TRAVEL AND INTERACTION IN THE GREEK AND ROMAN WORLD. A REVIEW OF SOME COMPUTATIONAL MODELLING APPROACHES

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1. Introduction

The inferential leap from static archaeological patterns to dynamic past behaviours is always challenging, especially in the case of the Greek and Roman world, where there is a real temptation to let the historical record do the heavy interpretative lifting. However, computational and quantitative techniques can also be of great assistance and this chapter offers a brief overview of their potential, as well as of some continuing problems associated with their effective application.

A useful preliminary question to pose, however, is simply this: why, in the first place, should we seek to build method and theory about travel and transport behaviours using classical archaeological datasets? In one sense, an easy but fairly narrow answer is: to understand better the dynamics of this particular period of the human past and this specific geographic area. However, it might also be argued that the Greek and Roman world offers a host of modelling advantages that have wider relevance, as exemplars in other, less evidentially advantaged archaeological and historical contexts worldwide. First, we can think of unusual levels of inter-regional connectivity (at several spatial scales) as being a defining feature of Mediterranean landscapes in particular.[1] Second, Mediterranean, European, and Middle Eastern paleo-landscapes are comparatively well understood and formal movement networks such as roads have been intensively studied, as well as less formally articulated ones such as patterns of pathways, transhumance, and migration. Third, we possess excellent evidence for the economic logistics of human and animal movement in certain parts of the classical world (*e.g.* Hellenistic and Roman Egypt) that allows us to speculate about the spatial and temporal scale of particular activities as well as their social and political structure. Fourth, we have a comparatively developed understanding of assisted movement technologies in this period and region, such as chariots, carts, boats, *etc*. Fifth, we have a good feeling for the contexts in which information about routing and territory are exchanged, such as via itineraries and geographic exegesis. Sixth and finally, there are clearly also interesting differences to be explored in terms of the directionality and scale of travel in this world (*e.g.* sailing *versus* rowing or paddling; grain fleets *versus* tramping trade;[2] individuals *versus* armies), as well as how it might change over time.

[1] For examples see: P. Horden and N. Purcell, *The corrupting sea* (Oxford 2000) 123-72.

[2] Paddling is a form of assisted human movement involving people in small- to medium-sized boats who face forward and propel their vessel via a one- or two-bladed flat paddle. It can be contrasted with rowing which often occurs in similar sized craft, but where people face backwards to the

These advantages argue for an approach to model-building in this region that does not circumscribe its relevance only to questions of interest to Greek and Roman specialists, but seeks to explore which features are contingent and which ones are of wider convergent interest. The discussion offered below cannot hope to address much of that larger agenda, but contributes by considering some cross-culturally relevant issues to do with modelling travel and transport, such as: (a) the range of cost-benefit trade-offs typically involved, and (b) the impact of different modes of travel. Thereafter, I will take a brief look at a range of different computational modelling approaches that all have complementary, spatially-explicit contributions to make for our understanding of past movement and interaction behaviours.

2. Some initial provisos: cost trade-offs and transport modes

One of several theoretical criticisms that might be levelled at the computational models of movement used by archaeologists so far, might be that most applications have been agnostic about, or willfully ignorant of, the kinds of costs and benefits they are seeking to measure. A good example, discussed in more detail below, is that of 'cost surface' analysis and 'least cost path' delineation. These two related methods typically either do not define the units with which they measure (*i.e.* costs have values without direct real-world correlates), or they blur several kinds of costs together, as if they were the same. In fact, it is useful to think explicitly about the interaction of at least three broad domains of movement cost: time, effort, and uncertainty. We can for example consider (a) the time taken to get from point A to point B (*e.g.* in hours), or (b) the effort (in terms of metabolic energy or money spent), or (c) the risk associated with such travel (in terms of uncertain travel conditions, personal danger, loss of cargo, unwanted observation by another party, *etc*.). As the above examples suggest, there are sub-divisions within these three broad domains of cost as well, and we cannot assume that any of them are related in a simple linear way to one another. Rather, we must seek to establish likely relationships and trade-offs between different kinds of cost cross-culturally and/or empirically from the archaeological and historical record. In some cases, we may still wish to offer a blurred, aggregate model of movement (*e.g.* via the circuit theory approaches introduced below), but we should at least do so as a part of a carefully thought-out and strategic choice, encouraged for example by an especially large spatial scale and/or *longue durée*.

In any case, where more specific routes and costs are to be modelled, it is important to understand the relevant trade-offs involved. Two good examples are the zig-zagging behaviour exhibited by both humans and animals on mountain paths[3] and the

direction of travel and use oars that are usually fixed into oarlocks. Sailing refers to the use of boats in which the dominant form of propulsion is a sail that harnesses the strength of the wind (though some sailing ships can still be rowed on occasion). Grain ships in the Roman and Byzantine period in particular could be very large vessels (*e.g.* the 50+m long *Isis* described in Lucian's *The ship, or the wishes*, even if there is some literary exaggeration) and followed very direct routes, typically from Carthage or Alexandria to Rome or Constantinople. In contrast, tramping involved much smaller sailing ships whose routes and destinations were far more irregular, multi-stop, and flexible to changing market conditions.

[3] See: A. Bevan, C. Frederick, and N. Krahtopoulou, 'A digital Mediterranean countryside: GIS approaches to the spatial structure of the post-Medieval landscape on Kythera (Greece)',

coast-hugging behaviour of many pre-modern sailing ships.[4] Zig-zagging is the propensity for mountain paths to follow indirect 'hairpin' routes. It is mainly engendered by the curve of a human's metabolic energy expenditure on slopes of different steepness, but important features of the problem also relate to the chosen velocity of the traveller. Likewise, sailing ships tack because of a relationship between optimum speed forward in the direction of the destination and the optimum direction for effectively harnessing the force of the wind.[5] In addition, it is worth noting the propensity for much Mediterranean shipping to hug the coastline for several reasons: (a) to trade off speed of more direct travel against potential risks incurred by travel out of sight of land, and (b) to trade off speed of more direct travel against the greater certainties often provided by diurnal shifts in near-shore breezes.

As the above should emphasize, there may not always be easy answers to these potentially complex cost-benefit calculations, and indeed at times the simpler rule-of-thumb decisions that any given individual might make could conceivably render much of the complexity irrelevant for the modeller (or at least dramatically change the modelling agenda). Even so, we should at the very least be far more explicit about these issues than we typically have been so far, either by declaring our starting assumptions, or by testing whether the implications of different costs are significant or not (*e.g.* by sensitivity analysis).

