Energy Harvesting Enabled MIMO Relaying Through Power Splitting

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***Abstract*—This paper considers a multiple-input multiple-output (MIMO) relay system with an energy harvesting relay node powered by harvesting energy from the source node using power splitting (PS). The rate maximization problem subject to power constraints at both the source and relay nodes is considered for two different scenarios. Firstly, the relay matrix and PS ratio are jointly optimized with uniform source precoding and then in the second scenario the source covariance is optimized as well. Iterative approaches and dual decomposition are proposed based on the structures of the optimal relay and source covariance matrices for the scenarios.**

***Index Terms*—MIMO, Relay, Energy Harvest, Power Splitting.**

I. Introduction

Cooperative communication using relay has been seen as one of the promising techniques to improve network coverage and throughput since 1970s [1]. Since then, considerable work has been done to explore cooperation strategies from various perspectives. To take the advantages of the multiple-input multiple-output (MIMO) technique, e.g. improving spectrum utilization and link reliability, MIMO relay networks were considered in [2]–[4] where the capacity maximization problem with source and relay transmit power thresholds was considered. In [2] and [3], relay only and joint source and relay optimization was considered. Then in [4], joint source and relay design was studied for MIMO-OFDM relay networks.

With green communication becoming an important tendency of next generation wireless communication, recently, researchers have started paying attention to the combination of energy harvesting technique and cooperative communication. In [5], the outage probability and the ergodic capacity were analyzed for one-way relaying system with energy harvesting while [6] focused on the power allocation strategies for multiple source-destination pair cooperative relay networks. In [7], the outage and diversity performance were investigated using stochastic geometry in energy harvesting enabled cooperative networks with spatially random relays. In [8], game theory was used to study simultaneous wireless information and power transfer (SWIPT) for interference relay channels. These works only considered single antenna and single-carrier relay networks. Later in [9], SWIPT was considered for a multi-antenna relay network with single antenna source and destination nodes. The transmit power at the relay was minimized subject to the signal-to-inference-plus-noise ratio (SINR) and energy harvesting constraints. [10] studied the optimal precoding for a two-hop decode-and-forward (DF) MIMO relay network with an energy harvesting receiver and an information decoding receiver. The achievable rate was maximized subject to independent transmit power and harvested energy constraints at the source and relay transmission phases, respectively, with fixed time switching (TS) ratio. The formulated problem was convex after semidefinite relaxation (SDR) and hence can be easily solved with existing software solvers.

In this paper, power-splitting based energy harvesting is considered for amplify-and-forward (AF) MIMO relay networks. In order to maximize the achievable rate with power constraints, we first consider uniform source precoding and jointly optimize relay matrix and PS ratio with extension to a more challenging scenario where the source covariance matrix needs to be optimized as well. Unlike the convex problem considered in [10], the noncovex problem here can not be solved using SDR and software solvers. Hence we provide the structures of the optimal source covariance and relay precoding matrix based on which iterative approaches are employed to derive the near-optimal results.

II. System Model

A two-hop MIMO relay network is considered with power splitting based energy harvesting at the relay node. The numbers of antennas for the source, relay and destination nodes are M, L, and N, respectively. Using half-duplex relaying, the signal transmission can be divided into two phases. As is shown in Fig. 1, the block length T is split equally into the two phases. In the first phase, information and energy are simultaneously transmitted from the source to the relay with ε of the received signal power used for energy harvesting and the rest used for information forwarding. Then in the second phase the relay forwards the rest received signal to the destination node using the energy harvested in the first phase.

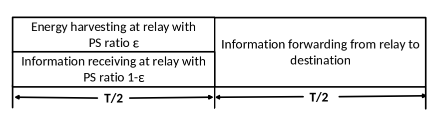

Fig. 1. The framework of the proposed PSR

To better clarify the expressions, we summarize some commonly used symbols. $\mathbf{s}$ is the source symbol vector subject to the maximum transmit power P. $\mathbf{H}_1$ and $\mathbf{H}_2$ give the channel matrices between the source and relay, and the relay and destination. $\mathbf{Q}$ is the source covariance matrix while $\mathbf{F}$ is the relay beamforming matrix. $\mathbf{n}_1$ denotes the Addictive White Gaussian Noise (AWGN) at relay with a variance of ${\sigma_1}^2$. And $\mathbf{n}_2$ is the AWGN at destination with a variance of ${\sigma_2}^2$.

