Integral Sliding Mode Control for Markovian Jump T-S Fuzzy Descriptor Systems Based on the Super-Twisting Algorithm

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Abstract: This paper investigates integral sliding mode control problems for Markovian jump T-S fuzzy descriptor systems via the super-twisting algorithm. A new integral sliding surface which is continuous is constructed and an integral sliding mode control scheme based on a variable gain super-twisting algorithm is presented to guarantee the well-posedness of the state trajectories between two consecutive switchings. The stability of the sliding motion is analyzed by considering the descriptor redundancy and the properties of fuzzy membership functions. It is shown that the proposed variable gain super-twisting algorithm is an extension of the classical single-input case to the multi-input case. Finally, a bio-economic system is numerically simulated to verify the merits of the method proposed.

1. Introduction

Sliding mode control originated from the former Soviet Union in the mid-1950s. It is an effective robust control method that tailors the dynamics of the system by an appropriate choice of switched control [1, 2, 3]. Since its inception, it has found wide practical application due to its attractive features including ease of implementation, fast response and insensitivity to matched uncertainties and disturbances. In general the design of a sliding mode control scheme involves two steps: the first step is to construct a sliding surface such that the dynamics when the state trajectories of the system are confined to the custom-defined sliding surface are stable and exhibit desirable performance (sliding phase); the second step is to design a control law to drive the state trajectories to the custom-defined sliding surface in finite time and and ensure they remain there subsequently (reaching phase). As the insensitivity to matched uncertainties only holds in the sliding phase, it is important to shorten or even eliminate the reaching phase. To this end, integral sliding mode control has been proposed to compensate exactly the matched uncertainties from the initial time instant. With increasing understanding of the real-world model informing model development, significant interest has been paid to the development of sliding mode control for diverse complex systems [4, 5, 6, 7, 8, 9]. Among this work, T-S fuzzy systems and Markovian jump systems are two representative classes of dynamical systems. Over the past decades, many meaningful results have been published on the sliding mode control of either T-S fuzzy systems [10, 11, 12, 13] or Markovian jump systems [14, 15, 16, 17, 18, 19, 20]. However, relatively little attention has been focused on research into sliding mode control for composite systems comprising the two classes of systems, namely Markovian jump T-S fuzzy systems [21, 22], despite their wide application [23, 24, 25, 26].

A descriptor system is a more compact and integrated description of a real-world model than the normal system ($E = I$) representation and its redundancy is widely exploited to improve the performance of dynamical systems [27, 28, 29]. Recently, sliding mode control has been studied for either T-S fuzzy descriptor systems [30] or Markovian jump descriptor systems [31, 32, 33]. There are, however, interesting issues outstanding. For example, neither Non-PDC sliding mode controller design for T-S fuzzy systems nor the analysis of the continuity of the switching function at the switching instants for Markovian jump descriptor systems have been mentioned. It is pointed out in [34] that the states of the Markovian jump descriptor systems will jump at the switching instants due to inconsistency of the initial conditions [35], which is a distinguishing feature of Markovian jump normal systems. The authors in [36] construct a continuous switching function for Markovian jump normal systems. But for Markovian jump descriptor systems, whether the states jump will affect the switching function and this phenomenon has not been considered. As shown in [37, 38, 39], Markovian jump T-S fuzzy descriptor systems have been widely used to model mechanical systems, biological systems, circuit systems and so on, where representative examples are the inverted pendulum controlled by a DC motor and constrained mechanical systems. Therefore, the first motivation is to construct an appropriate switching function for Markovian jump T-S fuzzy descriptor systems such that the sliding mode controller has a similar structure to the Non-PDC controller [40] and the effect of state jumps on the switching function can be explicitly analyzed.

Since the dynamical order of descriptor systems is usually less than the system dimension, the states will contain high-order derivatives of the control inputs. The states will therefore be impulsive unless each control input is continuously differentiable. Besides, a discontinuous control input may excite the inherent high-frequency chattering of the system, where especially for mechanical and biological systems, such chattering may be detrimental to the system performance and may even destroy the system. To circumvent this difficulty, [41] presents a high-order sliding mode control method for descriptor systems by artificially introducing integrators. It is noted that only linear descriptor systems with uncertainties are treated. Moreover, the high-order sliding mode controller developed is for single-input systems and the derivative of the switching function is required. It is well known that the super-twisting algorithm is the only second order sliding mode controller [42] which does not need the derivative of switching function. Since its introduction in [43], geometric, homogeneity and Lyapunov methods have been proposed to derive finite-time convergence and demonstrate robustness of the super-twisting algorithm. Among these, the Lyapunov method is popular because Lyapunov stability theory for linear systems can be recalled, and the finite-time convergence and robustness can also be achieved even when the super-twisting algorithm is not homogeneous [44]. Recently, a multivariable super-twisting algorithm has been presented [45] to extend the classical case [46] where the control gains are constant. In practice, if the disturbances are also bounded by functions of the states, then the constant gain super-twisting algorithm is not applicable. Furthermore, when the system is driven by multiple inputs, the variable gain super-twisting algorithm [44] cannot also be applied. As a result, the second motivation is to construct a variable gain super-twisting algorithm to guarantee that the state trajectories of descriptor systems with multiple inputs are well defined.

This paper is concerned with super-twisting controller design for Markovian jump T-S fuzzy descriptor systems via integral sliding modes. A new integral sliding surface is first defined to induce the Non-PDC sliding mode controller. Then the stability of the sliding motion is analyzed by making use of the descriptor redundancy and the properties of fuzzy membership functions. Furthermore, a variable gain super-twisting algorithm is developed for multi-input systems to ensure the well-posedness of the state trajectories of Markovian jump T-S fuzzy descriptor systems. Finally, a bio-economic system is provided to verify the results obtained. The main contributions are threefold: 1) a new continuous switching function is constructed which will result in a Non-PDC sliding mode controller; 2) a variable gain super-twisting algorithm is proposed for multi-input systems; 3) a method based on the equivalence of two sets is presented to remove the nonstrict matrix inequality often encountered in the stability analysis of descriptor systems.

The rest of this paper is organized as follows: Section 2 states the problem under consideration and presents some useful lemmas; in Section 3, the integral sliding mode control scheme is developed for Markovian jump T-S fuzzy descriptor systems based on the super-twisting algorithm; a bio-economic system is provided in Section 4 and Section 5 concludes the paper.

1.1. Notations

The notations used throughout this paper are quite standard. $\mathbb{R}^n$ represents the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ represents the set of all $m \times n$ real matrices. $\mathbb{N}^+$ denotes the set of positive integers. The symbol $(\Omega, \mathbb{F}, \{\mathbb{F}_t\}, \mathbb{P})$ is a complete probability space with a filtration $\{\mathbb{F}_t\}$ satisfying the usual conditions (i.e. it is right continuous and contains all $\mathbb{P}$-null sets) and $\mathbb{E}\{\cdot\}$ is the expectation operator. The superscripts T and -1 denote matrix transposition and matrix inverse respectively. $|\cdot|$ represents the absolute value of a scalar and $\|\cdot\|$ denotes the Euclidean norm of a vector or the induced norm of a matrix. The notation $P > 0$ ($P \geq 0$) implies that P is a real symmetric and positive definite (semi-positive definite) matrix. For any square matrix A, $\mathbf{He}A$ stands for $A + A^T$, furthermore, when A is symmetric, $\lambda_{min}(A)$ and $\lambda_{max}(A)$ denote the minimum and maximum eigenvalue of A respectively. $\mathrm{diag}\{A_1, A_2, \cdots, A_n\}$ denotes a block diagonal matrix with $A_1, A_2, \cdots, A_n$ in its diagonal position. The star $\star$ in a matrix block implies that it can be induced by symmetric position. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Preliminaries and Problem Formulation

Fix a probability space $(\Omega, \mathbb{F}, \mathbb{P})$ and consider a Markovian jump nonlinear descriptor system which can be represented by the following Markovian jump T-S fuzzy descriptor system

