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Developing an online cooperative police patrol routing strategy

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ABSTRACT

A cooperative routing strategy for daily operations is necessary to maintain the effects of hotspot policing and to reduce crime and disorder. Existing robot patrol routing strategies are not suitable, as they omit the peculiarities and challenges of daily police patrol including minimising the average time lag between two consecutive visits to hotspots, as well as coordinating multiple patrollers and imparting unpredictability to patrol routes. In this research, we propose a set of guidelines for patrol routing strategies to meet the challenges of police patrol. Following these guidelines, we develop an innovative heuristic-based and Bayesian-inspired real-time strategy for cooperative routing police patrols. Using two real-world cases and a benchmark patrol strategy, an online agent-based simulation has been implemented to testify the efficiency, flexibility, scalability, unpredictability, and robustness of the proposed strategy and the usability of the proposed guidelines.

1. Introduction

Patrolling is defined as "the act of walking or travelling around an area, at regular intervals, in order to protect or supervise it" (Abate, 1997, p 578). Police patrol occupies a central place in crime control efforts (Koper, 1995). As Cook (1980) stated, a visible police presence can increase the public's certainty of punishment, and a frequent police presence enhances potential criminals' perceptions of risk in the local area. In daily operations, approaches to police patrols range dramatically across varying contexts and cultures. Among them, one effective and promising approach is hotspot patrolling, or place-based patrolling, which focuses on crime hotspots, i.e., small geographical units with high crime intensity, such as street segments or small groups of street blocks (Braga, Papachristos, & Hureau, 2012). The effectiveness of hotspot patrolling in reducing crime has been proved by a range of experiments, such as the Minneapolis Hot Spots Patrol Experiment (Sherman & Weisburd, 1995) and the study conducted in Philadelphia (Ratcliffe et al., 2011). In patrolling, when there is more than one hotspot to cover, typically, police officers rotate randomly between hot spots, as in the field trial in Sacramento, California (Telep et al., 2014). However, the randomised strategy cannot be applied to situations where police resources are limited and there are many "hotspots" areas. Rather, the successful operation of patrolling to cover the "hotspots" requires a detailed patrol routing strategy. A relevant topic for policing is determining the efficient spatial distributions of police patrol areas to provide maximal and multiple coverage of incidents (Curtin et al., 2010). However, such strategies are focused on the location of centres of patrol areas, and do not consider a detailed routing strategy for patrol teams.

Designing a routing strategy for police patrolling is never a simple task due to several challenges. First, officers are required to cover hotspots regularly and repetitively (Curtin et al., 2010) as well as responding to emergencies. Thus, the performance of covering hotspots should not deteriorate significantly when emergencies occur and some patrollers are dispatched to handle them. Second, to cover the whole hotspot area effectively, police patrol requires cooperation among patrollers. Third, to confuse criminals and deter crime, the patrol routes should be somewhat difficult to predict. Additionally, hotspots may have different levels of importance and thus require different levels of attention. This problem is called the optimal design of patrol routes (ODPR) problem (Reis et al., 2006) or patrol route planning problem (Chen & Yum, 2010). This work focuses on designing patrol routes for foot patrol, rather than vehicle patrol.

All these challenges are very similar to the multi-agent patrolling problem (Almeida et al., 2004), or multi-robot patrolling problem (Portugal & Rocha, 2011), which focuses on surveillance tasks using multiple mobile robots to frequently visit important places in the environment. Here, we review the routing strategies in both police patrol and robot patrol because of their similarities. More importantly, the advances of multi-agent robot patrolling can benefit police patrol. Distinct solutions have been proposed to design patrol routes, which present different strategies in terms of routing, cooperation, evaluation, and other features. In general, they can be divided into pioneer strategies (Almeida et al., 2004; Machado et al., 2002; Portugal & Rocha, 2013a), operations research strategies (Chevaleyre, 2004; Elmaliach, Agmon, & Kaminka, 2009; Portugal & Rocha, 2010), alternative coordination strategies (Chen & Yum, 2010; Chu et al., 2007; Santana et al., 2004; Sempe & Drogoul, 2003), and interaction strategies (Reis et al., 2006; Tsai et al., 2010).

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Pioneer strategies use simple pioneer architectures to guide patrollers to visit places that have been visited less recently, and it would be convenient for them to consider other factors, such as distance and coordination. These simple and heuristics-based strategies have been shown to achieve good performance in covering hotspots and coordinating patrollers (Portugal & Rocha, 2013a). However, most of the pioneer strategies are developed in the context of robot patrol, and they neglect the aspects of being unpredictable in patrol routes and being robust to the influences of emergency response.

Operations research strategies use graph theory tools to compute low-cost cycles and efficient routes for each patroller. The tools include Travelling Salesman or the Hamilton cycle (Elmaliach et al., 2009; Pasqualetti, Franchi, & Bullo, 2012; Smith & Rus, 2010), spanning trees (Fazli & Davoodi, 2010; Gabriely & Rimon, 2001), and graph partitioning (Sak, Wainer, & Goldenstein, 2008; Stranders et al., 2013). These strategies guarantee high visit frequency on targets and efficient cooperation between patrollers, and they scale well with different numbers of patrollers. However, these strategies are naturally deterministic, which would more easily allow intelligent criminals to predict the patrol routes and take advantage of the idle time between the visits of patrollers. Additionally, Hamilton cycles and other algorithms have high computational complexity and are difficult to generalise to large numbers of targets. Moreover, these strategies would have to re-compute patrol routes if the number of patrollers were to change because of an emergency response.

Alternative coordination strategies aim to solve the routing problem using approaches such as task allocation (Sempe & Drogoul, 2003), reinforcement learning (Santana et al., 2004), cross entropy method (Chen & Yum, 2010), and swarm intelligence (Chu et al., 2007). However, strategies like reinforcement learning and the cross entropy method prove to be very complex in nature, so while they are suitable for designing patrol routes for a single patroller, it is difficult to extend them to cooperative patrol with multiple patrollers.

Interaction strategies have been derived from the interactions between officers and criminals, using agent-based simulation or game theory models. For example, Reis et al. (2006) designed patrol routes based on genetic algorithms and a multi-agent-based simulation, where a set of criminals frequently try to commit crimes and officers try to prevent crimes. Tsai et al. (2010) derived a strategy for police resource allocation based on modelling the interactions between police and terrorists as an attacker-defender Stackelberg game, where a player always predicts his opponent's behaviour and chooses the best response. These strategies can effectively prevent crime in crime hotspots, but only on the basis of a substantial knowledge of crime mechanisms in the area, and it is difficult to generalise these strategies to guiding police patrol in large areas and preventing multiple types of crimes.

In summary, existing approaches for patrol routing are not applicable to guide police patrol, as they omit the peculiarities and challenges of police daily patrol. To facilitate the design of an effective routing strategy for police patrol, the challenges mentioned above need to be specified and formulated using clear guidelines and need to be quantified by appropriate evaluation measures. To our knowledge, few studies have dealt with this issue. Therefore, there is a need for a comprehensive study of guidelines and evaluation measures for designing a routing strategy for police patrols.

In this work, we propose a set of guidelines for an effective police patrol routing strategy and the relevant evaluation measures, which are based on the characteristics and challenges of practical police patrol. Under such guidelines, we develop an effective routing strategy based on heuristics and Bayesian techniques, and subsequently quantify their effectiveness through realistic simulation tests and in comparison with a graph theory strategy.