III. Rate Maximization with Uniform Source Precoding

In the uniform source pre-coding case, i.e. $\mathbf{Q}=\mathbb{E}(\mathbf{s}\mathbf{s}^H)=\frac{P}{D}\mathbf{I}$, the harvested power at relay can be written as

$$\mathrm{tr}(\mathbf{y}_e{\mathbf{y}_e}^H)=\sigma_1^2\mathrm{tr}(\varepsilon\rho_1\mathbf{H}_1\mathbf{H}_1^H), \tag{1}$$

where D is the number of transmit data streams satisfying $D\le\min(M,L,N)$. We let the received signal for energy harvesting $\mathbf{y}_e=\sqrt{\varepsilon}\mathbf{H}_1 s$ and the signal-to-noise ratio (SNR) at the relay $\rho_1\triangleq\frac{P}{D\sigma_1^2}$. Here we assume that the antenna noise is relatively smaller and can therefore be ignored. The received signal at relay for information decoding is given by

$$\mathbf{y}_r=\sqrt{1-\varepsilon}\mathbf{H}_1\mathbf{s}+\mathbf{n}_1. \tag{2}$$

Then in the second phase, the receive signal at destination is

$$\mathbf{y}_d=\sqrt{1-\varepsilon}\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{s}+\mathbf{H}_2\mathbf{F}\mathbf{n}_1+\mathbf{n}_2. \tag{3}$$

In this case, the achievable rate can be written as

$$\begin{aligned}C=\frac{1}{2}\log_2\det\big(&\mathbf{I}_D+(1-\varepsilon)\rho_1\mathbf{H}_1\mathbf{H}_1^H\\&-(1-\varepsilon)\rho_1\mathbf{H}_1\mathbf{H}_1^H\mathbf{W}^{-1}\big),\end{aligned} \tag{4}$$

where $\mathbf{W}=\mathbf{I}_D+\mathbf{G}^H\mathbf{H}_2^H\mathbf{H}_2\mathbf{G}$, and $\mathbf{G}=\frac{\sigma_1}{\sigma_2}\mathbf{F}$. And the power constraint at relay is given by

$$\mathrm{tr}(\mathbf{G}(\mathbf{I}_D+(1-\varepsilon)\rho_1\mathbf{H}_1\mathbf{H}_1^H)\mathbf{G}^H)\le\eta\mathrm{tr}(\varepsilon\rho_2\mathbf{H}_1\mathbf{H}_1^H), \tag{5}$$

where $\rho_2\triangleq\frac{\mathrm{P}}{D{\sigma_2}^2}$. $0\le\eta\le1$ denotes the energy convention efficiency. Consequently, the problem of interest is

$$\max_{\mathbf{G},\varepsilon}\quad C\qquad \text{s.t.} \tag{6a}$$

$$\begin{aligned}&\mathrm{tr}(\mathbf{G}(\mathbf{I}_D+(1-\varepsilon)\rho_1\mathbf{H}_1\mathbf{H}_1^H)\mathbf{G}^H)\\&\qquad\le\eta\mathrm{tr}(\varepsilon\rho_2\mathbf{H}_1\mathbf{H}_1^H).\end{aligned} \tag{6b}$$

Obviously, the problem is not convex and hence can not been solved directly. Now define $\hat{\rho}_1=(1-\varepsilon)\rho_1,\hat{\rho}_2=\varepsilon\rho_2$ and fix ε. The problem then becomes similar to the one in [2]. We consider the singular value decompositions (SVDs) of the channel matrices shown below

$$\mathbf{H}_1=\mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_1^H, \tag{7}$$

