)A_{1N} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{i1}(k)B_{i1}F_1+R_{i1}(k)A_{i1} & \vdots & 0 & \cdots & P_{iN}(k)B_{iN}F_N+R_{iN}(k)A_{iN} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ P_{N1}(k)B_{N1}F_1+R_{N1}(k)A_{N1} & \cdots & P_{Ni}(k)B_{Ni}F_i+R_{Ni}(k)A_{Ni} & \cdots & 0 \end{bmatrix}}_{A^{pre}(k)} \times$$

$$\underbrace{\begin{bmatrix} e_1^{1pre}(k) \\ \vdots \\ e_i^{ipre}(k) \\ \vdots \\ e_N^{Npre}(k) \end{bmatrix}}_{e^{pre}(k)} + \underbrace{\begin{bmatrix} 0 & \cdots & \cdots \\ \vdots & \ddots & \vdots \\ R_{i1}(k)A_{i1} - R_{i1}{}^{ibuf}(k)A_{i1} + P_{i1}(k)B_{i1}F_1 - P_{i1}{}^{ibuf}(k)B_{i1}K_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ R_{N1}(k)A_{N1} - R_{N1}{}^{Nbuf}(k)A_{N1} + P_{N1}(k)B_{N1}F_1 - P_{N1}{}^{Nbuf}(k)B_{N1}K_1 & \cdots & \cdots \end{bmatrix}}$$

$$\underbrace{\begin{matrix} \cdots & R_{1N}(k)A_{1N} - R_{1N}{}^{1buf}(k)A_{1N} + P_{1N}(k)B_{1N}F_N - P_{1N}{}^{1buf}(k)B_{1N}K_N \\ \ddots & \vdots \\ \cdots & R_{iN}(k)A_{iN} - R_{iN}{}^{ibuf}(k)A_{iN} + P_{iN}(k)B_{iN}F_N - P_{iN}{}^{ibuf}(k)B_{iN}K_N \\ \ddots & \vdots \\ \cdots & 0 \end{matrix}\Bigg]}_{B^{pre}(k)} \underbrace{\begin{bmatrix} x_1^{1pre}(k) \\ \vdots \\ x_i^{ipre}(k) \\ \vdots \\ x_N^{Npre}(k) \end{bmatrix}}_{x^{pre}(k)}$$

Then the error dynamic equation of e^{buf} is obtained:

$$e^{buf}(k+1) = A^{buf}e^{buf}(k) + A^{pre}(k)e^{pre}(k) + B^{pre}(k)x^{pre}(k) \tag{25}$$

Rewrite (6) for $k \to k+1$:

$$\begin{aligned} x_i^{ipre}(k+1) &= A_{ii}\hat{x}_i(k) + B_{ii}u_i(k) \\ &\quad + \sum_{j=1, j\neq i}^{N} \{\lambda_{ij}{}^T[\hat{x}_i(k) - \chi_{ij}\hat{x}_j(k)]A_{ij}\hat{x}_j(k) + \delta_{ij}^T[u_i(k) - \sigma_{ij}u_j(k)]B_{ij}u_j(k)\} \end{aligned} \tag{26}$$

Subtracting (26) from (1):

$$\begin{aligned} e_i^{ipre}(k+1) &= x_i(k+1) - x_i^{ipre}(k+1) = A_{ii}\tilde{x}_i(k) \\ &\quad + \sum_{j=1, j\neq i}^{N} \{\lambda_{ij}{}^T[x_i(k) - \chi_{ij}x_j(k)]A_{ij}x_j(k) - \lambda_{ij}{}^T[\hat{x}_i(k) - \chi_{ij}\hat{x}_j(k)]A_{ij}\hat{x}_j(k)\} \\ &= A_{ii}\tilde{x}_i(k) + \sum_{j=1, j\neq i}^{N} [\lambda_{ij}{}^T\tilde{x}_i(k)A_{ij}\tilde{x}_j(k) + \lambda_{ij}{}^T\tilde{x}_i(k)A_{ij}\hat{x}_j(k) + \lambda_{ij}{}^T\hat{x}_i(k)A_{ij}\tilde{x}_j(k)] \\ &= A_{ii}\tilde{x}_i(k) + \sum_{j=1, j\neq i}^{N} \{[R_{ij}(k)A_{ij} - \hat{R}_{ij}(k)A_{ij}]\hat{x}_j(k) + R_{ij}(k)A_{ij}\tilde{x}_j(k)\} \end{aligned} \tag{27}$$

where $\hat{R}_{ij}(k) = \lambda_{ij}{}^T[\hat{x}_i(k) - \chi_{ij}\hat{x}_j(k)]$ is a scalar. Similarly to (24), for the whole system, e^{pre} has the following form:

$$\underbrace{\begin{bmatrix} e_1^{1pre}(k+1) \\ \vdots \\ e_i^{ipre}(k+1) \\ \vdots \\ e_N^{Npre}(k+1) \end{bmatrix}}_{e^{pre}(k+1)} = \underbrace{\begin{bmatrix} A_{11} & \cdots & R_{1j}(k)A_{1j} & \cdots & R_{1N}(k)A_{1N} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ R_{i1}(k)A_{i1} & \cdots & A_{ii} & \cdots & R_{iN}(k)A_{iN} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{N1}(k)A_{i1} & \cdots & R_{Nj}(k)A_{Nj} & \cdots & A_{NN} \end{bmatrix}}_{\tilde{A}^{pre}(k)} \underbrace{\begin{bmatrix} \tilde{x}_1(k) \\ \vdots \\ \tilde{x}_i(k) \\ \vdots \\ \tilde{x}_N(k) \end{bmatrix}}_{\tilde{x}(k)} \tag{28}$$

$$+ \underbrace{\begin{bmatrix} 0 & \cdots & R_{1i}(k)A_{1i} - \hat{R}_{1i}(k)A_{1i} & \cdots & R_{1N}(k)A_{1N} - \hat{R}_{1N}(k)A_{1N} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ R_{i1}(k)A_{i1} - \hat{R}_{i1}(k)A_{i1} & \cdots & 0 & \cdots & R_{iN}(k)A_{iN} - \hat{R}_{iN}(k)A_{iN} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R_{iN}(k)A_{iN} - \hat{R}_{iN}(k)A_{iN} & \cdots & R_{ij}(k)A_{ij} - \hat{R}_{ij}(k)A_{ij} & \cdots & 0 \end{bmatrix}}_{\hat{A}^{pre}(k)} \underbrace{\begin{bmatrix} \hat{x}_1(k) \\ \vdots \\ \hat{x}_i(k) \\ \vdots \\ \hat{x}_N(k) \end{bmatrix}}_{\hat{x}(k)}$$

where $\hat{R}_{ij}(k) = {\lambda_{ij}}^T[\hat{x}_i(k) - \chi_{ij}\hat{x}_j(k)]$ is a scalar. For the whole system, e^{pre} can be obtained as:

$$e^{pre}(k+1) = \tilde{A}^{pre}(k)\tilde{x}(k) + \hat{A}^{pre}(k)\hat{x}(k) \tag{29}$$

Substitute (29) into (25):

$$e^{buf}(k+1) = A^{buf}e^{buf}(k) + A^{pre}(k)[\tilde{A}^{pre}(k-1)\tilde{x}(k-1) + \hat{A}^{pre}(k-1)\hat{x}(k-1)] + B^{pre}(k)A^{pre}(k-1)x^{pre}(k-1) \tag{30}$$

Consider (6), (10) and (22). It follows that (30) can be written as:

$$\begin{aligned} e^{buf}(k+1) = {} & A^{buf}e^{buf}(k) + A^{pre}(k)\tilde{A}^{pre}(k-1)[\tilde{A}(k-2)\cdots\tilde{A}(0)]\tilde{x}(0) \\ & + A^{pre}(k)\hat{A}^{pre}(k-1)[\hat{X}(k-2)\cdots\hat{X}(0)]\hat{x}(0) + B^{pre}(k)[X^{pre}(k-1)\cdots X^{pre}(0)]x^{pre}(0) \end{aligned} \tag{31}$$

where $\tilde{A} \triangleq diag\{\tilde{A}_1, \tilde{A}_2\cdots, \tilde{A}_N\}$ and $\tilde{A}_i$ has been defined after (22),

$$X^{pre}(l) \triangleq \begin{bmatrix} A_{11}+B_{11}K_1 & \cdots & \hat{R}_{1i}(l)A_{1i}+\hat{P}_{1i}(l)B_{1i}K_i & \cdots & \hat{R}_{1N}(l)A_{1N}+\hat{P}_{1N}(l)B_{1N}K_N \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{R}_{i1}(l)A_{i1}+\hat{P}_{i1}(l)B_{i1}K_1 & \cdots & A_{ii}+B_{ii}K_i & \cdots & \hat{R}_{iN}(l)A_{iN}+\hat{P}_{iN}(k)B_{iN}K_N \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \hat{R}_{N1}(l)A_{N1}+\hat{P}_{N1}(l)B_{N1}K_1 & \cdots & \hat{R}_{Ni}(l)A_{Ni}+\hat{P}_{Ni}(l)B_{Ni}K_i & \cdots & A_{NN}+B_{NN}K_N \end{bmatrix}, \quad l = 0, 1, \cdots, k-1 \tag{32}$$

$$\hat{X}(l) \triangleq \begin{bmatrix} A_{11}+B_{11}K_1-L_1C_1 & \cdots & R_{1i}(l)A_{1i}+P_{1i}(l)B_{1i}K_i & \cdots & R_{1N}(l)A_{1N}+P_{1N}(l)B_{1N}K_N \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ R_{i1}(l)A_{i1}+P_{i1}(l)B_{i1}K_1 & \cdots & A_{ii}+B_{ii}K_i-L_iC_i & \cdots & R_{iN}(l)A_{iN}+P_{iN}(l)B_{iN}K_N \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ R_{N1}(l)A_{N1}+P_{N1}(l)B_{N1}K_1 & \cdots & R_{Ni}(l)A_{Ni}+P_{Ni}(l)B_{Ni}K_i & \cdots & A_{NN}+B_{NN}K_N-L_NC_N \end{bmatrix}, \quad l = 0, 1, \cdots k-2 \tag{33}$$

(32) and (33) can be rewritten as:

$$X^{pre}(l) = \breve{A}(l) + \breve{B}(l)K, l = 0, 1, \cdots, k-1 \tag{34}$$

$$\hat{X}(l) = \widehat{A}(l) + \widehat{B}(l)K, l = 0, 1, \cdots, k-2 \tag{35}$$

From (31), it can be seen that if $e^{buf}(k+1) = A^{buf}e^{buf}(k)$, according to the condition: $\|L_i\| > \frac{1-\|A_{ii}\|}{\|C_i\|}$, then $\left\|A^{buf}\right\| < 1$, $e^{buf}(k+1) = A^{buf}e^{buf}(k)$ is convergent. For the second term on the right of (31), according to Proposition 1, $\tilde{A}$ is a Schur matrix, if $\tilde{x}(0)$ is bounded, then, $A^{pre}(k)\tilde{A}^{pre}(k-1)[\tilde{A}(k-2)\cdots\tilde{A}(0)]\tilde{x}(0)$ is bounded. For the final two terms on the right of (31), according to the condition: $\|K\| < \min\{-\frac{1-\left\|\widehat{A}(l)\right\|}{\left\|\widehat{B}(l)\right\|}, -\frac{1-\left\|\widehat{A}(l)\right\|}{\left\|\widehat{B}(l)\right\|}\}$, then, $\left\|\hat{X}(l)\right\| = \left\|\widehat{A}(l) + \widehat{B}(l)K\right\| \leq \left\|\widehat{A}(l)\right\| + \left\|\widehat{B}(l)\right\| \|K\| < 1$ and $\|X^{pre}(l)\| = \left\|\breve{A}(l) + \breve{B}(l)K\right\| \leq \left\|\breve{A}(l)\right\| + \left\|\breve{B}(l)\right\| \|K\| < 1$, if $\hat{x}(0)$ and $x^{pre}(0)$ are bounded, then, $A^{pre}(k)\hat{A}^{pre}(k-1)[\hat{X}(k-2)\cdots\hat{X}(0)]\hat{x}(0)$ and $B^{pre}(k)[X^{pre}(k-1)\cdots X^{pre}(0)]x^{pre}(0)$ are bounded at k time. Hence $e^{buf}(k)$ is bounded. Furthermore, there exists a $\mathbb{C}$-set $\mathbb{E}_i^{ibuf}$ such that if $e_i^{ibuf}(k=0) \in \mathbb{E}_i^{ibuf}$ then $e_i^{ibuf}(k) \in \mathbb{E}_i^{ibuf}$ for all $k > 0$. Q.E.D.