A further issue that has hitherto been addressed in only a very limited way by archaeological modelling is the need to differentiate modes of travel involving different numbers of individuals (*e.g.* single person, large group), different kinds of human traveller (*e.g.* adult, child, female, male), different kinds of assisted travel and transport technology (*e.g.* on foot, on horseback, in a boat), different seasonal constraints, and different travelling agendas.[6] Related to this is also the issue of intermodal shifts – for example, the process of switching from boat to cart to foot transport – for which we have little, if any, coherent archaeological and ancient historical theorizing, in contrast, for example, to the attention this subject has received in modern transport planning.[7] In fact, it is to this critical threshold issue of 'transferral', or breakage-in-bulk for cargo, that future computational modelling will need to pay substantially greater attention, particularly with regard to the movement of goods. In addition, different group sizes (*e.g.* individual

Archeologia e Calcolatori 14 (2003) 227-29, and M. Llobera and T. J. Sluckin, 'Zigzagging: theoretical insights on climbing strategies', *Journal of Theoretical Biology* 247 (2007) 206–17.

[4] For example: for its relevance later in the Medieval period, see: F. Braudel, *The Mediterranean and the Mediterranean world in the age of Philip II* (London 1972).

[5] For a modelling initiative, see: A. Philpott and A. Mason, 'Optimising yacht routes under uncertainty', *Proceedings of the 15th Chesapeake Sailing Yacht Symposium* (2001).

[6] For examples of the latter two in Roman times see: R. Duncan-Jones, *Structure and scale in the Roman economy* (Cambridge 1990) 7-29, and C. van Tilburg, *Traffic and congestion in the Roman Empire* (Oxford 2007) 41-89.

[7] But also see some archaeological work on portage phenomena: A. Sherratt, 'Portages: a simple but powerful idea in understanding human history', in *The significance of portages. Proceedings of the first international conference on the significance of portages,* ed. C. Westerdahl (Oxford 2006) 1-13.

donkey or ship transport *versus* large caravans or fleets) obviously have different consequences for each of these different modes.

Table 1 provides some basic parameters for different kinds of travel speed and load, as relevant to Greek and Roman contexts.[8] It is surprising how rarely such variables as these are actually part of a modelling agenda (and indeed the computational examples provided later in the chapter do not do so either), but there are plenty of opportunities to so incorporate them in the future. Perhaps one of the best current examples is the work by Alberto Minetti on trade-offs between time, speed, and physiological stress on horses in equine postal services, where surprising cross-cultural regularities are present in the spacing of staging posts and the average adopted speed along the route.[9]

Type of Transport	**Speed (kmph)**	**Load (kg)**
Human pedestrian	4-5	n/a
Human porter	2.5-3	30-60
Pack donkey	2.5	60-100
Pack camel	3-4	150-250
Pack horse	4-8	80-180
Riding camel	15-30	n/a
Riding horse	10-30	n/a
Wagon	2-4	200-1000
Small rowboat/canoe	3-8	<1,000
Oared galley	7.5-15	<100,000?
Large riverboat	2-40	10,000-100,000
Sailing ship	2-7.5	10,000-1,000,000

Table 1. Some rough performance estimates for travel speed and load-carrying in the Greek and Roman world. Loads and speeds assume those sustainable over a full day's work, and could otherwise be higher for short trips. Speeds assume relatively flat terrain.[10]

3. Computational methods

Computational models can be more or less complex in their design and the number of parameters they consider. For archaeologists or historians, it is tempting to model as close a fit to observed reality as possible, building in all of the rich conceptual detail that we

[8] See more generally: B. Cotterell and J. Kaminga, *Mechanics of pre-industrial technology* (Cambridge 1992) 193-225 and C. Adams, *Land transport in Roman Egypt* (Oxford 2007) 49-69.

[9] A. E. Minetti, 'Efficiency of equine express postal systems', *Nature* 426 (2003) 785-86.

[10] Sources for the data: Adams, *Land Transport* (n. 8 above); J. Rennell, 'On the rate of travelling, as performed by camels; and its application, as a scale, to the purposes of geography', *Philosophical Transactions of the Royal Society of London* 81 (1791) 129-45; L. Casson, 'Speed under sail of ancient ships', *Transactions and Proceedings of the American Philological Association* 82 (1951) 136-48; P. J. Sijpesteijn, *Customs duties in Graeco-Roman Egypt* (Zutphen 1987); W. Habermann, 'Statistiche Datenanalyse an den Zolldokumenten des Arsinoites aus römischer Zeit II', *Münstersche Beiträge zur Antiken Handelsgeschichte* 9 (1990) 50-94; Cotterell and Kaminga, *Mechanics* (n. 8 above) table 8.1; A. E. Minetti, 'Efficiency' (n. 9 above) 1698-1703; van Tilburg, *Traffic and congestion* (n. 6 above).

believe is present in the real world case under study. With respect to human travel, this would involve, for example, assessing the full balance of cost and benefit calculations discussed above (travel time, effort, risk in myriad combinations and types), as well as any perceived cultural attitudes, special places in the landscape that might repel or attract, *etc.*[11] For many commentators therefore, the use of only very basic features of the landscape such as topography and access to roads, rivers, or the sea vastly under-appreciates the encultured nature of travel and transport decisions and ushers in a kind of casual environmental determinism. However, a reverse argument can also be made to the effect that, done well, there is an analytical elegance to highly simple models that is conceptually useful. They are, in theory, easy to understand, because only a limited number of parameters are involved, and easy to test, because we can offer robust predictions of how they should behave under controlled conditions. It is therefore often better, I would argue, to provide simple models of movement and interaction prior to exploring more complex ones, as the former often offer useful null hypotheses from which the real archaeological record can be observed to depart (or not) in interesting ways.

The rest of this chapter presents a review of the range of possible models that might be used for understanding movement and interaction, offering examples for some of them based on the landscapes in Greece, and spanning the Bronze Age, early Iron Age, and Graeco-Roman periods. While remaining aware of the need to build theory and explore method in the future with respect to types of travellers, cargoes, or agendas, the examples below are stripped free of such subtle differentiations, but do thereby remain simple to grasp.

3.1 Cost surfaces

Cost surfaces and so-called ‘least cost paths’ are by now very well established and closely related methods for exploring movement in a Geographical Information System (GIS), typically based on little more than a skeleton model of topography (with the latter usually being in a ‘raster’ or pixel-based format). The methodological stages involved are: (i) to define a set of costs for each cell in a raster map, (ii) to create a ‘cost surface’ by accumulating these costs out from a fixed point of departure (A), and (iii) if required, to trace a route from another point (B) back to the departure point (A) and thereby define a ‘least cost path’ between them. There are different kinds of cost surface. Some assume that (i) all of the costs incurred along the way are not altered by the direction of travel (*i.e.* they are isotropic, such as the cost of moving through different types of land cover) while others assume that (ii) costs are operating from a particular direction (partially anisotropic, such as the effect of a strong wind on a cyclist), (iii) costs are entirely direction dependent (*i.e.* fully anisotropic, such as the cost of moving across a slope which varies depending on the direction in which you walk across it), and/or (iv) that a combination of isotropic and anisotropic costs are in operation.

