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A new measure of resilience: An application to the London Underground

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ABSTRACT

The many varied views on resilience indicate that it is an important concept which has significance in many disciplines, from ecology to psychology to risk/disaster management. Therefore, it is important to be able to quantifiably measure the resilience of systems, and thus be able to make decisions on how the resilience of the system can be improved. In this paper we will work with the definition, due to Pimm (1991), that resilience is "how fast a variable that has been displaced from equilibrium returns to it." We will think of a system as being more or less resilient depending on the speed with which a system recovers from disruptive events or shocks. Here we consider systems which revert to an equilibrium state from shocks, and introduce a measure of resilience by providing a quantification of the rapidity of these systems' recovery from shocks.

We use a mean-reverting stochastic model to study the diffusive effects of shocks and we apply this model to the case of the London Underground. As a shock diffuses through the network, the human-flow in the network recovers from the shock. The speed with which the passenger counts return to normal is an indicator of how quickly the line is able to recover from the shock and thereafter resume normal operations.

1. Introduction

The urban world is a connected network of infrastructure systems, and urban communities depend on the functioning of these infrastructures. In times of natural or man-made disasters, it is not just the physical damage to infrastructure that is a cause for concern but also the cascading effect of problems along the network. Transportation systems provide one of the key infrastructure services to urban society. Ensuring that these services are resilient so as to reduce the level of disruption from disasters must be an important economic and social priority. In 2014, the UK government called for a review on transport resilience seeking views on planning, performance and response on the transport network to severe weather events (Department for Transport, 2014). Some of the issues the review highlighted were to identify the level of resilience that must be achieved depending on intensity of demand, the economic rationale for investing in resilience, impact of extreme weather on local, regional and national levels and the resulting cascading network of problems, etc.

A question that arises naturally at this point is 'how can one quantify resilience'? This would, for example, allow the identification of weak points in a transport network. Many authors (Tierney, 1997; Reggiani et al., 2002; Bruneau et al., 2003; Chang and Shinozuka, 2004; Rose, 2007) have studied system resilience. In particular, Rose, 2007, identified two main types of system resilience: static resilience and dynamic resilience. These stem from two alternative paradigms of resilience that

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exist in ecological literature; 'Holling resilience', which focuses on maintaining existence of function, possibly in a new regime from the original one, and 'Pimm resilience', which focuses on maintaining efficiency of function.

Within the transport context, we need to distinguish therefore not only the loss of transport assets but also the output losses, in other words, the disruptions of the services and interruptions of the operations which can have short term and long lasting consequences (Hallegate, 2014). Cox et al. (2011) examine transportation security and propose operational metrics to determine passenger transportation system resilience to terrorism. The approach is based on the comprehensive economic resilience metric proposed by Rose (2009, 2007). They applied this metric to a case study of the London July 2005 subway and bus bombings, measuring static resilience in terms of transportation mode shifts applied to passenger journeys.

In this paper we adopt Pimm's (1991) viewpoint that a system is more or less resilient depending on the rate at which it returns to equilibrium after a disturbance away from equilibrium. We propose a new quantitative measure of resilience using a mean-reverting stochastic model and we show that the model is able to capture the properties of systems with a wide variety of behaviours.

The work is thus structured has followed. Section 2 presents a literature survey of the concept of resilience in many different types of systems. Across many disciplines that examine resilience, from ecology to disaster management, much research is being done to quantify or measure the resilience of systems. We present some of these frameworks and measures of resilience, and discuss the shortcomings of these approaches. In Section 3, we present an innovative and new approach to measure resilience of systems. Our approach expresses the view that systems are continuously subjected to small levels of random shocks and perturbations. Therefore the state of the system exhibits stochastic behaviour. We measure the resilience of the system by the speed with which the state of system returns to historically normal levels after being disrupted by shocks. We use a stochastic mean-reverting model that can capture such behaviour. Assuming that the state of the system can be represented by an observable distribution, we fit the model to this data. The mean-reversion parameter then captures the rate of recovery of the system after being subjected to random shocks. In other words, the mean-reversion parameter measures the resilience of the system, thus providing us with a new, quantifiable, and versatile means of measuring resilience.

We use the London Underground as an example to illustrate the usefulness and versatility of our new measure in Section 4. Passenger counts on underground lines represent the state of the system. The simulations indicate that the model can be used usefully for widely differing behaviours, thus providing a means to measure the resilience of a wide variety of systems. Occasionally the system is subject a large shock that acutely disrupts the state of the system. In Section 5, we incorporate such large perturbations in the model, thereby further illustrating the versatility of the model.

2. Resilience of systems

In this section we examine a few definitions and general measures of resilience. Rose (2009) provides an excellent summary of the literature on resilience. Here we highlight a few of these approaches.

