Auctions and Information Acquisition: Sealed-bid or Dynamic Formats?*

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Abstract

Firms need to spend time and money to figure out how much assets for sale are worth to them. Which selling procedure is likely to generate better incentives for information acquisition? We show that multi-stage and in particular ascending price auctions with breaks (that allow for information acquisition) perform better than their static counterpart. This is because dynamic formats allow bidders to observe the number of competitors left throughout the selling procedure, hence to get a much better estimate of their chance of winning. Since information acquisition tends to generate higher revenues, our analysis provides a new rationale for using dynamic formats rather than sealed-bid ones. Our conclusion is reinforced when multiple objects are sold, because dynamic formats may not only provide bidders with estimates on whether competition is soft or not, but also on which object competition is softer.

Key words: auctions, private value, information acquisition, option value.

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1 Introduction

Assessing the value of an asset for sale is a costly activity, and it is often buyer specific. When a firm is being sold, each individual buyer has to figure out what is his own best use of the assets, which business unit to keep or resell, which site or production line to close. The resources spent can be very large when there is no obvious way for the buyer to combine the asset for sale with the assets that he already owns. Similarly, when acquiring a license for digital television, entrants have to figure out the type of program they will have a comparative advantage on, as well as the advertisement revenues they can expect from the type of program they wish to broadcast. Incumbents may also want to assess the economies of scale that can be derived from the new acquisition. All such activities are aimed at refining the assessment of the valuation of the license, and they are costly.

>From the seller's perspective, if the assets are auctioned to a set of potential buyers, the better informed the bidders are the higher the revenues, at least when the number of competitors is not too small. However, when information is costly to acquire, a potential buyer may worry about the possibility that he spends many resources, and yet ends up not winning the asset. Providing the bidders with incentives to acquire information is thus key for the seller.

This paper analyzes how various auction formats compare with respect to the buyers' incentives for information acquisition, and with respect to the revenues that they generate for the seller. Most of the literature on information acquisition in auctions restricts attention to static or sealed-bid auction formats in which information acquisition may only take place prior to the auction (see the literature review below).

But, what if the selling procedure allows the bidders to acquire information during the auction as well?

Multi-stage auctions are common examples of selling procedures that allow for information acquisition within the auction. Besides, by adjusting the lapse of time between stages, it is possible to allow for such information acquisition even when acquiring information takes time. More generally, the standard ascending auction format could be amended to allow for information acquisition by including breaks at pre-determined dates or events (say, each time a bidder drops out), precisely designed so that bidders have enough time to acquire

information, or by allowing any bidder to trigger such breaks. How do these selling procedures compare with their sealed bid counterparts, that only allow for information acquisition prior to the auction?

An important insight of this paper is that dynamic auction procedures are likely to generate more information acquisition and higher revenues than their static counterparts.¹ More precisely, we highlight a significant benefit induced by formats in which bidders gradually get to know the number of (serious) competitors they are facing, which in turn allow them to better adjust their information acquisition strategy.

To illustrate, suppose there is one good for sale and compare the sealed bid second price auction and the ascending price auction in which each bidder can decide at any time to acquire information.

In the sealed-bid static format, bidders are unlikely to decide to acquire extra information whenever there are potentially many competitors. The point is that the risk of ending up not buying the good (because it turns out that someone else has a higher value) is then so large that bidders prefer not to waste their money (or time) on getting extra information. In contrast, in the ascending price auction format, bidders get to obtain a better estimate of their chance of winning just by observing the number of bidders left. They can thus base their information acquisition strategy on better grounds, which in turn induce them to acquire the extra information more often. And the more information the bidders acquire, the higher the revenues (at least when the number of bidders is not too small).

While most of our formal analysis will be based on a single object problem, we will show that our basic insight is actually reinforced in multi-object auctions: not only do bidders have to decide whether or not to acquire extra information; but when they do so, they have to decide on which object to acquire information. Ascending formats that generate information on which object a bidder has better chances of winning are then likely to perform better than formats such as static ones that do not generate such information: when a bidder is not guided as to which object(s) to focus on, he takes poor information acquisition decisions, which in turn is likely to discourage him from acquiring any information. In contrast, when a bidder is guided as to which objects he should focus on, he makes good information acquisition

¹Though we focus here on revenues, efficiency may also be higher in the ascending format (see Compte-Jehiel 2000).

decisions, which in turn leads him to acquire extra information more often. (See Ausubel and Milgrom 2001 for an account of why this may be of practical importance in package auctions.)

In the above discussion we have emphasized the benefit of providing bidders with some estimate of the level of competition (through the number of competitors left). But, not all dynamic formats have the property of conveying such an estimate. For example, in the one-object ascending price auction with secret drop-out, bidders observe the current level of price, but not how many competitors are left.² We show that if in the static auction bidders prefer not to acquire extra information, the ascending price auction with secret drop out does not provide bidders with incentives to acquire information either.³ Thus, it is not merely the dynamic nature of the format that is key for our insight, but its property of conveying (for free) an estimate of the intensity of competition (through the information about the number of competitors left).

A similar comment holds for multi-objects auctions. Some formats used in practice leave open the possibility for bidders to switch from one auction to the other. This feature is often viewed as desirable because otherwise bidders who, say, only wish to acquire one object would be forced at an early stage to make up their mind about which object to bid on, and would possibly regret their choice later on. Our analysis however unveils a possible drawback of such formats when information is costly to acquire: the possibility that bidders switch from one object to the other makes the information concerning the numbers of serious bidders left in each auction less reliable. Hence such formats are not as good at guiding bidders as to which object to acquire information on.

Our work is related to that on research contests (Fullerton and McAfee (1998), and more recently Che and Gale (2001)). In research contests in which the winner gets a fixed prize, incentives to invest can be greater when there are fewer competitors. For the sponsor of a research contest (who values high investment in research), there are two ways to affect contestants' incentives to do research: either by reducing competition, or by increasing the

²Rezende (2001) studies this auction format in a symmetric setting.