[11] For a good discussion on this, see: M. Llobera, ‘Understanding movement: a pilot model towards the sociology of movement’, in *Beyond the map: archaeology and spatial technologies*, ed. G. Lock (2000) 71-75.

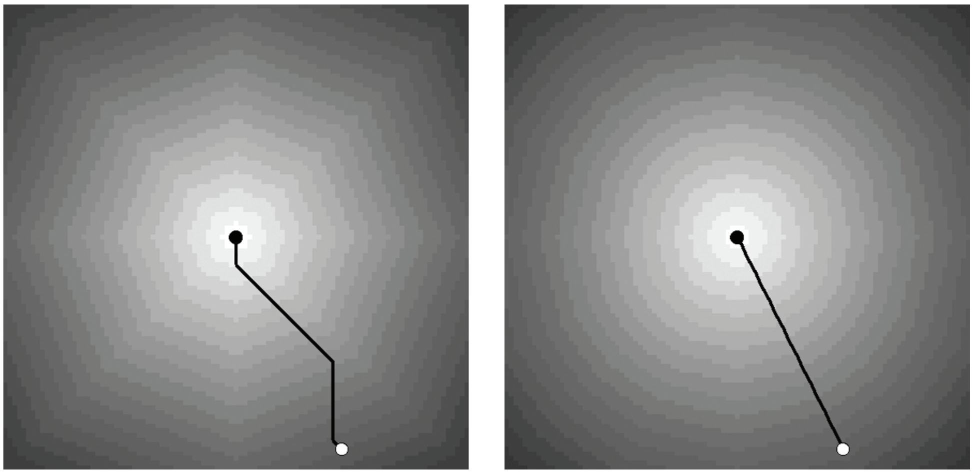

Figure 1 Simple models and cost surface problems. Cost surfaces and a least cost path calculated on a flat 100 x 100 surface, for two kinds of spreading algorithm: (a) D8 in ArcGIS, and (b) D16 in GRASS GIS.

Unfortunately, despite a great deal of early enthusiasm with archaeological applications of cost surfaces in the 1990s, there continue to be a host of difficulties associated with cost surface analysis.[12] We can list them as being: (i) a failure on the part of the vast majority of implementations to address anisotropic costs, (ii) the widely varying computational 'curves' used to measure the cost of passage across slopes of differing steepness in a landscape,[13] (iii) the specific kind of 'spreading' algorithm used to accumulate cost, and (iv) the degree to which the results are ever formally calibrated or validated.

A further, well-known but often-ignored problem is associated with the search neighbourhood used by a cost surface algorithm. The default in many software packages is a queen's case search neighbourhood (or 'D8', where the routine moves through each cell in the raster map, and for each one, checks the costs of the 8 immediately neighbouring cells, effectively allowing it to search in the manner that a queen on a chessboard would move).[14] If this method is applied to the simplest case of an entirely flat topography of uniform costs, however, the resulting surface from any point (*e.g.* A in Figure 1a) has a faceted appearance,

[12] See: D. H. Douglas, 'Least cost path in GIS using an accumulated cost surface and slope lines', *Cartographica* 31.3 (1994) 37-51; T. Bell and G. Lock, 'Topographic and cultural influences on walking the ridgeway in later prehistoric times', in *Beyond the map*, ed. Lock (n. 11 above) 85-100; W. Collischonn and V. Pilar, 'A direction dependent least cost path algorithm for roads and canals', *International Journal of Geographical Information Science* 14 (2000) 397-406; J. Conolly and M. Lake, *Geographical Information Systems in archaeology* (Cambridge 2006) 215-25; I. Herzog, 'Theory and practice of cost functions', in *Computer applications and quantitative methods in archeology. Computing applications in archaeology*, ed. F. Javier Melero and P. Cano (Granada 2010).

[13] For example: Herzog, 'Theory and practice' (n. 12 above) fig.1; and also A. E. Minetti, 'Optimum gradient of mountain paths', *Journal of Applied Physiology* 79.5 (1995) 1698–1703.

[14] See Conolly and Lake, *Geographical Information Systems* (n. 12 above) fig.10.11.

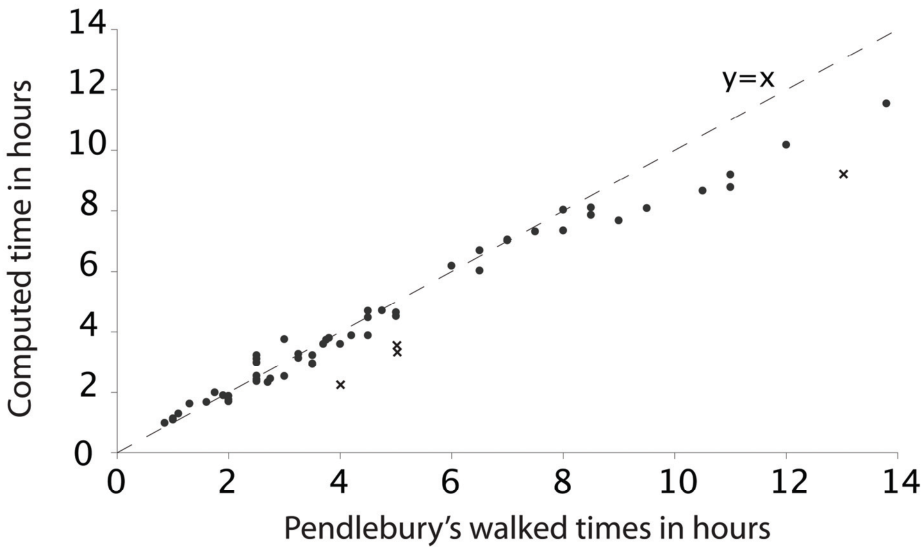

Figure 2 Comparison of the times recorded by John Pendlebury for his walks between Cretan sites in the 1930s (n=60 trips) and those computed by anisotropic cost surface analysis with GRASS *r.walk* using default parameters and D16 search (suggested outliers are marked as crosses).

when it should appear as perfectly concentric rings, and any path delineated to a second point B risks being a dog-leg, when it should be a straight line. One way to reduce this problem is to expand the search to a knight's case neighbourhood ('D16', where the routine searches 16 neighbouring cells, including both the queen's case cells and those to which a knight on a chessboard could theoretically move), or larger (Figure 1b). In any case, it is also surprising how rarely cost surface results have been tested against a known set of journey times. One such opportunity is provided for the Greek island of Crete by John Pendlebury, due to journey times he recorded for travel by foot during the 1930s, prior to major mechanized transport or modern road-building.[15] Figure 2 compares the times that Pendlebury recorded for sixty of these journeys with the results from anisotropic modelling with a D16 search (using GRASS GIS' *r.walk* module).[16] The only costs involved are the direction-specific ones imposed by terrain of varying steepness.[17]

The correlation between the two sets of times (observed and computed) is very good, particularly for journeys of less than about eight hours. In fact, such a correlation is

[15] J. D. S. Pendlebury, *The archaeology of Crete* (London 1939).