Let $\bar{e}_i(k) = \begin{bmatrix} \tilde{x}_i(k)^T & e_i^{ibuf}(k)^T \end{bmatrix}^T$ denote the estimation error for the ith subsystem at time step k, where the stability is addressed in Theorem 1.

Theorem 1: For each subsystem i, there exists a $\mathbb{C}$-set $\bar{\mathbb{E}}_i$ such that if $\bar{e}_i(k=0) \in \bar{\mathbb{E}}_i$, then $\bar{e}_i(k) \in \bar{\mathbb{E}}_i$, $\forall k \geq 0$. Furthermore, $\bar{\mathbb{E}}_i \subseteq \mathbb{E}_i$.

Proof:

Substitute $x_i(k+1)$ and $\hat{y}_i(k)$ into $\hat{x}_i(k+1)$:

$$\hat{x}_i(k+1) = A_{ii}\hat{x}_i(k) + B_{ii}u_i(k) + \sum_{j=1,j\neq i}^{N}[R_{ij}^{ibuf}(k)A_{ij}x_j^{ibuf}(k) + P_{ij}^{ibuf}(k)B_{ij}u_j^{ibuf}(k)] + L_iC_i\tilde{x}_i(k) + L_iv_i(k) \tag{36}$$

Subtracting the above equation from $x_i(k+1)$:

$$\tilde{x}_i(k+1) = A_{ii}\tilde{x}_i(k) + \sum_{j=1,j\neq i}^{N} A_{ij}e_j^{ibuf} - L_iC_i\tilde{x}_i(k) - L_iv_i(k) \tag{37}$$

Subtracting (8) from the equation obtained by rewriting $x_i(k+1)$ for the ith subsystem:

$$e_i^{ibuf}(k+1) = A_{ii}e_i^{ibuf}(k) + B_{ii}(u_i(k) - u_i^{ibuf}(k)) + \sum_{j=1,j\neq i}^{N} A_{ij}e_j^{ibuf}(k) + A_{ii}\tilde{x}_i(k) \tag{38}$$

For each subsystem i, the error dynamic equation is obtained from (37) to (38) in the form:

$$\bar{e}_i(k+1) = A_i\bar{e}_i(k) + \bar{\Omega}_i(k) \tag{39}$$

where $\bar{\Omega}_i(k) \triangleq B_i\Delta\bar{u}_i(k) + q_i(k)(L_iC_i\tilde{x}_i(k) + L_iv_i(k))$ is considered to be a disturbance where

$$q_i(k) = \begin{bmatrix} \underbrace{0 \quad \cdots \quad 0}_{i-1} & -1 & \underbrace{0 \quad \cdots \quad 0}_{n_{x_i}-i} \end{bmatrix}^T$$

$$\Delta\bar{u}_i^{ibuf}(k) = \begin{bmatrix} \underbrace{\Delta u_i{}^{1buf}(k) \quad \cdots \quad \Delta u_i{}^{(i-1)buf}(k)}_{i-1} & 0 & \underbrace{\Delta u_i{}^{(i+1)buf}(k) \quad \cdots \quad \Delta u_i{}^{Nbuf}(k)}_{N-i} \end{bmatrix}^T$$

As described previously, $\Delta u_i^{ibuf}(k)$ is bounded and there exists $\pi_{ij} > 0$ such that $\max_{k\geq 0}\left\|\Delta u_i^{ibuf}(k)\right\| \leq \pi_{ij}$, where $\pi_{ij} + u_i(k\,|k-1\,) \in \mathbb{U}_j$. There exists $r_i > 0$ such that $\Delta u_i^{ibuf}(k) \in r_i\mathbb{B}$ and $\Delta\bar{u}_i(k) \in \underbrace{r_i\mathbb{B} \times \cdots \times r_i\mathbb{B}}_{N-1}$. By using Proposition 1 and Proposition 2, $\bar{\Omega}_i(k)$ lies in the $\mathbb{C}$-set and Δ_i defined by $\Delta_i \triangleq B_i\left(\underbrace{r_i\mathbb{B} \times \cdots \times r_i\mathbb{B}}_{N-1}\right) \oplus q_i(k)(L_iC_i\tilde{\mathbb{X}}_i \oplus L_i\mathbb{V}_i)$. If A_i is Schur then there exists a $\mathbb{C}$-set $\bar{\mathbb{E}}_i$ that is RPI for (39). It follows that $A_i\bar{\mathbb{E}}_i \oplus \Delta_i \subseteq \bar{\mathbb{E}}_i$ and if $\bar{e}_i(k=0) \in \bar{\mathbb{E}}_i$ then $\bar{e}_i(k) \in \bar{\mathbb{E}}_i, \forall k \geq 0$. Furthermore, in order to reduce the upper set $\bar{\mathbb{E}}_i$ and consequently the bounds of $\bar{e}_i(k)$, RPI approximation of the minimal RPI (mRPI) can be used [31] and the upper bound of $\bar{e}_i(k)$ is made as small as possible by tuning parameters ϑ_i and r_i [26]. Then, under Proposition 2, the two sets $\tilde{\mathbb{X}}_i$ and $\bar{\mathbb{E}}_i$ are both contained in the origin set $\mathbb{E}_i$ such that $\tilde{\mathbb{X}}_i \oplus \bar{\mathbb{E}}_i \subseteq \mathbb{E}_i$. It follows that $\bar{\mathbb{E}}_i \subseteq \mathbb{E}_i$. Q.E.D.

IV. Robust output feedback distributed model predictive control for parallel systems

This section synthesizes a ROFDMPC that brings system (4) to a bounded target set. Since the real states $x_i(k)$ are unmeasured, the estimated states are adopted when determining the controller. The corresponding robust output feedback distributed model predictive control law can be designed as:

$$u_i(k) = F_i\hat{x}_i(k) \tag{40}$$

For subsystem i at time step k, to determine a robust output feedback distributed model predictive control law (40), the control objective function is defined as:

$$\min_{u_i(k)} \max_{w_i(k),v_i(k)} J_i(k) = \sum_{l=0}^{\infty}\left[\|\hat{x}_i(k+l)\|^2_{\Upsilon_i} + \|u_i(k+l)\|^2_{\varsigma_i}\right] + \sum_{j=1,j\neq i}^{N}\sum_{l=0}^{\infty}\left[\|\hat{x}_j(k+l)\|^2_{\Upsilon_j} + \|u_j(k+l)\|^2_{\varsigma_j}\right] \tag{41}$$

$$s.t.(4),(9)-(12), u_i(k+l)\in\mathbb{U}_i, x_i(k+l)\in\mathbb{X}_i, y_i(k+l)\in\mathbb{Y}_i \tag{42}$$

where u_i is the control law to be designed, for all $i \in 1,2\cdots,N$, Υ_i, and ς_i are symmetric positive definite weighting matrices.

The objective is to design a robust output feedback distributed model predictive controller for the parallel system with $w_i(k)$ and $v_i(k)$, calculate the competitive couplings (2), (3) in a timely fashion by using the pre-estimator, predictor and buffer, solve the $DMHE_i$ (9) in the presence of competitive constraints (12) to improve the estimate accuracy and finally, solve the control objective function (41) to determine a robust output feedback distributed model predictive control law (40) at every sampling time so that the uncertain parallel system meets the feasibility condition and achieves exponential stability.

A. Robust output feedback distributed model predictive control algorithm

The augmented closed loop system is :

$$\bar{x}_i(k+l+1) = \bar{T}_{ii}\bar{x}_i(k+l) + \sum_{j=1,j\neq i}^{N}\bar{T}_{ij}\bar{x}_j(k+l) + \bar{H}_i\bar{v}_i(k+l) \tag{43}$$

where

$$\bar{x}_i(k+l) = \begin{bmatrix} x_i(k+l)^T & x_i^{ibuf}(k+l)^T & \hat{x}_i(k+l)^T & e_i^{ibuf}(k+l)^T & \tilde{x}_i(k+l)^T \end{bmatrix}^T$$

$$\bar{v}_i(k+l) = \begin{bmatrix} 0 & v_i(k+l)^T & w_i(k+l)^T & e_i^{ibuf}(k+l)^T & \tilde{x}_i(k+l)^T \end{bmatrix}^T$$

$$\bar{T}_{ii} = \begin{bmatrix} A_{ii} & 0 & B_{ii}F_i & 0 & 0 \\ 0 & A_{ii}+B_{ii}K_i & 0 & 0 & 0 \\ 0 & 0 & A_{ii}+B_{ii}F_i & 0 & 0 \\ 0 & -B_{ii}K_i & B_{ii}F_i & A_{ii} & 0 \\ 0 & 0 & 0 & 0 & A_{ii} \end{bmatrix}$$

$$\bar{T}_{ij} = \begin{bmatrix} R_{ij}^{ibuf}A_{ij}+P_{ij}^{ibuf}B_{ij}K_i & 0 & 0 & 0 & 0 \\ 0 & R_{ij}^{ibuf}A_{ij}+P_{ij}^{ibuf}B_{ij}K_j & 0 & 0 & 0 \\ 0 & R_{ij}^{ibuf}A_{ij}+P_{ij}^{ibuf}B_{ij}K_j & 0 & 0 & 0 \quad 0 \\ 0 & 0 & 0 & A_{ij} & A_{ij} \\ 0 & 0 & 0 & A_{ij} & 0 \end{bmatrix}$$

$$\bar{H}_{ii} = \begin{bmatrix} 0 & L_i & D_i & 0 & L_iC_i \\ 0 & 0 & 0 & L_iC_i & 0 \\ 0 & L_i & D_i & 0 & L_iC_i \\ 0 & -L_i & 0 & 0 & -L_iC_i \\ 0 & -L_i & 0 & 0 & -L_iC_i \end{bmatrix}$$

Assume that there exists a matrix Q_i which satisfies $Q_i = Q_i^T > 0$ and a scalar γ_i which makes $J_i(k) \leq \gamma_i$:

$$\|\bar{x}_i(k+l)\|^2_{Q_i^{-1}} \leq 1 \tag{44}$$

$$\bar{T}_{ii}^T{Q_i}^{-1}\bar{T}_{ii} - {Q_i}^{-1} \leq -1/\gamma_i\Upsilon_i - 1/\gamma_i {F_i}^T\varsigma_1 F_i \tag{45}$$

Consider (44), with $l=0$, yields:

$$\begin{bmatrix} x_i(k)^T & x_i^{ibuf}(k)^T & \hat{x}_i(k)^T & e_i^{ibuf}(k)^T & \tilde{x}_i(k)^T \end{bmatrix} Q_i^{-1} \begin{bmatrix} x_i(k)^T & x_i^{ibuf}(k)^T & \hat{x}_i(k)^T & e_i^{ibuf}(k)^T & \tilde{x}_i(k)^T \end{bmatrix}^T \leq 1 \tag{46}$$

Theorem 2: Suppose there exist scalars γ_i, ε_i, matrices Q_{i12}, G_i, G_{i12}, G_{i2}, Y_i, M_i and symmetric matrices Q_{i11}, Q_{i22}, then (44) and (45) are satisfied when:

$$Q_i = \begin{bmatrix} Q_{i11} & {Q_{i12}}^T \\ Q_{i12} & Q_{i22} \end{bmatrix} \geq 0, F_i = Y_iG_i^{-1} \tag{47}$$

Proof: Let $\tilde{G}_i = \begin{bmatrix} G_i & 0 \\ G_{i12} & G_{i2} \end{bmatrix}$. The following LMIs are given:

$$\begin{bmatrix} G_i + {G_i}^T - Q_{i11} & * & * & * & * & * & * \\ G_{i12} - Q_{i12} & G_{i2} + {G_{i2}}^T - Q_{i22} & * & * & * & * & * \\ L_iC_i(G_i + G_{i12}) + M_i + B_{ii}Y_i & L_iC_iG_{i2} & Q_{i11} & * & * & * & * \\ (A_{ii} - L_iC_i)(G_1 + G_{i12}) - M_i + (B_{ij} - B_{ii})Y_i & (A_{ii} - L_iC_i)G_{i2} & Q_{i12} & Q_{i22} & * & * & * \\ {\Upsilon_i}^{1/2}G_i & 0 & 0 & 0 & \gamma_i I & * & * \\ {\Upsilon_i}^{1/2}G_{i12} & {\Upsilon_i}^{1/2}G_{i12} & 0 & 0 & 0 & \gamma_i I & * \\ {\varsigma_i}^{1/2}Y & 0 & 0 & 0 & 0 & 0 & \gamma_i I \end{bmatrix} \geq 0 \tag{48}$$

$$\begin{bmatrix} 1 & * & * \\ \hat{x}_i(k) & Q_{i11} & * \\ \varepsilon_i & Q_{i12} & Q_{i22} \end{bmatrix} \geq 0 \tag{49}$$

Multiply the left and right sides of (46) by $\tilde{G}_i^T$ and $\tilde{G}_i$, respectively, apply the Schur complement lemma and consider (48). Utilizing the fact that $\tilde{G}_i + \tilde{G}_i^T - Q_i \leq \tilde{G}_i^T {Q_i}^{-1} \tilde{G}_i$, it can be shown that (48) guarantees (45). Moreover, by applying (43) and the Schur complement lemma, it is shown that (47) guarantees (44).