³When drop-outs are secret, bidders only learn that the highest valuation still lies above the current price, which is bad news: bidders' incentives to acquire information do not increase as the auction progresses. In contrast, learning that the number of competitors left is small could increase dramatically the incentive to acquire information.

prize. Fullerton and McAfee identify situations in which it is better for the sponsor to reduce competition to just two competitors. In these situations, screening the best contestants is a major concern for the sponsor. Despite some resemblance with our problem (incentives to acquire information can be larger when there are fewer competitors, and the seller is concerned about screening the highest valuation buyers), the positive role of ascending formats does not appear in Fullerton McAfee (who discuss the merits of all-pay auctions). We will discuss further the difference between the two papers in Section 5.

Other related literature:

Our paper is related to various strands of literature in auction theory: the comparison of auction formats (and more precisely here the comparison of the second price and ascending price auction formats), the analysis of information acquisition in auctions and the literature on entry in auctions. To the best of our knowledge, our paper is the first attempt to analyze the issue of information acquisition in the ascending auction format.⁴ Our paper also provides a new explanation as to why the ascending format may generate higher expected revenue.

The non-equivalence between the second price auction and the ascending price auction has been noted in other contexts. In affiliated value settings (see Milgrom-Weber 1982), the two formats differ because the information on others' signals conveyed in equilibrium differ, hence the bidders' assessment of their valuation differ too. Milgrom-Weber (1982) consider a symmetric setup and show how, in the affiliated value paradigm, the ascending format may generate higher expected revenue.

In the context of auctions with negative externalities (see Jehiel-Moldovanu 1996), Das Varma (1999) has shown that the ascending format could (under some conditions) generate higher revenues than the sealed-bid (second-price) auction format (in the ascending format a bidder may be willing to stay longer, so as to combat a harmful competitor if he happens to be the remaining bidder).

In contrast to our work, the literature on information acquisition in auctions has focused on sealed-bid types of auction mechanisms, and it has essentially examined efficiency issues.

In a private value model, Hausch and Li (1991) show that first price and second price

⁴Rezende (2001) is an independent contribution that focuses solely on the ascending price auction with secret drop out.

auctions are equivalent in a symmetric setting (see also Tan 1992).⁵ Stegeman (1996) shows that second price auction induces an ex ante efficient information acquisition in the single unit independent private values case (see also Bergemann and Valimaki 2000). However, in Compte and Jehiel (2000), it is shown that the ascending price auction may induce an even greater level of expected welfare.

Models of information acquisition in interdependent value contexts (in static mechanisms) include Milgrom (1981) who studies second-price auctions, Matthews (1977), (1984) who studies first-price auctions and analyzes in a pure common value context whether the value of the winning bid converges to the true value of the object as the number of bidders gets large,⁶ Persico (1999) who compares incentives for information acquisition in the first price and second price auctions in the affiliated value setting, and Bergemann and Valimaki (2000) who investigate, in a general interdependent value context, the impact of ex post efficiency on the ex ante incentives for information acquisition.

Models with endogenous entry in auctions include McAfee-McMillan (1987), Harstad (1990) and Levin-Smith (1994).⁷ All these models assume the entry decision is made prior to the auction at a stage where bidders do not know their valuation. These models thus combine the idea of participation costs and the idea of information acquisition, as the decision to enter both allows the bidder to participate to the auction and to learn her valuation. This should be contrasted with our model in which there is no participation costs but only a cost to acquire information on the valuation.

⁵Engelbrecht-Wiggans (2001) also compares first price and second price auctions, but he examines the case where bidders acquire information on the number of competitors (rather than on their own valuation). These two formats are then not equivalent, since knowing the number of bidders is valuable in the first price auction only.

⁶See also Hausch and Li 1993 for an analysis of information acquisition in common value settings.

⁷These papers analyze the effect of entry fees or reserve prices on the seller's revenue. McAfee-McMillan (1987) show that in constrast with the case where the number of participants is given exogenously, the optimal reserve price may be zero (this insight is related to that of Bulow-Klemperer (1996) about the positive role of competition in symmetric setups). Levin-Smith (1994) (see also Harstad 1990) further analyze this issue by considering (symmetric) equilibria with possibly stochastic participation. They find that restricting the number of participants to equate the socially optimal number of bidders eliminates the coordination problem that would arise otherwise (if the number of bidders is larger than the socially optimal one, stochastic participation cannot be avoided and may result in no participation).

The rest of the paper is organized as follows. Section 2 describes the basic model, in which there is one object is for sale and only one bidder may acquire information. Section 3 extends our model to the case of multi-object auction. In Section 4, we analyze the case where more than one bidder may acquire information. Further discussion of our model appears in Section 5.

2 The Basic Model

In the basic model, there is one object is for sale, and among the potential buyers, one of them can acquire extra information on how much the object is worth to him. We will later on amend the basic model in two ways: we will examine a multi-object extension, and also an extension in which several buyers can acquire extra information.

One advantage of the basic model is that it allows for an easy characterization of the equilibrium behaviors, in particular regarding the information acquisition decision. When several buyers may acquire information, the decision to acquire information is affected by others' decisions to acquire information. This may result in other subtle strategic effects. But, our main insights will carry over to this case.

Formally, we consider n potential risk-neutral bidders indexed by $i \in N = \{1, ..., n\}$. Each bidder i = 1, ..., n has an expected valuation θ_i for the object. Even though θ_i need not exhaust all possible information on i's valuation, we assume that bidder i cannot acquire further information so that θ_i can be referred to as bidder i's valuation.

The valuations θ_i are assumed to be drawn from independent distributions. For simplicity, and in order to highlight comparative statics with respect to the state of competition through the number n of bidders, we consider the symmetric case in which the valuations θ_i , i = 1, ..., n are drawn from the same distribution. We will assume that this distribution has a density g with full support on $\left[\underline{\theta}, \overline{\theta}\right]$. It will be convenient to denote by $\theta^{(1)}, \theta^{(2)}, \theta^{(k)}$ the highest valuation, the second highest valuation, and the k^{th} highest valuation, respectively.

In addition to bidders i=1,...n, there is one extra risk-neutral bidder a who, in contrast with bidders i=1,...n, can acquire information about his valuation at some positive cost c. That is, bidder a has a valuation ω_a for the object, assumed to be drawn (independently from other bidders' valuations) from a distribution with density f and full support on $[\underline{\omega}_a, \overline{\omega}_a]$.

