# SWIPT in MISO Multicasting Systems

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*Abstract*—This letter considers simultaneous wireless information and power transfer (SWIPT) in multiple-input single-output (MISO) multicasting systems where each receiver is equipped with a power splitting device and can receive both information and energy from the base station (BS) continuously at the same time. We investigate the joint multicast transmit beamforming and receive power splitting problem for minimizing the transmit power of the BS subject to signal-to-noise ratio (SNR) and energy harvesting constraints at each receiver. Both scenarios of perfect and imperfect channel state information (CSI) at the BS are studied. Due to non convexity of the problems, we use semidefinite relaxation (SDR) technique to solve the problems. Interestingly, we show that the SDR is in fact tight in certain scenarios.

Index Terms—SWIPT, multicasting, beamforming, power splitting.

# I. INTRODUCTION

**S** INCE signals that carry energy can transport information at the same time, simultaneous wireless information and power transfer (SWIPT) has become an interesting new area of research and drawn upsurge of interest [1]–[5]. Through SWIPT, mobile users are provided with access to both energy and data at the same time which brings enormous prospects.

The concept of SWIPT was first introduced in [1], in which the fundamental tradeoffs between the rates at which energy and reliable information can be transmitted over a single noisy line were characterized. The work in [1] was later extended to frequency-selective channels in [2]. However, it was assumed in [1], [2] that the receiver is able to decode information and extract power simultaneously from the same received signal, which is not quite the case in practical designs.

To allow SWIPT at the receiver side, two practical schemes, namely, time switching (TS) and power splitting (PS) were proposed recently in [3], [4]. The scenario in [3] was broad-casting from a base station (BS) to two receivers taking turns for information decoding and energy harvesting (i.e., using TS). Though the scheme in [3] simplifies the receiver design, it compromises the efficiencies of SWIPT, and motivates the more practical PS architecture in [4]. Multi-antenna techniques can also be applied for increasing the wireless power transfer efficiency in SWIPT systems [3]. The work in [3] has also been extended to the case imperfect channel state information (CSI) at the transmitter in [5]. More recently in [6], multiple energy harvesting nodes were considered for the imperfect CSI case. However, the limitation is that each receiver either decodes information or harvests energy, but not both.

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The existing literature on SWIPT considered either point-topoint or point-to-multipoint systems with dedicated information for each of the receivers. However, in many practical communication systems, one transmitter needs to send a common message to a group of receivers simultaneously. These systems are referred to as multicasting systems [7], [8], which triggered great research interest due to the increasing demand for mobile applications such as streaming media, software updates, and location-based services involving group communications.

In this letter, we consider SWIPT in multiple-input singleoutput (MISO) multicasting systems where each mobile station (MS) has a power splitter that can receive both information and energy from the multi-antenna BS continuously at all time. Multicast transmit beamforming and adaptive receive power splitting is considered. Our aim is to minimize the BS transmit power while maintaining the signal-to-noise ratio (SNR) and energy harvesting thresholds at each MS, by jointly optimizing the beamforming vector at the BS and the MS power splitting parameters. Both perfect and imperfect CSI cases at the BS are considered and the problems are addressed using semidefinite relaxation (SDR) techniques. Remarkably, we show that SDR is tight in certain scenarios. Simulations are carried out to demonstrate the performance of the proposed algorithms for the multicast model.

## **II. SYSTEM MODEL**

Let us consider a MISO multicasting system with K receivers as illustrated in Fig. 1. It is assumed that the transmitter is equipped with  $N_{\rm s} > 1$  antennas and each receiver has single receiving antenna. All the receivers intend to simultaneously decode information and harvest energy from the received signal. We assume linear transmit precoding at the transmitter. Thus, the received signal at the kth receiver is given by

$$y_k = \mathbf{h}_k^H \mathbf{b}s + n_{\mathrm{A},k}, \text{ for } k = 1, \dots, K, \tag{1}$$

where  $\mathbf{h}_k$  is the conjugated complex channel vector between the transmitter and the *k*th receiver, **b** is the transmit beamforming vector, *s* is the transmitted data stream, and  $n_{A,k}$  is the additive Gaussian noise at the *k*th receiver's antenna.