Then, (44), (45) hold by parameterizing (47).Q.E.D.

The constraints need to be handled. Suppose there exist matrices Q_{i12}, G_i, G_{i12}, G_{i2}, Y_i, M_i and symmetric matrices Q_{i11}, Q_{i22}, Θ_i, Z_i, Γ_i such that (49) and the following LMIs are satisfied:

$$\begin{bmatrix} G_i + {G_i}^T - Q_{i11} & * & * \\ G_{i12} - Q_{i12} & G_{i2} + {G_{i2}}^T - Q_{i22} & * \\ (A_i - L_iC_i)(G_i + G_{i12}) - M_i + B_iY_i & (A_i - L_iC_i)G_{i2} & \Theta_i \end{bmatrix} \geq 0, \Theta_{ijj} \leq \tilde{x}_{ij}^2, j \in \{1, ..., n_{x_i}\} \tag{50}$$

$$\begin{bmatrix} G_i + {G_i}^T - Q_{i11} & * & * \\ G_{i12} - Q_{i12} & G_{i2} + {G_{i2}}^T - Q_{i22} & * \\ Y_i & 0 & Z_i \end{bmatrix} \geq 0, Z_{ijj} \leq {u_{ij}}^2, j \in \{1, ..., n_{u_i}\} \tag{51}$$

$$\begin{bmatrix} G_i + {G_i}^T - Q_{i11} & * & * \\ G_{i12} - Q_{i12} & G_{i2} + {G_{i2}}^T - Q_{i22} & * \\ \Psi_i(A_iG_i + A_iG_{i12} + B_iY_i) & \Psi_iA_iG_{i2} & \Gamma_i \end{bmatrix} \geq 0, \Gamma_{ijj} \leq {\psi_{ij}}^2, j \in \{1, ..., n_{y_i}\} \tag{52}$$

Then, (42) is guaranteed through the parameterization (49). By considering Theorem 2 and constraints handling, problem (41)-(42) can be solved by the following LMI optimization problem:

$$\min_{\gamma_i, M_i, Y_i, G_i, G_{i12}, G_{i2}, Q_{i11}, Q_{i12}, Q_{i22}, \Theta_i, \Gamma_i, Z_i} \gamma_i \qquad s.t. (48), (49), (50) - (52) \tag{53}$$

Based on the aforementioned analysis, the proposed output feedback DMPC can be summarized as the following step by step algorithm .

Algorithm 2:

Step 1 (Pre-estimation and buffer-update): At time step $k-1$, subsystem i receives the state sequences $x_j(k-1)$ and the predicted control sequences $u_j(k-1)$ from the other subsystems $j = 1, \cdots, N, j \neq i$, and stores its own $x_i(k-1)$ and $u_i(k-1)$. These valid packets are written in the $(k-1)$th part of $buffer_i$. The kth part of $buffer_i$ is updated according to Algorithm 1.

Step 2 (Estimation): The constrained optimization problem (9-12) is solved to obtain $\hat{x}_i(k - Ne_i)$ based on the state information in the kth part of $buffer_i$. An optimal sequence of the local states is obtained where the current optimal state estimate of subsystem i is denoted by $\hat{x}_i(k)$.

Step 3 (Robust DMPC): The constrained optimization problem (53) is solved to develop the control law $u_i(k)$ using LMIs.

Remark 4: Algorithm 2 computes output feedback controls for each subsystem. It has been validated that the performance of the state feedback robust DMPC is the same as the centralized robust MPC in [8] for parallel systems. Indeed, the proposed output feedback method can achieve a similar control performance as that of the state feedback developed in [8] as the accuracy and timeliness of the state estimation can be guaranteed. This will be verified experimentally in Section V.

B. Closed loop system stability analysis

Define he quadratic function:

$$\bar{V}_i(t, k) = \|\bar{x}_i(k + l\,|k)\|^2_{P_i(k)} \tag{54}$$

where $P_i(k) > 0$.

Theorem 3: For (4), under the output feedback control (40) which is given by Theorem 2, the closed loop system (43) will be asymptotically stable.

Proof: According to Theorem 1 and Theorem 2:

$$\left[\begin{array}{ccccc} x_i(k)^T & x_i^{ibuf}(k)^T & \hat{x}_i(k)^T & e_i^{ibuf}(k)^T & \tilde{x}_i(k)^T \end{array}\right] Q_i^{-1} \left[\begin{array}{ccccc} x_i(k)^T & x_i^{ibuf}(k)^T & \hat{x}_i(k)^T & e_i^{ibuf}(k)^T & \tilde{x}_i(k)^T \end{array}\right]^T \leq 1$$

$$\bar{T}_{ii}^T {Q_i}^{-1} \bar{T}_{ii} - {Q_i}^{-1} \leq -1/\gamma_i \Upsilon_i - 1/\gamma_i {F_i}^T \varsigma_1 F_i < 0$$

According to the Schur complement lemma:

$$\begin{array}{l} \Delta \bar{V}_i(k+l\,|k) = \bar{V}_i(k+l+1\,|k) - \bar{V}_i(k+l\,|k) = \|\bar{x}_i(k+l+1\,|k)\|_{P_i(k)}^2 - \|\bar{x}_i(k+l\,|k)\|_{P_i(k)}^2 \\ \leq -\bar{x}_i(k+l\,|k)^T (1/\gamma_i \Upsilon_i + 1/\gamma_i {F_i}^T \varsigma_i F_i) \bar{x}_i(k+l\,|k) < 0 \end{array}$$

After F_i, L_i have been implemented k times, the closed loop system (43) is asymptotically stable.Q.E.D.

V. Simulations and experimental results

Two simulation studies and an experimental implementation are undertaken in this section. Firstly, a general parallel system is simulated and the performance is compared with that achieved by other established approaches. Then, a parallel continuous stirred tank reactor (CSTR) simulation is undertaken. Finally, a parallel CSTR experiment is used to further validate the proposed approach and the results are compared with a state feedback method.

A. A general parallel system simulation

Consider the following parameterisation of the parallel system in (1):

$$\begin{array}{l} A_{11} = \left[\begin{array}{cc} 0.82 & -0.02 \\ 6.12 & 0.93 \end{array}\right], A_{12} = \left[\begin{array}{cc} 0.96 & -0.02 \\ -0.67 & 0.94 \end{array}\right], \\ A_{22} = \left[\begin{array}{cc} 0.88 & 0.04 \\ -0.94 & 0.99 \end{array}\right], A_{21} = \left[\begin{array}{cc} 0.89 & 0.01 \\ 2.77 & 0.88 \end{array}\right], \\ B_{11} = \left[\begin{array}{c} -0.01 \\ 0.16 \end{array}\right], B_{12} = \left[\begin{array}{c} -0.03 \\ 0.15 \end{array}\right], \\ B_{22} = \left[\begin{array}{c} -0.02 \\ 0.09 \end{array}\right], B_{21} = \left[\begin{array}{c} -0.02 \\ 0.17 \end{array}\right], \\ C_1 = \left[\begin{array}{cc} 1 & 0 \end{array}\right], C_2 = \left[\begin{array}{cc} 1 & 0 \end{array}\right] \end{array}$$

$$\begin{array}{llll} \sigma_{21} & = \left[\begin{array}{cc} 0.0156 & 0.0151 \\ 0.3254 & 0.0695 \end{array}\right], & \sigma_{12} & = \left[\begin{array}{cc} 0.0412 & 0.0651 \\ 0.2018 & 0.0511 \end{array}\right] \\ \chi_{21} & = \left[\begin{array}{cc} 0.0601 & 0.0628 \\ 0.0118 & 0.1010 \end{array}\right], & \chi_{12} & = \left[\begin{array}{cc} 0.0457 & 0.0258 \\ 0.0498 & 0.0854 \end{array}\right] \\ \delta_{21} & = \left[\begin{array}{cc} 0.1125 & 0.2021 \end{array}\right], & \delta_{12} & = \left[\begin{array}{cc} 0.2176 & 0.1892 \end{array}\right] \\ \lambda_{21} & = \left[\begin{array}{cc} 0.0601 & 0.0628 \end{array}\right], & \lambda_{12} & = \left[\begin{array}{cc} 0.0657 & 0.0854 \end{array}\right] \end{array}$$

Choose $L_1 = [0.51, 0.31]^T$ and $L_2 = [0.51, 0.41]^T$, A_{1L} and A_{2L} are Schur matrices and their eigenvalues are $(0.5784, 0.7862)$ and $(0.5864, 0.8962)$, respectively. $w_i(k)$ is a random disturbance satisfying the Gaussian distribution, $v_i(k)$ is a random noise satisfying the Gaussian distribution and for all simulation sets, the disturbance and noise effects are considered. In DMHE design, $Ne_i = 10$, $\vartheta_i = diag(100, 0.01)$ for $i = 1, 2$ are chosen. Assume that the outputs of the two subsystems are the flow rate. The desired outputs of the two subsystems are 5, the maximum output of the bus is 10 and the maximum input of the bus is 5. In the simulations, the proposed approach is compared with the following two methods:

Method 1. The pre-estimators are not used; all other parts are the same as the proposed approach.

Method 2. A distributed Luenberger framework is utilized to replace DMHE; other parts are the same as the proposed approach.

The pre-estimators are used in the proposed approach and in method 2 but are not used in method 1. The state estimation accuracy is tested by using a square wave response as the desired output trajectory for all three methods. The results are shown from Figure 4 to Figure 7.

It can be seen from Figure 4 to Figure 7 that the proposed approach has better estimation accuracy. The pre-estimators are not used in method 1, hence the tracking error is larger when the desired output trajectory is changed. Method 2 cannot solve the constraints effectively. The tracking error is larger than the DMHE when the desired output trajectory is not changed. The control performance is shown from Figure 8 to Figure 13.

