

1 On the effect of line current width and relative position
2 on the multi-spacecraft curlometer technique

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8 **Abstract**

The response of the multi-spacecraft curlometer technique to variations in the size and relative position of infinitely long line currents with radially varying current density is systematically investigated for spacecraft in a regular tetrahedral formation. It is shown that, for line currents with a width less than the spacecraft separation, there is significant variation in the returned current with position of that current within the tetrahedron. For infinitely thin line currents, the curlometer tends to detect approximately 20% of the input current. For increasingly wide line currents there is less variation of the curlometer results with position of the current and the percentage of current magnitude detected increases. When the width of the current system is half the spacecraft separation, the curlometer tends to detect approximately 80% of the input current. These results are discussed in the context of multi-scale, multi-spacecraft missions.

9 *Key words:*

10 **1. Introduction**

11 Currents are a critical part of any magnetised plasma environment, with
12 current sheets separating different plasma regimes through magnetic field shears
13 and current sheets, and line currents transporting energy along magnetic field-
14 lines. They are also intrinsically linked to electric and magnetic fields and as

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15 such, may play an important part in the reconnection of magnetic field lines.
16 In particular, the currents appear in two terms of the Generalised Ohm's Law,
17 the Hall term and the anomalous resistivity term, which may be important
18 at different scale sizes. As such, in order to understand fundamental physical
19 processes within plasmas, it is necessary to accurately compare currents detected
20 at different scale sizes.

21 The curlometer technique (Dunlop et al., 1988) combines magnetic field data
22 from four non-coplanar spacecraft, assuming linear variations of the field be-
23 tween the spacecraft, to determine the dyad $\nabla\mathbf{B}$. An estimate of the curl of the
24 magnetic field within the volume delimited by the spacecraft can be calculated
25 by combining the off-diagonal terms of this dyad. In the absence of high fre-
26 quency variations in the electric field, Ampère's law states that the curl of the
27 magnetic field is proportional to the current density through the volume, hence
28 from this estimate of $\nabla \times \mathbf{B}$, an estimate of the current density flowing through
29 the spacecraft tetrahedron can be found.

30 The curlometer technique also allows an estimate of $\nabla \cdot \mathbf{B}$ to be calculated
31 from the diagonal terms in $\nabla\mathbf{B}$. Due to a combination of the non-linear varia-
32 tions of the magnetic field within the spacecraft volume and uncertainties in the
33 measurements of magnetic field values and spacecraft position, the estimated
34 $\nabla \cdot \mathbf{B}$ is almost always non-zero. Given Gauss' Magnetic Field Law ($\nabla \cdot \mathbf{B} = 0$),
35 it is clear that any non-zero estimation of $\nabla \cdot \mathbf{B}$ is indicative of the limitations of
36 the technique, although it is not necessarily indicative of the uncertainty in the
37 estimate of the current density. Robert et al. (1998) showed that there was no
38 one-to-one correlation between the relative error in the current density, $\Delta j/j$,
39 and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ although statistically the two were similar.

40 The curlometer technique has been used to investigate physical processes in
41 the magnetosphere using data from the Cluster spacecraft. At a system level,
42 the Cluster spacecraft have been used to measure the magnetopause currents
43 (e.g. Dunlop et al., 2002; Dunlop and Balogh, 2005) and magnetotail current
44 sheet (e.g. Runov et al., 2003, 2005, 2006). Dynamical features such as flux
45 transfer events (Phan et al., 2004) and bursty bulk flows (Forsyth et al., 2008)

46 have also been examined. At a more physical level, the curlometer has been used
47 to compare the Hall and electron pressure tensor terms in the Generalised Ohm's
48 law (Henderson et al., 2006, 2008). For a more complete review of previous work
49 using the curlometer technique see the review paper by Dunlop and Eastwood
50 (2008).

51 Previous studies have investigated the curlometer's accuracy for a variety of
52 tetrahedron shapes, driven by the fact that, without significant manoeuvring of
53 the spacecraft during an orbit, the shape of a constellation of spacecraft will
54 evolve along that orbit (e.g. Robert and Roux, 1993; Dunlop and Balogh, 1993;
55 Coeur-Joly et al., 1995; Robert et al., 1995, 1998). The majority of these studies
56 suggested various quality parameters based on the shape of the tetrahedron
57 and examined them using the Tsyganenko (1987) magnetic field model. It was
58 noted, however, that local, transient variations in the Earth's magnetic field
59 that were not included in the model could invalidate these quality parameters
60 as the accuracy of the estimates of \mathbf{j} is dependent on both the shape of the
61 spacecraft tetrahedron and the configuration of the magnetic field.

62 Runov et al. (2005) modelled the response of the curlometer to a Harris-
63 type current sheet (Harris, 1962) in order to determine an appropriate limit for
64 $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ below which the currents returned by the curlometer are valid.
65 They showed that, for the Harris-type current sheets, $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ and $\Delta j / j$
66 did not vary linearly with changing scale sizes of the current sheet, although
67 both decreased with increasing scale size. $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ appeared to tend
68 towards 0.28 for current systems larger than the spacecraft tetrahedron.

69 As previous studies are concerned with planar currents, this study concen-
70 trates on the effects of the relative scale size of the tetrahedron and a line current,
71 and the effect of the position of the centre of this current system relative to the
72 spacecraft tetrahedron. Although the majority of currents encountered in the
73 magnetosphere will be in current sheets, field-aligned currents associated with
74 features such as bursty bulk flows, flux transfer events and flux ropes may be
75 more filamentary in nature or have a form such that the radial extent is of
76 the same order as the spacecraft separation. In the following sections we will

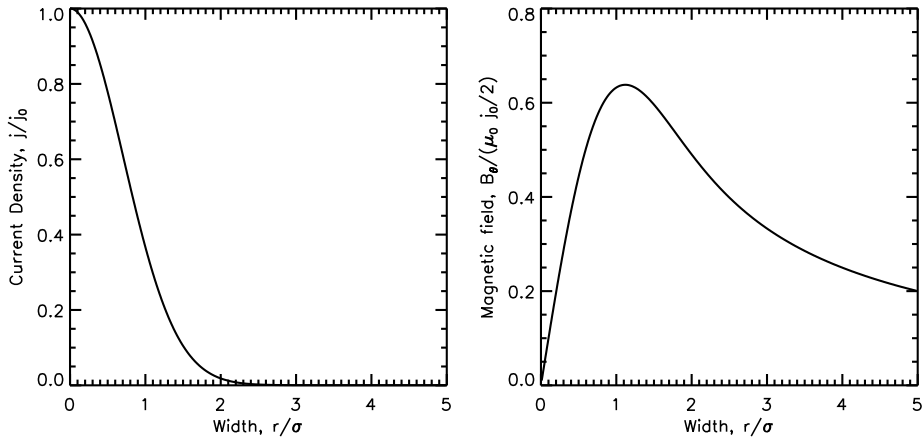


Figure 1: Plots of (a) j_{in}/j_0 and (b) $|B_\theta(r)|$ against radial position in units of the width of the current system, r/σ .

77 describe the methodology and the model line current system employed, show
 78 how the various parameters calculated by the curlometer vary with line current
 79 width and location, and discuss the considerations that need to be made when
 80 applying the curlometer technique to multi-scale observations and to different
 81 current structures.

