ARTICLE IN PRESS

Computers, Environment and Urban Systems xxx (2013) xxx–xxx

Contents lists available at ScienceDirect

Computers, Environment and Urban Systems

journal homepage: www.elsevier.com/locate/compenvurbsys

Beyond preference: Modelling segregation under regulation

Yiming Wang

The Bartlett School of Construction and Project Management, The Bartlett Faculty of Built Environment, University College London, United Kingdom

ARTICLE INFO

Article history:
Available online xxxx

Keywords:
Segregation
Cellular automaton
Preference
Institution

ABSTRACT

Segregation models often focus on private racial preference but overlook the institutional context. This paper represents an effort to move beyond the preference centricity. In this paper, an ideal Pigovian regulatory intervention is emulated and added into Schelling's (1971) classic spatial proximity model of racial segregation, with an aim to preserve collective welfare against the negative externalities induced by the changing local racial compositions after individual relocations. A key discovery from a large number of cellular automata is that the Pigovian regulation tends to result in less segregated but also less efficient (in terms of aggregate utility) residential patterns than *laissez faire*. This finding, albeit from a highly stylized model, bears intellectual relations to an important practical question: What are the potential racial effects of Pigovian local planning interventions, such as financially motivated anti-density zoning or the collection of a development impact fee? On top of its modest policy implications, this paper demonstrates a bottom-up computational modelling approach to reconcile the preference-based and institution-orientated academic perspectives regarding racial residential segregation.

1. Introduction

As a longstanding American urban phenomenon, residential segregation by race involves both preferential and institutional reasons. However, scholarship on this subject tends to fragment, eliciting a quite clear divide between the preference-based versus the institution-orientated perspectives. For instance, Schelling's (1971) classic segregation models[1] build entirely upon individual preference regarding the community level racial composition, with little attention paid to the local regulatory context. This feature seems to persist in Schelling's contemporary counterparts (Card et al., 2008; Chen et al., 2003; Clark, 1991; Fossett & Waren, 2005; Wang, 2011; Zhang, 2011). On the other hand, since the seminal work by Massey and Denton (1993), inquiries about the institutional causes of racial segregation have mushroomed, primarily relying on more qualitative methods coupled with in-depth case studies (Squires & Kubrin, 2005; Squires & O'Connor, 2001).

This paper may be seen as an attempt to move beyond the aforementioned preference-institution divide. Revisiting Schelling's (1971) classic spatial proximity model, this study explores the segregation outcomes in an artificial municipality which regulates individual agent actions for the sake of collective welfare in an ideal Pigou (1920) style, by explicitly accounting for and adjusting the potential negative externalities of all private transactions and relocations. In methodological terms, this piece resonates with earlier works by Webster and Wu (2001) and Heikkila and Wang (2009), among others, in trying to redress the micro-foundations of bottom-up computational models from an institutional economics perspective. While the word "institution", according to North (1990), can stand both for formal interventions and informal conventions, its first connotation is mainly referred to hereafter.

The relation between racial residential segregation and local planning regulation has for long been debated. For example, Nelson, Sanchez, et al. (2004) studied the data about major US metropolitan areas in the 1990s, but found no significant statistical relations between segregation measured in dissimilarity index and the adoption of such planning interventions as restrictive land use zoning, impact fee collection, and building permit caps. By contrast, another empirical study later by Rothwell and Massy (2009) identified financially motivated anti-density zoning as a significant factor in excluding, though covertly, ethnic minorities from White dominated residential suburbs.

Unlike the previous studies, this paper is intended to address the above debate from a more generic modelling perspective. Cellular automaton is employed in this study as a simplified agent-based simulation approach (Batty, 2009) to revisit Schelling's (1971) seminal spatial proximity model, which assumes a gridded urban spatial structure resembling a checkerboard that contains a finite number of locations. Each location can be easily represented by a cell, with every cell either accommodating a Black

E-mail address: yiming.wang@ucl.ac.uk

[1] Schelling (1971) actually contains two segregation models. The first is a spatial proximity model or the so called checkerboard model of segregation between neighbourhoods. The other is a bounded neighbourhood tipping model. This paper mainly addresses the former model.

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or a White family, as in the Schelling original. A household's satisfaction, measured in terms of utility, only depends on the racial composition of its neighbours, who live upon the nearest surrounding cells. When a household is unsatisfied with the local racial make-up, it seeks to move to another location. In Schelling (1971) relocation is essentially free insofar as the space allows. In the present model, however, any relocation imposing a net social cost upon the whole municipal community is prohibited. This could be framed as an ideal Pigou (1920) scenario, wherein the externalities of individual action are perfectly explicit and internalized through interventions, making the aggregate social cost tantamount to the accounted private cost; action stops when the cost is too high.

A large number of simulations show that a municipality which governs individual agents using the Pigovian mode of regulation eventually tends to generate less aggregate utility, but also becomes less racially segregated than in the Schelling original. Besides its quite intriguing, albeit admittedly modest, policy implications, this study illustrates the relevance of cellular automaton, as a kind of bottom-up computational modelling approach, to urban planning research in general and segregation studies in particular, given the profound spatial as well as institutional complexities therein contained (Batty, 2007).

The remainder of this paper is organized as follows. The next section reviews Schelling's (1971) spatial proximity model and points out that the model can be modified to account for Pigovian local regulation as a stylized institutional factor. This is followed by the set up of cellular automaton in this paper, with the simulation results reported and analyzed afterwards. The penultimate section discusses the policy and intellectual implications. Conclusions are drawn in the end, along with suggestions for future research.

2. Schelling and Pigou

2.1. Schelling's spatial proximity model

The phenomenon of residential segregation by race not only draws urban planners' attention, but also interests economists. Thomas Schelling (1971) is one of the first mainstream economists who have looked into the dynamics of segregation. From a game-theoretic perspective Schelling modelled segregation in a spatially explicit fashion. Schelling found that ethnic integration would eventually be improbable, even if individual agents have only mild racial preference and just avoid becoming ethnic minority in their local neighbourhoods.

Fig. 1 below illustrates the structure of private racial preference in Schelling's spatial proximity model. The two vertical axes in Fig. 1 measure, by utility (U), the degree of a White and a Black household's satisfaction respectively with housing when living in a neighbourhood. The horizontal x axis shows the percentage of neighbours who are African Americans.

