# On the Steady-State Harmonic Performance of Subsea Power Cables Used in Offshore Power Generation Schemes

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A Thesis Submitted for the Degree of Doctor of Philosophy

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ProQuest LLC 789 East Eisenhower Parkway P.O. Box 1346 Ann Arbor, MI 48106-1346 To my grandfather and in memory of my grandmother

.....and to those who believe that education is invaluable asset which can never be taken from us

## **Statement of Originality**

I, Chang-Hsin CHIEN, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis.

Chang-Hsin CHIEN University College London 20 June 2007

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### Abstract

This thesis reports upon investigations undertaken into the electrical performance of high power subsea transmission cables and is specifically focused upon their harmonic behaviour, an understanding of which is fundamental for developing accurate computer based models to evaluate the performance of existing or new offshore generation schemes.

A comprehensive literature search has been undertaken in the areas of offshore generation, offshore power transmission schemes and harmonic performance of subsea cable systems. Subsea cable configurations, types and anatomy are presented to give an appreciation of the arrangement of subsea power cables.

Mathematical equations and computer based algorithms have been developed to model subsea transmission system behaviour where the electrical parameters derived from natural physical phenomena such as skin effects, proximity effects and mutual couplings are included. Proximity effect is examined to determine the consequences of whether it needs to be considered for each subsea cable arrangement. Bonding solutions for subsea transmission are investigated to study the effect they have on resonance frequency and harmonic response for different cable lengths.

The resulting analysis for various cable arrangements explains how geometric arrangements affect the harmonic impedance and harmonic resonance. The harmonic distortion in HVAC offshore transmission systems is also studied to demonstrate the importance of considering all power components in a subsea power transmission system for harmonic evaluation. In addition, the harmonic distortions of the VSC-HVDC link and associated harmonic power loss are examined. The effects of switching frequency, smoothing capacitor bank size, cable materials and transmission method on harmonic performances of the VSC-HVDC system with varying cable lengths is discussed and therefore subsea power cable harmonic behaviour interacted with subsea transmission systems is investigated.

The novel contribution of this work is claimed to be in the development of superior models of subsea cables, transmission schemes and associated performance studies, which should lead to significant improvements over existing models and their results.

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## Nomenclatures

α	= attenuation constant	-
α'	= phase angle of measured longitudinal direction permeability $ \mu_l $	radian
β	= phase constant	-
β	= phase angle of $\mu_r$	radian
δ	= laying angle	radian
$\delta_{i,j}$	= the distance between cable i and cable j	m
$\Delta x$	= incremental section in a transmission line	m
$\boldsymbol{\varepsilon}_{0}$	= permittivity of free space	F/m
ε	= closed-loop feedback of power error signal	-
E <sub>ri</sub>	= permittivity of the insulation between conductor and sheath	F/m
E <sub>r2</sub>	= permittivity of the insulation between sheath and armour	F/m
E <sub>r</sub> 3	= permittivity of the insulation of jacket outside armour	F/m
E,	= closed-loop feedback of voltage error signal	-
γ	= propagation constant	-
γ.	= the constant 1.781	-
Г	= propagation constant matrix	-
κ	= earth return coefficient	-
λ	= wavelength at fundamental frequency	m
μ	= permeability of conducting layer	H/m
$\mu_{0}$	= permeability of free space	H/m

$\mu_a$	= relative permeability of armour	-
$\mu_{inst}$	= permeability of insulation	H/m
$\mu_l$	= permeability in longitudinal direction	H/m
$\mu_{sea}$	= permeability of the sea	H/m
$\mu_t$	= permeability in transversal direction or perpendicular direction	H/m
μ,	= permeability of conducting layer	H/m
θ	= angle between cables in respect to armour centre	radian
ρ	= resistivity of the conducting layer	Ω/m
$ ho_a$	= resistivity of the armour	Ω/m
$ ho_{sea}$	= resistivity of the sea	Ω/m
σ	= complex propagation constant in the conducting layer	-
$\sigma_{a}$	= complex propagation constant in armour	-
$\sigma_{c}$	= conductivity of the conductor	S/m
$\sigma_{\epsilon}$	= earth conductivity	S/m
$\sigma_{_{sea}}$	= complex propagation constant in the sea	-
ω	= angular velocity	m/s
$\omega_{0}$	= angular velocity at fundamental frequency	m/s
$\omega_{h}$	= harmonic angular velocity	m/s
A <sub>0</sub>	= constant of the solutions dependent on boundary condition	-
A <sub>n</sub>	= constant of the solutions dependent on boundary condition	-
Α	= magnetic vector potential	-
<b>B</b> <sub>0</sub>	= constant of the solutions dependent on boundary condition	-
B <sub>n</sub>	= constant of the solutions dependent on boundary condition	-
BW	= bandwidth as frequency separation between the two half-power frequencies	Hz
$C_1$	= constant of the solutions dependent on boundary condition	-
$C_2$	= constant of the solutions dependent on boundary condition	-
С	= capacitance	F
d	= wire diameter	m
<i>d</i> <sub>c</sub>	= distance between individual cable to the amour centre	m
d <sub>II</sub>	= distance between two conductors	m
D	= average diameter of the conducing layer	m
D <sub>ai</sub>	= internal diameter of the armour	m

D <sub>av</sub>	= external diameter of the armour	m
D <sub>cable</sub> out	= outside overall diameter of cable	m
D,	= external diameter of the conducting layer	m
D,	= internal diameter of the conducting layer	m
<i>D</i> <sub>11</sub>	= distance between conductor and the imagines of the other conductor	m
$D_{H_p}$	= distance between conductor and the image of the other conductor	
	under the earth taking account of complex depth	m
f(t)	= periodic function in complex Fourier series	-
$f_{0}$	= fundamental frequency	Hz
f <sub>h</sub>	= harmonic frequency	Hz
f,	= switching frequency of carrier signal	Hz
f <sub>res</sub>	= resonant frequency	Hz
$f_{r}$	= switching frequency of modulation signal	Hz
f <sub>Hres</sub>	= upper half-power frequency for determining bandwidth	Hz
$f_{l.res}$	= lower half-power frequency for determining bandwidth	Hz
<b>F</b> <sub>o</sub>	= DC term value in a harmonic domain	-
F	= vector with harmonic content for each order	-
h	= harmonic order	-
h <sub>resonance</sub>	= harmonic order where resonance occur	-
h <sub>sea</sub>	= depth from sea level at which the cable is laid	m
I	= current	Α
/0( <i>x</i> )	= modified zero order Bessel functions with a complex argument	-
l(x)	= modified first order Bessel functions with a complex argument	-
<i>I</i> 1	= current per unit length of loop 1	A/m
<i>I</i> <sub>2</sub>	= current per unit length of loop 2	A/m
Ι,	= current per unit length of loop 3	A/m
I armour	= current per unit length in armour	A/m
I core	= current per unit length in core	A/m
$I_n(x)$	= first kind modified Bessel's function, order n	-
$I_{n-1}(x)$	= first kind modified Bessel's function, order n-1	-
1 <sub>sheath</sub>	= current per unit length in sheath	A/m
I <sub>x</sub>	= current of transmission line at length $x$	Α

/ <sub>/X'10</sub>	= DC-term of $I_{DC1}$ harmonic vector	-
I <sub>IX:20</sub>	= DC-term of $I_{DC2}$ harmonic vector	-
I <sub>R</sub>	= current of transmission line at receiving end	Α
Is	= current of transmission line at sending end	Α
I.	= harmonic vector of current of the phase A	-
I <sub>b</sub>	= harmonic vector of current of the phase B	-
I,	= harmonic vector of current of the phase C	-
l <sub>dc</sub>	= the harmonic vectors of the DC link current	-
I <sub>DC1</sub>	= DC current harmonic vector of VSC 1	-
I DC2	= DC current harmonic vector of VSC 2	-
I <sub>R</sub>	= current of transmission line matrix at receiving end	-
I <sub>s</sub>	= current of transmission line matrix at sending end	-
JO	= Bessel function of the order zero	-
JI	= Bessel function of the order one	-
J	= current density vector	-
K0(x)	= modified zero order Kelvin functions with a complex argument	-
Kl(x)	= modified first order Kelvin functions with a complex argument	-
$K_n(x)$	= second kind modified Bessel's function, order n	-
L <sub>cc</sub>	= inductance per unit between core conductors	H/m
I,,,	= sum of depth from sea level of the cable i and cable j	m
$L_{1pe}$	= cable inductance at fundamental frequency with consideration	
	of proximity effect	H/m
L <sub>h</sub>	= harmonic inductance at each order	-
$L_G$	= inductance of AC voltage source generator	Н
L <sub>TX</sub>	= inductance of transformer of VSC-HVDC system	Н
L	= inductance	Н
m	= complex constant	-
m <sub>f</sub>	= frequency modulation ratio	-
n	= number of wires	-
p	= complex depth for earth return	-
<i>P</i> <sub>it</sub>	= laying pitch of the armour	-
Pmunual	= mutual potential coefficients between conductors	-

P <sub>ref</sub>	= reference power	W
P <sub>self</sub>	= self potential coefficients of conductor	-
P <sub>vsc</sub>	= active power of VSC power dispatcher	W
Р	= potential coefficient matrix of transmission line	-
Q	= quality factor	-
r <sub>ai</sub>	= inner radius of armour	m
r <sub>ao</sub>	= outer radius of armour	m
r <sub>cond</sub>	= radius of conductor	m
r <sub>inside</sub>	= inside radius of insulation layer	m
r <sub>insultain</sub>	= inside radius of the insulation between conductor and sheath	m
r <sub>insult-out</sub>	= outside radius of the insulation between conductor and sheath	m
r <sub>insul2-in</sub>	= inside radius of the insulation between sheath and armour	m
r <sub>insul2</sub> out	= outside radius of the insulation between sheath and armour	m
r <sub>insul3-in</sub>	= inside radius of the insulation of jacket outside armour	m
r <sub>insul3 out</sub>	= outside radius of the insulation of jacket outside amour	m
r <sub>ouiskk</sub>	= outside radius of insulation layer	m
r <sub>sea</sub>	= sea return path radius as the external radius of the cable	m
r,,,,	= outer radius of sheath	m
$R_{1pr}$	= resistance at fundamental frequency with consideration of proximity effect	Ω/m
R <sub>c</sub>	= resistance of core conductor including the skin effect	Ω/m
R <sub>k</sub>	= harmonic resistance at each order	Ω/m
R <sub>layer</sub>	= resistance of the conducting layer	Ω/m
R <sub>C</sub>	= DC resistance of transmission line	Ω/m
R <sub>G</sub>	= resistance of AC voltage source generator	Ω
R <sub>TX</sub>	= resistance of transformer of VSC-HVDC system	Ω
R	= resistance	Ω
S	= distance between conductors	m
<b>S</b>	= switching function matrix	-
S <sub>ab</sub>	= PWM switching function matrix	-
S <sub>be</sub>	= PWM switching function matrix	-
S <sub>ca</sub>	= PWM switching function matrix	-

s <sub>c</sub>	= switching functions for 3 by 1 VSC of three-phase transfer matrix	-
S <sub>R</sub>	= switching functions for 1 by 3 VSC of three-phase transfer matrix	-
l thickness	= thickness of the conducting layer	m
V	= voltage	V
<i>V</i> <sub>1</sub>	= voltage per unit length of loop 1	V/m
V <sub>2</sub>	= voltage per unit length of loop 2	V/m
V <sub>3</sub>	= voltage per unit length of loop 3	V/m
V <sub>armour</sub>	= voltage per unit length in armour	V/m
V <sub>core</sub>	= voltage per unit length in core	V/m
V <sub>ref</sub>	= reference voltage	v
V sheath	= voltage per unit length in sheath	V/m
V <sub>svclX</sub> .	= DC voltage of voltage regulator SVC	v
V <sub>x</sub>	= voltage of transmission line at length $x$	v
V <sub>R</sub>	= voltage of transmission line at receiving end	v
V <sub>s</sub>	= voltage of transmission line at sending end	v
V.	= harmonic vector of voltage phase A	-
V,	= harmonic vector of voltage phase B	-
V,	= harmonic vector of voltage phase C	-
V <sub>de</sub>	= harmonic vectors of the DC link voltage	-
V <sub>R</sub>	= voltage of transmission line matrix at receiving end	-
Vs	= voltage of transmission line matrix at sending end	-
x	= transmission line with length $x$	m
X <sub>2</sub>	= calculation coefficient	-
Х,	= calculation coefficient	-
У <sub>С</sub>	= height of the conductor above the ground	m
Y	= admittance of transmission line	S
Y	= admittance per unit length of the insulation between core and sheath	S/m
Y <sub>2</sub>	= admittance per unit length of the insulation between sheath and armour	S/m
Y <sub>3</sub>	= admittance per unit length of the insulation of jacket outside armour	S/m
Ycable	= equivalent admittance of cable	S/m
Y <sub>sp</sub>	= equivalent admittance for subsea cable under the single-point	
	bonding method	S/m

Y	= self admittance of phase A	S/m
Y <sub>AB</sub>	= mutual admittance between phase A and phase B	S/m
Y <sub>AC</sub>	= mutual admittance between phase A and phase C	S/m
Y <sub>BA</sub>	= mutual admittance between phase B and phase A	S/m
Y <sub>BB</sub>	= self admittance of phase B	S/m
Y <sub>BC</sub> .	= mutual admittance between phase B and phase C	S/m
Y <sub>CA</sub>	= mutual admittance between phase C and phase A	S/m
Y <sub>CAP</sub>	= admittance of capacitor bank	S/m
Y <sub>CB</sub>	= mutual admittance between phase C and phase B	S/m
Y <sub>cc</sub>	= self admittance of phase C	S/m
Y	= admittance matrix of transmission line	-
Y <sub>cable</sub>	= cable equivalent admittance matrix	-
Ya	= equivalent cable capacitance admittance matrix under different bonding	
	methods	-
Y <sub>CAP</sub>	= harmonic vector of admittance for the capacitor bank	-
YS	= shunt admittance of the cable	S
Ζ	= impedance of transmission line	Ω
$Z_{11}$	= self impedance per unit length of loop 1	Ω/m
Z <sub>12</sub>	= mutual impedance per unit length between loop 1 and loop 2	Ω/m
Z <sub>21</sub>	= mutual impedance per unit length between loop 1 and loop 2	Ω/m
Z <sub>22</sub>	= self impedance per unit length of loop 2	Ω/m
Z <sub>23</sub>	= mutual impedance per unit length between loop 2 and loop 3	Ω/m
Z <sub>32</sub>	= mutual impedance per unit length between loop 2 and loop 3	Ω/m
Z <sub>33</sub>	= self impedance per unit length of loop 3	Ω/m
Zaa	= self impedance of armour	Ω/m
Z <sub>ac</sub>	= mutual impedance between conductor and armour	Ω/m
Z <sub>as</sub>	= mutual impedance between sheath and armour	Ω/m
Z <sub>armour-ex</sub>	= external impedance per unit length of armour	Ω/m
Z <sub>armour</sub> -in	= internal impedance per unit length of armour	Ω/m
Zarmour~mutual	= mutual impedance per unit length of armour between loop 2 and loop 3	Ω/m
Z <sub>armour</sub> / sea ~ insulation	= impedance per unit length of the insulation between armour and sea	Ω/m

Z <sub>b</sub>	= equivalent impedance for subsea cable under solid bonding method	Ω/m
Z <sub>c</sub>	= characteristic impedance	Ω/m
<i>Z</i> <sub>ca</sub>	= mutual impedance between conductor and armour	Ω/m
Z <sub>cable</sub>	= equivalent impedance of cable	Ω/m
$Z_{cc}$	= self impedance of conductor	Ω/m
Z <sub>con</sub>	= connection impedance of inter and outer armour surface for each phase	Ω/m
Z <sub>core-ex</sub>	= external impedance per unit length of conductor	Ω/m
Z <sub>core / sheath-insulation</sub>	= impedance per unit length of the insulation between core and sheath	Ω/m
Z <sub>cs</sub>	= mutual impedance between conductor and sheath	Ω/m
Z <sub>earth, I</sub>	= self cable impedance of earth return path of each phase	Ω/m
Z <sub>earth, jk</sub>	= mutual cable impedance of earth return path between each phase	Ω/m
Z <sub>eq</sub>	= equivalent impedance of cable	Ω
Z <sub>layer ex</sub>	= external impedance per unit length in a conducting layer	Ω/m
Z <sub>layer in</sub>	= internal impedance per unit length in a conducting layer	Ω/m
Z <sub>layer mutual</sub>	= mutual impedance per unit length in a conducting layer	Ω/m
Z <sub>layer/layer</sub> insulation	= impedances of the insulation between two conducting layers	Ω/m
Z <sub>pe mutu, jk</sub>	= mutual cable impedance of proximity effect between each phase	Ω/m
Z <sub>pe_self</sub> , y	= self cable impedance of proximity effect of each phase	Ω/m
Z <sub>pr self.y</sub>	= self impedance with consideration of proximity effect with respect	
	to armour	Ω/m
Z <sub>promutu, jk</sub>	= mutual impedance with consideration of proximity effect with respect	
	to armour	Ω/m
Z <sub>sa</sub>	= mutual impedance between sheath and armour	Ω/m
$Z_{sc}$	= mutual impedance between conductor and sheath	Ω/m
Z <sub>sea~in,mutual</sub>	= self impedance per unit length of sea return path	Ω/m
Z <sub>sea-in,self</sub>	= mutual impedance between cables per unit length of sea return path	Ω/m
Z <sub>sheath-ex</sub>	= external impedance per unit length of sheath	Ω/m
Z <sub>sheath-in</sub>	= internal impedance per unit length of sheath	Ω/m
Z <sub>sheath-mutual</sub>	= mutual impedance per unit length of sheath between loop 1 and loop 2	Ω/m
Z sheath / armour ~ insulation	$_{m}$ = impedance per unit length of the insulation between sheath and armour	Ω/m

Z <sub>35</sub>	= self impedance of sheath	Ω/m
Z <sub>M</sub>	= self impedance of phase A	Ω/m
Z <sub>AB</sub>	= mutual impedance between phase A and phase B	Ω/m
Z <sub>AC</sub>	= mutual impedance between phase A and phase C	Ω/m
Z <sub>BA</sub>	= mutual impedance between phase B and phase A	Ω/m
Z <sub>BB</sub>	= self impedance of phase B	Ω/m
Z <sub>BC</sub>	= mutual impedance between phase B and phase C	Ω/m
$Z_{CA}$	= mutual impedance between phase C and phase A	Ω/m
$Z_{CB}$	= mutual impedance between phase C and phase B	Ω/m
$Z_{cc}$	= self impedance of phase C	Ω/m
Z <sub>G E</sub> mutual	= mutual impedance of geometric and earth return	Ω/m
Z <sub>G-E</sub> self	= self impedance geometric and earth return	Ω/m
Z	= impedance	Ω
Z	= impedance matrix of transmission line	•
Z,	= characteristic impedance matrix	-
Za	= equivalent cable impedance matrix under different bonding methods	-
Z <sub>cable</sub>	= cable equivalent impedance matrix	-
Z <sub>con</sub>	= connection impedance matrix of inter and outer armour surface for each ph	ase
Z <sub>coan</sub>	= connection impedance matrix of inner and outer surface of armour	-
Z <sub>earth</sub>	= cable impedance matrix of earth return path	-
Z <sub>earth,AA</sub>	= self cable impedance matrix of earth return path of phase A	-
Z <sub>earth, AB</sub>	= mutual cable impedance matrix of earth return path between phase A and E	3 -
Z <sub>earth, AC</sub>	= mutual cable impedance matrix of earth return path between phase A and C	C -
Z <sub>earth, BA</sub>	= mutual cable impedance matrix of earth return path between phase B and A	۸-
Z <sub>earth,BB</sub>	= self cable impedance matrix of earth return path of phase B	-
Z <sub>earth, BC</sub>	= mutual cable impedance matrix of earth return path between phase B and C	2 -
Z <sub>earth,CA</sub>	= mutual cable impedance matrix of earth return path between phase C and A	۱-
Z <sub>earth, CB</sub>	= mutual cable impedance matrix of earth return path between phase C and E	3 -
Z <sub>earth,CC</sub>	= self cable impedance matrix of earth return path of phase C	-
Z <sub>earth, jj</sub>	= self cable impedance matrix of earth return path of each phase	-

Z <sub>earth, jk</sub>	= mutual cable impedance matrix of earth return path between each phase	-
Z <sub>i</sub>	= internal cable impedance matrix	-
Z <sub>i,A</sub>	= self internal cable impedance matrix of phase A	-
Z <sub>i,B</sub>	= self internal cable impedance matrix of phase B	-
Z <sub>i,C</sub>	= self internal cable impedance matrix of phase C	-
Z <sub>i,j</sub>	= self internal cable impedance matrix for each phase	-
Z <sub>pe</sub>	= cable impedance matrix of proximity effect	-
Z <sub>pe - mutu, AB</sub>	= mutual cable impedance matrix of proximity effect between phase A and	В-
Z <sub>pe-mutu, AC</sub>	= mutual cable impedance matrix of proximity effect between phase A and	C -
Z <sub>pe mutu, BA</sub>	= mutual cable impedance matrix of proximity effect between phase B and	A -
Z <sub>pe mutu, BC</sub>	= mutual cable impedance matrix of proximity effect between phase B and	с -
Z <sub>pe mutu,CA</sub>	= mutual cable impedance matrix of proximity effect between phase C and	A -
Z <sub>pe_mutu,CB</sub>	= mutual cable impedance matrix of proximity effect between phase C and	В-
Z <sub>pe mutu, jk</sub>	= mutual cable impedance matrix of proximity effect between each phase	-
Z <sub>pe self,AA</sub>	= self cable impedance matrix of proximity effect of phase A	-
Z <sub>pe self,BB</sub>	= self cable impedance matrix of proximity effect of phase B	-
Z <sub>pe_self,CC</sub>	= self cable impedance matrix of proximity effect of phase C	-
Z <sub>pe-self,jj</sub>	= self cable impedance matrix of proximity effect of each phase	-
Z <sub>pi</sub>	= cable impedance matrix with consideration of proximity effect with	
	respect to armour	-
Z <sub>pi-mutu, AB</sub>	= mutual cable impedance matrix with consideration of proximity effect w	ith
	respect to armour between phase A and B	-
Z <sub>pi-mutu,AC</sub>	= mutual cable impedance matrix with consideration of proximity effect w	ith
	respect to armour between phase A and C	-
Z <sub>pi-mutu, BA</sub>	= mutual cable impedance matrix with consideration of proximity effect w	ith
	respect to armour between phase A and B	-
Z <sub>pi-mutu, BC</sub>	= mutual cable impedance matrix with consideration of proximity effect w	ith
	respect to armour between phase B and C	-
Z <sub>pi - mutu.CA</sub>	= mutual cable impedance matrix with consideration of proximity effect w	ith
• ••••	respect to armour between phase A and C	-
Z <sub>pi-mutu,CB</sub>	= mutual cable impedance matrix with consideration of proximity effect w	ith

	respect to armour between phase B and C	-
Z <sub>pi-mutu, jk</sub>	= mutual cable impedance matrix with consideration of proximity effect with	
	respect to armour between each phase	-
Z <sub>pi-self,AA</sub>	= self cable impedance matrix with consideration of proximity effect with	
	respect to armour of phase A	-
Z <sub>pi-self, BB</sub>	= self cable impedance matrix with consideration of proximity effect with	
	respect to armour of phase B	-
Z <sub>pi-self,CC</sub>	= self cable impedance matrix with consideration of proximity effect with	
	respect to armour of phase C	-
Z <sub>pi self,jj</sub>	= self cable impedance matrix with consideration of proximity effect with	
	respect to armour of each phase	-
Z <sub>skin</sub>	= impedance matrix due to the magnetic fluxes inside the conductors	
	of skin effect	-
Z <sub>G-E</sub>	= impedance matrix from the magnetic fluxes outside the	
	conductors including the impedance contribution due to earth return path	-
Z <sub>TX</sub>	= harmonic vector of impedance of transformers	-
ZS	= series impedance of cable	Ω

## Abbreviations

AC	alternating current
BW	bandwidth
CSI	current source inverter
DC	direct current
FACTS	flexible AC transmission systems
FFT	fast Fourier transform
FPSO	floating, production, storage and offshore loading system
GPS	global position system
GTO	gate turn off thyristor
HD	harmonic domain
HDF	harmonic derating factor
HTS	high temperature superconducting
HVAC	high voltage alterative current transmission
HVDC	high voltage direct current transmission
ICR	integrated conductor return
IFFT	inverse fast Fourier transform
IGBT	insulated gate bipolar transistor
IGCT	integrated gate commutated thyristor
IRR	internal rate return
MathCAD	MathCAD <sup>©</sup> software package
MIND	mass impregnated non-draining
MVSC	multi-terminal voltage source converter
MVSC-HVDC	multi-terminal voltage source converter based on HVDC transmission
PF	power factor

#### Abbreviations

PSCAD	PSCAD/EMTDC <sup>©</sup> software package
РТ	pipe type cable
PWM	pulse width modulation
Q	quality factor
SCFF	self contained fluid filled
SCOF	self contained oil filled
SVC	static VAR compensator
TCR	thyristor controlled reactor
THD	total harmonic distortion
VAR	voltage-ampere reactive
VSC	voltage source converter
VSC-HVDC	voltage source converter based on HVDC transmission
VSI	voltage source inverter
XLPE	cross-linked polyethylene

### 1

### Introduction

#### 1.1 Background

Offshore power generation is currently regarded as a possible solution for tackling the coming power shortage and growing demand for clean electricity. It is reported [1] that the world will double its energy consumption by 2030 requiring somewhere in the region of 30 trillion dollars investment on energy projects over the next twenty years to meet the demand. In Europe such as Germany, Denmark, Spain and Ireland, energy feed-in tariffs currently ensure that the local distribution companies must buy renewable power fed into the local distribution grid and this is also a key driver for accelerating the development of renewable energy markets [2]. In the United Kingdom, the Energy White Paper [3] recommends that 10 % of total electrical power generated in the UK is to come from renewable sources by 2010. It is also implied that much of the increase in generation capacity will need to be met offshore using new and renewable sources such as wind farms, shown in **Figure 1.1**, wave generation and underwater current turbines, and also possibly by exploiting marginal hydrocarbon reserves using floating offshore power stations shown in **Figure 1.2**. It is widely acknowledged that offshore power generation has huge potential to provide sustainable electricity to the energy market in the UK [4].

Offshore power generation encompasses diversified fields of engineering including power plant engineering, offshore structural engineering, marine engineering, naval architecture, and electrical power transmission engineering. Most offshore power generation systems are currently located close to shore with less than 100 MW capacity such as Kentish Flats Wind Farm rated 90 MW located 10 km off the Kent coast and installed in 2005, and Barrow Offshore Wind Farm rated 90 MW and located 7.5 km offshore and installed in 2006. However, in the future new generation systems are expected to move further offshore to allow higher powers and large commercial scale systems [4]. Traditional power transmission analysis has been used for the transportation of electrical energy for near shore systems due to their comparatively low voltage levels. Nevertheless, for a longer distance and higher power offshore generation system, there is little knowledge about its lively performance, which is of major concern. The key challenge is how to design a technically feasible electrical transmission system that can transmit power economically and also provide good quality power. Offshore power generation and transmission at a large scale will require a re-evaluation of existing contemporary solutions and the development of new technologies, particularly low-loss subsea electrical transmission systems [2] [4].



Figure 1.2 Offshore Gas Generations

Research at UCL into the feasibility of offshore electrical power generation is being led by Dr. Richard Bucknall. One project that has interest from gas and oil majors is the exploitation of gas from marginal offshore fields utilising an offshore electrical power generation and subsea transmission system. Other ongoing projects at UCL are concerned with the structure of deep sea offshore wind generators and wave and current energy systems. These projects have included investigations for vessel structural design to house an offshore power station, power station design options, and the construction of economic models to assess the feasibility of various system solutions.

The concept of an offshore power generation system involves the generation of electrical power offshore using renewable energy or remote hydrocarbon resources and transportation via subsea transmission cables to shore-based landing stations where the power is fed into an existing distribution system infrastructure. Power would need to be transmitted at an appropriate voltage to minimise energy losses and transmission cable size.

In terms of high power subsea transmission systems, understanding is limited to shore-to-shore links, e.g. Sellindge to Les Mandarins (UK-France) DC inter-connector [5] and to low power down-hole induction motor feeds as used in the oil industry. However, as increased numbers of offshore wind farm projects come on stream, subsea transmission is becoming more relevant. Nevertheless, the technology for subsea transmission systems suited to offshore generation systems is still immature requiring more detailed studies [2]. This calls for a thorough examination and analysis from a technical perspective to assess the most suitable offshore power transmission systems for a range of different applications and to specifically undertake detailed performance analysis.

Considering then, suitable schemes for offshore power transmission there are two main options, AC transmission schemes operating at 50 or 60 Hz and DC link schemes. Both are technically and economically feasible but it is accepted [6] that AC transmission schemes generally offer advantages for distances of less than 50 km, in part because there is no requirement for converter stations, which need significant capital investment, at either end of the transmission system. DC links on the other hand usually provide efficiency advantages over longer distances and have significantly reduced charging currents. Hence, the selection of which of the two options is most suitable for a particular application is highly dependent upon economical evaluations which should take account of initial investment, maintenance, electrical losses and long-term electricity prices.

From a cost point of view, subsea cables usually contribute a significant part of the total cost of an offshore generation project. According to [7], a project connecting the Nantucket Shoals site wind farm to the New England mainland power system with around 48 miles (76.8 km) of cable, of which 35 miles is subsea, then the submarine cable cost, including installation, is 75 % of the total cost for the AC

transmission option, or 23 % for the DC link option, where the DC converters are the greatest cost. Therefore, as the subsea cable contributes a high proportion of the cost of any subsea transmission project, hence it needs careful assessment at the design stage.

For both HVAC and HVDC offshore transmission systems, due to the harmonic generated by the power electronics switches in the converters/inverters and also in non-linear loads, it is important to consider waveform quality. Harmonics introduce distortion into the AC power circuit whose voltage and current waveforms are expected to meet the appropriate power quality regulations. The problem is recently emphasized because of the widespread application of power electronic devices used to improve efficiency and the power factor of electrical power systems but these devices also lead to unwanted distortion. Harmonics are composed of sinusoidal waves of different frequencies, which are often integer multiples of the fundamental frequency. Generally speaking, harmonic studies are conducted to investigate the impact of non-linear devices and to analyse certain harmonic situations. In other words, they are aimed at detecting resonance and calculating distortion factors. For long distance transmission, the resonance, taking place on transmission lines, should be understood simply because of waveform quality issues and possible damage to the equipment in the power system. Hence it is important to detect the resonance conditions by modelling such schemes which effectively is achieved using distributed inductance and capacitance techniques. Additionally, for offshore power transmission systems, long length subsea cables with complex configurations including conductor, sheath and armour can also affect harmonics, which requires a more complicated means of evaluation and prediction due to the need to consider mutual coupling, skin effect, proximity effect and long-line effects.

A thorough investigation of harmonic generation in offshore power transmission systems is therefore needed together with the harmonic behaviour estimation of subsea power transmission cables, which are all fundamental issues in the study of electrical waveform quality and transmission efficiency. This thesis has focused on establishing and verifying an analytical methodology for predicting cable resonances and quantifying harmonic distortion in such systems. This research has also provided detailed investigations to improve knowledge of offshore power transmission systems.

#### 1.2 Aims and Research Objectives

The aims and objectives can be specified and summarised as follows:

- 1 Undertake an investigation of current practices and immediate future design trends for subsea cables, connectors and inter-connectors of electrical power devices in offshore power transmission systems by an extensive literature search.
- 2 Study and modify existing applied electrical theories so they are suitable for application to subsea power cables and offshore power transmission systems. Then to develop subsea cable mathematical models which are able to comprehensively evaluate cable harmonic behaviour in subsea transmission systems. The models need to consider the effect of complicated multi-layers within subsea cables in the impedance calculations, which have generally been simplified in previous studies.
- 3 Verify the cable model using available tools such as existing data obtained from practical systems and computer simulation packages. The validated models can then be further developed for system harmonic analysis with greater confidence.
- 4 Carry out a detailed trend analysis on the influence of materials used in subsea cables on the harmonic distortion by considering a variety of scenarios including different materials for insulation and armour, length, bonding situations, and the arrangement of cables on the seabed.
- 5 Examine the harmonics and associated power losses in subsea cables in transmission systems along the cable length. The total harmonic distortion (THD) factor and power losses of a transmission system along the length of offshore subsea cables are investigated and the effects from cable itself are evaluated.
- 6 Study the impact of the transmission network components such as switching devices on harmonic behaviour in the subsea cable. The sources of harmonics in subsea transmission systems such as power converters and static voltage compensators, which are essential devices in long length transmission systems, are studied for their interaction with cable harmonics. This study will specify how power electronic power devices contribute to harmonic levels.

#### 1.3 Publications

The following publications were generated during the course of the research work.

#### **Journal Papers:**

- C. H. Chien and R. W. G. Bucknall, 'Analysis of Harmonics in Subsea Power Transmission Cables used in VSC-HVDC Transmission Systems Operating under Steady-State Conditions,' Accepted by *IEEE Transactions on Power Delivery* in December 2006 and will be published in October Issue 2007.
- C. H. Chien and R. W. G. Bucknall, 'Theoretical Aspects on the Harmonic Performance of Subsea AC Transmission Systems for Offshore Power Generation Schemes,' *IEE Proceeding*, *Generation Transmission and Distribution*, Vol. 153, No. 5, pp. 599-609, September 2006.
- 3. C. H. Chien and R. W. G. Bucknall, 'An Assessment of the Influence of Bonding on the Harmonic Performance of AC XLPE Insulated Subsea Cable,' *IMarEST Proceedings Part A: Journal of Marine Engineering and Technology*, pp. 19-26, No. A8, 2006.
- 4. C. H. Chien and R. W. G. Bucknall, 'Evaluation of the Proximity Effect upon the Impedance Characteristics of Subsea Power Transmission Cables,' WSEAS Transactions on Power Systems, Vol. 1, pp. 1141-1147, July 2006. (This paper was awarded for the Best Student Paper Award of The 6<sup>th</sup> WSEAS International Conference on Power Systems 2006 in Lisbon Portugal and published by WSEAS Journal)

#### **Conference Paper:**

 C. H. Chien, E. H. Yap, H. G. A. Martindale, and R. W. G. Bucknall, "Mechanical and Electrical Performance Analysis of High Power Subsea Transmission Cable," University Power Engineering Conference, Vol. 3, pp. 1258-1261, Bristol UK, 6-8 September 2004.

#### 1.4 Contributions

The main contributions of this research work are summarised as follows:

#### 1. Review of Harmonics in Offshore Power Transmission Systems

A comprehensive literature review in offshore power systems has been conducted to understand the current status and future technical challenges for offshore power generation. Harmonic distortion in subsea power transmission cables has been identified as being an important research subject.

#### 2. Development of Models to Determine Subsea Cable Harmonic Performances

The governing principles of pipe-like cylindrical geometric cable were studied and applied to calculate the electrical properties of cables and their concentric layers i.e. conductor, sheath and armour. A harmonic model of long subsea cable has been developed to appreciate harmonic resistance, inductance and resonance behaviours in the transmission system and also to identify the resonant frequencies. The harmonic domain transform matrix method was adopted to predict the harmonic resonance of a cable where nodal analysis was applied for system harmonic calculations.

#### 3. Skin Effect, Mutual Coupling and Proximity Effect

The significance of skin effect, mutual coupling and proximity effect for subsea cable parameter calculations are presented using cable models. A comparison of system results considering and not considering of these effects is demonstrated.

#### 4. Calculation of Cable Parameters under Different Bonding Conditions

A model for calculating the frequency dependent cable parameters has been established. The performance of resonances under different bonding conditions and lengths for subsea cable is presented.

#### 5. Modelling of Harmonic Distortion in HVAC using Deifferent Cables

The developed harmonic model of subsea cable has been used for a variety of AC subsea cables for comparison of their harmonic behaviour. Models of HVAC systems with a SVC, which was the harmonic source, have been studied and the distortion levels compared.

#### 6. Harmonic Performance in Offshore VSC-HVDC Link Schemes

The models for VSC-HVDC transmission using PWM techniques were developed for subsea cable harmonic studies. The harmonic behaviour interaction between converters and cables has been demonstrated and is discussed for different switching functions, capacitor bank sizes, cable materials and transmission schemes.

#### 1.5 Outline of Thesis

The thesis is divided into 9 chapters:

**Chapter 1** This chapter gives a background introduction to the research. It reviews the current trend and developments of offshore power transmission. The research objectives, author's publications and contributions are presented.

**Chapter 2** In this chapter, a comprehensive literature review of harmonics in offshore transmission systems is conducted where subsea cables and associated harmonic issues have been given attention and important areas highlighted for further research.

**Chapter 3** This chapter describes the design and construction of subsea cables for offshore power transmission with the state of the art technology.

**Chapter 4** This chapter presents a calculation method for the prediction of harmonic resonances in a transmission line and the electrical parameters in multi-layered subsea cable models for harmonic resistance and inductance. The validation process of the model is given at the end of this chapter.

**Chapter 5** This chapter explains the skin effect and proximity effect and discusses whether or not to include the proximity effect for different geometries of subsea cables in their mathematical models.

**Chapter 6** In this chapter, a model is developed to deal with different bonding conditions for offshore cable harmonic calculations. The resonant frequency under these bonding conditions is compared.

**Chapter 7** This chapter presents harmonic behaviours for HVAC transmission with different geometrical arrangements. A harmonic model for assessing the harmonic distortion of HVAC transmission with a SVC is also demonstrated and the results discussed.

**Chapter 8** This chapter presents models for a VSC-HVDC transmission system using a subsea cable. The results of harmonic performance using distinct switching frequencies, capacitor bank sizes, cable materials and transmission methods are presented.

**Chapter 9** General conclusions of the research work carried out in this thesis with suggestions for outlines future research work.
## 2

## **Literature Review**

#### 2.1 Introduction

To understand offshore transmission systems and to identify the issues which need to be taken forward for further research is intricate work because the tasks of assessment for any subsea transmission system for offshore power generation involves a range of professionals. A comprehensive review is needed to appreciate the offshore power transmission field in general and then a further survey to identify likely problems due to harmonics in the subsea transmission system. The following literature review starts with a brief history of offshore power generation before going into a critical review of previous work in the development of offshore power transmission systems. Finally, an investigation into the harmonic issues associated with subsea transmission schemes was undertaken to provide a firm background to develop further study.

#### 2.2 A Brief History of Offshore Power Generation

In recent years, interest in offshore power generation has developed rapidly in the UK and in other European countries due to the global trend demanding cleaner energy. New and renewable energy sources from the oceans cover in service and developing technologies including offshore wind, underwater current, wave energy, and gas to wire using remote offshore fields with the CO<sub>2</sub> re-injected. A review including a brief history of offshore power generation has been conducted and is summarised as follows.

#### 2.2.1 Offshore Renewable Energy

Offshore wind energy is the most common source of renewable energy and has grown considerably over the past two decades. In the United States, Heronemus and Manwell conducted a study for offshore wind energy development in Lake Ontario in 1981, comprising of an assessment of the wind resource, design of a wind farm, and its economic evaluation. Over the past 20 years, the US Federal Government has mainly focused on on-land based wind turbines, but it has also shown some interest in offshore wind energy, in particular in regions such as New England [8]. According to [9], Denmark is the offshore wind farm pioneer constructing the first offshore wind farm off the Port of Vineby in 1991. A further wind farm was built by the Danish in 1995 at Tunoe Knob. Denmark continues to be a leading player in developing offshore wind farms. A commercial scale offshore wind farm is currently operational in Middelgrunden between Denmark and Sweden. Additionally in 2002, a large scale offshore wind farm at Horns Reef came into service providing a total capacity of 150 MW. There are several projects coming into service in the near term, which are expected to add to the offshore wind power, to supply 8 % in total of Danish electrical demand by 2008 [9]. In northern Europe, because of the potential of higher wind speeds suitable offshore sites and good seabed condition, offshore wind power generation is believed to be a practical approach for providing clean energy. However, Germany, one of the most enthusiastic countries in developing wind energy, having run out of suitable sites for wind power projects on land [2], has dramatically expanded the wind power capacity offshore in recent years and is currently the leading country in offshore power capacity. According to the statistics in [10] and shown in Figure 2.1 and taking account of the planned installed capacity of offshore wind farms to 2008, Germany will have 2001 MW capacity representing 30 % of the global offshore power generating capacity which totals 6612 MW.



Figure 2.1 Forecast Global Capacity of Offshore Energy by Country up to 2008 (MW) [10]



Figure 2.2 Potential Offshore Wind Energy Resources by Country in Europe Union (%) [4]

Location	Capacity (MW)	No. of Turbines	Year of Installation
Blyth	4	2	2000
North Hoyle	60	30	2003
Scroby Sands	60	30	2004
Kentish flats	90	30	2005
Barrow	90	30	2006
Gunfleet Sands	-	30	2007-2008
Lynn Dowsing	-	60	2007-2008
Cromer	-	30	2007-2008
Burbo bank	-	30	2008-2009
Shell Flat	-	30	2008-2009

Table 2.1 Existing and Near Future Round 1 Wind Farm Projects in UK [3] [4]

The United Kingdom has naturally good conditions for wind power development and has a distinct advantage over other countries in offshore wind energy potential: According to [4], the potential offshore wind energy resource in the UK represents 52 % of European Union countries as shown in Figure 2.2. The UK launched the first offshore wind farm in Blyth in 2000. In 2003, the first major offshore wind farm was built in North Hoyle located 4-5 miles off the North Wales coast with a capacity of 60 MW. In 2004, another commercial offshore wind farm at Middle Scroby Sands adopting technology proven on land was completed, which is located 3 km offshore, also providing a capacity of 60 MW [11]. Two major projects with a greater capacity of 90 MW have been installed in 2005 and 2006 and located in the Kentish Flats and Barrow being 10 km and 7 km offshore, respectively. Further, there are other planned offshore wind farm projects under construction with some more expected to be completed in the near future as listed in Table 2.1 [3] [4]. According to the Energy White Paper (UK) [3] in 2003, the Department of

Trade and Industry has revealed a determination to develop technologies for offshore wind power on a large scale. With parallel development of North Sea oil and gas, the government's long term ambition is to set up a central unit not only to supply the domestic chain but also to compete for export market share [12].

Wave power used to be considered as an uneconomic energy source amongst the renewable energy community in early 1980s [12]. However, in recent years, due to technology advances, wave power has dramatically declined in cost. This progress has attracted the attention of some countries such as the United Kingdom, Canada, Australia and the United States. Some companies which have developed their own ocean wave technology have long-term cooperation with governments and are expected to be able to supply larger commercial-scale wave power in the near future [9].

Tidal current power has been used for a long time. The first site was built at La Rance, France in 1966. The main constraint of this technology is a lack of sites with a significant tidal range. Nevertheless, because of advances in technology, it is now possible to establish fence sizes to produce higher capacity of tidal power. The Severn Estuary has one of the world's highest tidal ranges and studies over the past 60 years have analysed the prospects for using this tidal range to generate electrical power [12]. Therefore, the UK government has proposed a plan at this site to look at the benefits of tidal power for future development [9].

#### 2.2.2 Offshore Gas Generation

In the past three decades, offshore oil exploitation has developed in the North Sea. However, as oil and gas production from offshore field declines, the cost of exploitation grows to unprofitable levels. As a consequence, some marginal fields are expected to be abandoned within the coming years. A solution aimed at tackling this problem has emerged. The idea is to utilise the gas in marginal fields to generate power and transmit it to an on-shore based grid or receiving stations and to the electricity market in UK.

Hill, Inozu, Wang, and Bergeron [13] in 2002 introduce Floating, Production, Storage and Offshore Loading (FPSO) system on a vessel which is used to store produced oil and export it by shuttle tankers. An innovative technology for the disposal of natural gas from remote deepwater developments is to generate electricity offshore and then transmit it to shore via subsea power cables. The paper indicates that there are both technical challenges and organisational challenges for the system of generation of power offshore and the transmission of the electricity. Technical challenges include the uncertainty of gas capacity to design the generation configuration, the electrical characteristics difference between the offshore generation and onshore power grid, the metering and transport functions of delivery of gas to the generation plant, and untried submarine cable technology for offshore generation project.

Bucknall, Martindale and the Offshore Power Research Group at UCL [14] from 2002 to 2005 have been studying a detailed investigation of the technical and economical feasibility of power generation and transmission offshore. As stated by the reports, several approaches have been summarised.

- 1. Using both combined cycle and simple cycle power plants to generate electricity, the capital return rates are highly dependent on the size of gas field and the distance from the shore.
- 2. Owing to the environmental regulations and demands of clean energy from the EU and UK governments, it is necessary to economically and technically assess  $CO_2$  sequestration which is a process to collect the  $CO_2$  generated from the power generation system and to inject it into disused wells in order to prevent the  $CO_2$  being released into the atmosphere. By survey of the efficiency and the cost of several existing and future schemes of sequestration, it points out that the cost of  $CO_2$  sequestration could render offshore power generation economically unfeasible without significant government subsidy.
- 3. Studying subsea cable designs in both AC and DC transmission systems shows that XLPE-insulated cable is favoured for AC transmission due to the lower cost. On the other hand DC transmission prefers to use the MIND paper insulated cable because its material is able to withstand high power transmission. In addition, Integrated Conductor Return (ICR) Cable is given attention due to its ability of carrying return current from outside layers under DC transmission.
- 4. Investigation of High Temperature Superconducting (HTS) cable technology has revealed that from a technical and economic point of view, it remains unsuitable for applications to offshore power systems and its future for subsea transmission is uncertain.
- 5. A methodology has been set up to investigate cable fatigue and fatigue life assessment, in which the fatigue life in the touch down region in 500m water depth for a cable with copper core and outer diameter 0.133 m is 355 years, much longer than the expected service life of 30 years.
- 6. The feasibility of offshore power generation including purchase costs, operating characteristics of power plant and devices, and cable performance and cost under AC or DC conditions is assessed by an economic model, in which achievable Internal Rate of Return (IRR) shows the maximisation of the return for a gas field selection.

Overall, it is concluded that offshore power generation for transmission to shore is feasible, although there are still concerns about uncertainties based upon present day technologies [15].

#### 2.3 A Review of AC and DC Offshore Power Link

An offshore power generation needs a reliable transmission system to bring the electricity back to the receiving stations on land thereby feeding the national grid to provide power to the market. Subsea power cables link the offshore power generation to an onshore grid or receiving station. The choice of the power cable link is generally complicated, depending upon varied requirements. To completely assess the feasibility of power link solutions, it is necessary to design a practical offshore transmission scheme. A literature survey of offshore power links has been conducted to review the availability and characteristics of power links for a diversity of offshore transmission systems.

Hauge, Normann Johnsen, Holte, and Bjorlow-Larson [16] in 1988 test the performance of 250 kV HVDC mass impregnated non-draining (MIND) paper insulated submarine cables deployed between Denmark and Norway operating since 1976. The examination of experimental data shows that despite the mechanical damage occurring due to fish trawling and repair in severe weather, these cables met all requirements electrically and mechanically. Additionally, the insulation migration which had been previously believed could take place and would have significant effect on cables, is proven to be less influential. This investigation indicates that these HVDC cables are not only reliable and economical but could also be designed for transmission applications higher than 300 kV, which was regarded as the limit for subsea cables at that time.

Hammons, Olsen, and Gudnundsson [17] in 1989 address the feasibility of a submarine power link from Iceland to supply hydro power to the UK. In terms of availability of a subsea cable, submarine environment, installation of cables and cable repair and the DC link technology, this paper suggests that there is no major obstacle for establishing this link. Moreover, Guthnason, Henje, Shepherd, and Valenza [18] in 1993, specified the possible type of submarine cables applied for this link and evaluated the present and future technology that could be adopted. They conclude that submarine cable technology was capable of supporting this unprecedented project.

Valenza and Cipollini [19] in 1995 review the developments of the technologies of HVDC subsea power cable systems. The paper clearly points out the advantages and constraint when comparing HVDC and HVAC cables. For HVAC cables, length is limited around 40 to 50 km and no more than 60 km because of significant additional losses and the need for network synchronisation. On the other hand, for HVDC, there is no length limit, no additional losses, and no requirement for synchronisation but converters and inverters are required, which are unnecessary in HVAC schemes. As a result, HVDC is considered as state of the art according to this study. There are also descriptions in detail of two case studies, Spain-Morocco and Italy-Greece when using state of the art and future technology to allow submarine cable upgrading.

This paper points out that the trend for future developments in HVDC cables is towards the improvement in conductor size and voltage ratings to increase power capacity.

Terashima and Suzuki [20] in 1998 tested DC cross-linked polyethylene (XLPE) cables which are normally used for AC transmission. Because of the advantages of dry insulation, without fluid-feeding equipment, and lower cost, XLPE cable is ideal for long distance, large capacity DC transmission. The tests and measurements of the performance of XLPE DC cables demonstrate that this cable can be used in the 250 kV DC class. Further developments of DC XLPE cables are suggested focusing on reducing insulation thickness.

Hammons, Woodford, Loughtan, Chamia, Donahoe, Povh, Bisewski and Long [21] in 2000 review the developments of HVDC transmission. The break-even distance for the cost of HVAC and HVDC in submarine transmission applications is found to be 50 km. The enhancement of cable technology has great effect upon extending the HVAC transmission distance and also increasing the voltage rating for HVDC schemes. In addition, the development of gate turn off thyristor (GTO) and insulated gate bipolar transistor (IGBT) enables the voltage source converter (VSC) converter technology to give independent control of active and reactive power in the transmission system. These developments will enable HVDC to integrate small and medium size generators to much larger power systems more economically. In addition, the HVDC is attractive to interconnect systems so as to strengthen existing AC systems to provide more power and to improve network control. The authors conclude that HVDC transmission will play a very important role for the future development of energy.

Attwood [22] in 2000 examine the enhancement of submarine power cable technology by reviewing offshore power transmission systems in operation. The report mentions that the installation of cables was advanced by underwater navigation system where dynamic position position system and Global Position System (GPS) technology allows cables to be installed more accurately. In addition, protection of submarine cables is to be stressed as an important issue owing to the fact that most damage and failures are caused by anchors and heavy fishing tools. It concludes that the environmental issues should play a more important role in terms of selecting technical solutions for future designs.

Wolff and Elberling [23] in 2000 introduce a successful installation of the Kontek HVDC link with a combination of a 106 km underground cable and a 52 km subsea cable. The high power cable was the only one in the world at that time that could provide a transmitting capacity of 600 MW with the power being transmitted between Denmark and Germany. It is essential for Denmark to be able to maintain an efficient electricity system. This HVDC link is also the first time that environmental considerations resulted in underground cable designs being selected rather than the initial design that used overhead lines. This

highlights the growing trend towards the use of environmentally friendly power systems to transmit power between nations.

Harvey, Stenseth, and Wohlmuth [24] in 2001 introduce an innovative design of HVDC power cable known as Integrated Conductor Return (ICR) cable applied in the Moyle HVDC inter-connector connecting south of Scotland and North Ireland. The main distinctive design of the cable is that the return conductor is integrated into the cable to form the return path for the current. It provides advantages such as:

- 1. Outstanding properties of transport and installation due to power and return conductors in one cable
- 2. Good mechanical properties in torsion and tensile strength
- 3. No external magnetic fields exist.

This Nexans designed ICR cable also offers a new option for offshore power transmission projects.

Kirby, Luckett, Xu and Seipmann [25] in 2002 investigate the use of HVDC transmission for offshore wind farms. The main advantages and constraint for HVDC transmission applied to offshore wind farms have been demonstrated and compared. Generally speaking, for lower power and shorter distances they are in favour of AC cable interconnections but for high power and long distances, they suggest HVDC transmission is probably the optimal solution. The paper introduces a new technology, the voltage source converter (VSC) based HVDC transmission system, might be suitable for lower power transmission over a short distance. It suggests that technology combinations such as VSC offshore and conventional HVDC onshore may produce an effective and economical transmission solution for offshore wind farms.

Ackermann [2] in 2002 points out that the current trend of large offshore wind farms located further away offshore will extend the transmission distance beyond the knowledge of existing systems. The need for a high power subsea cable will make HVDC more attractive than HVAC for offshore transmission in future developments. There are two options for HVDC: conventional HVDC and VSC-HVDC. The main advantage of conventional HVDC is that since it has been developed and applied to offshore power transmission for a long time the technology has a proven record track and can be used for long distance and high power ratings. On the other hand VSC-HVDC has only been available on the market for a short time hence it is not a mature technology. The disadvantage of conventional HVDC is that it requires large converters stations onshore and offshore. It also needs a service to commutate the converter devices. Conversely, for VSC-HVDC the converter station is half the size of conventional HVDC and there is no need of a service for a means to commute the converter devices. Further, VSC-HVDC is able to control active and reactive power independently. The report concludes that the installation and operation of offshore transmission should be kept as simple as possible.

Lu and Ooi [26] in 2003 simulate an offshore wind power transmission system that uses Multi-terminal Voltage-Source (MVSC) HVDC. The model is conducted by considering a scenario in the Baltic Sea area where there is a high potential for wind turbines 10-30 km offshore. The results demonstrate that MVSC-HVDC is an attractive solution for DC transmission for offshore wind farms, with the power being good quality meeting the standards for frequency, voltage regulation, and total harmonic distortion. In terms of technology, this work produces a sound and optimal solution for transmission of offshore wind power.

Brakelmann [27] in 2003 argues that despite the superior ability of power control and smaller offshore construction when using VSC-HVDC there is a higher investment since converters and inverters are needed that have relatively high losses typically 3 % to 5 % and these need to be taken into consideration when comparing with an HVAC transmission system, in which no converters are used. The XLPE-insulated subsea cables with lower capacitive currents will in the short term be available for 245 kV and will make HVAC more attractive. This paper demonstrates that for a long distance transmission of 100 km, the cable losses in HVDC are normally lower than in HVAC cables. However, the total loss rate of the HVDC system, including converters and inverters, rises to more than 10 % and is higher than the loss incurred in an HVAC system at 8 %.

Reidy and Watson [28] in 2005 also point out that the power loss in VSC-HVDC is the highest amongst the VSC-HVDC, HVAC and Conventional HVDC transmission methods under the same conditions of 1200 MW and 345 kV with a transmission line length of 40 km. However, during faulted system throughout simulation, VSC-HVDC demonstrate excellent recovery from both a long and short time fault and yet the HVAC can only slowly recover from a short time fault and is unable to recover from long time fault. This has added an advantage for VSC-HVDC in addition to the ability of independent power control.

Morton, Cowdroy, Hill, Halliday and Nicholson [29] in 2006 apply an economic model to evaluate the options of AC and DC transmissions for 6 GW for the UK Round Two offshore projects as proposed by the UK government. This model takes account of wind farm generation and transmission system including the onshore and offshore cables and redundant cable costs. Three transmission options are compared for the Round Two projects. They are 132 kV AC connection, 245 kV AC connection and 150 kV VSC-HVDC connection. The results indicate that generally 132 kV AC connection is the preferred option except for two projects which are both larger and further offshore. It also implies that whilst high power cable is used widely and VSC-HVDC is more available the cost will be reduced making the other two options, the 245 kV AC and the 150 kV VSC-HVDC, more economic.

#### 2.4 A Review of Harmonics on Offshore Power Transmission

It is well known that harmonics exist in power transmission systems and cause damage and distortions. Offshore power transmission systems, therefore, inevitably produce harmonics which need to be analysed by detailed models to predict resonances and distortions. A subsea cable, in particular, is a considerably complicated structure when used in an offshore transmission system, so it is essential to investigate the harmonics generated in the power system including those associated with subsea cable. The following literature review considers some of the most important studies undertaken to model harmonics in high power transmission and also highlights a number of research investigations on harmonic issues in subsea power transmission schemes.