This paper is a further development of, and substantial improvement on, a previous work (Chen, Cheng, & Wise, 2015). In addition to the broad background introduced above, the current paper is substantially improved in five aspects. First, only three guidelines were discussed in Chen et al. (2015), namely, efficiency, flexibility, and unpredictability. Here two more guidelines—scalability and robustness—are developed, which measure the general applicability of the routing strategy in different situations including different team size, hotspot areas, and emergencies, as this has not be discussed in any previous literature. Furthermore, the guideline of unpredictability is further quantified here, which was only conceptually discussed in Chen et al. (2015). Second, the Bayesian Ant-based Patrol Strategy (BAPS) is further developed in accommodating these guidelines. Third, an agent-based modelling is now implemented as an online mode that simulates the real-world police patrol with the interaction of the controller and patrollers. Fourth, the strategy is now tested to include the emergency scenario, which was not included in the previous paper. Finally, in order to test its applicability to different areas, a new case, namely, South Chicago, is added in addition to Camden. Furthermore, the Camden case is now conducted with different team sizes and more experiments to cover the five measures.

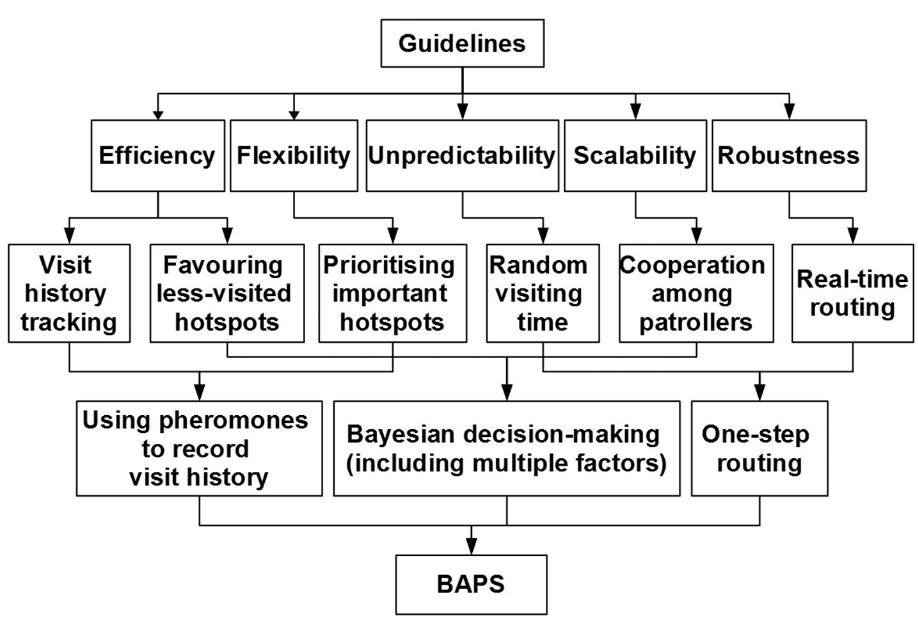

Fig. 1. From guidelines to BAPS.

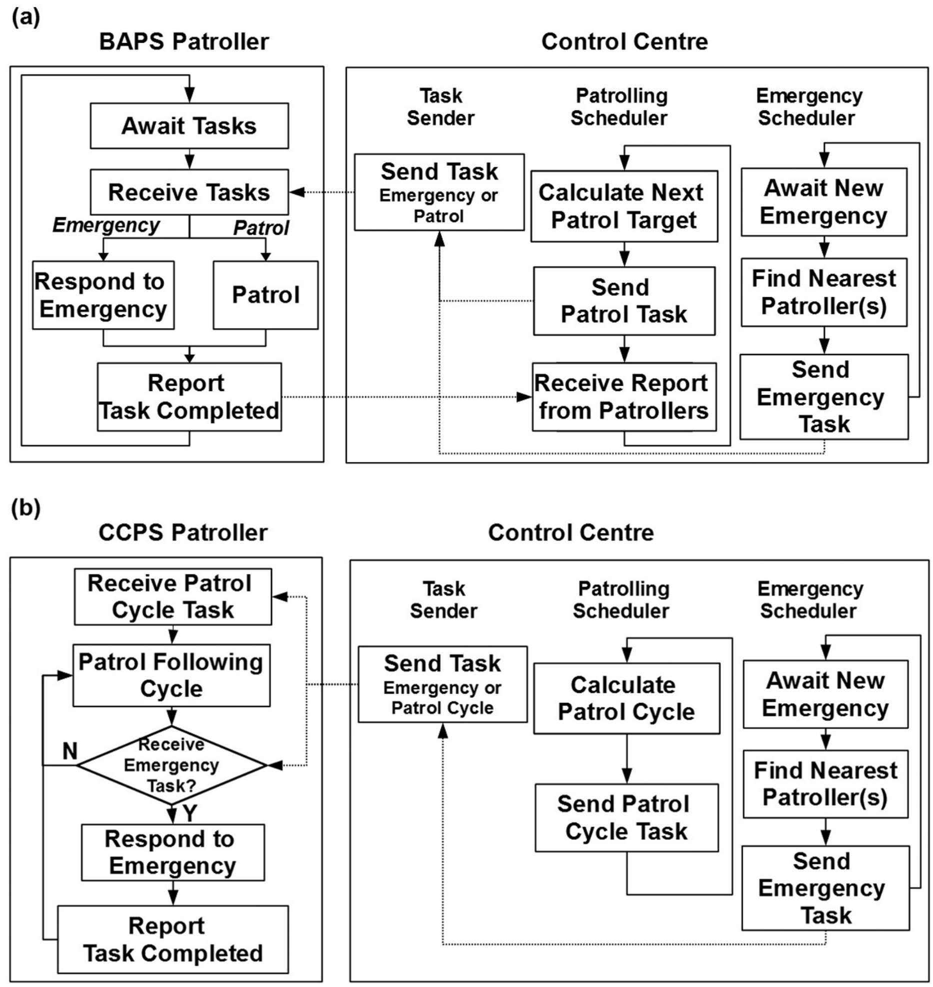

Fig. 2. Schematic diagram of patrolling strategies in the emergency scenario. (a) BAPS, (b) CCPS.

The remaining sections are organised as follows. In Section 2, we formulate the guidelines and evaluation measures for police patrol routing strategies, which is followed by a Bayesian Ant-based Patrolling Strategy (BAPS) proposed in Section 3. Section 4 develops an agent-based simulation of real-time police patrols, in order to test the effectiveness of the proposed routing strategies. To test the proposed

