A Dynamic Model of International Trade

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A thesis submitted for the degree of
Doctor of Philosophy

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June 2014

I, Hiral Patel confirm that the work presented in this thesis is my own. Where information is derived from other sources, I confirm that this has been indicated in the thesis.

Abstract

This thesis studies an international trade game modelled as a coordination game, where countries interact by following a simple behavioural rule of trying to reduce the gap between the maximal payoff and their own payoff. Countries are allowed to choose from one of two strategies – E (integrate into the world economy) or A (remain in autarky). D'Artigues and Vignolo [33] find that in this coordination game, the desire to imitate the leading country is frustrated by the impossibility of doing so. Hence, paradoxically, the desire of convergence into the world economy may lead to a more partitioned world economy. Envy is found to be the central motive behind developing countries opting for rejection of globalisation, and with sufficiently large heterogeneity in payoffs, spiteful behaviour overcomes imitation. The presence of envious people in a class of symmetric coordination games helps players to coordinate on a particular strict Nash equilibrium, which is always risk-dominant. This thesis first analyses the trade dynamics for a simple 2-country and a 3-country model and then derives conclusions on the more general n-country model. Results for the first trade model are based on the model output obtained for strict equilibria and strictly stable states of the trade game via numerical computations. This model is then extended by allowing countries to choose intermediate strategies, so that they can choose the level of integration with the world economy by a small strategy size $\delta > 0$. The trade dynamics for this continuous strategy trade game model are then derived for the 2-country scenario, which suggest that the lagging country can catch up with the leading country, leading to non-existence of any long-term equilibria.

Dedication

I would like to dedicate this thesis to my first supervisor, the late Prof. Rob Seymour, this thesis would not have been possible without his valuable guidance and support; and my beloved grandmother, the late Mrs. Pushpaben Babubhai Patel, this thesis would not have been possible without her blessings.

Acknowledgements

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Chapter 1

Introduction and Overview

World trade has been increasing over the centuries via discovery of trade routes and the advancements in communications, technology and transport. Globalization[1] began in the earliest nineteenth century driven by declines in transportation costs. While there seem to have been many advantages of economic interdependence such as efficient use of resources and reduction in international conflict, numerous criticisms have been made of globalization such as growing poverty and environmental damage; see Miles and Scott [88]. Theorists like Montesquieu and Immanuel Kant asserted that economic relations between nations pacify political interaction. According to Gartzke, Li and Boehmer [54], multiple studies, many identified with democratic peace, link interstate trade with reduction in militarized disputes or wars. However, there have been fears of growing inequality and poverty – Deardorff [35] uses his model to show that poor countries converge to a low steady state while rich countries converge to a high one[2]. Mountford [92] suggests international trade can cause the world economy to sort itself out into groups of fast and slow growing economies, while Sutcliffe [129] finds that overall inequality seems to have fallen while the ratio of the extremes have risen over the 20th century. Sala-i-Martin [114] also finds that global income inequality (reduction in the across-

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[1] In economic terms, globalisation occurs when barriers between national markets (for goods and services, capital or labour) disappear and a single market with a single price is created.

[2] The high and low steady states refer to the level of savings out of wages for the countries. Convergence to a high steady state means that the countries earn high savings out of wages, making them richer while convergence to a low steady state means that the countries earn low savings out of wages, making them poorer.

country disparities) has reduced in the recent years. Jaumotte, Lall and Papageorgiou [68] state that trade globalization is associated with a reduction in inequality whereas financial globalization is associated with an increase.

On average, trade liberalisation increases GDP growth, but not always (see Wacziarg and Welch [136]). According to Miles and Scott [88], econometric evidence suggests that increased openness to trade does raise the long-run rate of economic growth, although it is difficult to accurately isolate the impact that trade policy alone has on GDP, and although on average trade liberalization boosts growth, success is not guaranteed. Rodrik [110] says 'Globalization's soft underbelly is the imbalance between the national scope of governments and the global nature of markets. A healthy economic system necessitates a delicate compromise between these two. Go too much in one direction and you have protectionism and autarky. Go too much in the other and you have an unstable world economy'. Therefore, nations need a balance between integrating into the world economy and fostering trade within the nation. This thesis investigates whether it is advantageous for countries to fully integrate into the world economy, remain in autarky or to find a balance between the two.

Evolutionary games have considerable potential for modelling substantiative economics issues, which tells the applied economist what sorts of behaviour can evolve over time (see Friedman [50]). International trade and cooperation have been modelled as coordination games - even though they cannot capture all the complexities of trade between countries and economic growth, yet they can provide valuable insights into how countries interact with one another and how it affects international cooperation. Snidal [124] compares models of Prisoner's Dilemma games and Coordination games and finds that the latter better characterizes the issues between international political regimes and international cooperation.

This thesis studies an international trade game modelled as a coordination game as considered in D'Artigues and Vignolo [33], where countries interact by following a simple behavioural rule of trying to reduce the gap between the game's maximal payoff and their own payoff. In the first model, countries are allowed to choose

from one of two strategies – *E* (integrate into the world economy) or *A* (remain in autarky). D'Artigues and Vignolo [33] find that in this coordination game, the desire to imitate the leading country is frustrated by the impossibility of doing so. Hence, paradoxically, the desire of convergence into the world economy may lead to a more partitioned world economy. Envy is the central motive behind developing countries opting for rejection of globalisation, and with sufficiently large heterogeneity in payoffs, spiteful behaviour overcomes imitation. In his model, Vignolo [134] finds that the presence of envious people in a class of symmetric coordination games helps players to coordinate on a particular strict Nash equilibrium, which is always risk-dominant. This thesis first attempts to find the trade dynamics for a simple 2-country and a 3-country model and then derives conclusions on the more general *n*-country model. Numerical computations are then used to obtain the model output and subsequently the strict equilibria and the strictly stable states of the trade game are analysed. This model is then extended beyond the D'Artigues-Vignolo model by allowing countries to choose intermediate strategies, so that they can change the level of integration with the world economy by a small strategy size $\delta > 0$. The trade dynamics for this continuous strategy trade game model are then derived for the 2-country scenario.

The rest of this thesis is organised as follows:

Chapter 2 explores the sociological background of globalization and the economics behind international trade, degree of openness and economic growth. Globalization advantages and disadvantages are reviewed in order to find the winners and the losers of international trade. International policies that can be implemented in order for countries to gain from world trade are then discussed. The literature concerning international trade modelled as co-ordination games is reviewed and the long run equilibria of coordination games in general is also briefly discussed.

Chapter 3 re-defines the D'Artigues-Vignolo trade game modelled as a 2×2 coordination game. The game dynamics for the 2-country and the 3-country models are analysed and represented graphically, highlighting the possible strict equilibria and strictly stable states. The 4-country model is also briefly discussed, highlighting the

states that cannot be strict equilibria, with the rest of the states left to be explored via numerical computations in Chapter 4. For the more general n-country model, the notions of strict equilibria and strict stability are formalized and the complex conditions required to hold true for the homogeneous as well the heterogeneous states to be equilibria and strictly stable states are obtained.

In Chapter 4, a mathematical model is constructed and implemented in Mathematica and the model output obtained for the existence of strict equilibria and strictly stable states over a varied range of parameter values, for different models (ranging from the 3-country model to the 200-country model). The results for the 3-country and the 4-country models are graphically presented to analyse the likelihood of the varied heterogeneous states being strict equilibria. For $n > 4$, the model output is obtained for the fully integrated states (where all the countries are open to international trade and play strategy E) and the completely autarkic states (where none of the countries participate in world trade and choose to remain in autarky by playing strategy A). The results for up to 200 countries are graphically presented and then analysed to draw conclusions for the model.

Chapter 5 extends the trade game model considered in Chapter 3 to allow for intermediate strategies, so that countries can choose their level of integration into world trade. Instead of choosing between the fully integrated or completely autarkic strategies, countries are allowed to change their strategy by a small strategy size, $\delta > 0$. Unlike the previous model in Chapter 3, the countries also have varying degrees of openness to trade with varying countries, which is determined by the trade weights matrix. The complex dynamics for the fully integrated state, the all-in-autarky state and the heterogeneous states are analysed.

Chapter 6 analyses the 2-country trade game with strategy size δ. Based on the trade weights matrix and the payoffs to the countries, the dynamics obtained are presented graphically. The analysis suggests that the leadership of countries can show cyclic behaviour, where one country can overtake another country, and vice versa.

Chapter 7 concludes the thesis, highlighting the important results from both the models discussed. The first model predicts a greater likelihood of countries not participating in world trade but choosing to remain in autarky, possibly because of envious behaviour, which arises due to competition between countries and the lagging countries desire of imitating the leading country. The second model predicts that the lagging country can catch up with and overtake the leading country, which lends plausibility to the future world economy predictions where economies like China and India can overtake economies like the US and the UK. The thesis ends by discussing possible applications of analysis and results to other games and suggesting areas of further research.

Finally, note that the more technical material of many chapters is relegated to the appendices. These appear at the end of the corresponding chapters.

Chapter 2

Literature Review

Globalization refers to the way in which national economies are becoming increasingly interconnected with one another. In economic terms, barriers between national markets (whether for goods and services, or capital, or labour) disappear and a single market with a single price is created. This chapter reviews the relevant literature on globalization and lays the foundation of the trade game analysed in this thesis. Varied views on globalization are discussed along with predictions of the future world economy that tie up the results of the game model developed in the forthcoming chapters.

In the first section, some economic aspects of international trade are discussed and the literature on globalization is reviewed. First, a brief history of globalization is discussed – how trade between countries started and evolved over time. The effect of globalization on the world economies and the advantages and disadvantages of the removal of trade restrictions are then reviewed[3]. Finally, a forecast of the world economy in 50 years' time is analysed, taking into account potential global economic crises.

2.1 The Economics of International Trade

This section reviews various aspects surrounding the economics of international trade – from history of globalization to its advantages and disadvantages, and dis-

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[3] General ideas presented are taken from Miles and Scott [88], Portes [103] and Scott [119].

cusses policies for developed countries to make globalization work for all. The section ends with a brief insight into the future of international trade.

2.1.1 The History of Globalization

World trade has been increasing for centuries as explorers have discovered trade routes and the technology of transport has improved.

The first wave of globalization began in the early nineteenth century as transport costs fell, coming to an end in the early twentieth century due to World War I (WWI). After the end of WWI in 1918, trade resumed but collapsed dramatically during the early 1930's due to the Great Depression and protectionist measures. After World War II (WWII), most countries wanted to construct international institutions that would minimize the threat of conflict and foster international economic relations. Therefore, national governments coordinated to create international institutions such as the International Monetary Fund (IMF), the World Bank, and the General Agreement on Trade and Tariffs[4] (GATT). The resulting decline in trade tariffs initiated a second wave of globalization that accelerated in the 1980's and 1990's as increasing numbers of emerging markets adopted trade-oriented policies in an effort to boost their GDP growth. This lays the foundation for the trade game model developed in this thesis.

In 1990 total trade in goods and services (both exports and imports) amounted to 32% of GDP for Organization for Economic Cooperation and Development (OECD) economies[5] and 34% for emerging markets. By 2001, these numbers increased to 38% and 49%, respectively. In 1960, 15.6% of the countries in the world, representing 19% of its population, had open trade policies, in the sense

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[4] This was later transformed into the World Trade Organization (WTO).

[5] These economies include Australia, Austria, Belgium, Canada, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Iceland, Ireland, Italy, Japan, Korea, Luxembourg, Mexico, Netherlands, New Zealand, Norway, Poland, Portugal, Slovak Republic, Spain, Sweden, Switzerland, Turkey, United Kingdom and United States.

defined by Sachs and Warner [113][6]. In 2000, a total of 73% of the countries in the world, representing 47% of the world population[7], were open to international trade, see Figure 2.1. Therefore, an increasing number of world economies are opening up to international trade in order to raise their GDP.
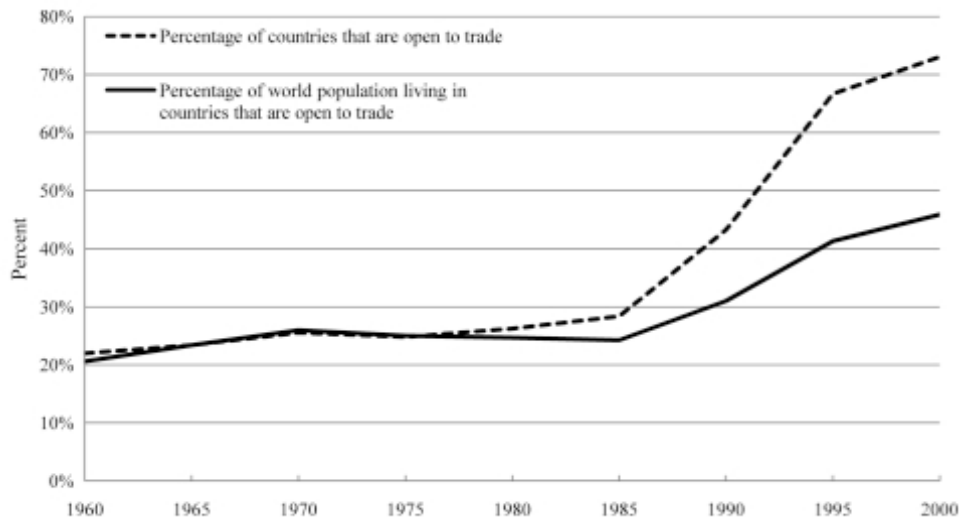
Figure 2.1. Increasing Globalization - Over the period 1960-2000, an increasing number of countries have opened up to trade (Source: Wacziarg and Welch [136]). Notes: Openness is defined according to Sachs and Warner [113] criteria (discussed later). The sample includes 141 countries.

In the 1950s, when many emerging markets were gaining independence, inward-looking strategies[8] dominated[9] to encourage the development of domestic indus-

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[6] A country is defined to have a closed trade policy if it has at least one of the following characteristics: non-tariff barriers covering 40% or more of trade; average tariff rates of 40% or more; a black market exchange rate that is depreciated by 20% or more relative to the official exchange rate.

[7] Wacziarg and Welch [136] find that the world's two most populated countries, China and India, remain closed as of 2000. This accounts for the discrepancy between the open trade countries and the percentage of world population living in open countries.

[8] Impose tariffs on manufactured imports and use revenue from primary exports to fund domestically oriented development, with extensive government intervention.

[9] For example, Latin American countries such as Argentina, Brazil and Mexico, Chile, Uruguay and Venezuela until the 1980s.

tries and *import substitution*[10]. However, import substitution did not work. Governments ended up paying too much in subsidies as the infant industry[11] argument was being applied to industries in all sectors rather than selectively to a few. This put a strain on fiscal finances and led to substantial rent seeking and inefficiencies in both industry and government. Governments had to extend *protectionism*[12] for less competitive industries as removing the subsidies and tariffs would cause the industry to suffer from global competition. Import substitution led to high capital goods prices and domestic firms having to use low-level technology. Protectionism resulted in large subsidies, low revenue, corruption and rent seeking, overvalued exchange rates that hindered exports. Once introduced, tariffs and controls were hard to abolish. Therefore, countries moved to externally oriented development programs following the evident success of South East Asia.

Starting in the 1970s, there was a huge shift towards outward-looking strategies. Brazil and other Latin American countries began liberalization of trade policies in the mid-1980s[13]. India on the other hand lowered its protectionist trade policies in the 1980s and experienced a fast growth rate[14]. Scott [119] finds that trade liberalization works as an important component of a wide range of policy reforms as a means of boosting GDP growth. Now international agencies advocate free trade and liberalization. For example, the World Bank aims to develop a global partner-

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[10] Substituting imports with domestically produced goods.

[11] Industries in developing and emerging markets that are protected against international competition by the governments, so that they can establish themselves.

[12] Protectionism refers to the notion of introduction of economic policies (such as tariffs on imported goods, restrictive quotas, etc.) by Governments to restrict international trade in order to protect the domestic economy. Protectionism can boost the domestic economy as the domestic firms have less competition due to a decrease in imported goods (as a direct result of tariffs imposed on imported goods). Unemployment rates also drop, as domestic firms require labour to produce goods that were previously being imported. However, protectionism implemented by one country may lead to retaliation from another country leading to hostility, which hampers the production of specialized goods in both countries, damaging their economies (see Altman [7]).

[13] However, Brazil did not show signs of growth after the 1990s due to lack of favourable macroeconomic and institutional environment (see Moreira [90]).

[14] However, the growth can be attributed to other factors too such as domestic consumption, increase in employment, and rise in investment.

ship for development amongst other goals, the IMF provides international monetary cooperation and facilitates expansion of balanced growth in trade, and the WTO leads global trade negotiations. Global integration now substitutes for development strategies. As increasing number of world economies open up to international trade, this thesis examines if engaging in world trade is equally beneficial to all the countries.

2.1.2 Advantages and Disadvantages of Globalization

Advantage - Economic Growth

Proponents of trade liberalization suggest emerging markets should adopt outward looking policies based around free trade, exports and encouraging inward Foreign Direct Investment (FDI). It has been claimed that globalization contributes to world peace by reducing the risk of war between nation states. For example, Montesquieu [7] asserts, "Commerce cures destructive prejudices". However, world trade was growing rapidly before WW1, but this did not prevent either WW1 or the Russian revolution of 1917. D'Artigues and Vignolo [33] also use their model to present terrorism as a result of competition between countries.

David Ricardo's theory of comparative advantage demonstrates that two nations without input factor mobility can produce more goods at lower costs through specialization and trade with one another than in isolation (see Formaini [48]). The theory of comparative advantage can be illustrated numerically as in Ricardo [108]. Consider two countries, England and Portugal, producing two commodities - wine and cloth, using labour as the sole input. England requires the labour of 100 men for a year to produce cloth and 120 men for a year to produce wine. Portugal on the other hand requires the labour of 80 men for a year to produce wine and 90 men for a year to produce cloth. It would be advantageous for England to produce cloth and import wine. Similarly, it would be advantageous for Portugal to produce wine and import cloth from England as it could get more cloth from England by exporting its wine produced by the labour than it could produce cloth on its own by diverting the wine producing labour to cloth production. Therefore, a country has a comparative

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advantage if it can produce that good at a lower *opportunity cost*[15] than another country. Thus, England would have the comparative advantage in cloth production relative to Portugal if it had to give up less wine to produce an extra unit of cloth than the amount of wine that Portugal would have to give up to produce an extra unit of cloth.

All countries can gain from the removal of trade restrictions if Ricardo's example above is extended to all countries[16]; for example see Figure 2.2. In other words, the international trade game is not a zero sum game. Countries can gain as long as they produce goods they are most efficient at producing and import other goods from abroad. The logic of comparative advantage implies that countries gain in varying amounts as shown in Figure 2.2. On an average, trade liberalization increases GDP growth, but not always[17]. Figure 2.3 shows improved growth performance for some emerging markets, however, there is considerable variation across countries in the success of trade liberalization. This variation can be viewed as an influential driver of destructive envy behaviour as defined by Wobker and Kenning [138] in the D'Artigues-Vignolo model.[18]

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[15] The amount of another good that must be given up in order to produce one more unit of the good, or in other words the cost of the alternative good that must be forgone to produce the good.

[16] The idea of static comparative advantage theory has been subject to a lot of criticisms. Some economists explain that agricultural products are characterized by low price income elasticities whereas manufacturing faces high-income elasticity, so as countries get richer, demand for manufactured goods rises faster and the price of agricultural goods falls relative to the price of manufactured goods. This is discussed in the section on disadvantages of globalization.

[17] Negative growth effects of liberalization can be attributed to various reasons across countries. For example, persistent amounts of debt and current account deficits despite implementing the IMF restructuring program in the 1990's in Hungary; macroeconomic instability and lack of structural reforms post liberalization during the late 1980's in Mexico.

[18] Note that comparative advantage is incorporated into the model presented in this thesis via adjustment in parameter values so that countries have added benefits of international trade.
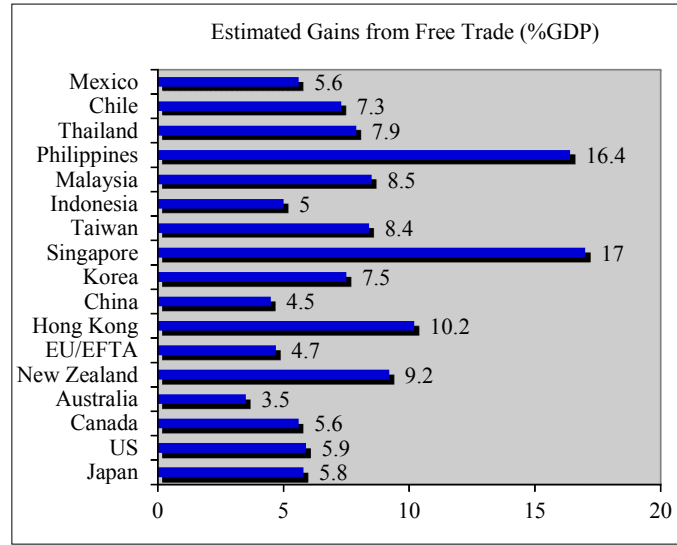
Figure 2.2. Estimated gains from comparative advantage from removing remaining post-Uruguay Round trade restrictions as a percentage of GDP (Source: Data taken from Brown, Deardorff and Stern [19]).

The Solow Model[19] assumes that GDP is produced according to an aggregate production function technology. Solow [125] finds that the growth rate of an economy depends only on the technological progress of the economy, the rate of capital stock growth and the rate of labour force growth. Within the Solow model, Total Factor Productivity (TFP)[20] is used as a measure of technological progress. The model is defined as:

$$Q = A_t K_t^{\alpha} L_t^{1-\alpha}, \ 0 < \alpha < 1,$$

where Q denotes the output, K denotes the capital input, L denotes the labour input and A_t denotes TFP, α is the production elasticity. This model reaches a steady

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[19] Also known as the Exogenous growth model or the Neo-classical growth model.

[20] TFP, in simple terms, is a measure of efficiency with which firms turn inputs into outputs.

state when Q and K grow at the same rate as L[21]. Once this steady state is reached, the growth in per-capita income can only be attributed to TFP[22]. One of the most important implications of this model is *absolute convergence* – countries with lower per-capita income will grow faster than richer countries, resulting in the convergence of the per-capita income of all countries.

The Endogenous growth theory[23] model allows the productivity increase to be generated endogenously. According to Fine [46], this can depend on increasing returns to scale in production function, or application of produced research and development (R&D), or production of human capital. This theory implies that economic growth can be promoted by economic policies that embrace openness, change and innovation, while slowing change in the form of protecting industries can result in slow growth. Romer [111] presents a neoclassical model with technological change, augmented to give an endogenous explanation of the source of the technological change. He suggests that free international trade can act to speed up growth.

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[21] At any given time t, let the output per worker be defined as follows: $q = \dfrac{Q}{L} = A\left(\dfrac{K}{L}\right)^{\alpha} L^{\alpha} L^{1-\alpha}$

$= Ak^{\alpha} = f(k)$, where k is the amount of capital per worker. Let c denote the amount of consumption, s denote the savings rate and y denote the output. Then the following equation is obtained: $c = (1-s)y$. If i denotes investment, then $y = c + i$ as all output is either consumed or invested. Substituting $c = (1-s)y$ in $y = c + i$ gives: $i = sy = sf(k)$. If n denotes the percentage by which the population increases, and there is a depreciation of δ (the rate at which the capital wears out), then new investment is required to offset this loss over time. This defines the steady state as a state where capital per worker is constant over time, so that $\Delta k = sf(k) - (\delta + n)k = 0$. Now if $sf(k) > (\delta + n)k$, then the capital stock will grow. If $sf(k) < (\delta + n)k$, then the capital stock will shrink. When the two are equal, no adjustment to capital will be required, and the steady state will be reached when

$k = \left[\dfrac{sA}{(\delta + n)}\right]^{1-\alpha}$. Since there is no change in k, the output per worker and capital per worker are constant.

[22] The model suggests that unless trade affects TFP it does not influence long-run growth.

[23] The Endogenous growth theory or the New growth theory was formed in the 1980's as a result of the criticism of the Neo-classical growth theory; see Aghion and Howitt [1].
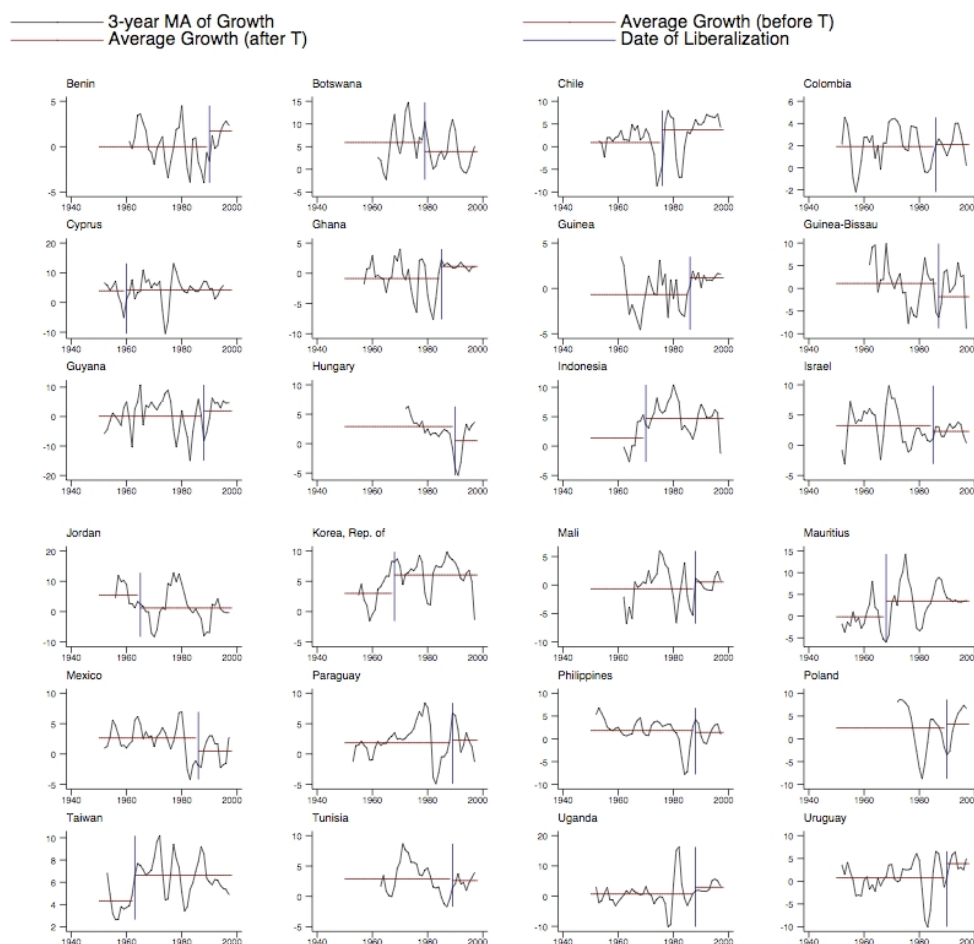
Figure 2.3. Impact of trade liberalization on economic growth in a sample of countries[24]. The vertical axes represent the growth rates[25]. Note that countries like Hungary, Israel, Mexico, etc. experience a negative growth effect of liberalization. Countries like Colombia and Philippines experience no growth rate due to trade liberalization, whereas countries like Ghana, Taiwan, Uruguay, etc. experience a positive growth effect of liberalization. (Source: Wacziarg and Welch [136]).

A vast majority of case studies, cross section and panel data studies show that economies that are more open grow faster. Parikh and Stirbu [100] find that liberal-

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[24] The vertical lines indicate the year of liberalization T, the left horizontal lines represent the average growth before T and the right horizontal lines represent the average growth after T.

[25] This is the real per capita growth rate of GDP per year.

ization contributes significantly to economic growth in their study of 42 developing countries spread over Asia, Africa and Latin America. Rao and Singh [107] find through their panel co-integration tests[26] that there is a well-defined long run relation between output, trade ratio and capital. They also find that openness to trade has made a significant contribution to the long-term growth of output by 1999-2003. Sheehey [120] measures trade openness by the rate of growth of the share of exports in GDP and finds that there is a close relationship between exports and growth in a cross section of countries. A positive relationship between growth and trade can be achieved if trade reform occurs simultaneously with domestic policy reforms.

Disadvantage - Increased Poverty

A common criticism of globalization and trade openness is that it leads to increased poverty especially in nations that are already poor. Leete, Mason, Naohiro, Mahmud and Chaudhury [78] presents the views of five panelists on the adverse effects of globalization on population and poverty - Chaudhury and Naohiro present the case that globalization is adversely affecting poverty; while Mason and Mahmud present the case that globalization is not affecting poverty. Chaudhury states that liberalized market policies[27] can erode household income and consequently give rise to poverty. He presents the case of Azerbaijan where the loss of income (due largely to the loss of jobs and price hikes) led to increased poverty and worsened income distribution led to increased inequality. He also includes evidence from East and South Asia[28] that indicates that globalization has exacerbated poverty and inequality. Naohiro discusses the case of Japan in the early 1990s, when the recession and Japan's poor economic growth performance led to unemployment and poverty. Mahmud gives examples of countries like South Korea and Malaysia that were able to increase human-development levels and significantly

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[26] The tests were based on selected East Asia countries - Hong Kong, Korea, Malaysia, Singapore, Thailand and the Philippines.

[27] For example, privatization of industries, which leads to the retrenchment of workers and unemployment, and instability of food prices.

[28] Countries include Bangladesh, Hong Kong, India, Indonesia and Pakistan, South Korea, Thailand.

reduce poverty levels through increased trade and economic growth. Mason states that poor countries benefit by globalization due to creation of jobs and improvement in the value of new ideas and incentives to develop them. Mason also implies that Asian countries that have participated in the global economy have successfully reduced poverty[29], whereas countries without global orientation have not[30]. There has been impressive growth performance in most developing economies except Sub-Saharan Africa, Eastern Europe and Central Asia, in terms of a decline in share of the population in poverty, decline in infant mortality rates, improvement in living standards and life expectancy. However, according to World Bank Staff [141], more than 40 developing countries with 400 million people have had negative or close to zero per capita income growth over the past thirty years. Moreover, the absolute number of poor has continued to increase in all regions except East Asia and the Middle East. The World Bank Staff also state that while there is now robust cross-country empirical evidence that growth is on average associated one-for-one with higher incomes of the poor (that is when an economy grows by 1%, the incomes of the poor also rise on an average by 1%), poverty is affected by other factors than globalization and growth, such as initial levels of income inequality. More recent estimates suggest that between 1970 and 1998, during a period of rapid globalization, there was a decline in world poverty (a decline from 16% to 5% in the world population living on less than $1 a day, and from 44% to 19% living on less than $2 a day), see Figure 2.4. Our model attempts to test if the competition between countries by means of openness to international trade leads to a more partitioned world economy.

Critics of comparative advantage theory have said that poor or developing nations cannot benefit from free trade in the long-term. The Prebisch-Singer hypothesis asserts that trade between developed countries and developing countries tends to deteriorate over the long-term as the relative prices for primary products (when compared to manufactured goods) decline. These theories suggest that comparative advantage may not always work in the case of developing countries as their pro-

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[29] For example, China, Hong Kong, Singapore, Taiwan and Thailand.

[30] For example, North Korea.

spects for growth are diminished due to the falling value of primary products. Cypher and Dietz [32] suggest that the basic Ricardian theory of comparative advantage is too static and find that for poor nations, the argument in favour of free trade policy cannot be sustained when the long-term historical trends of the terms of trade are taken into account. Violations to free trade are also proven beneficial to some nations in long term dynamic settings.
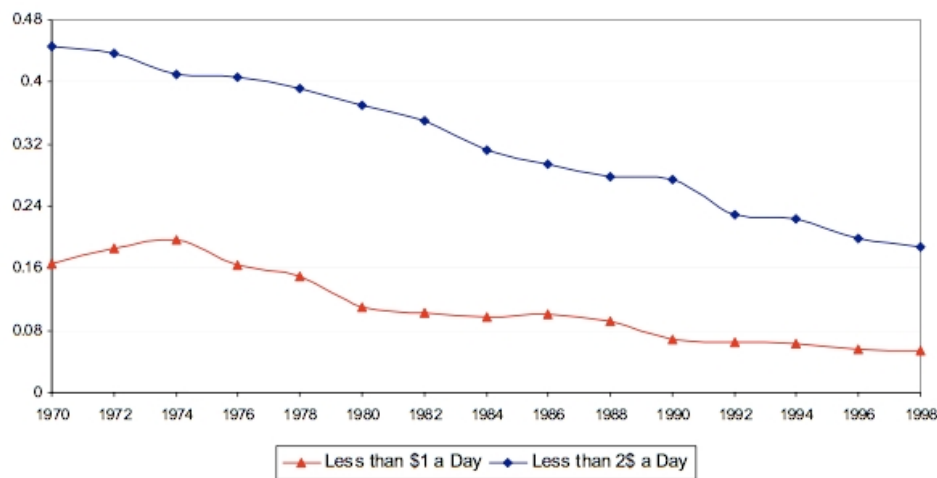
Figure 2.4. Poverty rates (Source: Sala-i-Martin [114]). Notes: The vertical axis represents the poverty rates, which are defined as the fraction of the world's population that live below the absolute poverty line (less than one dollar per day). The world distribution of income for this data was found by constructing the Gaussian kernel density functions for various years and poverty rates were then estimated by integrating the density functions below the poverty line.

Globalization has also been criticized for causing inequality in emerging markets. The World Bank Staff [141] find that the gap between the richest countries and the poorest countries has progressively widened.

As discussed above, globalization can indeed provoke negative consequences, which are typically a result of market failures, which are compounded by inadequate regulation, and bad domestic policies, for example, such happened in the Global Financial Crisis of 2007-08. Better international economic institutions as

well as domestic policies are required. Sachs and Warner [112] believe that economic growth and therefore economic convergence requires reasonably efficient economic institutions.

Sachs and Warner [112] show that all developing countries that have satisfied certain unexceptional conditions[31] on economic policy have experienced positive economic growth during the decades of the 1970s and the 1980s, and in almost all cases, these countries have shown a tendency to grow more rapidly than the developed economies, and thereby to converge. In the trade model derived from D'Artigues and Vignolo [33], it is found that an equilibrium can exist where the lagging countries trade with one another in order to increase their economic conditions (payoffs), while some of the leading countries choose to stay in autarky.

2.1.3 Openness to International Trade, Capital Mobility and Growth

Endogenous growth theories can produce a positive link between trade and growth, as shown previously. They can also produce a negative link between trade and growth. Sachs and Warner [113] sample 111 countries and approximately 98% of the non-communist world in 1970. They find that openness strongly supports growth. Open economies show convergence[32] to the leaders, while closed econo-

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[31] Sachs and Warner [112] establish *appropriate* policies based on property rights and openness to international trade. Three conditions are included with regard to the property rights test, first, the country should not have a socialist economic structure, second, the country should not have been involved in a civil war or major external war during the period 1970-1989 and third, there should be no extreme deprivation of civil or political rights. Three kinds of measures are included to test for openness, first, the country must not impose excessive quotas on imports, second, the country must not impose excessive quotas or state monopolies on exports and third, the country must maintain a reasonably convertible currency. For more details, see Sachs and Warner [112].

[32] Economic convergence is defined as the closure of the proportionate income gap between the richer countries and the poorer countries as a result of the poor countries growing more rapidly than the richer countries.

mies do not show economic convergence[33]. Many poor countries, particularly those in sub-Saharan Africa, not only fail to grow faster than the rich countries but also experience negative per capita growth. So the gap between these countries and the rich countries widens significantly. Sachs and Warner [113] also find that the open economies show convergence in terms of declining dispersion of GDP over time. Openness to trade and growth in terms of payoffs forms one aspect of the model explored in this thesis.

Global capital flows[34] have a profound impact on the financial market and economies of the world. Since 1990, global capital flows have grown faster than the value of trade. According to Farrell, Lund, Folster, Bick, Pierce and Atkins [43], in 2006, the annual value of global capital flows totalled $8.2 trillion, which represented an eightfold increase since 1990. On the one hand, they can seem completely uncontrolled and dangerous. They can allow countries to invest and consume more than they can produce. Capital inflow can lead to unsustainable foreign currency debts. Increased openness to capital flows has been associated with an increasing frequency of financial crises as shown by Bordo and Eichengreen [17]. Capital mobility can result in some poor countries being left out while the biggest capital inflow is to the richest country (the US). Alfaro, Kalemli-Ozcan and Volosovych [5] present an overview of general patterns in international capital mobility over the period 1970-2000 and show that most capital flows to rich countries. Figure 2.5 shows direct and portfolio equity investment inflows for 23 developed and 75 developing economies for the period 1970-2000. The stark difference between

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[33] In fact, there is not a single country in their sample that pursued open trade policies during the entire period 1970-89 and yet had per capita growth of less than 1.2% per year. Switzerland had the lowest growth, at 1.24%. Moreover, not a single open developing country grew at less than 2% per year, Greece at 2.38% being the lowest.

[34] Flows that include FDI, cross-border lending and deposits and purchases of foreign equity and debt securities.

them demonstrates the *Lucas Paradox*[35], which states that there is a lack of international capital flows from rich to poor countries.
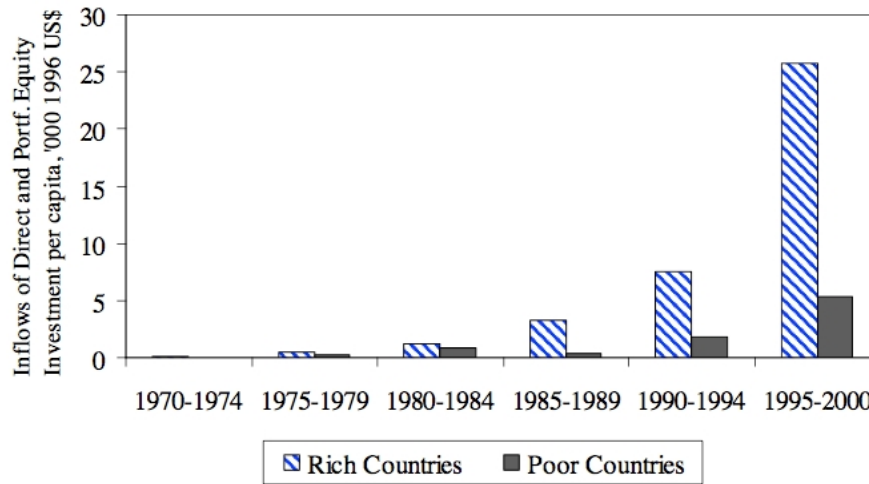
Figure 2.5. Total equity inflows per capita to rich and poor countries, 1970-2000 (Source: Alfaro, Kalemli-Ozcan and Volosovych [4]). Notes: Inflows of total equity (FDI[36] and portfolio equity investment[37]) divided by population are based on the IMF, IFS data in 1996 US$. Data represents 98 countries averaged over 5 year periods. Rich countries include 23 high GDP per capita countries that are classified as rich by the World Bank; poor countries denote the 75 remaining ones.

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[35] Lucas [83] examined international capital movements from the perspective of rich and poor countries. Under the standard assumptions, such as countries producing the same goods with the same constant returns to scale production function, the same factors of production, relating output to homogenous capital and labour inputs, and the same technology, new investment will occur only in the poorer economy where there is free capital mobility as the marginal product of capital will be higher in the less productive economy. And this will continue to be true until the returns to capital in every location are equalized. Hence, Lucas argued that the fact that more capital does not flow from rich countries to poor countries constitutes a paradox.

[36] FDI inflows correspond to direct investment in the domestic economy, which includes equity capital, reinvested earnings, other capital and financial derivatives associated with various intercompany transactions between affiliated enterprises.

[37] Portfolio equity inflows correspond to equity liabilities, which include shares, stocks participations and similar documents that usually denote ownership of equity.

As a recent example, there had been dangers associated with large investments made in the US and European markets by Asian and oil-exporting countries. By 2006, Germany, China and oil exporting nations were exporting large amounts of capital to the US leading the US to monopolize global excess savings, see McKinley [87]. By 2008, the US government and household sector sunk deeper into debt and the recession in the US has had adverse impact on the rest of the world. Birdsall, Rodrik and Subramanian [15], McKinley [87], Ricupero [109] and others suggest that the share of global excess savings should be recycled to poor countries in need of development finance. Low-income countries are also hit the hardest by global recession as surveyed by Burke [21] and thus need extra help to recover from the crises. On the other hand, many economists state that the flow of capital across nations can have benefits. Farrell et al. [43] state that companies around the world can tap larger pools of capital at better prices. Oil exporting nations are proving beneficial in injecting capital into US and European banks that are suffering from the fallout of the US subprime mortgage crisis. Demirguc-Kunt and Maksimovic [36] find that firms grow faster with the help of external funds in their sample of 30 developed and developing countries. Obstfeld [95] states that global capital flows allow countries to pool various risks and achieve more effective insurance than domestic arrangements would allow. He further states that developing countries with little capital can borrow to finance investment, thereby promoting economic growth. Moreover, a country suffering from a minor recession or a natural disaster can borrow abroad. Loungani and Razin [82] believe that global capital mobility contributes to the spread of the best practices in corporate governance, accounting rules and legal traditions, and limits the ability of governments to pursue bad policies. Edwards [41] concludes that in order to fund investment projects and adjustment programs, a country should devise strategies to attract foreign capital inflows, as it forms an important part of the country's strategy for recovery and growth.

2.1.4 The Future of International Trade

Trade is growing faster than GDP as illustrated in Figure 2.6. There has been a major shift towards financial globalization since the 1980's, with further acceleration in early 1990's, see Figure 2.7. Cross-border financial integration has accelerated; see Figure 2.9. Moreover, Integration is faster in finance than trade, and much faster for industrial countries, see Figure 2.8. Over the next 50 years, the fast growing developing economies (that are not current world leaders) can become a much larger force in the world economy, overtaking the US, the current world leader, see Figure 2.10. Developing economies like China and India can hence catch up with leading economies like the US and the UK and in fact overtake them to become world leaders due to world trade. Such a scenario where the lagging country overtakes the leading country and assumes leadership is also obtained in the continuous version of the trade game model considered in this thesis.
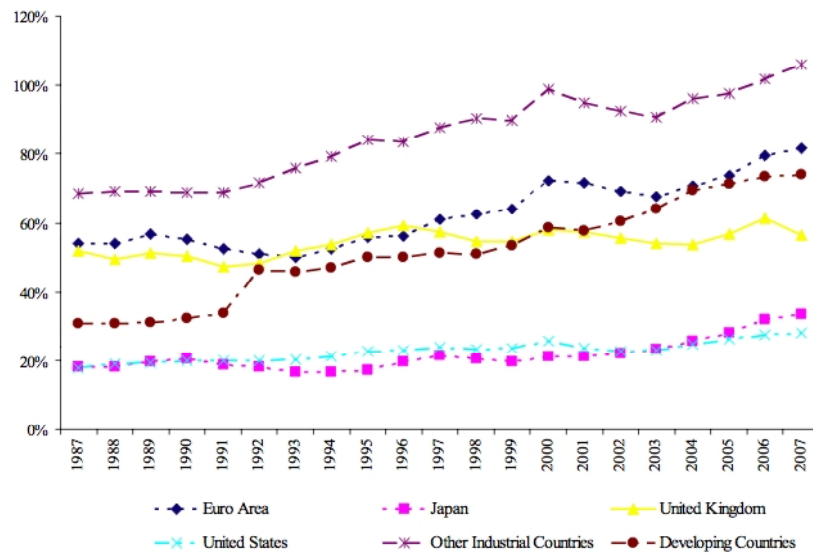
Figure 2.6. Trade (exports plus imports) as a percentage of GDP (Source: Papaioannou and Portes [99]).
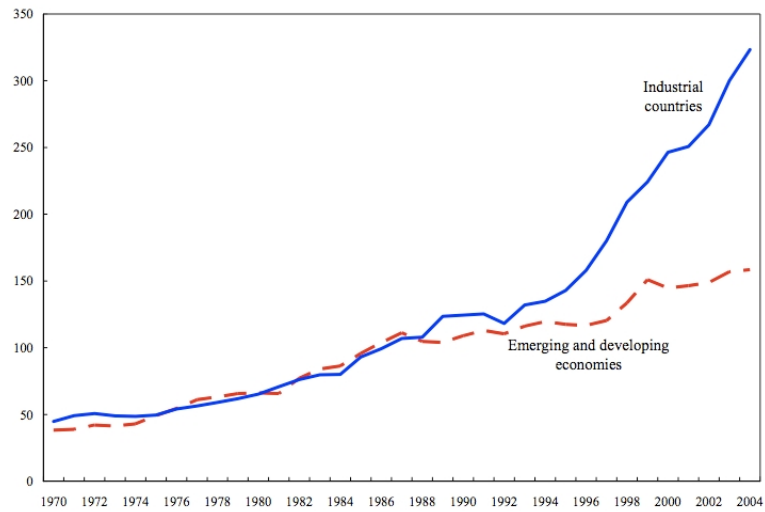
Figure 2.7. International Financial Integration: Industrial Group and Emerging Markets/Developing Countries Group, 1970-2004, showing the ratio of sum of foreign assets and liabilities to GDP (Source: Lane and Milesi-Ferretti [76]).
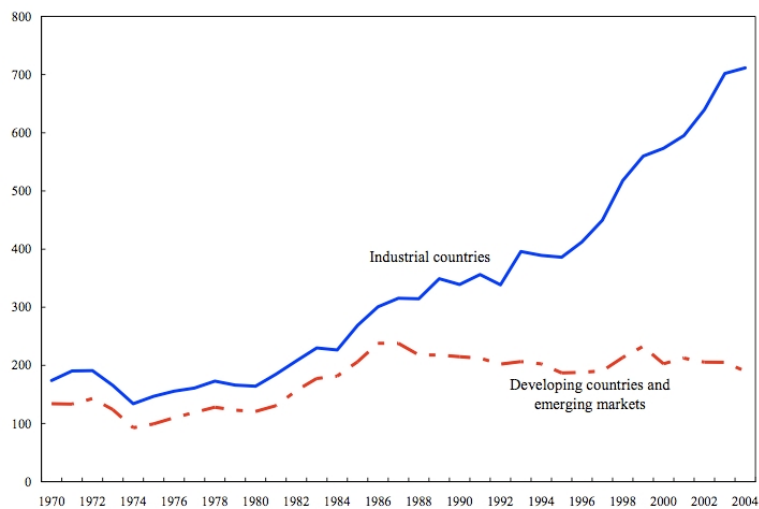
Figure 2.8. Financial Integration versus Trade Integration: Industrial Group and Emerging Markets/Developing Countries Group, 1970-2004, showing the sum of external assets and liabilities as a percentage of the sum of exports and imports (Source: Lane and Milesi-Ferretti [76]).
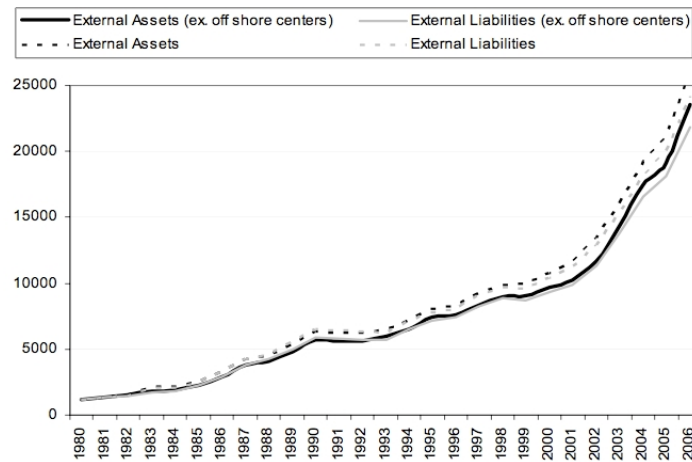
Figure 2.9. Bank external assets and liabilities in billion USD (Source: Portes [103]).
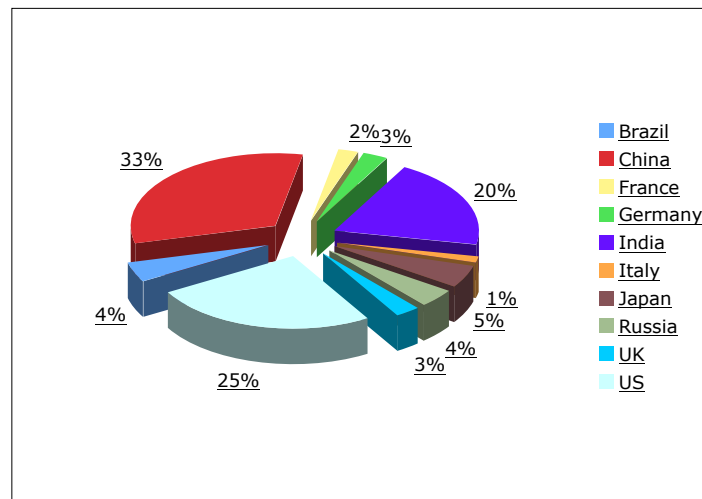
Figure 2.10. The world economy in 2050, showing a forecast of respective shares of total world GDP (Source: Purushothaman and Wilson [104]).

2.1.5 Making Globalization Work for All

As found in our model, world economies theoretically can benefit from openness to international trade, and a state where all the economies trade with one another can be a strict equilibrium provided certain restrictions on the parameters hold. Stern and Deardorff [127] examine the effects of trade liberalization on countries that do

not participate in it and find that excluded countries[38] are more likely to lose than gain, through improved terms of trade.

Some globalization problems are due to international externalities like tax, regulatory competition and environment. Summers [128] writes that one of the reasons for globalization causing some increase in inequality is the problem of *race to the bottom*[39] in corporate income to entice more business and the problem of tax havens to lure wealthy citizens. The international mobility of goods and capital puts competitive pressure on nation-states to redesign domestic market regulations, arising from either economic activities being shifted elsewhere or internal lobbying of industries, see Holzinger, Knill and Sommerer [64]. For example, if one country lowers its standards compared to others, then the others follow, leading the level of regulation to move to the *bottom*.

These are all big issues to which there are no easy answers. For example, WTO introduced the Doha Development Round in 2001 to lower trade barriers globally. It includes negotiations on various issues like agriculture, services, trade facilitation, market access for non-agricultural products, least-developed countries, special and differential treatment, trade-related aspects of intellectual property rights (TRIPS), etc.[40] According to James [67], the Doha round was the best prospect the world had for significant gains from freer trade and had enormous potential to bring growth and development to poorer countries. Studies have also shown that developing countries can gain from reduction or elimination of trade barriers in agricultural markets as a result of the implementation of the Doha round multilateral trade agreements, for example, Ferreira Filho and Horridge [45] analysed the case for Brazil and found a reduction in poverty with the implementation of the Doha round agreement. The Doha round negotiations came to a halt in 2008 large-

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[38] The excluded countries are defined as countries that have stayed out of preferential trade arrangements and have joined WTO/GATT whilst keeping their trade barriers high, or failed to join.

[39] Race to the bottom in this context is used to describe the situation where countries compete over tax regulation and the production of goods is moved to the country with the lowest tax rates.

[40] See Doha Development Agenda: Negotiations, implementation and development

(http://www.wto.org/english/tratop_e/dda_e/dda_e.htm#dohadeclaration) for the entire list of negotiations covered.

ly due to failure of reaching a consensus on agricultural proposals by the US and the EU, see Cho [25], Gallagher [52] and Griller [61]. Pereira, Teixeira and Raszap-Skorbiansky [101] study the impact of the Doha round on Brazil, China and India, and find that while Brazil and China show a high GDP growth rate, India shows a negative GDP growth rate. Furthermore, the GDP losses observed in the US and the EU make it difficult to reach an agreement at the Doha round. Literature on how the next Doha round could be more effective ranges from policies mainly focussing on agriculture, to policies more generally focusing on trade and technology. For example, agricultural policies can include developing countries retaining the policy flexibility necessary for development, providing appropriate incentives to their domestic agricultural sectors, increasing food security, shielding the poor from market failures that can affect their very survival, etc. (see Polaski [102]). Trade and technology policies can include using average tariffs to sequence certain industries into world markets; issuing compulsory licenses under TRIPs, requiring foreign firms to transfer technology, forming joint ventures and performing R&D in the host country (see Gallagher [52]).