Ecologists originally pioneered functional definitions of resilience involving system stability after a disturbance (Perrings, 1998). We consider two parallel definitions of resilience from the ecological literature. One definition, due to Pimm (1991), is the length of time that a system takes to return to equilibrium following a disturbance. Resilience could be estimated by a return time, the amount of time taken for the displacement to decay to some specified fraction of its initial value. A second definition due to Holling (1973) is a measure of the persistence of systems and of their ability to absorb change and disturbance and still maintain the same relationships between populations or state variables. It emphasises conditions far from any stable steady-state, where instabilities can flip a system into another regime of behaviour. In this case, resilience is measured by the magnitude of disturbance that can be absorbed before the system redefines its structure by changing the variables and processes that control behaviour.

These alternative meanings of resilience reflect different viewpoints about the stable states or equilibrium states of a system. On one hand, Pimm's resilience examines system behaviour near a known stable state and focuses on maintaining efficiency of function. On the other hand, Holling's resilience examines alternative stable states, the properties of boundaries between states, and focuses on maintaining existence of function. The differences in approaches impact the understanding and management of resilience of systems. These two paradigms of resilience are carried forward in the field of economic resilience (Rose and Krausmann, 2013) where the focus is on the flow of goods and services. They offer two definitions of economic resilience, namely: static and dynamic resilience.

Static economic resilience is the ability of a system to maintain function when shocked. It is thus aligned with the fundamental economic problem of efficient allocation of resources. The term static is used because resilience can be attained without repair and reconstruction activities. Dynamic economic resilience is based on the capacity to hasten the recovery from a shock. This refers to the efficient utilisation of resources for repair and reconstruction. It considers the speed at which an entity recovers from severe shock to achieve a desired state. It includes the concept of mathematical or system stability because it implies that the system is able to bounce back to a stable state after a shock.

Rose (2007) also emphasises the distinction between inherent and adaptive resilience within each context. Inherent resilience refers to aspects of resilience included in the system, while adaptive resilience refers to behavioural considerations after the event through ingenuity and extra effort. He also identifies three levels at which resilience can take place. At the microeconomic scale which includes individual behaviour of firms, households or organisations. At the mesoeconomic scale, in this scale we consider the economic sector, individual market, or cooperative group. At the macroeconomic scale, all individual units and markets combined, including interactive effects are examined. Moreover Rose defines economic resilience in flow terms in relation to economic output for a given time using the following mathematical definitions:

Direct static economic resilience (DSER) used at the micro and meso levels-

$$\mathrm{DSER} = \frac{\%\Delta DY^m - \%\Delta DY}{\%\Delta DY^m}$$

where $\%\Delta DY^m$ is the maximum percent change in direct output and $\%\Delta DY$ is the estimated percent change in direct output.

Total static economic resilience (TSER) used at the macro level-

$$\mathrm{TSER} = \frac{\%\Delta TY^m - \%\Delta TY}{\%\Delta TY^m} = \frac{M\%\Delta DY^m - \%\Delta DY}{M\%\Delta DY^m}$$

where M is the economy-wide input–output multiplier (to reflect the conditions in a non-resilient, inflexible economy), $\%\Delta TY^m$ is the maximum percent change in total output and $\%\Delta TY$ is the estimated percent change in total output.

Cox et al. (2011), use Rose's framework to introduce a discussion on transportation system resilience. Static transportation system resilience strategies include: maintaining service with fewer inputs (e.g., railroad cars, employees), shifting input combinations or transportation modes to achieve the same function, changing the site of business activity in terms of travel routes or end-user sites, etc. On the other hand, dynamic transportation system resilience strategies identify the initiatives that can speed recovery and include: removing operating impediments, managing effectively, speeding restoration through options such as alternative means of access to repair sites and incentive contracts. They apply this metric to a case study of the London July 2005 subway and bus bombings, measuring static resilience in terms of transportation mode shifts implemented to passenger journeys.

This approach contrasts nonetheless with the approach taken by Bruneau et al. (2003) in quantifying resilience. They adopt the view of resilience as the ability of the system to reduce the chances of a shock, to absorb a shock if it occurs (abrupt reduction of performance) and to recover quickly after a shock (re-establish normal performance). They argue that resilience has four dimensions:

- Robustness: strength, or the ability of elements, systems, and other units of analysis to withstand a given level of stress or demand without suffering degradation or loss of function.
- Redundancy: the extent to which elements, systems, or other units of analysis exist that are substitutable.
- Resourcefulness: the capacity to identify problems, establish priorities, and mobilise resources when conditions exist that threaten to disrupt some element, system, or other unit of analysis; resourcefulness can be further conceptualised as consisting of the ability to apply material (i.e., monetary, physical, technological, and informational) and human resources to meet established priorities and achieve goals.
- Rapidity: the capacity to meet priorities and achieve goals in a timely manner in order to contain losses and avoid future disruption.