Plant Rule i: IF $z_1(t)$ is F_{i1} and $z_2(t)$ is F_{i2} and $\cdots$ and $z_s(t)$ is F_{is} **THEN**

$$E\dot{x}(t) = A_{i(r(t))}x(t) + B_{r(t)}(u(t) + f_{r(t)}(x(t))) \tag{1}$$

where $i = 1, 2, \cdots, r$, $z_1(t), z_2(t), \cdots, z_s(t)$ are the premise variables, $F_{i1}, F_{i2}, \cdots, F_{is}$ are the fuzzy sets. $x(t) \in \mathbb{R}^n$ is the states, $u(t) \in \mathbb{R}^m$ is the inputs and $f_{r(t)}(x(t))$ is a unknown continuously differential vector function satisfying

$$\left\| \frac{df_{r(t)}(x(t))}{dt} \right\| \leq \rho_{r(t)}(x(t)) \tag{2}$$

where $\rho_{r(t)}(x(t))$ is a known continuous positive scalar function. $r(t)$ is a continuous-time Markov process whose states belong to a finite set $\ell = \{1, 2, \cdots, N\}$ with $N \in \mathbb{N}^+$. The transition probability between two consecutive switchings of $r(t)$ is defined by

$$\mathbb{P}\{r(t+\delta)=q|r(t)=p\}=\begin{cases}\lambda_{pq}\delta+o(\delta), & \text{when } r(t) \text{ jumps from } p \text{ to } q,\\ 1+\lambda_{pp}\delta+o(\delta), & \text{otherwise.}\end{cases} \tag{3}$$

where $\lambda_{pq}\geq 0, p\neq q$, $\lambda_{pp}=-\sum_{q\in\ell,q\neq p}\lambda_{pq}$ and $o(\delta)$ is an infinitesimal such that $\lim_{\delta\to 0}\frac{o(\delta)}{\delta}=0$. The generator matrix is defined by $\Lambda=[\lambda_{pq}]_{N\times N}$. Assume that the jumping instant sequence is $\{\tau_k, k=0,1,\cdots\}$ with $0=\tau_0<\tau_1<\cdots<\tau_k<\cdots$, then the mode $r(t)$ is a constant in each interval $[t_k,t_{k+1})$. To simply the notation, for a fixed $r(t)=p\in\ell$, matrices $A_{i(r(t))}$, $B_{i(r(t))}$ and function $f_{r(t)}(x(t))$ are respectively denoted as A_i^p, B^p, and $f^p(x(t))$, throughout this paper, and the remaining notation follows this symbolism. E, A_i^p and B^p are known matrices of compatible dimensions with $\mathrm{rank}E=r_e\leq n$ and B^p of full column rank. It is assumed that the Markov process $r(t)$ is recurrent, irreducible and aperiodic, then the solution of the differential equation $\dot{\pi}(t)=\Lambda^T\pi$ will asymptotically tend to a constant vector $\pi=[\pi_1\ \pi_2\ \cdots\ \pi_N]^T$, where the initial probability distribution is $\pi_0=[\pi_{01}\ \pi_{02}\ \cdots\ \pi_{0N}]^T$ and $\pi_{0i}=\mathbb{P}\{r(0)=i\}$.

By fuzzy blending, the overall Markovian jump T-S fuzzy system is inferred as follows

$$E\dot{x}(t)=\sum_{i=1}^{r}h_i(z(t))(A_i^px(t)+B^p(u(t)+f^p(x(t)))) \tag{4}$$

where $h_i(z(t))$ is the membership function which satisfies $h_i(z(t))\geq 0$ and $\sum_{i=1}^{r}h_i(z(t))=1$. In the sequel, unless it is indicated clearly, for a series of matrices C_i, $i=1,2,\cdots,r$, the notation $C(h)$ is used to denote $\sum_{i=1}^{r}h_i(z(t))C_i$.

Now, the following lemmas are recalled which will be essential to the following development.

Lemma 1. [47] If the following inequalities hold:

$$\Delta_{ii}<0,\ \frac{1}{r-1}\Delta_{ii}+\frac{1}{2}(\Delta_{ij}+\Delta_{ji})<0, i,j=1,2,\cdots,r,\ i\neq j$$

then the following matrix inequalities hold

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i(t)\alpha_j(t)\Delta_{ij}<0$$

where $\alpha_i(t)\geq 0$ and $\sum_{i=1}^{r}\alpha_i(t)=1$.

Lemma 2. [48] Suppose a piecewise continuous matrix $A(t)\in\mathbb{R}^{n\times n}$ and a matrix $X\in\mathbb{R}^{n\times n}$ satisfy the following inequality

$$A(t)^TX+X^TA(t)\leq-\alpha I$$

for all t and some positive number α. Then the following statements hold:

1) $A(t)$ is invertible;

2) $\|A^{-1}(t)\|\leq a$ for some $a>0$.

Lemma 3. [49] Let P be symmetric and $E = E_L E_R^T$ with $E_L \in \mathbb{R}^{n\times r_e}$ and $E_R \in \mathbb{R}^{n\times r_e}$ full column rank such that $E_R^T P E_R > 0$ and Φ is nonsingular, $\mathcal{U}^T \in \mathbb{R}^{n\times(n-r_e)}$ and $\mathcal{V} \in \mathbb{R}^{n\times(n-r_e)}$ have full column rank and respectively span the null space of E^T and E, then $PE^T + \mathcal{V}\Phi\mathcal{U}$ is nonsingular and its inverse is expressed by

$$\left(PE^T + \mathcal{V}\Phi\mathcal{U}\right)^{-1} = QE + \mathcal{U}^T\Psi\mathcal{V}^T \tag{5}$$

where Q is a symmetric matrix and Ψ is a nonsingular matrix with $\Psi = \left(\mathcal{U}\mathcal{U}^T\right)^{-1}\Phi^{-1}\left(\mathcal{V}^T\mathcal{V}\right)^{-1}$, $E_L^T Q E_L = \left(E_R^T P E_R\right)^{-1}$.

Lemma 4. Let U and V be two orthogonal matrices such that

$$UEV = \begin{bmatrix} \Sigma & 0 \\ 0 & 0 \end{bmatrix} \tag{6}$$

where $\Sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \cdots, \sigma_{r_e}\}$ with $\sigma_i > 0$, $i = 1, 2, \cdots, r_e$, $U = \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}$, $V = \begin{bmatrix} V_1 & V_2 \end{bmatrix}$. Define $\bar{U} = \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & I_{n-r_e} \end{bmatrix} U$ and the following sets

$$\mathcal{S}_1 = \{\mathcal{P} \in \mathbb{R}^{n\times n} : E\mathcal{P} = \mathcal{P}^T E^T \geq 0, \mathrm{rank}\mathcal{P} = n\}$$

$$\mathcal{S}_2 = \{\mathcal{P} \in \mathbb{R}^{n\times n} : \mathcal{P} = PE^T + V_2\Phi U_2, P = V\begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}V^T > 0, \mathrm{rank}\Phi = n - r_e\}$$

$$\mathcal{S}_3 = \{\mathcal{Q} \in \mathbb{R}^{n\times n} : E^T\mathcal{Q} = \mathcal{Q}^T E \geq 0, \mathrm{rank}\mathcal{Q} = n\}$$

$$\mathcal{S}_4 = \{\mathcal{Q} \in \mathbb{R}^{n\times n} : \mathcal{Q} = QE + U_2^T\Psi V_2^T, Q = \bar{U}^T\begin{bmatrix} Q_1 & Q_2^T \\ Q_2 & Q_3 \end{bmatrix}\bar{U} > 0, \mathrm{rank}\Psi = n - r_e\}$$

Then the following statements are satisfied:

1) $\mathcal{S}_1$ and $\mathcal{S}_3$ are respectively equivalent to $\mathcal{S}_2$ and $\mathcal{S}_4$;

2) for any $\mathcal{P} \in \mathcal{S}_1 (\mathcal{P} \in \mathcal{S}_2)$, there exists $\mathcal{Q} \in \mathcal{S}_3 (\mathcal{P} \in \mathcal{S}_4)$ such that $\mathcal{P}^{-1} = \mathcal{Q}$, and vice versa. Furthermore, $P_1^{-1} = Q_1$ and $\Phi^{-1} = \Psi$.