When he does acquire information, he learns the realization ω_a . Otherwise, his expected valuation from acquiring the object is:⁸

$$v_a \equiv \int \omega_a f(\omega_a) d\omega_a \tag{1}$$

Finally, the information structure is assumed to be common knowledge among all bidders.

Remark: The interpretation behind the definition of v_a (expression (1)) is that in case he wins, bidder a will learn the realization ω_a at no cost. Hence the only motive for spending resources on acquiring information is in checking that it is worthwhile to acquire the object. In some contexts (for example when bidders compete to acquire a firm), the resources that a bidder spends on acquiring information are best thought of as an investment that will have to be made anyway, in case that bidder wins.⁹ Such settings would be better captured by assuming that the expected valuation from acquiring the asset is equal to

$$\int \omega_a f(\omega_a) d\omega_a - c. \tag{2}$$

Our analysis would extend in a straightforward way to this alternative formulation, by equating v_a with the expression above; and our main result that the ascending format generates more revenues would even be stronger in that case. We will however stick to the previous formulation in which v_a is as shown in (1).

Auction formats:

Throughout the analysis, we will be mostly interested in the comparison between static and dynamic auction formats. In the basic model, we will compare the sealed bid second-price auction, the ascending price auction and the ascending price auction with secret drop out, in which bidders do not observe if and when bidders drop out until the auction ends (see below for the formal definition).

In the sealed bid second-price auction, bidder a decides prior to the auction whether or not to acquire information. Then, each bidder submits a bid. The object is allocated to the

⁸In Compte and Jehiel (2000), we analyze the more general case in which bidder a gets an imperfect signal about ω_a prior to the auction. No new insight is gained by doing so however.

⁹Such investments for example include the resources spent to assess the synergies with the assets already owned.

bidder whose bid is highest at a price equal to the second highest bid.¹⁰

In the ascending price auction,¹¹ the price starts at 0, at which each bidder is present. The price gradually increases. Bidder a is given one opportunity to stop the auction process and learn his valuation. This option may be exercised by bidder a at any moment. The auction process resumes after a lapse of time that is sufficient for bidder a to learn his valuation. In addition, at any moment where bidder a is not exercising his information acquisition option, any bidder i or a may decide to drop out of the auction. Bidders are assumed to observe when a bidder drops out or stops the auction. The auction ends when there is only one bidder left.¹² The object is allocated to that bidder at the current price.

Finally, the ascending price auction with secret drop-out is identical to the ascending price auction, except for the fact that bidders do not observe whether or not other bidders drop out until the auction gets to a complete end (i.e. until there is only one bidder left).

In each of these formats, bidder i = 1, ...n has a dominant strategy: bid θ_i in the secondprice auction, drop out at her valuation θ_i in the ascending formats. Similarly, (i) once he has acquired information, bidder a has a dominant strategy: bid ω_a in the second price auction, and drop out at ω_a (or immediately if the current price is above ω_a) in the ascending format; and (ii) conditional on never acquiring information, bidder a has a dominant strategy: bid v_a in the second price auction, or drop out at v_a in the ascending format.

Therefore, the only remaining question is if and when bidder a decides to acquire information. Before going into further details, we introduce functions that will be particularly useful.

It will be convenient to denote by h(p) the (ex ante) expected payoff bidder a would obtain if he were to learn (for free) his valuation ω_a and were offered to buy at price p:

$$h(p) = \Pr{\{\omega_a > p\}} E[\omega_a - p \mid \omega_a > p].$$

We also denote by H(k,p) the expected payoff (over ω_a) that bidder a obtains in the ascending

 $^{^{10}}$ In case of ties, each one of the highest bidder gets the object with equal probability.

¹¹We present here the continuous time/price version of the ascending price auction. This raises some technical difficulties regarding the definition of equilibria in undominated strategies. The equilibria we will refer to are the limits as $\varepsilon > 0$ tends to 0 of the equilibria in undominated strategies of the corresponding game in which time is discrete and after each round the price increases by the increment ε .

¹²In case all the remaining bidders quit at the same date, one of them is selected at random with equal probability to get the object. He then pays the current price.

format when (i) the current price is p, (ii) bidder a is still active, learns for free his valuation ω_a , and drops out at ω_a (or immediately in case $p > \omega_a$), (iii) there are still k other active bidders.

Since bidder a gets the object when $\omega_a > \theta^{(1)}$, and then pays $\theta^{(1)}$ for it, we have:

$$H(k, p) = E[h(\theta^{(1)}) \mid \theta^{(k)} > p > \theta^{(k+1)}]$$

As can be checked easily, the functions h and H are decreasing in their arguments.¹³

2.1 A preliminary comparison

We first wish to exhibit simple conditions under which bidder a would acquire information with positive probability in the ascending format, whereas he would not in the sealed bid format, illustrating the fact that the second-price auction provides poorer incentives for information acquisition. The derivation of these conditions will also illustrate a key feature of the ascending format, namely that it gives bidder a an option to postpone information acquisition until there are only few bidders left.

When he acquires information prior to the (second price) auction, bidder a's expected payoff is equal to

$$H(n,0) - c$$

So if

$$H(n,0) < c, (3)$$

for example because n is not small (or because c is significant), bidder a does not acquire information in the second price auction.

$$H(1,p) > Q^{(1)}h(p') + (1 - Q^{(1)})H(1,p') > H(1,p'),$$

and, by induction on k,

$$H(k,p) = EH(k-1,\theta^{(k)}) \mid \theta^{(k)} > p > \theta^{(k+1)}) < H(k-1,p)$$

$$H(k,p) \ge Q^{(k)}H(k-1,p') + (1-Q^{(k)})H(k,p') > H(k,p')$$

To see why observe that $h'(p) = -\Pr\{\omega_a > p\} < 0$. Now choose p' > p and let $Q^{(k)} = \Pr\{\theta^{(k)} \in (p, p') \mid \theta^{(k)} > p > \theta^{(k+1)}\}$. We have

Assume now that bidder a never acquires information in the ascending format. Then in case the price reaches v_a (which occurs whenever $\theta^{(1)} > v_a$), bidder a drops out and thus gets a profit equal to 0.