[16] GRASS: Geographical Resources Analysis Support System: <http://grass.fbk.eu>, free Geographical Information System (GIS) software.

[17] As measured on a 15m cell digital elevation model: N. Chrysoulakis, M. Abrams, H. Feidas, and D. Velianitis, 'Analysis of ASTER multispectral stereo imagery to produce DEM and land cover databases for the Greek islands: the REALDEMS project', in *e-Environment: Progress and Challenge*, ed. P. Prastacos, U. Cortés, J.-L. Díaz de León and M. Murillo (Mexico City 2004) 404-17.

potentially deceptive as even straight-line, 'as the crow flies' distances are already highly correlated with Pendlebury's estimates (r^2=0.88, *i.e.* sites further away as the crow flies will typically take longer to walk to as a matter of course). However, it is reassuring to note that the anisotropic calculations of less than eight hours offer significantly improved explanatory power (r^2=0.96, likely to be different from the above at $p < 0.005$). Thereafter, the predicted times for journeys over eight hours are still useful, but often a little too rapid, probably reflecting the fact that overnight travel requires extra time for rest-stops, the burden of extra baggage, *etc*.

Compared with many other methods of this kind, the one implemented in GRASS GIS offers two further advantages of: (i) working directly from rate of change in elevation values rather from a derived slope map, and (ii) being based on a well-established rule-of-thumb used by hikers (Naismith's rule)[18] and estimating costs explicitly as travel time. Some continuing theoretical problems with it, however, are: (i) that its function for estimating travel time is discontinuous,[19] (ii) that in some instances, its knight's-case moves potentially leapfrog intervening costs or barriers, and (iii) the fact that it still significantly under-estimates the kinds of times, and mis-delineates the kinds of hairpin route, that we might expect in extremely steep terrain (see below). Despite these provisos, however, the above results should give us confidence (albeit certainly not blind faith) that the times and paths modelled here offer interpretative added value when considering interaction across Crete's often tyrannically rugged landscape. I therefore make use of them in several examples below.

Figure 3a offers a useful, albeit earlier and prehistoric, example of how an anisotropic cost surface might be compared to documentary evidence to get an idea of possible travel and/or administrative thresholds. It takes as a departure point the Bronze Age centre of Knossos on Crete (although the shorter distance thresholds perhaps also provide relevant points of comparison for the territory of the later and smaller city-state located here in the Classical to Roman periods), and suggests the travel times out from this centre to all other parts of the island. Overlaid on these are the likely locations of toponyms found in the fourteeth- to thirteenth-century BC Linear B tablets from Knossos,[20] which suggest that this centre was organizing substantial activity across much of the island. The toponym groups shown here reflect those locations that regularly co-occur in the tablets and suggest qualitatively different kinds of interaction with Knossos in different regions and at different removes from the centre. Figure 3b then considers the possible effect of sea voyages (modelled here very grossly indeed as encouraging twice the speed of typical pedestrians). The key point to note here is both geographic and political: simply that political, economic, and social connections between Knossos and the far west or east of the island would be far better sustained via maritime travel that by terrestrial means.

[18] See: E. Langmuir, *Mountaincraft and leadership* (London and Edinburgh 1995) 39-43.

[19] See: Herzog, 'Theory and practice' (n. 12 above) fig.1.

[20] J. Bennet, 'The structure of the Linear B administration at Knossos,' *American Journal of Archaeology* 89.2 (1985) 231-49.

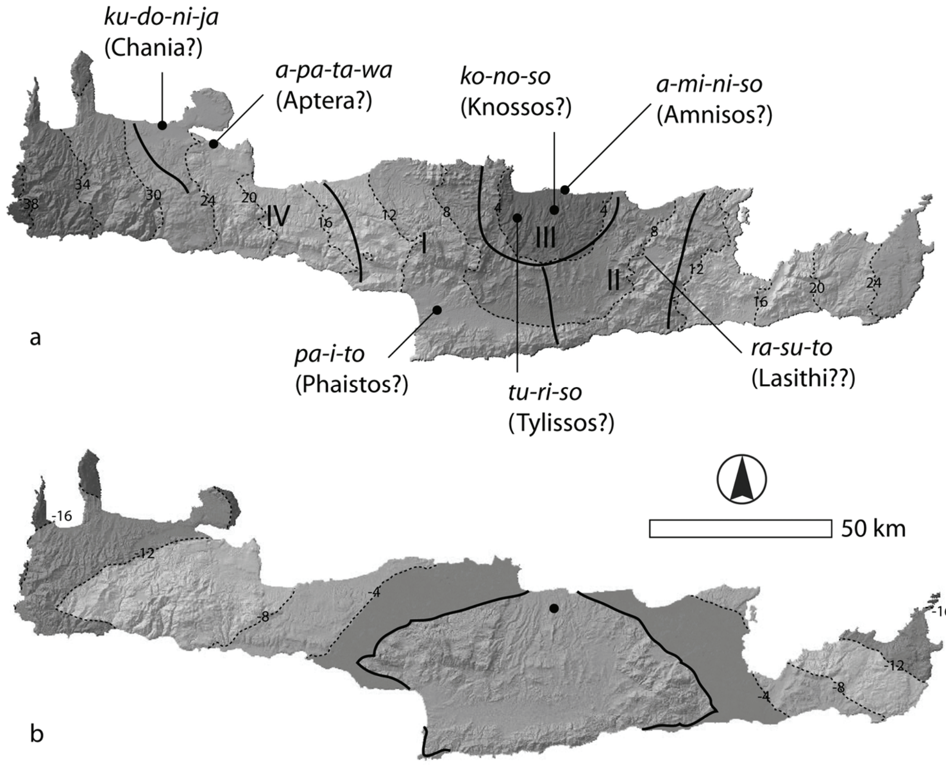

Figure 3 Anisotropic cost surfaces from Knossos: (a) terrestrial travel times (dotted lines are 4 hour contours outwards from Knossos), along with the toponym groups suggested by the Linear B archives (solid lines are very rough group divisions after Bennet 1985: fig.iii.4), (b) a rough impression of the time saved by including a maritime leg in the trip from Knossos. The negative values refer to the number of fewer hours needed to travel to that location by comparison to figure 3a above (assuming no embarkation delays). Dotted lines are 4 hour contours and the solid line marks the area with no change.