82 2. Model set up

The current system employed in this study is an infinitely long current system with a current density distribution of $j_{in}(\mathbf{r}) = j_0 e^{-r^2/\sigma^2} \hat{\mathbf{z}}$ where σ is a constant which we refer to as the width of the current system. The magnetic field from this current system is then given by

$$B_\theta(\mathbf{r}) = K(1 - e^{-r^2/\sigma^2}) + constant \quad (1)$$

83 where $K = \mu_0 \pi j_0 / 2\pi r$. The form of this current system and the associated
 84 magnetic field magnitude are shown in Fig. 1

85 Given that previous studies have shown that irregular spacecraft tetrahedra
 86 have an effect on the results of the curlometer, we distribute our test “spacecraft”
 87 in a regular tetrahedron, with three spacecraft in the XY plane and one below

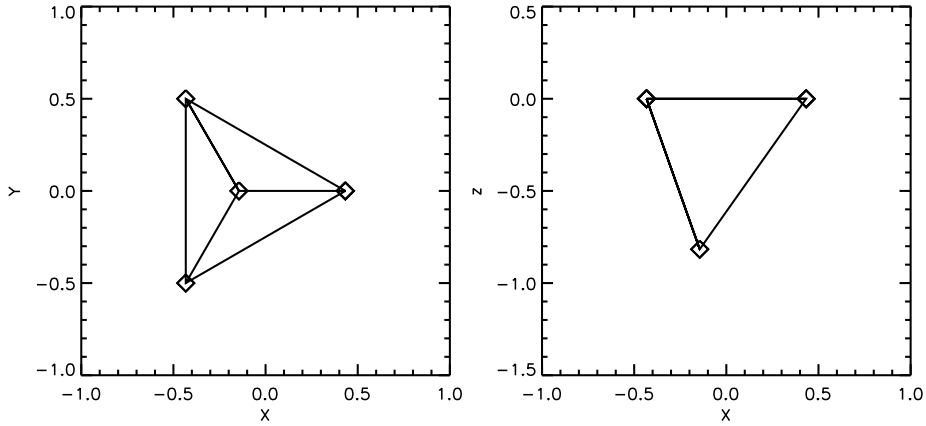


Figure 2: Plots of the locations of the test “spacecraft” in (a) the XY plane and (b) the XZ plane.

88 this plane (see Fig. 2). As such, throughout this study the current direction is
 89 always perpendicular to one plane of the spacecraft tetrahedron.

90 In order to examine the response of the curlometer technique to the model
 91 input current, $\nabla \times \mathbf{B}$ and $\nabla \cdot \mathbf{B}$ were calculated using the curlometer for currents
 92 centred in and around the spacecraft tetrahedron. $\nabla \times \mathbf{B}$ and $\nabla \cdot \mathbf{B}$ were calcu-
 93 lated for a 100×100 grid of locations with a resolution of $0.02 r_{sc} \times 0.02 r_{sc}$ where
 94 r_{sc} is the spacecraft separation. To compare the results of the model curlometer
 95 with the known input current, the current flowing through the spacecraft tetra-
 96 hedron was also calculated for each location of the current system centre. Given
 97 the form of the current, this is non-trivial. As a simplification, the current sys-
 98 tem was calculated in a 2-dimensional array with a resolution of $10^{-3} \sigma \times 10^{-3} \sigma$.
 99 The current flowing through the tetrahedron was estimated by summing the
 100 array elements that fall within the face of the tetrahedron perpendicular to the
 101 input current. It should be noted that the vector sum of the areas of the faces
 102 of the tetrahedron that are not perpendicular to the current is equivalent to the
 103 vector area of the face that is perpendicular to the current.

104 **3. Model results**

105 The limiting case of the model current system employed is an infinitely long,
 106 infinitely thin line current ($\sigma = 0$). In this case, the current density flowing
 107 through the tetrahedron can be calculated explicitly as the total current divided
 108 by the sum of the vector areas of three faces of the tetrahedron. Figure 3 shows
 109 the results from the curlometer technique for this current. The panels (a)-(d)
 110 show the current density (j) calculated by the curlometer scaled by π , $\nabla \cdot \mathbf{B}$
 111 calculated by the curlometer, the ratio of the current density calculated by the
 112 curlometer to the input current density flowing through the tetrahedron (j/j_{in}),
 113 and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ respectively against position of the centre of the current
 114 system in the XY plane. The colours show the value of the various parameters
 115 calculated by the curlometer when the current was centred on that point. For
 116 all points within the tetrahedron, the input current density is constant. Panels
 117 (e) and (f) show histograms of j/j_{in} and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ for points within the
 118 tetrahedron.

119 Panel (c) shows that the curlometer tends to under-estimate the currents
 120 flowing through the tetrahedron, detecting approximately 20% of the input cur-
 121 rent density, although for currents located close to the spacecraft (at the vertices
 122 of the tetrahedron), the current density is over-estimated. This is due to the
 123 magnetic field becoming increasingly large close to the location of the current for
 124 this particular current system ($\mathbf{B} = \mu_0 j_0 / 2\pi r \rightarrow \infty$ as $r \rightarrow 0$). The area over
 125 which this over-estimation occurs is largest about the out-of-the-plane space-
 126 craft. Panel (e) shows that both the modal current density (taken to be the
 127 peak of the histogram) and the mean current density were less than the input
 128 current density. The mean current is approximately 25% higher than the modal
 129 current due to the large current densities calculated when the currents lay close
 130 to a spacecraft.

131 Panel (a) shows that the curlometer calculates a curl of \mathbf{B} for currents that
 132 lie outside the tetrahedron and thus it infers currents inside the tetrahedron,
 133 although these are small. The magnitude of j decays more slowly along the