$$\mathbf{H}_2=\mathbf{U}_2\mathbf{\Sigma}_2\mathbf{V}_2^H, \tag{8}$$

where $\mathbf{\Sigma}_1,\mathbf{\Sigma}_2$ are diagonal matrices while others are unitary. According to [2], the optimal relay matrix satisfies $\mathbf{F}=\mathbf{V}_2\mathbf{\Lambda}_F\mathbf{U}_1^H$ where $\mathbf{\Lambda}_F$ is diagonal. Let $\mathbf{G}=\mathbf{V}_2\mathbf{X}^{\frac{1}{2}}(\mathbf{I}+(1-\varepsilon)\rho_1\mathbf{\Lambda}_1)^{-\frac{1}{2}}\mathbf{U}_1^H$ with $\mathbf{X}=\mathrm{diag}(x_1,x_2,...,x_D)$. In addition, we let $\mathbf{\Lambda}_1=\mathbf{\Sigma}_1^2$, $\mathbf{\Lambda}_2=\mathbf{\Sigma}_2^2$ be diagonal matrices with the vectors $\boldsymbol{\alpha}=[\alpha_1,...\alpha_D]$ and $\boldsymbol{\beta}=[\beta_1,...\beta_D]$ as the diagonal vectors, respectively. Problem (6) then becomes a scalar optimization problem:

$$\max_{0\le\varepsilon\le1,\{x_k\}}\quad f(\{x_k\},\varepsilon) \tag{9a}$$

$$\text{s.t.}\quad g(\{x_k\},\varepsilon)\triangleq\eta\sum_{k=1}^{D}\varepsilon\rho_2\alpha_k-\sum_{k=1}^{D}x_k\ge0, \tag{9b}$$

$$x_k\ge0,\quad\forall k \tag{9c}$$

where

$$\begin{aligned}f(\{x_k\},\varepsilon)\triangleq&\frac{1}{2}\Bigg[\sum_{k=1}^{D}\log_2(1+(1-\varepsilon)\rho_1\alpha_k)\\&+\sum_{k=1}^{D}\log_2\left(\frac{1+\beta_k x_k}{1+(1-\varepsilon)\rho_1\alpha_k+\beta_k x_k}\right)\Bigg].\end{aligned} \tag{10}$$

Considering the Lagrangian, the dual problem is

$$\max_{\substack{\{x_k\},\varepsilon,\\ \nu,\{\lambda_k\}}}\ \mathcal{L}\triangleq f(\{x_k\},\varepsilon)+\nu g(\{x_k\},\varepsilon)+\sum_{k=1}^{D}\lambda_k x_k \tag{11a}$$

$$\text{s.t.}\quad 0\le\varepsilon\le1,\nu\ge0,x_k\ge0,\lambda_k\ge0,\forall k. \tag{11b}$$

Using Karush-Kuhn-Tucker (KKT) conditions, we have

$$\nu g(\{x_k\},\varepsilon)=0, \tag{12a}$$

$$\lambda_k x_k=0,\forall k, \tag{12b}$$

$$\nabla_{x_k}\mathcal{L}=0,\forall k, \tag{12c}$$

$$\nabla_{\varepsilon}\mathcal{L}=0. \tag{12d}$$

Using (12c), we obtain

$$\begin{aligned}\frac{1}{2\ln2}\left(\frac{\beta_k}{1+\beta_k x_k}-\frac{\beta_k}{1+(1-\varepsilon)\rho_1\alpha_k+\beta_k x_k}\right)&\\-\nu+\lambda_k=0,\forall k.&\end{aligned} \tag{13}$$

Due to the fact that $\lambda_k\ge0$, it holds that

$$\nu\ge\frac{1}{2\ln2}\frac{\frac{(1-\varepsilon)\rho_1\alpha_k}{\beta_k}}{(x_k+\frac{1}{\beta_k})\left(x_k+\frac{(1-\varepsilon)\rho_1\alpha_k+1}{\beta_k}\right)},\forall k. \tag{14}$$

Considering (12b), we have

$$x_k\left[\nu-\frac{1}{2\ln2}\frac{\frac{(1-\varepsilon)\rho_1\alpha_k}{\beta_k}}{(x_k+\frac{1}{\beta_k})\left(x_k+\frac{(1-\varepsilon)\rho_1\alpha_k+1}{\beta_k}\right)}\right]=0. \tag{15}$$