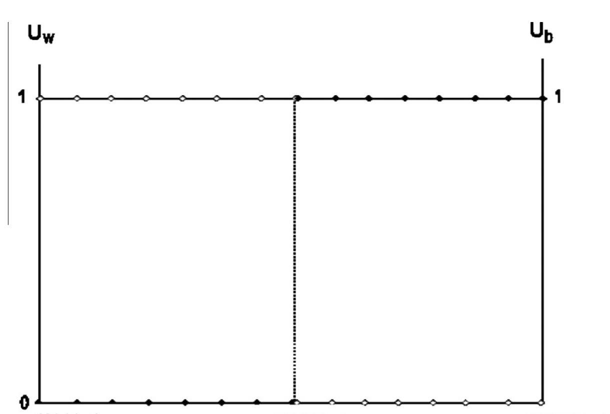

Fig. 1. Private racial preference in Schelling's spatial proximity model.

Schelling assumed that a household, whether Black or White, would dislike becoming racial minority within a neighbourhood and would want to move away immediately if that happened. Otherwise a household would stay happily at its current place and remain indifferent to its neighbours' ethnic identity. Let $U = 0$ denote dissatisfaction and $U = 1$ stand for satisfaction. Hence the utility function is binary for both a White and a Black household:

$$U_w = \begin{cases} 1, & \text{when, } x \leqslant 50\% \\ 0, & \text{when, } x > 50\% \end{cases} \tag{1}$$

$$U_b = \begin{cases} 1, & \text{when, } x \geqslant 50\% \\ 0, & \text{when, } x < 50\% \end{cases} \tag{2}$$

Eqs. (1) and (2) are represented respectively by two lines in Fig. 1, one marked with blank and the other with solid diamonds. The dotted line in Fig. 1 stands for a catastrophic transition in utility from 0 to 1 or *vice versa*.

Schelling defined neighbourhoods explicitly in spatial terms. He first examined locations on a line, upon which every point has eight neighbouring points, four on each side. He then moved to a two dimensional checkerboard model, in which a neighbourhood is usually made up of nine cells forming a 3 * 3 square, hence a typical Moore neighbourhood. This rule however does not apply to neighbourhoods in the urban periphery, which may contain less than nine cells given the boundary constraint.

For an illustrative example, in Fig. 2 above, there is a 3 * 3 square containing nine white cells, numbered from 1 to 9. Cell 9 in the center of this white square has eight neighbours, including all of the cells surrounding it. Note that this particular neighbourhood is unique for cell 9. In fact every cell in Schelling's checkerboard model perceives its neighbourhood in reference to the cell's own location; no cell shares exactly the same neighbourhood with another cell.

Schelling also allowed any discontent households to move freely to wherever they want, as long as the space is available. It should be noted that Schelling did presume a certain number of vacancies in the checkerboard so that an unsatisfied household could relocate to one of the preferred untaken spots and in the mean time leave a new vacancy available for other unhappy agents.

2.2. The Pigovian interventions

Schelling's free moving scheme exemplifies a typical institutional arrangement in most standard segregation models. But contrary to what is emulated in Schelling, a household's location choice, in reality, is often affected by the various forms of regulatory planning control. One primary example is residential zoning. In many affluent suburban communities, the local zoning

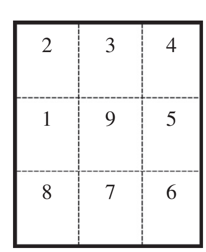

Fig. 2. An illustration of the neighbourhood in Schelling (1971).

ordinances often mandate minimum lot size and single family occupancy (Fischel, 2000, 2004). More price-based interventions include charging a one shot upfront impact fee toward a new residential development (Ihlanfeldt & Shaughnessy, 2004). Otherwise if a municipality is managed by a local homeowner association, a new participant is often required to deposit a considerable amount of membership dues (Mckenzie, 1994; Teaford, 1997).

Notwithstanding the diverse forms of planning regulations, economists consider most of these interventions essentially Pigovian, in the sense of aiming to internalize the negative externalities that private deals can cause to the local community as a whole (Fischel, 2000; Ihlanfeldt & Boehm, 1987). For example, an individual homeowner letting a single family house to multiple households usually means the entire community is burdened with extra demand for local infrastructures such as water and sewage services. This kind of private party transaction, in economics terms, reduces the utility of those who are not directly involved in and thus external to the deal, often ending up with net social cost. Anti-density zoning resolves such externality issues by restricting the supply of land, capping the number of residents per unit, and essentially raising the price of housing for potential home buyers and renters. Compared with zoning, collecting an impact fee is even more direct by charging the newcomers a sum of payment to offset the negative externalities of immigration. Interventions in the both cases are Pigovian, in the sense of forcing a concerned individual to be responsible for the social cost of private transactions.

Certainly, the exercise of any intervention in practice is always contingent upon a variety of political, cultural or even coincidental factors and thus much more complex than described above. Yet notwithstanding the real world complexity, it is both possible and reasonable to incorporate a stylized Pigovian intervention into Schelling's classic spatial proximity model of racial segregation. Note that the same externality issue actually also lurks in Schelling's model. Since every household cares about the racial make up of its neighbourhood, a household's utility might be affected adversely when the old neighbours move away and new families come in, which may result in an undesirable change in the racial composition of local neighbourhood (Pancs & Vriend, 2007). If the entire community ends up with a net loss in welfare due to the relocations of a few individual agents, a Pigovian regulatory intervention becomes justifiable, for the same reason that zoning and impact fees, for instance, are rationalized in practice.

2.3. Toward an integrated modelling approach

To date it remains controversial how the variegated local planning regulations can affect the pattern of racial residential segregation. The debate revolves particularly around the effect of zoning on local ethnic diversity. On one hand critics like Sager (1969) and Seitles (1998) have fiercely charged exclusionary zoning as *de facto* racial discrimination in the housing market. Their key claim is that the US housing market is imbued with racially discriminatory private preference. Zoning covertly institutionalizes such private discrimination in the name of protecting communal interest. On the other hand, researchers like Fischel (2004) insist that zoning does not directly target race, since lower income renters and homebuyers of any ethnic background can be priced out of the market due to such ordinances as density control. More empirical research also shows divergent results. For example, Nelson et al. (2004) suggests that there is no significant statistical relations between the degree of segregation and the exercise of planning interventions such as restrictive land use zoning, impact fee collection, or building permit caps in major US metropolitan areas during the 1990s. However, Rothwell and Massy (2009) discovered that anti-density zoning and impact fee collections did seem to significantly exclude African Americans from White dominated residential suburbs.

