Skewness variations of switching-current distributions in moderately damped Josephson junctions due to thermally induced multiple escape and retrapping

J. C. Fenton and P. A. Warburton

London Centre for Nanotechnology, 17–19 Gordon Street, London WC1H 0AH, UK
UCL, Department of Electronic and Engineering, Torrington Place, London WC1E 7JE, UK.

E-mail: `j.fenton@ucl.ac.uk`

Abstract. A crossover at a temperature T^* in the temperature dependence of the width σ of the distribution of switching currents of moderately damped Josephson junctions has been reported in a number of recent publications. For $T < T^*$, $\mathrm{d}\sigma/\mathrm{d}T$ is positive and the IV characteristics are associated with underdamped behaviour; for $T > T^*$, $\mathrm{d}\sigma/\mathrm{d}T$ is negative and the IV characteristics resemble overdamped behaviour. We have investigated in detail the behaviour of Josephson junctions around T^* by using Monte Carlo simulations including retrapping from the running state into the supercurrent state as given by a model due to Ben-Jacob *et al.* Thermally induced multiple escape and retrapping events play an important role for moderate damping. Around T^*, not just at lower $T < T^*$, the shape of the distribution and $\sigma(T)$ are largely determined by the shape of the conventional underdamped thermally activated switching distribution. The behaviour is more fully understood by considering *two* crossover temperatures. The skewness of the switching distribution parametrises its shape and becomes less negative, then positive, as T increases above the lower crossover temperature.

1. Introduction

In hysteretic Josephson junctions, the T dependence of the switching current I_{sw} and the width σ of its distribution are experimental parameters of much recent interest. In measurements of small Josephson junctions, Franz *et al.*[1] obtained IV curves characteristic of underdamped junctions below a crossover temperature and IV curves characteristic of overdamping above that temperature. A crossover in $\sigma(T)$ [2, 3, 4] at $T = T^*$ has been reported more recently, with positive $\mathrm{d}\sigma/\mathrm{d}T$ along with underdamped behaviour below T^*, and negative $\mathrm{d}\sigma/\mathrm{d}T$ along with overdamped behaviour above T^*. This was associated with a regime of moderate damping. The negative $\mathrm{d}\sigma/\mathrm{d}T$ region was associated with retrapping of the phase following escape. Krasnov *et al.*[4] demonstrated that such a crossover should be expected even for T-independent Q if the junctions are in the moderately damped regime ($Q \sim 5$) and derived an approximate quantitative formula with $T^* = T^*(Q)$, implying that T^* is a measure of the damping.

A resistively and capacitively shunted Josephson junction (RCSJ) is characterised by a damping parameter $Q = \omega_{\mathrm{P}} RC$, where R and C are the resistance and capacitance shunting the junction, $\omega_{\mathrm{P}} = \sqrt{2eI_{\mathrm{c}}/\hbar C}$ is the angular frequency of small oscillations at the bottom of the potential well at zero bias and I_{c} is the critical current. Hysteretic IV characteristics are obtained for $Q \gg 1$ and phase diffusion obtained for $Q \sim 1$. In a hysteretic junction at a finite

T, thermal fluctuations lead to switching from the supercurrent state to a state of finite voltage for $I < I_c$, and there arises experimentally a distribution in possible values of the switching current I_{sw}. A common experimental configuration is to ramp I up from zero at a constant rate dI/dt. In that case, the probability of a switch from I to $I + dI$ is $p(I)dI$, where[5]

$$p(I) = \frac{\Gamma_E}{dI/dt}\left[1 - \int_0^I p(I')dI'\right] = \frac{\Gamma_E}{\Gamma_I}.\frac{1 - \int_0^I p(I')dI'}{I}, \tag{1}$$

where Γ_E is the rate, at current I, of escape from the supercurrent state and we define a normalised current-ramp rate $\Gamma_I \equiv \frac{1}{I}dI/dt$. We define I_{EI} as the current at which $\Gamma_E = \Gamma_I$.

