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Tensions in the design of mathematical technological environments: Tools and Tasks

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Abstract

The design of tasks for the exploration of mathematical concepts involving technology can take several starting points. In many cases the 'tool' is predefined as an existing mathematics application with an embedded set of design principles that shape the mathematical tasks that are possible. In other cases, the tool and tasks are designed through a more dynamic process whereby designers and educators engage in a discourse that influences the resulting tasks. The chapter will begin with a brief description of a longitudinal study, and its theoretical framework that resulted in a rubric to inform the design of tasks that privilege the exploration of mathematical variants and invariants (Clark-Wilson and Timotheus 2013; Clark-Wilson 2010). This rubric is then used as a construct for the post-priori analysis of two tasks that introduced the concept of linear functions and that use different technologies. Conclusions will be drawn that highlight subtle tensions that relate to the mathematical knowledge at stake and the design principles of the underlying technology and task.

Key words: Mathematics, Digital technology, Task design,

Introduction

An important premise for the design of any task in mathematics education concerns the very nature of the mathematical knowledge that the task is intended to develop, which might encompass facts, skills, algorithms, relationships, notations etc. that are all situated within the particular mathematical culture. Alongside this, the diversity of the mathematical processes through which the user of the task might construct their mathematical knowledge could require her to represent, visualise, conjecture, notate, estimate, reason, justify, generalise and so on. I am in strong agreement with Mason, Graham and Johnston-Wilder's premise that 'a les-

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son without the opportunity for learners to express a generality is not in fact a mathematics lesson' (2005, p.297). That is, the core purpose of the tasks that we offer to learners of mathematics is to expand their frame of mind through an ongoing process of validating or refuting mathematical knowledge. Consequently, the work described in this chapter has emanated from two projects in which tasks have been designed within technology-mediated environments to privilege learners' first-hand dynamic exploration of mathematical variants and invariants within English mathematics classroom settings (11-16 years).

Theoretical framework

A longitudinal study that involved 15 English teachers of secondary school mathematics in the design, teaching and evaluation of 75 lesson activities resulted in a rubric for mathematical task design within dynamic multi-representational digital environments (Clark-Wilson 2010; Clark-Wilson and Timotheus 2013). In this study the teachers were designing tasks that required students to use the *TI-Nspire v1.8* handheld device or computer software (Texas Instruments 2007b), which at that time was a new digital environment for all concerned.
This research was framed within an activity-theoretic approach that interprets the Vygotskian notion of activity as a 'unit of analysis that included both the individual and his/her culturally defined environment' (Wertsch 1981). Verillon and Rabardel elaborated this earlier theory to develop the *instrumental approach* within technological environments (Verillon and Rabardel 1995). This construct has been further expanded by the mathematics education research community to include the notions: instrumentation; instrumentalisation; instrumental genesis; instrumental orchestration and documentational genesis. (Drijvers and Trouche 2008; Guin and Trouche 1999; Trouche 2004; Gueudet and Trouche 2009; Drijvers 2012; Haspekian 2014). These ideas concern the complex and interrelated processes of:

- Learning to use a new technology for purposeful mathematical activity;
- Designing tasks for students to initiate purposeful mathematical activity;
- Collating the various artefacts that comprise the 'document system' for the activity;
- Supporting students to learn to use technology for purposeful mathematical activity;
- Articulating the teacher's role in supporting the students to navigate their respective routes through the various artefacts that comprise the activity to include interaction with the technology.

It is important to comment that within the research contexts from which these notions have emanated, the chosen technologies fit Pierce and Stacey's description of 'mathematical analysis tools' (2008), which include technologies such as computer algebra software (CAS), dynamic geometry software (DGS), graphing software and spreadsheet software. The TI-Nspire technology used within my earlier research afforded a range of ‘applications’ that included: calculator; spreadsheet; dynamic geometry; function graphing; statistical calculation and graphing; built in commands i.e. *factor(n)*; and text editing. In all of these environments, the facility to save numeric outputs as variables supported the linking of variables within and between these different representations.

A second important theoretical construct of significance to the study was that of a multiple representational environment, which was postulated initially by Kaput (1986) in his vision for the way in which technology might support higher–level engagement with mathematics. In the intervening years, different genres of technologies have afforded opportunities to engage with mathematics dynamically by observing the simultaneous views of different representations, for example, the representations of a function, its graph and a table of its associated coordinate values. The development of ‘dragging’ an image through the interface of a mouse (or pen or finger) has afforded further forms of mathematical interaction.