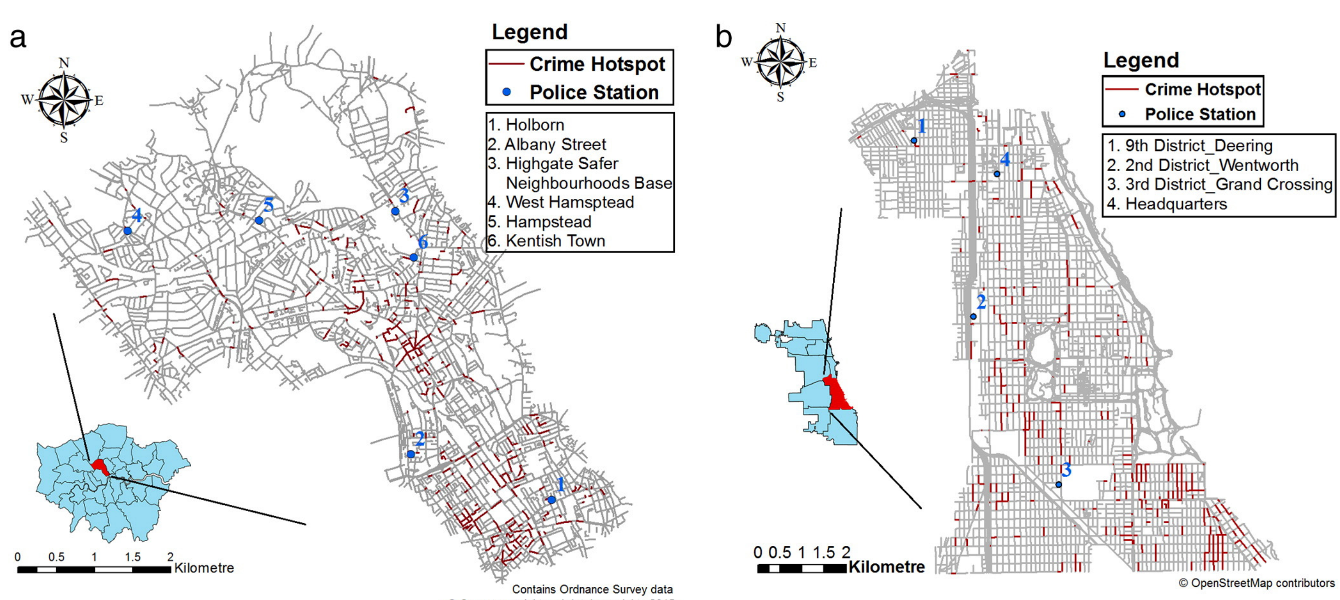

Fig. 3. Crime hotspot map. (a) in Camden, (b) in South Chicago.

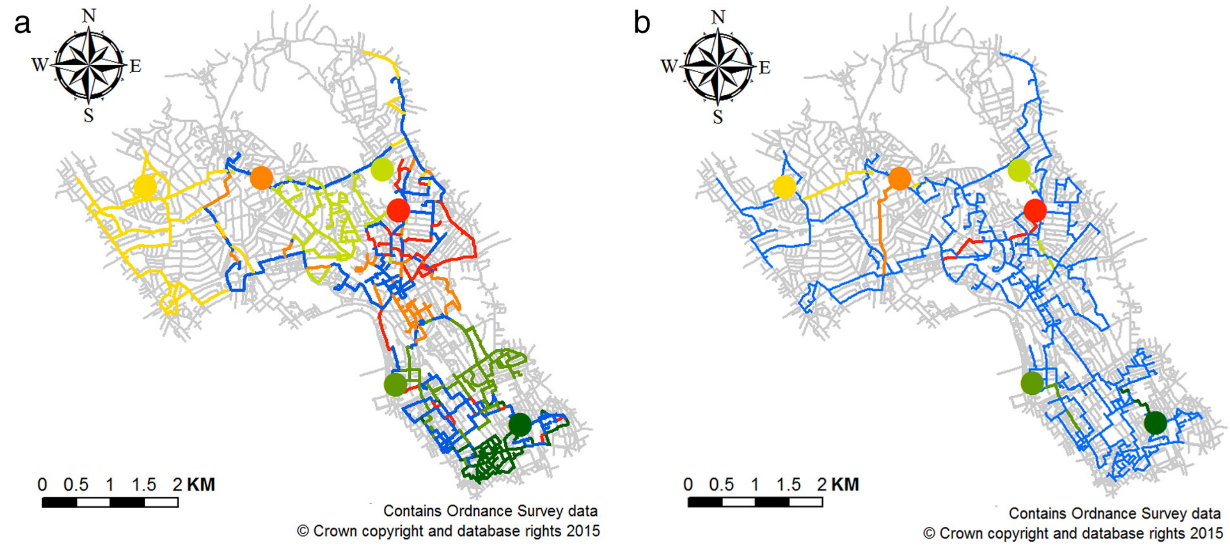

Fig. 4. Individual routes of six patrollers (the blue segments are overlaps among different routes). (a) using BAPS, (b) using CCPS. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

strategy vigorously, Section 5 presents two case studies of police patrol using realistic police and crime. Section 6 summarises the major findings and discusses topics for future study.

2. Guidelines and evaluation measures

Before we introduce the guidelines, it is necessary to describe the patrolling procedure. This study uses a simplified procedure of real-world police patrolling. The environment is the road network in an urban area. Certain road segments are identified as hotspots through crime mapping and prediction (Ratcliffe, 2010), and the n hotspots identified are denoted as $H=\{h_1,h_2, \ldots ,h_n\}$. The idleness of a hotspot is defined as the time duration between the two consecutive visits, and the average idleness of a hotspot is the average of the idleness sequence. Patrollers have full knowledge of the area and always travel to the next hotspot via the shortest path on the network. A control centre dispatches the patrolling tasks to the patrollers, and receives the response and feedback from patrollers. The control centre can use different routing strategy to guide the movements of patrollers and to affect how the hotspots are monitored.

Police patrol mainly aims at preventing and reducing potential crime. A fundamental question is what makes a good police patrol routing strategy. We claim that a good patrol routing strategy should follow the guidelines that are proposed in this study, which as a minimum, should include efficiency, flexibility, scalability, unpredictability, and robustness. Since the efficiency and flexibility have been discussed in Chen et al. (2015), we will briefly recap some of the main points here, with more focus on the new measures for scalability, unpredictability and robustness.

Several previous studies have provided inspirations for the guidelines and measures discussed here. Two basic concepts, namely, the idleness and global idleness of patrolled targets, were first introduced by Machado et al. (2002), and these are directly used in this study. Portugal and Rocha (2013a) proposed the measure of team scalability to quantify the impact of team size on robot patrolling, and the measure of robustness to consider the influence of communication errors in robot patrolling. These measures are adapted here to account for the impact of team size and emergency response in police patrol. To our knowledge, this is the first research to use the measures for team scalability and robustness in police patrol. The other measures, including flexibility, unpredictability, and spatial scalability, are first proposed in this study.

2.1. Efficiency

Police patrol requires every important place or hotspot to be regularly and repetitively visited. Thus, efficiency is the foremost requirement for police patrol, which means patrols should minimise the time lag between two visits to every hotspot (Chen et al., 2015).

The measure of efficiency has been systematically discussed in Chen et al. (2015), and the concepts are used in this study, except for the different notations. Efficiency is measured by global average idleness (*GAI*) (Chen et al., 2015). $GAI(t)$ is the global average idleness among all hotspots at time t, and is the defined as:

$$GAI(t) = \sum_{i=1}^{n} AIdl(h_i, t)/n \tag{1}$$

This study uses continuous time, and all the measures related to time use the time unit of second. Here, $AIdl(h_i,t)$ represents the average idleness of a hotspot h_i at time t, and n is the number of hotspots. $GAI(t)$ changes with time. When patrolling begins, *the idleness* of each hotspot is set as 0, as if it has just been visited (Chevaleyre, 2004) and then it gradually converges as the distribution of patrollers becomes stable. Empirically, $GAI(t)$ is regarded as converged if the relative difference of its value in two consecutive patrolling is within 1%. The converged $GAI(t)$ is denoted as *GAI*, and is used to measure efficiency.

2.2. Flexibility

The flexibility of a patrol routing strategy is to prioritise the more important hotspots so that they have higher visiting frequency or lower average idleness. Such flexibility is measured by weighted global average idleness (*WGAI*) (Chen et al., 2015), which is the converged

Table 1
Efficiency performance in Camden (values in seconds).