Proof. 1) On one hand, if $\mathcal{P} \in \mathcal{S}_1$, it can be calculated that

$$\begin{aligned} V^T\mathcal{P}\bar{U}^T &= \begin{bmatrix} P_1 & 0 \\ P_2 & P_4 \end{bmatrix} = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}\begin{bmatrix} I_{r_e} & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I_{n-r_e} \end{bmatrix} P_4 \begin{bmatrix} 0 & I_{n-r_e} \end{bmatrix} \\ &= \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix} V^T E^T \bar{U}^T + \begin{bmatrix} 0 \\ I_{n-r_e} \end{bmatrix} P_4 \begin{bmatrix} 0 & I_{n-r_e} \end{bmatrix} \end{aligned} \tag{7}$$

Since $P_1 > 0$, P_4 is nonsingular and P_3 is an arbitrary matrix, then $\begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix} > 0$ holds by appropriately choosing P_3. By setting $\Phi = P_4$, thus, $\mathcal{P} \in \mathcal{S}_2$.

On the other hand, when $\mathcal{P} \in \mathcal{S}_2$, from

$$E\mathcal{P} = \mathcal{P}^T E^T = EPE^T \geq 0$$

and

$$\text{rank}\mathcal{P} = \text{rank}V \begin{bmatrix} P_1 & 0 \\ P_2 & \Phi \end{bmatrix} \bar{U}^{-T} = n$$

it follows that $\mathcal{P} \in \mathcal{S}_1$. The equivalence of sets $\mathcal{S}_3$ and $\mathcal{S}_4$ can be derived analogously.

2) Based on (6), E_L and E_R can be selected as $E_L = \bar{U}^{-1} \begin{bmatrix} I_{r_e} \\ 0 \end{bmatrix}$ and $E_R = \begin{bmatrix} I_{r_e} & 0 \end{bmatrix} V^T$. For $\mathcal{P} \in \mathcal{S}_1 (\mathcal{P} \in \mathcal{S}_2)$, $E_R^T P E_R = P_1 > 0$, then

$$\mathcal{P}^{-1} = QE + U_2^T \Phi^{-1} V_2^T$$

where $\begin{bmatrix} I_{r_e} & 0 \end{bmatrix} \bar{U}^{-T} Q \bar{U}^{-1} \begin{bmatrix} I_{r_e} \\ 0 \end{bmatrix} = P_1^{-1}$. As a result, there exists a matrix $\mathcal{Q} \in \mathcal{S}_3 (\mathcal{P} \in \mathcal{S}_4)$ with $Q_1 = P_1^{-1}$ and $\Psi = \Phi^{-1}$ such that $\mathcal{P}^{-1} = \mathcal{Q}$. For any $\mathcal{Q} \in \mathcal{S}_3$, a matrix $\mathcal{P} \in \mathcal{S}_1$ can also be found such that $\mathcal{Q}^{-1} = \mathcal{P}$ by following the similar line above. $\square$

Remark 1. The advantages of Lemma 4 are two fold: 1) it is possible to remove the non-strict linear matrix inequalities $E\mathcal{P} = \mathcal{P}^T E^T \geq 0$ and $E^T \mathcal{Q} = \mathcal{Q}^T E \geq 0$ since $\mathcal{P}$ and $\mathcal{Q}$ can be respectively replaced by the corresponding elements in $\mathcal{S}_2$ and $\mathcal{S}_4$. Furthermore, the inverses of the elements in $\mathcal{S}_1$ ($\mathcal{S}_2$) and $\mathcal{S}_3$ ($\mathcal{S}_4$) are also explicitly described by virtue of orthogonal matrices and a diagonal matrix, which is numerically reliable; 2) the redundancy of descriptor system and the property of fuzzy membership functions can be exploited to introduce some extra matrices.

The objectives of this paper are to propose a variable gain super-twisting algorithm for multi-input systems and design an integral sliding mode control strategy for the Markovian jump T-S fuzzy descriptor system (4) such that the resultant closed-loop system is asymptotically stable.

3. Sliding Mode Controller Development

In this section, a multiple Lyapunov function method is introduced to analyze the stability of the sliding motion, and a variable gain super-twisting algorithm for multi-input systems is developed to generate a continuous control input.

3.1. Construction of Sliding Surface

The sliding surface for the Markovian jump T-S fuzzy descriptor system (4) is defined as

$$\mathcal{S} = \{x(t) : s(t) = 0\} \tag{8}$$

where the switching function is constructed in each time interval $[\tau_k, \tau_{k+1})$ with mode $r(t) = p$ by

$$s(t) = SEx(t) - SEx(\tau_k) - S \int_{\tau_k}^{t} \sum_{i=1}^{r} h_i(z(\psi))(A_i^p + B^p K_i^p (\sum_{i=1}^{r} h_i(z(\psi)) Y_i^p)^{-1}) x(\psi) d\psi \tag{9}$$

where $S \in \mathbb{R}^{m \times n}$ is a known matrix such that SB^p is nonsingular. K_i^p and Y_i^p are design coefficients to be determined in the sequel.

Remark 2. Let τ_k^- be the instant immediately before the switching instant τ_k, and τ_k^+ denote the instant immediately after τ_k. It is known from [34] that at the switching instants τ_k, $x(t)$ will jump to satisfy the consistent initial condition but $Ex(t)$ will satisfy $Ex(\tau_k) = Ex(\tau_k^-) = Ex(\tau_k^+)$. It is assumed that the qth mode is active in the time interval $[\tau_{k-1}, \tau_k)$ and the pth mode in time interval $[\tau_k, \tau_{k+1})$. When the ideal sliding mode is exhibited, it follows that

$$\begin{aligned}
s(\tau_k^-) &= SEx(\tau_k^-) - SEx(\tau_{k-1}) - S\int_{\tau_{k-1}}^{\tau_k^-} (A^q(h) + B^q K^q(h)(Y^q(h))^{-1})x(\psi)d\psi \\
&= SEx(\tau_k^-) - SEx(\tau_{k-1}) - S\int_{\tau_{k-1}}^{\tau_k^-} (A^q(h) + B^q K^q(h)(Y^q(h))^{-1})x(\psi)d\psi \\
&\quad - S\int_{\tau_k}^{\tau_k^+} (A^p(h) + B^p K^p(h)(Y^p(h))^{-1})x(\psi)d\psi \\
&= SEx(\tau_k^-) - SEx(\tau_k^-) - S\int_{\tau_k}^{\tau_k^+} (A^p(h) + B^p K^p(h)(Y^p(h))^{-1})x(\psi)d\psi \\
&= SEx(\tau_k^+) - SEx(\tau_k) - S\int_{\tau_k}^{\tau_k^+} (A^p(h) + B^p K^p(h)(Y^p(h))^{-1})x(\psi)d\psi \\
&= s(\tau_k^+)
\end{aligned} \tag{10}$$

which implies that the switching function is continuous at the switching instant τ_k. Therefore, it can be concluded that although the switching function is defined in each time interval $[\tau_k, \tau_{k+1})$ respectively, the overall switching function is continuous.

Based on (4) and (9), it follows that

$$\begin{aligned}
\dot{s}(t) &= SE\dot{x}(t) - S(A^p(h) + B^p K^p(h)(Y^p(h))^{-1})x(t) \\
&= SB^p(u(t) - K^p(h)(Y^p(h))^{-1}x(t) + f^p(x(t)))
\end{aligned} \tag{11}$$

When the ideal sliding mode occurs, it is necessary that $s(t) = 0$ and $\dot{s}(t) = 0$. It follows that the equivalent control law can be obtained from (11) as follows

$$u(t) = K^p(h)(Y^p(h))^{-1}x(t) - f^p(x(t)) \tag{12}$$

Therefore, the dynamics of the sliding motion can be obtained as

$$E\dot{x}(t) = (A^p(h) + B^p K^p(h)(Y^p(h))^{-1})x(t) \tag{13}$$

Remark 3. It is noted that $K^p(h)(Y^p(h))^{-1}$ is introduced in the switching function (9) instead of $K^p(h)$ in [30] so that the sliding motion behaves as for the system $E\dot{x}(t) = A^p(h)x(t) + B^p u(t)$ with the loop closed by the Non-PDC controller $u(t) = K^p(h)(Y^p(h))^{-1}x(t)$. It can be established from [40] that the introduction of $K^p(h)(Y^p(h))^{-1}$ enhances the solvability of the design coefficients in the switching function.