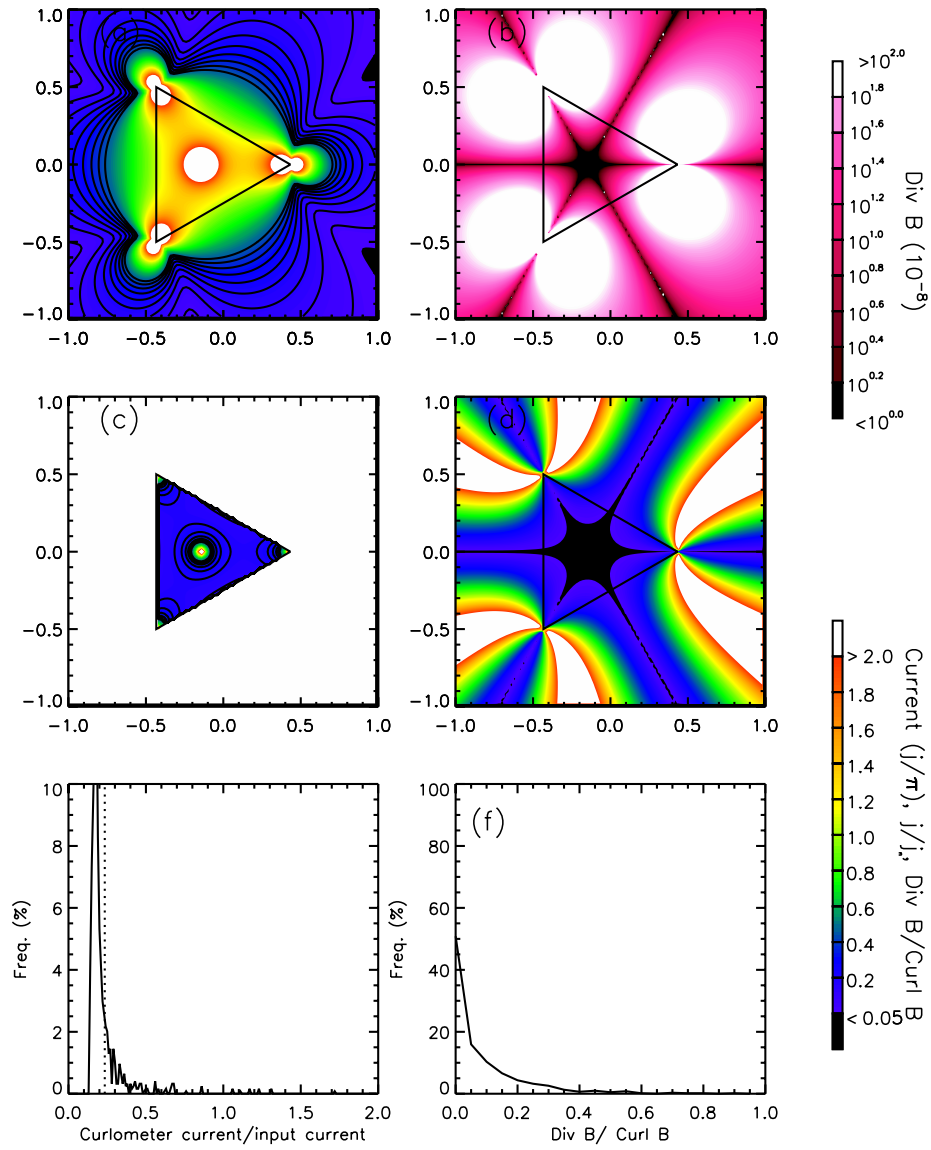


Figure 3: Plots of (a) j/π , (b) $\nabla \cdot \mathbf{B}$, (c) j/j_{in} , (d) $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ for different locations of the input current. The input current is an infinitely long, infinitely thin line current. White (black) indicates large (small) values that are off the given scale. Panel (e) and (f) show a histograms of the j/j_{in} and $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ within the tetrahedron. The dashed line in (e) shows the mean value of j/j_{in} for currents within the tetrahedron. (Colour figure for web only)

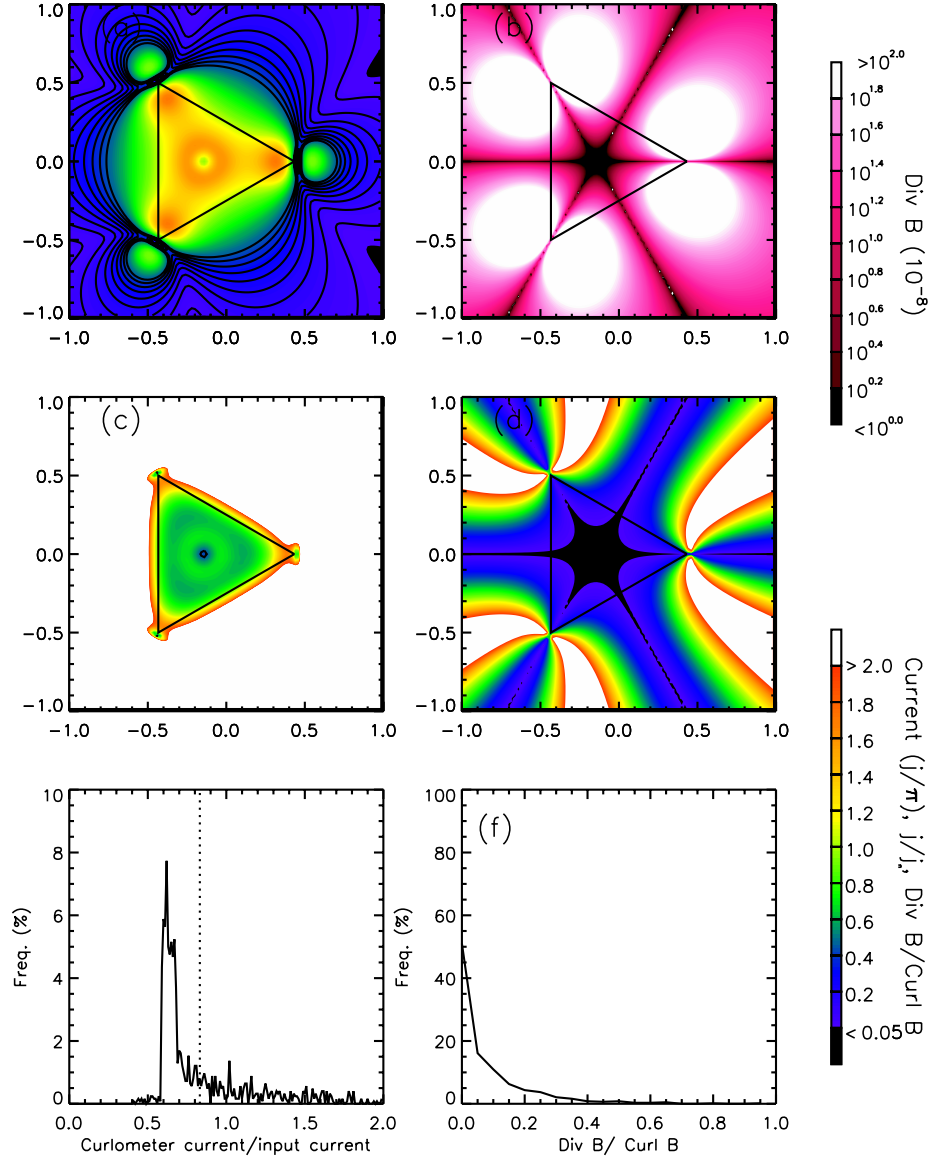


Figure 4: Plots of (a) j , (b) $\nabla \cdot \mathbf{B}$, (c) j/j_{in} , (d) $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ for different locations of the input current. The input current is an infinitely long line current with a radial distribution of $j = e^{-r^2/\sigma^2}/2 \hat{\mathbf{z}}$, where $\sigma = 0.1r_{sc}$. Panel (e) and (f) show a histograms of the j/j_{in} and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ within the tetrahedron. (Colour figure for web only)