According to [2], the optimal x_k can then be written as

$$\begin{aligned}x_k=&\frac{1}{2\beta_k}[\sqrt{(1-\varepsilon)^2\rho_1^2\alpha_k^2+\frac{2}{\ln2}(1-\varepsilon)\rho_1\alpha_k\beta_k\mu}\\&-(1-\varepsilon)\rho_1\alpha_k-2]^+,\end{aligned} \tag{16}$$

where $(a)^+=\max\{0,a\}$ and $\mu=\frac{1}{\nu}$ can be obtained from (12d). As such

$$\begin{aligned}l(\mu)=&\frac{\rho_1}{2\ln2}\sum_{k=1}^{D}\Bigg[\Bigg(\frac{1}{1+(1-\varepsilon)\rho_1\alpha_k+\beta_k x_k}\\&-\frac{1}{1+(1-\varepsilon)\rho_1\alpha_k}\Bigg)\alpha_k\Bigg]+\frac{1}{\mu}\eta\rho_2\alpha_k=0.\end{aligned} \tag{17}$$

Due to the inter-depended relationships among x_k, ε, and ν, it is difficult to derive the optimal closed form expressions for all the variables at the same time. To solve the problem, here we introduce an iterative method by firstly fixing ε.

Now we need to check the availability of root-searching for $l(\mu)=0$. Obviously, $l(\mu)$ is monotonic decreasing when $\mu \in \left[\max_k 2\ln 2\frac{1+(1-\varepsilon)\rho_1\alpha_k}{(1-\varepsilon)\rho_1\alpha_k\beta_k},\ \infty\right)$. Moreover, we notice that when $\mu \in \left[\min_k 2\ln 2\frac{1+(1-\varepsilon)\rho_1\alpha_k}{(1-\varepsilon)\rho_1\alpha_k\beta_k}, \max_k 2\ln 2\frac{1+(1-\varepsilon)\rho_1\alpha_k}{(1-\varepsilon)\rho_1\alpha_k\beta_k}\right]$, $l(\mu)$ still decreases monotonically since in this interval x_k either equals to 0 or increases with μ. Consequently, we have

$$l(\infty) \to -\frac{\rho_1}{2\ln 2}\sum_{k=1}^{D}\frac{\alpha_k}{1+(1-\varepsilon)\rho_1\alpha_k} < 0, \tag{18}$$

and

$$\begin{aligned} & l\left(\min_k 2\ln 2\frac{1+(1-\varepsilon)\rho_1\alpha_k}{(1-\varepsilon)\rho_1\alpha_k\beta_k}\right) \\ &= \max_k \frac{1}{2\ln 2}\frac{(1-\varepsilon)\rho_1\alpha_k\beta_k}{1+(1-\varepsilon)\rho_1\alpha_k}\sum_{k=1}^{D}\eta\rho_2\alpha_k > 0. \end{aligned} \tag{19}$$

In contrast, it is obvious that $x_k = 0, \forall k$, when $\mu \in \left(0, \min_k 2\ln 2\frac{1+(1-\varepsilon)\rho_1\alpha_k}{(1-\varepsilon)\rho_1\alpha_k\beta_k}\right)$, and thus we know that

$$l(u) = \frac{1}{\mu}\sum_{k=1}^{D}\eta\rho_2\alpha_k > 0. \tag{20}$$

Consequently, an optimal μ^* satisfying $l(\mu^*) = 0$ can always be found within $\left(\min_k 2\ln 2\frac{1+(1-\varepsilon)\rho_1\alpha_k}{(1-\varepsilon)\rho_1\alpha_k\beta_k}, \infty\right)$ by root-finding strategies such as bisection searching. Finally, we can calculate the optimal PS ratio using (12a) as follows

$$\varepsilon = \frac{\sum_{k=1}^{D} x_k}{\eta\rho_2\sum_{k=1}^{D}\alpha_k}. \tag{21}$$

The iteration framework is summarized in Algorithm 1.

Algorithm 1 Iteration scheme for uniform source precoding

1: **Initialization** let $\varepsilon = 0.001$
2: **while** $\varepsilon < 1$ **do**
3: find μ^* making $l(\mu^*) = 0$ using root-finding method
4: calculate x_k and ε^* using (16) and (21)
5: if $|\varepsilon^* - \varepsilon|$ is small enough, iteration terminates. Otherwise, $\varepsilon = \varepsilon + 0.001$
6: **end while**