Arrillaga and Watson have been studying harmonics in HVDC power transmission systems for a long time, contributing a number of technical papers. With co-author Arnold [30] in 1989, they point out that HVDC disturbances have significant effects on frequency dependent AC systems in dynamic HVDC simulations, notably the mutual coupling. By introducing a practical equivalent circuit of which the mutual impedances is presented, the result of dynamic simulations for an AC test system shows that there are considerable differences between a simple Thevenin equivalent and frequency dependent equivalent circuit. Further, more accurate frequency dependent models with mutual coupling representation are also illustrated in faulty conditions. They conclude that consideration of frequency dependence and mutual couplings at AC and DC transmission systems are necessary.

Sakis, Meliopoulos and Martin [31] in 1992 reveal a simple procedure for computing Ohmic losses in a cable when taking consideration of frequency dependent harmonics which will in turn increase the power losses. This paper indicates that when the magnitude of harmonic current exceeds 5 % to 10 % of the fundamental current, the harmonic losses can not be ignored and may require derating of the cable. This paper also provides the details of the calculation used for derating due to harmonics which are regarded as secondary cable losses.

Merhej and Nichols [32] in 1992 give some solutions and discussion to the problems of harmonics for the offshore industry. Based on the harmonic issue, twelve pulse converters are highly recommended to reduce the harmonics and results are presented in the paper. By utilising the filters for certain harmonic orders, the harmonics can be filtered. Nevertheless, due to the fact that the design of offshore power system is far more complicated than a series of combination of filters can cope with, system resonances should be discovered and estimated throughout the entire system. The natural frequency depends on the how many generators there are on line, how many transformers are connected to the subsea cables, and how many cables are connected to the platform etc, it is therefore very challenging to detect resonances

for the transmission system and that makes total harmonic distortion management difficult. Some interesting points have being made regarding subsea cables regarding the system harmonics:

1. Low frequency harmonics are increasing as they pass through the transmission line.

- 2. High frequency harmonics tend to be attenuated by the subsea cable.
- 3. Capacitance of the cable can lead to parallel resonances at some frequencies.

As a result, identification of parallel resonances in subsea cables is crucial in order to evaluate system harmonics. In addition, this paper also specifies the importance of a harmonic data manager to monitor and assist the operational system for observing the harmonic levels and warning when the distortions exceed set limits. It concludes that although it is too complex to make a simple rule to design a filter for all conditions in an offshore power system, it is possible to analyse harmonic levels for individual power components to understand the relationships between them, which can help by acquiring information for designing the filters.

A simulation regarding harmonic interaction between generation and transmission systems is discussed by Medina and Arrillaga [33] in 1993. The harmonic interaction between generator and transformer may affect the rest of transmission network. Hence the assumption should be that these two components are non-linear when assessing the ratings of a power transmission system. A general transmission model of a synchronous generator, which represents non-linear effects, combining with the Norton harmonic equivalent of a transformer, generating harmonics through a long transmission line into a passive load, are analysed and studied. A comparison when considering and when not considering harmonic interaction explains differences, of up to 30 percent for the third harmonic and smaller levels for fifth and seventh harmonics, are presented. This study implies that accurate models are required to predict the harmonic distortion in power transmission systems.

Mclean, Mcleay, and Sheldrake [34] in 1993 analyse the harmonic distortion on Total Oil Marine's North Alwyn platforms connecting to the new Dunbar platform via two 22.5 km subsea cables. The harmonic currents produced by the drilling converter alter the distortion levels on the drilling busbar and the impedance of the entire system, which needs to be re-calculated. The effect of subsea cable combinations, generators and step-up transformers changes the impedance and this can contribute to large resonances occurring. Some remedy for this problem has been suggested:

- 1. Good isolation of the source of distortion.
- 2. Injection of an anti-phase harmonic current.
- 3. Increasing the drilling converter pulse number.
- 4. Installation of a passive filter near to the drilling busbar.

A concept of filter design for this particular case is presented, and the paper also summarises that the frequency domain simulation is a convenient tool for identifying electrical resonances.

Hiranandani [35] in 1995 also points out the importance of taking account the presence of harmonics while calculating the ampacities and sizing of the cables. The effect of harmonic currents is the amplification of ohmic losses because of an increase in current-carrying components and an increase in conductor frequency-dependent resistance. Under steady state conditions, by calculating the harmonic derating factor (HDF), the data of how much the harmonics contribute derating for entire losses can be acquired. When the resonance frequency happens to be close to one of the harmonic frequencies, the result may be the appearance of high over-voltages which can fail the capacitive bank and breakdown the insulation of cables.

De Lima, Stephan, Pedroso and Mourente [36] in 1996 publish a paper to analyse the behaviour of an electrical submarine pump motor connected to a subsea cable where the motor works with voltage distortion as well as current distortion. According to the digital simulation which is verified by an experimental model, the motor inductances and the cable capacitances work as a harmonic filter for high frequency.

Raad, Henriksen, Raphael and Hadler-Jacobsen [37] in 1996 analyse a subsea motor drive system fed by a voltage source inverter which has the advantage of modifying the output waveform to avoid the resonances via a long subsea cable using simulation. In respects of cable calculations, the  $\pi$  equivalent circuit has to take account of natural distributions of cable parameters which are dependent on the serial impedance and shunt admittance. This paper indicates that the frequency range is crucial for operation because when significant harmonic resonances occur within range, they should be avoided to protect the motor and other electric devices in the system. Two important points regarding resonance in this transmission for both VSI (voltage source inverter) and CSI (current source inverter) have been revealed:

- 1. The peak value of voltage supplied by the VSI is a series resonance between the transformer inductance and the capacitance of the cable.
- 2. The peak resonance of current supplied by the CSI corresponds to a parallel resonance between the cable capacitance and the motor leakage inductance.

It also indicates that the load may be influential for the output of the inverter at the frequency range close to resonance.

Grotzbach and Schorner [38] in 1997 gives a comprehensive research on how current resonance is influential on current harmonics when close to the resonant frequency in a subsea transmission cable system fed by a CSI to drive a motor. The benefit of CSI is that there is no need to reduce the overvoltages caused by travelling waves along the cable. The calculation of resonance is determined by a transfer characteristic which is a series of corresponding connections presented in matrix form. The model shows that the skin effect, which is important for impedance calculation, could highly dampen the resonances in contrast to the result when skin effect is not considered. By inspecting the time domain

simulation, it clearly shows that there is a huge reduction of the current harmonics close to the resonance frequency. However, the most decisive harmonic is generally not located on the resonant frequency if the cable is longer than 20 km for the study case of six-pulse CSI. In short, the paper predicts that if a twelve-

pulse inverter is used, the limits of the CSI-fed cable transmission will be extended up to 35 km.

Castellanos and Marti [39] in 1997 introduce a new model (Z-line) for a frequency dependent transmission cable, which can accurately represent the frequency dependent harmonic loss matrix in the equation by precisely evaluating the wave propagation which has been visualised into 'inside' and 'outside' conductor parts. Propagation in the external field is ideal and travels at the speed of light. However, internal propagation, due to wave travelling in the conductors, has to take account of a time delay. By knowing the parameters such as the actual wave propagation rate and the skin effect of the line, the Z-line model can be used to simulate the losses under steady-state, transient conditions and asymmetrical configurations of the transmission line, which is not accessible for traditional frequency domain models.

Sutherland [40] in 1997 discusses different characteristics of resistance versus frequency from diverse diameters of cable conductors. The comparison is conducted by a harmonic analysis program. As a result, the paper concludes that the larger cables with larger diameter conductors have a greater increase of resistance against frequency.

Yao and Ooi [41] in 1998, point out that the larger capacitances in the submarine cables can be utilised to reduce the size of dc capacitance banks in HVDC transmission schemes. The case study is a DC subsea cable with a voltage-source gate turn off thyristor (GTO) converter stations located at both ends. The simulation shows that a cable length as short as 10 km can offer adequate capacitance to reduce the size of the capacitance bank. Another observation is that by placing a small inductance to impede the current pulses from entering the cable the current total harmonic distortion (THD) can be diminished.

Taylor [42] in 1998 describes the need for voltage regulation in subsea power applications for long motor leads. This paper indicates that the conceptual design of offshore applications should consider the voltage drop when long subsea cables are used.

Vendrusculo and Pomilio [43] in 1999 present a method to determine the cable parameters by measuring only the current at the inverter output terminals at a subsea power system consisting of an inverter, a subsea cable and an induction motor. This measurement can observe the oscillation frequency in a cable taking account of the skin effect. Hence, the cable parameter can be identified by the behaviour of the wave of the inverter output current.

Bathurst, Smith, Watson, and Arrillaga [44] in 1999 provide a solution for modelling an HVDC link within the harmonic domain using the full Newton solution. Its result convergence has been validated by studying the test cases provided by a CIGRE benchmark HVDC link model. This paper concludes that this model allows the harmonic domain to extend to mono-polar HVDC link instead of single converter busbar interactions with linear AC and DC systems.

Smith and Ran [45] in 1999 introduce the active filter which is regarded as an adjustable shunt capacitance or inductance that can move the system resonance away from the excitation source to deal with the harmonics caused by the growing electrical loads on offshore oil installations. The paper studies harmonic resonance in a system having two offshore platforms supplying non-linear controlled DC drives for drilling. By comparing the measured harmonic current at the supply end and the receiving end, the harmonic current at the receiving end was found to be amplified producing a resonance. It notes that a DC driver can not be heavy loaded as high amplitude currents would result in a system shut down. A model of a simplified harmonic circuit with a generator, a step-up transformer and one part of it including a subsea cable modelled predominately as a shunt capacitance, has been illustrated. The result shows that although skin effect could smooth the resonances, when the resonance is close to the exciting frequency, the harmonics will be significantly amplified. Further, a passive filter due to its large number of different combinations of generator and cables is not practical. Therefore an active filter is presented as a comparatively reliable device. The advantages of an active filter for offshore power installations are testified in detail in the paper and it concludes that the active filter is an excellent tool always able to attenuate the harmonic resonance.

Pomilio, de Souza, Matias, Peres and Bonatti [46] in 1999 give a report regarding the inverter switching strategy for the application of AC motors through a long cable offshore. A detailed modelling for each component of the system has been conducted comprising a distributed parameter model for the cable. One important observation is that it is essential to take account of other system components for cable resonance simulation owing to the use of the transformer which can reduce the current in the cable providing lower energy loss. This is because the critical frequency is mainly generated by the transformer inductance and cable capacitance. By obtaining the critical frequencies in the cable, the switching frequency of the inverter can be identified and a suitable waveform can be provided to the motor via the subsea cable. Furthermore, a long cable has been noticed to need special attention due to the first resonance occurring at low frequency, which makes it difficult to avoid the resonance coincidence between the inverter, motor and cable.

Montanari and Fabiani [47] in 1999 investigate the effect of harmonics on the intrinsic aging of cable insulating materials. They conclude that the consequence of the voltage distortion can be significant and

also highly dependent on the voltage peak. This paper specifies that the design of insulation of the cable has to be taken into account for the evaluation of the harmonic distortion.

Arrillaga [48] in 2000 reviewed the progress and modifications of simulations for accommodating the electronic devices in power systems. In terms of harmonic analysis, it presents the difficulty of conventional computer based programs to cope with the instability of harmonic prediction. It is therefore that Newton's iterative solution is adopted to forecast the harmonic interactions existing between non-linear power devices. The simulations in the harmonic domain are proven as an accurate solution due to presenting the coupling between frequencies of distortion. This paper emphasises that power simulation actually follows tightly with the developments of the hardware.

Bathurst, Watson and Arrillaga [49] in 2000 inspect the effect of including detailed representation of the mutual coupling effects of DC transmission lines. The governing effect of the mutual coupling is a shift in the resonant frequencies of the line impedances. This also implies that mutual coupling is a significant effect on the harmonics and must be considered at the design stage.

Carrescia, Profumo and Tataglia [50] in 2000, find out that a parallel connection of conductors can magnify or shrink the magnetic field amplitude depending on the harmonics order. They point out that the geometrical configuration and the distance between conductors influence inductance in transmission lines. The relationship of the conductor arrangements and the magnetic field and harmonic order and has been demonstrated in this paper.

Heiss, Balzer, Schmitt and Richter [51] in 2001 propose a method to avoid the power losses generated by the current of the sheath of a cable. Providing a surge arrester placed between sheath and earth at one end of the cable with grounding only one end of the cable, the power losses could be significantly reduced. The effect of the sheath current, particularly in long length transmission lines, has been demonstrated in the paper for power losses with harmonics presented.

Caramia, Carpinelli, La Vitola and Verde [52] in 2002 present a scheme to precisely optimise the economic selection of cables under non-sinusoidal conditions involving the consideration of the initial investment and cost of Joule losses, i.e. taking account of the current harmonics in the cable. This model considers the skin effect and proximity effect and the losses in the metallic sheaths and armours and their contribution to the conductor losses. This depends on the conductor cross section and computed frequency related to the harmonic order. By showing various sensitivity analyses, the losses caused by non-sinusoidal conditions increase somewhere to exceed 15 % in respect to the sinusoidal condition. Further investigation suggests that there is no solid correlation between harmonic distortion and the change of standard cross sections. In short, this paper indicates that the harmonic loss factor has to be carefully

estimated owing to the fact that a tiny change can cause rapid alteration of standard cross sections of the cable.

Shwehdi, Mantawy and Al-Bekhit [53] in 2002 investigate the harmonic problems created by adjustable speed drives in offshore oil field pumping schemes. They establish a model to simulate the harmonic distortion under different scenarios where 18-pulse drivers show the lowest level of distortion among the 6-pulse, 12-pulse and 18-pulse arrangements. Also when the subsea cable capacitance is presented, the harmonic distortion level is increased on the system when using 18-pulse driver as compared with the case without cable capacitance. Also the submarine cable capacitance could also produce multiple resonance points at all busbars and shall be taken into consideration during the harmonic analysis.

Yuan and Du [54] in 2003 give an experimental investigation into harmonic resistance and reactance of armoured cables which are widely used in underground and submarine transmission schemes. The bonding conditions of single-point bonding, solid bonding are considered as well as the cable arrangements such as flat and touching, flat and spaced and trefoil arrangements. The results show that the solid bonding method causes significant power losses when rich harmonic currents are presented but it is widely adopted for industry use for safety reasons. If safety is not an issue for concern, single-point bonding is recommended because of less power loss compared to the solid bonding method. Furthermore, the results also suggest that flat and spaced configurations are not a favourable arrangement since cable losses increase dramatically if the solid bonding method is applied.

M. Yin, G. Li, M. Zhou and Y. Liu [55] in 2005 use PSCAD/EMTDC to simulate the harmonic performance of wind farm incorporated VSC-HVDC, namely, WVSC-HVDC. The simulation demonstrates that the harmonic performance of AC grid passive network is not greatly affected by shut down of wind turbine but is largely affected by the transmission line faults. This emphasises the importance of detecting faults along the transmission system.

Papathanassiou and Papadopoulous [56] in 2006 present a case study of harmonic analysis in a power system with wind generation which is connected to a network with high voltage submarine cable lines. The harmonic sources are modelled as current injections with given magnitudes and angles for each harmonic frequency. It is noted that reactive compensators are installed at the ends of transmission cables to compensate for the high capacitance in this AC transmission system. The harmonic impedance is presented for each busbar where the resonances and corresponding frequency should be detected since the distortion would be significant if the resonant frequencies coincide with the harmonic excitation spectrum generated by wind turbine. This paper suggests that a detailed harmonic model is needed for modelling the transmission system and to evaluate the potential voltage distortions, which coincide with the system resonance frequencies.

#### 2.5 Summary

Undoubtedly offshore electrical power generation is now attracting greater attention and clearly needs to be explored. With concern over the environment, countries especially developed countries, are more likely to invest in clean and sustainable energy. Offshore renewable energies such as wind farms provide an alternative to being totally reliant on conventional energy such as fossil fuels which is considered unclean and unsustainable. On the other hand, although offshore gas generation with  $CO_2$  sequestration is not entirely 'green,' it offers an option to fully utilise the remaining gas in abandoned offshore fields which is not economically feasible to be extracted by pipeline.

For the review of the developments of AC and DC submarine power links in offshore high voltage transmission schemes, there are some key points can be summed up as follows:

- 1. The development of offshore new and renewable energy is highly dependent upon offshore power transmission links, which is an important subject that needs to be carefully studied.
- Offshore power transmission is generally categorised as being HVDC and HVAC. The applications of these two solutions are largely dependent on the transmission distances. For distances less than 50 km HVAC system are mainly employed while HVDC becomes more advantageous over longer distances.
- 3. The development of subsea cables is crucial for offshore power transmission since advanced materials and structures enable subsea cables to carry higher voltage which reduces the transmission losses and hence bringing down the cost.
- 4. Due to a number of advantages such as a smaller space needed, independent control of active and reactive power and no need of auxiliary service for active commutation techniques, the VSC-HVDC now are preferred over conventional HVDC for offshore transmission.
- 5. Considering power losses, although the transmission loss of HVAC cable is higher, the total loss of VSC-HVDC system will higher than an equivalent HVAC system where the losses of power devices and converters are included. However considering the power control aspect, VSC-HVDC provides excellent characteristics such as independent active and reactive power control, quick recovery from faults and ease of voltage regulation.

From the review of papers published on harmonic issues of high power transmission associated with subsea cables has shown this is an active area for research. Some important notes can be summarised as follows:

1. For offshore power transmission, harmonic issues need to be addressed due to increasing use of power electronic systems in transmission systems.

- 2. To develop a suitable harmonic model for transmission system, the fundamental mathematical analysis is essential to evaluate the electrical characteristics of the subsea cable and interactive behaviours between power devices in the system.
- 3. The harmonic calculation of a subsea cable is complicated because of the multi-conducting layers involved and this leads to extra losses due to circulating current in these layers.
- 4. Harmonic characteristics of subsea cables such as harmonic resistance and harmonic inductance are frequency dependent and affected by the cable geometrical arrangements.
- 5. Subsea power cables usually provide huge capacitance interactive with inductance devices such as generator, transformer and loads that contribute to establishing harmonic resonances, which may need to be avoided by any harmonic since its magnification could damage the power devices and transmission system.
- 6. The harmonic source for HVDC offshore transmission is mainly the converters. However for HVAC offshore transmission, due to a need for compensator devices, which are usually power electronic based, will generate harmonics in the transmission system.

Summarising the literature reviews and by considering the aims of the study, there are some areas that can be concluded as being possible research subjects:

- From the previous study, subsea cables which are complicated in structure have the potential to play an important part in harmonic behaviour in a transmission system but usually the analysis is simplified. Thus accurate mathematical harmonic models need to be developed that consider all electrical parameters of the subsea cable in order to study further its role in contributing to the harmonic performance of the entire system.
- 2. In order to accurately assess the electrical characteristics of subsea cables some important factors for calculations also need to be understood and evaluated such as skin effect, proximity effect and the mutual coupling between the cables.
- 3. According to review [54], subsea configurations, arrangements and bonding conditions may also influence the harmonic behaviour of subsea cables and ought to be further explored.
- 4. Subsea power links are crucial for HVAC and HVDC offshore transmission systems but yet there is no study so far to comprehensively explain how the subsea cable influences the harmonic behaviour in a transmission system and what the harmonic performance is when different types of cable are used.

## **Subsea Power Cable Structures**

#### 3.1 Introduction

Applications of subsea power cables have grown dramatically over the past four decades [57]. However, to design and then install a large and long cable in deep water has proven to be a challenging task. The length of subsea cables can range from a few hundred metres to hundreds of kilometres and generally they are heavy and awkward to manoeuvre and also difficult to observe and repair once installed at sea. In addition, subsea power cables usually consist of a number of layers in a complicated structure. Hence it is necessary to understand subsea power cable structures and their physical materials to acquire a sound understanding of their performance to aid further research and development.

#### 3.2 Subsea Power Cable Configurations

It is crucial to consider cable technology options by considering efficiency, reliability and power quality in any offshore transmission system. To choose the right subsea cable requires the understanding of practical issues and case experience. Generally speaking, AC submarine cables can be simply divided into either single or multiple core types in terms of their configuration. For AC transmission three conductors are needed to accommodate three-phase currents; three single-core cables or alternatively one three-core cable are widely applied as shown **Figure 3.1** in a trefoil arrangement. Apart from the trefoil arrangement, single-core cables can be also arranged in a flat formation where cables lay on the seabed in parallel. The significant difference of a single-core cable and a three-core cable type is how the armour layer surrounds the cable. For single-core, each individual conductor is surrounded by its own individual armour while for a three-core cable three inner conductors are surrounded by common armour. Considering laying, installation and cable costs then, three-core cables usually cost less when compared to three single-core cables. However, there is a physical limitation for three-core cables since as the load capacity increases the diameter of the conductor increases resulting in a cable with a large size and that is impossible to bend and install in a subsea application. Thus in general AC cable diameters less than 150 mm, designed for AC application up to 145 kV, can be either designated as single-core or three-core cables whilst cables designed to withstand higher voltages and carry higher capacity are almost always single-core [57]. It should be noted that efficient power transfer takes place at high voltage rather than at high current so high power capacity transmission systems are always single-core types because they can withstand higher voltages. In addition, in case of a need for redundancy and repair, single-core cables are much easier to replace and maintain.



Figure 3.1 Three Single-Core Cables and One Three-Core Cable in Trefoil Configuration

For subsea DC transmission systems, mono-polar transmission has been extensively used. Generally the cable is a single core and only one conductor is required to carry the DC current with the return current flowing through the sea bed i.e. sea path return current using electrodes located at the shoreline at both ends to transmit/receive the return current. Although this is the most economical solution because only one cable is needed, the corrosion of nearby equipments e.g. pipe-work and the effect on maritime creatures are of concern to environmentalist due to the sea path return current. Bipolar transmission, on the other hand, has two cables to accommodate both main and return currents and avoid the problems of the sea return current. The design of a bipolar transmission cable includes operating a two pole system where each pole carries the current in opposite directions using two cables. One of these carries the load current in one direction and the other cable carries the return current in the opposite direction. One of the main advantages of bipolar transmission is to be able operate if one of the cables is damaged. Of course, the cost of this transmission is considerably higher than a mono-polar transmission system since there is double the cable cost. However the metallic return cable which uses a small cable attached to the main cable and the ICR cable, which was described in chapter 2 of literature review with a conductor layer to accommodate return current integrated onto the outside of the main cable, are able to reduce cost because both methods achieve the function of carrying the main and return currents within one cable. For detailed description and figures of these two cables refer to [58].

#### 3.3 Subsea Power Cable Types

The main types of subsea cable can be categorised as being self-contained fluid-filled, mass impregnated non-draining and extruded insulation where cross-link polyethylene material is widely used, abbreviated as SCFF, MIND, and XLPE respectively. The type of cable selected for a subsea application is highly dependent upon the transmission method, the voltage level and power capacity and the surrounding environmental conditions. Thus it is of great important to understand the structure and characteristics of these cable types for further design of transmission systems.

SCFF: The fluid used to fill in this cable is usually oil hence it is also being called as SCOF cable. For a single-conductor cable, the centre of the conductor is the fluid channel but for a three-core cable, the slots between the conductors act as fluid oil ducts as shown in **Figure 3.2** where the insulations for the cables are paper impregnated with synthetic oil. The oil serves as a filler to eliminate voids in the insulation conductor expanding or contracting through loading. The pressure of fluid inside the cables is controlled by a fluid reservoir located at the joints or terminating points. The additional need of the auxiliary pumping equipments and close monitoring increases its cost over other types of cable. Further, leakage of oil is of major concern which may lead to the suspension of the entire transmission system and damage to the environment when a leak occurs. Therefore, this type of cable is not as attractive as other cable types and the use of such cables has declined worldwide since the technology development of other types of cable. Today, the application of this type of cable is limited to the transmission with extra high voltage and greater carrying capacity of current such as the Kii Channel crossing project in Japan with  $\pm$  500 kV and 2,800 MW in capacity [59].



Figure 3.2 Single-Core and Three-Core SCFF (SCOF) Cables

MIND: This type of cable has a central conductor and its insulation is usually lapped paper that is impregnated with a high viscosity non-draining compound such as micro-crystalline waxes or resins, which are liquid at the maximum operating temperatures of the cables but solidify at normal ambient temperature at 20°C. The non-draining compound provides an essential advantage because it prevents impregnated fluid draining to the lower part of the cable if the cable is laid in a seabed with high elevation difference [58]. The construction of MIND cables is similar to the SCFF cables but the difference is that MIND cables have no need for a fluid duct. The cable can be found in both AC and DC systems but mainly used for DC applications due to the high voltage withstanding capability and lower cost compared to its SCFF counterpart owing to no need for auxiliary equipments.

XLPE: Extruded insulation cables use XLPE (cross-liked polyethylene) as insulation material which has high insulation resistance, low dielectric constant. The advantages of this material are specific weight, clean, easy to joint and terminate, no problem with elevation differences, easy to repair and maintain. Furthermore, this type of cable is less expensive and easier to manufacture than SCFF and has lower capacitance than that in a mass impregnated cable, which in turn means the losses in this type of cable are lower. However, the main disadvantage of this extruded cable is that it breaks down easily under high DC currents so it is mainly used for AC transmission. Nevertheless, the progress of material and its development of technology led to improvements of this type of cable for high DC applications in the near future [20].

Based on the evaluations for application in offshore generation systems, for an AC scheme XLPE cables are considered the best option, as they have superior advantages in terms of economic consideration and electrical properties when compared to MIND and SCFF cables. For a DC link on the other hand, MIND cables are considered to be more suitable because of their ability to withstand high DC voltages and are lower cost than SCFF. However, the development of XLPE cables now makes these available for the DC transmission market and most offshore projects using VSC-HVDC are adopting XLPE cables, as usually these projects are not operating at very high voltage levels or large power capacities [60]. The details of submarine cable configurations, structure and applications for offshore transmission schemes have been carefully studied by the offshore group in UCL and described in Offshore Power Research Report [14]. This report offers important information for the understanding of fundamentals of subsea cable design and further study of offshore power transmission systems.

#### 3.4 Subsea Power Cable Anatomy

Subsea cables are made up from several component layers. Each component plays a different role in the cable and in turn their materials and characteristics need to be appreciated. Figure 3.3 shows the typical cross section of a subsea cable and in the following section the cable anatomy is explored and an explanation of each layer is given.



Figure 3.3 Cross Section of Typical Subsea Power Cable

#### 3.4.1 Conductor

The function of conductor is to carry the current. For long subsea cables, the size of conductor is usually large because it is normally required to have a great current carrying capacity. However the main limitation is the bending radius which may be large for big conductor diameter cables. Copper and aluminium are both widely used for offshore power cable conductors. Copper has lower resistance and provides greater tensile strength which leads to smaller cross section area of the conductor compared to an aluminium conductor. On the other hand, an aluminium conductor is lighter and costs less than a copper conductor. For high current and high voltage subsea transmission, it is usual to use a copper conductor as it requires a smaller cable diameter but for low current and low voltage ratings aluminium is still commonly used for economic reasons. Stranding is another characteristic of subsea cable conductors because it offers the necessary flexibility and reduces the size of conductors compared to a solid conductor. More wires and closer pitch will create a less flexible conductor [61].

#### 3.4.2 Insulation

As described in previous section, there are three types of insulation that are adopted for subsea cable structures. These are the fluid impregnated paper for SCFF cables, mass impregnated paper for MIND cables and extruded insulation for XLPE cables.

#### 3.4.3 Screening

A semi-conductive layer usually made from PE (Polyethylene) is used for screening because it reduces the electrical stress and ensures a complete bond between the two layers on either side. It is usually applied as the interface between the conductor and the insulation and between insulation and metallic sheath.

#### 3.4.4 Metallic Sheath

The metallic sheath serves as a protective layer for the insulation and conductors in case of an earth fault. It is also a safety and moisture barrier for subsea cables preventing the insulation from water ingress and other mechanical damage [58]. The significant concern associated with the sheath is that it could accommodate induced currents which lead to circulating current sheath losses. Lead is commonly used for sheath material for subsea power cable applications and has been proven to be superior over other material such as copper, aluminium and steel and has shown itself to be suitable for long-term service without any major problem [62].

#### 3.4.5 Armour

The outmost layer of a subsea cable is usually a jacket with armouring served as a mechanical protection and a water barrier for inner layers. The purpose of the armour is mainly to protect against external damage and to provide strength to the cable. In addition, like the metallic sheath, it also carries the fault currents and assists grounding of the cable. However, the same problem as experienced in the metallic sheath occurs in that the circulating induced current induced in the armour gives rise to increased losses. The armour is usually constructed by a series of metallic wires helically wound together. The common material for metallic wire is galvanised steel wires and its size and type is dependent on the size, weight and water depth of cable. Also, copper is an alternative material for the armour since it can reduce system losses due to magnetic induction and consequently increase capacity.

### 4

# Harmonic Calculation Models of Subsea Power Cable

#### 4.1 Introduction

Sinusoidal waveforms are preferred since a sine wave contains no harmonics thus minimises losses and resulting in an increased efficiency. Additionally, machine, transformer and electric appliance designs assume a sinusoidal supply thus simplifying design calculations. Nevertheless, a sinusoidal waveform is somewhat ideal and cannot be achieved in practise and will therefore always contain some harmonics. Electric utilities are always concerned about having a high power factor which has the advantages of reducing the required equipment ratings, minimising line losses and voltage drops, it is in turn accompanied by considerable use of electronic devices of voltage regulators which produce harmonic distortion of the current and voltage waveforms. Distorted waveforms caused by harmonics are always composed of sinusoidal waves at different harmonic frequencies, which are generally integer multiples of the waveform fundamental frequency [63].

The design of any subsea transmission scheme also needs to include an assessment of the expected power quality, which is in turn concerned with waveform harmonic distortion and the interaction of these harmonics with the natural resonant frequencies of the transmission system [64]. From the literature survey, it is well known that non-linear loads such as inverters and saturated magnetic cores can generate harmonic distortion, which can also be generated by the non-linear characteristics of the transmission system itself [44] [45]. For example, it is reported in [37] [46], that subsea transmission schemes using induction motors fed by long AC transmission cables experience substantial voltage and current waveform distortion because of the harmonics generated by the variable frequency drive and the response of the transmission system to these harmonics. Harmonic levels can be influenced by the non-linear characteristics of the transmission link configurations [49].

Offshore subsea power cables, unlike subterranean cables, need to be heavily armoured and are consequently complicated structures having many concentric layers of different materials as previously described in Chapter 3. Inductive couplings across each and every material boundary contribute to the overall cable impedance and these complex relationships consequently affect the level of voltage and current waveform distortion that will be experienced [65]. Additionally, according to [49], the configuration or arrangement of each cable relative to each other is another important factor that influences cable impedance.

Subsea cable designs vary considerably and a thorough investigation of harmonic performance is therefore required for each new offshore generation and transmission system. Commercial software packages such as **PSCAD/EMTDC**<sup>•</sup> and **MATLAB/SimPowerSystems**<sup>•</sup> are currently available to predict the harmonic response of electrical systems. However when considering subsea cable modelling and especially its harmonic performance, the limitations of such commercial simulation packages soon become apparent such as limited cable designs, no consideration of saturation effects for magnetic materials in the cable and restricted cable bonding conditions. Thus, it is important in subsea transmission system modelling to develop appropriate models of the subsea cables that include all these effects so as to obtain an accurate performance prediction. The commercially available packages can be usefully employed as a tool to validate any new models as these are developed, providing of course the appropriate validation conditions and model limitations are observed.

In this chapter, a fundamental understanding of a transmission line has been introduced and the evaluations of its electrical parameters for harmonic calculations are presented where the transfer matrices are applied for transmission line natural frequency prediction. Furthermore, computer based models for single-core multi-conducting layer subsea cables have been developed and investigations also have been implemented to determine its frequency response, the magnitudes of resistance and inductance, and the frequencies at which resonances occur. The importance of consideration of skin effect and mutual coupling between cables has also been demonstrated where the results with consideration and without consideration of these natural phenomena are compared. The results of electrical parameters from the developed model of the subsea cable provided by subsea cable manufacturers are compared to the hard data at fundamental frequency. A further validation for harmonic characteristics of subsea cables are carried out using PSCAD (**PSCAD/EMTDC<sup>®</sup>**) to verify the model under the same conditions, which are able to be simulated by both analytical model and PSCAD. This will allow this analytical model to be further developed with accurate results.

#### 4.2 Constraints of Software Packages

There are a number of software packages suitable for harmonic calculations of transmission cables such as **PSCAD/EMTDC**<sup>•</sup> and **MATLAB/SimPowerSystems**<sup>•</sup> which provide accurate calculations of electrical parameters of multi-layers cable. Therefore, they are commonly recognised as being powerful tools for harmonic analysis of power transmission systems. However, for specific purpose for application of subsea cable harmonic simulation, there are still some limitations which prevent the software serving as a universal tool for all conditions occurred in practice. Since PSCAD (**PSCAD/EMTDC**<sup>•</sup>) has been known for its strong ability in dealing with multi-layers cable both in transient and steady state simulations [66] [67], it is chosen as an objective for following discussion for its constraints to subsea cable calculations. The limitations include:

#### Generally, only a limited number of cable designs are available within the library of models.

For single-core subsea cable, PSCAD provides sufficient cable arrangements for simulation while there is no specific model in the library for three-core subsea cable which is constructed by three conductors with three sheath layers, each sheath shields a conductor within its insulation, and one common armour layer as the outmost conducting layer. Further the initial design for PSCAD is to assume each cable conducting layer is made by a cylindrical shape of conducting material but this is not representative of the subsea cable armour which is usually made using a series of helically wound metallic wires.

# The models do not take consideration of the magnetic effect from the materials that make up the cable e.g. steel armour.

As explained in previous chapters, steel wires are used to form the subsea cable armour. While transmitting high power current the magnetic saturation effects will become apparent and will need to be taken into account when determining the overall subsea cable harmonic performance. However, since the outer conducting layers are presumed to be formed from non-magnetic materials the cable models in PSCAD do not take account of saturation effects.

#### Limited bounding conditions, where commonly only the outermost conducting layer can be grounded, whereas in practice other conducting layers in subsea cables may also be grounded (e.g. sheath and armour).

By consulting ABB, a major manufacturer of subsea power cables, it is learnt that for safety reason subsea cable sheath and armour both should be well bonded to earth by bonding both the sheath layer and armour at the ends of the cable and then grounding these together at each end of the cable. Yet the cable parameters in PSCAD only allows the last metallic layer outside the cable to be grounded, which in turn can not meet the requirements of the boundary condition for subsea cable harmonic calculations.

#### 4.3 Equivalent Circuit of Transmission Line

Subsea power cables are regarded as a transmission line laying on the seabed for offshore transmission systems. Thus for identifying the electrical characteristics of subsea power cables, the first step is to understand the transmission line which has been considered as a connecting device providing a path for the power flow between several circuits in the system. For this reason, it is regarded as having a sending end and a receiving end with a series resistance and inductance and shunt capacitance and conductance as characteristic parameters. Hence, it is important to appreciate the electrical characteristics of a transmission line in order to analyse and understand its performance.

Considering the long transmission line, the shunt effect and line capacitance is not negligible. Consequently the representation of the transmission line as an equivalent circuit is needed to accurately calculate the parameters of the transmission line. The  $\pi$  circuit and T circuit can be employed to represent transmission lines. Since the  $\pi$  circuit is commonly used in the research [41], it is adopted for the following study. Figure 4.1 shows the  $\pi$  circuit of transmission line and the calculations of resistance, inductance and capacitance for both DC and AC transmission line is presented in details in Appendix A where the resistance of a DC transmission line is solely dependent on the temperature distribution of the conductor but in the case of an AC transmission line the influence of inductance and capacitance should also be taken into account where the electrical and magnetic field calculation for cylindrical geometries are applied to obtain their magnitudes.



Figure 4.1 Transmission Line Equivalent  $\pi$  Circuit

Due to the fact that the transmission line is long and can not be represented as a single  $\pi$  equivalent circuit, an iterative of circuits is connected and the value per unit of the parameters are used and multiplied by total length of the transmission line. Capacitance values are divided into half at the beginning and half at the end of the  $\pi$  circuit between the series connected resistances and inductances. The current and voltage in a transmission and their phase and changes under different load conditions can be estimated by using the  $\pi$  circuit network. In addition, the transmission line's electrical characteristic can be estimated and the harmonic performance of a transmission line can also be evaluated and harmonic resonances predicted.

#### 4.4 Harmonic Calculations of Overhead Transmission Line

To develop a model, it is essential to start with the basic understandings of fundamental concepts. For consideration of a subsea power cable harmonic model, the calculation notion is derived from basic harmonic modelling of overhead transmission line attributed to its extensive application in the power industry. Thus the following evaluations are based on harmonic modelling on overhead transmission line parameters. This will then allow the model to further develop for a subsea power cable harmonic model. When studying the harmonics in a transmission line, the transfer matrices of the transmission line parameters at harmonic frequencies is widely used. Several effects which are normally ignored at power frequencies such as frequency dependence, long-line effects, line imbalance and line transpositions and VAR (voltage-ampere reactive) compensation plant have to be included at harmonic calculations. The calculation of transmission line parameters are obtained from the geometric configuration of the transmission line taking into account the effect of the earth return and skin effects. The second is the evaluation of distributed parameters: Long line effects are added to the lumped parameters, to generate an exact model of the transmission line at harmonic frequencies [69].

#### 4.4.1 Evaluations of Lumped Parameters

The lumped series impedance and shunt admittance of a three-phase overhead transmission line could be described below [63] [69]:

$$\mathbf{Z} = \mathbf{Z}_{\mathbf{G}-\mathbf{E}} + \mathbf{Z}_{\mathsf{skin}} \tag{4.1}$$

$$\mathbf{Y} = j 2\pi \omega \varepsilon_0 \mathbf{P}^{-1} \tag{4.2}$$

Where, Z the impedance matrix of transmission line; Y is the admittance matrix of transmission line;  $Z_{G-E}$  is the impedance matrix from the magnetic fluxes outside the conductors [70] and unit is  $\Omega/m$ , including the impedance contribution due to earth return path;  $Z_{skin}$  is the impedance matrix due to the magnetic fluxes inside the conductors of skin effect; P is the potential coefficient matrix of transmission line;  $\varepsilon_0 = 8.854 \times 10^{-9}$  is the permittivity of free space and unit is F/m;  $\omega = \omega_0 h$  is the angular velocity, where  $\omega_0 = 2\pi f_0$  is the angular velocity at fundamental frequency;  $f_0$  is the fundamental frequency in Hz; h is the harmonic order.

The impedance and admittance matrices above are given by:

$$\mathbf{Z}_{\mathbf{G}-\mathbf{E}} = \begin{bmatrix} Z_{G-E\_self} & Z_{G-E\_mutual} & Z_{G-E\_mutual} \\ Z_{G-E\_mutual} & Z_{G-E\_self} & Z_{G-E\_mutual} \\ Z_{G-E\_mutual} & Z_{G-E\_mutual} & Z_{G-E\_self} \end{bmatrix}$$
(4.3)

$$\mathbf{Z}_{skin} = \begin{bmatrix} Z_{skin} & 0 & 0 \\ 0 & Z_{skin} & 0 \\ 0 & 0 & Z_{skin} \end{bmatrix}$$
(4.4)

$$\mathbf{P} = \begin{bmatrix} P_{self} & P_{mulual} & P_{mulual} \\ P_{mulual} & P_{self} & P_{mulual} \\ P_{mulual} & P_{mulual} & P_{self} \end{bmatrix}$$
(4.5)

The self impedance of geometric and earth return of  $Z_{G-E\_self}$  and mutual impedance  $Z_{G-E\_mutual}$  can be expressed by the following equations where the detailed calculations are developed in [71] [72].

$$Z_{G-E_{self}} = \frac{j\omega\mu_{0}}{2\pi} \ln \frac{2(y_{c} + p)}{r_{cond}}$$
(4.6)

$$Z_{G-E_{mutual}} = \frac{j\omega\mu_0}{2\pi} \ln \frac{D_{H_{o}P}}{d_{H}}$$
(4.7)

Where,  $y_c$  is the height of the conductor above the ground;  $r_{cond}$  is the radius of the conductor;  $p = \frac{1}{\sqrt{j\omega\mu_0\sigma_e}}$  is the complex depth and introduced in [71] as a tool to evaluate earth return path

impedance;  $\mu_0 = 4\pi \times 10^{-7}$  is the permeability of free space and unit is H/m;  $\sigma_e$  is the earth conductivity in S/m;  $d_{ll}$  is the distance between two conductors and the unit is  $1/\Omega m$ ;  $D_{ll_p}$  is the distance between conductor and the image of the other conductor under the earth taking account of complex depth [69] where the concept of imagining conductor underneath the ground can be found in [72] [73] [74].

Also, citation of Bessel functions for the impedance of skin effect calculation can be given as following equations and the detailed derivation can be found in [75]:

$$Z_{skin} = R_{c} \cdot \frac{\kappa \cdot r_{cond}}{2} \cdot \frac{J0(\kappa \cdot r_{cond})}{J1(\kappa \cdot r_{cond})}$$
(4.8)

$$\kappa = \sqrt{-j\omega\mu_0\sigma_c} \tag{4.9}$$

Where,  $R_c$  is the DC resistance of the transmission line in  $\Omega/m$ ;  $\kappa$  is constant coefficient;  $\sigma_c$  is the conductivity of the conductor in S/m; J0 is Bessel function of the order zero and J1 is the Bessel function of the order one.

For the potential coefficient matrix, the equations are defined as follows:

$$P_{self} = \ln(\frac{2y_{.}}{r_{cond}})$$
(4.10)

$$P_{munual} = \ln(\frac{D_{ll}}{d_{ll}}) \tag{4.11}$$

 $P_{self}$  is the self potential coefficients of conductor and  $P_{mutual}$  is the mutual potential coefficients between conductors where,  $D_{ll}$  is the distance between conductor and the imagines of the other conductor located in the earth described in [69] [74].

#### 4.4.2 Evaluations of Distributed Parameters

Because the propagation and attenuation of the voltage waves travelling along a transmission lines is dependent heavily on the length of the line and the number of phase involved, harmonic evaluation for long transmission lines has to take account of distributed parameters in order to accurately calculate the resonances in transmission line at harmonic frequencies. A representation of the transmission line in the form of ABCD matrix [76] has been introduced and provides an effective tool for modelling the harmonic behaviour in transmission line.

In a long transmission line the voltages and the currents existing in an incremental section  $\Delta x$  could be expressed as [63]:

$$\frac{\partial^2 V}{\partial x^2} = z \frac{dI}{dx} = zyV = \gamma^2 V \tag{4.12}$$

$$\frac{\partial^2 I}{\partial x^2} = y \frac{dV}{dx} = zyI = \gamma^2 I$$
(4.13)

 $\gamma$  is the propagation constant and  $\gamma = \alpha + j\beta$ 

 $\alpha$  is the attenuation constant

 $\beta$  is the phase constant

The propagation constant  $\gamma$  and the characteristic impedance  $Z_c$  are given by

$$\gamma = \alpha + j\beta = \sqrt{zy} \Longrightarrow \gamma = \sqrt{ZY}$$
(4.14)

$$Z_c = \sqrt{\frac{Z}{Y}}$$
(4.15)

Where, Z is the impedance of transmission line; Y is the admittance of transmission line. Solve V and I,

$$V_x = C_1 e^{jx} + C_2 e^{-jx}$$
(4.16)

$$I_{x} = \frac{C_{1}}{Z_{c}} e^{x} - \frac{C_{2}}{Z_{c}} e^{-x}$$
(4.17)

Where,  $V_x$  is the voltage of transmission line at length x;  $I_x$  is the current of transmission line at length x;  $C_1$  and  $C_2$  are the constants of the solutions dependent on the boundary condition.

Assume receiving end x = 0, then  $V_x = V_R$  and  $I_x = I_R$  at x = 0,

$$C_1 = \frac{V_R + I_R Z_c}{2}$$
(4.18)

$$C_2 = \frac{V_R - I_R Z_c}{2}$$
(4.19)

$$V_{x} = \frac{V_{R} + I_{R}Z_{c}}{2} \cdot e^{jx} + \frac{V_{R} - I_{R}Z_{c}}{2} \cdot e^{-jx}$$
(4.20)

$$I_{x} = \frac{V_{R}/Z_{c} + I_{R}}{2} \cdot e^{jx} + \frac{V_{R}/Z_{c} - I_{R}}{2} \cdot e^{-jx}$$
(4.21)

Where,  $V_R$  is the voltage of transmission line at receiving end;  $I_R$  is the current of transmission line at receiving end.

Using Euler's Equation:

$$\sinh(px) = \frac{e^{px} - e^{-px}}{2}$$
 (4.22)

$$\cosh(y_{x}) = \frac{e^{y_{x}} + e^{-y_{x}}}{2}$$
 (4.23)

Hence:

$$V_x = V_R \cosh(\gamma x) + I_R Z_c \sinh(\gamma x)$$
(4.24)

$$I_x = \frac{V_R}{Z_c} \sinh(\gamma x) + I_R \cosh(\gamma x)$$
(4.25)

The sending end of the line:

$$V_{S} = \cosh(\gamma I) V_{R} + Z_{c} \sinh(\gamma I) I_{R}$$
(4.26)

$$I_{S} = \frac{\sinh(\gamma)}{Z_{c}} V_{R} + \cosh(\gamma) I_{R}$$
(4.27)

Where,  $V_s$  is the voltage of transmission line at sending end;  $I_s$  is the current of transmission line at sending end.

In matrix form

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} \cosh(\gamma l) & Z_c \sinh(\gamma l) \\ Z_c^{-1} \sinh(\gamma l) & \cosh(\gamma l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$
(4.28)

Where, the ABCD transmission parameters are found to be:

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$
(4.29)

Where,

$$A = D = \cosh(\gamma l) \tag{4.30}$$

$$B = Z_c \sinh(\gamma t) \tag{4.31}$$

$$C = \frac{\sinh(\gamma l)}{Z_c}$$
(4.32)

Presenting as a function of harmonic order:

$$V_{S}(h) = V_{R}(h)\cosh(\gamma(h)) + I_{R}(h)Z_{c}(h)\sinh(\gamma(h))$$
  
=  $V_{R}(h)\cosh(\sqrt{Z(h) \cdot Y(h)} + I_{R}(h)\sqrt{\frac{z(h)}{y(h)}}\sinh(\sqrt{Z(h) \cdot Y(h)})$  (4.33)

$$I_{S}(h) = \frac{V_{R}(h)}{Z_{c}(h)} \sinh(\mathcal{A}(h)) + I_{R}(h) \cosh(\mathcal{A}(h))$$

$$= \frac{V_{R}(h)}{\sqrt{z(h)/y(h)}} \sinh(\sqrt{Z(h) \cdot Y(h)} + I_{R}(h) \cosh(\sqrt{Z(h) \cdot Y(h)})$$
(4.34)

#### 4.4.3 Resonances Calculations of A Three-Phase Overhead Transmission Line

The velocity of propagation of the voltage wave associated with a single line conductor will differ from those of a three-phase transmission line of the same length, as will its attenuation [69]. An overhead transmission line is usually designed as three-phase and the harmonic behaviour in each phase could be affected by the other phases since mutual couplings exist between the phases. To calculate multi-conductor transmission line in harmonics, matrix is used for parameters rather than solely scalar operations. Therefore, the ABCD transfer matrix is re-defined as matrix for three-phase transfer matrix operations as:

$$\begin{bmatrix} \mathbf{V}_{\mathbf{S}} \\ \mathbf{I}_{\mathbf{S}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{\mathbf{R}} \\ \mathbf{I}_{\mathbf{R}} \end{bmatrix}$$
(4.35)

Where,  $V_s$  and  $I_s$  are the sending end voltage and current vectors in matrix form for three phase,  $V_R$  and  $I_R$  are the receiving end voltage and current vectors in vector form for three phases. A, B, C and D are matrices of transfer where the mutual impedance and shunt between phase is taken account therefore the matrix is  $3 \times 3$  and is represented by:

$$\mathbf{A} = \mathbf{D} = \cosh(\Gamma l) \tag{4.36}$$

$$\mathbf{B} = \mathbf{Z}_{c} \sinh(\Gamma l) \tag{4.37}$$

$$\mathbf{C} = \mathbf{Z}_{e}^{-1} \sinh(\Gamma l) \tag{4.38}$$

Where,  $\Gamma = \sqrt{ZY}$  and  $Z_c = \sqrt{\frac{Z}{Y}}$  and the impedance Z is expressed as a matrix with self impedance in diagonal and others are mutual impedance between phases shown in the following equation:

$$\mathbf{Z} = \begin{bmatrix} Z_{AA} & Z_{AB} & Z_{AC} \\ Z_{BA} & Z_{BB} & Z_{BC} \\ Z_{CA} & Z_{CB} & Z_{CC} \end{bmatrix}$$
(4.39)

Where,  $Z_{AA}$ ,  $Z_{BB}$ ,  $Z_{CC}$  are the self impedance of phase A, B and C respectively;  $Z_{AB}$  and  $Z_{BA}$  are mutual impedance between phase A and B;  $Z_{AC}$  and  $Z_{CA}$  are the mutual impedance between phase C and A;  $Z_{BC}$  and  $Z_{CB}$  are the mutual impedance between phase B and C.

Applied the same form the shunt admittance matrix Y is expressed as:

$$\mathbf{Y} = \begin{bmatrix} Y_{AA} & Y_{AB} & Y_{AC} \\ Y_{BA} & Y_{BB} & Y_{BC} \\ Y_{CA} & Y_{CB} & Y_{CC} \end{bmatrix}$$
(4.40)

Where,  $Y_{AA}$ ,  $Y_{BB}$ ,  $Y_{CC}$  are the self admittance of phase A, B and C respectively;  $Y_{AB}$  and  $Y_{BA}$  are mutual admittance between phase A and B;  $Y_{AC}$  and  $Y_{CA}$  are the mutual admittance between phase C and A;  $Y_{BC}$  and  $Y_{BC}$  are the mutual admittance between phase B and C.



Figure 4.2 Overhead Transmission Line

In order to fully understand the resonance in a transmission line, a case study is used to demonstrate and the equations above are applied to predict the resonance frequency for an overhead transmission line.

**Case Study**: A three-phase overhead transmission system with an open ended 50 km length is excited by a one per unit balanced voltage source at each harmonic order with fundamental frequency of 50 Hz as shown in **Figure 4.2** where the three lines hang on towers 30 m height with 5 m intervals between lines. For the purpose of comparison of subsea cable calculation, the conductor diameter of these lines is chosen as 5 mm and material is copper but the lines consist of no outside layers such as insulation, sheath and armour which of course are needed for subsea cable.

When modelling power flows and faulty analysis, the sequence domain analysis would naturally be preferred but owing to the symmetrical nature of sources for harmonic propagation study that are discussed here, then it is preferable, for the sake of simplicity, to consider an exact three-phase model. For the case of harmonic voltage excitation at the sending end of the line and open-ended of receiving end  $I_{\mathbf{R}} = 0$ , the open ended voltage at receiving end is given by:

$$\mathbf{V}_{\mathbf{R}} = \mathbf{A}^{-1} \mathbf{V}_{\mathbf{S}} \tag{4.41}$$

By applying this equation, the resonance against frequency of receiving end can be acquired and evaluated.

The results of **Figure 4.3** using phase domain shows harmonic voltage against frequency up to 5000 Hz where the resonance peaks appear for each phase. It is at 1550 Hz for the first peak for all three phases and 4650 Hz for second peak for phase A and 4700 Hz for the second peak for Phase B and C. The magnitude of the first resonance for three phases is greater than 30 p.u. and greater than 15 p.u. for the second resonance. An electrical transmission system can magnify harmonic voltages or harmonic currents that happen to be at or near to a resonant frequency [63]. Therefore, to detect the response frequencies of resonances for a transmission line is essential since the system may experience harmonic voltage or current at these frequencies and there is a possibility that damage to the transmission system could occur. The characteristics of propagation and attenuation of voltage waves travelling along a transmission line are highly dependent on the length of the line and how many phases are involved and how they are arranged since mutual couplings between cables exist, that change the impedance and admittance of transmission lines, which in turns affect the resonance response. This clarifies that the resonance of each phase of this case of flat formation is dissimilar because the propagation rate and attenuations of each phase are different.



Figure 4.3 Overhead Line Harmonic Voltages (a) Receiving End Harmonic Resonance; (b) First Peak Resonance Using a Finer Resolution; (c) Second Peak Resonance Using a Finer Resolution
## 4.5 Skin Effect and Mutual Coupling

From the preceding section of transmission line modelling, it has been noticed that it is essential to take account of the skin effect of transmission line and the mutual coupling effects that occur between conducting lines. Skin effect is the phenomenon when the current tends to concentrate on the surface of a conductor instead of being uniformly distributed and in turn causes the increase of resistance and decrease in inductance as the frequency increases [77]. Mutual coupling occurs when the multi-conductors generate the mutual inductances and capacitances upon each other and in turn influence the harmonic propagation rate as demonstrated in the previous section.

According to previous studies [37] [38] [43] [46], skin effect has enormous impact on the resonances of along lead motor in an offshore power driving system. These papers mention that the result of resonance magnitudes could be amplified and the resonant frequencies could be shifted if skin effect is not included. Furthermore, modelling of high voltage transmission links in the harmonic domain [49] has pointed to the effect that cable mutual coupling must be taken into account in any transmission line modelling. Therefore, before modelling harmonics in a multi-layer subsea cable, the two elements skin effect and mutual coupling must be accounted for since they may affect the accuracy of the simulation. A simple model, therefore, is designed to clarify how skin effect and mutual coupling influence the resonance behaviour of a transmission line.

**Case Study**: Consider a simple transmission system consisting of an open ended three single-core XLPE insulated power cable of 50 km length in touching trefoil configuration when excited by a one per unit balanced voltage source at each harmonic order with fundamental frequency of 50 Hz, where the operating temperature of conductor is 90°C and cable configurations, dimensions and materials is shown in **Appendix B.1**. Frequency-phase domain modelling has been applied since the test model is symmetrical as the same conditions as previous case study for overhead transmission line. The corresponding matrix using equation (4.41) acquires the harmonic resonance response.

The simulation using **MathCAD<sup>©</sup>** (MathCAD) programming of pure mathematic calculations whose list is given in **Appendix C**. The results are shown in **Figure 4.4** which demonstrates the computer simulated frequency response of one of the three phases using frequency domain modelling techniques where the resonance peaks can clearly be seen. At resonance, any coincident waveform harmonics would create resonant excitation that would lead to increased voltage stress, which of course could potentially impair the insulation layers within the cable. This result agrees with [37] [38] [46] in that the resonances are seen to be significantly damped when the skin effect is included in the model as compared to a model in which the skin effect is omitted. Also, considering the effects of mutual coupling from [49], then this figure shows that mutual coupling between conductors cannot be ignored because the frequencies at which resonance occur are different when mutual coupling is included in the model as compared to when it is excluded.



Figure 4.4 Simple Cable Model Receiving End Harmonics

The computer based mathematical model developed so far has provided a satisfactory means to determine resonant frequencies in transmission cables. This result implies that the skin effect has to be taken into account while evaluating the harmonics. Also, if neglecting the mutual coupling effect between the conductors, the resonant frequencies were seen slightly different, which can not accurately express the cable harmonics. More importantly, it also points out that the consideration of skin effects and mutual coupling effects are necessary for harmonic calculations in transmission cables.

# 4.6 Subsea Power Cable Harmonic Model

From proceeding sections, it is noted that the harmonic impedance and harmonic admittance are the main factors of determining the harmonic behaviour in a transmission cable. Thus to establish a harmonic model for subsea power cable, the electrical parameters of power cable should be accurately calculated. Owing to the fact that submarine power cables are well armoured with multi-layer structure and designed for long distance connection, the calculations of their characteristics must be modified. Also as discussed earlier, the impedance of a transmission cable must take account skin effect within the conducting layers and the coupling effects between conductors, sheath, and armour in the cable. Hence the following equations are developed to assess the harmonic characteristic of single-core subsea power cables where the cable impedances and admittances are based on super-positions of loop equations. This evaluation will form the basis for further development of three-phase subsea power cable harmonic model.

