Objectives, stimulus and feedback in signal control of road traffic

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ABSTRACT

This paper identifies the prospective role of a range of intelligent transport systems technologies for the signal control of road traffic. We discuss signal control within the context of traffic management and control in urban road networks, and then present a control-theoretic formulation for it that distinguishes the various roles of detector data, objectives of optimisation, and control feedback. By reference to this, we discuss the importance of different kinds of variability in traffic flows, and review the state of knowledge in respect of control in the presence of different combinations of them. In light of this formulation and review, we identify a range of important possibilities for contributions to traffic management and control through traffic measurement and detection technology, and contemporary flexible optimisation techniques that use various kinds of automated learning.

Keywords: signal control, traffic control, optimisation, formulation

1. INTRODUCTION

Traffic signals are used to manage conflicting requirements for the use of road space – often at road junctions – by allocating right of way to different sets of mutually compatible traffic movements during distinct time intervals. This mode of allocating exclusive use of road space to different contending sets of movements in turn confers a special structure on the operational requirement for traffic signals: this has the discrete element of the order in which mutually incompatible movements receive right of way, and the continuous one of the duration for which this occurs. The objectives of signal control will vary in accordance with the prevailing policy of urban traffic management and control. However, for a specific objective, the control task is to calculate and implement a series of control decisions that promotes this objective, and ideally that optimises it. The resources that are available for this optimisation are data from detectors – either collected in advance or on-line or in some combination of these – and optimisation procedures to make use of these data to calculate the plan.

If traffic flows were constant over time and known in advance, then calculation of signal timing plans to optimise a specified objective would be a relatively straightforward matter. However, traffic flows vary in several distinct ways that make the control task substantially more complicated. In particular, flows vary stochastically from moment to moment due to fluctuations in demand and driver behaviour; flows in urban road networks vary cyclically over time due to upstream signals; flows vary systematically within each day due to peak periods; and flows change over protracted periods of time due to developing demand for travel. In practice, each of these separate reasons will apply to some degree. However, each of them affects the control of road traffic in different ways, so we will consider their treatment separately.

In this paper, we present a general formulation for the calculation of signal timing plans as a kind of optimal control problem (see, for example, Kamien and Schwartz, 1991). This provides a framework within which a wide literature is unified. This then highlights the roles of information that can be provided by detectors, of measures of performance, and of optimisers that calculate signal timings plans using the data that are available. From this review, we can identify the potential of techniques of intelligent transport systems (ITS) to support urban traffic management and control though signal control of road traffic by providing data and processes for their use.

2. SIGNAL CONTROL OF ROAD TRAFFIC

2.1 Introduction

We now consider in some detail the nature of signal control of road traffic. In particular, we identify the control decisions that are available, we discuss the objectives that can be promoted by signal control, and we consider in detail the data that are required for the calculation of control decisions under various circumstances.

2.2 **A model of signal control**

We consider the streams of traffic at a road junction that are controlled by signals. Suppose that each set of signals has displays that are determined by a signal controller at the junction that switches them between red and green, through various combinations including amber displays. The details of this operation will vary according to prevailing rules that specify, for example, minimum clearance times between mutually incompatible streams having green indications, the exact sequence and durations of combinations that include amber, minimum durations of green indications, and in some cases permissible orderings of mutually incompatible streams. The rules that specify these details vary between countries, and are generally intended to ensure safe operation that is consistent with local custom and practice.

In general terms, the control decisions to be taken are:

in what order should signals be switched to give green indications;

for how long should each green indication persist.

Two distinct styles of formulation have been developed for these decisions. The more straightforward one is to establish maximal sets of signals that control mutually compatible streams of traffic; these sets are known as *stages*. Each stream is included in at least one stage, and can appear in several of them. In the resulting stage-based formulation (see, for example, Allsop, 1971), the control decisions are taken for stages. This has the advantage of dividing time into a series intervals throughout each of which a single stage runs; these intervals are separated by interstage periods within which signals change between green and red.

The second style of formulation is in terms of minimal sets of mutually compatible streams that are necessarily switched simultaneously; these sets are known as *phases*. Each stream belongs to exactly one phase, but where different phases consist of streams that are all mutually compatible, they can receive right of way concurrently. In the resulting phase-based formulation (see, for example, Improta and Cantarella, 1984; Gallivan and Heydecker, 1988) the control decisions are taken for phases individually to specify their starting times and durations. This formulation is more flexible than is the stage-based one because the sequence of signal indications and the structure of the interstage periods are implicit endogenous variables. It can therefore yield substantially better control performance (Heydecker and Dudgeon, 1987), but this is achieved at the expense of requiring a greater number of variables and constraints.

In the remainder of this paper, we consider the effect of a sequence of control decision on an individual stream. The present analysis can be developed within either a stage-based or a phase-based formulation as required.

2.3 A model of traffic under signal control

Various models have been developed of the way in which road traffic responds to signal control. In this section, we present a widely used model of this kind in a form that can be identified with control theory (see, for example, Kamien and Schwartz, 1992). Whilst this is not the only possible model, and indeed others have been investigated that provide further detail, this model provides sufficient detail and information for most analyses of signal-controlled road junctions.

Let the mean arrival rate of traffic in stream i at time t be $q_i(t)$, and let $a_i(t) = q_i(t) + \varepsilon_i(t)$ be the actual number or arrivals at time t, where $\varepsilon_i(t)$ is a stochastic error for the arrival process with zero mean. In the case of continuous deterministic arrivals, $\varepsilon_i(t) = 0$. In the case of road junctions in dense urban networks, the mean arrival rate $q_i(t)$ will often vary quasi-cyclically with time t due to platooning of traffic arriving from upstream signal-controlled junctions. We consider this and other kinds of variability in traffic flows, their representation in the state equation, and approaches to exploiting knowledge of their nature in signal control in due course.

At times when the signals that control a steam display red, then no traffic departs and any arrivals join a queue. Let the number of vehicles in stream i that would have departed at time t but that have been prevented from doing so by current or earlier red signal indications be

$L_i(t)$: this corresponds to the amount of traffic in a notional *vertical queue* of vehicles at the stop-line. The variables $L_i(t)$ and $q_i(t)$ correspond to *state variables* for the stream.

Let $\chi_i(t)$ be a characteristic for effective green indications in stream i , so that

$$\chi_i(t) = \begin{cases} 1 & \text{if stream } i \text{ has effective green at time } t \text{ ,} \\ 0 & \text{otherwise.} \end{cases}$$

This quantity is used here to represent the control decisions. It is subject to constraints that embody the safety and operational limitations outlined in section 2.2, including ones that enforce safe resolution of the contention between different streams for green time. In this exposition we will treat this as a binary (and hence discontinuous) *control variable*, and will consider only combinations of such variables that satisfy all relevant constraints.

Suppose that when the signals are green, if $L_i(t) > 0$, then traffic will depart at a certain rate s_i know as the *saturation departure rate*, and that if $L_i(t) = 0$, then traffic departs as it arrives. This model relies upon a certain correction from displayed green indications to effective ones that is achieved by introducing start and end lags that typically have values of 2 and 3 seconds respectively; with this correction, accurate estimates can be made of both capacity and delay by use of the effective green time and the saturation departure rate.

This model of queue dynamics can be summarised in the *state equation* for queue length $L_i(t)$:

$$\frac{dL_i}{dt} = [q_i(t) + \varepsilon_i(t)]\,[1 + \chi_i(t)\,\{\Theta[L_i(t)] - 1\}] - s_i(t)\,\chi_i(t)\,\Theta[L_i(t)] \qquad (1)$$

where $\Theta(.)$ is the Heaviside step function (Lighthill, 1958), specified by $\Theta(x) = 0$ if $x \leq 0$, and $\Theta(x) = 1$ if $x > 0$. We note that this component of the state equation is non-linear for several reasons:

the Heaviside step function is discontinuous,

the equation includes the product $\chi_i(t)\ \Theta[L_i(t)]$ of a control variable and a function of a state variable, and

the equation also includes the further product $q_i(t)\ \chi_i(t)\ \Theta[L_i(t)]$.