3.2. *Stability of Sliding Motion*

In the sequel, descriptor redundancy and properties of fuzzy membership functions are utilized to derive stability conditions for the sliding motion from a switched system perspective [34, 52].

Theorem 1. The Markovian jump T-S fuzzy descriptor system (4) has an asymptotically stable sliding motion (13) if there exist positive definite matrices $P_1^p \in \mathbb{R}^{r_e \times r_e}$, $P_{i3}^p \in \mathbb{R}^{(n-r_e)\times(n-r_e)}$, nonsingular matrices $\Psi_i^p \in \mathbb{R}^{(n-r_e)\times(n-r_e)}$, $Y_i^p \in \mathbb{R}^{n\times n}$, matrices $P_{i2}^p \in \mathbb{R}^{(n-r_e)\times r_e}$, $K_i^p \in \mathbb{R}^{m\times n}$ $i = 1, 2, \cdots, r$, and positive scalars ϵ_p, α, β, $p = 1, 2, \cdots, N$ such that the following matrix inequalities hold for each $i, j = 1, 2, \cdots, r,\ i \neq j, p = 1, 2, \cdots, N$

$$\begin{bmatrix} P_1^p & \star \\ P_{i2}^p & P_{i3}^p \end{bmatrix} > 0 \tag{14}$$

$$\Delta_{ii}^p < 0,\ \frac{1}{r-1}\Delta_{ii}^p + \frac{1}{2}\left(\Delta_{ij}^p + \Delta_{ji}^p\right) < 0 \tag{15}$$

$$P_1^p \leq \beta P_1^q \tag{16}$$

$$\sum_{p=1}^{N}\left((\beta-1)\left(-\lambda_{pp}\right) - \alpha\right)\pi_p < 0 \tag{17}$$

where $\mathcal{P}_i^p = V \begin{bmatrix} P_1^p & \left(P_{2i}^p\right)^T \\ P_{i2}^p & P_{i3}^p \end{bmatrix} V^T E^T + V_2 \Phi_i^p U_2$, and U, V are orthogonal matrices satisfying (6),

$$\Delta_{ij}^p = \begin{bmatrix} \mathbf{He}\left(A_i^p Y_j^p + B^p K_i^p\right) & \star & \star \\ Y_i^p - \mathcal{P}_i^p + \epsilon_p\left(A_i^p Y_j^p + B^p K_i^p\right) & -\epsilon_p \mathbf{He}\mathcal{P}_i^p & \star \\ \begin{bmatrix} I_{r_e} & 0 \end{bmatrix} V^T Y_i^p & 0 & -\frac{1}{\alpha}P_1^p \end{bmatrix}.$$

Proof. Based on Lemma 1 and (15), appropriate matrix manipulation yields

$$\begin{bmatrix} \mathbf{He}\left(A^p(h)Y^p(h) + B^p K^p(h)\right) + \alpha\left(Y^p(h)\right)^T V \begin{bmatrix} \left(P_1^p\right)^{-1} & 0 \\ 0 & 0 \end{bmatrix} V^T Y^p(h) & \star \\ Y^p(h) - \mathcal{P}^p(h) + \epsilon_p\left(A^p(h)Y^p(h) + B^p K^p(h)\right) & -\epsilon_p \mathbf{He}\mathcal{P}^p(h) \end{bmatrix} < 0 \tag{18}$$

Due to $\mathcal{P}^p(h) = V \begin{bmatrix} P_1^p & \left(P_2^p(h)\right)^T \\ P_2^p(h) & P_3^p(h) \end{bmatrix} V^T E^T + V_2 \Phi^p(h) U_2 \in \mathcal{S}_2$, it follows from Lemma 4 that

$$\left(\mathcal{P}^p(h)\right)^{-1} = \bar{U}^T \begin{bmatrix} \left(P_1^p\right)^{-1} & * \\ * & * \end{bmatrix} \bar{U} E + U_2^T \left(\Phi^p(h)\right)^{-1} V_2^T \in \mathcal{S}_3 \tag{19}$$

where $*$ denotes the term that has no effect on the subsequent derivation.

From (19), it can be computed that

$$E^T\left(\mathcal{P}^p(h)\right)^{-1} = \left(\mathcal{P}^p(h)\right)^{-T} E \geq 0 \tag{20}$$

and

$$E^T\left(\mathcal{P}^p(h)\right)^{-1} = V \begin{bmatrix} \left(P_1^p\right)^{-1} & 0 \\ 0 & 0 \end{bmatrix} V^T \tag{21}$$

Noting (21), pre- and post-multiplying (18) by $\mathrm{diag}\{\left(Y^p(h)\right)^{-T}, \left(P^p(h)\right)^{-1}\}$ yields

$$\begin{gathered} \begin{bmatrix} \alpha E^T\left(\mathcal{P}^p(h)\right)^{-1} & \left(\mathcal{P}^p(h)\right)^{-T} \\ \left(\mathcal{P}^p(h)\right)^{-1} & 0 \end{bmatrix} \\ + \mathbf{He}\left(\begin{bmatrix} \left(Y^p(h)\right)^{-T} \\ \epsilon_p\left(\mathcal{P}^p(h)\right)^{-1} \end{bmatrix} \begin{bmatrix} A^p(h) + B^p K^p(h)\left(Y^p(h)\right)^{-1} & -I \end{bmatrix}\right) < 0 \end{gathered} \tag{22}$$

By Finsler's Lemma [51], it can be obtained that

$$\begin{bmatrix} y_1^T & y_2^T \end{bmatrix} \begin{bmatrix} \alpha E^T (\mathcal{P}^p(h))^{-1} & (\mathcal{P}^p(h))^{-T} \\ (\mathcal{P}^p(h))^{-1} & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} < 0 \tag{23}$$

for $\forall \begin{bmatrix} y_1^T \ y_2^T \end{bmatrix} \neq 0$ satisfying

$$\begin{bmatrix} A^p(h) + B^p K^p(h) (Y^p(h))^{-1} & -I \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = 0 \tag{24}$$

Substituting (24) into (23) yields

$$\mathbf{He}\left((\mathcal{P}^p(h))^{-T} \left(A^p(h) + B^p K^p(h) (Y^p(h))^{-1} \right) \right) + \alpha E^T (\mathcal{P}^p(h))^{-1} < 0 \tag{25}$$

Since U, V are orthogonal matrices satisfying (6), define

$$\begin{aligned} \bar{U} &= \begin{bmatrix} \Sigma^{-1} & 0 \\ 0 & I_{n-r_e} \end{bmatrix} U, \ x(t) = V \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \\ \bar{U} \left(A^p(h) + B^p K^p(h) (Y^p(h))^{-1} \right) V &= \begin{bmatrix} \mathcal{A}_1^p & \mathcal{A}_2^p \\ \mathcal{A}_3^p & \mathcal{A}_4^p \end{bmatrix} \end{aligned} \tag{26}$$

Then, (25) can be rewritten in the form of the following block matrix

$$\begin{bmatrix} * & * \\ * & \mathbf{He}\left((\mathcal{A}_4^p)^T (\Phi^p(h))^{-1} \right) \end{bmatrix} < 0$$

By Lemma 2, $\mathcal{A}_4^p$ is invertible and there exists a constant $0 < a < \infty$ satisfying $\| (\mathcal{A}_4^p)^{-1} \| \leq a$. Then $x_2(t) = - (\mathcal{A}_4^p)^{-1} \mathcal{A}_3^p x_1(t)$.