#### 4.6.1 Calculations of Single-Core Subsea Power Cable Impedances

The harmonic equations of a single-core subsea power cable can be expressed as loop equations [64] [65] as shown in **Figure 4.5** where Loop 1 is the loop of conductor core to sea, Loop 2 is the loop of sheath to armour, and Loop 3 is the loop of armour to sea. To express the relationships as loop equations:



Figure 4.5 Single-Core Subsea Cable Cross Sections

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & 0 \\ Z_{21} & Z_{22} & Z_{23} \\ 0 & Z_{32} & Z_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$
(4.42)

Where,  $V_1$ ,  $V_2$ ,  $V_3$ ,  $I_1$ ,  $I_2$  and  $I_3$  are the voltages and current per unit length for loops 1, 2 and 3, respectively.

$$Z_{11} = Z_{core-ex} + Z_{core/sheath-insulation} + Z_{sheath-in}$$
(4.43)

$$Z_{22} = Z_{sheath-ex} + Z_{sheath/armour-insulation} + Z_{armour-in}$$
(4.44)

$$Z_{33} = Z_{armour-ex} + Z_{armour/sea-insulation} + Z_{sea-in}$$
(4.45)

$$Z_{12} = Z_{21} = -Z_{sheath-mutual}$$
(4.46)

$$Z_{23} = Z_{32} = -Z_{armour-mutual}$$
(4.47)

Where,  $Z_{core-ex}$  is the external impedance per unit length of conductor;  $Z_{sheath-ex}$  is the external impedance per unit length of sheath;  $Z_{armowr-ex}$  is the external impedance per unit length of armour;  $Z_{sheath-in}$  is the internal impedance per unit length of sheath;  $Z_{armowr-in}$  is the internal impedance per unit length of armour;  $Z_{sea-in}$  is the internal impedance per unit length of sea;  $Z_{core/sheath-insulation}$  is the impedance per unit length of the insulation between core and sheath;  $Z_{sheath/armowr-insulation}$  is the impedance per unit length of the insulation between sheath and armour;  $Z_{armowr/sea-insulation}$  is the impedance per unit length of the insulation between armour and sea;  $Z_{sheath-mutual}$  is the mutual impedance per unit length of sheath between loop 1 and loop 2;  $Z_{armowr-mutual}$  is the mutual impedance per unit length of armour between loop 2 and loop 3.

The impedances can be formed in general equations given by:

$$Z_{kayer-in} = \frac{\rho\sigma}{\pi D_i H} \left[ I0(\frac{\sigma D_i}{2}) K I(\frac{\sigma D_{\epsilon}}{2}) + K 0(\frac{\sigma D_i}{2}) I1(\frac{\sigma D_{\epsilon}}{2}) \right]$$
(4.48)

$$Z_{layer-ex} = \frac{\rho\sigma}{\pi D_e H} \left[ I0(\frac{\sigma D_e}{2}) K1(\frac{\sigma D_e}{2}) + K0(\frac{\sigma D_e}{2}) I1(\frac{\sigma D_e}{2}) \right]$$
(4.49)

$$Z_{layer-mutual} = \frac{2\rho}{\pi D_{i} D_{e} H}$$
(4.50)

$$H = I1(\frac{\sigma D_{\epsilon}}{2})K1(\frac{\sigma D_{i}}{2}) - I1(\frac{\sigma D_{i}}{2})K1(\frac{\sigma D_{\epsilon}}{2})$$
(4.51)

Where,  $Z_{layer-in}$ ,  $Z_{layer-ex}$ ,  $Z_{layer-mulual}$  is the internal, external and mutual impedance per unit length in a conducting layer, respectively; I0(x), I1(x) is the zero order and first order of modified Bessel functions of 1<sup>st</sup> kind with a complex argument; K0(x), K1(x) is zero order and first order of the modified Bessel functions of 2<sup>nd</sup> kind as so called Kelvin functions with a complex argument;  $\sigma = \sqrt{j \frac{\omega \mu_r}{\rho}}$  is the complex propagation constant in the conducting layer;  $\rho$  is the resistivity of the conducting layer;  $\mu_r$  is the permeability of the conducting layer;  $D_i$  is the internal diameter of the conducting layer;  $D_e$  is the external diameter of the conducting layer.

While in some practical cases, the simplified expression for conducting layers' general equation can be made. For instance, the central conductor is usually not hollow but solid. Therefore the internal diameter of the conducting layer is set to zero  $(D_i = 0)$  and then the impedance equation can be reduced to:

$$Z_{layer-ex} = \frac{\rho\sigma}{\pi D_e} \frac{I0(\frac{\sigma D_e}{2})}{I1(\frac{\sigma D_e}{2})}$$
(4.52)

Also, in most cases for the subsea power cable, the thickness of each conducting layers are much less than the diameters where the internal and external impedances are effectively the same so the general equations can again be simplified as:

$$Z_{layer-ln} = Z_{layer-ex} = R_{layer} \rho \sigma \coth[\sigma t_{lhickness}]$$
(4.53)

$$Z_{layer mutual} = R_{layer} \frac{\sigma t_{lhickness}}{\sinh(\sigma t_{lhickness})}$$
(4.54)

Where,

 $R_{layer} = \frac{\rho}{\pi D t_{ihickness}}$  is the resistance of the conducting layer;  $D = \frac{D_e + D_i}{2}$  is the average diameter of the

conducing layer;  $t_{thickness} = \frac{D_e - D_t}{2}$  is the thickness of the conducting layer.

The impedance of the insulation between two conducting layers can be described by:

$$Z_{layer/layer-insulation} = j\omega \frac{\mu_{insl}}{2\pi} \ln(\frac{r_{outside}}{r_{inside}})$$
(4.55)

Where,  $\mu_{inst}$  is the permeability of insulation;  $r_{insule}$  is the inside radius of insulation layer;  $r_{outsule}$  is the outside radius of insulation layer.

According to [78], for single-core cables, both the self and mutual impedance of underground cables includes consideration of impedance to earth. Applying the earth return path self and mutual impedance developed originally by Wedepohl and Wilcox [79] and referring to the calculation matrix for the exact model in [67], then the impedance of sea return path of single-core subsea cables can be rewritten:

For the self-impedance of sea return path:

$$Z_{sea-in,self} = \frac{j\omega\mu_{sea}}{2\pi} \left[ -\ln(\frac{\gamma_e \sigma_{sea} r_{sea}}{2}) + \frac{1}{2} - \frac{4\sigma_{sea} h_{sea}}{3} \right]$$
(4.56)

For the mutual-impedance of sea return path:

$$Z_{sea-in,mutual} = \frac{j\omega\mu_{sea}}{2\pi} \left[ -\ln(\frac{\gamma_e \sigma_{sea} \delta_{i,j}}{2}) + \frac{1}{2} - \frac{2\sigma_{sea} l_{i,j}}{3} \right]$$
(4.57)

Where,  $\mu_{sea} = \mu_0$  is the permeability of the sea;  $r_{sea}$  is the sea return path radius as the external radius of the cable;  $\gamma_e$  is the constant 1.781 [67];  $h_{sea}$  is the depth from sea level at which the cable is laid;  $\delta_{i,j}$  is the distance between cable i and cable j;  $l_{i,j}$  is the sum of depth from sea level of the cable i and cable j;  $\sigma_{sea} = \sqrt{j \frac{\omega \mu_{sea}}{\rho_{sea}}}$  is the complex propagation constant in the sea;  $\rho_{sea}$  is the resistivity of the sea

which is  $1(\mu\Omega m)$  of sea water.

### 4.6.2 Calculations of Magnetic Armour Impedances

Usually submarine cables are designed with magnetic steel armour which consists of a number of wires. In the case of the magnetic layer it is also necessary to consider the saturation effects i.e. the permeability of the armour layers. Expressions for magnetic armour wire impedance have been previously derived and are given by Bianchi and Luoni [65] who derived relative permeability curves based on measurement of longitudinal permeability since it is difficult to compute the eddy currents and hysteresis losses for evaluating the permeability of round steel armour wires.



Figure 4.6 Components on Armour Wire Permeability

The armour wire permeability basically can be specified as shown in **Figure 4.6** where the permeability in circumferential magnetic direction is  $\mu_r$ , permeability in longitudinal direction is  $\mu_l$  and permeability in transversal direction or perpendicular direction is  $\mu_t$  which due to discontinuities between the wires it is usually of minor importance and can be roughly assumed as  $\mu_t = 10$  for wires are in contact to each other and  $\mu_t = 1$  for wires are separated [57]. The computation of electrical parameters of relative permeability of armour wire is to evaluate the circumferential direction permeability with respect to permeability of longitudinal and transversal directions. The equation can be expressed by [65]:

$$\mu_{r} = |\mu_{r}|e^{-j\beta^{r}} = \frac{\pi nd}{4p_{ii}}|\mu_{i}|e^{-j\alpha^{r}}\sin\delta + \mu_{r}\cos^{2}\delta$$
(4.58)

Where, *n* is the number of wires; *d* is the wire diameter;  $\delta$  is the laying angle;  $p_{ii}$  is the laying pitch of the armour;  $\beta'$  is the phase angle of  $\mu_r$ ;  $\alpha'$  is the phase angle of measured longitudinal direction permeability  $|\mu_l|^4$ .



Figure 4.7 Longitudinal Permeability [65]

The  $|\mu_i|$  is the measured value of permeability in the longitudinal direction and is a function of wire diameter and the intensity of the circumferential magnetic field as shown in **Figure 4.7**. It is therefore the circumferential permeability can be computed by input of the value of  $|\mu_i|$  and  $\alpha'$ . The result for steel wire of 5 mm diameter armour is shown in **Figure 4.8** where the circumferential permeability is a function of assumed perpendicular permeability, laying angle and the magnetic field strength.



Figure 4.8 Circumferential Permeability for Steel Wire Diameter of 5mm Armour [65]

#### 4.6.3 Boundary Condition

For subsea transmission systems, cables would normally be well bonded with both the sheath and armour layers being connected to earth at each end of the cable. According to [80], the armour of a submarine cable is usually of substantial thickness to prevent flux penetration and the armour can be reasonably assumed to be at ground potential at all points along its length. Similarly, the sheath voltages along a cable's length are insignificant when compared to the voltages of the conductors so these too may be reasonably assumed to be at earth potential along its length. Thus it is plausible to assume that the cables have good bonding at both ends and well earthed as:

$$V_{sheath} = V_{armour} = 0, \qquad (4.59)$$

$$V_1 = V_{corr} - V_{sheath}, \qquad (4.60)$$

$$V_2 = V_{sheath} - V_{armour}, \tag{4.61}$$

$$V_{3} = V_{armour}, \qquad (4.62)$$

$$I_1 = I_{corr}$$
, (4.63)

$$I_2 = I_{cure} + I_{sheath}, \qquad (4.64)$$

$$I_3 = I_{core} + I_{sheath} + I_{armour}$$

$$(4.65)$$

Therefore, equation (4.42) can be rewritten to

$$\begin{bmatrix} V_{core} \\ V_{sheath} \\ V_{armour} \end{bmatrix} = \begin{bmatrix} Z_{cc} & Z_{cs} & Z_{ca} \\ Z_{sc} & Z_{ss} & Z_{sa} \\ Z_{ac} & Z_{as} & Z_{aa} \end{bmatrix} \begin{bmatrix} I_{core} \\ I_{sheath} \\ I_{armour} \end{bmatrix}$$
(4.66)

 $Z_{cc} = Z_{11} + 2Z_{12} + Z_{22} + 2Z_{23} + Z_{33},$ (4.67)

$$Z_{cs} = Z_{sc} = Z_{12} + Z_{22} + 2Z_{23} + Z_{33}, \qquad (4.68)$$

$$Z_{ca} = Z_{ac} = Z_{sa} = Z_{23} + Z_{33}, \qquad (4.69)$$

$$Z_{ss} = Z_{22} + 2Z_{23} + Z_{33}, \qquad (4.70)$$

$$Z_{aa} = Z_{33},$$
 (4.71)

Where,  $I_{core}$ ,  $I_{sheath}$ ,  $I_{armour}$ ,  $V_{core}$ ,  $V_{sheath}$  and  $V_{armour}$  is the current per unit length in core, sheath, armour the voltage per unit length in core, sheath and armour. Applying boundary conditions to equations (4.59) to (4.65), the matrix (4.66) can be reduced as [64] where  $Z_{cable}$  is the equivalent impedance of the cable:

$$V_{core} = Z_{cable} I_{core}$$
(4.72)

#### 4.6.4 Calculations of Single-Core Subsea Power Cable Admittance

For calculations of a single-core subsea cable, similarly the corresponding harmonic shunt admittance matrix can be expressed as loop equations where there is no mutual capacitance between two adjacent layers since it is concentric layer and no potential difference between any two loops:

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} Y_1 & 0 & 0 \\ 0 & Y_2 & 0 \\ 0 & 0 & Y_3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$
(4.73)

Where,  $V_1$ ,  $V_2$ ,  $V_3$ ,  $I_1$ ,  $I_2$  and  $I_3$  are the voltages and current per unit length for loop 1, 2 and 3, respectively.

$$Y_{1} = j\omega \frac{2\pi\varepsilon_{r_{1}}}{\ln(\frac{r_{insul1-out}}{r_{insul1-in}})}$$
(4.74)

$$Y_{2} = j\omega \frac{2\pi\varepsilon_{r2}}{\ln(\frac{r_{insul,2}}{r_{insul,2}})}$$
(4.75)

$$Y_{3} = j\omega \frac{2\pi\varepsilon_{r3}}{\ln(\frac{r_{insul,3} \cdot out}{r_{insul,3} \cdot out})}$$
(4.76)

Where,  $Y_1$  is the admittance per unit length of the insulation between conductor and sheath;  $Y_2$  is the admittance per unit length of the insulation between sheath and armour;  $Y_3$  is the admittance per unit length of the insulation of jacket outside armour;  $\varepsilon_{r1}$  is the permittivity of the insulation between conductor and sheath;  $\varepsilon_{r2}$  is the permittivity of the insulation between sheath and armour;  $\varepsilon_{r3}$  is the permittivity of the insulation of jacket outside armour;  $r_{insul1-out}$  is the outside radius of the insulation between conductor and sheath;  $r_{insul1-in}$  is the insulation of the insulation between sheath and armour;  $\varepsilon_{r3}$  is the outside radius of the insulation between conductor and sheath;  $r_{insul1-in}$  is the insulation between sheath and armour;  $r_{insul2-in}$  is the outside radius of the insulation between sheath and armour;  $r_{insul2-in}$  is the insulation of jacket outside radius of the insulation of jacket outside amour;  $r_{insul3-in}$  is the insulation of jacket outside radius of the insulation of jacket outside armour;  $r_{insul3-in}$  is the insulation of jacket outside armour.

To adopt the boundary condition of well bonded with both the sheath and armour layers at both end of the cable as the previous section described. Use equation from equation (4.59) to equation (4.65) to substitute matrix (4.73) and the matrix is expressed as:

$$\begin{bmatrix} I_{corr} \\ I_{sheath} \\ I_{armour} \end{bmatrix} = \begin{bmatrix} Y_1 & -Y_1 & 0 \\ -Y_1 & Y_1 + Y_2 & -Y_2 \\ 0 & -Y_2 & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_{corr} \\ V_{sheath} \\ V_{armour} \end{bmatrix}$$
(4.77)

The system matrix again can be reduced as:

$$I_{core} = Y_{cable} V_{core}$$
(4.78)

Where,  $Y_{cable}$  is the equivalent admittance of the cable and  $Z_{cable} \neq \frac{1}{Y_{cable}}$ .

#### 4.6.5 Resonances Calculations of Three Phase Single-Core Subsea Power Cable

For three-phase single-core subsea power cables, the ABCD transfer matrix of equation (4.35) can be used to evaluate its harmonic resonances and to obtain the frequency response. The three-phase impedance and admittance matrices of equation (4.39) and (4.40) are adopted where the self impedance and self admittance of single-core subsea cable are demonstrated in the previous sections and mutual impedance of each phase in respect to other phases can be found using equation (4.57) mutual earth return path and mutual admittance can be defined as zero since it is noted that there is no relation between the distance of cables and the cable shunt capacitance for heavy armoured underground or submarine cables according to [81]. Once the impedance and admittance matrix have been determined, the propagation constant matrix  $\Gamma$  and the characteristic impedance matrix  $Z_e$  can be obtained and further evaluations undertaken to determine the resonance frequency response.

**Case Study**: Consider a simple subsea transmission system consisting of an open ended three single-core XLPE insulated typical subsea power cable of 50 km length designed by the cable manufacturer ABB in touching trefoil configuration lying on seabed of 50 m depth is excited by a one per unit balanced voltage source at each harmonic order with fundamental frequency 50 Hz where the operating temperature of conductor is 90°C and cable configurations, dimensions and materials is given in **Appendix B.2**. Again, since the cable has a symmetrical arrangement and the supplying source is balance, frequency-phase domain modelling has been applied from fundamental frequency up to 5000 Hz of 100 orders.

The detailed calculation process is being conducted using MathCAD programming and the list is shown in **Appendix D**. In addition to skin effect, circulating currents will flow in the metallic sheath and armour contributing to the total loss in the core. The harmonic resistance and inductance of the core does not have a linear relationship with frequency as is often assumed. The results shown in **Figure 4.9** demonstrate that the harmonic resistance and inductance curves of the cable per unit length of 1 km having nonlinear relationship with the frequency which is in accordance with the results in [82] where the harmonic resistance and harmonic inductance of armoured cables are shown as non-linear. The resistance increases as harmonic order is increased and appears to curve as expected. Also the inductance reduces as harmonic order is increased which is caused because of the cancellation of flux linkage of the conductor due to induced current in the sheath armour [54] [83] and the reduction of inductance is increases with a rise in

the frequency. Therefore the trend of the harmonic resistance agrees with the observation of [83] which harmonic resistance and inductance armoured cables under different conductor cross section are analysed and measured.



Figure 4.9 Harmonic Resistances and Inductances per km for Study Case 4.6

**Figure 4.10** shows the harmonic resonances at the cable receiving end plotted against the frequency up to 5000 Hz. The resonance peaks take place at 750 Hz, 2350 Hz and 3950 Hz with magnitude of 7.62 p.u., 3.11 p.u. and 2.07 p.u., respectively. The results imply that although the resonance magnitudes of subsea cable are less than those of overhead line of the same length of Figure 4.3 whose largest voltage resonance is greater than 30 p.u., the peak resonance is still a concern since the it is still over 7 times of the voltage at fundamental frequency for the first resonances at lower frequency. Furthermore, it can be found that the first resonance at lower frequency for the subsea power cable takes place at 750 Hz is lower than 1550 Hz for overhead line and the span between two adjacent resonances for subsea cable is shorter than the span for overhead line. Thus, it is noted that cable structure is a key factor to decide the natural resonances and frequency responses of a cable.



Figure 4.10 Harmonic Voltages at Receiving End for Study Case 4.6

# 4.7 Validation of Single-Core Subsea Power Cable Harmonic Model

For simulation models, one of the most critical steps is to verify the models using experimental data or another validated approach to give credit to the model. However harmonic data for subsea cables is not widely available and it is not a standard requirement for manufacturers to provide it. It is also uneconomic and practically difficult to measure subsea power cable harmonic data by experiment. For the validation process of this study, the analytical model results first are being compared with the existing measured data provided by manufacturers. Since the test data is not comprehensive enough to cover a wide range of harmonic frequency, commercial software programme, PSCAD, is introduced to produce results to compare to the analytical model under the same scenario for a wide range of frequencies. Due to some limitation of the software programme previously described, the scenario should be able to be simulated by this software programme. Should the both limited test data and the commercial software package verify the results generated by the analytical model, it can then be assured that the analytical model is correct and can be carried on for further development for harmonic analysis of subsea power cables.

For the harmonic test data, ABB provided a limited test data set for the Case Study 4.6 of resistance, inductance and capacitance per kilometre at fundamental frequency 50 Hz in sequence domain where the electrical data is the same in each sequence. Table 4.1 shows the results obtained from analytical model and ABB measured data with comparison of each other. It clearly demonstrates that the differences between the results using test data and model are tiny and the error is very small. This ensures that the analytical model is able to accurately evaluate the resistance, inductance and capacitance at fundamental frequency for subsea power cable.

	Sequence Domain (Zero, Positive and Negative)		
	Resistance at 50 Hz	Inductance at 50 Hz	Capacitance at 50 Hz
	(Ω/km)	(mH/km)	(µF/km)
ABB Data	0.0458	0.2145	0.25
Analytical Model	0.0445	0.2147	0.2547
Difference	2.8 %	0.09 %	1.88 %

Table 4.1 Comparison of Results from ABB Data and Analytical Model

The analytical model developed in the previous sections now needs to be verified using an independent approach for validation purposes and to give confidence in the results. PSCAD, providing transmission line frequency dependent models with consideration of propagation, is known to produce accurate results for multi-layers concentric cable in the frequency domain so is suited to undertake this task. However, due to the application restrictions as mentioned in the proceeding section, the input conditions in the analytical model will need to be adjusted to match those conditions that can be simulated within PSCAD. A case study that uses the same test conditions for the analytical model and PSCAD program was applied. The test cable was modified to adopt the same initial and boundary conditions, i.e. single-core cable type, grounding (earthing) arrangement and types of non-magnetic materials in its structure which is available for PSCAD simulation. The cable dimension and material of this verification study case is shown **Appendix B.3** where the cable is also laying on a seabed of 50 m depth with operational temperature of 90°C and excited by a balanced voltage source with one per unit at each harmonic order with fundamental frequency of 50 Hz.

The results of single-core subsea cables harmonic resonances at receiving end of the cable from the analytical model using pure mathematic MathCad programming gave a precise match with the results obtained from the PSCAD as shown by the example given in **Figure 4.11**. Such validation gives confidence in the mathematical analysis to this point allowing further development of the analytical model to proceed i.e. extending beyond the capabilities of the commercial simulation packages.



Figure 4.11 Comparisons of Harmonic Resonances Results from Analytical Model with the Results from PSCAD/EMTDC

## 4.8 Summary

In short, the theory of phase domain harmonic modelling of transmission lines has been presented where the  $\pi$  network circuit is employed and the electrical parameters of transmission line are evaluated to investigate the harmonic behaviour. Applying the inductance and capacitance calculation equations for cylindrical geometries the transmission line electrical parameters can be assessed.

To estimate the natural harmonic resonance both lumped and distribution parameters, which includes the propagation rate and attenuation needs to be accurately calculated. The model of an overhead transmission line is introduced to demonstrate the harmonic calculation process and illustrate the harmonic resonance frequency response.

The importance of mutual coupling and skin effect are also presented by simulating the harmonic resonance response for an insulted cable for different conditions including without consideration of skin effect, without consideration of mutual coupling and consideration of both effects. The results show that the resonance frequency can be shifted and the magnitude can be amplified if these effects are not included in the models.

A detailed harmonic calculation model of single-core multi-layer subsea cable is presented where superposition of loop equations are applied and the inter-layer skin effect, inductance and capacitance are taken into account to calculate the actual impedance and admittance for the conductor under both end well-bonded conditions. The evaluation of impedance of magnetic layer of armour wire has also been demonstrated and its relative permeability charts created by Bianchi and Luoni [65] which offers a greatly simplified solution, are also introduced to facilitate the calculation of steel armour wires.

Finally a validation is carried out by comparing with the hard data of impedance, inductance and capacitance of subsea cable at fundamental frequency and then PSCAD software is used to verify the harmonic characteristics of the cable. However, due to the constraints of PSCAD, not all scenarios can be verified by using PSCAD. Therefore, a scenario has been chosen that is modelled in the analytical model for validation must be able to be verified by PSCAD. From the results, the frequency responses for harmonic resonances showed the perfect match for both commercial package model and the analytical model. Thus the analytical model developed in this chapter can be proved as a correct model and able to further investigate the subsea power cable harmonic behaviours from this basis.

5

# Proximity Effect on Harmonic Impedance of Subsea Power Cable

# 5.1 Introduction

It is known from the previous chapter that it is necessary to have accurate harmonic impedance models of the subsea cables otherwise, it is impossible to predict system resonances reliably and to assess the effects of any generated harmonics such as those from power converters. To develop accurate harmonic impedance models, a good understanding of the physical phenomena that goes into the makeup of the cable impedance is necessary. As described in the proceeding chapter, the harmonic calculations of single-core subsea cables can be regarded as multi-concentric cylindrical layers and the computing equations have been demonstrated. However for three-core subsea cables the calculations have to involve the application of a pipe-type cable which has been widely adopted for underground cable calculations since the individual conductors are not in the centre of the cable, which leads to the further complication of harmonic impedance calculations.

Many studies have been conducted to determine the appropriate methods for calculating harmonic impedance of underground cables [84] [85] [86] [87] [88] [89]. This is not the case for subsea cables which are different in that they have a layer of heavy armour on the outside to give added strength both for laying and for protecting against mechanical damage e.g. fishing. Because of the heavy armour, the electromagnetic effects between the layers within the subsea cable need to be considered carefully when developing impedance models. Also, subsea cable arrangements and structures are diverse, which suggests that each cable type will generate a distinct impedance characteristic. There is therefore a need to develop appreciable equations to fully estimate the harmonic behaviour particularly for three-core subsea cables.

The proximity effect is a phenomenon that is seen when two conductors carrying alternating currents run parallel and close to each other. The current densities in the conductor layers on the near sides i.e. facing

each other are decreased and those on the rear sides are increased because of differences in the magnetic flux densities. As a consequence of proximity effect there is an increase in the conductor ac resistance [90].

For single-core cables having a core, a sheath and an armour layer, the impedance calculation should only need to consider the skin effect because of the concentred arrangement of the cable layers. However, when other cables, such as three-core cables are being modelled then the proximity effect should be included in the analysis since there is a strong possibility that the impedance of the conductors will be affected by it. The intriguing question is how influential is the proximity effect in contributing to the overall cable impedance. It may be that for single-core cables, due to both sheath and armour, the proximity effect will be considerably small. On the other hand, for three-core cables, the cable conductors are located within the armour so the proximity effect would certainly be expected to have an impact upon impedance.

This chapter presents an evaluation of the proximity effect upon the impedance characteristics of singlecore and three-core subsea cables. The proximity effect is often ignored in subsea cable models as used by computer based simulation packages that are widely used for establishing the performance of offshore electrical power systems. Two common types of subsea cables have been investigated in this chapter; single-core and three-core cables. Models have been developed and results presented which clearly show that the proximity effect has almost no effect on single-core cables but has a significant effect upon threecore cables. In addition, sequence domain harmonic impedance analysis also showed that the proximity effect is a significant factor for evaluation of harmonic impedance of three-core cable where a common armour is used as a common grounding return path. Nevertheless the proximity effect appears not to influence the zero-sequence harmonic impedance for both single-core subsea cables. The chapter concludes that when calculating the harmonic impedance of single-core subsea cables. The chapter concludes that when calculating the harmonic impedance of single-core subsea cables then only skin effect needs to be accounted for but for three-core cables then both skin effect and proximity effects must be considered. This is in the specific harmonic equations for three-core subsea cable calculations can be established where those physical phenomena should be included.

# 5.2 Proximity Effect

The key to determining the proximity effect in subsea cables is to develop and use equations for impedances of multi-layer cylinders. These equations are well established and have been introduced in **Chapter 4** but they need to be modified according to the subsea cable structure and the physical arrangement under consideration in order to evaluate accurately the impact of the proximity effect.

The general equation for magnetic potential from Faraday's law with flux linkage can be expressed by Maxwell's equations as follows [88] [91]:

$$\nabla^2 \cdot \mathbf{A} = \boldsymbol{\mu} \cdot \mathbf{J} \tag{5.1}$$

$$\mathbf{J} = \frac{j\omega}{\rho} \cdot \mathbf{A}$$
(5.2)

Where, A is magnetic vector potential; J is the current density vector;  $\mu$  is the corresponding permeability of the conducting layer;  $\rho$  is the corresponding resistivity of the conducting layer;  $\omega$  is the angular velocity.

Using cylindrical coordinates:

$$\frac{\partial^2 \mathbf{A}}{\partial r^2} + \frac{1}{r} \frac{\partial \mathbf{A}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \mathbf{A}}{\partial \theta^2} - j \frac{\mu \omega}{\rho} \mathbf{A} = 0$$
(5.3)

The general solution using Bessel's equations is given as:

$$\mathbf{A}(r,\theta) = \sum_{n=0}^{\infty} [A_n I_n(mr) + B_n K_n(mr)] \cos(n\theta)$$
(5.4)

Where the  $I_n(x)$  is first kind modified Bessel's function, order n;  $K_n(x)$  is second kind modified Bessel's function, order n;  $A_n$  and  $B_n$  are constants need to be determined using the boundary condition;

$$m = \sqrt{\frac{j\mu\omega}{\rho}}$$
 is the complex constant.

While considering the vector potential within the material conducting layer such as conductor, sheath and armour. Use the Bessel's equation (5.4) to derive the impedance [92] and to acquire the general solution:

$$\mathbf{E}(\mathbf{r},\theta) = Z \cdot \mathbf{I} = \rho \cdot \mathbf{J}(\mathbf{r},\theta) \tag{5.5}$$

Where,  $\mathbf{E}(r,\theta)$  is the vector of electric field.

$$Z = \rho [A_0 I_0(mr) + B_0 K_0(mr)] + \rho \sum_{n=1}^{\infty} (\frac{s}{r})^n [A_n I_n(mr) + B_n K_n(mr)] + j\omega \frac{\mu_0}{2\pi} (\frac{s}{r})$$
(5.6)

In this formula, the first term of the right hand side is due to the skin effect while the second term is due to the proximity effect and the third term is the external inductance between conductors. Where, s is the distance between conductors and the detailed derivation of constants  $A_0$ ,  $B_0$ ,  $A_n$  and  $B_n$  obtained from boundary condtin are fully described in [92] [93]. The solutions of the first term of the right hand side regarding skin effect of a cable has been given details and shown in Section 4.6 for harmonic impedance of single-core subsea cables.

For the purpose of determining the significance of the proximity effect in subsea cables, the single-core trefoil touching formation and the three-core cable have been chosen for following investigation shown in **Figure 5.1**. The single-core trefoil type is expected to be more influenced by the proximity effect as compared to the flat touching, trefoil and flat formations [94], thus the single-core trefoil touching cable is selected for further comparison to the three-core subsea cable. The calculations of single-core and three-core cables using Bessel's function are demonstrated in the following sections and the equations are developed according to the geometric arrangement of the layers in the cables.



b. Three-Core Trefoil Touching Formation Cable Structure

#### Figure 5.1 Single-Core Trefoil Touching and Tree-Core Trefoil Touching Cable Configurations

# 5.3 Calculations of Harmonic Impedance of Single-Core Subsea Cables with Proximity Effect

For single-core subsea cable, the impedance matrix is described in [95].

$$\mathbf{Z}_{cable} = \mathbf{Z}_{i} + \mathbf{Z}_{earth}$$
(5.7)

Where,  $\mathbb{Z}_{cable}$  is the cable equivalent impedance matrix (Note:  $\mathbb{Z}_{cable} \neq \frac{1}{Y_{cable}}$ );  $\mathbb{Z}_i$  is the internal cable impedance matrix;  $\mathbb{Z}_{earth}$  is the cable impedance matrix of earth return path.

For a three-phase system, the cable impedance matrix can be express as:

$$\mathbf{Z}_{cable} = \begin{bmatrix} \mathbf{Z}_{i,A} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_{i,B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{Z}_{i,C} \end{bmatrix} + \begin{bmatrix} \mathbf{Z}_{earth,AA} & \mathbf{Z}_{earth,AB} & \mathbf{Z}_{earth,AC} \\ \mathbf{Z}_{earth,BB} & \mathbf{Z}_{earth,BC} \\ \mathbf{Z}_{earth,CA} & \mathbf{Z}_{earth,CB} & \mathbf{Z}_{earth,CC} \end{bmatrix}$$
(5.8)

Where,  $Z_{i,A}$ ,  $Z_{i,B}$  and  $Z_{i,C}$  is the self internal cable impedance matrix of phase A, B and C, respectively;  $Z_{earth,AA}$ ,  $Z_{earth,BB}$  and  $Z_{earth,CC}$  is the self cable impedance matrix of earth return path of phase A, B and C, respectively;  $Z_{earth,AB}$  and  $Z_{earth,BA}$  are the mutual cable impedance matrices of earth return path between phase A and B;  $Z_{earth,AC}$  and  $Z_{earth,CA}$  are the mutual cable impedance matrices of earth return path between phase A and C;  $Z_{earth,BC}$  and  $Z_{earth,CB}$  are the mutual cable impedance matrices of earth return path phase B and C;  $Z_{earth,BC}$  and  $Z_{earth,CB}$  are the mutual cable impedance matrices of earth return path

The first term of the right hand side of the equation is the internal impedance matrix of one single-core cable of a three-phase transmission system with the conductor, sheath and armour layers considered and carefully explained in [65] [95]. The detailed derivations are demonstrated for single-core cable harmonic impedance calculations in Section 4.6 where the sub-matrix  $Z_{i,j}$ , as the representative of  $Z_{i,A}$ ,  $Z_{i,B}$  and  $Z_{i,C}$ , taking account of impedance of three conducing layers of conductor, sheath and armour, are presented in matrix form as:

$$\mathbf{Z}_{\mathbf{i},\mathbf{j}} = \begin{bmatrix} Z_{cc} & Z_{cs} & Z_{ca} \\ Z_{sc} & Z_{ss} & Z_{sa} \\ Z_{ac} & Z_{as} & Z_{aa} \end{bmatrix}$$
(5.9)

Where,  $Z_{cc}$ ,  $Z_{ss}$  and  $Z_{aa}$  is the self impedance of conductor, sheath and armour, respectively;  $Z_{cs}$  and  $Z_{sc}$  are the mutual impedances between conductor and sheath;  $Z_{sa}$  and  $Z_{as}$  are mutual impedance between sheath and armour;  $Z_{ca}$  and  $Z_{ac}$  are mutual impedance between conductor and armour.

The second term of the right hand side of equation (5.8) is the earth return impedance matrix of cable. Since internal impedance matrices of each phase  $Z_{i,j}$  are  $3 \times 3$ , the sub-matrices  $Z_{earth,jj}$ , as the representative of self cable impedance matrix of earth return path of each phase, and  $Z_{earth,jk}$ , as the representative of mutual cable impedance matrix of earth return path between each phase, are also  $3 \times 3$  and can be expressed as:

$$\mathbf{Z}_{earth,jj} = \begin{bmatrix} Z_{earth,jj} & Z_{earth,jj} & Z_{earth,jj} \\ Z_{earth,jj} & Z_{earth,jj} & Z_{earth,jj} \\ Z_{earth,jj} & Z_{earth,jj} & Z_{earth,jj} \end{bmatrix}$$
(5.10)

$$\mathbf{Z}_{earth,jk} = \begin{bmatrix} Z_{earth,jk} & Z_{earth,jk} & Z_{earth,jk} \\ Z_{earth,jk} & Z_{earth,jk} & Z_{earth,jk} \\ Z_{earth,jk} & Z_{earth,jk} & Z_{earth,jk} \end{bmatrix}$$
(5.11)

Where, the self-impedance  $Z_{earth,j}$  and mutual-impedance  $Z_{earth,jk}$  of earth return path using Wedepohl approach [79] are shown in equation (4.56) and (4.57).

Due to the symmetrical arrangement within a single-core cable, the proximity effect will be zero when considering the internal impedance alone. However, outside the cable core there are eddy currents flowing in the conducting layers due to the presence of other phases and the proximity effect will influence these, especially the outer most circulating current loops, thereby affecting the armour and sea return impedance matrix. The cable impedance matrix should therefore include the proximity effect  $Z_{pe}$  as:

$$\mathbf{Z}_{cable} = \mathbf{Z}_{i} + \mathbf{Z}_{pe} + \mathbf{Z}_{earth}$$
(5.12)

Where, to express  $\mathbf{Z}_{pe}$  in three-phase matrix form as:

$$Z_{pe} = \begin{bmatrix} Z_{pe-self,AA} & Z_{pe-mutu,AB} & Z_{pe-mutu,AC} \\ Z_{pe-mutu,BA} & Z_{pe-self,BB} & Z_{pe-mutu,BC} \\ Z_{pe-mutu,CA} & Z_{pe-self,CC} \end{bmatrix}$$
(5.13)

Where,  $Z_{pe-setf,AA}$ ,  $Z_{pe-setf,BB}$  and  $Z_{pe-setf,CC}$  is the self cable impedance matrix of proximity effect of phase A, B and C, respectively;  $Z_{pe-mutu,AB}$  and  $Z_{pe-mutu,BA}$  are mutual cable impedance matrices of proximity effect between phase A and B;  $Z_{pe-mutu,AC}$  and  $Z_{pe-mutu,CA}$  are mutual cable impedance matrices of proximity effect between phase A and C;  $Z_{pe-mutu,BC}$  and  $Z_{pe-mutu,CB}$  are mutual cable impedance matrices of proximity effect between phase B and C. The sub-matrices, self impedance matrix of proximity effect  $Z_{pe-self,jj}$  which is the representative of self cable impedance matrix of proximity effect of each phase, and mutual impedance  $Z_{pe-mutu,jk}$  which is the representative of mutual cable impedance matrix of proximity effect between each phase, are dependent on the internal impedance of  $Z_i$  which is shown in equation (5.9) as  $3 \times 3$  matrix since there are three conducting layers of the cable contributing internal impedance: conductor, sheath and armour. Therefore, the sub-matrix of the self impedance and the mutual impedance are  $3 \times 3$  matrices presented as:

$$\mathbf{Z}_{pe-self,jj} = \begin{bmatrix} Z_{pe-self,jj} & Z_{pe-self,jj} & Z_{pe-self,jj} \\ Z_{pe-self,jj} & Z_{pe-self,jj} & Z_{pe-self,jj} \\ Z_{pe-self,jj} & Z_{pe-self,jj} & Z_{pe-self,jj} \end{bmatrix}$$
(5.14)

$$\mathbf{Z}_{pe-mutu,jk} = \begin{bmatrix} Z_{pe-mutu,jk} & Z_{pe-mutu,jk} & Z_{pe-mutu,jk} \\ Z_{pe-mutu,jk} & Z_{pe-mutu,jk} & Z_{pe-mutu,jk} \\ Z_{pe-mutu,jk} & Z_{pe-mutu,jk} & Z_{pe-mutu,jk} \end{bmatrix}$$
(5.15)

Where,  $Z_{pe-self,y}$  is the self cable impedance of proximity effect and  $Z_{pe-mutu,jk}$  is the mutual cable impedance of proximity effect.

To obtain the self-impedance of proximity effect  $Z_{pe-self,j}$  and mutual impedance of proximity effect  $Z_{pe-self,j}$  when including the proximity effect of a single-core subsea cable, the general solutions of equation (5.6) can be modified [92] as specific solutions as follows:

$$Z_{pe-self,y} = 2\sum_{n=1}^{\infty} \frac{j\omega\mu_0 r_{ao}^{n-1}}{\pi} (\frac{r_{ao}}{s})^n (\frac{1}{s})^n \cdot \frac{I_n(r_{ao}\sigma_a)}{\frac{n}{r_{ao}} I_n(r_{ao}\sigma_a) + \frac{m_a}{\mu_a} I_{n-1}(r_{ao}\sigma_a)}$$
(5.16)

$$Z_{pe-mutu,jk} = \sum_{n=1}^{\infty} \frac{j\omega\mu_0 r_{ao}^{n-1}}{\pi} (\frac{r_{ao}}{s})^n (\frac{1}{s})^n \cdot \frac{I_n(r_{ao}\sigma_a)}{\frac{n}{r_{ao}} I_n(r_{ao}\sigma_a) + \frac{m_a}{\mu_a} I_{n-1}(r_{ao}\sigma_a)}$$
(5.17)

Where,  $\sigma_a = \sqrt{j \frac{\omega \mu_a \mu_0}{\rho_a}}$  is the complex constant for armour;  $\mu_a$  is the relative permeability of armour;

 $\rho_a$  is the resistivity of the armour;  $r_{ao}$  is the outer radius of armour;  $I_{n-1}(x)$  is first kind modified Bessel's function, order n-1.

# 5.4 Calculations of Harmonic Impedance of Three-Core Cables with Proximity Effect

When considering the impedance of three-core cables, the equations for Pipe-Type (PT) cable can be utilised. According to [96], because of non-concentric of cables the proximity effect needs to be taken into account and the impedance matrix, therefore, is expressed as [95]:

$$\mathbf{Z}_{cable} = \mathbf{Z}_{i} + \mathbf{Z}_{pi} + \mathbf{Z}_{conn} + \mathbf{Z}_{earth}$$
(5.18)

Where  $Z_i$  is the single-core internal impedance matrix;  $Z_{pi}$  is the cable impedance matrix with consideration of proximity effect with respect to armour;  $Z_{cons}$  is the connection impedance matrix of inner and outer surface of armour;  $Z_{carth}$  is the earth return impedance matrix. To preset in three-phase form:

$$Z_{cable} = \begin{bmatrix} Z_{i,A} & 0 & 0 \\ 0 & Z_{i,B} & 0 \\ 0 & 0 & Z_{i,C} \end{bmatrix} + \begin{bmatrix} Z_{pi-self,AA} & Z_{pi-sutu,AB} & Z_{pi-sutu,AC} \\ Z_{pi-sutu,BA} & Z_{pi-self,BB} & Z_{pi-sutu,BC} \\ Z_{pi-sutu,CA} & Z_{pi-self,CC} \end{bmatrix} + \begin{bmatrix} Z_{con} & Z_{con} \\ Z_{con} & Z_{con} & Z_{con} \\ Z_{con} & Z_{con} & Z_{con} \end{bmatrix} + \begin{bmatrix} Z_{earth,AA} & Z_{earth,AB} & Z_{earth,AC} \\ Z_{earth,BA} & Z_{earth,BC} \\ Z_{earth,CA} & Z_{earth,BC} \end{bmatrix}$$
(5.19)

Where,  $Z_{pi-self,AA}$ ,  $Z_{pi-self,BB}$  and  $Z_{pi-self,CC}$  is the self cable impedance matrix with consideration of proximity effect with respect to armour of phase A, B and C, respectively;  $Z_{pi-mutu,AB}$  and  $Z_{pi-mutu,BA}$  are the mutual cable impedance matrix with consideration of proximity effect with respect to armour between phase A and B;  $Z_{pi-mutu,AC}$  and  $Z_{pi-mutu,CA}$  are the mutual cable impedance matrix with consideration of proximity effect with respect to armour between phase A and C;  $Z_{pi-mutu,BC}$  and are the mutual cable impedance matrix with consideration of proximity effect with respect to armour between phase B and C;  $Z_{con}$  is the connection impedance matrix of inter and outer armour surface for each phase.

Due to the fact that the there are only two conducting layers of a single-core cable as conductor and sheath formulate the concentric individual cables inside the armour layer, the internal impedance  $\mathbb{Z}_{i,j}$  is made by  $2 \times 2$  matrix shown as following equations and consequently the  $\mathbb{Z}_{pi-self,jj}$ , as representative of the self cable impedance matrix with consideration of proximity effect with respect to armour of each phase,  $\mathbb{Z}_{pi-mutu,jk}$ , as representative of the mutual cable impedance matrix with consideration of proximity effect with respect to armour between each phase,  $\mathbb{Z}_{cen}$ ,  $\mathbb{Z}_{earth,jj}$  and  $\mathbb{Z}_{earth,jk}$  are also  $2 \times 2$  matrices. The armour is regarded as the pipe outside these single-core cables where the matrices  $\mathbb{Z}_{pi-self,jj}$  and  $\mathbb{Z}_{pi-mutu,jk}$ , taking account of proximity effect of inner cables with respect to the armour, need to be evaluated for their contribution on harmonic impedance of the cable.

$$\mathbf{Z}_{\mathbf{i},\mathbf{j}} = \begin{bmatrix} Z_{cc} & Z_{cs} \\ Z_{sc} & Z_{ss} \end{bmatrix}$$
(5.20)

$$\mathbf{Z}_{con} = \begin{bmatrix} Z_{con} & Z_{con} \\ Z_{con} & Z_{con} \end{bmatrix}$$
(5.21)

$$\mathbf{Z}_{\mathbf{pi-self},\mathbf{jj}} = \begin{bmatrix} Z_{\mathbf{pi-self},\mathbf{jj}} & Z_{\mathbf{pi-self},\mathbf{jj}} \\ Z_{\mathbf{pi-self},\mathbf{jj}} & Z_{\mathbf{pi-self},\mathbf{jj}} \end{bmatrix}$$
(5.22)

$$\mathbf{Z}_{\mathbf{p}i-\mathbf{mutu},\mathbf{j}\mathbf{k}} = \begin{bmatrix} Z_{pi-\mathbf{mutu},j\mathbf{k}} & Z_{pi-\mathbf{mutu},j\mathbf{k}} \\ Z_{pi-\mathbf{mutu},j\mathbf{k}} & Z_{pi-\mathbf{mutu},j\mathbf{k}} \end{bmatrix}$$
(5.23)

$$\mathbf{Z}_{earth,jj} = \begin{bmatrix} Z_{earth,jj} & Z_{earth,jj} \\ Z_{earth,jj} & Z_{earth,jj} \end{bmatrix}$$
(5.24)

$$\mathbf{Z}_{earth,jk} = \begin{bmatrix} Z_{earth,jk} & Z_{earth,jk} \\ Z_{earth,jk} & Z_{earth,jk} \end{bmatrix}$$
(5.25)

The single-core impedance equations of  $Z_{cc}$ ,  $Z_{cs}$ ,  $Z_{sc}$  and  $Z_{ss}$  can be found in Section 4.6 and again the self-impedance  $Z_{earth, jj}$  and mutual-impedance  $Z_{earth, jk}$  of earth return are shown in equation (4.56) and (4.57). The connection impedance equation  $Z_{con}$  of inter and outer surface of armour referring to the equations (4.45) and (4.70) as the outmost loop of the cable and take account of the mutual impedance of armour can be re-written as:

$$Z_{con} = Z_{armour-ex} - 2 \cdot Z_{armour-mutual} + Z_{armour/sea-insulation}$$
(5.26)

Where,

$$Z_{armour-ex} = \frac{\rho_a \sigma_a \left[ I0(\frac{\sigma_a D_{ao}}{2}) K1(\frac{\sigma_a D_{al}}{2}) + K0(\frac{\sigma_a D_{ao}}{2}) I1(\frac{\sigma_a D_{al}}{2}) \right]}{\pi D_{ao} I1(\frac{\sigma_a D_{ao}}{2}) K1(\frac{\sigma_a D_{al}}{2}) - I1(\frac{\sigma_a D_{al}}{2}) K1(\frac{\sigma_a D_{ao}}{2}) K1($$

$$Z_{armour-multual} = \frac{2\rho}{\pi D_{al} D_{ao} / 1(\frac{\sigma_a D_{ao}}{2}) K l(\frac{\sigma_a D_{al}}{2}) - / 1(\frac{\sigma_a D_{al}}{2}) K l(\frac{\sigma_a D_{ao}}{2})}$$
(5.28)

$$Z_{armour/sea-insulation} = j\omega \frac{\mu 0}{2\pi} \ln(\frac{D_{cable-out}}{D_{ao}})$$
(5.29)

Where,  $Z_{armour-ex}$ ,  $Z_{armour-mutual}$ ,  $Z_{armour/sea-insulation}$  and is the external impedance, mutual impedance and impedance of jacket insulation outside of armour per unit length, respectively;  $D_{ai}$  is the internal diameter of the armour;  $D_{ao}$  is the external diameter of the armour;  $D_{cable-out}$  is the outside overall diameter of the cable.

For the cable impedance matrix with consideration of proximity effect with respect to armour, self impedance  $Z_{pi-self,j}$  and mutual impedance  $Z_{pi-mutu,jk}$  are given by [95] [96]:

$$Z_{pr-self,j} = \frac{j\omega\mu_{0}\mu_{a}}{2\pi} \left[ \frac{K_{0}(\sigma_{a}r_{ai})}{\sigma_{a}r_{ai} \cdot K_{1}(\sigma_{a}r_{ai})} \right] + \frac{j\omega\mu_{0}}{2\pi} \ln\left[\frac{r_{ai}}{r_{so}}(1-(\frac{d_{c}}{r_{ai}})^{2})\right] + \frac{j\omega\mu_{0}\mu_{a}}{r_{ai}} \sum_{n=1}^{\infty} \frac{(\frac{d_{c}}{r_{ai}})^{2n} \cdot \cos(n\theta)}{n(1+\mu_{a}) + \sigma_{a}r_{ai}K_{n-1}(\sigma_{a}r_{ai})/K_{n}(\sigma_{a}r_{ai})}$$

$$Z_{pr-mulu,jk} = \frac{j\omega\mu_{0}\mu_{a}}{2\pi} \left[\frac{K_{0}(\sigma_{a}r_{ai})}{\sigma_{a}r_{ai} \cdot K_{1}(\sigma_{a}r_{ai})}\right] + \frac{j\omega\mu_{0}}{2\pi} \left[\ln\left[\frac{r_{ai}}{s}\right] - \sum_{n=1}^{\infty} \frac{(\frac{d_{c}}{r_{ai}})^{2n} \cdot \cos(n\theta)}{n}\right]$$
(5.30)

$$+\frac{j\omega\mu_{0}\mu_{a}}{\pi}\sum_{n=1}^{\infty}\frac{\left(\frac{d_{c}}{r_{a}}\right)^{2n}\cdot\cos(n\theta)}{n(1+\mu_{a})+\sigma_{a}r_{ai}K_{n-1}(\sigma_{a}r_{ai})/K_{n}(\sigma_{a}r_{ai})}$$
(5.31)

Where,  $r_{ai}$  is the inner radius of armour;  $r_{so}$  is the radius of the conducting layer right inside the armour as outer radius sheath;  $d_c$  is the distance between individual cable to the amour centre;  $\theta$  is the angle between cables in respect to armour centre.

The first term and second term of the right hand side of equations (5.30) and (5.31) represent impedance including skin effect according to the geometric arrangements. The proximity effect is the third term on the right hand sides of equations (5.30) and (5.31). This term will be included when the proximity effect needs to be considered. If proximity effect is not to be taken into account then this term can be ignored.

# 5.5 Proximity Effect on Single-Core and Three-Core Subsea Cables

#### 5.5.1 Harmonic Analysis of Proximity Effect using Phase Domain

As previous described, for high power three-phase AC transmission, cable systems are usually designed as either three single-core cables or as a single three-core cable. Because of recent developments in crosslinked polyethylene (XLPE) insulation for high power transmission [57], then XLPE-insulated cables have been chosen for analysis in this study.

**Case Study**: The cable configurations of three single-core trefoil touching cables and three-core cables are shown as **Figure 5.1**. The cables are considered to be lying on the sea bed at a depth of 50 m and the conductor temperature is 90 °C when operational. For safety reasons, solid-bonding of both the sheath and armour have been adopted. The parameters and size of 150 kV rating single-core and three-core subsea cable for this case are given in the **Appendix B.4**.

The harmonic resistance and inductance of the single-core and the three-core subsea cables per phase are plotted against frequency from the fundamental frequency, 50Hz, up to 30 orders in Figures 5.2 and Figure 5.3. For simplicity of comparison, the figures of harmonic resistance and inductance magnitudes are expressed as a ratio to the magnitudes found at the fundamental frequency when considering the proximity effect as  $R_h / R_{1pe}$  and  $L_h / L_{1pe}$ . Where,  $R_h$  is the harmonic resistance at each order;  $R_{1pe}$  is the resistance at fundamental frequency with consideration of proximity effect;  $L_h$  is the harmonic inductance at each order and  $L_{1pe}$  is the cable inductance at fundamental frequency with consideration of proximity effect.



Figure 5.2 Harmonic Resistances and Inductances of a Steel Armour Single-Core Subsea Cable

The significance of the proximity effect in both types of cables is demonstrated in these results. In general, the resistance increases as the frequency increases due to the skin effect. However because losses occur due to the circulating currents between the layers, the curve is non-linear. Also, the inductance of the conductor due to cancellation of flux linkage between layers tends to decrease as frequency increases.

The results in **Figure 5.2** compare the harmonic resistance and inductance of a steel armour single-core subsea cable. The harmonic resistance curves with and without inclusion of the proximity effect term can be seen to overlap each other. This is also seen for the inductance curves. The proximity effect only affects the outer most current loop i.e. the circulating current that flows between the armour and earth. The proximity effect has significantly reduced thereby affecting the impedance of the conductor, located within the sheath screen.



Figure 5.3 Harmonic Resistances and Inductances of a Steel Armour Three-Core Subsea Cable

However, in Figure 5.3 which compares the harmonic resistance and inductance of a steel armour threecore cable with and without the proximity effect term, than it is seen that there are differences between the curves. This is because three-core cables are located within a common armour shield. The phases are much closer to each other with only a sheath screen surrounding individual conductors. Observing in more detail, Figure 5.4 shows the differences in the results obtained with and without the proximity effect. The differences in the harmonic resistances and inductances are expressed as a percentage of the value obtained with proximity effect included. Again, for the single-core subsea power cable, there is nearly no difference between the resistance and inductance. On the other hand, for three-core subsea power cable, there are differences of up to 19 % at fundamental frequency. Also, it is higher up to 48 % difference at the third order (150 Hz) for the two methods. This clearly shows that for harmonic impedance evaluations then for three-core cables the consideration of proximity effect is necessary.



Figure 5.4 Differences of two methods (with and without proximity effect)

The analysis has clarified that due to single-core subsea cable has not only screen sheath layer but also armour layer shielding the conductor, the proximity effect due to other cables through sea return path on the conductor is significantly reduced and can be neglected. This explains why the calculations of cable impedance of proximity effect on impedance through sea or earth return path of single-core multi-layer cables buried underground or sea in [65] [67] [79] are not particularly detailed derived. Instead these papers adopt simplified equations which are proved to be able to present all practical cases of underground or subsea cables. For three-core subsea power cable, the results are in full accordance with previous work such as [84] [85] [86] [87] [88] [91] [96] which demonstrate that when the pipe-type cable is concerned, it is crucial to take into consideration the proximity effect from other cables with respect to the common pipe which can be regarded as the armour layer for a three-core subsea power cable.

#### 5.5.2 Harmonic Analysis of Proximity Effect using Sequence Domain

In order to understand the complex relationship between proximity effect on the earth return path or under unbalance system, the sequence domain needs to be used as an analysis tool. Zero sequence can be adopted as the comparison for phase domain in order to study the proximity effect behaviour of a subsea cable with common armour where grounding return path is important in respect to three-phase conductors inside the cable. Positive and negative sequences can be used to investigate how the proximity effect affects the harmonic impedance performance under asymmetrical conditions. The harmonic resistance and inductance of the single-core and the three-core subsea cables for sequence analysis are presented in **Figures 5.5**, **Figures 5.6**, **Figure 5.7**, **Figure 5.8**, **Figure 5.9** and **Figure 5.10** where the harmonic resistance and inductance are plotted against frequency at fundamental and h = 3n + 1order for positive sequence, at h = 3n - 1 for negative sequence and at h = 3n for zero sequence. Where, *h* is the harmonic order; h = 3n + 1 = 1, 4, 7, 10, 13, 16, 19, 22, 25, 28, 31 represent positive sequence harmonics; h = 3n - 1 = 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32 represent positive sequence harmonics; h = 3n = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33 represent positive sequence harmonics. Also the figures of harmonic resistance and inductance magnitudes are expressed as a ratio to the magnitudes found at the fundamental frequency taking consideration of proximity effect as  $R_h / R_{1m}$  and  $L_h / L_{1m}$ .

Figure 5.5, Figure 5.6 and Figure 5.7 shows that harmonic resistance and inductance of a single-core cable with and without proximity effect considerations in positive, negative and zero sequence, respectively. It clearly identify that the proximity effect has no influence on harmonic impedance in sequence domain analysis for single-core subsea cable. Also, although the harmonic resistance and inductance are plotted against different frequency, e.g. harmonic order at 3n+1 for positive sequence, 3n-1 for negative sequence and 3n for zero sequence, the trends of harmonic resistance and inductance in positive, negative and zero are similar, which are also indistinct from those curves of phase analysis in Figure 5.2. This implies that the single-core subsea cable can be regarded as a symmetrical component where harmonic characteristics are unchanged in positive, negative and zero sequence analysis.