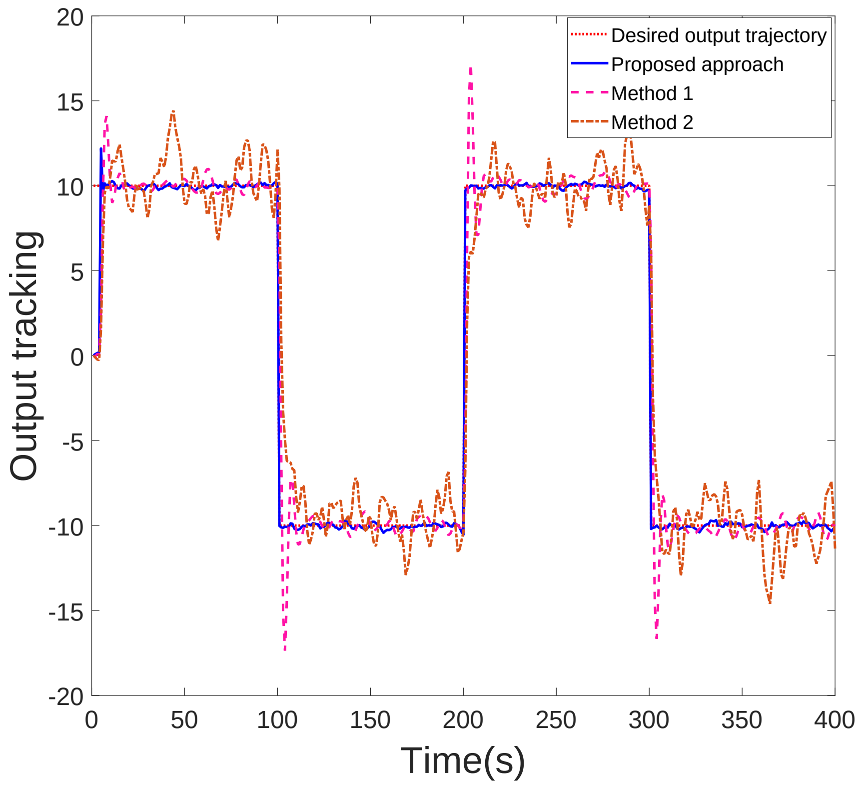

Fig. 4: The output tracking of subsystem 1 for the proposed method, method (1) and method (2)

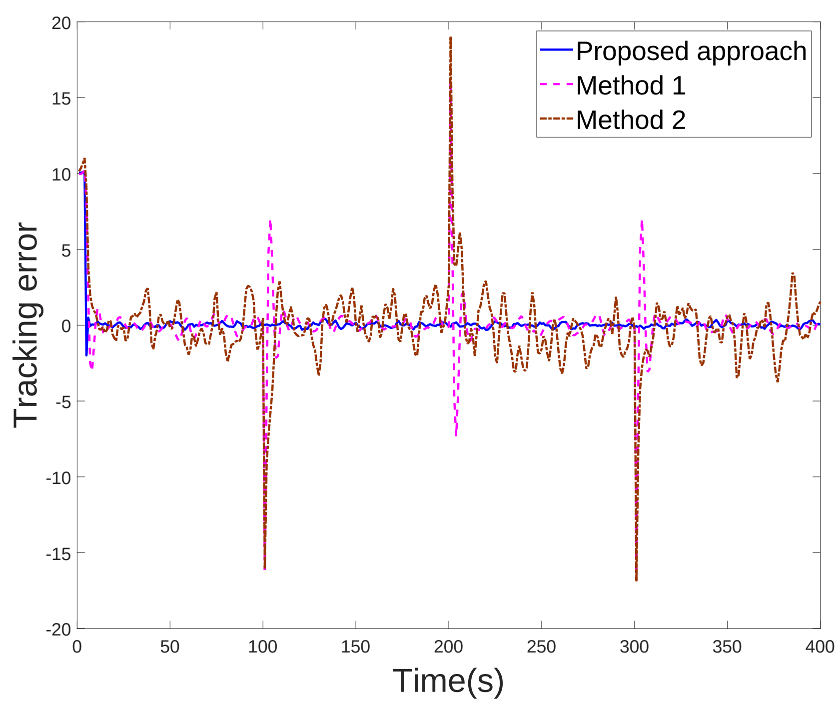

Fig. 5: The tracking error of subsystem 1 for the proposed method, method (1) and method (2)

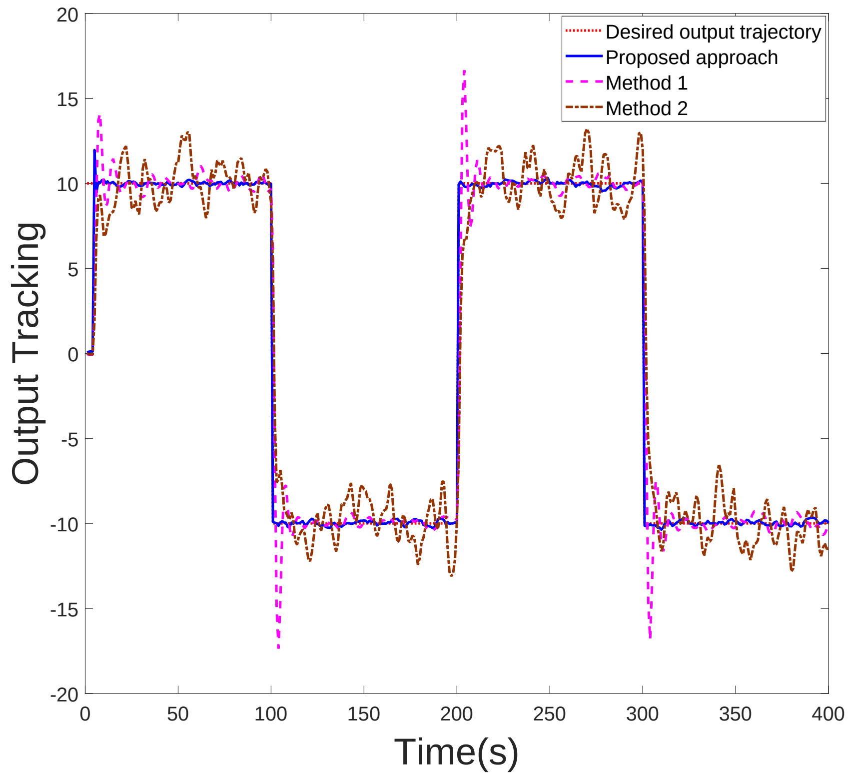

Fig. 6: The output tracking of subsystem 2 for the proposed method, method (1) and method (2)

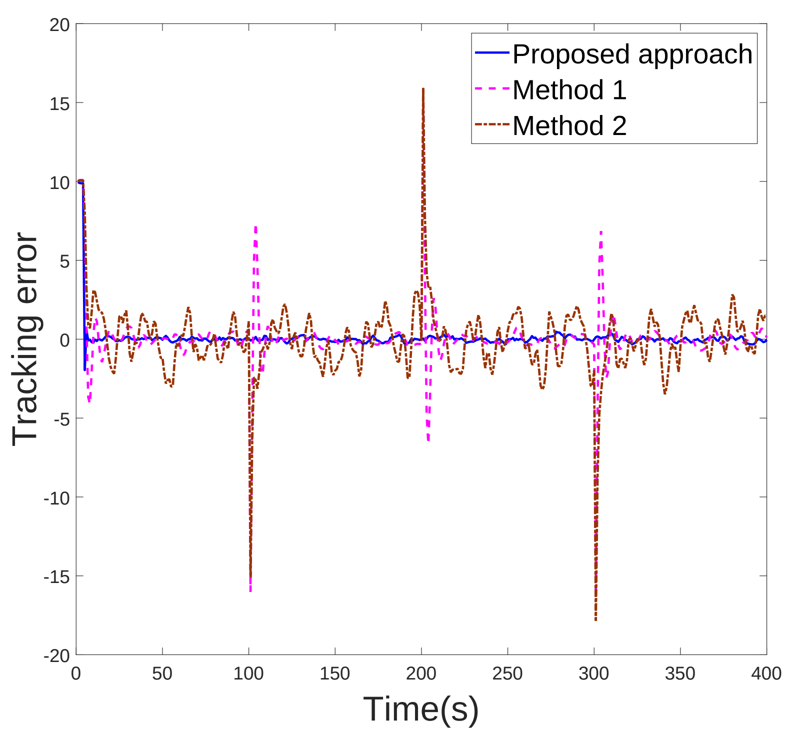

Fig. 7: The tracking error of subsystem 2 for the proposed method, method (1) and method (2)

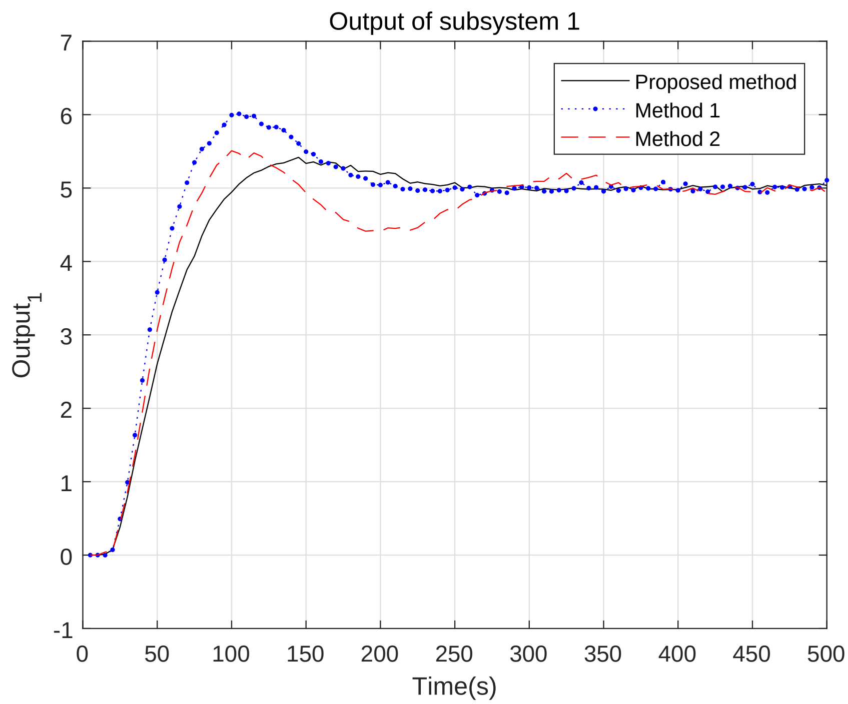

Fig. 8: The output of subsystem 1 for the proposed method, method (1) and method (2)

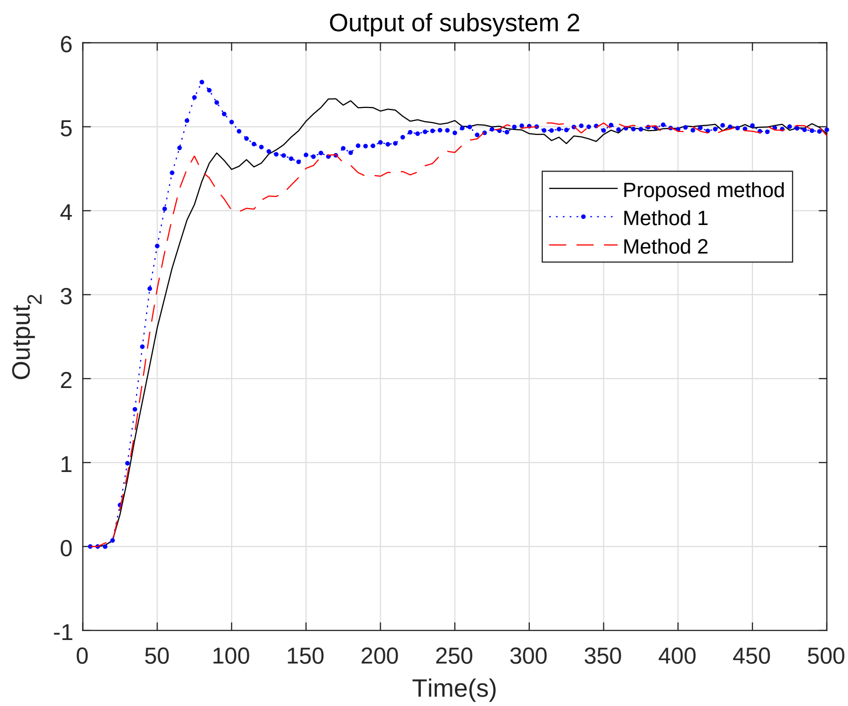

Fig. 9: The output of subsystem 2 for the proposed method, method (1) and method (2)