Policies for Developed Countries

In this section, the enhancement of policies to be implemented by developed countries in order to help developing and least developed economies to boost economic growth and reduce poverty worldwide is discussed.[41]

Global markets should open further to exports of developing countries, especially in the areas of agriculture and textiles. As discussed above, poor countries stand to lose the most from the remaining protectionism in rich countries. For example, Anderson and Valenzuela [10] state that most governments restrict international trade to some extent especially in agricultural goods, especially advanced economies, in order to protect domestic producers from import competition. However, agricultural growth and food related aid is essential for the poorer countries to prosper. Goyal [60] studies the impact of globalization in developed countries and

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[41] Note that the recommendations come from a wide range of literature on economic policies and discussions on the world economy with economic experts at London Business School.

lists boosting agricultural growth through diversification and development of agro-processing as one of the future challenges for the Indian economy. Bhalla [12] writes that the most important problem arising in global agricultural trade is large subsidies given by developed countries to certain products, which make the exports from developing countries non-competitive. In order to address this issue, developing countries like India should urge developed countries to reduce tariffs on all commodities, especially on agricultural commodities, and negotiate on the easing of market access. Von Braun [135] also states that the remaining agricultural subsidies and trade-distorting policies in developed countries should be eliminated, as the poor countries are unable to match them. Anderson and Martin [8] suggest moving to free global merchandise trade in order to boost real net farm incomes in sub-Saharan Africa and Southeast Asia, thereby helping to reduce poverty. Anderson, Martin and van der Mensbrugghe [9] suggest rewarding developing countries' commitment to greater trade reform with an expansion of trade-facilitating aid, to be provided by a major expansion of the current *Integrated Framework*[42] for less developed countries (LDCs) (see Hoekman [63]). Bouet, Fontaigné and Jean [18] agree that this may provide a path for developing countries to trade their way out of poverty, and for developed countries to assist low-income nations efficiently.

Developed countries should aim at protecting workers while allowing for reallocations, between sectors or within sectors between firms. Blanchard [16] agrees that a social insurance system should *protect workers, not jobs*. He says that a good system should include some employment protection such as a firm, which lays off workers, contributing towards the benefits paid to the laid-off workers. Countries like Denmark and the Netherlands have benefited from *flexicurity*[43], which follows the policy of protecting workers instead of jobs, by combining high unemployment benefits with low job protection and high participation rate; see Ahn, Garcia and Jimeno [2]. Lipsey and Chrystal [80] write that unemployment is a major problem of our time and economic policies that protect workers instead of jobs are needed.

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[42] A process established in 1996 to support LDCs in trade capacity building and integrating trade issues into national development strategies.

[43] Flexicurity combines security for workers with labour market flexibility so that both the workers' and the employers' needs are taken care of.

For example, policies can couple unemployment benefits with measures to help the unemployed find jobs; employment protection policies can internalize social costs, etc. Vermeylen [133] suggest that the society and the economy on the whole, including social systems, should be involved in shaping and implementing labour markets' and workers' policies such as flexicurity.

Developed countries should raise *development aid*[44], and improve its effectiveness. Speth [126] summarizes that poverty is increasing rapidly in developing countries and the West must increase development aid and political investment. Minoiu and Reddy [89] attempt to separate two components of aid, a developmental component that consists of growth-promoting expenditures and a non-developmental component that consists of all other expenditures. They find that developmental aid has a positive, large and robust effect on economic growth while non-developmental aid is mostly growth-neutral. World leaders pledged to increase development aid in 2000 in order to reduce extreme poverty in the world by half by 2015[45], see Ismail [66]. However, Bulir and Hamann [20] find that the positive impact of foreign aid is limited by the erratic behaviour of aid flows. Corruption is one of the main factors that reduce the effectiveness of aid in developing countries and Schudel [118] finds that as a result, donor countries allocate less bilateral aid to recipient countries with high levels of corruption. The development aid funds need to target the poor directly and the problem of corruption needs to be tackled, for example, by adding some levels of government that are truly responsive to the public, see Andersson et al. [57], or allocating through multilateral channels, see Schudel [118]. In order to stimulate economic growth, developed countries need to invest in physical, human and social capital, including creation of opportunities for the less advantaged, see Gibson, Andersson, Ostrom and Shivakumar [57]. Implementation of key reforms and social services to the poor must be monitored, see Thomas [132]. A better way for information sharing, planning and coordination is

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[44] Aid given by governments and other agencies to support the economic, social and political development of developing countries (development aid is considered to be different from humanitarian aid).

[45] Birdsall [14] estimates $1 trillion must be unlocked for developing countries to cope with the financial crises alone.

required according to Kharas [73], along with a revised approach towards the allocation of aid based on the effectiveness of the development contribution, not on politics.

Debt relief is advantageous to heavily indebted and underdeveloped countries as it can increase the amount of financial resources available to poor countries (see Weiss [137]). In order to cancel, or reduce, external debt payment to sustainable levels, the Heavily Indebted Poor Countries Initiative (HIPC) was formed in 1996 by the IMF and the World Bank. Of the 40 countries[46] that qualified for the HIPC initiative, 28 have reached *completion point* and received debt relief. The HIPC initiative has had beneficial effects on education, see Cuaresma and Vincelette [31]; health, see Dessy and Vencatachellum [37]; and the earning prospects of companies operating in HIPCs, see Raddatz [106]. The Multilateral Debt Relief Initiative (MDRI) was formed in 2005 by the IMF, the World Bank and the African Development Fund, as an extension to the HIPC initiative, in order to reduce further the debts of HIPCs and to help countries expand development programs fast enough to meet the Millennium Development Goals (MDGs). Between the 1990s and mid-2008, there have been significant reductions in public indebtedness of low-income countries and improvements in countries' external positions (improvements in balance of payments and increase in foreign exchange reserves), see Liu, Prasad, Rowe and Zeikate [81]. Debt relief can have its largest impact if it is accompanied by a broader package of reforms, like growth enhancing changes in national policies (see Hornbeck [65]). Long-term response may entail changes to the system of international aid, for example, establishment of minimum accountability requirements for governments managing aid money, see Thomas [132].

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[46] Countries include Benin, Bissau, Bolivia, Burkina, Burundi, Cameroon, Central African Republic, Chad, Comoros, Democratic Republic of Congo, Republic of Congo, Côte d'Ivoire, Eritrea, Ethiopia, Faso, Guinea, Guyana, Haiti, Honduras, Kyrgyz Republic, Liberia, Madagascar, Malawi, Mali, Mauritania, Mozambique, Nicaragua, Rwanda, Sierra Leone, São Tomé & Príncipe, Senegal, Somalia, Sudan, Tanzania, The Gambia, Togo, Uganda, and Zambia. See IMF Factsheet on Debt Relief Under HIPC Initiative (http://www.imf.org/external/np/exr/facts/pdf/hipc.pdf) for further details on countries.

Policies for Financial Integration

In this section, it is shown how financial integration policies around the world can have a positive impact on economies. Many studies (Arteta, Eichengreen and Wyplosz [11], Klein [74], Klein and Olivei [75]) have shown that capital account liberalization can positively affect economic development through financial development. However, careful sequencing of financial sector policies is extremely important especially when the issue of capital account liberalization is concerned. Caprio and Honohan [22] suggest that mistaken sequencing for capital account liberalization contributed to the speed and severity of the 1997 Asian financial crisis. Capital account liberalization should be sequenced in an integrated manner so that it reinforces domestic financial liberalization and allows for institutional capacity building to manage the additional risks, see Johnston and Sundararajan [69] and World Bank Staff [139]. Capital account liberalization should start with the liberalization of FDI (as this helps in importing superior technology and management expertise needed to implement operational reforms in financial institutions), and closely complement the domestic development market strategy (for example, money and exchange market development can benefit from safeguarded short-term capital flows).

Policies that strengthen financial supervision and regulation are essential as bank failures not only affect taxpayers and financial institutions, but also the economy as a whole. Major failures in this area were the fundamental causes of the current financial crisis. Banks should be closely involved with financial supervision and regulation, especially to limit systemic risks, see Geraats [56] and Quintyn, Ramirez and Taylor [105]. In response to the financial crisis in 2009, the Leaders of the Group of Twenty (G20) agreed on strengthening financial regulation and supervision to minimize risk across the financial system and dampen, rather than amplify, the financial and economic cycle. They also agreed that regulators and supervisors must play a crucial role in avoiding adverse impact on other countries, reducing the scope for regulatory arbitrage, protecting investors and consumers, keeping pace with innovation in the marketplace and supporting dynamism and competition. According to Tarullo [130], the Federal Reserve has begun to evaluate

regulatory and supervisory changes that can help reduce the incidence and severity of future financial crises.

Policies that oppose extreme cases of *economic patriotism*[47] are required. Durand [40] explains with the example of energy mergers in Spain and France[48], how the energy markets in Europe were unable to produce their desired outcome in terms of prices, security of supply and environment sustainability due to economic patriotism. Studies like Joongi [71] show that economic patriotism through protectionism will not be a sustainable alternative to global economic integration. As a healthy balance, liberal economic patriotism must operate within limits of supranational and global regulation as suggested by Clift [27].

Capital controls[49] can be used to alter the composition of inflows, for example, Chile and Columbia effectively changed the composition of inflows toward less vulnerable liability structures via such controls, see De Gregorio, Edwards and Valdes [34] and Cardenas and Barrera [23]. Coelho and Gallagher [28] also find that capital controls[50] were instrumental in stemming asset bubbles in the case of Colombia and Thailand. Magud and Reinhart [84] find that capital controls on inflow make monetary policy more independent and reduce real exchange rate pressures. Empirical results by Ostry, Ghosh, Habermeier, Chamon, Qureshi, and Reinhardt [97] find that capital controls aimed at achieving a less risky external liability structure have helped in reducing financial fragility in the current crisis. As explained in Chang and Grabel [24], well-designed capital controls can promote

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[47] This notion has emerged as a counterforce to constrain foreign ownership.

[48] Gas Natural, Spain's largest gas supplier, launched a takeover bid for Endesa, Spain's largest electricity producer to prevent a counter-bid by German electricity and gas giant E.ON in 2006-2007. However, Endesa now forms a subsidiary of the Italian utility company Enel. In 2006, the French government announced the merger between Gaz de France and Suez, a French utility company, in order to prevent a takeover of Gaz de France by Italy's Enel.

[49] Capital controls can limit a country's ability to access foreign funds; however, well-designed capital controls on inflows and outflows can lengthen maturities and limit destabilizing flows.

[50] Developing countries used unremunerated reserve requirements (URR) as a response to massive amounts of capital inflows that their financial systems were unable to absorb. Colombia and Thailand deployed URR in 2006-2007 with positive results.

economic stability and prevent economic and social devastation associated with economic crises.

Policies for institutional development

Better policies are needed that can cope better with cross-border spillovers and international prisoners' dilemmas. For example, Corsetti, Meier and Muller [30] analyse the international spillover effects of short-term fiscal stimulus and find that coordinated short-term stimulus policies are most effective when coupled with credible medium-term consolidation plans featuring at least some spending restraint. The global financial crisis that originated in 2007 affected almost all developed economies and emerging markets, some of which were mainly affected through rapid financial spillovers. Claessens, Dell'Ariccia, Igan and Laeven [26] suggest closer cooperation and greater coordination among regulators and supervisors to forestall policy measures that have adverse spillovers, and better international liquidity provision to both financial institutions and countries.

The international financial architecture needs to be reformed for orderly resolution of debt crises and multilateral efforts to deal with global imbalances. According to Akyuz [3], there is a need for an independent assessment of debt sustainability; a dispute-settlement body placed beyond the reach of the IMF and its major shareholders; and protection of debtors through an internationally sanctioned stay on litigation. Tasks to be undertaken for an orderly unwinding of global imbalances can include steps to boost national saving in the US including fiscal consolidation; further progress on growth-enhancing reforms in Europe; further structural reforms including fiscal consolidation in Japan; reforms to boost domestic demand in emerging Asia, together with greater exchange rate flexibility in a number of surplus countries; and increased spending consistent with absorptive capacity and macroeconomic stability in oil producing countries, as stated in Akyuz [3][51].

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[51] Taken from Communiqué of the International Monetary and Financial Committee of the Board of Governors of the International Monetary Fund, October 20, 2007.

The international institutional framework needs to be strengthened to deal with environment, international transmission of disease, intellectual property rights (IPR), global competition policy issues and terrorism, with emphasis on the needs of poor countries.

2.2 Models of International Trade

This section provides a brief background on coordination games and compares models of international trade that have been analyzed in literary studies.

The Coordination Game Model

A coordination game is a game in which players benefit from mutual cooperation but only by making mutually consistent decisions. A 2×2 coordination game can be represented by the following payoff matrix:

	s_1	s_2
s_1	A, a	C, c
s_2	B, b	D, d

where for player 1: $A > B$, $C < D$ and for player 2: $a > c$, $d > b$. In this game, s_1 is the coordinated strategy whereas s_2 is the non-coordinated strategy. The strategy profile is denoted by $x = (x_1, x_2)$, where $x_i \in \{s_1, s_2\}$ for $i = 1, 2$ and x_i represents the strategy played by player i. Following the payoff matrix above, player 1 obtains payoff A by playing strategy s_1, or payoff B by playing s_2, when player 2 plays strategy s_1; and payoff C by playing strategy s_1, or payoff D by playing s_2, when player 2 plays strategy s_2. Player 2 obtains payoff a by playing strategy s_1, or payoff c by playing s_2, when player 1 plays strategy s_1; and payoff b by playing strategy s_1, or payoff d by playing s_2, when player 1 plays strategy s_2. Note that in this game, (s_1, s_1) is the coordinated outcome and (s_2, s_2) is the non-coordinated outcome. This game model can be extended to n players, with the strategy profile defined as $x = (x_1, \ldots, x_n)$, where $x_i \in \{s_1, s_2\}$ for $i = 1, \ldots, n$ and the payoff function

defined as $f = \{f_1(x),\dots,f_n(x)\}$. This definition of the *n*-player model is used in Chapter 3.

A Nash equilibrium is defined as a set of strategies such that no player can do better by unilaterally changing its strategy. In the coordination game above, (A,a) and (D,d) are pure strategy Nash equilibria. (A,a) is a Nash equilibrium since player 1 cannot obtain a better payoff by switching to s_2 when player 2 plays s_1 (since $A > B$), and player 2 cannot obtain a better payoff by switching to s_2 when player 1 plays s_1 (since $a > c$). (D,d) is a Nash equilibrium since player 1 cannot obtain a better payoff by switching to s_1 when player 2 plays s_2 (since $C < D$), and player 2 cannot obtain a better payoff by switching to s_1 when player 1 plays s_2 (since $d > b$).[52] For the *n*-player coordination described above, a strategy profile x^* is a Nash equilibrium if $f_i(x_i^*, x_{-i}^*) \geq f_i(x_i, x_{-i}^*)$, for all $x_i \in \{s_1, s_2\}$ and all *i*. Note that here x_{-i}^* represents the strategy profile of all players other than player *i*. If a strict inequality holds in the above inequality, then the equilibrium is defined as a strict Nash equilibrium.

Payoff dominance and risk dominance are defined by Harsanyi and Selten [62] as follows. A payoff dominant outcome is an outcome that yields the highest payoffs to all other outcomes so that when given a choice between outcomes, players choose the payoff dominant outcome as it gives each player at least as much payoff as other outcomes. A risk dominant outcome or the least risky outcome is an outcome that players choose if they are unsure of other player's strategies. In the coordination game above, (A,a) payoff dominates or Pareto dominates (D,d) since $A > D$ and $a > d$. (D,d) risk dominates (A,a) if the product of the deviation loss-

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[52] Note that this game also has a mixed strategy Nash equilibria where player 1 plays s_1 with probability p, and s_2 with probability $1 - p$, where $p = \dfrac{d - b}{a - c + d - b}$. Player 2 plays s_1 with probability q, and s_2 with probability $1 - q$, where $q = \dfrac{D - C}{A - B + D - C}$.

es is highest for (D, d), i.e. $(D - C)(d - b) \geq (A - B)(a - c)$ (see Harsanyi and Selten [62]).

The Prisoners' Dilemma game can be defined using the same payoff matrix as the coordination game above, but with the conditions on payoffs as follows - for player 1: $B > A > D > C$, and for player 2: $c > a > d > b$. In this game, mutual cooperation (s_1, s_1) yields a better payoff than mutual defection (s_2, s_2) since $A > a$ for player 1 and $D > d$ for player 2. However, strategy s_2 is the dominant strategy for both players since $B > A$, $D > C$ for player 1 and $c > a$, $d > b$ for player 2. Mutual defection (s_2, s_2) is the only Nash equilibrium in this game.

Ricardian Model of Comparative Advantage

The standard Ricardian model of comparative advantage can be presented from Dornbusch, Fischer and Samuelson [39] as follows[53]. Consider a world with two countries C_1, C_2 with one factor of production. Let L_1, L_2 denote the endowments of labour in countries C_1, C_2, respectively. Let w_1, w_2 denote the wages in countries C_1, C_2, respectively. The continuum of goods can be indexed by $z \in [0, 1]$. Constant returns to scale with respect to labour endowments can be defined for C_1, C_2 as $a_1(z)$, $a_2(z)$, respectively. Goods can be ordered so that $S(z) = \dfrac{a_2(z)}{a_1(z)}$ and $S'(z) < 0$. Hence C_1 has comparative advantage in the low-z goods. If $p(z)$ denotes the price of good z in both countries under free trade, profit is maximized in both countries when:

$p(z) - w_1 a_1(z) \leq 0$ (in this case $p(z) - w_1 a_1(z) = 0$ if z is produced in C_1),

$p(z) - w_2 a_2(z) \leq 0$ (in this case $p(z) - w_2 a_2(z) = 0$ if z is produced in C_2),

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[53] Definitions and proofs derived from [38].

then there exists $z^* \in [0,1]$ such that C_1 produces all goods $z < z^*$ and C_2 produces all goods $z > z^*$.[54] Thus, C_1 should produce the goods in which it has a comparative advantage.

Next, let the relative wage rate be denoted by:

$$\omega := \frac{w_1}{w_2} = S(\hat{z}).$$ (2.1)

According to Dornbusch et al. [39], the equilibrium condition of this model requires the production of goods to be in the country where it is cheaper to do so, conditional on wages. Let the share of expenditure on good z be denoted by:

$E(z) \in (0,1)$, such that $E(z) = \frac{p(z)c_1(z)}{w_1 L_1} = \frac{p(z)c_2(z)}{w_2 L_2}$, where $c_1(z)$, $c_2(z)$ are

consumptions in countries C_1, C_2, respectively. By definition, shares of expenditure satisfy: $\int_0^1 E(z)dz = 1$. If the fraction of income spent on goods produced in C_1

is denoted by $\vartheta(\hat{z}) := \int_0^{\hat{z}} B(z)dz$, then balance of trade requires:

$\vartheta(\hat{z})w_2 L_2 = (1 - \vartheta(\hat{z}))w_1 L_1$, where $\vartheta(\hat{z})w_2 L_2$ denotes C_1's exports, and

$(1 - \vartheta(\hat{z}))w_1 L_1$ denotes C_1's imports. Re-arranging the balance of trade equation gives:

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[54] The proof is obtained as follows: Suppose there exists a good $\hat{z} < z$ such that z is produced in C_1 and $\hat{z}$ is produced in C_2. Then:

$p(\hat{z}) - w_1 a_1(\hat{z}) \le 0$, $p(z) - w_1 a_1(z) = 0$,

$p(z) - w_2 a_2(z) \le 0$, $p(\hat{z}) - w_2 a_2(\hat{z}) = 0$.

This gives: $p(z) = w_1 a_1(z)$, $p(\hat{z}) = w_2 a_2(\hat{z})$, $p(\hat{z}) \le w_1 a_1(\hat{z})$, $p(z) \le w_2 a_2(z)$.

Hence, $w_1 a_1(z) w_2 a_2(\hat{z}) \le w_1 a_1(\hat{z}) w_2 a_2(z)$, which can be written as $\frac{a_2(\hat{z})}{a_1(\hat{z})} \le \frac{a_2(z)}{a_1(z)}$. This contradicts

the fact that $S'(z) < 0$. Therefore, there exists $z^* \in [0,1]$ such that C_1 produces all goods $z < z^*$ and

C_2 produces all goods $z > z^*$.   □

$$\omega = \frac{\vartheta(\hat{z})}{\left(1-\vartheta(\hat{z})\right)} \frac{L_2}{L_1} =: D(z). \tag{2.2}$$

Equilibrium for this Ricardian model can be summarized by $(\hat{z}, \omega)$, which is obtained from (2.1) and (2.2).[55]

2.2.1 International Trade and Coordination

International trade and cooperation have been modelled as coordination games in order to provide valuable insights into how countries interact with one another and how it affects international cooperation. Snidal [124] uses models of Prisoners' Dilemma games and Coordination games to analyse strategic interactions between countries and finds that in comparison with the Prisoners' Dilemma model, the Coordination model better characterizes important issues surrounding variations within strategic structures and international political regime characteristics. Lee, Park and Cho [77] model international trade markets using an agent-based modeling and evolutionary game theory approach to show that economies show more defecting behaviours for the powerful countries (against the cooperative nations) whereas countries with similar economic power cooperate with one another. The trade game in this thesis (derived from the Artigues-Vignolo model) also obtains a similar result whereby it is more favourable for countries with similar payoffs to trade with one another as opposed to all the countries including the leading countries to participate in international trade. Friedman and Fung [51] analyze the effects of trade between two countries (in inputs as well as outputs) and find that evolutionary pressures do not necessarily lead to efficient outcomes. This is similar to the notion of a country choosing less efficient strategies (in terms of absolute payoffs) in order to reduce the gap between the leading country and itself in our model. Cordella and Gabszewicz [29] find that even though there are mutual benefits to free trade between two countries, the autarkic outcome turns out to be the unique oligopoly equilibrium on the world market. This gives some credibility to the autarkic out-

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[55] Note that the curves $S(z)$ and $D(z)$ are essentially the labour supply and the labour demand curves, respectively.

come being the most favourable strict equilibrium in the model analysed in Chapter 3. In a stark contrast, Fisher and Vikas [47] find that comparative advantage predicts trade (that enhances economic welfare) between countries facing severe trading frictions. This thesis analyzes the trade dynamics and possible equilibria of the trade game derived from the Artigues-Vignolo model, and extends the model in order to incorporate intermediate strategies in order to find long run behaviour of trade between two countries.

The D'Artigues-Vignolo model considered in Chapter 3 is derived incorporating ideas from Kandori, Mailath and Rob [72]. The definition of payoffs to countries is also obtained from Kandori et al. [72]. They consider two models of matching, first – where each player is matched with each of the remaining players exactly once as in a tournament, and second – where there are an infinite number of random matches in time period t, so that each player's average payoff in that period is the expected payoff. A non-negligible number of mutations occur in their second version of the model as independent shocks as a result of individual behaviour, which results in a stochastic component in the dynamics. They find that the equilibrium selected in the long run depends on the payoff structure of the underlying game instead of the adjustment process. In this thesis, countries are matched exactly once with each of the remaining countries in a given unit time and mutations are presented as *one-shot* small perturbations of equilibria (as considered in Maynard, Smith and Price [86] and Taylor and Jonker [131]). The equilibrium selected in the long run in this thesis also depends on the parameter values that determine the payoffs, including the strategy size.

The concept of payoff dominance versus risk dominance (as defined by Harsanyi and Selten [62][56]) with regards to the D'Artigues-Vignolo model is also briefly discussed. According to Harsanyi and Selten [62], if the payoff dominance selection and the risk dominance criteria yield different outcomes for equilibrium selec-

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[56] A payoff dominant equilibrium is an equilibrium (based on collective rationality) which yields every player a strictly higher payoff than all other equilibrium payoffs. A risk dominant equilibrium is an equilibrium (based on individual rationality) which yields every player the least risky payoff, as players are uncertain about each other's actions.

tion, then the payoff dominant outcome should be the first one to be considered, making the risk dominant outcome irrelevant. Ostrom, Schmidt, Shupp and Walker [96] also find that players select the payoff dominant strategy more often than not. However, changes in risk dominance significantly affect the play of the subjects, whereas changes in the level of payoff dominance do not. Their findings demonstrate that players are affected by the risk-dominance characteristics of games, even in the presence of a payoff dominant equilibrium. In the D'Artigues-Vignolo model considered in Chapter 3, the fully integrated equilibrium (where all the countries are fully engaged in world trade) is the payoff dominant equilibrium, and this outcome can be achieved even if gains from trade differ between the countries. D'Artigues and Vignolo [33] find that this payoff dominant equilibrium is the long run equilibrium if the countries are symmetric. However, in a world of asymmetric countries, a country j chooses the autarkic strategy when all other countries are fully integrated into the world economy, provided certain restrictions hold on its payoff parameter values. This result captures the essence of *mimetic rivalry* (see Girard [58]) which explains why country j deliberately chooses a strategy that degrades the payoff to the leading country. The role of envy is discussed further in the next subsection.

2.2.2 The Role of Envy

Several empirical studies have emphasized the importance of *envy* as a motive for Pareto-optimality[57] rejection (see Elster [42]; Kim and Smith [123]; Schoeck [117]). The destructive envy behaviour is *Pareto-damaging* as it results in players lowering both their own payoffs and others' payoffs (see Frank [49]; Schimmel [116]). This negative emotion is incorporated into the framework of the D'Artigues-Vignolo model by constructing a psychological game in the sense of Geanakoplos, Pearce and Stacchetti [55]. The strategy changes involve the countries' own payoff and their relative payoff so that the envious country suffers if the

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[57] Pareto optimality, a measure of the efficiency of a game, is defined as the outcome where no player can obtain a better payoff without reducing at least one other player's payoff (note that this is a minimal notion of efficiency and does not always result in the socially desired outcomes).

opponent obtains a higher payoff and this results in the non-Pareto optimal outcome. Vignolo [134] finds that this happens as a result of the deviating player obtaining a higher status relative to his opponent in the non-coordinated outcome than in the Pareto-optimal equilibrium. Thus, envious behaviour causes the dynamics to move away from the payoff dominant equilibrium. According to Wobker and Kenning [138], *frustration* is one of the factors that influences the destructive envy behaviour. The D'Artigues-Vignolo model also presents terrorism as a result of competition between countries, when the desire of imitating the leading country is frustrated by the impossibility of doing so. These results of envious behaviour can be incorporated into the model in the following chapters to lend plausibility to the fact that the all-in-autarky state is a more favourable strict equilibrium than the other states (the fully integrated state and the heterogeneous states).

The next chapter defines the D'Artigues-Vignolo model and analyses the n-country scenario in order to obtain conditions required to hold true for strict equilibria and strictly stable equilibria.

Chapter 3

The Trade Game with Pure Strategies

Integration into the world economy and economic competitiveness are generally viewed as ways to generate positive gains[58] for all countries participating in international trade. However, even if global integration is a Pareto-dominant state[59], it can lead to rivalrous behaviour[60], where countries deliberately try to degrade the position of the leading country, when the desire to imitate the leading country is frustrated by the impossibility of doing so. This chapter introduces an evolutionary trade game model, as considered in D'Artigues and Vignolo [33], where countries choose between two strategies: integrating into the world economy, or not, that is staying in autarky; and describes the long-run behaviour of n countries.

Section 3.1 defines the D'Artigues-Vignolo trade model with n countries, playing a 2×2 coordination game, and defines the payoff to countries using a simple behavioural rule. The trade dynamics using a simple 2-country and a 3-country model are then derived, highlighting the strict equilibria and strictly stable states. The 4-country model is also briefly discussed. Section 3.2 defines the n-country trade model, formalizing the behavioural rule using new notation. The selection and mutation mechanisms are then defined and the conditions required for the homogeneous and heterogeneous states to be strict equilibria are analysed. Section 3.3 is an

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[58] Gains can be thought of as economic and social benefits arising from cross-country trade.

[59] Pareto-dominant state is the payoff dominant state, where given a choice, all the countries play strategies that give them the maximum possible payoff, as defined in Harsanyi and Selten [62].

[60] Using the theory of mimetic rivalry (see Girard [58]), competition between countries can lead to negative behaviour in the form of terrorism.

appendix containing the conditions required for strategy changes in the 3-country model.

3.1 The D'Artigues-Vignolo Trade Model

Consider a population of asymmetric[61] countries denoted by *C*. The players (countries) are boundedly rational[62] agents using past experience and a simple behavioural rule of trying to reduce the gap between the maximal payoff and their own payoff. Time is measured discretely (indexed by $t = 1, 2, 3, \ldots$) and in each time period[63], countries are matched exactly once with one another to play the game and adjust their behaviour over time. The model is summarized by a 2×2 coordination game in which each country chooses between two strategies *E – engaging in world trade* and *A – staying in autarky*, and plays the same strategy in a given time period, where 1 time period = 1 unit time.

In each unit time, with a small fixed probability, a country receives an opportunity to update its strategy. So the length of the time period does not matter, just the sequence in which the countries are chosen and the path of the trade dynamics leading to a steady state. In this chapter, it is assumed that only one country is given an opportunity to update its strategy in a unit time.

International trade between countries *i* and *j* is summarized by the following asymmetric coordination game:

	E	A
E	a_i, a_j	$0, \lambda_j$
A	$\lambda_i, 0$	λ_i, λ_j

where $0 < \lambda_i < a_i \; \forall i \in C$, a_i is the payoff to country i for complete integration into the world economy and λ_i the payoff for remaining in complete autarky.[64] Assume $a_1 > a_2 > \ldots > a_n$, so that country 1 is the natural leader. The fully integrated state (E, E) and the all-in-autarky state (A, A) are two strict Nash equilibria[65] in this coordination game[66].

The evolutionary dynamics that describe the long run behaviour of the countries are driven by the selection mechanism[67]. The selection mechanism in this model defines the way the countries choose their strategies using a variation of the *satisficing dynamics*[68] defined by Smallwood and Conlisk [122]. In this model, it is as-

[64] Integration and economic competition generate positive gains for all participants of international trade. Countries gain if they trade with one another as they can produce goods they are more efficient at producing and import the rest. However, if a country is willing to trade with another country i, and invests in resources towards production of goods for international trade, and country j chooses to not trade, then country i loses out on its investment while country j receives its autarkic payoff, whereby it utilizes its resources to become a self-sufficient economy. Note that the autarkic payoff is not the optimal payoff for country j as it can obtain higher payoff by engaging in trade with country i and sharing resources. This interaction between the countries is classified by a coordination game, as defined previously in Section 2.2.

[65] The fully integrated Nash equilibrium Pareto-dominates the all-in-autarky Nash equilibrium, so the payoff dominant outcome can be reached even if the gains from world trade differs between the countries.

[66] Note that a country's optimal strategy depends on its expectation of what the other country may do.

[67] Note that D'Artigues and Vignolo [33] derive the model from Kandori et al. [72], and they both consider long run behaviour of the countries driven by the selection mechanism and the mutation mechanism. Mutations in this chapter are only considered for testing stability.

[68] Smallwood and Conlisk [122] consider a market scenario where consumers buy a certain product, from a given number of brands. If they are satisfied with a particular brand in one time period, they continue to use the same brand over consequent periods. However, if the product breaks down (or if they are dissatisfied with a brand they are currently using), then they choose another brand in the next period. They find that the adaptive consumer behaviour can lead to a wide diversity of possible outcomes, for both the weakly dissatisfied consumer and the strongly dissatisfied consumer. Outcomes

sumed that the countries are myopic and adaptive, so that they do not form expectations about the future course of the play and simply take into account the decisions made in the past. The mutation mechanism takes into account a small probability with which each country plays a non-optimal strategy[69], so that the selection mechanism is perturbed[70].

In each time period t, a country plays a pure strategy $s \in \{E, A\}$. Let x_t be the number of countries playing strategy E at time t. The average payoff of country i playing strategy s, denoted by $\pi_i(s)$, is given by (see D'Artigues and Vignolo [33]):

$$\pi_i(E) = \frac{x_t - 1}{n - 1} a_i{}^{[71]},$$
$$(3.1)$$

$$\pi_i(A) = \frac{x_t}{n - 1} \lambda_i + \frac{n - x_t - 1}{n - 1} \lambda_i = \lambda_i{}^{[72]}.$$
$$(3.2)$$

Note that if a country $i \in C$ plays strategy A, then its payoff is λ_i, so the autarkic payoff is independent of the population distribution, where as if it chooses strategy

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[69] A mutation can be viewed as a deliberate experimentation of a new strategy by a country or exit of a country replaced with a new one knowing less or nothing about the game and so choosing a strategy at random.

[70] The mutation phenomenon can be viewed as a residual capturing of whatever has been excluded when modelling selection, see Samuelson [115].

[71] Note that in the n-country scenario, when country i plays strategy E against country j, it receives a payoff of a_i if j plays E and a payoff of 0 if j plays A. If there are x_t countries playing strategy E including country i, then it receives $(x_t - 1)a_i$ against $(x_t - 1)$ countries playing E, and 0 against $(n - (x_t - 1))$ countries playing A. Therefore, the average payoff to country i is: $\frac{x_t - 1}{n - 1} a_i$.

[72] Note that when country i plays strategy A against country j, it receives a payoff of λ_i regardless of what j plays. If there are x_t countries playing strategy E, then it receives $x_t \lambda_i$ against x_t countries playing E, and $(n - x_t - 1)\lambda_i$ against $(n - x_t - 1)$ countries playing A (excluding country i). Therefore, the average payoff to country i is: $\frac{x_t}{n - 1} \lambda_i + \frac{n - x_t - 1}{n - 1} \lambda_i$.

E, then its payoff depends on x_t, the number of countries participating in world trade. Note that two models of matching can generate the above payoffs – each country matched with each of the remaining countries exactly once, or an infinite number of random matches within period t so that each country's average payoff in that period is equal to the expected payoff, see Kandori et al. [72]. Each country's actions are assumed to be fixed within a period.

3.1.1 The Selection Mechanism

Let $s \in \{E, A\}$ denote the current strategy of country i. Each country is assumed to observe μ - the maximal payoff[73] over all countries that is attained by one or several countries in a given unit time. It then compares the maximal payoff to its average payoff earned from its current strategy s and chooses the strategy that reduces the gap between the maximal payoff and its own payoff[74]. This behavioural rule[75] is formally expressed as:

$$s' \in \arg\min_{s \in \{E, A\}} \left(\mu - \pi_i (s) \right), \tag{3.3}$$

where s' denotes the strategy played in the next period and $\pi_i(s)$ denotes the payoff obtained if country i plays s when other countries play the strategies from the current period.

Evolutionary game theory considers that countries do not simultaneously adjust their strategies following the behavioural rule (3.3) at each period. Rather, there is some inertia[76] in this model so that countries do not update their strategies frequently. So the strategies that prove to be effective for a country, at a given time

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[73] This will be defined more formally in terms of the n-country model in section 3.2.

[74] Note that the payoff from engaging in world trade can be the same as the autarkic payoff for country i - $\pi_i(E) = \pi_i(A)$, when $(x_t - 1)a_i = (n-1)\lambda_i$, where x is the number of countries playing strategy E in a n-country scenario. However, it is later assumed that the behavioural rule gives a unique best reply and the parameters are chosen such that the degenerate cases are omitted.

[75] This rule defines the selection mechanism for this model.

[76] See Kandori et al. [72].

period, are likely to remain effective in the subsequent time periods. In this model, countries *switch to* a different strategy in a period if the payoff realized by doing so is strictly greater than the payoff obtained from the strategy from the previous period. Otherwise, they *continue to play* their strategies from the previous period. A set of strategies constitutes an *equilibrium* if no country can do better under (3.3) by unilaterally changing its current strategy and a *strict equilibrium* if each country does explicitly worse by changing its strategy. Here *strict* refers to strict inequality in the equilibrium conditions so that each country is playing a strategy that is a unique best reply under (3.3).

For the analysis of the 2-country, 3-country and 4-country models, the following definition of *states* is used.

Definition 3.1. A *state* (or *strategy profile*) in the *n*-country scenario is defined as $(s_1, s_2, \ldots, s_n)$, where $s_i \in \{E, A\}$ and s_i is the strategy played by country *i*, for $i = 1, \ldots, n$ (at a given unit time).[77]

Definition 3.2. A state $Q = (s_1, s_2, \ldots, s_n)$, where either $s_i = E$ for all *i* or $s_i = A$ for all *i* is defined as a *homogeneous state*[78] in the *n*-country scenario. The rest of the states are defined as *heterogeneous states*[79].

Definition 3.3. A strategy profile $(s_1, s_2, \ldots, s_n)$ is an *equilibrium* when no country has any incentive to update its strategy when given an opportunity to do so.

Definition 3.4. A strategy profile $(s_1, s_2, \ldots, s_n)$ is a *strict equilibrium* when it is an equilibrium and each country does explicitly worse by changing its strategy.

The simple 2-country scenario, consisting of four states is analysed in Section 3.1.2. The 3-country scenario, consisting of eight states is analysed in Section

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[77] Note that there are 2^n states for the *n*-country model.

[78] Note that there are two homogeneous states in the *n*-country scenario: the fully integrated state $(E, \ldots, E)$ and the all-in-autarky state $(A, \ldots, A)$.

[79] Note that there are $2^n - 2$ heterogeneous states in the *n*-country scenario.

3.1.3. As each country chooses between the pure strategies – full integration (*E*) and complete autarky (*A*) in each scenario, the trade dynamics are obtained and presented graphically. The 4-country scenario is briefly examined in Section 3.1.4 and the heterogeneous states that cannot be strict equilibria are highlighted. The homogeneous states and the rest of the heterogeneous states in the 4-country scenario are left to be explored via numerical computations in the next chapter.

Note that one-shot mutations (non-optimal strategies in the sense of the behavioural rule (3.3)) are introduced in this model once it reaches a strict equilibrium, in order to measure stability. A mutant country playing strategy *E* or *A* switches to strategy *A* or *E*, respectively, while all other countries continue playing their original strategies. A strict equilibrium is stable with respect to such mutations if the mutant country is playing with a strictly worse strategy after the mutation, while the other countries have no better strategies and stick with their original strategies.

Definition 3.5. Let $Q = (s_1, s_2, \ldots, s_n)$ be an equilibrium strategy profile and $Q^{(k)}$ be a mutant strategy profile such that country k switches its strategy from s_k. Then, Q is a *stable equilibrium* if each country apart from *k* has no incentive to update its strategy when given an opportunity to do so, and country *k* updates its strategy to its original non-mutant strategy, and this must be true for each *k*.

Definition 3.6. Let $Q = (s_1, s_2, \ldots, s_n)$ be a stable equilibrium. Then, Q is *strictly stable* if each country apart from the mutant country does explicitly worse by changing its strategy while the mutant country does explicitly better by changing back to its non-mutant strategy.

The 2-country model, the 3-country model and the 4-country model is analysed in the following subsections.

3.1.2 The 2-Country Model

In this model, there are 4 states: (E, E), (E, A), (A, E), (A, A), each of which is analysed below.

Consider the fully integrated state (E,E). If country 1 is given the opportunity to update its strategy (in the next period), then it assumes that country 2 will continue to play strategy E and calculates its own payoff for strategies E and A. It then compares $\pi_1(E)$ and $\pi_1(A)$ with the maximal payoff realized, and chooses the strategy that reduces the difference between the maximal payoff realized and its own payoff. The payoffs are calculated as: $\pi_1(E) = a_1$, $\pi_2(E) = a_2$, $\mu = a_1$, so country 1 continues to play E as it yields the maximal payoff. This dynamic is represented by a circular arrow around state (E,E) in Figure 3.1, as country 1 has no incentive to update its strategy. On the other hand, if country 2 is given an opportunity to update its strategy, it calculates $\pi_2(E)$, $\pi_2(A)$ (which is a_2, λ_2, respectively) and the maximal payoff in each case (which is a_1, λ_2, respectively). The maximal payoff realised when country 2 plays E is $\mu = \max\{\pi_1(E), \pi_2(E)\} = \max\{a_1, a_2\} = a_1$; and when it plays A: $\mu = \max\{\pi_1(E), \pi_2(A)\} = \max\{0, \lambda_2\} = \lambda_2$. Hence, country 2 realizes the maximal payoff by switching to strategy A.[80] This dynamic is represented by an arrow from state (E,E) to state (E,A) in Figure 3.1, as country 2 updates its strategy to A when given an opportunity to do so.

Consider the all-in-autarky state (A,A). If country 1 is given the opportunity to update its strategy (in the next period), then it assumes that country 2 will continue to play strategy A (obtaining λ_2 as its payoff) and calculates its own payoff for strategies E and A as $\pi_1(E) = 0$ and $\pi_1(A) = \lambda_1$, respectively. The behavioural rule (3.3) then implies that country 1 will continue to play A ($\mu = \lambda_2$ when it plays E, and $\mu = \max\{\lambda_1, \lambda_2\}$ when it plays A, and $\lambda_2 - 0 > \max\{\lambda_1, \lambda_2\} - \lambda_1$). Similarly, when country 2 is given the opportunity to update its strategy, it calculates its payoffs for strategies E and A as 0 and λ_2, respectively, and continues to play A using (3.3). These dynamics are represented by two circular arrows around state (A,A)

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[80] Note that the payoff difference is $\mu - \pi_2(E) = a_1 - a_2$ when country 2 plays strategy E, and the payoff difference is $\mu - \pi_2(A) = \lambda_2 - \lambda_2 = 0$ when it plays strategy A. So it chooses strategy A following the behavioural rule (3.3).

in Figure 3.1, as neither country has incentive to switch from strategy A to E when the other country continues to play A. Hence, the all-in-autarky state (A, A) is a strict equilibrium with respect to the behavioural rule (3.3).

Next, consider the heterogeneous state (E, A). If country 1 is given the opportunity to update its strategy, then it switches to strategy A under the behavioural rule (3.3) , as continuing to play E (when country 2 plays A, obtaining a constant payoff of λ_2) yields a payoff of 0, whereas switching to strategy A yields a payoff of λ_1. This dynamic is represented by an arrow from state (E, A) to state (A, A) in Figure 3.1. On the other hand, if country 2 is given the opportunity to update its strategy, then continuing to play strategy A yields the maximum payoff of λ_2 while country 1 receives 0, and switching to strategy E yields a payoff of a_2 while country 1 receives a_1. Therefore, country 2 continues to play strategy A as it yields the maximum payoff in that round. This dynamic is represented by a circular arrow around state (E, A) in Figure 3.1.

Finally, consider the heterogeneous state (A, E). If country 1 is given the opportunity to update its strategy, then it switches to strategy E as it yields the maximum payoff of a_1 whereas continuing to play A would only yield a payoff of λ_1. This dynamic is represented by an arrow from state (A, E) to state (E, E) in Figure 3.1. If country 2 is given the opportunity to update its strategy, then it switches to strategy A under the behavioural rule (3.3) as $\pi_2(A) = \lambda_2$ and $\pi_2(E) = 0$, while country 1 receives a constant payoff of λ_1 by playing strategy A. This dynamic is represented by an arrow from state (A, E) to state (A, A) in Figure 3.1 as it is beneficial for country 2 to switch to strategy A when country 1 plays A.

The only strict equilibrium from the above analysis is the all-in-autarky state (A, A) in the sense that neither country 1 nor country 2 can do better by switching (unilaterally) to strategy E.

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Consider a single mutation away from the all-in-autarky state (A, A) to the hetero-geneous state (E, A). Then, the trade dynamics either remain at (E, A) (as country 2 sticks with strategy E) or move back to (A, A) (as country 1 updates to strategy A). Similarly, a single mutation from (A, A) to (A, E) leads the trade dynamics back to (A, A) if country 2 is given the opportunity to update its strategy, otherwise the dynamic $(A, E) \rightarrow (E, E) \rightarrow (E, A) \rightarrow (A, A)$ follows a full circle back to the all-in-autarky state. Thus, (A, A) is also strictly stable with respect to single muta-tions[81] in the long run in the sense that a random deviation one state away from the all-in-autarky equilibrium will eventually lead the dynamics back to the strict equi-librium (A, A) under the behavioural rule (3.3). Therefore, the all-in-autarky state is the long-run strict equilibrium for the 2-country model.
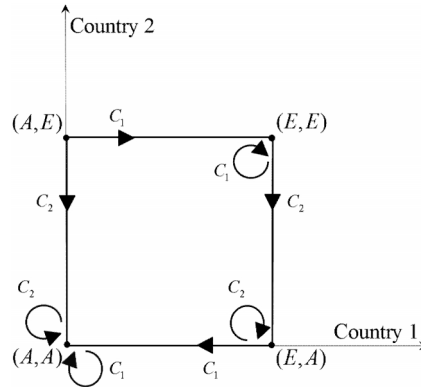
Figure 3.1. Trade dynamics of the 2-country model.

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[81] In this case mutations refer to countries randomly updating their strategies, which can be thought of as experimentation in order to achieve better payoffs so that the gap is reduced, or exit of one of the countries from the trade game, replaced by a new country with less or no knowledge about the game and thus choosing a strategy at random. These mutations occur after the completion of the learning adjustment by the selection mechanism (3.3), so that countries are already playing strategies that min-imize the gap between the maximal payoff and their own payoffs when mutations occur.
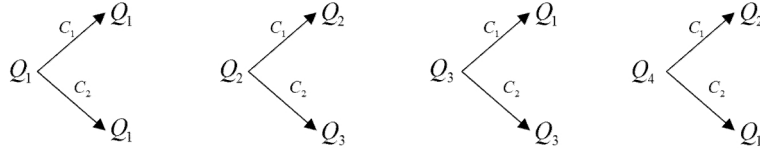
Figure 3.2. Tree diagram representing a complete set of possible dynamics from each of the eight states of the 2-country model as countries 1 and 2 update their strategies, where $Q_1 = (A,A)$, $Q_2 = (E,E)$, $Q_3 = (E,A)$ and $Q_4 = (A,E)$.

3.1.3 The 3-Country Model

In this model, there are 8 states: $Q_1 \equiv (E,E,E)$, $Q_2 \equiv (A,E,E)$, $Q_3 \equiv (A,A,E)$, $Q_4 \equiv (E,A,E)$, $Q_5 \equiv (E,A,A)$, $Q_6 \equiv (E,E,A)$, $Q_7 \equiv (A,E,A)$, $Q_8 \equiv (A,A,A)$[82], each of which is analysed below.

The term *switches to* refers to the *strict inequality* conditions and *continues to play* refers to the *non-strict inequality* conditions, as countries update their strategies only if they obtain a strictly greater payoff than obtained from the strategies from the previous period.

First, consider the fully integrated state $Q_1 \equiv (E,E,E)$.

Country 1 will not update its strategy when given the opportunity to do so, as it receives the maximal payoff of a_1 by playing E.

Country 2 when given an opportunity to update its strategy, will calculate the pay-offs using (3.1) and (3.2) to be $\pi_2(E) = a_2$, $\mu = a_1$ when it plays E, and $\pi_2(A) = \lambda_2$, $\mu = \max\left\{\dfrac{a_1}{2}, \lambda_2\right\}$[83] when it plays A. It will only continue playing E if:

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[82] In each state, the first entry represents country 1's strategy, the second entry represents country 2's strategy and the third entry represents country 3's strategy.

$$a_1 - a_2 \leq \max\left\{\frac{a_1}{2}, \lambda_2\right\} - \lambda_2, \tag{3.4}$$

using the behavioural rule (3.3). If $\frac{a_1}{2} \leq \lambda_2$, then (3.4) cannot be satisfied as $a_1 > a_2$ and country 2 will switch to strategy A as it will yield the maximal payoff. So $\frac{a_1}{2} > \lambda_2$ is a necessary condition for country 2 to continue playing E. Condition (3.4) then simplifies to $a_2 - \lambda_2 \geq \frac{a_1}{2}$.

Country 3 will continue playing E if:

$$a_1 - a_3 \leq \max\left\{\frac{a_1}{2}, \lambda_3\right\} - \lambda_3.^{84} \tag{3.5}$$

If $\frac{a_1}{2} \leq \lambda_3$, then country 3 will switch to strategy A as it will yield the maximal pay-off. So $\frac{a_1}{2} > \lambda_3$ is a necessary condition for country 3 to continue playing E. Condition (3.5) then simplifies to $a_3 - \lambda_3 \geq \frac{a_1}{2}$. Hence, the fully integrated state Q_1 is a strict equilibrium when the following conditions hold:

$$a_2 - \lambda_2 \geq \frac{a_1}{2} > \lambda_2, \ a_3 - \lambda_3 \geq \frac{a_1}{2} > \lambda_3. \tag{3.6}$$

Next, consider the heterogeneous state $Q_2 \equiv (A, E, E)$.

If country 1 receives the opportunity to update its strategy, then it switches to E as it obtains the maximal payoff of a_1 using (3.1). So Q_1 is not a strict equilibrium and

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[83] $\mu = \max\left\{\frac{a_1}{2}, \lambda_2, \frac{a_3}{2}\right\} = \max\left\{\frac{a_1}{2}, \lambda_2\right\}$ as $a_1 > a_3$.

[84] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2) as follows: $\pi_1(E) = a_1$, $\pi_2(E) = a_2$, $\pi_3(E) = a_3$, $\mu = a_1$ when country 3 plays E and

$\pi_1(A) = \frac{a_1}{2}$, $\pi_2(E) = \frac{a_2}{2}$, $\pi_3(A) = \lambda_3$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_3\right\}$ when country 3 plays A.

the trade dynamics move from Q_2 to $Q_1 \equiv (E,E,E)$. The conditions required for countries 2 and 3 to continue playing strategy E are listed in Table 3.1.

Next, consider the heterogeneous state $Q_3 \equiv (A,A,E)$.

When country 3 receives the opportunity to update its strategy, it assumes countries 1 and 2 will continue to play A and calculates the difference between the maximal payoff and its own payoff for strategies E and A. The maximal payoff is realized if it plays strategy E is $\max\{\lambda_1, \lambda_2\}$ and its own payoff is $\pi_3(E) = 0$. The maximal payoff realized if it plays strategy A is $\max\{\lambda_1, \lambda_2, \lambda_3\}$ and its own payoff is $\pi_3(A) = \lambda_3$. Since $\max\{\lambda_1, \lambda_2, \lambda_3\} - \lambda_3 < \max\{\lambda_1, \lambda_2\} - 0$ [85], country 3 will uncondi-tionally update its strategy from E to A.

Similarly, if the heterogeneous state $Q_7 \equiv (A,E,A)$ is considered, country 2 will update its strategy from E to A when given an opportunity to do so [86].

Moreover, if state $Q_5 \equiv (E,A,A)$ is considered, then country 1 will update its strat-egy from E to A when given an opportunity to do so [87]. These dynamics ($Q_3 \to Q_8$, $Q_7 \to Q_8$, $Q_5 \to Q_8$) are represented by double-arrows in Figure 3.5 as no extra conditions are required for countries 1, 2, 3 to switch to A when in states Q_3, Q_5, Q_7, respectively.

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[85] If $\max\{\lambda_1, \lambda_2, \lambda_3\} = \lambda_3$, then country 3 realizes maximal payoff by switching to strategy A. If $\lambda_3 < \max\{\lambda_1, \lambda_2\}$, then the maximal payoff is $\mu = \max\{\lambda_1, \lambda_2\}$ and country 3 reduces the gap by switching to strategy A as $\mu - \lambda_3 < \mu - 0$ (playing strategy E yields 0 as payoff).

[86] If country 2 plays E, then $\pi_1(A) = \lambda_1$, $\pi_2(E) = 0$, $\pi_3(A) = \lambda_3$, $\mu = \max\{\lambda_1, \lambda_3\}$. If country 2 plays A, then $\pi_1(A) = \lambda_1$, $\pi_2(A) = \lambda_2$, $\pi_3(E) = \lambda_3$, $\mu = \max\{\lambda_1, \lambda_2, \lambda_3\}$. Since $\max\{\lambda_1, \lambda_3\} - 0 > \max\{\lambda_1, \lambda_2, \lambda_3\} - \lambda_2$, the behavioural rule (3.3) implies that country 2 updates it strategy from E to A.

[87] If country 1 plays E, then $\pi_1(E) = 0$, $\pi_2(A) = \lambda_2$, $\pi_3(A) = \lambda_3$, $\mu = \max\{\lambda_2, \lambda_3\}$. If country 1 plays A, then $\pi_1(A) = \lambda_1$, $\pi_2(A) = \lambda_2$, $\pi_3(A) = \lambda_3$, $\mu = \max\{\lambda_1, \lambda_2, \lambda_3\}$. Since $\max\{\lambda_2, \lambda_3\} - 0 > \max\{\lambda_1, \lambda_2, \lambda_3\} - \lambda_1$, the behavioural rule (3.3) implies that country 1 updates it strategy from E to A.

Next, consider the heterogeneous state $Q_4 \equiv (E, A, E)$.

For Q_4 to be a strict equilibrium, countries 1, 2, 3 are required to stick to strategies E, A, E, respectively, when given opportunities to update their strategies.

Country 1 will continue playing E if:

$$\max\left\{\frac{a_1}{2}, \lambda_2\right\} - \frac{a_1}{2} \leq \max\{\lambda_1, \lambda_2\} - \lambda_1.^{88} \tag{3.7}$$

If $\lambda_1 \geq \lambda_2$, then country 1 switches to A as it yields the maximal payoff. So $\lambda_1 < \lambda_2$ is a necessary condition for (3.7) to hold. If $\frac{a_1}{2} \geq \lambda_2$, then (3.7) holds. If $\frac{a_1}{2} < \lambda_2$, then for (3.7) to hold, the following conditions need to hold: $\lambda_1 < \lambda_2$ and $-\frac{a_1}{2} \leq -\lambda_1$. Hence, in order to satisfy (3.7), one of these two conditions need to hold: $\lambda_1 < \lambda_2 \leq \frac{a_1}{2}$ or $\lambda_1 \leq \frac{a_1}{2} < \lambda_2$.