Team size	12	18	24	30	36	42	48
GAI_CCPS	5079	3477	2605	2039	1709	1443	1254
GAI_BAPS	4407	2833	2128	1700	1401	1223	1075
Relative change (%)	−13.2	−18.5	−18.3	−16.6	−18.0	−15.2	−14.3

Table 2
Efficiency performance in South Chicago (values in seconds).

Team size	8	12	16	20	24	28	32
GAI_CCPS	11,133	7425	5613	4418	3711	3152	2762
GAI_BAPS	11,725	6899	4981	3917	3253	2824	2478
Relative change (%)	5.32	−7.08	−11.26	−11.34	−12.34	−10.41	−10.28

WGAI(*t*). *WGAI*(*t*) is the weighted *GAI*(*t*):

$$WGAI(t) = \sum_{i=1}^{n} W(h_i) \times AIdl(h_i, t) / \left[\sum_{i=1}^{n} W(h_i)\right] \quad (2)$$

Here $W(h_i)$ is the weight of hotspot h_i. The higher crime density level of a hotspot, the larger weight, which means the higher priority for patrolling. $AIdl(h_i, t)$ is the average idleness of h_i at time t.

Similar to *GAI(t)*, *WGAI*(*t*) changes with time and converges after a certain period in patrolling, and the converged value is denoted as *WGAI*.

2.3. Scalability

A promising patrol strategy should be applicable to different areas and with different numbers of patrollers; this is described as scalability. There are two types of scalability for a patrol routing strategy: *team scalability* and *spatial scalability*.

Team scalability is related to how well the strategy performs as the number of patrollers increases (Portugal & Rocha, 2013b). A scalable patrolling strategy can adapt to different team sizes without severe performance degradation.

Team scalability can be evaluated by a classical metric, called Balch's speedup measure (Balch & Arkin, 1994). In the patrolling problem, Balch's speedup measure reveals how much more efficient a group of patrollers is than just one patroller in completing the patrolling task, and is defined as follows:

$$v(R) = GAI(1)/[R \times GAI(R)] \quad (3)$$

where *GAI*(*R*) and *GAI*(1) are the *GAI* value of patrolling by *R* patrollers and by one single patroller, respectively.

If a group of *R* patrollers are more efficient and achieve a low *GAI* value, the resultant $v(R)$ would be >1.0, and this performance is said to be superlinear. Linear performance is equal to 1.0, which means equal performance, and sublinear performance is <1.0, corresponding to the lower efficiency (Balch & Arkin, 1994). Since it is uncommon for one patroller to patrol a large area, the measure is modified using a small size *S* as the reference, and the modified measure is named as ST (Scalability of Team Size):

$$ST(R) = [S \times GAI(S)]/[R \times GAI(R)] \quad (4)$$

Spatial scalability of a patrol routing strategy concerns its performance in different space areas, including the layout of the area, the density of crime hotspots, and the distribution of police officers. Unlike team scalability, the various factors of spatial scalability are difficult to quantify. Generally, spatial scalability can be measured by comparing the efficiency of the designed strategy with the benchmark strategy. To our knowledge, no previous study has considered the spatial scalability of routing strategies in the context of police patrol.

Here we consider the influence of crime hotspot density level, as one example of spatial scalability. For convenience, the notation *crime density level at x%* represents that the total length of crime hotspots cover x% of the total segment length of the road network. The measure to compare the performance in two hotspot density levels is named as SS (Spatial Scalability):

$$SS(L_i) = [GAI(L_i) - GAI(L_B)]/[GAI(L_B)] \quad (5)$$

Here, $SS(L_i)$ refers to relative change in *GAI* performance due to the change of hotspot density from the baseline level L_B (e.g. level at 5%) to L_i (e.g. level at 10% or 15%).

2.4. Unpredictability

If potential criminals can easily deduce the patrol routes or the visits to hotspots, they would commit a crime within the time between two visits, thus rendering police patrol ineffective (Sak et al., 2008). Therefore, it is important to keep the patrolling strategy unpredictable

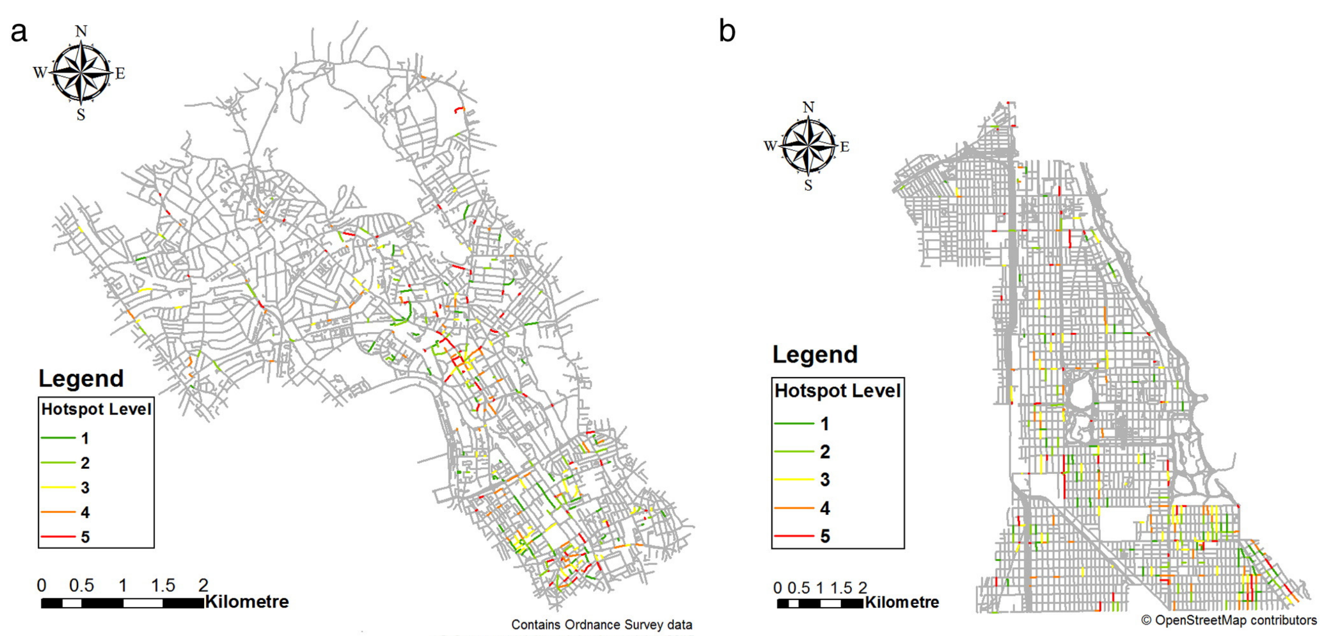

Fig. 5. Crime hotspot maps with different risk levels. (a) in Camden, (b) in South Chicago.

Table 3
Flexibility performance of patrolling on hotspots with priority in Camden (all values in seconds).

Strategy	WGAI	GAI	GAI of each level				
			1	2	3	4	5
CCPS	2030	2040	2075	2048	2088	2039	2004
BAPS	1653	1671	1718	1725	1697	1700	1607
WBAPS	1623	1712	2054	1872	1720	1620	1456

(Sherman et al., 2014; Yin et al., 2012). The greater is the uncertainty of police visits, the greater is the risk perceived by potential criminals (Sherman, 1990).