In this letter, it is assumed that each receiver is equipped with a PS device to coordinate the processes of information decoding and energy harvesting from the received signal. In particular, the received signal at the *k*th receiver is split such that an  $\rho_k \in [0, 1]$  portion of the signal power is fed to the information decoder (ID) and the remaining  $1 - \rho_k$  portion of the power to the energy harvester (EH) of the receiver. Thus the signal split to the ID of the *k*th receiver is given by

$$y_{I,k} = \sqrt{\rho_k} \left( \mathbf{h}_k^H \mathbf{b} s + n_{A,k} \right) + n_{P,k}, \text{ for } k = 1, \dots, K,$$
 (2)

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Fig. 1. A MISO SWIPT multicasting system.

where  $n_{P,k}$  is the additional processing noise at the ID of the kth receiver. Also, the signal split to the EH is expressed as

$$y_{\mathrm{E},k} = \sqrt{1 - \rho_k} \left( \mathbf{h}_k^H \mathbf{b} s + n_{\mathrm{A},k} \right), \text{ for } k = 1, \dots, K.$$
 (3)

Accordingly, the SNR of the ID at the kth receiver is

$$\Gamma_k = \frac{\rho_k \mathbf{h}_k^H \mathbf{b} \mathbf{b}^H \mathbf{h}_k}{\rho_k \sigma_{\mathrm{A},k}^2 + \sigma_{\mathrm{P},k}^2}, \text{ for } k = 1, \dots, K,$$
(4)

and the power harvested by the EH is given by

$$\Upsilon_k = \xi_k (1 - \rho_k) \left( \mathbf{h}_k^H \mathbf{b} \mathbf{b}^H \mathbf{h}_k + \sigma_{\mathbf{A},k}^2 \right), \text{ for } k = 1, \dots, K,$$
(5)

in which  $\xi_k \in (0, 1]$  is the energy conversion efficiency of the EH at the *k*th receiver. For convenience, we assume, without loss of generality, that  $\xi_k = 1 \forall k$  in this letter.

Several PS schemes have been considered in the literature including i) uniform PS (UPS) with  $\rho_k = \frac{1}{2}, \forall k$ ; ii) on-off PS (OOPS) with  $\rho_k \in \{0, 1\}, \forall k$ , meaning that the receiving nodes can switch between the EH and ID modes, commonly referred to as TS, or *binary PS*; and iii) optimal PS (OPS) (our aim). Note that UPS and OOPS can be regarded as special forms of OPS, and thus in general OPS achieves better rateenergy transmission trade-offs than UPS and OOPS [3], [4].

## III. MULTICAST BEAMFORMING WITH PERFECT CSI

In this section, we address the joint multicast beamforming and receive PS problem assuming that the instantaneous CSI is perfectly known at the BS transmitter. We consider the OPS scheme that allows each receiver to decode information and harvest energy simultaneously.

To ensure a continuous information transfer, each ID needs its received SNR to be above a given threshold at all times. Also, the harvested power at each EH should also be above a given threshold so that a useful level of harvested energy is reached. Hence, we formulate the joint transmit beamformer and receive PS ratio optimization problem as a minimization problem of the total transmit power of the BS, i.e.,

$$\min_{\mathbf{b}, \{0 < \rho_k < 1\}_{\forall k}} \mathbf{b}^H \mathbf{b} \quad \text{s.t.}$$
(6a)

$$\frac{p_k \mathbf{h}_k^H \mathbf{b} \mathbf{b}^H \mathbf{h}_k}{k \sigma_{\mathrm{A},k}^2 + \sigma_{\mathrm{P},k}^2} \ge \gamma_k, \ \forall k, \qquad (6b)$$

$$(1-\rho_k)\left(\mathbf{h}_k^H\mathbf{b}\mathbf{b}^H\mathbf{h}_k+\sigma_{\mathbf{A},k}^2\right) \ge \eta_k, \forall k.$$
 (6c)

Here  $\gamma_k > 0$  and  $\eta_k > 0$  are the minimum protection ratios of the ID and EH at the *k*th receiver. According to [9, Theorem III], the problem is feasible if and only if the SNR targets satisfy the condition  $\sum_{k=1}^{K} \frac{\gamma_k}{1+\gamma_k} \leq \operatorname{rank}(\mathbf{H})$ , where  $\mathbf{H} \triangleq$ 