Country 2 will continue playing A if:

$$a_1 - a_2 \geq \max\left\{\frac{a_1}{2}, \lambda_2\right\} - \lambda_2.^{89} \tag{3.8}$$

If $\frac{a_1}{2} \leq \lambda_2$, then country 2 receives the maximal payoff by playing A. If $\frac{a_1}{2} > \lambda_2$, then for (3.8) to hold, the following condition needs to hold: $a_2 - \lambda_2 \leq \frac{a_1}{2}$.

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[88] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2) as follows: $\pi_1(E) = \frac{a_1}{2}$, $\pi_2(A) = \lambda_2$, $\pi_3(E) = \frac{a_3}{2}$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_2\right\}$ when country 1 plays E and $\pi_1(A) = \lambda_1$, $\pi_2(A) = \lambda_2$, $\pi_3(E) = 0$, $\mu = \max\{\lambda_1, \lambda_2\}$ when country 1 plays A.

[89] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2) as follows: $\pi_1(E) = a_1$, $\pi_2(E) = a_2$, $\pi_3(E) = a_3$, $\mu = a_1$ when country 2 plays E, and $\pi_1(E) = \frac{a_1}{2}$, $\pi_2(A) = \lambda_2$, $\pi_3(E) = \frac{a_3}{2}$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_2\right\}$ when country 2 plays A.

Country 3 will continue playing E if:

$$\max\left\{\frac{a_1}{2}, \lambda_2\right\} - \frac{a_3}{2} \leq \max\{\lambda_2, \lambda_3\} - \lambda_3 .^{90} \tag{3.9}$$

If $\lambda_2 \leq \lambda_3$, then country 3 realizes the maximal payoff by switching to strategy A.

Thus, $\lambda_2 > \lambda_3$ and (3.9) can be simplified to $\frac{a_3}{2} - \lambda_3 \geq \max\left\{\frac{a_1}{2}, \lambda_2\right\} - \lambda_2$. If $\frac{a_1}{2} \leq \lambda_2$,

then (3.9) simplifies to $\frac{a_3}{2} \geq \lambda_3$. If $\frac{a_1}{2} > \lambda_2$, then (3.9) simplifies to $\frac{a_3}{2} - \lambda_3 \geq \frac{a_1}{2} - \lambda_2$.

State Q_4 is a strict equilibrium if the following conditions hold:

$$\left\{\left[\lambda_1 < \lambda_2 \leq \frac{a_1}{2}\right] \text{ or } \left[\lambda_1 \leq \frac{a_1}{2} < \lambda_2\right]\right\}, \left\{\left[\frac{a_1}{2} \leq \lambda_2\right] \text{ or } \left[\lambda_2 < \frac{a_1}{2}, \ a_2 - \lambda_2 \leq \frac{a_1}{2}\right]\right\},$$

$$\left\{\left[\frac{a_1}{2} \leq \lambda_2, \ \lambda_3 \leq \frac{a_3}{2}, \ \lambda_3 < \lambda_2\right] \text{ or } \left[\lambda_3 < \lambda_2 < \frac{a_1}{2}, \ \frac{a_1}{2} - \lambda_2 \leq \frac{a_3}{2} - \lambda_3\right]\right\}.^{91} \tag{3.10}$$

The conditions in (3.10) can be re-written as:

$$\left[\lambda_3 < \lambda_2, \ \lambda_1 < \lambda_2 \leq \frac{a_1}{2}, \ a_2 - \lambda_2 \leq \frac{a_1}{2}, \ a_1 - 2\lambda_2 \leq a_3 - 2\lambda_3\right] \text{ or }$$

$$\left[\lambda_3 < \lambda_2, \ \lambda_1 \leq \frac{a_1}{2} < \lambda_2, \ \lambda_3 < \frac{a_3}{2}\right]. \tag{3.11}$$

Hence if either of the conditions in (3.11) hold, then once the trade dynamics are in state Q_4, countries 1, 2, 3 will have no incentive to change from strategies E, A, E, respectively, making Q_4 a strict equilibrium.

Next, consider the heterogeneous state $Q_6 \equiv (E, E, A)$.

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[90] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2)

as follows: $\pi_1(E) = \frac{a_1}{2}$, $\pi_2(A) = \lambda_2$, $\pi_3(E) = \frac{a_3}{2}$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_2\right\}$ when country 3 plays E, and

$\pi_1(E) = 0$, $\pi_2(A) = \lambda_2$, $\pi_3(A) = \lambda_3$, $\mu = \max\{\lambda_2, \lambda_3\}$ when country 3 plays A.

[91] Three conditions need to hold, each represented by $\{\cdot\}$, and either sub-condition needs to hold

within each condition $\{\cdot\}$.

For Q_6 to be a strict equilibrium, countries 1, 2, 3 are required to stick to strategies E, E, A, respectively, when given opportunities to update their strategies.

Country 1 will continue playing E if:

$$\max\left\{\frac{a_1}{2}, \lambda_3\right\} - \frac{a_1}{2} \leq \max\left\{\lambda_1, \lambda_3\right\} - \lambda_1.\text{[92]} \tag{3.12}$$

If $\lambda_1 \geq \lambda_3$, then country 1 will realize its maximal payoff by switching to strategy A. Hence $\lambda_1 < \lambda_3$, which gives $\max\left\{\lambda_1, \lambda_3\right\} = \lambda_3$. So (3.12) can be simplified as $\max\left\{\frac{a_1}{2}, \lambda_3\right\} - \frac{a_1}{2} \leq \lambda_3 - \lambda_1$. If $\frac{a_1}{2} \geq \lambda_3$, then (3.12) is satisfied as $\lambda_1 < \lambda_3$.

If $\frac{a_1}{2} < \lambda_3$, then (3.12) holds if $\lambda_3 - \frac{a_1}{2} \leq \lambda_3 - \lambda_1 \Leftrightarrow \lambda_1 \leq \frac{a_1}{2}$. Hence, (3.12) is satisfied if one of these two conditions hold: $\lambda_1 < \lambda_3 \leq \frac{a_1}{2}$ or $\lambda_1 \leq \frac{a_1}{2} < \lambda_3$.

Country 2 will continue playing E if:

$$\max\left\{\frac{a_1}{2}, \lambda_3\right\} - \frac{a_2}{2} \leq \max\left\{\lambda_2, \lambda_3\right\} - \lambda_2.\text{[93]} \tag{3.13}$$

If $\lambda_2 \geq \lambda_3$, then country 2 will realize the maximal payoff by switching to strategy A. Hence $\lambda_2 < \lambda_3$ and this simplifies (3.13) to $\max\left\{\frac{a_1}{2}, \lambda_3\right\} - \frac{a_2}{2} \leq \lambda_3 - \lambda_2$. If $\frac{a_1}{2} > \lambda_3$, then (3.13) further simplifies to $\frac{a_1}{2} - \lambda_3 \leq \frac{a_2}{2} - \lambda_2$. If $\frac{a_1}{2} \leq \lambda_3$, then (3.13)

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[92] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2) as follows: $\pi_1(E) = \frac{a_1}{2}$, $\pi_2(E) = \frac{a_2}{2}$, $\pi_3(A) = \lambda_3$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_3\right\}$ when country 1 plays E, and $\pi_1(A) = \lambda_1$, $\pi_2(E) = 0$, $\pi_3(A) = \lambda_3$, $\mu = \max\left\{\lambda_1, \lambda_3\right\}$ when country 1 plays A.

[93] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2) as follows: $\pi_1(E) = \frac{a_1}{2}$, $\pi_2(E) = \frac{a_2}{2}$, $\pi_3(A) = \lambda_3$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_3\right\}$ when country 2 plays E, and $\pi_1(E) = 0$, $\pi_2(A) = \lambda_2$, $\pi_3(A) = \lambda_3$, $\mu = \max\left\{\lambda_2, \lambda_3\right\}$ when country 2 plays A.

can be simplified to $\frac{a_2}{2} \geq \lambda_2$. Hence, (3.13) is satisfied if one of these two conditions hold:

$$\left[\frac{a_1}{2} > \lambda_3 > \lambda_2, \frac{a_1}{2} - \lambda_3 \leq \frac{a_2}{2} - \lambda_2\right] \text{ or } \left[\lambda_2 < \lambda_3, \frac{a_1}{2} \leq \lambda_3, \lambda_2 \leq \frac{a_1}{2}\right].$$

Country 3 will continue playing A if:

$$a_1 - a_3 \geq \max\left\{\frac{a_1}{2}, \lambda_3\right\} - \lambda_3 .^{94} \tag{3.14}$$

If $\frac{a_1}{2} \leq \lambda_3$, then country 3 realizes the maximal payoff if it continues to play A. If $\frac{a_1}{2} > \lambda_3$, then (3.14) simplifies to $\frac{a_1}{2} \geq a_3 - \lambda_3$. State Q_6 is stable if the following conditions hold:

$$\left\{\left[\frac{a_1}{2} \geq \lambda_3\right] \text{ or } \left[\lambda_1 \leq \frac{a_1}{2} < \lambda_3\right]\right\}, \left\{\left[\frac{a_1}{2} \geq \lambda_3 > \lambda_2\right] \text{ or } \left[\lambda_3 > \lambda_2, \frac{a_1}{2} < \lambda_3, \frac{a_2}{2} \geq \lambda_2\right]\right\},$$

$$\left\{\left[\frac{a_1}{2} \leq \lambda_3\right] \text{ or } \left[\lambda_3 < \frac{a_1}{2}, a_3 - \lambda_3 \leq \frac{a_1}{2}\right]\right\}.^{95} \tag{3.15}$$

Conditions in (3.15) can be re-written as:

$$\left[\frac{a_1}{2} > \lambda_3 > \lambda_2, a_3 - \lambda_3 < \frac{a_1}{2}\right] \text{ or } \left[\lambda_1 \leq \frac{a_1}{2} < \lambda_3, \lambda_2 < \lambda_3, \frac{a_2}{2} \geq \lambda_2\right] \text{ or }$$

$$\left[\lambda_1 < \lambda_3, \lambda_2 < \lambda_3 = \frac{a_1}{2}\right]. \tag{3.16}$$

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[94] Condition obtained using the behavioural rule (3.3) and the payoffs calculated using (3.1) and (3.2) as follows: $\pi_1(E) = a_1$, $\pi_2(E) = a_2$, $\pi_3(E) = a_3$, $\mu = a_1$ when country 3 plays E, and

$\pi_1(E) = \frac{a_1}{2}$, $\pi_2(E) = \frac{a_2}{2}$, $\pi_3(A) = \lambda_3$, $\mu = \max\left\{\frac{a_1}{2}, \lambda_3\right\}$ when country 3 plays A.

[95] Three conditions need to hold, each represented by $\{\cdot\}$, and either sub-condition needs to hold within each condition $\{\cdot\}$.

Hence if either of the conditions hold in (3.6), (3.11), (3.16), then once the trade dynamics are in state Q_6, countries 1, 2, 3 will have no incentive to change from strategies E, E, A, respectively, making Q_6 a strict equilibrium.

Next, consider the all-in-autarky state $Q_8 \equiv (A, A, A)$.

If country 1 receives the opportunity to update its strategy, then switching to E yields a payoff of 0, while a payoff of λ_1 is realized if it continues to play A. If $\mu = \max\{\lambda_1, \lambda_2, \lambda_3\} = \lambda_1$, then λ_1 is the maximal payoff, and it continues to play A. However, if the maximal payoff $\mu = \max\{\lambda_2, \lambda_3\}$, then (3.3) implies strategy A is favourable over strategy E. Similarly, if countries 2 and 3 receive opportunities to update their strategies at different time periods, then they continue to play A as it either yields the maximal payoff in the given time period, or it minimizes the difference between the maximal payoff and their own payoff. Thus, the trade dynamics move from states Q_3, Q_7, Q_5 to Q_8 as countries 3, 2, 1, respectively, switch to strategy A when they receive an opportunity to do so. As neither of the countries have an incentive to switch to E once in state (A, A, A), Q_8 is a strict equilibrium.

Table 3.1 lists the conditions required for the countries to continue playing the same strategies, which essentially form conditions for the states to be strict equilibria. A quick glance at this table shows that states Q_2, Q_3, Q_7, Q_5 cannot be strict equilibria as countries 1, 3, 2, 1 update their strategies when in these states, respectively. This recaptures the analysis obtained in Subsection 3.1.3. The conditions required for countries to update their strategies are essentially the reverse conditions of those listed in Table 3.1. These conditions are used as a basis for constructing the tree diagram in Figure 3.3, which shows the dynamics moving from one state to another when a country is given an opportunity to update its strategy.

States	Country (receiving the opportunity to update strategy)		
	C_1	C_2	C_3
$Q_1 \equiv (E,E,E)$	Continue to play E	Continue to play E if $a_2 - \lambda_2 \geq \dfrac{a_1}{2} > \lambda_2$	Continue to play E if $a_3 - \lambda_3 \geq \dfrac{a_1}{2} > \lambda_3$
$Q_2 \equiv (A,E,E)$	Switch to E	Continue to play E if $\left[\dfrac{a_2}{2} - \lambda_2 \geq \beta - \lambda_1, \ \lambda_1 > \lambda_2 \right]$	Continue to play E if $\left[\dfrac{a_3}{2} - \lambda_3 \geq \beta - \lambda_1, \ \lambda_1 > \lambda_3 \right]$
$Q_3 \equiv (A,A,E)$	Continue to play A if $\left[\dfrac{a_1}{2} - \lambda_1 \leq \alpha - \lambda_2, \ \lambda_1 < \lambda_2 \right]$ or $\left[\lambda_1 \geq \lambda_2 \right]$	Continue to play A if $\left[\dfrac{a_2}{2} - \lambda_2 \leq \beta - \lambda_1, \ \lambda_1 > \lambda_2 \right]$ or $\left[\lambda_1 < \lambda_2 \right]$	Switch to A
$Q_4 \equiv (E,A,E)$	Continue to play E if $\left[\dfrac{a_1}{2} - \lambda_1 \geq \alpha - \lambda_2, \ \lambda_1 < \lambda_2 \right]$	Continue to play A if $a_2 - \lambda_2 \leq a_1 - \alpha$	Continue to play E if $\dfrac{a_3}{2} - \lambda_3 \geq \alpha - \lambda_2$
$Q_5 \equiv (E,A,A)$	Switch to A	Continue to play A if $\left[\dfrac{a_2}{2} - \lambda_2 \leq \gamma - \lambda_3, \ \lambda_2 < \lambda_3 \right]$ or $\left[\lambda_2 > \lambda_3 \right]$	Continue to play A if $\left[\dfrac{a_3}{2} - \lambda_3 \leq \alpha - \lambda_2, \ \lambda_2 > \lambda_3 \right]$ or $\left[\lambda_2 < \lambda_3 \right]$
$Q_6 \equiv (E,E,A)$	Continue to play E if $\left[\dfrac{a_1}{2} - \lambda_1 \geq \gamma - \lambda_3, \ \lambda_1 < \lambda_3 \right]$	Continue to play E if $\left[\dfrac{a_2}{2} - \lambda_2 \geq \gamma - \lambda_3, \ \lambda_2 < \lambda_3 \right]$	Continue to play A if $a_3 - \lambda_3 \leq a_1 - \gamma$
$Q_7 \equiv (A,E,A)$	Continue to play A if $\left[\dfrac{a_1}{2} - \lambda_1 \leq \gamma - \lambda_3, \ \lambda_1 < \lambda_3 \right]$ or $\left[\lambda_1 > \lambda_3 \right]$	Switch to A	Continue to play A if $\left[\dfrac{a_3}{2} - \lambda_3 \leq \beta - \lambda_1, \ \lambda_1 > \lambda_3 \right]$ or $\left[\lambda_1 < \lambda_3 \right]$
$Q_8 \equiv (A,A,A)$	Continue to play A	Continue to play A	Continue to play A
where, $\alpha = \max\left\{ \lambda_2, \dfrac{a_1}{2} \right\}$, $\ \beta = \max\left\{ \lambda_1, \dfrac{a_2}{2} \right\}$, $\ \gamma = \max\left\{ \lambda_3, \dfrac{a_1}{2} \right\}$			

Table 3.1. Conditions required for countries in the 3-country model to stick with their strategies (if it is advantageous), causing the dynamics to remain in the states given in the left column. (Note that countries update their strategies if the reverse strict inequalities exist above. Countries are said to stick with their current strategies in case of equality however these cases are not considered due to degeneracy.)
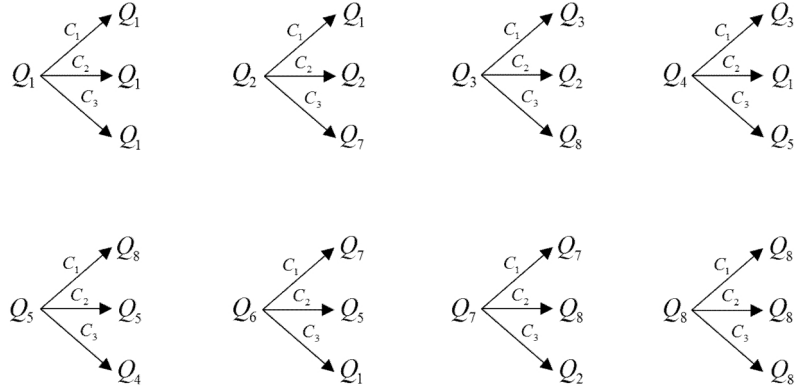
Figure 3.3. Tree diagram representing a complete set of possible dynamics from each of the eight states of the 3-country model, when the fully integrated state and the all-in-autarky states are equilibria of the trade game.[96]

Lemma 3.1. In the 3-country model, the all-in-autarky state is always a strict equilibrium, and there can exist at most one other strict equilibrium.

Proof. The analysis above shows that states Q_2, Q_3, Q_7, Q_5 cannot be strict equilibria. It has also been shown previously in this subsection that the all-in-autarky state is a strict equilibrium. The rest of the states: Q_1, Q_4, Q_6 are strict equilibria if the strict inequality conditions in (3.6), (3.11), (3.16) hold, respectively.[97] Note that states Q_4 and Q_6 cannot both be strict equilibria simultaneously as a necessary condition for state Q_4 to be a strict equilibrium is $\lambda_2 > \lambda_3$ while a necessary condition for state Q_6 to be an equilibrium is $\lambda_2 < \lambda_3$. Furthermore, when Q_1 is a strict equilibrium, both the conditions in (3.6) hold. Then, Q_4 cannot be a strict equilibrium as neither of the following strict inequality conditions from (3.11) hold:

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[96] Consider the game to be in state Q_2. If country 1 is given an opportunity to update its strategy, then referring to Table 3.1, it switches to E, and this dynamic is represented by $Q_2 \xrightarrow{C_1} Q_1$. The condition required for country 2 to switch to A needs to hold in Table 3.1 for the dynamic represented by $Q_2 \xrightarrow{C_2} Q_3$. Similarly, the condition required for country 3 to switch to A needs to hold for the dynamic represented by $Q_2 \xrightarrow{C_3} Q_7$. The rest of the dynamics are obtained in a similar way.

[97] Note that for a given state to be a strict equilibrium, strict inequality needs to hold for the conditions listed for that state.

$a_2 - \lambda_2 < \dfrac{a_1}{2}$, $\dfrac{a_1}{2} \le \lambda_2$. Similarly, Q_6 cannot be a strict equilibrium as neither of the strict inequality conditions from (3.16) hold: $a_3 - \lambda_3 < \dfrac{a_1}{2}$, $\dfrac{a_1}{2} \le \lambda_3$. Hence, there are two strict equilibria at most, one of which is always the all-in-autarky state. □

Lemma 3.2. In the 3-country model, based on topology, no two neighbouring states (that are connected by an edge) can be strict equilibria in the same unit time.[98]

Proof. It has already been shown previously in this subsection that Q_3, Q_7, Q_5 cannot be strict equilibria. For topological consideration, see Figure 3.5 - Q_8 is connected via edges to states Q_3, Q_7, Q_5. And since Q_8 is a strict equilibrium, the trade dynamics move from states Q_3, Q_7, Q_5 to Q_8 and so Q_3, Q_7, Q_5 cannot be strict equilibria. Similarly, for all the other vertices that represent the states, no two vertices can be strict equilibria at the same time. □

Consider the case where Q_1 and Q_8 are the strict equilibria in the trade game.

The conditions required for both countries 2 and 3 to continue playing strategy E when in state Q_1 are obtained from Table 3.1 as: $a_2 - \lambda_2 > \dfrac{a_1}{2} > \lambda_2$, $a_3 - \lambda_3 > \dfrac{a_1}{2} > \lambda_3$. This means that country 2 switches to strategy E when the game is in state Q_4, and country 3 switches to strategy E when the game is in state Q_6, see strategy changes in Table 3.1. This gives the following trade dynamics: $Q_4 \to Q_1$, $Q_6 \to Q_1$. These dynamics are represented by single arrows[99] in Figure 3.5. If a further assumption of $\lambda_1 > \lambda_2 > \lambda_3$ is made, so that the natural leader (country 1) is also relatively more self-sufficient than country 2 and 3, then the reverse conditions from Table 3.1 give the following dynamics from states Q_6 and Q_4 (the country given an opportunity to update its strategy is denoted above the arrows for each dynamic): $Q_6 \xrightarrow{C_1} Q_7$,

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[98] Note that this lemma can be extended to the general *n*-country model.

[99] Single arrows are used for these dynamics to differentiate them from the ones that hold true for all the a_i's and the λ_i's – represented by double arrows.

$Q_6 \xrightarrow{C_2} Q_5$, $Q_6 \xrightarrow{C_3} Q_1$ (as obtained previously in this subsection), $Q_4 \xrightarrow{C_1} Q_3$, $Q_4 \xrightarrow{C_2} Q_1$ (as obtained previously in this subsection), $Q_4 \xrightarrow{C_3} Q_5$ if $\frac{a_3}{2} - \lambda_3 < \frac{a_1}{2} - \lambda_2$.

Next, consider state Q_2. It has already been obtained from previous analysis that country 1 switches to strategy E, moving the trade dynamics to state Q_1. The condition required to hold for country 2 to update its strategy to A, that can potentially move the trade dynamics to state Q_3, is obtained using the reverse conditions from Table 3.1 as follows: $\frac{a_2}{2} - \lambda_2 < \max\left\{\lambda_1, \frac{a_2}{2}\right\} - \lambda_1$. If $\lambda_1 \geq \frac{a_2}{2}$, then this condition simplifies to $\frac{a_2}{2} < \lambda_2$ which is not true. So $\lambda_1 < \frac{a_2}{2}$ becomes a necessary condition for country 2 to update its strategy to A. However, this gives $\lambda_1 < \lambda_2$ which is again not true (by the assumption above). So country 2 continues playing E. A similar analysis is worked out for country 3 and conditions required in order to update to strategy A are obtained. The dynamics from state Q_2 are obtained as follows: $Q_2 \xrightarrow{C_1} Q_1$ (as previously obtained in this subsection) and $Q_2 \xrightarrow{C_3} Q_7$ if $\lambda_1 < \frac{a_2}{2}$, $\frac{a_3}{2} - \lambda_3 < \frac{a_2}{2} - \lambda_1$.

Next, consider state Q_3. Country 1 does not update its strategy to E, as $\lambda_1 > \lambda_2$ (by the assumption above). From Table 3.1, country 2 updates its strategy from A to E if $\frac{a_2}{2} - \lambda_2 > \max\left\{\lambda_1, \frac{a_2}{2}\right\} - \lambda_1$ and this is easily satisfied for both $\lambda_1 \geq \frac{a_2}{2}$ and $\lambda_1 < \frac{a_2}{2}$. Country 3 updates its strategy to A and this is already obtained in previous analysis. The dynamics from state Q_3 are obtained as follows: $Q_3 \xrightarrow{C_2} Q_2$, $Q_3 \xrightarrow{C_3} Q_8$.

Next, consider state Q_5. Country 1 switches to A when given an opportunity to update its strategy, as obtained from previous analysis. Country 2 does not switch to

E as $\lambda_2 > \lambda_3$ (by the assumption above). Country 3 updates its strategy to E if the following condition holds from Table 3.1: $\frac{a_3}{2} - \lambda_3 > \max\left\{\lambda_2, \frac{a_1}{2}\right\} - \lambda_2$. The dynamics from state Q_5 are obtained as follows: $Q_5 \xrightarrow{C_1} Q_8$, $Q_5 \xrightarrow{C_3} Q_4$ if $\frac{a_3}{2} - \lambda_3 > \frac{a_1}{2} - \lambda_2$.

Finally, consider state Q_7. Country 1 does not update its strategy to E as $\lambda_1 > \lambda_3$ (by the assumption above). Country 2 switches to strategy A as already obtained from previous analysis. Country 3 switches to strategy E if the following condition holds from Table 3.1: $\frac{a_3}{2} - \lambda_3 > \max\left\{\lambda_1, \frac{a_2}{2}\right\} - \lambda_1$. The dynamics from state Q_7 are obtained as follows: $Q_7 \xrightarrow{C_2} Q_8$, $Q_7 \xrightarrow{C_3} Q_2$ if $\left[\lambda_1 \geq \frac{a_2}{2}\right]$ or $\left[\frac{a_3}{2} - \lambda_3 > \frac{a_2}{2} - \lambda_1\right]$.

These dynamics are summarized by means of a tree diagram in Figure 3.4.
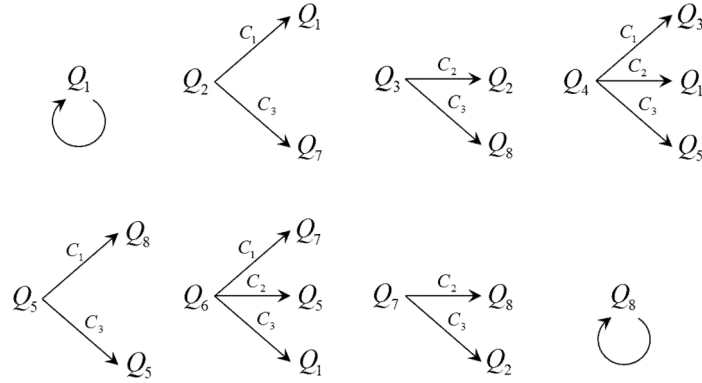
Figure 3.4. Tree diagram for trade dynamics converging to Q_1 and Q_8. The circular arrows around states Q_1 and Q_8 denote that no country can improve their position by changing their strategy from these states, hence the trade dynamics do not move away from these states, where as for all other (non-equilibrium) states, an arrow from state Q_j to Q_k indicates movement away from state Q_j when country i receives the opportunity to update its strategy.

In this figure, there are two arrows leading away from state Q_1, to states Q_2 and Q_6, denoted by C_1 and C_3, respectively. This means that when in state Q_1, country 1 updates its strategy leading the trade dynamics to move to Q_2 if given the opportunity, and country 3 updates its strategy leading the trade dynamics to move to Q_8 if given the opportunity, and hence Q_1 is not an equilibrium as well as not a strict equilibrium. Note that for this figure, the conditions required for Q_1 to be an equilibrium need to hold as well as other conditions listed in the paragraph above.

Country 3

Q_3 C_1 Q_4

C_2

Q_2 Q_1 C_2

C_1

C_3 C_3

C_3 C_3

Q_8 C_1 Q_5 Country 1

C_2

Q_7 Q_6 C_2

C_1
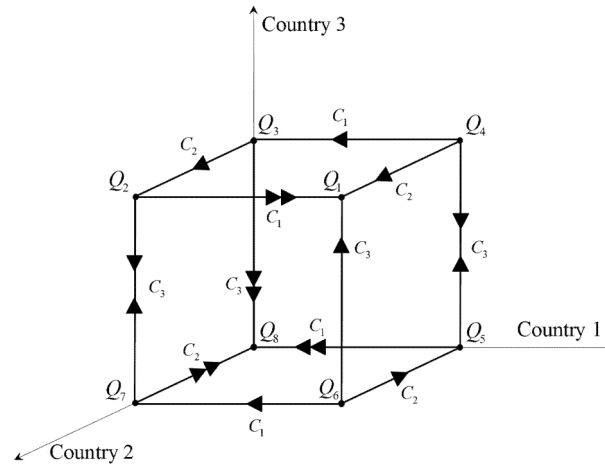
Country 2

Figure 3.5. Trade dynamics of the 3-country model when states Q_2 and Q_5 are strict equilibria, with the assumption of $\lambda_1 > \lambda_2 > \lambda_3$. Double arrows indicate unconditional transitions while single arrows are subject to conditions on the a_i's and λ_i's, here $a_1 > a_2 > a_3$ and $\lambda_1 > \lambda_2 > \lambda_3$. In the case of the edge $Q_8 - Q_4$, there are two arrows as country 3 can update its strategy at both states (leading to the other)[100]. In the cases of the edges where a single arrow is drawn, a country changes its strategy leading to another state, as by doing so, it obtains a better payoff in terms of rule (3.3).

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[100] Note that only one dynamic can hold for $Q_2 \rightleftarrows Q_7$ and $Q_4 \rightleftarrows Q_5$, as mentioned previously. A simplified version of this figure is shown in the next figure.

Figure 3.6. Trade dynamics of the 3-country model converging to states Q_1 and Q_8.[101] Note that the edges $Q_4 - Q_5$ and $Q_2 - Q_7$ now have one arrow each, as the following conditions hold: $\dfrac{a_3}{2} - \lambda_3 > \max\left\{\lambda_1, \dfrac{a_2}{2}\right\} - \lambda_1$ and

$$\frac{a_3}{2} - \lambda_3 > \max\left\{\lambda_1, \frac{a_2}{2}\right\} - \lambda_1.$$

One-shot mutations are now considered, in order to define the notion of strict stability of the equilibria[102]. A country randomly updates its strategy once the equilibrium has been reached, and then the trade dynamics are allowed to proceed in their usual way under the behavioural rule (3.3). These mutations can be viewed either as a way of countries choosing a strategy at random as a result of experimentation, or as the exit of an existing country from the trade game followed by the entry of a new country with little knowledge of the game.

Let Q_1 and Q_8 be strict equilibria as per Figure 3.6. Consider a single mutation away from Q_1 to Q_2. Then, the trade dynamics lead back to Q_1 as country 1 updates its strategy. However if country 2 mutates away from Q_1 to Q_4, then the trade dynamics can either lead back to Q_1 via paths $Q_4 \rightarrow Q_1$ or

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[101] Note that this is just one possible version of the trade dynamics converging to the fully integrated and all-in-autarky states, other versions can be obtained with different assumptions on autarkic payoffs λ_is and further assumptions on dynamics $Q_2 \rightleftarrows Q_7$ and $Q_4 \rightleftarrows Q_5$.

[102] Note that a more formal definition follows in the *n*-country model discussed in Section 3.2.

$Q_4 \to Q_3 \to Q_2 \to Q_1$, or lead to Q_8 via path $Q_4 \to Q_3 \to Q_8$. Similarly a mutation away from Q_1 to Q_6 can either lead the trade dynamics back to Q_1 via paths $Q_6 \to Q_7 \to Q_2 \to Q_1$ or $Q_6 \to Q_1$, or lead to Q_8 via path $Q_6 \to Q_7 \to Q_8$. Therefore, depending on the order of the countries given the opportunity to update their strategy after a mutation occurs, the fully integrated state is a strictly stable equilibrium or an *interchangeably stable*[103] equilibrium. A similar analysis for state Q_8 gives the following: a mutation away from Q_8 to Q_3 leads the trade dynamics back to Q_8 via path $Q_3 \to Q_8$, or to Q_1 via path $Q_3 \to Q_2 \to Q_1$; a mutation away from Q_8 to Q_7 leads the trade dynamics back to Q_8 via path $Q_7 \to Q_8$, or to Q_1 via path $Q_7 \to Q_2 \to Q_1$; a mutation away from Q_8 to Q_5 leads the trade dynamics back to Q_8 via paths $Q_5 \to Q_4 \to Q_3 \to Q_8$ or $Q_5 \to Q_8$, or to Q_1 via paths $Q_5 \to Q_4 \to Q_1$ or $Q_5 \to Q_4 \to Q_3 \to Q_2 \to Q_1$. Again, depending on the order of the countries given the opportunity to update their strategy after a mutation occurs, the all-in-autarky state is a strictly stable equilibrium or an interchangeably stable equilibrium. A quick glance at the paths followed by single mutations away from states Q_1 and Q_8 shows that Q_1 is a strictly stable equilibrium[104] *with a higher probability*[105] than Q_8.

The 3-country trade dynamics converging to the all-in-autarky state and a heterogeneous state, either Q_4 or Q_6, are obtained in Figure 3.7 and Figure 3.8. Note that the double arrows $\rightleftarrows$ actually narrow down to single arrows once a set of conditions surrounding the a_is and λ_is are fulfilled in Table 3.1 (similar to the previous analysis of Q_{12}, Q_7 and Q_4, Q_5, when the trade dynamics converge to either the fully integrated equilibria or the all-in-autarky equilibria).

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[103] Interchangeable stability refers to the dynamics converging to either of the two given equilibria in the presence of single mutations, but each mutation leading away from one equilibria and converging to the other. For example, a single mutation from Q_5 to Q_6 or Q_4 leads the trade dynamics (following the behavioural rule) to converge to the strict equilibrium Q_1 instead of Q_8.

[104] Note that the conditions required for Q_1 to be a strict equilibrium need to hold, as per Figure 3.6.

[105] Note that out of three possible mutations from state Q_1 to states Q_1, Q_4, Q_6, the mutation to Q_2 leads the dynamics to converge to Q_1. Whereas all possible mutations from state Q_8 can lead the dynamics to converge to state Q_1.
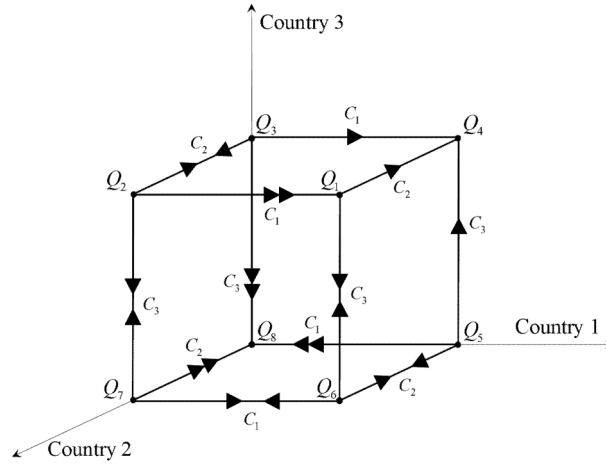
Figure 3.7. The 3-country trade dynamics converging to states Q_8 and Q_4, when either set of the following conditions holds:

$$\left[\lambda_3 < \lambda_2, \ \lambda_1 < \frac{a_1}{2} \leq \lambda_2, \ \lambda_3 < \frac{a_3}{2} \right] \text{ or}$$

$$\left[\lambda_3 < \lambda_2, \ \lambda_1 < \lambda_2 \leq \frac{a_1}{2}, \ a_2 - \lambda_2 < \frac{a_1}{2}, \ a_1 - 2\lambda_2 < a_3 - 2\lambda_3 \right].$$
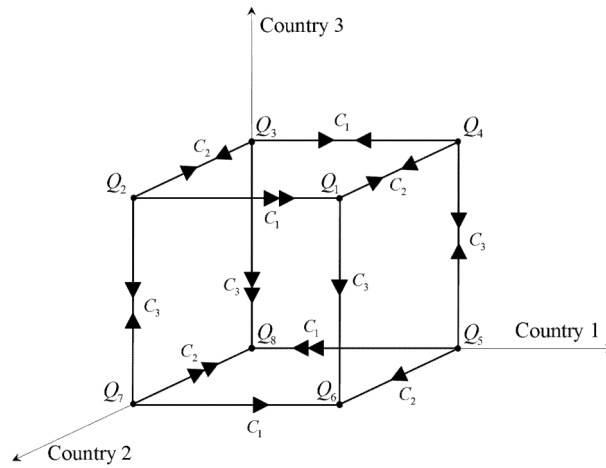
Figure 3.8. The 3-country trade dynamics converging to states Q_8 and Q_6 if either set of the following conditions holds:

$$\left[\lambda_1 < \frac{a_1}{2} < \lambda_3, \ \lambda_2 < \lambda_3, \ \frac{a_2}{2} < \lambda_2 \right] \text{ or } \left[\frac{a_1}{2} > \lambda_3 > \lambda_2, \ a_3 - \lambda_3 < \frac{a_1}{2} \right] \text{ or}$$

$$\left[\lambda_1 < \lambda_3, \ \lambda_2 < \lambda_3 = \frac{a_1}{2} \right].$$

3.1.4 The 4-Country Model

In this model, there are 16 states: (E,E,E,E), (E,E,E,A), (E,E,A,A), (E,A,A,A), (E,A,E,A), (E,A,A,E), (E,A,E,E), (E,E,A,E), (A,E,E,E), (A,A,E,E), (A,A,A,E), (A,E,A,E), (A,E,A,A), (A,E,E,A), (A,A,E,A), (A,A,A,A), some of which are analysed below.

First, consider the heterogeneous state (E,A,A,A). Country 1 receives a payoff of 0 while countries 2, 3, 4 receive payoffs λ_2, λ_3, λ_4, respectively. The maximal payoff μ in this case is $\max\{\lambda_2,\lambda_3,\lambda_4\}$ and country 1 reduces the behavioural gap by switching to strategy A as it yields payoff λ_1, thereby reducing the gap, as $\mu - \lambda_1 < \mu$ (where $\mu = \max\{\lambda_2,\lambda_3,\lambda_4\}$). Therefore, the heterogeneous state (E,A,A,A) can never be a strict equilibrium. Similarly, heterogeneous states (A,A,A,E), (A,E,A,A), (A,A,E,A) cannot be strict equilibria as countries 4, 2, 3 switch their strategies from E to A, respectively, when given an opportunity to do so.

Next, consider the heterogeneous state (A,E,E,E). Country 1 updates its strategy to E when given an opportunity, as it yields the maximal payoff of a_1. Therefore, this heterogeneous state can never be a strict equilibrium.

The rest of the states are analysed via numerical computations in the next chapter.

3.2 The *n*-Country Trade Model

Revisit the *n*-country scenario: There are $n \geq 2$ countries $C_1,\ldots,C_n$, which potentially participate in the world economy. In any unit time of the *n*-player trade game, each country plays one of two strategies: E – *engage in world economy* or A – *remain in autarky*. The payoffs in this *n*-player game are defined below.

Consider a strategy vector[106] $z = (z_1, \ldots, z_n)$, where $z_i = 1$ if C_i plays E, and $z_i = 0$ if C_i plays A.

Definition 3.7. $|z| = \sum_{i=1}^{n} z_i$ is defined as the number of countries that are integrated into the world economy; i.e. that play E.

Definition 3.8. The *payoff* to country i is:

$$\pi_i(z) = z_i \sigma(|z|) a_i + (1 - z_i) \lambda_i, \qquad (3.17)$$

where $0 < \lambda_i < a_i$, and $\sigma(m)$ is the function defined on the non-negative integers by:

$$\sigma(m) = \max\left\{ 0, \frac{m-1}{n-1} \right\}, \ for \ 0 \le m \le n. \qquad (3.18)$$

Thus, if C_i plays E, then its payoff is $\sigma(|z|) a_i$, which depends on the number of other countries engaged in the world economy. If C_i plays A, then its payoff is λ_i, which is independent of what other countries do. Note that if all other countries play E, then it is better for C_i to play E, giving payoff a_i, than it is to remain in autarky (play A) giving payoff λ_i. At the other extreme, if no other countries play E, then it is better for C_i to remain in autarky giving payoff λ_i, than it is to open up to world trade giving payoff 0. This is because such a country would have no trade partners: formally, $\sigma(1) = 0$. Thus, this game has two pure, strict Nash equilibria – either all countries play E or all countries play A[107].

Subsection 3.2.1 defines the behavioural rule and strategy changes using new notation. Subsection 3.2.2 formalizes the conditions required for any state to be a strict equilibrium. Subsection 3.2.3 formalizes the conditions required for a strict equilib-

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[106] This has previously been referred to as a state, and from here on will be interchangeably used as a state or strategy profile.

[107] Note that this corresponds to the previous definition of Nash equilibrium used for the 2×2 coordination game in Section 3.1.

rium to be stable with respect to single mutations. Subsection 3.2.4 analyses the all-in-autarky state and lays the conditions for its strict stability. Subsection 3.2.5 analyses the fully integrated state and lays the conditions for its strict stability. Subsection 3.2.6 investigates the existence of any heterogeneous state strictly stable equilibria.

3.2.1 Strategy Changes

At any given time period, a country is chosen at random and given the opportunity to change its current strategy. When it does this, it tries to minimize the difference between the payoff it gets and the maximum payoff to any country in that time period. Let the current strategy profile be $z = (z_1, \ldots, z_n)$ and let country i be chosen at random so that it has an opportunity to change its strategy.

Let z_{-i} denote the $n-1$ vector of strategies used by countries other than country i, let $\pi_i(s|z_{-i})$ denote the payoff to C_i from using strategy s, determined by (3.17), when other countries use z_{-i}.

Definition 3.9. The *maximum payoff* μ is defined as the highest payoff realized by one or several countries and is formally expressed as:

$$\mu(z) := \max_{j \in \{1, \ldots, n\}} \{\pi_j(z)\}.$$

When C_i uses strategy s and z_{-i} represents the strategy profile for all other countries, C_i calculates the maximal payoff as:

$$\mu(s|z_{-i}) = \max_{j \in \{1, \ldots, n\}} \{\pi_j(s|z_{-i})\}.$$

Definition 3.10. The *strategy change* is defined as $z_i \to z_i' = B_i(z)$, where:

$$B_i(z) = \arg\min_{s \in \{0,1\}} \{\mu(s|z_{-i}) - \pi_i(s|z_{-i})\}. \tag{3.19}$$

Again, assume $a_1 > a_2 > \cdots > a_n$, so that country 1 is the natural leader.

First, consider the special case $|z_{-i}| = 0$. Then, z_{-i} is a vector of zeroes, so that every other country is currently in autarky and country i will always choose to remain in autarky as per the lemma below.

Lemma 3.3. The all-in-autarky state is a strict equilibrium.

Proof. The all-in-autarky state is $z = (0, \ldots, 0)$, which gives: $|z_{-i}| = 0$ as z_{-i} is a vector of zeroes. Thus, $\pi_i(1|z_{-i}) = 0$ and $\mu(1|z_{-i}) = \lambda_{-i}^* := \max_{j \neq i} \{\lambda_j\}$ if C_i plays E, and on the other hand, $\pi_i(0|z_{-i}) = \lambda_i$ and $\mu(0|z_{-i}) = \lambda^* := \max_j \{\lambda_j\}$ if C_i plays A. Now if $\lambda_i \leq \lambda_{-i}^*$, then $\lambda_{-i}^* = \lambda^*$ and $\mu(1|z_{-i}) - \pi_i(1|z_{-i}) = \lambda^*$, $\mu(0|z_{-i}) - \pi_i(0|z_{-i}) = \lambda^* - \lambda_i$. Since $\lambda^* - \lambda_i < \lambda^*$, it follows from the rule (3.19) that C_i will update its strategy to $z_i' = 0$. If $\lambda_i > \lambda_{-i}^*$, then $\lambda^* = \lambda_i$ and $\mu(1|z_{-i}) - \pi_i(1|z_{-i}) = \lambda_{-i}^*$, $\mu(0|z_{-i}) - \pi_i(0|z_{-i}) = 0$. Since $\lambda_{-i}^* > 0$, it follows from the rule (3.19) that C_i will update its strategy to $z_i' = 0$. Thus, if $z_{-i} = 0$, country i will always choose to remain in autarky. $\square$

Now consider the general case, $0 \leq |z_{-i}| \leq n-1$. Then, $\pi_i(1|z_{-i}) = \sigma(|z_{-i}| + 1)a_i$ and $\pi_i(0|z_{-i}) = \lambda_i$. In order to compute $\mu(1|z_{-i})$ and $\mu(0|z_{-i})$, the following notation is introduced:

$$j_{-i}^* := \arg\min_{j \neq i} \{z_j = 1\}, \qquad j_{+i}^* := \min\{i, j_{-i}^*\},$$
$$a_{-i}^* := a_{j_{-i}^*}, \qquad a_{+i}^* := a_{j_{+i}^*},$$
$$\lambda_{-i}^* := \max_{j \neq i} \{(1 - z_j)\lambda_j\}, \qquad \lambda_{+i}^* := \max\{\lambda_i, \lambda_{-i}^*\}.\text{[108]} \tag{3.20}$$

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[108] The term j_{-i}^* gives the lowest country index that plays strategy E excluding country i, while j_{+i}^* includes the country index of C_i. The maximal *integrated* payoff excluding country i is given as a_{-i}^*, while a_{+i}^* includes the integrated payoff to country i. The maximal *autarkic* payoff is given as λ_{-i}^*, while λ_{+i}^* includes the autarkic payoff to country i.

The maximal payoff when C_i plays strategies A ($z_i = 0$) and E ($z_i = 1$), respectively, can now be re-defined as follows:

$$\mu\left(0|z_{-i}\right) = \max\left\{\sigma\left(|z_{-i}|\right)a^*_{-i}, \lambda^*_{+i}\right\}, \quad \mu\left(1|z_{-i}\right) = \max\left\{\sigma\left(|z_{-i}|+1\right)a^*_{+i}, \lambda^*_{-i}\right\}. \tag{3.21}$$

It follows that C_i should update its strategy to $z'_i = 1$ if:

$$\max\left\{\sigma\left(|z_{-i}|+1\right)a^*_{+i}, \lambda^*_{-i}\right\} - \sigma\left(|z_{-i}|+1\right)a_i < \max\left\{\sigma\left(|z_{-i}|\right)a^*_{-i}, \lambda^*_{+i}\right\} - \lambda_i.$$

Moreover, it should update its strategy to $z'_i = 0$ if this strict inequality is reversed. If equality holds in this relation, then C_i is indifferent between playing E or A. However, the parameters are chosen such that this case does not arise. Thus, the condition for C_i to engage in the world economy (play E) is:

$$\max\left\{\sigma\left(|z_{-i}|+1\right)a^*_{+i}, \lambda^*_{-i}\right\} + \lambda_i < \max\left\{\sigma\left(|z_{-i}|\right)a^*_{-i}, \lambda^*_{+i}\right\} + \sigma\left(|z_{-i}|+1\right)a_i. \tag{3.22}$$

For example, if $|z_{-i}| = 0$, so that every other country is in autarky, then (3.22) reduces to $\lambda^*_{-i} + \lambda_i < \lambda^*_{+i}$. It is clear that the reverse of this strict inequality is always satisfied, and hence C_i should always go into autarky. This reclaims the previously obtained special case of autarky as per the lemma below.

Lemma 3.4. The conditions required for C_i to engage in the world economy (play E) can be summarized as:

$$\begin{cases} \lambda_1 < \dfrac{n-1}{n-2}a_2 \; for \; i = 1. \\[2mm] \lambda_i < \dfrac{n-1}{n-2}a_1 \; and \; \dfrac{1}{n-1}a_1 < a_i - \lambda_i \; for \; i > 1. \end{cases}$$

Proof. Suppose that every other country is already engaged in the world economy. From (3.18): $|z_{-i}| = n-1$, $\sigma\left(|z_{-i}|\right) = \dfrac{n-2}{n-1}$, $\sigma\left(|z_{-i}|+1\right) = 1$. Further, from (3.20): $\lambda^*_{-i} = 0$ and $\lambda^*_{+i} = \lambda_i$. Thus, (3.22) reduces to:

$$a^*_{+i} - a_i < \max\left\{\dfrac{n-2}{n-1}a^*_{-i}, \lambda_i\right\} - \lambda_i. \tag{3.23}$$

Clearly, a necessary condition for this to be possible is:

$$\lambda_i < \frac{n-1}{n-2} a_{-i}^*, \tag{3.24}$$

in which case (3.23) reduces to:

$$a_{+i}^* - \frac{n-2}{n-1} a_{-i}^* < a_i - \lambda_i. \tag{3.25}$$

If (3.24) does not hold, then the right hand side of (3.23) is 0, and hence either the strict inequality is reversed - in which case C_i will go into autarky, or equality holds - in which case C_i is indifferent. However, equality is not considered in this thesis to avoid complexity in equations.

Given (3.24), there are two cases.

First, $i = 1$. Then, from (3.20), $a_{+i}^* < a_1$ and $a_{-i}^* < a_2$. Thus, (3.23) holds if and only if: $\lambda_1 < \frac{n-1}{n-2} a_2$.

Second, $i > 1$. Then, $a_{-i}^* = a_{+i}^* = a_1$, and (3.23) holds if and only if: $\lambda_i < \frac{n-1}{n-2} a_1$ and

$$\frac{1}{n-1} a_1 < a_i - \lambda_i. \qquad \qquad \square$$

The conditions obtained in Lemma 3.4 recapture the reverse result of Proposition 2 of D'Artigues and Vignolo [122], which states that a country j chooses strategy A from the equilibrium in which all the countries are engaged in world trade if:

$$a_j - \lambda_j < \frac{\mu}{n-1}.$$

3.2.2 Equilibria

The following notation is introduced in order to define strict equilibria using the *n*-country model:

$$\Phi_i(z) = \max\left\{\sigma(|z_{-i}|)a_{-i}^*, \lambda_{+i}^*\right\} + \sigma(|z_{-i}|+1)a_i, \tag{3.26}$$

$$\Psi_i(z) = \max\left\{\sigma(|z_{-i}|+1)a_{+i}^*, \lambda_{-i}^*\right\} + \lambda_i. \tag{3.27}$$

Then, condition (3.22) for the strategy change $z_i \to 1$ is $\Psi_i(z) < \Phi_i(z)$, and the condition for a strategy change $z_i \to 0$ is $\Psi_i(z) > \Phi_i(z)$. For z to be a strict equilibrium of the trade game under the updating rule (3.19), no country should have an incentive to change its strategy when given the opportunity to do so. Thus, if $z_i = 1$, then $\Psi_i(z) < \Phi_i(z)$ is required, and if $z_i = 0$, then $\Psi_i(z) > \Phi_i(z)$ is required.

Definition 3.11. A strategy profile z^* is an *equilibrium* when no country has any incentive to update its strategy when given an opportunity to do so. This can be formally expressed as: $z_i^* = B_i(z^*)$, *for all i*; i.e. z^* is a fixed point of the map $B: \{0,1\}^n \to \{0,1\}^n$.[109]

Definition 3.12. A strategy profile z^* is a *strict equilibrium* if $z_i^* = B_i(z^*)$, for all i, and each country i does explicitly worse by changing its strategy from z_i^*.

A necessary and sufficient condition for z^* to be a strict equilibrium of the dynamic trade game (as defined above) can be formally expressed in terms of Φ and Ψ as:

$$\left(2z_i^* - 1\right)\left(\Phi_i(z^*) - \Psi_i(z^*)\right) > 0, \tag{3.28}$$

for all i. Consider the all-in-autarky state: $z = (0, \ldots, 0)$, so that every country is in autarky, then $\Phi_i(z) = \lambda_{+i}^*$ and $\Psi_i(z) = \lambda_{-i}^* + \lambda_i$. It is clear that $\Psi_i(z) > \Phi_i(z)$ for all i, and hence that (3.28) holds. Thus, it is shown that $z = (0, \ldots, 0)$ is an equilibrium, as obtained previously in Lemma 3.3.

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[109] Note that by this definition, each country does explicitly worse by changing its strategy when given an opportunity to do so.

A quick look at the n-country model from a topological point of view shows that the all-in-autarky state is connected via n edges to n vertices, where each vertex represents a state z with the property: $|z| = 1$, in the n-dimensional space.

Lemma 3.5. Two neighbouring states in the n-country model cannot be strict equilibria.

Proof. The n-country model has 2^n vertices that represent the states and $2^{n-1}n$ edges that represent possible change from one state to another as one country updates its strategy from either E to A or from A to E. If one vertex is a strict equilibria, then all the edges connected to it have trade dynamics moving towards it, and away from the neighbouring n connected vertices. Therefore, the neighbouring n vertices cannot be strict equilibria in the same unit time.   □

Note that Lemma 3.5 holds for equilibria in general as well.

Lemma 3.6. The total number of equilibria cannot exceed $2^n - n - 1$ for the n-country model.