On the basis of (20) and (21), the Lyapunov function can be defined as follows

$$V_{r(t)}(x_1(t)) = x_1^T(t) \left(P_{1(r(t))} \right)^{-1} x_1(t) = x^T(t) E^T \left(\mathcal{P}_{r(t)}(h) \right)^{-1} x(t) \tag{27}$$

When $t \in [\tau_k, \tau_{k+1})$, then the time derivative of Lyapunov function (27) along with the system (4) is

$$\begin{aligned} \dot{V}_{r(t)}(x_1(t)) &= \dot{x}^T(t) E^T \left(\mathcal{P}_{r(t)}(h) \right)^{-1} x(t) + x^T(t) E^T \left(\mathcal{P}_{r(t)}(h) \right)^{-1} \dot{x}(t) \\ &\quad + x^T(t) \frac{d}{dt} \left(E^T \left(\mathcal{P}_{r(t)}(h) \right)^{-1} \right) x(t) \\ &= \dot{x}^T(t) E^T \left(\mathcal{P}_{r(t)}(h) \right)^{-1} x(t) + x^T(t) \left(\mathcal{P}_{r(t)}(h) \right)^{-T} E \dot{x}(t) \\ &= x^T(t) \left(\mathbf{He}\left(\left(\mathcal{P}_{r(t)}(h) \right)^{-T} \left(A_{r(t)}(h) + B_{r(t)} K_{r(t)}(h) \left(Y_{r(t)}(h) \right)^{-1} \right) \right) \right) x(t) \\ &\leq -\alpha V_{r(t)}(x_1(t)) \end{aligned} \tag{28}$$

where (21) and (25) respectively are used in the second equation and the last inequality.

For $t \in [\tau_k, \tau_{k+1})$, integrating (28) from τ_k to t implies

$$V_{r(t)}(x_1(t)) \leq e^{-\alpha(t-\tau_k)} V_{r(t)}(x_1(\tau_k)) \tag{29}$$

When the mode switches at time instant τ_k, it follows from (16) that

$$V_{r(\tau_k)}(x_1(\tau_k)) \leq \beta V_{r(\tau_k^-)}(x_1(\tau_k^-)) \tag{30}$$

Based on (17), (29) and (30), the following can be obtained by following the arguments in [34]

$$\mathbb{E}\{\|x_1(t)\|^2\} \leq \frac{\max_{p\in\ell} \lambda_{max}((P_1^p)^{-1})}{\min_{p\in\ell} \lambda_{min}((P_1^p)^{-1})} e^{(\sum_{i=1}^{N}((\beta-1)(-\lambda_{pp})-\alpha)\pi_p)t}\|x_1(0)\|^2 \tag{31}$$

Due to $\| (\mathcal{A}_4^p)^{-1} \| \leq a$ and $x_2(t) = -(\mathcal{A}_4^p)^{-1} \mathcal{A}_3^p x_1(t)$, the following can be obtained

$$\mathbb{E}\{\|x_2(t)\|^2\} \leq a^2 \max_{p\in\ell} \|\mathcal{A}_3^p\|^2 \frac{\max_{p\in\ell} \lambda_{max}((P_1^p)^{-1})}{\min_{p\in\ell} \lambda_{min}((P_1^p)^{-1})} e^{(\sum_{i=1}^{N}((\beta-1)(-\lambda_{pp})-\alpha)\pi_p)t}\|x_1(0)\|^2 \tag{32}$$

As t tends to ∞, by (31) and (32), it follows that $\lim_{t\to\infty} \mathbb{E}\{\|x(t)\|^2\} = 0$. As a consequence, the sliding motion (13) is asymptotically stable. □

Remark 4. Although a switched system method [34, 52] is utilized to deal with the switching of the sliding motion (13) in Theorem 1, it is straightforward to obtain the corresponding stability results by tackling the switching with the method from [25, 26].

Remark 5. From the proof of Theorem 1, it can be seen that both descriptor redundancy and properties of fuzzy membership functions are exploited to derive the asymptotic stability of the sliding motion (13). Moreover, the utilization of descriptor redundancy and properties of fuzzy membership function can be understood from two perspectives: on the one hand, based on the redundancy of the matrix E, matrices $P_2^p(h)$, $P_3^p(h)$, $\Phi^p(h)$ can be selected freely and the matrix $E^T(\mathcal{P}^p(h))^{-1}$ only depends on V and P_1; on the other hand, due to the properties of fuzzy membership functions, P_1 is set to be independent of the fuzzy membership function to avoid the derivative of the fuzzy membership function appearing. Matrices $P_2^p(h)$, $P_3^p(h)$, $\Phi^p(h)$ are selected to involve one sum of fuzzy membership functions [50] to maintain a trade-off between reducing conservatism and convenience of calculation. Discussion of the concept of the multiple sum and the reduction in conservatism can be found in [50], and is omitted since it is not the main concern of this paper. The merit of the utilization of descriptor redundancy and the properties of fuzzy membership functions is that more slack matrices can be introduced. This will enhance the solvability of the matrix inequalities in Theorem 1. To be clear, if the methods in [26, 30] are used where the descriptor redundancy and properties of fuzzy membership functions are not exploited, then the Markovian jump T-S fuzzy descriptor system (4) has an asymptotically stable sliding motion if there exist positive definite matrices $\tilde{P}_1^p \in \mathbb{R}^{r_e\times r_e}$, $\tilde{P}_3^p \in \mathbb{R}^{(n-r_e)\times(n-r_e)}$, nonsingular matrix $\tilde{\Psi}^p \in \mathbb{R}^{(n-r_e)\times(n-r_e)}$, matrices $\tilde{P}_2^p \in \mathbb{R}^{(n-r_e)\times r_e}$, $Z_i^p \in \mathbb{R}^{m\times n}$ $i = 1, 2, \cdots, r$, $p = 1, 2, \cdots, N$ and positive scalar $\tilde{\alpha}$, $\tilde{\beta}$, such that the following matrix inequalities hold for each $i, j = 1, 2, \cdots, r,\ i \neq j$, $p = 1, 2, \cdots, N$

$$\tilde{P}^p = \begin{bmatrix} \tilde{P}_1^p & \star \\ \tilde{P}_2^p & \tilde{P}_3^p \end{bmatrix} > 0 \tag{33}$$

$$\mathbf{He}\left(A_i^p\mathcal{P}^p + B^p Z_i^p\right) + \alpha(\mathcal{P}^p)^T E^T < 0 \tag{34}$$

$$\tilde{P}_1^p \leq \beta \tilde{P}_1^q \tag{35}$$

$$\sum_{p=1}^{N}\left(\left(\beta - 1\right)\left(-\lambda_{pp}\right) - \alpha\right)\pi_p < 0 \tag{36}$$

where $\mathcal{P}^p = VP^pV^TE^T + V_2\Phi^pU_2$, and U, V are orthogonal matrices satisfying (6). To show the benefits of the utilization of descriptor redundancy and properties of fuzzy membership functions, the following corollary is presented.

Corollary 1. If the matrix inequalities (33)-(36) are solvable with respect to $\tilde{P}^p$, $\tilde{\Psi}^p$, Z_i^p, $\tilde{\alpha}$, $\tilde{\beta}$, then there must exist positive definite matrices P_1^p, P_{i3}^p, nonsingular matrices Ψ_i^p, Y_i^p, matrices P_{i2}^p, K_i^p $i = 1, 2, \cdots, r$, and positive scalars ϵ_p, α, β, $p = 1, 2, \cdots, N$ satisfying matrix inequalities (14)-(17).

Proof. If matrix inequalities (33)-(36) have solutions, define $P_i^p = Y_i^p = \begin{bmatrix} \tilde{P}_1^p & \left(\tilde{P}_2^p\right)^T \\ \tilde{P}_2^p & \tilde{P}_3^p \end{bmatrix}$, $\Psi_i^p = \tilde{\Psi}^p$, $K_i^p = Z_i^p$, $\alpha = \tilde{\alpha}$, $\beta = \tilde{\beta}$. Then there must exist a sufficiently small positive scalar ϵ_p satisfying matrix inequalities (14)-(17). Therefore, the utilization of descriptor redundancy and properties of fuzzy membership functions will benefit the solvability of the matrix inequalities. □

Remark 6. When $Y^p(h) = \mathcal{P}^p(h)$, the conditions in Theorem 1 are also satisfied with $\Delta_{ij} = \mathbf{He}\left(A_i^p\mathcal{P}_j^p + B^pK_i^p\right) + \alpha\mathcal{P}_i^TE^T$.

3.3. Design of the Sliding Mode Controller

Based on (26), define $\bar{U}B^p = \begin{bmatrix} B_1^p \\ B_2^p \end{bmatrix}$, for $t \in [\tau_k, \tau_{k+1})$ and $r(t) = p$, the system (4) is equivalent to

$$\begin{aligned} \dot{x}_1(t) &= \mathcal{A}_1^p x_1(t) + \mathcal{A}_2^p x_2(t) + B_1^p\left(v(t) + f^p(V[x_1^T(t)\ x_2^T(t)])\right) \\ 0 &= \mathcal{A}_3^p x_1(t) + \mathcal{A}_4^p x_2(t) + B_2^p\left(v(t) + f^p(V[x_1^T(t)\ x_2^T(t)])\right) \end{aligned} \tag{37}$$

where $\mathcal{A}_4^p$ is nonsingular and $v(t)$ is defined in (38).