This vertical queueing model will generally underestimate the number of vehicles queueing in a stream of traffic because it makes no account of the space that the queue occupies, though it will certainly be 0 when there are no vehicles queueing. The main limitation of a model based upon a vertical queue is that because it does not have any spatial content it cannot inform directly on blocking back between adjacent junctions: the importance of this for control or road networks has been emphasised by Lo (1999) and Lo, Chang and Chan (2000).

2.4 Objectives of signal control

Signal control can be used to promote the objectives of urban traffic management and control in many different ways (Wood, 1993), including both tactical considerations and more strategic ones. The general purpose of tactical traffic management includes ensuring good operation of the junction and network with current and expected arrivals of traffic. The purpose of strategic traffic management is broader, and includes possibilities such as prioritisation and promotion of different groups of travellers such as pedestrians or bus passengers by provision of appropriate facilities, and limitation of capacity for motor vehicles to manage traffic growth.

In this section, we show how a range of traffic management objectives can be expressed within the framework of optimal control theory using the control and state variables introduced above. According to Kamien and Schwartz, the standard form of objective of optimal control is

$$A(\boldsymbol{\chi}, \mathbf{L}, \mathbf{q}) = \int_t f\left[\boldsymbol{\chi}(t), \mathbf{L}(t), \mathbf{q}(t)\right] e^{-\alpha t}\, dt \qquad (2)$$

for some scalar function $f(.)$ of the control and state variables, and discount rate α. In case the evaluation horizon ends at time $t = T$, then an additional terminal cost $B[\boldsymbol{\chi}(T), \mathbf{L}(T), \mathbf{q}(T)]$ can be associated with the final control condition and state in view of the omission of evaluation at any later time.

At heavily loaded junctions, it is often desirable to match the capacity of each of the most heavily loaded streams to the mean arrival rate in it so as to maximise the reserve capacity or, if the junction is overloaded, to minimise the degree of this (Allsop, 1976). This objective can be expressed in terms of the state and control variables introduced above in the formulation

$$\underset{\boldsymbol{\chi}}{\text{Minimise}}\ \underset{i}{\text{Max}}\ \frac{q_i}{E_t\left[\chi_i(t)\right] s_i} \qquad (3)$$

where $E_t(.)$ represents mathematical expectation operator over time t .

In cases where junctions are in close proximity, management of the spatial length of queues occurring at them can be important so that capacity is not lost through queues forming at one junction extending upstream to block exits and stop-lines at adjacent ones (Quinn, 1992). Other queue-related objectives include minimising the number of vehicles that remain in the queue at the end of an effective green period, which is known as the *overflow* queue. Formulations of signal control relating to objectives of this kind can be expressed directly in terms of the control variables $\boldsymbol{\chi}(t)$ and the state variables $\mathbf{L}(t)$.

The number of vehicles in the vertical queue associated with a stream of traffic can be interpreted as the mean rate of delay in that stream and provides an objective that bears direct economic interpretation (Allsop, 1971). Where junctions are operating within capacity and traffic growth is not considered to be an issue, minimisation of the total mean rate of delay at a junction is often adopted as an objective for traffic control. This can be expressed as

$$\underset{\chi}{\text{Minimise}} \sum_i E_t\left[L_i(t)\right] \tag{4}$$

where the relationship between the state variables $\mathbf{L}(t)$ and the control variables $\boldsymbol{\chi}$ is governed by the state equation (1). In cases where for some or other reason the future queue lengths are uncertain, a discount can be applied within the objective of optimisation according to (2) to reflect this. This can be expressed as

$$\underset{\chi}{\text{Minimise}} \sum_i \int_t L_i(t)\, e^{-\alpha t} dt\,, \tag{5}$$

which corresponds to optimisation of objective (2) with $f\,[\boldsymbol{\chi}(t), \mathbf{L}(t), \mathbf{q}(t)] = \sum_i L_i(t)$. This formulation emphasises the importance of decisions to be implemented in the near future that affect the state promptly relative to those that are implemented later and that affect states afterwards on the basis that there will be a possibility to reconsider them in the future.

Other objectives can also be represented using these control and state variables. For example, Robertson, Lucas and Baker (1980) showed that rate of fuel usage F in a stream of traffic can be estimated from the total rate of travel Q , the rate of delay L and rate at which vehicles stop S according to $F = \alpha_Q\, Q + \alpha_L\, L + \alpha_S\, S$ where the parameters $\boldsymbol{\alpha}$ are determined experimentally. The total rate of travel in a network with fixed demand is insensitive to variations in signal control. The instantaneous rate at which vehicles in a stream stop to join the vertical queue can be found using the state and control variables of the present traffic model as

$$S_i(t) = [q_i(t) + \varepsilon_i(t)][1 + \chi_i(t)\{\Theta[L_i(t)] - 1\}]. \tag{6}$$

Accordingly, the present analysis can encompass minimisation of fuel usage within its framework.

2.5 Variability in traffic flows

We now consider ways in which traffic flows and their variability can be represented within the present framework. The simplest case is one in which the flows are constant and deterministic: $q_i(t) = \bar{q}_i \;\; \forall t$, so that $\varepsilon_i(t) = 0$ and the corresponding component of the state equation is $dq_i/dt = 0$. Beyond this case, variability in traffic flows can arise in several different ways. Here we consider four different kinds of variability in traffic flows and show how these can each be represented in the state equation.

The first case of variability that we consider is that in which traffic flow varies stochastically around a constant mean rate. In this case, the arrival rate $a_i(t)$ in stream i at time t can be expressed as $a_i(t) = \bar{q}_i + \varepsilon_i(t)$. In the case of continuous traffic, $\varepsilon_i(t)$ is continuous in time and represents deviation in the arrival rate from the constant mean value $\bar{q}_i$. In the case of discrete traffic in which vehicular arrivals form a point process at instants $t_n^i \;\; (n \in \mathrm{N})$, the error can be expressed as $\varepsilon_i(t) = \sum_n \delta(t - t_n^i) - \bar{q}_i$, where $\delta(.)$ is the Dirac delta function (Lighthill, 1958). If arrivals are regular, then the inter-arrival headways will be determined by the flow as $t_n^i - t_{n-1}^i = 1/\bar{q}_i \;\; (n \in \mathrm{N})$; if the arrivals are stochastic, then the headways will vary around this value. This can be used to represent short-term variability in arrivals due to random fluctuations around a constant mean arrival rate. In the case of constant mean flow with random variations, the corresponding component of the state equation is again $dq_i/dt = 0$. However, in this case the mean flow $\bar{q}_i$ is a hidden variable that cannot be observed directly: only the sum $a_i(t) = \bar{q}_i + \varepsilon_i(t)$ can be observed from this the mean can be estimated by repeated measurement of $a_i(t)$.

We now consider a second kind of variation, which is the platooning of flows in road networks due to upstream traffic signals. This effect gives rise to quasi-cyclic profiles of flow, typically with high arrival rates for vehicles that passed through upstream junctions early during the green period there. In cases where the profile of flow $e_i(t)$ that enters link i is known, either by measurement or by modelling, the flow $q_i(t)$ arriving at the downstream end of the link can be estimated using the undelayed travel time ϕ_i for the link as $q_i(t) = e_i(t - \phi_i)$. Robertson (1969) showed how this estimation could be improved by smoothing the flows according to a platoon dispersion model, which in discrete time with step Δt can be written as

$$q_i(t) = F\, e_i(t - \phi_i) + (1 - F)\, q_i(t - \Delta t) \tag{7}$$

where F is a smoothing parameter that Robertson found varied with the free-flow travel time ϕ_i according to $F = 1/(1 + \beta\phi_i)$ for some parameter β.

The third kind of variation is that arising from systematic changes in demand for travel over a timescale of many minutes, as occurs within a morning peak period. This kind of variation can affect either discrete or continuous traffic arrivals, whether or not they are subject to random variations. We therefore consider it separately; its effect and treatment can be combined with those for other variations that are present. In this case, the mean flow $q_i(t)$ follows a regular profile $P_i(t)$ over time – for example, repeating each weekday. In this case, the flow component of the state equation is $dq_i/dt = P_i'(t)$: because of the periodic nature of the regular profile, $P_i(t) = P_i(t+p)$ for some period p. Whether or not the mean flow is hidden by the presence of stochastic variations in addition to the periodic ones, it can be estimated by repeated measurement, in this case using measurements at time t to estimate $P_i\,[t\ (\mathrm{mod}\ p)]$. A further possibility arises in this case that the profile differs in detail between repetitions in the sense that similar mean flows occur but at times that do not correspond exactly: this can be represented as $q_i(t) = P_i[t+\tau_i(t)]$, where $\tau_i(t)$ represents the stochastic deviation in the timing of the profile.

