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Evolution and turnover in scaling systems[†]

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Scaling has been discovered in the long tails of size distributions characterizing a variety of diverse systems, many of which evolve in terms of the size of their components through competition [1]. Such time-invariant macro distributions, however, often obscure the micro-dynamics of change, such as continual turnover in the rank order of the constituents. Here we show how a model drawn from evolutionary theory can explain this change, such that the time spent in the top ranked constituents is finite and also characterized by long-tailed distributions. To show the broad applicability of this model, we compare typical model runs to real-world examples including US boys' names, UK Number One for pop albums, journal article keywords, and city sizes.

In the social sciences, scaling in distributions of income and wealth [2] and city sizes has been known for a long time, at least since Pareto [3] Many other size distributions such as the structure of the internet [4, 5], author citations [6, 7] the number of sexual partners [8], the size of firms [9] and the distribution of their extinctions [10, 11], have been explored more recently but all indicate the effects of scaling that reflects decisions which lead to competitive growth. An obvious issue is the extent to which any scaling inherent in the tail of such distributions is stable over time. Much more importantly, however, macro-outcomes such as those cited above, even when apparently time-invariant, can obscure considerable variability at the micro-level. For example, whilst at any point in time the distribution of city sizes is approximated in the upper tail by a Pareto distribution, there is considerable turnover in the ranking of individual cities [12]. The same degree of turnover, perhaps even more, can be seen, for example, in the popularity of film stars, books or music [13, 14, 15].

Explaining such change is also a key challenge for network science in which "dynamical problems lie at the forefront." [16]. Diverse phenomena are increasingly envisaged as networks, with interacting entities (e.g., Web pages, firms, individuals) seen as a 'nodes', and their influences on each other (e.g., linked Web pages, allied

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firms, collaborators/friends) articulated as incoming and outgoing links [16]. Because the formulation of a network presupposes a structure to interactions, the modelling of continual change tests the limits of the network analogy. In most network models, the connections of today determine (often strongly) what will happen tomorrow, such that change must be implemented as a modification of the existing network. Often in social and cultural evolution, however, interactions of influence can be quite different from one time period to the next. Change is not just a modification; it is the essence of the process.

Change, in fact, is the essence of evolution, and evolutionary theory [17] has much to offer the newer science of evolving dynamic networks. Specifically, through an adaptation of the neutral model of population genetics [18, 19, 20], we propose a general model for scaling phenomena that exhibits aspects of both preferential attachment [5], the characteristic feature of "scale-free" network models, and turnover, which these models struggle to account for. In contrast to preferential attachment, the neutral model exhibits both stable right-skew distributions at the macro-level, and considerable turnover at the micro-level. Further, it is more general in that different choices of the parameters of the model can lead to a range of distributions, such as power law over the whole sample, power law only in the tail, and winner-take-all.

Consider a model populated initially by N agents located in some space such as the real line. The model proceeds in a series of steps. In each step, between 1 and N new agents enter the model. This number, ν say, is a parameter of the model and is fixed at the outset for each solution. With probability $(1-\mu)$, an agent copies the choice of location from that of an existing agent within the previous m time steps, or else with probability μ the agent innovates by choosing a unique new location at random. In other words, the agent either copies an existing agent from the last m steps, or invents a new location.

If an agent decides to co-locate with an existing agent, the process is equivalent to preferential attachment, because the probability for each location is proportional to the number of agents that have chosen it within the last m steps. This can produce a power law distribution of location popularity [18, 21], as we show for a particular run of the model in Figure 1(a). Although many models can produce power laws [22], the challenge is to do so in a model with correct level of turnover in the elements that comprise the power law, such that new entries can overtake old ones, and the "rich" do not always get richer, but occasionally become extinct.