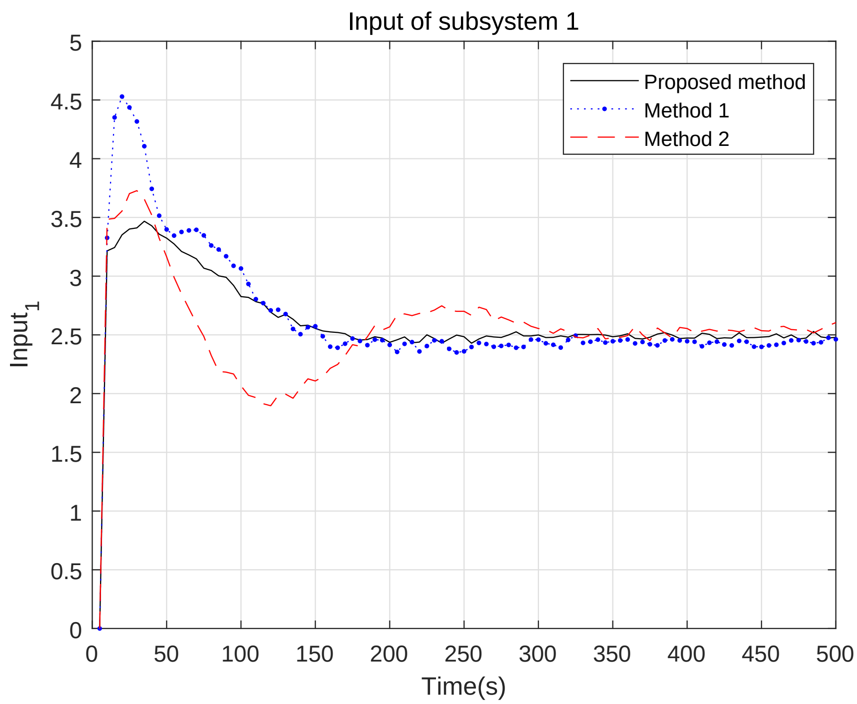

Fig. 10: The input of subsystem 1 for the proposed method, method (1) and method (2)

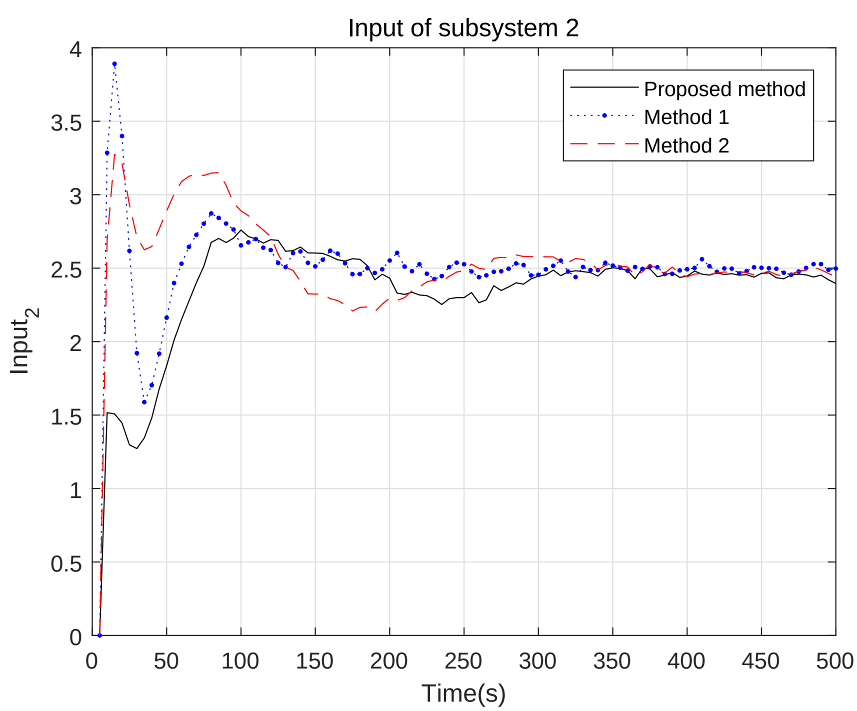

Fig. 11: The input of subsystem 2 for the proposed method, method (1) and method (2)

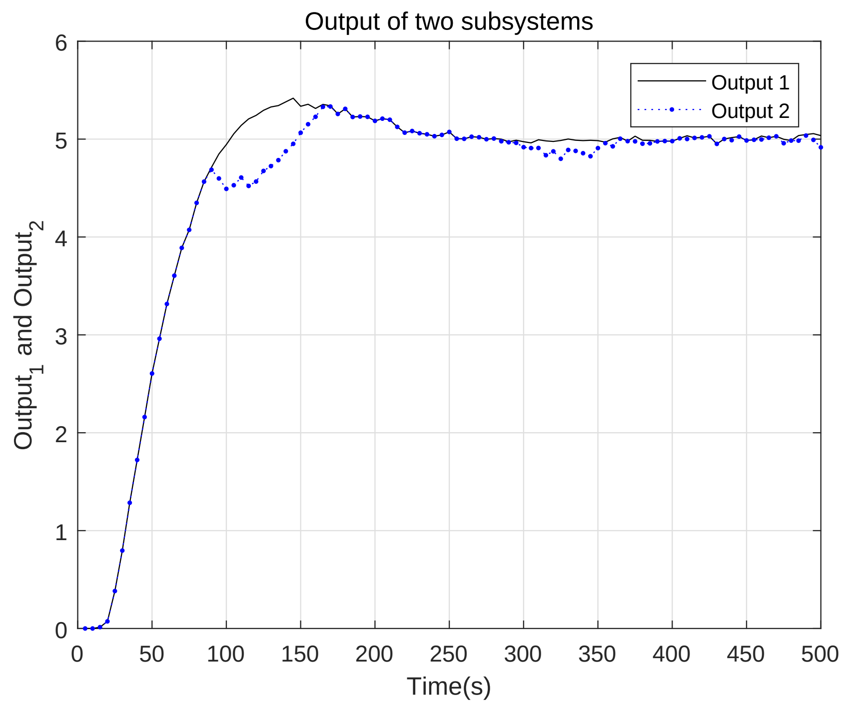

Fig. 12: The competitive coupling of the output for the proposed method

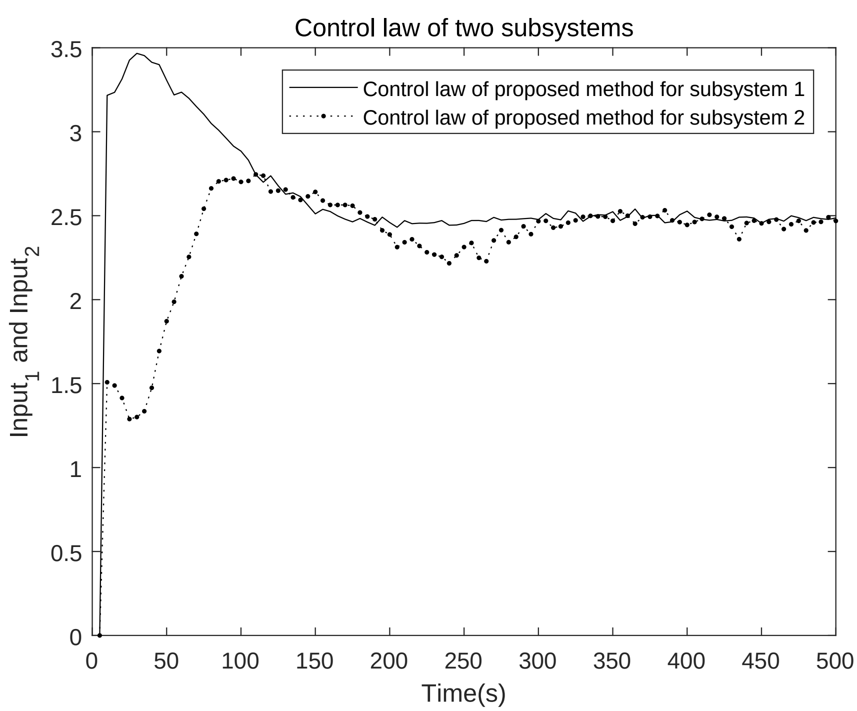

Fig. 13: The competitive coupling of the input for the proposed method

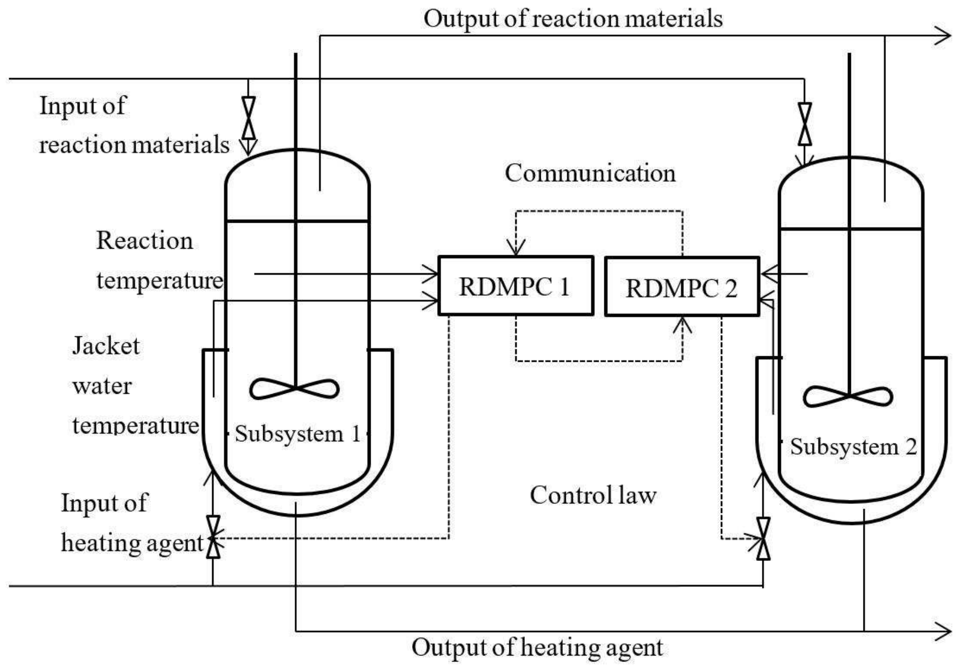

Fig. 14: Simplified physical model of parallel CSTR system.

The simulation results show that the output feedback RDMPC for parallel systems can effectively deal with problems with unmeasured states. Figure 8 shows that the performance of the proposed approach is better than the other two methods. The proposed method (solid line) with pre-estimator has a faster response. This is because the pre-estimator can locate the eigenvalues closer to the origin and then a faster response results. The pre-estimator is not used in Method 1 (dotted line). Method 1 has a higher overshoot. A distributed Luenberger framework is utilized instead of the DMHE in Method 2 (dashed line). The pre-estimator is used in this method and the overshoot is smaller than Method 1. However the Luenberger framework cannot deal with the competitive constraints effectively; hence Method 2 has a larger oscillation. Figure 10 and Figure 11 indicate that the proposed approach needs smaller control effort. The computing time of the proposed approach is 1.87s while that of method 1 is 3.21s and method 2 is 2.96s. Figure 12 and Figure 13 show the output and robust DMPC control action for both subsystems. This clearly demonstrates how the competitive characteristics in parallel systems are accommodated by the proposed method: when u_1 and y_1 increase, u_2 and y_2 correspondingly decrease to accommodate the competitive coupling and competitive constraints.

B. Parallel CSTR system simulation

The parallel CSTR system is simulated before proceeding to implementation on the experiment. A simplified physical model of a parallel CSTR system is given in Figure 14. The reaction is a temperature control reaction. If the temperature inside the reactor is low, it will affect the depth and conversion rate of the reaction. This affects the quality of the product. In order to stabilize the temperature inside the reactor, it is necessary to heat the jacket. The temperature of the material in the reactor can be controlled to meet the requirements of the process by adjusting the flow rate of the heat agent flowing into the jacket. The heat agent is water, which is supplied by one water heater. When the two CSTRs are connected in parallel, the system exhibits competitive coupling and there is a need to consider competitive constraints; when the flow of hot water in one jacket exceeds a certain amount, the flow of water in the other jacket must be be reduced. The heat agent flowing into the jacket must be less than or equal to the total supply of the water heater.