The fourth kind of variation that we consider arises because of long-term development in mean flows due to urban development and changes in travel patterns. In this case, the flow component of the state equation is $dq_i/dt = \eta_i(t)$ for some $\eta_i(t)$ that represents the variation over time of the mean flow – this is nominally an error term in the state equation. The distinction between this case and the previous one is that here the variation in flow is not periodic, so cannot be estimated in the same way by repeated measurement: if both changes in mean flow and stochastic variations around the mean are present, then estimation of the mean flow is a problem of statistical filtering to estimate the separate effects of stochastic variation $\varepsilon_i(t)$ about the mean and variation $\eta_i(t)$ of the mean (see, for example, Brown and Hwang, 1992 for a general treatment of this topic, and Heydecker and Bressaud, 1992 for one specific to the analysis of data from traffic detectors).

These four kinds of variability have been introduced separately. However, all sixteen ($16 = 2^4$) possible combinations of them can arise and can be analysed by appropriate combination of the specific analyses presented here.

2.6 Data requirements for optimisation

According to the analysis presented in this section, knowledge of the state variables $\mathbf{L}$ and $\mathbf{q}$ is sufficient for optimisation of signal control within an optimal control framework. Control actions taken at any time t_0 will influence future states $\mathbf{L}(t)$, $t > t_0$ as expressed in the state equation (1) for $\mathbf{L}$. Furthermore, because of the rules of operation of signal controllers, control actions made at any time will affect the range of possible actions at future times. The evaluation of a control decision is made on the basis of estimated performance during the future, which will be affected by each of these considerations as well as by details of the arrivals. For this reason, some further information such as the realisation of a stochastic arrivals process $\mathbf{a}(t)$ $(t > t_0)$ can be used to advantage in the control.

The data requirements for optimal control are then on-line estimates of the queue lengths $\mathbf{L}$ and of the mean flows $\mathbf{q}$. Provided that the state $\mathbf{L}(t_0)$ is known at some time t_0 , the state equation (1) can in principle be used to calculate values at all future times from the control $\boldsymbol{\chi}(t)$ and the mean flows $\mathbf{q}(t)$. Furthermore, the queue length will generally return to zero from time to time, which if identified correctly provides an opportunity to reset the estimates of that component of the state. However, in practice measurement errors and stochastic variations in arrivals around the mean flow $\mathbf{q}$ will lead to accumulation of errors so that additional information will be conferred by measurements that relate to the queue lengths $\mathbf{L}$.

3. ESTABLISHED RESULTS ON OPTIMISATION

3.1 Introduction

A substantial literature exists on optimisation of control decisions for traffic signals. In this section, we review these approaches within the framework that was presented in section 2. The intention of this is to show how each of these approaches addresses one of the four different kinds of variability that were introduced in section 2.5, and what their data requirements are. We consider these approaches in turn according to the kind of variability that it admits.

3.2 Control of constant mean flows

A large proportion of the literature on optimal control of road traffic presupposes that the mean flow of traffic in each steam is constant and known. Here, we review results from the literature, considering in turn results that apply to deterministic arrivals at the mean rate, and then stochastic arrivals with each of fixed and responsive control.

Consider first the case of constant uniform arrivals at a junction where the initial queue lengths $\mathbf{L}(0)$ are known and possibly large. Grafton and Newell (1965) explored use of the rule of running each stage until queues had dissipated and then switching to the next stage: the argument in support of this is that until the queue dissipates, flow at the stopline will be at the saturation rate s whereas after that, it will fall to the arrival rate q and so will use green time less efficiently. They showed that under a wide range of circumstances, this rule provides control that minimises the mean rate of future discounted delay, corresponding to formulation (5). The exceptions that they noted were for cases of large queues on high capacity approaches occurring and small queues on low capacity ones. This rule conforms to the concept of *feedback control* because it specifies the optimal control rule in terms of the current value of the state variables $\mathbf{L}(t)$ and $\mathbf{q}(t)$ (Kamien and Schwartz, p262).

In the case that the arrivals are stochastic with constant mean rate, then two distinct styles of operation are available: the first is to calculate and implement fixed timings that will accommodate stochastic variation in arrivals, whilst the second is to calculate timings on-line in a way that responds to detected arrivals and states. We consider these in turn.

Following Miller (1963a) and others, the mean rate of delay in a signal cycle starting at time $t = 0$ with initial queue length $L(0)$ that consists of a red period of duration r followed by a green period of duration g, is given to good approximation by

$$D = \frac{s}{2(s-q)} \frac{\left[q\,\mathrm{E}\left(r^2\right) + 2\,\mathrm{E}\left[L(0)\,r\right]\right]}{\mathrm{E}(r+g)} \; . \tag{8}$$

When the red and green times are fixed, stochastic arrivals can give rise to non-zero initial queue length $L(0)$, even if the green time is adequate for mean arrivals during the cycle: this formula shows that any *overflow queue* of this kind will lead to substantial additional delay. The effect of overflow on the queue length during the subsequent cycle is shown for three different cases in Figure 1.

Webster (1958) introduced a delay formula for fixed-time signals that avoids quantification of the overflow, and is therefore convenient to use. Webster showed by analysis that simple formulae could be used to achieve an approximate solution of (4) by calculating a fixed cycle time and then calculating the duration of stage times within this cycle that solves (3). Allsop (1971) formulated (4) as a convex non-linear programming problem using Webster's simplified delay formula in the objective. This formulation has been extended by Improta and Cantarella (1984), and Heydecker and Dudgeon (1987) to incorporate the flexibility of phase-based calculations, including some elements of sequencing.

The feedback rule of ending green when the queue length falls to zero has attracted several approaches for traffic-responsive operation of signals: if successful, this approach has the effect of eliminating the second term of the delay expression (8). An important issue in implementing a method of this kind is identifying when indeed the queue has dissipated, and this is often done by the a proxy measure such as identifying when flow at the stopline falls from saturation s to arrival q rate. In Britain, this is achieved by seeking time gaps in the combined output of an array of vehicle detectors located at 12m, 25m and 40m upstream of the stop-line on the approach to a junction (the *System D* configuration, DETR, 1997). The corresponding Dutch system has the additional sophistication of phase-based control and logic to determine which phases are selected when some are terminated (Van Zuylen, 1976). The Australian SCAT system (Lowrie, 1982; 1991) uses detectors at the stopline to measure flow there and hence to identify the fall from saturation to arrival flow. The profile of flows at the stop-line corresponding to a queue length without overflow is shown in Figure 2: this shows that the number of departures in each 6 second time interval falls from saturation to the number of arrivals when the queue dissipates – in this case at time $t = 84\text{s}$, which can be taken as an indirect indication that the queue has dissipated.

Formal optimisation approaches have been developed for this based upon *feedforward* of detector data to estimate details of future arrivals $a_i(t)$. Because of the proximity of detectors to the junction, the can provide estimates of arrivals for only a short time into the future – for example, a detector at 40m gives about 4s future information; for this reason the SCOOT (Hunt, Robertson, Bretherton and Winton, 1981) detection loops are placed as far upstream as possible. Beyond this detection period, only lower quality estimates of arrivals are generally available.

Miller (1963b) calculated delay based upon imminent arrivals estimated from detector data supplemented by an exponentially weighted moving average estimate of mean flow for use after the detection period: he developed a discrete-time control rule based upon minimisation of delay, calculating at each decision point whether or not to extend the current stage according to whether there is a prospect of achieving lower delay by doing so. This showed that with stochastic arrivals, extending green after the end of saturation flow can be justified if a group of vehicles is approaching the stop-line as would occur if $a_i(t)$ is sufficiently great in the near future. This analysis has been developed into the fully practical MOVA system (Vincent and Peirce, 1988), which combines a stage-extension decision strategy with rules that prevent a stage from being terminated before queues have cleared. Robertson and Bretherton (1974) devised a backward dynamic programming formulation in which they used hypothetical knowledge of individual arrivals over a 600 second horizon. Although they recognised that this is an impractical data requirement, they showed that the optimal decisions in the short run were insensitive to variations in traffic arrivals at times after 25 seconds into the future. Gartner (1983) developed the OPAC rolling optimisation procedure that uses a direct search method over estimates of delay based upon detected arrivals for the short-term future and estimated arrivals thereafter.