IV. Rate Maximization with Arbitrary Source Precoding

In this section, a more general scenario with arbitrary source covariance matrix is considered in which the source covariance matrix, relay beamforming matrix and PS ratio need to be jointly optimized. In this case, the achievable rate can be expressed as

$$C = \frac{1}{2}\log_2\det\left(\mathbf{I}_D + (1-\varepsilon)\frac{\mathbf{H}_2\mathbf{F}\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H\mathbf{H}_2^H}{\sigma_2^2\mathbf{I}_D + \sigma_1^2\mathbf{H}_2\mathbf{F}\mathbf{F}^H\mathbf{H}_2^H}\right). \tag{22}$$

Then the problem of interest can be written as

$$\begin{aligned} \max_{\mathbf{F},\mathbf{Q},\varepsilon}\quad & C & \text{(23a)}\\ \text{s.t.}\quad & \mathrm{tr}(\mathbf{Q}) \le P, & \text{(23b)}\\ & \mathrm{tr}(\sigma_1^2\mathbf{F}\mathbf{F}^H + (1-\varepsilon)\mathbf{F}\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H\mathbf{F}^H) & \\ & \qquad \le \varepsilon\eta\mathrm{tr}(\mathbf{H}_1\mathbf{Q}\mathbf{H}_1^H). & \text{(23c)} \end{aligned}$$

To make use of the results in the previous section, we introduce an equivalent channel $\widehat{\mathbf{H}}_1 = \mathbf{H}_1\mathbf{Q}^{\frac{1}{2}}$ and find that the structure of the optimal relay beamforming still works, i.e. $\widehat{\mathbf{F}} = \mathbf{V}_2\widehat{\mathbf{\Sigma}}_F\widehat{\mathbf{U}}_1^H$. Meanwhile, $\widehat{\mathbf{\Sigma}}_F$ is diagonal, and $\widehat{\mathbf{U}}_1$ comes from the SVD of $\widehat{\mathbf{H}}_1 = \widehat{\mathbf{U}}_1\widehat{\mathbf{\Sigma}}_1\widehat{\mathbf{V}}_1^H$. Because the objective function and the transmit power constraint at relay only depend on $\widehat{\mathbf{\Sigma}}_1$ but not on $\widehat{\mathbf{U}}_1$, it was claimed in [3] that the optimal $\mathbf{Q}$ must require the least transmit power. Moreover, it can be easily proved that the structures of the optimal source covariance and relay beamforming matrices in [3] still work after considering energy harvesting, by defining a new variable $\widehat{\rho}_1 = (1-\varepsilon)\rho_1$. So the structures of the optimal source and relay matrices in (23) can be written as

$$\mathbf{F} = \mathbf{V}_2\mathbf{\Sigma}_F\mathbf{U}_1^H, \tag{24}$$

$$\mathbf{Q} = \mathbf{V}_1\mathbf{\Lambda}_Q\mathbf{V}_1^H, \tag{25}$$

where $\mathbf{U}_1, \mathbf{V}_1, \mathbf{U}_2, \mathbf{V}_2$ have been introduced in (7) and (8). $\mathbf{\Lambda}_Q, \mathbf{\Sigma}_F$ are diagonal matrices with $\mathbf{\Lambda_Q} = \mathrm{diag}(q_1, q_2, ..., q_D)$, and $\mathbf{\Lambda_F} = \mathbf{\Sigma}_F^2 = \mathrm{diag}(f_1, f_2, ..., f_D)$. Now let $d_k = f_k((1-\varepsilon)\alpha_k q_k + \sigma_1^2), \forall k$, and rewrite the optimization problem (23) as

$$\begin{aligned} \max_{\varepsilon,\{d_k\},\{q_k\}}\quad & \tilde{f}(\varepsilon,\{d_k\},\{q_k\}) & \text{(26a)}\\ \text{s.t.}\quad & \sum_{k=1}^{D} q_k \le P, & \text{(26b)}\\ & \tilde{g}(\varepsilon,\{d_k\},\{q_k\}) \ge 0, & \text{(26c)}\\ & q_k \ge 0, d_k \ge 0, \forall k & \text{(26d)}\\ & 0 \le \varepsilon \le 1, & \text{(26e)} \end{aligned}$$

where we let

$$\begin{aligned} & \tilde{f}(\varepsilon,\{d_k\},\{q_k\}) \\ & \triangleq \frac{1}{2}\sum_{k=1}^{D}\log_2\frac{\left(1+(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k\right)\left(1+\frac{\beta_k}{\sigma_2^2}d_k\right)}{1+(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k+\frac{\beta_k}{\sigma_2^2}d_k}, \end{aligned} \tag{27}$$

$$\tilde{g}(\varepsilon,\{d_k\},\{q_k\}) \triangleq \varepsilon\eta\sum_{k=1}^{D}\alpha_k q_k - \sum_{k=1}^{D} d_k. \tag{28}$$

Now problem (23) with matrix variables has been reformulated into (26) which involves only scalar variables but still non-convex. It is difficult to obtain a closed-form solution. Thus a local optimal iterative algorithm is proposed in the following.