 $[\mathbf{h}_1, \dots, \mathbf{h}_K]$ . The problem is non-convex due to the coupled beamforming vector **b** and PS ratios  $\{\rho_k\}$  in (6b) and (6c). However, defining  $\mathbf{X} \triangleq \mathbf{b}\mathbf{b}^H$ , (6) can be reformulated as

$$\min_{\mathbf{X} \succeq \mathbf{0}, \{0 < \rho_k < 1\}_{\forall k}} \operatorname{trace}(\mathbf{X}) \quad \text{s.t.}$$
(7a)

$$\mathbf{h}_{k}^{H} \mathbf{X} \mathbf{h}_{k} \geq \gamma_{k} \left( \sigma_{\mathrm{A},k}^{2} + \frac{\sigma_{\mathrm{P},k}^{2}}{\rho_{k}} \right), \ \forall k, \qquad (7b)$$

$$\mathbf{h}_{k}^{H} \mathbf{X} \mathbf{h}_{k} \geq \frac{\eta_{k}}{(1-\rho_{k})} - \sigma_{\mathrm{A},k}^{2}, \ \forall k, \tag{7c}$$

$$\operatorname{ank}\left(\mathbf{X}\right) \leq 1.$$
 (7d)

The problem is still non-convex due to the rank constraint (7d). Hence, we drop the rank constraint and formulate the relaxed problem as a semidefinite programming (SDP) problem:

 $\mathbf{r}$ 

$$\min_{\mathbf{X} \succeq \mathbf{0}, \{0 < \rho_k < 1\} \forall k} \operatorname{trace}(\mathbf{X}) \quad \text{s.t.}$$
(8a)

$$\mathbf{h}_{k}^{H} \mathbf{X} \mathbf{h}_{k} \geq \gamma_{k} \left( \sigma_{\mathrm{A},k}^{2} + \frac{\sigma_{\mathrm{P},k}^{2}}{\rho_{k}} \right), \forall k, \qquad (8b)$$

$$\mathbf{h}_{k}^{H} \mathbf{X} \mathbf{h}_{k} \geq \frac{\eta_{k}}{(1-\rho_{k})} - \sigma_{\mathbf{A},k}^{2}, \forall k.$$
(8c)

Now the problem (8) is convex and can be efficiently solved by the disciplined convex programming toolbox CVX [10], where interior-point method-based solvers such as SeDuMi or SDPT3 are called internally, at a complexity cost that is at most  $O\left((N_s^2 + 2K)^{3.5}\right)$  [11] and is usually much less.

Due to the relaxation, any optimal solution  $X_{opt}$  obtained by solving (8) is not necessarily rank one in general. If it is, its principal eigenvector is the optimal solution to the original problem (6). Otherwise, one may need to apply some alternative technique to obtain a near-optimal solution. Researchers in the optimization community have long recognized the value of rank relaxation for obtaining approximate solutions to hard nonconvex problems, and have developed suitable procedures for converting the solution of the relaxed problem into an approximate solution of the original problem.

Fortunately, recent results on Hermitian matrix rank-one decomposition techniques [12] can obtain exact optimal solution to (6) in certain scenarios. The following lemma defines the necessary conditions for the existence of a rank-one matrix solution to a system of four linear matrix equations.

Lemma 1: [12, Theorem 2.3] For  $n \ge 3$ , let  $\mathbf{A}_1, \mathbf{A}_2, \mathbf{A}_3$ ,  $\mathbf{A}_4 \in \mathcal{H}^n$  denote complex Hermitian matrices,  $\mathbf{X} \in \mathcal{H}^n_+$  be a nonzero Hermitian positive semidefinite matrix of rank r, and for two complex matrices  $\mathbf{Y}$  and  $\mathbf{Z}$ , their inner product is defined as  $\mathbf{Y} \bullet \mathbf{Z} = \operatorname{Re} (\operatorname{trace}(\mathbf{Y}^H \mathbf{Z}))$ . Suppose that

$$(\mathbf{A}_1 \bullet \mathbf{Y}, \mathbf{A}_2 \bullet \mathbf{Y}, \mathbf{A}_3 \bullet \mathbf{Y}, \mathbf{A}_4 \bullet \mathbf{Y}) \neq (0, 0, 0, 0), \quad (9)$$

for any nonzero matrix  $\mathbf{Y} \in H^n_+$ . If  $r \ge 3$ , then one can find in polynomial-time a nonzero vector  $\mathbf{y} \in \operatorname{range}(\mathbf{X})$  such that

$$\mathbf{A}_i \bullet \mathbf{y}\mathbf{y}^H = \mathbf{A}_i \bullet \mathbf{X}, \text{ for } i = 1, \dots, 4.$$
 (10)

If r = 2, for any  $\mathbf{z} \notin \text{range}(\mathbf{X})$ , there exists  $\mathbf{y}$  in the linear subspace spanned by  $\mathbf{z}$  and  $\text{range}(\mathbf{X})$ , such that (10) holds.