In this model, with limited memory, there is always a non-zero chance that a location will be forgotten and not copied again. In the most limited memory case $(m = 1)$, the probability of a location of not being selected in a particular time step is just $(1-\phi)^{N(1-\mu)}$, where ϕ is the fraction of times the location was chosen in the last time step. Hence while more popular locations are less likely to be forgotten, as long as there remain multiple choices, any location can be forgotten with finite probability. Multiple choices are ensured when $\mu > 0$ in that new unique locations are continually injected into the pool of options. Even though these new locations enter with the lowest possible popularity (i.e., are chosen once), they introduce turnover at all popularities through drift, yielding a long-tailed distribution of life-spans which we show for the Top 10 in Figure 1(b). In other words, there is turnover even among the very top-ranked locations. Indeed, for $m = 1$, the model exhibits continual turnover in the top y highest-ranked locations, at a rate approximately proportional to $y\sqrt{\mu}$ [23].

A rich variety of distributional outcomes can be generated by the model, including a power law probability distribution in the number of copies among the locations (Figure 1(a)), along with continual turnover in the topmost ranked locations yielding a long-tailed distribution of life-spans, for example in the Top 10 (Figure 1(b)). The form of these distributions – number of lifespans and time spent in the list of top ranked items – is comparable to many real-world systems which display scaling in the right tail of their size distributions. Figures 1(c) and (d) show the distribution of boys' names in the US over the last century, keywords published within a particular academic paradigm [24], and number one pop albums in the UK over the last half-century. Figures 1(e) and (f) show the average scaling of the top 100 cities by population in the US over the last 160 years and the associated turnover in cities that comprise this order. These examples illustrate the diversity of scaling in human attributes, elements and groups that make up social systems at different spatial scales and suggest that power laws are an even more ubiquitous phenomena than our current knowledge suggests.

Up until now, other models have not naturally been able to account for flux in the constituents of this rank-size distribution [25], either when growth is one of strict preferential attachment or even when growth is proportionate to a stochastic rate independent of size [26]. This model is highly generalisable, as the 'locations' can represent anything that might be copied among agents – from dog breeds to baby names, pottery styles, etc. [18]. Similarly, the agents might represent anything from individuals to large groups. In the case of re-location, such groups could include divided villages in non-western societies [27], or the self-interested firms of western economies [28].

Our model captures two fundamental motivations, copying others versus novel invention, each of which is adaptive: copying carries the potential advantages of efficiency (not having to learn a behaviour from scratch), social acceptance and/or alliance formation, whereas innovation potentially offers freedom from competition, temporary local monopoly and (if the innovation becomes successful) high status or wealth. Both these micro-processes are required to understand and predict the dynamics of scaling distributions which are frequently observed on the macro-scale.