As the study progressed, an element of the teachers’ epistemological development was related to their realisation that expressing generality was a very important aspect of the tasks that they went on to design, although this was not necessarily realised at the time. Other elements of the teachers’ professional learning concerned increasing attention to the way that the digital environment supported or hindered the expression of generality, the design of the associated supporting resources, and the teacher’s role in mediating the associated classroom discourse. The evidence from the study suggested that the process of designing tasks that utilise such environments to privilege explorations of variance and invariance is a highly complex process, which requires teachers to carefully consider how variance and invariance might be manifested within any given mathematical topic. The relevance and importance of the initial example space and how this might be productively expanded to support learners towards the desired generalisation is a crucial aspect of task design. For example, the example space might need to be flexible enough to enable the students to explore and generate different example sets, which might be accomplished by dragging an on-screen object that drives a variant property.

The starting point for any classroom task is its initial design, and the following set of questions, generated as a result of this study, offer a research-informed approach to the design process:

- What is the generalisable property within the mathematics topic under investigation?
- How might this property manifest itself within the multi-representational technological environment – and which of these manifestations is at an accessible level for the students concerned?
- What forms of interaction with the multi-representational technology will reveal the desired manifestation?
- What labelling and referencing notations will support the articulation and communication of the generalisation that is being sought?

- What might the ‘flow’ of mathematical representations (with and without technology) look like as a means to illuminate and make sense of the generalisation?
- What forms of interaction between the students and teacher will support the generalisation to be more widely communicated?
- How might the original example space be expanded to incorporate broader related generalisations? (Clark-Wilson 2010, p.242-3)

These questions will be used as a rubric later in this chapter.

Discussions during the ICMI Study Conference 22 on Task Design (Margolinas 2013) led Paul Drijvers to contribute a further question: *How do you know that this generalisation is true for all cases? (Can it be proved?)*

However, these questions only become useful as one begins to consider the study of a particular mathematical topic in relation to the teaching context, that is the age, prior attainment and prevalent teaching and learning culture for the students for whom a task is being designed.

What follows are two examples of mathematical tasks that have been designed as early introductions to linear functions within lower secondary mathematics. The first example is a task designed by one of the participating teachers within the original research study that used *TI-Nspire* computer software (the teacher) and handheld technology (the teacher and students). The second task was designed to be accessed within a web browser for the more recent *Cornerstone Maths* project (a brief description of which is provided below). These examples are used both to provide a deeper discussion of the task design rubric and to highlight some of the tensions within the process of task design.

Task examples: Introducing linear functions

When these tasks were designed, the English National Curriculum (Department for Children Schools and Families 2007) stipulated the following content knowledge related to linear functions for students aged 11-14 years[1]: *The study of mathematics should include linear equations, formulae, expressions and identities* (2007 p. 145), which was exemplified by the attainment target:

> They [the students] formulate and solve linear equations with whole-number coefficients. They represent mappings expressed algebraically, and use Cartesian coordinates for graphical representation interpreting general features (2007 p. 150).

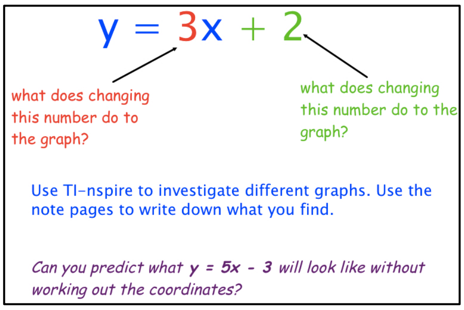
It is from this starting point that teachers make decisions about the tasks they design and adapt, which requires a consideration of the particular mathematical and pedagogic starting points alongside the finer mathematical progression of the stipulated content knowledge.

[1] This was replaced by a new National Curriculum in 2013 (Department of Education 2013)

Task 1 - Investigating straight lines

The task that follows was designed by an experienced teacher who was confident with a range of existing mathematical technologies and chose to use the TI-Nspire PC software with a group of 11-12 year olds to meet her mathematical learning objective 'To be able to discover the gradient and intercept and how they connect to the [linear] equation'. She displayed the task instructions as shown in Fig. 1. and informed the students to: 'work with a partner; be systematic; use lots of different pages to record your findings; and experiment with different layouts'.