Based on *Lemma 1*, we describe the following proposition to obtain a rank-one solution to problem (8), and hence to (6) as well, with up to four linear matrix constraints.

**Proposition 1:** For  $N_{\rm s} \geq 3$ , if  $\operatorname{rank}(\mathbf{X}_{\rm opt}) \geq 3$  and  $\operatorname{trace}(\mathbf{X}_{\rm opt}\mathbf{h}_k\mathbf{h}_k^H) \neq 0$ , for k = 1, 2, then there always exists a rank-one solution  $\mathbf{b}_{\rm opt}$  for  $\mathbf{X}_{\rm opt}$  such that  $\mathbf{b}_{\rm opt} \in \operatorname{range}(\mathbf{X}_{\rm opt})$ . In addition, if  $\operatorname{rank}(\mathbf{X}_{\rm opt}) = 2$ , then for any  $\mathbf{z} \notin \operatorname{range}(\mathbf{X}_{\rm opt})$ , there exists a rank-one solution  $\mathbf{b}$  in the linear subspace spanned by  $\mathbf{z}$  and  $\operatorname{range}(\mathbf{X}_{\rm opt})$ . Both  $N_{\rm s} \geq 3$  and  $\operatorname{trace}(\mathbf{X}_{\rm opt}\mathbf{h}_k\mathbf{h}_k^H) \neq 0$ , for  $k = 1, \ldots, K$ , are the necessary conditions for a rank-one solution to exist.

**Proof:** Similar to that of [12, Theorem 2.3]. The detailed procedure for obtaining such a rank-one decomposition of  $\mathbf{X}_{opt}$  can be obtained in [12]. For problems with more than four linear matrix constraints, we may resort to alternative techniques such as randomization to obtain a (suboptimal) **b** from  $\mathbf{X}_{opt}$  [7]. Note that the beamforming vector obtained by randomization may not satisfy some of the constraints in the original problem (6) with equality. However, a feasible transmit beamforming vector can be computed by simply scaling **b** so that all the constraints are satisfied.

#### **IV. ROBUST MULTICAST BEAMFORMING**

The assumption of perfect CSI in Section III is not always practical due to the time-varying nature of wireless propagation channels and the mobility of the users. Therefore, in this section, we develop a robust algorithm for joint beamforming and PS ratio optimization in the case of erroneous CSI which uses the concept of worst-case design.

We consider a deterministic model for the imperfect CSI case. In particular, we assume that the actual channels  $\{\mathbf{h}_k\}$  lie in the neighbourhood of the estimated channels  $\{\hat{\mathbf{h}}_k\}$  available at the BS. Hence, the actual channels are modeled as

$$\mathbf{h}_k = \mathbf{h}_k + \Delta \mathbf{h}_k, \text{ for } k = 1, \dots, K, \tag{11}$$

in which  $\Delta \mathbf{h}_k$  represents the channel uncertainties, which are assumed to be bounded such that

$$\|\Delta \mathbf{h}_k\|_2 = \|\mathbf{h}_k - \hat{\mathbf{h}}_k\|_2 \le \varepsilon_k, \text{ for some } \varepsilon_k \ge 0, \qquad (12)$$

where  $\varepsilon_k$  depends on the accuracy of the CSI estimates.

As such, the robust formulation problem (6) becomes

$$\min_{\mathbf{b}, \{0 < \rho_k < 1\} \forall k} \|\mathbf{b}\|_2^2 \quad \text{s.t.}$$
(13a)

$$\min_{\|\Delta \mathbf{h}_k\| \le \varepsilon_k} |\mathbf{b}^H(\mathbf{h}_k + \Delta \mathbf{h}_k)|^2 \ge \gamma_k \left(\sigma_{\mathrm{A},k}^2 + \frac{\sigma_{\mathrm{P},k}^2}{\rho_k}\right), \forall k,$$
(13b)

$$\min_{\|\Delta \mathbf{h}_k\| \le \varepsilon_k} |\mathbf{b}^H \left(\mathbf{h}_k + \Delta \mathbf{h}_k\right)|^2 \ge \frac{\eta_k}{(1 - \rho_k)} - \sigma_{\mathbf{A},k}^2, \forall k.$$
(13c)

It can be seen from (13b) and (13c) that both the SNR and the harvested power constraints are satisfied for all realizations of the channel error vectors  $\{\Delta \mathbf{h}_k\}$ . As a result, statistical information about the channel error vectors is not required in this approach, and the minimal knowledge of the upper-bound of channel error vector norms is sufficient.