There are two kinds of randomness in the patrolling problem: randomness of patrol routes and randomness of visits to hotspots. The former can be evaluated using the entropy of a patrol strategy, as proposed by Chen and Yum (2010). However, the entropy quantifies the dissimilarity of different patrol routes but fails to measure the randomness of police visits to a given place. An experienced burglar waiting around a potential target for a time when no patrols are nearby would be more concerned about predicting the time of the next visit rather than the routes of the patrol team. Here, the randomness of visitations to hotspots is evaluated by the average of the standard deviation of idleness on each hotspot:

$$ASDI(t) = \sum_{i=1}^{n} SDI(h_i, t)/n \tag{6}$$

where $SDI(h_i, t)$ is the standard deviation of idleness of hotspot h_i at time t, and $ASDI(t)$ is the average of $SDI(h_i, t)$ for all hotspots, with total number n.

Similarly, the converged $ASDI(t)$ is denoted as $ASDI$. The higher value of $ASDI$ is favoured, as it means higher unpredictability in the patrol routes and so it is less likely to be predicted by offenders. This is the first time that the unpredictability is quantified for police patrol.

Besides the random visiting time, there are other methods to impart unpredictability to patrolling, such as accessing a long hotspot segment randomly from both ends.

2.5. Robustness

Because officers are also responsible for dealing with emergency calls during patrolling, it is necessary to use a robust strategy, i.e., one that remains effective even if some patrollers are dispatched for emergencies. Here, the measure of robustness is the relative increase of the GAI value in emergency scenario in comparison with the GAI value in non-emergency scenario, which is represented as:

$$RI\_GAI = (GAI\_Emerg - GAI\_Norm)/(GAI\_Norm) \times 100\% \tag{7}$$

where RI_GAI is the relative increase of GAI, and GAI_Emerg and GAI_Norm are the GAI in the emergency scenario and normal scenario respectively. The routing strategy with low RI_GAI is preferable, as it is less influenced by emergencies.

To summarise, these guidelines describe five requirements for an effective police patrol routing strategy to minimise the idleness and its change in different situations. These requirements are efficiency (global idleness), flexibility (weighted hotspots), scalability (change of the number of patrollers), unpredictability (randomness of idleness), and robustness (dealing with emergencies). It is difficult to define the general threshold values of each evaluation criteria to identify an effective strategy, as this depends on the situation, which will be shown in the case studies in Section 4.

Table 4
Flexibility performance of patrolling hotspots with priority in South Chicago (all values in seconds).

Strategy	WGAI	GAI	GAI of each level				
			1	2	3	4	5
CCPS	4442	4361	4358	4380	4460	4346	4550
BAPS	3890	4032	4046	4035	4000	3869	3753
WBAPS	3888	4214	4762	4466	3999	3715	3554

Table 5
Performance of team scalability in Camden (GAI values in seconds).

Team size	12	18	24	30	36	42	48
GAI_CCPS	5079	3477	2605	2039	1709	1443	1254
GAI_BAPS	4407	2833	2128	1700	1401	1223	1075
TS_CCPS	1.000	0.974	0.975	0.996	0.991	1.006	1.013
TS_BAPS	1.000	1.037	1.035	1.037	1.049	1.030	1.025

3. An online Bayesian ant-based patrolling strategy

This section describes how the aforementioned guidelines are transferred into a patrol routing strategy. The strategy to achieve the necessary efficiency and flexibility has been described in Chen et al. (2015); here, we give details of how to turn other guidelines into the same Bayesian Ant-based Patrolling Strategy (BAPS). This will provide a new perspective to understand the routing strategy and an example to transfer the guidelines to a practical strategy.

3.1. Transferring guidelines to strategy

To follow the guidelines, one possible "bottom-up" approach is to turn each guideline into implementable modules, which are then assembled to form a complete routing strategy (see Fig. 1). First, the efficiency requires fair and frequent visits on each hotspot, without any hotspot being neglected for a long time, which requires tracking of the visit history of each hotspot ("history tracking" for short), and decision-making that favours less-visited hotspots. Second, the flexibility requires prioritising important hotspots in the "history tracking". Third, the unpredictability of patrol routes would prefer irregular or random visiting times to hotspots, rather than the repetition of a predefined route. Fourth, the scalability calls for the cooperation among patrollers, and the cooperation should be adaptable to different hotspot distributions and different team sizes. That is, the route of one patroller should take account of the routes and distribution of other patrollers. Fifth, the robustness requires real-time routing, which considers the current hotspot distribution and activity of patrollers.

The next question is how to implement and assemble these modules to form a strategy. The modules of visit history tracking and prioritising important hotspots can be implemented by the pheromone mechanism, with different decaying rates for different weights if needed. The requirements of preference on less-visited hotspots and cooperation among patrollers can be satisfied by the Bayesian decision-making process. Moreover, the random visiting time and real-time routing is achieved via the one-step routing, which calculates only the next patrol target in each step.

Table 6
Performance of team scalability in South Chicago (GAI values in seconds).

Team size	8	12	16	20	24	28	32
CCPS	11,133	7425	5613	4418	3711	3152	2762
BAPS	11,725	6899	4981	3917	3253	2824	2478
ST_CCPS	1.000	1.000	0.992	1.008	1.000	1.009	1.008
ST_BAPS	1.000	1.133	1.177	1.197	1.201	1.186	1.183

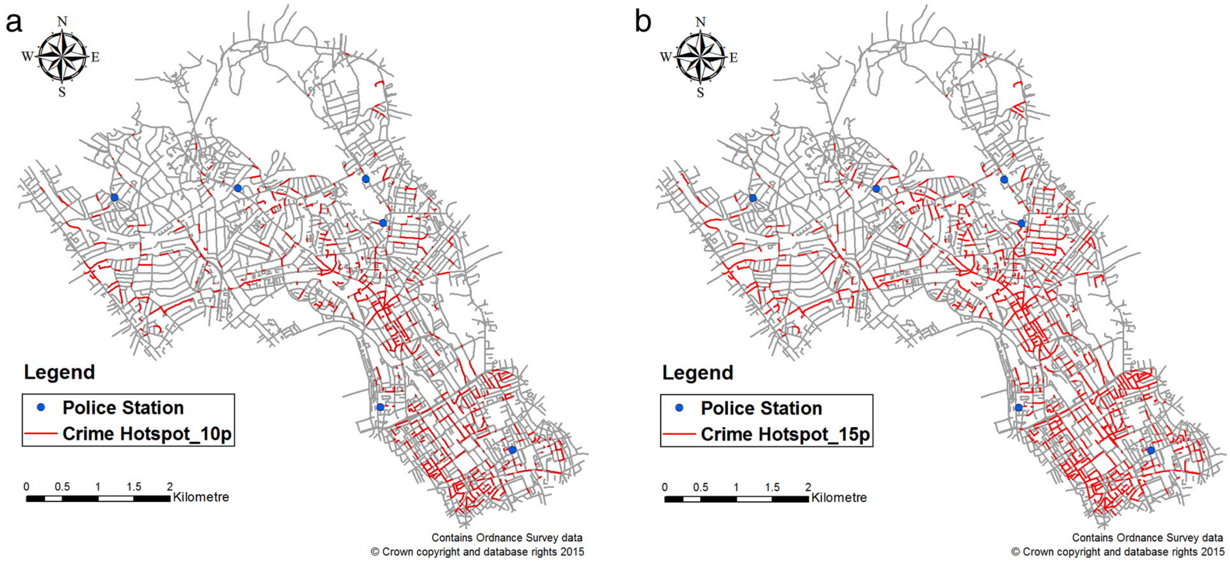

Fig. 6. Crime hotspot maps with denser hotspots. (a) in Camden with 10% of total road length, (b) in Camden with 15% of total road length, (c) in South Chicago with 10% of total road length, (d) in South Chicago with 15% of total road length.