3.3 **Cyclic variations in flow**

In co-ordinated signal systems, the cyclic variations in flow due to platooning of traffic can be exploited to achieve good control. Provided that adjacent junctions operate on closely related cycle times, the time at which green is indicated at the downstream end of a link can be arranged relative to control at the upstream junction according to the cyclic profile of traffic arrivals that results. In order to achieve good coordination, additional control variables, known as *offsets*, can be used to manage the relative times at which green indications start at successive junctions along a route: this is known to have a substantial influence on control performance in road networks, but can generally only be achieved by striking a balance between the requirements of different routes where they conflict.

The model relationship (7) can be used to estimate arrivals of traffic $q_i(t)$ throughout a time interval of duration ϕ_i into the future on the basis of flow measurements $e_i(t)$ made upstream. However, the duration of this interval is generally less than that of a complete signal cycle, which might be considered reasonable minimum planning period so that some supplementary estimates are usually required for arrivals after this interval. These are provided in the SCOOT system (Hunt, Robertson, Bretherton and Winton) by estimates based on flow profiles during earlier cycles, and in the network form of the OPAC system (Gartner, Kaltenbach and Miyamoto, 1983) by ones based upon the planned control and hence future outflow at upstream junctions. The ways in which these different sources of data can be combined to provide information for feedforward control are shown schematically in Figure 3.

3.4 Systematic variation in flow

The third cause of variation in flow identified above, namely systematic within-day variation (peak periods etc) can be accommodated within fixed-time traffic control systems by having available a library of appropriate plans each pre-calculated for typical flow patterns either at a certain time of day, or for special events such as market days or sports match days. The most appropriate plan can then be identified and implemented according to one of several possible criteria. This approach is known as *plan selection*.

The matter then arises of how to select the most appropriate plan for use at each time. Various strategies are available for this, including according to:

1. time of day, day of week, or special event,
2. measured flows,
3. estimated performance.

The first of these is straightforward and has the important merit that a plan can be implemented before the traffic conditions for which it is appropriate arise. This can be especially important for peak period plans, which ideally would be in place before the start of the peak period (Bell, 1983; Bell, Gault and Taylor, 1983) so that there is no need to change signal plans at times of heavy flow. However, the success of this strategy depends on accurate advance knowledge of the flows that will arise, and hence on the repeatability of the flow patterns.

The second and third of these strategies both depend on the availability of some on-line measures of flow. In the second strategy, good agreement is sought between measured flows and those for which a plan has been calculated. This can be implemented using relatively few observations of flow, generally in important parts of the network. The third strategy is to implement the plan in the library that performs best with the observed flows; this has the advantage of selecting between signal plans directly according to their estimated performance. However, in order to implement this, observations of flow are required on many or most links of the network.

An appealing idea for cases in which extensive on-line data are available is to generate a signal plan accordingly so as to be optimal for to the current flows. This approach is known as *plan generation*. Despite the attractiveness of this approach, it has been found not to work well in practice (Holroyd, 1972). As with the second strategy, this has extensive requirements for on-line data. Even so, the plans that are implemented according to them will lag behind the development of flows because switching will not occur until after new flows have been identified.

All plan-switching strategies suffer from the problem that even if a new plan is optimal for the current flows, transient conditions that arise during the changeover between plans can cause substantial delays. This effect is particularly marked in coordinated systems where even a temporary mismatch in offsets can cause queues, the effects of which can persist. No entirely satisfactory method has been established for achieving a smooth transition between plans (Bretherton, 1979; Bell, 1983; Bell, Gault and Taylor, 1983). The SCOOT (Hunt, Robertson, Bretherton and Winton, 1981) on-line control system avoids abrupt plan changes by maintaining a plan that is varied gradually according to observations of traffic, and implements timings in each cycle that are allowed to deviate from this plan within certain limits according to detected arrivals.

A different approach to management of varying flows was developed by Ribeiro (1994), who sought the best single fixed-time plan to accommodate either a wide range or a known temporal profile of flows. He established robust signal plans by optimising their performance when implemented with a variety of flows. This approach has the particular advantages that no on-line data are required and no switching penalties are incurred.

3.5 Long-term developments of flow

Bell and Bretherton (1986) found that because of long-term changes in mean flows, the performance of a fixed-time plan would degrade at a rate of about 3 per cent each year. This phenomenon of *ageing* of fixed-time plans means that in order to achieve good performance, traffic surveys should be conducted and signal plans revised accordingly within every few years: the cost and effort of this survey work is substantial. One of the advantages of traffic responsive control systems is therefore that by tracking long-term developments in mean flows, they can overcome the need for extensive resurvey. Traffic-responsive control systems that calculate timings according to imminent arrivals and current state offer some prospect of achieving this: the SCOOT mechanism for on-line plan modification is an example of this.

4. ITS AND SIGNAL CONTROL

4.1 Introduction

The review of approaches to signal control of road traffic presented in section 3 shows that explicit optimisation formulations of signal control depend on information about the current and future state of traffic. The quantities that were identified there as appropriate state variables at time t_0 for signal control of road traffic were the current queue lengths $\mathbf{L}(t_0)$ and the mean arrival rates $\mathbf{q}(t)$ $(t \geq t_0)$, with some further information being provided by details of arrivals $\mathbf{a}(t)$ $(t \geq t_0)$, which are generally available only for a short time into the future: we denote the collection of these state variables as $\mathbf{x}(t)$. However, the data, which we denote as $\mathbf{z}(t)$, that are provided by detector systems do not generally provide direct observations of the traffic state $\mathbf{x}(t)$: rather they provide observations of quantities that can be represented as functions $\mathbf{h}(.)$ of the traffic state and the control state $\boldsymbol{\chi}(t)$. Alongside this formulation of observations through detector systems, the dynamics of the traffic under signal control can be expressed in terms of the non-linear state equation (1) augmented by an appropriate description of the development of flows as discussed in section 3: this dependence is represented by the state function, here denoted as $\mathbf{g}(.)$. Together, these lead to the non-linear state and observation system that depends on the control $\boldsymbol{\chi}$:

$$\left.\begin{aligned} \frac{d\mathbf{x}}{dt} &= \mathbf{g}[\mathbf{x}(t), \boldsymbol{\chi}(t)] + \boldsymbol{\eta}(t) \\ \mathbf{z}(t) &= \mathbf{h}[\mathbf{x}(t), \boldsymbol{\chi}(t)] + \boldsymbol{\varepsilon}(t) \end{aligned}\right\} \tag{9}$$

In this formulation, estimation of the state $\mathbf{x}(t)$ from the observations $\mathbf{z}(t)$ corresponds to a non-linear filtering problem, to which the extended Kalman filter (Hwang and Brown, p357) can be applied.

We now consider ways in which the information that is provided by various kinds of detector systems, and in particular those based on the techniques of intelligent transport systems (ITS), can be used to support signal control of road traffic. A key element in this is identification of the function $\mathbf{h}(.)$ that relates the observations $\mathbf{z}$ to the state $\mathbf{x}$ that is to be estimated.

4.2 Point detectors

Point traffic detectors, such as inductive loops, are widely used to provide traffic information for signal control. Depending on the location of the detectors and the time during the cycle, they can provide different kinds of information, which should therefore be interpreted and used accordingly. The main reason for this is that when queues form, they will extend upstream, occupying progressively more space and covering the field of any point detectors there. Once a detector field is covered in this way, traffic behaviour at that location will be dominated by the queue and departures from it until the time of passage of the last vehicle that was affected by presence of the queue. During this period, the detector will provide information about the current state of the queue, modified according to the departure pattern that is determined by signal control; this information can therefore be interpreted for use in feedback control. After the queue clears the position of the detector, it will provide information about individual arrivals of vehicles until it is next covered by a queue; this information can therefore be interpreted more directly in terms of imminent arrivals and used in feedforward control.