To simplify (13), we modify the inequality constraints (13b) and (13c) using an approach similar to the one developed in [13]. From the triangle inequality, it follows that

$$|\mathbf{b}^{H}(\mathbf{h}_{k} + \Delta \mathbf{h}_{k})| \ge |\mathbf{b}^{H}\mathbf{h}_{k}| - |\mathbf{b}^{H}\Delta \mathbf{h}_{k}|.$$
(14)

Here, we assume that  $|\mathbf{b}^H \mathbf{h}_k| \ge |\mathbf{b}^H \Delta \mathbf{h}_k|$  which essentially means that the errors  $\{\Delta \mathbf{h}_k\}$  are sufficiently small.

Applying the Cauchy–Schwarz inequality, we have

$$|\mathbf{b}^{H}\Delta\mathbf{h}_{k}| \leq \|\mathbf{b}\|_{2} \|\Delta\mathbf{h}_{k}\|_{2} \leq \varepsilon_{k} \|\mathbf{b}\|_{2},$$
(15)

where  $\|\Delta \mathbf{h}_k\| \leq \varepsilon_k$  has been used. Thus we obtain

$$\max_{\Delta \mathbf{h}_k \parallel \le \varepsilon_k} |\mathbf{b}^H \Delta \mathbf{h}_k| = \varepsilon_k \|\mathbf{b}\|_2.$$
(16)

Substituting (16) back into (14), we obtain

$$\min_{\|\Delta \mathbf{h}_k\| \le \varepsilon_k} |\mathbf{b}^H \mathbf{h}_k + \Delta \mathbf{h}_k|^2 \ge \left( |\mathbf{b}^H \mathbf{h}_k| - \varepsilon_k \|\mathbf{b}\|_2 \right)^2.$$
(17)

Expanding the right-hand-side of (17), we have

$$( |\mathbf{b}^{H}\mathbf{h}_{k}| - \varepsilon_{k} ||\mathbf{b}||_{2})^{2}$$

$$= |\mathbf{b}^{H}\mathbf{h}_{k}|^{2} + \varepsilon_{k}^{2} ||\mathbf{b}||_{2}^{2} - 2\varepsilon_{k} ||\mathbf{b}||_{2} ||\mathbf{b}^{H}\mathbf{h}_{k}|$$

$$\ge |\mathbf{b}^{H}\mathbf{h}_{k}|^{2} + \varepsilon_{k}^{2} ||\mathbf{b}||_{2}^{2} - 2\varepsilon_{k} ||\mathbf{b}||_{2}^{2} ||\mathbf{h}_{k}||_{2}$$

$$= |\mathbf{b}^{H}\mathbf{h}_{k}|^{2} + \varepsilon_{k}(\varepsilon_{k} - 2||\mathbf{h}_{k}||_{2}) ||\mathbf{b}||_{2}^{2} = \mathbf{b}^{H}\tilde{\mathbf{H}}_{k}\mathbf{b},$$
(18)

where  $\tilde{\mathbf{H}}_k \triangleq \mathbf{h}_k \mathbf{h}_k^H + \varepsilon_k (\varepsilon_k - 2\sqrt{\mathbf{h}_k^H \mathbf{h}_k}) \mathbf{I}_{N_s}$ . Consequently, the left-hand-sides of (13b) and (13c) are lower bounded by

$$\min_{\|\Delta \mathbf{h}_k\| \le \varepsilon_k} |\mathbf{b}^H \mathbf{h}_k + \Delta \mathbf{h}_k|^2 \ge \mathbf{b}^H \tilde{\mathbf{H}}_k \mathbf{b}.$$
 (19)

Using the above results, (13) can be simplified as

$$\min_{\mathbf{b}, \{0 < \rho_k < 1\} \forall k} \|\mathbf{b}\|_2^2 \quad \text{s.t.}$$
(20a)

$$\mathbf{b}^{H}\tilde{\mathbf{H}}_{k}\mathbf{b} \geq \gamma_{k}\left(\sigma_{\mathrm{A},k}^{2} + \frac{\sigma_{\mathrm{P},k}^{2}}{\rho_{k}}\right), \forall k, \qquad (20b)$$

$$\mathbf{b}^{H}\tilde{\mathbf{H}}_{k}\mathbf{b} \geq \frac{\eta_{k}}{(1-\rho_{k})} - \sigma_{\mathrm{A},k}^{2}, \forall k.$$
(20c)