3.2 Network analysis

Thinking of the world in terms of network connectivity is a very popular approach nowadays, prompted by its conceptual simplicity in one sense, and potentially great analytical and interpretative complexity in another. Networks, can be used to think about social relationships (*e.g.* 'small worlds'),[21] physical relationships (*e.g.* terrain morphology),[22] or conceptual ones (*e.g.* ancient itineraries,[23] and religious ideas),[24] to name

[21] For example: 'small worlds', D. J. Watts and S. H. Strogatz, 'Collective dynamics of "small-world" networks', *Nature* 393 (1998) 440-42.

[22] See: J. Wood, 'Constructing weighted surface networks for the representation and analysis of surface topology', in *5th international conference on GeoComputation*, (2000): www.soi.city.ac.uk/~jwo/Geocomputation00.

[23] S. Graham, and J. Steiner, 'Travellersim: growing settlement structures and territories with agent-based modelling', in *Digital discovery: exploring new frontiers in human heritage. CAA 2006.*

but a few applications. In a sense, modern network science is a triangulation of many established approaches such as space syntax, graph theory, and social network theory.[25] At its heart is the idea of a set of nodes as an abstraction of real-world sources of interaction, with a set of edges as connectors among them. One key issue with network analysis is the degree to which it is or is not sensitive to uncertainty in: (a) the location or number of defined nodes, and (b) the connection matrix (in terms of which nodes are connected, in what directions, and with what weights).[26] All of these issues can in theory be explored by considering a range of different scales of connection, by assessing a range of input parameters (*e.g.* parameter sweeps), and by randomized Monte Carlo perturbation of the network (*e.g.* slightly moving existing nodes, altering connectivity, and/or addition or subtraction of nodes). A further possible route is to consider not observed nodes (*i.e.* not observed archaeological settlements if we are talking about a settlement network), but hypothetical candidates (*e.g.* plausible settlement locations).

Many commonly used networks are single-mode (*i.e.* only one kind of connection is being considered at any one time), unweighted (*i.e.* where edges all have the same cost of passage along them) and undirected (*i.e.* the passage from A to B and from B to A is equivalent), but we can make them more realistic (without necessarily overcomplicating) by adding edge weights based on likely travel times. Just to take an example, settlement on the island of Crete has, over the last nine thousand years or so, clearly been affected by environmental affordances (*e.g.* access to better land for certain activities, to preferred trans-insular routes, or to off-island maritime connections) as well as by a range of historically contingent and culturally specific influences. A useful first step is to consider some very simple baseline models of likely demographic connectivity across the island. For example, we can start by dropping 100 random settlements down onto the Cretan landscape (taking inspiration from the classical tradition of 'hundred-citied Crete'),[27] with a preference for more agriculturally-favourable parts of the island (Figure 4a). For our purposes here, a map of such favourable areas was formally defined as follows: (a) identify all cells in the map that are ≤ 10° slope and hence might initially be preferred for agriculture (*i.e.* without terracing), (b) exclude all such flatland cells that are more than 1000m above sea-level (*i.e.* the approximate tree line), and (c) for each cell in the map (*i.e.* a neighbourhood operator), calculate the mean number of such flatland cells that are

Computer applications and quantitative methods in archaeology. Proceedings of the 34th conference, Fargo, United States, April 2006, ed. J. T. Clark and E. M. Hagemeister (Budapest 2006) 49-59; L. Isaksen, 'The application of network analysis to ancient transport geography: a case study of Roman Baetica', *Digitial Medievalist* 4 (2008).

[24] A. Collar, *Networks and religious innovation in the Roman Empire* (Unpublished PhD Thesis, University of Exeter, 2008).

[25] For a good discussion of its relevance to archaeology, see: T. Brughmans, 'Connecting the dots: towards archaeological network analysis', *Oxford Journal of Archaeology* 29.3 (2010) 277-303.

[26] See also: M. Zanin, 'Uncertainty in complex network,' *International Journal of Complex Systems in Science* 1 (2011) 78-82.

[27] P. Perlman, 'One hundred-citied Crete and the "Cretan *Politeia*"', *Classical Philology* 87.3 (1992) 193-205.