Fig. 1. Task instructions as displayed to the students



The students, a homogenous[2] class of 30 boys and girls, worked in small groups of twos and threes around laptop computers in their normal mathematics classroom. During the one hour lesson, the teacher moved around the classroom, interacting with groups of students to support them to: get started on the task; overcome technological issues (how to input functions, how to split the page to enable them to record their learning notes alongside their graphs, etc.); and to question them about their choice of functions and provide motivational encouragement. The teacher did not choose to convene a whole-class discussion at any point during this particular lesson. Instead, the students posted their task conclusions to the school's virtual learning environment, which the teacher reviewed and responded to after the lesson. In the subsequent lesson, the students worked further on the task before each group presented their findings to the whole class.

The work of one pair of students (a boy and a girl), which is typical of the work produced by the class in terms of its content, layout, and the informal language within the learning notes, is shown below in Figs. 2-5 in the order in which they presented to the class.

[2] The students in most English state schools are organised into setted mathematics classes, according to their prior attainment.

Fig. 2.

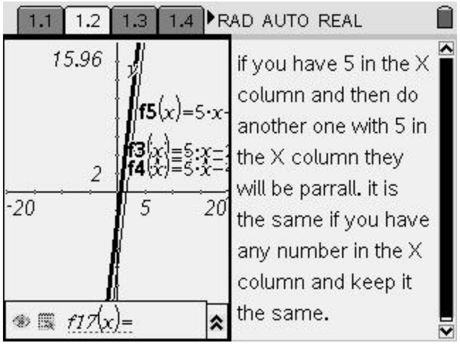


Fig. 3.

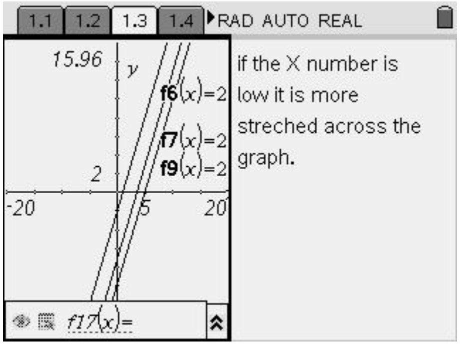


Fig. 4.

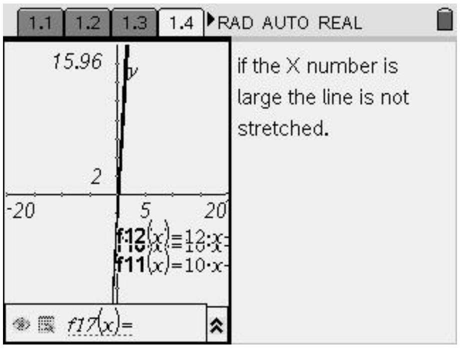
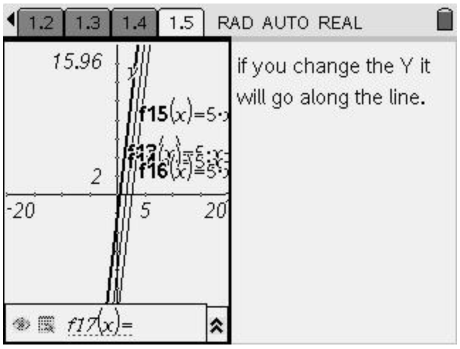


Fig. 5.



In her detailed evaluation of the students' activity during the lesson, the teacher concluded the following:

> Previously they had plotted coordinates and joined them up to make a straight line. They were first of all quite amazed that they could just type in the equation and the line would appear automatically. They began to realise that changing the equation affected the graph in different ways, and that the number before the x affected it differently from changing the number added on.
> They first of all were very unsure about what to do and needed some prompting, especially on how to fix one variable and alter the other so they did not end up with loads of random looking lines on the page. Mostly it was just an idea of where to start and what equation to type in initially, and then once they had told me what they wanted to fix and what they wanted to change they were fine. Most students managed to reach the conclusion that as the number before the x gets bigger the line gets steeper, and that the number on the end moves the graph and you can make parallel lines. Hardly any had yet managed to generalise to look at fractions or negatives.

This task design is highly typical of tasks that introduce the gradient and intercept properties of linear functions using mathematical technologies, which have prevailed in English classrooms since the late 1980s[3]. Although the student use of technology to explore mathematical concepts is still under-reported in English secondary school practice (Office for Standards in Education 2008, 2012), research continues to report similar approaches (Ruthven and Hennessy 2003; Godwin and Sutherland 2004).