Similar to the perfect CSI case before, (20) can be relaxed to a convex problem applying the so-called SDR and the fact that  $\mathbf{b}^H \tilde{\mathbf{H}}_k \mathbf{b} = \text{trace}(\mathbf{b}\mathbf{b}^H \tilde{\mathbf{H}}_k)$ . Therefore, we have

$$\min_{\mathbf{X} \succeq \mathbf{0}, \{0 < \rho_k < 1\}_{\forall k}} \operatorname{trace}(\mathbf{X}) \quad \text{s.t.}$$
(21a)

trace
$$(\mathbf{X}\tilde{\mathbf{H}}_k) \ge \gamma_k \left(\sigma_{\mathrm{A},k}^2 + \frac{\sigma_{\mathrm{P},k}^2}{\rho_k}\right), \forall k,$$
 (21b)

trace
$$(\mathbf{X}\tilde{\mathbf{H}}_k) \ge \frac{\eta_k}{(1-\rho_k)} - \sigma_{\mathbf{A},k}^2, \forall k,$$
 (21c)

where  $\mathbf{X} \triangleq \mathbf{bb}^{H}$  is introduced. This SDP problem is convex and can be efficiently solved by interior point methods. The tightness conditions for (8) also apply to (21) and identical techniques can be used to obtain rank-one approximations.

## V. SIMULATION RESULTS

In this section, we study the performance of the proposed algorithms in MISO SWIPT systems through numerical simulations. We assume that there are K = 3 single-antenna MSs simultaneously harvesting energy and decoding the same information. For simplicity, it was considered that  $\gamma_k = \gamma$ ,  $\forall k$ ,  $\eta_k = \eta$ ,  $\forall k$ , and  $\varepsilon_k = \varepsilon$ ,  $\forall k$ . We simulated a flat Rayleigh fading environment where the channel vectors have entries with zero mean and variance  $1/N_{\rm s}$ . Both perfect and imperfect CSI cases were evaluated. For the case of imperfect CSI, the error vector is uniformly and randomly generated in a sphere



Fig. 2. Transmission power versus target SNR  $\gamma$  for the case with  $N_{\rm s}=5$  and K=3.



Fig. 3. Transmission power versus energy threshold  $\eta$  for the case with  $N_{\rm s}=5$  and K=3.

centered at zero with the radius  $\varepsilon = 0.001$ . All simulation results were averaged over 500 independent channel realizations.

In Fig. 2, we investigate the performance of the proposed non-robust (with perfect CSI at the BS) and the robust algorithms versus the SNR threshold  $\gamma$  (dB) with various fixed harvested power constraint  $\eta$  (dB). We see that the BS needs more power with the increase in the required SNR threshold for both non-robust and robust algorithms. Also, the increased harvested power constraints demand more power to be transmitted. In all cases, the robust algorithm achieves comparable performance with the perfect CSI algorithm. However, as  $\gamma$  increases, their performance gaps become insignificant.

Results in Fig. 3 are provided for the same setting, but with the transmission power against the energy harvesting threshold  $\eta$  (dB) with various fixed SNR constraint  $\gamma$  (dB). As we can see, the required transmit power increases with the increase in the required harvested power threshold for both non-robust and robust algorithms. Also, the increased SNR constraints impose more transmitting power to be applied. Thus the transmission power requirement versus the harvested power constraint is comparable to that versus the SNR protection threshold.

In the last figure, we analyze the PS ratios between the EH and ID receivers. Fig. 4 illustrates the value of  $\rho_k$  (where k is randomly chosen from 1, 2, ..., K) versus the SNR protection threshold  $\gamma$  (dB) with fixed harvested power constraint  $\eta$  (dB). Interestingly, the robust algorithm puts less power to the ID receiver compared to the non-robust algorithm. One possible



Fig. 4. Receive PS ratio versus energy threshold  $\gamma$  for the case with  $N_{\rm s}=5$  and K=3.

reason is that the ID receiver is more sensitive to channel uncertainties compared to the EH receiver and hence requires less power when robust design is considered.

## VI. CONCLUSIONS

This letter studied multicasting MISO systems for SWIPT and proposed transmit beamforming and PS ratio optimization algorithms for both perfect and imperfect CSI cases, utilizing SDR techniques. We showed that the relaxation is in fact tight in some particular scenarios. Simulation results are provided to demonstrate the effectiveness of the proposed algorithms.

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