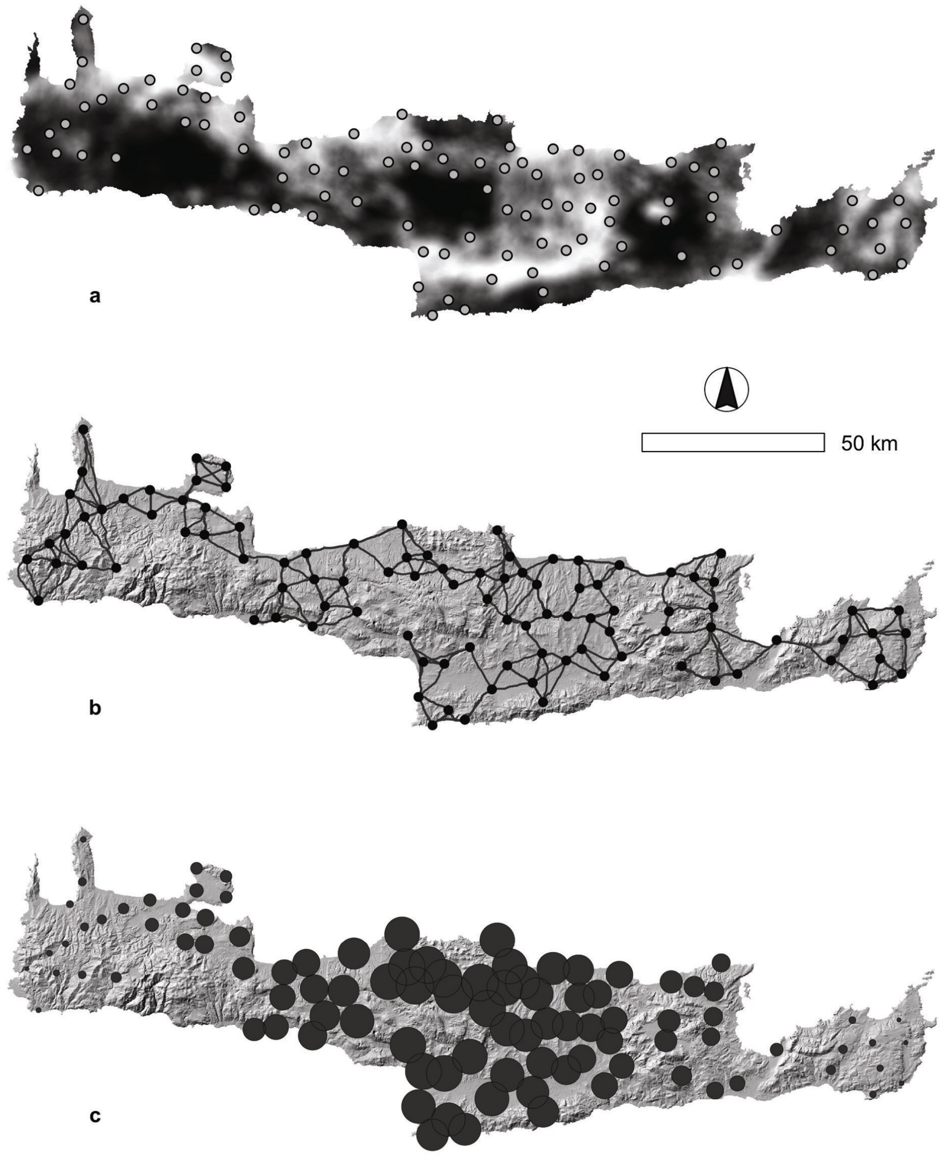

Figure 4 Cretan landscape affordances: (a) a set of 100 random points allocated preferentially on areas with better access to flat land (the latter shown as lighter shades of grey in the underlying intensity map), (b) an anisotropic path network linking each random point to its three nearest neighbours based on pedestrian travel time, (c) closeness centrality measures for the same sites, but based on a full weighted network (*i.e.* number of out-nodes = 99, weights are travel time).

found within a 2.5km radius (*i.e.* about an hour's roundtrip and typical of a wide cross-cultural range of human daily travel budgets).[28]

This map was then used as an intensity surface on which to simulate 100 points while maintaining a minimum distance of 5km between them.[29] We can then link up these hypothetical sites into a network via computed paths that seem optimal in terms of travel time by a single pedestrian (*i.e.* calculated anisotropically as above). Figure 4b shows an example where we have retained only the three nearest neighbouring links out from each site. Already with this kind of modelled connectivity, we can see a propensity for the island to break itself up into smaller regional network components. However, archaeologically and historically we have no reason to assume that three paths is a valid level of connection in any given time and place. A less judgemental approach might be simply to work with a complete set of connections among sites (*i.e.* site 1 is connected to each of sites 2-100 via 99 direct paths and so on), but one in which the connecting edges of the network are weighted by the varying estimated travel time along them. Although this network is full, weighted, and directional, we can nonetheless calculate traditional network metrics.[30]

As an example, figure 4c shows a network measure known as 'closeness centrality'[31] for each hypothetical settlement across the island: it is wholly unsurprising to note that sites in the middle of the island prove to be more central within the network (in this case shown as larger circles). Likewise, a fuller analysis would ideally consider: (a) the degree to which these results vary over multiple point simulation runs and under variable densities of points, as well as (b) a wider range of network metrics. However, an important insight is the fact that this measure of centrality shifts abruptly at certain points across the island due to a variety of topographical pinch-points. In other words, there is not a continuous gradient of change to the far west and the far east of the island, but a built-in propensity to generate regional spheres of interaction, some of which potentially exist in near isolation from the rest, at least in terms of their terrestrial connectivity. Were we to consider the movement of larger groups of travellers or heavier cargo explicitly, this situation is only likely to become more extreme. If such regions were therefore ever to be integrated politically and economically, then we can probably assume that it was maritime linkages that often most easily achieved it. Indeed, in historical times, the island has only been under a unified authority when it has been occupied by external powers with strong navies (*e.g.* Roman, Venetian, and Ottoman),[32] and prior to this, the classical sources

[28] J. H. Ausubel and C. Marchetti, 'The evolution of transport', *The Industrial Physicist* 7.2 (2001) 20-24.

[29] For intensity-based point simulations, see: J. Baddeley and R. Turner, 'spatstat: an R Package for analyzing spatial point patterns', *Journal of Statistical Software* 12.6 (2005) 33-35.

[30] For examples, see: T. Opsahl, F. Agneessens, and J. Skvoretz, 'Node centrality in weighted networks: generalizing degree and shortest paths', *Social Networks* 32.3 (2010) 245-51.

[31] For an introduction to this, see: M. Newman, *Networks. An introduction* (Oxford 2010) 181-85.

[32] J. Bennet, 'Knossos in context: comparative perspectives on the Linear B administration of LM II-III Crete', *American Journal of Archaeology* 94.2 (1990) 193-211.

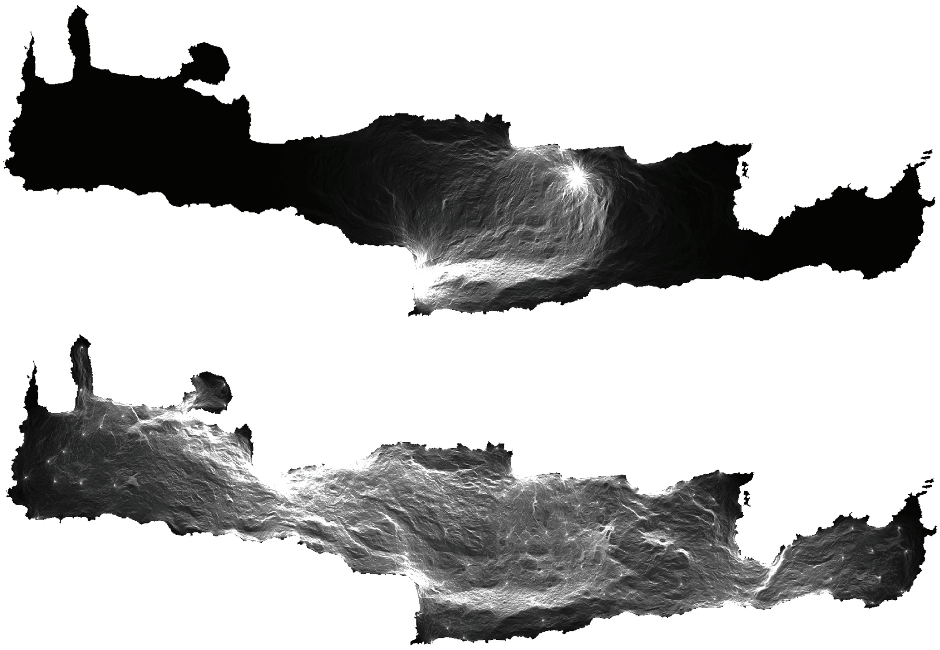

Figure 5 Current maps based on using terrain slope as a measure of electrical resistance: (a) between two random sites, and (b) aggregated for all sites.