Proof. In the n-country model, there are 2^n states represented by 2^n vertices and there are n vertices connected via n edges to the all-in-autarky state. Since the all-in-autarky state $z = (0,\ldots,0)$ is an equilibrium, the neighbouring n vertices cannot be strict equilibria as the trade dynamics move towards the all-in-autarky state. Furthermore, the state $(0,1,\ldots,1)$ also cannot be a strict equilibrium since switching from strategy A to strategy E leads country 1 to obtain the maximal payoff a_1 as all other countries are playing strategy E. Hence, a total of $n+1$ states cannot be strict equilibria, which means the total number of equilibria cannot exceed $2^n - n - 1$ in the n-country scenario.   □

3.2.3 Strict Stability

Suppose z is an equilibrium strategy profile. Consider a single mutation away from z which changes the strategy of country k while leaving all other strategies un-

changed. Denote the mutant strategy by $z^{(k)}$. Thus, $z_i^{(k)} = z_i$ for $i \neq k$ and $z_k^{(k)} = 1 - z_k$. For z to be stable with respect to such single mutations, the behavioural rule (3.19) should always provide an incentive to return to the equilibrium strategy: $z_k \to z$. This is the case if and only if, when in state $z^{(k)}$, countries $i \neq k$ will not change their strategies when given the opportunity, but country k will change. In the notation (3.26) and (3.27), these requirements can be formally expressed as follows.

Definition 3.13. Let z be an equilibrium strategy profile and $z^{(k)}$ be a mutant strategy profile. Then, z is a *stable equilibrium* if each country apart from k has no incentive to update its strategy when given an opportunity to do so, and country k updates its strategy from $z_k^{(k)}$ back to z_k, and this must be true for each k. This can be formally expressed as: $z_i^{(k)} = B_i\left(z^{(k)}\right)$, *for $i \neq k$* and $z_k^{(k)} \to z_k = B_k\left(z^{(k)}\right)$, for all i and all k.[110]

Definition 3.14. Let z be an equilibrium strategy profile and $z^{(k)}$ be a mutant strategy profile. Then, z is a *strictly stable equilibrium* if $z_i^{(k)} = B_i\left(z^{(k)}\right)$, *for $i \neq k$* and $z_k^{(k)} \to z_k = B_k\left(z^{(k)}\right)$, and each country $i \neq k$ does explicitly worse by changing its strategy from z_i, while country k does explicitly better by changing its strategy from $z_k^{(k)}$ to z_k.

A necessary and sufficient condition for z to be a strictly stable equilibrium of the dynamic trade game (as defined above) can be formally expressed in terms of Φ and Ψ as:

$$(2z_i - 1)\left(\Phi_i\left(z^{(k)}\right) - \Psi_i\left(z^{(k)}\right)\right) > 0, \tag{3.29}$$

for all i and all k. Note that for a given k, (3.29) must hold for all i, including $i = k$. This must be true for each k.

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[110] Note that by this definition, each country apart from k does explicitly worse by changing its strategy when given an opportunity to do so.

If the convention $z^{(0)} = z$, so that no country mutates, then (3.29) includes (3.28). Thus, (3.29) specifies the conditions for $z^{(0)}$ to be a strictly stable equilibrium.

In the following subsections, explicit conditions for equilibria and their stability are derived for the all-in-autarky state, the fully integrated state and the heterogeneous states.

3.2.4 The All-In-Autarky State

In this subsection, explicit conditions for the stability of the all-in-autarky equilibrium are derived.

The all-in-autarky equilibrium is $z = 0$, when all countries play A. The requirement (3.29) for strict stability is $\Phi_i\left(z^{(k)}\right) < \Psi_i\left(z^{(k)}\right)$ for $1 \leq k \leq n$ (this has already been shown for $k = 0$: that $z = 0$ actually is a strict equilibrium).

For $i = k$, (3.26) and (3.27) give: $\Phi_k\left(z^{(k)}\right) = \lambda^*_{+k}$ and $\Psi_k\left(z^{(k)}\right) = \lambda^*_{-k} + \lambda_k$.

Clearly, $\Phi_k\left(z^{(k)}\right) < \Psi_k\left(z^{(k)}\right)$, as required for strict stability.

For $i < k$, if follows from (3.20) that $a^*_{-i} = a_k$, $a^*_{+i} = a_i$ and
$\lambda^*_{-i} = \max\left\{\lambda_1, \ldots, (\lambda_i), \ldots, (\lambda_k), \ldots, \lambda_n\right\}$, where $(\cdot)$ indicates that this term has been removed.

Thus, from (3.26) and (3.27):
$$\Phi_i\left(z^{(k)}\right) = \lambda^*_{+i} + \sigma(2)a_i \text{ and } \Psi_i\left(z^{(k)}\right) = \max\left\{\sigma(2)a_i, \lambda^*_{-i}\right\} + \lambda_i.$$

The strict stability criterion (3.29) is:
$$\sigma(2)a_i < \max\left\{\sigma(2)a_i, \lambda^*_{-i}\right\} + \lambda_i - \lambda^*_{+i}, \text{ for all } i < k.$$

Clearly, this holds if and only if $\sigma(2)a_i < \lambda^*_{-i} + \lambda_i - \lambda^*_{+i}$, i.e.

$$\frac{1}{n-1}a_i < \min\left\{\lambda_i, \max\left\{\lambda_1, \ldots, (\lambda_i), \ldots, (\lambda_k), \ldots, \lambda_n\right\}\right\}, \tag{3.30}$$

for all $1 \le i < k \le n$.

For $i > k$, it follows from (3.20) that $a_{-i}^* = a_{+i}^* = a_k$, with λ_{-i}^* as above. Thus, from (3.26) and (3.27):

$$\Phi_i\left(z^{(k)}\right) = \lambda_{+i}^* + \sigma(2)a_i \text{ and } \Psi_i\left(z^{(k)}\right) = \max\left\{\sigma(2)a_k, \lambda_{-i}^*\right\} + \lambda_i.$$

If $\lambda_i \ge \lambda_{-i}^*$, then $\lambda_{+i}^* = \lambda_i$, and hence $\Phi_i\left(z^{(k)}\right) < \Psi_i\left(z^{(k)}\right)$ if and only if:

$\sigma(2)a_i < \max\left\{\sigma(2)a_k, \lambda_{-i}^*\right\}$, which is always true since $a_i < a_k$.

On the other hand, if $\lambda_i < \lambda_{-i}^*$, then $\lambda_{+i}^* = \lambda_{-i}^*$. Thus, if $\max\left\{\sigma(2)a_k, \lambda_{-i}^*\right\} = \lambda_{-i}^*$, then the following needs to hold: $\sigma(2)a_i < \lambda_i$. If $\max\left\{\sigma(2)a_k, \lambda_{-i}^*\right\} = \sigma(2)a_k$, then the following needs to hold: $\sigma(2)a_i < \sigma(2)a_k - \lambda_{-i}^* + \lambda_i$. A sufficient condition for this to hold is again $\sigma(2)a_i < \lambda_i$.

Thus, the strict stability criterion will hold if:

$$\begin{aligned} &\textit{Either: } \lambda_i \ge \max\left\{\lambda_1, \ldots, (\lambda_k), \ldots, (\lambda_i), \ldots, \lambda_n\right\}, \\ &\quad \textit{Or: } \frac{1}{n-1}a_i < \min\left\{\lambda_i, \max\left\{\lambda_1, \ldots, (\lambda_i), \ldots, (\lambda_k), \ldots, \lambda_n\right\}\right\}, \end{aligned} \tag{3.31}$$

for $1 \le k < i \le n$.

For example, conditions (3.30) and (3.31) hold if:

$$\frac{1}{n-1}a_1 < \min_i\left\{\lambda_i\right\}. \tag{3.32}$$

The trade game can be pulled out from the all-in-autarky equilibrium if two countries decide to open up to trade, one of which is the leading country (with fully integrated payoff of a_1), such that the average payoff to the leading country is greater than the autarkic payoffs to all the countries remaining in autarky. Therefore, the

gains to the leading country from trading with merely a single country will make the all-in-autarky equilibrium unstable.

3.2.5 The Fully Integrated State

In this subsection, explicit conditions for the existence of the fully integrated equilibrium and its stability are derived.

The fully integrated state is attained when every country plays E, and is defined by the strategy profile $z = e = (1,1,\ldots,1)$. Then, $|z_{-i}| = n-1$, and from (3.20), $a_{+i}^* = a_1$, $a_{-1}^* = a_2$ and $a_{-i}^* = a_1$ for $i > 1$. Also, $\lambda_{-i}^* = 0$ and $\lambda_{+i}^* = \lambda_i$.

From (3.28), the criterion for $z = e$ to be a strict equilibrium is $\Psi_i(z) < \Phi_i(z)$, for all i.

From (3.26) and (3.27):
$$\Phi_1(z) = \max\{\sigma(n-1)a_2, \lambda_1\} + a_i, \ \Phi_i(z) = \max\{\sigma(n-1)a_1, \lambda_i\} + a_i \ for \ i > 1,$$
$$\Psi_i(z) = a_1 + \lambda_i.$$

For $i = 1$, clearly, $\Psi_1(z) < \Phi_1(z)$ if and only if: $\lambda_1 < \sigma(n-1)a_2$.

For $i > 1$, $\Psi_1(z) < \Phi_1(z)$ if and only if: $\lambda_i < \sigma(n-1)a_1$ and $a_i - \lambda_i > \dfrac{1}{n-1}a_1$.

These are conditions already found in Lemma 3.4.

Given that $z = e$ is a strict equilibrium, the condition (3.29) for its strict stability is:
$$\Psi_i\left(z^{(k)}\right) < \Phi_i\left(z^{(k)}\right), \ for \ 1 \le k \le n.$$

For $i = k$, $|z_{-i}| = n-1$, and from (3.20):
$$\lambda_{-i}^* = 0 \ and \ \lambda_{+i}^* = \lambda_i.$$

Strict stability for $i = k$ holds if $\Phi_k\left(z^{(k)}\right) > \Psi_k\left(z^{(k)}\right)$. This holds if and only if:

$$a^*_{+i} - a_i < \max\left\{\frac{n-2}{n-1}a^*_{-i}, \lambda_i\right\} - \lambda_i.$$

This is already obtained in (3.23). Hence, the condition for strict stability for $i = k$ is condition (3.23).

For $i < k$ and $i > k$, $|z_{-i}| = n - 2$, and from (3.20):
$$\lambda^*_{-i} = \lambda_k \text{ and } \lambda^*_{+i} = \max\{\lambda_i, \lambda_k\}.$$

Strict stability for $i < k$ and $i > k$ holds if $\Phi_i\left(z^{(k)}\right) > \Psi_i\left(z^{(k)}\right)$. This holds if and only if:

$$\max\left\{\sigma(n-2)a^*_{-i}, \lambda_i, \lambda_k\right\} + \sigma(n-1)a_i > \max\left\{\sigma(n-1)a^*_{+i}, \lambda_k\right\} + \lambda_i. \qquad (3.33)$$

If $\max\left\{\sigma(n-2)a^*_{-i}, \lambda_i, \lambda_k\right\} = \lambda_i$, then for strict stability the following condition needs to hold: $\sigma(n-1)a_i > \max\left\{\sigma(n-1)a^*_{+i}, \lambda_k\right\}$. This is impossible as $\sigma(n-1)a_i \leq \sigma(n-1)a^*_{+i}$. So a necessary condition for (3.33) to hold is:

$$\lambda_i < \min\left\{\sigma(n-2)a^*_{-i}, \lambda_k\right\}. \qquad (3.34)$$

Given (3.34), condition (3.33) can be simplified to:

$$\max\left\{\sigma(n-2)a^*_{-i}, \lambda_k\right\} + \sigma(n-1)a_i > \max\left\{\sigma(n-1)a^*_{+i}, \lambda_k\right\} + \lambda_i. \qquad (3.35)$$

Conditions (3.34) and (3.35) are necessary and sufficient for strict stability of the fully integrated equilibrium for $i < k$ and $i > k$.

3.2.6 Biheterogeneous States

In this subsection, explicit conditions for the existence of heterogeneous equilibria and their stability are derived for the special cases of *biheterogeneous states*, where

the rich countries find it more beneficial to trade amongst themselves while the lagging countries choose to remain in autarky.

Definition 3.15. A *biheterogeneous state* is a state z where $z_i = 1$ for $1 \leq i \leq m$ and $z_i = 0$ for $m + 1 \leq i \leq n$. If $m \geq 1$, there is at least one country playing strategy E, and if $m < n$, then there is at least one country in autarky.

Consider a potential biheterogeneous strict equilibrium z as per Definition 3.15.

Consider $z^{(k)}$.

If $1 \leq k \leq m$, then $z^{(k)}$ has $m - 1$ ones[111] and $n - m + 1$ zeroes[112].

If $k > m$, then $z^{(k)}$ has $m + 1$ ones and $n - m - 1$ zeroes.

Case I: $k = 0$.

It needs to be shown that the strict equilibrium conditions in (3.28) hold.

Suppose $i = 1$. Then, $|z_{-i}| = m - 1$, and from (3.20):

$a_{-1}^* = a_2$ and $a_{+1}^* = a_1$.

Also: $\lambda_{-1}^* = \hat{\lambda} := \max\{\lambda_{m+1}, \ldots, \lambda_n\}$ and $\lambda_{+1}^* = \max\{\lambda_1, \hat{\lambda}\}$.

Condition (3.28) for $i = 1$ is: $\Psi_1(z) < \Phi_1(z)$, which gives:

$$\max\{\sigma(m)a_1, \hat{\lambda}\} + \lambda_1 < \max\{\sigma(m-1)a_2, \lambda_1, \hat{\lambda}\} + \sigma(m)a_1. \tag{3.36}$$

If $m = 1$, this holds only if $\hat{\lambda} + \lambda_1 < \max\{\lambda_1, \hat{\lambda}\}$, which is clearly impossible. So a necessary condition for (3.36) to hold is $m \geq 2$.

Suppose $1 < i \leq m$. Then, $|z_{-i}| = m - 1$ and from (3.20):

$a_{-i}^* = a_{+i}^* = a_1$.

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[111] Representing countries playing E.

[112] Representing countries playing A.

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Condition (3.28) for $1 < i \le m$ is: $\Psi_i(z) < \Phi_i(z)$, which gives:

$$\max\left\{\sigma(m)a_1, \hat{\lambda}\right\} + \lambda_i < \max\left\{\sigma(m-1)a_1, \lambda_i, \hat{\lambda}\right\} + \sigma(m)a_i. \qquad (3.37)$$

Again, a necessary condition for this to hold is $m \ge 2$.

Suppose $i > m$. Then, $|z_{-i}| = m$ and from (3.20):
$$a^*_{-i} = a^*_{+i} = a_1.$$

On the other hand, $\lambda^*_{-i} = \hat{\lambda}_{-i} = \max\left\{\lambda_{m+1}, \dots, (\lambda_i), \dots, \lambda_n\right\}$ and $\lambda^*_{+i} = \hat{\lambda}$.

Condition (3.28) for $i > m$ is: $\Phi_i(z) < \Psi_i(z)$, which gives:

$$\max\left\{\sigma(m)a_1, \hat{\lambda}\right\} + \sigma(m+1)a_i < \max\left\{\sigma(m+1)a_1, \hat{\lambda}_{-i}\right\} + \lambda_i. \qquad (3.38)$$

The strict equilibrium conditions in (3.28) obtained for the three cases: $i = 1$, $1 < i \le m$, $i > m$ above give the following result: Conditions (3.36), (3.37), (3.38) are necessary and sufficient conditions for a biheterogeneous state z to be a strict equilibrium.

Case II: $1 \le k \le m$.

In this case, the mutant country k, assumed to be playing strategy E mutates to strategy A. Hence $1 \le k \le m$ and for strict stability, it needs to be shown that the conditions in (3.28) hold for $1 \le k \le m$.

Suppose conditions (3.36), (3.37), (3.38) hold so that the heterogeneous state z is a strict equilibrium.

For $i = k$, $|z_{-i}| = m - 1$, and from (3.20):
$a^*_{+i} = a_1$ for all i,
$a^*_{-1} = a_2$ and $a^*_{-i} = a_1$ for $i > 1$.
Also, $\lambda^*_{-i} = \hat{\lambda}$ and $\lambda^*_{+i} = \max\left\{\lambda_i, \hat{\lambda}\right\}$.

For strict stability when $i = k = 1$, the following must hold: $\Phi_1\left(z^{(1)}\right) > \Psi_1\left(z^{(1)}\right)$.

This inequality holds if and only if:

$$max\left\{\sigma(m-1)a_2, \lambda_1, \hat{\lambda}\right\} + \sigma(m)a_1 > max\left\{\sigma(m)a_1, \hat{\lambda}\right\} + \lambda_1. \tag{3.39}$$

If $m = 1$, then condition (3.39) holds if and only if $\max\left\{\lambda_1, \hat{\lambda}\right\} > \hat{\lambda} + \lambda_1$, which is

clearly impossible. So a necessary condition for (3.39) to hold is $m \geq 2$.

When $i = k > 1$, the following condition needs to hold: $\Phi_k\left(z^{(k)}\right) > \Psi_k\left(z^{(k)}\right)$.

This holds if and only if:

$$max\left\{\sigma(m-1)a_1, \lambda_k, \hat{\lambda}\right\} + \sigma(m)a_k > max\left\{\sigma(m)a_1, \hat{\lambda}\right\} + \lambda_k. \tag{3.40}$$

Again, a necessary condition for this to hold is $m \geq 2$.

For $i < k$, $|z_{-i}| = m - 2$, and from (3.20):

$a_{-i}^* = a_{+i}^* = a_3$ for $i = 1$, $k = 2$,

$a_{-i}^* = a_2$, $a_{+i}^* = a_1$ for $i = 1$, $k > 2$, and,

$a_{-i}^* = a_{+i}^* = a_1$ for $i > 1$.

Also, $\lambda_{-i}^* = \max\left\{\lambda_k, \hat{\lambda}\right\}$ and $\lambda_{+i}^* = \max\left\{\lambda_i, \lambda_k, \hat{\lambda}\right\}$.

For strict stability when $i = 1$, $k = 2$, the following needs to hold:

$\Phi_1\left(z^{(2)}\right) > \Psi_1\left(z^{(2)}\right)$.

This holds if and only if:

$$max\left\{\sigma(m-2)a_3, \lambda_1, \lambda_2, \hat{\lambda}\right\} + \sigma(m-1)a_1 > max\left\{\sigma(m-1)a_3, \lambda_2, \hat{\lambda}\right\} + \lambda_1. \tag{3.41}$$

Strict stability for $i = 1$, $k > 2$ holds if and only if:

$$max\left\{\sigma(m-2)a_2, \lambda_1, \lambda_k, \hat{\lambda}\right\} + \sigma(m-1)a_1 > max\left\{\sigma(m-1)a_1, \lambda_k, \hat{\lambda}\right\} + \lambda_1. \tag{3.42}$$

Strict stability for $i > 1$ holds if and only if:

$$\max\left\{\sigma(m-2)a_1,\lambda_i,\lambda_k,\hat{\lambda}\right\}+\sigma(m-1)a_i > \max\left\{\sigma(m-1)a_1,\lambda_k,\hat{\lambda}\right\}+\lambda_i. \qquad (3.43)$$

A necessary condition for (3.41)-(3.43) to hold is $m\geq 3$.

For $k<i\leq m$, $|z_{-i}|=m-2$, and from (3.20):

$a_{-i}^*=a_3$, $a_{+i}^*=a_2$ for $k=1$, $i=2$,

$a_{-i}^*=a_{+i}^*=a_2$ for $k=1$, $i>2$ and,

$a_{-i}^*=a_{+i}^*=a_1$ for $k>1$.

Also, $\lambda_{-i}^*=\max\left\{\lambda_k,\hat{\lambda}\right\}$ and $\lambda_{+i}^*=\max\left\{\lambda_k,\lambda_i,\hat{\lambda}\right\}$.

For strict stability when $i=1$, $k=2$, the following needs to hold:
$\Phi_2\left(z^{(1)}\right)>\Psi_1\left(z^{(1)}\right)$. This holds if and only if:

$$max\left\{\sigma(m-2)a_3,\lambda_1,\lambda_2,\hat{\lambda}\right\}+\sigma(m-1)a_2 > \max\left\{\sigma(m-1)a_2,\lambda_1,\hat{\lambda}\right\}+\lambda_2. \qquad (3.44)$$

Strict stability for $k=1$, $i>2$ holds if and only if:

$$max\left\{\sigma(m-2)a_2,\lambda_1,\lambda_i,\hat{\lambda}\right\}+\sigma(m-1)a_i > \max\left\{\sigma(m-1)a_2,\lambda_1,\hat{\lambda}\right\}+\lambda_i. \qquad (3.45)$$

Strict stability for $k\geq 2$ holds if and only if:
$$max\left\{\sigma(m-2)a_1,\lambda_k,\lambda_i,\hat{\lambda}\right\}+\sigma(m-1)a_i > \max\left\{\sigma(m-1)a_1,\lambda_k,\hat{\lambda}\right\}+\lambda_i.$$

This has already been obtained in (3.43). Hence (3.43) is also the condition for strict stability when $2\leq k<i\leq m$.

Again, a necessary condition for (3.44) and (3.45) to hold is $m\geq 3$.

For $i>k$ and $i>k$, $|z_{-i}|=m-1$, and from (3.20):

$a_{-1}^*=a_2$ and $a_{-i}^*=a_1$ if $k>1$.

$a_{+i}^*=a_{-i}^*$.

Also, $\lambda_{-i}^*=\max\left\{\lambda_k,\hat{\lambda}_{-i}\right\}$ and $\lambda_{+i}^*=\max\left\{\lambda_k,\hat{\lambda}\right\}$.

For strict stability when $k = 1$, the following needs to hold: $\Phi_i\left(z^{(1)}\right) > \Psi_i\left(z^{(1)}\right)$.

This holds if and only if:

$$max\left\{\sigma(m-1)a_2, \lambda_1, \hat{\lambda}\right\} + \sigma(m)a_i > max\left\{\sigma(m)a_2, \lambda_1, \hat{\lambda}_{-i}\right\} + \lambda_i. \qquad (3.46)$$

Note that a necessary condition for this to hold is $m \geq 2$.

Strict stability for $k > 1$ holds if and only if:

$$max\left\{\sigma(m-1)a_1, \lambda_k, \hat{\lambda}\right\} + \sigma(m)a_i > max\left\{\sigma(m)a_1, \lambda_k, \hat{\lambda}_{-i}\right\} + \lambda_i. \qquad (3.47)$$

A necessary condition for this to hold is $m \geq 2$ as $k \geq 2$.

Case III: $m < k \leq n$.

In this case, the mutant country k, assumed to be playing strategy A, mutates to strategy E. Hence $m < k \leq n$ and for strict stability, it needs to be shown that the conditions in (3.28) hold for $m < k \leq n$.

Again assume that conditions (3.36), (3.37), (3.38) already hold so that the heterogeneous state z is a strict equilibrium.

For $i = k$, $|z_{-i}| = m$, and from (3.20):
$a_{-i}^* = a_{+i}^* = a_1$.
Also, $\lambda_{-i}^* = \hat{\lambda}_{-i}$ and $\lambda_{+i}^* = \hat{\lambda}$.

For strict stability when $i = k$, the following must hold: $\Phi_i\left(z^{(k)}\right) < \Psi_i\left(z^{(k)}\right)$.

This inequality holds if and only if:

$$max\left\{\sigma(m)a_1, \hat{\lambda}\right\} + \sigma(m+1)a_i < max\left\{\sigma(m+1)a_1, \hat{\lambda}_{-i}\right\} + \lambda_i. \qquad (3.48)$$

For $m < i < k$, $|z_{-i}| = m$, and from (3.20):

$a_{-i}^* = a_{+i}^* = a_1$.

Also, $\lambda_{-i}^* := \hat{\lambda}_{-ik} = \max\left\{\lambda_{m+1}, \ldots, (\lambda_i), \ldots, (\lambda_k), \ldots, \lambda_n\right\}$ and $\lambda_{+i}^* = \hat{\lambda}_{-k}$.

Strict stability holds if and only if:

$$\max\left\{\sigma(m+1)a_1, \hat{\lambda}_{-k}\right\} + \sigma(m+2)a_i < \max\left\{\sigma(m+2)a_1, \hat{\lambda}_{-ik}\right\} + \lambda_i. \tag{3.49}$$

For $i < k$ and $i \leq m$, $|z_{-i}| = m$, and from (3.20):

$a_{-i}^* = a_2$, $a_{+i}^* = a_1$ for $i = 1$, and,

$a_{-i}^* = a_{+i}^* = a_1$ for $i > 1$.

Also, $\lambda_{-i}^* = \hat{\lambda}_{-k}$ and $\lambda_{+i}^* = \max\left\{\lambda_i, \hat{\lambda}_{-k}\right\}$.

For strict stability when $i = 1$ the following needs to hold: $\Phi_1\left(z^{(k)}\right) > \Psi_1\left(z^{(k)}\right)$. This holds if and only if:

$$\max\left\{\sigma(m)a_2, \lambda_1, \hat{\lambda}_{-k}\right\} + \sigma(m+1)a_1 > \max\left\{\sigma(m+1)a_1, \hat{\lambda}_{-k}\right\} + \lambda_1. \tag{3.50}$$

For strict stability when $i > 1$ the following needs to hold:

$$\max\left\{\sigma(m)a_1, \lambda_i, \hat{\lambda}_{-k}\right\} + \sigma(m+1)a_i > \max\left\{\sigma(m+1)a_1, \hat{\lambda}_{-k}\right\} + \lambda_i. \tag{3.51}$$

For $i > k$, $|z_{-i}| = m+1$, and from (3.20):

$a_{-i}^* = a_{+i}^* = a_1$.

Also, $\lambda_{-i}^* = \hat{\lambda}_{-ki}$ and $\lambda_{+i}^* = \hat{\lambda}_{-k}$.

For strict stability when $i > k$ the following needs to hold:

$$\max\left\{\sigma(m+1)a_1, \hat{\lambda}_{-k}\right\} + \sigma(m+2)a_i < \max\left\{\sigma(m+2)a_1, \hat{\lambda}_{-ki}\right\} + \lambda_i.$$

This is already obtained in (3.49). Hence (3.49) is also the condition for strict stability when $i > k$.

Theorem 3.1. The following conditions are necessary and sufficient for z to be a strictly stable equilibrium: (3.39), (3.41), (3.42), (3.43), (3.44), (3.45), (3.46), (3.47), (3.48), (3.49), (3.50), (3.51).

As these conditions cannot be simplified further, numerical computations are run and results tabulated in the next chapter, in order to understand these conditions better and make conclusions on the trade dynamics.

3.3 Conclusions

Based on the analysis in the previous sections in this chapter, the following conclusions are obtained for the trade game with pure strategies:

The 2-country model has only one strict equilibrium – the all-in-autarky state (A, A), where both countries choose to not trade with one another. It is also strictly stable with respect to single mutations and hence is the long-run equilibrium for the 2-country model.

In the 3-country model, 4 states can be strict equilibria – the all-in-autarky state (A, A, A), the fully integrated state (E, E, E), and the heterogeneous states (E, A, E) and (E, E, A). However, there can only exist a maximum of 2 strict equilibria for a given set of parameter constraints (at any given time), one of which is always the all-in-autarky state.

The 4-country model is more complex to analyse and while the heterogeneous states that cannot be strict equilibria are briefly discussed, the other states are left to be analysed via numerical computations in the next chapter.

The conditions for the existence of the fully integrated equilibrium and the all-in-autarky equilibrium are obtained for the n-country model. The complex conditions for the heterogeneous equilibria are also obtained. However, the analysis of these conditions is difficult due to parameter constraints. Hence, the states are analysed via numerical computations in the next chapter.

Chapter 4

Numerical Computations

In this chapter, the model introduced for n countries in the previous chapter is programmed in Mathematica and using the definitions of strict equilibria and strict stability defined in Chapter 3, the statistical likelihood of the existence of fully integrated strict equilibria, heterogeneous strict equilibria and strictly stable all-in-autarky equilibria is analysed for up to 200 countries[113].

In a similar analysis, Gardner and Ashby [53] examine critical values of stability for linear systems and suggest that all large complex dynamic systems can show stability up to a critical level of *connectance*[114], and can suddenly go unstable as connectance increases beyond this critical point. May [85] looks at the likelihood of socially stable equilibria for random interaction matrices in ecological networks, and finds that for sufficiently large complex networks, the probability of persisting is close to zero. For a more recent analysis, Allesina and Tang [6] derive stability criteria for unstructured networks in which species interact at random in competitive pairs and find that *weak interactions*[115] are destabilizing for competitive networks. The analysis in this chapter is in the spirit of these papers.

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[113] There are 206 countries around the world at present (16 of whose sovereignty is disputed).

[114] The linear system is defined by a vector of random n random variables: $x = (x_1, \ldots, x_n)$ and changes in time by: $\dot{x} = Ax$. It comprises of n variables of the form x_i, and two variables x_i, x_j are connected if the entry a_{ji} is non-zero, and not-connected if a_{ji} is 0.

[115] Species effects are defined as *weak* when the addition or removal of a species does not cause a statistically discernible mean change in the abundance of target species (see Berlow [13]).

An exponential distribution is used to generate the values of a_i's [116], which denote the fully integrated payoffs for the countries [117]. The rate parameter of this exponential distribution is denoted by η and an upper bound $U = a_1 - 1$, where a_1 is the payoff to country 1 – the natural leader. The λ_i's are uniformly distributed pseudorandom [118] real numbers in the range of 1 and the corresponding a_i's [119], which denote the autarkic payoffs for the countries. The values of the λ_i's are generated using the Random[] function in Mathematica. The notation introduced for the *n*-country model in Chapter 3 (Subsection 3.2.1) is re-defined using Mathematica functions. The strict equilibria conditions and the strict stability conditions are then simulated for the *n*-country model, using two different Mathematica programs and a single state (fully integrated state, autarkic state, or any heterogeneous state) is analysed for its equilibrium property and strict stability, respectively. Each program independently gives the model output on any given state per 100,000 runs, so that the result when divided by the number of runs gives the fraction of that particular state being a strict equilibrium or a strictly stable equilibrium.

Note that strict stability of equilibria is only considered for the all-in-autarky cases for all the *n*-country scenarios simulated, as it has already been shown in Chapter 3 that it is always a strict equilibrium, regardless of parameter values. The numerical computations for strict stability of the fully integrated state and the heterogeneous states were found to be negligible over a varied range of parameter values, yielding

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[116] The exponential distribution is used to generate the values of a_i's as the gains from trade is assumed to be significantly higher for richer countries than for poorer countries (as stated by dependency theorists, for more on dependency theory see Ferraro [44] or Namkoong [94]. Furthermore, among the developing nations, the advanced Asian economies gain significantly from international trade while the poorer African economies do not gain much; see Page and Davenport [98]). Also in terms of GDP, the number of middle-income countries and poor countries is relatively higher than the number of rich countries (see World Bank [140]).

[117] See Subsection 4.4.1 for a derivation of the exponential distribution used.

[118] A set of statistically random numbers, derived from a known starting point.

[119] The uniform distribution is used to generate the values of λ_i's as the autarkic payoffs for countries are assumed to be more randomly distributed, having an amount of *uncertainty* associated with them, so the distribution is justified in the sense that it makes fewer assumptions than other distributions.

zero for all scenarios, so the model output for these was not presented graphically. Note that May [85] and Allesina and Tang [6] do not consider the question of equilibrium existence. The study of stability conditional on existence of a strict equilibrium given a certain set of parameters can be a topic of further research.

Section 4.1 lists the pseudocode for the Mathematica programs by defining the variables, the for-loops and the functions used. Section 4.2 contains a graphical presentation for the 3-country and 4-country models, and lists the results for the more general n-country model. Section 4.3 forms conclusions based on the results obtained. Section 4.4 is an appendix that lists the exponential distribution derivation, discusses the scaling of payoff values and provides an insight into the Mathematica programs built to evaluate the likelihood of a state being a strict equilibrium and the likelihood of an equilibrium being strictly stable. It also contains tabulated the model output for reference purposes (for up to a maximum of 200 countries) with varying levels of a_1 and η.

4.1 Description of Mathematica Program

This section lays out the pseudocode for the Mathematica program listed in Section 4.4.

4.1.1 The Strict Equilibrium Numerical Computations

The input variables are defined as:

n: the number of countries playing the trade game[120].
a_1: the payoff to the leading country[121].
η: the rate parameter of the exponential distribution generating the values of a_i's,

such that $\eta = \dfrac{1}{0.2U}$ or $\dfrac{1}{0.5U}$ or $\dfrac{1}{0.8U}$, where U is the upper bound for the exponential distribution, defined below. (Note that decreasing the rate parameter η

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[120] The minimum number of countries considered is 3, while the maximum is 200.

[121] The values of a_1 used in the numerical computations are 10, 100, 1000, 2000, 4000, 6000, 10000.

leads to a more even spread of the values of a_i's so that the gains from trade for countries is not too varied and the difference between the a_i's is relatively low.)

Next, other variables for calculations are defined:

listofz: the strategy vector or the state[122] $z = (z_1, \ldots, z_n)$, where $z_i = 1$ if country i plays E and $z_i = 0$ if country i plays A.

noofstates: a variable that keeps the count of z being a strict equilibrium.

U: the upper bound for the exponential distribution that generates a_is, such that $U = a_1 - 1$.

K: a variable used for generating the values of a_i's, such that $K = \dfrac{\eta}{1 - e^{-\eta U}}$.

Next, the functions required in order to calculate the payoffs (as previously defined in Chapter 3) are defined:

countz: a function that counts the number of countries playing strategy E, such that

$$countz = |z| = \sum_{i=1}^{n} z_i .$$

sigmafn: a function defined on non-negative integers to calculate payoffs, such that

$$sigmafn = \sigma(m) = \max\left\{0, \frac{m-1}{n-1}\right\}, \text{ for } 0 \le m \le n .$$

jminusistar: $j_{-i}^* = \arg\min_{j \neq i} \{z_j = 1\}$.

jplusistar: $j_{+i}^* = \min\{i, j_{-i}^*\}$.

aminusistar: $a_{-i}^* = a_{j_{-i}^*}$.

aplusistar: $a_{+i}^* = a_{j_{+i}^*}$.

lminusistar: $\lambda_{-i}^* = \max_{j \neq i}\{(1 - z_j)\lambda_j\}$.

lplusistar: $\lambda_{+i}^* = \max\{\lambda_i, \lambda_{-i}^*\}$.

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[122] A random heterogeneous state $z = (z_1, \ldots, z_n)$ is generated using the command:

Table[Random[Integer], {n}] while the fully integrated state and the all-in-autarky state are obtained using the commands: Table[1, {n}] and Table[0, {n}], respectively.

The for-loop *counter*, with 100000 iterations counts the number of times z is a strict equilibrium as follows:

First, the values of $a_2, \ldots, a_n$, using the exponential distribution and variables defined above are generated.

Next, the values of $\lambda_1, \ldots, \lambda_n$, using the uniform distribution (and the for-loop *j*) are generated.

The value *true* is assigned to the variable *boolean* (this remains true if the strict equilibrium condition is satisfied by all countries).

The for-loop *i* (within the for-loop *counter*), with *n* iterations checks the strict equilibrium condition for each country *i* as follows:

First, the function *phifn* is defined:
$$\Phi_i(z) = \max\left\{\sigma\left(\left|z_{-i}\right|\right)a^*_{-i}, \lambda^*_{+i}\right\} + \sigma\left(\left|z_{-i}\right| + 1\right)a_i.$$

Next, the function *psifn* is defined: $\Psi_i(z) = \max\left\{\sigma\left(\left|z_{-i}\right| + 1\right)a^*_{+i}, \lambda^*_{-i}\right\} + \lambda_i$.

Next, the equilibrium condition is checked: $\left(2z_i - 1\right)\left(\Phi_i(z) - \Psi_i(z)\right) > 0$ and if this condition holds, the value *true* is assigned to *boolean*, *false* otherwise.

Outside of the for-loop *i*, *noofstates* keeps a count of the equilibrium/strictly stable states (based on the value returned by *boolean*).

Outside of the for-loop *counter*, *listofz* - the state (or the strategy vector) under consideration, along with *noofstates* - the number (out of 100000) of strict equilibria, for given values of n, a_1, η is printed.

4.1.2 The Strict Stability Numerical Computations

Following the pseudocode above, a for-loop: *k* is introduced before the for-loop *i*.

The for-loop k, with n iterations defines the mutant strategy $z^{(k)}$ played by country k as follows:

The entry k in *listofz* is replaced to reflect the mutant strategy $z^{(k)}$, defined in Chapter 3 as $z_i^{(k)} = z_i$ for $i \neq k$, and $z_k^{(k)} = 1 - z_k$.

The for-loop i (within the for-loop k), with n iterations checks the strict stability condition for each country i as follows:

First, the functions *phifn* and *psifn* are defined as above.

Next, the condition required to hold for strict stability is checked: $\left(2z_i - 1\right)\left(\Phi_i\left(z^{(k)}\right) - \Psi_i\left(z^{(k)}\right)\right) > 0$ and if this condition holds, assign the value *true* to *boolean*, *false* otherwise.

Outside of the for-loop i, *listofz* is restored to its original value so that another country can play the mutant strategy.

Outside of the for-loop k, *noofstates* keeps a count of the strictly stable states (based on the value returned by *boolean*).

Outside of the for-loop *counter*, *listofz* - the state (or the strategy vector) under consideration, along with *noofstates* - the number (out of 100000) of strictly stable equilibria, for given values of n, a_1, η is printed.

The Mathematica programs for the strict equilibrium calculations and the strict stability calculations are listed in the Appendix section of this chapter.

4.2 Results

In this section, the results of the model output obtained using the Mathematica programs are depicted graphically for the 3-country model, the 4-country model and the more general n-country model. Note that the scaling of the a_i's and the λ_i's by a common (positive) factor yields exactly the same results via numerical computa-

tions (see Subsection 4.4.2), however when these parameter values are not scaled by the same common factor, varied results are obtained (see Subsection 4.2.3).

4.2.1 The 3-Country Model

The 3-country model was analysed in Chapter 3 and it was shown that there exist two strict equilibria at most, one of which is always the all-in-autarky state. Hence the number of times (out of 100000) the following states individually are strict equilibria is obtained via the first Mathematica program: the fully integrated state $(1,1,1)$, and the heterogeneous states $(1,0,1)$ and $(1,1,0)$ (see Subsection 4.4.3). The number of times (out of 100000) the all-in-autarky state $(0,0,0)$ is a strictly stable equilibrium is obtained via the second Mathematica program (see Subsection 4.4.4). In the figures to follow, $\eta_1 = \dfrac{1}{0.2A}$, $\eta_2 = \dfrac{1}{0.5A}$ and $\eta_3 = \dfrac{1}{0.5A}$.

(a) Fully integrated state $(1,1,1)$.

(b) Heterogeneous state $(1,0,1)$.

(c) Heterogeneous state $(1,1,0)$.
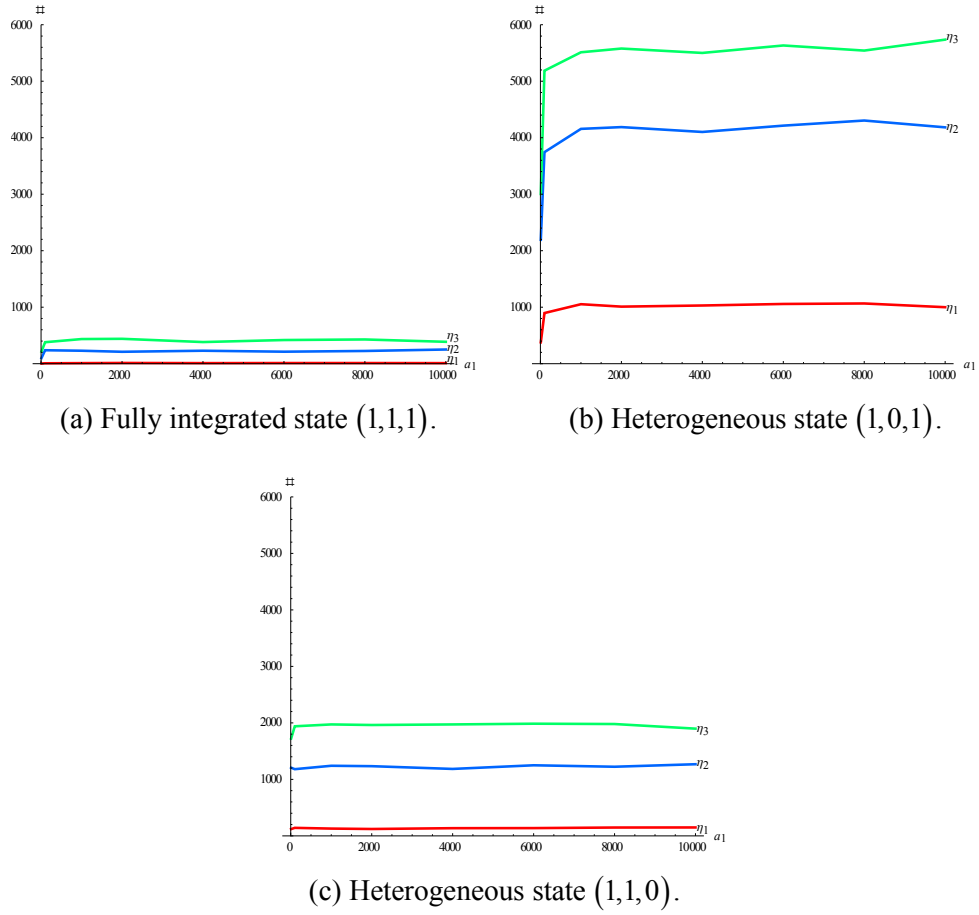
Figure 4.1. The model output for the 3-country model - strict equilibria. The horizontal axes represent the values of a_1 (payoff to the leading country) and the vertical axes represent the number of times out of 100000 that the given state in the 3-country model is a strict equilibrium (for three different η values). It can be deduced from Figures (a), (b) and (c) that the heterogeneous state $(1,0,1)$ is more likely to be a strict equilibrium than the fully integrated state $(1,1,1)$ or the heterogeneous state $(1,1,0)$.

Figure 4.2. The model output for the 3-country model all-in-autarky equilibrium - strict stability.[123] The vertical axis represents the number of times out of 100000 that the all-in-autarky state $(0,0,0)$ is a strictly stable equilibrium (for three different η values). The low values imply that though $(0,0,0)$ is always a strict equilibrium, it is unlikely to be a strictly stable equilibrium.

Based on the graphs presented in Figure 4.1, the heterogeneous state $(1,0,1)$ is significantly more likely to be an equilibrium than the fully integrated state and the other heterogeneous state - $(1,1,0)$. This shows that country 2 prefers to not trade when country 1 engages in world trade, possibly as a deliberate attempt to reduce the leading country's payoff thereby improving its own position in the world economy. In addition, the number of times each state is a strict equilibrium increases with decreasing η_i $(i=1,2,3)$, as gains from trade is more uniform for countries with decreasing η_i (since the spread of a_i's is lower) and hence countries are more likely to trade with one another. In general, the number of times each state is a strict equilibrium as shown in Figure 4.1 is quite low[124]. This is in contrast with the all-in-autarky state, which is always a strict equilibrium. Based on the graph presented in Figure 4.2, it can only be deduced that the probability of the all-in-autarky equilibrium being a strictly stable is extremely low. This shows that coun-

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[123] Note that the all-in-autarky state is the only state considered for testing stability via numerical computations as mentioned previously in this chapter.

[124] The number of strictly stable equilibria for each of these states is negligible, hence the data for the all-in-autarky state as a strictly stable equilibrium is the only one shown with regards to strict stability.

tries are unlikely to trade with one another (since the all-in-autarky state is always a strict equilibrium unlike the fully integrated state and the heterogeneous states) and even if they do, country 2 is more likely to stay in autarky while countries 1 and 3 open up to world trade.

4.2.2 The 4-Country Model

The 4-country model was briefly analysed in Chapter 3 and it was shown that the heterogeneous states $(0,1,1,1)$, $(1,0,0,0)$, $(0,1,0,0)$, $(0,0,1,0)$ and $(0,0,0,1)$ can never be strict equilibria. The model output is obtained for the rest of the states in this chapter. The number (out of 100000) of the fully integrated equilibria $(1,1,1,1)$, and the number of heterogeneous equilibria for the following states - $(1,1,0,0)$, $(1,0,1,0)$, $(1,0,0,1)$, $(0,1,1,0)$, $(0,1,0,1)$, $(0,0,1,1)$ is obtained via the first Mathematica program (see Subsection 4.4.3). The number (out of 100000) of strictly stable all-in-autarky equilibria $(0,0,0,0)$ is obtained via the second Mathematica program (see Subsection 4.4.4).
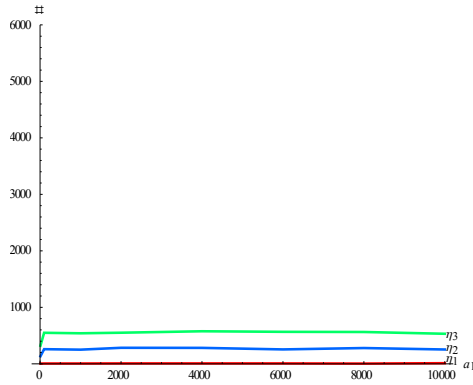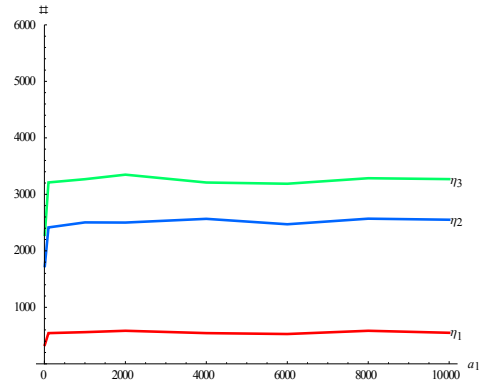
(a) Fully integrated state $(1,1,1,1)$. (b) Heterogeneous state $(1,1,0,0)$.
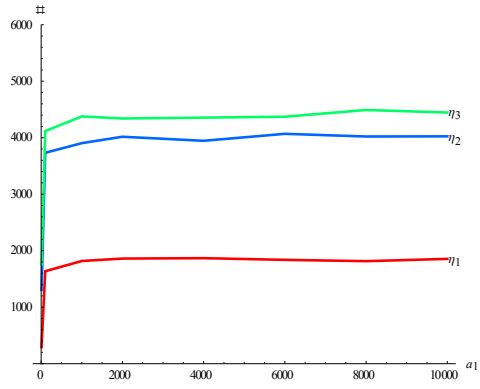
(c) Heterogeneous state $(1,0,1,0)$.
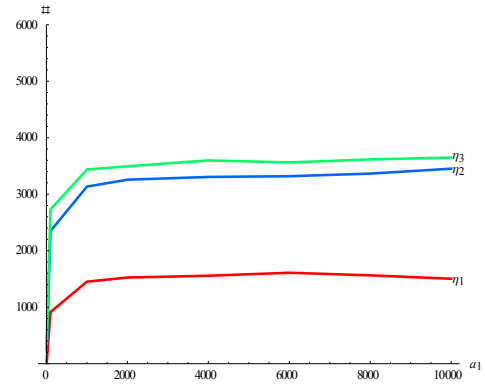
(d) Heterogeneous state $(1,0,0,1)$.
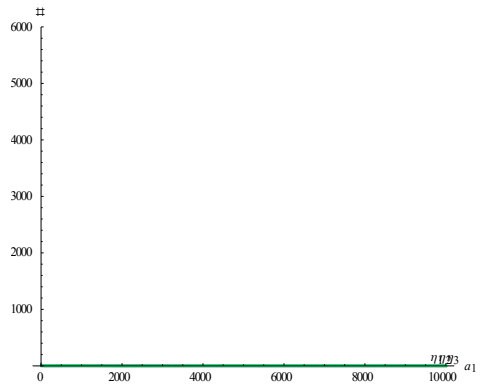
(e) Heterogeneous $(0,1,1,0)$ eq.
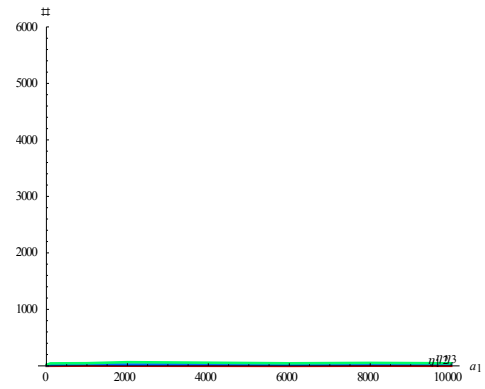
(f) Heterogeneous $(0,1,0,1)$ eq.
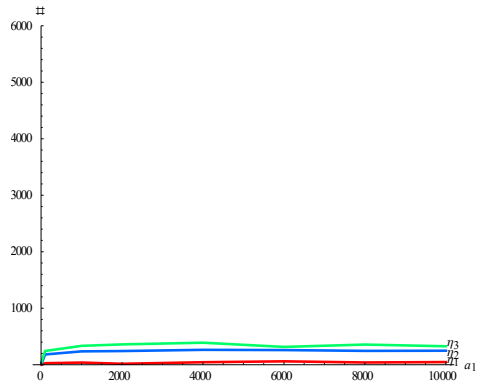
(g) Heterogeneous state $(0,0,1,1)$.

(h) Heterogeneous state $(1,1,1,0)$.

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(i) Heterogeneous state $(1,1,0,1)$.
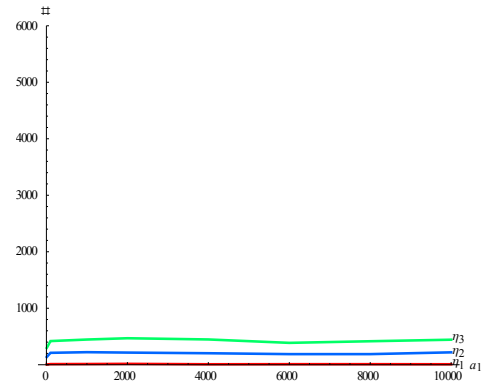
(j) Heterogeneous state $(1,0,1,1)$.
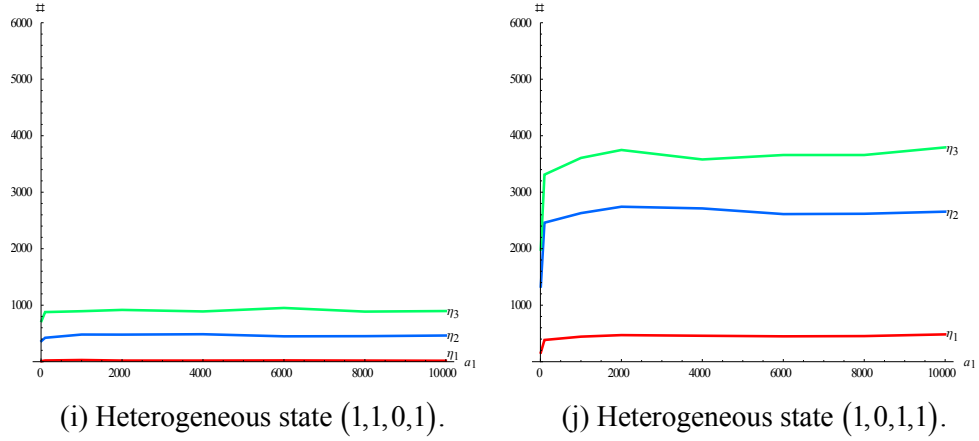
Figure 4.3. The model output for the 4-country model - strict equilibria. The horizontal axes represent the values of a_1 (payoff to the leading country) and the vertical axes represent the number of times out of 100000 that the given state in the 4-country model is a strict equilibrium (for three different η values). It can be deduced from Figures (a) - (j) that the heterogeneous states $(1,0,1,0)$ and $(1,0,1,1)$ are more likely to be strict equilibria than the fully integrated state $(1,1,1,1)$ or other heterogeneous states.
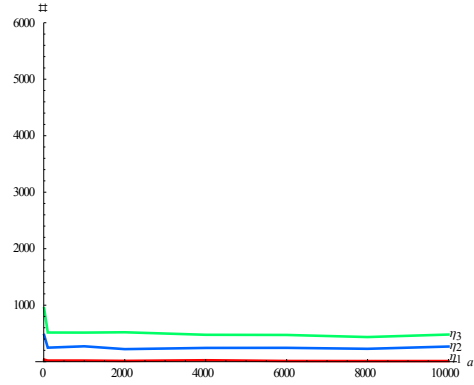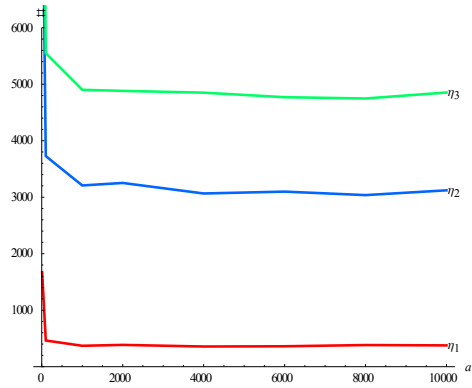
Figure 4.4. The model output for the 4-country model all-in-autarky equilibrium – strict stability.[125] The horizontal axis represents the values of a_1 (payoff to the leading country) and the vertical axis represents the number of times out of 100000 that the all-in-autarky state $(0,0,0,0)$ is a strictly stable equilibrium (for three different η values).