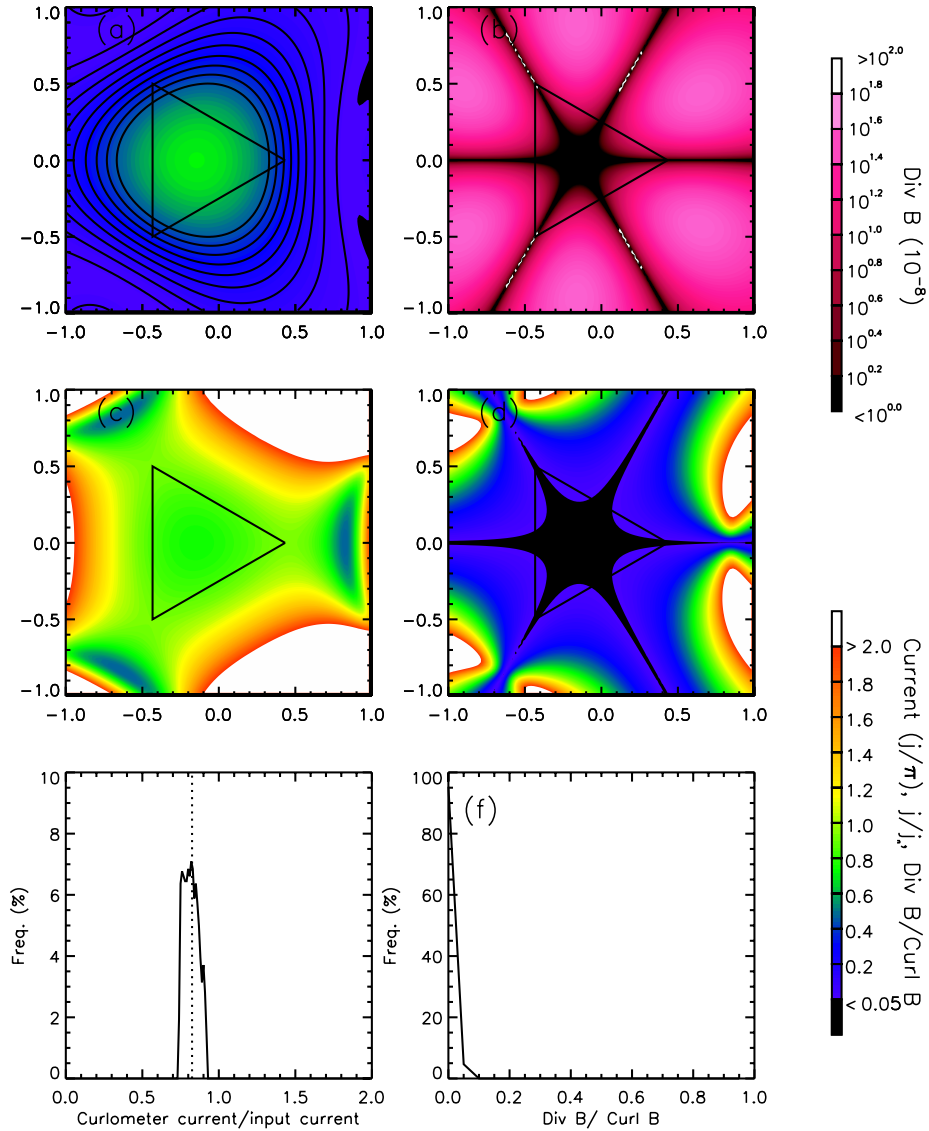


Figure 5: Plots of (a) j , (b) $\nabla \cdot \mathbf{B}$, (c) j/j_{in} , (d) $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ for different locations of the input current. White (black) indicates large (small) values that are off the given scale. The input current is an infinitely long line current with a radial distribution of $j = e^{-r^2/\sigma^2}/2 \hat{\mathbf{z}}$, where $\sigma = 0.5r_{sc}$. Panel (e) and (f) show a histograms of the j/j_{in} and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ within the tetrahedron. (Colour figure for web only)

134 perpendicular bisectors of the faces of the tetrahedron. Along these bisectors,
 135 $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}| < 0.4$, which is comparable with the limits used to indicate reliable
 136 curlometer results from spacecraft data for a current sheet (e.g. Runov et al.,
 137 2005), although is significantly higher than the values determined for currents
 138 passing through the tetrahedron ($|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}| < 0.05$).

139 Figures 4 and 5 show similar plots to Fig. 3 for current systems with widths
 140 σ of 0.1 and 0.5 times the spacecraft separation respectively. Given that the
 141 current system has an extent in XY plane, j_{in} is calculated as the portion of
 142 the current system passing through the tetrahedron, as described in Section 2.

143 Comparing Fig. 4 with Fig. 3 shows that the peaks in j in the vicinity of the
 144 spacecraft are no longer present, replaced by local minima. This is because the
 145 magnetic field tends to infinity close to the infinitely thin line current, whereas
 146 the magnetic field is zero at the centre of a distributed current system (Eq. 1).
 147 Panels (c) and (e) show that $j/j_{in} \approx 0.6$ within the tetrahedron and that there
 148 is variation in j/j_{in} across the tetrahedron. Panel (c) shows that the largest
 149 j/j_{in} are along the edges of the tetrahedron and (to a lesser extent) in a ring
 150 around the out-of-the-plane spacecraft.

151 Comparing Fig. 5 with Fig. 4 shows that the variations of j/j_{in} within the
 152 tetrahedron are reduced for larger current systems (Panel c). In particular, the
 153 high j/j_{in} tail has been removed such that there are no points with $j/j_{in} > 1$
 154 within the tetrahedron. Panel (e) shows that the mean j/j_{in} (dotted line) is
 155 close to the peak of the distribution for the $\sigma = 0.5$ case. Comparing Panel (d)
 156 between Figs. 5 and 3 shows that $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ has similar pattern in both
 157 figures, suggesting that the pattern for $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ is scalable.

158 For currents centred outside the tetrahedron, the curlometer is able to es-
 159 timate the current density of the portion of the current flowing through the
 160 tetrahedron. Panel (c) shows that when a current centre is less than $\sigma/2$ from
 161 one of the faces of the tetrahedron, the curlometer results are comparable to
 162 those from currents passing through the tetrahedron. With increasing distance
 163 from the tetrahedron, the curlometer over-estimates the current density, as in
 164 the line current case. It is interesting to note, however, that close to the vertices

165 of the tetrahedron, but for currents centred outside the tetrahedron, there are
166 small regions in which the current passing through the tetrahedron is under-
167 estimated.

168 Figure 6 shows the variation of j/j_{in} and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ with the width of the
169 current system relative to the spacecraft separation. The black lines indicate
170 the mean value within the tetrahedron. The red line in panel (a) shows the
171 modal value (taken to be the peak of a histogram of the current density with a
172 bin size of 0.01). The error bars indicate the standard deviation.

173 As the current system width increases relative to the separation of the space-
174 craft, the curlometer tends to return current densities closer to the input current
175 density flowing through the tetrahedron. The modal and mean current densi-
176 ties converge such that above $\sigma/r_{sc} = 0.6$ they are approximately equal and
177 the standard deviation decreases. Whilst the modal value of j/j_{in} increases
178 fairly steadily with σ/r_{sc} , the mean current densities show a decrease between
179 $\sigma/r_{sc} = 0.1$ and $\sigma/r_{sc} = 0.3$. The mean value of $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ decreases steadily
180 with σ/r_{sc} .

181 Below $\sigma/r_{sc} = 0.5$, both the mean and modal j/j_{in} show deviation away
182 from the smooth curve evident above $\sigma/r_{sc} = 0.5$. We suggest that this devi-
183 ation is due to the limitations of estimating the input current density from a
184 finite resolution square grid across a triangular face of the tetrahedron. This is
185 supported by the fact that $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$, which is determined from the explicitly
186 calculated magnetic field, varies smoothly with σ/r_{sc} . Note, however, that the
187 values for $\sigma/r_{sc} = 0$ are explicitly calculated and therefore not subject to this
188 error.