claim a period of earlier political unification associated with the legendary figure of king Minos and underpinned by a Minoan 'thalassocracy'.[33]

3.3 Circuit theory and isolation by resistance

Further models of interaction can be generated via an approach inspired by the behaviour of electrical circuits.[34] In circuit theory terms, if we choose one of our random sites on Crete (as used above) and connect it to the ground, while connecting another site to the equivalent of a 1 amp electrical source, we can then model the behaviour of current flow over the intervening space with, for example, steepness of slope acting as a form of resistance. Figure 5a offers an example where current is allowed to flow from a site in central Crete (in electrical circuit terms, the 'source', and a stand-in for a major settlement such as Cnossus/Knossos in Bronze Age through to Roman periods) to one in the Mesara valley in central Crete (the 'ground', and perhaps a stand-in for a site such as Gortyn or Phaistos). Such a depiction is useful for three reasons: first, it suggests a broad corridor of likely movement between the two areas rather than a discrete single pathway or straight line. It also suggests that interaction via the Pediadha upland and the eastern end of the Mesara may

[33] For relevant archaeological discussion and caution over this later mythical tradition, see: C. Broodbank, 'Minoanisation: beyond the loss of innocence', *Proceedings of the Cambridge Philological Society* 50 (2004) 46-91; A. Bevan, 'Political geography and palatial Crete', *Journal of Mediterranean Archaeology* 23.1 (2010) 27-54.

[34] B. H. McRae, 'Isolation by resistance', *Evolution* 60 (2006) 1551-61.

have been just as important as a direct linkages northeast-southwest. Second, we might think of it as a more plausible, less deterministic model of possible connectivity than either an 'as-the-crow-flies', Euclidean distance or a single least cost path. For example, if we take three sites roughly equidistant apart, but with the connectivity of A and B benefitting from several possible alternative routes, then a least cost path approach will typically model just one of these in each case and assume interactions between A, B, and C are roughly the same, whereas multiple routes between A and B should in theory engender greater linkage. Third, the theoretical equivalence of a circuit theory approach to two-dimensional diffusion models and random walk models makes it an attractive 'null hypothesis' for interaction over a spatially heterogeneous space. At present it has mainly been used, with promising results, to model patterns of genetic variation with real-world distance.[35]

Figure 5b shows a cumulative current map produced by grounding one site at a time, then mapping the current from all ninety-nine other sites as individual sources, and then iterating for all sites. The map emphasizes certain short, sharp corridors (light grey filaments) of connection, for example between Chania and Rethymnon along the northwestern coast, north-south in the Ierapetera region, as well as broader zones of important linkage, for example between the Mesara plain and western Crete via the Amari valley.

Circuit theory offers an attractive way of exploring aggregate patterns of large-scale movement via multiple pathways. Resistances are likely to become an alternative model of distance effects to least cost paths or Euclidean measures and can be combined effectively with several of the other methods described below, such as network analysis or spatial interaction models (Section 3.4 below).

3.4 Evolving models

So far, we have considered methods for characterizing conditions for movement and interaction that are, perhaps slightly counter-intuitively, rather static in nature. While some network methods do also consider the dynamic properties of such configurations,[36] the last three types of model discussed briefly here are all promising ways to understand evolutionary patterns of movement, growth, decay, and interaction over time. To begin with, it is worth reconsidering an approach successfully introduced into archaeology and history some time ago by Tracey Rihll and Alan Wilson,[37] but rarely heard of since,[38]

[35] For examples: B. H. McRae and P. Beier, 'Circuit theory predicts gene flow in plant and animal populations', *Proceedings of the National Academy of Sciences of the USA* 104 (2007) 19885-90; J. van Etten, and R. J. Hijmans, 'A geospatial modelling approach integrating archaeobotany and genetics to trace the origin and dispersal of domesticated plants', *PLoS ONE* 5.8 (2010) e12060.

[36] For example: T. Evans, C. Knappett, and R. Rivers, 'Using statistical physics to understand relational space: a case study from Mediterranean prehistory', in *Complexity Perspectives on Innovation and Social Change*, ed. D. Lane, S. Van der Leeuw, D. Pumain, and G. West (New York 2009) 451-79.

[37] T. E. Rihll, and A. G. Wilson, 'Spatial interaction and structural models in historical analysis: some possibilities and an example', *Histoire et Mesure* 2.1 (1987) 5-32, and T. E. Rihll, and A. G. Wilson, 'Modelling settlement structures in ancient Greece: new approaches to the polis', in *City and country in the ancient world*, ed. J. Rich and A. Wallace-Hadrill (London 1991) 59-95.

despite its thoroughly established role in urban geography.[39] ‘Spatial interaction models’, as they are often known, are developed versions of gravity models and usually have at their heart an equation of the kind:

$$S_{ij} = \frac{O_i W_j^{\alpha} e^{-\beta c_{ij}}}{\sum_k W_k^{\alpha} e^{-\beta c_{ik}}}$$

where:

S_{ij} is a matrix recording the quantity of resources flowing from each site i to each other site j;

O_i is a measure of the size of flow originated at site i;

W_j is the attractiveness of site j;

C_{ij} is the distance from i to j;

α is a parameter used to model the advantages of concentrating resources in one place;

β is a parameter used to model the ease of communication over a distance;

e is an exponential function.

A model set up in this manner can then be run iteratively, updating the attractiveness (W_j) and size (O_i) of each site until collectively these all reach a point of equilibrium, with no remaining imbalances in modelled inflow to and outflow from sites ($\sum S_{ij}$). In an extreme case, and one particularly relevant to archaeological situations, the only necessary input for such a model is a set of point locations, some measure of the distances between them, and perhaps a rough idea of conceivable outcomes that might suggest a suitable range of α and β values to explore. Although further details of these methods will not be considered here, they certainly offer a rich set of analytical possibilities because: (a) they can support full, weighted, and directed spatial distance matrices in which very few assumptions are made *a priori* about the nature of the connectivity involved, and (b) they are at least potentially evolutionary in concept (*e.g.* the method can be used to explore the emergence of central places over different time-steps).

At present, spatial interaction models of this kind are useful for considering the hierarchy of population centres or of commercial outlets, but only include the effect of distance in relatively simple ways (typically as negative exponential isotropic decay). One fruitful future possibility would be to combine these with either least cost paths onto which the spatial interaction model flows can be loaded and mapped, separately from the centres (such as an example from Geometric Greece shown in Figure 6), or resistance distances (see above). Likewise, pathways are themselves subject to evolutionary forces: for example, without further guidance, pedestrians take the most direct route from A to B, B to C, and C to A, but once visible paths are established, they (a) act to encourage

[38] But now see: T. Evans, R. Rivers, and C. Knappett, ‘Interactions in space for archaeological models’, *Advances in Complex Systems* 85 (2011) 1150009.