More specifically, they summarise that a resilient system is one that shows the following:

- Reduced failure probabilities.
- Reduced consequences from failures, in terms of lives lost, damage, and negative economic and social consequences.
- Reduced time to recovery (restoration of a specific system or set of systems to their "normal" level of performance).

Bruneau et al. (2003) use an illustration of a Resilience Triangle to illustrate the key features of their definition of resilience (see Fig. 1).

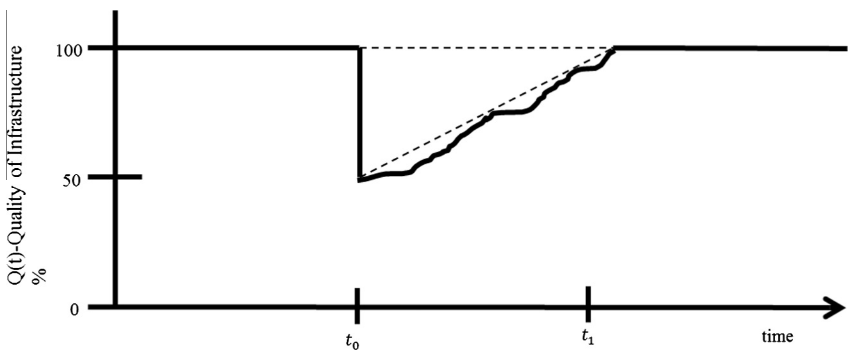

Fig. 1. Resilience triangle (adapted from Bruneau et al. (2003)).

They used this approach to primarily measure infrastructure resilience in the event of natural disasters like earthquakes. It plots the quality or functionality and the performance of infrastructure after a 50% loss. The triangle represents the loss of functionality from damage and disruption, as well as the pattern of restoration and recovery over time. It is used to measure the functionality of a system after a disaster, and also the time it takes for a system to return to pre-disaster levels of performance. Hence, community earthquake loss of resilience, R, with respect to that specific earthquake, can be measured by the size of the expected degradation in quality (probability of failure), over time (that is, time to recovery). Mathematically, it is defined by

$$R = \int_{t_0}^{t_1} [100 - Q(t)]dt$$

We observe that reduction the triangle area, but with the same recovery length at time t_1 represents static resilience while reduction in the triangle through recovery time earlier than t_1 represents dynamic resilience (Rose, 2014).

3. New approach

In this work we suggest an alternative perspective in the study of resilience. We suggest a new framework for examining the resilience of systems, drawing on the definitions and frameworks that were presented above. Using this new framework, we model the behaviour of the system, and use a model parameter to quantify the resilience of the system.

For the rest of this paper, we adopt the following definition for resilience:

The speed at which a system returns to equilibrium after a disturbance away from equilibrium.

In our framework, we therefore argue that a system is more or less resilient depending on whether it recovers rapidly or slowly from disruptive events or shocks. We assume that the state of the system can be measured by some quantifiable quantity that exhibits stochastic behaviour. We also adopt the new perspective that the shocks that disrupt the functioning of the system are assumed to be random in nature, and the disruption caused by the shock in the next time interval has a Gaussian distribution with variance equal to the square root of the length of the interval.

Given the above assumptions, we use a stochastic model of the form

$$\text{System state} = \text{Random shock} + \text{Recovery}$$

and attempt to quantify the rate of recovery, thus providing a measure for resilience of the system.

Furthermore we assume that as the shock dissipates and is absorbed in the system, it always aims to recover or revert back to normal functioning, thereby exhibiting a phenomenon called mean-reversion. The standard mean-reversion model that exists in literature is the Ornstein and Uhlenbeck (Uhlenbeck and Ornstein, 1930) model.

$$dX(t) = \lambda(\mu(t) - X(t))dt + \sigma dW(t)$$

where

$X(t)$ is the measurable quantity of the system,
λ is the mean-reversion rate which describes the speed with which the process return to the mean value,
$\mu(t)$ is the mean-reversion level or time-dependent average level, and
σ is the volatility.
$W(t)$ is Brownian motion

Observe that the drift term (first term) in the equation is governed by the mean-reversion level and the mean-reversion rate. This term captures the ability of a system to recover from random shocks. If the value of $X(t)$ exceeds the mean-reversion level $\mu(t)$, the drift term is negative and pulls the process down towards the mean-reversion level, at a rate determined by λ, the mean-reversion rate. Conversely if the value of $X(t)$ is less than the mean-reversion level $\mu(t)$, the drift term is positive and pushes the process upwards towards the mean-reversion level, again at a rate determined by λ. We note that the parameter λ, measures the rate at which the system is able to revert back to normal after a perturbation or a shock. The parameter λ is therefore our innovative measure of the 'resilience' of the system to shocks.