Task 2 – Controlling characters with equations

The example that follows 'Controlling characters with equations' is a task from a sequence of tasks focused on linear functions - one of three curriculum units developed during the Cornerstone Maths project (2011-2014). Cornerstone Maths (CM) is a collaborative design research project involving colleagues at the London Knowledge Lab, UCL Institute of Education and SRI International, USA that has the particular aim to widen student access to dynamic mathematical technology in lower secondary classrooms across England to support the teaching of 'hard to teach topics'. (For a fuller description of this project and its research outcomes, see (Clark-Wilson, Hoyles, Noss, Vahey, and Roschelle 2015; Hoyles, Noss, Vahey, and Roschelle 2013) . Central to its design is the 'curriculum activity system', which incorporates digital resources, pupil workbooks, teacher guides and teacher professional development (Vahey, Knudsen, Rafanan, and Lara-Meloy 2013). The digital resources for each of the curriculum units have been developed in html5 to enable wider access by students through a web browser and so overcome the need for software to be installed and maintained on school computer networks, a known barrier to technology use in English mathematics classrooms. In each case a rapid prototyping methodology was adopted to the design of the web-based software by taking the desirable features of existing software that had already been shown to enhance students' mathematical learning. In the case of the CM curriculum unit on linear functions, its software antecedent was *SimCalc*, for which a body of research exists (Hegedus and Roschelle 2013; Kaput and Schorr 2008). The curriculum unit includes 14 separate tasks for students, not all of which require access to technology. This particular task has been selected as its learning objectives most closely align with those of the earlier example.

> Equations are a form of mathematical representation. Graphs and tables are other forms. Equations can be written based on tables or graphs. You can "translate" between graphs, tables and equations. Time, distance and speed are represented differently in these three representations. For equations of the form $y = mx$, in motion contexts, m is the speed of a moving object. (SRI International and Institute of Education 2013, p. iv)

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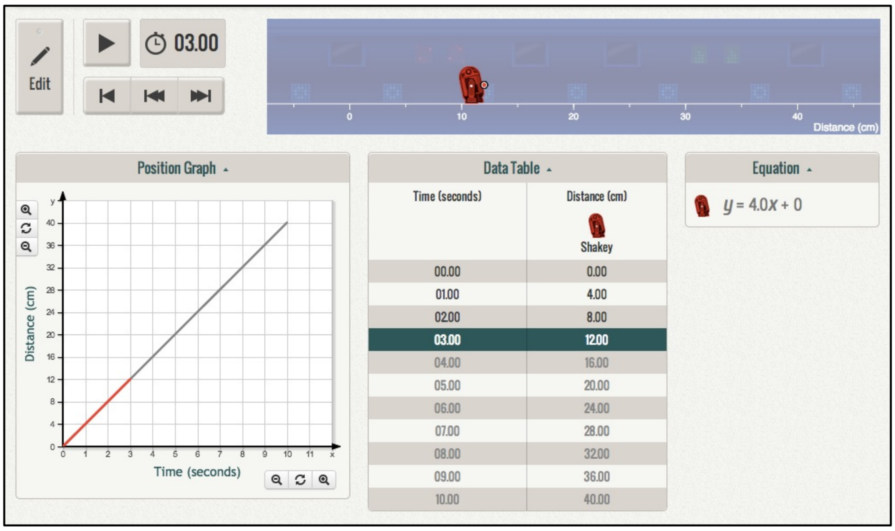
[3] From graphing calculators and software packages such as Mouseplotter (BBC Micro), Coypu (Acorn/PC), Omnigraph (PC) and Autograph (PC/Mac/iPad).

Fig. 6. below shows the digital resource that accompanies the task. When the play button is activated, the character (Shakey the robot) moves along the horizontal number line, and Shakey's position and time are highlighted simultaneously on the position-time graph and within the table (using colour). In Fig. 6. the animation has been paused at t=3 seconds.