The rate of thermally activated escape from a metastable potential minimum and the rate of thermally induced retrapping are given respectively by [6, 7]

$$\Gamma_E = a_t \frac{\omega_a}{2\pi} \exp\left(-\frac{\Delta U_E}{kT}\right), \qquad \Gamma_R = \frac{I - I_r}{I_c}\omega_P \sqrt{\frac{E_J}{2\pi kT}} \exp\left[-\frac{E_J Q^2 (I - I_r)^2}{2kTI_c^2}\right], \tag{2}$$

where ΔU_E is the height of the energy barrier from the metastable potential minimum to the adjacent maximum, a_t is a damping-dependent pre-factor and the quantities a_t, ω_a and ΔU_E are all I-dependent, with $\omega_a = \omega_P(1-(I/I_c)^2)^{1/4}$ and, close to I_c, $\Delta U_E \approx \frac{4\sqrt{2}}{3}E_J(1-I/I_c)^{3/2}$ where the Josephson energy $E_J = \hbar I_c/2e$ and $I_r = I_r(Q) \approx 4I_c/\pi Q$. Combining Eqns. 1 and 2 gives a characteristic asymmetric distribution of switching currents for such junctions. Note that the retrapping rate is strongly dependent on the damping through Q.

2. The multiple switch-retrapping regime

As I is increased from zero, at sufficiently low I, $\Gamma_R \gg \Gamma_E, \Gamma_I$; no escape events occur for $\Gamma_E \ll \Gamma_I$. When I is increased to around I_{EI}, an escape event becomes likely. For an underdamped junction, when $I \gtrsim I_{EI}$, we find $\Gamma_R \ll \Gamma_E$. For more strongly damped junctions, an important current is I_{ER} at which $\Gamma_E = \Gamma_R$. While Γ_E is only weakly dependent on Q through the pre-factor a_t (Eqn. 2), Γ_R is exponentially dependent on Q (Eqn. 2); when the damping is sufficiently large that $I_{ER} > I_{EI}$ then, for $I \sim I_{EI}$, $\Gamma_R \gg \Gamma_E$. Escape events occur, but retrapping occurs shortly afterwards; the phase increases in fits and starts and the time-averaged voltage across the junction is non-zero, but close to zero. As I increases, there is a gradual increase in the time-averaged voltage. Γ_E and Γ_R become more and more similar, so the proportion of time the junction spends in the escaped state increases and the time-averaged junction voltage also increases. At $I \sim I_{ER}$, the junction spends a similar amount of time in the zero-voltage and escaped states, so an experiment is likely to measure a switch event. I_{sw} is greater, and so thermal fluctuations suppress I_{sw} less, than for an otherwise equivalent underdamped junction at the same temperature.

3. Temperature dependence, the crossover regime and T^*

In our Monte Carlo simulations[8], for a current I, the probability in a short time interval δt of a transition between the metastable and running states is given by $\Gamma_E(I)\delta t$ for escape from the metastable state or $\Gamma_R(I)\delta t$ for retrapping from the running state (with Γ_E and Γ_R given by Eqn. 2). A bias current is ramped up at a constant rate in order to generate switching-current distributions for junctions with a number of different parameters. We neglect the T dependence of I_c and Q. A switch is counted when the junction spends more than half the time in the running state over some time period. We use the term "escape" to describe any (possibly short-lived) escape from the instantaneous zero-voltage state, and reserve the term "switch" to describe an experimentally measured switch to the running state.

Results of simulations of these dynamics over a range of T are shown in Fig. 1. At lower T, σ and I_{sw} follow the conventional underdamped thermal behaviour (with $\sigma \sim T^{2/3}$). At higher T, σ passes through a maximum and then falls, matching experimental observations. The mean $I_{\rm sw}$, $\bar{I}_{\rm sw}$, flattens out at around T^* and reaches an approximately constant value $\bar{I}_{\rm sw} \approx I_{\rm ER}$ well above T^*. Fig. 1c shows, as a function of T, the mean number of escape events before eventual switching, where each escape event, other than the final one, is followed by a retrapping event. At low T, a single escape event results in a switch being counted, but for higher T a significant number ($\sim 10^3 - 10^4$ above 25 K) of escape events occurs before a switch is counted. The shape of the switching distribution also changes as T is increased (Fig. 1e–h), departing from the conventional underdamped thermally activated distribution (UTAD). The skewness γ_1 (the ratio of the third moment about the mean to the standard deviation) gives a simple one-parameter description of the shape of the distribution; a symmetrical distribution has $\gamma_1 = 0$. For the UTAD, $\gamma_1 \approx -1$ over the range of T shown in Fig. 1. Fig. 1d shows the variation of γ_1 of the simulated distribution around T^*. Somewhat below T^*, γ_1 begins to depart from its thermal value, becoming progressively less negative and then positive.