Thus the above model along with the parameter λ provides a new technique to measure the resilience of systems that exhibit stochastic behaviour.

Using methods from stochastic calculus, it is possible to derive an analytic solution for the Ornstein–Uhlenbeck process

$$dX_t = \lambda(\mu_t - X_t)dt + \sigma dW_t$$

We assume that W_t is standard Brownian motion, $\lambda > 0$ and σ are constants, and μ_t is a deterministic function.

We make the substitution $Y_t = e^{\alpha t}X_t$, then using Ito's Lemma we obtain

$$dY_t = \alpha e^{\alpha t}X_t dt + e^{\alpha t}dX_t$$

$$dY_t = \alpha e^{\alpha t} X_t dt + e^{\alpha t}(\lambda(\mu_t - X_t)dt + \sigma dW_t)$$

$$dY_t = e^{\alpha t}(\lambda\mu_t - \lambda X_t + \alpha X_t)dt + e^{\alpha t}\sigma dW_t$$

Let $\alpha = \lambda$. Then

$$dY_t = e^{\lambda t}\lambda\mu dt + e^{\lambda t}\sigma dW_t$$

$$Y_T - Y_0 = \int_0^T e^{\lambda t}\lambda\mu_t dt + \int_0^T e^{\lambda t}\sigma dW_t$$

The first integral is a purely deterministic integral. The second integral is an Ito integral. Using stochastic calculus we know that (Shreve)

$$\int_0^T e^{\lambda t}\sigma dW_t = \lim \sum_{j=1}^{N} e^{\lambda t_{j-1}}\sigma N(0, t_j - t_{j-1})$$

Therefore $Y_{t+1} - Y_t$ has a normal distribution with

$$\text{Mean} = \int_t^{t+1} e^{\lambda t}\lambda\mu_t dt, \text{and}$$

$$\text{Variance} = \int_t^{t+1} e^{2\lambda t}\sigma^2 dt$$

Or equivalently,

$$Y_{t+1} = Y_t + \int_0^T e^{\lambda t}\lambda\mu_t dt + \surd\left(\int_t^{t+1} e^{2\lambda t}\sigma^2 dt\right)\epsilon$$

where $\epsilon \sim N(0, 1)$.

A mean-reverting model such as the Ornstein–Uhlenbeck process is often used in finance in commodity pricing (Schwartz, 1997), and forecasting volatility (Heston, 1993), as it has been empirically observed that these processes exhibit mean reversion. Commodity prices are believed to be driven mostly by underlying economic fundamentals, but with a stochastic component representing short-term supply and demand imbalances overlaid on this. When prices are high, supply will increase since higher cost producers enter the market putting a downward pressure on prices. Conversely, when prices are low, supply will decrease since some of the higher cost producers will exit the market, putting upward pressure on prices. Therefore the imbalances induce mean reversion in commodity prices. Mean reversion models are also used in forecasting volatility, since otherwise the distribution of future prices would have unfeasibly large variance. In Batabyal et al. (2003) the Ornstein–Uhlenbeck process has also been used to model the behaviour of a variable useful in the study of lake ecosystems, by computing the probability of the given variable lying within a certain range. This range corresponds to the stability domain of a particular state of the lake, and is used to study a Holling type resilience.

We note that in these cases, the model is used to estimate the distribution of a variable at certain times, based on its distribution and behaviour at other times. Our new approach is to fit the model to an observable distribution and to use the mean-reversion parameter as a measure of resilience. This parameter captures the rate of recovery of the system after being subjected to random shocks, thereby providing a measure of the resilience of the system. In the next section we will test our new approach on London Underground network.

4. Application to London Underground network

In the context of London Underground, we want to examine the resilience of the system to shocks such as delays or disruptions in the underground service. A disruption to an underground line leads to the diversion of passenger flow and/or crowding on platforms. This affects the passenger counts on the underground lines. The counts could either go up or down, depending on the type of disruption. Furthermore depending on the severity of the disruption, the spikes in passenger counts could either be gradual or steep. The effect of the shock eventually dissipates as passengers respond to the shock, and passenger flows return to normal expected levels.

For example, if the Victoria line is running with severe delays, there is a sharp fall in passenger counts on the Victoria line, and consequently there will be an increase in passenger counts on lines which run parallel or North–South through London, perhaps causing minor delays on these lines too. The passenger counts will eventually return to normal levels once the shock dissipates.

We use our model to understand the resilience of these underground lines. The passenger count time series reflects the state of the system. We propose to fit our model to this data, and then use the mean-reversion parameter in the model to understand the resilience of the underground line.

We will not use this model to study adaptation, in the Rose (2007) sense. We propose this model to study short-term changes in the behaviour of the London Underground; over these time scales we make the modelling assumption that the system will return to its original operating behaviour, as determined by equilibrium with the other available forms of transportation.