In the event where

$$\theta^{(2)} < v_a < \theta^{(1)}$$

however, there is only one bidder (other than a) who remains active at price v_a . Compared to the situation prior to the auction where bidder a had n potential competitors, bidder a has much higher chances of winning, and may therefore be more inclined to acquire information. Indeed, by acquiring information at v_a , bidder a would obtain an expected payoff equal to

$$H(1, v_a) - c,$$

which may be positive even when (3) holds (see below). Besides, if $\underline{\theta} < v_a < \overline{\theta}$, the event $\{\theta^{(2)} < v_a < \theta^{(1)}\}$ has positive probability. So we have proved the following Proposition:

Proposition 1 Assume that $H(n,0) < c < H(1,v_a)$ and $\underline{\theta} < v_a < \overline{\theta}$. Then bidder a does not acquire information in the second price auction, whereas he acquires information with positive probability in the ascending format.

To fix ideas about when the conditions of Proposition 1 hold, assume that the distributions of θ_i and ω_a have the same support $[\underline{\theta}, \overline{\theta}]$. Then when the number of bidders increases, chances of winning get arbitrarily small when the number of bidders increase, hence H(n,0) gets arbitrarily small. Thus, as soon as the number of bidders is not too small, the conditions of Proposition 1 hold for a significant range of costs. When θ_i and ω_a are drawn from uniform distributions with same support, the minimum number of bidders required for the conditions of Proposition 1 to hold for some range of costs c is 4.

2.2 Equilibrium behavior

We now derive in more detail the equilibrium behavior, and in particular the information acquisition strategy of bidder a in the ascending price auction. Throughout this Section, we assume that the conditions of Proposition 1 hold (and in particular, $H(1, v_a) > c$), which ensures that bidder a acquires information with positive probability in equilibrium.

As mentioned before, a key feature of the ascending price auction is that it gives bidder a an option to postpone the decision to acquire information. It is this feature that explains why bidder a never acquires information when there is more than one other bidder left.

We first define:

Definition 1 Let p^* be the price such that when there is only one other bidder left, bidder a is indifferent between dropping out and acquiring information at p^* .

By definition, the price p^* satisfies

$$H(1, p^*) = c, (4)$$

which implies, since $H(1, v_a) > c$ and since $H(\cdot, \cdot)$ is a decreasing function of p,

$$p^* > v_a$$
.

When the current price is above p^* and bidder a has not acquired information yet, it cannot be optimal for bidder a to acquire information,¹⁴ and since $p^* > v_a$, it is then optimal for bidder a to drop out.

Consider now a current price p below p^* . Assume that there are still k other active bidders, with k > 1, and that bidder a has not acquired information yet. We wish to compare the payoff obtained by bidder a when he acquires information, with the payoff he would obtain by:

- (i) dropping out at p^* without acquiring information in case there is still more than one other active bidder (at price p^*),
 - (ii) and otherwise acquiring information as soon as there is only one other bidder left.

Under the second event (which occurs when $\theta^{(2)} < p^*$), the two strategies yield the same payoff. Under the first however,¹⁵ the first strategy (of immediate information acquisition at p) yields at most¹⁶ $H(2, p^*) - c < 0$, while the second yields 0.

¹⁴This follows again from the monotonicity of $H(\cdot, \cdot)$ with respect to k and p implying that for all $k \ge 1$ and $p > p^*$, $H(k, p) < H(1, p^*) = c$.

¹⁵This event has positive probability because otherwise, $\theta^{(2)}$ would always be smaller than p^* so that we would have $H(n,0) = EH(1,\theta^{(2)}) > c$, contradicting the premise that H(n,0) < c.

¹⁶by the monotonicity of H w.r.t k and the definition of p^* .

So the first strategy (acquiring information at p) cannot be optimal. Besides, since the second strategy yields a strictly positive payoff,¹⁷ dropping out at p cannot be optimal either.

We have thus proved:

Proposition 2 In equilibrium, as long as there is more than one other bidder left, bidder a does not acquire information and he remains active until the price reaches p^* .

To complete the description of equilibrium behavior, we need to examine the events where there is only one other bidder left.

Consider a subgame where there remains only one other active bidder, say bidder 1, where bidder a is still active and has not acquired information yet, and where the current price is p.

Will bidder a choose to acquire information immediately or will be postpone his decision and until when?

To answer this question, we first define:

Definition 2 Let p^{**} be the price for which if bidder a were given the option to buy at p^{**} , he would be indifferent between acquiring information and not acquiring information.

The price p^{**} is uniquely defined by 18

$$h(p^{**}) - c = v_a - p^{**}, (5)$$

and satisfies

$$p^{**} < v_a$$
.

Besides, for prices above p^{**} , bidder a would rather acquire information, while for prices below p^{**} , he would not.¹⁹

¹⁷This is because $\underline{\theta} < v_a < p^*$ (hence the event $\theta^{(2)} < p^*$ has positive probability), and because H(1,x) > c for $x < p^*$.

¹⁸ Indeed, it is readily verified that the function h(p) + p is strictly increasing in p on $[\underline{\omega}_a, \bar{\omega}_a]$. Besides, for $p = \underline{\omega}_a$, $h(p) + p = v_a$, and for $p = v_a$, our assumption that $H(1, v_a) > c$ implies (since $h(v_a) > H(1, v_a)$) that $h(p) + p > c + v_a$, thus guaranteeing that p^{**} is uniquely defined and satisfies $p^{**} < v_a$.

¹⁹Under the alternative formulation where v_a is set equal to $Ew_a - c$, for any price $p > \underline{\omega}_a$, bidder a always prefers to acquire information. So we would set $p^{**} = \underline{\omega}_a$.

Assume that bidder a postpones information acquisition until the price reaches $p + \varepsilon$ (rather than acquiring information at p), where ε should be thought of as a small increment. Under the event where bidder 1 does not drop out before $p + \varepsilon$, bidder a obtains the same expected payoff. Under the event where bidder 1 drops out before $p + \varepsilon$, bidder a 's expected payoff is equal to

$$E[h(\theta_1) - c \mid p < \theta_1 < p + \varepsilon]$$

if he acquires information at p, while it is equal to

$$v_a - E[\theta_1 \mid p < \theta_1 < p + \varepsilon]$$

if he had planned to acquire information at $p + \varepsilon$.