A. Updating $\boldsymbol{d}$ and ε^* with fixed $\boldsymbol{q}$

To start with, we let $\boldsymbol{q} = [q_1, q_2, \ldots, q_D]^T$, and $\boldsymbol{d} = [d_1, d_2, \ldots, d_D]^T$. Then we fix $\boldsymbol{q}$ subject to (26b) and update

$\boldsymbol{d}$ and ε accordingly. Considering the Lagrangian of (26), we formulate the following dual problem

$$\max_{\substack{\varepsilon,\{d_k\},\\ \nu,\{\lambda_k\}}} \mathcal{L} \triangleq \tilde{f}(\varepsilon,\{d_k\}) + \nu\tilde{g}(\varepsilon,\{d_k\}) + \sum_{k=1}^{D}\lambda_k d_k \tag{29a}$$

$$\text{s.t. } 0 \le \varepsilon \le 1, d_k \ge 0, \nu \ge 0, \lambda_k \ge 0, \forall k. \tag{29b}$$

The corresponding KKT conditions can be written as

$$\nu\tilde{g}(\varepsilon,\{d_k\}) = 0, \tag{30a}$$

$$\lambda_k d_k = 0, \forall k, \tag{30b}$$

$$\nabla_{d_k}\mathcal{L} = 0, \forall k, \tag{30c}$$

$$\nabla_{\varepsilon}\mathcal{L} = 0. \tag{30d}$$

Following similar approach in the uniform source procoding case, the optimal d_k can be derived as

$$d_k = \frac{\sigma_2^2}{2\beta_k}\left(\sqrt{(1-\varepsilon)^2(\frac{\alpha_k}{\sigma_1^2}q_k)^2 + 2(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2\ln 2}q_k\beta_k\mu} - (1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k - 2\right)^+, \tag{31}$$

where $\mu = \frac{1}{\nu}$ can be obtained using (30d). Thus we have

$$l(\mu) = -\frac{1}{2\ln 2}\left[\sum_{k=1}^{D}\frac{\alpha_k}{\sigma_1^2}q_k\left(\frac{1}{1+(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k} - \frac{1}{1+(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k + \frac{\beta_k}{\sigma_2^2}d_k}\right)\right] + \frac{1}{\mu}\eta\sum_{k=1}^{D}\alpha_k q_k = 0. \tag{32}$$

Note that in this case, both ε and μ are needed to calculate d_k. Here we introduce an initial ε, and then search for the optimal μ by bisection method using (31) and (32). With μ known, we calculate d_k using (31) and then ε^* can be derived using (30a) as follows

$$\varepsilon^* = \frac{\sum_{k=1}^{D} d_k}{\eta\sum_{k=1}^{D}\alpha_k q_k}. \tag{33}$$

The iterative framework is similar to Algorithm 1 and hence ignored to avoid redundancy.

B. Updating $\boldsymbol{q}$ with fixed $\mathbf{d}$ and ε

Now fixing $\boldsymbol{d}$ and ε, we update q_k. Considering the Lagrangian of problem (26), the dual problem is defined as

$$\max_{\substack{\varepsilon,\{q_k\},\nu_1,\\ \nu_2,\{\lambda_k\}}} \mathcal{L} = \tilde{f}(\varepsilon,\{q_k\}) + \nu_1(P - \sum_{k=1}^{D}q_k) + \nu_2\tilde{g}(\rho,\{q_k\}) + \sum_{k=1}^{D}\lambda_k q_k \tag{34a}$$

$$\text{s.t. } 0 \le \varepsilon \le 1, \nu_1 \ge 0, \nu_2 \ge 0, \lambda_k \ge 0, q_k \ge 0, \forall k. \tag{34b}$$