4.1. Characteristics of passenger count processes

We first study the properties of passenger flow time series. To begin an analysis of the response of passenger flows to shocks in the system, we must grasp the essential statistical features of the time series representing passenger counts on underground lines. The passenger count data was taken from Transport for London's Online Syndicated Feeds in August 2012:

Passenger counts collects information about passenger numbers entering and exiting London Underground stations, largely based on the Underground ticketing system gate data. Thus this feed gives entries and exits by station and quarter hour, for Weekday, Saturday and Sunday. Counts data is obtained during the autumn of each year and does not necessarily reflect whole-year annual demand. The data is adjusted to remove the effect of abnormal circumstances that may affect demand such as industrial action. We used the Weekday entry data to estimate the passenger load on each line. Passenger loads on a line at a station were allotted in proportion to the number of lines servicing that particular station (see Fig. 2).

Five features can be highlighted in the passenger count behaviour:

1. Fluctuating time series

As we are particularly interested in the diffusive nature of shocks, and given the fluctuating behaviour of the time series, a stochastic process would be appropriate to describe the evolution of passenger counts through the day. Brownian motion was first used to describe the zigzag nature of pollen grains suspended in water. Since then it has been used in multiple fields, and is commonly known as the random walk. The main properties of such a process are:

- Changes in passenger counts are independent of each other.
- Changes in passenger counts are normally distributed with constant mean and volatility.

Brownian motion provides a good building block to model the fluctuating behaviour of passenger counts through the day. However, it is necessary to add a drift component to incorporate rush hour effects on passenger counts.

2. Rush hour effects

As most weekday commuters hold regular 9–5 jobs, there is an expected significant increase in passengers at the start and end of the working day. This time-dependent drift through the day is clearly evident as the two peaks in each time series. Brownian motion has an expected value of zero, which means that if the initial value of the process is zero, then after time Δt, the expected value of the process remains zero. While using Brownian motion to model processes in which the expected change in value after time Δt is non-zero, it is necessary to incorporate a drift term that is deterministic.

$$X(t + \Delta t) - X(t) = \text{Drifteffect(non-random)} + \text{Brownian motion(random)}$$

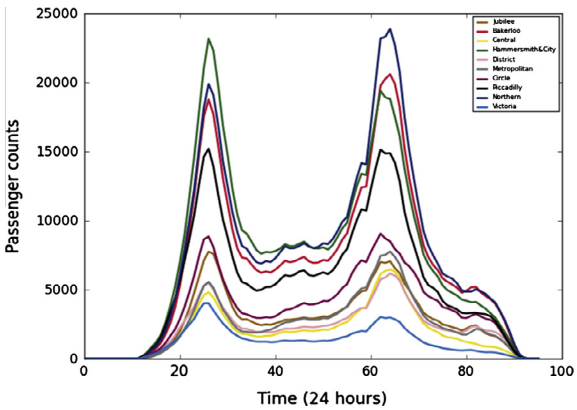

Fig. 2. Sample one days' data showing passenger count by line.

3. Mean-reversion

Passenger counts tend to fluctuate, but drifts over time to values determined by the average demand for the service at that time of the day, and that day of the week. Mean reversion accounts for this drift. Mathematically this behaviour is incorporated into a stochastic process in the following manner:

$$X(t+\Delta t) - X(t) = \lambda(\mu(\mathrm{t}) - X(t)) + \text{Brownian motion(random)}$$

$\mu(t)$ is the mean-reversion level. Thus if $X(t) > \mu(t)$ the drift term of the process is negative (given $\lambda > 0$) which drags the process down towards $\mu(t)$ and vice versa.

λ is the mean-reversion rate which describes the speed with which the process return to the mean value

4. Occasional sharp increases or decreases

Large changes in passenger counts are attributed to major shocks (e.g. station/line closures, severe delays…). Although Brownian motion with mean reversion is useful to model processes which fluctuate around a historical mean value, and in fact can also model the way in which prices diffuse back towards the historical equilibrium after a jump event (shock to the system), it fails to capture the jump event itself. These jumps could be incorporated into the Brownian motion with mean reversion by adding Poisson jump models. We will briefly examine this model at the end of this section.

5. We also assume that the passenger levels of the Underground system are in equilibrium with the other available means of transportation in the city. Thus passenger levels may be higher or lower than average depending on the performance of the underground system relative to the alternatives

4.2. Stochastic model

At the first instance, it seems that we can fit the observable behaviour of the passenger count process to an Ornstein–Uhlenbeck model described in Section 3. This model captures the fluctuating behaviour of the passenger count process, as well as its tendency to gravitate towards a time-dependent equilibrium level. We then propose the use model parameters to measure the rate of recovery of an underground line after a shock, thereby providing a measure of the resilience of the line.