As shown in the second equation of (37), the term $v(t)$ should be at least continuous in order to ensure the continuity of the states $x_2(t)$ in each time interval $[\tau_k, \tau_{k+1})$. To this end, a multivariable super-twisting algorithm which is absolutely continuous will be proposed.

Theorem 2. For the matrices K_i^p and Y_i^p obtained in Theorem 1, a scalar $k_3 > 0$, the following sliding mode controller

$$\begin{aligned} u(t) &= \sum_{i=1}^{r} h_i(z(t))K_i^p(\sum_{i=1}^{r} h_i(z(t))Y_i^p)^{-1}x(t) + v(t) \\ v(t) &= -(SB^p)^{-1}\left(k_1^p(x(t))\left(\frac{s(t)}{\|s(t)\|^{1/2}} + k_3 s(t)\right) - \varpi(t)\right) \\ \dot{\varpi}(t) &= -k_2^p(x(t))\left(\frac{1}{2}\frac{s(t)}{\|s(t)\|} + \frac{3}{2}k_3\frac{s(t)}{\|s(t)\|^{1/2}} + k_3^2 s(t)\right) \end{aligned} \tag{38}$$

can drive the system (4) to the sliding surface (8) and keep a sliding motion for all subsequent time if $\begin{bmatrix} q_1 & q_2 \\ q_2 & 1 \end{bmatrix} > 0$, $q_2 < 0$, $k_1^p(x(t)) > -\dfrac{\|SB^p\|^2\rho_p^2(x(t))}{(q_2 + \theta)(q_1 - q_2^2)} - \dfrac{q_2(q_1 + 2\|SB^p\|\rho_p(x(t)))}{q_1 - q_2^2} + \dfrac{\theta}{q_1 - q_2^2}$ and $k_2^p(x(t)) = q_1 - k_1^p(x(t))q_2$.

Proof. Substituting (38) into (11) yields

$$\begin{aligned}\dot{s}(t) &= -k_1^p(x(t))\phi_1(s(t)) + \varpi(t)\\ \dot{\varpi}(t) &= -k_2^p(x(t))\phi_2(s(t)) + SB^p\frac{df^p(x(t))}{dt}\end{aligned}\tag{39}$$

where $\phi_1(s(t)) = \frac{s(t)}{\|s(t)\|^{1/2}} + k_3 s(t),\ \phi_2(s(t)) = \frac{1}{2}\frac{s(t)}{\|s(t)\|} + \frac{3}{2}k_3\frac{s(t)}{\|s(t)\|^{1/2}} + k_3^2 s(t)$.

Choose the following Lyapunov function

$$V(s(t),\varpi(t)) = \zeta^T(t)Q\zeta(t) = \begin{bmatrix}\phi_1^T(s(t)) & \varpi^T(t)\end{bmatrix}\begin{bmatrix}q_1 I & q_2 I\\ q_2 I & I\end{bmatrix}\begin{bmatrix}\phi_1(s(t))\\ \varpi(t)\end{bmatrix}\tag{40}$$

which is positive definite and radially unbounded. Furthermore, it can be proved that the Lyapunov function (40) is continuous, but only differential except on the subspace $\Omega_1 = \{[s^T(t)\ \varpi^T(t)] \in \mathbb{R}^{2m} :\ s(t) = 0\}$. If the time derivative of the Lyapunov function (40) along the system (39) is negative definite for $[s^T(t)\ \varpi^T(t)] \notin \Omega_1$ and the trajectories of the system (39) can not stay on the subspace $\Omega_1 \setminus \{0\}$, then the stability of the system (39) can be determined. As in [44, 45], the trajectories of the system (39) will cross and not stay on the subspace $\Omega_1 \setminus \{0\}$. Thus, the stability of the system (39) can be guaranteed if the time derivative of $V(s(t),\varpi(t))$ in (40) along with the system (39) is negative definite for $[s^T(t)\ z^T(t)] \notin \Omega_1$.

It can be calculated that

$$\begin{aligned}\dot{V}(s(t),\varpi(t)) = {} & q_1\frac{s^T(t)\dot{s}(t)}{\|s(t)\|} + 3k_3q_1\frac{s^T(t)\dot{s}(t)}{\|s(t)\|^{1/2}} + 2k_3^2q_1 s^T(t)\dot{s}(t) + 2q_2\frac{s^T(t)\dot{\varpi}(t)}{\|s(t)\|^{1/2}}\\ & + 2q_2\frac{\varpi^T(t)\dot{s}(t)}{\|s(t)\|^{1/2}} - q_2\frac{(s^T(t)\varpi(t))(s^T(t)\dot{s}(t))}{\|s(t)\|^{5/2}} + 2k_3q_2 s^T(t)\dot{\varpi}(t)\\ & + 2k_3q_2\varpi^T(t)\dot{s}(t) + 2\varpi^T(t)\dot{\varpi}(t)\end{aligned}\tag{41}$$

By (39), $\dot{V}(s(t),\varpi(t))$ in (41) can be further arranged as

$$\begin{aligned}\dot{V}(s(t),\varpi(t)) = {} & -k_4^p(x(t))\|s(t)\|^{1/2} - 4k_3k_4^p(x(t))\|s(t)\| - 5k_3^2k_4^p(x(t))\|s(t)\|^{3/2}\\ & - 2k_3^3k_4^p(x(t))\|s(t)\|^2 + 2q_2\frac{\|\varpi(t)\|^2}{\|s(t)\|^{1/2}} + 2k_3q_2\|\varpi(t)\|^2\\ & - q_2\frac{\|s^T(t)\varpi(t)\|^2}{\|s(t)\|^{5/2}} + (q_1 - k_1^p(x(t))q_2 - k_2^p(x(t)))\frac{s^T(t)\varpi(t)}{\|s(t)\|}\\ & + 3k_3(q_1 - k_1^p(x(t))q_2 - k_2^p(x(t)))\frac{s^T(t)\varpi(t)}{\|s(t)\|^{1/2}}\\ & + 2k_3^2(q_1 - k_1^p(x(t))q_2 - k_2^p(x(t)))s^T(t)\varpi(t)\\ & + 2q_2\frac{s^T(t)}{\|s(t)\|^{1/2}}SB^p\frac{df^p(x(t))}{dt} + 2k_3q_2 s^T(t)SB^p\frac{df^p(x(t))}{dt}\\ & + 2\varpi^T(t)SB^p\frac{df^p(x(t))}{dt}\\ \leq {} & -(k_4^p(x(t)) + 2\|SB^p\|\rho_p(x(t))q_2)\|s(t)\|^{1/2}\\ & - 4k_3(k_4^p(x(t)) + \frac{1}{2}\|SB^p\|\rho_p(x(t))q_2)\|s(t)\| - 5k_3^2k_4^p(x(t))\|s(t)\|^{3/2}\end{aligned}\tag{42}$$

$$
\begin{aligned}
&- 2k_3^3 k_4^p(x(t))\|s(t)\|^2 + q_2\frac{\|\varpi(t)\|^2}{\|s(t)\|^{1/2}} + 2k_3q_2\|\varpi(t)\|^2\\
&+ (|q_1 - k_1^p(x(t))q_2 - k_2^p(x(t))| + 2\|SB^p\|\rho_p(x(t)))\|\varpi(t)\|\\
&+ 3k_3|q_1 - k_1^p(x(t))q_2 - k_2^p(x(t))|\|s(t)\|^{1/2}\|\varpi(t)\|\\
&+ 2k_3^2|q_1 - k_1^p(x(t))q_2 - k_2^p(x(t))|\|s(t)\|\|\varpi(t)\|
\end{aligned}
$$

where $k_4^p(x(t)) = k_1^p(x(t))q_1 + k_2^p(x(t))q_2$.