[39] For more details on this see: A. G. Wilson, *Complex spatial systems. The modelling foundations of urban and regional analysis* (Harlow 2000).

Figure 6 An example of a spatial interaction model for which minimum travel times over land and sea have been used as distances, and modelled flows have then been mapped back onto least cost routes, indicating major hubs of activity and preferred corridors of interaction (darker paths have higher flow). This is a preliminary reinvestigation (by Bevan, Dearden, and Wilson) of the Rihll-Wilson model of Greek geometric sites.

subsequent travel along roughly the same route, but (b) also can be shown gradually to adjust their respective course and 'bundle' into a more centralized set of paths over time.[40]

Two further approaches to movement modelling, arguably at opposite ends of the strategic spectrum, are diffusion-reaction equations and agent-based modelling. Diffusion-reaction models have not, as yet, received much consideration in classical archaeology but have been used to model the dispersal of farming practices into Europe,[41] or the southward spread of Palaeolithic hunter-gatherers into the Americas.[42] They are highly relevant to understanding how, for example, innovative ideas spread out across a population, where the latter has a clear spatial structure (*e.g.* is clumped into certain regions and into certain

[40] For example see: D. Helbing, J. Keltsch, and P. Molnar, 'Modelling the evolution of human trail systems', *Nature* 388 (1997) 47-50. See also: A. Bevan and A. Wilson, 'Models of settlement hierarchy based on partial evidence', *Journal of Archaeological Science* 40.5 (2013) 2415-27.

[41] A. J. Ammerman and L. L. Cavalli-Sforza, 'Measuring the rate of spread of early farming in Europe', *Man* 6 (1971) 674-88.

[42] For a full review of such models, see J. Steele, 'Human dispersals: mathematical models and the archaeological record', *Human Biology* 81.2-3 (2009) 121-40.

settlements). They are therefore also of potential relevance, for example, for understanding the spread of technological innovations or novel religious practices across the Graeco-Roman world. The differential equations that typically underpin them have much potential synergy with the spatial interaction models described above, and the opportunities for considering them all as a complementary family of models, of broadly Boltzmann-Lotka-Volterra type, has been recently emphasized.[43]

Agent-based modelling (ABM), on the other hand, attempts to model behaviour as a set of interactions among discrete agents who typically each have their own set of individual behaviours. ABM approaches potentially facilitate the exploration of the following in useful ways:[44] (a) emergence – whether higher-levels of structure in the model can be shown to emerge without being imposed in some way from the outset; (b) adaptation and fitness – which aspects of agent behaviour respond to changes in the overall environment and how does this affect their pursuit of particular objectives (*e.g*. successful reproduction, greater wealth, a particular destination, *etc*.); (c) sensory capacities – for example, the impact of an agent's local *versus* global knowledge about the surrounding environment; (d) interaction and collectivity – the degree to which agents interact in important ways with one another and/or form larger groups; and (e) stochasticity – the effect of inserting random chance into the model.

There have been several ABMs in archaeology that consider spatial interaction and/or movement explicitly. For example, Mark Lake has emphasized the degree to which we should consider how agents might share imperfect and partial spatial information about their surroundings (*e.g*. mental maps of resources in the landscape).[45] Likewise, it is possible for an ABM to consider the impact of localized knowledge on the navigational decisions made by sailors.[46] In contrast, Graham and Steiner demonstrate that it is also possible to consider some of the same issues of spatial interaction among human settlements in terms of real flows of travelling agents.[47] More precisely, they have developed an agent-based model that considers whether the random walks of individual agents from 'parent' sites to other sites in a region could approximate, stochastically and from the bottom up, the top-down spatial interaction models of settlement interaction suggested by Rihll and Wilson (see above). While both the exactness of their match with Rihll and Wilson's methods and the validity of the proposed model set-up might be argued about, their overall ABM-led approach to the issue is undeniably an attractive one.

[43] See A. G. Wilson, 'Boltzmann, Lotka and Volterra and spatial structural evolution: an integrated methodology for some dynamical systems', *Journal of the Royal Society. Interface* 5 (2008) 865-71.

[44] Following: V. Grimm, U. Berger, F. Bastiansen, S. Eliassen, V. Ginot, J. Giske, J. Goss-Custard, T. Grand, S. K. Heinz, G. Huse, A. Huth, J. U. Jepsen, C. Jørgensen, W. M. Mooij, B. Muller, G. Pe'er, C. Piou, S. F. Railsback, A. M. Robbins, M. M. Robbins, E. Rossmanith, N. Ruger, E. Strand, S. Souissi, R. A. Stillman, R. Vabø, U. Visser, and D. L. DeAngelis, 'A standard protocol for describing individual-based and agent-based models', *Ecological Modelling* 198 (2006) 115-26.

[45] M. W. Lake, 'The use of pedestrian modelling in archaeology, with an example from the study of cultural learning', *Environment and Planning B* 28.3 (2001) 385-403.

[46] G. Indruszewski and C. M. Barton, 'Simulating sea surfaces for modeling Viking Age seafaring in the Baltic Sea', in *Digital discovery*, ed. J. T. Clark and E. Hagemeister (n. 23 above) 616-30.

[47] S. Graham and J. Steiner, 'Travellersim' (n. 23 above).

All three of the above approaches – spatial interaction models, diffusion-reaction equations and ABM – have their advantages and disadvantages, as well as their conceptual overlaps, but all offer possibilities for modelling evolutionary trajectories and for factoring in patterns of geographic interaction in an explicit way, if necessary. The choice as to which one is most appropriate in a given modelling context is very much a strategic one, driven by the specific objectives in mind, and often some careful, combined exploration of several is wise.

4. Conclusion

This brief review has sought to consider the range of computational perspectives on travel and geographic interaction that are currently in use within archaeology, history, and geography, and to underscore some common problems that often still remain. Overall, it has emphasized the need (without demonstrating as yet the means by which) to develop models that are more aware of different priorities behind different kinds of travel, and different practicalities behind transitions from one transport mode to another, to name just two amongst a range of future objectives. Increasingly, the methodological and disciplinary boundaries between different kinds of spatial-analytical method are being eroded to the extent that we can integrate them in highly complementary ways. In any event, if we want to allow Mediterranean archaeology and history to contribute as effectively as it should to wider debates about the character of complex adaptive systems where humans play a role, then well-informed modelling will need to make an important contribution.

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