We model the passenger counts on every underground line (there are 11) using the mean-reverting stochastic process suggested in Section 3.

$$dX(t) = \lambda(\mu(t) - X(t))dt + \sigma dW(t)$$

where

$X(t)$ is the number of passengers on the line.

λ is the mean-reversion rate which describes the speed with which the process return to the mean value. λ is assumed to be positive.

$\mu(t)$ is the mean-reversion level or time-dependent average level. Note that the mean-reversion level is a function of time, not a constant.

σ is the volatility. This controls the amount of randomness in the passenger numbers.

The parameters of the model are λ, the mean-reversion rate, $\mu(t)$, the mean-reversion level, and σ, the volatility of the process. These parameters can be estimated from data using techniques from (Damiano Brigo, Antonio Dalessandro, Matthias Neugebauer, Fares Triki). We then define.

*the **resilience** of the line to be the mean-reversion rate parameter, λ.*

We expect that when there is shock to the system, the perturbation will be reflected in the passenger counts. A shock causes the process to deviate from the long running average level. The speed with which the passenger counts return to normal is an indicator of how quickly the line is able to recover from the shock, and resume normal functioning. This provides a way to quantitatively measure and compare the resilience of underground lines.

4.3. Simulations

In this section we simulate the passenger count process using the solution from Section 3, with a range of parameter values, to explore the capacity of the model to capture a wide range of behaviours. We use this example to therefore show that we have a versatile tool that can be used to measure the resilience of vastly differing systems.

In this simulation, mean levels of passenger count data was taken from TfL Online's Syndicated Feeds, as above in Section 4.1. Passenger counts collects information about passenger numbers entering and exiting London Underground stations, largely based on the Underground ticketing system gate data. Thus this feed gives entries and exits by station and quarter hour, for an average Weekday. Counts data is obtained during the autumn of each year and does not necessarily reflect whole-year annual demand. We also note that the available data was adjusted at the source to remove the effect of abnormal circumstances that may affect demand such as industrial action. We used the Weekday entry data to estimate the passenger load on each line. Passenger loads on a line at a station were allotted in proportion to the number of lines servicing that particular station. The plot in Fig. 2 represents the mean level of passenger counts on an average weekday. We use a mean-reverting process to simulate the passenger counts.

$$X(t) = \lambda(\mu(t) - X(t))dt + \sigma dW(t)$$

The parameters of the model are:
λ – mean-reversion rate
σ – volatility
$\mu(t)$ – mean level

We therefore use the passenger counts estimated from TfL Online data to represent the mean level, and choose a range of values for λ and σ to simulate passenger counts. For the simulation we divide the day into 2000 time intervals. The model can capture a wide range of behaviours as seen below:

1. Low volatility and low mean reversion

Volatility is amount of fluctuation in the passenger count process, and therefore low volatility implies that the process will typically be determined by drift, and it is unlikely that there will be large changes in value in successive time intervals. On the other hand, if the process also exhibits low mean reversion (Fig. 3) and it starts to slowly drift away from historic mean values, then it may be difficult to control the drift, and steer the process back to the long standing mean values. A system which gives rise to such processes is deceptively stable, because if it is subject to a major shock (albeit with very low probability) it will be unable to recover quickly from this shock as the mean reversion parameter indicates that the system has low resilience.

2. Low volatility and high mean reversion

As in the previous case, low volatility implies no big swings in the passenger count process. On the hand high mean reversion, implies that the any small drifts in the passenger count process are quickly corrected. We note here that the simulated graph is almost identical to the plot of a sample day's passenger count process (Fig. 4) which we have taken to be the historical mean level. A system which gives rise to such a process is highly resilient, and capable of recovering quickly from shocks.

3. High volatility and low mean reversion

A process which has high volatility and low mean reversion is essentially just Brownian motion, as is indicated in the simulated graph (Fig. 5). Such a system will have low resilience and highly unpredictable behaviour.

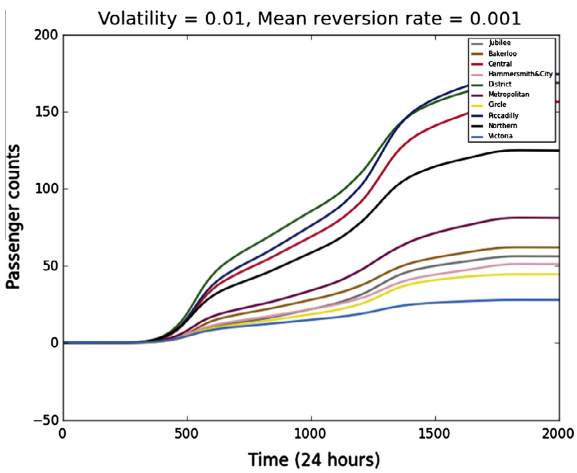

Fig. 3. Low volatility, low mean-reversion.