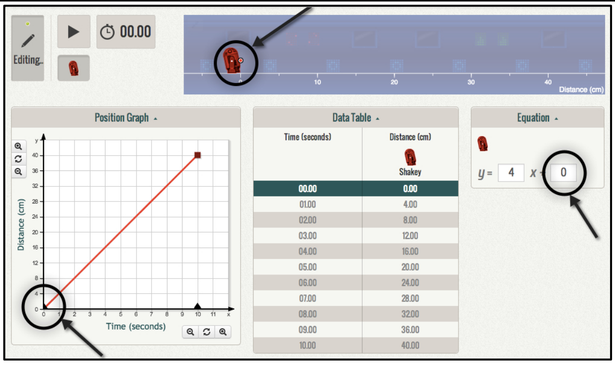
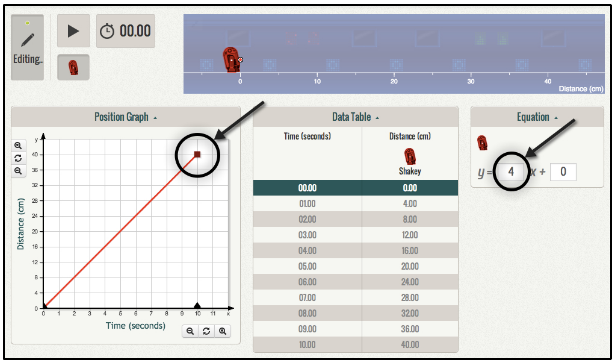
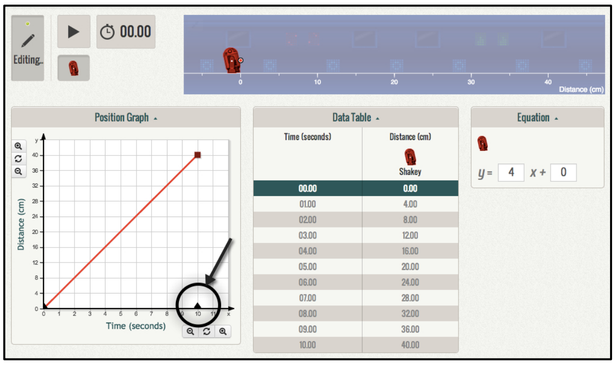
Fig. 6. The dynamic digital environment that accompanies the task 'Controlling characters with equations'.



In the task, the students adopt the role of a digital games designer. The task narrative informs them that they are learning the underlying mathematics to enable them to design interesting computer games for mobile devices. The pupils are asked to edit[4] the scenario in Fig. 6 to meet different mathematical constraints (i.e. to make 'Shakey' move slower and faster) and to record the resulting graph, table of values and equation in their workbooks. The process of editing the software, which has the effect of altering the starting position, speed and overall travel time, is accomplished in the following ways:

The starting position of the character is varied by: dragging the character; dragging the point representing	**Fig. 7. Varying the charater's start position.**

[4] The pupils are not given guidance on how to do this in the pupil workbook. Also, during their initial professional development teachers are discouraged from demonstrating the different ways to edit the software to pupils before pupils have had an opportunity to explore the editing functionality for themselves.

<table>
<tr>
<td>t=0 on the position-time graph in a vertical direction; and/or inputting a numeric value representing ‘c’ in the equation.</td>
<td>
</td>
</tr>
<tr>
<td>The gradient/speed is varied by: dragging the end-point of the line segment in a vertical direction and/or inputting a numeric value representing ‘m’ in the equation.</td>
<td>Fig. 8. Varying the speed/gradient.

</td>
</tr>
<tr>
<td>Dragging the position of the ‘hot spot’ on the x-axis in a horizontal direction varies the overall travel time.</td>
<td>Fig. 9. Varying the travel time.

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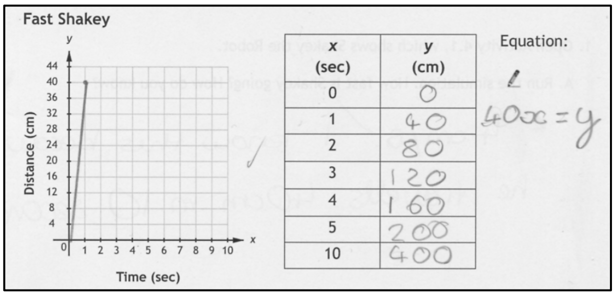
</table>

Due to the scale of the Cornerstone Maths project, over 180 teachers have taught all or some of the linear functions unit to approximately 6000 students. A scrutiny of the pupil workbooks of one particular gender mixed class of twenty-eight 12-13 year olds revealed that all of the students were able to edit the graph and sketch the graphs to meet the given constraints (slower and faster). Furthermore, sixteen students gave a written description of the mathematical differences between the two scenarios that they had created, which used language such as ‘steeper’, ‘more shallow’, ‘more gradient’ to explain the differences between the slower and faster scenarios. There was also a great variety in the notations that pupils used to record the ‘equation’ and their interpretations of the decimal notation that the software displayed. For example, most students recorded the equation exactly as it was displayed, i.e. y=8.0x+0, whereas others recorded it as y=8x+0 or y=8x. One student represented the equation in a way that was consistent with the table of values, recording the equation as 40x=y (see Fig. 10.).