Again the KKT conditions are given by

$$\nu_1(P - \sum_{k=1}^{D}q_k) = 0, \tag{35a}$$

$$\nu_2\tilde{g}(\varepsilon,\{q_k\}) = 0, \tag{35b}$$

$$\lambda_k q_k = 0, \forall k, \tag{35c}$$

$$\nabla_{q_k}\mathcal{L} = 0, \forall k, \tag{35d}$$

$$\nabla_{\varepsilon}\mathcal{L} = 0. \tag{35e}$$

Then according to (35d), we have

$$\frac{(1-\varepsilon)\alpha_k}{2\ln 2\ \sigma_1^2}\left(\frac{1}{1+(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k} - \frac{1}{1+(1-\varepsilon)\frac{\alpha_k}{\sigma_1^2}q_k + \frac{\beta_k}{\sigma_2^2}d_k}\right) - \nu_1 + \nu_2\varepsilon\eta\alpha_k + \lambda_k = 0. \tag{36}$$

Using (35d) and defining $\hat{\nu}_k = 2(\nu_1 - \nu_2\varepsilon\eta\alpha_k)$, we then have

$$q_k = \frac{\sigma_1^2}{2\alpha_k(1-\varepsilon)}\left(\sqrt{(\frac{\beta_k}{\sigma_2^2}d_k)^2 + 4\frac{(1-\varepsilon)\alpha_k\beta_k}{\ln 2\sigma_1^2\sigma_2^2}d_k\hat{\mu}_k} - \frac{\beta_k}{\sigma_2^2}d_k - 2\right)^+, \tag{37}$$

where $\hat{\mu}_k = \frac{1}{\hat{\nu}_k}$. Since each q_k depends on unique dual variable $\hat{\mu}_k$, it is difficult to find all the dual variables by searching. Instead, here we use the dual decomposition method proposed in [11] to derive the optimal solution to the dual problem (34). The key idea is to find the optimal dual variables ν_1 and ν_2 by alternating searching and then use them to calculate the corresponding q_k. The framework is presented in Algorithm 2.

C. Iterative Optimization

Algorithm 3 shows the framework of iteration to solve (26).

V. Numerical Results

In this section, the performance of the proposed schemes are analyzed via simulations. We let $M = N = L = 4$ and $P = 1$. $\mathbf{H_1}$ and $\mathbf{H_2}$ are modeled with a set of independent zero-mean complex Gaussian random variables with a variance of 10dBm. Unless otherwise stated, the iteration threshold is 10^{-3}. Case I and Case II denote the uniform and arbitrary source precoding scenarios, respectively. Note that for comparison the naive amplify-and-forward (NAF) scheme is also considered. In the NAF scheme, we let $\mathbf{Q} = \frac{P}{D}\mathbf{I}$ and $\mathbf{F} = \sqrt{\chi}\mathbf{I}$ where χ is the scalar which makes the equality in (5) holds and the optimal PS ratio ε is obtained by exhaust searching.

Fig. 2 presents how the value of the noise variance σ_1^2 at relay decides the maximum rate. Let σ_1^2 vary from -20dBm to 20dBm and σ_2^2 fixed at $\sigma_2^2 = -20\text{dBm}$. As can be observed, the joint optimization of source, relay and PS ratio in Case II yields the highest rate followed by the iterative scheme in Case I while the NAF scheme shows the worst performance.

Fig. 3 shows the maximum rate versus the value of the noise variance σ_2^2 at destination. Here we let σ_2^2 vary from -40dBm to 0dBm with σ_1^2 fixed at 0dBm. According to Fig. 3, the proposed scheme in Case II outperforms the joint optimization of relay and PS ratio in case I as well as the NAF scheme with a higher gain at a lower σ_2^2.