Due to $q_1 - k_1^p(x(t))q_2 - k_2^p(x(t)) = 0$ and $\phi_1^T(s(t))\phi_1(s(t)) = \|s(t)\| + 2k_3\|s(t)\|^{3/2} + k_3^2\|s(t)\|^2$, it can be obtained that

$$
\begin{aligned}
\dot{V}(s(t),\varpi(t)) \leq\ & -\frac{1}{\|s(t)\|^{1/2}}\,((k_4^p(x(t)) + 2\|SB^p\|\rho_p(x(t))q_2)\|s(t)\|\\
&+ 2k_3(k_4^p(x(t)) + \frac{1}{2}\|SB^p\|\rho_p(x(t))q_2)\|s(t)\|^{3/2} + k_3^2k_4^p(x(t))\|s(t)\|^2\\
&-q_2\|\varpi(t)\|^2 - 2\|SB^p\|\rho_p(x(t))\|s(t)\|^{1/2}\|\varpi(t)\|\big)\\
&- 2k_3\bigg((k_4^p(x(t)) + \frac{1}{2}\|SB^p\|\rho_p(x(t))q_2)\|s(t)\| + 2k_3k_4^p(x(t))\|s(t)\|^{3/2}\\
&+k_3^2k_4^p(x(t))\|s(t)\|^2 - q_2\|\varpi(t)\|^2\big)
\end{aligned}
\tag{43}
$$

Furthermore, it follows from (43) that

$$
\begin{aligned}
\dot{V}(s(t),\varpi(t)) \leq\ & -\frac{\theta}{\|s(t)\|^{1/2}}\|\zeta(t)\|^2 - 2k_3\theta\|\zeta(t)\|^2\\
\leq\ & -\theta\frac{\lambda_{min}^{1/2}(Q)}{\lambda_{max}(Q)}V^{1/2}(s(t),\varpi(t)) - 2k_3\theta\frac{1}{\lambda_{max}(Q)}V(s(t),\varpi(t))
\end{aligned}
\tag{44}
$$

if $k_1^p(x(t))$ can be selected such that

$$
\begin{bmatrix} k_1^p(x(t))(q_1 - q_2^2) + q_1q_2 + 2\|SB^p\|\rho_p(x(t))q_2 - \theta & -\|SB^p\|\rho_p(x(t)) \\ -\|SB^p\|\rho_p(x(t)) & -q_2 - \theta \end{bmatrix} > 0 \tag{45}
$$

Under the conditions of Theorem 2, it is straightforward to verify that (45) is satisfied and thus (44) holds. By following similar arguments to [44, 45], the finite time convergence of $s(t)$, $\dot{s}(t)$, $z(t)$ can be derived. Since the integral sliding mode paradigm is used in this paper, the sliding surface (8) can be reached from the initial time. □

Remark 7. When the unperturbed dynamics with the variable gain super-twisting algorithm is considered, the algorithm (39) can be regarded as an extension of the variable gain super-twisting algorithm in [44] to the multi-input case and an extension of the constant gain multivariable super-twisting algorithm in [45] to the variable gain case. In the multi-input case, although it is also possible to implement the variable gain super-twisting algorithm in [44] by using the decoupling method [2], the variable gain super-twisting algorithm (39) is more elegant and convenient since it is inherently multivariable.

4. Example

Consider the following bio-economic system [26]

$$\begin{aligned}\dot{x}_1(t) &= 0.15x_2(t) - 0.5x_1(t) - 0.5x_1(t) - 0.01x_1^2(t) - E(t)x_1(t) + u_1(t)\\ \dot{x}_2(t) &= 0.5x_1(t) - 0.1x_2(t)\\ 0 &= E(t)(p_{r(t)}x_1(t) - 50) + u_2(t)\end{aligned} \tag{46}$$

where $x_1(t)$ and $x_2(t)$ represent the population density of immature species and mature species, respectively. $E(t)$ is the harvest effort on the immature population. $u_1(t)$ represents the introduction or fishing of an immature population and $u_2(t)$ denotes government regulation of a biological resource (via a tax or subsidy). $p_{r(t)}$ is a price coefficient per the individual population. $r(t)$ is a Markov process taking values in $\{1, 2\}$ with transition rate matrix $\Lambda = \begin{bmatrix} -0.3 & 0.3 \\ 0.2 & -0.2 \end{bmatrix}$. From [26], $p_1 = 1$, $p_2 = 1.2$.

By translating the positive equilibriums to zero as in [26], the bio-economic system (46) is transformed to the following system

$$\begin{aligned}\dot{z}_1(t) &= a_{r(t)}z_1(t) + 0.15z_2(t) + b_{r(t)}z_3(t) - 0.01z_1^2(t) - z_1(t)z_3(t) + u_1(t)\\ \dot{z}_2(t) &= 0.5z_1(t) - 0.1z_2(t)\\ 0 &= c_{r(t)}z_1(t) + p_{r(t)}z_1(t)z_3(t) + u_2(t)\end{aligned} \tag{47}$$

where $a_1 = -1.25$, $a_2 = -1.6667$, $b_1 = -50$, $b_2 = -41.6667$, $c_1 = -0.75$, $c_2 = -0.8$.

By the sector nonlinearity approach in [53], the system (47) can be represented exactly in the set $\{z(t) : -10 \le z_1(t) \le 10\}$ by the following Markovian jump T-S fuzzy descriptor system

$$E\dot{z}(t) = \sum_{i=1}^{r} h_i(z_1(t))A_i^p z(t) + B(u(t) + f(z(t))) \tag{48}$$

with membership functions $h_1(z_1(t)) = 0.5(1 - 0.1z_1(t))$ and $h_2(z_1(t)) = 0.5(1 + 0.1z_1(t))$, $z(t) = \begin{bmatrix} z_1(t) \\ z_2(t) \\ z_3(t) \end{bmatrix}$, $u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$, $E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $A_1^p = \begin{bmatrix} a_p + 0.1 & 0.15 & b_p + 10 \\ 0.5 & 0.1 & 0 \\ c_p & 0 & -10p_p \end{bmatrix}$, $A_2^p = \begin{bmatrix} a_p - 0.1 & 0.15 & b_p - 10 \\ 0.5 & 0.1 & 0 \\ c_p & 0 & 10p_p \end{bmatrix}$, $p = 1, 2$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$, $f(z(t)) = \begin{bmatrix} 0.1\sin(z_2(t)) \\ 0.1\cos(z_2(t)) \end{bmatrix}$.

Fig.1-2 depict the time responses of the mode signal and the states of the bio-economic system (48) operating open-loop with the initial condition $z(t) = [0.9 \ -0.4 \ 0]^T$. This shows that the bio-economic system (48) is unstable when no control is implemented. It is desirable to establish control schemes to stabilize the system (48).

From the transition rate matrix, it can be computed that $\pi_1 = 0.4$ and $\pi_2 = 0.6$. By solving the matrix inequalities (14)-(17) in Theorem 1 with $\alpha = \epsilon_1 = \epsilon_2 = 1$ and $\beta = 1.1$, the design