The spatio-temporal zones within which traffic activity is dominated by downstream conditions and upstream ones are depicted in Figure 4. Calculation of these zones will generally require use of a kinematic model of traffic to augment the point queue: a prime example of this is the Lighthill-Whitham (1955) model from which several others have been derived.

Where detectors are placed at the stop-line, they will provide information about the current state of the queue that can be interpreted directly in the vertical queueing model. Thus if a vehicle is present, then the vertical queue is non-empty: presence detectors at the stop-line can be used for variable sequencing, for example to call for a special stage for priority vehicles or for offside turning traffic that would otherwise be opposed by oncoming traffic. Furthermore, when the flow at a detector at this location remains at the saturation rate s during green, it indicates that a queue is still discharging, corresponding to a non-zero vertical queue; the Australian SCAT system (Lowrie, 1982; 1991) uses this kind of detection for feedback control.

For a point detector that is located on the approach upstream of the stop-line, the transition from saturation flow s to arrival flow q indicates the imminent exhaustion of the vertical queue at a time that can be estimated from the travel time to the stopline. Detectors in upstream positions are often used to provide feedforward data on imminent vehicular arrivals, either for heuristic control rules such as System D or for optimisation approaches such as SCOOT, OPAC or MOVA, each of which provides for extension logic of some kind in respect of high rates of imminent arrivals. Because the duration of the estimated arrival period corresponds to the travel time to the stopline, the further upstream such a detector is located, the greater the extent of the information.

4.3 Above-ground detection

Detectors such as Doppler effect microwave and video-based systems that are located above ground have several advantages over inductive loops. Some of these arise from reduced installation and maintenance costs, whilst others arise from their operational characteristics. A typical above-ground detector will respond to traffic on a region of the approach to the junction, often (but not always) starting at the stop-line and extending upstream. Because traffic in a region of this kind will not usually all be either queueing or flowing freely, the data correspond exactly to neither feedback nor feedforward concept, but rather to some combination of them.

The characteristics of Doppler-effect microwave detectors are that they will respond to any vehicle that is approaching the junction within the detection region. The output is binary, indicating either presence of at least one such vehicle or absence of any. This kind of detection has the advantage over inductive loops that it can discriminate between approaching vehicles and any departing ones that are present in the detection zone. It is well suited to extension logic of the kind embodied in System D vehicle actuation. However, it has no spatial resolution within the detection region so that detailed interpretation of the output is difficult.

Video image analysis detection systems offer the prospect of more detailed information capture. A suitably mounted video camera in favourable circumstances can acquire a view of several hundred metres of approach to a junction. The level of detail in the data extracted from a video image depends on the sophistication of the image processing that is undertaken, and can range from identification of presence of traffic to reporting of positions and speeds of individual approaching vehicles. A special feature of video image detection is that tall vehicles will tend to occlude shorter ones and hence prevent their detection; this becomes an increasing problem as distance from the camera increases, and can only be ameliorated by multiple camera installations and increasing mounting heights. Preliminary investigations (Ali et al, 1999) with a system that estimates the number of vehicles visible in the detection region on each approach showed that using a simple switching rule could achieve substantial performance advantages over System D vehicle actuated control. Further investigation showed that an appropriate choice of distance to the far end of the detection region was about 100m, and that performance deteriorated if an exact count of vehicles was used rather than the count of visible ones that omits occluded vehicles suggesting an advantage of implicit discounting of future delays. These results establish the potential for using video data for signal control and suggest that more sophisticated use could offer further performance advantages.

4.4 Vehicle tracking

Several technologies are emerging that can be used to track the progress of individual vehicles through the road network. Whilst some of these are intended specifically for this purpose, in other cases it is incidental to their primary purpose but can still contribute information that is relevant for use in signal control. Most systems of this kind provide information on a sample of vehicles within the traffic stream, and can be interpreted to provide trajectory information for those vehicles that are equipped. Ways in which data of this kind can be integrated into signal control systems have been discussed by Bell (1992).

These technologies include geographical positioning systems (GPS) and route guidance systems that provide information to a mobile device on its location. This information can be exported, either directly or in summary form, from participating vehicles to traffic control systems. Similarly, mobile telephones that are switched on, in standby mode, can provide locational information that is updated frequently; this is of lower resolution than GPS data, but can be collected from base stations rather than requiring contribution from the subscribers. Finally, congestion charging and other road pricing systems collect data on vehicles that can be used to estimate current flows and queue lengths and, where data are matched through sections of the network, on travel times: an important feature of these systems is that they are intended to collect data on all vehicles rather than a sample of them.

4.5 Automatic learning systems

The approach of optimal control theory to signal control of road traffic is based upon idealised formalisations of traffic management objectives and the data that are available to address them. Whilst this can provide insights into the formulation of optimisation methods, experience shows that heuristic methods of traffic control can often outperform analytical ones. Reasons for this include that the data that are available often do not conform to the ideal requirements of optimal control theory – for example, they are observations of neither current nor future state, but rather relate to combinations of them.

A contrasting approach to optimisation of traffic control systems is to apply an automated learning system to generate control rules that work well using the data that are available. Several approaches are available to this, including artificial neural networks (Hertz, Krogh and Palmer, 1991) and genetic algorithms (GA) (Holland, 1975) to generate rules within a rule-base that is then tested – usually by simulation. This approach can accommodate a wide range of different kinds of detector data and of candidate performance criteria for traffic management in a flexible manner so that urban traffic management and control can be implemented without the need to develop explicit optimisation processes. Automated learning achieves this by using reinforcement learning with performance fed back from simulation of traffic: this has the advantage that it is not specific to any particular objective or form of primary data. The purpose of this is to develop an optimisation formulation that can achieve good traffic performance flexibly according to any of a range of possible criteria using data from a variety of kinds of traffic detectors (see, for example, Sayers, Anderson and Clement, 1996).

The automated learning process that is common to these approaches is depicted in Figure 5. The automated learning system (ALS) has the function of generating rules of operation for traffic control: this entails devising rules that use the detector information that is available to determine the signal operation. The learning process uses feedback in the form of a reward value that is calculated – most usually from simulation – according to the traffic operation that results when these rules are applied, and is used to discriminate between the different rules. Ultimately, the operation of the system is evaluated by reference to the traffic performance calculated according to traffic management objectives. In some circumstances, these systems can outperform traditional ones such as vehicle-actuated control. An interesting feature of this approach is that the good rules can be generated by using a reward quantity that differs from the performance objectives: for example, use of instantaneous queue length at the end of a stage, corresponding to overflow, can lead to lower mean rate of delays than using the mean rate of delay itself as the basis for calculation of the reward (Sha'Aban, Tomlinson, Heydecker, and Bull, 1992). We note with reference to the present context of optimal control theory that this choice of reward corresponds to an instantaneous observation of the traffic state whilst the objective of optimisation corresponds to the time-averaged value of it. Thus the formulation using feedback in which the instantaneous state is monitored seems to be more effective that one in which recent values of the objective itself are used as the basis of control.

5. CONCLUSIONS

This paper has formulated signal control for road traffic within the framework of optimal control theory. By reference to this general formulation, the importance is discussed of appropriate treatments for different kinds of variability in traffic. A review of the literature shows how various optimisation approaches can be identified within this formulation as corresponding to treatments of combinations of these kinds of variability.

The kinds of traffic detector data that are currently available and for which there is a reasonable prospect do not generally correspond to direct observations of the state variables of the traffic system. There is an established record of heuristic traffic control systems that use the raw data in operational rules that succeed in controlling road traffic. However, detector data require careful interpretation if they are to be used in optimisation formulations. As a distinct alternative approach, the detector data that are available can be used in raw form together with rules that are generated automatically to achieve good control performance. The way in which each of these approaches is developed as further detector data become available will influence the style, if not the objectives, of signal control operations in the future.

The optimal control framework that is presented here has the feature that it can be applied using data in varying quantity and quality, according to their availability. On general grounds, one could reasonably expect that the greater the quantity and quality of data, the better the performance that can be achieved. To a certain extent, any formal optimisation procedure can be identified within this framework. The interesting issue arises in practical application of whether with a certain availability of data, the performance that can be achieved by an explicit optimal control formulation exceeds that that can be achieved by either a heuristic approach or an automated learning one. This is a matter for investigation, with the expectation that as the quantity and quality of available data increases, so the balance of traffic control will develop from heuristics towards optimal control formulations.