Based on the graphs presented in Figure 4.3, the heterogeneous states $(1,0,1,0)$ and $(1,0,1,1)$ are more likely to be strict equilibria than the fully integrated state and the other heterogeneous states. This suggests that country 2 prefers to remain in autarky, when country 1 (and other countries) engage in world trade, as a deliberate attempt to reduce the leading country's payoff. Based on the graph presented in Figure 4.4, the likelihood of the all-in-autarky equilibrium being strictly stable is significantly higher for the 4-country model when compared to the 3-country model. It appears that in this model, countries find it more advantageous to remain in autarky than to trade with one another. A possible explanation for this behaviour is the *lagging countries* not wanting to participate in world trade in order to degrade the situation of the leading country (which inevitably obtains the maximal payoff of a_1 by engaging in world trade). However, the leading country opts out of world trade in the long run, thereby increasing its own payoff from 0 to λ_1, as it views the reluctance of other countries to engage in world trade.

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[125] Note that the all-in-autarky state is the only state considered for testing stability via numerical computations as mentioned previously in this chapter.

4.2.3 The *n*-Country Model

The *n*-country model was analysed in Chapter 3 and the all-in-autarky state is always a strict equilibrium. Conditions were laid out for the fully integrated state and heterogeneous states to be strict equilibria. Conditions for strict stability of all states were also obtained. The number (out of 100000) of fully integrated strict equilibria is now obtained for up to 200 countries (see Subsection 4.4.7); along with the number (out of 100000) of strictly stable all-in-autarky equilibria for up to 200 countries (see Subsection 4.4.8).

In the plots below, the *x*-axis represents the values of a_1, varying between 10 and 100000; the *y*-axis represents the values of η, where 1 represents $\eta_1 = \dfrac{1}{0.2U}$, 2 represents $\eta_2 = \dfrac{1}{0.5U}$ and 3 represents $\eta_3 = \dfrac{1}{0.8U}$; the *z*-axis represents # - the number (out of 100000) of strict equilibria (in Figure 4.5) or the number of strictly stable equilibria (in Figure 4.6).
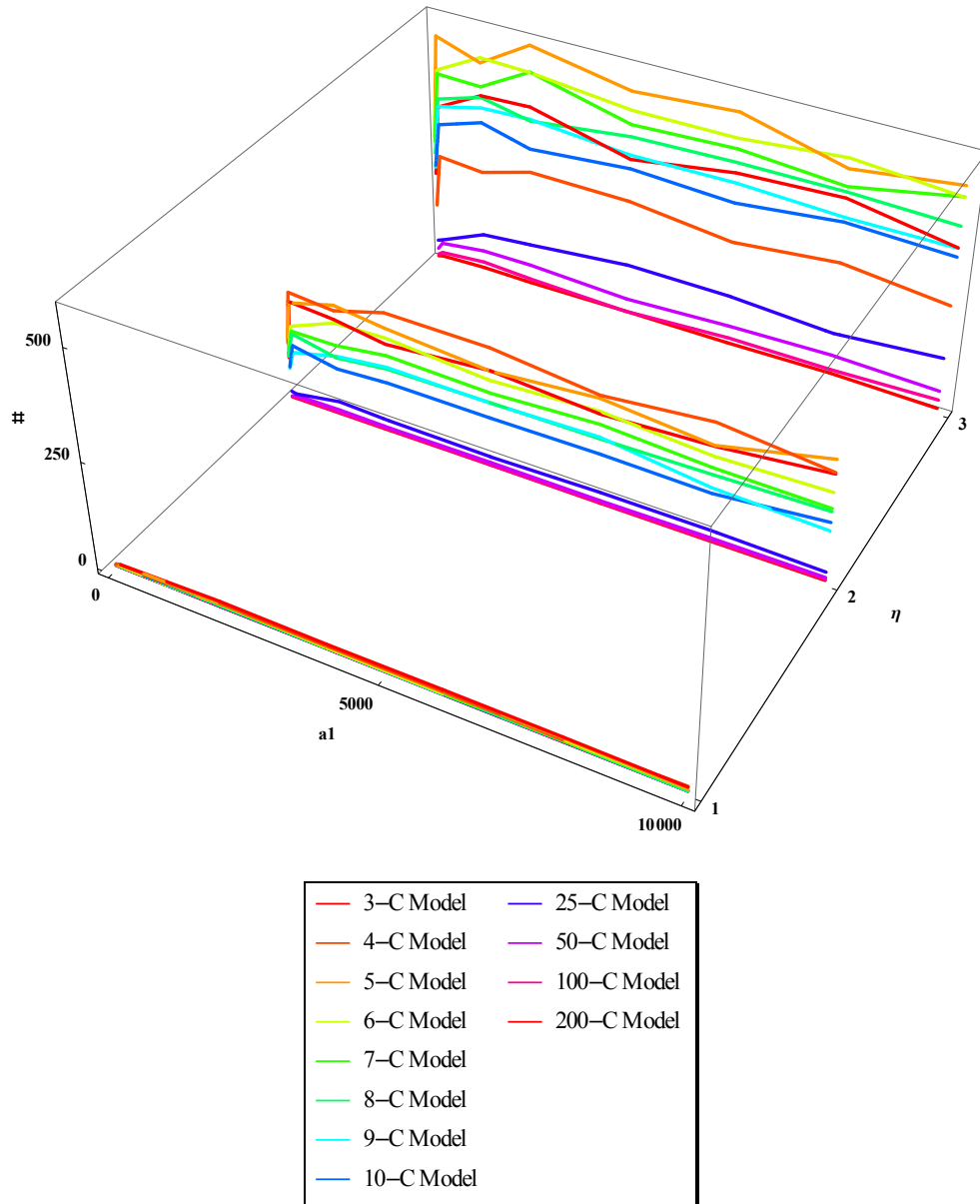
Figure 4.5. The model output for the *n*-country model fully integrated state - strict equilibria. The two horizontal axes represent the values of a_1 (payoff to the leading country) and η (1, 2, 3 represent η_1, η_2, η_3, respectively) and the vertical axis represents the number of times out of 100000 that the fully integrated state in the *n*-country model is a strict equilibrium, where *n* varies from 3 to 200.
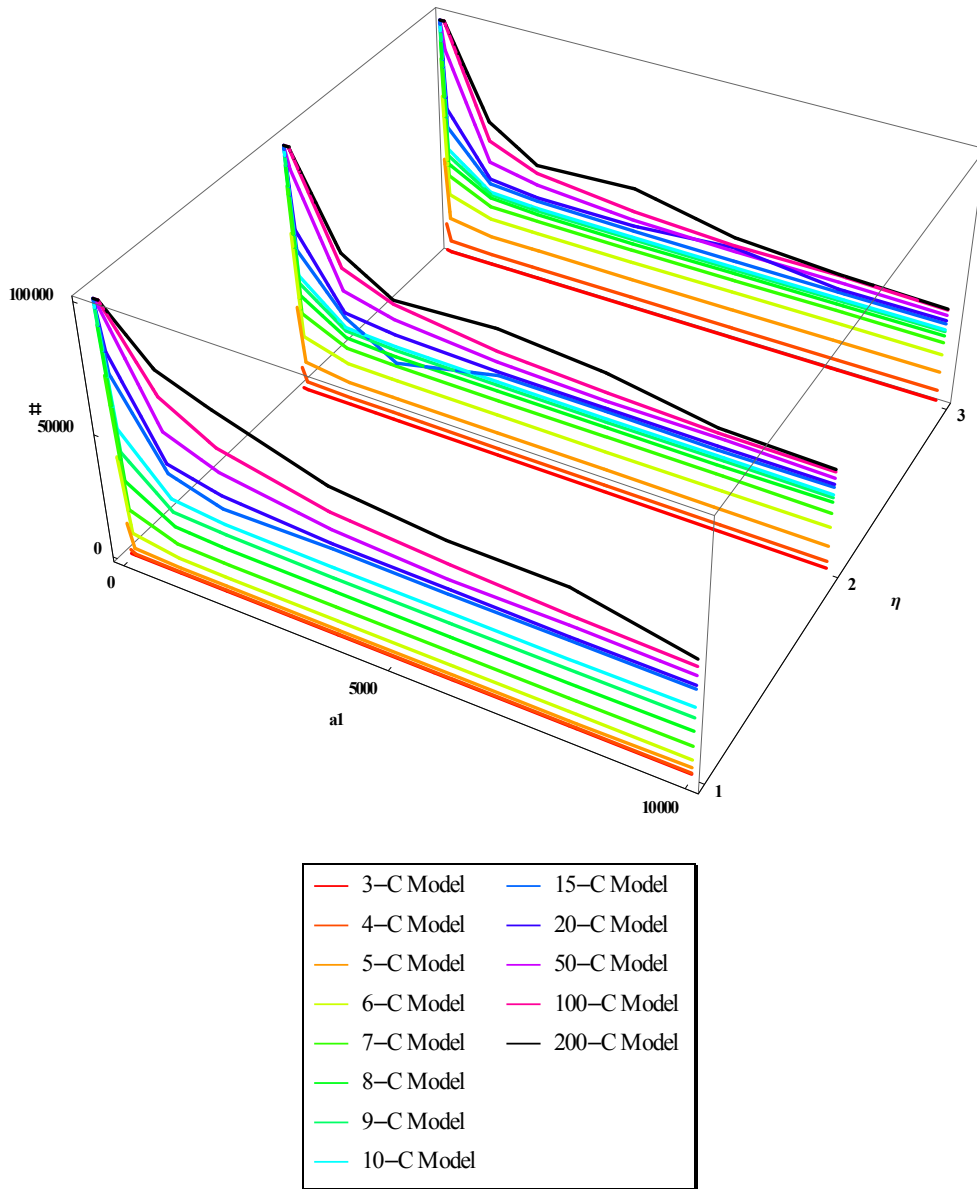
Figure 4.6. The model output for the *n*-country model all-in-autarky equilibrium - strict stability.[126] The two horizontal axes represent the values of a_1 (payoff to the leading country) and η (1, 2, 3 represent η_1, η_2, η_3, respectively) and the vertical axis represents the number of times out of 100000 that the all-in-autarky equilibrium in the *n*-country model is strictly stable.

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[126] Note that the all-in-autarky state is the only state considered for testing stability via numerical computations as mentioned previously in this chapter.

Based on the numerical computations output (see Section 4.4), the following results and general trends are noted for the n-country model:

Result 4.1. The probability of the fully integrated state $(1,\ldots,1)$ being a strict equilibrium over a varied range of parameter values:

(i)   Is extremely low.[127]

(ii)   Increases with decreasing η for a given number of countries.[128]

(iii)   Decreases with increasing number of countries (4-country model and higher)[129].

(iv)   Tends to 0 for $\eta = \dfrac{1}{0.2U}$ for 50 countries and higher.

(v)   Tends to 0 for all η for 200 countries.

Result 4.2. The probability of the all-in-autarky equilibrium $(0,\ldots,0)$ being strictly stable over a varied range of parameter values:

(i)   Increases with decreasing η for a given number of countries.[130]

(ii)   Decreases with increasing number of countries.[131]

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[127] Reaching up to a maximum of ~ 0.006.

[128] For example, consider the 5-country model: the probabilities of the fully integrated state $(1,1,1,1,1)$ being a strict equilibrium for $a_1 = 10$ and η values of $1/0.2U$, $1/0.5U$, $1/0.8U$ are 0.00001, 0.00148, 0.00305, respectively.

[129] For example, for 5, 6, 7, 8, 9, 10 countries, with $\eta = 1/0.5U$ and $a_1 = 10000$, the probabilities of the fully integrated state being a strict equilibrium are 0.00283, 0.00208, 0.00171, 0.00164, 0.00119, 0.00111, respectively.

[130] For example, consider the 6-country model: for $a_1 = 6000$ and η values of $1/0.2U$, $1/0.5U$, $1/0.8U$, the probabilities of the strict equilibrium $(0,0,0,0,0,0)$ being strictly stable are 0.05940, 0.17260, 0.20477, respectively.

(iii)   Tends to 1 as a_1 tends to n (see Subsection 4.4.8).

Result 4.3. For the heterogeneous state, two cases were evaluated over a varied range of parameter values:

(i)   6 countries: The probability of the heterogeneous state $(1,0,1,1,1,1)$ being a strict equilibrium is higher than that of other states like $(1,1,0,1,0,1)$ and $(1,1,0,1,1,1)$.

(ii)   10 countries: The probability of the heterogeneous states $(1,1,1,0,0,0,0,0,0,1)$ and $(1,0,1,0,1,0,0,0,1,0)$ being strict equilibria is 0. The probability of the existence of heterogeneous equilibria $(1,0,1,1,1,1,1,1,1,1)$ is higher than that of other heterogeneous equilibria like $(1,1,1,1,1,1,0,1,0,1)$ and $(1,0,1,1,1,1,1,1,0,1)$.

4.3 Conclusions

Based on the results obtained in this chapter, the following conclusions are obtained for the *n*-country model:

It is rather surprising to find that countries individually find it more advantageous to remain in autarky when given an option to choose between complete autarky and full integration into the world economy, especially when the number of countries participating in the world trade increases. A way to view this behaviour would be that a *weaker* country chooses not to participate in world trade (even though doing so may help it to obtain a higher payoff in absolute terms) just so that the leading countries are not able to substantially benefit from world trade, thereby making the abstaining country's own *relative* position stronger. In addition, when all the other countries are fully integrated in international trade, a non-leading country finds it advantageous to choose the autarkic strategy over the fully integrated strategy even

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[131] The 200-country model has highest probability of the all-in-autarky equilibrium being strictly stable.

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though the former yields a greater payoff, as by doing so it can reduce the leading country's payoff thereby reducing the behavioural gap (as defined in the previous chapter)[132]. This recaptures the main theory of terrorism or negative behaviour in that the position of the leading countries is deliberately reduced.

However, complete autarkies do not exist today and countries limit their growth potential if they choose to not open up to international trade[133]. International trade and investment are major factors contributing towards economic growth[134] and realistically speaking, the level with which countries engage in world trade varies from country to country, whereby each country chooses an intermediate strategy as opposed to a *fully integrated* or *completely autarkic* strategy. Hence, this trade game model is extended to a continuous version in the next chapter, where countries can update their strategies by a small strategy size $\delta > 0$, allowing them to integrate into the world economy by different amounts.

4.4 Appendix

This section contains the derivation of the exponential distribution used for generating the values of the a_i's, the Mathematica programs used to evaluate the number of strict equilibria and strictly stable equilibria and tabulated the model output obtained for the 3-country model, the 4-country model and the n-country model.

4.4.1 Exponential Distribution

In this appendix subsection, the derivation of the exponential distribution used for generating the values of the a_i's (for $i > 1$) is shown. Note that the value of a_1 is fixed before generating the list of the a_i's.

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[132] This is a direct result of competition and envious behaviour as obtained in other literary papers discussed in Chapter 2.

[133] For example, the incomes of the least globalized countries like Iran, North Korea, and Pakistan have declined or remained static over the last few decades, see Manzella [98].

[134] For example, developing countries with open economies grew by 5% in the 1970s and the 1980s while those with closed economies grew by less than 1%, see Manzella [98].

Let X be an exponential variable (which represents the value of a_i) with parameter η. Then, by definition:

$$\text{Prob}(a \le X < b) = K\int_a^b e^{-\eta(t-1)}dt$$

$$= \frac{K}{\eta}\left[e^{-\eta(a-1)} - e^{-\eta(b-1)}\right].$$

Thus,

$$\text{Prob}(X < x) = K\int_1^x e^{-\eta(t-1)}dt$$

$$= \frac{K}{\eta}\left[1 - e^{-\eta(x-1)}\right].$$

Let $y = \frac{K}{\eta}\left[1 - e^{-\eta(x-1)}\right]$.

If $y = f(x)$, then $x = f^{-1}(y)$.

Thus, re-arranging: $x = 1 - \frac{1}{\eta}\ln\left(1 - \frac{\eta y}{K}\right)$.

Lemma 4.1. The probability density function is $\dfrac{\eta e^{-\eta(a_1-1)}}{1 - e^{-\eta(a_1-1)}}$, where a_1 is the payoff to the leading country.

Proof. Let U be the upper bound for the distribution, such that $U = a_1 - 1$ (since the values of a_i's generated should be strictly less than a_1). Then,

$$1 = K\int_1^{U+1} e^{-\eta(a-1)}da$$

$$= K\int_0^U e^{-\eta t}dt$$

$$= -\frac{K}{\eta}\left[e^{-\eta t}\right]_o^U$$

$$= \frac{K}{\eta}\left[1 - e^{-\eta U}\right].$$

Thus, $K = \dfrac{\eta}{1 - e^{-\eta U}}$.

This gives the probability density function as $\dfrac{\eta e^{-\eta U}}{1 - e^{-\eta U}} = \dfrac{\eta e^{-\eta(a_1 - 1)}}{1 - e^{-\eta(a_1 - 1)}}$.   □

The list of the a_i's is obtained using a tabulated form of $1 - \dfrac{1}{\eta} \ln\left(1 - \dfrac{\eta y}{K}\right)$, where

$K = \dfrac{\eta}{1 - e^{-\eta(a_1 - 1)}}$ and y is a uniformly distributed real number in the range 0 to 1.

4.4.2 Scaling payoff values

In this appendix subsection, it is shown that the results are independent of the scaling of the a_i's and the λ_i's by a common (positive) factor.

From Chapter 3, the condition required for a state or strategy profile z^* to be a strict equilibrium is:

$\left(2z_i^* - 1\right)\left(\Phi_i\left(z^*\right) - \Psi_i\left(z^*\right)\right) > 0$, for all i.

This is also the condition used in the Mathematica program to test (within a single for-loop i) if a given state is a strict equilibrium.

The strict equilibrium condition above can be expressed using the a_i's and the λ_i's for all i as follows:

$$\left(2z_i^* - 1\right)\left[\left(\max\left\{\sigma\left(|z_{-i}|\right)a_{-i}^*, \lambda_{+i}^*\right\} + \sigma\left(|z_{-i}| + 1\right)a_i\right) - \left(\max\left\{\sigma\left(|z_{-i}| + 1\right)a_{+i}^*, \lambda_{-i}^*\right\} + \lambda_i\right)\right]$$
$$> 0$$

$$(4.1)$$

It needs to be shown that scaling the a_i's and the λ_i's by a common (positive) factor does not change the above inequality for all i. Let $A_i = \alpha a_i$ and $L_i = \alpha \lambda_i$ for all i, where $\alpha > 0$. The strict equilibrium criteria for a strategy profile with payoffs A_is and L_is can be written as:

$$\left(2z_i^* - 1\right)\left[\left(\max\left\{\sigma\left(|z_{-i}|\right)A_{-i}^*, L_{a+i}^*\right\} + \sigma\left(|z_{-i}| + 1\right)A_i\right) - \left(\max\left\{\sigma\left(|z_{-i}| + 1\right)A_{+i}^*, L_{-i}^*\right\} + L_i\right)\right]$$
$$> 0.$$

Using the notation introduced in Chapter 3, the above inequality can be simplified as follows:

$$\left(2z_i^* - 1\right)\left[\left(\max\left\{\sigma\left(|z_{-i}|\right)\alpha a_{-i}^*, \alpha\lambda_{a+i}^*\right\} + \sigma\left(|z_{-i}| + 1\right)\alpha a_i\right) - \left(\max\left\{\sigma\left(|z_{-i}| + 1\right)\alpha a_{+i}^*, \alpha\lambda_{-i}^*\right\}\right.\right.$$
$$\left.\left. + \alpha\lambda_i\right)\right] > 0.$$

This inequality can be simplified as:

$$\left(2z_i^* - 1\right)\alpha\left[\left(\max\left\{\sigma\left(|z_{-i}|\right)a_{-i}^*, \lambda_{a+i}^*\right\} + \sigma\left(|z_{-i}| + 1\right)a_i\right) - \left(\max\left\{\sigma\left(|z_{-i}| + 1\right)a_{+i}^*, \lambda_{-i}^*\right\} + \lambda_i\right)\right]$$
$$> 0.$$

As $\alpha > 0$, this inequality is essentially the same as (4.1). Thus, scaling of payoff values (a_i's and λ_i's) does not make a difference to the number of times a given strategy profile is a strict equilibrium when the program is run.

From Chapter 3, the condition required for a state or strategy profile to be a strictly stable equilibrium is:

$$\left(2z_i^{(k)} - 1\right)\left(\Phi_i\left(z^{(k)}\right) - \Psi_i\left(z^{(k)}\right)\right) > 0, \text{ for all } i \text{ and all } k. \tag{4.2}$$

Now, $\Phi_i\left(z^{(k)}\right) = \left(\max\left\{\sigma\left(\left|z_{-i}^{(k)}\right|\right)a_{-i}^*, \lambda_{a+i}^*\right\} + \sigma\left(\left|z_{-i}^{(k)}\right| + 1\right)a_i\right)$ and

$\Psi_i\left(z^{(k)}\right) = \left(\max\left\{\sigma\left(\left|z_{-i}^{(k)}\right| + 1\right)a_{+i}^*, \lambda_{-i}^*\right\} + \lambda_i\right)$. Note that the expressions for both

$\Phi_i\left(z^{(k)}\right)$ and $\Psi_i\left(z^{(k)}\right)$ can be simplified to the form $Aa_i + B\lambda_i$, where $A, B \geq 0$. If

the a_i's and the λ_i's are scaled by $\alpha > 0$, then the above expressions for $\Phi_i\left(z^{(k)}\right)$

and $\Psi_i\left(z^{(k)}\right)$ can be simplified to the form $A\left(\alpha a_i\right) + B\left(\alpha\lambda_i\right) = \alpha\left(Aa_i + B\lambda_i\right)$. The

strict stability criteria (4.2) can be written for the scaled values as follows:

$$\left(2z_i^{(k)}-1\right)\alpha\left(\Phi_i\left(z^{(k)}\right)-\Psi_i\left(z^{(k)}\right)\right)>0, \text{ for all } i \text{ and all } k.$$

This is satisfied if the original criteria (4.2) is satisfied as $\alpha>0$. Thus, scaling of payoff values (the a_i's and the λ_i's) does not make a difference to the number of times a given strategy profile is a strictly stable equilibrium when the program is run.

4.4.3 Mathematica Program for a Strict Equilibrium

The *n*-country model for evaluating the number of times (out of 100000) a random heterogeneous state is a strict equilibrium (defined by the variable *listofz*) is programmed using Mathematica as follows:

```
(* MATHEMATICA PROGRAM FOR EVALUATING THE NUMBER TIMES (OUT
OF 100000) A GIVEN HETEROGENEOUS STATE IS A STRICT EQUILIBRIA
FOR THE n-COUNTRY MODEL *)

ClearAll[n,listofz,U,eta,a1,K,countz,x,sigmafn,m,
jminusistar,z,i,j,k,jplusistar,aminusistar,aplusistar,lminusi
star,lplusistar,counter,boolean,data,datasorted,lsitofa,listo
fl,noofstates,phifn,psifn];

Off[General::spell1]

(* Input data needed for the n-country model:
     n  - number of countries
     a1 - leading country's payoff when it plays strategy  E
          against an opponent playing E
      u- rate parameter of the exponential distribution
          generating the values of a's *)

n=3;
a1=10;
eta=1/(0.2 U);

(* Other variables needed for calculations:
     noofstates - counts the number of strictly stable
                    autarky states
     listofz    - strategy vector, to be tested for strict
                    stability [z=(Subscript[z,
                    1],...,Subscript[z, n])]
     U          - upper bound used for exponential
                    distribution
     K          - variable (equal to   u/(1-e^- u A)) used in
                    generating values of a's *)
```

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```
noofstates=0;
listofz=Table[Random[Integer],{n}];
U=a1-1;
K=eta/(1-Exp[-eta U]);

(* Function countz to count the number of countries playing
strategy E(1):
      |z|=Sum(i=1,n)zi *)

countz[x_]:=Count[x,1];

(* Function sigmafn to determine the fraction of countries
playing strategy E:
       (m)=max{0,(m-1)/(n-1)} for 0<=m<=n *)

sigmafn[m_]:=Max[0,(m-1)/(n-1)];

(* Functions required to define the variables needed for
calculating payoffs: define *)

jminusistar[z_,i_]:=If[Count[z,1]==0,0,If[i==Min[Position[z,1
]],If[Delete[Position[z,1],1]=={},0,Min[Delete[Position[z,1],
1]]],Min[Position[z, 1]]]];
jplusistar[z_,i_]:=Min[i,jminusistar[z,i]];
aminusistar[z_,i_]:=If[jminusistar[z,
i]==0,0,listofa[[jminusistar[z, i]]]];
aplusistar[z_,i_]:=If[jminusistar[z,i]==0,0,listofa[[jplusist
ar[z,i]]]];
lminusistar[z_,i_]:=Max[Delete[(1-z)listofl,i]];
lplusistar[z_,i_]:=Max[listofl[[i]],  minusistar[z,i]];

For[counter=1,counter<=100000,counter++,boolean=True;

     (* Generating random values of a's using exponential
      distribution *)

    data=Table[y=Random[];1-(1/eta)Log[1-eta y/K],{n-1}];
    datasorted=Sort[data,#1>#2&];
    listofa=Prepend[datasorted,a1];

     (* generating random values of   's using uniform
      distribution *)

    listofl={};

     (* The i-loop defines the functions phifn and psifn and
      checks the equilibrium criterion *)

    For[j=1,j<=n,j++,listofl=Append[listofl,Random[Real,{1,
     listofa[[j]]}]]]]

    For[i=1,i<=n,i++,

        (* define phifn *)
```

```
        phifn[z_,i_]:=Max[sigmafn[countz[Delete[z,i]]]
        aminusistar[z,i],lplusistar[z,i]]+sigmafn[countz[D
        elete[z,i]]+1] listofa[[i]];

        (* define phifn *)

        psifn[z_,i_]:=Max[sigmafn[countz[Delete[z,i]]+1]
        aplusistar[z,i],lminusistar[z,i]]+listofl[[i]];

        (* checking the equilibrium condition: define *)

        boolean=boolean && ((2(listofz[[i]])-1)
        (phifn[listofz,i]-psifn[listofz,i])>0);

        If[boolean==False,Break[]];

    ];

     If[boolean==True,noofstates++];

];

(* Printing a state and the number of times (out of 100000)
it is strict equilibrium *)

Print[listofz,noofstates]
```

4.4.4 Mathematica Program for a Strictly Stable Equilibrium

The *n*-country model for evaluating the number (out of 100000) a random hetero-
geneous equilibria is strictly stable (defined by the variable *listofz*) is programmed
using Mathematica as follows:

```
 (* MATHEMATICA PROGRAM FOR EVALUATING THE NUMBER OF TIMES
(OUT OF 100000) A GIVEN STATE IS A STRICTLY STABLE
EQUILIBRIUM FOR THE n-COUNTRY MODEL, BUILDING UP ON THE
PROGRAM ABOVE BY DEFINING A MUTANT STRATEGY AND CONDITIONS
REQUIRED TO HOLD FOR STRICY STABILITY *)

ClearAll[n,listofz,U,eta,a1,K,countz,x,sigmafn,m,
jminusistar,z,i,j,k,jplusistar,aminusistar,aplusistar,lminusi
star,lplusistar,counter,boolean,data,datasorted,lsitofa,listo
fl,noofstates,phifn,psifn];

Off[General::spell1]
```

```
n=3;
a1=10;
eta=1/(0.2 U);

noofstates=0;
listofz=Table[Random[Integer],{n}];
U=a1-1;
K=eta/(1-Exp[-eta U]);

countz[x_]:=Count[x,1];

sigmafn[m_]:=Max[0,(m-1)/(n-1)];

jminusistar[z_,i_]:=If[Count[z,1]==0,0,If[i==Min[Position[z,1
]],If[Delete[Position[z,1],1]=={},0,Min[Delete[Position[z,1],
1]]],Min[Position[z, 1]]]];
jplusistar[z_,i_]:=Min[i,jminusistar[z,i]];
aminusistar[z_,i_]:=If[jminusistar[z,
i]==0,0,listofa[[jminusistar[z, i]]]];
aplusistar[z_,i_]:=If[jminusistar[z,i]==0,0,listofa[[jplusist
ar[z,i]]]];
lminusistar[z_,i_]:=Max[Delete[(1-z)listofl,i]];
lplusistar[z_,i_]:=Max[listof [[i]], lminusistar[z,i]];

For[counter=1,counter<=100000,counter++,boolean=True;

    data=Table[y=Random[];1-(1/eta)Log[1-eta y/K],{n-1}];
    datasorted=Sort[data,#1>#2&];
    listofa=Prepend[datasorted,a1];

    listof ={};

     For[j=1,j<=n,j++,listofl=Append[listofl,Random[Real,{1,
     listofa[[j]]}]]]]

     (* The k-loop defines the mutant strategy z^(k), where
     k denotes the country deviating away from strategy z *)

    For[k=1,k<=n,k++,listofz[[k]]=1-listofz[[k]];

        For[i=1,i<=n,i++,

          phifn[z_,i_]:=Max[sigmafn[countz[Delete[z,i]]]
          aminusistar[z,i],lplusistar[z,i]]+sigmafn[countz[
          Delete[z,i]]+1] listofa[[i]];

          psifn[z_,i_]:=Max[sigmafn[countz[Delete[z,i]]+1]
          aplusistar[z,i],lminusistar[z,i]]+listofl[[i]];

          (* checking the strict stability condition:
          (2Subscript[z, i]-1)(Subscript[ ,i](z^(k))-
          Subscript[ ,i](z^(k)))>0 for all i and all k *)

          If[i==k,boolean=boolean && ((2(listofz[[i]])-
```

```
        1)(phifn[listofz,i]-
        psifn[listofz,i])<0),boolean=boolean &&
        ((2(listofz[[i]])-1)(phifn[listofz,i]-
        psifn[listofz, i]) > 0)];

        If[boolean==False,Break[]];

      ];

      listofz[[k]]=1-listofz[[k]];

      If[boolean==False,Break[]];

    ];

    If[boolean==True,noofstates++];

];

(* Printing a state and the number of times (out of 100000)
it is a strictly stable equilibrium *)