3.2. BAPS

The resultant routing strategy BAPS includes two components: pheromone mechanism and Bayesian decision. These two components will be described in sequence.

The pheromone is used to track the visiting history of a hotspot. The pheromone level of a hotspot is affected by two procedures, including deposit and decay.

Firstly, deposit occurs when a hotspot is visited at time t and the pheromone level is increased:

$$\mathrm{Phe}(h_i,\mathrm{t}) = \mathrm{Phe}(h_i,\mathrm{t}-1) + \mathrm{Phe\_Dep}(h_i) \tag{8}$$

where $\mathrm{Phe}(h_i,\mathrm{t})$ is the pheromone level of h_i at time t, and $\mathrm{Phe\_Dep}(h_i)$ is the amount of pheromone deposit on h_i after a visit.

Secondly, the pheromone levels decay exponentially at each hotspot. The decaying from time t_0 to t is formulated as:

$$\mathrm{Phe}(h_i,\mathrm{t}) = \mathrm{Phe}(h_i,t_0) \times \lambda(h_i)^{t-t_0} \tag{9}$$

where $\lambda(h_i)$ is the decay rate at hotspot h_i, and $\lambda(h_i)\in(0,1)$.

Overall, the pheromone level at a hotspot is affected by the time and frequency of visits, and is controlled by multiple parameters: the decay rate and the amount of pheromone per deposit. These parameters can be adjusted to prioritise certain hotspots.

The Bayesian decision determines which hotspot to patrol in the next stage. For n hotspots, the decision is applied independently n times to calculate the posterior possibility, and the hotspot with the largest possibility is the next target. Factors influencing the posterior possibility include the pheromone level, the distance from the hotspot to the patroller, and the moving direction of other patrollers. Therefore, the posterior possibility of patrolling hotspot h_i is defined as:

$$P(\mathrm{patrol}(h_i)|\mathrm{G}(h_i),\mathrm{S}(h_i))P(\mathrm{patrol}(h_i)) \times \left[\frac{P(\mathrm{G}(h_i)|\mathrm{patrol}(h_i))}{P(\mathrm{G}(h_i))}\right] \times \left[\frac{P(\mathrm{S}(h_i)|\mathrm{patrol}(h_i))}{P(\mathrm{S}(h_i))}\right] \tag{10}$$

For simplification, the term of time is omitted in Eq. (10). $P(\mathrm{patrol}(h_i)|\mathrm{G}(h_i),\mathrm{S}(h_i))$ and $P(\mathrm{patrol}(h_i))$ are the posterior and prior possibility of patrolling h_i $(i=1,n)$, respectively. $\mathrm{G}(h_i)$ represents the gain of patrolling h_i. $P(G(h_i))$ is the prior probability of the gain, and $P(\mathrm{G}(h_i)|\mathrm{patrol}(h_i))$ is the probability of gain $\mathrm{G}(h_i)$ on the condition that h_i is patrolled.$\mathrm{S}(h_i)$ represents the number of patrollers that are going to patrol h_i. $P(\mathrm{S}(h_i))$ is the prior probability of $\mathrm{S}(h_i)$, and $P(\mathrm{S}(h_i)|\mathrm{patrol}(h_i))$ is the probability of $\mathrm{S}(h_i)$ on the condition that h_i is patrolled. In this study $P(\mathrm{patrol}(h_i))$ is defined as uniform among every hotspot, and is omitted in the computation for simplification.

The gain of patrolling h_i, $\mathrm{G}(h_i)$ is defined as $\mathrm{G}(h_i)=1/[\mathrm{Phe}(h_i,\mathrm{t})\times \mathrm{NormDist}(\mathrm{p},h_i)]$, where $\mathrm{NormDist}(\mathrm{p},h_i)$ is the normalised distance from p, the current position of the patroller, to the hotspot h_i. The normalisation is done to avoid local optima in which patrollers repeatedly patrol hotspots in a small cluster and neglect other hotspots. The distribution of $\mathrm{G}(h_i)$ needs to be defined. Without loss of generalisation, $\mathrm{G}(h_i)$ is defined as a continuous random variable with a probability density function f(g):

$$f(g) = \frac{1}{M} \times \ln\left(\frac{1}{L}\right) \times e^{\ln\left(\frac{1}{L}\right)\times\frac{g}{M}} \tag{11}$$

where L and M are constants, and $L>0$, $M>0$. L controls the probability values for zero gain and M is the gain saturation. Empirically, the value of L is selected as close to 0, and M is the maximum of gain when the

Table 7
Performance of spatial scalability in denser hotspot maps (values in seconds).

HotspotMap_TeamSize	Camden_10p_30	Camden_15p_30	SouthChicago_10p_20	SouthChicago_15p_20
GAI_CCPS	2806	3415	6492	8035
GAI_BAPS	2489	3132	6358	8392
Relative change (%)	−11.30	−8.29	−2.06	4.44
SS_CCPS (%)	37.62	67.48	46.94	81.87
SS_BAPS (%)	46.41	84.24	62.32	114.25

lower bound of the pheromone level and the lower bound of the normalised distance are used.

In Eq. (10), $P(\mathrm{G}(h_i))$ is treated as a normalisation factor and omitted in the computation for simplification (Jensen & Nielsen, 2007). $P(\mathrm{G}(h_i)|\mathrm{patrol}(h_i))$ is defined as $P(\mathrm{G}(h_i)|\mathrm{patrol}(h_i)) = f(\mathrm{G}(h_i))$.

$P(\mathrm{S}(h_i)|\mathrm{patrol}(h_i))$ and $P(\mathrm{S}(h_i))$ are used to coordinate the multiple patrollers. The idea behind is that a patroller should avoid patrolling the same hotspots as other teammates. The distribution of $\mathrm{S}(h_i)$ can be defined as $\mathrm{f}(s) = \frac{2^{m-(s+1)}}{2^m - 1}$, where m is the number of patrollers. Like $P(\mathrm{G}(h_i))$, $P(\mathrm{S}(h_i))$ is a normalisation term and can be omitted in the computation for simplification, and $P(\mathrm{S}(h_i)|\mathrm{patrol}(h_i))$ equals $\mathrm{f}(s = \mathrm{patrol}(h_i))$.

Overall, the hotspot to patrol next is the one with the highest posterior probability:

$$h_{next} = \underset{h_i}{argmax}\, P(patrol(h_i)|G(h_i), S(h_i)) \tag{12}$$

If more than one hotspot has the equal and highest probability, the hotspot is randomly selected from these candidates.

Notice that BAPS is a greedy strategy, as it searches only for the optimal choice of the next patrol target, instead of building the optimal patrol route for over a long period. A greedy strategy is used to find the locally optimal choice at each step, in the hope that these steps will lead to a globally optimal solution (Thomas et al., 2009). It is useful when obtaining a globally optimal solution is infeasible in a reasonable time.

4. Agent-based modelling of cooperative police patrols

This section presents a multi-agent modelling framework to test the effectiveness of the routing strategy. Agent-based modelling (ABM) is a simulation technique that seeks to capture how individual behavioural units interact with each other and with the environment, allowing higher-order behaviours to emerge from these interactions (Epstein & Axtell, 1996). Chen et al. (2015) tested BAPS in an agent-based simulation, but only in a non-emergency scenario. This study extends the framework to incorporate the emergency response.