189 4. Summary and Discussion

190 In the previous section we have shown that the results from the curlometer
191 technique can vary with respect to position and width of a line current relative
192 to the observing spacecraft tetrahedron. For the model line current examined,
193 the results show:

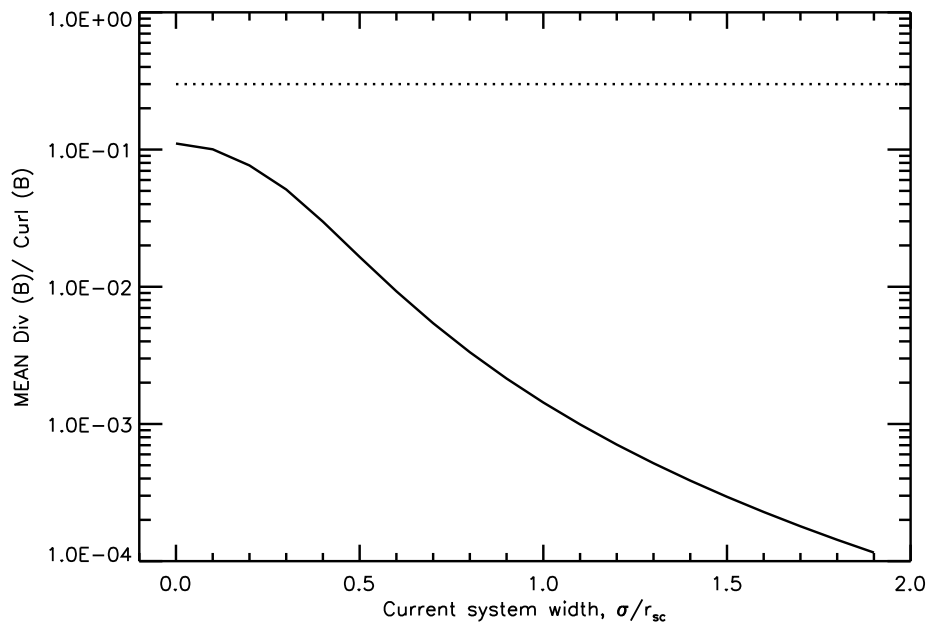
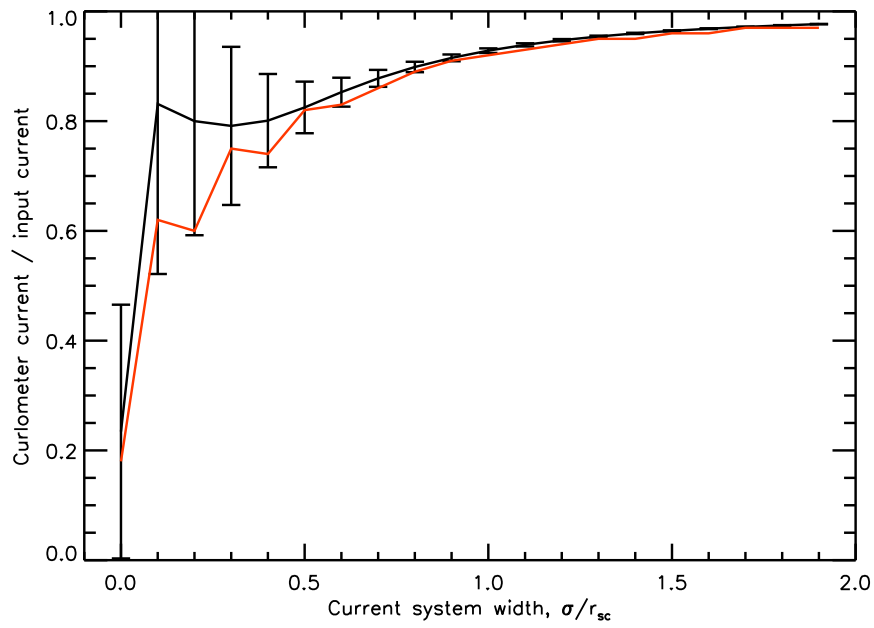


Figure 6: Plots of (a) mean (black) and modal (red) j/j_n and (b) $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ against current system width for currents systems with their centre passing through the spacecraft tetrahedron. (Colour figure for web only)

- 194 1. In the limit of an infinitely thin, infinitely long line current, the curlometer
195 tends to return 20% of the current density on average, but the value returned
196 is strongly dependant on the position of the current within the tetrahedron,
197 2. The curlometer tends to under-estimate the current density for currents flow-
198 ing through the tetrahedron, although this under-estimation reduces as line
199 current width increases,
200 3. The curlometer detects the effects of currents centred outside the tetrahe-
201 dron. For those line currents which are less than $\sigma/2$ from the edge of the
202 the tetrahedron, j/j_{in} is comparable to those currents with centres within
203 the tetrahedron. Outside of this, the curlometer over-estimates the current
204 flowing through the tetrahedron.
205 4. $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ is small (< 0.1) for currents passing through the tetrahedron
206 and is larger for currents passing outside the tetrahedron, although along the
207 perpendicular bisector of the faces of the tetrahedron, $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ remains
208 relatively low,

209 In this section we discuss these results in the context of multi-scale, multi-
210 spacecraft missions and previous results.

211 In the model used, there were no uncertainties in the position of the observing
212 spacecraft and the magnetic field values. As such, the non-zero calculation of
213 $\nabla \cdot \mathbf{B}$ is due to non-linear variations of the magnetic field. For the current systems
214 tested, $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ was significantly less than values typically calculated
215 from Cluster data (e.g. Forsyth et al., 2008) and from the model results of
216 Runov et al. (2005). We suggest that the higher values of $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ often
217 calculated from Cluster data are indicative of uncertainties in the spacecraft
218 positions and magnetic field values, although are also dependant on the form of
219 the current being examined. As previously discussed by Robert et al. (1998),
220 there is not necessarily a dependence between the uncertainties in the diagonal
221 and off-diagonal terms in the $\nabla \mathbf{B}$ dyad, although we note that both the error
222 in the current returned by the curlometer and $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ decreased with
223 increasing current system width. One must be aware that for a multi-scale,

224 multi-tetrahedra mission, the factors that determine $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ will vary
225 between the scale sizes (non-linearity will be more important than uncertainties
226 in position at large scale sizes and vice-versa). One must also be aware that the
227 results presented here are valid for the current system tested but are current
228 system dependant (see e.g. Runov et al., 2005)

229 Figure 6 shows that as the scale size of the current increases, the mean
230 value of $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ for currents passing through the tetrahedron becomes
231 increasingly small. Previous studies have used $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ to determine
232 the quality of the results from the curlometer, with lower values indicating
233 more reliable results, despite pre-Cluster studies suggesting that $\nabla \cdot \mathbf{B}$ is not
234 a reliable indicator of quality (Robert and Roux, 1993; Dunlop and Balogh,
235 1993; Coeur-Joly et al., 1995; Robert et al., 1995, 1998). However, one must
236 consider that alternative indicators of quality rely on a priori knowledge of the
237 current system being investigated or do not consider local variations in the
238 magnetic field. As such, comparing model current systems and spacecraft data
239 is increasingly important for multi-scale missions.