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[24] We define the paradigm as the set of all papers that cited a certain highly-cited, seminal work. Among several examples shown in our online supplementary data, here we use *Outline of a Theory of Practice*, by Pierre Bordieu (Cambridge University Press, 1977), which has been cited by over 2,700 journal articles listed by the ISI Web of Knowledge database. For these 2,700 articles, all words from titles and author-chosen keywords were sorted by publication year, with the following common words removed: a, an, and, as, by, for, from, in, its, of, on, the, to, using, and with. No other common words significantly affected the top 10 most popular or the frequency distributions.
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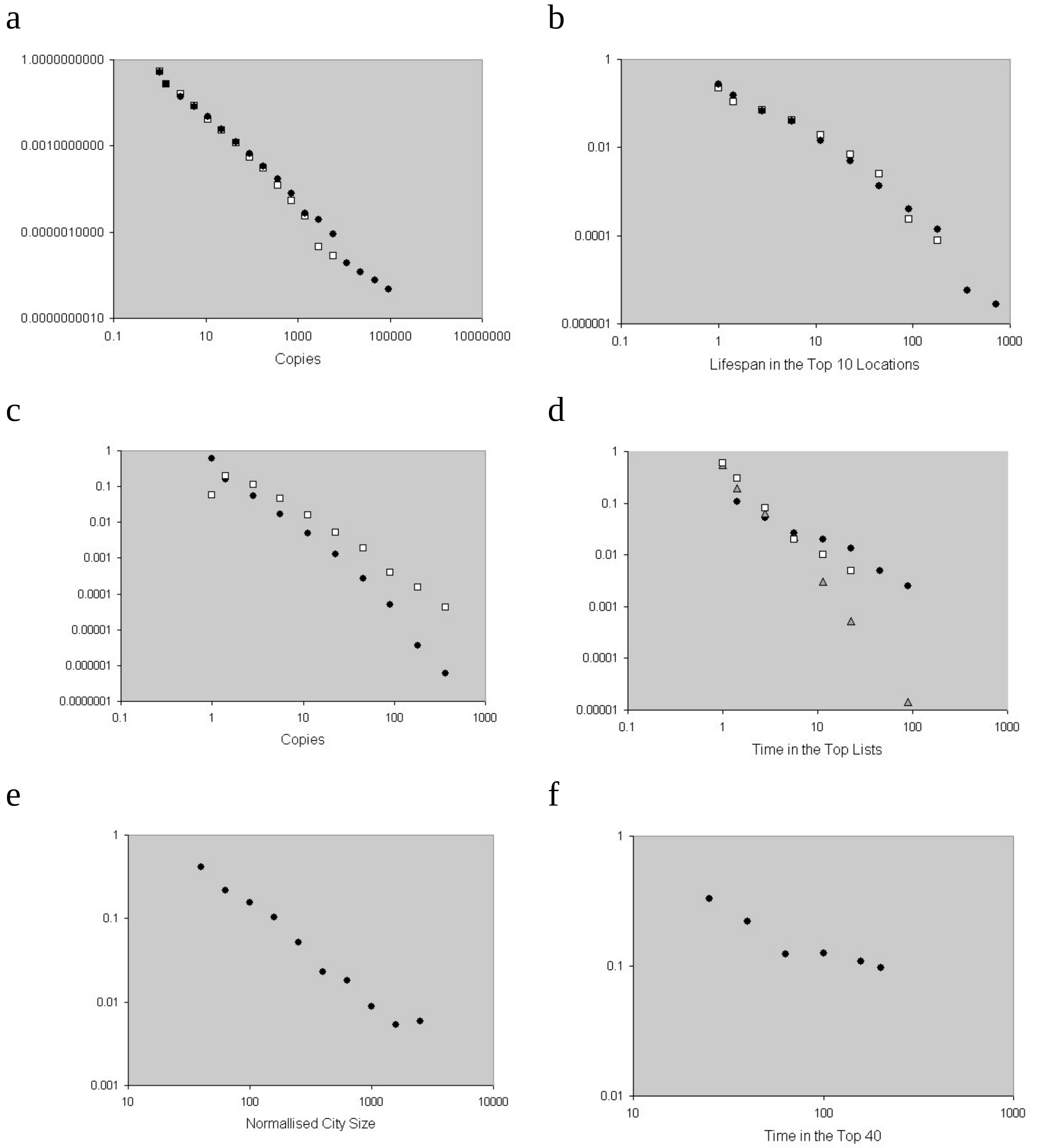

Figure 1. **(a)** Probability distribution of location copies for two characteristic runs of 200 time steps with $m = 1, \nu = 1, \mu = 0.03$ *and* $N = 200$ (filled circles) and $N = 250$ (white squares); (**b)** Probability distribution of time spent in the Top 10 locations for the same two runs; **(c)** Probability distribution of various real data sets, including boys' name frequencies (filled circles) and keywords within an academic paradigm [24], 1994-2007 (white squares); (**d)** Life-spans of individual variants, including years in the Top 5 US boys' names, 1907-2006 (filled circles), weeks at UK Number One for pop albums, 1956-2007 (grey triangles), half years among the Top Ten journal article keywords within an academic paradigm, 1994-2007 (white squares); (**e**) Averaged probability distribution of the Top 100 city sizes in the US between 1840 and 2000; and (**f**) the number of years spent in the Top 40 US City Size Ranks from 1840 to 2000.