Fig. 10. One student’s own graph of ‘Fast Shakey’



The diversity of the students’ responses exemplifies the software tool’s role as a *semiotic mediator* supporting the students’ personal constructions of meaning related to linear functions and related notations. (Bartolini Bussi and Mariotti 2008).

Post-priori analysis

The task design rubric is now used to analyse the two tasks, paying attention to both the designer’s perspective and the implications of this supported by the evidence of the students’ responses to the tasks in the respective classrooms.

Table 1. A post-priori analysis of the two tasks

Question	**Task 1: Investigating straight line graphs**	**Task 2: Controlling characters with equations**
What is the generalisable property within the mathematics topic under investigation?	Designers' intentions: There were two generalizable properties: The value of m defines the gradient/steepness of a linear function y = mx + c; and the value of c defines the position of the intercept on the y-axis. In reality the students were required to choose which of these properties they would investigate first. They were also free to choose the type of numbers that they input for 'm' and 'c'. (i.e. positive/negative, integer/fraction/decimal)	Designers' intentions: For equations of the form y = mx, in motion contexts, m is the speed of a moving object.
How might this property manifest itself within the multi-representational technological environment? – *and which of these manifestations is at an accessible level for the students concerned?*	The gradient property: The appearance of the 'steepness' of the line within the graph domain. *This is accessible to the students*. The value of 'm' as displayed in the equation. *This was visible to the students, although as multiple lines were on the screen, students would need to remember the creation of each line to link it to its respective equation*. The increase in y-value for a unit increase in x-value. *This is accessible to students through the Table view, however, in the lesson concerned, the students were not made aware of this functionality. Although the value of 'm' defines the gradient, it is not clear whether students were able to connect the value of 'm' in a numeric sense to the particular graph that they were looking at as the resolution of the screen did not allow for accurate interpretations*. The intercept property	The real-time speed with which the character moves, the line segment is highlighted on the graph and the corresponding rows are highlighted in the table. *Students' written responses suggest that that most students were able to make sense of these different representations*. The value of 'm' as displayed in the equation. *This was visible to the students*. The numeric increase in y-value for unitary increases in x-value as highlighted within the table of values. *This was visible to the students*.

	The position of the intercept on the y-axis. *This was accessible to students*. The value of 'c' as displayed in the equation. *This was visible to the students*. The y-value when x=0, which can be observed in the Table view. *This functionality was not used by the students during the lesson*.	
What forms of interaction with the multi-representational technology will reveal the desired manifestation?	Having decided whether to vary the value of m or c, students could vary these values by: Inputting the right hand side of equations in the form 'mx + c' (the left hand syntax is given automatically by the tool, i.e. f*n*(x)= where *n* increases by one to define each new function.) Dragging the position of the line using one of two 'hotspots. A rotate hotspot - to vary the gradient around the point (0, c) and a translate hotspot to vary the value of c, whilst maintaining the gradient. *In the lesson concerned, the students did not interact with the graphs in this way*.	Editing the graph to vary the gradient/speed and overall travel time, as shown in Figs. 7 and 8. Editing the equation to vary the gradient/speed.
What labelling and referencing notations will support the articulation and communication of the generalisation that is being sought?	Multiple functions were visible at the same time, each with its own reference, i.e. f3(x)=2x+3. This could result in a 'pile-up' of representations as seen in Figures 2-5.	The representations referred to a single animation of the character (Shakey). The initial animation represented the reference point to which the animations of 'Slow Shakey' and 'Fast Shakey' could be compared.
What might the 'flow' of mathematical representations (with and without technology) look like as a means to illuminate and make sense of the generalisation?	Function input (using algebraic notation). Observation of resulting graph in graphics view. Further input(s) of function(s) and observation/comparison of resulting graphs in graphics view.	Playing of animation. Observation of: the emergent trace of the line segment; the highlighted values in the table; and the overall appearance of the graph on the given axes. Varying the animation by editing