Since ε can be chosen arbitrarily small, we get that when $p > p^{**}$ the former option (immediate acquisition) is preferable, and that when $p < p^{**}$, the later option (delayed acquisition) is preferable.

This concludes the derivation of equilibrium behavior:

Proposition 3 When bidder a has not acquired information yet and there is only one other active bidder, bidder a acquires information if only if the current price lies between p^{**} and p^{**} are defined by (4) and (5), respectively.

Based on Propositions 1-2-3, we can now make precise under what events information acquisition occurs in the ascending-price auction.

Proposition 4 Assume that $H(n,0) < c < H(1,v_a)$. Then, in the ascending price auction, bidder a acquires information whenever $\theta^{(1)} > p^{**}$ and $\theta^{(2)} < p^{*}$.

2.3 Ascending-price auction with secret drop-out

An important feature of the ascending price auction is that in the course of the auction, bidder a learns about the number of bidders who are still interested in the object. To illustrate why this feature of the ascending format is important, we now briefly examine the ascending price auction with secret drop out in which bidders no longer observe whether and when other bidders drop out until the auction gets to a complete end. Bidder a still has the option to

postpone information acquisition, but in general, this option is not as valuable, as we now show.

Indeed, for any current price p (at which the auction is still going on), bidder a now only learns that $\theta^{(1)}$ is larger than p. Hence the expected payoff that bidder a would obtain by acquiring information is

$$E[h(\theta^{(1)}) \mid \theta^{(1)} > p] - c$$

This expression coincides with H(n,0) - c at p = 0, and since h is decreasing in p, it is also decreasing in p. So, if acquiring information yields a negative profit at 0, it also yields a negative profit at any other price. Intuitively, learning that $\theta^{(1)}$ is larger than p is bad news for bidder a, hence this decreases his expected payoff from information acquisition. To summarize:

Proposition 5 If H(n,0) < c, then bidder a does not acquire information in the secondprice auction. Nor does he acquire information in the ascending price auction with secret drop out.

Other variants of the ascending price auction include the possibility (see Harstad-Rothkopf 2000) that bidders might re-enter after dropping out.²⁰ In such formats, the observation that there are few bidders around may not be as reliable as in the format we analyzed in section 2.2.²¹ Yet, if in equilibrium this number is sufficiently informative that competition is not too tough, then the ascending format is likely to generate more information acquisition (under the conditions of Proposition 4).

2.4 Revenues

What is the effect of information acquisition on the seller's revenues? In general, the effect of information acquisition on revenues is ambiguous.

When $\theta^{(1)} < v_a$ for example, the seller would prefer that bidder a does not acquire information. Without information acquisition the selling price is $\theta^{(1)}$, while with information acquisition, the selling price never exceeds $\theta^{(1)}$ and sometimes gets below $\theta^{(1)}$ (when bidder a learns that $\omega_a < \theta^{(1)}$).

²⁰ Alternatively, bidders may sometimes have the possibility to hide that they are still around.

²¹This would be the case if in equilibrium bidders sometimes use the option to drop out and re-enter.

In contrast, when $\theta^{(2)} > v_a$, the seller would prefer that bidder a acquires information. Without information acquisition the selling price is $\theta^{(2)}$, while with acquisition, the selling price never gets below $\theta^{(2)}$ and sometimes exceeds $\theta^{(2)}$ (when bidder a learns that $\omega_a > \theta^{(2)}$).

When the number of bidders is not too small, the event $\{p^* > \theta^{(2)} > v_a\}$ is much more likely than the event $\{\theta^{(2)} < v_a\}$, and as a consequence, the second effect dominates:

Proposition 6 When the number of bidders is not too small, the ascending price auction generates more revenues than the sealed-bid second price auction.

To illustrate, we quantify this effect in a simple numerical example.

2.5 A numerical example

We assume that θ_i and ω_a are drawn independently from the uniform distribution on [0, 1]. We choose a cost of information acquisition equal to c = 0.025. For this cost, the conditions of Proposition 1 are satisfied as soon as there are at least 5 bidders (in addition to bidder a). To fix ideas, we first give, as a function of the number of bidders, the probability that bidder a acquires information in the ascending price auction.

$$n$$
 5 6 7 8 9
Pr{bidder a acquires information} 0.36 0.25 0.16 0.12 0.08

Information acquisition is therefore significant, at least when the number of bidders is not too large. Bidder a does not acquire information with probability one however, essentially because there are events where $\theta^{(2)} > p^*$, in which case bidder a prefers to drop out without acquiring information: competition is too tough.²²

The next table gives, as a function of the number of bidders, the percentage of increase in revenues (denoted $\bar{\Delta}$) that would be generated if bidder a were to acquire information with probability 1. These numbers give an idea of the range of increase in revenues that we can expect, at best.

²²In our numerical example, p^* is equal to 0.6.

Finally, we give, as a function of the number of bidders, the percentage of increase in revenues (denoted Δ) generated by the ascending price auction (as compared to the second price auction). Because bidder a does not acquire information with probability one however, these numbers do not match the maximum percentage of increase $\bar{\Delta}$. Still, compared to $\bar{\Delta}$, the percentage of increase Δ is significant, as the following table shows.

n 5 6 7 8 9
$$\Delta = \frac{R^{ascending} - R^{second price}}{R^{second price}}$$
 2.5% 1.9% 1.3% 0.9% 0.6% $\Delta/\overline{\Delta}$ 57% 50% 42% 35% 28%

Note that $\Delta/\overline{\Delta}$ is significant even though the probabilities of information acquisition are not very large. The reason is that bidder a acquires information precisely in those events where $\theta^{(2)}$ is not very large, hence in events where information acquisition is most likely to have a big impact on revenues.

We shall return to this numerical example after examining the case where more than one bidder can acquire information, in which case the increase in revenues generated by the ascending price auction is even larger.