The above controversy can be partly attributed to the gap between what is modelled and what is happening in reality. Conventional segregation models tend to place an overwhelming weight on private preferences regarding the community level racial composition, while largely neglecting the local regulatory context (e.g., Card et al., 2008; Chen, 2003; Clark, 1991; Fossett & Waren, 2005; Wang, 2011; Zhang, 2011). A more integrated approach would not only look at the individual's racial preferences, but also account for the institutional factors that affect agent's decisions and actions. Recent years have seen a mounting number of publications in this direction (e.g., Benenson et al., 2009; Grauwin, Bertin, et al., 2009; Pancs & Vriend, 2007).

A strengthened notion about the interplay between private choices and public institutions, arguably, also features many contemporary developments in the general field of computational urban spatial modelling. For instance, Webster and Wu (2001) used a cellular simulation to justify localized land use control from a Coasian efficiency perspective. Heikkila and Wang (2009) also deliver an agent-based model of polycentic urban form and illustrate the implicit yet critical influence of social institutions on urban spatial structure. The both works, among many others, demonstrate a renewed appreciation of the microeconomic foundations of bottom-up computational urban modelling, aiming perceptively at emulating spontaneous agent actions given certain types of institutional structures. Bottom-up computational modelling deployed in such fashion, according to Batty (2007), has arisen as an essential approach to understanding and unravelling social and behavioural complexities underpinning urban spatial structures. In a similar spirit, the next section of this paper presents a cellular automaton as a simplified agent-based model (Batty, 2009), whereby an individual agent's freedom of action in Schelling's spatial proximity model is regulated and restrained for the sake of collective welfare.

3. A cellular automaton approach

A cellular automaton is coded in Visual Basic after a substantial modification and secondary programming based on Teknomo's (2001) original codes. The specific model setup is detailed below.

3.1. Defining neighbourhood

There is a municipal space which contains $N * N$ cells as land parcels available for housing. The definition of neighbourhood mostly follows the Schelling (1971) original, with one difference. While Schelling allows a certain number of cells to be vacant, in the revised model all cells are occupied, either by a Black or a White household. This setting is necessary because the revised model involves a moving-by-swapping algorithm as detailed below.

3.2. The moving algorithm

Since there is no vacant cell available, a discontent household can move only by exchanging its current location with another cell. Adapted from Zhang's (2011), a moving-by-swapping algorithm is employed in this paper, guided by the principle of Pareto optimization. Specifically, imagine two households from a same racial group who could swap their locations. Yet since the both agents have the same utility function, a trade would make neither better off; both would remain unhappy. In contrast a transaction between two

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racially different households would at least make neither worse off and possibly increase the total payoffs. Given Eqs. (1) and (2), Eq. (3) holds for any individual location:

$$U_w + U_b = 1 \Rightarrow \begin{cases} U_b = 1, & \text{if, } U_w = 0 \\ U_w = 1, & \text{if, } U_b = 0 \end{cases} \tag{3}$$

Eq. (3) has the following intuition: A household will deal with another only if, (a) they are from different racial group, and, (b) one of the two is unhappy at its present location. In other words, a household would always end up satisfied by dealing with an unhappy household, insofar as the two households are from different ethnic groups. Note that this does not necessarily mean the former would have to be better off, since an agent can be content already before it switches. On the other hand, a discontent household can never become any further worse off; swapping location is nevertheless possible (yet not guaranteed) to make the unsatisfied happy again. In summary an exchange between two racially different households must be Pareto improving (Zhang, 2011). Thus given the principle of Pareto improving, two households from different racial backgrounds would always agree to a deal, as long as at least one of them is unsatisfied with the existing condition.

An implicit assumption underlying this model concerns the vision of individual agent. An agent is assumed to only see the immediately adjacent peers, in the same way as it delineates its own neighbourhood. However one may also consider a trade between two faraway agents made up of multiple rounds of local transactions as the two agents move closer and closer throughout the process of spatial simulation. Now assume that spatial proximity decides the priority in deal-making. According to Eqs. (1) and (2), a discontent family must have more than half of its neighbours from the alternative ethnic group, so it has to choose which neighbour gets priority. In this model a household is supposed to start looking from the cell immediately to the left of it and then move clockwise until it finds the first possible dealmaker within its neighbourhood. For an illustrative example, a household upon cell 9 in Fig. 2 would search from cell 1 to 8, following the numerical sequence clockwise, until it identifies a suitable neighbour to trade the locations. Similar rule applies to a household located in the urban periphery with less than eight neighbours.

3.3. The rule of transition without regulation

As there are only two kinds of households inhabiting the municipal space, every cell can exhibit only two possible racial states, or formally, $s \in \{w, b\}$, where w denotes White and b stands for Black. Also note that an unhappy agent can always find a racially different neighbour who is willing to exchange locations. Thus a Markovian transition function for any single cell may be expressed as the follows:

$$s^{i,t+1} = \begin{cases} w, & \text{if, } U_b^{i,t} = 0, \quad \text{or, } U_w^{j,t} = 0 \\ b, & \text{if, } U_w^{i,t} = 0, \quad \text{or, } U_b^{j,t} = 0 \end{cases} \tag{4}$$

$s^{i,t+1}$ denotes the state of cell i at time $t + 1$, which depends on the utility of a household occupying cell i at time t. Given the moving-by-swapping algorithm, an unhappy household will always switch with a neighbour located at cell j from the other racial group. Otherwise a happy family would simply stay. Another possibility is that a household, whether happy or not, has to swap its home with a nearby unhappy agent who is racially different and taking cell j at time t. (A caveat is that cell j's relative location to cell i is not fixed in a spatial sense. Instead one may assume that an agent upon cell j always gets the priority to deal with an agent upon cell i, as long as there is a deal to make.) This would also trigger a transition in cell i's racial state from time t to $t + 1$. Finally the sojourn between t and $t + 1$ is defined as the length of time during which every cell has to update its ethnic state once as per Eq. (4).