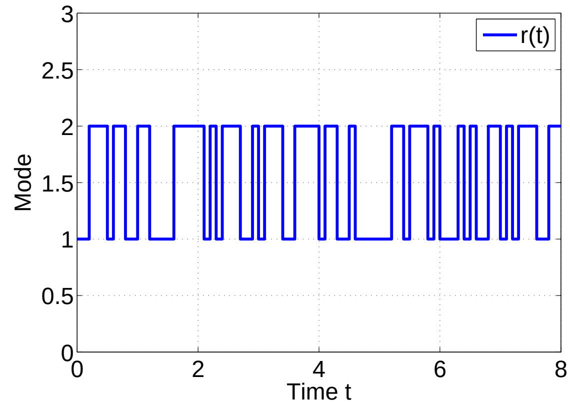

Fig. 1. *Time response of the mode signal* $r(t)$

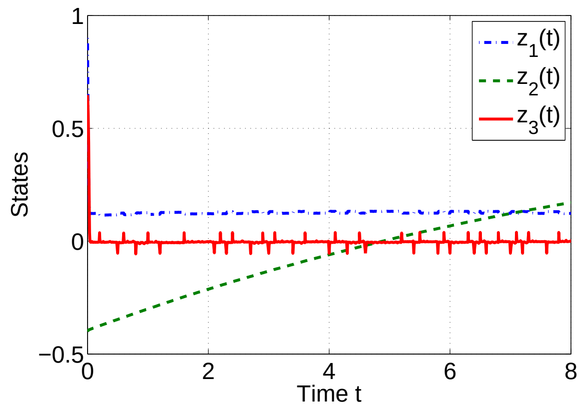

Fig. 2. *Time response of the open-loop bio-economic system*

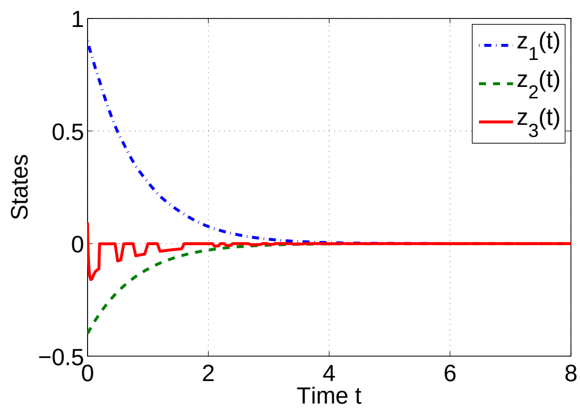

Fig. 3. *Time response of the closed-loop system*

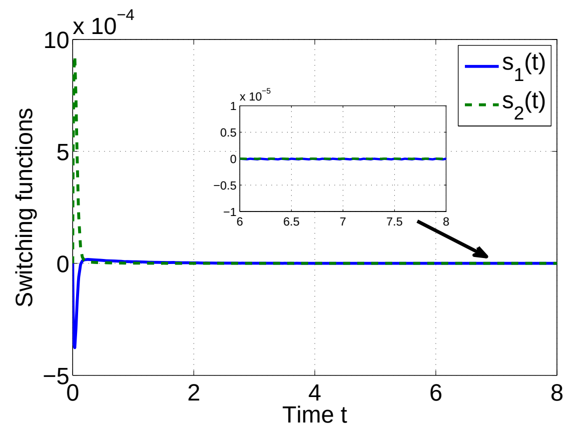

Fig. 4. *Time response of the switching functions*

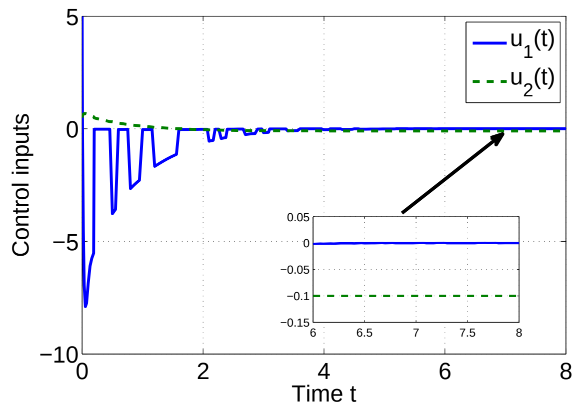

Fig. 5. *Time response of the control inputs*

coefficients can be obtained as follows

$$
\begin{aligned}
K_1^1 &= \begin{bmatrix} -8.2163 & 0.4696 & 191.6006 \\ 0.6452 & -0.9225 & 44.2590 \end{bmatrix}, \ K_1^2 = \begin{bmatrix} -1.0860 & -2.0433 & 2.4234 \\ 2.5228 & -1.7502 & -1.2121 \end{bmatrix}, \\
K_2^1 &= \begin{bmatrix} -3.5331 & -0.9872 & 269.3905 \\ 3.1267 & -1.5804 & -47.9597 \end{bmatrix}, \ K_2^2 = \begin{bmatrix} 0.6842 & -4.0170 & 1.6095 \\ 1.1497 & -0.8854 & -2.0965 \end{bmatrix}, \\
Y_1^1 &= \begin{bmatrix} 2.6425 & -1.7038 & -0.0458 \\ -1.0449 & 1.3844 & 0.0250 \\ -0.1882 & 0.0499 & 4.7660 \end{bmatrix}, \ Y_1^2 = \begin{bmatrix} 2.7050 & -1.7028 & -0.1142 \\ -1.1066 & 1.3963 & 0.0719 \\ -0.0122 & -0.0204 & 0.0624 \end{bmatrix}, \\
Y_2^1 &= \begin{bmatrix} 2.7305 & -1.7263 & 0.0495 \\ -1.0901 & 1.3962 & -0.0270 \\ -0.0578 & 0.0151 & 4.4722 \end{bmatrix}, \ Y_2^2 = \begin{bmatrix} 2.7782 & -1.8056 & 0.1184 \\ -1.0809 & 1.4138 & -0.0532 \\ 0.0077 & -0.0415 & -0.0050 \end{bmatrix}.
\end{aligned} \tag{49}
$$

It follows from (48) that $\rho(z(t)) = |0.5z_1(t) - 0.1z_2(t)|$. With $k_3 = 2$, $q_1 = 3.0484$, $q_2 = -0.22$, $\theta = 0.11$, it can be calculated that $k_1(z(t)) = 12.2368 + 0.0207|0.5z_1(t) - 0.1z_2(t)| + 0.0606(0.5z_1(t) - 0.1z_2(t))^2$, $k_2(z(t)) = 3.484 + 0.22k_1(z(t))$.

Now the integral sliding mode control scheme (9) and (38) with $S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ can be designed to stabilize the Markovian jump T-S fuzzy descriptor system (48). Under the initial conditions $z(t) = [0.9\ -0.4\ 0]^T$, $\varpi(0) = [0\ 0]^T$ and the same model signal in Fig.1. The time responses of the mode signal, the states of the closed-loop system, the switching functions and the control inputs are shown in Fig.3-5 respectively. It is seen that the resultant closed-loop system is asymptotically stable and the switching functions are continuous.

Since $z_1(t)$ and $z_2(t)$ are the state variables of the differential equations, they are always continuous. The state variable of the algebraic equation, $z_3(t)$, however may jump at each switching instant if the initial conditions are inconsistent. This is a common phenomenon in switched descriptor systems [34]. The state jump of $z_3(t)$ will not arise at the switching instants until all the states $z_1(t)$, $z_2(t)$ and $z_3(t)$ converge to zero. This fact is illustrated in Fig.3. Since $u_1(t)$ and $u_2(t)$ are functions of the state $z_3(t)$, $u_1(t)$ and $u_2(t)$ may also undergo state jumps at each switching instant. This coincides with Fig.5 where the control input $u_1(t)$ has instantaneous jumps at the switching instants before the states converge to zero. As proved in Remark 2, it is also shown from Fig.3 and Fig. 4 that although the state $z_3(t)$ is subject to state jumps at each switching instant, the switching functions are always continuous. The reason why the control input $u_2(t)$ converges to -0.1 is that when the ideal sliding mode occurs, the control inputs are required to exactly compensate the unknown vector function $f(z(t))$ and $\lim_{z \to 0} f(z(t)) = [0\ 0.1]^T$ will be satisfied.

From this example, it is verified that the proposed method can stabilize the bio-economic system. From the biological viewpoint, the sustainable development of an ecosystem can be guaranteed by the method presented. As a result, the results obtained in this paper are useful for management agencies concerned with governing ecological resources.

5. Conclusion

This paper has studied the stabilization problems for Markovian jump T-S fuzzy descriptor systems via second order integral sliding modes. A new integral-type switching function was first defined and was shown to be continuous. Then, a new variable gain super-twisting algorithm was developed for multi-input systems. The integral sliding mode control scheme was based on the proposed super-twisting algorithm and Non-PDC control method. It has been shown that the closed-loop system formed by the proposed control scheme is continuous in the time interval between two consecutive switchings.

Since each local subsystem of the Markovian T-S fuzzy descriptor system (4) in this paper shares the same input distribution matrix $B_{r(t)}$ and time delay is not considered [54], future work will focus on designing sliding mode control schemes for Markovian T-S fuzzy descriptor systems when the input distribution matrices of each local system are no longer the same [11, 13, 22] and where time delay is introduced into the switching function to improve the system performance [55]. Extension of the proposed method to semi-Markovian jump systems [56, 57] is an interesting and challenging avenue for future work.

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