3 Multi-object auctions

Does our basic insight carry over to settings where there is more than one object for sale? Intuitively, our insight should be reinforced: if the ascending format reveals information about the number of bidders interested in each object, not only should this help bidder a to decide whether or not to acquire information, but it should also help him in his decision as to which object to acquire information on.

In order to illustrate this effect, we modify our basic model to allow for two objects A and B. One could think of these two objects as licenses to operate a public service in two distinct areas. We will assume that a buyer is not allowed to buy both licenses.

There are n potential risk neutral bidders, i=1,...,n, only interested in object A. These bidders are referred to as type A bidders. And there are also n potential risk neutral bidders j=1,...,n, only interested in object B. These bidders are referred to as type B bidders. In the spirit of the basic model, we assume that each bidder i has a valuation θ_i^A for object A, and each bidder j has a valuation θ_j^B for object B. All θ_i^A , θ_j^B are assumed to be drawn

from independent and identical distributions. We will denote by $\theta_A^{(k)}$ (respectively. $\theta_B^{(k)}$) the k- highest θ_i^A (respectively. θ_i^B).

There is one additional bidder, bidder a, interested in either one but only one of the two objects (his marginal valuation for the second object is equal to 0). Bidder a's valuations for objects A and B are denoted by ω_a^A and ω_a^B , respectively where ω_a^A and ω_a^B are assumed to be drawn from independent and identical distributions. So ex ante bidder a values the objects identically, and this expected valuation is denoted by v_a .

Bidder a is given the option to acquire information on one object of his choice (A or B), and learn, for that object, the true valuation.

Because of the multi-object character of the problem, several ascending formats might be considered. For simplicity, but also to highlight the effect on information acquisition, we consider the following ascending format.

The simultaneous ascending price format

The prices for each object simultaneously increase at the same rate, until for one of the objects, say A, only one bidder, say i, remains active. This object A is then allocated at the current price to bidder i. Then the auction for object B resumes, and the price rises until only one bidder is left, as for the one object case. Bidders may a priori be active on both objects. But, once they drop out from one auction, they can no longer be active on that auction.²³ We also assume that once a bidder wins an auction, he is required to drop out of the other auction (because a bidder is not allowed to buy both licenses).

In addition, throughout the auction, bidder a may stop the process once and then decide to learn his valuation for one of the objects at cost c. When a bidder drops out of an auction, this is observed by other bidders.

We wish to compare this ascending format with its sealed-bid counterpart.

The sealed-bid format

In the sealed-bid format, two second price auctions (one for each object) are held at the same time (with the request that each bidder can only bid on one of the auctions). Bidder

²³This assumption may induce undesirable inefficiencies properties, but this is not the subject *per se* of this paper.

a decides (ex ante) whether or not to acquire information and on which object to acquire information on. Each bidder then decides on which auction to participate, and submits a bid in that auction. The objects are allocated to the highest bidders in each auction, at a price equal to the second highest bid (in that auction).

Comparing incentives to acquire information

We will show that bidder a has greater incentives to acquire information in the ascending format than in the sealed bid format. In our arguments, we will restrict attention to equilibria in which each bidder i (respectively j) is active in auction A (respectively B) only and remains active until the price in that auction reaches his valuation θ_i^A (respectively θ_i^B).

Note first that if bidder a had to decide prior to the auction on which auction to participate, then the analysis would be mostly identical to the single object case: once the participation decision is made, incentives to acquire information are identical to those discussed in the basic model. What makes the analysis different from the previous one is that both the decision on information acquisition and the decision as to which auction to drop out from can be postponed in the ascending format (while they have to be made immediately in the sealed bid format).

At any moment of the simultaneous ascending price auctions, bidder a observes the number n_A of type A bidders who are still active in auction A, and the number n_B of type B bidders who are still active in auction B. As in the basic model, we define the function $H^A(n_A, n_B, p)$ as the expected equilibrium payoff of bidder a (over the realization ω_a^A) when (i) he is still active at p in both auctions, (ii) he learns for free his valuation ω_a^A for object A, and (iii) there are n_A type A bidders active in auction A, and n_B type B bidders active in auction B. The function $H^B(n_A, n_B, p)$ is defined similarly. Note that when $p \geq v_a$, $H^A(1, n_B, p)$ coincides with H(1, p) where the function H corresponds to the function introduced in the basic model (in which the sole object for sale is A and the sole bidders are i and a).²⁴

By construction, the expected payoff from acquiring information on A in the sealed bid

The reason is that once $p > v_a$ and bidder a acquires information on object A (hence not on object B), it is optimal for bidder a to drop out from auction B (as his expected valuation of B, v_a , lies below the current price).

format is $H^A(0,n,n)-c$ (which by symmetry is also $H^B(n,n,0)-c$). Hence if

$$H^A(n, n, 0) < c,$$

it cannot be optimal for bidder a to acquire information (neither on A nor on B) in the sealed-bid format.

Consider now the ascending format. Suppose that bidder a never acquires information. In the event where

$$\theta_A^{(1)} > v_a > \theta_A^{(2)} \text{ and } \theta_B^{(1)} > v_a,$$

the price reaches v_a in both auctions. At this price, bidder a should drop out of both actions. However, if

$$H(1, v_a) > c,$$

bidder a strictly prefers to acquire information on object A and then drop out as soon as the current price exceeds or equals ω_a^A .

Assume that the condition $H(1, v_a) > c$ holds, and define p^* as in the basic model. Also assume that at the current price p, bidder a has not yet acquired information. As in the basic model, it is easy to check that whenever

$$v_a \le p < p^*, n_A > 1$$
, and $n_B > 1$,

dropping out at the current price, or acquiring information immediately are strategies that are dominated by the strategy that consists in remaining active until the price reaches p^* at most (as in the basic model) and acquiring information as soon as the number of competitors for a particular object drops down to one.