3.4. Pigovian regulation

Regulation in this model targets the aggregate household utility within the municipality averaged by the number of households. The following Eq. (5) assesses the average utility, denoted as ψ, at time t across the municipal space.

$$\psi_t = \sum_{i=1}^{N*N} U_{s^{i,t}}^{i,t} / (N * N) \tag{5}$$

Instead of maximizing ψ_t, regulation in this model aims to ensure that no location swapping would ever diminish ψ_t. To formalize this type of regulation, let $\xi^{i\leftrightarrow j,t}$ denote the net social profit or aggregate efficiency gain that a location swap between two households at cell i and cell j at time t can produce for the entire local municipality. As indicated in Eq. (6) below, $\xi^{i\leftrightarrow j,t}$ is assumed to be made up of two parts. One part is the total private pay-offs for the two households involved in the exchange, denoted as $P^{i\leftrightarrow j,t}$. The other part is, $E^{i\leftrightarrow j,t}$, the externality this transaction imposes upon the other local households which are not directly engaged in the deal. $E^{i\leftrightarrow j,t} < 0$ implies a negative externality induced by the private transaction.

$$\xi^{i\leftrightarrow j,t} = P^{i\leftrightarrow j,t} + E^{i\leftrightarrow j,t} \tag{6}$$

If $P^{i\leftrightarrow j,t} + E^{i\leftrightarrow j,t} < 0$, or in other words, $\xi^{i\leftrightarrow j,t} < 0$, the transaction entails an overall inefficiency or a net social cost and will be forbidden under Pigovian regulation for the sake of infringing collective interest. This may also be understood as a scenario wherein the two private parties cannot make sufficient profits to afford a Pigovian impact fee that is intended to make up for the induced negative externality. Otherwise the exchange will be endorsed. In this vein, the following relationship is readily deducible:

$$\psi_{t+1} = \psi_t + \sum_{j=1}^{N}\sum_{i=1}^{N} \xi^{i\leftrightarrow j,t} / (N * N), \text{only if, } \xi^{i\leftrightarrow j,t} \geq 0, \tag{7}$$

$$\Rightarrow \psi_{t+1} \geqslant \psi_t \forall t$$

Eq. (7) says that the regulation tends to improve or, at least, would never reduce the overall household utility in a municipality. To ensure that this condition holds, a backtracking algorithm is applied when coding the model in Visual Basic (Gurari, 1999).

3.5. The rule of transition under regulation

Incorporating Pigovian regulation into the model requires the rule of transition aforementioned in Eq. (4) to be accordingly modified as illustrated in Eq. (8) below:

$$s^{i,t+1} = \begin{cases} W, & \text{if, } U_b^{i,t} = 0, \quad \text{or, } U_w^{j,t} = 0, \quad \text{and, } \xi^{i\leftrightarrow j,t} \geqslant 0 \\ B, & \text{if, } U_w^{i,t} = 0, \quad \text{or, } U_b^{j,t} = 0, \quad \text{and, } \xi^{i\leftrightarrow j,t} \geqslant 0 \end{cases}$$

Recall that $\xi^{i\leftrightarrow j,t}$ only exists when there is at least one unhappy agent either taking cell i or cell j at time t. Otherwise no transaction would happen between two satisfied agents.

3.6. The stopping conditions

Because cellular automaton is a simulation approach, some stopping conditions need to be specified in advance. In the case of this model a key interest lies with the average housing utility, ψ_t. If the value of ψ_t remains constant for N iterations, or formally,

$$\psi_{T+N} = \psi_{T+N-1} = \cdots = \psi_T \tag{9}$$

then the simulation is assumed to reach a stable endpoint at time T. Otherwise the simulation should also stop after a sufficiently large number of iterations, which equals to the total number of cells involved in the model:

$$T = N * N \tag{10}$$

3.7. The initial segregation pattern

An initial racial residential pattern needs to be placed upon the municipal space as a starting point of the simulation. As in the Schelling original, two types of distribution are generated randomly by computer as the initial patterns. The first involves a random distribution of an equal number of White and Black households. In the second case there is no control on the ratio between the White and Black population, as long as the aggregate population equal to $N * N$. However it should be noted that, in the both cases, the number of Blacks and Whites, once after a simulation has started, would remain fixed until the end of the simulation.

3.8. Assessing the segregation pattern

Recall Schelling's key discovery: Segregation would always arise, even though individual agents only have a very mild racial preference. Will this discordance between "Micromotives and Macrobehaviours" (Schelling, 1978) persist in a regulated Schelling model? To answer this question, one needs to monitor the segregation pattern. For this purpose an indicator is adapted from Moran's I to specifically measure the overall degree of segregation:[2]

$$|I_t| = \left| \sum_{i=1}^{N*N} \sum_{k=1}^{n} [X^{k,t} - \sum_{i=1}^{N*N} X^{i,t}/(N*N)]/[X^{i,t} - \sum_{i=1}^{N*N} X^{i,t}/(N*N)] \right| \tag{11}$$

where

$$X^{i,t} = \begin{cases} 0, & \text{if, } s^{i,t} = w \\ 1, & \text{if, } s^{i,t} = b \end{cases} \tag{12}$$

and cell k is one of cell i's neighbours, including but not limited to cell j. n is the total count of neighbours and for most cells, $n = 8$. n might be less than eight for cells at the municipal borders.

Given the statistical properties of Moran's I, the value of I_t falls in the range of $[-1, 1]$ (Odland, 1988). In the context of this model, $I_t = 1$, if the entire municipality sees a complete separation between White and Black households. In another extreme, $I_t = -1$, if a household, wherever it lives within the municipality, always finds all of its neighbours to be from the other ethnic group. Finally, $I_t = 0$, if every household is located in a racially mixed neighbourhood, with half of the neighbours being White and the other half being Black. Thus, $|I_t|$, the absolute value of I_t, has a value range as [0, 1]. A larger value in $|I_t|$ indicates a racially segregated pattern overall, whereas a smaller value suggests the municipality to be relatively more integrated in racial terms.

4. Results

The model results are presented below in two ways. The first involves a case study of sample simulations, by comparing the simulation outcomes with the initial statuses and monitoring the transition trajectories of ψ_t and $|I_t|$. The primary goal here is to replicate the classic findings by Schelling (1971).

[2] The more conventional segregation indices such as the dissimilarity index are not suitable in this case, wherein the boundary of neighbourhood is self-referenced (given a cell's own location) rather than delineated exogenously as in Massey and Denton (1989), for example.

The second way of presentation concentrates on the general difference that the Pigovian regulatory control can make to the simulation outcomes. The target measures are still ψ_t and $|I_t|$. Their values at the end of 200 simulations, half involving regulation and half not, are compared using paired sample t-test, to see whether there is a significant difference in their population means.