We identify three different regimes of behaviour. At low T, conventional thermal underdamped behaviour is observed. At some higher $T < T^*$, γ_1 and, in detail, also σ and $I_{\rm sw}$ depart from the underdamped thermal values. Above this T, γ_1 and σ vary rapidly. At a higher T, there is a crossover to a different regime in which $\bar{I}_{\rm sw}$ is approximately constant, and γ_1 and σ are slowly decreasing as T is increased. We label the two boundaries between these three regimes $T^*_{\rm low}$ and $T^*_{\rm high}$, where $T^*_{\rm low} < T^* < T^*_{\rm high}$. We define $T^*_{\rm low}$ ($T^*_{\rm high}$) quantitatively as the T at which $I_{\rm ER}$ coincides with the bottom (top) of the UTAD (see Fig. 2a&c).

As T increases, $I_{\rm EI}$ decreases, while $I_{\rm ER}$ remains almost the same. (This is expected by inspection of Eqns. 2. The exponential factors both have the same T dependence and hence $I_{\rm ER}$ is approximately independent of T too.) For $T > T^*_{\rm low}$, escape events occur for $I_{\rm EI} \lesssim I \lesssim I_{\rm ER}$ and are followed by retrapping events; they do not result in the count of a switch. Escape events leading to *switching* only occur at $I \gtrsim I_{\rm ER}$. Fig. 2b shows that the distribution of switching currents begins to depart from the UTAD. When a part of the underdamped thermal