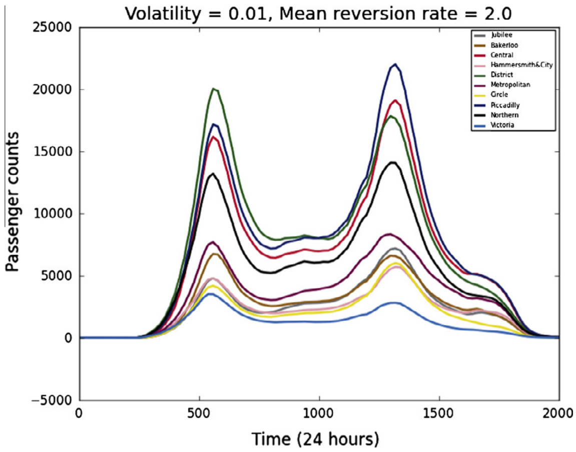

Fig. 4. Low volatility, high mean-reversion.

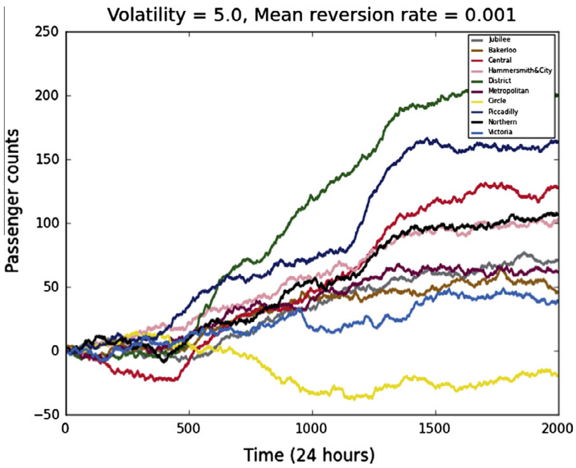

Fig. 5. High volatility, low mean-reversion.

4. High volatility and high mean reversion

Finally a process with high volatility that fluctuates wildly can be controlled if the process also has high mean reversion (Fig. 6). The mean reverting parameter coerces the process back to the historic mean when it begins to drift away. The model parameters indicate that such a system is in fact very resilient. It performs predictably and is resilient.

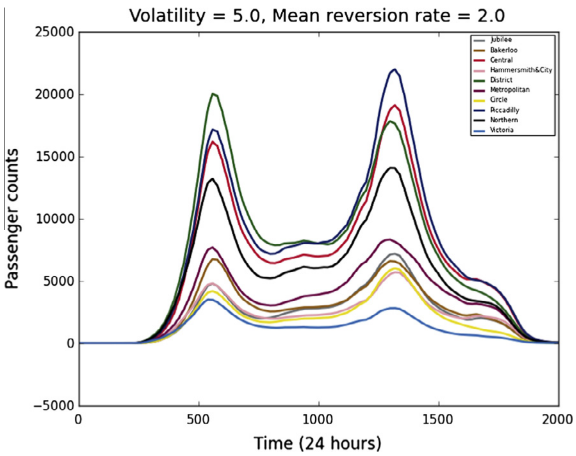

Fig. 6. High volatility, high mean-reversion.

Thus we see that we can vary the parameters of this model to represent a wide variety of qualitatively different behaviours, with different levels of volatility and resilience. The model, once implemented, could then be used as a predictive tool to determine the resilience of systems. In the case of the application to the London Underground transportation system, it could for example, highlight the Underground lines which have low resilience.

5. Alternate model with jumps

Systems can sometimes be subject to sharp and unexpected shocks. The model that we have described in the previous sections is suitable for handling systems subject to small unpredictable variations, however is not able to model sudden and large shocks. In the example of passenger counts on an underground line, this would perhaps be a severe disruption to the service, thereby drastically affecting the passenger count. The framework that we have developed for capturing the resilience of systems, can further be extended to systems that are subject to such sudden abrupt shocks by adding jumps or spikes to the model.

We can incorporate these spikes by extending the mean-reversion model, and including Poisson processes to model the jumps.

$$dX(t) = \lambda(\mu(t) - X(t))dt + \sigma dW(t) + dJ_t$$

where the jumps J are defined as:

$$J(t) = \sum_{j=1}^{N(t)} Y_j, dJ(t) = Y_{N(t)} dN(t)$$

where $N(t)$ is a Poisson process with intensity λ and Y_j are independent and identically distributed random variables modelling the size of the j-th jump, independent of N and W. This model introduces multiple new parameters for the jump component in addition to the mean-reversion parameters.

We can now apply the new analytical formulae for sharp shocks in the case of the passenger count process in the case of the London Underground transport system.