4.1. A case study of simulation results

Fig. 3 below illustrates a sample comparison between two simulations that both start from the same random pattern shown in Fig. 3(1), when $N = 10$ and an equal number of Black and White households (i.e., 50 each) are involved in the model. The simulation involving no regulatory control stops when $t = 29$ and ends up with the pattern shown in Fig. 3(2). Although the average utility has increased from $\psi_0 = 0.600$ to $\psi_{29} = 0.950$, the final pattern becomes more racially segregated, given the rise from $|I_0| = 0.055$ to $|I_{29}| = 0.452$. This result resonates with Schelling's classic finding that segregation would eventually dominate notwithstanding individuals' general indifference to their neighbours' racial identities.

Compared with Fig. 3(2) and (3) displays a more racially integrated residential pattern. The simulation involving regulation stops at $t = 28$, with $\psi_{28} = 0.900$ and $|I_{28}| = 0.200$. Although the final residential pattern is still more segregated than the initial one, the exercise of regulation has prevented further segregation which otherwise would become the pattern as shown in Fig. 3(2).

Figs. 4 and 5 below compare the trend of ψ_t and $|I_t|$ between the two simulations. The horizontal axes in the both figures count iterations, t, while the vertical axes respectively show the value of ψ_t and $|I_t|$. In Fig. 4 the trend of ψ_t for the unregulated simulation is marked with blank squares. For the regulated simulation, the marks are solid squares. Conceivably the former regulated simulation eventually attains a higher level of collective utility, though with a bit of fluctuations in the value of ψ_t over the course. Yet also notice that ψ_t has never declined in the second simulation but rather kept rising until it stabilizes at the level of 0.900. This finding seems to confirm the analytical insights carried in Eq. (7) above, namely that the average utility never declines.

In Fig. 5 the trend of $|I_t|$ for the unregulated simulation is marked with blank triangles. For the regulated simulation, the marks are the solid triangles. Both trends consist of ups and downs, while the trajectory for the regulated simulation seems a bit more stable and eventually results in a lower level of segregation.

A large number of trial simulations, including those with a random number (i.e., not necessarily 50:50) of Blacks and Whites, are also experimented with. The pattern summarized in the above case study however seems fairly robust. Almost all of the automations end up with a more segregated pattern and yet also reaches a higher level of aggregate utility compared with the initial status. This finding is essentially consistent with that by Schelling some 40 years ago.

Schelling of course did not run a simulation that involves regulatory control, so he lacked an opportunity to compare the results of regulated versus unregulated simulations. The above case however suggests that the two types of simulations may engender very different consequences, whether in terms of utility or in terms of the spatial pattern of segregation. While the structural differences illustrated in Figs. 4 and 5 above seem to be quite commonplace and almost constantly recurring, there is indeed a small number of observations that do not follow suit. Hence an important question arises, that whether or not there is a generalizable difference in ψ_t and $|I_t|$ between the regulated and non-regulated simulations? The next section addresses this question using inferential statistics.

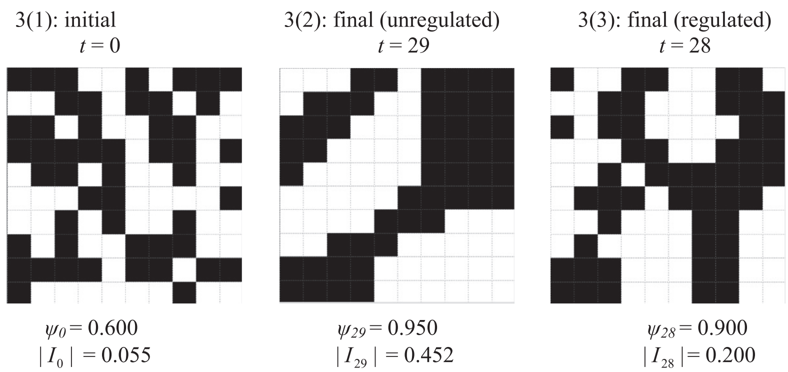

Fig. 3. A sample comparison between two simulations, $N = 10$.

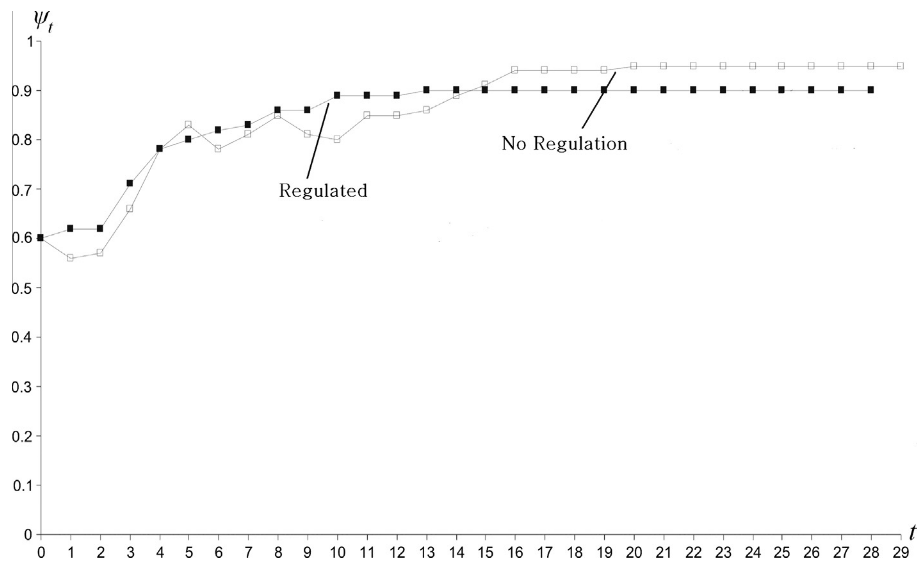

Fig. 4. Comparing the trends of ψ_t between two simulations.