It follows that in any event where $\theta_A^{(2)} \in (v_a, p^*)$ or $\theta_B^{(2)} \in (v_a, p^*)$, either bidder a has acquired information before v_a , or the conditions above hold, implying that bidder a acquires information at $\min(\theta_A^{(2)}, \theta_B^{(2)})$.²⁵ We thus have:

Proposition 7 Suppose that $H^A(n, n, 0) < c < H(1, v_a)$. Bidder a never acquires information in the sealed bid format and he acquires information in the ascending format whenever $\theta_A^{(2)} \in (v_a, p^*)$ or $\theta_B^{(2)} \in (v_a, p^*)$.

 $[\]theta_A^{(2)} \in (v_a, p^*)$ or $\theta_B^{(2)} \in (v_a, p^*)$.

25 In our symmetric setting, as soon as, say, $n_A = 1$, it cannot be optimal to further delay information acquisition. In the case of asymmetric distributions, it might be worthwile for bidder a to delay a bit further information acquisition, in the hope that competition will shrink on auction B too.

Compared to the analysis of the basic model, the incentive to acquire information is reinforced in the ascending format because it suffices now that either $\theta_A^{(2)}$ or $\theta_B^{(2)}$ lies in (v_a, p^*) to justify the information acquisition.

Our argument also highlights a key advantage of the ascending format: competition is softer on the object for which the number of competitors drops down to one earlier, and bidder a can wait and see on which object competition is softer before choosing the object on which to acquire information.

4 Information acquisition by multiple bidders

We have focused on the case where only one bidder could acquire information. What if more than one bidder can acquire information?

One extra difficulty is that the incentives of a bidder to acquire information may depend on the other bidders' decisions to acquire information. Most of the conclusions of Section 2 however would not be affected.

To fix ideas suppose that in addition to the n bidders who cannot acquire further information, there are now two bidders, bidders a and b, who each can acquire information and learn their true valuations ω_a and ω_b , respectively. For simplicity, we assume that θ_i , i = 1, ..., n, ω_a and ω_b are drawn from the same distributions. Expected valuations for bidders a and b are thus identical, and denoted by v. In the ascending format, we assume that each bidder a and b has one opportunity to stop the auction, leaving him some time to acquire information if he is willing to.

As in the basic model, we define the function H(k, p) for each bidder a or b assuming the other bidder b or a is not present, and we assume that

In the sealed bid auction, then, as before, neither bidder a nor b has an incentive to acquire information.

In the ascending format, bidders still have the option to wait until there is only one other bidder left to acquire information, but this option cannot be as good as before for both bidders a and b. In the event where bidders a and b are the two remaining active bidders, both would

simultaneously acquire information, and they would both be willing to drop out in the event where both realizations ω_a and ω_b are below the current price. One bidder would be selected however, and end up buying the object at a price above his current valuation.

The consequence is that at least one of the two bidders, say b, will have to adopt a different strategy, and either drop out without acquiring information before the price p^* is reached (see (3) in Section 2), or acquire information earlier. Note that in case bidder b adopts the latter strategy, then for bidder a, the situation becomes identical to that of the basic model, and we may therefore expect that in some events, both bidders will actually acquire information.

To make the formal analysis a bit simpler, and avoid the possibility of mixed strategy equilibria (with bidders mixing on when to acquire information), we will assume that prior to the auctions, bidders are ranked, and that whenever all remaining active bidders drop out simultaneously, the object is allocated at the current price to the bidder with lowest rank (among those who remain active). Without loss of generality, we shall assume that bidder b has a rank lower than that of bidder a.

Because a is ranked higher than b, bidder a is never picked in case of ties with bidder b, so for bidder a, the option to wait until there is only one other bidder left (or until $p = p^*$) is as good as in the basic model, and it dominates both, dropping out and immediate information acquisition.

We wish to check whether bidder b has incentives to acquire information. Let us denote by $\widetilde{H}(p)$ the expected gain of bidder b when (i) the current price is p, (ii) bidders a, b and one other bidder, say 1, are still active, (iii) bidder b acquires information (for free), and remains active as long as the current price remains below ω_b , (iv) bidder a follows the strategy described in the basic model.

We choose $p < p^*$ and compare $\tilde{H}(p)$ with the cost c. We distinguish three events.

- (1) In the event $\{\theta_1 > p^*\}$, bidder a does not acquire information and drops at p^* , hence bidder b's expected payoff (under that event) is equal to $H(1,p^*)$, which is equal to c, by definition of p^* .
- (2) In the event $\{\theta_1 < p^*, \omega_a < \theta_1\}$, bidder b obtains the object at price θ_1 if and only if $\omega_b > \theta_1$. Hence his expected payoff under that event, and for a particular realization θ_1 , is equal to $h(\theta_1)$, which is larger than c since $h(\theta_1) > H(1, \theta_1)$ and $\theta_1 < p^*$.
 - (3) In the event $\{\theta_1 < p^*, \omega^a > \theta_1\}$, bidder b obtains the object at price ω_a if and only if

 $\omega_b > \omega_a$, hence, his expected payoff under that event, and for a particular realization θ_1 , is equal to $E[h(\omega_a) \mid \omega_a > \theta_1]$. Given our assumption that ω_a and θ_i are drawn from the same distribution, this payoff coincides with $H(1, \theta_1)$ which is also larger than c since $\theta_1 < p^*$.

We thus conclude that for $p < p^*$,

$$\widetilde{H}(p) > c$$
,

which implies that as soon as the number of active bidders (other than a and b) is equal to 1, and the current price is below p^* , bidder b prefers to acquire information rather than dropping out.

Could it be that bidder b prefers to postpone information acquisition?

By an argument similar to that in the basic model, it can be checked that if $p > p^{**}$ (where p^{**} is defined as in Section 2), bidder b does not want to postpone this decision.²⁶ We have thus proved:

Proposition 8 Assume H(1,v) > c > H(n,0). In the ascending price format, with b ranked below a, in any event where $\{\theta^{(1)} > p^{**} \text{ and } \theta^{(2)} < p^*\}$ bidder b acquires information, and in any event where $\{\theta^{(1)} > p^{**}, \theta^{(2)} < p^*, \text{ and } \omega_b < p^*\}$, **both** bidders a and b acquire information.

Note that Proposition 8 assumes that H(1,v) > c. In particular, even if H(2,v) < c, bidder b has incentives to acquire information even though two other bidders (say i and a) are present. The reason is that bidder a will acquire information only in the favorable event where θ_i is not too large.