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References

Ali, AT, Crosta, DA, Allsop, RE, Heydecker, BG, Robertson, DI and Vincencio-Silva, M-A (1999) Improved traffic signal control using image analysis. Paper presented to the 31st UTSG Annual Conference, University of York, January 1999 (unpublished).

Allsop, RE (1971) Delay minimizing settings for fixed-time traffic signals at a single road *junction. Journal of the Institute of Mathematics and its Applications*, **8**(2), 164-85.

Allsop, RE (1976) SIGCAP: a computer program for assessing the traffic capacity of signal-controlled road junctions. *Traffic Engineering and Control*, **17**(8/9), 338-41.

Bell, MGH (1992) Future directions in traffic signal control. *Transportation Research*, **26A**(4), 303-13.

Bell, MC (1983) A survey of the methods used to define and change signal plans in the UK. *Research Report* **50**. Transport Operations Research Group, University of Newcastle upon Tyne.

Bell, MC and Bretherton, RD (1986) Ageing of fixed-time traffic signal plans. *Proceedings* of the 2nd IEE Conference on Road Traffic Control. London: IEE.

Bell, MC, Gault, HE and Taylor, IG (1983) Plan change algorithms for area traffic control systems. *Research Report* **51**. Transport Operations Research Group, University of Newcastle upon Tyne.

Bretherton, RD (1979) Five methods of changing fixed-time traffic signal plans. Transport and Road Research Laboratory *Report*: **LR 879**. Crowthorne: TRL.

Brown, RG and Hwang PYC (1992) Introduction to random signals and applied Kalman filtering. Chichester: Wiley.

DETR (1997) Junction principles of control by traffic signals. Design Manual for Roads and Bridges, **8.1**, **TA 16/81**. London: TSO.

Gallivan, S and Heydecker, BG (1988) Optimising the control performance of traffic signals at a single junction. *Transportation Research*, **22B**(5), 357-70.

Gartner NH (1983) OPAC: a demand-responsive strategy for traffic signal control. Transportation Research Record, **906**, 75-84.

Gartner NH, Kaltenbach MH and Miyamoto MM (1983) Demand-responsive decentralized urban traffic control, part II: network extensions. Report **DOT/OST/P**-34/85/009.

Gazis, DC and Potts, RB (1963) The over-saturated intersection. *Proceedings* of the Second International Symposium on the Theory of Road Traffic Flow (ed J Almond), Paris: OECD, 221-37.

Grafton, RB and Newell, GF (1965) Optimal policies for the control of an undersaturated intersection. **In**: Vehicular Traffic Science (eds LC Edie, R Herman and R Rothery). New York: Elsevier, 239-57.

Hertz, J, Krogh, A and Palmer, R (1991) Introduction to the theory of neural computation. London: Addison Wesley.

Heydecker, BG and Bressaud, X (1992) Filtering detector data for real-time traffic control systems. *Note* from the Transport Studies Group, University College London.

Heydecker, BG and Dudgeon, IW (1987) Calculation of signal settings to minimise delay at a junction. **In**: *Transportation and Traffic Theory* (eds NH Gartner and NHM Wilson). New York: Elsevier, 159-78.

Holland JH (1975) Adaptation in Natural and Artificial Systems, University of Michigan Press.

Holroyd, J (1972) Comparison between a dynamic signal plan generation system of area traffic control and a fixed-time system. Transport and Road Research Laboratory *Report*: **LR 487**. Crowthorne: TRL.

Hunt, PB, Robertson, DI, Bretherton, RD and Winton, RI (1981) SCOOT – a traffic responsive method of coordinating signals. Transport and Road Research Laboratory *Report* **LR 1014**. Crowthorne: TRL.

Improta, G and Cantarella, GE (1984) Control system design for an individual signalised junction. *Transportation Research*, **18B**(2), 147-67.

Kamien, MI and Schwartz, NL (1991) Dynamic optimisation: the calculus of variations and optimal control in economics and management. London: North-Holland.

Lighthill, MJ (1958) Introduction to Fourier analysis and generalised functions. Cambridge University Press.

Lighthill, MJ and Whitham, GB (1955) On kinematic waves II. A theory of traffic flow on long crowded roads. *Proceedings of the Royal Society*, **229A**, 317-45.

Lo, H (1999) A Novel Traffic Signal Control Formulation. *Transportation Research*, **33A**(6), 433-448.

Lo, H, Chang, E and Chan, YC (2000) Dynamic Intersection Signal Control Optimization (DISCO): Numerical Results for Argyle Street. *Hong Kong Institution of Engineers Transactions*, **6**(3), 1-7.

Lo, HK and Chow, HF (2003) Adaptive traffic control: control strategy, prediction resolution, and accuracy. *Journal of Advanced Transportation*, **36**(3), 323-47.

Lowrie, PR (1982) The Sydney co-ordinated adaptive traffic system – principles, methodology, algorithms. *Proceedings* of the IEE International Conference on Road Traffic Signalling, Conference Publication **207**. London: IEE, 67-70.

Lowrie, PR (1991) SCATS: Sydney co-ordinated adaptive traffic system – a traffic responsive method of controlling urban traffic. Sydney: Roads and Traffic Authority.

Miller, AJ (1963a) Settings for fixed-cycle traffic signals. *Operational Research Quarterly*, **14**(4), 373-86.

Miller, AJ (1963b) A computer control system for traffic networks. *Proceedings* of the Second International Symposium on the Theory of Road Traffic Flow (ed J Almond), Paris: OECD, 200-20.

Quinn, D (1992) A review of queue management strategies. *Traffic Engineering and Control*, **33**(11), 600-5.

Ribeiro, PCM (1994) Handling traffic fluctuations with fixed-time plans calculated by TRANSYT. *Traffic Engineering and Control*, **35**(10), 362-6.

Robertson, DI (1969) TRANSYT: a traffic network study tool. Transport and Road Research Laboratory Report: **LR 253**. Crowthorne: TRL..

Robertson, DI and Bretherton, RD (1974) Optimum control of an intersection for any known sequence of vehicular arrivals. *Proceedings* of the 2nd IFAC-IFIP-IFORS Symposium on Traffic Control and Transportation Systems.

Robertson, DI, Lucas, CF and Baker, RT (1980) Coordinating traffic signals to reduce fuel consumption. Transport and Road Research Laboratory *Report*: **LR 934**. Crowthorne: TRL.

Sayers, T, Anderson, J and Clement, S (1996) The multi-objective optimisation of a traffic control system. **In**: *Transportation and Traffic Theory* (ed A Ceder). Haifa: The Technion, 153-76.

Sha'Aban, J, Tomlinson, A, Heydecker, BG and Bull, L (2002) Addressing policy objectives of traffic control using evolutionary algorithms. *Proceedings* of the European Transport Conference, Seminar on Making Better Use of the Network. London: PTRC.

Van Zuylen, H (1976) Acyclic traffic controllers. *Note* from the Transport Studies Group, University College London.

Vincent, RA and Peirce, JR (1988) MOVA: traffic responsive, self-optimising signal control for isolated intersections. Transport and Road Research Laboratory *Report* **RR 170**. Crowthorne: TRL.

Webster, FV (1958) Traffic signal settings. Road Research *Technical Paper* **39**. London: HMSO.

Wood, K (1993) Urban traffic control, systems review. Transport and Road Resaerch Laboratory *Report* **PR 41**. Crowthorne: TRL.

Biography

Benjamin Heydecker has a degree in mathematics from Cambridge University, and a doctorate in transport studies from University College London. He worked as Research Office at the University of Leeds, and then returned to a lectureship at University College London where he is now Professor of Transport Studies. Heydecker's research centres on the application of mathematical and statistical analyses in transport studies, and has included statistical analysis of road accident data, transport modelling – especially dynamic traffic assignment, and traffic management and control – especially aspects of signal control.