The nonlinear dynamic equations of the plant model which is described in [32] can be reformulated to include the parallel characteristics as follows:

$$\dot{T}_1 = \frac{F_0}{V_1}(T_0 - T_1) + \frac{F_r}{V_2}(T_2 - T_1) + G_1(T_1 - \frac{\rho_2 C_2}{V_2}T_2)T_1 + R_1(Q_1 - \frac{\Delta H_2 k_2}{\rho_2 C_2}Q_2)Q_1 + \frac{Q_{r1}}{\rho_s c_p V_1} \tag{55}$$

$$\dot{C}_{A1} = \frac{F_0}{V_1}(C_{A0} - C_{A1}) + \frac{F_r}{V_1}(C_{A2} - C_{A1}) - R_1(T_1)C_{A1} - R_2(T_1)C_{A3} \tag{56}$$

$$\dot{T}_2 = \frac{F_1}{V_2}(T_{A0} - T_{A1}) + \frac{F_3}{V_2}(T_{03} - T_2) + G_2(T_2 - \frac{\rho_1 C_1}{V_1}T_1)T_2 + R_2(Q_2 - \frac{\Delta H_1 k_1}{\rho_1 C_1}Q_1)Q_2 + \frac{Q_{r2}}{\rho_s c_p V_2} \tag{57}$$

TABLE I: Definitions of the variables corresponding to the process parameters

Variables	Definitions
F_0	Flow rate of fresh material A
F_r	Flow rate of recycled material A from reactor 2
F_3	Flow rate of additional fresh stream feeding pure A
F_1, F_2	Effluent flow rate from reactors 1, 2
C_{A1}, C_{A2}	Molar concentration of material A in reactors 1, 2
T_1, T_2	Temperatures in reactors 1, 2
T_0, T_{03}	Feed stream temperatures to reactor 1, 2
Q_{r1}, Q_{r2}	Heat input rate into reactors 1, 2
C_{A0}, C_{A3}	Inlet reactant concentration of reactors 1, 2
V_1, V_2	Reactor volume of reactors 1, 2
ΔH_j, k_j, E_j	Enthalpies, pre-exponential constants and activation energies of the reaction
ρ_s, R, c_p	Heat capacity, gas constant and density of fluid in the reactor

TABLE II: Values of the process parameters

$F_0 = 5.04m^3/h$	$T_0 = 0^oC$	$k_{30} = 3\times 10^5 h^{-1}$	$C_{A1}^s = 1.67kmol/m^3$
$F_1 = 50m^3/h$	$T_{03} = 0^oC$	$E_1 = 5\times 10^4 kJ/kmol$	$T_2^s = 10^oC$
$F_3 = 30m^3/h$	$\Delta H_1 = -5\times 10^4 kJ/kmol$	$E_2 = 6.5\times 10^4 kJ/kmol$	$C_{A2}^s = 0.63kmol/m^3$
$F_r = 35m^3/h$	$\Delta H_2 = -5.2\times 10^4 kJ/kmol$	$E_3 = 5.5\times 10^4 kJ/kmol$	$Q_1^s = 1.2\times 10^6 kJ/h$
$V_1 = 1.0m^3$	$\Delta H_3 = -5.4\times 10^4 kJ/kmol$	$c_p = 0.88kJ/kgK$	$C_{A0}^s = 0.8kmol/m^3$
$V_2 = 1.0m^3$	$k_{10} = 3\times 10^5 h^{-1}$	$\rho_s = 1000kg/m^3$	$Q_2^s = 1.6\times 10^6 kJ/h$
$R = 8.314kJ/kmolK$	$k_{20} = 3\times 10^5 h^{-1}$	$T_1^s = 10^oC$	$C_{A01}^s = 0.4kmol/m^3$

$$\dot{C}_{A2} = \frac{F_1}{V_2}(C_{A0} - C_{A1}) + \frac{F_3}{V_2}(C_{A03} - C_{A2}) - R_1(T_2)C_{A2} - R_2(T_2)C_{A0} \tag{58}$$

where $R_i(T_j) = k_{i0}\exp(-E_i/RT_j)$ and $G_i(T_i) = ((-\Delta H_i)/\rho_s c_p)R_i(T_j)$ for $j = 1, 2$. The definitions for the variables and the values of the process parameters can be found in Tables 1 and 2, respectively.

Choose a sampling interval of $T_s = 0.0025h$. Considering (55-58), the nominal discrete time linear state space model of the plant around the mentioned steady state points has the form:

$$\begin{aligned}
\bar{x}_1(k+1) &= A_{11}\bar{x}_1(k) + B_{11}u_1(k) + {\lambda_{12}}^T(\bar{x}_1 - \chi_{12}\bar{x}_2)A_{12}\bar{x}_2(k) + {\delta_{12}}^T(u_1 - \sigma_{12}u_2)B_{12}u_2(k), \bar{y}_1(k) = C_1\bar{x}_1(k) \\
\bar{x}_2(k+1) &= A_{22}\bar{x}_2(k) + B_{22}u_2(k) + {\lambda_{21}}^T(\bar{x}_2 - \chi_{21}\bar{x}_1)A_{21}\bar{x}_1(k) + {\delta_{21}}^T(u_2 - \sigma_{21}u_1)B_{21}u_1(k), \bar{y}_2(k) = C_2\bar{x}_2(k)
\end{aligned} \tag{59}$$

where x_i and u_i are the (dimensionless) state and manipulated input vectors for the ith CSTR, respectively:

$$\bar{x}_1(k) = \begin{bmatrix} \frac{T_1 - {T_1}^s}{{T_1}^s} \\ \frac{C_{A1} - {C_{A1}}^s}{{C_{A1}}^s} \end{bmatrix}, \bar{x}_2(k) = \begin{bmatrix} \frac{T_2 - {T_2}^s}{{T_2}^s} \\ \frac{C_{A2} - {C_{A2}}^s}{{C_{A2}}^s} \end{bmatrix}$$

$$u_1(k) = \begin{bmatrix} \frac{Q_{r1} - {Q_{r1}}^s}{{Q_{r1}}^s} \\ \frac{C_{A0} - {C_{A0}}^s}{{C_{A0}}^s} \end{bmatrix}, u_2(k) = \begin{bmatrix} \frac{Q_{r2} - {Q_{r2}}^s}{{Q_{r2}}^s} \\ \frac{C_{A03} - {C_{A03}}^s}{{C_{A03}}^s} \end{bmatrix}$$

The other constant matrices are given by

$$\begin{aligned}
A_{11} &= \begin{bmatrix} 0.9600 & 0.0039 \\ -0.2488 & 0.8902 \end{bmatrix}, A_{12} = \begin{bmatrix} 0.0722 & 0.0002 \\ -0.0134 & 0.0773 \end{bmatrix}, \\
A_{22} &= \begin{bmatrix} 0.8312 & 0.0024 \\ -0.0235 & 0.5627 \end{bmatrix}, A_{21} = \begin{bmatrix} 0.0657 & 0.0002 \\ -0.0201 & 0.0645 \end{bmatrix}, \\
B_{11} &= \begin{bmatrix} 0.0072 & 0.0001 \\ -0.0009 & 0.0265 \end{bmatrix}, B_{12} = \begin{bmatrix} 0.0722 & 0.0002 \\ -0.0134 & 0.0773 \end{bmatrix}, \\
B_{22} &= \begin{bmatrix} 0.0097 & 0.0001 \\ -0.0005 & 0.0738 \end{bmatrix}, B_{21} = \begin{bmatrix} 0.0247 & 0.0003 \\ -0.0200 & 0.0824 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}
\end{aligned}$$

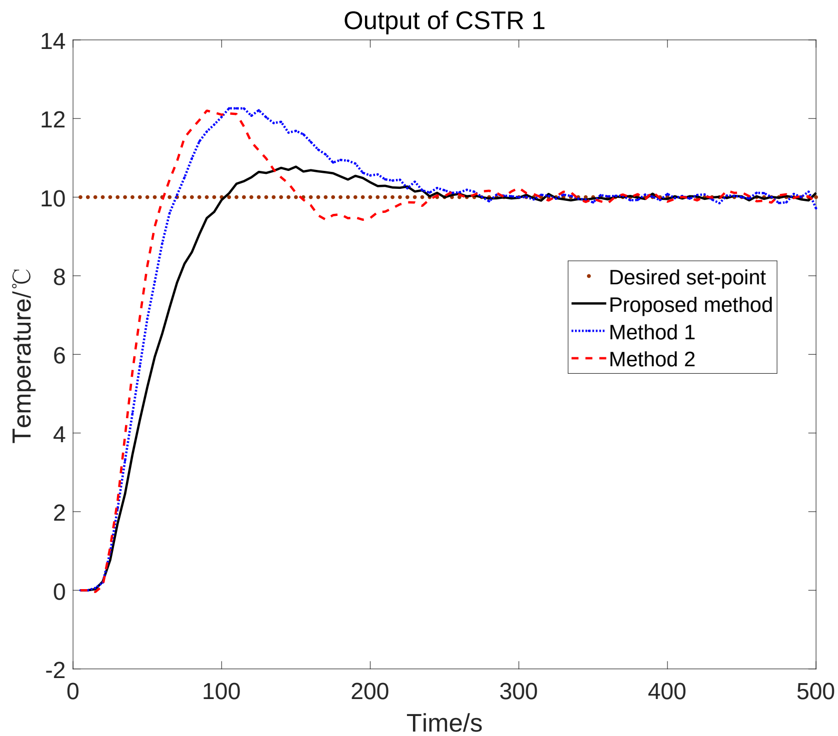

Fig. 15: The output temperature of the first CSTR for the proposed method, method (1) and method (2)

$$\begin{aligned}
\sigma_{21} &= \begin{bmatrix} 0.0238 & 0.0412 \\ 0.1428 & 0.0981 \end{bmatrix}, \quad \sigma_{12} = \begin{bmatrix} 0.0351 & 0.0741 \\ 0.1024 & 0.0427 \end{bmatrix} \\
\chi_{21} &= \begin{bmatrix} 0.0416 & 0.0597 \\ 0.0736 & 0.1328 \end{bmatrix}, \quad \chi_{12} = \begin{bmatrix} 0.0842 & 0.0718 \\ 0.0981 & 0.0889 \end{bmatrix} \\
\delta_{21} &= \begin{bmatrix} 0.1243 & 0.2004 \end{bmatrix}, \quad \delta_{12} = \begin{bmatrix} 0.2181 & 0.1411 \end{bmatrix} \\
\lambda_{21} &= \begin{bmatrix} 0.1024 & 0.0971 \end{bmatrix}, \quad \lambda_{12} = \begin{bmatrix} 0.0518 & 0.0449 \end{bmatrix}
\end{aligned}$$

A local pre-estimator is designed for each CSTR. Choosing $L_1 = [0.6, 0.28]^T$ and $L_2 = [0.5, 0.4]^T$, A_{1L} and A_{2L} are Schur matrices and their eigenvalues are $(0.3639, 0.8863)$ and $(0.3430, 0.8163)$, respectively. In the DMHE design, $Ne_i = 10$ and $\vartheta_i = diag(100, 0.01)$ for $i = 1, 2$ are chosen. $w_i(k)$ is random disturbance satisfying the Gaussian distribution and $v_i(k)$ is a random noise satisfying Gaussian distribution. For all simulation sets, the disturbance and noise are present. In the simulations, the proposed approach is compared with the two different methods. The desired set-points of the two subsystems are both 10 degrees Celsius, and the initial temperatures of both subsystems are 0 degree Celsius. The control objective is to increase the temperature of the jacket from 0 degree Celsius to 10 degrees Celsius by manipulating the flow of water in the jacket. The results are shown from Figure 15 to Figure 19.

The simulation results show that the method proposed in this paper can effectively deal with problems relating to parallel CSTR systems. Figure 15, Figure 16, Figure 17 and Figure 18 show that the performance of the proposed approach is better than the other two methods and the control effort of the proposed approach is smaller. The computing time of the proposed approach is 2.21s while that of method 1 is 4.32s and method 2 is 3.95s. Figure 19 shows the robust DMPC control action for both subsystems. This clearly demonstrates how the competitive characteristics in parallel systems are accommodated by the proposed method: when u_1 increases, u_2 correspondingly decreases to accommodate the competitive coupling and competitive constraints. This is different from the parallel system considered in subsection 4.1; the outputs of the CSTR are the temperature and do not have material couplings between them.