In this ABM framework, the environment is a street network in the urban area, and crime hotspots are the street segments with a high crime risk. There are two types of agents, namely, patrollers and the control centre. Foot patrols with uniform skills and speed are dispatched either to patrol or to deal with an emergency. The control centre records the system state (idleness and visiting history of hotspots, etc.), communicates with patrollers, calculates patrol routes, and sends tasks to patrollers.

The framework is used to model BAPS and a benchmark strategy Christofides Cyclic Patrolling Strategy (CCPS), which is a deterministic and cyclic patrolling strategy from graph theory (Chen et al., 2015). CCPS is used as benchmark, as the real-world patrol strategy is confidential and difficult to obtain. Moreover, the cyclic strategies are classical algorithms for patrolling problems and perform well in different situations (Chevaleyre, 2004).

CCPS is fundamentally different from BAPS. CCPS firstly compute the shortest cyclic route that covers every hotspot at least once. This problem is known as the Rural Postman Problem, which can be solved by the Christofides Algorithm (Christofides et al., 1981). Then, the patrollers are distributed evenly on the route, and they begin to patrol following the same direction on the cycle. Thus, CCPS strives to achieve a regular and fair visit on each hotspot. In contrast, patrol routes in BAPS are built in real time, which requires patrollers to communicate with the control centre after they finish the patrol task, and to wait for the command of the next patrol target.

In the emergency scenario, patrollers in both BAPS and CCPS would be interrupted if an emergency occurred. The officers in the neighbourhood of the emergency would stop patrolling and head for the emergency site. They would resume patrolling after dealing with the emergency. To our knowledge, this is the first time that a cyclic patrolling strategy has been tested in the emergency scenario. We believe this will give a fair comparison for both strategies.

Processes of the two strategies are presented in Fig. 2. The control centre consists of the patrol route scheduler and the emergency scheduler. The patrol route scheduler calculates the next patrol target for BAPS patrol (Fig. 2a), or the cycle for CCPS patrol (Fig. 2b), and the emergency scheduler sends out the emergency task to patrollers close to the emergency sites. Patrollers are assigned to patrol or to respond to emergency calls. Whenever a patroller receives the emergency task, s/he stops patrolling and responds to the emergency. In BAPS, whenever a patroller finishes the emergency or patrolling task, s/he sends out "Task completed" to the control centre and awaits the next task. However, in CCPS, patrollers follow the cycle in their patrolling. The spatial differences of the two strategies will be demonstrated in the case studies. The normal scenario is a simplification of the emergency scenario in which no emergency occurs and the emergency scheduler is not used.

Table 8
Measure of unpredictability in Camden (all values in seconds).

Team size	12	18	24	30	36	42	48
ASdIdl_CCPS	939	1233	1599	497	753	583	493
ASdIdl_BAPS	4349	3368	2590	2055	1649	1422	1197

5. Case studies

To test the applicability of the guidelines and BAPS, two case studies were conducted, one in the London Borough of Camden, London, United Kingdom, and the other in South Side, Chicago, Illinois, the United States of America. For convenience, they are called Camden and South Chicago. The Camden case in this study is distinguished from that of the previous study (Chen et al., 2015) by different team size and more experiments to cover the five measures.

The data used are provided by various agencies. Details of the Camden data can be found in Chen et al. (2015), and some original crime incidents were aggregated to the centre of grids when they were recorded. Emergency calls data are added here to test the robustness. In the South Chicago case, the street network data are obtained from OpenStreetMap. The locations of police stations and the crime dataset from 2001 to present are from the City of Chicago data portal (https://data.cityofchicago.org). The time duration of crimes used is from 2011-03-01 to 2012-03-01, and the crime types include theft, burglary, homicide, battery, arson, motor vehicle theft, assault, and robbery. Crime counts of each segment are computed by the same method as the Camden case. Due to the lack of emergency call data and police dispatch data for South Chicago, the robustness is tested using hypothetical emergency calls, whose location and time is generated from a uniform distribution in the area and time period. Crime hotspots are identified as the street segments with the highest crime density and covering 5% of the total road length (Chen et al., 2015). There are 311 and 289 crime hotspots in Camden and South Chicago, respectively. Fig. 3 shows the hotspot maps.

The agent-based framework is built using Java and the MASON simulation toolkit (Luke et al., 2005). The simulation is updated on a temporal scale of 5 s per simulation step. Simulation results are analysed using R language and environment (R Core Team, 2015). BAPS and CCPS were tested with different sizes of patrollers (2–8 officers per police station).

Table 9
Measure of unpredictability in South Chicago (all values in seconds).

Team size	8	12	16	20	24	28	32
ASdIdl_CCPS	2218	1309	1843	1441	955	1132	1231
ASdIdl_BAPS	7836	5696	4735	4148	3696	3410	3010

Table 10
Robustness performance in Camden in emergency scenario.

Patrollers per emergency	0	1	2	3	4
BAPS_18	0.0%	1.8%	4.5%	4.7%	10.2%
BAPS_30	0.0%	3.0%	3.3%	3.9%	5.4%
BAPS_48	0.0%	−0.2%	0.3%	1.0%	2.4%
CCPS_18	0.0%	1.1%	0.9%	1.6%	4.1%
CCPS_30	0.0%	3.2%	4.1%	3.4%	5.5%
CCPS_48	0.0%	4.7%	4.8%	5.7%	6.1%

GAI or *WGAI* is considered to converge when its value after any patrol cycle converges with no >1% difference from the previous cycle. Each simulation ran for at least 11 patrol cycles, after which, each hotspot had been visited at least 11 times. This number was selected experimentally to guarantee the convergence of *GAI* or *WGAI*. The parameters of BAPS were selected to minimise the *GAI* in the trial experiments. For example, in selecting the pheromone decaying rate, different values (0.9999, 0.99991, 0.99992, 0.99993, etc.) were tested in a typical simulation (Camden case, 30 patrollers, 311 hotspots) with other parameters fixed, and 0.99993 was selected, as it resulted in the lowest GAI. The parameter settings of Camden were directly applied to South Chicago without further experimenting, as they led to good performance in South Chicago.

The computational efficiency of BAPS was tested. The simulations were run on a Dell machine, with a 3.60 GHz Intel Core i7-4790 processor, 32.0 GB RAM and 64-bit Windows 7 operating system. In the experiment of 48 officers covering 311 hotspots in Camden, the simulation lasted 82 s, with 7000 times of determining the next patrol target and thus about 0.01 s cost for each determination. The computational efficiency in the large-scale problem or dynamic situations is subject to further experiments.

We used the Camden case to demonstrate the spatial differences of the two strategies. Fig. 4 shows the coverage and route of six patrollers using BAPS and CCPS after patrolling for 5760 steps, corresponding to 8 h in the real world, which is a typical shift for policing. The colour of the route is consistent with the station where the patroller started, except that the blue colour represents the route overlap among patrollers. Under BAPS, each patroller had its distinct route, and mainly patrolled a small and different area. The level of route overlap, which is the ratio of segments that had been traversed by over one patroller to segments that had been traversed, is about 35%. However, under CCPS, patrollers travelled on the same cycle that traverses every hotspot, and the level of route overlap is 92%. This indicates that BAPS provides different routes for different patrollers, while CCPS enforces the same patrolling route on all the patrollers involved. The evaluation and comparison of BAPS and CCPS are presented in order according to the guidelines.