	[Away from the technology] Justification of why a higher/lower value of m (or c) affects the graph in particular ways. Reveal the Table view for particular equations to identify key features that relate to particular values of 'm' and 'c'.	one of the variables as described in Figs. 7 to 9, which would lead to multiple 'flows' of representations, dependent on the focus of attention.
What forms of interaction between the students and teacher will support the generalisation to be more widely communicated?	Discussion to ensure that students focus on varying either 'm' or 'c' in the first instance. Discussion about the notation Discussion to highlight how particular equations generate particular lines and of their distinctive features. This would involve direct interaction with the software, including inputting equations and dragging lines to new positions. Discussion to relate the distinctive features of the lines and equation to the distinctive features of the related function machine and table of values.	Discussion to relate the distinctive features of the initial animation to the graph, table of values and equation. This would involve direct interaction with the software. Discussion to highlight how the different hotspots on the graph affect the animation, graph, table of values and equation. This would involve direct interaction with the software to edit the graph and/or equation.
How might the original example space be expanded to incorporate broader related generalisations?	The flexibility of the tool would enable the students to: • Reveal the table of values in order to make generalisations about the relationship between the function and its displayed values. • Input other families of functions, such as quadratics and cubics. • Explore how 'zooming' in and out of the graphing window affects the gradient both visually and numerically.	The flexibility of the tool would enable the students to: • Edit the scenario to vary the start position of the character (i.e. to vary 'c') and to show backward motion, i.e. graphs of negative gradient. • Explore how 'zooming' in and out of the graphing window affects the gradient both visually and numerically.

Conclusions

The analysis of the two task examples, which had both been designed to introduce the concept of linear functions to lower secondary students in a dynamic technological environment reveals subtle tensions that relate to the mathematical knowledge that is being addressed by the task and aspects of the process of task design.

Mathematical knowledge

It is clear that different aspects of mathematical knowledge concerning linear functions were being addressed by the two tasks, with the first task adopting a pure mathematical context whereas the second task involved a realistic motion context. However, both tasks sought to involve students in explorations of the variant and invariant properties of linear functions.

The analysis does indicate that the cognitive load for Task 1 is higher as it not only requires the students to begin by choosing which variable they will focus on but the task also relies on them making sense of:

- which line relates to which function;
- the particular notation adopted by the tool (i.e. **f17**(x)= 5•x+2);
- and the range of values of x that is automatically plotted.

Consequently, although the task enables students to be 'successful' in that they can notice the most obvious generalizations – that the value of 'm' controls the appearance of the graph and that greater values of 'm' results in steeper lines – the task did not provide opportunities for students to link this with other representations in the technology, in particular the table of values, which would have enabled a deeper justification[5].

By contrast, even though Task 2 involved more representations on the screen that were dynamically linked, each animation generated an example for which the links between the representations had been made visually explicit. By looking at fewer linear functions in a greater depth, it is possible for students to recognize key features within each representation and, therefore be better placed to be able to see the connections between them. This presents a tension for the designer. Does she offer a task environment that gives a global view of the mathematical domain using multiply-linked representations and then 'zoom' in on particular features to reveal particular variant and invariant properties or does she offer a fo-

[5] The teacher did begin the lesson by reminding the students that, in order to generate each graph, the computer used a 'function machine' - an idea and representation that was familiar to the students.

cused view and subsequently 'pan-out' to support the student to connect the variant and invariant properties within the dynamically connected representations.

This highlights the complexity of the initial task design process in defining the mathematical domain that a task is intended to address, the nature of the initial example space and the intended user pathway through this space that incorporates different 'instrument utilization schema' (Verillon and Rabardel 1995). Increasingly, when designing tasks within dynamic mathematical environments, designers are including follow-on tasks away from the technology that support students to make more explicit links with the formal paper and pencil methods. For example, in the technology-mediated aspects of Task 2, which emphasizes the mathematical content of position-time graphs and the concept of speed, it was necessary to provide accompanying tasks away from the computer. These tasks required students to work flexibly from different mathematical starting points to develop a complete set of mathematical representations and support them to work fluently between these representations. For example, given some key values within the table of values, could they construct the related equation and graph?