C. *Parallel CSTR system experiment*

The effectiveness of the robust DMPC algorithm for parallel CSTR systems has been verified by simulation. The effectiveness will be further validated by two experiments. The Process Modelling and Control Group at the China University of Petroleum (East China) have developed an experimental rig which is shown in Figure 20. The operation interface of the rig is shown in Figure 21. The four reactors, labelled R101, R102, R103, R104, can be connected in numerous ways for controller validation

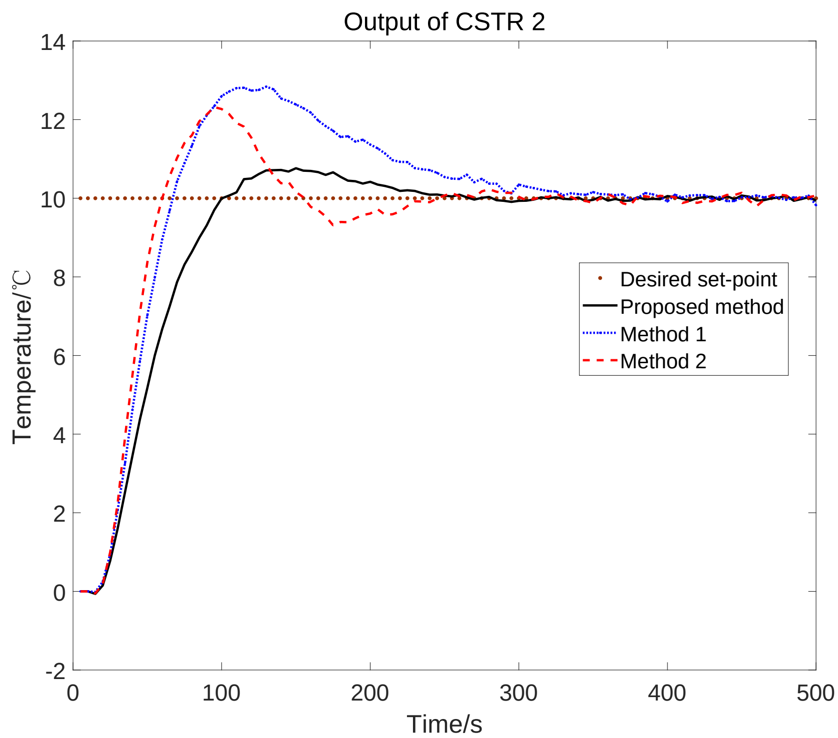

Fig. 16: The output temperature of the second CSTR for the proposed method, method (1) and method (2)

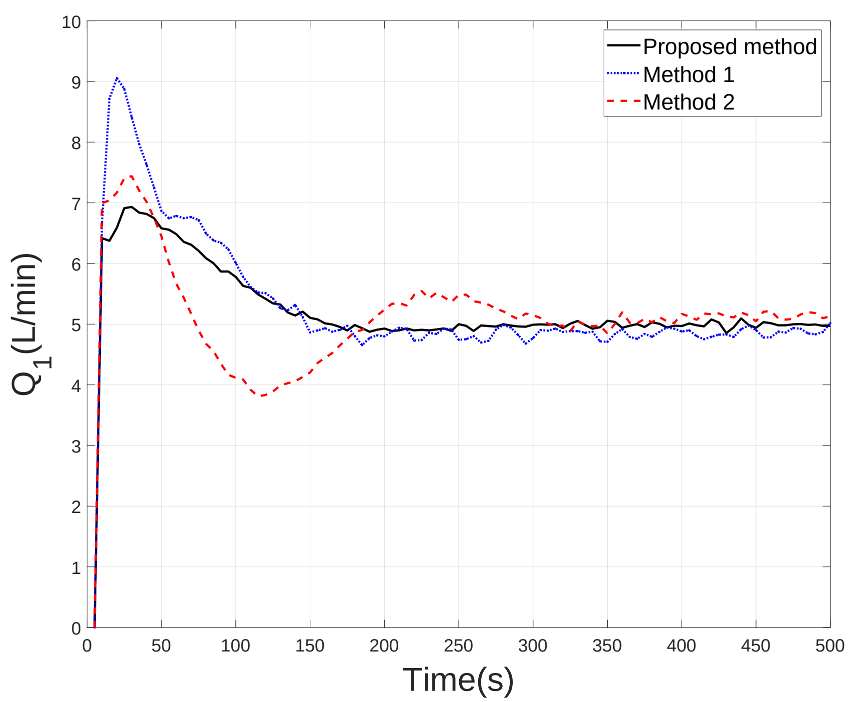

Fig. 17: The inputs of the first CSTR for the proposed method, method (1) and method (2)

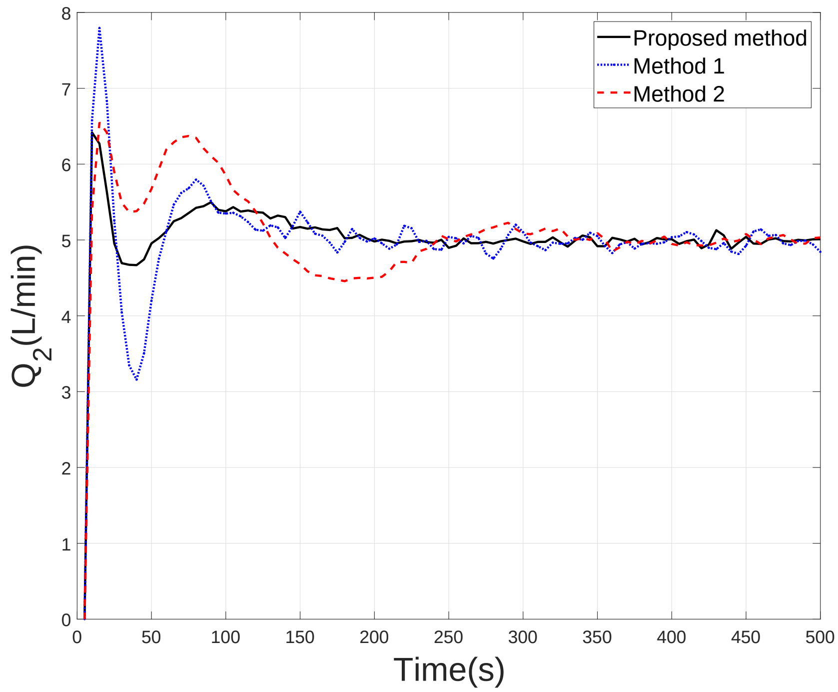

Fig. 18: The inputs of the second CSTR for the proposed method, method (1) and method (2)

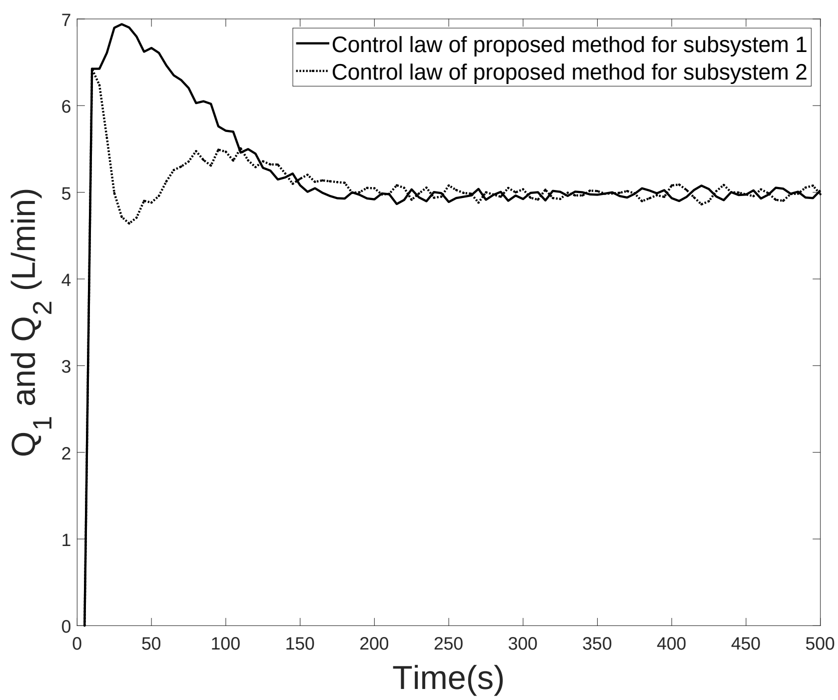

Fig. 19: Control signal for both subsystems when controlled using the proposed method

Fig. 20: Experimental equipment

and testing (series, parallel, series and parallel). The chemical reaction is carried out after feeding. The process can implement continuous operation as well as enable measurement and control of the flow, liquid level and temperature. V111 is the header tank which contains acetic ether and V112 is the header tank containing sodium hydroxide. These raw materials are processed in the CSTR at the same time [8]. Unlike the previous simulation, this reaction is an exothermic reaction. Only two reactors are used in these experiments. The output of the first experiment is temperature. For the reactors R101 and R102, the desired set-points are both 30 degrees Celsius. The coolant flow in the jacket is used as the control variable. The initial temperatures of R101 and R102 are 24.8 degrees Celsius and 25.2 degrees Celsius, respectively. The control objective is to increase the temperature of the jacket from the initial temperatures to 30 degrees Celsius by manipulating the flow of water in the jacket. The output of the second experiment is concentration of product C, the desired set-points are both $0.8kmol/m^3$. The initial product concentrations of R101 and R102 are are $0.46kmol/m^3$ and $0.61kmol/m^3$, respectively. The control objective is to increase the concentration of product C from the initial concentration to a desired set-point. The proposed approach is used for control of the system. The temperature tracking performance is shown in Figure 22 and the product concentration tracking performance is shown in Figure 23, which further validate the proposed approach. Figure 24 shows the control effect of the classical DMPC algorithm when the states are measurable. Comparing Figure 22 with Figure 24, the output feedback robust DMPC proposed in this paper can achieve a similar performance to the method proposed in [8], in which the states are measurable.

VI. Conclusion

This paper has proposed an on-line algorithm to implement an output feedback robust DMPC strategy that explicitly accommodates the characteristics of parallel systems with state estimation. The main contribution of this paper can be summarized as follows: (1) new buffer, predictor and pre-estimator are presented to solve the competitive couplings and competitive constraints under the condition that the states cannot be measured; (2) an output feedback robust DMPC algorithm has been proposed for parallel systems and the stability of the closed loop system has been analyzed. The subsystem performance takes into account the state estimation, competitive couplings and competitive constraints in order to achieve optimization of the whole system. The problem can be converted into N convex problems which can be expressed as linear matrix inequalities and solved iteratively by using the method of successive iteration to ensure rapid convergence. The simulations show that the proposed approach can effectively deal with the constraints for parallel systems and can achieve better performance than the OFDMPC without pre-estimator. The results of an experimental trial further illustrate that the proposed approach is suitable for control of parallel systems in process networks when the states are unmeasured.

Acknowledgement

This work is partially supported by the National Natural Science Foundation of China (61973315) and (61473312).

References

[1] P. Garcia-Herreros, A. Agarwal, J. M. Wassick, I. E. Grossmann, Optimizing inventory policies in process networks under uncertainty, Computers & Chemical Engineering 92 (2016) 256–272.

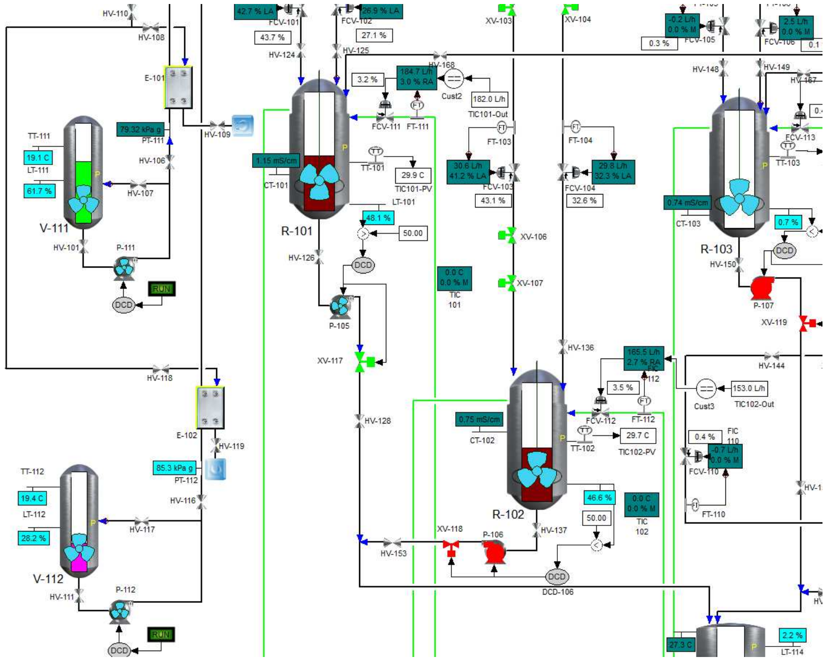

Fig. 21: Parallel R101 and R102.