Figure 5.5 Positive Sequence Resistances and Inductances for Single-Core Subsea Cable



Figure 5.6 Negative Sequence Resistances and Inductances for Single-Core Subsea Cable



Figure 5.7 Zero Sequence Resistances and Inductances for Single-Core Subsea Cable

When considering sequence domain analysis, the harmonic impedance in pipe-type cable such as threecore subsea power cable is an intriguing subject since cables located inside the armour which provide the common earth return path for the cables. Figure 5.8, Figure 5.9 and Figure 5.10 shows the harmonic resistance and harmonic inductance of a three-core subsea cable with and without proximity effect presented as positive, negative and zero sequence, respectively.



Figure 5.8 Positive Sequence Resistances and Inductances for Three-Core Subsea Cable



Figure 5.9 Negative Sequence Resistances and Inductances for Three-Core Subsea Cable



Figure 5.10 Zero Sequence Resistances and Inductances for Three-Core Subsea Cable

The proximity effect appears to play an important part for positive and negative sequence analysis since the curves of harmonic resistance and inductance with consideration of the proximity effect are different from those without consideration of proximity effect. These observations seem to indicate that the proximity effect can indeed play a significant role in calculating harmonic resistance and inductance in three-core subsea cable, which agrees to the previous discovery of phase domain analysis in **Figure 5.3** where the curve trends are also similar to those in positive and negative sequences analysis.

On the other hand, from zero-sequence analysis for a three-core subsea cable the curve of harmonic resistance and inductance including consideration of proximity effect and without consideration of proximity effect appear to have very little difference from each other and the curves trends are also appeared to be distinct from those of phase domain analysis in **Figure 5.3**. This points to the effect that the sea return path due to circulating current between armour and grounding path is not greatly affected by the proximity effect which mainly caused by the cables which are located inside the armour. Furthermore, the trend of the harmonic resistance and inductance curves of positive and negative sequences are different from those of zero sequence. This explains why the zero sequence impedance for pipe-type cable needs to be particularly evaluated referring to [97] since that its harmonic characteristic is distinct from those in positive and negative sequences. From this point of view, a three-core subsea cable can be regarded as an asymmetrical component for calculating the harmonic behaviours and this should be taken consideration before harmonic impedance calculations are undertaken.

# 5.6 Summary

In this chapter an evaluation of the proximity effect on subsea cables has been presented. The evaluation has been undertaken using the theory of superposition for determining the resistance and the inductance of the conductor for single-core and three-core subsea cables.

This study has demonstrated that the impedance for single-core subsea power cable is unaffected by the proximity effect and can therefore be neglected. However, the proximity effect is a significant factor in three-core subsea power cables and it should therefore be taken into account when calculating harmonic impedance. Also, from sequence domain analysis for single-core and three-core subsea power cable, proximity effect is also a significant factor of evaluation of harmonic impedance for three-core subsea cables. However, the proximity effect is not a concern for both zero sequence harmonic impedance for both single-core and three-core as the circulating current between grounding path and armour is not affected by the proximity effect between the cables.

The study presented here extends current knowledge of the proximity effect on modelling of harmonic characteristic of subsea power cables where the geometric arrangements and structures of cable are essential for determining the consideration of proximity effect in calculations. How conductors are shielded, the layer arrangements in respect to each cable and relative to each phase are critical features for deciding whether or not the proximity effect is influential. For the purposes of harmonic analysis of the subsea cable transmission system, then these issues need to be carefully considered at the very early stages of modelling to avoid significant errors.

# Harmonic Assessment on Bonding Methods of Subsea Power Cable

# 6.1 Introduction

The number and size of proposed offshore generation projects [3] is growing and future plans include 'far offshore' as well as more 'near shore' generation schemes. Long distances electrical power transmission technologies and cables are still being developed however their expected operation must be properly understood for reasons of power quality, system stability and efficiency. For high power and long distances, the use of a metallic sheath and armour is necessary in submarine cable designs to guard against cable damage to the insulation and conductor layers by external means e.g. anchor drops, fishing, etc. However, the metallic characteristics of the sheath and armour layers give rise to additional power loss in the cable. Also, these layers tend to be bonded for reasons that will be discussed later, using solid bonding, single-point bonding, or cross bonding methods, these being the three basic bonding methods currently in use [90]. The bonding method influences the power loss in the cable but additionally waveform quality, induced voltage, circulating currents, safety and economics are also important considerations.

From a harmonic point of view, the different bonding arrangements imply that the cable's electrical parameters will be different for each bonding method due to the distinct impedance contributed by the current and voltage of return path in each of the layers [78] described in the following section. It can be reasonably expected that the cable resonant frequencies will also be distinct for each type of bonding arrangement and also that harmonic performance will change with cable length

This chapter presents results from computer based simulations of the harmonic performance of high power XLPE-insulated cables in subsea transmission systems as used by offshore generators (e.g. wind farms). The particular focus of this analysis is on how the cable harmonic performance is influenced by the type of bonding method such as solid, single-point and cross bonding for the sheath and armour layers in the

cable. The methods used to analyse the frequency response and to determine the harmonic resonances for each bonding arrangement are fully explained. Computer simulations have been used to carry out investigations into the frequency harmonic response that occur in metallic armoured subsea cables in power transmission systems when using different bonding arrangements where the effect are presented and compared with a particular study on their performance at harmonic resonance. In the conclusions, the results obtained for the different transmission arrangements are discussed with due consideration of their impact for cable designers.

### 6.2 Bonding Methods

For safety reasons, submarine cable sheaths and armours are bonded to earth [90]. Three main types of bonding can be considered for submarine cables: Solid bonding, single-point bonding, and cross bonding as shown in Figure 6.1, Figure 6.2 and Figure 6.3, respectively. Solid bonding is where the sheath and armour are bonded together and to earth at both ends of the subsea transmission cable. This method is commonly adopted in submarine transmission schemes since it eliminates any induced voltages [80]. However the closed 'earth' circuit allows circulating currents to flow in the sheath and armour layers, which in turn diminish the current carrying capacity of the cable. To overcome this, single-point bonding can be used where the bonding of the sheath and armour to earth occurs at only one end of the cable, the other end being left open circuit. Although circulating currents in the sheath and armour layers are now eliminated, the induced voltage and potential differences along the sheath and armour can pose a danger especially near the free-end. Cross bonding is where the cable is divided into three separate sections with cross connecting of the sheaths and armour layers within the cable and these layers are also being earthed at each end [98]. The induced sheath and armour voltages in each section of each phase are equal in magnitude and  $\frac{2}{3}\pi$  degree out of phase so that each sheath and armour circuit contains one of the three sections from each phase and therefore the total voltage in each sheath and armour circuit sums to zero. At the end of the cable the sheath and amour are bonded and earthed and then the net voltage and circulating currents in the loops will be zero. Although this method eliminates both the potential voltage and circulating currents, some practical points must be addressed such as high voltage can take place on the sheath and armour joint during switching or transients and the sheaths and armour have to be carefully insulated. This method is comparative costly when compared to the other two methods.



Figure 6.1 Arrangements for Solid Bonding

The conductors (thick line) are not earthed, but the sheath and armour (thin lines) are bonded and earthed at both ends.



Figure 6.2 Arrangements for Single-Point Bonding

The conductors (thick line) are not earthed, but the sheath and armour (thin lines) are bonded and earthed at one end only.



Figure 6.3 Arrangements for Cross Bonding

The conductors (thick lines) are not earthed, but the sheath and armour (thin lines) are divided into three sections and cross connected and bonded and earthed at each end.

# 6.3 Harmonic Considerations of Bonding Methods

Bonding methods are known to influence power loss in subsea power cables [90] but for long distance offshore power transmission, it is also important to quantify the impact of harmonics in a power cable for the purposes of determining power quality, which would be expected to be influenced by the different bonding arrangements as well as cable length. It is well known [69] that cable parameters and cable length are the main factors that influence the cable's frequency response and consequently the frequencies at which harmonics are most likely to occur. The cable impedances and admittances are dependent upon the method of bonding being used and consequently the cable will experience different induced voltages, circulating currents, frequency response and harmonic resonances depending upon the cable's bonding method. However the influence of bonding on the frequency response of a cable is at present poorly understood. There is a need to develop the theory to understand a cable's frequency response and to examine the expected harmonic resonances of the cable, under the different bonding conditions and cable lengths, to gain an understanding of the influence of bonding in offshore transmission systems. Computer simulation offers the means to do this.

Long distance transmission systems will usually employ single-core cables for practical reasons [57]. Solid bonding solutions due to economic and safety reasons have been widely adopted in many submarine transmission systems. Nevertheless, as already mentioned one of the main disadvantages of this method is that there are circulating currents produced by electro-magnetic fluxes, which result in additional power loss. Therefore, it is usual to employ cables of the 'close touching' trefoil formation to minimise these circulating currents [54]. Hence, the three-phase single-core XLPE-insulated close touching trefoil formation cable arrangement has been selected for analysis and simulation in this chapter.

Harmonics in mutually coupled three-phase lines can be described in matrix form [64]. Impedance calculations of submarine cables require special consideration of their multi-layer construction and sea return current paths, which can be referred in [65] and also **chapter 4** and **chapter 5**, where detailed calculation of subsea cable impedance is given. As this study is to examine the influence of the different types of bonding on subsea cable frequency and harmonic response, the calculations of the harmonic impedance and admittance in loop equations in a variety of bonding solutions is an important aspect of the work presented here.

Impedance relationships may be obtained from [64] and [65] to include saturation effects in magnetic materials such as those that occur in the armour layer. According to [78], the general form for impedance matrix can be expressed as:
$$\begin{bmatrix} V_{core} \\ V_{sheath} \\ V_{armour} \end{bmatrix} = \begin{bmatrix} Z_{cc} & Z_{cs} & Z_{ca} \\ Z_{sc} & Z_{ss} & Z_{sa} \\ Z_{ac} & Z_{as} & Z_{aa} \end{bmatrix} \begin{bmatrix} I_{core} \\ I_{sheath} \\ I_{armour} \end{bmatrix}$$
(6.1)

Where,  $V_{core}$ ,  $V_{sheath}$ ,  $V_{armour}$  is the voltage per unit length in core, sheath and armour, respectively;  $I_{core}$ ,  $I_{sheath}$ ,  $I_{armour}$  is the current per unit length in core, sheath and armour, respectively;  $Z_{cc}$  is the self impedance of core;  $Z_{cs}$  is the mutual impedance between core and sheath;  $Z_{ca}$  is the mutual impedance between core and sheath;  $Z_{ss}$  is the self impedance of sheath;  $Z_{ss}$  is the mutual impedance between sheath and armour;  $Z_{ac}$  is the self impedance between sheath and armour;  $Z_{ac}$  is the self impedance between sheath and armour;  $Z_{ac}$  is the self impedance between sheath and armour;  $Z_{ac}$  is the self impedance of armour;

Similarly by adopting the same method to form harmonic admittance equation:

$$\begin{bmatrix} I_{core} \\ I_{sheath} \\ I_{armour} \end{bmatrix} = \begin{bmatrix} Y_1 & -Y_1 & 0 \\ -Y_1 & Y_1 + Y_2 & -Y_2 \\ 0 & -Y_2 & Y_2 + Y_3 \end{bmatrix} \begin{bmatrix} V_{core} \\ V_{sheath} \\ V_{armour} \end{bmatrix}$$
(6.2)

Where,  $Y_1$  is the admittance per unit length of the insulation between core to sheath;  $Y_2$  is the admittance per unit length of the insulation between sheath to armour;  $Y_3$  is the admittance per unit length of the insulation of jacket outside armour

However, since the bonding method can be expected to affect the cable impedance, the impedance and admittance equations must include an appropriate modification.

# 6.3.1 Harmonic Equations for Solid Bonding

For solid bonding method since the both end of sheath and armour cable is earthed and therefore the voltage of these metallic layers can be assumed as grounded.

$$V_{sheath} = V_{armour} = 0 \tag{6.3}$$

The equations for harmonic impedance calculations in each layer for solid bonding in subterranean cables are described in [90] where consideration of the earth return path was found to be necessary. Applying this method to a submarine cable, the earth return path can be regarded as being the sea return and the appropriate calculation process using Wedepohl approach [79] and the detailed equations of sea return are shown in Section 4.6.

Using boundary condition of equation (6.3), the system matrix of harmonic impedance equation (6.1) can be reduced to:

$$V_{core} = Z_b \cdot I_{core} \tag{6.4}$$

Where,  $Z_b$  is the equivalent impedance for subsea cable under the solid bonding method and is given as:

$$Z_{b} = Z_{cc} + Z_{cs} \cdot X_{2} + Z_{ca} \cdot X_{3}$$
(6.5)

Where,  $X_2$  and  $X_3$  is the calculation coefficient and can be expressed as:

$$X_{2} = \frac{Z_{sa} \cdot Z_{ac} - Z_{sc} \cdot Z_{aa}}{Z_{ss} \cdot Z_{aa} - Z_{sa} \cdot Z_{as}}$$
(6.6)

$$X_{3} = \frac{Z_{sc} \cdot Z_{as} - Z_{ss} \cdot Z_{ac}}{Z_{ss} \cdot Z_{aa} - Z_{sa} \cdot Z_{as}}$$
(6.7)

However, the induced sheath and armour voltages are zero so the admittance matrix equation (6.2) can be simplified:

$$I_{corr} = Y_1 \cdot V_{corr} \tag{6.8}$$

# 6.3.2 Harmonic Equations for Single-Point Bonding

For single-point bonding, since there is no circulating current flowing in the sheath and armour, the boundary condition can be presented as:

$$I_{shealh} = I_{armour} = 0 \tag{6.9}$$

By calculating the matrix equation (6.1) with the condition of equation (6.9) and with sea return considered, the impedance equation can be modified to:

$$V_{corre} = Z_{cc} \cdot I_{corre} \tag{6.10}$$

For the single-point bonding condition, magnetic flux linkage can be induced by the conductors. The self impedance of conductor core  $Z_{cc}$  is then given as:

$$Z_{cc} = R_c + j\omega_h \cdot L_{cc} \tag{6.11}$$

Where  $R_c$  is the resistance of core conductor including the skin effect;  $L_{cc}$  is the inductance between core conductors is given as:

$$L_{cc} = \frac{\mu_0}{2\pi} \ln(\frac{s}{r_{cond}})$$
(6.12)

Where, s is the distance between conductors;  $r_{cond}$  is the radius of conductor.

$$\omega_h = 2\pi f_h \tag{6.13}$$

Where,  $\omega_h$  is the harmonic angular velocity;  $f_h$  is the harmonic frequency.

The admittance calculations for single bonding condition are complicated by the fact that the sheath and armour induced charges will generate electric fields within the cable. Applying terminal conditions, matrix equation (6.2) can be developed as:

$$I_{core} = Y_{sp} \cdot V_{core} \tag{6.14}$$

Where,  $Y_{ip}$  is the equivalent admittance for subsea cable under the single-point bonding method and is given as:

$$Y_{sp} = Y_1 \cdot \left[ 1 - \frac{Y_1 \cdot (Y_2 + Y_3)}{Y_1 \cdot Y_2 + Y_1 \cdot Y_3 + Y_2 \cdot Y_3} \right]$$
(6.15)

# 6.3.3 Harmonic Equations for Cross Bonding

For cross bonding method, if the lengths and cable spacing of each section is identical the losses from circulating current in sheath and armour of the cable can be reasonably assumed as zero [90]. Also the induced voltage in the sheath and armour of the cable, due to both bonded to the earth, can also be presumed as having no potential difference between two ends of the cable [57]. The boundary condition of cross bonding method can now be defined as:

$$V_{sheath} = V_{armour} = 0 \text{ and } I_{sheath} = I_{armour} = 0 \tag{6.16}$$

Therefore, for cross bonding the equation (6.11) can be developed as an impedance equation, and equation (6.8) can be developed as admittance equation.

Now by applying known cable characteristics to the calculations as previously described in Section 4.6 and by employing the ABCD transmission line transfer matrix illustrated in Section 4.4, the frequency or harmonic response of the cable to different to bonding conditions can be established and the cable resonant frequencies can be identified.

# 6.4 Simulation of Subsea Cable Harmonics under Different Bonding Methods

The multi-layer subsea cable impedance and admittance equations previous described must also account for the skin effect and the mutual coupling between cables for each bonding method. In order to understand the effect of bonding methods on harmonic behaviour of subsea power cable, a test model in **Figure 6.4** is set to test the harmonic characteristic under different bonding method. The cable bonding for this case can be adjusted using different bonding conditions and cable lengths as required.



Figure 6.4 Subsea Power Cable Harmonic Test System under Different Bonding Methods

**Case Study:** Consider a simple balanced three-phase transmission system, as shown in **Figure 6.4**, consisting of three open-circuit subsea single-core XLPE-insulated power cables excited by a one per-unit balanced three-phase voltage source at each harmonic order having a fundamental transmission frequency of 50 Hz and rating of 145 kV. Because of the need to meet the boundary condition as described for cross bonding in the previous section where the spacing between cables is assumed to be equal, the cables of this study case are set to be arranged in a trefoil configuration having a conductor diameter size of 26.4 mm, XLPE insulation thickness of 16 mm, which is surrounded by a copper sheath and an armour layer having 60 steel round wires. The dimensions of the cable are shown in **Appendix B.5**.

Due to the balanced system and single-core trefoil touching configuration of the cable, one of the threephase will be presented for the following equations and results for simplicity purpose. The detailed harmonic resonance calculation for a transmission line is presented in Section 4.4 so following equations are to simply show how the calculation is conducted for this case. Harmonic resonances are calculated by applying the cable harmonic calculation matrix given by equation (6.17), which determines the harmonic voltage at receiving end as ABCD transfer matrix [69] where the detailed calculation processes can be found in Section 4.4.

$$\begin{bmatrix} \mathbf{V}_{s} \\ \mathbf{I}_{s} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{r} \\ \mathbf{I}_{r} \end{bmatrix}$$
(6.17)

Where,  $\mathbf{A} = \cosh(\frac{\Gamma l}{2})$ ;  $\mathbf{B} = \mathbf{Z}_{e} \sinh(\frac{\Gamma l}{2})$ ;  $\mathbf{C} = \frac{1}{\mathbf{Z}_{e}} \sinh(\frac{\Gamma l}{2})$ ;  $\mathbf{D} = \cosh(\frac{\Gamma l}{2})$ ;  $\mathbf{V}_{a}$  and  $\mathbf{V}_{r}$  is the voltage matrix of sending end and receiving end, respectively.  $\mathbf{I}_{a}$  and  $\mathbf{I}_{r}$  is the current matrix of sending end and receiving end, respectively. And where  $\Gamma$ ,  $\mathbf{Z}_{e}$  and l is the wave propagation matrix, characteristic impedance matrix, and length of the cable, respectively. The wave propagation and the characteristic impedance are given by:

$$\Gamma = \sqrt{\mathbf{Z}_{\mathbf{c}\mathbf{r}} \cdot \mathbf{Y}_{\mathbf{c}\mathbf{r}}} \tag{6.18}$$

$$\mathbf{Z}_{\mathbf{c}} = \sqrt{\frac{\mathbf{Z}_{\mathbf{cx}}}{Y_{\mathbf{cx}}}} \tag{6.19}$$

Where,  $Z_{ex}$  is the equivalent cable impedance matrix under different bonding methods and  $Y_{ex}$  is the equivalent cable capacitance admittance matrix under different bonding methods acquired from the equations in previous section and  $Z_{ex} \neq \frac{1}{Y_{ex}}$ .

Because it is an open-end circuit, the receiving end current can be set as zero as:

$$\mathbf{I}_{\mathbf{r}} = \mathbf{0} \tag{6.20}$$

Therefore, matrix equation can be simplified as:

$$\mathbf{V}_{\mathbf{r}} = \mathbf{A}^{-1} \mathbf{V}_{\mathbf{s}} \tag{6.21}$$

It is now important to validate the test model under different bonding methods in order to further analyse the harmonic resonance different between each method. However, due to lack of hard data, other studies may be adopted as references for verifying the model which has been developed in this chapter. It has been experimental investigated that the armoured cable under solid and single-point bonding conditions [54] where the harmonic resistance of solid bonding method is higher than those under single-point method as frequency increased. In addition, the harmonic inductance reduction against frequency for the solid bonding method is much stronger than those under the single-point method. Since there is only the harmonic impedance data for solid and single-point bonding methods available, the impedance results of these two from the model has been chosen for validation purpose.

Figure 6.5 shows the results of harmonic resistance and inductance of solid and single-point methods plotting against frequency from fundamental frequency 50 Hz to 5000 Hz using the analytical calculations

introduced in Chapter 5. For comparison reason, the harmonic resistance is presented as the ratio of the DC resistance of the cable against the frequency as  $R_h / R_{dc}$  and the harmonic inductance is presented as the ratio of the harmonic inductance under single-point method at fundamental frequency as  $L_h / L_{lip}$ . Where,  $R_h$  is the harmonic resistance at each order;  $R_{dc}$  is the DC resistance DC of the cable;  $L_h$  harmonic inductance at each order;  $R_{dc}$  is the DC resistance DC of the cable;  $L_h$  harmonic inductance at each order;  $L_{lip}$  is the cable inductance at fundamental frequency under single-point bonding method.

The results show that the similar trends of harmonic resistance and inductance against frequency compared to the experimental data of cable harmonic figures of solid boding method and single-point bonding method shown in [54]. Hence the accordance of the trends gives confidence in the analytical calculations allowing further analysis to be carried on. For cross bonding method due to there being no available data so far for harmonic impedance trend study particularly for subsea multi-layer power cable, it is difficult to validate. Nevertheless it is believed that the trend of the harmonic resistance and inductance curves against frequency will be similar to single-point method [57] since there is minimal induced current in the sheath and armour having very little affect on the harmonic resistance and inductance of the cable.



Figure 6.5 Cable Harmonic Resistances and Inductances under Different Bonding Methods

# 6.5 Harmonic Resonance Analysis of Subsea Cable under Different Bonding Methods

For a XLPE-insulated submarine cable transmission test system described in [27], the transmission length is represented as a function of transmission voltage up to 200 km. Therefore, 50 km, 100 km and 150 km have been chosen as representative cases in this study to allow the relationship of the subsea cable harmonic performance to the transmission length to be evaluated.

During the investigations, exactly the same frequency responses and harmonic resonances were found to occur in each phase of the single core trefoil arrangement, therefore only the results from one phase are presented in the figures that follow. The frequency response for a XLPE-insulated trefoil formation single-core cable in solid bonding, single-point bonding and cross bonding for 50 km, 100 km and 150 km are shown in **Figures 6.6** to **6.14**, respectively, where the receiving end voltages have been plotted against frequency at 50 Hz intervals from a fundamental frequency of 50 Hz to 5,000 Hz. In these figures, the plots correspond as follows: solid line is for solid bonding method; dash line is for single-point bonding method.

# 6.5.1 Resonance Magnitude Analysis

**Figure 6.6, 6.7** and **6.8** show the harmonic voltage at the receiving end under solid, single-point and cross bonding methods for 50 km. It is noted from these results that the frequency response and harmonic resonances do, as expected, depend upon the bonding method used. For 50 km distance the single-point bonding contributes the greatest magnitude of resonance (first resonance is 34.80 p.u. at 950 Hz). For solid bonding cable inductance is reduced by the circulating currents that flow in the sheath and armour thereby giving the lowest magnitude of resonance (first resonance is 5.77 p.u. at 800 Hz). When using cross bonding the results show that the resonance magnitude is less than single-point bonding but greater than solid bonding (first resonance is 15.06 p.u. at 550 Hz).

Now consider the results using the different bonding methods for transmission lengths of 100 km of **Figure 6.9, 6.10** and **6.11** and 150 km of **Figure 6.12, 6.13** and **6.14** where a similar phenomenon to that at 50 km is seen to occur. The greatest difference for resonance magnitude of using different length is that as the length increased the resonance magnitude is reduced and this phenomenon is applied to all three bonding methods. To compare these three methods under different length, it is noted that the single-end bonding method generates the greatest resonance peak, solid bonding has the lowest magnitude of resonance and the cross bonding method creates the resonance magnitude in between of single-end bonding method and solid bonding method.



Figure 6.6 Harmonic Voltage of 50 km Cable under Solid Bonding Method



Figure 6.7 Harmonic Voltage of 50 km Cable under Single-Point Bonding Method



Figure 6.8 Harmonic Voltage of 50 km Cable under Cross Bonding Method



Figure 6.9 Harmonic Voltage of 100 km Cable under Solid Bonding Method



Figure 6.10 Harmonic Voltage of 100 km Cable under Single-Point Bonding Method



Figure 6.11 Harmonic Voltage of 100 km Cable under Cross Bonding Method



Figure 6.12 Harmonic Voltage of 150 km Cable under Solid Bonding Method



Figure 6.13 Harmonic Voltage of 150 km Cable under Single-Point Bonding Method



Figure 6.14 Harmonic Voltage of 150 km Cable under Cross Bonding Method

# 6.5.2 Resonance Frequency Analysis

Figures 6.6 to Figure 6.14 show the relationship between the resonance peaks and frequency. Comparing the results for the three bonding methods, the cross bonding method has the shortest frequency span between any two adjacent resonant frequency peaks when compared to the solid and single end bonding methods.

Now consider the effect of the length on the resonant frequency and peak amplitude using Figures 6.6 through to Figure 6.14. The results indicate that as the length of cable increases the frequency span between any two adjacent resonance frequencies reduce i.e. becomes narrower. Further, the single-point bonding method contributes the least resonances within a frequency rage. For example: for 100 km cases the single methods generates 5 resonances form 0 to 5000 Hz compared to 6 for solid bonding method and 9 for cross bonding method. This implies that the cross bonding method has more difficulty in avoiding resonance frequencies in a operational transmission system since there are more resonances in a frequency rage need to be avoided than the other two methods.

An equation commonly used to predict the harmonic order at which resonant peaks occur in a simple transmission line under voltage excitation is given in (6.22) [69]. Due to the complicated structure of subsea cable, this equation is not able to accurately predict the harmonic resonance of subsea cable but the equation indicates that the harmonic resonance is inversely proportional to the length of transmission line.

$$h_{resonance} = \frac{\lambda}{4l} \tag{6.22}$$

Where

 $h_{resonance}$  is the harmonic order where resonance occur  $\lambda$  is the wavelength at fundamental frequency l is the length of the transmission line

### 6.5.3 Q Factor Analysis

Quality factor (Q Factor) is widely used to assess the 'quality' of the resonant phenomena. The Q factor is the geometric mean of the two half-power frequencies; the upper half-power frequency and the lower half-power frequency as shown in **Figure 6.15**.



Figure 6.15 Two Half-Power Frequencies

The half-power frequencies are the corresponding frequencies where the power is half of the power that occurs at the resonant frequency. The resonant frequency is centrally located between the lower and upper half-power frequencies. The Q factor can be expressed as:

$$Q = \frac{f_{res}}{f_{Hres} - f_{Lres}}$$
(6.23)

Where,  $f_{res}$  is the resonant frequency, and  $f_{Hres}$  is the upper half-power frequency and  $f_{Lres}$  is the lower half-power frequency.

The frequency separation between the two half-power frequencies is known as bandwidth (BW):

$$BW = f_{Hres} - f_{Lres} \tag{6.24}$$

Therefore, when the Q factor is high, it implies that the frequency response at resonance is sharp and narrow with significant peak amplitude. On the other hand, when Q factor of a resonance is low, the resonance is damped with low peak amplitude.

Now, consider the Q-factor, the resonances and the bandwidths for the three bonding conditions. An improved finer resolution was used in the models to acquire harmonic data and the results obtained are given in **Table 6.1** for the first resonance for each bonding method in a 50 km cable:

	First Resonance Frequency (Hz)	Bandwidth (Hz)	Q Factor
Solid	809	187	4.33
Single-Point	943	33	28.58
Cross	531	24	22.13

Table 6.1 Resonance and Q factor data of the first resonance for various bonding methods

It is evident that the highest Q factor occurs when the single-point bonding method is used, which is 6.6 times of the solid bonding method and 1.29 times of the cross bonding method. This method also has the greatest magnitude of response of the three different bonding methods. For cross bonding methods, the first resonance takes place at a relatively low frequency at 531 Hz compared to 809 Hz for solid bonding method and 943 for single-point bonding method. This method also has a higher Q factor than solid bonding but less than that of the single-point bonding arrangement. For solid bonding method due to the much wider bandwidth as 187 Hz compared to those bandwidths in the other two methods, 33 Hz for single-point method and 24 Hz for cross bonding method, it has the lowest Q-factor and resonance occurs at the highest frequency for the first cable natural resonance.

In short, it should be noted that the large capacitance in a submarine cable may be used to reduce the size of any capacitor banks, which are often utilised in filter arrangements to reduce harmonic distortion [41]. This implies that harmonic response for solid bonding and cross bonding solutions, which provide greater capacitance than single-end bonding due to the admittance reduction in the conductor through the interaction of electrical fields between layers in single-end bonding arrangements, tend to offer more capacitance to the system thereby reduce such harmonic distortion. Considering the Q factor, the single-point bonding method generates the highest value of all three methods. This implies that the single-point method will give the greatest amplitude response to an exciting source at or near to this frequency. In terms of the resonant frequency, cables using the cross-bonding method have the shortest span between adjacent resonant peaks thereby potentially being more difficult to avoid excitation where a broad harmonic source such as a switching converter is used.

# 6.6 Summary

This chapter has reported on investigations, using computer based simulation methods, of harmonic resonances that may occur in XLPE-insulated armour submarine power transmission cables when using solid, single-point and cross bonding methods for distances of 50 km, 100 km and 150 km.

Solid bonding gives rise to additional power losses caused by the circulating currents that flow in the sheath and armour layers. It also generates less harmonic voltage resonance at the receiving end compared with the other methods. On the other hand, the low loss single-point bonding method creates large magnitude resonant peaks which have the potential to lead to significant damage should the cables be excited by a harmonic near to or at one of its resonant frequencies. The cross-bonding method, with no circulating currents and no induced voltages in the sheath and armour layers, produces resonances whose magnitudes are in between the resonance magnitudes generated by solid bonding and single-point bonding method.

The Q-Factor results show that the solid bonding method has the lowest value i.e. damped response and the single-point bonding method has the highest value, i.e. un-damped response of three methods considered. The length of the transmission line has a significant impact upon the span of adjacent resonant frequencies. The cross-bonding method generates the shortest span of the resonant frequency period between any two adjacent resonances of the three methods considered.

This study contributes to the knowledge of the natural frequency response of subsea cable transmission system under different bonding methods. The results presented in this study demonstrate conclusively that the frequency response and harmonic resonances are highly dependent upon the bonding method used. From the resonance response point of view, solid-bonding provides the lowest magnitude at harmonic resonance. This bonding method is currently widely adopted for offshore power transmission systems. It is essential to appreciate the influence of harmonics for the different bonding methods at the design stage to avoid any possibility of cable damage caused by any resonant harmonics, i.e. knowledge of its frequency response and the harmonics in the power system are equally important.

# 7

# Harmonic Performances of Subsea Power Cables in HVAC Transmission Systems

# 7.1 Introduction

All offshore generation schemes require subsea transmission cables to transmit generated electrical power from the offshore generator to a shore-based landing station where the power can be fed into an existing shore-based power distribution system infrastructure. Two methods of subsea electrical transmission are currently considered to be technically and economically feasible; DC link schemes and AC transmission schemes operating at either 50 or 60 Hz. It is generally accepted that DC link schemes provide efficiency advantages over AC schemes over long distances whilst AC transmission schemes are more cost effective over distances of less than 50 km [6], in part because they do not require converter and inverter stations at each end of the transmission system. To date, most offshore wind power generators have adopted AC schemes because so far most existing and near future projected wind farms are located near shore with small capacity [4] [29].

Large offshore wind farms have generally been located 'near shore' but plans for 'far offshore' generators are well advanced. Due to the charging current, higher concutor losses and limitation of insulation for high votage stress, AC subsea transmission schemes have traditionally been regarded as less attractive than equivalent DC transmission schemes for medium and long distance transmission. A HVDC transmission scheme would normally be favoured for long distance transmission using converters and inverters at each end of the transmission line to rectify the generated power into DC and invert the DC power back into the ac grid network but nevertheless, the high-cost of converters/inverters that accompany HVDC transmission schemes and their need for appropriate platform space at sea and ashore is a big concern to wind farm operators [2] [27] [28].

Because of the material technology developments in AC XLPE-insulated cables, then the drawback of high voltage stress has been overcome [99], making AC XLPE-insulated cables available for over distances of up to 100 km. It is reported [27] that Cross Linked Polyethylene (XLPE) insulated submarine cables are expected to be available for AC voltages up to 245 kV in the near term. Thus, high voltage AC transmission distance is possible to be extended beyond the current limit of around 80 km, thereby making HVAC schemes more competitive against HVDC schemes. Also, it has been shown [27] using simulation methods, that for an HVAC and HVDC offshore transmission having the same capacity and distance, the efficiency of HVDC transmission is actually lower than for HVAC system with the converter stations accounting for most of the loss. Furthermore, AC transmission systems are considered to be far more manageable since paralleling multiple generators, voltage step-up and step-down for integration into national grid networks is much simpler.

As described in **Chapter 4** the design of any new AC subsea transmission scheme needs to consider the harmonic effect which could be generated by power devices and influenced by transmission system itself. Power transmission cables playing an important part of the transmission scheme will affect the harmonics in the system. Offshore subsea power cables need to be heavily armoured and are consequently complicated structures having many concentric layers of different materials. Inductive couplings across each and every material boundary contribute to the overall cable impedance and these complex relationships consequently affect the level of voltage and current waveform distortion that will be experienced [65]. Additionally, according to [49] the configuration or arrangement of each cable relative to each other is another important factor that influences cable impedance.

AC subsea cable designs vary considerably and a thorough investigation of harmonic performance is therefore required for each new offshore generation and transmission system. It is important in subsea transmission system modelling to develop appropriate models of the subsea cables that includes all these effects so as to obtain the accurate performance prediction. In this chapter, computer based models for different AC subsea cable lay arrangements with different layered structures have been developed and investigations have been carried out to determine their frequency response, the magnitudes of resistance and inductance, and the frequencies at which resonance will occur. Results from harmonic performance studies, in which an offshore AC generation and transmission scheme supplies a SVC (static VAR compensator) used to compensate for reactive power in the AC transmission system, are also considered.

# 7.2 Arrangements of AC Subsea Power Cable

As mentioned in **Chapter 3**, subsea cables designed for AC transmission may be diverse in their construction but they are usefully divided into single or multiple core types. It is widely accepted that efficient power transfer takes place at high voltage [57]. For high power three-phase AC transmission, cable systems are usually designed as three single-core cables or alternatively as a single three-core cable.

In terms of construction as described in **Chapter 3**, subsea cables can be divided into three types; selfcontained fluid-filled, mass-impregnated non-draining, and extruded cross-linked polyethylene insulated types, which are abbreviated as SCFF, MIND, and XLPE respectively. Again, owing to a number of advantages discussed in [57] and current trends, the XLPE cables are carried forward for further analysis in this chapter. However, the same analysis methodology can also be applied to other types of subsea power cable.

From a cable configuration point of view, the geometry and the position of a subsea cable transmission system will influence its electrical characteristics and its frequency response. The resistance, inductance and capacitance characteristics are dependent upon the conductor, sheath and armour material characteristics and their geometrical arrangement and also the spacing between cables. Consequently, cable designs having different construction and transmission lay arrangements will possess their own distinct frequency and harmonic response characteristics.

For AC single-core armoured cable arrangement, there are three configurations that are extensively used in the industry: These are trefoil touching, flat touching and flat spacing configuration. Trefoil touching configuration is shown as **Figure 7.1 (a)** where three cables are arranged as a trefoil formation and touching each other. Flat touching configuration is shown in **Figure 7.1 (b)** and the cables are laid on seabed where the central cable are in contact with the other two cables either side. Flat spacing configuration is the arrangement where the cables are laid flat but there is space between adjacent cables. **Figure 7.1 (c) (d)** shows the flat spacing configuration where there are 1 m distances between adjacent cables for (c) and 10 m distance for (d). For AC three-core armoured cable arrangement, a common arrangement is shown as **Figure 7.1 (e)** where the three sheathed cables are trefoil arrangement in contact and within common armour. At the present time, a three-core cable has a limit in manufacturing for cables of large diameters as there are difficulty in winding around the drum [57] so for a subsea power system of voltage rating higher than 150 kV this cable is regarded as not practical.





c. Single-Core Flat Arrangement Cables with 1m Intervals



d. Single-Core Flat Arrangement Cables with 10m Intervals



e. Three-Core Trefoil Arrangement Cables

Figure 7.1 AC Subsea Cable Configurations

An electrical transmission system can magnify harmonic voltages or harmonic currents that happen to be at or near to a characteristic resonant frequency [63]. In **chapter 6**, it has been proved that the harmonic characteristics of cable are indeed influenced by the bonding methods and the length of the cable. In addition, according to [37], the resonant frequencies in transmission cables are highly influenced by the cable's electrical characteristics, which are in turn dependent upon their geometrical arrangement and material layer structures. It is therefore essential to understand how the cable configurations impact on the harmonic performance for a subsea power transmission cable.

**Case Study:** This study adopts typical designs suggested by ABB with 1000 mm<sup>2</sup> copper cross-section conductor for single-core cable and 630 mm<sup>2</sup> copper cross-section conductors for three-core cable. The thickness and material of the insulation and sheath layers were selected to give comparable dielectric strengths. All the cables considered were well armoured. The different arrangements of XLPE cables for AC subsea transmission systems are illustrated in **Figure 7.1** for the study purpose of comparison. The cables are considered to be lying on the sea bed at a depth of 50 m and the conductor operational temperature is 90 °C. Due to safety reasons, the solid bonding arrangement for both sheath and armour has been adopted and a cable of break even length for HVAC and HVDC of 50 km has been chosen. The parameters and size of typical 150 kV rating single-core and three-core subsea cable are given in **Appendix B.6**.

As discussed earlier in Chapter 4 and according to [64] [65], the impedance of a transmission cable must take account of the coupling effects between conductors, sheath, and armour in each phase and also between phases. The impedance and admittance of each layer can be expressed as loop equations as described in Section 4.5 and referred to Figure 4.5. Since the inside structure for each single-core cable configuration as Figure 7.1 (a) (b) (c) and (d) are effectively the same, the loop equations using superposition equation (4.42) to (4.55) are applicable to these cases where the values of cable size and material are identical for each case. However, the major distinction for three-phase single-core subsea cables harmonic impedance calculations under different configuration is the outmost earth return path between cables where the value for the distance between cables are different from each case, which in turn the equations of equation (4.57) of sea return path mutual impedance for three phase matrix derivation should take different value of cable distance  $\delta_{i,j}$  for each case. In other words, to use the matrix of three phase expression of equation (5.8) in Chapter 5 the sea return path of self impedance of  $Z_{earth,AA}$ ,  $Z_{earth,BB}$  and  $Z_{earth,CC}$  are essentially the same throughout the single-core cable case of (a) (b) (c) and (d) but the sea return path of mutual impedance of  $Z_{earth,AB}$ ,  $Z_{earth,AC}$ ,  $Z_{earth,BA}$ ,  $Z_{earth,CA}$  will be different from case to case. For three-core trefoil subsea cable configuration of case (e), the detailed calculations of harmonic impedance are introduced in **Chapter 5** where the cables located inside the common armour can be considered as single-core cable with conductor and sheath conducting layers only and the calculation processes can also refer to **Section 4.5**. The equation (5.18) to (5.30) can be applied as the three-phase matrix equations for three-core subsea cable of the harmonic impedance calculations including the considerations of proximity effect.

# 7.3.1 Comparison of Wedepohl and Wilcox's Appraoch and Bianchi and Luoni's Approach for Sea Return Path Impedance

From the subsea cable arrangement point of view, it is known that the main difference of harmonic impedance calculations of harmonic impedance for each configuration of single-core subsea cables is the outmost loop of sea return path as described in the previous section. The main difference between single-core subsea cable case of (a) (b) (c) and (d) is the geometrical lay arrangement. In each case, the cable impedance is influenced by other existing cables on harmonic behaviours through sea return path where the self impedance of earth return is caused by cable itself and the mutual impedance of sea return path influenced by other existing cables.

The common approach taken to calculate the earth return path, which in our case is the sea return, was originally developed by Wedepohl and Wilcox and is widely used in multi-layer cable impedance calculations and consequently has been adopted for this study. This approach is a simplified solution for a complicated equation of electromagnetic-wave propagation and the detailed derivations can be found in [79] where the equation (4.56) is the self impedance of earth return path and equation (4.57) is the mutual impedance of earth return path. However, Bianchi and Luoni also derived simple equations for multi-layer cable harmonic impedance calculations and these equations are the specific solutions using Bessel's functions and are particularly developed for single-core heavy armoured subsea power cable. The detailed equations can be found in [65] and shown as follows:

For the self-impedance of sea return path in Bianchi and Luoni's approach:

$$Z_{sea-in,self} = \frac{\sigma_{sea}\rho_{sea}}{2\pi r_{sea}} \frac{K0(\sigma_{sea}r_{sea})}{K1(\sigma_{sea}r_{sea})}$$
(7.1)

For the mutual-impedance of sea return path in Bianchi and Luoni's approach:

$$Z_{sea-in,mutual} = \frac{\sigma_{sea}\rho_{sea}}{2\pi r_{sea}} \frac{K0(\sigma_{sea}\delta_{i,j})}{K1(\sigma_{sea}r_{sea})}$$
(7.2)

Where,  $r_{sea}$  is the sea return path radius as the external radius of the cable;  $\delta_{i,j}$  is the distance between cable i and cable j;  $\sigma_{sea} = \sqrt{j \frac{\omega \mu_{sea}}{\rho_{sea}}}$  is the complex propagation constant in the sea;  $\rho_{sea}$  is the resistivity of the sea which is 1 ( $\mu\Omega$ m) of sea water;  $\mu_{sea}$  is the permeability of the sea.



Figure 7.2 Harmonic Resistances and Inductance of Case (a) Using Two Approaches for Sea Return



Figure 7.3 Harmonic Resistances and Inductance of Case (b) Using Two Approaches for Sea Return



Figure 7.4 Harmonic Resistances and Inductance of Case (c) Using Two Approaches for Sea Return



Figure 7.5 Harmonic Resistances and Inductance of Case (d) Using Two Approaches for Sea Return



Figure 7.6 Harmonic Resistances and Inductance of Case (e) Using Two Approaches for Sea Return

These two approaches adopting different equations to estimate the sea return impedance need to be clarified prior to further study. In addition, for three-core subsea cable harmonic impedance, although the outmost loop of sea return path is the relationship between armour and the surrounding sea only but not involved of the mutual impedance between cables due to cables all locating inside the pipe-like armour, both approaches for the sea return self impedance of a three-core subsea power cable are still different from each other: equation (4.57) for Wedepohl and Wilcox's approach and equation (7.1) for Bianchi and Luoni's approach. Therefore, it is still interesting to see how different sea return calculation approaches have the effect on the evaluations of three-core subsea cable.

Figure 7.2 to Figure 7.6 show the harmonic resistance and inductance under different subsea cable configuration using the two approaches: Figure 7.2 for case (a), Figure 7.3 for case (b), Figure 7.4 for case (c) and Figure 7.5 for case (d). The results are obtained from phase domain plotted against frequency at 50 Hz intervals to 5,000 Hz. The unit for harmonic resistance is  $\Omega/km$  and mH/km for harmonic inductance.

From the results from Figure 7.2 to Figure 7.6 of harmonic resistance and inductance, the curves of both approaches are perfectly matched. This suggests that for harmonic impedance evaluation of subsea power cable, either the Wedepohl and Wilcox's approach or Bianchi and Luoni's approach can be adopted for sea return calculations since the results for the cable impedances are the same. According to [65], the sea return path current can be neglected for subsea cable calculations since the value is too small compared to currents in metallic conducting layers. The results in this study also comply with this point that the sea return path is insignificant for subsea cable impedance so that both approaches have no great effect on the results of harmonic impedance of subsea cables.

# 7.3.2 Harmonic Resistance and Inductance for Subsea Power Cables in Each Case

It is evident that the resistance and the inductance of cables are dependent upon frequency [100]. Different cable designs with different material layers and arrangements will have mutual-coupling and skin effects that will produce different inductive characteristics. Figures 7.7 show the harmonic resistances and inductances of subsea cable types (a), (b), (c), (d) and (e) as described in Figure 7.1. Since armour has been addressed as an important component for subsea power cable structure and could generate circulating currents which in turn contribute to the harmonic impedance of conductor, a comparable scenario is made for subsea power cable without armour to study the influence of armour on the harmonic impedance of subsea power cables. The results are shown in Figure 7.8 with the harmonic resistances and inductances of subsea types (a), (b), (c), (d) and (e) designed without surrounding armour.



Figure 7.7 Harmonic Resistances and Harmonic Inductances of All Cases with Steel Armour



Figure 7.8 Harmonic Resistances and Harmonic Inductances of All Cases without Armour

The harmonic resistance results shown in Figure 7.7 demonstrate that for single-core cables (a), (b), (c) and (d), the resistance increases as the harmonic order increases with this phenomenon being attributed to the skin effect, which is well known to be significant at high frequencies [54]. However the harmonic resistance in Figure 7.8 of cable designs without armour experience a more rapid rise of resistance with harmonic order as compared to the cable designs with armour. Consider the trefoil formation cable, case (a) in Figure 7.8, without armour, the resistance at the 15<sup>th</sup> harmonic is 11 times the resistance at the fundamental frequency whereas the same cable with armour in Figure 7.7 has only rised to 2.3 times in resistance across the same harmonic order range. This result, and similarly for other cable designs, suggests the outside conducting layers (sheath and armour) play a significant role in reducing resistance as the harmonic order increases.

For cable designs without armour in Figure 7.8, because of the absence of the armour resistance affect, the mutual impedance from other phase cables through the earth-return path directly contributes to the sheath impedance; therefore conductor resistance responds accordingly i.e. increases rapidly compared to Figure 7.7, cables with steel armour. Harmonic resistance in cable case (b), flat formation, is seen to be slightly higher than cable case (a), trefoil formation, and as the spacing between cables is increased, cable cases (c) and (d), the harmonic resistances increase so becoming greater than cable cases (a) and (b) at the higher harmonic orders. The three-core cable case (e) without armour protection, has a similar geometrical arrangement to cable case (a), trefoil formation, so it is reasonable to expect that the increase in resistance with harmonic order would be similar for both cables. The differences seen in the results given in Figure 7.8 are attributed to the differences in cable size and also to the distances between phases being different.

For the cable cases having steel armour in **Figure 7.7**, the resistance of the single-core cable cases (a), (b), (c) and (d) are almost identical as the harmonic order increases. This is because the mutual impedance of the earth-return path from the other adjacent phase cables does not affect the conductor in a significant way essentially because there are two conducting layers (sheath and steel armour) so the mutual effect is significantly reduced. The different geometric arrangements of the single-core cable cases do not appear to significantly affect harmonic resistance at any harmonic order except for the three-core cable case. The difference is attributed to the armour that surrounds the three-phase cables and although the conductor resistance is actually slightly less than single-core cable cases at fundamental frequency, it increases more rapidly with increasing frequency as compared to the single-core cable cases. This is because higher levels of induced currents are experienced as the harmonic order increases in this cable configuration. These results imply that the harmonic resistances will differ in single-core and three-core cable systems and this phenomenon can be attributed to how the armour surrounding the conductor is arranged, i.e. no armour layer, individual phase armour layers, or a common single layer that encapsulates all phases.

The results of cable inductance versus harmonic frequency in both Figure 7.7 and Figure 7.8 show declining impedance curves as the harmonic order increases. It is noted in [54], that induced currents circulating within the cable conductor, sheath and armour will cancel part of the flux linkage within the cable conductor itself, which leads to a reduced inductance. This is the phenomenon seen in the results of Figure 7.7 and Figure 7.8.

For those cables without armour in Figure 7.8, the cancellation of inductance is caused by only one conducing layer i.e. the sheath. The single-core trefoil and flat without spacing, cases (a) and (b), generate almost identical inductances at all harmonic orders. The single-core flat with spacing, cases (c) and (d), have greater harmonic inductance reduction than in cases (a) and (b) at fundamental frequency, but as frequency is increased harmonic inductance reduces at a steadily declining rate. When spacing is increased, induced currents increase so that when spacing is provided between two cables such as case (c) and (d), the inductance has reduced to a greater degree than in those cases that do not have spacing between the cables e.g. cases (a) and (b).

However, for the cables with steel armour in Figure 7.7 the inductances for single-core cable cases (a), (b), (c) and (d) diverge at the fundamental frequency but converge to form a single curve from the second order harmonic up to the higher orders. This is attributed to the fact that the influence of the mutual impedance of the earth-return path from the other phase cables is insignificant on the single-core cables. On the other hand, for the three-core cable, case (e), the mutual impedance is directly contributing to the sheath impedance within the armour itself. This in turn leads to different harmonic inductance curve trends from the single-core cases. This diversity can also be attributed to the different armour arrangements.

These observations imply that the arrangements of armoured single-core cables have very minor impact on the harmonic resistance and inductance for subsea power cable. The survey is in full accordance of the conclusion in [65] where the results of armoured single-core subsea cable current distribution which is associated with the cable impedance is very slightly influenced by the spacing between subsea cables.

# 7.3.3 Harmonic Resonance for Subsea Power Cables in Each Case

To analyse the natural frequency resonance of the cables, the simple test condition used for the previous chapters is applied where the cables are excited by one per-unit balanced three-phase voltage source at each harmonic order having a fundamental frequency of 50 Hz. By applying known cable characteristics to the calculations described previously and by employing the transmission cable matrix equation (4.35), the response of a transmission system arrangement to frequency variation can be established and the cable natural frequencies identified. Subjecting the AC subsea cables having steel armour as illustrated in **Figure 7.1** to the test conditions as described in the case study and then the frequency response of each

cable transmission system can be obtained. The results obtained from this analysis are given in Figure 7.9 in which the magnitude of the mean voltage and its frequency at the receiving end are presented. The fundamental frequency is 50 Hz and results have been plotted at 50 Hz intervals up to 5,000 Hz.



Figure 7.9 Harmonic Resonances of Cables with Steel Armour for All Cases

Comparing the magnitudes at the resonant frequencies for the single-core cables cases (a), (b), (c) and (d), the resonances, which are dependent upon the cable impedance, appear to be very similar. This may be attributed to the similar resistances and inductances found for these cases, described in the previous section, but also because the solid bonding solution for the sheath and armour have been applied, so the admittance for each cable is identical.

Consider now the results for the three-core cable case (e). Compared to the single-core cable cases (a), (b), (c) and (d) the magnitude of the resonance differs with the magnitude at the resonant frequencies for the three core case being slightly higher than the those in single core cases at the first resonance but at the second and the third resonances the magnitude for three core case (e) are slightly less than those in singlecore cases. In terms of the actual frequencies at which the resonances occur then the three-core cable cases are slightly shifted from the single-core cable cases. This implies that the armour arrangement also plays an important part in determining the cable resonant characteristics.

# 7.4 Harmonic Model for Offshore Power HVAC Transmission Systems

To evaluate the influence of the different subsea cable arrangements upon a transmission system's harmonic response an appropriate model containing all of the transmission components is needed. For offshore wind farm transmission systems, an HVAC interconnected system is commonly used with an SVC (Static VAR compensator) at the receiving end of the transmission system to compensate reactive power flow in the cable [28] [101]. Due to SVC having power semiconductor switches (thyristors etc.) within, then the switched output waveforms are rich in harmonics. The effect of these harmonics needs to be analysed and in particular an investigation carried out to understand their interaction with the AC subsea transmission system. A schematic of the test system used is given in Figure 7.10, in which a simple inductive SVC of thyristor-controlled reactor (TCR), already widely used in offshore generation schemes [101], is modelled.



Figure 7.10 A HVAC Harmonic Test System Configuration

**Case Study:** A HVAC subsea power transmission system shown in **Figure 7.10** consists of a wind turbine three-phase power generator of voltage source of 33 kV, a 360 MVA three-phase delta-star connection 33/150 kV power transformer, three single-core subsea power cable, a 250 MVA three-phase delta-star connection 150/220 kV power transformer connecting to a passive load with 100 MVA and power factor of 0.9. A 250 MVA TCR of static VAR compensator is connected on the end of the cable as shown in the figure. According to [102] [103] [104], the study case adopted three single-phase TCR in delta-connection which is usually used for a three-phase system. The harmonic current in the TCR reactor can be controlled by firing angle. Since maximum fifth harmonic current will be injected when the firing angle is close to 110° for TCR shown in [69] [105], a firing delay angle of 110° is set for the case study. Steady state condition is applied in order to evaluate the harmonic distortion level at each node of the circuit. The transformers are assumed to be ideal without situation effect so the TCR is the only source to generate harmonics in the system. Since the study is focused on how subsea cables influence the harmonic

behaviour of a HVAC system, cables in different configuration described in Figure 7.1 are used for analysis.

Using nodal analysis, the electrical components transfer matrix for the test system can be written in admittance form so the system equations representing the HVAC test system network can be expressed by:

$$\begin{bmatrix} I_{1} \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{T1,11} & Y_{T1,12} & 0 & 0 \\ Y_{T1,21} & Y_{T1,22} + Y_{C,11} & Y_{C,12} & 0 \\ 0 & Y_{C,21} & Y_{C,22} + Y_{T2,11} + Y_{TCR} & Y_{T2,12} \\ 0 & 0 & Y_{T2,21} & Y_{T2,22} + Y_{load} \end{bmatrix} \begin{bmatrix} V_{1} \\ V_{2} \\ V_{3} \\ V_{4} \end{bmatrix}$$
(7.3)

Representation of the transmission cable is given in the form of a transfer admittance matrix so the corresponding cable matrix in equation (4.35) is rewritten as follows:

$$\begin{bmatrix} \mathbf{I}_{S} \\ \mathbf{I}_{R} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{C,11} & \mathbf{Y}_{C,12} \\ \mathbf{Y}_{C,21} & \mathbf{Y}_{C,22} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{S} \\ \mathbf{V}_{R} \end{bmatrix}$$
(7.4)

Where,  $V_s$  and  $I_s$  are the sending end voltage and current vectors in three phase,  $V_R$  and  $I_R$  are the receiving end voltage and current vectors in three phase.  $Y_{C,11}$ ,  $Y_{C,12}$ ,  $Y_{C,21}$  and  $Y_{C,22}$  are matrices of the cable transformation in admittance shunt formation. Each variable above, due to three-phase calculation, is represented as a  $3 \times 3$  matrix where mutual impedance and shunt between phases is taken into account.

Where,

$$Y_{C,11} = Y_{C,22} = Z_{\text{series}}^{-1} + \frac{1}{2} Y_{\text{shunt}}$$
(7.5)

$$Y_{C,12} = Y_{C,21}^{T} = -Z_{series}^{-1}$$
(7.6)

$$\mathbf{Z}_{\text{series}} = \mathbf{Z}_{\text{cable}} \Gamma^{-1} \sinh(\Gamma l)$$
(7.7)

$$\mathbf{Y}_{\text{shunt}} = 2\Gamma^{-1} \tanh(\frac{1}{2}\Gamma/)\mathbf{Y}_{\text{cable}}$$
(7.8)

Where,  $\Gamma = \sqrt{Z_{cable} Y_{cable}}$  is the propagation rate matrix of the transmission line;  $Z_c = \sqrt{\frac{Z_{cable}}{Y_{cable}}}$  is the characteristic impedance of the transmission line; *l* is the length of the cable; the equivalent cable impedance matrix  $Z_{cable}$  is expressed as a matrix with self impedance in diagonal and others are mutual

impedance between phases and the equivalent cable admittance matrix Y<sub>cable</sub> are expressed in Chapter 4

and where  $\mathbb{Z}_{\text{cable}} \neq \frac{1}{Y_{\text{cable}}}$ .