Design principles – technology and tasks

It is important to (re)state that Task 1 was designed by a teacher for use in her own classroom as part of her early experiences with a new software tool. This example has been selected here as it typifies a genre of tasks that have been prevalent within technology use in English classrooms. However, although prevalent, this task approach has not been widely used and, on reflection, the post-priori analysis provided by the task design rubric may offer some insight into why such tasks have not become embedded within localised schemes of work. The very open nature of the task coupled with the resulting display of the software may have been sufficient for students to draw a broad conclusion, but there may have been insufficient direction in the task design to draw students' direct attention to key features, that is to support them to 'notice' important aspects of the graph. Interestingly, the functionality is present in the software to support this further work, for example, to reveal the table of values. However, in the teacher's early lesson design, she was either unaware or chose not to use this representation within the task.

By comparison, Task 2 was designed by a team that involved software designers, researchers and teachers over several years, and the task included multiple dynamic representations of several particular functions. The analysis of Task 2 using the design rubric, supported by evidence of students' outcomes suggest that the cognitive load of Task 2 was manageable for the students. In some ways this could be seen as an example of *discrepancy potential*, as described by Leung and Bolite-Frank (for publication 2015), whereby the limitations of one tool might be complemented by the affordances of another. The more contained example space in

Task 2 enabled students to be successful in their early work, but it may also have constrained some students from making other mathematical insights, for example exploring non-linear functions.

This highlights a tension when designers choose whether designing a task in an existing mathematical technological environment or to create a completely new digital space that is wholly 'bespoke' for its intended mathematical purposes. Given the multitude of existing tools that might be suited to early explorations of linear functions (Graphing calculators, Geogebra, Autograph, TI-Nspire, The Geometer's Sketchpad, SimCalc..), why create another environment?

The task design rubric includes an important consideration that seems critical to this early consideration in the design process, 'ascertaining the forms of interaction with the tool that reveal the desired variant/invariant properties'. This requires a deep knowledge of the tool's mathematical affordances and constraints. For example, if, as in Task 1, we choose to explore linear functions using TI-Nspire handhelds, there are many alternative tasks that could be designed to explore gradient and intercept properties. The software file could be pre-written with 'm' and 'c' predefined and the table of values visible. Students could have a more directed task in which they are instructed to change particular values (i.e. vary the value of 'm' by dragging an on-screen slider) to meet certain constraints and to observe particular features.

The resolution of this dilemma involves many considerations that include the need for the designer's deep understanding of the mathematical content appropriate to the students, its representations and connections alongside a level of familiarity with affordances and constraints of existing software tools. However, repeated research studies have shown over many years that, as it is a teacher's principle role to be a task designer, it is important that teachers have opportunities to work alongside more experienced colleagues, researchers and task designers to develop this aspect of their role (Noss, Sutherland, and Hoyles 1991; Clark-Wilson 2008; Artigue 1998).

Implications and further research

In concluding this chapter, it is important to highlight one aspect of the task design rubric for which there was insufficient data from the post-priori analysis of the two tasks to draw any substantial conclusions. This concerns the forms of interaction between teacher and students to support mathematical generalizations. These interactions might be evident in teachers' lesson designs (i.e. a 'lesson plan'), but can only be robustly researched through lesson observations and interviews. Within English teaching practices, it is most common for teachers to share teaching resources with each other and, in the case of digital lesson resources this is usually the software file and/or the task 'idea'. It is far less common for these resources to include the teacher's narrative to accompany how a task is introduced, developed and assessed. Hence the 'blank' page start with a digital tool adopted by the first example is a common one. It required little advanced preparation, that is, the software file did not need to be made available to students via the school network for the beginning of the lesson.

Other studies have revealed the subtleties of the teacher's role within technology-mediated lessons of this type as it demands a high level of teacher interaction, not just with the students but also involving the software tool itself (Aldon 2011; Clark-Wilson 2010). The teacher is required to have a depth of instrumentalisation with the software such that she can select particular cases, change and display key features in order to support the discourse such that generalizations can emerge, be formalised and ultimately proven.

The nature of teachers' 'mathematical pedagogic practices' whilst designing and teaching technology-mediated lessons is a research focus for a 3-year study in England, funded by the Nuffield Foundation[6] and co-directed by Celia Hoyles and myself. Set in the context of the Cornerstone Maths project, we have a adopted a lesson study approach to the design, implementation and evaluation of 'landmark activities' (Clark-Wilson, Hoyles, and Noss 2015) within each curriculum unit that aims to articulate teachers' practices.

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[6] http://www.nuffieldfoundation.org/developing-teachers-mathematical-knowledge-using-digital-technology

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