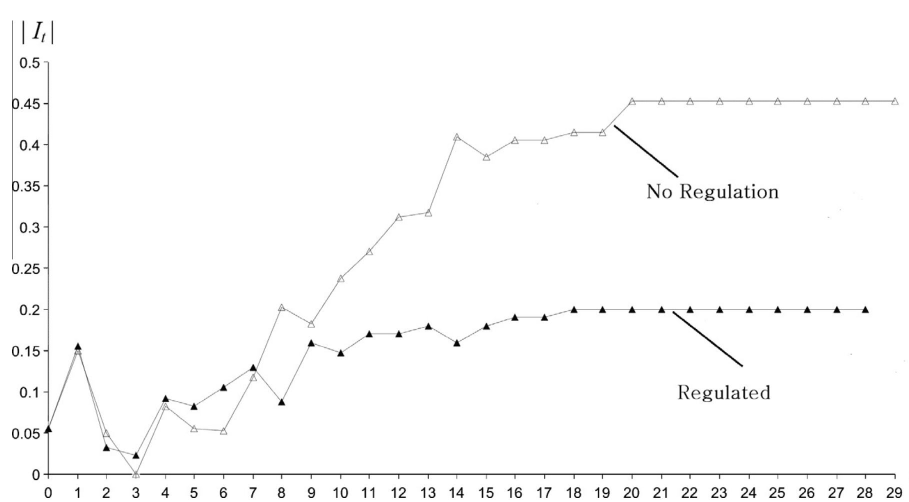

Fig. 5. Comparing the trends of $|I_t|$ between two simulations.

4.2. Comparing the general impacts of regulation versus non-regulation

Since the computer randomizes the initial pattern for every simulation and each simulation is an independent data generation procedure, the simulation results are deployable for some standard inferential statistical tests. Although, in theory, a sufficiently large number of computerised simulations can exhaust all of the possible model outcomes, which then can be compared simply using conventional descriptive statistics, the interest here is however in whether one can see a significant difference in a relatively small number (e.g., 100) of simulations. This thus calls for comparing the general impacts of planning regulation versus no regulation in a more systematic and rigorous fashion than individual case studies.

To generate data 100 unregulated simulations and 100 regulated automations are run in a pairwise fashion, firstly based on some random initial states involving 50 White and 50 Black agents. The simulation outcomes are compared using paired sample*t*-test and the test results are summarized in the following tables. Tables 1 and 2 below basically suggest that, in this setting, Pigovian regulation tends to generate a significantly lower level of average utility than *laissez faire*. By contrast, Tables 3 and 4 suggest that the regulation tends to result in a significantly less segregated racial residential pattern compared with the *laissez faire* in the Schelling (1971) original.

Almost essentially the same results arise even if the number of Whites and Blacks are allowed to be uneven at the beginning of the simulations. However N must remain fixed and in this case $N = 10$ for all the simulations. Otherwise the simulation results would become incomparable. For instance the value of $|I_t|$ would be systematically smaller for larger N, according to Fotheringham and Wong's (1991) seminal study on the modifiable areal unit problem of spatial autocorrelation.

Tables 5 and 6 are the counterparts of Tables 1 and 2 above, while the ratio between Whites and Blacks is purely stochastic. Again regulation seems to result in a significantly lower level of average utility compared with no regulation. Similarly Tables 7 and 8 convey essentially the same message as Tables 3 and 4. That is, Pigovian regulation tends to prevent racial segregation which otherwise would be inevitable in a world devoid of regulatory control.

Table 1
Comparing the outcome average utility (50 Whites and 50 Blacks).

	Regulated?	Simulations	Mean	Std. Deviation	Std. Error Mean
Outcome average utility (ψ_t)	No	100	.954	.015	.002
	Yes	100	.938	.022	.002

Table 2
Difference in the mean of average utility (50 Whites and 50 Blacks).

Paired differences in outcome average utility (ψ_t)					
Mean	Std. Deviation	T	df	Sig. (2-tailed)	Std. Error Mean
0.016	.028	5.796	99	.000	.003

Table 3
Comparing the outcome degree of segregation (50 Whites and 50 Blacks).

	Regulated?	Simulations	Mean	Std. Deviation	Std. Error Mean
Outcome degree of segregation ($\|I_t\|$)	No	100	.434	.087	.009
	Yes	100	.383	.100	.010

Table 4
Difference in the mean degree of segregation (50 Whites and 50 Blacks).

Paired differences in the outcome degree of segregation ($\|I_t\|$)					
Mean	Std. Deviation	T	df	Sig. (2-tailed)	Std. Error Mean
0.051	.103	4.924	99	.000	.010

Table 5
Comparing the outcome average utility (random Black-White ratio).

	Regulated?	Simulations	Mean	Std. Deviation	Std. Error Mean
Outcome average utility (ψ_t)	No	100	.956	.022	.002
	Yes	100	.931	.021	.002

Table 6
Difference in the mean of average utility (random Black/White).

Paired differences in outcome average utility (ψ_t)					
Mean	Std. Deviation	T	df	Sig. (2-tailed)	Std. Error Mean
0.025	.032	7.722	99	.000	.003

Table 7
Comparing the outcome degree of segregation (random Black/White).

	Regulated?	Simulations	Mean	Std. Deviation	Std. Error Mean
Outcome degree of segregation ($\|I_t\|$)	No	100	.434	.106	.011
	Yes	100	.346	.092	.010

Table 8
Difference in the mean degree of segregation (random Black/White).

Paired differences in the outcome degree of segregation ($\|I_t\|$)					
Mean	Std. Deviation	T	df	Sig. (2-tailed)	Std. Error Mean
0.088	.119	7.384	99	.000	.012

5. Discussion

5.1. Why regulation results in less segregation?

The outcomes of cellular automata clearly suggest that the exercise of Pigovian regulation tends to result in a less segregated residential pattern than the *laissez faire* scenario modelled in the Schelling original. This can be understood, first of all, by recognising that the Pigovian regulation in this model actually favours the local residents and tends to discourage moving. In other words, whether the initial pattern is an integrated or segregated one, the regulation, in effect, tends to preserve the status quo compared with the free moving mechanism set in the original Schelling (1971) model; every household simply tends to have less degree of freedom under regulation.

Secondly, recall that in this paper all of the initial states are randomized by the computer. The probability of generating the first group of neighbouring cells which all accommodate same-race households is actually very small, given Eq. (13) below:

$$\Pr = (r-1)/(N*N-1)*(r-2)/(N*N-2)\ldots*(r-n)/(N*N-n) \tag{13}$$

r in Eq. (13) stands for the subtotal population within a particular racial group, such as Black or White. $N * N$ gives the aggregate municipal population. n counts the number of neighbours for a household living upon a specific cell. Given the definition of neighbourhood in this study $n = 8$ for most cells that are not located at the boarders. Suppose 50 Whites and 50 Blacks upon a 10 * 10 municipal space, as studied above. In that case, *Pr*, the probability of

Please cite this article in press as: Wang, Y. Beyond preference: Modelling segregation under regulation. *Computers, Environment and Urban Systems* (2013), http://dx.doi.org/10.1016/j.compenvurbsys.2013.09.003

utter racial segregation in the first randomly generated neighbourhood is less than 0.003. The probability for each and every neighborhood to be completely segregated would actually be even smaller than that. In this vein, it is simply much more likely to start a simulation from a mixed beginning pattern, which later tends to be preserved by the Pigovian regulation set up in the model.