The detailed descriptions of the functions and harmonic characteristics of TCR are well studied in [102] [103] [104] [106] where the specific harmonic equations for TCR are also introduced. The transfer matrices representing the three-phase transformers, passive load and TCR have already been developed in [69]. The admittance matrices representing the transformers:  $Y_{T1,11}$ ,  $Y_{T1,12}$ ,  $Y_{T1,21}$ ,  $Y_{T2,21}$ ,  $Y_{T2,11}$ ,  $Y_{T2,12}$ ,  $Y_{T2,21}$  and  $Y_{T2,22}$ , passive load:  $Y_{load}$  and TCR:  $Y_{TCR}$  for the test system are given in Appendix E.

The solution for equation (7.3) can be given as a matrix of the function of  $V_1$ :

$$\begin{bmatrix} \mathbf{V}_{2} \\ \mathbf{V}_{3} \\ \mathbf{V}_{4} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{\mathsf{T}1,22} + \mathbf{Y}_{\mathsf{C},11} & \mathbf{Y}_{\mathsf{C},12} & \mathbf{0} \\ \mathbf{Y}_{\mathsf{C},21} & \mathbf{Y}_{\mathsf{C},22} + \mathbf{Y}_{\mathsf{T}2,11} + \mathbf{Y}_{\mathsf{TCR}} & \mathbf{Y}_{\mathsf{T}2,12} \\ \mathbf{0} & \mathbf{Y}_{\mathsf{T}2,21} & \mathbf{Y}_{\mathsf{T}2,22} + \mathbf{Y}_{\mathsf{load}} \end{bmatrix}^{-1} \begin{bmatrix} -\mathbf{Y}_{\mathsf{T}1,21} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \cdot \mathbf{V}_{\mathsf{I}}$$
(7.9)

Since the busbar  $V_3$  is of the interest for this study where the static VAR compensator is connected, the equation then is derived as follows:

$$\mathbf{V}_{3} = -[\mathbf{Y}_{C,12} - (\mathbf{Y}_{T1,22} + \mathbf{Y}_{C,11}) \cdot \mathbf{Y}_{C,21}^{-1} \cdot (\mathbf{Y}_{C,22} + \mathbf{Y}_{T2,11} + \mathbf{Y}_{TCR} - \mathbf{Y}_{T2,12} \cdot (\mathbf{Y}_{T2,22} + \mathbf{Y}_{load})^{-1} \cdot \mathbf{Y}_{T2,21})]^{-1} \cdot \mathbf{Y}_{T1,21} \cdot \mathbf{V}_{1}$$
(7.10)

Now by applying those cable types having steel armour as given in **Figure 7.1** within the test system, the impact of the distortion produced by the static VAR compensator and the influence of subsea cable characteristics on the resultant harmonic distortion can be determined.

# 7.4.1 Harmonic Resonances of the HVAC Test System

According to [37] [46] [107] [108], it is important to estimate the system resonances in a submarine transmission system, particularly when long distance subsea cables are involved. The same test condition as the previous case as the HVAC system is excited by one per unit balanced voltage source from fundamental frequency 50 Hz up to 5000 Hz with 50 Hz interval is used in order to study the system harmonic resonances. The harmonic voltage and harmonic current of the HVAC system with different types of cables at the  $V_3$  busbar where the TCR is connected to the system is demonstrated in Figure 7.11 and Figure 7.12. The cable natural resonances of harmonic voltage are also shown with the system resonances as comparisons.

The results shown in the figures demonstrate that the system resonances with cable type of (a), (b), (c), and (d) overlap to each other and exactly the same and only system resonances with cable type (e) is distinct from the others. This is attributed that the natural frequencies of the single-core cables cases of (a), (b), (c) and (d) were found to be identical in previous section, then it is expected that the system harmonics with cable type (a), (b), (c) and (d) should be identical. However, in the previous section the three-core cable case (e) has shown different behaviour from other single-core cases, then the system resonances with the cable type (e) is also expected to be distinct from the system resonances, which are simulated using other single-core cable types.



Figure 7.11 Harmonic Voltages and Harmonic Current of Test Systems for Single-Core Subsea Cable Types (a) (b) (c) and (d)



Figure 7.12 Harmonic Voltages and Harmonic Current of Test Systems for Three-Core Subsea Cable Types (e)

The magnitudes of harmonic voltages of the first resonance for single-core cable types (a) (b) (c) (d) and three-core cable type (e) is 3.521 p.u. and 3.81 p.u. respectively shown in the Figure 7.11 and Figure 7.12 but the resonant frequency is the same at 750 Hz while the system voltage harmonics for the first resonances are 5.7 p.u. at 300 Hz for single-core cables and 7 p.u. at 350 Hz for the three-core cable, respectively. When considering the second and third resonances, then these appear to be peaky shaped for resonances of natural cable harmonics in single-core and three-core subsea power cable types. On the other hand, the second and third resonances of harmonic voltages for the test system are significantly damped when the interactions between electrical components are considered. These results agree with [37] [46] that demonstrate when considering transformers and other electrical components, such as the induction motor, the resonances of the transmission system have high magnitudes at low frequencies as compared to the transmission cable's natural frequency. This study also confirms the work of [107] that also suggests the employing different types of power cable or power components will influence system resonances in offshore transmission systems.

By contrast, the harmonic current for systems with single-core cable cases and three-core cable case in **Figure 7.11** and **Figure 7.12** show the first resonances are 1.036 p.u. at 300 Hz and 1.088 p.u. at 350 Hz, respectively, which are merely 3.6 % and 8.8 % higher than fundamental current. This is unlike first harmonic voltage resonances which are 5.7 times greater and 7 times greater than fundamental voltage for single-core cables cases and three-core cable case, respectively. The harmonic current reduced as the frequency increased and there are no second and third resonances in the spectrums. It is then predicted that the current harmonics which are generated by the TCR will not be greatly affected by the system resonances and will be demonstrated in the following section.

#### 7.4.2 Harmonic Distortions of the HVAC Test System

It is now essential to analyse the harmonic distortion results from the harmonic source, the TCR, for different types of cables. The detailed programmes using  $MATLAB^{TM}$  to obtain the harmonic voltage and current within the test system in each node are listed as demonstrated in Appendix F where the harmonic characteristics of the TCR switching functions, transformers and cable type (a) are also listed in the subroutines.

Time and frequency domain results up to the 15<sup>th</sup> order for the three-phase HVAC system voltage and current using single-core subsea cable type (a), (b), (c) and (d) are given in Figure 7.13 and Figure 7.14 and time and frequency domain results of system harmonic voltages and currents using three-core subsea cable type (e) are shown in Figure 7.15 and Figure 7.16. The time domain results are constructed using the inverse fast Fourier transform (IFFT) of results of frequency domain. Since the steady state condition is assumed, one waveform period time range from 0 to 0.02 second is shown to represent the waveforms.

The values of one waveform period windowing rectangle in time domain Figure 7.13 and Figure 7.15 are obtained based on the information of 15 harmonics from discrete frequency spectrum of Figure 7.14 and Figure 7.16, respectively. These figures show the voltage and current distortion at the TCR at  $V_3$  busbar when using different types of subsea cable in the transmission system.



Figure 7.13 Time Domain Harmonic Voltage and Current of Test System with Single-Core Subsea Power Cable Type (a), (b), (c) and (d)



Figure 7.14 Frequency Domain Harmonic Voltage and Current of Test System with Single-Core Subsea Power Cable Type (a), (b), (c) and (d)



Figure 7.15 Time Domain Harmonic Voltage and Current of Test System with Three-Core Subsea Power Cable Type (e)



Figure 7.16 Frequency Domain Harmonic Voltage and Current of Test System with Three-Core Subsea Power Cable Type (e)

As expected, the three-phase voltage and current waveforms are balanced, this being because of the symmetrical arrangements of the two cables. The characteristic harmonics of the delta-connection TCR under balanced condition shown in **Figure 7.14** and **Figure 7.16** are also seen, i.e. 5<sup>th</sup>, 7<sup>th</sup>, 11<sup>th</sup> and 13<sup>th</sup>, which can be verified in [102] [104] [106] where the triplen harmonic currents (third, ninth, fifteenth, etc) circulate in the delta connected TCR do not enter the power system i.e. zero sequence harmonics.

Referring to Figure 7.11 and Figure 7.12, then there are significant harmonic voltage resonances close to  $6^{th}$  and  $7^{th}$  of the system with single-core subsea power cables (a), (b), (c) and (d) and three-core subsea cable (e), which give rise to the distorted voltage waveforms at the TCR busbar. Figure 7.13 and 7.14 shows that for single-core cables type (a), (b), (c) and (d) the 5<sup>th</sup> and 7<sup>th</sup> voltage harmonics are 4.96 % and 5.45 % respectively whilst the system resonance is located at the 6<sup>th</sup> order (300 Hz) i.e. in between the 5<sup>th</sup> and 7<sup>th</sup>. Consequently, this close relationship has increased the harmonic distortion of both the 5<sup>th</sup> and 7<sup>th</sup> harmonics produced by the TCR. Figure 7.15 and 7.16 shows that for the system with the three-core subsea cable type (e) the 7<sup>th</sup> order voltage harmonic has a relative magnitude of 8.51 % and is distinctly greater than the 5<sup>th</sup> order voltage harmonic which has a magnitude of 4.66 % this being because the system resonance is located at the 7<sup>th</sup> (350 Hz) harmonic which can magnify the system harmonic generated at this frequency. On the other hand, as discussed in the previous section, the magnitude of first harmonic current resonances are small for the single-core cable cases and also for the three-core cable case so the system current harmonics are not being greatly affected by harmonic current resonances. Figure 7.13 to Figure 7.16 show that the time and frequency domains of the harmonic current waveforms and harmonic current spectrums are similar between the case using single-core cables and three-core cable due to insignificance of harmonic current resonances.

Consider now the total voltage harmonic distortion (THD<sub>v</sub>) and total current harmonic distortion (THD<sub>l</sub>). The system employing three-core subsea cable type (e) has 9.7 % THD<sub>v</sub> and this is greater than that when using single-core cable type (a), (b), (c) and (d) where it is measured as being 7.4 %. Therefore, when the system resonances of harmonic voltage are exactly located at the characteristic harmonics of voltage generated by the TCR, then the total voltage harmonic distortion of the system will be greater than when they are not. However, the total current harmonic distortions of system for single-core subsea cable type are similar implying that the current resonances of system for these two cases will not greatly affect the current harmonics as seen at the TCR. Again, as predicted in the precious section, there are only small magnitudes of system harmonic resonances of currents at TCR for the single-core cable cases and the three-core cable case so the current harmonics generated by TCR show no particular distinct of the total harmonic current distortion between single-core cable cases of 8.5 % THD<sub>1</sub>. This observation can again reinforce the point that for subsea transmission systems, the cables have the effect of shifting the system resonances, which will in turn influence the total harmonic distortion within the subsea transmission system itself.

# 7.5 Summary

This chapter presents an analytical investigation of harmonic phenomena in power cables designed for use in HVAC subsea transmission systems. A range of commercially available AC subsea power cables have been introduced with different configurations and arrangements. Methods for calculating the frequency response of the subsea cables and for determining the harmonic resonance frequencies are adopted from previous chapters' analytical models. Sea return path impedance calculations using two approaches, Wedepohl and Wilcox's appraoch and Bianchi and Luoni's approach, have been discussed and the results of harmonic resistances and inductances with different AC subsea power cable designs clearly indicated that both approaches are applicable for sea return path impedance evaluations and have a trivial effect of the harmonic impedance of subsea cables.

The contribution of this study is that to establish accurate mathematical models to quantify the harmonic resistances and harmonic inductances and also to predict the harmonic resonant frequencies under different configurations and lay arrangements of AC subsea power cable designs. For the cables without armour, the harmonic resistance is much higher than in those cables employing armour and the resistance is influenced by the geometrical arrangement of the cables. For a steel armour subsea cable bonded at both ends, the resistance and inductance were seen to be unaffected by the cable configuration but were influenced by the cable lay arrangements i.e. single-core or three-core arrangements. Also, frequency domain analysis of subsea cables has shown that the frequency response and magnitude of the resonances are not greatly affected by the different geometrical structures but differences are seen when comparing the natural frequencies of single-core and three-core subsea power cables.

This study also contributes to the knowledge of understanding and evaluating the influence attributed by the HVAC transmission system arrangements upon harmonic performance when employing a TCR as a static VAR compensator. The results for system resonance and harmonic distortion in the test HVAC offshore power transmission system have demonstrated conclusively that the subsea cable type will influence the system resonances and this in turn will influence harmonic distortion within the transmission system when a harmonic source such as static VAR compensator is present. The mathematical analysis demonstrated in this chapter has shown that AC subsea power cables need to be designed with due consideration of the harmonics that may be present in the HVAC transmission system. This implies that the harmonic distortion is dependent upon the interactions and combinations between the electrical components such as static VAR compensator, transformers and subsea cables rather than solely on the harmonic source itself.
## 8

# Harmonic Analysis of Subsea Power Cables in VSC-HVDC Transmission Systems

#### 8.1 Introduction

High voltage alternating current (HVAC) subsea transmission schemes are commonly used by offshore wind farms e.g. Horns Rev, Denmark. However the distances over which AC transmission systems can be used are limited, mainly because of their charging current requirements. High voltage direct current (HVDC) schemes offer an alternative means for subsea transmission [7]. There are two categories of HVDC scheme currently available; conventional HVDC and VSC-HVDC (Voltage Source Converter – High Voltage Direct Current). Conventional subsea HVDC transmission schemes employ line commutated thyristors arranged a rectifier and a inverter in back to back Graetz bridge networks which has been described in [109] [110]. These have been installed at many different locations worldwide mainly for interconnecting two shore-based AC networks e.g. Sellindge to Les Mandarins, the UK-France inter-connector. A comparison of advantages and disadvantages of conventional HVDC and VSC-HVDC has been conducted and presented in the literature review in **Chapter 2**.

For the transmission of power generated offshore e.g. generated by an offshore wind farm, the conventional HVDC scheme has several limitations and undesirable characteristics including being physically large. The VSC-HVDC technology, which utilizes modern semi-conductor technologies such as the gate turn-off thyristor (GTO) and the insulated gate bipolar transistor (IGBT), overcomes the disadvantages associated with conventional HVDC transmission for low power transmission applications and it is therefore more attractive for use with power generation schemes situated far offshore. In VSC-

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HVDC transmission schemes, pulse width modulation (PWM) techniques are used, which allows independent control of active and reactive power, a useful feature in AC networks [2] [55].

Subsea power cables where the DC current is transmitted through are a critical component of a VSC-HVDC transmission scheme. Modulated upon this DC current however are ripples which originate from the switching behaviour of both the VSC stations, which in turn generates harmonics in the system. Harmonic performance of the transmission system is dependent upon the interactions between the subsea cable, the power converters and other system components such as smoothing capacitors. Therefore, it is important to understand how subsea power cables contribute and influence the harmonic behaviour in a VSC-HVDC system.

In previous computer based harmonic analysis [41] [111] of the voltage and current waveforms in a VSC-HVDC subsea transmission system, the cables have been described as having linear impedance characteristics with frequency. However offshore transmission cables have a multi-layered structure and are heavy armoured which gives rise to non-linear effects, which need to be accounted for. For example, it is reported [54] that circulating currents in the conducting layers (sheath and armour) give rise to losses in the main conductor and these losses are seen to increase with increases in the circulating current frequency but furthermore the increase is non-linear. Additionally, inductance in the conducting layers (sheath and armour) reduces the inductance characteristic of the main conductor and again the relationship is nonlinear with frequency. The non-linear relationship of harmonics against frequency of subsea power cables can be observed through out previous studies in **Chapter 4**, **Chapter 5**, **Chapter 6** and **Chapter 7**.

In order to accurately evaluate how the cable structure influences the harmonic performance characteristics in a VSC-HVDC subsea transmission system, accurate electrical characteristics of the cable are required. An initial approach might be to examine cable models provided in commercially available software packages such as PSCAD/EMTDC that provides powerful analysis of multi-layer cable models. However there are limitations with the cable models used by such commercial software packages as discussed in Section 4.2. For example, it is a common feature that only non-magnetic material layers are considered; only the outermost conducting layers are bonded to ground and limited cable arrangements with often only single-core cable models being available for simulation purposes. The focus of the work presented here has been therefore to investigate the steady-state harmonic behaviour of a long distance VSC-HVDC system when accurate harmonic models for the cables are used. This study also examines the effects of different subsea cables upon the performance of a VSC-HVDC system. Results presented here show harmonic resonances and those harmonics present in the waveforms for different operating conditions, e.g. different switching functions, capacitor bank size, cable materials, and transmission methods – mono-polar and bipolar arrangements.

#### 8.2 DC Subsea Cable Harmonic Calculations using Different Materials

As mentioned in **Chapter 3**, a subsea power cable has a multi-layered structure and is generally heavily armoured such as shown in **Figure 8.1**. For DC subsea cables, the insulation is usually designed to withstand higher voltage strengths as compared to an equivalent AC subsea cable. The majority of DC cables in subsea HVDC installations are of the impregnated paper insulated type [57] however recent developments in insulation materials has seen the introduction of a PE-based (Polyethylene-based) polymeric insulated cables as tested in a VSC-HVDC offshore transmission scheme [60], which makes XLPE insulated cable more attractive than other types of cables. These cable types are now considered as 'first choice' by the offshore power generation industry. A multi-layer cylindrical construction has been used to determine the subsea cable impedance and admittance [95] with the analysis accounting for the magnetic saturation of the steel armour wires as described in **Section 4.6** and the conducting layers (e.g. sheath and armour) when bonded at both ends, which is common practice.



Conductor (Diameter: 37.9 mm) Conductor Screen (Thickness: 1.7 mm) Insulation (XLPE or Paper-Impregnated) (Thickness: 17 mm) Insulation Screen (Thickness: 1 mm) Metallic Sheath (Thickness: 2.5 mm) Armour Wires (Steel  $\mu_r$ =10 or  $\mu_r$ =100

or Copper) (Thickness: 5 mm)

Jacket (Thickness: 2mm)

#### Figure 8.1 Cross Section of a Typical 150 kV DC Subsea Power Cable

The detailed equations describe mathematically the relationships between the core, metallic sheath and armour layers have been developed in Section 4.6 where the loop equations for each layers are presented. However, some modifications of these equations or changes of the cable properties may be needed in order to calculate the harmonic impedance and admittance for different DC subsea power applications.

Apart from XLPE insulated cables, the paper-impregnated insulated cables have also been widely applied in the HVDC subsea industry because of the ability of this material to withstand high voltage stress. These two types are both chosen for comparison of insulation material effects upon harmonic in a VSC-HVDC transmission system. Therefore, for the equation for calculating admittance of subsea cable under solid bonding condition as expressed as equation (8.1), the value of permittivity of insulation should be substituted according to the insulation materials e.g.  $\varepsilon_r = 2.5 \times \varepsilon_0$  for XLPE insulation and  $\varepsilon_r = 3.6 \times \varepsilon_0$  for paper-impregnated insulations where  $\varepsilon_0$  is the permittivity of air.

$$Y_{cable} = j\omega \frac{2\pi\varepsilon_r}{\ln(\frac{r_{insul-out}}{r_{insul-in}})}$$
(8.1)

Where,  $Y_{cable}$  is the equivalent admittance of the cable,  $r_{insul-out}$  is the outside radius of the insulation and  $r_{insul-in}$  is the inside radius of the insulation and  $\omega$  is the angular velocity.

In terms of armour material, the permeability is a key factor of deciding the harmonic impedance where the equations are derived from equation (4.48) to equation (4.51) and given by:

$$Z_{armour-in} = \frac{\rho_a \sigma_a}{\pi D_{ai} H} \left[ I0(\frac{\sigma_a D_{ai}}{2}) K1(\frac{\sigma_a D_{ao}}{2}) + K0(\frac{\sigma_a D_{ai}}{2}) I1(\frac{\sigma_a D_{ao}}{2}) \right]$$
(8.2)

$$Z_{armowr-ex} = \frac{\rho_a \sigma_a}{\pi D_{ao} H} \left[ I0(\frac{\sigma_a D_{ao}}{2}) K I(\frac{\sigma_a D_{ai}}{2}) + K 0(\frac{\sigma_a D_{ao}}{2}) II(\frac{\sigma_a D_{ai}}{2}) \right]$$
(8.3)

$$Z_{armour-mutual} = \frac{2\rho_a}{\pi D_{al} D_{ao} H}$$
(8.4)

$$H = I1(\frac{\sigma_a D_{aa}}{2})K1(\frac{\sigma_a D_{aa}}{2}) - I1(\frac{\sigma_a D_{aa}}{2})K1(\frac{\sigma_a D_{aa}}{2})$$
(8.5)

Where,  $Z_{armour-in}$  is the internal impedance per unit length of armour;  $Z_{armour-ex}$  is the external impedance per unit length of armour; I0(x) and I1(x) are modified zero order and first order Bessel functions with a complex argument; K0(x) and K1(x) are modified zero order and first order Kelvin functions with a complex argument;  $\sigma_a = \sqrt{j \frac{\omega \mu_0 \mu_a}{\rho_a}}$  is the complex propagation constant in armour and where  $\rho_a$  is the resistivity of the armour;  $\omega$  is the angular velocity;  $\mu_a$  is the relative permeability of the armour;  $D_{a_i}$  is the internal diameter of the armour; and  $D_{a_i}$  is the external diameter of the armour.

The permeability of steel armour is dependent on the alignment of steel wires since only longitudinal laid wires contribute to the permeability of the cable [65]; whether or not these steel wires are touching each other, and the type of steel used. Since a steel armour layer is magnetic, then the magnetic saturation effects need to be accounted for. The relative permeability can be found using longitudinal permeability diagrams of steel wires as explained in **Section 4.6.2**. Also in general practice, copper wire amour is also commonly adopted for offshore industry and can be selected as an alternative to steel armour. The relative permeability of steel wires can range from 10 to 300 [65]. For the purpose of comparison and simplicity,

steel armour with 10 and 100 relative permeability e.g.  $\mu_r = 10$  and  $\mu_r = 100$ , and copper armour are used for evaluation of subsea cable armour material effects on harmonics of VSC-HVDC transmission system.

Since it has been demonstrated in Section 7.3 that both approaches of Wedepohl and Wilcox's equations and Bianchi and Luoni's equations can be adopted for calculating sea return path impedance, Bianchi and Luoni's equations which are simplified for subsea cable application is chosen for this study. It is important to note that for a VSC-HVDC transmission system, bipolar transmission is usually preferred [2] [112], i.e. using two cables, one carrying the load current and the other carrying the return current. The bipolar cable arrangement, one cable current will affect the current in the other, simply because of the currents are flowing in opposite directions. The sea return path self-impedance  $Z_{sea-in,self}$  and mutual impedance  $Z_{sea-in,mutual}$  can be expressed as:

$$Z_{sea-in,self} = \frac{\sigma_{sea}\rho_{sea}}{2\pi r_{sea}} \frac{K0(\sigma_{sea}r_{sea})}{K1(\sigma_{sea}r_{sea})}$$
(8.6)

$$Z_{sea in,mulual} = \frac{-\sigma_{sea}\rho_{sea}}{2\pi r_{sea}} \frac{K0(\sigma_{sea}\delta_{i,j})}{K1(\sigma_{sea}r_{sea})}$$
(8.7)

Where, the negative mutual impedance of the sea return  $Z_{sea-in,mutual}$  is attributed to the current flowing in the opposite direction in the other cable;  $r_{sea}$  is the sea return path radius as the external radius of the cable;  $\delta_{i,j}$  is the distance between cable i and cable j;  $\sigma_{sea} = \sqrt{j \frac{\omega \mu_{sea}}{\rho_{sea}}}$  is the complex propagation constant in the sea;  $\rho_{sea}$  is the resistivity of the sea;  $\mu_{sea}$  is the permeability of the sea.

**Case Study:** This study again adopts typical designs of ABB with 1000 mm<sup>2</sup> copper cross-section conductor. The cable sizes and dimensions are selected to give comparable results for harmonic resistance, harmonic inductance and harmonic admittance using different materials for the insulation and armour. The different material type used for AC subsea transmission systems are illustrated in **Figure 8.1** for the study purpose of comparison. The cables are considered to be lying on the sea bed at a depth of 50 m and the conductor operational temperature is 90 °C. Due to safety reasons, solid bonding arrangement for both sheath and armour have been adopted. The parameters and size of typical 150 kV rating DC single-core subsea cable are given in **Appendix B.7**.

Since the skin effect in the conducting layers, circulating currents will flow in the metallic sheath and armour contributing to the total loss. The harmonic resistance and inductance of the core does not have a linear relationship with frequency as is so often assumed. Figure 8.2 shows the resistance and inductance curves plotted against frequency for 150 kV rated DC cables where different armour materials are used

e.g. steel armour  $\mu_r = 10$ ; steel armour  $\mu_r = 100$  and copper armour. The results show that there is an obvious difference of harmonic resistance using different armour materials. As the relative permeability of steel armour is increased the resistance also increases and if copper armour is used it will have less harmonic resistance than steel armour cable. In terms of harmonic inductance the difference is not obvious particularly at high frequency where the three curves are close to each other but nevertheless copper armoured cable has relatively low inductance at fundamental frequency compared to steel armour cables. **Figure 8.3** shows the harmonic admittance for XLPE insulated and paper-impregnated cables. Since the difference of permittivity of these two cables the harmonic admittance curves of these two cables are diverged as the frequency increases. These differences will also contribute to distinct harmonic behaviours in a VSC-HVDC transmission system when different cable materials are used.



Figure 8.2 Harmonic Resistances and Inductances per km of DC Subsea Cable



Figure 8.3 Harmonic Admittances per km of DC Subsea Cable

#### 8.3 Offshore VSC-HVDC Transmission Systems

#### 8.3.1 Characteristics of VSC-HVDC

An offshore power VSC-HVDC transmission scheme consists of two Voltage Source Converter (VSC) stations linked together by DC subsea cables. Referring to the offshore VSC-HVDC transmission described in [113], a simplified transmission diagram is shown in **Figure 8.4** for convenience where two AC systems are connected via transformers to the supply (generator) and load (network). The development of high speed gate turn-on and turn-off power electronic switching devices such as the IGBT, integrated gate commutated thyristor (IGCT) and Gate turn off (GTO) thyristors, has resulted in PWM switching techniques being adopted by the converters to control both the output voltage and output frequency. The converter switching frequency usually is substantially higher than the fundamental supply current frequency of 50 Hz for a GTO controlled VSC. For an IGBT controlled VSC the switching frequency can be even higher as 15 to 38 times [115].



Figure 8.4 Offshore Power VSC-HVDC Transmission Configuration

The PWM switching techniques control the magnitude and the phase of the output voltage of the VSC by determining the points of turn-on and turn-off and hence the conduction periods of each semiconductor switch. The switching points are effectively determined by the points of intersection of a carrier waveform (typically a triangular waveform) and modulation signal within the converter control system [116] [117]. It is the switching of the semiconductor switches that generates the harmonics in the transmission system. For the purpose of harmonic study, the harmonic domain (HD) vector is adopted to represent harmonic equations for further analysis. The harmonic domain is built using the complex Fourier series as a periodic function f(t) given as:

$$f(t) = \sum_{h=-\infty}^{\infty} F_h e^{jh\omega_0 t}$$
(8.8)

Where, h is the harmonic order and  $\omega_0$  is the angular velocity at fundamental frequency.

To represent in the harmonic domain using a vector approach:

$$\mathbf{F} = \begin{bmatrix} \dots & F_{-3} & F_{-2} & F_{-1} & F_{0} & F_{1} & F_{2} & F_{3} & \dots \end{bmatrix}^{T}$$
(8.9)

Where, **F** is the vector with harmonic content for each order including a DC term  $F_0$ .

For a three-phase voltage supply and output current for the VSC, use harmonic domain to present then the equations can be expressed as:

$$\mathbf{V}_{\mathbf{a}} = \mathbf{S}_{\mathbf{a}\mathbf{b}}\mathbf{V}_{\mathbf{d}\mathbf{c}}, \mathbf{V}_{\mathbf{b}} = \mathbf{S}_{\mathbf{b}\mathbf{c}}\mathbf{V}_{\mathbf{d}\mathbf{c}}, \mathbf{V}_{\mathbf{c}} = \mathbf{S}_{\mathbf{c}\mathbf{a}}\mathbf{V}_{\mathbf{d}\mathbf{c}}$$
(8.10)

$$\mathbf{I}_{dc} = \mathbf{S}_{ab}\mathbf{I}_{a} + \mathbf{S}_{bc}\mathbf{I}_{b} + \mathbf{S}_{ca}\mathbf{I}_{c}$$
(8.11)

Where,  $V_{de}$  and  $I_{de}$  are the harmonic vectors of the DC link voltage and current respectively. The operation of the PWM switching functions  $S_{ab}$ ,  $S_{be}$  and  $S_{ea}$  are Toeplitz matrices containing the harmonic contents for the three-phase six-pulse VSC converters as described in [69] [111].  $V_a$ ,  $V_b$  and  $V_e$  are the harmonic vectors of the phase voltages and  $I_a$ ,  $I_b$ , and  $I_e$  are the harmonic vectors of the phase currents.

#### 8.3.2 Steady-State Control

The control strategy usually adopted in a VSC-HVDC system is to use one of the VSCs as the power dispatcher and the other as the voltage regulator. The power dispatcher VSC 2 in Figure 8.4 supplies the active power  $P_{vsc}$  for a given reference power  $P_{ref}$  with the closed-loop feedback error signal  $\varepsilon_p$  applied to vary the power until zero error is achieved.

$$\varepsilon_p = P_{vsc} - P_{ref} \tag{8.12}$$

The other VSC is used as a voltage regulator as VSC 1 in Figure 8.4 to maintain the DC voltage  $V_{svelX}$  for a given reference voltage  $V_{ref}$  and the feedback error signal  $\varepsilon_v$  is applied until zero error is achieved.

$$\varepsilon_{v} = V_{ref} - V_{vsclx}.$$
(8.13)

In practice, the steady-state voltage of the DC link is achieved by adjusting the conducting periods of the VSC switching devices and under steady state conditions no DC current will flow into the capacitor banks as explained in [111]:

$$I_{DC10} + I_{DC20} = 0 \tag{8.14}$$

Where,  $I_{DC10}$  is the DC-term of  $I_{DC1}$  harmonic vector and  $I_{DC20}$  is the DC-term of  $I_{DC2}$  harmonic vector.

Obtaining the conduction periods for the steady-state conditions mathematically becomes an iterative process. The relationship between phase angle and DC current can be considered to be a linear one [118].

#### 8.3.3 Calculations Model for Harmonic Analysis

The harmonic calculation model for the VSC-HVDC station is developed in [118] and validated by PSCAD/EMTDC. For the transmission system, the model can be regarded as being a combination of two VSC converter station models with a cable linking the converters [118] [119] since the system is symmetrical where one of the half of the system can be modelled first and the other part can be simply implemented by reflecting the first part. A complete VSC-HVDC subsea mono-polar transmission model is represented in **Figure 8.5**, where the subsea cable model is divided into two half length equivalent  $\pi$  circuits.



Figure 8.5 Combination of Two Models of VSC with Two half length of Cables

The harmonic equations that represent the VSC-HVDC transmission system shown in Figure 8.5 are:

$$\begin{bmatrix} \mathbf{V}_{ST} \\ \mathbf{V}_{RT} \end{bmatrix} = \begin{bmatrix} SA & SB \\ SC & SD \end{bmatrix} \begin{bmatrix} \mathbf{I}_{S} \\ \mathbf{I}_{R} \end{bmatrix}$$
(8.15)

Where, SA, SB, SC and SD present the system ABCD transfer matrix.

$$V_{ST} = V_{S} - Z_{G}I_{S} - S_{C1}V_{IX'10}$$
(8.16)

$$\mathbf{V}_{\mathbf{R}\mathbf{T}} = \mathbf{V}_{\mathbf{R}} - \mathbf{Z}_{\mathbf{G}}\mathbf{I}_{\mathbf{R}} - \mathbf{S}_{\mathbf{C}\mathbf{2}}\mathbf{V}_{JX'20} \tag{8.17}$$

Where,  $V_s$  and  $V_R$  are the voltages vectors of the sending end and receiving end;  $I_s$  and  $I_R$  are the current vectors of the sending and receiving end;  $Z_G$  are the harmonic vectors of initial impedance of the AC source;  $V_{IX'10}$  and  $V_{DX'20}$  are the dc-terms of  $V_{DC1}$  and  $V_{DC2}$  harmonic vectors respectively.

Also, by taking account of the power losses in the cable then:

$$V_{JX'20} = V_{JX'10} - R_{cable} I_{JX'10}$$
(8.18)

Where,  $R_{cable}$  is the resistance of the cable.

The switching functions for VSC1 and VSC2 of three phase transfer matrices  $S_{c}$  and  $S_{R}$  can be expressed:

$$\mathbf{S}_{\mathbf{C}} = \begin{bmatrix} \mathbf{S}_{\mathbf{a}\mathbf{b}} \\ \mathbf{S}_{\mathbf{b}\mathbf{c}} \\ \mathbf{S}_{\mathbf{c}\mathbf{a}} \end{bmatrix}$$
(8.19)

$$\mathbf{S}_{\mathbf{R}} = \begin{bmatrix} \mathbf{S}_{\mathbf{a}\mathbf{b}} & \mathbf{S}_{\mathbf{b}\mathbf{c}} & \mathbf{S}_{\mathbf{c}\mathbf{a}} \end{bmatrix}$$
(8.20)

The SA, SB, SC and SD in system ABCD transfer matrix of equation (8.15) are given:

$$SA = Z_{TX} + \frac{1}{2}S_{C1}A_{C}(Y_{CAP}A_{C} + C_{C})^{-1}S_{R1} + \frac{1}{2}S_{C1}B_{C}(D_{C} + Y_{CAP}B_{C})^{-1}S_{R1}$$
(8.21)

$$SB = \frac{1}{2}S_{C1}A_{C}(Y_{CAP}A_{C} + C_{C})^{-1}S_{R2} - \frac{1}{2}S_{C1}B_{C}(D_{C} + Y_{CAP}B_{C})^{-1}S_{R2}$$
(8.22)

$$SC = \frac{1}{2}S_{C2}A_{C}(Y_{CAP}A_{C} + C_{C})^{-1}S_{R1} - \frac{1}{2}S_{C2}B_{C}(D_{C} + Y_{CAP}B_{C})^{-1}S_{R1}$$
(8.23)

$$SD = Z_{TX} + \frac{1}{2}S_{C2}A_{C}(Y_{CAP}A_{C} + C_{C})^{-1}S_{R2} + \frac{1}{2}S_{C2}B_{C}(D_{C} + Y_{CAP}B_{C})^{-1}S_{R2}$$
(8.24)

Where,  $Z_{TX}$  is the harmonic vector of impedance of transformers,  $Y_{CAP}$  is the harmonic vector of admittance for the capacitor banks.

The cable transformation functions  $A_c$ ,  $B_c$ ,  $C_c$  and  $D_c$  are given as follows:

$$A_{c} = \cosh(\gamma l/2) \tag{8.25}$$

$$\mathbf{B}_{c} = \mathbf{Z}_{c} \cosh(\gamma l/2) \tag{8.26}$$

$$C_{c} = \frac{1}{Z_{c}} \sinh(\gamma l/2)$$
(8.27)

$$\mathbf{D}_{\mathrm{C}} = \cosh(\gamma l / 2) \tag{8.28}$$

Where, *l* is the length of the cable,  $\gamma = \sqrt{Z_{cable} Y_{cable}}$  is the harmonic vector of cable propagation constant,  $Z_{c} = \sqrt{Z_{cable}/Y_{cable}}$  is the harmonic vector of the characteristic impedance and the cable impedance per unit length in harmonic domain  $Z_{cable}$  and cable admittance per unit length in harmonic domain  $Y_{cable}$  are acquired using the equations for subsea cables as presented in Chapter 4 and  $Z_{cable} \neq \frac{1}{Y_{cable}}$ .

#### 8.4 Resonance Evaluations of VSC-HVDC Subsea Transmission Systems

Case Study: The VSC-HVDC system used for the harmonic study has the parameters given as follows: AC-voltage sources are 150 kV having R-L parallel equivalent initial impedance where  $R_G = 1 \Omega$ and  $L_G = 0.01$  H. The two transformers have series impedances described as  $R_{TX} = 5 \Omega$  and  $L_{TX} = 0.2$  H. Both capacitor banks are 50 µF. For two VSCs with a fundamental frequency of 50 Hz, three-level and six-pulse VSCs are used with a switching frequency of 250 Hz, which is adopted here after referring to [111], a study of PWM switching techniques in VSC-HVDC transmission systems. Should higher switching frequencies be employed then the methods reported here remain appropriate and could equally be used to analyze harmonic performance. A modulation index of 0.9 which is adopted in [111] has been chosen where VSC 1 is selected as the voltage regulator to maintain the DC link at 150 kV and VSC 2 is selected as a 150 MW power dispatcher. Conductive period in both converters are assumed to be ideal and instantaneous without delay so the overlap of current transfer can be ignored. DC subsea cable parameters and size are given in Appendix B.7 whose resistance under operating temperature 90 °C is 0.02  $\Omega$  / km. It is noted that the calculation equations of the subsea cable may be subject to modification or use of proper material property if different cable materials are applied as earlier presented in the Section 8.2. The sinetriangular modulation technique [69] is used for PWM switching where the switching frequency of carrier signal is  $f_r$  and the frequency of modulation signal is  $f_s$  as 50 Hz for this case study and the frequency modulation ratio  $m_f$  is given as:

$$m_f = \frac{f_r}{f_s} \tag{8.29}$$

When considering a VSC-HVDC transmission system, the harmonic amplitudes will be influenced by the cable harmonic resonant characteristics [120]. The equivalent impedance of the DC side of the transmission system needs to take account of both the cable and the DC capacitors, which constitute part of the admittance shunt. The equivalent impedance of the DC cable  $Z_{eq}$  is given as:

$$Z_{eq} = \frac{Y_p \cdot (ZS + \frac{1}{Y_p})}{Y_p + (ZS + \frac{1}{Y_p})}$$
(8.30)

Where,

$$Y_p = Y_{CAP} + YS \tag{8.31}$$

 $ZS = Z_C \cdot \sinh(\gamma t) \tag{8.32}$ 

$$YS = \frac{1}{Z_c} \tanh\left(\frac{\gamma l}{2}\right)$$
(8.33)

Where, ZS is series impedance of the cable; YS is shunt admittance of the cable;  $\gamma$  is propagation constant; *l* is the cable length;  $Y_{CAP}$  is the admittance of bank.;  $Z_C$  is characteristic impedance of the cable.



Figure 8.6 DC Side of VSC-HVDC Equivalent Impedance Response

The equivalent impedance from the sending end is shown in Figure 8.6 where the resonances of frequency response for the different lengths of the DC cable are identified and the resonances against cable length for different harmonics are also demonstrated. For resonances of frequency response for cable length of 10 km, 30 km, 50 km, 70 km and 90 km, the greatest values of equivalent impedance are at the fundamental frequency and the amplitude reduces as frequency increases as would be expected. Harmonic impedance resonances are seen to take place as the frequency is increased but the magnitudes are not as high as the value at fundamental frequency. Comparing the harmonic impedance resonances for different lengths of cable, then generally the resonance occurs at lower frequencies as the cable is lengthened. The magnitudes of the resonances of frequency response are generally less in the shorter cables. The harmonic impedance resonance of frequency response of the DC transmission side is less significant for subsea cables than those in over-head transmission lines [49], which usually have high peaks at high frequencies. For resonances of harmonic impedance against cable length at 6<sup>th</sup>, 12<sup>th</sup>, 18<sup>th</sup>, 24<sup>th</sup> and 30<sup>th</sup> harmonic which are the harmonics expected to be generated on DC side of a six pulse VSC [118], the 6<sup>th</sup> harmonic has highest harmonic impedance peak at 57 km of 13.6  $\Omega$  but while the harmonic order increased the peak takes place at shorter length of cable and less magnitude such as 17 km for  $12^{th}$  of 8.5  $\Omega$ , 8 km for 18<sup>th</sup> of 6.6  $\Omega$ , 4 km for 24<sup>th</sup> of 5.4  $\Omega$  and 3 km for 30<sup>th</sup> of 4.9  $\Omega$ . This observation implies that with the different cable length the DC side harmonics which is generated by VSC may also perform differently because the harmonics could interact with cable resonances and create different harmonic distortions levels and vary the power losses of the system.

#### 8.5 Harmonic Analysis of VSC-HVDC Subsea Transmission Systems

The next step is to study harmonic behaviour for different arrangements so as to understand how subsea cables influence the harmonic behaviour in a VSC-HVDC transmission system. The simulations using **MATLAB<sup>TM</sup>** are detailed listed in **Appendix G**.



#### 8.5.1 Switching Frequency Effect

Figure 8.7 Current of AC Side and Voltage of DC Side on Sending End

Simulation results for the study case are acquired in the harmonic domain hence the inverse fast Fourier transform (IFFT) has been used in order to obtain both the waveforms on the AC and DC sides. It should be noted that had the higher order harmonics been included in the analysis i.e. up to  $100^{th}$  order, the represented waveform would be more representative. However the  $30^{th}$  order is sufficiently accurate and computationally efficient. Figure 8.7 shows the waveforms of the DC voltage  $V_{DC1}$  and the AC current  $I_s$  containing harmonics to the  $30^{th}$  order in the DC link at the sending end of the transmission system. The frequency modulation ratio  $m_f$  was set to 5 i.e. the switching frequency was 250 Hz and the cable length is set to be 50 km. As expected there are ripples in the DC waveform that are generated by low order harmonics with the voltage regulator VSC1 keeping the average voltage at 150 kV with fluctuation of  $\pm 10 \text{ kV}$ . Also evident are the harmonic distortions present in the AC current which can be attributed to the converter switching, which are close to the  $5^{th}$  order at 250 Hz.



Figure 8.8 Current Harmonics on AC side and Voltage Harmonics on DC side on Sending End under Different Switching Frequency

When the switching frequency is increased then the harmonics shift to higher frequencies but the principles of analysis remain the same. The switching functions in the VSC are the main source of harmonics on both sides of the VSC. Figure 8.8 shows the response to the 30<sup>th</sup> harmonic order with the switching frequency being varied as  $m_f = 5, 7, 9, 11$  and 13 for 250, 350, 450, 550 and 650 Hz, respectively and with the harmonics on both the AC and DC sides changing accordingly. The results show that the AC supply current harmonics have the greatest magnitude when the harmonics are at the switching frequency; For instance, the greatest harmonics under different switching frequencies are: the 5<sup>th</sup> harmonic for  $m_f = 5$ , the 7<sup>th</sup> harmonic for  $m_f = 7$ , the 11<sup>th</sup> harmonic for  $m_f = 11$ , and the 13<sup>th</sup> harmonic for  $m_f = 13$ . However, when  $m_f = 9$  then due to characteristic harmonics not being generated at this frequency, the nearest harmonics 7<sup>th</sup> and 11<sup>th</sup> are shown to be greatest. To consider the harmonics on the DC side then, there are 6n (n = 1, 2, 3...) orders of harmonic generated in Figure 8.8. The same phenomenon is seen to occur in the voltage on the DC side of the VSC where the harmonic magnitudes are higher when they become close to the switching frequency. It is evident therefore that the PWM switching frequency is a key factor that determines the amplitudes and frequencies of the harmonics generated on both the AC and DC sides of an offshore VSC-HVDC transmission system and the response of the system. This indicates that the switching frequency should be chosen carefully for the design of a system.

#### 8.5.2 Capacitor Bank Effect

The capacitance of a subsea cable is considerable and it provides a capacitance shunt that can be expected to reduce the size of DC capacitor bank required for providing DC voltage stability and for filtering out the switching noise in an HVDC transmission system [41]. Figure 8.9 shows the DC and AC side THD at the sending end and also power losses against the length of cable for different sizes of capacitor banks. For DC side voltage THD, it is obvious that as the capacitor bank is increased the THD significantly reduces, implying that the capacitor bank stabilizes the DC voltage, agreeing with the observations given in [41]. For DC side current THD then as the capacitor bank is increased, the resonances, which are produced by the interaction of the DC current harmonics with the system itself, are gradually reduced and eventually become insignificant. The results indicate that the resonances on the DC side are attributed to the cable and capacitor banks these diminished as the capacitor bank size is increased. For the AC side current harmonics, then when using a capacitor bank  $c = 50 \mu F$ , the resonance takes place at 50 km. However, when the capacitor bank is increased the resonance becomes damped such as is seen for values of c = 100 $\mu$ F and c = 500  $\mu$ F. In terms of magnitude, the range for these cases extends from 16.9 % up to 17.8 %, and does not significantly fluctuate along the cable length. Power losses, which are the major concern for efficiency, need to account for the harmonic resonances against the cable length. The peak for  $c = 50 \ \mu F$  is 45.4 % at 54 km, which is 12.5 % higher than the peak for  $c = 100 \ \mu$ F which is 32.9 % at 27 km. Also, as

the capacitor bank size is increased, the resonances are generally more damped and losses reduced. These results suggest that the losses are not only contributed by cable resistance and converter operation but are also due to the level of harmonics. It is important that the resonances of the power loss for a specific cable length are detected to avoid excessive loss.



Figure 8.9 AC Side and DC side THD and Power Losses for Different Sizes of Capacitor Bank

#### 8.5.3 Cable Material Effect



Figure 8.10 AC Side and DC side THD and Power Losses under Different Materials of Cable

Subsea cables use a diverse range of materials. As mentioned in proceeding sections, there are different materials of cable types can be used for offshore transmission applications. In terms of insulations for subsea cables, there are XLPE insulated and paper-impregnated insulated cables are widely adopted in the industry. In terms of armour, steel amour and copper amour both are popular for subsea cable outside layer protection. Therefore, the difference of materials can affect the system harmonic behaviours. Again,

for the purpose of comparison, steel armour with 10 and 100 relative permeability was used in the simulations;  $\mu_r = 10$  and  $\mu_r = 100$ . Analysis was also carried out using cables with XLPE insulation and paper insulation and copper armour and their effects on HVDC system harmonics were investigated. Figure 8.10 shows that generally the harmonics on both the AC and DC sides and the transmission power losses for the paper-impregnated insulation cable, whose relative permittivity is 3.6, is not much different from the XLPE insulation cable, whose relative permittivity is 2.5. This is because of the change of the permittivity, which in turn produces a difference in capacitance in the two cable designs, is relatively insignificant when contrasted to the capacitance provided by the capacitance banks. However, in terms of the permeability, the cable with steel armour of  $\mu_r = 100$  has less amplitudes of resonance along the cable length as compared to the cable with steel armour of  $\mu_r = 10$ , on power losses. Furthermore, when the armour material is changed from steel to copper, the harmonic amplitudes on both the AC and DC sides tend to reduce and the power losses steadily increase as the cable length increases, with no obvious resonance, which apparently occurs in the case of the cable with steel armour wires. Due to armour impedance is affected by the permeability and large impedance differences between the steel armour and copper armour, these in turn influence harmonic resistance and inductance of the layers, the cable characteristics interact with the transmission system and provide distinct harmonic resonances for variations in cable length.

#### 8.5.4 Bipolar Transmission Effect

As described in the proceeding section, bipolar transmission is extensively applied for VSC-HVDC transmissions. Thus it is also a need to evaluate the influence of bipolar transmission on harmonic performances of the system. For the purposes of analysis, a 150 kV transmission system designed as a bipolar transmission system with ± 75 kV rated cable using two identical cables placed 5 m apart; one carrying the load current and the other the return current. To allow comparison with the mono-polar arrangement, the cross-section of two cable conductors were designed with the same cross sectional area of 1000 mm<sup>2</sup> but with the insulation thickness being 10 mm instead of 17 mm as used in the 150 kV rated mono-polar cable because of the reduced insulation requirement. The cable parameters and dimensions are described in Appendix B.8. Considering the results of the analysis in Figure 8.11, then there are clear differences in harmonic behaviour and power loss when the two transmission methods are compared. For the bipolar cable, there are two cables carrying current in opposite directions which influences the mutualimpedance of the sea return path in equation (8.7). The bipolar arrangement and the thinner cable insulation have an effect on the system harmonics giving a different response to that found in the monopolar system. The harmonic resonances of the bipolar arrangement when plotted against cable length tend to be less sharp. Also the curve of power loss against the cable length for bipolar transmission is damped in comparison with the mono-polar arrangement because the losses from harmonics are less in bipolar transmission.



Figure 8.11 AC Side and DC side THD and Power Losses of Mono-polar and Bipolar Transmission

#### 8.6 Summary

Harmonic performance of the system is dependent upon the interactions between the subsea cable, the power converters and other system components such as smoothing capacitors. In this chapter, a mathematical model of a HVDC-VSC transmission system is developed and its harmonic performance investigated for steady-state operating conditions. The results suggest that the design of the subsea transmission cable has important effects on harmonic levels in the voltage and current waveforms in the cable and also upon power loss within the transmission system. The study demonstrates that it is always important to consider interactions between all the system components when predicting harmonic performance in a VSC-HVDC transmission system.

In this chapter, investigations using computer simulation of harmonic performance of VSC-HVDC models under steady-state condition have been investigated using improved cable models. The study has shown that accurate subsea cable models are necessary, and appropriate mathematical models are presented here which are used to predict the resonance of the cable along its length in a VSC-HVDC system. From the switching frequency aspect, then if the switching frequency avoids the characteristic resonance harmonics on both AC and DC side, then the level of harmonics will be reduced. From the capacitor bank point of view, an increase of capacitor bank size dampens the resonance. By changing insulation and armour material of the DC cable, the harmonic response was much less distinct between the XLPE-insulation and paper-impregnated insulation. However, the permeability of steel wires is an essential factor influencing harmonic performance, and also there is a distinct difference of harmonic performance between steel armoured cables and copper armoured cables. For bipolar transmission, due to thinner insulation requirements and a second cable carrying the return current in the opposing direction, the harmonics are damped and there is reduced power loss within subsea transmission system.

This study contributes a novel aspect towards an insight of how subsea cables influence the harmonic of VSC-HVDC system in a transmission system. It implies that a subsea cable model will need to be carefully designed in order to represent the harmonic behaviour in subsea VSC-HVDC transmission system. Also, to understand the harmonic behaviour and harmonic distortion, power loss must be considered as this is critical to the design of offshore power transmission systems.

### 9

# **Conclusions and Further Work**

#### 9.1 Conclusions

The aim of the work has been to evaluate the harmonic performances of subsea power cable in offshore power transmission systems. A subsea power cable is a complicated structure with multi-conducting layers so the harmonic impedance and admittance have a non-linear relationship against frequency because of the effects of skin effect, proximity effect, mutual coupling and induced currents in the conducting layers under loaded conditions. Interactions of subsea power cable harmonic characteristics with harmonics generated by power devices influence the harmonic behaviour of a subsea transmission system.

The innovated contributions of this study are further investigating the harmonic characteristics of subsea cables beyond current understanding and also to analyse the harmonic behaviour of HVAC and VSC-HVDC offshore transmission systems by considering their interaction with subsea cables. The detailed mathematical models of subsea cable harmonic evaluations including the consideration of physical effects of skin effect, mutual coupling and proximity effect have been established where the importance of these effects have been discussed. Examination of subsea cable harmonic behaviour under different bonding conditions, cable length and different geometrical arrangements have been conducted. The HVAC and VSC-HVDC system harmonic models with harmonic sources have been developed to evaluate the transmission system resonances and harmonic distortions with subsea cables.

It is shown in the open literature that in the design of a subsea generation and transmission system, one of the most critical issues is the subsea power transmission cable. The technical problems of subsea power cables involve a number of challenges particularly in electrical performance including harmonics and distortion losses. At present, subsea cable harmonic models are usually simplified and considered as being linear in nature and a simple part of a transmission system. Therefore, they are not able to accurately represent harmonic propagation, resonances and characteristics of the subsea power cable. Further, there are concerns over a lack of research to comprehensively study the harmonic interaction of subsea cables with the rest of the power network, which of course influences overall harmonic behaviour in a transmission system.

The research issue of great importance in this thesis was to develop harmonic models which were capable of accurately predicting propagation rates in subsea cables and to evaluate offshore transmission system behaviour. It is known that there are a range of programs available on the market for electric power engineering simulation. However, these software programs have constraints such as limited cable designs, non-consideration of saturation effect for magnetic material of cables and restricted cable bonding conditions, which limit the ability to examine all conditions in offshore power transmission and may not be as precise as a study may require. Therefore, a thorough analytical model is developed in this thesis to evaluate the harmonic characteristics of subsea power cables to overcome these drawbacks. The theories of frequency domain harmonic modelling of transmission lines, including lump and distribution parameter transformation matrices, are being employed to investigate the harmonic propagation and harmonic behaviours. A simple model of harmonic voltage of a three-phase overhead transmission line has shown peak resonances when considered using a frequency domain spectrum. The mutual coupling and skin effect of multi-layered cable were also given due attention, which is demonstrated in the analysis such that factors can not be ignored as the main features of electrical properties for harmonic calculations of cables.

The governing equations of pipe-like cylindrical geometric cables were adopted to understand the concepts and to identify fundamental principles to determine subsea cable parameters. Evaluations of each layer in a subsea cable are being studied in detail where the loop equations with superposition are used to work out the harmonic impedance and harmonic admittance of the cable. The frequency dependent model has also been validated by hard data and shown a perfect match of harmonic frequency response with the results obtained from the software program. The results of harmonic voltage against frequency have shown that the resonances for a three-phase single-core subsea cable have less resonance peaks compared to those for a three-phase single-core transmission line. With accurate harmonic propagations, the model may find useful application on natural resonance prediction and harmonic analysis of subsea cables.

The study of proximity effect on subsea cable have shown that the proximity effect has no great effect on harmonic impedance for single-core subsea power cable but for three-core subsea power cables it is a significant factor. In addition, the proximity effect is not a concern in both zero sequence harmonic impedance for both single-core and three-core because the circulating current between grounding path and armour is not affected by the proximity effect between cables. The results have contributed the knowledge of how geometric arrangements and structures of subsea cables influence the proximity effect.

The observation of harmonic resonance behaviours of subsea power cables against frequency under different bonding condition; solid bonding, single-point and cross bonding methods for different cable length has pointed out that using solid bonding method subsea cables generate less harmonic voltage resonance of magnitude compared with the other methods but subsea cables create largest magnitude of resonance peaks of three methods using single-point bonding method. For cross-bonding method subsea cables produce a frequency response with the shortest span between resonant frequencies of the three methods considered. Also from Q-factor analysis, the solid bonding method has the most damped response and the single-point bonding method has the sharpest response of three methods considered. Also, as length of cable increases the span of resonance frequency period of any two adjacent resonances is shortened but the resonances magnitude are decreased. The results have demonstrated that the frequency response and harmonic resonances for subsea power cables are highly dependent upon the bonding method and cable length used.

The investigation into harmonic behaviours with different configurations and arrangements of AC subsea cables have been conducted where sea return path impedance calculations using two approaches, Wedepohl and Wilcox's approach and Bianchi and Luoni's approach, have shown that both approaches are appropriate and applicable for computation. The results using analytical models to quantify the harmonic resistances and harmonic inductances and to predict the harmonic resonant frequencies under different configurations and arrangements of AC subsea power cable designs have pointed out that for armoured subsea cables bonded at both ends, the harmonic resistance, inductance and frequency response of resonances were unaffected by the cable configuration but were influenced by the cable lay arrangements i.e. single-core or three-core arrangements. In addition, the results of resonance and harmonic distortion for a HVAC system employing different AC subsea cables have demonstrated conclusively that the subsea cables contribute and shift the system resonances have great influence on harmonic distortion within the transmission system when a harmonic source such as static VAR compensator is present. The study concluded that for an offshore power transmission system the harmonic distortion is dependent upon the interaction and combination between the power components such as static VAR compensator, transformers and subsea cables rather than solely on the harmonic source itself.