240 Comparing the width of the line currents relative to the separation of the
241 spacecraft (σ/r_{sc}) and the proportion of the input current returned gives an
242 indication that one can set a lower limit to the relative width of the line cur-
243 rent to the spacecraft separation for which the curlometer provides meaningful
244 results. For line currents with a $\sigma/r_{sc} \geq 0.5$ the curlometer detects approxi-
245 mately $80 \pm 15\%$ of the input current. In terms of multi-tetrahedra missions,
246 this indicates that one must take care when comparing currents from various
247 scale sizes of tetrahedron and that one can set limits on the required accuracy of
248 the curlometer such that the results from larger tetrahedra can be disregarded
249 for current systems below a given relative scale size.

250 Clearly, in order to use the curlometer technique for a multi-tetrahedron
251 spacecraft mission, one must be aware that the location of the current system
252 under investigation within the tetrahedra could have a significant effect on the
253 results returned by the various tetrahedra, especially if the current system is
254 small compared to one or more of the spacecraft tetrahedra. One would expect

255 that, for most situations, the smallest tetrahedra will give the most accurate re-
256 sults and the largest the least accurate, although this does not take into account
257 uncertainties in the measurement of spacecraft position and magnetic field, the
258 effect of which is likely to be greater for smaller tetrahedra rather than larger
259 ones.

260 The majority of previous studies into the accuracy and response of the cur-
261 lomometer have concentrated on current sheets, either of the Harris type or intrinsic
262 to global magnetic field models. However, Robert et al. (1998) used simple cur-
263 rent tube models (line currents) to examine how the results of the curlometer
264 varied with varying tetrahedron shape in order to determine a parameter of the
265 tetrahedron shape that could be used as an indicator of quality for the current
266 density measurements. In their study the barycentre of the tetrahedra were
267 at a fixed location and the tetrahedra were normalised to have the same mean
268 spacecraft separation and hence no attempt was made to examine the effect of
269 the location or width of the line current on the curlometer.

270 It is not clear that the effects of variations in the tetrahedron shape and
271 size on the results of the curlometer for line and planar currents are directly
272 comparable. It is expected that the curlometer will return more accurate results
273 when the current systems is much larger than the spacecraft tetrahedron as
274 variations in the field can be better approximated as linear. For the same
275 reason, variations in the shape of the tetrahedron are also expected to cause
276 variations in the curlometer results. Studies have shown that this is true for
277 planar current systems (Robert and Roux, 1993; Dunlop and Balogh, 1993;
278 Coeur-Joly et al., 1995; Robert et al., 1995) , but there has previously been
279 no direct comparison between the results for line currents and planar currents.
280 However, by comparing our results with the results of Runov et al. (2005), it
281 can be seen that $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ is much smaller in the case of a line current and
282 appears to drop continuously with increasing current system width, whereas
283 $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ tends towards ≈ 0.28 for the current sheet case. Also, the relative
284 errors of the currents appear to tend towards similar values for both line and
285 planar current systems larger than the spacecraft separation.

286 Given that accuracy of the curlometer is dependant not only on the relative
287 size and shape of the spacecraft tetrahedron but also on the current system
288 under observation, it is useful to determine whether or not the observed current
289 system is similar to a planar or line current structure. These two current systems
290 should be discernible from the magnetic field data. For a planar current system,
291 the magnetic field rotates in one direction, the direction perpendicular to the
292 current and the current sheet normal. In the line current case, the magnetic
293 field parallel to the current direction is invariant such that the field varies in
294 the plane perpendicular to the current. It should be noted that in order to
295 determine if the field is invariant in the direction perpendicular or parallel to
296 the current direction requires accurate determination of the current direction,
297 an issue which we do not address in this paper.

298 Although the results presented are specific to the line currents tested, they
299 clearly show that in order to compare results from the curlometer across various
300 scale sizes one has to take into account the scale sizes being examined and the
301 location of the current system. This is clearly a non-trivial task. We suggest,
302 however, that by comparing the curlometer results from multiple scales with a
303 model of the current system under examination may prove insightful in most
304 cases.

305 **5. Conclusions**

306 The results of the curlometer technique have been examined for an infinitely
307 long line current with a given radial distribution of current density for a range
308 of positions relative to the observing spacecraft tetrahedron and for a range
309 of scale sizes (i.e. line current widths) of the radial variation of the current
310 density. The magnetic field associated with these current systems was explicitly
311 calculated at the locations of four test “spacecraft”, and these data were then
312 processed using the curlometer technique. The results show:

- 313 1. For small-scale line currents, the proportion of the input current returned by
314 the curlometer varies with the location of the centre of the current system

- 315 through the spacecraft tetrahedron,
- 316 2. For small-scale line currents, the curlometer under-estimates the current den-
317 sity by up to 80%, although for current systems with $\sigma/r_{sc} \geq 0.5$, the cur-
318 lometer tends to return 80% of the current,
- 319 3. The curlometer will detect the effects of currents located entirely outside the
320 limits of the spacecraft tetrahedron, although these values are small compared
321 to the input current,
- 322 4. $|\nabla \cdot \mathbf{B}/\nabla \times \mathbf{B}|$ can be low for currents detected outside the spacecraft tetra-
323 hedron and as such, is not always a useful indicator of the quality of the
324 determination of j .

325 The results presented here are specific to the line currents examined, al-
326 though our results suggest that in order to meaningfully compare the results
327 from the curlometer at various scale sizes, one should compare spacecraft data
328 with a model of the current system under examination.