5.2. Why regulation results in less utility?

By economic theory, a Pigovian intervention should improve efficiency. Yet the Pigovian regulation modelled in this paper seems to produce less collective utility than *laissez faire*. One possible explanation is associated with Eq. (7) and related to the issue of path dependency (Arthur, 1994). According to Eq. (7), the average collective utility, ψt, would never decline in the course of any simulation insofar as there is regulatory control. However one should note that this is neither a necessary nor sufficient condition to maximize the overall utility at the end of the simulations. In fact Fig. 4 suggests that a higher level of utility can be reached eventually without regulation, even though the level of utility may fluctuate during the course of simulation. This finding is intriguing because it suggests that long run utility gains may be locked off by short term maximization behaviour.

Another, and perhaps even more pertinent, explanation is that the Pigovian regulation modelled here only deals with negative externalities (i.e., $E^{i\leftrightarrow j,t} < 0$ in Eq. (6)) which end up as social cost (i.e., $\xi^{i\leftrightarrow j,t} < 0$) for the entire municipality. However, there is indeed a possibility that a privately unprofitable (i.e., $P^{i\leftrightarrow j,t} < 0$) relocation may improve collective welfare (i.e., $\xi^{i\leftrightarrow j,t} > 0$), in which case, a subsidybecomes necessary to compensate for the substantial positive externalities (i.e., $E^{i\leftrightarrow j,t} > \xi^{i\leftrightarrow j,t} > 0$). What is missing in the current Pigovian regulation is thus a reward or incentive scheme that encourages individual transactions and relocations that may seem unviable from a private accounting perspective, but do yield net social benefits for all. From an economics perspective, this may explain why the public sector in practice often takes inclusionary planning measures, such as rent control, affordable housing or inclusionary zoning, to subsidize lower income ethnic minorities and encourage them to move into White dominated middle class communities.

5.3. Reconciling preferences and institutions

On top of the admittedly mild policy implications discussed above, the cellular automata conducted in this study exemplify an integrated perspective to study preferential as well as institutional factors underlying the phenomenon of racial residential segregation. Many recent inquiries have already moved in this direction (e.g., Benenson et al., 2009; Grauwin et al., 2009; Nelson et al., 2004; Pancs & Vriend, 2007; Rothwell & Massey, 2009). Following suit, this paper employs a bottom-up computational modelling approach, featuring a large number of cellular automata of which the programming is guided by some classic microeconomic principles.

This paper, alongside many other similar efforts in recent years, also has important implications for the general discipline of urban planning. In his recent paper entitled "Should planners start playing computer games", Devisch (2008) posits that seemingly game-like urban computational models can enable planners to better understand the complex process of social and spatial evolution. Compared with the traditional and more deterministic methods such as regression or input-output analysis, bottom-up computational models are uniquely sensitive to myopic agent behaviour, bounded rationality, path dependency, social and spatial interactions, all of which are particularly relevant to the practice of urban planning (Rittel & Webber, 1973; Schon, 1983).

6. Conclusion

Employing a cellular automaton approach, this paper revisits Schelling's (1971) spatial proximity model of racial segregation. As in the Schelling original, racial integration appears to be systematically untenable insofar as the agents are allowed to move freely between neighbourhoods. However, the simulation results also suggest that a Pigovian regulation to preserve collective welfare may alleviate the degree of eventual segregation, which otherwise would be much more substantial under the *laissez faire* emulated in Schelling (1971). Results from inferential statistical tests confirm the findings. These discoveries bear intellectual relations to an important policy question in practice regarding the potential racial effects of planning interventions. On top of that, this paper showcases a bottom-up computational modelling approach which attempts to reconcile the preference-based and institution-orientated perspectives in the contemporary segregation research.

Like many other computational models, the cellular automata presented in this paper are also faced with potential challenges in terms of model verification and validation (Crooks et al., 2008; Xiang et al., 2005). "Verification is the process of making sure that an implemented model matches its design. Validation is the process of making sure that an implemented model matches the real-world." (Crooks et al., 2008, p. 419). Given the relatively straightforward model setup in this study, validation is perhaps a more pressing issue than verification. Several improvements can be made in that respect.

Firstly, a "parameter variability-sensitive analysis" (Xiang et al., 2005, p.48) can be conducted by inputting into the model alternative initial segregation conditions or/and different kinds of racial preference. In fact, Schelling (1971) did explore a variety of individual preferential structures in his tipping model, even though that happened only after he presented his checkerboard model. Given the very fast speed of computerized cellular simulation, a large set of alternative parameters, either based on Schelling (1971) or other sources, can be experimented with in future to test the robustness of simulation results reported in this study.

Secondly, the cellular automaton model included in this paper is conceivably hypothetical, partly because of the Schelling original it builds upon. However, a lot of efforts have been made in recent years to link the Schelling style spatial process model with large-scale empirical spatio-economic dataset, using such apparatuses as geographic information systems (GIS) (e.g., Benenson et al., 2009). This offers a potential opportunity to test, calibrate and even resurrect the assumptions of Schelling original in reference to empirical observations. Indeed an inferential statistical test framework is applied in this paper upon pseudo data generated by computerized cellular automata. The same thing can thus be done with actual demographic data that are becoming more and more accessible to the general public.

Thirdly, the kind of model studied in this paper is rather stylized in terms of the setting of agent behaviour. As in the Schelling original, the definition of neighbourhood is so homogenous that, even though every family defines its own neighbourhood in reference to the household's own location, the way of self-mapping is identical for everyone. However, Coulton et al.'s (2001) empirical study suggests that people tend to perceive the geography of their neighbourhood in very different fashions. Such cognitive heterogeneity could possibly be accounted for by using an online GIS survey platform which allows users to map their own neighbourhoods.

Acknowledgements

I am grateful to Eric Heikkila, Philip Viton and Chris Webster for reviewing several draft versions of this paper. My gratitude also goes to CEUS editors and two anonymous reviewers for guiding me through the peer review process. All errors lurking in the paper are mine.