5.1. Efficiency

Efficiency is measured by *GAI*. Tables 1 and 2 present the result of *GAI* and the relative change (Bennett & Briggs, 2005) of *GAI*:

$$RelativeChange = (GAI\_BAPS - GAI\_CCPS)/(GAI\_CCPS) \times 100\% \qquad (13)$$

In Camden, BAPS has lower *GAI* and consequently better performance than CCPS. The relative change varies slightly with the team size, reaching the maximum when team size is 18 and 24. In South Chicago, the *GAI* values in BAPS are lower than in CCPS by around 10%, except for the team size of 8. This might be because with a smaller patrol team, officers have to travel longer distances to cover the whole area, which results in the degeneracy of BAPS efficiency.

Table 11
Robustness performance in South Chicago in emergency scenario.

Patrollers per emergency	0	1	2	3	4
BAPS_12	0.0%	1.7%	13.7%	21.2%	30.9%
BAPS_20	0.0%	4.8%	7.4%	11.4%	18.8%
BAPS_32	0.0%	−1.3%	0.3%	3.1%	5.2%
CCPS_12	0.0%	2.9%	7.3%	12.1%	31.2%
CCPS_20	0.0%	3.5%	2.5%	2.7%	0.7%
CCPS_32	0.0%	3.8%	4.1%	4.5%	5.5%

5.2. Flexibility

The flexibility of routing strategies is measured by Weighted *GAI* (*WGAI*). Here, the hotspots are evenly divided into five levels, with crime density and weight decreasing from Level 5 to Level 1. Fig. 5 shows the hotspot map with different risk levels in both cases.

The decay rates are selected experimentally (Chen et al., 2015). The decay rates from Level 5 to 1) are: 0.99989, 0.99990, 0.99991, 0.99992, and 0.99993. Table 3 compares the performance of three strategies (BAPS, WBAPS, and CCPS) in patrolling hotspots of multiple levels using 30 patrollers in Camden. BAPS or WBAPS have a superior performance to CCPS in terms of *WGAI*, *GAI* and *GAI* of each risk level. In comparison with BAPS, WBAPS reduces the *WGAI* by about 1.8%, at the cost of a slight rise (2.4%) in *GAI*. Moreover, the *GAI* at Level 4 and Level 5 hotspots is reduced moderately by 4.7% and 9.4% when WBAPS is used. WBAPS provides an easy and effective approach to highlighting hotspots of higher levels, which shows the advantage of BAPS in that it can be tuned for specific aims. A similar trend is observed in Table 4 in the South Chicago case with 20 patrollers. WBAPS and BAPS have lower *GAI* and *WGAI* compared with CCPS. WBAPS has slightly lower *GAI* and higher *WGAI* than BAPS, as well as lower *GAI* in the prioritised hotspots of Level 4 and 5. The result verifies the flexibility of BAPS to patrol hotspots of varied levels by using varied decay rates.

5.3. Scalability

To test the team scalability, the ST metric (see Eq. (4)) is calculated for different sizes (see Tables 5 and 6). Table 5 reveals that in Camden, for all tested team sizes, BAPS systems present a superlinear performance as the speedup is >1.0, while the performances of CCPS systems are sublinear when team size is between 18 and 36. On every tested team size, the speedup performance of BAPS outperforms CCPS, indicating the better scalability of BAPS. Moreover, the scalability of CCPS is achieved by setting all patrollers as evenly distributed in time and space (Pasqualetti et al., 2012; Smith & Rus, 2010), which means the starting positions have to be recalculated for each size, while in BAPS starting positions have little influence on its performance. Similarly, in South Chicago, BAPS has a superlinear performance and outperforms CCPS on every team size (see Table 6).

Spatial scalability (SS) was tested by changing the crime hotspot density level from 5% to 10% and 15%. The higher density level requires better cooperation between patrolling to cover all hotspots. The corresponding hotspot maps are shown in Fig. 6, and the results are presented in Table 7. For example, *Camden_10p_30* represents the experiment of covering the Camden hotspot map of 10% of the total road length with 30 patrollers. The SS values in Table 7 use 5% as the baseline density level, with other factors fixed, including the patrol area, strategy, and team size. Overall, BAPS outperforms CCPS in all hotspot density levels, except for the 15% South Chicago hotspot map with 20 patrollers. Furthermore, the SS value of BAPS is consistently larger than CCPS, indicating that BAPS is more affected by the hotspot density level. In summary, regarding SS, BAPS has better performance on different hotspot density levels, but it is more sensitive to the high hotspot density levels.

5.4. Unpredictability

ASdIdl is measured to evaluate the unpredictability of patrolling. In Camden, for different team sizes, the *ASdIdl* values of BAPS are higher than those of CCPS (see Table 8). The low standard deviation in CCPS can be explained by the even distribution of patrollers on the cycle and the same patrolling cycle used by all patrollers. In contrast, the high deviation of idleness in BAPS and the high randomness of patrol routes would create a perceived "omnipresence" of the police that would deter crime in crime hotspots (Sherman & Eck, 2002). Likewise, in South Chicago, for every team size, BAPS has higher *ASdIdl* values, compared with CCPS (Table 9).

5.5. Robustness

The experiments of robustness were conducted using the real-world emergency records in Camden (March of 2011) and hypothetical emergency calls in South Chicago. When an emergency is reported, the nearest m patrollers stop patrolling and respond. Due to insufficient details of the emergency responses (time length, number of patrollers dispatched, etc.), different settings were attempted, including the total number of patrollers and the number of patrollers per emergency. Tables 10 and 11 show the robustness performance in Camden and South Chicago. For example, *BAPS_18* represents the BAPS simulation with 18 patrollers. The percentages represent relative changes in comparison with the non-emergency scenario (0 patrollers per emergency). In Camden (Table 10), the performance of both BAPS and CCPS deteriorated slightly or moderately as the number of patrollers required per emergency increased. Evidently, the higher number of patrollers needed by an emergency, the more affected the patrolling performance is. Further, holding constant the patrolling strategy and the number of patrollers per emergency means the emergency response has a more prominent impact on the performance when the patrolling group is smaller. Comparatively, with the team size of 18, the BAPS patrol was more affected by the emergency response than was the CCPS patrol. However, when the group size increased to 48, the influence on the BAPS patrol was less prominent than that on the CCPS patrol. A similar comparison exists in South Chicago (see Table 11), where the BAPS patrol was more influenced than the CCPS patrol when the group size was 12 or 20, and was more robust than CCPS when the group size increased to 32. The result supports the robustness of BAPS against emergency responses.

6. Conclusions and future work

This research developed a set of guidelines for real-world police activities, in particular a real-time cooperative police patrol routing. Five quantitative measures have been developed for the guidelines: efficiency, flexibility, unpredictability, scalability, and robustness. Under these guidelines, an online Bayesian Ant-based Patrolling Strategy (BAPS) has been developed. This strategy accounts for multiple factors that affect patrol, and it adopts a probabilistic computational framework, resulting in effective patrolling. As illustrated in the two real-world case studies, BAPS generally outperforms CCPS in terms of multiple measures. Thus, BAPS has great potential for real-time cooperative police patrol and other related applications.

The major contributions of this study include the developments of the relevant guidelines and measures for a police patrol routing strategy, the development of the BAPS routing strategy following these guidelines, and the verification of the strategy using agent-based simulations.

Future work will aim at including the relevant dynamics of police activity, such as the coordination between foot patrol and vehicle patrol, which will provide insight into a more practical patrol routing strategy. To carry out the research, it is necessary to automatically derive realistic police-patrol behaviours from GPS tracks of patrols and combine them into the patrol strategy, which will be another challenge. Other interesting directions would include customising patrolling strategy for alleviating specific crime types or focusing on improving the visibility of policing at public places, which will need to combine multiple sources of data, such as geodemographics.

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