Print[listofz,noofstates]
```

4.4.5 Data for the 3-country Model

All the model output given in this subsection is per 100000 runs.

The number of fully integrated $(1,1,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	2	6	9	12	10	11	11	9
$1/0.5U$	98	237	229	210	229	212	224	250
$1/0.8U$	214	380	435	440	382	418	429	387

The number of heterogeneous state $(1,0,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	378	896	1053	1010	1029	1057	1065	999
$1/0.5U$	2194	3744	4156	4188	4102	4214	4304	4184
$1/0.8U$	3004	5186	5513	5578	5501	5633	5542	5736

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The number of heterogeneous state $(1,1,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	120	142	129	121	135	137	147	149
$1/0.5U$	1206	1180	1241	1234	1184	1249	1224	1268
$1/0.8U$	1722	1940	1972	1963	1972	1985	1980	1898

The number of strictly stable all-in-autarky $(0,0,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	36	19	20	14	25	12	11	11
$1/0.5U$	473	247	272	222	243	244	228	268
$1/0.8U$	947	516	514	520	476	473	436	480

4.4.6 The model output for the 4-country Model

All the model output given in this subsection is per 100000 runs.

The number of fully integrated $(1,1,1,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	4	5	5	7	6	7	4	11
$1/0.5U$	134	259	250	284	282	255	279	253
$1/0.8U$	321	549	540	550	576	566	563	532

The number of heterogeneous state $(1,1,0,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	337	543	560	585	543	528	585	549
$1/0.5U$	1732	2415	2503	2500	2566	2471	2569	2550
$1/0.8U$	2284	3211	3268	3349	3210	3188	3285	3270

The number of heterogeneous state $(1,0,1,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	294	1637	1818	1860	1867	1838	1814	1854
1/0.5U	1306	3733	3904	4017	3946	4069	4021	4025
1/0.8U	1754	4117	4378	4340	4356	4371	4490	4446

The number of heterogeneous state $(1,0,0,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	40	913	1451	1525	1556	1609	1564	1503
1/0.5U	261	2347	3134	3257	3306	3318	3364	3452
1/0.8U	356	2730	3437	3494	3598	3562	3615	3648

The number of heterogeneous state $(0,1,1,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	0	0	0	0	0	0	0	0
1/0.5U	2	0	1	0	0	0	0	0
1/0.8U	3	3	3	0	3	4	4	2

The number of heterogeneous state $(0,1,0,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	0	1	3	2	1	0	2	1
1/0.5U	8	16	32	23	29	20	32	33
1/0.8U	19	44	47	65	55	45	53	45

The number of heterogeneous state $(0,0,1,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	1	27	38	17	44	59	39	47
1/0.5U	13	181	236	242	263	259	245	248
1/0.8U	50	243	333	359	389	315	357	326

The number of heterogeneous state $(1,1,1,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	3	11	12	15	8	10	9	8
1/0.5U	137	210	222	214	203	187	187	219
1/0.8U	299	419	445	468	447	387	416	443

The number of heterogeneous state $(1,1,0,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	9	21	30	19	19	22	19	16
1/0.5U	365	422	480	478	486	449	451	463
1/0.8U	719	877	894	916	889	950	886	896

The number of heterogeneous state $(1,0,1,1)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	162	384	442	470	458	449	453	483
1/0.5U	1331	2460	2630	2743	2713	2612	2618	2655
1/0.8U	1984	3311	3607	3747	3579	3659	3658	3704

The number of strictly stable all-in-autarky $(0,0,0,0)$ equilibria:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	1672	463	369	385	356	361	382	376
1/0.5U	9408	3728	3208	3252	3066	3099	3037	3122
1/0.8U	12842	5548	4901	4882	4851	4771	4748	4854

4.4.7 The Fully Integrated State – Strict Equilibrium Model output

5 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	1	5	7	6	5	5	9	5
1/0.5U	148	234	263	248	228	248	226	283
1/0.8U	305	547	512	582	539	556	495	526

6 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	1	2	0	7	2	2	5	5
1/0.5U	116	178	221	225	206	218	201	208
1/0.8U	318	468	524	520	496	497	519	500

7 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	2	1	2	4	0	2	2	1
1/0.5U	108	166	167	183	176	188	174	171
1/0.8U	294	459	456	521	463	473	455	501

8 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	1	0	4	2	2	1	4	2
1/0.5U	101	160	137	151	153	154	158	164
1/0.8U	264	399	431	407	435	442	443	436

9 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	3	1	0	2	0
$1/0.5U$	71	114	141	155	152	158	127	119
$1/0.8U$	229	381	406	411	392	395	385	386

10 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	1	1	3	1	0	0	1	0
$1/0.5U$	75	131	110	117	116	118	114	111
$1/0.8U$	234	337	371	340	360	348	374	366

25 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	14	11	29	24	21	26	26	21
$1/0.8U$	43	47	93	103	125	123	108	127

50 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	2	7	8	6	7	6	8	8
$1/0.8U$	22	38	51	51	39	49	54	46

100 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	1	2	1	2	1	1	2	4
$1/0.8U$	7	14	22	15	7	19	18	24

200 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	0	0	0	0	1	0	0	1
$1/0.8U$	2	5	8	6	7	6	9	3

4.4.8 The All-In-Autarky State – Strict Stability Model output

From the analysis of the n-country model in Chapter 3, it is already shown that the all-in-autarky state is a strict equilibrium. The strict stability of this equilibrium remains to be explored. The Mathematica program is run in order to find the number (out of 100000) of strictly stable all-in-autarky equilibria for a maximum of 200 countries.

5 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	12759	2969	2197	2193	2092	2147	2026	2668
$1/0.5U$	35245	12362	9874	9847	9803	9840	9624	9495
$1/0.8U$	41668	15839	13073	13077	12752	12706	12772	12826

6 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	40013	9303	6324	5999	5971	5940	5957	5814
1/0.5U	65671	23459	18029	17724	17541	17260	17330	17416
1/0.8U	68683	26621	21014	20690	20384	20477	20469	20318

7 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	71621	19161	12437	11845	11675	11521	11337	11388
1/0.5U	84462	33276	24620	23967	23646	23657	23558	23149
1/0.8U	84361	34998	26508	26154	25718	25590	25750	25500

8 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	90495	30942	19452	18374	17991	17550	17334	17473
1/0.5U	92540	40790	29094	27924	27724	27532	27302	27747
1/0.8U	91806	40174	30462	29159	28546	28627	28488	28362

9 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	96758	42569	25472	24401	23498	23140	22917	22811
1/0.5U	96175	45904	32014	30655	30149	29715	29749	29540
1/0.8U	95700	43753	31986	30759	30478	30159	30167	30204

10 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	98754	51762	30801	28585	27767	27413	26905	27029
1/0.5U	98528	49034	33408	32234	31547	31070	31018	30737
1/0.8U	98412	46622	33010	31833	31383	31157	31033	30948

15 countries:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	100000	72954	40970	34781	35360	34642	34445	34048
1/0.5U	100000	59576	37610	25306	34282	34087	33318	33769
1/0.8U	100000	56044	36637	35005	34190	33825	33551	33429

20 countries:

η				a_1				
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	100000	81209	44810	40063	37381	36703	35827	35539
1/0.5U	100000	67629	39624	37204	35771	35347	34916	34963
1/0.8U	100000	63840	38756	36503	36369	39493	34586	34801

50 countries:

η				a_1				
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	100000	97223	57332	49213	43015	40493	40311	39304
1/0.5U	100000	91476	48667	43496	39778	38317	38299	37392
1/0.8U	100000	88420	46083	42051	38841	37958	37761	37030

100 countries:

η				a_1				
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	100000	99987	70672	58665	49671	46558	44221	42804
1/0.5U	100000	99983	57936	49782	44139	41793	40143	39953
1/0.8U	100000	99976	55223	47049	42563	40361	40556	39224

200 countries:

η				a_1				
	10	100	1000	2000	4000	6000	8000	10000
1/0.2U	100000	100000	80791	72991	59781	55397	54783	45602
1/0.5U	100000	100000	63965	51426	53368	49605	42154	40925
1/0.8U	100000	100000	63289	50453	52256	43741	40976	39641

Note from the tables above that the number of times the all-in-autarky state is a strictly stable equilibrium reaches 100000 as the value of a_1 tends to the number of countries n. Hence the probability of the all-in-autarky state being a strictly stable equilibrium $\to 1$ as $a_1 \to n$ from above. Note that condition (3.32) required for the all-in-autarky equilibrium to be strictly stable is summarized in Chapter 3 as:

$\frac{1}{n-1} a_1 < \min_i \{\lambda_i\}$. As $a_1 \to n$, the condition essentially requires the smallest λ_i to

be greater than 1, which is always the case. Hence as $a_1 \to n$, the numerical computations give 100000 strictly stable all-in-autarky equilibria per 100000 runs. Also note that as a_1 increases over the value of n, the number of all-in-autarky equilibria per 100000 runs decreases as the smallest λ_i needs to be significantly larger than 1 for condition (3.32) to hold. Generally speaking, increasing n increases stability as condition (3.32) is easily satisfied for larger n.

4.4.9 Heterogeneous States – Strict Equilibrium Model output

6 countries – Heterogeneous state $(1,1,0,1,0,1)$:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	1	1	2	1	0	2	2	2
$1/0.8U$	0	3	6	5	5	3	5	8

6 countries – Heterogeneous state $(1,0,1,1,1,1)$:

η	a_1							
	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	1	3	3	1	0	2	1	3
$1/0.5U$	35	52	51	56	55	58	72	68
$1/0.8U$	63	128	153	139	132	130	135	154

6 countries – Heterogeneous state $(1,1,0,1,1,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	11	17	29	23	25	27	32	22
$1/0.8U$	23	80	65	78	66	65	69	66

10 countries – Heterogeneous state $(1,1,1,0,0,0,0,0,0,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	0	0	0	0	0	0	0	0
$1/0.8U$	0	0	0	0	0	0	0	0

10 countries – Heterogeneous state $(1,0,1,0,1,0,0,0,1,0)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	0	0	0	0	0	0	0	0
$1/0.8U$	0	0	0	0	0	0	0	0

10 countries – Heterogeneous state $(1,1,1,1,1,1,1,1,0,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	8	18	24	25	22	22	22	21
$1/0.8U$	31	79	71	81	82	70	76	67

10 countries – Heterogeneous state $(1,1,1,1,1,1,0,1,0,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	0	4	7	2	2	1	10	3
$1/0.8U$	3	14	15	11	5	8	11	16

10 countries – Heterogeneous state $(1,0,1,1,1,1,1,1,1,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	1	0	0	0	0	1	0
$1/0.5U$	15	2	34	34	28	39	43	49
$1/0.8U$	57	83	107	114	111	84	99	99

10 countries – Heterogeneous state $(1,0,1,1,1,1,1,1,0,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	0	0	0	0	0	0	0
$1/0.5U$	0	7	3	3	2	8	4	6
$1/0.8U$	2	14	17	24	15	16	14	10

10 countries – Heterogeneous state $(1,0,1,1,1,1,1,1,1,1)$:

	a_1							
η	10	100	1000	2000	4000	6000	8000	10000
$1/0.2U$	0	1	0	0	0	0	1	0
$1/0.5U$	15	2	34	34	28	39	43	49
$1/0.8U$	57	83	107	114	111	84	99	99

Note from the tables above that the number of times the heterogeneous states of the form $(1,0,1,1,\ldots,1)$ are strict equilibria is greater than for the other heterogeneous states for η_2, η_3. This implies that even though it is more advantageous for country 2 to play E (in terms of obtaining a higher fully integrated payoff as opposed to the autarkic payoff) when all other countries are fully integrated into the world economy, the competition between countries (modelled by the behavioural rule in the previous chapter) results in envious behaviour leading to country 2 choosing strategy A over strategy E, in order to reduce the leading country's payoff (so that the gap in payoffs is reduced). These particular heterogeneous states were chosen to highlight this envious behaviour.

Chapter 5

The Trade Game with Strategy Size δ

In this chapter, an extension to the trade game model of the previous chapters is formulated, in order to model the trade dynamics when countries can only change their strategy by a small strategy size as opposed to switching between the pure strategies (engaging in world trade or remaining in autarky).

Section 5.1 presents the continuous strategy trade game model, defining the payoff to countries in terms of their level of engagement with the world economy. Section 5.2 defines the strategy changes that enable the countries to reach their *aspiration level*. Sections 5.3 and 5.4 state the conditions for a state to be a strict equilibrium and a strictly stable equilibrium, respectively. These conditions are then simplified for the all-in-autarky and fully integrated states. Specializations to the 2-country and 3-country scenarios are briefly discussed.

5.1 The δ-Model

Definition 5.1. A *strategy* for a country i is a level of engagement with the world economy, $z_i \in I = \{\zeta_0, \zeta_1, \ldots, \zeta_K\}$, where $0 = \zeta_0 < \zeta_1 < \ldots < \zeta_k < \ldots < \zeta_K = 1$, and in particular, $\zeta_k = \zeta_{k-1} + \delta$, where $\delta = 1/K$, and is called the *strategy size*.[135]

The strategy z_i for country i is also allowed to vary continuously between 0 and 1 in the following chapter. The strategy size can be thought of as a measure by which a country can adjust its economic policy in a given unit time, in order to benefit

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[135] Note that $\delta = 1/K$ is defined as the smallest increment in investment in international trade.

from cross-border trade as well as trade at home. In this chapter, δ is considered to be of the form $1/K$, where $K > 1$ and $K \in \mathbb{N}$. Note that $\delta = 1$ gives us the original model from Chapter 3, hence only $\delta \leq 1/2$ is considered in this chapter.

Definition 5.2. A *strategy profile,* or simply a *state* for n countries is a vector $z = (z_1, \ldots, z_n) \in I^n$.

Definition 5.3. The *trade weights matrix* is a matrix of probability weights $W = (w_{ij})$, with $0 \leq w_{ij} \leq 1$ and $\sum_j w_{ij} = 1$ for each country i, where w_{ij} measures the relative degree of openness country i has to trade with country j, when country i is fully engaged in world trade (i.e. when $z_i = 1$).

The weights w_{ij} can be thought of as defining barriers to trade in the sense that they measure the relative degree of openness a country has to trade with other countries. So $w_{ij} = 0$ means that country i does not trade with country j at all, i.e. it remains in autarky relative to country j, and $w_{ij} > w_{ik}$ means that country i is more open to trade with country j than it is with country k. These barriers may relate to geography, for example through transport costs, or to the fact that country i gains more or less benefit from trading with country j because of more or less complementarities between their economies, or the greater or lesser size of the economy of country j. These barriers can also relate to language and culture, or deliberate impediments to commercial intercourse. Goodfriend and McDermott [59] define the notion of *familiarities* to account for asymmetries between countries and attribute unfamiliarity of a country to barriers towards cultural or commercial interaction. Trade barriers discourage the general expansion of trade between the countries. On the timescale of strategy changes, the trade weights w_{ij} (for all i and all j) are assumed to be constant.

Definition 5.4. Given a strategy profile z, the *payoff* to country i is:

$$\pi_i(z) = a_i z_i \sum_j w_{ij} z_j + (1 - z_i) \lambda_i. \tag{5.1}$$

Here, λ_i is the payoff received when country i is in autarky ($z_i = 0$), and a_i is the (maximum) payoff that country i can receive, realised when all countries are fully engaged in world trade ($z_j = 1$ for all j). It is assumed that $a_i > \lambda_i$ for all i, so that countries obtain a greater payoff by integrating into the world economy than by remaining in complete autarky. For $0 < z_i < 1$, country i's payoff is a weighted sum of these two effects, and depends on the levels of integration of the other countries through the factor:

$$W_i(z) = \sum_{j=1}^{n} w_{ij} z_j. \tag{5.2}$$

Note that, if $w_{ii} > 0$, then there is a gain to internal trade yielding a payoff $a_i w_{ii} z_i^2$. This can be thought of as a result of stimulation to growth of internal markets either as a country's primary objective or as a secondary effect of international trade. For example, the secondary effect can be R&D spillovers from international trade that boost domestic markets (see Lichtenberg and Van Pottlesberghe [79]).

The previous versions of this trade game that have been considered in Chapter 3 use the weights: $w_{ii} = 0$ and $w_{ij} = \dfrac{1}{n-1}$ for all $i \neq j$. This means there is no secondary stimulation of the internal economy due to world trade, and the contribution of any other country to country i's per capita GDP depends only on its own degree of openness, and hence is the same for any country with the same degree of openness, so there are no exogenous restrictions to trade.

At the other extreme is the theoretical possibility that $w_{ii} = 1$ and $w_{ij} = 0$ for $i \neq j$. In this case, the stimulation of the domestic economy is the primary objective of the country. Country i can then achieve its maximum payoff by taking $z_i = 1$ without any direct contribution from any other country. However, this is not the same as country i being in autarky ($z_i = 0$ with payoff λ_i), since generally country i contributes to the gains other countries receive from engagement in world trade (if $w_{ji} > 0$ for some j). In the very extreme case $w_{ii} = 1$ for all i, every country is in a *maximal payoff autarky*. This can be thought of as a situation in which enlightened

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economic policies by governments (i.e. $z_i = 1$) lead to maximum extraction of economic value from the country's internal economy. The reduced payoff λ_i from true autarky can then be regarded as the payoff obtained not just from lack of openness to world trade, but from poor economic policies at home (i.e. $z_i = 0$).

Note that when $w_{ii} = 1$, country i can obtain its maximum possible payoff a_i by choosing to play $z_i = 1$, regardless of other countries' strategies. However, when $0 < w_{ii} < 1$, country i can only obtain its maximal payoff of a_i by playing $z_i = 1$ when all other countries j choose to play $z_j = 1$.

For most countries (with the exception of a small number of Asian Tigers), even when (apparently) completely open to world trade, cross-border trade accounts for a relatively small proportion of per capita GDP. Hence, it is not unreasonable to suppose that w_{ii} is often large. Nevertheless, it is generally not considered to take the value $w_{ii} = 1$, as countries do not solely depend on domestic trade and enlightened trade policies at home to boost economic growth. International trade is essential for economic development and economic survival of countries today. With $w_{ii} < 1$, there are *some* additional gains from cross-border trade that are not available just from enlightened economic policies at home. For example Murshid [93] states that cross-border trade is used as a vehicle to promote increased regional integration and as a way to prepare for a much more liberal and open trading regime in the CLTV[136] neighbourhood, which is one of the fastest growing sub-regions in the world. The more advanced economies of Thailand and Vietnam obtain greater benefits from opening up to cross-border trade as they have clear, well-focussed policies (for example, Thailand entered into bilateral free trade agreements with various countries including Japan, USA, China, India and is directed towards creation of new market clusters for its exports of processed goods. Vietnam successfully developed a border trade zone with China at Mong Cai, and also entered into a bilateral free trade agreement with USA in 2001 to boost exports. Economic liberalization generated its rapid growth and helped reduce poverty).

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[136] Cambodia, Laos, Thailand and Vietnam.

Note that countries observe the maximal payoff obtained in a given time period and seek to minimize the difference between the maximal payoff and their own payoff. It is assumed that degeneracies do not occur in this model, so that countries have a unique best reply that minimizes the payoff difference. The non-generic cases are therefore not considered in this model.

5.2 Strategy Change Rules

Definition 5.5. Given a current strategy profile $z = (z_1, \ldots, z_n)$, the *strategy profile of the countries other than i* is defined as the $n - 1$ vector:

$$z_{-i} := (z_1, \ldots, (z_i), \ldots, z_n).^{137}$$

The countries participating in international trade compete with one another and have the common desire of imitating the country with the maximum payoff.

Definition 5.6. The *maximum payoff* μ is defined as the highest payoff realized by one or several countries and is formally expressed as:

$$\mu(z) := \max_{j \in \{1, \ldots, n\}} \{\pi_j(z)\}.$$

When C_i uses strategy s and z_{-i} represents the strategy profile for all other countries, then it calculates the maximum payoff as:

$$\mu_i(s|z_{-i}) = \max_{j \in \{1, \ldots, n\}} \{\pi_j(s|z_{-i})\}.$$

Given an opportunity to adjust its current strategy (economic policy), country i changes its level of commitment to engagement in world trade from z_i to $z_i' = B_i^\delta(z)$, where:

$$B_i^\delta(z) := \arg\min_{s \in \Delta(z_i)} \{\mu(s|z_{-i}) - \pi_i(s|z_{-i})\}, \tag{5.3}$$

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[137] The brackets around z_i means that it is omitted.

and $\Delta(r) = [r - \delta, r + \delta] \cap I$, for $r \in I$, and $\delta > 0$ a fixed constant. Thus, country i can only adjust its level of integration in the world economy by a maximum amount δ in any one move. Note that the optimal strategy $B_i^{\delta}(z)$ is always unique as degeneracies do not occur.[138]

$B_i^{\delta}(z)$ determines a strategy s that attempts to minimize the difference between country i's current economic state prowess and the quantity $\mu(s|z_{-i})$ that represents an aspiration held by country i, *which itself depends on country i's strategy as well as the strategies of all the remaining countries*. This aspiration can be modelled in one of (at least) three possible ways:

(i) $\mu(s|z_{-i})$ is the payoff to the *leading country* given the modified strategy profile, that is $\mu(s|z_{-i})$ is the payoff when country i chooses $s \in \Delta(z_i)$ $= \pi_i(s, z_{-i})$[139].

(ii) $\mu(s|z_{-i})$ is the *average payoff* of those countries that are currently doing better (achieving a higher payoff) than country i.

(iii) $\mu(s|z_{-i})$ is the payoff of the country whose payoff is currently *closest to*, but better than, country i's.

In all these cases, the leading country (the country with the highest payoff) always seeks to maximize its payoff. Case (i) is the case that has been considered so far, but the other two cases are also worth some attention.

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[138] One interpretation of this assumption could be – country i sticks with its current strategy s if more than one strategy minimizes the behavioural gap (5.3).

[139] Note that for each country i, the difference between $\mu(s|z_{-i})$ and its own current state prowess $\pi_i(s|z_{-i})$ is unique so that country i has a unique best strategy $s \in \Delta(z_i)$ when the other countries play z_{-i}.

5.3 Strict Equilibria for the *n*-Country Model

A strict equilibrium in this model means no country can improve its payoff by adjusting its strategy by size δ.

Definition 5.7. A state z^* is an *equilibrium* of the trade game under the updating rule (5.3), if no country has an incentive to change its strategy when given the opportunity to do so. This can be formally expressed as: $z_i^* = B_i^\delta \left(z^* \right)$.

Definition 5.8. A state z^* is a *strict equilibrium*[140] or a $\delta - equilibrium$ of the trade game if $z_i^* = B_i^\delta \left(z^* \right)$ and each country i does explicitly worse by changing its strategy from z_i^*.

Based on the *aspiration rule* (5.3), for a state z to be a strict equilibrium, the following condition is required to hold: For each i,

when $0 < z_i < 1$[141]:

$$\mu\left(z_i | z_{-i} \right) - \pi_i \left(z_i | z_{-i} \right) < \mu\left(z_i \pm \delta | z_{-i} \right) - \pi_i \left(z_i \pm \delta | z_{-i} \right), \tag{5.4}$$

when $z_i = 0$:

$$\mu\left(0 | z_{-i} \right) - \pi_i \left(0 | z_{-i} \right) < \mu\left(\delta | z_{-i} \right) - \pi_i \left(\delta | z_{-i} \right), \tag{5.5}$$

when $z_i = 1$:

$$\mu\left(1 | z_{-i} \right) - \pi_i \left(1 | z_{-i} \right) < \mu\left(1 - \delta | z_{-i} \right) - \pi_i \left(1 - \delta | z_{-i} \right). \tag{5.6}$$

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[140] Note that the equilibrium is classified as a strict equilibrium as the inequality required to hold is a strict inequality.

[141] Note that when $z_i = 1$, country i can only update its strategy to $1 - \delta$ when given an opportunity to do so and when $z_i = 0$, an updated strategy can only be of the form $z_i = \delta$, hence separate conditions are required for these cases.

The conditions in (5.4) can be rearranged to obtain the strict equilibrium condition for $0 < z_i < 1$ as:

$$\mu\left(z_i \pm \delta \middle| z_{-i}\right) - \mu\left(z_i \middle| z_{-i}\right) > \pi_i\left(z_i \pm \delta \middle| z_{-i}\right) - \pi_i\left(z_i \middle| z_{-i}\right) \text{ for all } i.$$

The right hand side of the inequality can be simplified as:

$$\pi_i\left(z_i \pm \delta \middle| z_{-i}\right) - \pi_i\left(z_i \middle| z_{-i}\right)$$

$$= a_i\left(z_i \pm \delta\right)\left(\sum_{j \neq i} w_{ij} z_j + w_{ii}\left(z_i \pm \delta\right)\right) + \left(1 - \left(z_i \pm \delta\right)\right)\lambda_i - \pi_i\left(z_i \middle| z_{-i}\right)$$

$$= \left(a_i z_i \pm a_i \delta\right)\left(\sum_j w_{ij} z_j \pm w_{ii} \delta\right) + \left(1 - z_i\right)\lambda_i \mp \lambda_i \delta - \pi_i\left(z_i \middle| z_{-i}\right)$$

$$= \left[a_i z_i \sum_j w_{ij} z_j + \left(1 - z_i\right)\lambda_i\right] \pm \delta\left(a_i \sum_j w_{ij} z_j + a_i w_{ii} z_i \pm a_i w_{ii} \delta - \lambda_i\right) - \pi_i\left(z_i \middle| z_{-i}\right)$$

$$= \pm\delta\left(a_i \sum_j w_{ij} z_j - \lambda_i + a_i w_{ii}\left(z_i \pm \delta\right)\right).$$

Similarly, the conditions in (5.5) can be rearranged to obtain the equilibrium condition for $z_i = 0$ as: $\mu\left(\delta \middle| z_{-i}\right) - \mu\left(0 \middle| z_{-i}\right) > \pi_i\left(\delta \middle| z_{-i}\right) - \pi_i\left(0 \middle| z_{-i}\right)$, and the right hand side of this inequality can be simplified to: $\delta\left(-\lambda_i + a_i w_{ii} \delta\right)$ (using the simplified form of the expression $\pi_i\left(z_i \pm \delta \middle| z_{-i}\right) - \pi_i\left(z_i \middle| z_{-i}\right)$ obtained above).

The conditions in (5.6) can also be rearranged to obtain the equilibrium condition for $z_i = 1$ as: $\mu\left(1 - \delta \middle| z_{-i}\right) - \mu\left(1 \middle| z_{-i}\right) > \pi_i\left(1 - \delta \middle| z_{-i}\right) - \pi_i\left(1 \middle| z_{-i}\right)$, and the right hand side of this inequality can be simplified to: $-\delta\left(a_i - \lambda_i + a_i w_{ii}\left(1 - \delta\right)\right)$ (again using the simplified form of the expression $\pi_i\left(z_i \pm \delta \middle| z_{-i}\right) - \pi_i\left(z_i \middle| z_{-i}\right)$ obtained above).

A necessary and sufficient condition for z to be a strict equilibrium[142] of the continuous strategy trade game (as defined above in condition (5.4)) can be formally expressed as:

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[142] Note that $0 < z_i < 1$ in this case.

$$\mu\left(z_i \pm \delta \mid z_{-i}\right) - \mu\left(z_i \mid z_{-i}\right) > \pm\delta\left(a_i \sum_j w_{ij} z_j - \lambda_i + a_i w_{ii}\left(z_i \pm \delta\right)\right), \text{ for all } i. \qquad (5.7)$$

5.3.1 The All-In-Autarky State for the *n*-Country Model

For the all-in-autarky state, $z_i = 0$, for all i. So when given the opportunity, country i can either stick to strategy $z_i = 0$ or update its strategy to $z_i = \delta$. Then, $\mu\left(0 \mid z_{-i}\right) = \lambda^*$ and $\mu\left(\delta \mid z_{-i}\right) = \max\left\{a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right), \lambda^*_{-i}\right\}$, where $\lambda^* = \max_j\left\{\lambda_j\right\}$ and $\lambda^*_{-i} = \max_{i \neq j}\left\{\lambda_j\right\}$.

The equilibrium condition (5.5) for the all-in-autarky state can now be written as:

$$\max\left\{a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right), \lambda^*_{-i}\right\} - \lambda^* > \delta\left(a_i w_{ii} \delta - \lambda_i\right)^{[143]}, \text{ for all } i. \qquad (5.8)$$

The left hand side of condition (5.8) can be simplified to either of the four follow-ing expressions when λ^* is re-written as $\max\left\{\lambda_i, \lambda^*_{-i}\right\}$:

(i) $a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right) - \lambda_i$ [144]: This expression simplifies to $\delta\left(a_i w_{ii} \delta - \lambda_i\right)$ and hence (5.8) cannot hold.

(ii) $a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right) - \lambda^*_{-i}$ [145]: This expression simplifies to $\delta\left(a_i w_{ii} \delta - \lambda_i\right) + \lambda_i - \lambda^*_{-i}$, but $\lambda_i < \lambda^*_{-i}$ in this case (see footnote [145] below). Hence (5.8) cannot hold.

---

[143] The simplified expression on the right hand side of this inequality has been derived previously in Section 5.3.

[144] This expression is obtained when $\max\left\{a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right), \lambda^*_{-i}\right\} = a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right)$ and $\max\left\{\lambda_i, \lambda^*_{-i}\right\} = \lambda_i$.

[145] This expression is obtained when $\max\left\{a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right), \lambda^*_{-i}\right\} = a_i w_{ii} \delta^2 + \lambda_i\left(1 - \delta\right)$ and $\max\left\{\lambda_i, \lambda^*_{-i}\right\} = \lambda^*_{-i}$.

(iii) $\lambda_{-i}^* - \lambda_i$[146]: This expression is less than 0 since $\lambda_i > \lambda_{-i}^*$ (see footnote [146] be-

low). A necessary and sufficient condition required for (5.8) becomes:

$\lambda_{-i}^* - \lambda_i - \delta\left(a_i w_{ii}\delta - \lambda_i\right) > 0$. This can be simplified to: $w_{ii} < \dfrac{\lambda_{-i}^* + \lambda_i\left(\delta - 1\right)}{a_i \delta^2}$.

Since $w_{ii} \geq 0$, $\dfrac{\lambda_{-i}^* + \lambda_i\left(\delta - 1\right)}{a_i \delta^2} > 0$ gives $\delta > \dfrac{\lambda_i - \lambda_{-i}^*}{\lambda_i}$. Moreover, since

$\lambda_i > \lambda_{-i}^*$, the equilibrium condition can be satisfied for a range of δ-values:

$$\left(0, 1 - \frac{\lambda_{-i}^*}{\lambda_i}\right] \cap I_K = \left\{\frac{1}{K}\middle| K \in \mathbb{N}, K > 1\right\}.$$

(iv) $\lambda_{-i}^* - \lambda_{-i}^*$[147]: This expression simplifies to 0 and so a necessary and sufficient

condition required for (5.8) to hold becomes: $a_i w_{ii}\delta - \lambda_i < 0$. This can be

simplified to: $w_{ii} < \dfrac{\lambda_i}{a_i \delta}$, for all i.

5.3.2 The Fully Integrated State for the *n*-Country Model

For the fully integrated state, $z_i = 1$, for all i. Therefore, when given the opportuni-

ty, country i can either stick to strategy $z_i = 1$ or update its strategy to $z_i = 1 - \delta$.

Country 1 has no incentive to change its strategy by δ as it obtains the maximal

payoff of a_1 when it plays $z_1 = 1$. This result is true for all n and stated as Lemma

5.1. In this case, $\mu\left(1|z_{-i}\right) = a_1$ and $\mu\left(1 - \delta|z_{-i}\right) = \max\left\{\pi_i\left(1 - \delta|z_{-i}\right), \mu_i^*\left(\delta\right)\right\}$, where

$\mu^*\left(\delta\right) = \max\limits_{j \neq i}\left\{a_j\left(1 - w_{ji}\delta\right)\right\}$ [148].

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[146] This expression is obtained when $\max\left\{a_i w_{ii}\delta^2 + \lambda_i\left(1 - \delta\right), \lambda_{-i}^*\right\} = \lambda_{-i}^*$ and $\max\left\{\lambda_i, \lambda_{-i}^*\right\} = \lambda_i$.

[147] This expression is obtained when $\max\left\{a_i w_{ii}\delta^2 + \lambda_i\left(1 - \delta\right), \lambda_{-i}^*\right\} = \lambda_{-i}^*$ and $\max\left\{\lambda_i, \lambda_{-i}^*\right\} = \lambda_{-i}^*$.

[148] When country i switches to strategy $1 - \delta$, country j receives a payoff of:

$a_j z_j \sum\limits_k w_{jk} z_k + \left(1 - z_j\right)\lambda_j$, where $z_j = 1$ for $j \neq i$ and $z_i = 1 - \delta$. This can be simplified as:

$a_j \sum\limits_{k \neq i} w_{jk} + a_j\left(1 - \delta\right)w_{ji} = a_j - a_j w_{ji}\delta$.

Lemma 5.1. In the n-country model, when the game is in the fully integrated state, country 1 does not update its strategy from $z_1 = 1$ as it yields the maximal payoff of a_1.

Proof. Consider the n-country model. In the fully integrated state, $z_i = 1$ for all $i \in \{1, \ldots, n\}$. And the payoffs to countries $1, \ldots, n$ is $a_1, \ldots, a_n$, respectively. Since $a_1 > \ldots > a_n$, the maximal payoff obtained is a_1 and hence country 1 has no incentive to update its strategy when given an opportunity to do so. □

The equilibrium condition (5.6) for the fully integrated state can now be re-written as:

$$\max \left\{ \pi_i \left(1 - \delta \middle| z_{-i} \right), \mu^* (\delta) \right\} - a_1 > a_i w_{ii} \delta^2 + \left(\lambda_i - a_i - a_i w_{ii} \right) \delta^{\,149}, \text{ for } i > 1. \tag{5.9}$$

If $\mu \left(1 - \delta \middle| z_{-i} \right) = \pi_i \left(1 - \delta \middle| z_{-i} \right)$, then country i obtains the maximal payoff by switching to strategy $z_i = 1 - \delta$. Therefore, the fully integrated state cannot be a strict equilibrium. Hence, it is required that:

$$\mu \left(1 - \delta \middle| z_{-i} \right) = \mu^* (\delta), \tag{5.10}$$

so that (5.9) can be re-written as:

$$\mu^* (\delta) > a_i w_{ii} \delta^2 + \left(\lambda_i - a_i - a_i w_{ii} \right) \delta + a_1, \text{ for } i > 1. \tag{5.11}$$

Condition (5.10) holds if and only if:

$$\mu^* (\delta) > \pi_i \left(1 - \delta \middle| z_{-i} \right).$$

Now $\pi_i \left(1 - \delta \middle| z_{-i} \right) = a_i w_{ii} \delta^2 + \left(\lambda_i - a_i - a_i w_{ii} \right) \delta + a_i{}^{150}$. So (5.10) can be simplified as:

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[149] The simplified expression on the right hand side of this inequality has been derived previously in Section 5.3.

$$\mu^*(\delta) > a_i w_{ii} \delta^2 + (\lambda_i - a_i - a_i w_{ii})\delta + a_i. \tag{5.12}$$

A quick comparison of conditions (5.11) and (5.12) implies that condition (5.11) is necessary and sufficient for the fully integrated state to be a strict equilibrium (i.e. (5.11)$\Rightarrow$(5.12)), since $a_1 > a_i$ for $i > 1$.

The right hand side of condition (5.11) (which is the only inequality required to address since (5.11)$\Rightarrow$(5.12)) can be analysed as a quadratic function of δ (provided $w_{ii} > 0$[151]), in order to find the parameter ranges for which the fully integrated state is a strict equilibrium.

Let $f_i(\delta) = a_i w_{ii}\delta^2 + (\lambda_i - a_i - a_i w_{ii})\delta + a_1$. Then,

$f_i'(\delta) = 2a_i w_{ii}\delta + (\lambda_i - a_i - a_i w_{ii})$ and $f_i''(\delta) = 2a_i w_{ii} > 0$. Thus, $f_i(\delta)$ has a minimum point at $\delta_{\min} = \dfrac{a_i + a_i w_{ii} - \lambda_i}{2a_i w_{ii}}$[152], and $f_i(\delta_{\min}) = a_1 - \dfrac{(a_i + a_i w_{ii} - \lambda_i)^2}{4a_i w_{ii}}$[153].

---

[150] This payoff is obtained using (5.1) as follows:

$$
\begin{aligned}
\pi_i(1 - \delta | z_{-i}) &= a_i(1 - \delta)\left(\sum_{j \neq i} w_{ij} z_j + w_{ii}(1 - \delta)\right) + (1 - 1 + \delta)\lambda_i \\
&= a_i(1 - \delta)\left(\sum_{j \neq i} w_{ij} - w_{ii}\delta\right) + \delta\lambda_i \\
&= a_i(1 - \delta)(1 - w_{ii}\delta) + \delta\lambda_i \\
&= a_i w_{ii}\delta^2 + (\lambda_i - a_i - a_i w_{ii})\delta + a_i.
\end{aligned}
$$

[151] Note that the right hand side of (5.11) is linear if $w_{ii} = 0$.

[152] The δ value at the minimum point is obtained as follows: $f_i'(\delta) = 2a_i w_{ii}\delta + (\lambda_i - a_i - a_i w_{ii}) = 0$.

Hence, $\delta_{\min} = \dfrac{a_i + a_i w_{ii} - \lambda_i}{2a_i w_{ii}}$.

If $\delta_{\min}$ is re-arranged as $\dfrac{a_i - \lambda_i}{2a_i w_{ii}} + \dfrac{1}{2}$, then $\delta_{\min} > \dfrac{1}{2}$ (since $a_i > \lambda_i$).

Note that $\delta_{\min} < 1$ if and only if $a_i - \lambda_i < a_i w_{ii}$, which gives $w_{ii} > 1 - \dfrac{\lambda_i}{a_i}$.

The conditions required for the fully integrated state to be a strict equilibrium for the 2-country model and the 3-country model are obtained below. The 2-country model is further explored in the next chapter, linking together results from this chapter in the context of trade dynamics.

5.3.3 Specialisations to $n = 2,3$

The 2-Country Model

In the 2-country model, country 1 obtains the maximal payoff of a_1 by playing $z_1 = 1$, hence it does not update its strategy. The conditions required for country 2 to not change its strategy by δ (to $1 - \delta$) and thus allowing $(z_1, z_2) = (1,1)$ to be a strict equilibrium is obtained below.

In this case, $f_2(\delta) = a_2 w_{22} \delta^2 + (\lambda_2 - a_2 - a_2 w_{22})\delta + a_1$.

The minimum point of the quadratic curve $f_2(\delta)$ is obtained as: $\delta_{\min} = \dfrac{1}{2} + \dfrac{a_2 - \lambda_2}{2a_2 w_{22}}$

and $f_2(\delta_{\min}) = a_1 - \dfrac{(a_2 + a_2 w_{22} - \lambda_2)^2}{4a_2 w_{22}}$.

---

[153] The function value at the minimum point is obtained as follows:

$$f(\delta_{\min}) = a_i w_{ii}\left(\frac{a_i + a_i w_{ii} - \lambda_i}{2a_i w_{ii}}\right)^2 + (\lambda_i - a_i - a_i w_{ii})\left(\frac{a_i + a_i w_{ii} - \lambda_i}{2a_i w_{ii}}\right) + a_1$$

$$= \frac{(a_i + a_i w_{ii} - \lambda_i)^2}{4a_i w_{ii}} - \frac{(a_i + a_i w_{ii} - \lambda_i)(a_i + a_i w_{ii} - \lambda_i)}{2a_i w_{ii}} + a_1$$

$$= \frac{(a_i + a_i w_{ii} - \lambda_i)^2 - 2(a_i + a_i w_{ii} - \lambda_i)^2}{4a_i w_{ii}} + a_1$$

$$= a_1 - \frac{(a_i + a_i w_{ii} - \lambda_i)^2}{4a_i w_{ii}}.$$

The values of the quadratic curve $f_2(\delta)$ at $\delta = 0$, $\delta = 1$ and $\delta = 1/2$ are obtained as: $f_2(0) = a_1$, $f_2(1) = a_1 - a_2 + \lambda_2$ and $f_2\left(\dfrac{1}{2}\right) = \dfrac{4a_1 + 2(a_2 - \lambda_2) - a_2 w_{22}}{4}$, respectively.[154]

Note that $a_2 - \lambda_2 > 0$ and $a_1 > a_2 w_{22}$ (since $a_1 > a_2$ and $0 \le w_{22} \le 1$), so $f_2\left(\dfrac{1}{2}\right) > 0$.

The maximum payoff obtained by the opponent country (country 1) is calculated as: $\mu_2^*(\delta) = \max\limits_{j \ne 2}\left\{a_j\left(1 - w_{j2}\delta\right)\right\} = a_1\left(1 - w_{12}\delta\right)$.

Condition (5.11) - $\mu_i^*(\delta) > f_2(\delta)$ is necessary and sufficient for the fully integrated state to be a strict equilibrium, as obtained above and for the 2-country model, this can be written as:

$$a_1(1 - w_{12}\delta) > a_2 w_{22}\delta^2 + (\lambda_2 - a_2 - a_2 w_{22})\delta + a_1,$$

which gives:

$$\left[a_2 w_{22}\delta + (\lambda_2 - a_2 - a_2 w_{22} + a_1 w_{12})\right]\delta < 0.$$

Therefore, a necessary and sufficient condition required for $z = (1,1)$ to be a strict equilibrium is:

$$\delta < \frac{a_2 + a_2 w_{22} - a_1 w_{12} - \lambda_2}{a_2 w_{22}}. \tag{5.13}$$

The curves for $f_2(\delta)$ and $\mu^*(\delta)$ are now obtained to analyse condition (5.11) further for all values of δ in the range $\left(0, \dfrac{1}{2}\right]$.

$\mu^*(0) = a_1$ and $\mu^*(1) = a_1 - a_1 w_{12} \ge 0$.

---

[154] Note that either $f_2\left(\dfrac{1}{2}\right) \ge f_2(1)$ or $f_2\left(\dfrac{1}{2}\right) < f_2(1)$.

If $f_2(\delta_{\min}) > 0$, then the curves obtained for $f_2(\delta)$ and $\mu^*(\delta)$ are as shown in Figure 5.1 (a) and (b), and if $f_2(\delta_{\min}) < 0$, then the curves obtained are as shown in Figure 5.1 (c). In order for country 2 to not update its strategy by δ, it is required that $\mu^*(\delta) > f_2(\delta)$.

(a) $\delta_{\min} < 1$, $f_2(\delta_{\min}) > 0$.　　　　(b) $\delta_{\min} > 1$, $f_2(\delta_{\min}) > 0$.

(c) $\delta_{\min} < 1$, $f_2(\delta_{\min}) < 0$.[155]
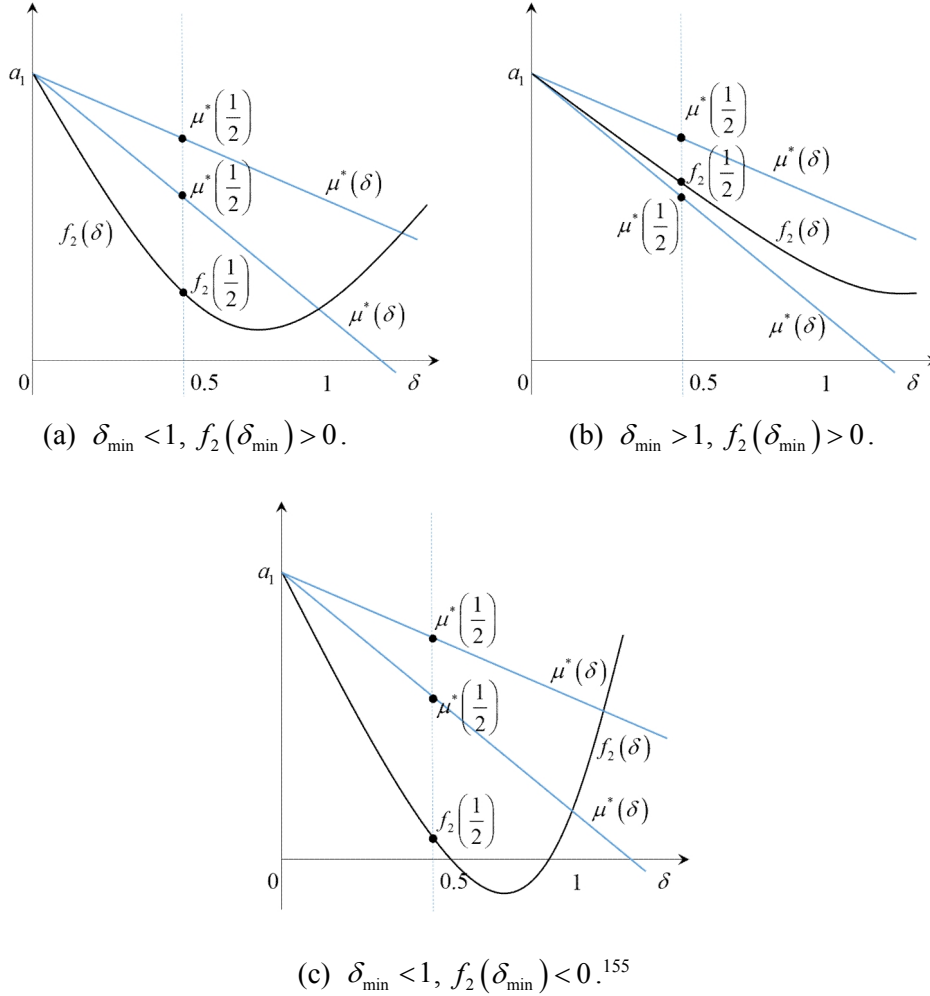
Figure 5.1. Graphical representation of $f_2(\delta)$ and $\mu^*(\delta)$ for the 2-country model. Note that condition (5.11) holds if $\mu^*\left(\frac{1}{2}\right) > f_2\left(\frac{1}{2}\right)$.

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[155] Note that $\delta_{\min}$ cannot be greater than 1 when $f_2(\delta_{\min}) < 0$ as $f_2(1) > 0$.

From Figure 5.1, a necessary condition for (5.11) to hold for $0 < \delta \leq \frac{1}{2}$ is obtained

as: $\mu^* \left(\frac{1}{2} \right) > f_2 \left(\frac{1}{2} \right)$.

This can be simplified as:

$$a_1 - \frac{a_1 w_{12}}{2} > -\frac{1}{4} a_2 w_{22} - \frac{1}{2}(a_2 - \lambda_2) + a_1$$

$$\Leftrightarrow -a_1 w_{12} > -\frac{1}{2} a_2 w_{22} - (a_2 - \lambda_2).$$

Thus, a necessary condition for (5.11) to hold for $0 < \delta \leq \frac{1}{2}$ is:

$$a_1 w_{12} - \frac{1}{2} a_2 w_{22} < a_2 - \lambda_2. \tag{5.14}$$

Condition (5.14) states that for the fully integrated state to be a strict equilibrium for the 2-country model, the difference between the fully integrated payoff and the autarkic payoff for country 2 should strictly offset the difference between the gains from trade with country 1 (that is open to trade) and half the self-generating capacity of country 2. Note that for $\delta = 1$, Condition (5.14) reduces to $a_1 w_{12} < a_2 - \lambda_2$, which means that for the fully integrated state to be a strict equilibrium for the 2-country model (where countries either play strategy 0 or strategy 1), the difference in the fully integrated payoff and the autarkic payoff for country 2 should be strictly greater than the gains from trade with country 1. The 2-country model is analysed further in the next chapter in the context of trade dynamics.

The 3-Country Model

In the 3-country model, country 1 does not update its strategy from $z_1 = 1$ as it yields the maximal payoff of a_1. The conditions required for country 2 and country 3 to not update their strategies by δ, meaning $(z_1, z_2, z_3) = (1,1,1)$ is a strict equilibrium, are obtained below.

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First, consider country 2.

$$f_2(\delta) = a_2 w_{22} \delta^2 + (\lambda_2 - a_2 - a_2 w_{22})\delta + a_1.$$

The expressions for δ_{min}, $f_2(\delta_{min})$, $f_2(0)$, $f_2(1)$, $f_2\left(\dfrac{1}{2}\right)$ are the same as obtained for the 2-country model above. The expression for $\mu^*(\delta)$ for $\delta \in (0,1)$ can be written as:

$$\begin{aligned}\mu^*(\delta) &= \max_{j \neq 2}\left\{a_j(1 - w_{j2}\delta)\right\} \\ &= \max\left\{a_1(1 - w_{12}\delta), a_3(1 - w_{32}\delta)\right\}.\end{aligned}$$

The curve for $\mu^*(\delta)$ can be evaluated at $\delta = 0$ and $\delta = 1$ as:

$$\mu^*(0) = a_1, \quad \mu^*\left(\frac{1}{2}\right) = \max\left\{a_1\left(1 - \frac{w_{12}}{2}\right), a_3\left(1 - \frac{w_{32}}{2}\right)\right\}.$$

The curve for $\mu^*(\delta)$ is thus either linear or piecewise linear as shown in Figure 5.2.

$\mu^*(\delta)$ is linear.   (b) $\mu^*(\delta)$ is piecewise linear.
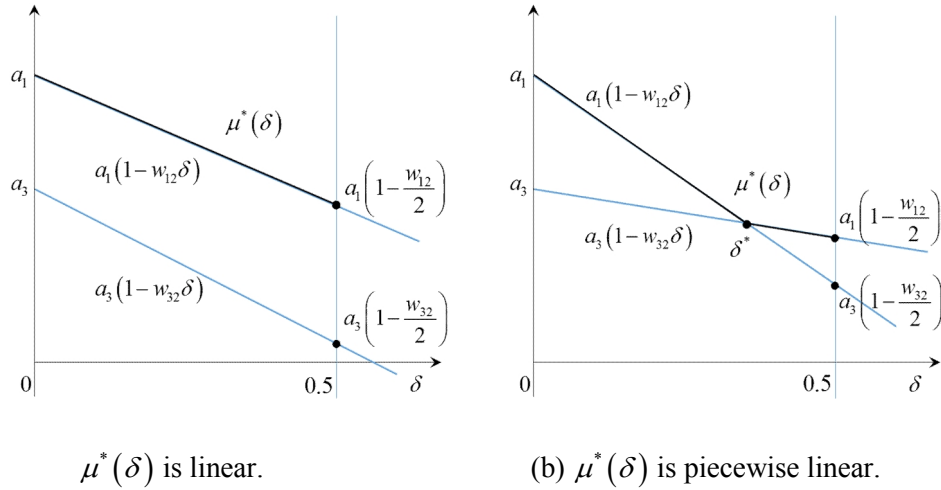
Figure 5.2. Graphical representation of $\mu^*(\delta)$ for the 3-country model. Unlike the 2-country model, the maximal payoff function here can be linear or piecewise linear.

In Figure 5.2 (b), $\delta^* = \dfrac{a_1 - a_3}{a_1 w_{12} - a_3 w_{32}}$ [156].

If $a_1\left(1 - \dfrac{w_{12}}{2}\right) > a_3\left(1 - \dfrac{w_{32}}{2}\right)$, then $\mu^*(\delta)$ is linear[157] and $\mu^*(\delta) = a_1(1 - w_{12}\delta)$.