The passenger count process occasionally exhibits jumps or spikes. These are a result of severe disruption in the underground service. The effect of the disruption on the passenger count process could either be sudden drop in passenger counts owing to a station closure or a sudden increase in the passenger load resulting from long delays or disruptions on other underground lines. After the disruption has been dealt with, passenger counts return to normal levels.

We argue that this extension to our initial model, as presented in Section 4 can provide a more accurate representation of the passenger count process. As before, the mean-reverting parameter provides a measure of the "resilience of the line".

We simulate some disruptions on an underground line (data for mean values taken for the Northern line) using the model above. In both graphs the volatility is taken to be high value and held constant. The parameters controlling the shocks have been adjusted so that only a few shocks of large magnitude occur. In both graphs, at least two large and abrupt disruptions occur. However after the disruption, the passenger count process in Fig. 7 takes a significantly longer time to revert back to the long-term average level as compared to the process in Fig. 8. This is because the simulation in Fig. 7 is run with a low mean-reversion parameter, and would model a system that has low resilience. As expected such a system would take longer to revert back to normal levels after a disruption.

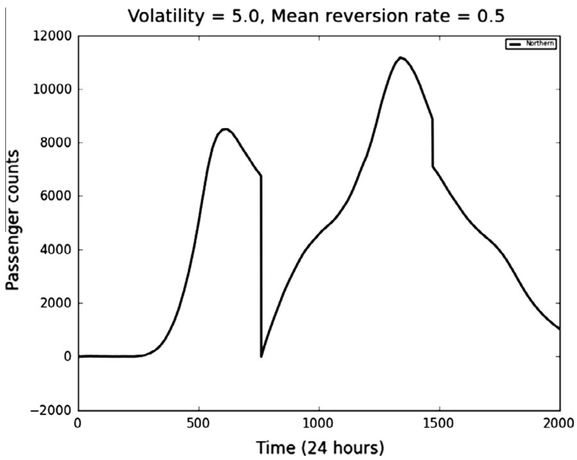

Fig. 7. Jumps with low mean-reversion.

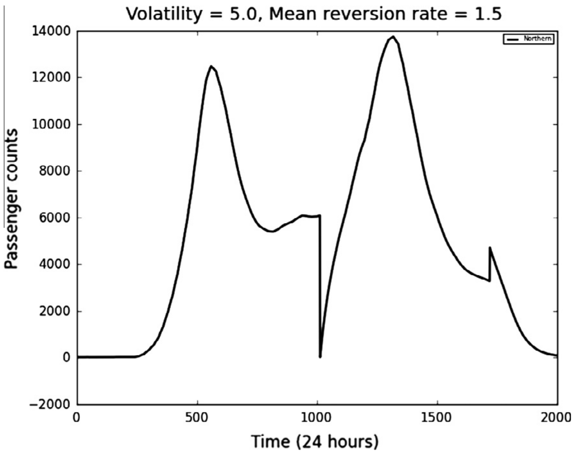

Fig. 8. Jumps with high mean-reversion.

We can summarise the results by observing that incorporating a jump process in the model provides a more realistic and accurate representation of the passenger count process, due to its discontinuous and asymmetric nature. In other words, while Brownian motion provides continuous and symmetrical stochasticity in the passenger count process, introducing a jump process adds a discontinuous and asymmetrical source of disruption.

6. Conclusions

Quantifying the resilience of a system leads to a better understanding of the strength and resilience of the system, thereby making it possible to determine how best to improve the system to withstand shocks in the future. This is particularly important in urban infrastructure such as transport where we observe high level of interdependency. This work proposes a new approach to quantifying the resilience of systems. We draw on the vast literature that currently exists on studying resilience of systems in different contexts, and by so doing we define a flexible and general approach that can be used to measure resilience.

Our new measure of resilience is defined as the mean-reverting parameter in a specified stochastic mean-reverting model. This parameter captures the rate of recovery of the system after it is subjected to random shocks.

We see that the model we suggest is useful and interesting because of several factors. It can capture the behaviour of a wide range of systems, from low to high volatility (the up-and-down variation from the equilibrium value) and from low to high mean reversion (the speed with which a system recovers from a shock). We also see that including jump processes in the model would enable it to capture the response of the system to sharp Poisson shocks, thus capturing the behaviour of the system under acute disruptions. Therefore a mean-reversion model with jumps, once implemented, would provide a powerful predictive tool to assess the resilience of systems.

For example, in the case of the London Underground transport system, such a model could be useful to assess the resilience of the Underground lines to shocks. One could obtain a comparative study of all Underground Lines, and ascertain which lines are more or less resilient. Further using the mean-reversion model with jumps, one could also study how resilient a particular Underground line is to small shocks versus large shocks. These studies could assist in making investment decisions on improvements to the Underground Lines.

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