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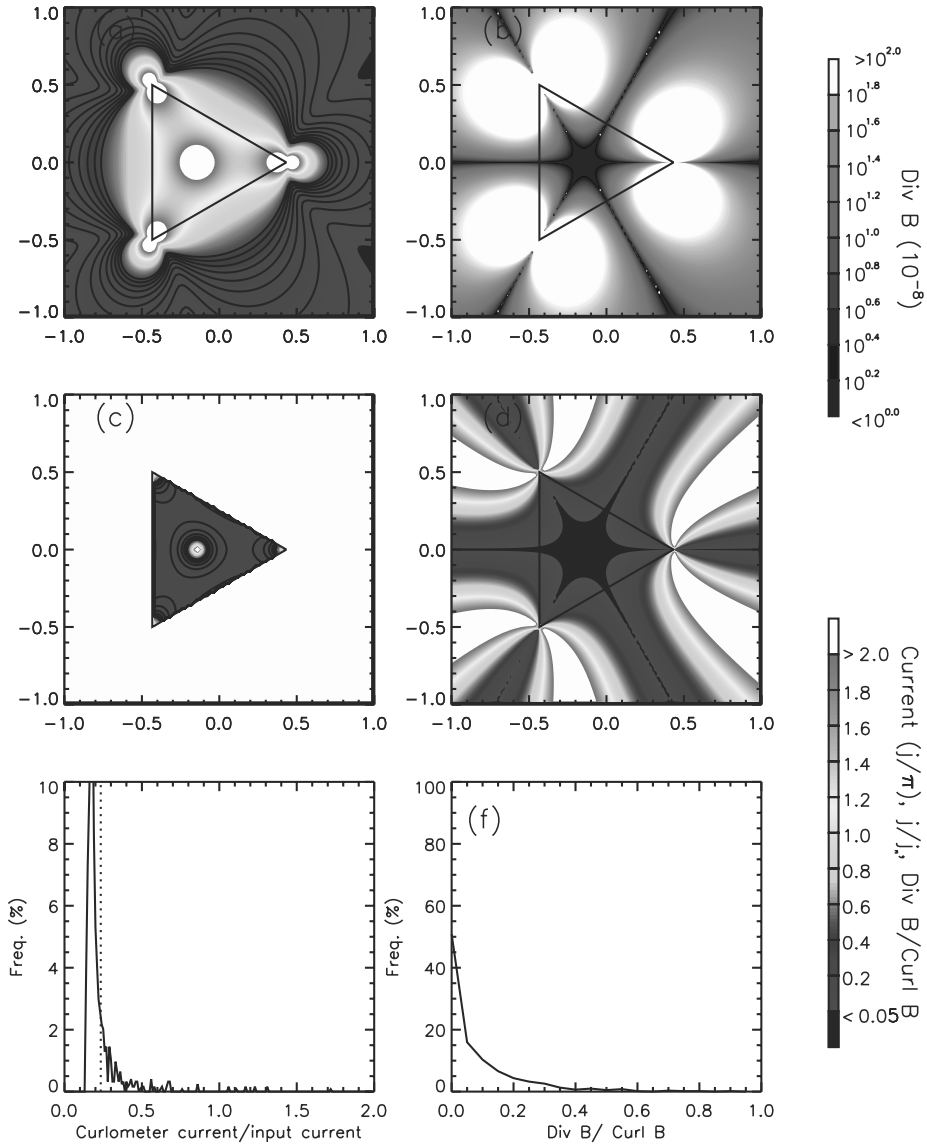


Figure 7: Plots of (a) j/π , (b) $\nabla \cdot \mathbf{B}$, (c) j/j_{in} , (d) $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ for different locations of the input current. The input current is an infinitely long, infinitely thin line current. White (black) indicates large (small) values that are off the given scale. Panel (e) and (f) show a histograms of the j/j_{in} and $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ within the tetrahedron. The dashed line in (e) shows the mean value of j/j_{in} for currents within the tetrahedron. (Print copy figure)

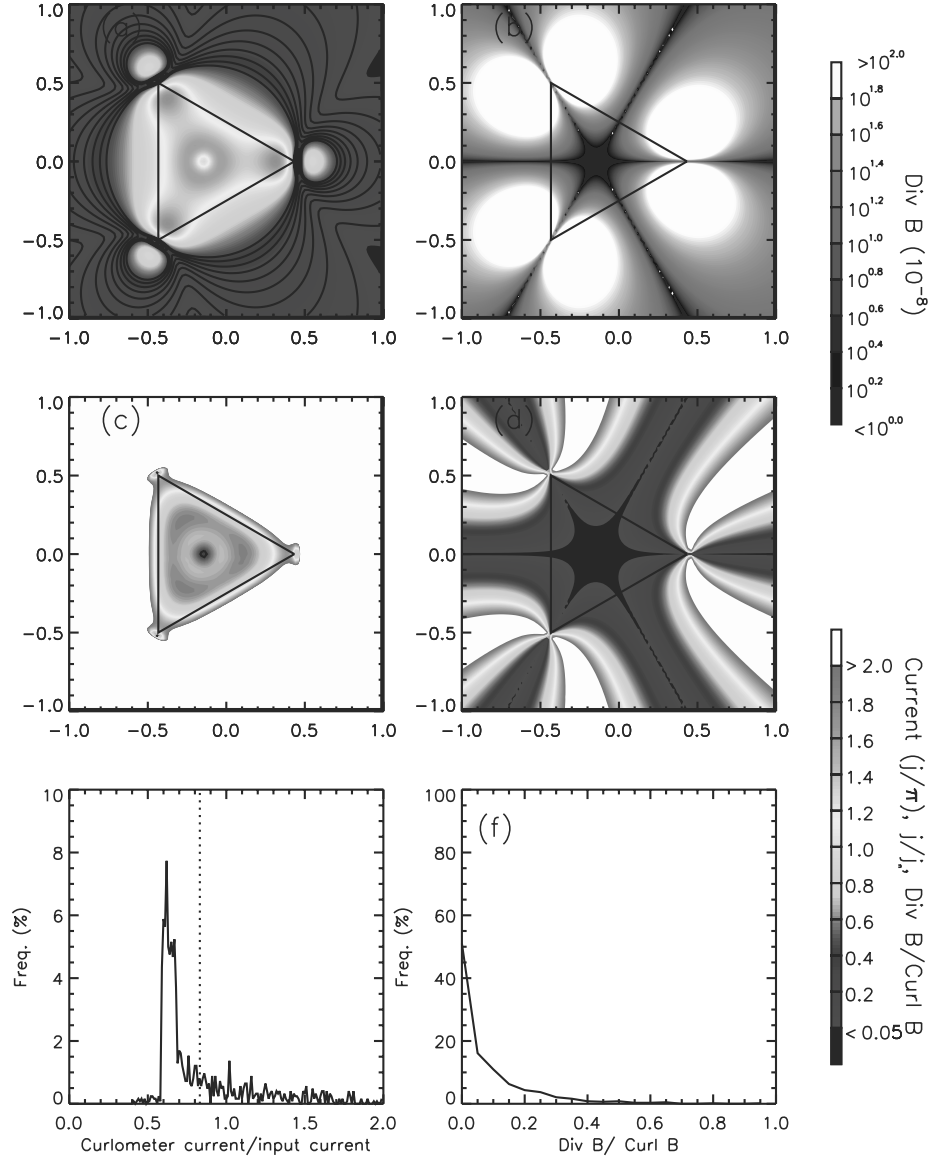


Figure 8: Plots of (a) \mathbf{j} , (b) $\nabla \cdot \mathbf{B}$, (c) j/j_{in} , (d) $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ for different locations of the input current. The input current is an infinitely long line current with a radial distribution of $j = e^{-r^2/\sigma^2}/2 \hat{\mathbf{z}}$, where $\sigma = 0.1r_{sc}$. Panel (e) and (f) show a histograms of the j/j_{in} and $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ within the tetrahedron. (Print copy figure)