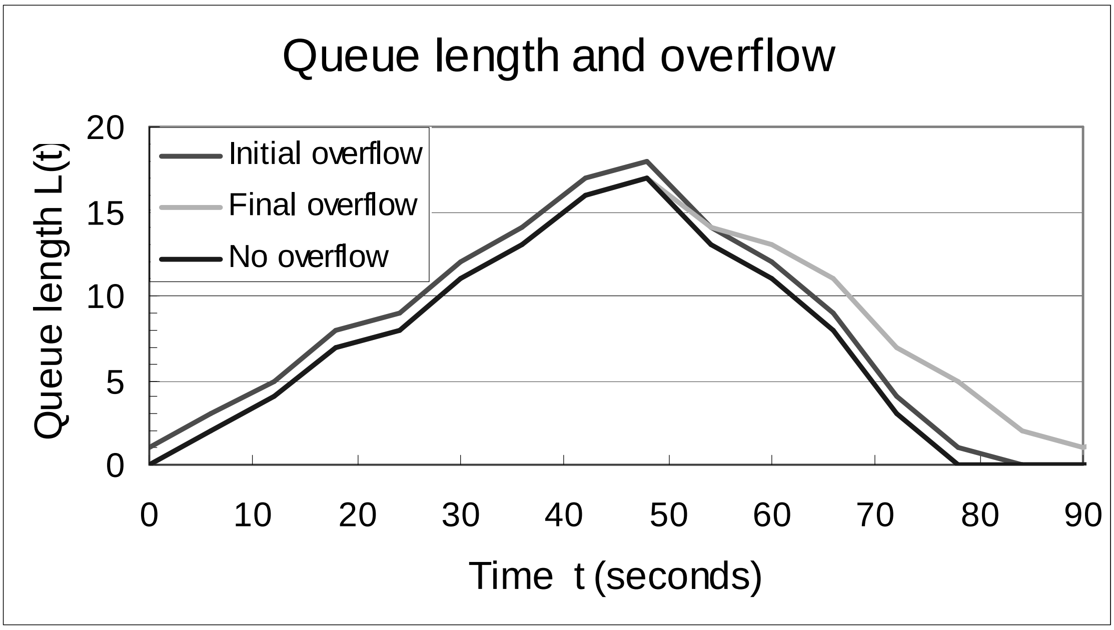


Figure 1: Queue length profiles illustrating various overflow possibilities

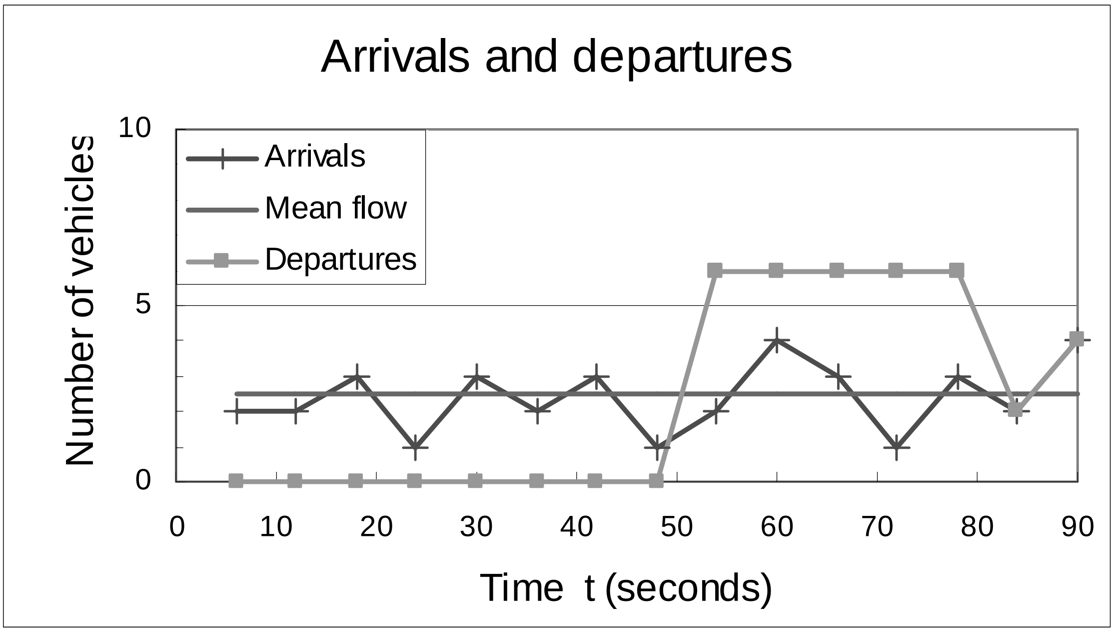
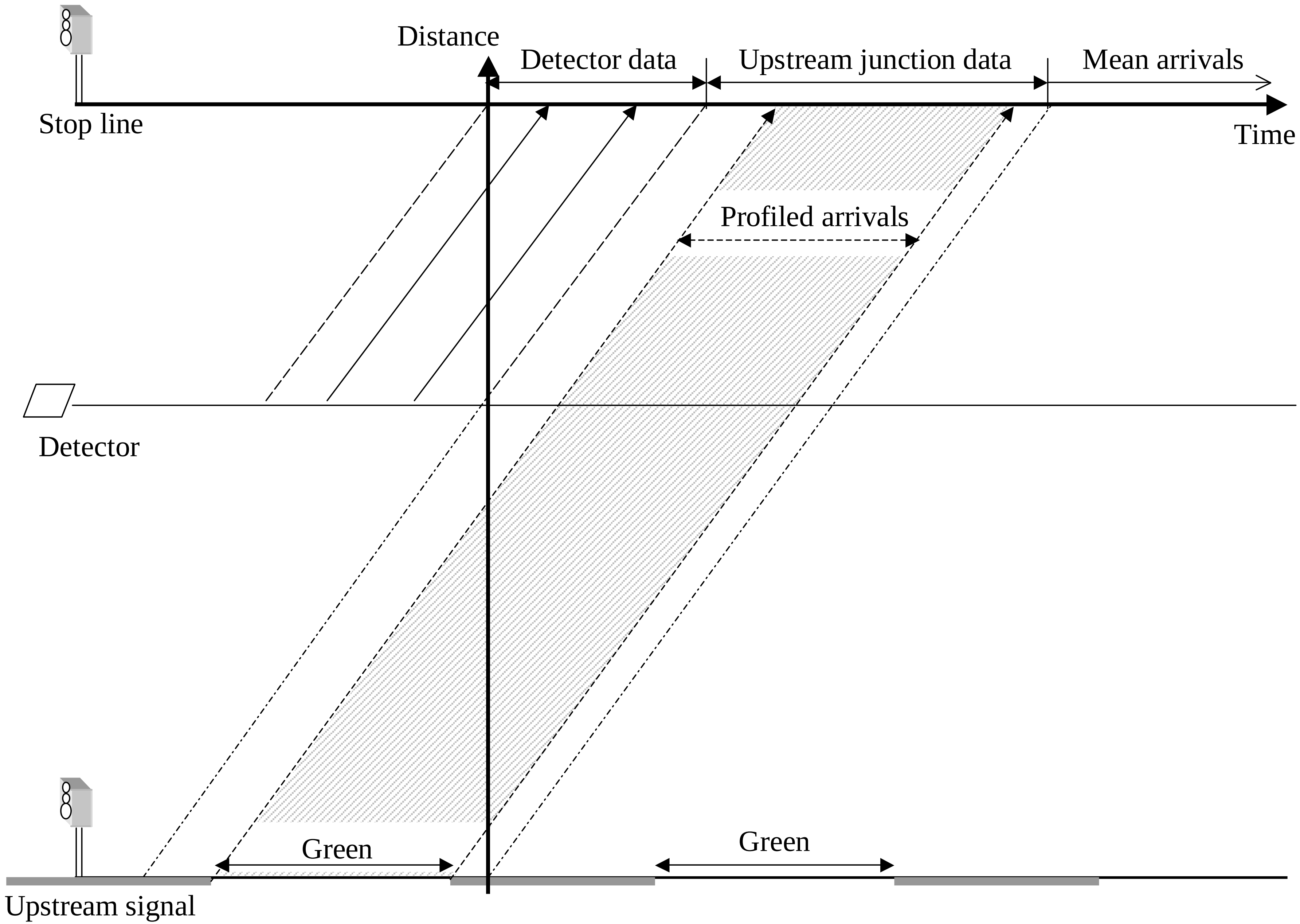