The investigations of commonly used DC subsea cables, each having different material structures showed that, due to their response characteristics being unique, each responded differently and to produce a distinct harmonic pattern. The analysis of harmonic performance of an offshore VSC-HVDC transmission system under steady-state operating conditions has further demonstrated the voltage and current harmonic distortion and its associated power losses along the subsea cable length. From analysis of the results some points can be made as follows:

1. To reduce the level of harmonics, the switching frequency should avoid the characteristic resonance harmonics on both AC and DC side.

2. An increase of capacitor bank size dampens the resonance.

- 3. The system harmonic response was much less influenced by switching between the XLPE-insulated and paper-impregnated insulated DC cables but the permeability of steel armour is an important factor of determining harmonic performance. Also there is a distinct difference of system harmonic performance between steel armoured and copper armoured cables.
- 4. Since thinner insulation requirements and a second cable carrying the return current in the opposing direction, the harmonic distortion along the cable length for bipolar transmission are damped and power loss is reduced within the subsea transmission system as compared to the mono-polar transmission method.

This study has given sound conclusions with the main contribution being the knowledge that harmonic interactions of DC subsea transmission cables with the VSC-HVDC transmission system need to be carefully assessed by considering the effects of the switching function, capacitor bank size, cable material and construction, transmission methods and cable lengths from which the harmonic levels of voltage and current in the system and associated power loss can be accurately evaluated.

#### 9.2 Further Work

The models already developed can easily be extended to look at further cases of harmonic interaction with subsea power cables and other power components in any transmission system. As illustrated in **Chapter 7** if an extra component such as wind turbine generator [121] [122] or filter is connected to the network, either nodal voltage analysis or nodal current analysis can be applied to examine the harmonic performance of the node which is of interest. The influence of subsea cables on harmonic behaviour and interaction with the wind turbine generator or filter can be an intriguing area for further research.

In Chapter 8 of the VSC-HVDC harmonic model, the ideal conductive condition of transistors of VSC is assumed so there is no overlap of current transfer in the converters and consequently no notch in the waveforms. However, in reality the overlap is likely to take place and affect the waveforms of the VSC. This contributes additional harmonics in the system. For a subsea VSC-HVDC transmission system the notches in the waveforms interact with the long subsea cable and is an area that needs to be investigated further. Since the PWM is used in VSC systems it is important that prior to the harmonic investigation of notched waveform on a VSC-HVDC subsea cable transmission system, a study needs to be conducted to understand notching currents in VSCs as these are not as well understood as they are in a conventional HVDC transmission system.

Offshore power transmission systems usually include high power transformers. It is therefore predictable that these transformers, due to saturation effects of magnetic, non linear characteristics may generate harmonic currents during steady-state operation as well as transient conditions [69]. The ferromagnetic effect of steel armour can also be further studied using Finite Element computation tools to understand the magnetic effects under different cable arrangements and high power transmission to accurately predict the core losses of the steel armoured cable. Another interesting research area to explore is the harmonic interaction of steel armoured subsea cable with the magnetic saturation of high power transformers.

There are some oil fields in the North Sea coming to the end of the production in the near future so many oil pipes will be abandoned. It is suggested by ABB that subsea high power transmission cables could be installed in these abandoned pipes because they provide extra protection for the cables. However, extra losses due to circulating current in the pipe and rise of the temperature of the cables are of major concern. Also it is predicted that harmonic characteristics of the cables installed in these pipes will be different from those cables laid on the sea bed without extra protection of the oil pipes because these pipes are regarded as an extra conducting layer of the cables, which affects the harmonic impedance and harmonic admittance values. This requires a comprehensive evaluation and feasibility assessment where the harmonic behaviour of the cables needs to be addressed.

A transient study is essential for any offshore generation system. While supplying power via long distance transmission offshore under constant load, the harmonics resonances tend to be stable. However, the power load will change because of varying demand or fluctuation of wind in a wind generating scheme which will provide an unstable power supply. Therefore, the harmonics under transient conditions are expected to be time varying. Also since long cables are applied in the system the harmonic behaviours under faulty condition can be very different from steady-state condition [123] [124]. This needs to be studied, particularly sudden harmonics interacting with cable resonances which may seriously harm the power devices at which frequency the system is operating. Also, how influential the transient load is on the system harmonic distortion through subsea needs to be estimated. Therefore, time domain analysis should be employed and dynamic harmonic response analysis need to be adopted to investigate the impact.

Since more and more wind farms are being built offshore, it is expected that the offshore inter-connection systems need more complicated designs and structures to link these network and to transmit power back to the mainland. It is reported [26] [125] that multi-terminal voltage source converter HVDC (MVSC-HVDC) system provides a solution to link all the networks and control the power quality. Instead of one cable, multiple numbers of subsea cables are applied in such systems and connected to substations where the power is regulated and transmitted via main cables back to land. Since there are a number of subsea power cables, converters, transformers, capacitor banks involved, it can be a challenging area of study to evaluate how these cables affect the harmonic resonances and their harmonic behaviours interacting with multiple numbers of harmonic sources.

# References

- T. J. Hammons, B. Richardson, M. Abaza, K. L. Browne and G. Lindahl, 'Energy and Technology: Sustaining World Development into the Next Millennium,' *IEEE Power* Engineering Review, May 1999, pp. 3-8.
- 2. T. Ackermann, 'Transmission Systems for Offshore Wind Farms,' *IEEE Power Engineering Review*, Vol. 22, No. 12, December 2002, pp. 23-27.
- Department of Trade and Industry UK, 'Our Energy Future-Creating a Low Carbon Economy,' Energy White Paper, February 2003.
- Renewable UK 'The World Offshore Renewable Energy Report 2002-2007,' A Report by Douglas-Westwood Limited for Renewable UK, Report No. 197-02, October 2002.
- Carlton, J., 'International Interconnections-National Grid's Perspective,' CIGRE/IEE Conference, July 2004.
- Zeus Development Corporation, 'Offshore Transmission: Assessing the Issues, Economics and Opportunities for the E&P Industry,' Offshore Transmission Joint- Industry Research Project, Houston USA, 2002.
- 'Limitation of Long Transmission Cables for Offshore Wind Farms, Appendix 3-C: Transmission Issues for Offshore Wind Farms with Specific Application to Sitting of the Proposed Cape Wind Project' copyright produced by ESS, Inc., consultants to Cap Wind Associates, LLC., 2003.
- 8. J. F. Manwell, A. L. Rogers, and J. G. McGowan, 'Status of Offshore Wind Energy in the United States,' *IEEE Power Engineering Society Summer Meeting*, Vol. 1, 2001, pp. 10-13.
- 9. A. T. Jones and W. Rowley, 'Recent Developments and Forecasts for Renewable Ocean Energy Systems,' *MTS/IEEE Conference and Exhibition*, Vol. 1, November 2001, pp. 575-578.
- Renewable UK 'The World Offshore Renewable Energy Report for 2004-2008,' A Report by Douglas-Westwood Limited for Renewable UK, June 2004.

- L. Sharpe, 'Offshore Generation Looks Set to Take Off,' *IEE Review*, Vol. 48, May 2002, pp. 24-25.
- T. J. Hammons and A. G. Geddes, 'Assessment of Alternative Energy Source for Generation of Electricity in the UK Following Privatisation of the Electricity Supply Industry,' *IEEE Transactions on Energy Conversion*, Vol. 3, No. 4, December 1990, pp. 609-615.
- P. J. Hill, B. Inozu, T. Wang and J. J. Bergeron, 'Offshore Power Generation Using Natural Gas From Remote Deepwater Developments,' *Offshore Technology Conference*, OTC 14289, Houston Texas USA, May 2002
- R. W. G. Bucknall H. G. A. Martindale, 'Remote Area Hydrocarbon Exploitation Power Generations and Transmission,' Offshore Power Research Report, Phase 1, Work Package 1-5, University College London, June 2003.
- E. H. Yap, H. G. A. Martindale, T. Zachos, and R. W. G. Bucknall, 'Exploitation of Remote Area Hydrocarbon Reserves through Offshore Electric Power Generation: A Technical and Economical Evaluation,' 38<sup>th</sup> International University Power Engineering Conference, Vol. 1, pp. 229-232, Greece, September 2003.
- O. Hague, J. Normann Johnsen, T. A. Holte and K. Bjorlow-Larson, 'Performance of the ± 250 kV HVDC Skagerrak Submarine Cables. Further Development of the HVDC Paper-Insulated, Mass-Impregnated (Solid Type) Submarine Cable,' *IEEE Transactions on Power Delivery*, Vol. 3, No. 1, January 1998, pp. 1-15.
- T. J. Hammons, A. Olsen and T. Gudnundsson, 'Feasibility of Iceland/United Kingdom HVDC Submarine Cable Link,' *IEEE Transactions on Energy Conversion*, Vol. 4, No. 3, January 1989, pp. 414-424.
- E. Guthnason, J. Henje, P. Shepherd and D. Valenza, 'A 550 MW HVDC Submarine Cable Link. Iceland-UK-Continental Europe,' Power Cables and Accessories 10kV-500kV, November 1993, pp. 220-224.
- D. Valenza and G. Cipollini, 'HVDC Submarine Power Cables Systems State of the Art and Future Developments,' *Proceedings of Energy Management and Power Delivery*, International Conference, Vol. 1, November 1995, pp. 283-287.
- K. Terashima and H. Suzuki, 'Research and Development of ± 250 kV DC XLPE Cables,' *IEEE Transactions on Power Delivery*, Vol. 13, No. 1, January 1998.
- T. J. Hammons, D. Woodford, J. Loughtan, M. Chamia, J. Donahoe, D. Povh, B. Bisewski and W. Long, 'Role of HVDC Transmission in Future Energy Development,' *IEEE Power Engineering Review*, February 2000, pp. 10-25.
- J. R. Attwood, 'Cable Design for Subsea Power Links,' *IEEE Power Engineering Review*, Vol. 20, pp. 13-14 and 21, September 2000.
- C. Wolff and T. Elberling, 'Kontek HVDC Cable Link,' *IEEE Power Engineering Review*, Vol. 20, October 2000, pp. 16-17.

- 24. C. Harvey, K. Stenseth and M. Wohlmuth, M, 'The Moyle HVDC Interconnector Project Considerations, Design and Implementation,' *AC-DC Power Transmission*, Seventh International Conference, No. 485, November 2001, pp. 145-149.
- N. M. Kirby, L. Xu, M. Luckett, and W. Siepmann, 'HVDC Transmission for Large Offshore Wind Farm,' *Power Energy Journal*, Vol. 16, No. 3, June 2002, pp. 135-141.
- W. Lu and B. T. Ooi, 'Optimal Acquisition and Aggregation of Offshore Wind Power by Multiterminal Voltage-Source HVDC,' *IEEE Transactions on Power Delivery*, Vol. 18, No. 1, January 2003
- 27. H. Brakelmann, 'Efficiency of HVAC Power Transmission from Offshore-Windmills to the Grids,' *IEEE Bologna PowerTech Conference*, Italy, June 2003.
- A. Reidy and R. Watson, 'Comparison of VSC based HVDC and HVAC Interconnections to a Large Offshore Wind Farm,' *IEEE Power Engineering Society General Meeting*, June 2005, pp.71-78.
- A. B. Morton, S. Cowdroy, J. R. A. Hill, M. Halliday and G. D. Nicholson, 'AC or DC? Economic of Grid Connection Design for Offshore Wind Farms,' *The 8<sup>th</sup> IEEE International Conference on AC and DC Power Transmission*, March 2006, pp. 236-240.
- N. R. Watson, J. Arrillaga and C. P. Arnold, 'Simulation of HVDC System Disturbances with Reference to the Frequency Dependence of the AC system,' *IEE Proceedings Pt. C*, Vol. 136, No. 1, January 1989, pp. 9-14.
- 31. A. P. Sakis Meliopoulos and Jr. M. A. Martin, 'Calculation of Secondary Cable Losses and Ampacity in the Presence of Harmonics,' *IEEE Transactions on Power Delivery*, Vol. 7, No. 2, April 1992, pp. 451-459.
- 32. S. Merhej and B. Nichols, 'Harmonic Filtering for Offshore Industry,' *Petroleum and Chemical Industry Conference*, Industry Applications Society 39th Annual, September 1992, pp. 111-120.
- 33. A. Mdeina and J. Arrillaga, 'Harmonic Interaction between Generation and Transmission Systems,' *IEEE Transactions on Power Delivery*, Vol. 8, No. 4, October 1993, pp. 1981-1987.
- A. Mclean, K. Mcleay and A. Sheldrake, 'Harmonic Suppression Filter for Offshore Interconnected Power System,' Three Phases LV Industrial Supplies: Harmonic Pollution and Recent Developments in Remedies, IEE Colloquium, June 1993, pp. 6/1-6/6.
- A. Hiranandani, 'Calculation of Ampacities and Sizing of Line and Neutral Conductors in the Presence of Harmonics,' *IEEE Industrial and Commercial Power Systems Technical Conference*, May 1995, pp. 49-58.
- A. C. S. De Lima, R. M. Stephan, A. Pedroso and J. Mourente, 'Analysis of a Long Distance for an Induction Motor,' *IEEE International Electronic Proceedings, IEEE International* Symposium, Vol. 2, June 1996, pp. 867-872.

- R. O. Raad, T. Henriksen, H. B. Raphael and A. Hadler-Jacobsen, 'Converter-Fed Subsea Motor Drivers,' *IEEE Transactions on Industry Applications*, Vol. 32, No. 5, September 1996, pp. 1069-1079.
- M. Grotzbach and J. Schorner, 'CSI-Fed Subsea Cable Transmission for Motor Drive Applications,' *IEEE Industrial and Commercial Power Systems Technical Conference*, May 1997, pp. 116-121.
- F. Castellanos and J. R. Marti, 'Full Frequency-Dependent Phase-Domain Transmission Line Model,' *IEEE Transactions on Power Systems*, Vol. 12, No.3, August 1997, pp. 1331-1339.
- 40. P. E. Sutherland, 'Modelling of Impedance vs. Frequency in Harmonic Analysis Programs,' *IEEE Industry Applications Conference*, Vol. 3, October 1997, pp. 2243-2247.
- 41. Z. Yao and B. T. Ooi, 'Utilization of Cable Capacitance in GTO-HVDC Transmission,' *IEEE Transactions on Power Delivery*, Vol. 13, No. 3, July 1998, pp. 945-951.
- 42. M. F. Taylor, 'Conceptual Design for Subsea Power Supplies for Extremely Long Motor Lead Applications,' *Petroleum and Chemical Industry Conference*, Industry Applications Society 45<sup>th</sup> Annual, September 1998, pp. 119-128.
- 43. E. A. Vendrusculo and J. A. Pomilio, 'Power Cable Parameters Estimation in Long Distance Driving of Electrical Machines,' *Electric Machines and Drives International Conference*, May 1999, pp. 410-412.
- 44. G. N. Bathurst, B. C. Smith, N. R. Watson and J. Arrillaga, 'Modelling of HVDC Transmission systems in the Harmonic Domain,' *IEEE Transactions on Power Delivery*, Vol. 14, No. 3, July 1999, pp. 1075-1080.
- 45. K. S. Smith and L. Ran, 'Active Filter Used as a Controlled Reactance to Prevent Harmonic Resonance in Interconnected Offshore Power Systems,' *IEE Proceedings Generation, Transmission, and Distribution,* Vol. 146, No.4, July 1999, pp. 393-399.
- J. A. Pomilio, C. R. de Souza, Matias, P. L. D. Peres and I. S. Bonatti, 'Driving AC Motors through a Long Cable: The Inverter Switching Strategy,' *IEEE Transactions on Energy Conversion*, Vol. 14, No. 4, December 1999, pp. 1441-1447.
- G. C. Montanari and D. Fabiani, 'The Effect of Non-Sinusoidal Voltage on Intrinsic Aging of Cable and Capacitor Insulating Materials,' *IEEE Transactions on Dielectrics and Electrical Insulation*, Vol. 6, No. 6, pp. 798-802, December 1999.
- 48. J. Arrillaga, 'The Evolution of Power Transmission Models to Accommodate Power Electronics,' IEEE Power System Technology, International Conference Proceedings, 2000.
- G. N. Bathurst, N. R. Watson and J. Arrillaga, 'Modelling of Bipolar HVDC Links in the Harmonic Domain,' *IEEE Transactions on Power Delivery*, Vol. 15, No. 3, July 2000, pp. 1034-1038.

- M. Carrescia, F. Profumo and M. Tartaglia, 'Prediction of Magnetic Fields in Multiconductor Systems with Significant Harmonic Currents,' *IEEE Transactions on Industry Applications*, Vol. 36, No. 5, September/October 2000, pp. 1206-1211.
- A. Heiss, G. Balzer, O. Schmitt and B. Richter, 'Surge Arresters for Cable Sheath Preventing Power Losses in M.V. Networks,' *IEE Conference of Electricity Distribution, Part 1:* Contributions. Vol. Summaries (IEE Conference Publish No.482), 2001.
- P. Caramia, G. Carpinelli, A. La Vitola and P. Verde, 'On the Economic Selection of Medium Voltage Cable Sizes in Nonsinusoidal Conditions,' *IEEE Transactions on Power Delivery*, Vol. 17, No. 1, January 2002, pp. 1-7.
- 53. M. H. Shwehdi, A. H. Mantawy and H. H. Al-Bekhit, 'Solving the Harmonic Problems Produced from the Use of Adjustable Speed Drives in Industrial Oil Pumping Field,' *IEEE Power System Technology, International Power Conference Proceedings*, Vol. 1, October 2002, pp. 86-92.
- 54. Z. H. Yuan and Y. Du, 'Harmonic Impedance of Single-Core Armoured Cables,' *IEEE/PES* Transmission and Distribution Conference and Exposition, Vol. 1, September 2003, pp.45-48.
- 55. M. Yin, G. Li, M. Zhou and Y. Liu, 'Analysis and Control of Wind Farm Incorporated VSC-HVDC in Unbalanced Conditions,' *IEEE/PES Transmission and Distribution Conference and Exhibition: Asia and Pacific*, August 2005, pp.1-6.
- S. A. Papathanassiou and M. P. Papadopoulous, 'Harmonic Analysis in a Power System with Wind Generation,' *IEEE Transactions on Power Delivery*, Vol. 21, No. 4, October 2006, pp. 2006-2016.
- 57. O. I. Gilbertson, 'Electrical Cables for Power and Signal Transmission,' John Wiley & Sons, USA, 2000.
- T. Zachos, 'Technical and Economical Evaluation of Power Transmission Options for Gas to Wire Project,' MSc Thesis, Department of Mechanical Engineering, University College London, December 2002.
- S. Koyama, H. Uno, K. Fujii, S. Gouda, Y. Nakamura, S. Simiyia, Y. Shimura and I. Shigetoshi, 'DC±500kV Oil-Filled Submarine Cable Crossing Kii-Channel,' Fujicura Technical Review, 2001, pp. 45-55.
- B. Normark and E. K. Nielsen, 'Advanced Power Electronics for Cable Connection of Offshore Wind,' presented at Copenhagen Offshore Wind, 2005.
- B. M. Brown, 'Review of Structural Configurations and Future Options Definition for High Power Subsea Power Cables,' MSc Thesis, Department of Mechanical Engineering, University College London, September 2001.
- 62. R. Bartnikas and K. Shrivastava, 'Power and Communication Cable, Theory and Applications.' IEEE Press, New Jersey, 2000.
- 63. G. J. Wakileh, 'Power System Harmonics: Fundamentals, Analysis and Filter Design,' Springer, Germany, 2001.

- 64. J. Arrillaga, N. R. Watson and S. Chen, 'Power System Quality Assessment,' John Wiley & Sons, England, 2000.
- G. Bianchi and G. Luoni, 'Induced Currents and Losses in Single-core Submarine cables,' *IEEE Transactions on Power Apparatus and Systems*, PAS-95, No.1, January/ February 1976, pp. 49-57.
- J. Guo, J. He, S. CHEN, R. Zeng and Z. Guan, 'Equivalent Simplified Parallel Multi-Conductors Model of Shield Control Cable in Substation,' *IEEE International Symposium on Electromagnetic Compatibility*, pp. 87-90, May 2002.
- 67. S. Sörensen, and H. Nielsen, 'Underground Cables and Overhead Lines Earth Return Path Impedance Calculations with Reference to Single Line to Ground Faults,' *Nordic Distribution and Asset Management Conference*, Espoo, Finland, August 2004.
- 68. S. A. Nasar and F. C. Trutt, 'Electric Power Systems,' CRC Press LLC, USA, 1999.
- 69. E. Acha and M. Madrigal, 'Power Systems Harmonics: Computer Modelling and Analysis,' John Wiley & Sons, England, 2001.
- 70. G. Lucca, 'Mutual Impedance between an Overhead and a Buried Line with Earth Return,' *IEE* International Conference on Electromagnetic Compatibility, pp. 80-86, September 1994.
- A. Deri, G. Tevan, A. Semlyen and A. Castanheira, 'The Complex Ground Return Plane: A Simplified Model for Homogeneous and Multi-Layer Earth Return.' *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-100, No. 8, pp. 3686-3693, August 1981.
- A. Semlyen and M. H. Abdel-Rahman, 'Transmission Line Modelling by Rational Transfer Functions,' *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-101, No. 9, pp. 3576-3584, September 1982.
- A. Semlyen and A. Deri, 'Time Domain Modelling of Frequency Dependent Three-Phase Transmission Line Impedance,' *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-104, No. 6, pp. 1549-1555, June 1985.
- R. G. Olsen and T. A. Pankaskie, 'On the Exact, Carson and Image Theories for Wires at or Above the Earth' Interface,' *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-102, No. 4, pp. 769-778, April 1983.
- 75. W. L. Weeks, 'Transmission and Distribution of Electrical Energy,' Harper & Row, Publishers, New York, 1981.
- S. Galli, 'Exact Conditions for the Symmetry of a Loop,' *IEEE Communications Letters*, Vol. 4. No. 10, pp. 307-309, October 2000.
- A. A. Ghandakly and R. L. Curran, 'A Model to Predict Current Distributions in Heavy Current Parallel Conductor Configurations,' *IEEE Transactions on Industry Applications*, Vol. 30, No. 2, March/April 1994, pp. 240-244.

- 78. T. T. Nguyen, 'Earth-Return Path Impedances of Underground Cables. Part 1: Numerical Integration of Infinite Integrals,' *IEE Proceedings Generations, Transmission and Distributions*, Vol. 145, No. 6, November 1998, pp. 621-626.
- 79. L. M. Wedepohl, and D. J. Wilcox, 'Transient Analysis of Underground Power-Transmission Systems; System Model and Wave-Propagation Characteristics,' *IEE Proceedings on Generation, Transmission and Distribution*, Vol. 120, No. 3, February 1973, pp. 253-260.
- 80. B. Gustavsen, J. A. Martinez, and D. Durbak, IEEE PES Task Force on Data for Modeling System Transients of IEEE PES Working Group on Modeling and Analysis of Systyem Transients Using Digital Simulation (General Systems Subcommittee), 'Parameter Determination for Modeling System Transients- Part II: Insulated Cables,' *IEEE Transactions on Power Delivery*, Vol. 20, No. 3, July 2005, pp. 2045-2050.
- J. Vargas, A. Guzman and J. Robles, 'Underground/ Submarine Cable Protection Using a Negative- Sequence Directional Comparison Scheme,' Schweitzer Engineering Laboratories Technical Papers, Washington USA, 1999.
- G. J. Cokkinides and A. P. S. Meliopoulos, 'Harmonic Sequence Impedance of MultiPhase Underground Cables,' *ICHPS International Conference on Harmonics in Power Systems*, pp. 96-101, September 1992.
- Y. Du and Z. H. Yuan, 'Harmonic Impedance of Armour Cables for Low-voltage Power Distribution in Buildings,' *IEEE Industry Application Society Conference, IAS 40<sup>th</sup> Annual Meeting*, Vol. 3, October 2004, pp. 2259-2263.
- R. Schinzinger and A. Ametani, 'Surge Propagation Characteristics of Pipe Enclosed Underground Cables,' *IEEE Transactions on Power Apparatus and System*, Vol. PAS-97, No. 5, Sept/Oct 1978, pp. 1680-1687.
- Y. Yin and H. W. Dommel, 'Calculation of Frequency-Dependent Impedances of Underground Power Cables with Finite Element Method,' *IEEE Transactions on Magnetics*, Vol. 25, No. 4, July 1989, pp. 3025-3027.
- J. A. Palmer, R. C. Degeneff, T. M. McKernan and T. M. Halleran, 'Pipe-Type Cable Ampacities in the Presence of Harmonics,' *IEEE Transactions on Power Delivery*, Vol. 8, No. 4, October 1993, pp. 1689-1695.
- 87. Y. Yang, J. Ma, and F. P. Dawalibi, 'Computation of Cable Parameters for Pipe-Type Cables with Arbitrary Pipe Thickness,' *IEEE/PES Transmission and Distribution Conference and Exposition*, Vol. 2, 2001, pp. 659-662.
- N. Amekawa, N. Nagaoka, and A. Ametani, 'Impedance Derivation and Wave Propagation Characteristics of Pipe-Enclosed and Tunnel-Installed Cables,' *IEEE Transactions on Power Delivery*, Vol. 19, No.1, January 2004, pp. 380-386.
- T. Imai, 'Exact Equation for Calculation of Sheath Proximity Loss of Single-Conductor Cables,' *Proceedings of the IEEE*, Vol. 56, No. 7, pp. 1172-1181. July 1968.

- 90. G. J. Anders, 'Rating of Electric Power Cables,' Chapter 7, IEEE Press and McGraw-Hill, 1997.
- 91. E. E. Kriezis and J. A. Tegopoulos, 'Transient Eddy Current Distribution in Cylindrical Shells,' *IEEE Transactions on Magnetics*, Vol. 1, No.5, September 1975, pp. 1529-1531.
- 92. M. Kane, A. Ahmad and P. Auriol, 'Multiwire Shield Cable Parameter Computation,' *IEEE Transactions on Magnetics*, Vol. 31, No.3, May 1995, pp. 1646-1649.
- 93. M. Kane and Ph. Auriol, 'Analytical Modelling of Frequency Parameters of Lines,' International Conference on Computation in Electromagnetics, pp. 239-242, April 1994.
- 94. K. Ferkal and E. Dorisoni, 'Proximity Effect and Eddy Current Losses in Insulated Cables,' *IEEE Transactions on Power Delivery*, Vol. 11, No.3, July 1996, pp. 1171-1178.
- A. Ametani, 'A General Formulation of Impedance and Admittance of Cables,' IEEE Transactions on Power Apparatus and Systems, Vol. PAS-99, No.3, May/June 1980, pp. 902-910.
- 96. G. W. Brown and R. G. Rocamora, 'Surge Propagation in Three-phase Pipe-Type, Part 1-Unsaturated Pipe,' *IEEE Transactions on Apparatus and Systems*, Vol. PAS-95, No.1, January/February 1976, pp. 89-95.
- X. B. Xu, G. Liu, and P. Chow, 'A Finite-Element Method Solution of the Zero-Sequence Impedance of Underground Pipe-Type Cable,' *IEEE Transactions on Power Delivery*, Vol. 17, No.1, January 2002, pp. 13-17.
- 98. A. Trageser, L. L. Lai and R. Yokoyama, 'Analysis of Sheath Transients in Specially Bonded Cable Systems Using EMTP,' *IEEE International Conference on Energy Management and Power Delivery: Proceedings of EMPD*, Vol. 2, pp. 694-697, March 1998.
- 99. S. Katakai, 'Design of XLPE Cables and Soundness Confirmation Methods to Extra High Voltage XLPE Cables,' Asia Pacific IEEE/PES Transmission and Distribution Conference and Exhibition, Vol. 2, October 2002, pp 1411 – 1415.
- Y. Du and J. Burnett, 'Experimental Investigation into Harmonic Impedance of Low-Voltage Cables' IEE Proceedings Generations, Transmission and Distributions, Vol. 147, No. 6, November 2000, pp. 322-328.
- 101. K. Eriksson, P. Halvarsson, D. Wensky and M. Hausler, 'System Approach on Designing an Offshore Windpower Grid Connection,' Fourth International Workshop on Wind Power and Transmission Networks for Offshore Wind Farm, Billund, Denmark, October 2003.
- 102. L. Gyugyi, 'Power Electronics in Electric Utilities: Static Var Compensators,' Proceedings of the IEEE, Vol. 76, No.4, April 1988, pp. 483-494.
- 103. J. J. Rico, E. Acha, T. J. E. Miller, 'Harmonic Domain Modelling of Three Phase Thyristor-Controlled Reactors by Means of Switching Vectors and Discrete Convolutions,' *IEEE Transactions on Power Delivery*, Vol. 11, No. 3, July 1996, pp. 1678-1684.

- 104. Task Forces on Harmonics Modeling and Simulation, IEEE PES Harmonic Working Group, 'Characteristics and Modeling of Harmonic Sources-Power Electronic Devices,' IEEE Transactions on Power Delivery, Vol. 16, No, 4, October 2001, pp. 791-800.
- 105. Y. H. Chung, G. H. Kwon, T. B. Park and G. Y. Lim, 'Voltage Sag and Swell Generator with Thyristor Controlled Reactor,' *International Conference on Power System Technology*, Vol. 3, pp. 1933-1937, October 2002.
- L. J. Bohmann and R. H. Lasseter, 'Harmonic Interactions in Thyristor Controlled Reactor Circuits,' *IEEE Transactions on Power Delivery*, Vol. 4, No. 3, July 1999, pp. 1919-1926.
- 107. K. S. Smith, P. Brogan and L. Wilson, 'Measurements and Simulation of DC Drive Harmonic on an Interconnected Offshore System with Long Subsea Cables,' 7<sup>th</sup> European Power Electronics Conference, September 1997, pp. 4.946-4.951.
- 108. N. R. Watson and J. Arrillaga, 'Harmonic Assessment using Electromagnetic Transient Simulation and Frequency-Dependent Network Equivalents,' *IEE Proceeding, Generation Transmission and Distribution*, Vol. 150, No. 6, pp. 641-650, November 2003.
- 109. J. W. Dixon, 'Three-Phase Controlled Rectifies', Chapter 12 in Power Electronics Handbook, Academic Press, pp. 12-1-12-63, August 2001.
- 110. C. W. Lander, 'Power Electronics,' Third Edition, McGraw-Hill Book, 1993.
- 111. M. Madrigal and E. Acha, 'Harmonic Modelling of Voltage Source Converters for HVDC Stations,' *IEE AC-DC Power Transmission Conference*, Publication No. 485, pp. 169-174, November 2001.
- 112. D. M. Larruskain, I. Zamora, A. J. Mazon, O. Abarrategui and J. Monasterio, 'Transmission and Distribution Networks: AC versus DC,' Spanish-Portuguese Congress on Electrical Engineering, Marbella, June and July, 2005.
- 113. M. Hyttinen and K. Bentzen, 'Operating Experiences with a Voltage Source Converter (HVDClight) on the Gas Platform Troll A,' in *Energex 2006 International Conference and Exhibition*, June 2006.
- 114. A. Lindberg and L. Lindberg, 'High Power Voltage Source Converter Control Response at Large AC Voltage Phase Shifts' *IEEE International Conference on Power Electronics and Devices* Systems, Vol. 2, pp. 719-725, Feb 2005.
- 115. D. Krug, S. Bernet and S. Dieckerhoff, 'Comparison of State-of the Art Voltage Source Converter Topologies for Medium Voltage Applications,' *Industry Applications Conference Annual Meeting*, Vol. 1, pp. 168-175, October 2003.
- W. Lu, and B. T. Ooi, 'Optimal Acquisition and Aggregation of Offshore Wind Power by Multiterminal Voltage-Source HVDC,' *IEEE Transactions on Power Delivery*, Vol. 18, No. 1, pp. 201-206, January 2003.

- 117. K. H. Chan, J. A. Parle, N. Johnson, and E. Acha, 'Real-Time Implementation of a HVDC-VSC Model for Application in a Scaled-down Wind Energy Conversion System,' *IEE AC-DC Power Transmission Conference*, Publication No. 485, pp. 169-174, November 2001.
- M. Madrigal, 'Modeling of Power Electronics Controllers for Harmonic Analysis in Power Systems,' PhD Thesis, University of Glasgow, pp. 47-60, December 2001.
- 119. K. R. Padiyar and N. Parbhu, 'Modelling, Control Design and Analysis of VSC Based HVDC Transmission Systems,' *IEEE International Conference on Power System Technology*, pp. 774-779, November 2004.
- 120. S. Poullain, F. heliodore, A. Henni, J. L. Thomas and E. Courbon, 'Modelling of the Dynamic Characteristics of the DC Line for VSC Transmission Scheme,' *IEE AC-DC Power Transmission Conference*, pp. 305-310, November 2001.
- 121. F. Soares dos Reis, J. A. V. Ale, F. D. Adegas, R. Tonkoski, S. Slan and K. Tan, 'Active Shunt Filter for Harmonic Mitigation in Wind Turbines Generators,' *IEEE Power Electronic Specialists* Conference, pp. 1-6, June 2006.
- 122. D. Xiang, R. Li, J. R. Bumby, P. J. Tavner and S. Yang, 'Coodinated Control of an HVDC Link and Doubly Fed Induction Generators in a Large Offshore Wind Farm,' *IEEE Transactions on Power Delivery*, Vol. 21, No. 1, pp. 463-471, January 2006.
- 123. X. Wu, L. Holdsworth, N. Jenkins and G. Strbac, 'Integrating Renewables and CHP into the UK electricity System: Investigation of the Impact of Network Faults on the Stability of Large Offshore Wind Farms,' Tyndall Centre for Climate Change Research, April 2003.
- 124. K. Sobrink, D. Woodford, R. Belhomme and E. Joncquel, 'AC Cable Versus DC Cable Transmission for Offshore Wind Farms, a Study Case,' International Workshop on Large-Scale Integration of Wind Form and Transmission Networks for Offshore Wind Farms, pp. 1-6, October 2003.
- W. Lu and B. T. Ooi, 'Premium Quality Power Park Based on Multi-Terminal HVDC,' IEEE Transactions on Power Delivery, Vol. 20, No. 2, pp. 978-983, April 2005.
# A

# Electrical Parameters of Transmission Lines

#### A.1 Introduction

The transmission line has been considered as a connecting device providing path for the power flow between several circuits in the system. For this reason, it is regarded as having a sending end and a receiving end with a series resistance and inductance and shunt capacitance and conductance as characteristic parameters. The follows are the electrical characteristics for both AC and DC transmission lines referring to [57] [68].

#### A.2 DC Transmission Line Parameters

The DC resistance R is given by

$$R = \rho \frac{l}{A} \tag{a.1}$$

Where,  $\rho$  is the resistivity of the conductor, *l* is the conductor length, and *A* is the conductor area of cross section in.

The resistance of a conductor is greatly affected by the operating temperature of the conductor, linearly increasing with temperature.

$$R_T = R_0(1 + \alpha_0 T) \tag{a.2}$$

The resistance  $R_T$  at a temperature  $T^{\circ}C$  is related to the resistance  $R_0$  at  $0 \circ C$ . Therefore, the losses contributed by the resistance are:

$$P = I^2 R \tag{a.3}$$

Based on the equation above, the accurate operating temperature is essential for assessing cable losses. In order to investigate the temperature distribution in the cable, the cable structure is regarded as a cylinder. According to the Fourier equation for cylindrical systems, heat flow is:

$$q = -KA\frac{dT}{dr} = -K(2\pi rL)\frac{dT}{dr}$$
(a.4)

Where, A is the area perpendicular to the direction of heat transfer, K is the thermal conductivity, L is the length. Specifically, we can estimate each part of the temperature distribution within the cable in order to calculate the accurate resistance.

The equation can be reformed by applying boundary conditions.

$$q = \frac{2\pi L K (T_2 - T_1)}{\ln(r_2 / r_2)}$$
(a.5)

The equation above could be regarded as:  $T_2 - T_1$  is the voltage difference, q is the current and  $\frac{\ln(r_2/r_1)}{2\pi LK}$  is the thermal resistance.

#### A.3 AC Transmission Line Parameters

#### A.3.1 Resistance

The resistance of AC is influenced by the skin and the proximity effect. While operating on AC, the current-density distribution across the conductor cross section becomes non-uniform, and is a function of the AC frequency. This phenomenon is known as the skin effect, and therefore, the AC resistance of a conductor is higher than its DC resistance. When the distribution of the AC current density is not uniform because of other conductors of other cables nearby, this phenomenon is the proximity effect. However, compared to skin effect, the proximity effect is usually negligible. The main variable cause of resistance is skin effect.

$$R_{ac} = R_{dc}(1 + y_{skin}) \tag{a.6}$$

Concerning losses, the axial induction currents in the metallic sheath are one of the important causes. The current increases due to the induction sheath voltage that is contributed because of the adjacent axial conductors carrying the current which produces a magnetic field leading to the voltage drop of the sheath. In addition, other causes of losses are the eddy current losses and hysteresis loss whose quantities depend upon the material of the conductors.

#### A.3.2 Inductance

The inductance may be determined either by finding the flux linkage or from magnetic field energy storage concept. The following explanation is adopted using the former concept. First, the case is simplified into a solid single round conductor. Applying the defining equation, determination of the inductance of the conductor whose cross section is shown in **Figure A.1** can be found as follows.



Figure A.1 Magnetic Fields Internal and External to a Conductor

Inductance= flux linkage per ampere:

$$L = \frac{\lambda}{I}$$
(a.7)

Applying Ampere's Law, the magnetic fields in the conductor and the external to the conductor can be decided.

$$\oint H \cdot dl = I \tag{a.8}$$

In other words, around a closed path the line integral of the magnetic field intensity H equals the total current I enclosed by the path. Consideration of conduction, if r is the radius of the conductor and it carries a current I, the field at some radius x is given by:

$$H_{\phi}(2\pi x) = I_{x} \quad 0 < x < r \tag{a.9}$$

Where,  $I_x$  is the total current through a cross section  $\pi x^2$ .  $I_x$  can be written as:

$$I_x = \frac{I}{\pi r^2} (\pi x^2) \tag{a.10}$$

Combining the above two equations by assuming there is a uniform current density through the conductor:

$$H_{\phi} = \frac{lx}{2\pi r^2} \quad 0 < x < r \tag{a.11}$$

Because B and H is given by  $B = \mu H = \mu_0 H$  (assuming the conductor being non-magnetic)

$$B_{\phi} = \frac{\mu_0 I x}{2\pi r^2} \quad 0 < x < r \tag{a.12}$$

Thus, the circumferential flux within the annular cylinder of thickness dx per-unit length of the cylinder is:

$$d\phi = B_{\phi}dx = \frac{\mu_0 I}{2\pi r^2} x dx \tag{a.13}$$

Based on the equation above, estimation of the flux linkage within the conductor is the product of the flux and the fraction of the current linked.

$$\lambda_{in} = \int_{0}^{r} \frac{\pi x^{2}}{\pi r^{2}} d\phi = \frac{\mu_{0} I}{2\pi r^{4}} \int_{0}^{r} x^{3} dx = \frac{\mu_{0} I}{8\pi}$$
(a.14)

Leading to

$$L = \frac{\lambda_{in}}{I} = \frac{\mu_0}{8\pi}$$
(a.15)

In order to find the inductance from the flux linkage external to the conductor, the circumferential flux within a cylinder bounded by the outer radius  $D_2$  and inner radius  $D_1$  as shown in Figure A.1 can be determined. Outside of the conductor, Ampere's law yields:

$$H_{\phi} = \frac{l}{2\pi x} \qquad r < x < \infty \tag{a.16}$$

$$B_{\phi} = \mu_0 H_{\phi} = \frac{\mu_0 I}{2\pi x} \qquad r < x < \infty \tag{a.17}$$

Thus

$$d\phi = B_{\phi}dx = \frac{\mu_0 I}{2\pi} \frac{dx}{x}$$
(a.18)

We could estimate the flux linkage between the radius  $D_2$  and  $D_1$ 

$$\lambda_{ex} = \frac{\mu_0 I}{2\pi} \int_{D_1}^{D_2} \frac{dx}{x} = \frac{\mu_0 I}{2\pi} \ln \frac{D_2}{D_1}$$
(a.19)

The inductance will be

$$L = \frac{\mu_0}{2\pi} \ln \frac{D_2}{D_1}$$
(a.20)

#### A.3.3 Capacitance

The transmission parameter of particular importance for long length transmission line is the capacitance between various conductors of the transmission line. Define the capacitance as charge per-unit volt as flux linkage per ampere for inductance:

$$C = \frac{Q}{V}$$
(a.21)

Referring Figure A.2, define Q as the charge (in coulomb) on one of the conductors and V as the voltage (potential difference) between the two conductors, which will be used as the basic symbols for determination of capacitance of various line configurations.



Employ Gauss' law to find the electric field caused by the charge and then determine the voltage from the electric field. According to Gauss' law, the total electric flux outward from a closed surface equals the charge enclosed by the surface, in which the electric flux density D is related to the electric field E by

$$D = \varepsilon E \tag{a.22}$$

In the equation above,  $\varepsilon$  is defined as the permittivity of the material (in F/m) where D and E exist. From Figure A.2, we could imagine a cylinder of unit length and of radius  $R^+$ , concentric with the conductor a. The total outward flux from the surface of the cylinder is given by

$$\varphi_r = D_r 2\pi R^+ \tag{a.23}$$

In which  $D_r$  is the flux density at the surface of the cylinder. If Q is the total charge on a unit length of the conductor, with Gauss' law combined with  $D = \varepsilon E$  and the equation above, a reforming equation as following can be obtained:



$$Q = \varphi_e = D_r 2\pi R^+ = 2\pi \varepsilon_0 E_r R^+ \tag{a.24}$$

 $\varepsilon_0$  =permittivity of free space =  $10^{-9}/36\pi$  (F/m). Eventually, reform again for the electric field in terms of the charge,

$$E_r = \frac{Q}{2\pi\varepsilon_0 R^+}$$
(a.25)

Now, determine the voltage  $V_{op}^{+} = -\int_{R_{0^{+}}}^{R_{+}} E_{r} dr = -\frac{Q}{2\pi\varepsilon_{0}} \int_{R_{0^{+}}}^{R_{+}} \frac{1}{R^{+}} dR^{+} = \frac{Q}{2\pi\varepsilon_{0}} \ln \frac{R_{0}^{+}}{R^{+}}$  (a.26)

Due to the charge Q on conductor a, the voltage can be acquired. Repeating the calculation process once more for the charge –Q on conductor b,

$$V_{op} = -\frac{Q}{2\pi\varepsilon_0} \ln \frac{R_0}{R}$$
(a.27)

Combining the positive and negative voltages, yields

$$V_{op} = V_{op}^{+} + V_{op}^{-} = \frac{Q}{2\pi\varepsilon_0} \ln \frac{R_0^+ R^-}{R^+ R_0^-}$$
(a.28)

Let P come to the surface of a and O to the surface of b, and such  $R^+ = r_a$ ,  $R_0^- = r_b$ , and  $R_0^+ = d = R^-$ . Thus, reform and substitute these to the equation above:

$$V_{op} = \frac{Q}{\pi \varepsilon_0} \ln(\frac{d}{\sqrt{r_a r_b}})$$
(a.29)

Hence, from the equation (a.21) and this equation above, and assuming the two conductors are of the same radius in the particular case  $r = r_a = r_b$ , the capacitance per-unit length of the line is given by

$$C = \frac{\pi \varepsilon_0}{\ln(d/r)}$$
(a.30)

The effect of earth may be simulated by image charge since earth may be considered to be an equipotential surface as shown in Figure A.3.



Figure A.3 Image of Earth Effect

#### A.3.4 Shunt Conductance

There is a small leakage of current through the insulator, when a voltage is applied between a pair of conductors. This is the so called "shunt current" which is usually negligible. Therefore, in some cases, the shunt conductance per-unit length could be defined as the ratio of the shunt current that flows from one conductor to the other in a unit length to the voltage between. No separate calculation of shunt conductance is needed if the capacitance per unit length is known. The shunt is represented as the equation following:

$$Y = \frac{\sigma}{\varepsilon} C \tag{a.31}$$

The capacitance needs to be multiplied by the ratio of conductivity to permittivity of the dielectric in order to determine the shunt Y.

# В

# Cable Dimensions and Materials for Case Studies

## B.1 Cable Dimensions and Materials of Case Study 4.5



Figure B.1 Three Insulated Cables in Trefoil Touching Configuration

Copper Conductor Diameter:	10mm
XLPE Insulation Thickness:	17mm
Permittivity of XLPE	2.5 × Air Permittivity
Outer Diameter of Cable:	44mm
Length:	50km

# B.2 Cable Dimensions and Materials of Case Study 4.6



## Figure B.2 Three Single-Core Subsea Cables Trefoil Touching Configuration

Copper Conductor Diameter:	37.9mm
Conductor Screen Thickness:	1.7mm
XLPE Insulation Thickness:	15mm
Insulation Screen Thickness:	1mm
Bedding Thickness:	0.6mm
Lead Sheath Thickness:	2.5mm
Inn Sheath Thickness:	2.2mm
Bedding Thickness:	0.15
Armour Copper Wire Diameter:	5mm
Copper Wire Number:	51
Polypropylene Yarn Thickness:	2mm
Permittivity of XLPE	2.5 × Air Permittivity
Outer Diameter of Cable:	100mm
Length:	50km
Sea Depth:	50m

#### B.3 Cable Dimensions and Materials of Case 4.7 Validation of PSCAD/EMTDC



### Figure B.3 Three Insulated Cables with Sheath in Trefoil Touching Configuration

Copper Conductor Diameter:	20mm
XLPE Insulation Thickness:	38mm
Permittivity of XLPE	$2.5 \times Air Permittivity$
Lead Sheath Thickness:	2.5mm
Outer Diameter of Cable:	44mm
Distance between Cables:	50mm
Length:	50km
Sea Depth:	50m

## B.4 Cable Dimensions and Materials of Case Study 5.2

For Single-Core Cables:	
Copper Conductor Diameter:	37.9 mm
XLPE Insulation Thickness:	17 mm
Relative Permittivity of XLPE:	2.5
Diameter over Insulation:	78.5 mm
Lead Sheath Thickness:	2.5 mm
Outer Diameter of Cable:	102.2 mm
Steel wire Number:	54
Steel wire Diameter:	5mm
For Three-Core AC cables:	
Copper Conductor Diameter:	29.8 mm
XLPE Insulation Thickness:	17 mm

Relative Permittivity of XLPE:2.5Diameter over Insulation:69 mmMetallic Cable Sheath Thickness:2.4 mmOuter Diameter of Cable:187 mmSteel wire Number:106

Steel wire Diameter: 5mm

# B.5 Cable Dimensions and Materials of Case Study 6.4

Copper Conductor Diameter:	26.4 mm
XLPE Insulation Thickness:	16 mm
Relative Permittivity of XLPE:	2.3
Diameter over Insulation:	58.4 mm
Copper Sheath Screen Thickness:	0.5 mm
Outer Diameter of Cable:	72 mm
Steel wire Number:	60
Steel wire Diameter:	3.4 mm

## B.6 Cable Dimensions and Materials of Case Study 7.3

Single-Core AC cables:	
Copper Conductor Diameter:	37.9 mm
XLPE Insulation Thickness:	17 mm
Permittivity of XLPE	2.5 x Air Permittivity
Diameter over Insulation:	78.5 mm
Lead Sheath Thickness:	2.5 mm
Outer Diameter of Cable:	102.2 mm
Steel wire Number:	54
Steel wire Diameter:	5mm
Length:	50 km
Three-Core AC cables:	
Copper Conductor Diameter:	29.8 mm
XLPE Insulation Thickness:	17 mm
Permittivity of XLPE	2.5 x Air Permittivity
Diameter over Insulation:	69 mm
Metallic Cable Sheath Thickness:	2.4 mm
Outer Diameter of Cable:	187 mm
Steel wire Number:	106
Steel wire Diameter:	5mm
Length:	50 km

# B.7 Cable Dimensions and Materials of Case Study 8.2

Dimension of the 150 kV DC Cable:	
Copper Conductor Diameter:	37.9 mm
Insulation Thickness:	17 mm
Insulation Material:	XLPE
Permittivity of XLPE:	2.5 x Air Permittivity
Diameter over Insulation:	78.5 mm
Lead Sheath Thickness:	2.5 mm
Outer Diameter of Cable:	102.2 mm
Armour Material:	Steel
Armour wire Number:	54
Armour wire Diameter:	5mm

# B.8 Cable Dimensions and Materials of Case Study 8.5.4

Dimension of the 75 kV DC Cable:	
Copper Conductor Diameter:	37.9 mm
XLPE Insulation Thickness:	10 mm
Permittivity of XLPE	2.5 x Air Permittivity
Diameter over Insulation:	61.3 mm
Lead Sheath Thickness:	2.5 mm
Outer Diameter of Cable:	88.2 mm
Steel wire Number:	49
Steel wire Diameter:	5mm

(m)

(m)

# С

# Modelling of Skin Effect and Mutual Coupling using MathCAD

The following lists of the program are constructed using MathCAD<sup>®</sup> v. 11

#### Constants

Geometric position of these cables is:

Cable A: coordinate (0, 0.0254)

Cable B: coordinate (-0.022,-0.0127) (m)

Cable C: coordinate (0.022,-0.0127) (m)

Input the coordinate data:

Coordinate :=

	0	1
0	0	0.0254
1	-0.022	-0.0127
2	0.022	-0.0127

x := Coordinate

# y := Coordinate (1)

The coordinate of cable A is $(x_0, y_0)$	) (m)

- The coordinate of cable B is  $(x_1, y_1)$
- The coordinate of cable C is  $(x_2, y_2)$  (m)

Distance between cables:

$$\mathbf{S} := \sqrt{\left|\mathbf{x}_{1} - \mathbf{x}_{0}\right|^{2} + \left|\mathbf{y}_{1} - \mathbf{y}_{0}\right|^{2}}$$
(m)

Distance to neutral point

$SC := \frac{S}{2}$	
$\sqrt{3}$	(m)
External radius of copper conductor:	
rad_ex := 0.005	(m)
Outer radius of xlpe insulation:	
r_ex_xipe := 0.022	(m)
Permeability of free space:	
μ0 := 4·π·10 <sup>-7</sup>	(H/m)
Permittivity of free space:	
s0 := 8.854 10 <sup>-12</sup>	(F/m)
Permittivity of xlpe:	
ar_xipe := 2.5·s0	(F/m)
Resistance of 10mm diameter copper at 90°C:	
R_c := 0.00028	(Ohm/m)
Coefficient of conductivity at 90°C:	
$\sigma = 4.5 \cdot 10^{7}$	(S/m)
Fundamental frequency:	
<b>10</b> = <b>5</b> 0	(Hz)
Fundamental angular velocity:	
w0 := 2.π·f0	(m/s)
Length of cable:	
long := 50000	(m)
Variables:	
Harmonic order from 1 to 100:	
<b>n</b> := 1 100	
Harmonic frequency:	
f <sub>n</sub> := n·f0	(Hz)
Harmonic angular velocity:	
$\mathbf{w}_{\mathbf{n}} \coloneqq \mathbf{n} \cdot \mathbf{w} 0$	(m/s)

### Calculation of Admittance:

Potential coefficient of three cables:

$$P := \begin{pmatrix} \ln\left(\frac{r\_ex\_xtpe}{rad\_ex}\right) & 0 & 0 \\ 0 & \ln\left(\frac{r\_ex\_xtpe}{rad\_ex}\right) & 0 \\ 0 & 0 & \ln\left(\frac{r\_ex\_xtpe}{rad\_ex}\right) \end{pmatrix}$$

Three phase admittance matrix:

$$\mathbf{Y}_{\mathbf{n}} \coloneqq \mathbf{j} \cdot \mathbf{w}_{\mathbf{n}} \cdot 2 \cdot \boldsymbol{\pi} \cdot \mathbf{er}_{\mathbf{n}} \mathbf{x} \mathbf{p} \mathbf{e} \cdot \mathbf{P}^{-1}$$

#### Calculation of Impedance including skin effect and mutual coupling

The impedance including skin effect of transmission line can be expressed as followings:

$$k_{\mathbf{n}} \coloneqq \sqrt{-j \mathbf{w}_{\mathbf{n}} \cdot \boldsymbol{\mu} \boldsymbol{\Omega} \cdot \boldsymbol{\sigma}}$$

Impedance including skin effect

$$Zskin_{\underline{n}} := R_c \cdot \frac{k_{\underline{n}} \cdot red_{\underline{ox}}}{2} \cdot \frac{J0(k_{\underline{n}} \cdot red_{\underline{ox}})}{J1(k_{\underline{n}} \cdot red_{\underline{ox}})}$$

Where, k is the coefficient of Bessel function, J0 is Bessel function of the order zero, and J1 is Bessel function of the order one.

Therefore, calculation of impedance for three phase including skin effect and mutual coupling are:

Self impedance of phase A

$$2SKIN00_{R} = 2skin_{R} + \frac{j \cdot w_{R} \cdot \mu 0}{2 \cdot \pi} \cdot \ln \left( \frac{SC}{red_{ex}} \right)$$

Mutual impedance between phase A and phase B

$$2SKIN01_{R} = \frac{-j \cdot w_{R} \cdot \mu 0}{2 \cdot \pi} \cdot \ln \left( \frac{S}{SC} \right)$$

Mutual impedance between phase A and phase C

$$2SKIN02_{\mathbf{n}} \coloneqq \frac{-\mathbf{j} \cdot \mathbf{w}_{\mathbf{n}} \cdot \mathbf{\mu} \mathbf{0}}{2 \cdot \mathbf{\pi}} \cdot \ln\left(\frac{S}{SC}\right)$$

Mutual impedance between phase B and phase A

$$2SKIN10_{n} := \frac{-j \cdot w_{n} \cdot \mu 0}{2 \cdot \pi} \cdot \ln \left( \frac{S}{SC} \right)$$

Self impedance of phase B

$$2SKIN11_{\mathbf{n}} = 2skin_{\mathbf{n}} + \frac{j \cdot \mathbf{w}_{\mathbf{n}} \cdot \mu 0}{2 \cdot \pi} \cdot ln \left(\frac{SC}{rad\_sx}\right)$$

Mutual impedance between phase B and phase C

$$ZSKIN12_{\mathbf{n}} \coloneqq \frac{-j \cdot \mathbf{w}_{\mathbf{n}} \cdot \mu \mathbf{0}}{2 \cdot \pi} \cdot \ln \left( \frac{S}{SC} \right)$$

Mutual impedance between phase C and phase A

$$2SKIN20_{n} = \frac{-j \cdot w_{n} \cdot \mu 0}{2 \cdot \pi} \cdot \ln \left(\frac{S}{SC}\right)$$

.