Otherwise $\mu^*(\delta)$ is piecewise linear[158] and $\mu^*(\delta) = \begin{cases} a_1(1 - w_{12}\delta) & \text{if } \delta \leq \delta^* \\ a_3(1 - w_{32}\delta) & \text{if } \delta > \delta^* \end{cases}$.

If $\mu^*(\delta)$ is linear, then the curves for $\mu^*(\delta)$ and $f_2(\delta)$ are obtained as in Figure 5.1 previously. So (5.14) is a necessary condition for country 2 to stick with its current strategy $z_2 = 1$.

If $\mu^*(\delta)$ is piecewise linear, then the curves for $\mu^*(\delta)$ and $f_2(\delta)$ are obtained as in Figure 5.3.

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[156] At $\delta = \delta^*$, $a_1(1 - w_{12}\delta) = a_3(1 - w_{32}\delta)$, hence $\delta^* = \dfrac{a_1 - a_3}{a_1 w_{12} - a_3 w_{32}}$.

[157] In this case, either $\delta^* < 0$ or $\delta^* > \dfrac{1}{2}$.

[158] In this case $0 < \delta^* \leq \dfrac{1}{2}$, which requires $a_3 w_{32} < a_1 w_{12}$ (since $\delta^* > 0$) and $2(a_1 - a_3) > a_1 w_{12} - a_3 w_{32}$ (since $\delta^* \leq \dfrac{1}{2}$).

(a) $\delta_{min} < 1$, $f_2(\delta_{min}) > 0$.

(b) $\delta_{min} > 1$, $f_2(\delta_{min}) > 0$.

(c) $\delta_{min} < 1$, $f_2(\delta_{min}) < 0$.[159]
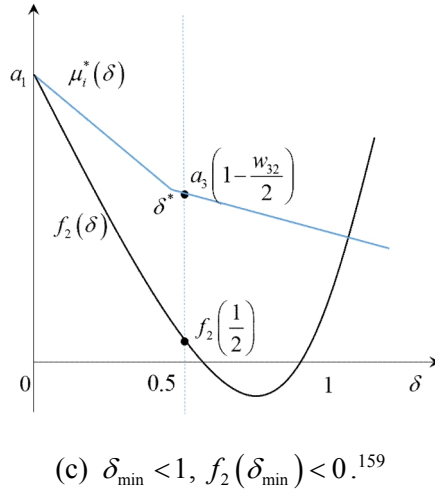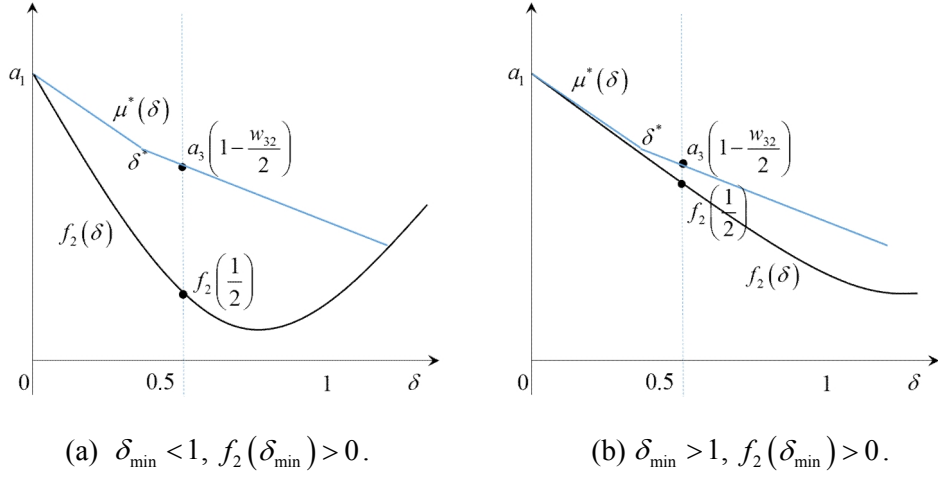
Figure 5.3. Graphical representation of $f_2(\delta)$ and $\mu^*(\delta)$ for the 3-country

model. Note that in all the cases, condition (5.11) holds if $\mu^*\left(\dfrac{1}{2}\right) > f_2\left(\dfrac{1}{2}\right)$.

From Figure 5.3, the necessary condition required for (5.11) to hold is obtained for

$\delta \le \delta^*$ and $\delta > \delta^*$, for piecewise linear $\mu^*(\delta)$ as: $\mu^*\left(\dfrac{1}{2}\right) > f_2\left(\dfrac{1}{2}\right)$.[160]

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[159] Note that δ_{min} cannot be greater than 1 when $f_2(\delta_{min}) < 0$ as $f_2(1) > 0$.

[160] Note that for $\delta = 1$, this condition reduces to: $a_3(1 - w_{32}\delta) > \lambda_2 - a_2 + a_1$.

This can be simplified as:

$$a_3\left(1-\frac{w_{32}}{2}\right) > -\frac{1}{4}a_2 w_{22} + \frac{1}{2}(\lambda_2 - a_2) + a_1$$

$$\Leftrightarrow 2(a_1 - a_3) < \frac{1}{2}a_2 w_{22} - a_3 w_{32} + (a_2 - \lambda_2).$$

Hence, a necessary and sufficient condition required for country 2 to stick with its fully integrated strategy when the maximal payoff function is piecewise linear is:

$$2(a_1 - a_3) - (a_2 - \lambda_2) < \frac{1}{2}a_2 w_{22} - a_3 w_{32}. \tag{5.15}$$

Conditions for country 3 can be analysed by simply swapping the indices 2 and 3 from the analysis of country 2 above. Condition (5.14) for linear $\mu^*(\delta)$ and (5.15) for piecewise linear $\mu^*(\delta)$ can be re-written for country 3 as:

$$a_1 w_{13} - \frac{1}{2}a_3 w_{33} < a_3 - \lambda_3, \tag{5.16}$$

$$2(a_1 - a_2) - (a_3 - \lambda_3) < \frac{1}{2}a_3 w_{33} - a_2 w_{23}. \tag{5.17}$$

The conditions necessary and sufficient for country 2 to stick with strategy $z_2 = 1$ can now be consolidated as: (5.10) and (5.14) if $\mu^*(\delta)$ is linear, otherwise (5.10) and (5.15) for piecewise linear $\mu^*(\delta)$. The necessary and sufficient conditions for country 3 to stick with strategy $z_3 = 1$ can now be consolidated as: (5.10) and (5.16) if $\mu^*(\delta)$ is linear, otherwise (5.10) and (5.17) for piecewise linear $\mu^*(\delta)$.

The fully integrated state $(1,1,1)$ is a strict equilibrium if the necessary and sufficient conditions consolidated above for countries 2 and 3 to stick with their strategies $z_2 = 1$ and $z_3 = 1$, respectively, hold true.

5.4 Strict Stability for *n* countries

Suppose z is a strict equilibrium strategy. Suppose that country k randomly decides to upgrade its strategy by a small amount δ. Then, a *mutant strategy* is a strategy profile where country k upgrades its strategy by δ while all the remaining countries continue playing their original strategies. Now consider a mutant strategy denoted by $z^{(k)}$, such that $z_i^{(k)} = z_i$ for $i \neq k$, and $z_k^{(k)} = z_k \pm \delta$. Then, for z to be stable with respect to such single mutations, it would be required for country k to find it advantageous to revert back to its original strategy, while all other countries continue playing their original strategies.

Definition 5.9. An equilibrium strategy z is said to be a *stable equilibrium* with respect to single mutations $z^{(k)}$, if $z^{(k)} \to z$ under the updating rule (5.3), so that each country apart from k has no incentive to update its strategy when given an opportunity to do so, and country k updates its strategy from $z_k^{(k)}$ back to z_k (so that $z_k' = z_k$), and this must be true for each k. This can be expressed formally as:

$$z_i^{(k)} = B_i^\delta \left(z^{(k)} \right), \textit{for } i \neq k \text{ and } z_k = B_k^\delta \left(z^{(k)} \right), \text{ for all } i \text{ and all } k.$$

Definition 5.10. An equilibrium strategy z is said to be a *strictly stable equilibrium* or a $\delta - stable\,quilibrium$ with respect to single mutations $z^{(k)}$, if $z_i^{(k)} = B_i^\delta \left(z^{(k)} \right)$, for $i \neq k$ and $z_k = B_k^\delta \left(z^{(k)} \right)$, for all i and all k, and each country $i \neq k$ does explicitly worse by changing its strategy from z_i while country k does explicitly better by changing its strategy from $z_k^{(k)}$ to z_k.

This definition of strict stability requires the following conditions to hold for all k:

$$\mu \left(z_i \pm \delta \Big| z_{-i}^{(k)} \right) - \pi_i \left(z_i \pm \delta \Big| z_{-i}^{(k)} \right) > \mu \left(z_i \Big| z_{-i}^{(k)} \right) - \pi_i \left(z_i \Big| z_{-i}^{(k)} \right) \text{ for all } i \neq k \,,$$

$$\mu\left(z_k^{(k)}\middle|z_{-k}^{(k)}\right)-\pi_i\left(z_k^{(k)}\middle|z_{-k}^{(k)}\right)>\mu\left(z_k^{(k)}\mp\delta\middle|z_{-k}^{(k)}\right)-\pi_i\left(z_k^{(k)}\mp\delta\middle|z_{-k}^{(k)}\right)^{161}.$$

Since $z_k^{(k)}=z_k\pm\delta$, $\mu\left(z_k^{(k)}\middle|z_{-k}^{(k)}\right)-\pi_i\left(z_k^{(k)}\middle|z_{-k}^{(k)}\right)$ can be re-written as

$\mu\left(z_k\pm\delta\middle|z_{-k}^{(k)}\right)-\pi_k\left(z_k\pm\delta\middle|z_{-k}^{(k)}\right)$ and $\mu\left(z_k^{(k)}\mp\delta\middle|z_{-k}^{(k)}\right)-\pi_i\left(z_k^{(k)}\mp\delta\middle|z_{-k}^{(k)}\right)$ can be re-

written as $\mu\left(z_k\middle|z_{-k}^{(k)}\right)-\pi_k\left(z_k\middle|z_{-k}^{(k)}\right)$.

Hence, a necessary and sufficient condition for z to be a strictly stable equilibrium of the dynamic trade game (as defined above) can be formally expressed as:

$$\mu\left(z_i\pm\delta\middle|z_{-i}^{(k)}\right)-\pi_i\left(z_i\pm\delta\middle|z_{-i}^{(k)}\right)>\mu\left(z_i\middle|z_{-i}^{(k)}\right)-\pi_i\left(z_i\middle|z_{-i}^{(k)}\right), \text{ for all } i \text{ and all } k. \quad (5.18)$$

If condition (5.18) is re-arranged as:

$$\mu\left(z_i\pm\delta\middle|z_{-i}^{(k)}\right)-\mu\left(z_i\middle|z_{-i}^{(k)}\right)>\pi_i\left(z_i\pm\delta\middle|z_{-i}^{(k)}\right)-\pi_i\left(z_i\middle|z_{-i}^{(k)}\right),$$

for all i and all k, then the inequality is hard to analyse in general because of the complex nature of the maximal payoffs that appear on the left-hand side.

In the next chapter, only the 2-country model is analysed in full detail and results are deduced based on this model.

5.5 Conclusions

Based on the analysis in the previous sections of this chapter, the following conclusions are obtained for the trade game with strategy size δ:

In the 2-country model, the fully integrated state is a strict equilibrium if the difference in the fully integrated and autarkic payoffs for the subordinate country (country 2) is strictly greater than the self-generating capacity of the leading country (country 1).

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[161] Note that $z_k^{(k)}=z_k\pm\delta$, so $z_k^{(k)}\mp\delta$ leads the mutant strategy back to the more advantageous strategy z_k for country k.

In the 3-country model, a range of parameter constraints are required to hold for the fully integrated state to be an equilibrium.

For the general n-country model, the parameter constraints required for the all-in-autarky state are obtained. Strict stability is difficult to test in this case due to the complex analysis of the maximal payoffs in the inequality for the strictly stable states. Hence, the 2-country model with strategy size δ is analysed in detail in the next chapter, obtaining several equilibria and strictly stable equilibria.

Chapter 6

The 2-Country Model with strategy size δ

This chapter discusses the 2-country model with strategy size δ in detail, first using the maximal payoff functions and the payoff functions for countries 1 and 2 as obtained in the previous chapter, and then analysing strategies of the countries individually where the leading country always seeks to maximize its payoff function.

Parameter regimes are obtained for the values of the strategy size for which the all-in-autarky and fully integrated states can be strict equilibria. The trade dynamics are then found along with any additional equilibria, including any heterogeneous states.

The best response dynamics of the trade game indicate that there exist *leadership cycles*[162] between the countries in the case of the 2-country scenario, in which the countries overtake each other and become the leaders alternately. In this case, it is easier for the country with the *intrinsically* bigger economy to overtake the lagging country in case the lagging country assumes the role of leadership, than vice versa. Additional strict equilibria with respect to the small strategy size are also obtained.

The strategy profile for a world with two countries C_1 and C_2 is $z = (z_1, z_2)$. The all-in-autarky state and the fully integrated state are first analysed in Section 6.1 and Section 6.2, respectively. Section 6.3 briefly considers the effect of varying aspiration level in the 2-country scenario. Section 6.4 analyses the trade dynamics

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[162] In the 2-country model, country 2 (the subordinate country) can catch-up with country 1 (the leading country) and then assume the role of leadership. Country 1 can then catch-up with country 2 and assume the role of leadership, and vice versa.

in order to obtain the equilibria and strictly stable states by assuming a continuous strategy size. Section 6.5 illustrates two examples and discusses the strategies chosen by the two countries, the trade dynamics and the strict equilibria. Section 6.6 concludes the chapter, highlighting important results from the 2-country scenario. Section 6.7 is an appendix containing strategy moves by country 1 when country 2 becomes the leader.
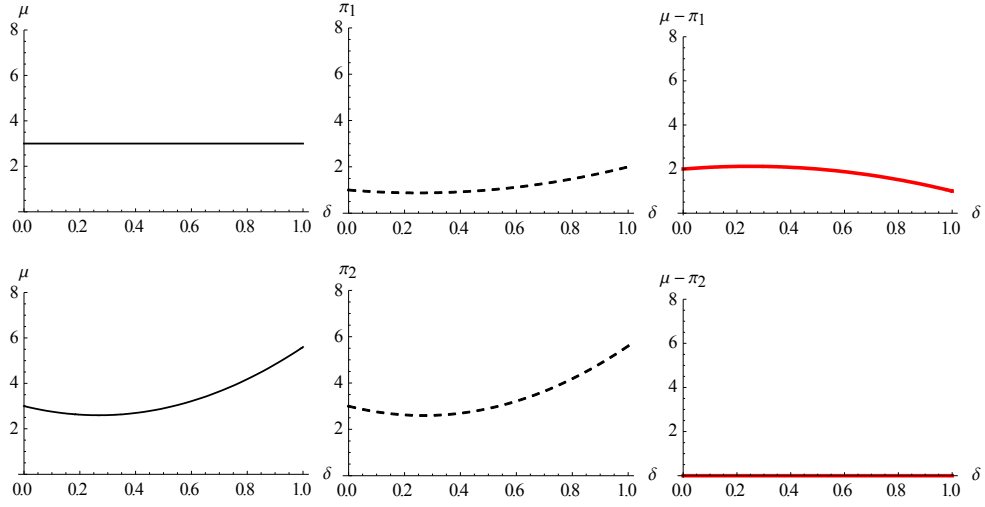
6.1 The All-In-Autarky State $(0,0)$

The all-in-autarky state $z = (0,0)$ is a strict equilibrium if:

$$\mu(\delta|z) - \pi_i(\delta|z) > \mu_i(0|z) - \pi_i(0|z), \text{ for } i = 1,2. \tag{6.1}$$

A selection of sample curves for $\mu(\delta|z)$, $\pi_i(\delta|z)$, $\mu(\delta|z) - \pi_i(\delta|z)$ are presented below in Figure 6.1 in order to visualize condition (6.1), for countries 1 and 2.
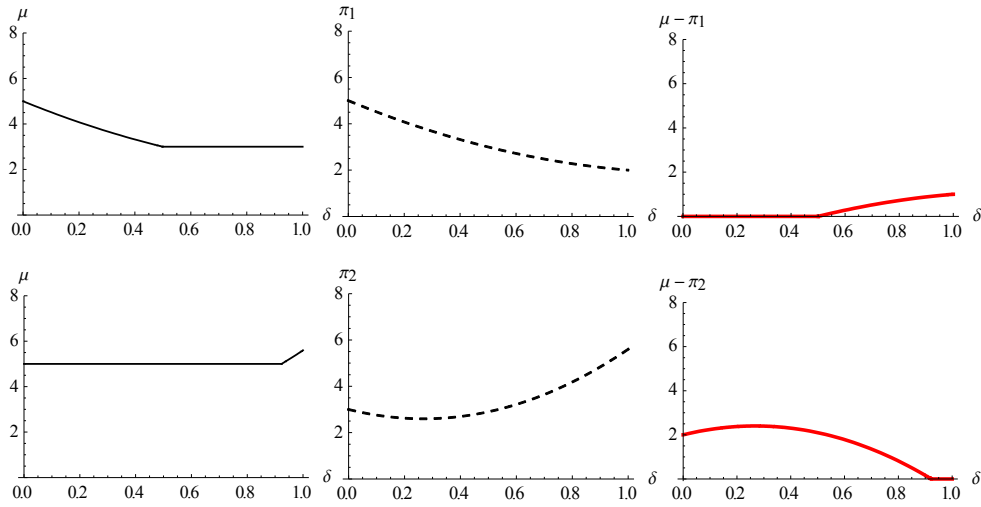
(a) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 5$, $\lambda_2 = 3$, $w_{11} = 0.2$, $w_{22} = 0.8$.

(b) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 1$, $\lambda_2 = 3$, $w_{11} = 0.2$, $w_{22} = 0.8$.
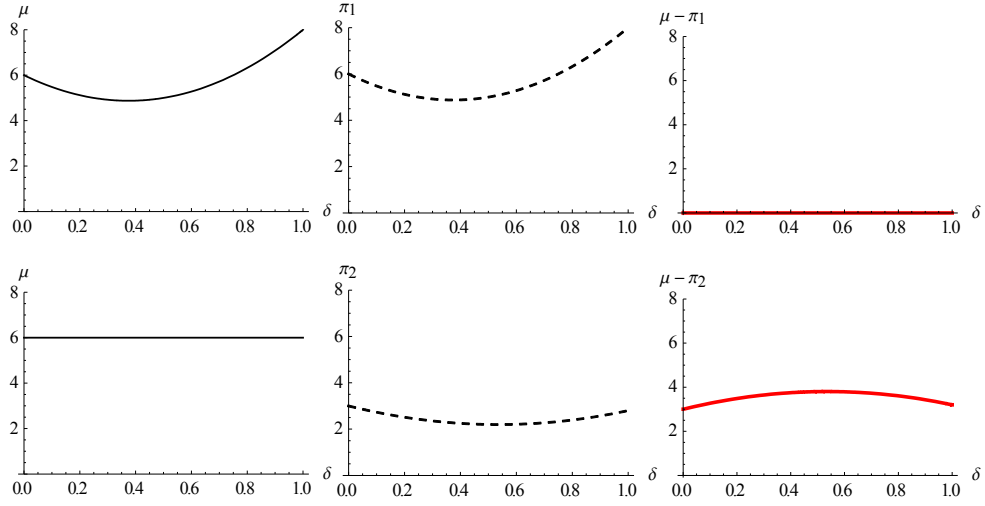
(c) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 6$, $\lambda_2 = 3$, $w_{11} = 0.8$, $w_{22} = 0.4$.
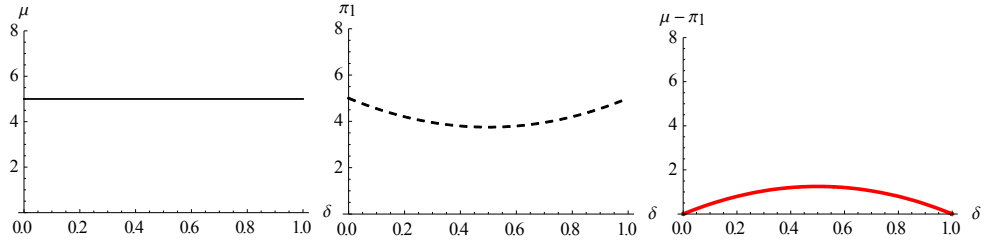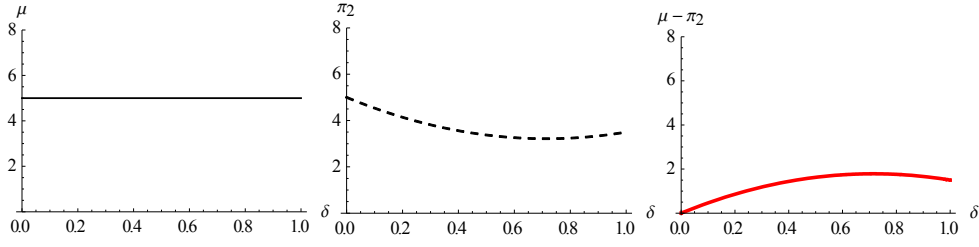
(d) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 5$, $\lambda_2 = 5$, $w_{11} = 0.5$, $w_{22} = 0.5$.
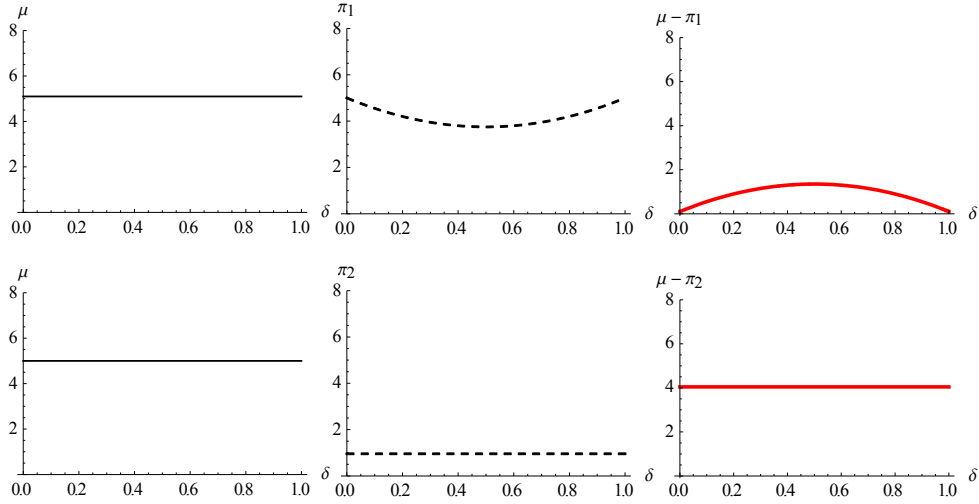
(e) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 5$, $\lambda_2 = 1$, $w_{11} = 0.7$, $w_{22} = 0.8$.

Figure 6.1. Comparison of $\mu(\delta|z)$, $\pi_i(\delta|z)$ and $\mu(\delta|z) - \pi_i(\delta|z)$, for $i = 1, 2$, where $z = (0, 0)$. Note that the parameter values ensure that the all-in-autarky state is a strict equilibrium for all values of δ only in (d) as the payoff difference for both countries by increasing level of integration by δ is greater than the payoff difference by playing strategy A, that is $z_i = 0$.

For $i = 1$: $\mu(\delta|z) = \max\{\pi_1(\delta|z), \lambda_2\}$, $\pi_1(\delta|z) = a_1 w_{11} \delta^2 + (1-\delta)\lambda_1$,

$\mu(0|z) = \max\{\lambda_1, \lambda_2\}$, $\pi_1(0|z) = \lambda_1$ [163]. Condition (6.1) can be re-written as:

$$\max\{\pi_1(\delta|z), \lambda_2\} - a_1 w_{11}\delta^2 + \delta\lambda_1 > \max\{\lambda_1, \lambda_2\}\text{ [164]}. \tag{6.2}$$

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[163] When country 1 updates its strategy by δ, country 2 continues to play $z_2 = 0$, receiving payoff λ_2. So $\mu(\delta|z) = \max\{\pi_1(\delta|z), \lambda_2\}$.

If $\max\left\{\pi_1(\delta|z), \lambda_2\right\} = \pi_1(\delta|z)$, then country 1 realizes its maximal payoff by switching to strategy $z_1 = \delta$, so $\max\left\{\pi_1(\delta|z), \lambda_2\right\} = \lambda_2$ is a necessary condition required for the all-in-autarky state to be a strict equilibrium.

Now, $\max\left\{\pi_1(\delta|z), \lambda_2\right\} = \lambda_2 \Leftrightarrow a_1 w_{11}\delta^2 + (1-\delta)\lambda_1 < \lambda_2$.

This gives:

$$w_{11} < \frac{\lambda_2 - (1-\delta)\lambda_1}{\delta^2 a_1}. \tag{6.3}$$

With $\max\left\{\pi_1(\delta|z), \lambda_2\right\} = \lambda_2$, condition (6.2) can be simplified as:

$\lambda_2 - a_1 w_{11}\delta^2 + \lambda_1\delta > \max\left\{\lambda_1, \lambda_2\right\}$.

If $\lambda_1 > \lambda_2$ (recall that unlike the a_i's, the λ_i's are not necessarily ordered), then $w_{11} < \dfrac{\lambda_2 - (1-\delta)\lambda_1}{a_1\delta^2}$ and this has already been obtained in (6.3), so a necessary condition for country 1 to stick with its strategy $z_1 = 0$ is (6.3).

If $\lambda_1 \leq \lambda_2$, then a necessary condition for country 1 to stick with its strategy $z_1 = 0$ is:

$$w_{11} < \frac{\lambda_1}{a_1\delta}. \tag{6.4}$$

For $i = 2$: The conditions for this case can be obtained by simply swapping the indices for the Case $i = 1$.

Conditions (6.3) and (6.4) can be re-written with swapped indices as:

$$w_{22} < \frac{\lambda_1 - (1-\delta)\lambda_2}{a_2\delta^2}, \tag{6.5}$$

---

[164] Condition (6.1) gives: $\max\left\{\pi_1(\delta|z), \lambda_2\right\} - a_1 w_{11}\delta^2 - (1-\delta)\lambda_1 > \max\left\{\lambda_1, \lambda_2\right\} - \lambda_1$.

$$w_{22} < \frac{\lambda_2}{a_2 \delta}. \tag{6.6}$$

The equilibrium conditions for the all-in-autarky state can be summarized as per below.

Lemma 6.1. The conditions required for the all-in-autarky state to be a strict equilibrium are:

$$w_{11} < \frac{\lambda_2 - (1-\delta)\lambda_1}{a_1 \delta^2} \text{ and } w_{22} < \frac{\lambda_2}{a_2 \delta} \text{ for } \lambda_1 > \lambda_2, \tag{6.7}$$

$$w_{11} < \frac{\lambda_1}{a_1 \delta} \text{ and } w_{22} < \frac{\lambda_1 - (1-\delta)\lambda_2}{a_2 \delta^2} \text{ for } \lambda_1 \leq \lambda_2, \tag{6.8}$$

where $\delta = 1/K$, such that $K > 1$ and $K \in \mathbb{N}$, as defined previously.

Proof. Conditions (6.3), (6.4), (6.5), (6.6) are necessary and sufficient for countries 1 and 2 to stick to their strategies $z_1 = 0$, $z_2 = 0$, respectively. These can be consolidated as conditions (6.7) and (6.8).[165]   □

Note that the conditions obtained in Lemma 6.1 have also been obtained as conditions for the all-in-autarky state to be a strict equilibrium for the general n-country model in Subsection 5.3.1. The strength of these conditions remains to be analysed, as does how advantageous it would be for a leading country to engage in trade with the other country.

If equations (6.7) and (6.8) are compared, then it is found that the condition for w_{22} is weaker in (6.7) than in (6.8), and the condition for w_{11} is weaker in (6.8) than in (6.7), in the sense that the condition for w_{22} is more easily satisfied in (6.7) while the condition for w_{11} is more easily satisfied in (6.8). This is true since for small δ, the w_{11}-condition in (6.8) and the w_{22}-condition in (6.7) necessarily hold as

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[165] The conditions obtained in Lemma 6.1 can be used to determine which set of parameter values used in the Figure 6.1 illustrations indicate $z = (0,0)$ is a strict equilibrium

$w_{11}, w_{22} < 1$, whereas the w_{11}-condition in (6.7) and the w_{22}-condition in (6.8) require parameter restrictions on not only the value of δ, but also on the values of a_i's and λ_i's.

In order to analyse and compare the strength of the w_{ii}-conditions in (6.7) and (6.8) via diagrams, two functions of δ, $g_i(\delta)$ and $h_i(\delta)$, are defined below.

Let $g_i(\delta) = \dfrac{\lambda_j - \lambda_i}{a_i \delta^2} + \dfrac{\lambda_i}{a_i \delta}$ and $h_i(\delta) = \dfrac{\lambda_i}{a_i \delta}$ for $i = 1, 2$ and $j \neq i$.

Then, for Figure 6.2, $h_2(\delta) = \dfrac{\lambda_2}{a_2 \delta}$, $\lim\limits_{\delta \to 0} h_2(\delta) \to +\infty$ and $h_2(1) = \dfrac{\lambda_2}{a_2}$,

and, $g_1(\delta) = \dfrac{\lambda_2 - \lambda_1}{a_1 \delta^2} + \dfrac{\lambda_1}{a_1 \delta}$, $\lim\limits_{\delta \to 0} g_1(\delta) \to -\infty$ and $g_1(1) = \dfrac{\lambda_2}{a_1}$.

The curve $g_1(\delta)$ has a maximum point at $\delta_{\max} = 2\left(1 - \dfrac{\lambda_2}{\lambda_1}\right)$[166]. Note that $\lambda_1 > \lambda_2$ for condition (6.7), hence $0 < \delta_{\max} < 2$.

Let δ' be the solution of $g_1(\delta) = 0$ and δ^*, δ^{**} be the solutions of $w_{11} = g_1(\delta)$.

At δ': $g_1(\delta) = \dfrac{\lambda_2 - \lambda_1}{a_1 \delta^2} + \dfrac{\lambda_1}{a_1 \delta} = 0$. Hence $\delta' = 1 - \dfrac{\lambda_2}{\lambda_1}$. Note that $0 < \delta' < 1$.

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[166] At the stationary point: $g_1'(\delta) = 0 \Leftrightarrow \dfrac{-2(\lambda_2 - \lambda_1)}{a_1 \delta^3} - \dfrac{\lambda_1}{a_1 \delta^2} = 0$, which gives $\delta = 2\left(1 - \dfrac{\lambda_1}{\lambda_2}\right)$. Now

$g_1''(\delta) = \dfrac{6(\lambda_2 - \lambda_1)}{a_1 \delta^4} + \dfrac{2\lambda_1}{a_1 \delta^3}$ and $\delta = 2\left(1 - \dfrac{\lambda_1}{\lambda_2}\right)$, so

$g_1''\left(\dfrac{2(\lambda_1 - \lambda_2)}{\lambda_1}\right) = \dfrac{6\lambda_1^4(\lambda_2 - \lambda_1)}{16 a_1 (\lambda_1 - \lambda_2)^4} + \dfrac{2\lambda_1^4}{8 a_1 (\lambda_1 - \lambda_2)^3}$

$= \dfrac{-6\lambda_1^4}{16 a_1 (\lambda_1 - \lambda_2)^3} + \dfrac{2\lambda_1^4}{8 a_1 (\lambda_1 - \lambda_2)^3}$

$= \dfrac{-2\lambda_1^4}{16 a_1 (\lambda_1 - \lambda_2)^3}$.

Since $\lambda_1 > \lambda_2$, $(\lambda_1 - \lambda_2)^3 > 0$ and hence $\delta_{\max} = 2\left(1 - \dfrac{\lambda_1}{\lambda_2}\right)$.
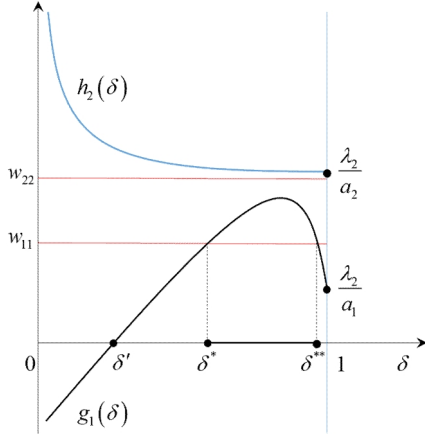
At δ^*, δ^{**}: $w_{11} = g_1(\delta) = \dfrac{\lambda_2 - \lambda_1}{a_1 \delta^2} + \dfrac{\lambda_1}{a_1 \delta}$. This gives the following quadratic in δ:

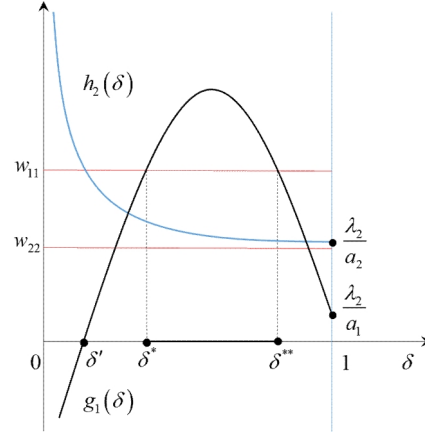$$a_1 w_{11} \delta^2 - \lambda_1 \delta + (\lambda_1 - \lambda_2) = 0^{167}.$$

Thus, $\delta^* = \dfrac{\lambda_1 - \sqrt{\lambda_1^2 - 4a_1 w_{11}(\lambda_1 - \lambda_2)}}{2a_1 w_{11}}$ and $\delta^{**} = \dfrac{\lambda_1 + \sqrt{\lambda_1^2 - 4a_1 w_{11}(\lambda_1 - \lambda_2)}}{2a_1 w_{11}}$.

Note that δ^* and δ^{**} are real if either $\lambda_1 \le \lambda_2$ or $\lambda_1^2 > 4a_1 w_{11}(\lambda_1 - \lambda_2)$ when $\lambda_1 > \lambda_2$. Using conditions in (6.7), a necessary condition for the solution of $w_{11} = g_1(\delta)$ is $\lambda_1^2 > 4a_1 w_{11}(\lambda_1 - \lambda_2)$.
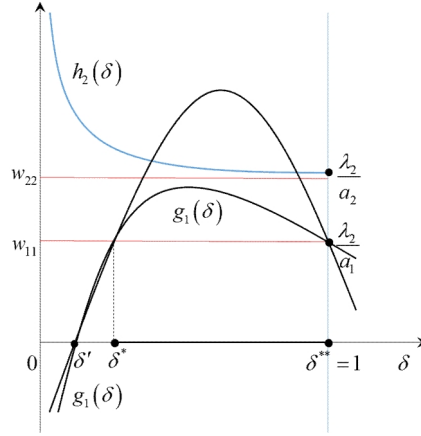

(a) $h_2(\delta) > g_1(\delta)$ for $0 \le \delta \le 1$, $\delta^{**} < 1$.

(b) $h_2(\delta) < g_1(\delta)$ for some $0 < \delta < 1$, $\delta^{**} < 1$.

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[167] $w_{11} = \dfrac{\lambda_2 - \lambda_1}{a_1 \delta^2} + \dfrac{\lambda_1}{a_1 \delta} \Leftrightarrow a_1 w_{11} \delta^2 = \lambda_2 - \lambda_1 + \lambda_1 \delta \Leftrightarrow a_1 w_{11} \delta^2 - \lambda_1 \delta + (\lambda_1 - \lambda_2) = 0$.

(c) $\delta^{**} = 1$.

Figure 6.2. Graphical representation for $\lambda_1 > \lambda_2$ - Condition (6.7) holds for

$\delta^* \leq \delta \leq \delta^{**}$. Note that $h_2(\delta) = \dfrac{\lambda_2}{a_2 \delta}$, $g_1(\delta) = \dfrac{\lambda_2 - \lambda_1}{a_1 \delta^2} + \dfrac{\lambda_1}{a_1 \delta}$,

$$\delta^* = \frac{\lambda_1 - \sqrt{\lambda_1^2 - 4 a_1 w_{11} (\lambda_1 - \lambda_2)}}{2 a_1 w_{11}} \text{ and } \delta^{**} = \frac{\lambda_1 + \sqrt{\lambda_1^2 - 4 a_1 w_{11} (\lambda_1 - \lambda_2)}}{2 a_1 w_{11}}.$$

For Figure 6.3, $h_1(\delta) = \dfrac{\lambda_1}{a_1 \delta}$, $\lim\limits_{\delta \to 0} h_1(\delta) = +\infty$ and $h_1(1) = \dfrac{\lambda_1}{a_1}$,

and, $g_2(\delta) = \dfrac{\lambda_1 - \lambda_2}{a_2 \delta^2} + \dfrac{\lambda_2}{a_2 \delta}$, $\lim\limits_{\delta \to 0} g_2(\delta) = -\infty$ and $g_2(\delta) = \dfrac{\lambda_1}{a_2}$.

The curve $g_2(\delta)$ has a maximum point at $\delta_{\max} = 2\left(1 - \dfrac{\lambda_1}{\lambda_2}\right)$. Note that $\lambda_1 \leq \lambda_2$ for

condition (6.7), hence $0 \leq \delta_{\max} < 2$.

At δ', $g_2(\delta) = \dfrac{\lambda_1 - \lambda_2}{a_2 \delta^2} + \dfrac{\lambda_2}{a_2 \delta} = 0$. Hence, $\delta' = 1 - \dfrac{\lambda_1}{\lambda_2}$. Note that $0 < \delta' < 1$.

At δ^+, δ^{++}: $w_{22} = g_2(\delta) = \dfrac{\lambda_1 - \lambda_2}{a_2 \delta^2} + \dfrac{\lambda_2}{a_2 \delta}$. This gives the following quadratic in δ:

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$$a_2 w_{22} \delta^2 - \lambda_2 \delta + (\lambda_2 - \lambda_1) = 0\,^{168}.$$

Thus, $\delta^+ = \dfrac{\lambda_2 - \sqrt{\lambda_2^2 - 4a_2 w_{22}(\lambda_2 - \lambda_1)}}{2a_2 w_{22}}$ and $\delta^{++} = \dfrac{\lambda_2 + \sqrt{\lambda_2^2 - 4a_2 w_{22}(\lambda_2 - \lambda_1)}}{2a_2 w_{22}}$.

Note that δ^+ and δ^{++} are real if either $\lambda_2 \le \lambda_1$ or $\lambda_2^2 > 4a_2 w_{22}(\lambda_2 - \lambda_1)$ when $\lambda_2 > \lambda_1$. Also, δ^+ and δ^{++} are real and positive if $\lambda_1 > a_2 w_{22}$, in which case $\lambda_1 > a_2 w_{22}\delta$ for all $\delta \in (0,1]$.
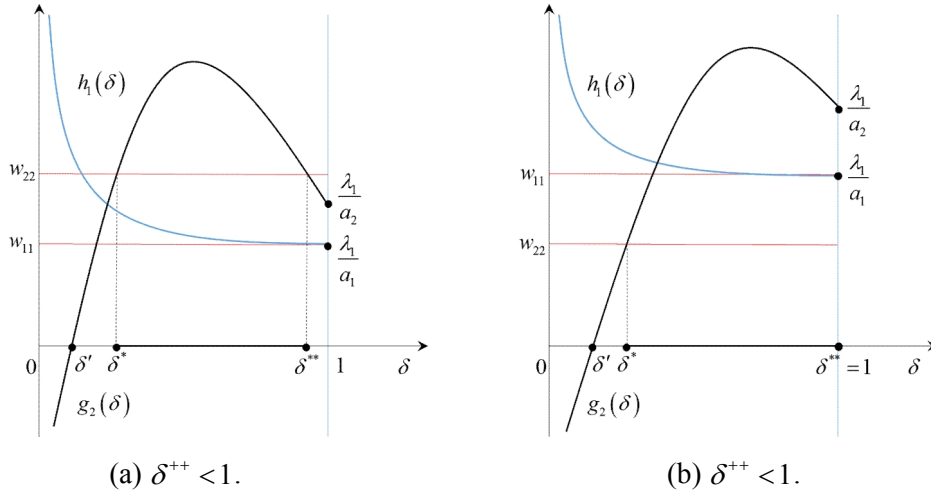

(a) $\delta^{++} < 1$.　　　　　　　　(b) $\delta^{++} < 1$.

Figure 6.3. Graphical representation for $\lambda_1 \le \lambda_2$ - Condition (6.8) holds for $\delta^+ \le \delta \le \delta^{++}$. Note that $h_1(\delta) = \dfrac{\lambda_1}{a_1 \delta}$ and $g_2(\delta) = \dfrac{\lambda_1 - \lambda_2}{a_2 \delta^2} + \dfrac{\lambda_2}{a_2 \delta}$,

$$\delta^+ = \dfrac{\lambda_2 - \sqrt{\lambda_2^2 - 4a_2 w_{22}(\lambda_2 - \lambda_1)}}{2a_2 w_{22}} \text{ and } \delta^{++} = \dfrac{\lambda_2 + \sqrt{\lambda_2^2 - 4a_2 w_{22}(\lambda_2 - \lambda_1)}}{2a_2 w_{22}}.$$

From Figure 6.2, the following observation is made: If the condition $w_{22} < \dfrac{\lambda_2}{\delta a_2}$ holds true, then w_{11} is required to be in the range $\left[\delta^*, \delta^{**}\right]$ for (6.7) to hold, so that the all-in-autarky state is then an equilibrium for $\lambda_1 > \lambda_2$. Similarly, from Figure

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[168] $w_{11} = \dfrac{\lambda_2 - \lambda_1}{a_1 \delta^2} + \dfrac{\lambda_1}{a_1 \delta} \Leftrightarrow a_1 w_{11} \delta^2 = \lambda_2 - \lambda_1 + \lambda_1 \delta \Leftrightarrow a_1 w_{11} \delta^2 - \lambda_1 \delta + (\lambda_1 - \lambda_2) = 0$.

6.3: If the condition $w_{11} < \dfrac{\lambda_1}{\delta a_1}$ holds true, then w_{22} is required to be in the range $\left[\delta^+, \delta^{++}\right]$ for (6.8) to hold, so that the all-in-autarky state is a strict equilibrium for $\lambda_1 \leq \lambda_2$.

Looking at Figure 6.2 and Figure 6.3, it can be seen that condition (6.7) and (6.8) can be satisfied for either large values of δ, or an intermediate range $\delta^* \leq \delta \leq \delta^{**}$, $\delta^+ \leq \delta \leq \delta^{++}$. This means that a leading country can pull the world economy out of the all-in-autarky state if the strategy size δ is small enough.

However, if the self-generating economic capacities of countries 1 and 2, w_{11} and w_{22}, respectively, lie in the critical range $\left[\delta^*, \delta^{**}\right]$ and $\left[\delta^+, \delta^{++}\right]$, then both countries 1 and 2 will remain in autarky, and neither country will have an incentive to engage in trade with the other country.

Lemma 6.2. A necessary and sufficient condition for the all-in-autarky state to be a strict equilibrium is:

$$w_{ii} < \frac{\lambda_j - \max\{\lambda_i, \lambda_j\}}{\delta^2 a_i} + \frac{\lambda_i}{\delta a_i} \text{ for } i = 1,2 \text{ and } j \neq i. \tag{6.9}$$

Proof. Combining conditions (6.7) and (6.8) gives:

$$w_{11} < \frac{\lambda_2 - \lambda_1}{\delta^2 a_1} + \frac{\lambda_1}{\delta a_1} < \frac{\lambda_1}{\delta a_1} \text{ if } \lambda_1 > \lambda_2 \text{ and } w_{22} < \frac{\lambda_1 - \lambda_2}{\delta^2 a_2} + \frac{\lambda_2}{\delta a_2} < \frac{\lambda_2}{\delta a_2} \text{ if } \lambda_1 < \lambda_2,$$

which can be consolidated as condition (6.9). □

The conditions obtained in Lemma 6.2 for the all-in-autarky state to be a strict equilibrium are illustrated in Figure 6.4.

Condition (6.9) is easily satisfied for $\delta = 1$ and $w_{ii} = 0$ (and for relatively small w_{ii}), $i = 1,2$. This implies that both countries tend to remain in the autarkic state if the internal economic activities do not contribute significantly towards the payoffs. However, as discussed in Chapter 5, it is not unreasonable to assume w_{ii} is often

large, as cross-border trade generally accounts for a relatively small proportion of GDP for a country. This assumption implies it is rather unlikely for countries to remain in the autarkic state. So countries integrate into the world economy and this gives plausibility to the fact that complete economic autarkies are rare today.

The trade dynamics previously obtained in Chapter 3 for $\delta = 1$, $w_{ii} = 0$, $i = 1, 2$ also show that the all-in-autarky state is a strict equilibrium, see Figure 3.1.
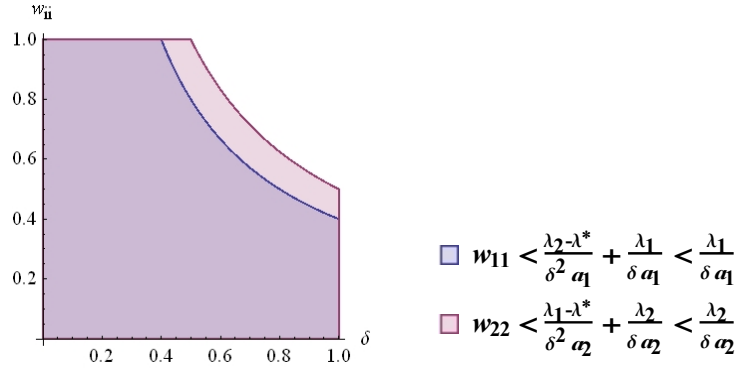
It remains to be investigated if the two countries will continue to take part in world trade if the trade dynamics start off with the fully integrated state and if this can be a strict equilibrium, for a small strategy size. This is explored in the next section.
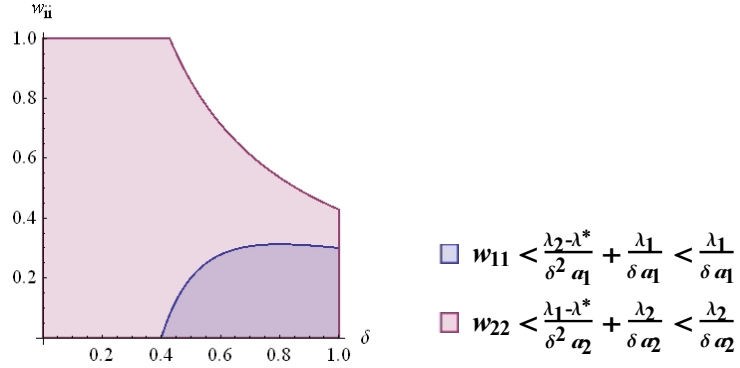
(a) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 5$, $\lambda_2 = 3$.
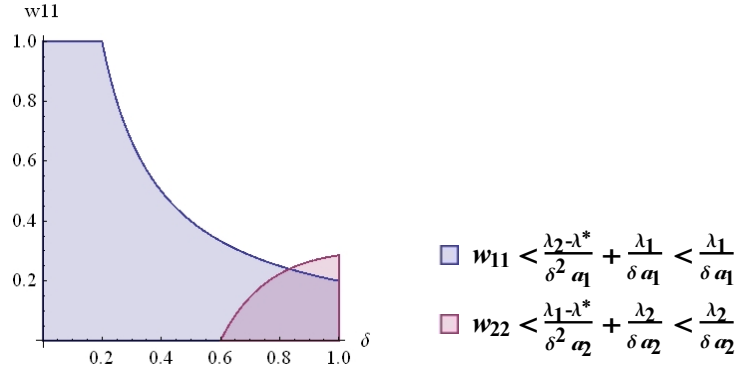
(b) Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 2$, $\lambda_2 = 5$.

(c) Parameter values: $a_1 = 10$, $a_2 = 8$, $\lambda_1 = 4$, $\lambda_2 = 4$.

Figure 6.4. The regions of the $\delta - w_{ii}$ plot for which the all-in-autarky state is an equilibrium, i.e. condition (6.9) holds true: $w_{ii} < \dfrac{\lambda_j - \lambda^*}{\delta^2 a_i} + \dfrac{\lambda_i}{\delta a_i}$, for $i = 1, 2$ and $j \neq i$, where $\lambda^* = \max\{\lambda_i, \lambda_j\}$. Note: The horizontal axes represent the δ-values, whereas the vertical axes represent the w_{ii}-values.

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6.2 The Fully Integrated State $(1,1)$

It has been shown that in the 2-country model, both countries cannot be fully integrated if $\delta = 1$ and $w_{ii} = 0$, $i = 1,2$ (from the above analysis of countries choosing to remain in autarky under such conditions and Figure 3.1).

Now, consider $0 < \delta < 1$.

First, consider country 1. It already obtains the maximal payoff of a_1 by playing strategy $z_1 = 1$, so it does not update its strategy to $z_1 = 1 - \delta$, as obtained previously in Lemma 5.1.

Next, consider country 2. As obtained previously in Chapter 5 - condition (5.13), the condition required for country 2 to stick with its fully integrated strategy $z_2 = 1$ is: $\delta < \dfrac{a_2 + a_2 w_{22} - a_1 w_{12} - \lambda_2}{a_2 w_{22}}$, and a necessary and sufficient condition required for the fully integrated state $z = (1,1)$ to be a strict equilibrium for all δ in the range $(0,1]$ is $a_1 w_{12} < a_2 - \lambda_2$, as obtained previously in Chapter 5 - condition (5.14).

If condition (5.14) is not satisfied, then the fully integrated strict equilibrium does not exist for $\delta < \dfrac{a_2 + a_2 w_{22} - a_1 w_{12} - \lambda_2}{a_2 w_{22}}$. It remains to be examined if a heterogeneous equilibrium $z^* = \left(z_1^*, z_2^*\right)$, such that $0 < z_1^*, z_2^* < 1$ exists for strategy size δ.

6.3 Varying Aspiration Level

In the 2-country scenario so far, $\mu\left(s \middle| z_{-i}\right)$ has been considered to be the payoff to the *leading country* given the modified strategy profile. If other cases are considered, where $\mu\left(s \middle| z_{-i}\right)$ is the average payoff of those countries which are currently achieving a higher payoff than country *i*, or $\mu\left(s \middle| z_{-i}\right)$ is the payoff of the country whose payoff is currently closest to, but better than, country *i*'s, then the conditions

remain the same for the 2-country trade game (existence of only one opponent implies the country with better the payoff is the *leading country*).

For example, consider the all-in-autarky state. When C_1 tries to update its strategy, $\mu(\delta|0) = \max\{a_1 w_{11}\delta^2 - \lambda_1\delta + \lambda_1, \lambda_2\}$ and $\mu(0|0) = \max\{\lambda_1, \lambda_2\}$ for all three cases above. However, for $n > 2$ countries, the situation is more complex and is not considered further.

6.4 The Trade Dynamics

Recall that the strategy profile for a world with two countries C_1 and C_2 is $z = (z_1, z_2)$.

The trade weights matrix is $W = \begin{pmatrix} w_{11} & 1 - w_{11} \\ 1 - w_{22} & w_{22} \end{pmatrix}$, where $0 \le w_{11}, w_{22} \le 1$.

From (5.1), the payoff to C_1 is:

$$\pi_1(z) = a_1 z_1 \left[w_{11} z_1 + (1 - w_{11}) z_2 \right] + (1 - z_1)\lambda_1. \tag{6.10}$$

The payoff to C_2 is:

$$\pi_2(z) = a_2 z_2 \left[(1 - w_{22}) z_1 + w_{22} z_2 \right] + (1 - z_2)\lambda_2. \tag{6.11}$$

Then, from (5.1), $a_i w_{ij}$ is the *maximum gain from trade* to C_i obtained from completely open trade with C_j (i.e. when $z_i = z_j = 1$). When $i = j$, this is the maximum gain that C_i can extract from its own economy (discounting any possible gains from trade with other countries), as a direct result of international trade.

In this section, it is assumed that the leading country always seeks to maximize its payoff (given the subordinate country's strategy). The strategy size δ is assumed to be varied arbitrarily in order to visualize how countries can catch up with one another to assume the leading position in international trade.

The calculations below help determine the trade dynamics, using the fact that the leading country seeks to maximize its payoff and the subordinate country tries to minimize the gap between the leading country and itself.

Assume that C_1 is the leading country. Then, the payoff to C_1 is greater than the payoff to C_2, which can be written as:

$$\pi_1(z) - \pi_2(z) \geq 0.$$

Equation (6.10) can be written as:

$$\pi_1(z_1, z_2) = a_1 w_{11} z_1^2 + \left[a_1 w_{12} z_2 - \lambda_1 \right] z_1 + \lambda_1.$$

This convex surface is shown (for varying z_2) graphically in Figure 6.5.

Country 1 will attempt to maximize its payoff. Since this quadratic is a convex function of z_1 (for fixed z_2), it follows that it is maximized either at $z_1^* = 0$ or at $z_1^* = 1$, or possibly both, see Figure 6.6.
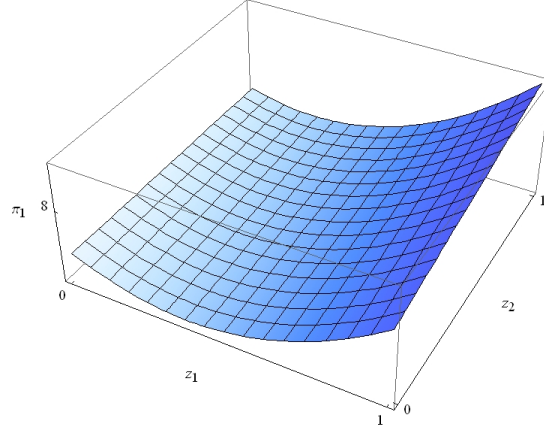

Figure 6.5. The payoff function surface for country 1. Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 6$, $\lambda_2 = 3$, $w_{11} = 0.8$, $w_{22} = 0.7$.
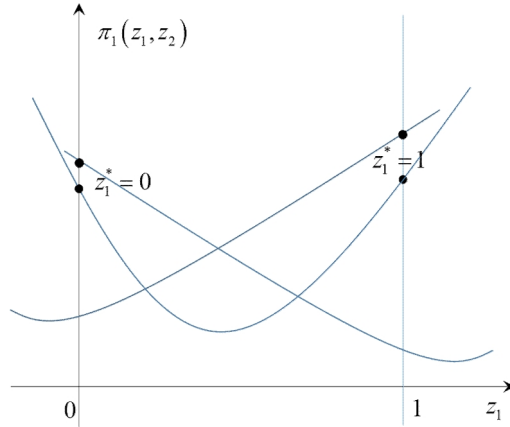
Figure 6.6. Curves for $\pi_1(z_1, z_2)$, for a fixed z_2, showing the payoff function to country 1 is maximised either at $z_1^* = 0$ or $z_1^* = 1$, or both.[169]

Case I: $z_2^* = 0$

This case finds the range of parameter values for which the autarkic strategy is the best strategy for country 1 (the leading country). The best reply of country 2 (the subordinate country) given the leading country plays the autarkic strategy is also obtained.

In this case, country 1 is in autarky and receives payoff λ_1. This occurs if and only if the fully integrated payoff is less than the autarkic payoff, that is: $\pi_1(1, z_2) < \pi_1(0, z_2)$. This holds if and only if:

$$a_1 w_{11} + a_1 w_{12} z_2 < \lambda_1. \tag{6.12}$$

This is possible for non-negative z_2 only if:

$$a_1 w_{11} < \lambda_1. \tag{6.13}$$

---

[169] Note that the minimum point for the curve $\pi_1(z_1, z_2)$ is at $(z_1)_{\min} = \dfrac{\lambda_1 - a_1 w_{12} z_2}{2 a_1 w_{11}}$, which implies either $(z_1)_{\min} < 0$, or $(z_1)_{\min} > 1$, or $0 \le (z_1)_{\min} \le 1$. In the first two cases, since $\pi_1(z_1, z_2)$ is convex, $z_1^* = 0$ or $z_1^* = 1$, and in the last case $z_1^* = 0$ or $z_1^* = 1$, or both.

That is, the maximum gain C_1 can extract from its own economy by opening to trade with C_2 is less than the payoff it receives from remaining in autarky. This means that gains from trade with country 2 ($a_1 w_{12}$) have to be sufficiently large to pull country 1 out of autarky, otherwise autarky becomes the more favourable strategy for country 1.

Given (6.13), condition (6.12) holds only for:

$$z_2 < \hat{z}_2 = \frac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}. \qquad (6.14)$$

The assumptions $a_1 > \lambda_1$ and $w_{11} + w_{12} = 1$ imply that $\hat{z}_2 < 1$[170]. Hence, this case cannot arise if z_2 is sufficiently large. That is, C_2 can force C_1 out of autarky by itself investing in open trade to a large enough extent.[171]

This analysis must be compatible with the underlying assumption that C_1 is the leading country. That is, when $z_1 = z_1^* = 0$, the payoff to country 1 must be greater than the payoff to country 2. Thus, let:

$$q_1(z_2) = \pi_1(0, z_2) - \pi_2(0, z_2).$$

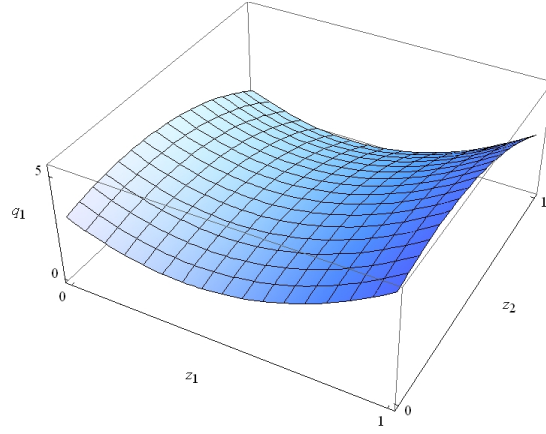The surface generated by this function is represented diagrammatically in Figure 6.7.

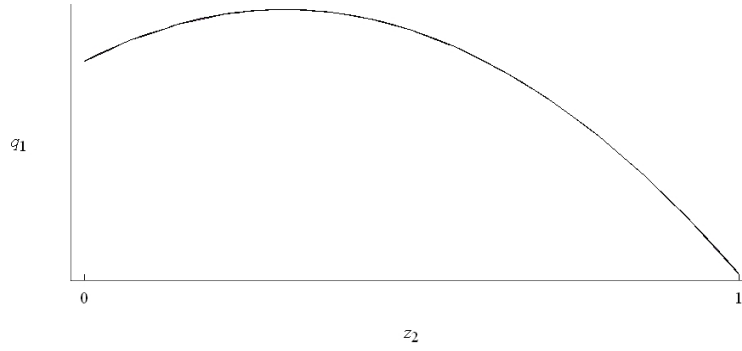---

[170] Since $w_{11} = 1 - w_{12}$,

$$\hat{z}_2 = \frac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}$$

$$= \frac{\lambda_1 - a_1(1 - w_{12})}{a_1 w_{12}}$$

$$= 1 - \frac{a_1 - \lambda_1}{a_1 w_{12}}.$$

Now $a_1 > \lambda_1$, hence $\hat{z}_2 < 1$.

[171] A lagging nation may be able to *leapfrog* a rival country as a reaction to major exogenous change in technology, see Brezis, Krugman and Tsiddon [3].

(a) Saddle surface of $\pi_1(z) - \pi_2(z)$.

(b) Curve of $q_1(z_2)$.[172]

Figure 6.7. Surface generated by payoff difference $\pi_1(z) - \pi_2(z)$.

Note that $z_1^* = 0$ in (b) so country 1 is in autarky and receives payoff λ_1,

and the curve in (b) is a cross-section of the surface in (a).

Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 6$, $\lambda_2 = 3$, $w_{11} = 0.8$, $w_{22} = 0.7$.

Then, the following condition is required to hold for country 1 to be the leading country for all z_2:

$$q_1(z_2) = (\lambda_1 - \lambda_2) + \lambda_2 z_2 - a_2 w_{22} z_2^2 \geq 0. \tag{6.15}$$

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[172] $q_1(z_2) = \pi_1(0, z_2) - \pi_2(0, z_2)$ as introduced previously in this section.

For example, when $z_2 = 0$, C_1 is the leading country only when its autarky payoff λ_1 is greater than the autarky payoff λ_2 of C_2. When $z_2 = 1$, C_1 is the leading country only when:

$$\lambda_1 \geq a_2 w_{22}. \tag{6.16}$$

That is, the maximum gain that C_2 can obtain from its own economy as a result of openness to trade when C_1 is in autarky, is not greater than C_1's autarky payoff λ_1.

More generally, consider the discriminant for $q_1(z_2)$:

$$D = \lambda_2^2 + 4a_2 w_{22}(\lambda_1 - \lambda_2). \tag{6.17}$$

Then, $q_1(z_2)$ has *no* real roots if and only if $D < 0$. This can occur only if $\lambda_2 > \lambda_1$ and $a_2 w_{22}$ is sufficiently large. That is, C_2's autarky payoff must be greater than that of C_1, and the maximum gain that C_2 can obtain from its own economy must be sufficiently large (i.e. $a_2 w_{22} > \lambda_1$). Then, $q_1(z_2) < 0$ for all z_2. In this case, C_1 *cannot* be the leading country by remaining in autarky, *whatever* C_2 does. Thus, $D < 0$ contradicts the assumption that C_1 is the leading country.

Suppose $D > 0$. That is, either $\lambda_1 \geq \lambda_2$ or $\lambda_2 > \lambda_1$ and $a_2 w_{22}$ is not too large. Then, $q_1(z_2)$ has two real roots since it is concave:

$$z_2^{\mp} = \frac{1}{2a_2 w_{22}} \left\{ \lambda_2 \mp \sqrt{\lambda_2^2 + 4a_2 w_{22}(\lambda_1 - \lambda_2)} \right\}, \tag{6.18}$$

in addition, condition (6.15) holds for z_2 in the range:

$$\max\left\{0, z_2^-\right\} \leq z_2 \leq \min\left\{z_2^+, 1\right\}. \tag{6.19}$$

For example, if $\lambda_1 \geq \lambda_2$, then the left-hand limit in (6.19) is always $z_2 = 0$, and the right-hand limit is $z_2 = 1$ provided $a_2 w_{22} \leq \lambda_1$[173]. In this case, C_1 *is* the leading country while remaining in autarky, *whatever* C_2 does. The constraint on $z_1^* = 0$ being a best strategy against z_2 is determined by (6.14); so it is only required that $0 \leq z_2 < \hat{z}_2$.

If $a_2 w_{22} > \lambda_1$, then C_2's maximum gain from its own open economy is larger than C_1's autarky payoff. In this case, still assuming that $\lambda_1 \geq \lambda_2$, the following is obtained: $z_2^+ < 1$. Thus, C_1 is the leading country while remaining in autarky provided C_2 uses a strategy in the range $z_2 \leq z_2^+$. Thus, for $z_1^* = 0$ to be a best strategy for the leading country C_1 requires $0 \leq z_2 < \min\{z_2^+, \hat{z}_2\}$.

Finally, if $\lambda_2 > \lambda_1$ but $D \geq 0$, then $z_2^- > 0$, and so there is a range of *near-autarky* strategies for C_2, for which C_1 ceases to be the leading country when it remains in autarky.

Given that C_1 is the leading country playing in autarky, a look at what C_2 should do is now examined. C_2 is required to choose a best strategy against $z_1^* = 0$ by minimizing the payoff difference $q_1(z_2)$ while retaining the constraint (6.15) - $q_1(z_2) \geq 0$; i.e. that C_1 should remain the leading country. Now $q_1(z_2)$ is a concave function of z_2, see Figure 6.8, and there are various possibilities to consider as per below.

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[173] The right-hand limit is 1 if and only if $z_2^+ = \dfrac{1}{2a_2 w_{22}}\left\{\lambda_2 + \sqrt{\lambda_2^2 + 4a_2 w_{22}(\lambda_1 - \lambda_2)}\right\} \geq 1$. Simplifying this:

$$\frac{1}{2a_2 w_{22}}\left\{\lambda_2 + \sqrt{\lambda_2^2 + 4a_2 w_{22}(\lambda_1 - \lambda_2)}\right\} \geq 1$$
$$\Leftrightarrow \sqrt{\lambda_2^2 + 4a_2 w_{22}(\lambda_1 - \lambda_2)} \geq 2a_2 w_{22} - \lambda_2$$
$$\Leftrightarrow \lambda_2^2 + 4a_2 w_{22}\lambda_1 - 4a_2 w_{22}\lambda_2 \geq 4a_2^2 w_{22}^2 + \lambda_2^2 - 4a_2 w_{22}\lambda_2$$
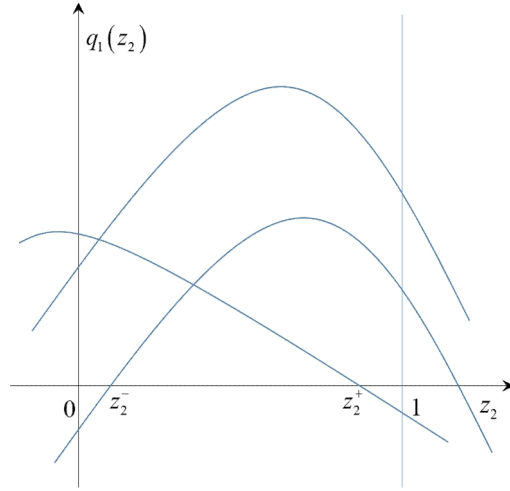$$\Leftrightarrow \lambda_1 \geq a_2 w_{22}.$$

Figure 6.8. The concave function $q_1(z_2)$.

There are several possibilities:

(i) $0 < z_2^+ < \hat{z}_2$: In this case, since $z_2^+ < \hat{z}_2$, it follows from (5.7) and (5.18) that $z_2 = z_2^+$ is an allowable strategy, and by definition that $q_1(z_2^+) = 0$. Hence, C_2 can *catch up* with C_1 (reduce the payoff difference to 0), and any further development by C_2 (i.e. increase in z_2 above z_2^+) will lead to it assuming the role of leading country.

Figure 6.9. Country 2's strategy when $0 < z_2^+ < \hat{z}_2$: Increase its level of trade integration to z_2^+ and catch up with country 1. Any further increase in z_2 leads to $q_1(z_2) < 0$, which means country 2 becomes the leading country.

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(ii) $0 < z_2^- < \hat{z}_2$: In this case, (6.14) and (6.19) imply that $z_2 = z_2^-$ is an allowable strategy, and by definition $q_1\left(z_2^-\right) = 0$. Again, C_1 can *catch up* with C_1 (reduce the payoff difference to 0), and any further development by C_2 (i.e. decrease in z_2 below z_2^-) will lead to its assuming the role of leading country.
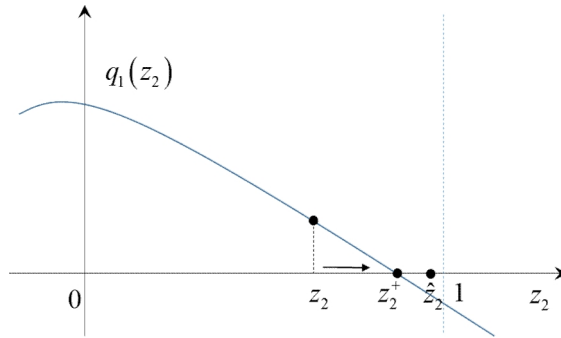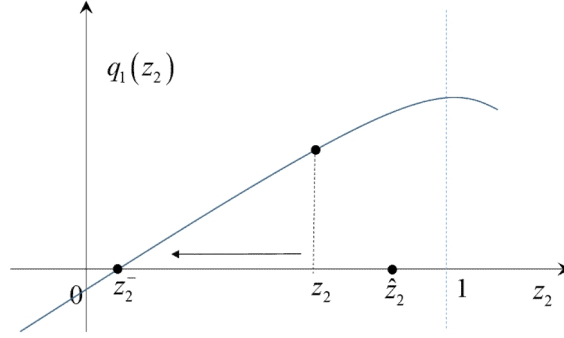

Figure 6.10. Country 2's strategy when $0 < z_2^- < \hat{z}_2$: Decrease its level of trade integration to z_2^- and catch up with country 1. Any further decrease in z_2 leads to $q_1\left(z_2\right) < 0$, which means country 2 becomes the leading country.

(iii) $0 < z_2^- \le z_2^+ < \hat{z}_2$: In this case, both (i) and (ii) hold simultaneously, and it is possible for C_2 to *catch up* with C_1 either by playing a more integrative strategy z_2^+, or a more autarkic strategy z_2^-. Note that the choice of strategy in this case depends on the side of the maximum point $\left(z_m, q_1\left(z_m\right)\right)$ of $q_1\left(z_2\right)$ the trade dynamics for C_2 start from. If $z_2 > z_m$, then C_2 increases its level of integration; and if $z_2 < z_m$, then C_2 decreases its integration to z_2^-. If the dynamics start from $\left(z_m, q_1\left(z_m\right)\right)$, then either of the strategies $z_2 = z_2^+$ or $z_2 = z_2^-$ are feasible options.
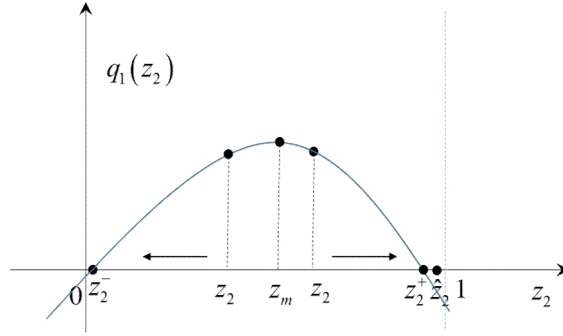
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Figure 6.11. Country 2's strategy when $0 < z_2^- \le z_2^+ < \hat{z}_2$: Decrease its level of trade integration to z_2^- or increase its level to z_2^+, depending on the starting point of the dynamics for C_2 (any further decrease or increase, respectively, in z_2 leads to $q_1(z_2) < 0$, which means country 2 becomes the leading country).

(iv) $z_2^- = 0$: In this case, C_2 can catch up with C_1 by itself reverting to autarky. From (6.18), this case can only arise when $\lambda_1 = \lambda_2$.

(v) $z_2^- < 0 < \hat{z}_2 \le z_2^+$: In this case, the allowable range of C_2 strategies, $0 \le z_2 < \hat{z}_2$, contains no root of $q_1(z_2)$, and hence $q_1(z_2) > 0$ for any z_2 in this range. It follows that $q_1(z_2)$ is minimized at an extreme point; i.e., either by taking $z_2 = 0$ or by taking $z_2 \to \hat{z}_2$. This is explored further below.
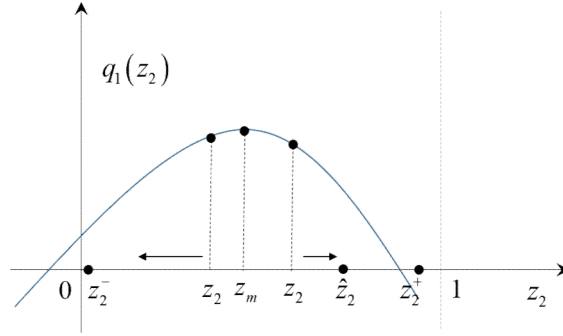
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Figure 6.12. Country 2's strategy when $z_2^- < 0 < \hat{z}_2 \leq z_2^+$: Decrease its strategy to $z_2 = 0$ or increase its level to $z_2 = \hat{z}_2$, (depending on the starting point of the dynamics for C_2).

Consider the function:

$$R(\hat{z}_2) = q_1(0) - q_1(\hat{z}_2) = \hat{z}_2 \left(a_2 w_{22} \hat{z}_2 - \lambda_2 \right).$$

Then, $z_2 = 0$ is the unique optimal C_2 strategy if $\hat{z}_2 < \dfrac{\lambda_2}{a_2 w_{22}}$, and $z_2 = \hat{z}_2$ is the unique optimal strategy if $\hat{z}_2 > \dfrac{\lambda_2}{a_2 w_{22}}$. If $\hat{z}_2 = \dfrac{\lambda_2}{a_2 w_{22}}$, then C_2 is indifferent between these two strategies.

Thus, from (6.14), the non-autarkic strategy $z_2 = \hat{z}_2$ is an optimal strategy provided:

$$\lambda_1 a_2 w_{22} - \lambda_2 a_1 w_{12} \geq (a_1 w_{11})(a_2 w_{22}).$$

It is the unique optimal strategy if this inequality is strict.

Note from (6.18) that the condition $z_2^- < 0$ can hold only if $\lambda_1 > \lambda_2$ and $w_{22} > 0$.

Now consider the condition $\hat{z}_2 \leq z_2^+$. By definition, this holds if and only if $q_1(\hat{z}_2) \geq 0$. That is:

$$\left(\lambda_1 - \lambda_2\right) + \lambda_2 \left(\frac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}\right) - a_2 w_{22} \left(\frac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}\right)^2 \geq 0.$$

In this case, $\left(z_1, z_2\right) = \left(0, \hat{z}_2\right)$ is a Nash equilibrium.[174]

Case II: $z_2^* = 1$

This case finds the range of parameter values for which the fully integrated strategy is the best strategy for country 1 (the leading country). The best reply of country 2 (the subordinate country) given the leading country plays the fully integrated strategy is also obtained.

C_1 is fully integrated into the world economy and receives payoff $a_1 w_{11} + a_1 w_{12} z_2$. The first contribution is from maximizing the payoff from its own economy, and the second is obtained from gains in trade with C_2. For $z_1^* = 1$ to be C_1's unique best reply to z_2, the following condition is required to hold: $\pi_1\left(1, z_2\right) > \pi_1\left(0, z_2\right)$, which gives:

$$a_1 w_{11} + a_1 w_{12} z_2 > \lambda_1. \tag{6.20}$$

This always holds for $z_2 = 1$ since $w_{11} + w_{12} = 1$ and $a_1 > \lambda_1$. On the other hand, it holds for $z_2 = 0$ only if $a_1 w_{11} > \lambda_1$; i.e. C_1's maximum gain from its own economy is at least as big as its autarky payoff.

If this is not the case – i.e. if $a_1 w_{11} \leq \lambda_1$ – and condition (6.13) holds, then (6.20) can hold only if $z_2 > \hat{z}_2$, with $\hat{z}_2$ as in (6.14). When $z_2 = \hat{z}_2$, the inequalities (6.12) and (6.20) are replaced by an equality, and C_1 is then indifferent between autarky ($z_1^* = 0$) and full engagement ($z_1^* = 1$).

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[174] Country 1 is the leading country playing in autarky. By the very definition of this case, country 1 maximizes its payoff against country 2 by doing so. Country 2 minimizes the payoff difference at $z_2 = \hat{z}_2$. Hence, $\left(z_1, z_2\right) = \left(0, \hat{z}_2\right)$ is a Nash equilibrium.

It remains to be determined whether $z_1^* = 1$ is compatible with C_1 being the leading country. To determine this, consider:

$$r_1(z_2) = \pi_1(1, z_2) - \pi_2(1, z_2).$$

Then, C_1 is the leading country only if $r_1(z_2) \geq 0$. Thus, the following condition is required to hold:

$$r_1(z_2) = (a_1 w_{11} - \lambda_2) + [a_1 w_{12} - a_2 w_{21} + \lambda_2] z_2 - a_2 w_{22} z_2^2 \geq 0. \tag{6.21}$$

For example, when $z_2 = 0$ this can hold only if $a_1 w_{11} \geq \lambda_2$. At the other extreme, when $z_2 = 1$: $r_1(1) = a_1 - a_2$, and this is positive if $a_1 \geq a_2$; i.e. when C_1's economy is *intrinsically* bigger than C_2's economy, so that C_1 is a *natural* leader. However, as it has been seen in Case I, when C_1 stays in autarky, in some circumstances, C_2 can *catch up* with it, and assume the role of leading country itself. Thus, if $a_1 < a_2$ then C_2 would be the natural leader, and could become so by playing a strategy z_2 which is sufficiently close to $z_2 = 1$.
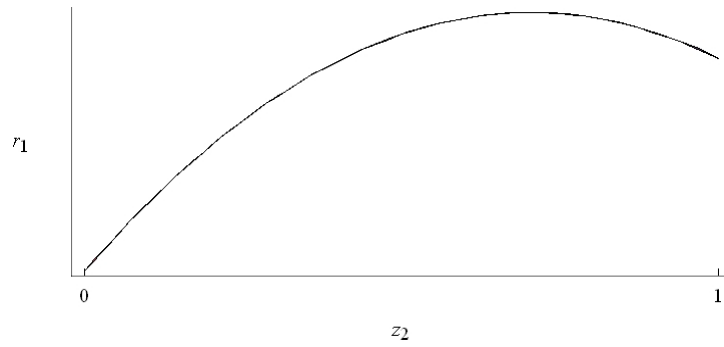

Figure 6.13. Curve of $r_1(z_2)$. Parameter values: $a_1 = 10$, $a_2 = 7$, $\lambda_1 = 6$, $\lambda_2 = 3$, $w_{11} = 0.4$, $w_{22} = 0.7$.

Consider the discriminant for $r_1(z_2)$:

$$E = (a_1 w_{12} - a_2 w_{21} + \lambda_2)^2 + 4(a_1 w_{11} - \lambda_2) a_2 w_{22}. \tag{6.22}$$

Clearly this is always non-negative if $a_1 w_{11} \geq \lambda_2$.

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For E to be negative requires $\lambda_2 > a_1 w_{11}$ and $a_2 w_{22}$ sufficiently large. That is, the autarky payoff to C_2 must be greater than C_1's maximum gain from its own economy, while the maximum gain to C_2 from its own economy must be sufficiently large. These are an unlikely set of circumstances. However, were they to occur, then $r_1(z_2)$ has no real roots, and hence, from (6.21), $r_1(z_2) < 0$ for all z_2, which is not compatible with C_1 being the leading country.

It can therefore be assumed that $E > 0$, so that $r_1(z_2)$ has two real roots:

$$z_2^{\pm} = \frac{1}{2a_2 w_{22}}\left\{(a_1 w_{12} - a_2 w_{21} + \lambda_2) \pm \sqrt{(a_1 w_{12} - a_2 w_{21} + \lambda_2)^2 + 4(a_1 w_{11} - \lambda_2)a_2 w_{22}}\right\}$$

(6.23)

and condition (6.21) holds if and only if:

$$\max\{0, z_2^-\} \le z_2 \le \min\{z_2^+, 1\}.$$

(6.24)

Now, note that it is always the case that $z_2^+ \ge 0$. Clearly this is the case if $\lambda_2 \le a_1 w_{11}$ and $w_{22} \ge 0$. If $\lambda_2 > a_1 w_{11}$ and $w_{22} > 0$ (but $z_2^{\pm}$ are real), then z_2^+ can be negative only if $(a_1 w_{12} - a_2 w_{21} + \lambda_2) < 0$. However:

$$a_1 w_{12} - a_2 w_{21} + \lambda_2 = a_1(1 - w_{11}) - a_2 w_{21} + \lambda_2$$
$$= a_1 - a_2 w_{21} + (\lambda_2 - a_1 w_{11})$$
$$\ge (a_1 - a_2) + (\lambda_2 - a_1 w_{11})$$
$$> 0.$$

This yields a contradiction. Thus, it is always the case that $z_2^+ \ge 0$.

Clearly, $z_2^- \le 0$ if $\lambda_2 \le a_1 w_{11}$ and $w_{22} \ge 0$ (with equality if and only if $\lambda_2 = a_1 w_{11}$ or $w_{22} = 0$). However, if $\lambda_2 > a_1 w_{11}$ and $w_{22} > 0$ (but $z_2^{\pm}$ are real), then the above argument shows that $z_2^- > 0$.

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However, it can be shown that it is always the case that $z_2^- < 1$. Suppose not. Then, from the above discussion the following conditions hold: $\lambda_2 > a_1 w_{11}$ and $w_{22} > 0$. Thus, from (6.23), $z_2^- \geq 1$ if and only if:

$$a_1 w_{12} - a_2 w_{21} + \lambda_2 - 2a_2 w_{22} \geq \sqrt{E}.$$

Since $w_{21} + w_{22} = 1$, the left-hand side can be written as $(a_1 w_{12} - a_2 w_{22}) - (a_2 - \lambda_2)$. If this is negative, then clearly the above inequality cannot hold. If it is non-negative, then the above inequality holds if and only if:

$$\left[(a_1 w_{12} - a_2 w_{22}) - (a_2 - \lambda_2) \right]^2 \geq E.$$

Using (6.22) and $w_{21} = 1 - w_{22}$, E can be re-written as:

$$E = \left[(a_1 w_{12} + a_2 w_{22}) - (a_2 - \lambda_2) \right]^2 + 4(a_1 w_{11} - \lambda_2) a_2 w_{22}$$
$$= (a_1 w_{12} + a_2 w_{22})^2 + (a_2 - \lambda_2)^2 - 2(a_2 - \lambda_2)(a_1 w_{12} + a_2 w_{22}) + 4a_2 w_{22}(a_1 w_{11} - \lambda_2).$$

From above,

$$z_2^- \geq 1 \Leftrightarrow \left[(a_1 w_{12} - a_2 w_{22}) - (a_2 - \lambda_2) \right]^2 \geq E$$
$$\Leftrightarrow (a_1 w_{12} - a_2 w_{22})^2 + (a_2 - \lambda_2)^2 - 2(a_2 - \lambda_2)(a_1 w_{12} - a_2 w_{22}) \geq E.$$

Using the expression for E, this inequality holds true if and only if:

$$-2a_1 w_{12} a_2 w_{22} - 2(a_2 - \lambda_2)(a_1 w_{12} - a_2 w_{22}) \geq$$
$$\qquad 2a_1 w_{12} a_2 w_{22} - 2(a_2 - \lambda_2)(a_1 w_{12} + a_2 w_{22}) + 4a_2 w_{22}(a_1 w_{11} - \lambda_2)$$
$$\Leftrightarrow 2(a_2 - \lambda_2)(a_1 w_{12} + a_2 w_{22} - a_1 w_{12} + a_2 w_{22}) \geq$$
$$\qquad 4a_2 w_{22} a_1 w_{12} + 4a_2 w_{22}(a_1 w_{11} - \lambda_2)$$
$$\Leftrightarrow 4a_2 w_{22}(a_2 - \lambda_2) \geq 4a_2 w_{22}(a_1 w_{12} + a_1 w_{11} - \lambda_2)$$
$$\Leftrightarrow 4a_2 w_{22}(a_2 - \lambda_2) \geq 4a_2 w_{22}(a_1 - \lambda_2)$$
$$\Leftrightarrow 4a_2 w_{22}(a_2 - a_1) \geq 0.$$

Thus, it can be concluded that $z_2^- \geq 1$ if and only if $a_2 \geq a_1$, which contradicts the basic assumption: $a_1 > a_2$. Thus, $z_2^- < 1$ must hold true, as claimed.

On the other hand, it can also be claimed that it is always true that $z_2^+ > 1$. Since z_2^+ is always non-negative, it follows from (6.23) that $z_2^+ > 1$ if and only if:

$$E > 2a_2 w_{22} - (a_1 w_{12} - a_2 w_{21} + \lambda_2) = (a_2 - \lambda_2) - (a_1 w_{12} - a_2 w_{22}).$$

If the right-hand-side is negative, this is always true. Otherwise, it is true if and only if:

$$E^2 > \left[(a_2 - \lambda_2) - (a_1 w_{12} - a_2 w_{22}) \right]^2.$$

As in the previous argument, it is now easy to show that this holds if and only if $a_1 > a_2$. It can therefore be concluded that $z_2^+ > 1$, as claimed.

It now follows from (6.24) that C_1 is the leading country provided:

$$\max \{ 0, z_2^- \} \le z_2 \le 1.$$

Furthermore, $\max \{ 0, z_2^- \} = 0$ if $\lambda_2 \le a_1 w_{11}$ and $w_{22} \ge 0$, and $\max \{ 0, z_2^- \} = z_2^- > 0$ if $\lambda_2 > a_1 w_{11}$ and $w_{22} > 0$.

It remains to be found what C_2 should do given that $z_1^* = 1$ and C_1 remains the leading country. Thus, in choosing an optimal reply, C_2 must minimize $r_1(z_2)$ subject to the constraint $r_1(z_2) \ge 0$.

There are several possibilities:

(i) $a_1 w_{11} > \lambda_1$ and $a_1 w_{12} > a_2 - \lambda_2$. Then, the full range $0 \le z_2 \le 1$ is compatible with (6.21). Also, $r_1(1) - r_1(0) = a_1 w_{12} - a_2 + \lambda_2$.

     Hence, in this case $r_1(1) > r_1(0)$, and hence $z_2^* = 0$ provided $r_1(0) \ge 0$; i.e. provided $a_1 w_{11} \ge \lambda_2$ (this is necessarily the case if $\lambda_1 \ge \lambda_2$). In this case, there is a unique Nash equilibrium $(z_1, z_2) = (1, 0)$ in which C_1 is fully engaged in the world economy but C_2 remains in autarky.

Figure 6.14. Country 2's strategy when $a_1 w_{11} > \lambda_1$ and $a_1 w_{12} > a_2 - \lambda_2$.

On the other hand, if $a_1 w_{11} < \lambda_2$ (and hence $\lambda_2 > \lambda_1$), $r_1(0) < 0$ and $z_2^- > 0$. In this case, $z_2^* = z_2^-$, and by playing this strategy, C_2 can catch up with C_1.
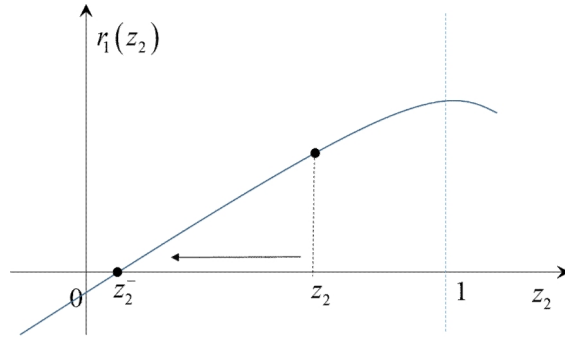
Figure 6.15. Country 2's strategy when $a_1 w_{11} < \lambda_2$, $r_1(0) < 0$ and $z_2^- > 0$.

(ii) $a_1 w_{11} > \lambda_1$ and $a_1 w_{12} \leq a_2 - \lambda_2$. Again, the full range $0 \leq z_2 \leq 1$ is compatible with (6.21). However, now $r_1(1) \leq r_1(0)$. If this inequality is strict, then $z_2^* = 1$ is the unique best reply, and so $(z_1, z_2) = (1,1)$ is the unique Nash equilibrium in which both countries are fully engaged in the world economy.
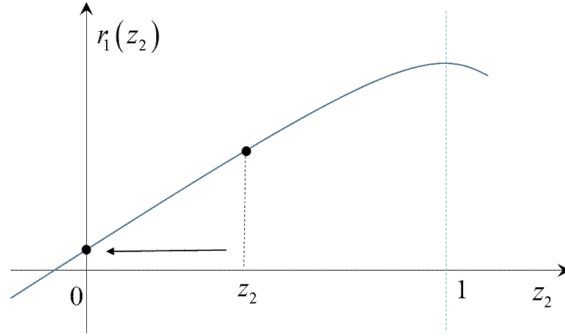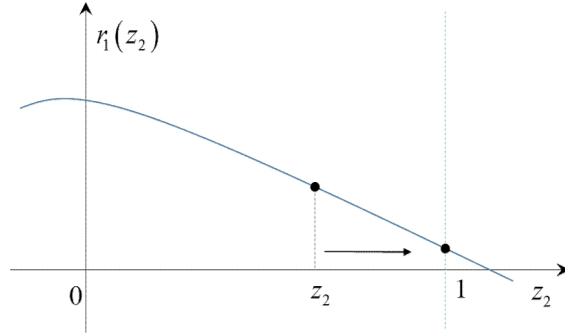
Figure 6.16. Country 2's strategy when $a_1 w_{11} > \lambda_1$ and $a_1 w_{12} \leq a_2 - \lambda_2$.

If $a_1 w_{12} = a_2 - \lambda_2$, then $r_1(1) = r_1(0)$ and C_2 is indifferent between autarky and full engagement. However, this is a very non-generic scenario.

(iii) $a_1 w_{11} \leq \lambda_1$ and $r_1(\hat{z}_2) \leq 0$. In this case, only the range $\hat{z}_2 < z_2 \leq 1$ is compatible with (6.21). Further, $z_2^- > 0$. Since $r_1(\hat{z}_2) \leq 0$, it must be true that $z_2^- \geq \hat{z}_2$, and hence C_2 can catch up with C_1 by playing $z_2^* = z_2^-$. Strategy change is similar to that shown in Figure 6.15.

(iv) $a_1 w_{11} \leq \lambda_1$ and $r_1(\hat{z}_2) > 0$. In this case $z_2^- < \hat{z}_2$ and it is necessarily the case that $r_1(z_2) > 0$ for all z_2 in the range $\hat{z}_2 \leq z_2 \leq 1$. Thus, C_2 should play $z_2^* = 1$ if $r_1(1) < r_1(\hat{z}_2)$ (strategy change in this case is similar to that shown in Figure 6.16), and should play $z_2^* = \hat{z}_2$ if $r_1(1) > r_1(\hat{z}_2)$. These give Nash equilibria $(z_1, z_2) = (1,1)$ and $(z_1, z_2) = (1, \hat{z}_2)$, respectively. If $r_1(1) = r_1(\hat{z}_2)$ then C_2 is indifferent between these two strategies. The conditions for $r_1(1) > r_1(\hat{z}_2) > 0$, leading to the unique Nash equilibrium $(z_1, z_2) = (1, \hat{z}_2)$ are:

$$0 < (a_1 w_{11} - \lambda_2) + (a_1 w_{12} - a_2 w_{21} + \lambda_2)\left(\frac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}\right) - a_2 w_{22}\left(\frac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}\right)^2 < a_1 - a_2.$$

Now if C_2 assumes the role of the leading country, then:

$$\pi_2(z) - \pi_1(z) \geq 0. \tag{6.25}$$

Also:

$$\pi_2(z_1, z_2) = a_2 w_{22} z_2^2 + \left[a_2 w_{21} z_1 - \lambda_2 \right] z_2 + \lambda_2.$$

As the leading country, C_2 will now attempt to maximize its payoff, either at $z_2^* = 0$ or at $z_2^* = 1$, or possibly both.

Case III: $z_2^* = 0$

Similar results are obtained to the case when C_2 becomes the leading country and it attempts to maximize its payoff. So, the indices can be swapped between 1 and 2 from the previous analysis where C_1 is the leading country, in order to obtain conditions for C_2's strategy changes as the leading country, see Section 6.7 (appendix).

Case IV: $z_2^* = 1$

It remains to be examined if C_1 does catch up with C_2, and assumes the leadership role[175]. Similar results are obtained by swapping indices 1 and 2 from the previous analysis where C_1 is the leading country. The strategy changes for C_1 and C_2, obtained in Section 6.7 (appendix), show that C_1 can also catch up with C_2 and gain back its leadership.

The main results from this section are summarized in the tables below.

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[175] It is assumed that $a_1 > a_2$, so that C_1 is the natural leader.

	$z_1^* = 0$		$z_1^* = 1$	
C_1 -strategy				
C_1 maximizes payoff if:	$a_1 w_{11} < \lambda_1$ and $z_2 < \hat{z}_2$ [176]		$a_1 w_{11} > \lambda_1$ when $0 \leq z_2 \leq 1$, but $z_2 > \hat{z}_2$ when $a_1 w_{11} < \lambda_1$	
C_1 remains leading country if:	$0 \leq z_2 < \min\{z_2^+, \hat{z}_2\}$ [177]		$a_1 w_{11} \geq \lambda_2$ if $0 \leq z_2 \leq 1$, $a_1 \geq a_2$ if $z_2 = 1$, $\lambda_2 \leq a_1 w_{11}$ and $w_{22} > 0$ when $0 \leq z_2 \leq 1$, $\lambda_2 > a_1 w_{11}$ and $w_{22} > 0$ when $z_2^- \leq z_2 \leq 1$ [178]	
C_2's optimal strategy	$z_2^* = z_2^+$ if $0 < z_2^+ < \hat{z}_2$	C_2 catches up with C_1 and becomes the leading country by playing $z_2^* > z_2^+$	$z_2^* = 0$ if $a_1 w_{11} \geq \lambda_2$, $a_1 w_{11} > \lambda_1$ and $a_1 w_{12} > a_2 - \lambda_2$	C_2 does *not* catch up with C_1, $(1,0)$ is a unique Nash equilibrium

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[176] $\hat{z}_2 = \dfrac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}.$

[177] $z_2^+ = \dfrac{1}{2 a_2 w_{22}}\left\{\lambda_2 + \sqrt{\lambda_2^2 + 4 a_2 w_{22}\left(\lambda_1 - \lambda_2\right)}\right\}.$

[178] $z_2^- = \dfrac{1}{2 a_2 w_{22}}\left\{\left(a_1 w_{12} - a_2 w_{21} + \lambda_2\right) - \sqrt{\left(a_1 w_{12} - a_2 w_{21} + \lambda_2\right)^2 + 4 a_2 w_{22}\left(a_1 w_{11} - \lambda_2\right)}\right\}.$

C_2's optimal strategy	$z_2^* = z_2^-$ if $0 < z_2^- < \hat{z}_2$ [179]	C_2 catches up with C_1 and becomes the leading country by playing $z_2^* < z_2^-$	$z_2^* = 1$ if $a_1 w_{11} > \lambda_1$ and $a_1 w_{12} < a_2 - \lambda_2$	C_2 does *not* catch up with C_1, $(1,1)$ is the unique Nash equilibrium
C_2's optimal strategy	$z_2^* = z_2^+$ or $z_2^* = z_2^-$ if $0 < z_2^- \le z_2^+ < \hat{z}_2$	C_2 catches up with C_1 and becomes the leading country by playing $z_2^* > z_2^+$ or $z_2^* < z_2^-$	$z_2^* = z_2^-$ if $a_1 w_{11} \le \lambda_1$ and $r_1(\hat{z}_2) \le 0$ [180]	C_2 catches up with C_1 and becomes the leading country by playing $z_2^* > z_2^-$
C_2's optimal strategy	$z_2^* = 0$ if $z_2^- < 0 < \hat{z}_2 \le z_2^+$ and $\lambda_1 a_2 w_{22} - \lambda_2 a_1 w_{12} < (a_1 w_{11})(a_2 w_{22})$	C_2 does *not* catch up with C_1, $\left(z_1^*, z_2^*\right) = (0,0)$ is a Nash equilibrium	$z_2^* = 1$ if $r_1(1) < r_1(\hat{z}_2)$, $a_1 w_{11} \le \lambda_1$ and $r_1(\hat{z}_2) > 0$	C_2 does *not* catch up with C_1, $(1,1)$ is the unique Nash equilibrium
C_2's optimal strategy	$z_2^* = \hat{z}_2$ if $z_2^- < 0 < \hat{z}_2 \le z_2^+$, $\lambda_1 a_2 w_{22} - \lambda_2 a_1 w_{12} > (a_1 w_{11})(a_2 w_{22})$ and $(\lambda_1 - \lambda_2) + \lambda_2 \hat{z}_2 - a_2 w_{22} \hat{z}_2^2 \ge 0$	C_2 does *not* catch up with C_1, $\left(z_1^*, z_2^*\right) = (0, \hat{z}_2)$ is a Nash equilibrium	$z_2^* = \hat{z}_2$ if $r_1(1) > r_1(\hat{z}_2) > 0$ and $a_1 w_{11} \le \lambda_1$	C_2 does *not* catch up with C_1, $(1, \hat{z}_2)$ is the unique Nash equilibrium

Table 6.1. C_1 is the leading country, $\lambda_1 > \lambda_2$.

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[179] $z_2^- = \dfrac{1}{2 a_2 w_{22}}\left\{\lambda_2 - \sqrt{\lambda_2^2 + 4 a_2 w_{22}\left(\lambda_1 - \lambda_2\right)}\right\}$.

[180] $r_1\left(z_2\right) = \left(a_1 w_{11} - \lambda_2\right) + \left[a_1 w_{12} - a_2 w_{21} + \lambda_2\right] z_2 - a_2 w_{22} z_2^2$.

C_1-strategy	$z_1^* = 0$		$z_1^* = 1$	
C_1 maximizes payoff if:	$a_1 w_{11} < \lambda_1$ and $z_2 < \hat{z}_2$ [181]		$a_1 w_{11} > \lambda_1$ when $0 \le z_2 \le 1$, but $z_2 > \hat{z}_2$ when $a_1 w_{11} < \lambda_1$	
C_1 remains leading country if:	C_1 *cannot* be the leading country by remaining in autarky[182]		$a_1 w_{11} \ge \lambda_2$ when $0 \le z_2 \le 1$, $a_1 \ge a_2$ if $z_2 = 1$, $\lambda_2 \le a_1 w_{11}$ and $w_{22} > 0$ when $0 \le z_2 \le 1$, $\lambda_2 > a_1 w_{11}$ and $w_{22} > 0$ when $z_2^- \le z_2 \le 1$ [183]	
C_2's optimal strategy	$z_2^* = 0$	C_2 becomes the leading country	$z_2^* = z_2^-$ if $\lambda_1 < a_1 w_{11} < \lambda_2$ and $a_1 w_{12} > a_2 - \lambda_2$	C_2 catches up with C_1

Table 6.2. C_1 is the leading country, $\lambda_1 < \lambda_2$.

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[181] $\hat{z}_2 = \dfrac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}$.

[182] There is a range of *near-autarky* strategies for C_2, for which it assumes the role of leading country

[183] $z_2^- = \dfrac{1}{2 a_2 w_{22}} \left\{ \left(a_1 w_{12} - a_2 w_{21} + \lambda_2 \right) - \sqrt{ \left(a_1 w_{12} - a_2 w_{21} + \lambda_2 \right)^2 + 4 a_2 w_{22} \left(a_1 w_{11} - \lambda_2 \right) } \right\}$.

C_2-strategy	$z_2^* = 0$		$z_2^* = 1$	
C_2 maximizes payoff if:	$a_2 w_{22} < \lambda_2$ and $z_1 < \hat{z}_1$ [184]		$a_2 w_{22} > \lambda_2$ when $0 \leq z_1 \leq 1$, but $z_1 > \hat{z}_1$ when $a_2 w_{22} < \lambda_2$	
C_2 remains leading country if:	C_2 *cannot* be the leading country by remaining in autarky[185]		$z_1 \neq 1$, $\lambda_1 \leq a_2 w_{22}$, $w_{11} \geq 0$, $0 \leq z_1 \leq z_1^+$ [186] and $\hat{z}_1 < z_1^+$	
C_1's optimal strategy	$z_1^* = 0$	C_1 becomes the leading country	$z_1^* = z_1^+$ if $a_2 w_{22} > \lambda_2$ ($0 \leq z_1 \leq z_1^+$ is compatible with $z_2^* = 1$ being C_2's unique best reply)	C_1 catches up with C_2 and becomes the leading country by playing $z_1^* > z_1^+$

Table 6.3. C_2 is the leading country, $\lambda_1 > \lambda_2$.

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[184] $\hat{z}_1 = \dfrac{\lambda_2 - a_2 w_{22}}{a_2 w_{21}}$.

[185] There is a range of "near-autarky" strategies for C_1, for which it assumes the role of leading country

[186] $z_1^+ = \dfrac{1}{2 a_1 w_{11}} \left\{ \left(a_2 w_{21} - a_1 w_{12} + \lambda_1 \right) + \sqrt{\left(a_2 w_{21} - a_1 w_{12} + \lambda_1 \right)^2 + 4 a_1 w_{11} \left(a_2 w_{22} - \lambda_1 \right)} \right\}$.

C_2-strategy	$z_2^* = 0$		$z_2^* = 1$	
C_2 maximizes payoff if:	$a_2 w_{22} < \lambda_2$ and $z_1 < \hat{z}_1$ [187]		$a_2 w_{22} > \lambda_2$ when $0 \leq z_1 \leq 1$, but $z_1 > \hat{z}_1$ when $a_2 w_{22} < \lambda_2$	
C_2 remains leading country if:	$0 \leq z_1 < \min\{z_1^+, \hat{z}_1\}$ [188]		$z_1 \neq 1$, $\lambda_1 \leq a_2 w_{22}$, $w_{11} \geq 0$, $0 \leq z_1 \leq z_1^+$ [189] and $\hat{z}_1 < z_1^+$	
C_1's optimal strategy	$z_1^* = z_1^+$ if $0 < z_1^+ < \hat{z}_1$	C_1 catches up with C_2 and becomes the leading country by playing $z_1^* > z_1^+$	$z_1^* = z_1^+$ if $a_2 w_{22} \leq \lambda_2$ ($\hat{z}_1 < z_1 \leq z_1^+$ is compatible with $z_2^* = 1$ being C_2's unique best reply)	C_1 catches up with C_2 and becomes the leading country by playing $z_1^* > z_1^+$

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[187] $\hat{z}_1 = \dfrac{\lambda_2 - a_2 w_{22}}{a_2 w_{21}}$.

[188] $z_1^+ = \dfrac{1}{2 a_1 w_{11}} \left\{ \lambda_1 + \sqrt{\lambda_1^2 + 4 a_1 w_{11} (\lambda_2 - \lambda_1)} \right\}$.

[189] $z_1^+ = \dfrac{1}{2 a_1 w_{11}} \left\{ (a_2 w_{21} - a_1 w_{12} + \lambda_1) + \sqrt{(a_2 w_{21} - a_1 w_{12} + \lambda_1)^2 + 4 a_1 w_{11} (a_2 w_{22} - \lambda_1)} \right\}$.

C_1's optimal strategy	$z_1^* = z_1^-$ if $0 < z_1^- < \hat{z}_1$ [190]	C_1 catches up with C_2 and becomes the leading country by playing $z_1^* < z_1^-$	
C_1's optimal strategy	$z_1^* = z_1^+$ or $z_1^* = z_1^-$ if $0 < z_1^- \leq z_1^+ < \hat{z}_1$	C_1 catches up with C_2 and becomes the leading country by playing $z_1^* > z_1^+$ or $z_1^* < z_1^-$	
C_1's optimal strategy	$z_1^* = 0$ if $z_1^- < 0 < \hat{z}_1 \leq z_1^+$ and	C_1 does *not* catch up with C_2, $\lambda_2 a_1 w_{11} - \lambda_1 a_2 w_{21} < (a_1 w_{11})(a_2 w_{22})$ is a Nash equilibrium	
C_1's optimal strategy	$z_1^* = \hat{z}_1$ if $z_1^- < 0 < \hat{z}_1 \leq z_1^+$, $\lambda_2 a_1 w_{11} - \lambda_1 a_2 w_{21} > (a_1 w_{11})(a_2 w_{22})$ and $(\lambda_2 - \lambda_1) + \lambda_1 \hat{z}_1 - a_1 w_{11} \hat{z}_1^2 \geq 0$	C_1 does *not* catch up with C_2, $(z_1^*, z_2^*) = (\hat{z}_1, 0)$ is a Nash equilibrium	

Table 6.4. C_2 is the leading country, $\lambda_1 < \lambda_2$.

[190] $z_1^- = \dfrac{1}{2a_1 w_{11}}\left\{\lambda_1 - \sqrt{\lambda_1^2 + 4a_1 w_{11}(\lambda_2 - \lambda_1)}\right\}$.

6.5 Example Models

6.5.1 Example 1

Consider parameter values: $a_1 = 10, a_2 = 7, \lambda_1 = 6, \lambda_2 = 3, w_{11} = 0.8, w_{22} = 0.7$.
The surfaces for $\pi_1(z)$ and $\pi_2(z)$ can be obtained as in Figure 6.17.

Figure 6.17. Example 1 - Payoff functions to countries 1 and 2. Note
that the top surface represents $\pi_1(z)$ and the bottom surface represents
$\pi_2(z)$.

In this example, country 1 has a higher payoff than country 2 for all z_1 and all z_2. It
therefore seeks to maximize its own payoff, which occurs at $z_1^* = 1$. In addition,
given this strategy is chosen by country 1 as its best strategy, country 2 seeks to
minimize the difference in the maximal payoff (which is the payoff to country 1)
and its own payoff. From Figure 6.17, this occurs at $z_2^* = 1$. So in this case, $(1,1)$ is
a δ-equilibrium for all feasible δ. Therefore, countries 1 and 2 find it advanta-
geous to trade with one another, which either maximises its own payoff (as for
country 1) or reduces the behavioural gap as discussed in the previous chapters (as
for country 2).

6.5.2 Example 2

Consider parameter values: $a_1 = 10, a_2 = 7, \lambda_1 = 3, \lambda_2 = 5, w_{11} = 0.8, w_{22} = 0.7$. The surfaces for $\pi_1(z)$ and $\pi_2(z)$ can be obtained as in Figure 6.18.
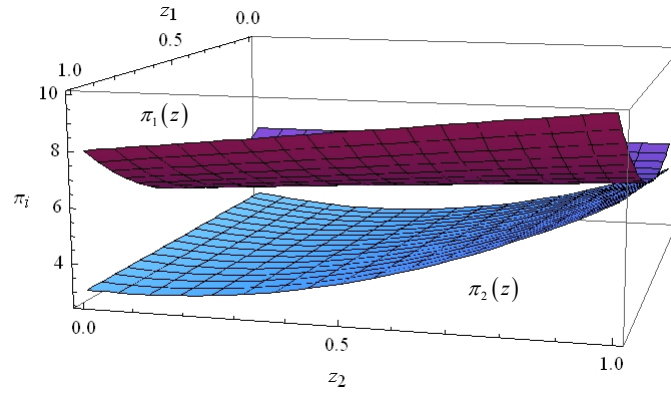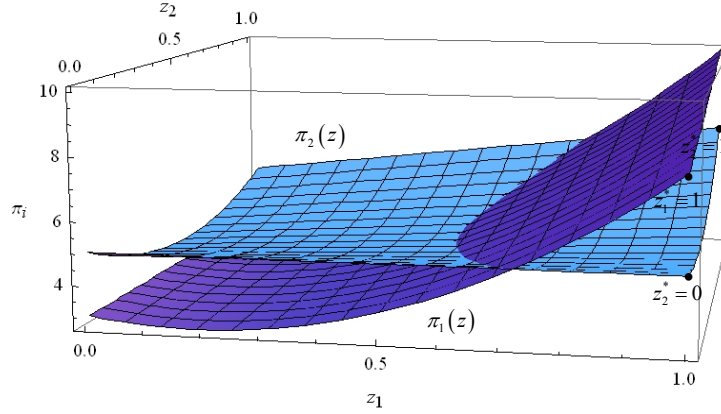

Figure 6.18. Example 2 - Payoff functions to countries 1 and 2. Note that the payoff function surface for country 1 lies below the surface for country 2 for lower values of z_1.

In this example, country 1 increases its level of integration into world trade – either to reduce the behavioural gap (as the payoff to country 2 is greater than its own payoff) for lower values of z_1, or to increase its payoff function for higher values of z_1 (for which the payoff to country 2 is less than its own payoff). Country 1 maximizes its payoff at $z_1^* = 1$. Note that $\pi_2(0,0) = \pi_2(0,1) = 3$. Hence, country 2 is indifferent between $z_2^* = 0$ and $z_2^* = 1$ and its strategy choice depends on its starting position, i.e. to the left or right, respectively, of the minimum point on its payoff curve at $z_1^* = 1$. In this case either $(1,0)$ or $(1,1)$ is a δ-equilibrium. If the dynamics for country 2 start at the minimum point of its payoff curve, then both $(1,0)$ and $(1,1)$ are δ-equilibria.

6.6 Conclusions

Conclusion 6.1. If C_1 is assumed to be a natural leader, then C_2 can catch-up with C_1 under some conditions (see Table 6.1 and Table 6.2), and subsequently C_1 can catch up and assume the role of leadership again (see Table 6.3 and Table 6.4).

One analysis of this trade game model reveals some sort of cyclic dynamics operating in the 2-country scenario, where it is easier for C_1 (being a natural leader) to catch up if it lags behind C_2, than vice versa.

The strategy size δ plays an interesting role in these dynamics as the equilibrium and strict stability conditions require constraints on the strategy size. For example, if $\delta < \dfrac{a_2 + a_2 w_{22} - a_1 w_{12} - \lambda_2}{a_2 w_{22}}$, then the fully integrated state is a strict equilibrium for the 2-country trade game with strategy size δ, see condition (5.13). This condition requires the strategy size to be small enough so that country 2 cannot obtain a better payoff (when compared with the payoff to the leading country) by decreasing its openness to cross-border trade. Country 1 already has no incentive to move away from its fully integrated strategy in this case as it yields the maximal payoff so a small strategy size can ensure a strictly stable fully integrated equilibrium.

The δ-equilibria and the δ-stable equilibria are found using the tables in the previous section. Let C_1 be the leading country playing $z_1^* = 0$. Consider the curve for $q_1(z_2)$, for $\lambda_1 > \lambda_2$ and $a_2 w_{22} > \lambda_1$, see Figure 6.19. If the z_2-coordinate of its maximum point is labelled as z_2^m, then the stability of the states $(0,0)$ and $(0,1)$ depends on the starting point of the dynamics. If the trade dynamics start on the left side of the maximum point, then $(0,0)$ is a δ-stable equilibrium. However, if the trade dynamics start on the right side of the maximum point, then $(0,1)$ is a δ-stable equilibrium.

Similarly, the curves can be obtained for all other cases defined in Table 6.1, Table 6.2, Table 6.3, Table 6.4, and the δ-stable equilibria are obtained as $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$.[191]

Conclusion 6.2. $(0,0)$, $(0,1)$, $(1,0)$ and $(1,1)$ are δ-stable equilibria, whereas $(1,\hat{z}_2)$ and $(\hat{z}_1,1)$ are δ-equilibria, for a given set of parameter values.
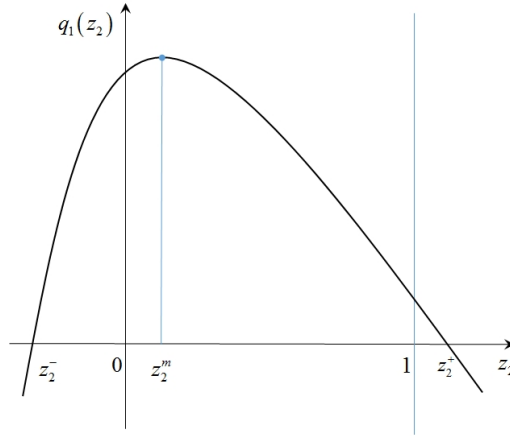
Figure 6.19. $(0,0)$ and $(0,1)$ are both δ-stable equilibria. If the trade dynamics start to the left of the point $\left(z_2^m, q\left(z_2^m\right)\right)$, then they converge to $(0,0)$; if the trade dynamics start to the right of the point $\left(z_2^m, q\left(z_2^m\right)\right)$, then they converge to $(0,1)$.

Therefore, the purely autarkic and fully integrated strategies, along with the *pure heterogeneous strategies*[192] are stable when countries are allowed to change their

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[191] Note that the states $(1,\hat{z}_2)$ and $(\hat{z}_1,1)$ are δ-equilibria, but they are not stable, as by deviating from the strategies $\hat{z}_1$ and $\hat{z}_2$, respectively, C_1 and C_2 can reduce the payoff difference between the countries.

[192] $(0,1)$ and $(1,0)$ in the 2-country model.

strategy by size δ. The analysis of *intermediate states*[193] is more complicated and can be explored as further research to this thesis.

6.7 Appendix

This section is an appendix that shows that if country 2 assumes the role of leadership and plays a purely autarkic strategy ($z_2^* = 0$) or fully integrated strategy ($z_2^* = 1$), then country 1 uses strategies similar to those used by country 2 when country 2 was the lagging country (and trying to catch up with country 1).

Case: $z_2^* = 0$

Country 2 assumes the role of the leading country and plays $z_2^* = 0$.

So C_2 is in autarky and receives payoff λ_2. This occurs if and only if $\pi_2(z_1, 0) > \pi_2(z_1, 1)$. That is, if and only if:

$$a_2 w_{22} + a_2 w_{21} z_1 < \lambda_2. \tag{6.26}$$

This is possible for non-negative z_1 only if:

$$a_2 w_{22} < \lambda_2. \tag{6.27}$$

That is, the maximum gain C_2 can extract from its own economy by opening to trade with C_1 is less than it receives from remaining in autarky.

Given (6.13), condition (6.26) holds only for:

$$z_1 < \hat{z}_1 = \frac{\lambda_2 - a_2 w_{22}}{a_2 w_{21}}. \tag{6.28}$$

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[193] $(1, \hat{z}_2)$ and $(\hat{z}_1, 1)$ in the 2-country model.

The assumptions $a_2 > \lambda_2$ and $w_{21} + w_{22} = 1$ imply that $\hat{z}_1 < 1$. Hence, this case cannot arise if z_1 is sufficiently large. That is, C_1 can force C_2 out of autarky by itself investing in open trade to a large enough extent.

An analysis similar to Section 6.4 – Case I gives the following possibilities:

(i) $0 < z_1^+ < \hat{z}_1$. In this case, C_1 can catch up with C_2 (reduce the payoff difference to 0), and any further development by C_1 (i.e. increase in z_1 above z_1^+) will lead to its assuming the role of leading country.

(ii) $0 < z_1^- < \hat{z}_1$. Again, C_1 can catch up with C_2 (reduce the payoff difference to 0), and any further development by C_1 (i.e. decrease in z_1 below z_1^-) will lead to its assuming the role of leading country.

(iii) $0 < z_1^- \leq z_1^+ < \hat{z}_1$. In this case, it is possible for C_1 to catch up with C_2 either by playing a more integrative strategy z_1^+, or a more autarkic strategy z_1^-.

(iv) $z_1^- = 0$. Then, C_1 can catch up with C_2 by itself reverting to autarky.

(v) $z_1^- < 0 < \hat{z}_1 \leq z_1^+$. In this case, $(z_1, z_2) = (\hat{z}_1, 0)$ is a Nash equilibrium.

Case: $z_2^* = 1$

In this case, C_2 is fully integrated into the world economy and receives payoff $a_2 w_{22} + a_2 w_{21} z_1$. The first contribution is from maximizing the payoff from its own economy, and the second is obtained from gains in trade with C_2. For $z_2^* = 1$ to be C_2's unique best reply to z_1, the following inequality is required to hold: $\pi_2(z_1, 1) > \pi_2(z_1, 0)$, which gives:

$$a_2 w_{22} + a_2 w_{21} z_1 > \lambda_2. \tag{6.29}$$

This always holds for $z_1 = 1$ since $w_{21} + w_{22} = 1$ and $a_2 > \lambda_2$. On the other hand, it holds for $z_1 = 0$ only if $a_2 w_{22} > \lambda_2$; i.e. C_2's maximum gain from its own economy is at least as big as its autarky payoff.

If this is not the case – i.e. if $a_2 w_{22} \le \lambda_2$ and condition (6.27) holds, then (6.29) can hold only if $z_1 > \hat{z}_1$, with $\hat{z}_1$ as in (6.28). When $z_1 = \hat{z}_1$, the inequalities (6.26) and (6.29) are replaced by an equality, and C_2 is then indifferent between autarky ($z_2^* = 0$) and full engagement ($z_2^* = 1$).

It remains to be determined whether $z_2^* = 1$ is compatible with C_2 being the leading country. To determine this, consider:

$$r_2(z_1) = \pi_2(z_1, 1) - \pi_1(z_1, 1).$$

Then, C_2 is the leading country only if $r_2(z_1) \ge 0$. Thus, the following condition is required to hold:

$$r_2(z_1) = (a_2 w_{22} - \lambda_1) + (a_2 w_{21} - a_1 w_{12} + \lambda_1) z_1 - a_1 w_{11} z_1^2 \ge 0. \tag{6.30}$$

For example, when $z_1 = 0$ this can hold only if $a_2 w_{22} \ge \lambda_1$. At the other extreme, when $z_1 = 1$, then $r_2(1) = a_2 - a_1$, and this is negative if $a_1 > a_2$; i.e. when C_1's economy is *intrinsically* bigger than C_2's economy, so that C_1 is a *natural* leader.

Consider the discriminant for $r_2(z_1)$:

$$E = (a_2 w_{21} - a_1 w_{12} + \lambda_1)^2 + 4(a_2 w_{22} - \lambda_1) a_1 w_{11}. \tag{6.31}$$

Clearly this is always non-negative if $a_2 w_{22} \ge \lambda_1$.

For E to be negative, the conditions required are: $\lambda_1 > a_2 w_{22}$ and $a_1 w_{11}$ sufficiently large. That is, the autarky payoff to C_1 must be greater than C_2's maximum gain from its own economy, while the maximum gain to C_1 from its own economy must be sufficiently large. These are an unlikely set of circumstances. However, were they to occur, then $r_2(z_1)$ has no real roots, and hence, from (6.30), $r_2(z_1) < 0$ for all z_1, which is not compatible with C_2 being the leading country.

It can therefore be assumed that $E > 0$, so that $r_2(z_1)$ has two real roots:

$$z_1^{\pm} = \frac{1}{2a_1 w_{11}} \left\{ \left(a_2 w_{21} - a_1 w_{12} + \lambda_1 \right) \pm \sqrt{\left(a_2 w_{21} - a_1 w_{12} + \lambda_1 \right)^2 + 4a_1 w_{11} \left(a_2 w_{22} - \lambda_1 \right)} \right\},$$

(6.32)

and condition (6.30) holds if and only if:

$$\max \left\{ 0, z_1^- \right\} \leq z_1 \leq \min \left\{ z_1^+, 1 \right\}.$$

(6.33)

Now, note $z_1^+ \geq 0$ if $\lambda_1 \leq a_2 w_{22}$ and $w_{11} \geq 0$. If $\lambda_1 > a_2 w_{22}$ and $w_{11} > 0$ (but $z_1^{\pm}$ are real), then z_1^+ can be negative only if $\left(a_2 w_{21} - a_1 w_{12} + \lambda_1 \right) < 0$. However:

$$a_2 w_{21} - a_1 w_{12} + \lambda_1 = a_2 \left(1 - w_{22} \right) - a_1 w_{12} + \lambda_1$$
$$= a_2 - a_1 w_{12} + \left(\lambda_1 - a_2 w_{22} \right)$$
$$\geq \left(a_2 - a_1 \right) + \left(\lambda_1 - a_2 w_{22} \right).$$

Thus, it *not* is always the case that $z_1^{\pm} \geq 0$[194]. In fact, $z_1^+ > 0$ when $\lambda_1 > a_2 w_{22}$ and $w_{11} > 0$ (but $z_1^{\pm}$ are real) only if $\lambda_1 > a_1 w_{12} - a_2 w_{21}$[195]. But, if $\lambda_1 < a_1 w_{12} - a_2 w_{21}$, then $z_1^+ < 0$ and $r_2 \left(z_1 \right) < 0$ for all z_1, so that C_2 no longer remains the leading country, whatever C_1 does.

Clearly, $z_1^- \leq 0$ if $\lambda_1 \leq a_2 w_{22}$ and $w_{11} \geq 0$[196]. However, if $\lambda_1 > a_2 w_{22}$ and $w_{11} > 0$ (but $z_1^{\pm}$ are real), then the above argument shows that $z_2^- > 0$ only if $\lambda_1 > a_1 w_{12} - a_2 w_{21}$.

It can be shown that $z_1^- \geq 1$ if $\lambda_1 > a_1 w_{12} - a_2 w_{21}$. Suppose not. Then, from the above discussion it must be true that $\lambda_1 > a_2 w_{22}$ and $w_{11} > 0$. Thus, from (6.32), $z_1^- < 1$ if and only if:

$$a_2 w_{21} - a_1 w_{12} + \lambda_1 - 2a_1 w_{11} < \sqrt{E}.$$

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[194] $z_1^+ = 0$ only if $\lambda_1 = a_1 w_{12} - a_2 w_{21}$ and either $\lambda_1 = a_2 w_{22}$ or $w_{11} = 0$.

[195] Note that when $\lambda_1 > a_2 w_{22}$, $w_{11} > 0$ and $\lambda_1 = a_1 w_{12} - a_2 w_{21}$, then $r_2 \left(z_1 \right)$ has no real roots.

[196] Equality holds if and only if $\lambda_1 = a_2 w_{22}$ or $w_{11} = 0$.

Since $w_{11} + w_{12} = 1$, the left-hand side can be written as $(a_2 w_{21} - a_1 w_{11}) - (a_1 - \lambda_1)$. If this is negative, then clearly the above inequality holds. If it is non-negative, then the above inequality holds if and only if:

$$\left[(a_2 w_{21} - a_1 w_{11}) - (a_1 - \lambda_1) \right]^2 < E.$$

From (6.31), E can be written as:

$$E = \left[(a_2 w_{21} - a_1 w_{11}) - (a_1 - \lambda_1) \right]^2 + 4(a_2 w_{22} - \lambda_1) a_1 w_{11}.$$

By expanding the $[\cdot]^2$, it can be concluded that $z_1^- < 1$ if and only if $a_2 > a_1$, which contradicts the basic assumption: $a_1 > a_2$. Thus, it must be true that $z_1^- \geq 1$ when $\lambda_1 > a_1 w_{12} - a_2 w_{21}$. In this case $r_2(z_1) < 0$ for all z_1, i.e. C_1 becomes the leading country.

Similarly it can be shown that $z_1^+ \leq 1$ if $\lambda_1 \leq a_2 w_{22}$ and $w_{11} \geq 0$. But if $\lambda_1 > a_2 w_{22}$ and $w_{11} > 0$, then $z_1^+ > 1$ only if $\lambda_1 > a_1 w_{12} - a_2 w_{21}. \lambda_1 > a_1 w_{12}$

If $a_2 w_{22} \leq \lambda_2$ and $\hat{z}_1 \geq z_1^+$, then $r_2(\hat{z}_1) < 0$ and C_2 no longer remains the leading country for $z_1 > \hat{z}_1$. Therefore, the following condition must hold: $\hat{z}_1 < z_1^+$.

It now follows from (6.33) that C_2 is the leading country provided:

$$0 \leq z_1 \leq z_1^+, \ \lambda_1 \leq a_2 w_{22}, \ w_{11} \geq 0 \text{ and } \hat{z}_1 < z_1^+. \tag{6.34}$$

In choosing an optimal reply, C_1 must minimize $r_2(z_1)$ subject to the constraint $r_2(z_1) \geq 0$.

There are two possibilities:

(i) $a_2 w_{22} > \lambda_2$. Then, the range $0 \leq z_1 \leq z_1^+$ is compatible with (6.29). Also,
        $$r_2(z_1^+) - r_2(0) = \lambda_1 - a_2 w_{22} < 0.$$

Hence, in this case C_1 catches up with C_2 by playing $z_1 = z_1^+$, and any further development by C_1 (i.e. increase in z_1 above z_1^+) will lead to its assuming the role of leading country.

(ii) $a_2 w_{22} \leq \lambda_2$. In this case, the range $\hat{z}_1 < z_1 \leq z_1^+$ is compatible with (6.29). Again C_1 can catch up with C_2 by playing $z_1 = z_1^+$.

Chapter 7

Summary and Conclusions

This chapter summarizes the main conclusions and results obtained in both versions of the trade models. The chapter is concluded with possible areas of further research.

7.1 Summary and Results

The original trade game model proposed by D'Artigues and Vignolo [33] states that when countries try to imitate the leading country (in the sense of minimizing the gap between the maximal payoff and their own payoff), the desire of convergence may paradoxically lead to a more partitioned world economy. Instead of choosing a strategy that increases its payoff so that the behavioural gap is reduced, a country may deliberately choose a strategy that degrades the situation of the leading country.

The trade game model considered in Chapter 3 defined clearly the maximal payoff obtained in each unit time and examined the complex dynamics underlying the model. The 2-country scenario verified the result obtained by D'Artigues and Vignolo [33] in the sense that the lagging country finds it more advantageous to not trade with the leading country. However, the leading country also found it advantageous to not engage in trade with the lagging country, making the all-in-autarky state (where countries do not trade with one another) a strict equilibrium. In the case of 3 countries, four out of eight states could be strict equilibria (subject to parameter constraints). But only two out of these four could coexist for a given set of

parameters at any given time period, one of which was always the all-in-autarky state. Further analysis via numerical computations in Chapter 4 revealed that a heterogeneous state where the leading country and the lagging country trade with one another while the country in the second position chooses to stay in autarky (which can be viewed as a deliberate attempt to degrade the situation of the leading country) is a more favourable strict equilibrium than a heterogeneous state where the two leading countries trade with one another while the lagging country chooses to remain in autarky. Looking at the model output for the 3-country model, it was deduced that a state where a single country chose to not trade with the others was more favourable to be a strict equilibrium over the fully integrated state (where all countries trade with one another). In the 4-country model, similar results were obtained when the heterogeneous states that could potentially be strict equilibria were considered, where the country in the second position chose to not trade with the rest of the countries, while the leading country traded with the lagging countries. In addition, these heterogeneous states were also more likely to be strict equilibria than the fully integrated state[197]. A general result for n countries was that the all-in-autarky state was always a strict equilibrium, regardless of parameter constraints. The numerical computations output for the all-in-autarky state revealed that the probability of the all-in-autarky state being a strictly stable equilibrium increased as the number of countries participating in the trade game increased, and this probability approached 1 as the number of countries reached 200. On the other hand, the probability of the fully integrated state being a strict equilibrium decreased as the number of countries participating in the trade game increased, and this probability approached 0 as the number of countries reached 200. This meant countries found it individually more advantageous to remain in autarky as opposed to trading with all or some of the other countries. This recaptured the original idea presented by D'Artigues and Vignolo [33] in the sense of negative behaviour of weaker countries by not participating in world trade so that the leading countries are not able to substantially benefit from opening up to world trade. However, since complete autarkies don't exist today, this model was extended so that countries could choose intermediate strategies as opposed to fully integrated or autarkic strategies.

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[197] Based on the results obtained for the 3-country and the 4-country models in Chapter 4.

The new model incorporated a strategy size $\delta > 0$, and introduced the concept of trade weights so that countries had different levels of openness to trade with different countries. In the 2-country model, country 1 was assumed to be the natural leader with the aim of maximising its payoff. Country 2 was assumed to be the lagging country, trying to reduce the gap between its payoff and country 1's payoff. It was shown via the analysis of the trade dynamics that it was possible for country 2 to catch up with country 1 and become the leading country. Moreover, once country 2 assumed the role of leadership, it was possible for country 1 to catch up with country 2 and overtake it to reassume its leading position.

The 2-country model with pure strategies had only one strict equilibrium (regardless of the parameter values): the all-in-autarky state, where both countries found it advantageous to not trade with one another. In this model, even though country 1 was assumed to be the natural leader of international trade (since $a_1 > a_2$), yet country 2 could become the leader with the trade dynamics in its equilibrium state, if $\lambda_1 < \lambda_2$. In the 2-country model with strategy size δ, both countries could adjust their strategies by small amounts, instead of switching between the pure strategies (full integration or complete autarky). Several equilibria were obtained over a range of parameter values. All pure states: $(z_1, z_2) = (1,1)$, $(1,0)$, $(0,1)$, $(0,0)$ were found to be strict equilibria depending on the starting point of the trade dynamics and the parameter values. In addition, two more equilibria of the form $(1, \hat{z}_2)$ and $(\hat{z}_1, 1)$ were obtained, where $\hat{z}_1 = \dfrac{\lambda_2 - a_2 w_{22}}{a_2 w_{21}}$ and $\hat{z}_2 = \dfrac{\lambda_1 - a_1 w_{11}}{a_1 w_{12}}$, given certain restrictions on the parameter values.

7.2 Further Research

In this thesis, the trade game with pure strategies has been analysed for *n* countries, in order to find strict equilibria. However, the trade game with strategy size δ has only been analysed for 2 countries. The analysis of complex conditions required for the more general *n*-country model with strategy size δ is left for further research. The non-generic cases of countries having the option of choosing between strate-

gies (by assigning probabilities to each strategy) that are best replies (under the behavioural rule that minimizes the payoff difference between the countries) could also be explored as a topic of further research.

In this thesis, δ is assumed to be a small increment to investment in international trade, such that $\delta = 1/K$, where $K \in \mathbb{N}$ and $K > 1$. The strategy size δ is also varied continuously in Chapter 6 to find δ-equilibria and δ-stable equilibria. Another work on future research could investigate the model in the limit $\delta \to 0$, which has not been covered in this thesis.

In this thesis, the parameters are assigned values as a primary step in the determination of the trade dynamics. With the assumption of non-degeneracy, once a player is chosen to update its strategy, the best move is determined without ambiguity. Each player has an equal chance of being the chosen one to change. A unique absorbing state is reached in the long run since the sequence of countries that makes the system reach the absorbing state will eventually occur after a long period of time. This thesis is concerned with the possible equilibria and their local stability. In future work, mutations could be added in each time step and it would be interesting to see whether the equilibrium selection worked.

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