References

Arthur, B. (1994). *Increasing returns and path dependence in the economy*. Ann Arbor, Michigan: University of Michigan Press.
Batty, M. (2007). *Cities and complexity: understanding cities with cellular automata, agent-based models, and fractals*. The MIT Press.
Batty, M. (2009). ''Urban Modelling''. In Thrift, N., & Kitchin, R. (Eds.), *International encyclopaedia of human geography* (pp. 51–58). Oxford: Elsevier.
Benenson, I. E. et al. (2009). From schelling to spatially explicit modeling of urban ethnic and economic residential dynamics. *Sociological Methods and Research, 37*(4), 463–497.
Card, D. et al. (2008). Tipping and the dynamics of segregation. *Quarterly Journal of Economics, 123*(1), 177–218.
Chen, K. et al. (2003). The emergence of racial segregation in an agent-based model of residential location: The role of competing preferences. *Computational and Mathematical Organization Theory, 11*(4), 333–338.
Clark, W. A. V. (1991). Residential preferences and neighborhood racial segregation: A test of the schelling segregation model. *Demography, 28*(1), 1–19.
Coulton, C. J. et al. (2001). Mapping residents' perception of neighborhood boundaries: A methodological note. *American Journal of Community Psychology, 29*(2), 371–383.
Crooks, A. et al. (2008). Key challenges in agent-based modelling for geo-spatial simulation. *Computers, Environment and Urban Systems, 32*(6), 417–430.
Devisch, O. (2008). Should planners start playing computer games? Arguments from Simcity and second life. *Planning Theory and Practice, 9*(2), 209–226.
Fischel, W. (2000). Zoning and land use regulation. *Encyclopedia of Law and Economics, 2*, 403–423.
Fischel, W. (2004). An economic history of zoning and a cure for its exclusionary effects. *Urban Studies, 41*(2), 317–340.
Fossett, M., & Waren, W. (2005). Overlooked implications of ethnic preferences for residential segregation in agent-based models. *Urban Studies, 42*(1), 1893–1917.
Fotheringham, A. S., & Wong, D. W. S. (1991). The modifiable areal unit problem in multivariate statistical analysis. *Environment and Planning A, 23*(7), 1025–1044.
Grauwin, S., Bertin, E., et al. (2009). Competition between collective and individual dynamics. *Proceedings of the National Academy of Sciences, 106*(49), 20622–20626.
Gurari, E. (1999). *Backtracking Algorithms CIS 680: Data Structures: Chapter 19: Backtracking Algorithms* [Online]. Ohio State University. <http://www.cse.ohio-state.edu/~gurari/course/cis680/cis680Ch19.html#QQ1-51-128> Accessed 23.03.08.
Heikkila, E., & Wang, Y. (2009). Fujita and ogawa revisited: An agent-based modeling approach. *Environment and Planning B: Planning and Design, 36*(4), 741–756.
Ihlanfeldt, K., & Boehm, T. (1987). Government intervention in the housing market: An empirical test of the externalities rationale. *Journal of Urban Economics, 22*(3), 276–290.
Ihlanfeldt, K. R., & Shaughnessy, T. M. (2004). An empirical investigation of the effects of impact fees on housing and land markets. *Regional Science and Urban Economics, 34*(6), 639–661.
Massey, D. S., & Denton, N. A. (1993). *American apartheid: Segregation and the making of the underclass*. Cambridge, MA: Harvard University Press.
Mckenzie, E. (1994). *Privatopia: Homeowner associasions and the rise of residential private government*. New Haven Co., USA: Yale Univeristy Press.
Nelson, A. C., Sanchez, T. W., et al. (2004). The effect of urban containment and mandatory housing elements on racial segregation in US Metropolitan Areas, 1990–2000. *Journal of Urban Affairs, 26*(3), 339–350.
North, D. (1990). *Institutions, Institutional Change and Economic Performance*. Cambridge: Cambridge University Press.
Odland, J. (1988). *Spatial autocorrelation*. Beverly Hills, CA, USA: Sage.
Pancs, R., & Vriend, N. J. (2007). Schelling's spatial proximity model of segregation revisited. *Journal of Public Economics, 91*(1–2), 1–24.
Pigou, A. (1920). *The economics of welfare*. London: Macmillan.
Rittel, H., & Webber, M. (1973). Dilemmas in a general theory of planning. *Policy Sciences, 4*, 155–169.
Rothwell, J., & Massey, D. (2009). The effects of density zoning on racial segregation in US urban areas. *Urban Affairs Review, 44*(6), 779–806.
Sager, L. G. (1969). Tight Little Islands: Exclusionary zoning, equal protection, and the indigent. *Stanford Law Review, 21*(4), 767–800.
Schelling, T. C. (1971). Dynamic models of segregation. *Journal of Mathmatical Sociology, 1*, 143–186.
Schelling, T. C. (1978). *Micromotives and macrobehaviors*. New York, NY, USA: W.W. Norton & Company.
Schon, D. (1983). *The reflective practitioner: How professionals think in action*. New York, USA: Basic Books.
Seitles, M. (1998). The Perpetuation of Residential Racial Segregation in America: Historical Discrimination, Modern Forms of Exclusion, and Inclusionary Remedies. *Journal of Land Use and Environmental Laws, 14*, 89.
Squires, G. D., & Kubrin, C. E. (2005). Privilged Places: Race, Uneven Development and the Geography of Opportunity in Urban America. *Urban Studies, 24*(1), 47–68.
Squires, G. D., & O'Connor, S. (2001). *Color and Money: Politics and Prospects for Community Reinvestment in Urban America*. Albany: NY State University of New York Press.
Teaford, J. C. (1997). *Post-Suburbia: Government and Politics in the Edge Cities*. Baltimore, ML: Johns Hopkins University Press.
Teknomo, K. (2001). CAM by Kardi Teknomo. <http://www.planetsourcecode.com/vb/scripts/ShowCode.asp?txtCodeId=26432&lngWId=1> Accessed 01.09.05.
Wang, Y. (2011). White Flight in Los Angeles County, 1960–1990: a Model of Fuzzy Tipping. *Annals of Regional Science, 47*(1), 111–129.
Webster, C., & Wu, F. (2001). Coase, Spatial Pricing and Self-Organizing Cities. *Urban Studies, 38*(11), 2037–2054.
Xiang, X., et al. (2005). Verification and validation of agent-based scientific simulation models. In *Proceedings of the 2005 agent-directed simulation symposium*.
Zhang, J. (2011). Tipping and Residential Segregation: a Unified Schelling Model. *Journal of Regional Science, 51*(1), 167–193.