Mutual impedance between phase C and phase B

$$2SKIN21_{\mathbf{n}} = \frac{-j \cdot \mathbf{w}_{\mathbf{n}} \cdot \mu \mathbf{0}}{2 \cdot \pi} \cdot \ln \left(\frac{S}{SC}\right)$$

Self impedance of phase C

$$ZSKIN22_{\underline{n}} := Zskin_{\underline{n}} + \frac{j \cdot w_{\underline{n}} \cdot \mu 0}{2 \cdot \pi} \cdot ln \left(\frac{SC}{rad\_ex}\right)$$

Impedance matrix including skin effect is:

$$Z_{\mathbf{n}} := \begin{pmatrix} ZSKIN00_{\mathbf{n}} & ZSKIN01_{\mathbf{n}} & ZSKIN02_{\mathbf{n}} \\ ZSKIN10_{\mathbf{n}} & ZSKIN11_{\mathbf{n}} & ZSKIN12_{\mathbf{n}} \\ ZSKIN20_{\mathbf{n}} & ZSKIN21_{\mathbf{n}} & ZSKIN22_{\mathbf{n}} \end{pmatrix}$$

#### Calculation of Distributed Parameters including skin effect and mutual coupling

Once the lump parameter such as impedance and admittance acquired, as previous described, distributed parameters can also be obtained by following equations:

Eigenvectors of  $Z_n$  and  $Y_n$ 

$$\mathbf{Ev}_{\mathbf{R}} \coloneqq \mathbf{eigenvecs}_{\mathbf{R}} \mathbf{Z}_{\mathbf{R}} \mathbf{Y}_{\mathbf{R}}$$

Eigenvectors of  $Y_n$  and  $Z_n$ 

Diagonal impedance matrix

$$Z\mathbf{m}_{\mathbf{n}} := (E\mathbf{v}_{\mathbf{n}})^{-1} \cdot Z_{\mathbf{n}} \cdot E\mathbf{i}_{\mathbf{n}}$$

Diagonal admittance matrix

$$\mathbf{Ym}_{\mathbf{n}} \coloneqq (\mathbf{Ei}_{\mathbf{n}})^{-1} \cdot \mathbf{Y}_{\mathbf{n}} \cdot \mathbf{Ev}_{\mathbf{n}}$$

Inverse matrix of Y<sub>m</sub>

$$\mathbf{Yf}_{\mathbf{n}} \coloneqq (\mathbf{Ym}_{\mathbf{n}})^{-1}$$

Diagonal matrix of propagation constants

$$\gamma_n := \overline{\sqrt{(Zm_n \cdot Ym_n)}}$$

Diagonal matrix of characteristic impedance

.

$$Zc_n := \sqrt{Zm_n \cdot Yf_n}$$

The representation transform matrix of transmission line in the form of ABCD is:

$$\begin{pmatrix} \mathbf{V} \mathbf{A} \mathbf{B} \mathbf{C} \\ \mathbf{I} \mathbf{A} \mathbf{B} \mathbf{C} \end{pmatrix} := \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{V} \mathbf{a} \mathbf{b} \mathbf{c} \\ -\mathbf{I} \mathbf{a} \mathbf{b} \mathbf{c} \end{pmatrix}$$

$$\underbrace{\mathbf{A}}_{\mathbf{R}} := (\mathbf{E} \mathbf{v}_{\mathbf{R}}) \cdot \left( \overbrace{\mathbf{tanh}^{i} \gamma_{\mathbf{R}} \cdot \mathbf{long}^{i}} \right)^{-1} \xrightarrow{\mathbf{sinh}^{i} \gamma_{\mathbf{R}} \cdot \mathbf{long}^{i}} \underbrace{\mathbf{E} \mathbf{v}_{\mathbf{R}}}^{-1}$$

$$\underbrace{\mathbf{B}}_{\mathbf{R}} := (\mathbf{E} \mathbf{v}_{\mathbf{R}}) \cdot \mathbf{Z} \mathbf{c}_{\mathbf{R}} \cdot \overrightarrow{\mathbf{sinh}^{i} \gamma_{\mathbf{R}} \cdot \mathbf{long}^{i}} + \underbrace{\mathbf{E}}_{\mathbf{R}}^{-1}$$

$$\underbrace{\mathbf{C}}_{\mathbf{R}} := (\mathbf{E} \mathbf{i}_{\mathbf{R}})^{-1} \mathbf{Z} \mathbf{c}_{\mathbf{R}}^{-1} \cdot \overrightarrow{\mathbf{sinh}^{i} \gamma_{\mathbf{R}} \cdot \mathbf{long}^{i}} + \underbrace{\mathbf{E}}_{\mathbf{R}}^{-1}$$

$$\underbrace{\mathbf{D}}_{\mathbf{R}} := \mathbf{A}_{\mathbf{R}}^{T}$$

#### Calculation of Impedance without skin effect

Use the same equation forms from the previous calculations of skin effect. The difference is that the self impedances take account of only resistance of conductor instead of skin effect: Self impedance of phase A

$$ZNOSK00_{\mathbf{h}} \coloneqq R_c + \frac{j \cdot \mathbf{w}_{\mathbf{h}} \cdot \mu 0}{2 \cdot \pi} \cdot \ln \left( \frac{SC}{rad_ex} \right)$$

Mutual impedance between phase A and phase B

$$ZNOSK01_{\mathbf{R}} := \frac{-j \cdot \mathbf{w}_{\mathbf{R}} \cdot \boldsymbol{\mu} \mathbf{0}}{2 \cdot \pi} \cdot \ln \left(\frac{\mathbf{S}}{\mathbf{SC}}\right)$$

Mutual impedance between phase A and phase C

$$ZNOSK02_{\mathbf{R}} := \frac{-j \cdot \mathbf{w}_{\mathbf{R}} \cdot \boldsymbol{\mu} \mathbf{0}}{2 \cdot \boldsymbol{\pi}} \cdot \ln \left( \frac{\mathbf{S}}{\mathbf{SC}} \right)$$

Mutual impedance between phase B and phase A

$$ZNOSK10_{\mathbf{n}} = \frac{-j \cdot \mathbf{w}_{\mathbf{n}} \cdot \mu \mathbf{0}}{2 \cdot \pi} \cdot \ln \left(\frac{S}{SC}\right)$$

Self impedance of phase B

$$ZNOSK11_{n} := R_c + \frac{j \cdot w_n \cdot \mu}{2 \cdot \pi} \cdot \ln\left(\frac{SC}{rad_ex}\right)$$

Mutual impedance between phase B and phase C

$$ZNOSK12_{\underline{n}} := \frac{-j \cdot \underline{w}_{\underline{n}} \cdot \mu \underline{0}}{2 \cdot \pi} \cdot \ln \left( \frac{S}{SC} \right)$$

.

Mutual impedance between phase C and phase A

$$ZNOSK20_{\mathbf{R}} := \frac{-j \cdot \mathbf{w}_{\mathbf{R}} \cdot \boldsymbol{\mu} \mathbf{0}}{2 \cdot \boldsymbol{\pi}} \cdot \ln \left(\frac{\mathbf{S}}{\mathbf{SC}}\right)$$

Mutual impedance between phase C and phase B

$$2NOSK2i_{n} = \frac{-j \cdot w_{n} \cdot \mu \theta}{2 \cdot \pi} \cdot \ln \left(\frac{S}{SC}\right)$$

self impedance of phase C

$$ZNOSK22_{n} := R_c + \frac{j \cdot w_n \cdot \mu 0}{2 \cdot \pi} \cdot ln \left(\frac{SC}{rad_{ex}}\right)$$

Impedance matrix without skin effect is

$$Z1_{n} := \begin{pmatrix} ZNOSK00_{n} & ZNOSK01_{n} & ZNOSK02_{n} \\ ZNOSK10_{n} & ZNOSK11_{n} & ZNOSK12_{n} \\ ZNOSK20_{n} & ZNOSK21_{n} & ZNOSK22_{n} \end{pmatrix}$$

#### Calculation of Distributed Parameters without skin effect

Adopt the same equations from previous equations and the difference is the impedance matrix is Z1 instead of Z.

$$Evl_{n} := eigenvecs(Zl_{n} Y_{n})$$

$$Eil_{n} := eigenvecs(Y_{n} Zl_{n})$$

$$Zcl_{n} := (Evl_{n})^{-1} Zl_{n} Eil_{n}$$

$$Ycl_{n} := (Eil_{n})^{-1} Y_{n} Evl_{n}$$

$$Yfl_{n} := (Ycl_{n})^{-1}$$

$$\gamma l_{n} := \sqrt{Zcl_{n} Ycl_{n}}$$

$$Zml_{n} := \sqrt{Zcl_{n} Yfl_{n}}$$

$$Al_{n} := (Evl_{n}) (tenh) \gamma l_{n} long)^{-1} (teil_{n})^{-1}$$

$$Bl_{n} := (Evl_{n}) Zml_{n} (teil_{n})^{-1}$$

$$Bl_{n} := (Evl_{n}) Zml_{n} (teil_{n})^{-1}$$

$$Dl_{n} := Al_{n}^{T}$$

#### Calculation of Impedance without mutual coupling

Use the same equation forms from the previous calculations of skin effect and the difference is that the mutual impedances are set to be zero:

Self impedance of phase A

$$ZNOMU00_{R} := Zskin_{R} + \frac{j \cdot w_{R} \cdot \mu 0}{2 \cdot \pi} \cdot ln \left(\frac{SC}{rad_{ex}}\right)$$

Self impedance of phase B

$$ZNOMU11_{\mathbf{R}} := Zskin_{\mathbf{R}} + \frac{j \cdot \mathbf{w}_{\mathbf{R}} \cdot \mu 0}{2 \cdot \pi} \cdot ln \left(\frac{SC}{rad\_ex}\right)$$

Self impedance of phase C

$$ZNOMU22_{n} = Zskin_{n} + \frac{j \cdot w_{n} \cdot \mu l}{2 \cdot \pi} \ln \left( \frac{SC}{rad_{ex}} \right)$$

Mutual impedance

## ZNOMUAN<sub>n</sub> := 0

Impedance matrix without mutual coupling is

$$Z2_{n} := \begin{pmatrix} ZNOMU00_{n} & ZNOMUAN_{n} & ZNOMUAN_{n} \\ ZNOMUAN_{n} & ZNOMU11_{n} & ZNOMUAN_{n} \\ ZNOMUAN_{n} & ZNOMUAN_{n} & ZNOMU22_{n} \end{pmatrix}$$

#### Calculation of Distributed Parameters without mutual coupling

Adopt the same equations from previous equations and the difference is the impedance matrix is Z2 instead of Z.

$$Ev2_{n} := eigenvecs_{1}Z2_{n}Y_{n}$$

$$Ei2_{n} := eigenvecs_{1}Y_{n}Z2_{n}$$

$$Zc2_{n} := |Ev2_{n}|^{-1}Z2_{n}Ei2_{n}$$

$$Yc2_{n} := |Ei2_{n}|^{-1}Y_{n}Ev2_{n}$$

$$Yf2_{n} := |Yc2_{n}|^{-1}$$

$$y2_{n} := \sqrt{|Zc2_{n}Yc2_{n}|}$$

$$Zm2_{n} := \sqrt{|Zc2_{n}Yf2_{n}|}$$

$$A2_{\mathbf{n}} := |Ev2_{\mathbf{n}}| \cdot \left( \overbrace{\operatorname{tanh}(\gamma 2_{\mathbf{n}} \cdot \log i)}^{-1} \cdot \overbrace{\operatorname{sinh}(\gamma 2_{\mathbf{n}} \cdot \log i)}^{-1} \cdot \overbrace{\operatorname{sinh}(\gamma 2_{\mathbf{n}} \cdot \log i)}^{-1} \cdot Ev2_{\mathbf{n}}^{-1} \right)$$

$$B2_{\mathbf{n}} := |Ev2_{\mathbf{n}}| \cdot |Zm2_{\mathbf{n}} \cdot \overrightarrow{\operatorname{sinh}(\gamma 2_{\mathbf{n}} \cdot \log i)} \cdot |Ev2_{\mathbf{n}}|^{-1}$$

$$C2_{\mathbf{n}} := |Ei2_{\mathbf{n}}| \cdot |Zm2_{\mathbf{n}}|^{-1} \cdot \overrightarrow{\operatorname{sinh}(\gamma 2_{\mathbf{n}} \cdot \log i)} \cdot |Ev2_{\mathbf{n}}|^{-1}$$

$$D2_{\mathbf{n}} := A2_{\mathbf{n}}^{T}$$

#### Calculations of Harmonic Voltage at receiving end of the Cable

Input voltage is:

Input voltage of phase A

Input voltage of phase B

$$\mathbf{VB} := \frac{-\mathbf{VA}}{2} - \mathbf{j} \cdot \mathbf{VA} \cdot \frac{\sqrt{3}}{2}$$

Input voltage of phase C

$$\mathbf{VC} := \frac{-\mathbf{VA}}{2} + \mathbf{j} \cdot \mathbf{VA} \cdot \frac{\sqrt{3}}{2}$$

Input voltage matrix

$$\mathbf{V}\mathbf{A}\mathbf{B}\mathbf{C} := \begin{pmatrix} \mathbf{V}\mathbf{A} \\ \mathbf{V}\mathbf{B} \\ \mathbf{V}\mathbf{C} \end{pmatrix}$$

Output voltage:

Output voltage including skin effect and mutual coupling

Vabc<sub>n</sub> := 
$$(A_n)^{-1}$$
 VABC

Output voltage without skin effect

$$Vabc1_n := (A1_n)^{-1} VABC$$

Output voltage without mutual coupling

$$Vabc2_n := (A2_n)^{-1} \cdot VABC$$

Due to comparative purpose of this model, phase A of each case above are calculated for the data comparison

$$\forall dist_n := \sqrt{\operatorname{Re}_i \forall \operatorname{abc}_n^2 + \operatorname{Im}_i \forall \operatorname{abc}_n^2}$$

Calculation of harmonic voltage magnitude at receiving end of phase A including skin effect and mutual coupling

$$\mathbf{M}_{\mathbf{R}} := \nabla \operatorname{dist}_{\mathbf{R}}^{\mathrm{T}}$$

$$\nabla_{\mathbf{R}} := \left| \left| \mathbf{M}_{\mathbf{R}} \right|^{(\mathbf{0})} \right|$$

$$\nabla \operatorname{distl}_{\mathbf{R}} := \sqrt{\operatorname{Re} \left| \nabla \operatorname{abcl}_{\mathbf{R}} \right|^{2} + \operatorname{Im} \left| \nabla \operatorname{abcl}_{\mathbf{R}} \right|^{2}}$$

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Calculation of harmonic voltage magnitude at receiving end of phase A without skin effect

$$M1_{n} := \forall dist1_{n}^{T}$$
  
$$\forall noskin_{n} := \left| M1_{n}^{(0)} \right|$$
  
$$\forall dist2_{n} := \sqrt{Re(\forall abc2_{n})^{2} + Im(\forall abc2_{n})^{2}}$$

Calculation of harmonic voltage magnitude at receiving end of phase A without mutual coupling

$$M2_{n} := \forall dist2_{n}^{1}$$
  
$$\forall nomutual_{n} := \left| (M2_{n})^{(0)} \right|$$



(m)

(m)

(m)

# D

# Modelling of Harmonic Impedance of Single-Core Subsea Cable using MathCAD

## The following lists of the program are constructed using MathCAD<sup>®</sup> v. 11

#### Constants

Geometric position of these cables is:

Cable A: coordinate (0,-49.8634)

Cable B: coordinate (-0.05,-49.95) (m)

Cable C: coordinate (0.05,-49.95) (m)

Input the coordinate data:

Coordinate :=

$\square$	0	1
0	0	-49.8634
1	-0.05	-49.95
2	0.05	-49.95

x := Coordinate

# y := Coordinate (1)

The coordinate of cable A is $(x_0, y_0)$	(m)

The coordinate of cable B is  $(x_1, y_1)$ (m)The coordinate of cable C is  $(x_2, y_2)$ (m)

external radius of lead sheath

#### r\_cond := 0.01895

internal radius of lead sheath

#### r\_isheath := 0.03725

average radius of lead sheath

r_sheath := 0.0385	(m)
external radius of lead sheath	
r_osheath := 0.03975	(m)
thickness of lead sheath	
t_sheath := 0.0025	(m)
internal radius of copper armour	
r_iarm := 0.0421	(m)
average radius of copper armour	
r_arm := 0.0446	(m)
external radius of copper armour	
r_oama := 0.0471	(m)
thickness of copper armour	
t_arm := 0.005	(m)
out radius of cable	
r_out := 0.05	(m)
internal radius of XLPE insulation	
r_in_xipe := 0.02065	(m)
external radius of XLPE insulation	
r_ex_zzpe := 0.03565	(m)
permeability of free sapce	
$\mu 0 := 4 \pi \cdot 10^{-7}$	(H/m)
permittivity of free space	
$s0 := 8.854 \cdot 10^{-12}$	(F/m)
permittivity of XLPE	
er_xtpe := 2.5 e0	(F/m)
resistivity of conductor at 90 degree C	
$p \text{ cond} := 0.02244176 \cdot 10^{-6}$	(Ohm m)
resistivity of sheath at 70 degree C	
psheath := 0.2568·10 <sup>-6</sup>	(Ohm m)
number of armour wires	
nama := 51	

resistivity of armour wires at 70 degree C

201

(Hz)

(m)

$\rho arm := 0.02062 \cdot 10^{-6}$	
cross section area of armour wire	
$CAecm := \frac{\pi}{4} \cdot t_{ecm}^2$	(m2)
laying angle of armour wires	
$\delta arm := \frac{\pi}{9}$	(radius)
resistance of armour wires at 70 degree C	
$R_arm := \frac{\rho arm}{narm \cdot CA arm \cdot \cos(\delta arm)}$	(Ohm)
resistivity of sea	
psea = 1	(Ohm m)
fundamental frequency	
f0 := 50	(Hz)
fundamental angular velocity	
w0 := 2.π·10	(m/s)

distance between cables

$$S_{\text{min}} = \sqrt[3]{\sqrt{|x_1 - x_0|^2 + |y_1 - y_0|^2} \sqrt{|x_2 - x_0|^2 + |y_2 - y_0|^2} \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}}$$
(m)

#### Variables

harmonic order from 1 to 100

n := 1.. 100

harmonic frequency

$$f_n := n \cdot f 0$$

$$\mathbf{w}_{\mathbf{n}} \coloneqq \mathbf{n} \cdot \mathbf{w} \mathbf{0} \tag{m/s}$$

length of cable

### long := 50000

complex parameter of conductor

$$\operatorname{ccond}_{\mathbf{h}} := \sqrt{j \cdot \frac{\mathbf{w}_{\mathbf{h}} \cdot \boldsymbol{\mu} \mathbf{0}}{\rho \operatorname{cond}}}$$

complex parameter of sheath

$$\sigma$$
sheath<sub>n</sub> :=  $\sqrt{j \cdot \frac{\mathbf{w}_n \cdot \boldsymbol{\mu} \boldsymbol{\Omega}}{\rho sheath}}$ 

complex parameter of armour

$$\sigma arm_{\mathbf{R}} := \sqrt{j \cdot \frac{\mathbf{w}_{\mathbf{R}} \cdot \boldsymbol{\mu} \mathbf{0}}{\rho arm}}$$

complex parameter of sea

$$\sigma sea_{\mathbf{R}} := \sqrt{j \cdot \frac{\mathbf{w}_{\mathbf{R}} \cdot \boldsymbol{\mu} \mathbf{0}}{\rho sea}}$$

#### **Calculation of Admittance**

Three phase admittance matrix

$$Y_{\mathbf{a}} := \mathbf{j} \cdot \mathbf{w}_{\mathbf{a}} \cdot 2 \cdot \pi \cdot \mathbf{e} \mathbf{r}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e} \cdot \begin{pmatrix} \ln\left(\frac{\mathbf{r}_{\mathbf{e}} \mathbf{e} \mathbf{x}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e}}{\mathbf{r}_{\mathbf{e}} \mathbf{i} \mathbf{n}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e}} \right) & \mathbf{0} \\ 0 & \ln\left(\frac{\mathbf{r}_{\mathbf{e}} \mathbf{e} \mathbf{x}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e}}{\mathbf{r}_{\mathbf{e}} \mathbf{i} \mathbf{n}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e}} \right) & \mathbf{0} \\ 0 & 0 & \ln\left(\frac{\mathbf{r}_{\mathbf{e}} \mathbf{e} \mathbf{x}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e}}{\mathbf{r}_{\mathbf{e}} \mathbf{i} \mathbf{n}_{\mathbf{a}} \mathbf{x} \mathbf{p} \mathbf{e}} \right) \end{pmatrix}^{-1}$$

#### **Calculation of Conductor Impedance**

external impedance of conducotr

$$Z^{*}core\_out_{n} := \frac{\rho cond \sigma cond_{n}}{2 \cdot \pi \cdot r\_cond} \cdot \left( \frac{10(\sigma cond_{n} \cdot r\_cond)}{11(\sigma cond_{n} \cdot r\_cond)} \right)$$

inductance of insulation between concutor and sheath

$$Z'core\_insul_{\mathbf{n}} \coloneqq \frac{\mathbf{j} \cdot \mathbf{w}_{\mathbf{n}} \cdot \mathbf{\mu} \mathbf{0}}{2 \cdot \pi} \cdot \ln \left( \frac{\mathbf{r}\_isheath}{\mathbf{r}\_cond} \right)$$

#### **Calculation of Sheath Impedance**

 $Hisheath_{n} \coloneqq Hir_osheath \sigma sheath_{n} (Kl)r_osheath_{n} (Kl)r_$ 

internal impedance of sheath

 $Z'sheath\_in\_ = \frac{\rho sheath\_orsheath\_n}{2 \cdot \pi \cdot r\_isheath\_H'sheath\_} + 10 \cdot r\_isheath\_orsheath\_n + K0 - r\_isheath\_orsheath\_n + K0 - r\_isheath\_orsheath\_n + K0 - r\_ishaath\_n + K0$ 

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external impedance of sheath

 $Z'sheath\_out\_= \frac{psheath\_\sigmasheath\_}{2\cdot \pi \cdot r\_osheath\_H'sheath\_} + 10 \cdot r\_osheath\_\sigmasheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheath\_Gsheatgsheath\_Gsheath_Gsheath]GsheatGsheath_GsheatgsheatGsha$ 

mutual impedance of sheath

Z'sheath\_mu\_n :=  $\frac{-\rho \text{sheath}}{2 \cdot \pi \cdot r_o \text{sheath} \cdot r_i \text{sheath} \cdot H$ 'sheath\_n

inductance of insulation between sheath and armour

$$Z'sheath\_insul_{n} := \frac{j \cdot w_{n} \cdot \mu 0}{2 \cdot \pi} \cdot \ln \left( \frac{r\_iarm}{r\_osheath} \right)$$

#### **Calculation of Armour Impedance**

internal impedance of armour

## Z'arm\_in\_ = R\_arm garma t\_arm coth garma t\_arm

external impedance of armour

mutual impedance of armour

$$2^{t}arm_{n} := R_{arm} \cdot \sigma arm_{n} \cdot \frac{-t_{arm}}{\sinh(\sigma arm_{n} \cdot t_{arm})}$$

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inductance of insulation between armour and cable jacket

$$Z^{\text{term}}_{\mathbf{n}} := \frac{\mathbf{j} \cdot \mathbf{w}_{\mathbf{n}} \cdot \mathbf{\mu} \mathbf{j}}{2 \cdot \mathbf{\pi}} \cdot \ln \left( \frac{\mathbf{r}_{\text{out}}}{\mathbf{r}_{\text{oerm}}} \right)$$

**Three Phase Impedance Matrix** 

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$$\begin{aligned} Zcore\_out_{\mathbf{n}} \coloneqq \begin{pmatrix} Z'core\_out_{\mathbf{n}} & 0 & 0 \\ 0 & Z'core\_out_{\mathbf{n}} & 0 \\ 0 & 0 & Z'core\_out_{\mathbf{n}} \end{pmatrix} \\ Zcore\_insul_{\mathbf{n}} \coloneqq \begin{pmatrix} Z'core\_insul_{\mathbf{n}} & 0 & 0 \\ 0 & Z'core\_insul_{\mathbf{n}} & 0 \\ 0 & 0 & Z'core\_insul_{\mathbf{n}} \end{pmatrix} \\ Zsheath\_in_{\mathbf{n}} \coloneqq \begin{pmatrix} Z'sheath\_in_{\mathbf{n}} & 0 & 0 \\ 0 & 0 & Z'sheath\_in_{\mathbf{n}} \end{pmatrix} \end{aligned}$$

$$\begin{aligned} & \text{Zsheath\_out}_{\mathbf{n}} := \begin{pmatrix} 2^{\text{zsheath\_out}}_{\mathbf{n}} & 0 & 0 \\ 0 & 2^{\text{zsheath\_out}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zsheath\_out}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zsheath\_out}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zsheath\_insul}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zerm\_insul}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zerm\_insul}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zerm\_out}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zerm\_out}}_{\mathbf{n}} & 0 \\ 0 & 0 & 2^{\text{zerm\_insul}}_{\mathbf{n}} & 0 \\ 0$$

Sea Return Impedance in Three Phase Matrix

$$Z_{\text{stem}_{in}} = j \frac{\mathbf{w}_{in}^{-1} \mathbf{i} \mathbf{0}}{2 \cdot \mathbf{x}} \left[ -\frac{\ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{x}_{n} - \cot t}{2}\right) + \frac{1}{2} + \frac{-4 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{y}_{0} - \mathbf{y}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{S}}{2}\right) + \frac{1}{2} + \frac{-2 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3}\right) - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3}\right) - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3}\right) - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3}\right) - \ln \left(\frac{1.781 \cdot \cos \mathbf{e}_{in} \cdot \mathbf{w}_{0}^{+}}{3} - \ln \left(\frac{1.781 \cdot \cos \mathbf{e$$

\_\_\_\_

Loop Equations of Cable Impedance  

$$211_n := 2core\_out_n + 2core\_insul_n + 2sheath\_in_n$$
  
 $222_n := 2sheath\_out_n + 2sheath\_insul_n + 2arm\_in_n$   
 $233_n := 2arm\_out_n + 2arm\_insul_n + 2sea\_in_n$   
 $2cc_n := 211_n + 2\cdot2sheath\_mu_n + 222_n + 2\cdot2arm\_mu_n + 233_n$   
 $2sc_n := 2sheath\_mu_n + 222_n + 2\cdot2arm\_mu_n + 233_n$   
 $2cs_n := 2sheath\_mu_n + 222_n + 2\cdot2arm\_mu_n + 233_n$   
 $2cs_n := 2sheath\_mu_n + 233_n$   
 $2ac_n := 2arm\_mu_n + 233_n$   
 $2as_n := 2arm\_mu_n + 233_n$ 

Adopt boundary condition of both bonding where the voltage of sheath and armour is zero, soluation can be expressed as:

$$X2_{h} = \frac{-i2sc_{h} \cdot 2aa_{h} - 2sa_{h} \cdot 2ac_{h}}{2ss_{h} \cdot 2aa_{h} - 2sa_{h} \cdot 2as_{h}}$$

$$X3_{h} = \frac{i2sc_{h} \cdot 2aa_{h} - 2sa_{h} \cdot 2as_{h}}{2ss_{h} \cdot 2aa_{h} - 2sa_{h} \cdot 2ac_{h}}$$

$$Z_{h} = 2cc_{h} + 2cs_{h} \cdot X2_{h} + 2ca_{h} \cdot X3_{h}$$

Harmonic Resistance and Inductance per km of Cable

Input voltage is:

input voltage of phase A

### **VA** := 1

.

input voltage of phase B

$$\nabla \mathbf{B} := \frac{-\nabla \mathbf{A}}{2} - \mathbf{j} \cdot \nabla \mathbf{A} \cdot \frac{\sqrt{3}}{2}$$

input voltage of phase C

$$\mathbf{VC} := \frac{-\mathbf{VA}}{2} + \mathbf{j} \cdot \mathbf{VA} \cdot \frac{\sqrt{3}}{2}$$

input voltage matrix

$$\nabla ABC := \begin{pmatrix} \nabla A \\ \nabla B \\ \nabla C \end{pmatrix}$$
$$ZW_{a} := Z_{a} \cdot \nabla ABC$$

$$Z\mathbf{w}_{\mathbf{n}} = Z\mathbf{W}_{\mathbf{n}}$$

harmonic resistance per km

$$RZw_{n} := \frac{Re_{i}Zw_{n}}{10^{-3}}$$

harmonic inductance per km

$$LZw_{n} = \frac{Im(Zw_{n})}{10^{-6}}w_{n}$$

#### Harmonic Resistance and Inductance Results



#### **Calculation of Distributed Parameters**

Once the lump parameter such as impedance and admittance acquired, as previous described, distributed parameters can also be obtained by following equations:

eigenvectors of Znand Yn

$$Ev_n = eigenvecs_N Z_N Y_n$$

eigenvectors of Y<sub>n</sub>and Z<sub>n</sub>

diagonal impedance matrix

$$Zm_{\mathbf{n}} := Ev_{\mathbf{n}}^{-1} Z_{\mathbf{n}} E$$

diagonal admittance matrix

$$\mathbf{Ym}_{\mathbf{n}} \coloneqq (\mathbf{Ei}_{\mathbf{n}})^{-1} \cdot \mathbf{Y}_{\mathbf{n}} \cdot \mathbf{Ev}_{\mathbf{n}}$$

inverse matrix of  $Y_m$ 

$$\mathbf{Yf}_{\mathbf{n}} := (\mathbf{Ym}_{\mathbf{n}})^{-1}$$

diagonal matrix of propagation constants

$$\gamma_{\mathbf{n}} := \overline{\sqrt{Z\mathbf{m}_{\mathbf{n}} \cdot Y\mathbf{m}_{\mathbf{n}}}}$$

diagonal matrix of characteristic impedance

$$Zc_{\mathbf{n}} = \overline{\sqrt{2m_{\mathbf{n}} Yf_{\mathbf{n}}}}$$

The representation transform matrix of transmission line in the form of ABCD is:

$$\begin{pmatrix} \mathbf{V} \mathbf{A} \mathbf{B} \mathbf{C} \\ \mathbf{I} \mathbf{A} \mathbf{B} \mathbf{C} \end{pmatrix} := \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \mathbf{V} \mathbf{e} \mathbf{b} \mathbf{c} \\ -\mathbf{I} \mathbf{e} \mathbf{b} \mathbf{c} \end{pmatrix}$$

$$\mathbf{A}_{\mathbf{n}} := (\mathbf{E} \mathbf{v}_{\mathbf{n}}) \cdot (\overrightarrow{\operatorname{tenh}} (\gamma_{\mathbf{n}} \cdot \log_{1})^{-1} \cdot \overrightarrow{\operatorname{sinh}} (\gamma_{\mathbf{n}} \cdot \log_{1}) \cdot (\mathbf{E} \mathbf{v}_{\mathbf{n}})^{-1} \cdot \overrightarrow{\operatorname{sinh}} (\gamma_{\mathbf{n}} \cdot \log_{1})^{-1} \cdot \overrightarrow{\operatorname{sinh}} (\gamma_{\mathbf{n}} \cdot \log_{1}) \cdot (\mathbf{E} \mathbf{v}_{\mathbf{n}})^{-1} \cdot \mathbf{E} \mathbf{v}_{\mathbf{n}} \cdot \mathbf{E} \mathbf{v}_{\mathbf{n}} = 1$$

$$\mathbf{B}_{\mathbf{n}} := (\mathbf{E} \mathbf{v}_{\mathbf{n}}) \cdot (\mathbf{Z} \mathbf{c}_{\mathbf{n}})^{-1} \cdot \overrightarrow{\operatorname{sinh}} (\gamma_{\mathbf{n}} \cdot \log_{1}) \cdot (\mathbf{E} \mathbf{v}_{\mathbf{n}})^{-1} \cdot \mathbf{E} \mathbf{v}_{\mathbf{n}} = 1$$

$$\mathbf{C}_{\mathbf{m}} := (\mathbf{E} \mathbf{i}_{\mathbf{n}}) \cdot (\mathbf{Z} \mathbf{c}_{\mathbf{n}})^{-1} \cdot \overrightarrow{\operatorname{sinh}} (\gamma_{\mathbf{n}} \cdot \log_{1}) \cdot (\mathbf{E} \mathbf{v}_{\mathbf{n}})^{-1}$$

$$\mathbf{D}_{\mathbf{n}} := \mathbf{A}_{\mathbf{n}}^{T}$$

#### Calculations of Harmonic Voltage at receiving end of the Transmission Cable

Output voltage

$$Vebc_n := iA_n^{-1} \cdot VABC$$

Due to comparative purpose of this model, phase A of each case above are calculated for the data comparison

$$\forall dist_{n} := \sqrt{\operatorname{Re}_{i} \forall \operatorname{abc}_{n}^{2} + \operatorname{Im}_{i} \forall \operatorname{abc}_{n}^{2}}$$

Calculation of harmonic voltage magnitude at receiving end of phase A

$$\mathbf{M}_{\mathbf{n}} \coloneqq \mathbf{V} \mathbf{dist}_{\mathbf{n}}^{\mathrm{T}}$$
$$\underbrace{\mathbf{V}}_{\mathbf{n}} \coloneqq \left[ \mathbf{M}_{\mathbf{n}}^{(0)} \right]$$

#### Harmonic Resonances Results



# Ε

# Transfer Matrices in Admittance Form for HVAC System Components

#### E.1 Transfer Matrix of Power Transformers in Admittance Form

The impedance of Power Transformer 1 is set as:  $Z_{TX1} = 0.1 + j10\%$ The impedance of Power Transformer 2 is set as:  $Z_{TX2} = 0.5 + j15\%$ The admittance matrix for  $\Delta$  -Y Transformers with Y grounded can be expressed as:

$$\begin{bmatrix} \mathbf{Y}_{\mathsf{T1},\mathsf{11}} & \mathbf{Y}_{\mathsf{T1},\mathsf{12}} \\ \mathbf{Y}_{\mathsf{T1},\mathsf{21}} & \mathbf{Y}_{\mathsf{T1},\mathsf{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{\mathsf{T2},\mathsf{11}} & \mathbf{Y}_{\mathsf{T2},\mathsf{12}} \\ \mathbf{Y}_{\mathsf{T2},\mathsf{21}} & \mathbf{Y}_{\mathsf{T2},\mathsf{22}} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{\mathsf{S}} & \mathbf{Y}_{\mathsf{SD}} \\ \mathbf{Y}_{\mathsf{SD}}^{\mathsf{T}} & \frac{1}{3} \mathbf{Y}_{\mathsf{D}} \end{bmatrix}$$
(E.1)

Where,

$$\mathbf{Y}_{S} = \begin{bmatrix} Y_{TX} & 0 & 0\\ 0 & Y_{TX} & 0\\ 0 & 0 & Y_{TX} \end{bmatrix}$$
(E.2)

$$\mathbf{Y}_{\mathbf{D}} = \begin{bmatrix} 2Y_{TX} & -Y_{TX} & -Y_{TX} \\ -Y_{TX} & 2Y_{TX} & -Y_{TX} \\ -Y_{TX} & -Y_{TX} & 2Y_{TX} \end{bmatrix}$$
(E.3)

$$\mathbf{Y}_{SD} = \begin{bmatrix} -\frac{1}{\sqrt{3}} Y_{TX} & \frac{1}{\sqrt{3}} Y_{TX} & 0\\ 0 & -\frac{1}{\sqrt{3}} Y_{TX} & \frac{1}{\sqrt{3}} Y_{TX} \\ \frac{1}{\sqrt{3}} Y_{TX} & 0 & -\frac{1}{\sqrt{3}} Y_{TX} \end{bmatrix}$$
(E.4)

Where,

$$Y_{TX} = \frac{1}{R_{TX}\sqrt{n} + jnX_{TX}}$$
(E.5)

 $R_{TX}$  is the resistance of transformer

 $X_{TX}$  is the reactance of transformer

n is harmonic order

#### E.2 Transfer Matrix of Passive Load in Admittance Form

The three-phase passive load in admittance form:

$$\mathbf{Y}_{load} = \begin{bmatrix} y_{load} & 0 & 0 \\ 0 & y_{load} & 0 \\ 0 & 0 & y_{load} \end{bmatrix}$$
(E.6)

Where

$$y_{load} = \frac{1}{R_{load} + X_{load}}$$
(E.7)

 $R_{load}$  is the resistance of load

 $X_{load}$  is the reactance of load

### E.3 Transfer Matrix of TCR in Admittance Form

The three-phase delta-connection TCR admittance model can be simply addressed as:

$$\mathbf{Y}_{\mathbf{TCR}} = \begin{bmatrix} y^{ab}_{TCR} + y^{ca}_{TCR} & -y^{ab}_{TCR} & -y^{ca}_{TCR} \\ -y^{ab}_{TCR} & y^{bc}_{TCR} + y^{ab}_{TCR} & -y^{bc}_{TCR} \\ -y^{ca}_{TCR} & -y^{bc}_{TCR} & y^{ca}_{TCR} + y^{bc}_{TCR} \end{bmatrix}$$
(E.8)

Where,

 $y^{ab}_{TCR}$  is the SVC admittance  $y_{TCR}$  of line voltage ab

 $y^{bc}_{TCR}$  is the SVC admittance  $y_{TCR}$  of line voltage bc

 $y^{ca}_{TCR}$  is the SVC admittance  $y_{TCR}$  of line voltage ca

$$y_{TCR} = \frac{1}{X_{TCR}}S$$

Where,

 $X_{TCR}$  is the reactance of the TCR

S is the switching function of TCR and the detailed derivation can be referred to [69]

(E.9)

# F

# Simulation of HVAC System Harmonics using MATLAB

The following lists of the program are constructed using MATLAB<sup>®</sup> 6.1

F.1 List of Main Program

% cablesvc.m clear all

% Impedances in p.u. for transformers, TCR and load

Xtrans1	=0.1;	% tranformer 1 reactance
Rtrans1	=0.01;	% tranformer 1 resistance
Xtrans2	=0.216;	% tranformer 2 reactance
Rtrans2	=0.0072:	% tranformer 2 resistance
Xrea	=1.44;	% TCR compensator reactance
Rload	=3.33:	% passive load resistance
Xload	=1.612;	% passive load reactance
% Firing Angle	of TCR	
alpha a	=110*pi/180:	% firing angle of phase A of TCR
alpha b	=110*pi/180:	% firing angle of phase B of TCR
alpha_c	=110*pi/180;	%firing angle of phase C of TCR
% Cable calc cable tref	oilZY:	
h	=15;	% harmonic order
% Load		
Yload	=inv(form Zm(Rload Xload h)).	% harmonic values for passive load
Yload	=[Yload Yload*0 Yload*0	vo narmonie values foi passive loud
nodu	Vioad*0 Vioad Vioad*0	
	Yload*0 Yload*0 Yload ];	% harmonic transfer matrix for passive load
% Three-phase	balanced voltage source	
Vap1	=1*sart(2/3):	% input voltage of phase A

Vbp1 =1\*sqrt(2/3); % input voltage of phase B % input voltage of phase C Vcp1 =1\*sqrt(2/3); % harmonic values for input voltage [Va1,Vb1,Vc1] =source\_h(Vap1,Vbp1,Vcp1,0,-120,120,h); =[Va1;Vb1;Vc1]; % harmonic transfer matrix for input voltage V1 % Initail guess value of three-phase voltage in the system at each busbar =1\*sqrt(2/3);% initial guess voltage of phase A at busbar 2 Vap2 =1\*sqrt(2/3); % initial guess voltage of phase B at busbar 2 Vbp2 =1\*sqrt(2/3); % initial guess voltage of phase C at busbar 2 Vcp2 % harmonic values for initial guess voltage at busbar 2 [Va2,Vb2,Vc2] =source\_h(Vap2,Vbp2,Vcp2,0,-120,120,h); % harmonic transfer matrix for initial guess voltage at busbar 2 =[Va2;Vb2;Vc2]; V20 =1\*sqrt(2/3); % initial guess voltage of phase A at busbar 3 Vap3 =1\*sqrt(2/3); % initial guess voltage of phase B at busbar 3 Vbp3 =1\*sqrt(2/3); % initial guess voltage of phase C at busbar 3 Vcp3 % harmonic values for initial guess voltage at busbar 3 of SVC [Vasvc,Vbsvc,Vcsvc]=source\_h(Vap3,Vbp3,Vcp3,0,-120,120,h); % harmonic transfer matrix for initial guess voltage at busbar 3 =[Vasvc;Vbsvc;Vcsvc]; V30 =1\*sqrt(2/3); % initial guess voltage of phase A at busbar 4 Vap4 Vbp4 =1\*sqrt(2/3); % initial guess voltage of phase B at busbar 4 =1\*sqrt(2/3); % initial guess voltage of phase C at busbar 4 Vcp4 % harmonic values for initial guess voltage at busbar 4 [Vaload, Vbload, Vcload]=source\_h(Vap4, Vbp4, Vcp4, 0, -120, 120, h); %harmonic values for initial guess voltage at busbar 4 =[Vaload;Vbload;Vcload]; V40 % initial guess voltage of busbar 2, 3, 4 V2340 =[V20;V30;V40]; % Recall sub routine of Star Delta connection transformer % start delta transformer 1 harmonic transfer matrix [Yt1a, Yt1b, Yt1c, Yt1d]=transf bank(Rtrans1, Xtrans1, h, 'Ss-D'); % start delta transformer 2 harmonic transfer matrix [Yt2a, Yt2b, Yt2c, Yt2d]=transf\_bank(Rtrans2, Xtrans2, h, 'Ss-D'); % Iternation process for obtaining the voltage value at each busbar =1: % initial error error =1: % initial iteration number iter % if the error is greater than 0.00000001 then ineration process continues while error>0.00000001 % recall sub routine of three phase TCR harmonic matrix =calc\_TCR\_ThreePhase(Vasvc,Vbsvc,Vcsvc,alpha a,alpha b, Ytcr\_delta alpha\_c,1,Xrea,h,0); Ysvc =Ytcr delta; % let SVC matrix equal to TCR matrix 00 =zeros(6\*h+3,6\*h+3); % transfer admittance matrix in equatin (7.9) **Y1** =[Yt1c 00 00 ]; Y2 =[Yt1d+Ac Bc 00 Cc Dc+Yt2a+Ysvc Yt2b
	00	Yt2c	Yt2d+Yload];
% calculation of	of output volta	ge of busbar 2, 3,	4
V234	= inv(Y2)*Y	1 <b>*</b> V1;	
% calculation of	of output volta	ge error of busbar	2, 3, 4
error	= norm(V23	4-V2340)	
% iteration pro	ess number		
iter	= iter+1		
% voltage and	current value	at each busbar	
Va2	= V234(1:2*	′h+1);	
Vb2	= V234(2*h·	+2:4*h+2);	
Vc2	= V234(4*h·	+3:6*h+3);	
Vasvc	= V234(6*h-	+4:8*h+4);	
Vbsvc	= V234(8*h-	+5:10*h+5);	
Vcsvc	= V234(10*i	1+6:12*h+6);	
Vsvc	= V234(6*h-	+4:12*h+6);	
Vaload	= V234(12*)	n+/:14"h+/);	
Vbload	= V234(14*1	1+8:16"n+8);	
Vcload	$= V234(16^{-1})$	1+9:18"n+9);	
Vload	$= V234(12^{-1})$	1+7:18"n+9); *\/ove:	
Itcr	$=$ <b>ficr_delta</b>	I <sup>~</sup> VSVC;	
later	$= \text{Itcr}(1:2^{n+1})$	*1); ·4*6+2)·	
IDICI	= 101(2 11+2)	.4 11™∠), ·6*h+2)·	
lood	= Xload*\/lo	.o 11.0), ad <sup>.</sup>	
lolood	= 10au vio	au, h+1) <sup>,</sup>	
lbload	= lload(1.2	-2·4*h+2)·	
Icload	= lload(2*h+	-3·6*h+3)	
% let the initia	- nous(+ n+	age of busbar 2.3.	4 equal to new value
V2340	$= \sqrt{234}$		
end	·=• ·,		
U.I.d			
VsvcHD	= zeros(h-2,	,3);	
ItcrHD	= zeros(h-2,	(3);	
VloadHD	= zeros(h-2,	,3);	
lloadHD	= zeros(h-2,	,3);	
% place the val	ues into harm	onic domain mat	rices for voltage and current of TCR and Load
for m	=1:h-2		
VsvcHD(m,1)	$=$ abs(real(\	/asvc(h+3+m)+V	asvc(h-1-m))+i*(imag(Vasvc(h+3+m)-Vasvc(h-1-
VavaUD(m 2)	m)))) 100/a	/boyo(b+2+m)+\/	$2) \neq vasvc(n) \neq i (mag(vasvc(n+2)-vasvc(n))));$
vsvcnD(III,2)	- abs(real()	he(real(\/hevc/h+	$2 + \sqrt{b} + \frac{1}{1} + 1$
VeveHD(m 3)	= abs(real()	/csvc(h+3+m)+\/	2/7 VDSVC(11)/71 (111ag(VDSVC(11+2)-VDSVC(11))), csvc(h-1-m))+i*(imag(V/csvc(h+3+m)-V/csvc(h-1))),
vsvci iD(iii,5)	m)))*100/a	hs(real(Vcsvc(h+	2)+V/csvc(h)+i*(imag(V/csvc(h+2)-V/csvc(h))))
			2/7 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5 = 0.5
ltcrHD(m 1)	= abs(real()	atcr(h+3+m)+late	r(h-1-m))+i*(imag(later(h+3+m)-later(h-1-
	m))))*100/a	hs(real(later(h+2)	)+later(h))+i*(imag(later(h+2)-later(h))))
ltcrHD(m 2)	= abs(real()	btcr(h+3+m)+1btc	$r(h_1-m))+i*(imag(later(h+3+m))-later(h_1)));$
	m))))*100/a	bs(real(lbtcr(h+2)	)+lbtcr(h))+i*(imag(lbtcr(h+2)-lbtcr(h)))
ltcrHD(m.3)	= abs(real(l)	ctcr(h+3+m)+lctc	r(h-1-m)+i*(imag(lctcr(h+3+m)-lctcr(h-1-
	m))))*100/a	bs(real(lctcr(h+2)	)+lctcr(h))+i*(imag(lctcr(h+2)-lctcr(h)))
	,,,,,	······································	
VloadHD(m.1)	= abs(real()	/aload(h+3+m)+	/aload(h-1-m))+i*(imaa(Vaload(h+3+m)-Vaload(h-1-
	m))))*100/a	bs(real(Vaload(h	+2)+Vaload(h))+i*(imag(Vaload(h+2)-Vaload(h)))
VloadHD(m,2)	= abs(real()	/bload(h+3+m)+	Vbload(h-1-m))+i*(imag(Vbload(h+3+m)-Vbload(h-1-
、 , - <b>,</b>	m))))*100/a	bs(real(Vbload(h	+2)+Vbload(h))+i*(imag(Vbload(h+2)-Vbload(h))))
		•	

```
VloadHD(m,3) = abs(real(Vcload(h+3+m)+Vcload(h-1-m))+i*(imag(Vcload(h+3+m)-Vcload(h-1-
               m))))*100/abs(real(Vcload(h+2)+Vcload(h))+i*(imag(Vcload(h+2)-Vcload(h))));
               = abs(real(laload(h+3+m)+laload(h-1-m))+i*(imag(laload(h+3+m)-laload(h-1-
lloadHD(m,1)
               m))))*100/abs(real(laload(h+2)+laload(h))+i*(imag(laload(h+2)-laload(h))));
               = abs(real(lbload(h+3+m)+lbload(h-1-m))+i*(imag(lbload(h+3+m)-lbload(h-1-
lloadHD(m,2)
               m))))*100/abs(real(lbload(h+2)+lbload(h))+i*(imag(lbload(h+2)-lbload(h))));
IloadHD(m,3)
               = abs(real(lcload(h+3+m)+lcload(h-1-m))+i*(imag(lcload(h+3+m)-lcload(h-1-
               m))))*100/abs(real(lcload(h+2)+lcload(h))+i*(imag(lcload(h+2)-lcload(h))));
end
% Plot of harmonic voltage and current at busbar 3 of TCR
harmonic
               =3:h;
subplot(1,1,1)
bar(harmonic, VsvcHD)
title 'Vtcr'
subplot(1,1,1)
bar(harmonic, ItcrHD)
title 'ltcr'
```

#### F.2 List of Sub-Routine of Cable Harmonic Impedance and Admittance for

#### Cable Type (a) Single-Core Trefoil Subsea Power Cable

#### % To obtain cable impedance and admittance matrices

% calc_cable	_trefoilZY.m	
f –	=50;	% fundamental frequency in Hz
long	=50000;	% cable length in m
h	=15;	% harmonic orders
% cable laye	rs dimensions, materials and pro	operties
x	= [0 -0.0511 0.0511];	% cable position in x coordinate in m
у	= [-49.8604 -49.9489 -49.9	489]; % cable position in y coordinate in m
% cable distar	nces between each phase	
distance1	$= sqrt((x(1)-x(2))^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2}+((y(1)-y(2))^{2})^{2})^{2}+((y(1)-y(2))^{$	<b>/(2))^2));</b>
distance2	$= sqrt((x(2)-x(3))^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2}+((y(2)-y(3))^{2})^{2})^{2}+((y(2)-y(3))^{$	/(3)) <sup>2</sup> );
distance3	$= sqrt((x(3)-x(1))^{2}+((y(3)-y)^{2})^{2})^{2}$	/(1)) <sup>2</sup> );
r_c	= 0.01895;	% conductor radius in m
res_c	= 0.02244176*1e-6;	% conductor resistivity in $\Omega$ .m
r_ex_xlpe	= 0.03765;	% XLPE insulation outside radius in m
r_in_xlpe	= 0.02065;	% XLPE insulation inside radius in m
r_sheath	= 0.0405;	% average sheath radius in m
r_isheath	= 0.03925;	% sheath inside radius in m
r_osheath	= 0.04175;	% sheath outside radius in m
t_sheath	= 0.0025;	% sheath thickness
res_s	= 0.214*1e-6;	% sheath resistivity in $\Omega$ .m
% armour di	mension and property:	
r_arm	= 0.0466;	% average armour radius in m
r_iarm	= 0.0441;	% armour inside radius in m
r_oarm	= 0.0491;	% armour outside radius in m

t_arm	= 0.005;	% armour thickness in m
dtangle_a	= 1*pi/9;	% armour wire laying angle
afangle_a	= 1*pi/4;	% armour wire longitudinal angle
mt	= 10;	% transversal permeability of amour wire
n_arm	= 54;	% number of armour wires
res_a	= 0.1386*1e-6;	% resistivity of armour wire in $\Omega$ .m
A_arm	= t_arm^2*pi/4;	% area of armour wires
% DC resistance	of armour wires	
R_arm	= res_a/(n_arm*A_arm*cos(dtar	igle_a));
p_arm	= 2*pi*r_arm/tan(dtangle_a);	% pitch of armour wires
% relative perme	ability of steel armour	
mr_arm	<pre>= n_arm*A_arm/(p_arm*t_arm)* +mt*cos(dtangle_a)^2;</pre>	me*exp(-i*afangle_a)*sin(dtangle_a)
r_out	= 0.0511	% overall radius of the cable
res_sea	= 1;	% resistivity of sea in $\Omega$ .m

# % cable impedance base value and establish matrices $Z150b = 625^{\circ}$

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+1); +1); +1);

% cable harmonic impedance calculation

for n=0:h

% calculation for non-DC harmonic impedance

if n~=0		
w0	= 2*pi*f;	% fundamental angular velociay
3	= w0*n;	% harmonic angular velociay
mO	= 4*pi*1e-7;	% permeability of air
eO	= 8.854*1e-12;	% permittivity of air
e xlpe	= 2.5*e0;	% relative permittivity of XLPE Insulation
cmx_c	= sqrt(i*w*m0/res_c);	% propagation complex number of conductor
cmx s	= sqrt(i*w*m0/res_s);	% propagation complex number of sheath
cmx_a	= sqrt(i*w*m0*mr_arm/res_a);	% propagation complex number of armour
cmx_sea	= sqrt(i*w*m0/res_sea);	% propagation complex number of sea
% conductor	impedance	

= res\_c\*cmx\_c/(2\*pi\*r\_c)\*(besseli(0,(cmx\_c\*r\_c))/besseli(1,(cmx\_c\*r\_c))); zvcore\_out

= i\*w\*m0/(2\*pi)\*log(r\_isheath/r\_c); % conductor-sheath insulation impdeance zvcs\_insul

% sheath impedance

Hsheath	= besseli(1,(cmx_s*r_osheath))*besselk(1,(cmx_s*r_isheath))-
zvsheath_in	(besseli(1,(cmx_s*r_isneatn))*besselk(1,(cmx_s*r_osneatn))); = res_s*cmx_s/(2*pi*r_isheath*Hsheath)*(besseli(0,(cmx_s*r_isheath))*
	besselk(1,(cmx_s*r_osheath))+besselk(0,(cmx_s*r_isheath))*
zvsheath_ou	ucesen( ו, עוואב י_טפורפעון)). it = res_s*cmx_s/(2*pi*r_osheath*Hsheath)*(besseli(0,(cmx_s*r_osheath))* בבבביועל ליבייי ביבי יהרביניע עבביי ביבי ביבי ביבי ביבי ביבי ביב

```
'r_osheath))
    ່ທ
besselk(1,(cmx_s*r_isheath))+besselk(0,(cmx_
besseli(1,(cmx_s*r_isheath)));
= -res_s/(2*pi*r_isheath*r_osheath*Hsheath);
```

```
zvsheath_m
```

```
% sheath-armour insulation impdeance
zvsa_insul = i*w*m0/(2*pi)*log(r_iarm/r_osheath);
```

% armour impedance

Harm	= besseli(1,(cmx_a*r_oarm))*besselk(1,(cmx_a*r_iarm))-
zvarm_in	(besseli(1,(cmx_a*r_iarm))*besselk(1,(cmx_a*r_oarm))); = res_a*cmx_a/(2*pi*r_iarm*Harm)*(besseli(0,(cmx_a*r_iarm))*
	besselk(1,(cmx_a*r_oarm))+besselk(0,(cmx_a*r_iarm))* besseli(1,(cmx_a*r_oarm)));
zvarm_out	= res_a*cmx_a/(2*pi*r_oarm*Harm)*(besseli(0,(cmx_a*r_oarm))*
	besselk(1,(cmx_a*r_iarm))+besselk(0,(cmx_a*r_oarm))*
	Dessell(1,(cmx_a^rf_larm)));
zvarm_m	= -res_a(∠'pi'r_larm'r_oarm'Harm);
% arm-sea ins	ulation impedance

'≥ 11

'm0/(2\*pi)\*log(r\_out/r\_oarm); zvasea\_insul

% sea impedance

```
i*w*m0/(2*pi)*((-log(1.781*cmx_sea*r_out/2))+(1/2)-(4*cmx_sea*(-y(1))/3));
= i*w*m0/(2*pi)*((-log(1.781*cmx_sea*r_out/2))+(1/2)-(4*cmx_sea*(-y(2))/3));
= i*w*m0/(2*pi)*((-log(1.781*cmx_sea*r_out/2))+(1/2)-(4*cmx_sea*(-y(3))/3));
= i*w*m0/(2*pi)*((-log(1.781*cmx_sea*distance1/2))+(1/2)-(2*cmx_sea*(-y(1)-
)
                                                                                                                                                                      = i*w*m0/(2*pi)*((-log(1.781*cmx_sea*distance2/2))+(1/2)-(2*cmx_sea*(-y(2)-
    zvsea_sel1
zvsea_sel2
zvsea_sel3
zvsea_mu12
                                                                                                                                                                      zvsea_mu23
y(3))/3));
                                                                                                                                       y(2))/3));
```

```
zvsea_mu13 = i*w*m0/(2*pi)*((-log(1.781*cmx_sea*distance3/2))+(1/2)-(2*cmx_sea*(-y(1)-
y(3))/3));
```

```
% insulation shunt
yv_insul = 0.0000000003+i*w*2*pi*e_xlpe/log(r_ex_xlpe/r_in_xlpe);
```