Figure 2: arrival and departure profiles during a signal cycle



Distance
Detector data
Upstream junction data
Mean arrivals
Stop line
Time
Profiled arrivals
Detector
Green
Green
Upstream signal

Figure 3: Sources of data for feedforward control of traffic signals

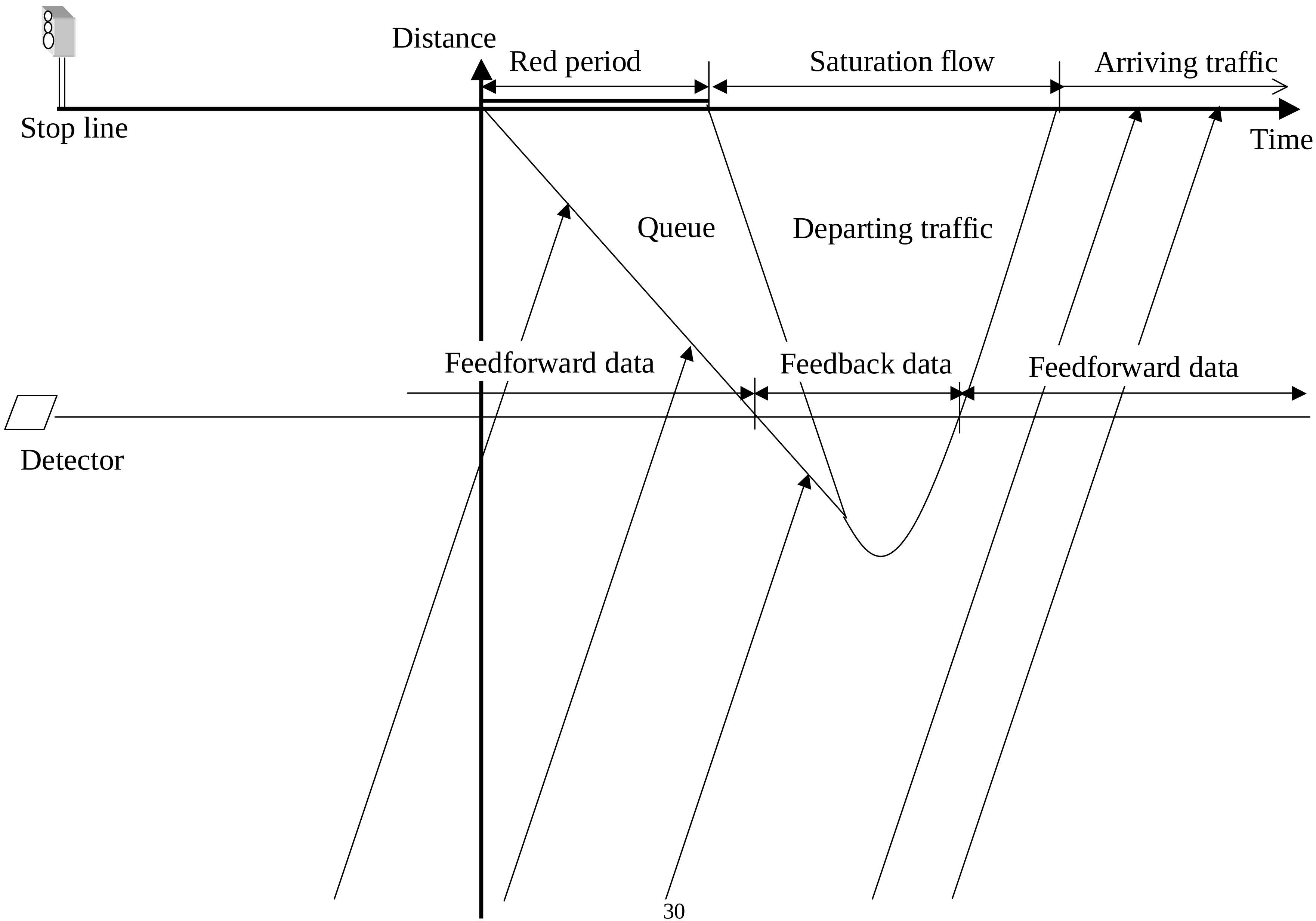

Distance
Red period
Saturation flow
Arriving traffic
Stop line
Time
Queue
Departing traffic
Feedforward data
Feedback data
Feedforward data
Detector

Figure 4: Nature of data from an upstream detector.

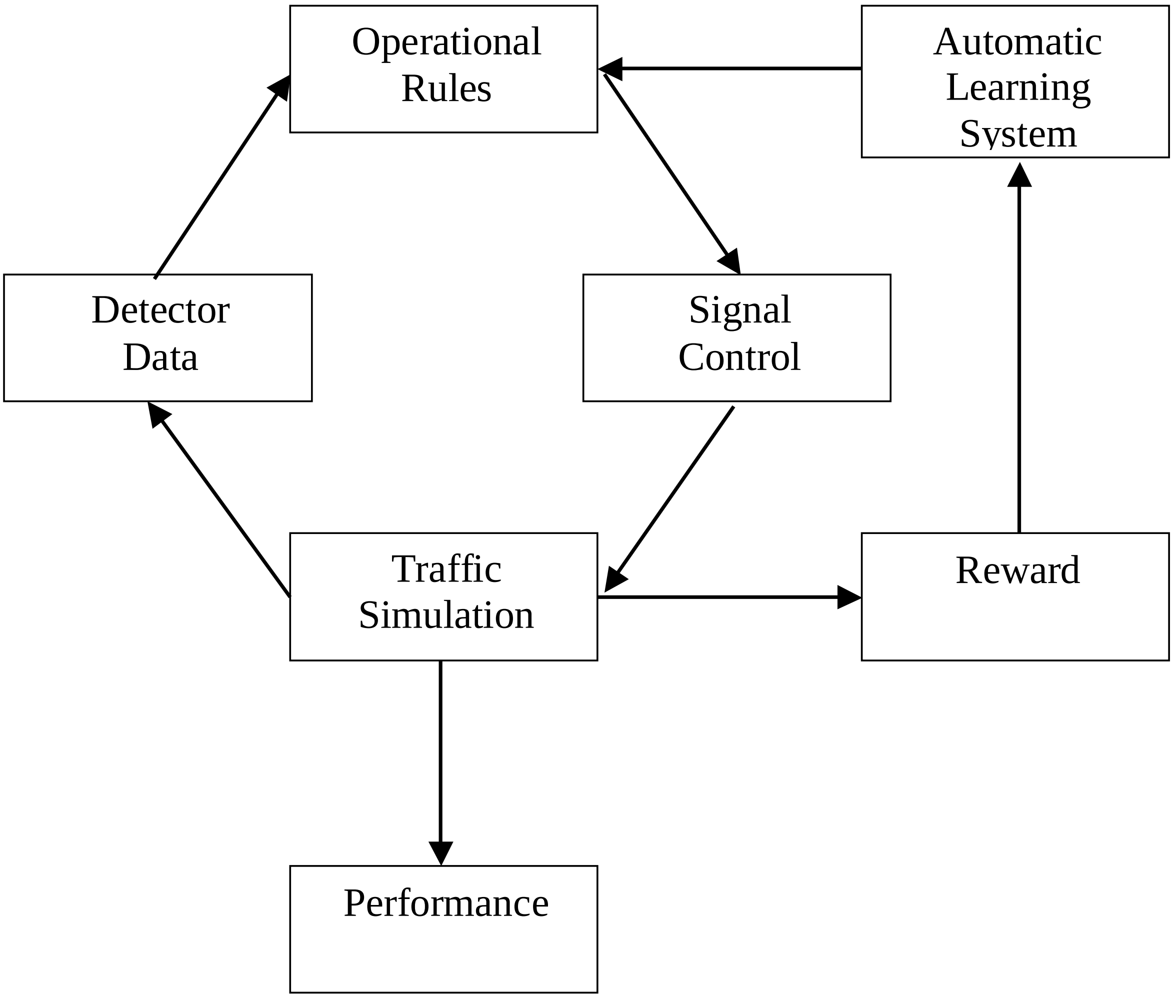


Figure 5: Structure of an automated learning system for traffic signal control