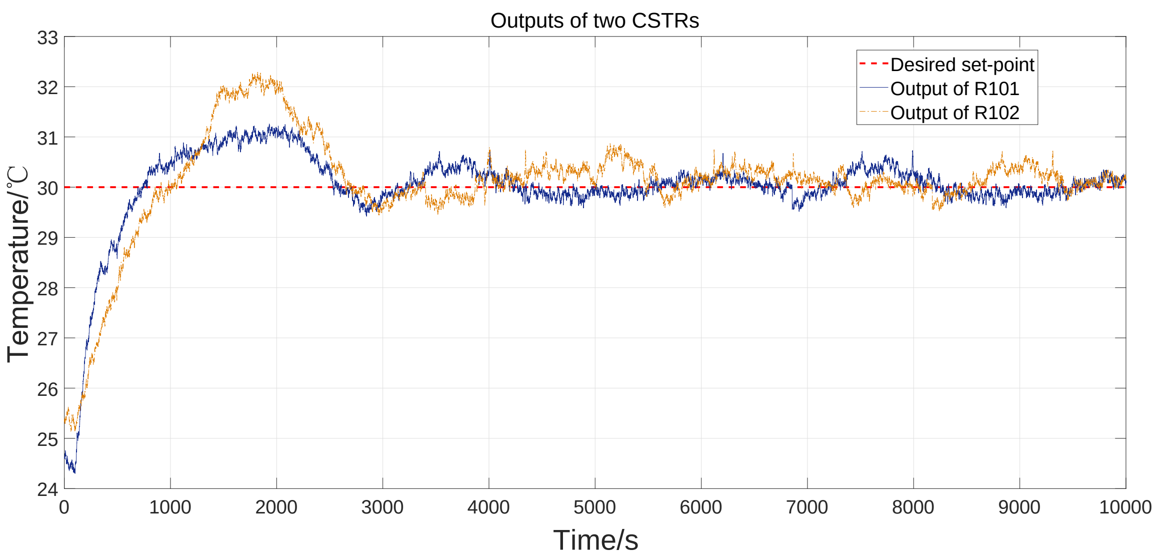

Fig. 22: Temperature tracking performance when the states are unmeasured and the CSTR are controlled using the OFRDMPC.

[2] J. J. Downs, E. F. Vogel, A plant-wide industrial process control problem, Computers & chemical engineering 17 (3) (1993) 245–255.
[3] Q. Chen, S. Li, Y. Xi., Distributed decoupling predictive control of a kind of cascade processes, Control and Design 19 (6) (2005) 647–654.
[4] Y. Zhang, S. Li, Networked model predictive control based on neighbourhood optimization for serially connected large-scale processes, Journal of Process Control 17 (1) (2007) 37–50.
[5] X. Cai, L. Xie, H. Su, Y. Gu., Distributed model predictive control based on cascade processes, Aata automatica Sinica 39 (5) (2013) 510–518.
[6] Q. Liu, H. S. Abbas, J. M. Velni, An lmi-based approach to distributed model predictive control design for spatially-interconnected systems, Automatica 95 (2018) 481–487.
[7] L. Zhang, J. Wang, Y. Ge, B. Wang, Robust distributed model predictive control for uncertain networked control systems, IET Control Theory & Applications 8 (17) (2014) 1843–1851.
[8] S. Zhang, D. Zhao, S. K. Spurgeon, Robust distributed model predictive control for systems of parallel structure within process networks, Journal of Process Control 82 (2019) 70–90.
[9] L. Liu, B. Huang, S. Dubljevic, Model predictive control of axial dispersion chemical reactor, Journal of Process Control 24 (11) (2014) 1671–1690.
[10] D. B. Pourkargar, A. Almansoori, P. Daoutidis, Distributed model predictive control of process networks: impact of control architecture, IFAC-PapersOnLine 50 (1) (2017) 12452–12457.
[11] D. Q. Mayne, S. Raković, R. Findeisen, F. Allgöwer, Robust output feedback model predictive control of constrained linear systems, Automatica 42 (7) (2006) 1217–1222.

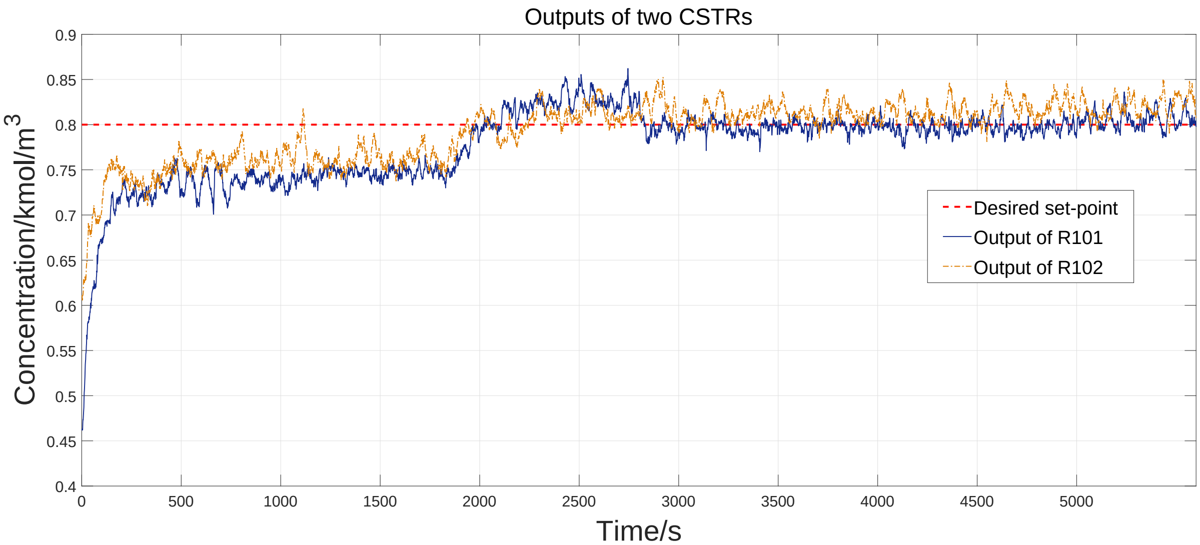

Fig. 23: Product concentration tracking performance when the states are unmeasured and the CSTR is controlled using the proposed OFRDMPC.

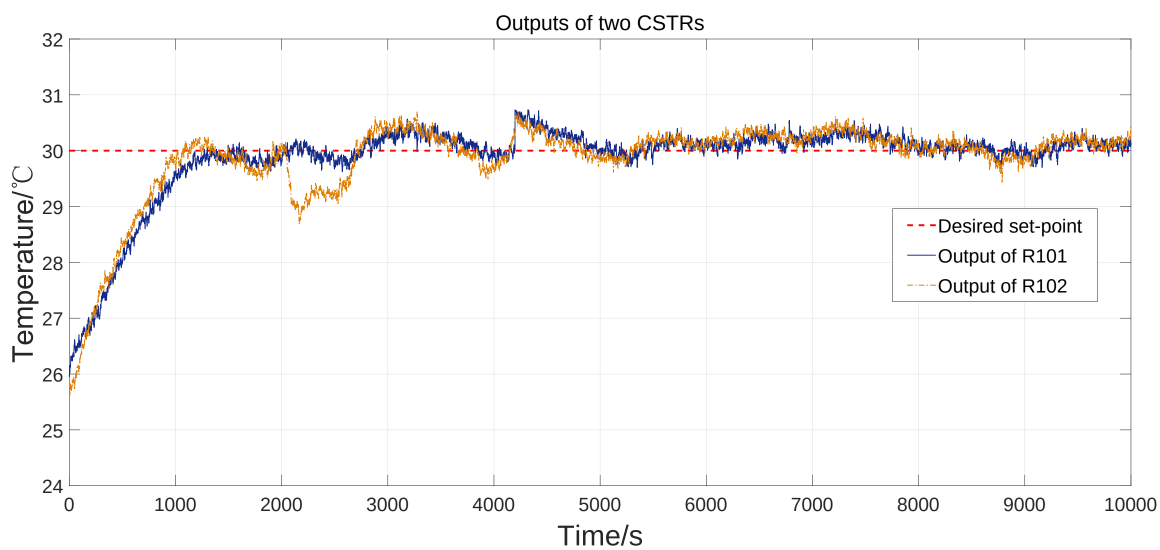

Fig. 24: Tracking performance when the states can be measured and the CSTR are controlled using classical DMPC.

[12] D. Mayne, S. Rakovic, R. Findeisen, F. Allgower, Robust output feedback model predictive control of constrained linear systems: Time varying case, Automatica 45 (9) (2009) 2082–2087.

[13] A. Rahideh, M. Shaheed, Constrained output feedback model predictive control for nonlinear systems, Control Engineering Practice 20 (4) (2012) 431–443.

[14] J. Yan, R. R. Bitmead, Incorporating state estimation into model predictive control and its application to network traffic control, Automatica 41 (4) (2005) 595–604.

[15] Y. Sun, K. K. Tan, T. H. Lee, Tube based quasi-min-max output feedback mpc for lpv systems, IFAC Proceedings Volumes 45 (15) (2012) 186–191.

[16] C. Xia, M. Wang, Z. Song, T. Liu, Robust model predictive current control of three-phase voltage source pwm rectifier with online disturbance observation, IEEE Transactions on Industrial Informatics 8 (3) (2012) 459–471.

[17] C. Martín, M. R. Arahal, F. Barrero, M. J. Durán, Five-phase induction motor rotor current observer for finite control set model predictive control of stator current, IEEE Transactions on Industrial Electronics 63 (7) (2016) 4527–4538.

[18] C. V. Rao, J. B. Rawlings, D. Q. Mayne, Constrained state estimation for nonlinear discrete-time systems: Stability and moving horizon approximations, IEEE transactions on automatic control 48 (2) (2003) 246–258.

[19] A. G. Beccuti, S. Mariéthoz, S. Cliquennois, S. Wang, M. Morari, Explicit model predictive control of dc-dc switched-mode power supplies with extended kalman filtering, IEEE Trans. Industrial Electronics 56 (6) (2009) 1864–1874.

[20] M. Cannon, Q. Cheng, B. Kouvaritakis, S. V. Raković, Stochastic tube mpc with state estimation, Automatica 48 (3) (2012) 536–541.

[21] D. Luenberger, Observers for multivariable systems, IEEE Transactions on Automatic Control 11 (2) (1966) 190–197.

[22] C. C. Pantelides, J. Renfro, The online use of first-principles models in process operations: Review, current status and future needs, Computers & Chemical Engineering 51 (2013) 136–148.

[23] P. D. Christofides, R. Scattolini, D. M. de la Pena, J. Liu, Distributed model predictive control: A tutorial review and future research directions, Computers & Chemical Engineering 51 (2013) 21–41.

[24] J. Zhang, J. Liu, Distributed moving horizon state estimation for nonlinear systems with bounded uncertainties, Journal of Process Control 23 (9) (2013) 1281–1295.

[25] J. Zeng, J. Liu, Distributed moving horizon state estimation: Simultaneously handling communication delays and data losses, Systems & Control Letters 75 (2015) 56–68.

[26] D. Sui, T. A. Johansen, L. Feng, Linear moving horizon estimation with pre-estimating observer, IEEE Transactions on Automatic Control 55 (10) (2010) 2363–2368.

[27] Y. Song, G. Wei, S. Liu, Distributed output feedback mpc with randomly occurring actuator saturation and packet loss, International Journal of Robust and Nonlinear Control 26 (14) (2016) 3036–3057.

[28] Z. Razavinasab, M. M. Farsangi, M. Barkhordari, State estimation-based distributed model predictive control of large-scale networked systems with communication delays, IET Control Theory & Applications 11 (15) (2017) 2497–2505.

[29] J. Liu, H. Xiao, Distributed output feedback model predictive control for a team of coupled linear subsystems, IET Control Theory & Applications 11 (11) (2017) 1807–1812.

[30] A. Bemporad, M. Morari, V. Dua, E. N. Pistikopoulos, The explicit linear quadratic regulator for constrained systems, Automatica 38 (1) (2002) 3–20.

[31] S. V. Rakovic, E. C. Kerrigan, K. I. Kouramas, D. Q. Mayne, Invariant approximations of the minimal robust positively invariant set, IEEE Transactions on Automatic Control 50 (3) (2005) 406–410.

[32] Y. Sun, N. H. El-Farra, Quasi-decentralized model-based networked control of process systems, Computers and Chemical Engineering 32 (9) (2008) 2016–2029.