#### else

```
% calculation for DC harmonic impedance
 % conductor impedance
 zvcore_out = 0.0176*1e-3;
 % conductor-sheath insulation impdeance
 zvcs_insul
               = 0;
 % sheath impedance
 zvsheath_in = res_s/(2*pi*r_sheath*t_sheath);
 zvsheath_out = res_s/(2*pi*r_sheath*t_sheath);
 zvsheath_m = 0;
  % sheath-armour insulation impdeance
 zvsa_insul
              = 0:
  % armour impedance
               = res a/(n arm*pi*r_arm^2*cos(dtangle a));
 zvarm in
               = res_a/(n_arm*pi*r_arm^2*cos(dtangle_a));
 zvarm out
               = 0;
 zvarm m
  % arm-sea insulation impedance
 zvasea_insul = 0;
 % sea impedance
 zvsea_sel1
              = 0:
 zvsea_sel2 = 0;
 zvsea_sel3 = 0;
 zvsea_mu12 = 0;
 zvsea_mu23 = 0;
 zvsea_mu13 = 0;
 % insulation shunt
 yv_insul
               = 0.000000003;
end
% place the harmonic values into the harmonic domain matrices
 zcore_out(n+1)
                  = zvcore_out;
 zcs_insul(n+1)
                  = zvcs_insul;
 zsheath_in(n+1) = zvsheath_in;
 zsheath_out(n+1) = zvsheath_out;
 zsheath_m(n+1) = zvsheath_m;
 zsa_insul(n+1)
                  = zvsa_insul;
 zarm_in(n+1)
                  = zvarm_in;
 zarm_out(n+1)
                  = zvarm_out;
 zarm_m(n+1)
                  = zvarm_m;
 zasea_insul(n+1) = zvasea_insul;
 zsea_sel1(n+1)
                  = zvsea_sel1;
 zsea_sel2(n+1)
                  = zvsea_sel2;
 zsea_sel3(n+1)
                  = zvsea sel3;
zsea_mu12(n+1) = zvsea_mu12;
```

```
zsea_mu23(n+1) = zvsea_mu23;
 zsea_mu13(n+1) = zvsea_mu13;
 y_insul(n+1)
                   = yv_insul;
 end
k
       = 1;
for j=1:2*h+1
  if j<=h
                   = 2*real(zcore_out(h+2-j))-zcore_out(h+2-j);
    zzcore_out
                   = 2*real(zcs_insul(h+2-j))-zcs_insul(h+2-j);
    zzcs_insul
                    = 2*real(zsheath_in(h+2-j))-zsheath_in(h+2-j);
    zzsheath_in
                   = 2*real(zsheath_out(h+2-j))-zsheath_out(h+2-j);
    zzsheath_out
                   = 2*real(zsheath_m(h+2-j))-zsheath_m(h+2-j);
    zzsheath_m
    zzsa_insul
                   = 2*real(zsa_insul(h+2-j))-zsa_insul(h+2-j);
    zzarm in
                   = 2*real(zarm_in(h+2-j))-zarm_in(h+2-j);
                   = 2*real(zarm_out(h+2-j))-zarm_out(h+2-j);
    zzarm out
                   = 2*real(zarm_m(h+2-j))-zarm_m(h+2-j);
    zzarm m
                   = 2*real(zasea_insul(h+2-j))-zasea_insul(h+2-j);
    zzasea insul
                   = 2*real(zsea_sel1(h+2-j))-zsea_sel1(h+2-j);
    zzsea sel1
                   = 2*real(zsea_sel2(h+2-j))-zsea_sel2(h+2-j);
    zzsea sel2
                   = 2*real(zsea_sel3(h+2-j))-zsea_sel3(h+2-j);
    zzsea sel3
    zzsea_mu12
                   = 2*real(zsea_mu12(h+2-j))-zsea_mu12(h+2-j);
                   = 2*real(zsea_mu23(h+2-j))-zsea_mu23(h+2-j);
    zzsea_mu23
    zzsea_mu13
                   = 2*real(zsea_mu13(h+2-j))-zsea_mu13(h+2-j);
                   = 2*real(y_insul(h+2-j))-y_insul(h+2-j);
    yy_insul
  else
                   = zcore_out(j-h);
    zzcore_out
                   = zcs_insul(j-h);
    zzcs insul
                   = zsheath_in(j-h);
    zzsheath_in
    zzsheath_out
                   = zsheath_out(j-h);
    zzsheath_m
                   = zsheath_m(j-h);
    zzsa_insul
                   = zsa_insul(j-h);
    zzarm_in
                   = zarm_in(j-h);
                   = zarm_out(j-h);
    zzarm_out
                   = zarm_m(j-h);
    zzarm_m
    zzasea_insul
                   = zasea_insul(j-h);
                   = zsea_sel1(j-h);
    zzsea_sel1
    zzsea_sel2
                   = zsea_sel2(j-h);
    zzsea_sel3
                   = zsea_sel3(j-h);
    zzsea_mu12
                   = zsea_mu12(j-h);
    zzsea_mu23
                   = zsea_mu23(j-h);
    zzsea_mu13
                   = zsea_mu13(j-h);
    yy_insul
                   = y_insul(j-h);
 end
 ZZcore_out(k,k)
                   = zzcore_out;
 ZZcs_insul(k,k)
                   = zzcs_insul;
 ZZsheath_in(k,k) = zzsheath_in;
 ZZsheath_out(k,k) = zzsheath_out;
 ZZsheath_m(k,k) = zzsheath_m;
 ZZsa_insul(k,k)
                   = zzsa_insul;
 ZZarm_out(k,k)
                   = zzarm_out;
 ZZarm_in(k,k)
                   = zzarm_in;
 ZZarm_m(k,k)
                   = zzarm_m;
 ZZasea_insul(k,k) = zzasea_insul;
 ZZsea_sel1(k,k)
                   = zzsea_sel1;
 ZZsea_sel2(k,k)
                   = zzsea_sel2;
```

ZZsea_sel3( ZZsea_mu12 ZZsea_mu13 ZZsea_mu13 YY_insul(k,k k=k+1; end	k,k) = zzsea_sel3; 2(k,k) = zzsea_mu1; 3(k,k) = zzsea_mu2; 3(k,k) = zzsea_mu1; 3(k,k) = yy_insul;	2; 3; 3;	
% three-phase	e matrices for cable la	yers in loop equa	tions
Zcore_out		ZZcore out	0
	0	0	ZZcore outl
Zcs insul	=[ZZcs_insul	õ	0
	0	ZZcs insul	Ō
	Ō	0	ZZcs_insul];
Zsheath_in	=[ZZsheath_in	0	0
_	0	ZZsheath_in	0
	0	0	ZZsheath_in];
Zsheath_out	=[ZZsheath_out	0	0
	0	ZZsheath_out	
Zahaath m	–IZZsheath m	0	
zsneaun_m		77sheath m	0
	õ	0	ZZsheath ml:
Zsa insul	=[ZZsa_insul	0	0
-	o	ZZsa_insul	0
	0	0	ZZsa_insul];
Zarm_in	=[ZZarm_in	0	0
	0	ZZarm_in	0
7	U =[77erm_out	0	ZZarm_inj;
Zarm_out	-[ZZarm_out	77arm out	0
	õ	0	ZZarm outl:
Zarm m	=[ZZarm m	õ	0
	0	ZZarm_m	0
	0	0	ZZarm_m];
Zasea_insul	=[ZZasea_insul	0	0
	0	ZZasea_insul	0
<b>7</b>	0	0	ZZasea_insul];
Zsea_in			ZZSea_mu13
		$ZZSEd_SEIZ$	77sea_0131 <sup>.</sup>
Y insul	=[YY insul	0	0
	0	YY insul	õ
	0	o <sup>–</sup>	YY_insul];
% calculation	of cable equivalent in	npedance Z and e	quivalent admittance Y
Z11	= Zcore_out +	∠cs_insul +	∠sheath_in;
ZZZ 733	= Zsneath_out +	Zsa_insul +	∠arm_in; Zeop_in:
200 7cc	= Zann_out +	$2*7$ sheath m $\pm$	-∋ca_III, 700 + 0*7arm m
Zsc	= Zsheath m +	22 +	2*Zarm m + 733
Zcs	= Zsheath m +		2*Zarm m + Z33:
Zca	= Zarm_m +	Z33;	,
Zac	= Zarm_m +	- <b>Z33</b> ;	
Zsa	= Zarm_m +	· Z33;	

+ Z33;

Zas = Zarm m + Z33: = <u>Z22</u> + 2\*Zarm\_m + Z33: Zss = Z33: Zaa = -(Zsc\*Zaa-Zsa\*Zac)/(Zss\*Zaa-Zsa\*Zas); X2 = -(Zss\*Zac-Zsc\*Zas)/(Zss\*Zaa-Zsa\*Zas); Х3 = Y insul: Yabc + X2\*Zcs + X3\*Zca; = Zcc Zabc % transfer to p.u. system  $= Y_{insul}(1/Z150b);$ Yabc = (Zcc + X3\*Zca)/Z150b; Zabc + X2\*Zcs % calculation of cable ABCD transform matrix = eig(Zabc\*Yabc); [Tv,Av] = transpose(inv(Tv)); Ti = inv(Tv)\*Zabc\*Ti; Zm = inv(Ti)\*Yabc\*Tv; Ym = sqrt(Zm\*Ym); % propagation rate in the cable p\_c Zc = sqrt(Zm\*inv(Ym)); % characteristic impedance of the cable = Zabc\*Ti\*inv(p\_c)\*sinh(p\_c\*long)\*inv(Ti); Zseries = 2\*Ti\*inv(p\_c)\*tanh(p\_c\*long/2)\*inv(Ti)\*Yabc; Yshunt Ac = inv(Zseries)+1/2\*Yshunt; % A of ABCD transform matrix in admittance = -inv(Zseries); % B of ABCD transform matrix in admittance Bc = transpose(Bc); % C of ABCD transform matrix in admittance Cc = Ac: % D of ABCD transform matrix in admittance Dc

#### F.3 List of Sub-Routine for Obtaining Transformer Impedance Matrices [69]

# % To calculate the equivalent impedance matrices for transformer with different connection function[YABC, YABCabc, YabcABC, Yabc]=transf\_bank(R,Xt,h,connection)

%R	: resistance of the single-phase transformer
%Xt	: reactance of the single-phase transformer
%h	: number of harmonics
%connection	: Ss-Ss Ss-Sf D-D Ss-D Sf-D
%Sf	: star with floating connection to ground
%Ss	: star with solid connection to ground
%D	: delta connection
Yt	=zeros(2*h+1,2*h+1);
0	=zeros(2*h+1,2*h+1);
k =1;	
for n=-h:h	
if n~=0	
yt	=1/(R*sqrt(abs(n))+j*Xt*n);
else	
yt	=1/R;
end	
Yt(k,k)	=yt;
K	=k+1;
End	
YD	$= \begin{bmatrix} 2^*Yt & -Yt & -Yt \end{bmatrix}$
	-Yt 2*Yt -Yt

	-Yt	-Yt	2*Yt];
YSD	= [ -Yt	Yt	0
	• •	-Yt	Yt
	Yt	0	-Ytl/sart(3);
YS	= [Yt	Ō	0
	ι Ω	Ŷt	Ō
	õ	0	Yt1:
Switch connect	tion	•	],
case 'Ss-Ss'			
YABC	= YS <sup>.</sup>		
YABCabc	=-YS		
YahcABC	=-YS		
Vaho	= YS:		
	- 10,		
	- VD/3		
VARCaba	-10/3,		
	-10/3,		
YabcABC	=-10/3;		
	= 10/3;		
case 'D-D',			
YABC	= YD;		
YABCabc	=-YD;		
YabcABC	=-YD;		
Yabc	= YD;		
case 'Ss-D',			
YABC	= YS;		
YABCabc	= YSD;		
YabcABC	= transpo	se(YSD);	
Yabc	= YD/3;		
case 'Sf-D',			
YABC	= YD/3;		
YABCabc	= YSD;		
YabcABC	= transpo	se(YSD);	
Yabc	= YD/3;		
End			

# F.4 List of Sub-Routine for Obtaining TCR Switching Functions [69]

% To calculate the equivalent admittance matrix for delta and star connected TCR function Ytcr =calc\_TCR\_ThreePhase(Va,Vb,Vc,alpha\_a,alpha\_b,alpha\_c,w,Ltcr,h,connection)

% alpha's in rad	
% connection 1	: star connection
% connection 0	: delta connection
if connection	==1
Ytcr_a	=calc_TCR(Va,alpha_a,h,w,Ltcr);
Ytcr_b	=calc_TCR(Vb,alpha_b,h,w,Ltcr);
Ytcr_c	=calc_TCR(Vc,alpha_c,h,w,Ltcr);
Ytcr	=[Ytcr_a Ytcr_a*0 Ytcr_a*0
	Ytcr_a*0 Ytcr_b Ytcr_a*0
	Ytcr_a*0 Ytcr_a*0 Ytcr_c];
else	
Ytcr_ab	=calc_TCR(Va-Vb,alpha_a,h,w,Ltcr);
Ytcr_bc	=calc_TCR(Vb-Vc,alpha_b,h,w,Ltcr);

Ytcr_ca	=calc_TCR(Vc-Va,alpha_c,h,w,Ltcr);
Ytcr	=[Ytcr_ab+Ytcr_ca -Ytcr_ab -Ytcr_ca
	-Ytcr_ab Ytcr_bc+Ytcr_ab -Ytcr_bc
	-Ytcr_ca -Ytcr_bc Ytcr_ca+Ytcr_bc]/3;
end	
% To generate t	he single-phaseTCR admittance
function Ytcr	=calc_TCR(V,alpha,h,w,L)
[sigma1,sigma2	t, thetaxj=1 hy_turn_on_off(V,aipna,n);
5V	=caic_5(sigma1,sigma2,thetax,n);
3	$= \operatorname{Calc}_{\operatorname{Fm}}(\operatorname{Sv}, \operatorname{n});$
D Vtor	-10/11_2/11(0,w,1),
r tCr	
% To obtain val function[sigma1	ues for conducting angles which give the conduction periods of both thyristors ,sigma2,thetax]=Thy_turn_on_off(V,alpha,h)
% V	:harmonic vector V=[-h1 0 1h]
%alpha	: firing angle in radians
%h	:number of harmonics
%sigma1	:first thyristor conduction angle in radians
%sigma2	second thyristor conduction angle in radians
%thetax	:angle in sigma1/2;
%initial condition	is as follows
theta00	=pi/2-angle(V(h+2)):
thetab10	=theta00+pi-alpha;
thetab20	=thetab10+pi;
error=1;	
while error>1e-1	12
Vtcr=0; A1=	:0; A2=0;
J11=0; J12	=0; J13=0;
J21=0; J22	=0; J23=0;
J31=0; J32	=0; J33=0;
for n=-h:h	
if n~=0	
Vtcr	=V(n+h+1)*exp(i*n*theta00)+Vtcr;
thetaa1	=theta00-(pi-alpha);
thetaa2	=thetaa1+pi;
A1	=1/(i*n)*V(n+h+1)*(exp(i*n*thetab10)-exp(i*n*thetaa1))+A1;
A2	=1/(i*n)*V(n+h+1)*(exp(i*n*thetab20)-exp(i*n*thetaa2))+A2;
J11	=i*n*V(n+h+1)*exp(i*n*theta00)+J11;
J12	=0;
J13	
J21	$=V(n+n+1)^{-1}exp(1^{-n}(theta00-p)+alpha))+J21;$
JZZ	=v(n+n+1)*exp(1*n*thetab10)+J22;
J23	=0; $=1/(n+h+1)t_{0}(n+h+1)t_{0}(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n+h+1)(n$
122	$-\infty$
133	-0, - $1/(n+h+1)*ava/i*n*thatab20)+122.$
and	= v(1+1+1) exp(1+1) (1e(a) z 0)+355,
end	
fun	=[\/ter: \[ 1 \ \ 2] \
Jac	=[.11,.12,.13,
theta0	=[theta00:thetab10:thetab20]
theta1	=theta0-inv(Jac)*fun;

```
error
                =norm(theta1-theta0);
  theta00
                =theta1(1);
  thetab10
                =theta1(2);
  thetab20
                =theta1(3);
end
theta01
                = theta1(1);
thetab11
                = theta1(2);
thetab21
                = theta1(3);
                = real(theta01-pi+alpha);
ta1
ta2
                = real(theta01+alpha);
sigma1
                = real(thetab11-ta1);
sigma2
                = real(thetab21-ta2);
thetax
                = ta1+sigma1/2;
```

```
% To obtain harmonic content of the switching function
function S =calc_S(sigma1,sigma2,thetax,h)
```

```
% S: switching harmonic vector S=[-h...-1 0 1...h]
S
                =zeros(2*h+1,1);
Ċ
                =1;
for n=-h:h
  if n~=0
                =1/(n*pi)*(sin(n*sigma2/2)*cos(n*pi)+sin(n*sigma1/2))*exp(-i*n*thetax);
     S(c)
  else
                =(sigma1+sigma2)/(2*pi);
     S(c)
  end
                =c+1;
  С
end
```

#### F.5 List of Other Sub-Routine for Obtaining Harmonic Content [69]

```
% To obtain harmonic content using convolutions in the Fourier harmonic domain
                =calc_Fm(Fv,h)
function Fm
                =zeros(2*h+1,2*h+1);
Fm
for k=-h:h
  for j=-h:h
     if abs(k-j)<=h
       pos
                =(h+1)+(k-j);
                =(h+1)+k;
       х
                =(h+1)+j;
       у
       Fm(x,y) = Fv(pos);
     end
   end
end
% To generate a linear impedance matrix
function Zm
                =form_Zm(R,X,h);
% R resistance in ohms
% X reactance in ohms (+ inductive, - capacitive)
% h harmonic
Zm
                =zeros(2*h+1,2*h+1);
```

n=1; for k=-h:h Zm(n,n) =R+i\*k^(sign(X))\*X; n=n+1; end Zm(h+1,h+1) =R+1e-9;

#### % To obtain a three-phase sinusoidal voltage source function [Vah,Vbh,Vch]=source\_h(Va,Vb,Vc,fa,fb,fc,h)

%Va,Vb,Vc %fa,fb,fc %h %V	<ul> <li>: pick value of the sinusoidal waveform</li> <li>: phase angle in degrees</li> <li>: number of harmonics</li> <li>: three-phase voltage source</li> </ul>
Vah	=zeros(2*h+1,1);
Vah(h)	=i*Va/2*exp(-i*fa*pi/180);
Vah(h+2)	=-i*Va/2*exp(i*fa*pi/180);
Vbh	=zeros(2*h+1,1);
Vbh(h)	=i*Vb/2*exp(-i*fb*pi/180);
Vbh(h+2)	=-i*Vb/2*exp(i*fb*pi/180);
Vch	=zeros(2*h+1,1);
Vch(h)	=i*Vc/2*exp(-i*fc*pi/180);
Vch(h+2)	=-i*Vc/2*exp(i*fc*pi/180);

# Simulation of VSC-HVDC System Harmonics using MATLAB

The following lists of the program are constructed using MATLAB<sup>®</sup> 6.1

### G.1 List of Main Program

% electrical p	roperty of VSC-HVDC	
clear all		
f	= 50;	% fundamental frequency in Hz
h	= 30;	% harmonic orders
х	= 2*h+1;	
0	= zeros(2*h+1,2*h+1);	
re1	= 5;	% resistance for transformer 1
xe1	= 62.8;	% reactance for transformer 1
re2	= 5;	% resistance for transformer 2
xe2	= 62.8;	% reactance for transformer 2
с	= 50 <b>e-</b> 6;	% capacitor bank capacitance
Edc1	= 150;	% DC voltage
VS	= 150*sqrt(2);	% AC peak voltage
[Va Vb Vc]	= source_h(VS,VS,VS,0,-1	20,120,h); % voltage source 1 in harmonic form
[VA VB VC]	= source_h(VS,VS,VS,0,-1	20,120,h); % voltage source 2 in harmonic form
Vabc	= [Va;Vb;Vc];	% Three phase voltage source
VABC	= [VA;VB;VC];	% Three phase voltage source
% electrical p	roperty of VSC 1	
nc1	= 1;	% number of PWM converters
f1	= 50;	% fundamental frequency
m1	= 0.9;	% modulation index
hr1	= 5;	% frequency modulation ratio
% electrical p	property of VSC 2 as constant a	angle
nc2	= 1;	% number of PWM converters
f2	= 50;	% fundamental frequency
m2	= 0.9;	% modulation index

hr2	= 5;	% frequency modulation ratio
р	=0;	
Ze1 Ze2	=form_Zm(re1,xe1,h); =form_Zm(re2,xe2,h);	% harmonic matrix for transformer 1 % harmonic matrix for transformer 2
% electrical pro	perty of generator	
Ra	=1;	% resistance of generators
Xa	=3.14;	% reactance of generators
Ra	=form $Zm(Rg,0,h)$ ;	% harmonic matrix for resistance
RĞ	=[Rg O O O Rg O O O Ra]:	% three phase harmonic matrix for resistance
Χα	=form Zm(0,Xg,h);	% harmonic matrix for reactance
xĞ	=[Xg O O O Xg O O O Xal:	% three phase harmonic matrix for reactance
ZG	=(RG*XG)/(RG+XG);	% equivalent impedance for generators
% cable		
ilona	=1000;	% initial cable length
dlong	=1000;	% interval of cable length
flong	=100000;	% cable length is up to 100 km
for long P	=ilong:dlong:flong =p+1;	
if long~=0 [Ac,Bc,Cc,Dc,R end	l]=calc_cable_DC(f,h,long);	% recall the cable harmonic calculations function
% initial conditi Ph2_0	ons for Ph2 of the VSC 2 =45*pi/180;	% initial value of voltage phase in VSC 2
[labc,IABC,Ph1 ZG,nc1,f1,m1,h IA PAi_0	_new0,V01,V02,Zcap,I1,I2] = cal r1,Edc1,nc2,f2,m2,hr2,c,xe1,re1, = IABC(1:x); = sum(VA.*conj(IA));	lc_PWM_vsc1(Vabc,VABC,Ph2_0,Rl,long, ,xe2,re2,Ac,Bc,Cc,Dc,h,f,x);
Ph2_1 [Iabc,IABC,Ph1 ZG,nc1,f1,m1,h IA PAi_1	=Ph2_0+0.1; _new1,V01,V02,Zcap,I1,I2] = cal r1,Edc1,nc2,f2,m2,hr2,c,xe1,re1 = IABC(1:x); = sum(VA.*conj(IA));	% second value of voltage phase in VSC 2 lc_PWM_vsc1(Vabc,VABC,Ph2_1,RI,long, ,xe2,re2,Ac,Bc,Cc,Dc,h,f,x);
% iterative proc errorV2=1;iterV while errorV2>1 M2 Ph2_new	eess to find the angle Ph2, the solu 2=1; e-6 = (PAi_1-PAi_0)/(Ph2_1-Ph2_0) = (150-PAi_1)/M2+Ph2_1;	tion is obtained when P=150MW );
if Ph2_new>pi/ Ph2_new end if Ph2_new<-pi	2 = Ph2_1*0.9; /2	

Ph2\_new = Ph2\_1\*0.9; end if long~=0 [Iabc,IABC,Ph1 new,V01,V02,Zcap,I1,I2] = calc\_PWM\_vsc1(Vabc,VABC,Ph2\_new,RI,long, ZG,nc1,f1,m1,hr1,Edc1,nc2,f2,m2,hr2,c,xe1,re1,xe2,re2,Ac,Bc,Cc,Dc,h,f,x); else [labc,IABC,Ph1\_new,V01,V02,Zcap,I1,I2]=calc\_PWM\_vsc\_back\_to\_back(Vabc,VABC,Ph2\_new ,ZG,nc1,f1,m1,hr1,Edc1,nc2,f2,m2,hr2,c,xe1,re1,xe2,re2,h,f,x); end IA = IABC(1:x);PAi\_new = sum(VA.\*conj(IA)); PAi\_0 = PAi\_1; PAi\_1 = PAi\_new; Ph2\_0 = Ph2\_1; Ph2\_1 = Ph2\_new; errorV2 = abs(150-PAi\_new) iterV2 = iterV2+1 end % end iterative process [iterV2 errorV2] % results data IA = IABC(1:x);IB  $= IABC(1+x:2^*x);$ IC = IABC(1+2\*x:3\*x);la = labc(1:x); lb  $= labc(1+x:2^*x);$ Ic = labc(1+2\*x:3\*x);Va1 =Va-Ze1\*la; VA1 =VA-Ze2\*IA; Vdc1 =Zcap\*I1+V01; Vdc2 =Zcap\*l2+V02;

 $\label{eq:sa(p),Pa(p),Qa(p),Da(p),Vrmsa(p),Irmsa(p),PFa(p),Phasea(p),Despa(p),VTHDa(p),ITHDa(p),P1a(p),Q1a(p),Vpa(p),I1a(p),V1a(p),Idc1THD(p),Vdc1THD(p),SA(p),PA(p),QA(p),DA(p),VrmsA(p),IrmsA(p),PFA(p),PhaseA(p),DespA(p),VTHDA(p),ITHDA(p),P1A(p),Q1A(p),VpA(p),I1A(p),V1A(p),Idc2THD(p),Vdc2THD(p)]=spqd(Va,Ia,h,I1,Vdc1,VA,IA,I2,Vdc2);$ 

[Sa1(p), Pa1(p), Qa1(p), Da1(p), Vrmsa1(p), Irmsa1(p), PFa1(p), Phasea1(p), Despa1(p), VTHDa1(p), ITHDa1(p), P1a1(p), Q1a1(p), Vpa1(p), I1a1(p), V1a1(p), Idc1THD1(p), Vdc1THD1(p), SA1(p), PA1(p), QA1(p), DA1(p), VrmsA1(p), IrmsA1(p), PFA1(p), PhaseA1(p), DespA1(p), VTHDA1(p), ITHDA1(p), P1A1(p), Q1A1(p), VpA1(p), I1A1(p), V1A1(p), Idc2THD1(p), Vdc2THD1(p)]=spqd(Va1, Ia, h, I1, Vdc1, VA1, IA, I2, Vdc2);

```
Phx1(p) =real(Ph1_new)*180/pi;
Phx2(p) =real(Ph2_new)*180/pi;
end
```

% plot the results length =ilong:dlong:flong subplot(2,2,1) plot(length,ITHDa,'-',length,ITHDA,'-.') title 'ITHD'

subplot(2,2,2) plot(length,Idc1THD,'-',length,Idc2THD,'-.') title 'IDCTHD'

subplot(2,2,3) plot(length,Vdc1THD,'-',length,Vdc2THD,'-.') title 'VDCTHD'

Ploss=Pa+PA; subplot(2,2,4) plot(length,Ploss,'-') title 'Power Loss'

#### G.2 List of Sub-Routine of Cable Harmonic Impedance and Admittance for DC

#### Subsea Cable

function [Ac,Bc,Cc,Dc,Rl]=calc\_cable\_DC(f,h,long)

% cable layers dimensions, materials and properties		
x	= [0];	% cable position in x coordinate in m
У	= [-50];	% cable position in y coordinate in m
% conductor dim	ension and property	
r c	= 0.01895:	% conductor radius in m
res c	= 0.02244176*1e-6:	% conductor resistivity in $\Omega$ .m
% insulation prov	nerty	
r ex xine	$= 0.03765^{\circ}$	% XLPE insulation outside radius in m
r in xine	= 0.02065	% XLPE insulation inside radius in m
% sheath dimens	ion and property	
r sheath	$= 0.0405^{\circ}$	% average sheath radius in m
r_isheath	= 0.03925	% sheath inside radius in m
r_osheath	= 0.04175	% sheath outside radius in m
t_sheath	= 0.0025	% sheath thickness
	$= 0.214*1e_{-6}$	% sheath resistivity in O m
% armour dimen	sion and property	/o sheath resistivity in sz.m
r arm	$= 0.0466^{\circ}$	% average armour radius in m
r_iarm	= 0.0400,	% armour inside radius in m
	= 0.0491	% armour outside radius in m
t_oann	= 0.005	% armour thickness in m
dtangle a	= 0.000, = 1*ni/Q:	% armour wire laving angle
ofonglo_o	= 1  pi/s, = $1 \text{ tri/4}.$	% annour wire lapsitudinal angle
alaliyie_a	= 1 pi/4, = 10;	% annour wire longitudinal angle
n orm	- 10, - 54:	% transversal permeability of amour wire
n_ann	- 54,	% number of armour wires
res_a	= 0.1386-1e-6;	% resistivity of armour wire in $\Omega$ .m
A_arm	= t_arm^2^pi/4;	% area of armour wires
% DC resistance	of armour wires	
R_arm	= res_a/(n_arm*A_arm*cos(dtar	ngle_a));
p_arm	= 2*pi*r_arm/tan(dtangle_a);	% pitch of armour wires
% relative perme	ability of steel armour	
mr_arm	= n_arm*A_arm/(p_arm*t_arm)*	'me*exp(-i*afangle_a)*sin(dtangle_a)

+mt\*cos(dtangle\_a)^2; % cable outer dimension: r out = 0.0511;% sea and property: res\_sea = 1; % establish matrices ZZcore out = zeros(2\*h+1,2\*h+1); ZZcs insul = zeros(2\*h+1,2\*h+1); ZZsheath\_in = zeros(2\*h+1,2\*h+1); ZZsheath\_out = zeros(2\*h+1,2\*h+1); ZZsheath\_m = zeros(2\*h+1,2\*h+1); ZZsa insul = zeros(2\*h+1,2\*h+1); ZZarm\_out = zeros(2\*h+1,2\*h+1);ZZarm\_in  $= zeros(2^{h+1},2^{h+1});$ ZZarm m = zeros(2\*h+1,2\*h+1);ZZasea\_insul = zeros(2\*h+1,2\*h+1); ZZsea\_sel1  $= zeros(2^{h+1},2^{h+1});$ YY\_insul = zeros(2\*h+1,2\*h+1);0 = zeros(2\*h+1,2\*h+1); zcore out = zeros(h+1,1); zcs insul = zeros(h+1,1); zsheath in = zeros(h+1,1);zsheath out = zeros(h+1,1); zsheath m = zeros(h+1,1); zsa insul = zeros(h+1,1);zarm out = zeros(h+1,1);zarm in = zeros(h+1,1); zarm\_m = zeros(h+1,1);zasea insul = zeros(h+1,1);zsea\_sel1 = zeros(h+1,1); y\_insul = zeros(h+1,1); % cable harmonic impedance calculation for n=0:h w0 = 2\*pi\*f; % fundamental angular velociay = w0\*n; % harmonic angular velociay W m0 = 4\*pi\*1e-7; % permeability of air = 8.854\*1e-12; % permittivity of air **e**0 e\_xlpe = 2.5\*e0; % relative permittivity of XLPE Insulation = sqrt(i\*w\*m0/res\_c); cmx\_c % propagation complex number of conductor cmx\_s = sqrt(i\*w\*m0/res\_s); % propagation complex number of sheath cmx\_a = sqrt(i\*w\*m0\*mr\_arm/res\_a); % propagation complex number of armour cmx\_sea = sqrt(i\*w\*m0/res\_sea); % propagation complex number of sea % calculation for non-DC harmonic impedance if n~=0 % conductor impedance = res\_c\*cmx\_c/(2\*pi\*r\_c)\*(besseli(0,(cmx\_c\*r\_c))/besseli(1,(cmx\_c\*r\_c))); zvcore\_out % conductor-sheath insulation impdeance zvcs\_insul = i\*w\*m0/(2\*pi)\*log(r\_isheath/r\_c); % sheath impedance Hsheath = besseli(1,(cmx\_s\*r\_osheath))\*besselk(1,(cmx\_s\*r\_isheath))-(besseli(1,(cmx\_s\*r\_isheath))\*besselk(1,(cmx\_s\*r\_osheath)));

```
= res_s*cmx_s/(2*pi*r_isheath*Hsheath)*(besseli(0,(cmx_s*r_isheath))*
zvsheath_in
                besselk(1,(cmx_s*r_osheath))+besselk(0,(cmx_s*r_isheath))*
                besseli(1,(cmx_s*r_osheath)));
               = res_s*cmx_s/(2*pi*r_osheath*Hsheath)*(besseli(0,(cmx_s*r_osheath))*
zvsheath_out
                besselk(1,(cmx_s*r_isheath))+besselk(0,(cmx_s*r_osheath))*
                besseli(1,(cmx_s*r_isheath)));
                = -res_s/(2*pi*r_isheath*r_osheath*Hsheath);
zvsheath_m
% sheath-armour insulation impdeance
zvsa_insul
                = i*w*m0/(2*pi)*log(r_iarm/r_osheath);
% armour impedance
                = besseli(1,(cmx_a*r_oarm))*besselk(1,(cmx_a*r_iarm))-
Harm
                (besseli(1,(cmx_a*r_iarm))*besselk(1,(cmx_a*r_oarm)));
                = R arm*cmx a*t arm*coth(cmx a*t arm);
zvarm in
                = R_arm*cmx_a*t_arm*coth(cmx_a*t_arm);
zvarm out
               = -R_arm*cmx_a*t_arm/sinh(cmx_a*t_arm);
zvarm_m
% arm-sea insulation impedance
zvasea_insul = i*w*m0/(2*pi)*log(r_out/r_oarm);
% sea impedance
               = i*w*m0/(2*pi)*((-log(1.781*cmx_sea*r_out/2))+(1/2)-(4*cmx_sea*(-y(1))/3));
zvsea_sel1
% insulation shunt
                = 0.0000000003+i*w*2*pi*e_xlpe/log(r_ex_xlpe/r_in_xlpe);
yv_insul
else
% calculation for DC harmonic impedance
% conductor impedance
zvcore_out
               = res_c/(r_c^2*pi);
zvcs insul
                = 0:
zvsheath in
                = res_s/(2*pi*r_sheath*t_sheath);
zvsheath out
               = res_s/(2*pi*r_sheath*t_sheath);
zvsheath m
               = 0:
zvsa insul
               = 0:
zvarm_in
                = res_a/(n_arm*pi*r_arm^2*cos(dtangle_a));
zvarm_out
                = res_a/(n_arm*pi*r_arm^2*cos(dtangle_a));
zvarm_m
               = 0:
zvasea_insul
               = 0:
zvsea_sel1
               = 0:
yv_insul
               = 0.000000003;
end
% place the harmonic values into the harmonic domain matrices
zcore_out(n+1)
                   = zvcore out;
zcs_insul(n+1)
                   = zvcs_insul;
                   = zvsheath_in;
zsheath_in(n+1)
zsheath_out(n+1) = zvsheath_out;
zsheath_m(n+1)
                   = zvsheath_m;
zsa_insul(n+1)
                   = zvsa_insul;
zarm_in(n+1)
                   = zvarm_in;
zarm_out(n+1)
                   = zvarm_out;
zarm_m(n+1)
                   = zvarm_m;
zasea_insul(n+1)
                  = zvasea_insul;
zsea_sel1(n+1)
                   = zvsea_sel1;
```

y_insul(n+1) end	= yv_insul;
k for j=1:2*h+1	= 1;
if j<=h zzcore_out zzcs_insul zzsheath_in zzsheath_out zzsheath_m zzsa_insul zzarm_in zzarm_out zzarm_m zzasea_insul zzsea sel1	<pre>= 2*real(zcore_out(h+2-j))-zcore_out(h+2-j); = 2*real(zcs_insul(h+2-j))-zcs_insul(h+2-j); = 2*real(zsheath_in(h+2-j))-zsheath_in(h+2-j); = 2*real(zsheath_out(h+2-j))-zsheath_out(h+2-j); = 2*real(zsa_insul(h+2-j))-zsa_insul(h+2-j); = 2*real(zarm_in(h+2-j))-zarm_in(h+2-j); = 2*real(zarm_out(h+2-j))-zarm_out(h+2-j); = 2*real(zarm_m(h+2-j))-zarm_out(h+2-j); = 2*real(zaram_m(h+2-j))-zarm_m(h+2-j); = 2*real(zase_insul(h+2-j))-zase_insul(h+2-j); = 2*real(zsea_sel1(h+2-j))-zsea_sel1(h+2-j);</pre>
yy_insul else	= 2*real(y_insul(h+2-j))-y_insul(h+2-j);
zzcore_out zzcs_insul zzsheath_in zzsheath_out zzsheath_out zzsheath_m zzsa_insul zzarm_in zzarm_out zzarm_m zzasea_insul zzsea_sel1 yy_insul end ZZcore_out(k,k) ZZcs_insul(k,k) ZZsheath_in(k,k) ZZsheath_out(k,k) ZZsheath_out(k,k) ZZsa_insul(k,k) ZZarm_out(k,k) ZZarm_in(k,k) ZZarm_m(k,k) ZZasea_insul(k,k)	<pre>= zcore_out(j-h); = zcs_insul(j-h); = zsheath_out(j-h); = zsheath_out(j-h); = zsa_insul(j-h); = zarm_out(j-h); = zarm_out(j-h); = zarm_m(j-h); = zasea_insul(j-h); = zsea_sel1(j-h); = y_insul(j-h); = zzcore_out; = zzcore_out; = zzcs_insul; = zzsheath_in; = zzsheath_out; = zzsheath_m; = zzsa_insul; = zzarm_out; = zzarm_out; = zzarm_in; = zzarm_m; = zzasea_insul;</pre>
ZZsea_sel1(k,k)	= zzsea_sel1;
k=k+1; end	, <u>, , , , , , , , , , , , , , , , , , </u>
Zcore_out Zcs_insul Zsheath_in Zsheath_out Zsheath_m Zsa_insul Zarm_in Zarm_out	= ZZcore_out; = ZZcs_insul; = ZZsheath_in; = ZZsheath_out; = ZZsheath_m; = ZZsa_insul; = ZZarm_in; = ZZarm_out;

= ZZarm m; Zarm\_m Zasea\_insul = ZZasea\_insul; = ZZsea\_sel1; Zsea in Y\_insul = YY\_insul; % calculation of Z and Y = Zcore out + Zcs insul + Zsheath\_in; Z11 = Zsheath\_out + Zsa\_insul + Zarm\_in; Z22 = Zarm\_out + Zasea\_insul + Zsea\_in; Z33 = Z11 + 2\*Zsheath m + Z22 + 2\*Zarm\_m + Z33; Zcc = Zsheath\_m + Z22 + 2\*Zarm\_m + Z33; Zsc = Zsheath\_m + Z22 + 2\*Zarm\_m + Z33; Zcs Zca = Zarm\_m + Z33; = Zarm\_m Zac + Z33; = Zarm\_m Zsa + Z33; = Zarm\_m Zas + Z33; Zss = Z22 + 2\*Zarm\_m + Z33; Zaa = Z33: X2 = -(Zsc\*Zaa-Zsa\*Zac)/(Zss\*Zaa-Zsa\*Zas); X3 = -(Zss\*Zac-Zsc\*Zas)/(Zss\*Zaa-Zsa\*Zas); Yabc = Y\_insul; Zabc = Zcc + X2\*Zcs + X3\*Zca; Zabc(h+1,h+1) =res\_c/(r\_c^2\*pi); % calculation of cable ABCD transform matrix p\_c = sqrt(Zabc\*Yabc); = sqrt(Zabc\*inv(Yabc)); Zc = Zabc\*inv(p\_c)\*sinh(p\_c\*long/2); Zseries = 2\*inv(p\_c)\*tanh(p\_c\*long/4)\*Yabc; Yshunt = inv(tanh(p\_c\*long/2))\*sinh(p\_c\*long/2); Ac = Zc\*sinh(p\_c\*long/2); Вс = inv(Zc)\*sinh(p\_c\*long/2); Сс Dc = transpose(Ac);

#### RI = Zabc(h+1,h+1);

#### G.3 List of Sub-Routine of Iteration Process for Obtaining Phase Angle under

#### Steady-State

function

 $\label{eq:label} $ [label{label} ABC,Ph1_new,V01,V02,Zcap,I1,I2] = cale_PWM_vsc1(Vabel,VABC,Ph2,RI,long,ZG,nc1,f1,m1,hr1,Edc1,nc2,f2,m2,hr2,c,xe1,re1,xe2,re2,Ac,Bc,Cc,Dc,h,f,x); $ \label{eq:label} $ \l$ 

#### %initial conditions for Ph1 of the VSC 1

IS	=Vabc;
IR	=VABC;
IsysX	=[IS;IR];
error11	=1;
iter11	=1;
l1x	=1;
Ph1 0	= -Ph2

while error11>1e-6

```
Edc2
               =Edc1-RI*I1x*long;
               =1;
error12
               =1;
iter12
while error12>1e-6
VG1
               =ZG*lsysX(1:3*x);
VG2
               =ZG*lsysX(3*x+1:6*x);
[Zeq,Qs1,Qs2,Vabc_eq,VABC_eq,V01,V02,Zcap]= calc_HVDC(Ph1_0,nc1,f1,m1,hr1,Edc1,Ph2,
nc2,f2,m2,hr2,Edc2,c,xe1,re1,xe2,re2,Ac,Bc,Cc,Dc,h,f);
               = inv(Zeq)*[Vabc-Vabc_eq-VG1;VABC-VABC_eq-VG2];
lsys
labc
               = sqrt(3)*lsys(1:3*x);
IABC
               = sqrt(3)*lsys(3*x+1:6*x);
11
               = Qs1*labc;
12
               = Qs2*IABC;
dl 0
               = real(11(h+1)+12(h+1));
               =norm(lsys-lsysX)
error12
               =iter12+1;
iter12
IsysX
               =lsys;
end
110
               = real(11(h+1));
error11
               = abs(110-11x)
               = 110:
I1x
               = iter11+1;
iter11
end
               =1:
error21
iter21
               =1:
               = Ph1_0+0.1;
Ph1_1
while error21>1e-6
Edc2
               =Edc1-Ri*i1x*long;
error22
               =1:
iter22
               =1;
while error22>1e-6
VG1
               =ZG*lsysX(1:3*x);
VG2
               =ZG*lsysX(3*x+1:6*x);
[Zeq,Qs1,Qs2,Vabc_eq,VABC_eq,V01,V02,Zcap]= calc_HVDC(Ph1_1,nc1,f1,m1,hr1,Edc1,Ph2,
nc2,f2,m2,hr2,Edc2,c,xe1,re1,xe2,re2,Ac,Bc,Cc,Dc,h,f);
               = inv(Zeq)*[Vabc-Vabc_eq-VG1;VABC-VABC_eq-VG2];
Isys
labc
               = sqrt(3)*lsys(1:3*x);
IABC
               = sqrt(3)*lsys(3*x+1:6*x);
11
               = Qs1*labc:
12
               = Qs2*IABC:
dl_1
               = real(l1(h+1)+l2(h+1));
error22
               =norm(lsys-lsysX)
iter22
               =iter22+1;
IsysX
               =lsys;
end
110
               = real(I1(h+1));
error21
               = abs(I10-I1x)
l1x
               =110;
iter21
               =iter21+1;
end
```

#### % end of initial conditions

```
% iterative process to find the angle Ph1, the solution is obtained when I1(dc)+I2(dc)=0
error=1;iter=1;
while error>1e-6
                = (dl_1-dl_0)/(Ph1_1-Ph1_0);
 Μ
 Ph1_new
                = -di_1/M+Ph1_1;
 if Ph1_new>pi/2
 Ph1_new
                = Ph1_1*0.9;
 end
 if Ph1_new<-pi/2
 Ph1_new
                = Ph1_1*0.9;
 end
error31
                =1;
                =1;
iter31
while error31>1e-6
                =Edc1-RI*I1x*long;
Edc2
                =1;
error32
                =1;
iter32
while error32>1e-6
                =ZG*lsysX(1:3*x);
VG1
                =ZG*lsysX(3*x+1:6*x);
VG2
[Zeq,Qs1,Qs2,Vabc_eq,VABC_eq,V01,V02,Zcap] = calc_HVDC(Ph1_new,nc1,f1,m1,hr1,Edc1,
Ph2,nc2,f2,m2,hr2,Edc2,c,xe1,re1,xe2,re2,Ac,Bc,Cc,Dc,h,f);
                = inv(Zeq)*[Vabc-Vabc_eq-VG1;VABC-VABC_eq-VG2];
 lsys
                = sqrt(3)*isys(1:3*x);
 labc
 IABC
                = sqrt(3)*lsys(3*x+1:6*x);
                = Qs1*labc;
11
12
                = Qs2*IABC;
error32
                =norm(lsys-lsysX)
iter32
                =iter32+1;
lsysX
                =lsys;
end
110
                = real(I1(h+1));
error31
                = abs(I10-I1x)
                =110;
I1x
iter31
                =iter31+1;
end
dl new
                = real(11(h+1)+12(h+1));
dl 0
                = dl_1;
dl 1
                = dl new;
 Ph1 0
                = Ph1_1;
Ph1_1
               = Ph1_new;
error
               = abs(dl_new)
iter
               = iter+1
end
% end iterative process
[iter error]
```

# G.4 List of Sub-Routine for Obtaining Equivalent Impedance of VSC Stations

% To model VSC-HVDC equivalent impedance

function

[Zeq,Qs1,Qs2,Vabc\_eq,VABC\_eq,V01,V02,Zcap]=calc\_HVDC(Ph1,nc1,f1,m1,hr1,Edc1,Ph2,nc 2,f2,m2,hr2,Edc2,c,xe1,re1,xe2,re2,Ac,Bc,Cc,Dc,h,f)

% Ph1	: Phase in radians of the modulation signal of the VSC1
% nc1	: Number of VSC per phase to form the VSC1
% fl	: Frequency of the modulation signal of the VSC1
% m1	: Modulation index of the modulation signal of the VSC1
% hr 1	: Harmonic of the carrier signal of the VSC1 in function of the frequency fl
% Edc1	: DC voltage in the DC-capacitor of the VSC1
%с	: Capacitance of the DC capacitor in farads of both VSCs
% xe	: Reactance of the transformer connected to the VSCs in ohms
% re	: Resistence of the transformer connected to the VSCs in ohms
% R1	: Series resistance of the cable in ohms/km
% Ll	: Series inductance of the cable in H/km
% Gl	: Shunt conductance of the cable in S/km
% Cl	: Shunt capacitance of the cable in F/km
% long	: Cable length
% h	: Harmonics
% I 0/ The served	: Frequency of the system (use to compute the impedance of the capacitor) solution is when $11(d_0)+12(d_0)=0$
% The correct	solution is when $\prod(dc) + 12(dc) = 0$
% Where II-Q	$a_{1} = a_{1} = b_{1} = b_{1$
70 [ V abc- V abc	
U	=form Zm(1.0.h):
D	=form Zm(0,1,h);
X1	=form_Zm(re1,xe1,h);
Xe1	=[X1 X1*0 X1*0
	X1*0 X1 X1*0
	X1*0 X1*0 X1 ];
X2	=form_Zm(re2,xe2,h);
X A	
Xe2	
7	X2"U X2"U X2 ]; /m/0_4//st0t=it0_h);
zcap=torm_z	.m(∪,-1/(C <sup>-</sup> 2 <sup>-</sup> pi <sup>-</sup> t),n);
%	VSC 1
V01	=zeros(2*h+1,1);
V01(h+1)	=Edc1;
[Sab,Sbc,Sca	a,Sabt1]=calc PWM unipolar pscad(Ph1,f1,m1,hr1,h);
Sab	=delete_even_harm(Sab,h);
Sbc	=delete_even_harm(Sbc,h);
Sca	=delete_even_harm(Sca,h);
Sab	=calc_Fm(Sab,h);
Sbc	=calc_Fm(Sbc,h);
Sca	=calc_Fm(Sca,h);
Ps1	=[Sab;Sbc;Sca];
Qs1	=[Sab Sbc Sca];

% V	SC 2
V02	=zeros(2*h+1,1);
V02(h+1)	=Edc2;
[Sab,Sbc,Sca,S	abt1]=calc_PWM_unipolar_pscad(Ph2,f2,m2,hr2,h);
Sab	<pre>=delete_even_harm(Sab,h);</pre>
Sbc	=delete_even_harm(Sbc,h);
Sca	=delete_even_harm(Sca,h);
Sab	=calc_Fm(Sab,h);
Sbc	=calc_Fm(Sbc,h);
Sca	=calc_Fm(Sca,h);
Ps2	=[Sab;Sbc;Sca];
Qs2	=[Sab Sbc Sca];
% VSC-H	VDC
Yc	=inv(2cap);
YAC	=inv(Yc*Ac+Cc);
DYB	=inv(Dc+Yc*Bc);
A1c2	=Xe1+Ps1*Ac*YAC*Qs1/2+Ps1*Bc*DYB*Qs1/2;
B1c2	=Ps1*Ac*YAC*Qs2/2-Ps1*Bc*DYB*Qs2/2;
C1c2	=Ps2*Ac*YAC*Qs1/2-Ps2*Bc*DYB*Qs1/2;
D1c2	=Xe2+Ps2*Ac*YAC*Qs2/2+Ps2*Bc*DYB*Qs2/2;
Vabc_eq	=Ps1*V01;
VABC_eq	=Ps2*V02;
Zeq	=[A1c2 B1c2;C1c2 D1c2];

## G.5 List of Sub-Routine for Obtaining Switching Functions of VSC Stations

## % To model and represent the VSC given by PSCAD function [Sab,Sbc,Sca,sabt]=calc\_PWM\_unipolar\_pscad(Ph,f,m,hr,h) global g1 g2 g3 g4 g5 g6 fr fs sabt sbct scat t

% Ph :	is the phase shift angle in rad
%f :	is the frequency in Hz of the modulation signal fs.
% m	: is the modulacion index, i.e. fs=m*sin(wt+Fis)
% hr	: is the harmonic frequency of the carrier signal, e.g for 250 Hz hr=5 for f=50 Hz.
% h	: is the number of harmonics to use in the vector Spwm
% Sab	: is a harmonic vector of the obtained switching function
% [-h1	0 1 h]'
% sabt	: is the switching function in time domain
Ph	=Ph+pi/6;
p_trian	=63; %63 is only to use an odd number per triangle in the carrier signal
tri1	=triang(p_trian);
tri1	=[0;tri1];
fr	=tri1;
for L-1. Other 1	N N N N N N N N N N N N N N N N N N N
for $K = 1.(2^{-n} r - 1)$	) 
Tr	=[fr;(-1)^(K)~trl1];
ena	
points	=size(fr):
points	=points(1):
F	

desp =round(points/3); =0:1/f/(points-1):1/f; t fs =m\*sin(2\*pi\*f.\*t); for k=1:points if fs(k)>fr(k) g1(k) =1; else g1(k)=0; end end for k=1:points if fs(k)<fr(k) =1; g4(k) else =0; g4(k) end end g1\_1 g1\_2 =g1(1:desp); =g1(desp+1:2\*desp); g1\_3 =g1(2\*desp+1:points); =[g1\_3 g1\_1 g1\_2]; g3 g5 =[g1\_2 g1\_3 g1\_1]; =g4(1:desp); g4\_1 g4\_2 =g4(desp+1:2\*desp); =g4(2\*desp+1:points); g4\_3 g2 =[g4\_2 g4\_3 g4\_1]; =[g4\_3 g4\_1 g4\_2]; g6 sabt =g1.\*g6-g4.\*g3; sbct =g3.\*g2-g6.\*g5; scat =g5.\*g4-g2.\*g1; =sabt; Upwm UpwmF =fft(Upwm)/points; UpwmF =fftshift(UpwmF); X =round(points/2)-points/2; if x==0.5 centre =round(points/2); else centre =round(points/2)+1; end Spwm =UpwmF(centre-h:centre+h); Sab =Spwm; for n=-h:h =Sab(n+h+1)\*exp(-i\*n\*Ph); Sab(n+h+1) Sbc(n+h+1) =Sab(n+h+1)\*exp(-i\*n\*(2\*pi/3)); Sca(n+h+1) =Sab(n+h+1)\*exp(-i\*n\*(-2\*pi/3)); end

# G.6 List of Sub-Routine for Supplement of VSC-HVDC Harmonic Calculations

% Most of the sub-routines for obtaining harmonic content can be found in Appendix F.5 and the following sub-routine is to define the harmonic distortions of voltage, current and power of the system

function

[Sa,Pa,Qa,Da,Vrmsa,Irmsa,PFa,Phasea,Despa,VTHDa,ITHDa,P1a,Q1a,Vpa,I1a,V1a,Idc1THD,		
Vdc1THD,SA,PA,QA,DA,VrmsA,IrmsA,PFA,PhaseA,DespA,VTHDA,ITHDA,P1A,Q1A,VpA,I1A,V		
1A,Idc2THD,Vc	lc2THD] = spqd(Va,Ia,h,I1,Vdc1,VA,IA,I2,Vdc2)	
Vrmsa	= sqrt(sum(abs(Va).^2));	
VrmsA	= sqrt(sum(abs(VA).^2));	
Irmsa	= sqrt(sum(abs(la).^2));	
IrmsA	= sqrt(sum(abs(IA).^2));	
Sa	= Vrmsa*Irmsa;	
SA	= VrmsA*irmsA;	
Pa	= sum(Va.*conj(la));	
PA	= sum(VA.*conj(IA));	
PFa	= Pa/Sa;	
PFA	= PA/SA:	
Phasea	= sign(angle(Va(h+2))-angle(la(h+2)));	
PhaseA	= sign(angle(VA( $h+2$ ))-angle(la( $h+2$ )));	
fia	= angle( $Va(h+2)$ )-angle( $la(h+2)$ ):	
fiA	= angle( $VA(h+2)$ )-angle( $IA(h+2)$ ):	
Despa	$= \cos(fia)$ :	
DespA	$= \cos(fiA)$	
Oa	= $sort(sum(abs(Va)^2 *abs(la)^2 - Va *coni(la) *Va *coni(la)))$	
QA QA	= $sqrt(sum(abs(VA))^2 *abs(IA)^2 - VA *coni(IA) *VA *coni(IA)))$	
Da	$= sart(Sa^{2}-Pa^{2}-Qa^{2})$	
	$= \operatorname{sort}(S\Delta^2 - P\Delta^2 - O\Delta^2)^{\circ}$	
VTHDa	= sort(sum(abs(Va(1 <sup>-</sup> h-1)) ^2)/abs(Va(h) ^2))*100 <sup>-</sup>	
ITHDa	= sort(sum(abs( a(1:h-1)), 2)/abs( a(h), 2))*100;	
VTHDA	$= sart(sum(abs(VA(1:h-1)) ^2)/abs(VA(h) ^2))*100$	
ITHDA	= sart(sum(abs( A(1:h-1)), 2)/abs( A(h), 2)) *100,	
IIIIOA	$= \operatorname{Sqr}(\operatorname{Sqr}(\operatorname{Sqr}(\operatorname{In}(1), \mathbb{Z})) \operatorname{Sqr}(\operatorname{In}(1), \mathbb{Z})) = \operatorname{Sqr}(\operatorname{Sqr}(\operatorname{Sqr}(\mathbb{Z})) = \operatorname{Sqr}(\operatorname{Sqr}(\mathbb{Z})) = \operatorname{Sqr}(\mathbb{Z}) = \operatorname{Sqr}(\operatorname{Sqr}(\mathbb{Z})) = \operatorname{Sqr}(\mathbb{Z}) = $	
V1a	= sort(abs(Va(h)),^2+abs(Va(h+2)),^2);	
l1a	$= sart(abs(la(h)), ^2+abs(la(h+2)), ^2);$	
P1a	$= V1a^{11a^{1}}(as(fia))$	
Q1a	= V1a*l1a*sin(fia):	
Vpa	= sum(abs(Va))	
VIA	= sort(abs(VA(h)) $^{2}$ +abs(VA(h+2)) $^{2}$	
11A	$= sart(abs(IA(h)) ^2 + abs(IA(h+2)) ^2)$	
P1A	$= \sqrt{1} \Delta^{*}(1)^{*} \Delta^{*}(1)^{*}$	
014	$= \sqrt{1} \Delta^* (1/\sqrt{10})$	
	$= e_{1} + i \Delta e_{1} + i \Delta e_{2}$	
vpA	- sum(abs(vA)),	
	$=$ sort(sum(abs/l1(1:b)) ^2)/abs(l1(b+1) ^2))*100:	
	= sqrt(sum(abs(1)(1.1)), 2)(abs(1)(1+1), 2)) 100, = sqrt(sum(abs(1)(dc1(1:b)) (2))(abs(1)(dc1(b+1) (2)))*100;	
	$= \operatorname{syntaum}(\operatorname{aus}(\operatorname{vuc}(1,1)), \operatorname{z}(\operatorname{aus}(\operatorname{vuc}(1 T ), 2))   UU,$	
Idc2THD	$= sart(sum(abs(12(1:b))^2)/abs(12(b+1)^2))*100$	
Vdc2THD	$= \operatorname{sqrt}(\operatorname{sum}(\operatorname{abs}(1/(d_2)(1\cdot h)) \wedge 2)/\operatorname{abs}(1/(d_2)(h+1) \wedge 2)) * 100,$	
VULLINU	$= \operatorname{sqrt}_{\operatorname{sqrt}} \operatorname{sqrt}_$	