Algorithm 2 Dual Decomposition for PS Relaying

Main Function
Fix $\boldsymbol{d}$ and ε
$\boldsymbol{q} = optimize\ \nu_1(\boldsymbol{d}, \varepsilon)$
Function $\boldsymbol{q} = optimize\ \nu_1(\boldsymbol{d}, \varepsilon)$
$\phi_k = \frac{(1-\varepsilon)\alpha_k\beta_k d_k}{2\sigma_1^2 \ln 2(\sigma_2^2+\beta_k d_k)}$, $\nu_{1min} = \nu_{1max} = \max_k(\boldsymbol{\phi})$
while $\sum_{k=1}^{D} q_k \geq P$ **do**
  $\nu_{1max} = \nu_{1max} + 10^{-4}$
  $q = optimize\ \nu_2(\nu_{1max}, \boldsymbol{d}, \varepsilon)$
end while
while $|\nu_{1max} - \nu_{1min}| > \varepsilon$ **do**
  $\nu_1 = \frac{\nu_{1max}+\nu_{1min}}{2}$
  $q = optimize\ \nu_2(\nu_1, \boldsymbol{d}, \varepsilon)$
  if $\sum_{k=1}^{D} q_k \geq P$, $\nu_{1min} = \nu_1$; otherwise, $\nu_{1max} = \nu_1$
end while
Function $\boldsymbol{q} = optimize\ \nu_2(\nu_1, \boldsymbol{d}, \varepsilon)$
$\theta_k = \frac{\nu_1 - \phi_k}{\eta\varepsilon\alpha_k}$, $\nu_{2min} = \nu_{2max} = \min_k(\boldsymbol{\theta})$
while $\eta\varepsilon \sum_{k=1}^{D} \alpha_k q_k - \sum_{k=1}^{D} d_k \leq 0$ **do**
  $\nu_{2max} = \nu_{2max} + 10^{-4}$
  $\boldsymbol{q} = optimize(\nu_1, \nu_{2max}, \boldsymbol{d}, \varepsilon)$
end while
while $|\nu_{1max} - \nu_{1min}| > \varepsilon$ **do**
  $\nu_1 = \frac{\nu_{1max}+\nu_{1min}}{2}$
  $\boldsymbol{q} = optimize\ \nu_2(\nu_1, \boldsymbol{d}, \varepsilon)$
  if $\eta\varepsilon \sum_{k=1}^{D} \alpha_k q_k - \sum_{k=1}^{D} d_k \leq 0$, let $\nu_{2min} = \nu_2$; otherwise, $\nu_{2max} = \nu_2$
end while
Function $\boldsymbol{q} = optimize(\nu_1, \nu_2, \boldsymbol{d}, \varepsilon)$
Calculate $\boldsymbol{q}$ according to (37)

Algorithm 3 Iteration Framework for PS Relaying

1: **Initialization** Let $\boldsymbol{q}$ satisfying (26b)
2: Calculate optimal $\boldsymbol{d}$ and ε with fixed $\boldsymbol{q}$ according to Algorithm 1
3: Re-optimize $\boldsymbol{q}$ with the obtained $\boldsymbol{d}$ and ε via dual decomposition method in **Algorithm 2**
4: Return to Step 2 until convergence

VI. Conclusion

This paper investigated wireless information and power transfer for MIMO power splitting relaying. To maximize the rate subject to the power constraints, we firstly considered uniform source precoding and optimized the relay matrix and PS ratio. Then the source covariance was optimized as well in the second case. Iterative schemes which yielded near-optimal solutions were proposed for the two cases.

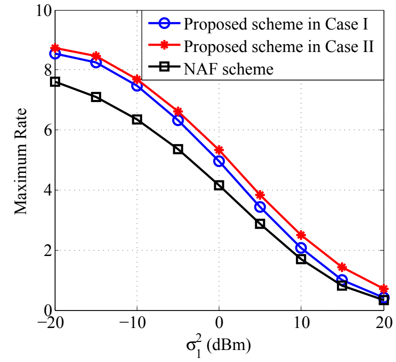

Fig. 2. Maximum rate versus σ_1^2 with $\sigma_2^2 = -20$dBm.

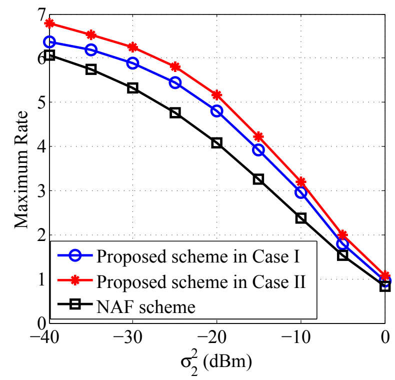

Fig. 3. Maximum rate versus σ_2^2 with $\sigma_1^2 = 0$dBm.

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