Generalizing Proposition 8 to the case where there are many bidders, say k, who may acquire information is straightforward. When the current price is above v, a bidder acquire information as soon as there is only one other bidder left among those who cannot acquire further information and he has the lowest rank among those who can still acquire information.

The reason is as follows. If at the current price p, bidder b decides to postpone information acquisition until $p + \varepsilon$, the outcome differs from immediate information acquisition only in events where $\theta_1 \in (p, p + \varepsilon)$ and $\omega_a . As in the basic model, under these events, bidder <math>b$ gets approximately h(p) - c in case of immediate acquisition, and approximately v - p in case of delayed acquisition. (Note that bidder b could also acquire information as soon as bidder 1 drops out, without waiting to see if bidder a remains active or not, however given our ranking assumption, bidder b prefers not to do so.)

Returning to our numerical example with n set equal to 5, c = 0.025 and valuations distributed uniformly on [0,1], we can compute the increase in revenues generated from moving from the second-price format to the ascending format, when k is large. In the second-price format, neither of these k bidders acquire information and the outcome is thus as if these k bidders were replaced by a single one with a valuation equal to v = 0.5. In the ascending format, the outcome is as if, in the event where $\{\theta^{(1)} > p^{**} \text{ and } \theta^{(2)} < p^{*}\}$, these k bidders were replaced by a single one with a valuation uniformly distributed on $[p^{*}, 1]$.

Computations show that the increase in revenue is equal to 10%.

5 Discussion

(i) Two phase-mechanisms

When information takes a long time to acquire, the seller may not afford to have the auction stopped several times. In such settings however, the seller may still manage to implement a two-phase mechanism that outperforms the sealed-bid second price auction, by stopping the auction once there remain only, say n^* , active bidders, and then give these bidders the option to acquire further information: The first phase can be interpreted as a screening phase in which non-serious bidders drop out; this first phase also sets a reserve price for the second phase, and between the first and the second phases 'serious bidders' (the n^* bidders who are still active) are given time to acquire information if they are willing to.

How should the seller set n^* ? In the case of our basic model (where only one bidder can acquire information), it is easy to check that setting $n^* = 2$ is best for the seller (this mechanism actually yields the same outcome as the one derived in Section 2). In more general settings however, it is not clear whether setting n^* equal to 2 would be optimal for the seller. The seller wants n^* small enough to induce information acquisition, yet maybe not too small: the previous section where two bidders a and b could acquire information suggests that if $n^* = 2$ and both bidder a and b end-up being selected, they each run the risk of acquiring the object at a price above their valuation, hence one of them will probably want to drop out earlier without acquiring information. In contrast, if $n^* = 3$, it may be that whenever bidders a and b are selected for the second phase, they both decide to acquire information. A more general analysis of these two-phase mechanisms deserves further research.

(ii) Screening among agents who may acquire information about their types.

>From a more general perspective, we have dealt with a setting in which a principal (the seller) attempts to screen among agents (potential buyers) who may acquire information about their types (valuations), or invest so as to affect their types (as in the alternative formulation – see (2)), prior to the principal-agent interaction (the sale) or the signing of a contract. As in our sale example, the principal may find it desirable to provide agents with incentives to acquire information or invest prior to signing a contract, because this improves the chance of selecting a more able agent; that is, an agent with a better type. In this context, our analysis suggests that a dynamic screening procedure, that leaves the competing agents some time to acquire information about their types, would outperform static screening procedures.²⁷

Another example along these lines is the case of a sponsor who wishes to induce potential contestants (and possibly the ablest one) to exert high research effort. When research outcomes are not measurable or contractible, one option for the sponsor is to organize a tournament in which the winner gets a fixed prize. As mentioned in the introduction, inducing high research effort is sometimes more economically achieved by reducing the number of contestants (rather than increasing the prize). In such settings, it is important for the sponsor to screen among potential contestants so as to induce the participation of the ablest ones only.²⁸

How should one organize screening?

This issue is of primary importance, as failing to screen good contestant may jeopardize the success of the tournament, and Fullerton and McAfee have identified why some screening procedures (that would auction rights to participate in the tournament) could fail to screen

²⁷Cremer-Khalil (1992) consider a principal-agent setup in which the agent can learn his type before signing the contract. Most of their analysis bears on the one-agent case in which no screening is needed (their main insight is that the principal should not induce the agent to learn his type before the contract is signed). In the multi-agent section of their paper, Cremer-Khalil restrict attention to contracts in which the agents do not have incentives to learn their type. However, (unlike in the one-agent case) such contracts need not be optimal.

²⁸When contestants have identical abilities (as in Che-Gale), this can easily be achieved by a random selection fo a subset of contestants. However, when they are not ex ante identical, a finer screening device is required.

properly.²⁹

Though screening is often thought of as a pure adverse selection problem, it seems plausible that some contestants may only have a rough idea of how successful their research effort will be, and that by investing a bit *prior to the tournament*, they could have a much better idea of how able they are. Besides, the sponsor could clearly benefit from such investments, since this should help him select the *truly* ablest contestants. Here again, a dynamic screening procedure would outperform static screening procedures.³⁰

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²⁹They suggest to consider all-pay auctions.

 $^{^{30}}$ To illustrate, and in order to abstract from the issues raised in Fullerton McAfee, assume that the outcome of research is measurable, and that it can be contracted on. Formally, the outcome of research for contestant i is a random variable x_i , and we assume that the sponsor auctions (through an ascending auction) a contract that pays P(x) for the outcome x. The cost of research is assumed to be identical across contestants, and equal to γ . The bidders differ in how likely they are to produce good research, that is, contestant i's distribution over research outcome is $f(\cdot \mid \theta_i)$ where θ_i is interpreted as contestant i's ability. When all bidders are aware of their ability, the auction selects the bidder for which $E[P \mid \theta_i]$ is highest. What if some bidders do not know θ_i precisely, but only have a rough idea of it, and if at some cost c, they could get to know θ_i ? This is precisely the setting we have analyzed, with the conclusion that it is generally worthwhile to design a procedure that allows participants to take some time to acquire information during the process.

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