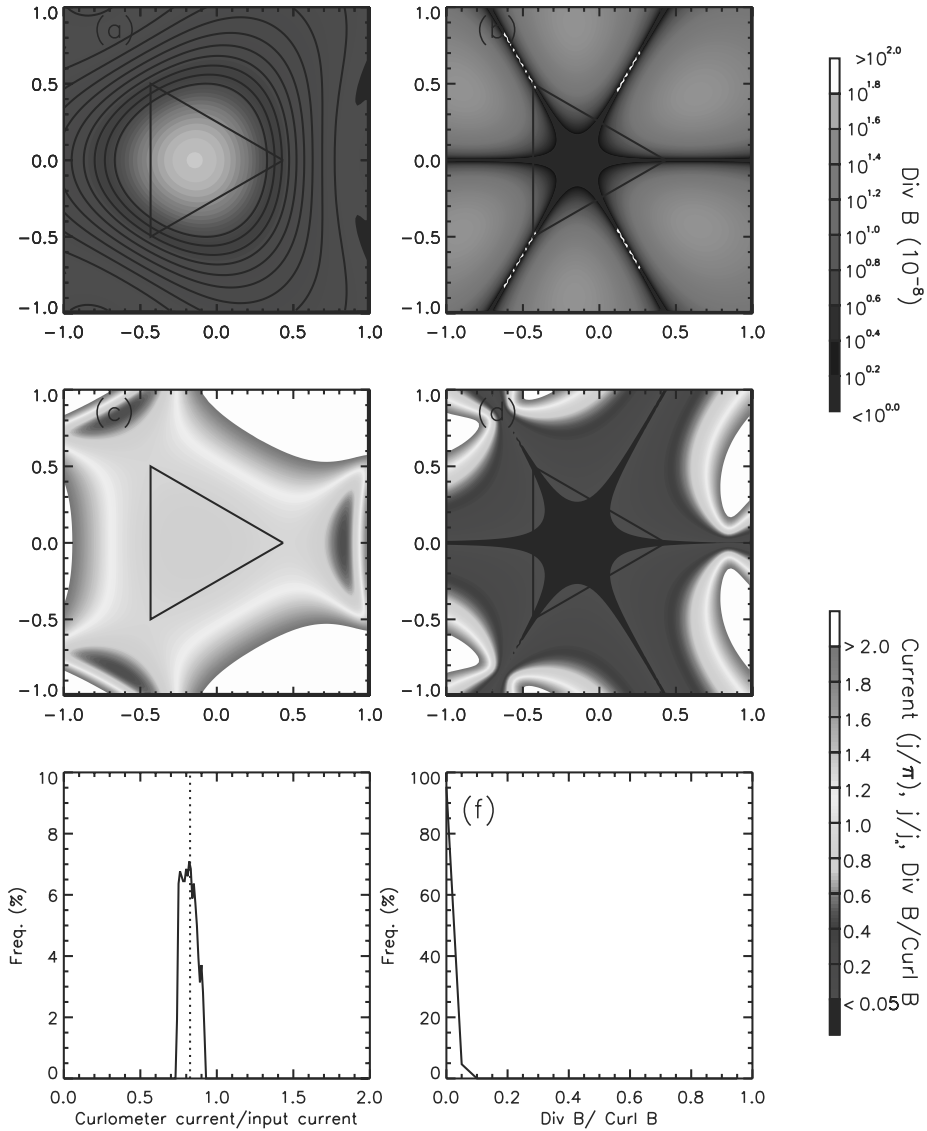


Figure 9: Plots of (a) j , (b) $\nabla \cdot \mathbf{B}$, (c) j/j_{in} , (d) $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ for different locations of the input current. White (black) indicates large (small) values that are off the given scale. The input current is an infinitely long line current with a radial distribution of $j = e^{-r^2/\sigma^2}/2 \hat{\mathbf{z}}$, where $\sigma = 0.5r_{sc}$. Panel (e) and (f) show a histograms of the j/j_{in} and $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ within the tetrahedron. (Print copy figure)

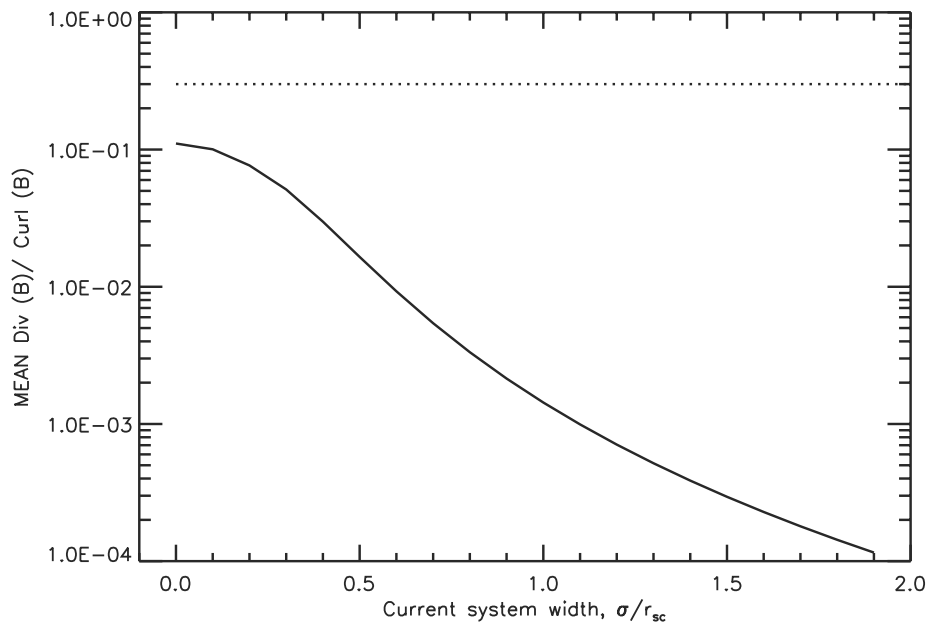
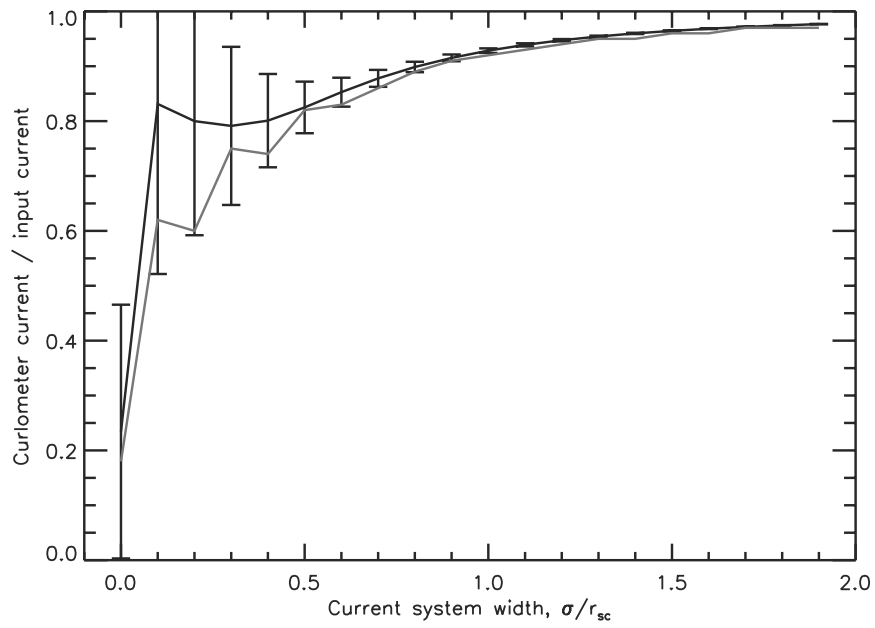


Figure 10: Plots of (a) mean (black) and modal (red) $j/j_i n$ and (b) $|\nabla \cdot \mathbf{B} / \nabla \times \mathbf{B}|$ against current system width for currents systems with their centre passing through the spacecraft tetrahedron. (Print copy figure)

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