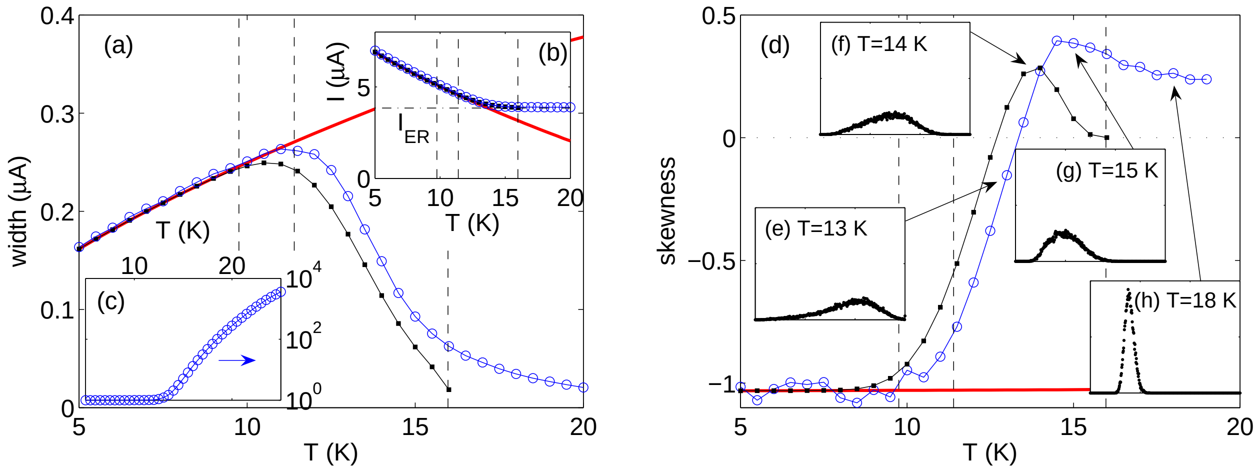

Figure 1. (a) $\sigma(T)$ for $Q = 7$. (b) $\bar{I}_{\rm sw}(T)$ for $Q = 7$. Open circles, blue line: simulation results based on 25000 switching events (C=5.9 fF, $I_{\rm c}$=10 μA, $\dot{I} = 10$ mAs^{-1}); thick red line: UTAD; small closed circles, black line: UTAD truncated to above $I_{\rm ER}$; dash-dotted line: $I_{\rm ER}$; broken lines: $T^*_{\rm low}$, T^* and $T^*_{\rm high}$. (c) Variation with T of the mean number of escape events before a switch is counted in simulations. Above $T^*_{\rm high}$, the distribution in this value is smaller than the symbols. (d) $\gamma_1(T)$. (e)–(h) Switching distributions at selected T. Note the scales on these insets are the same, with the range of $I = 3.5 - 5$ μA and $p(I) = 0 - 10$ $(\mu{\rm A})^{-1}$.

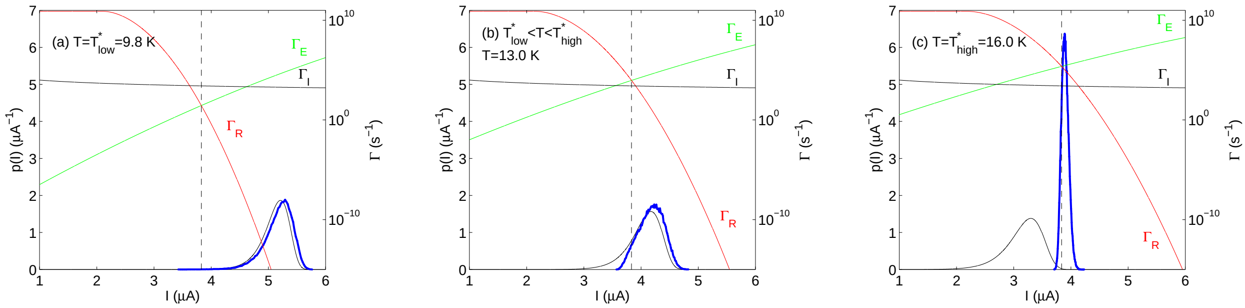

Figure 2. (a)-(c) Switching distribution and variation of characteristic rates with I at three temperatures for $Q = 7$. Heavy blue points: simulations based on 100000 switching events (C, I_c and $\dot{I}$ as Fig. 1); lower black curve: UTAD; broken black line: $I_{\rm ER}$.

distribution lies at $I \gtrsim I_{\rm ER}$, one might naïvely expect that σ, $\bar{I}_{\rm sw}$ and the shape of the switching distribution would be approximately the same as those for the part of the UTAD lying at $I > I_{\rm ER}$. Fig. 1 illustrates that $\bar{I}_{\rm sw}$ in the simulations closely matches $\bar{I}_{\rm sw}$ of the truncated UTAD, and $\sigma(T)$ and $\gamma_1(T)$ in the simulations also follow the respective variations in σ of the truncated distribution, up to T approaching $T^*_{\rm high}$. The shape of the part of the UTAD with $I > I_{\rm ER}$ largely determines the shape of the switching distribution for $T^*_{\rm low} < T < T^*_{\rm high}$, *i.e.*, the conventional thermal behaviour is only followed below $T^*_{\rm low}$, but the variations in $\bar{I}_{\rm sw}$ and σ remain analytically determinable for T up to $T^*_{\rm high}$. For $T \gtrsim T^*_{\rm high}$, the behaviour is not associated with the UTAD. Fig. 1b shows that, for $T > T^*_{\rm high}$, $\bar{I}_{\rm sw}$ approaches $I_{\rm ER}$.

The crossover temperatures are set by the shape of the UTAD, which depends on the current-ramp rate $\mathrm{d}I/\mathrm{d}t$. For $\mathrm{d}I/\mathrm{d}t = 10^{-4} - 10^{-1}$ As^{-1}, we find a range for $T^*_{\rm low} = 8.2 - 10.8$ K and $T^*_{\rm high} = 12.2 - 18.9$ K, *i.e.*, varying $\mathrm{d}I/\mathrm{d}t$ can have a significant effect on T^*.

4. Conclusions

In moderately damped Josephson junctions, there is a regime in which the junction repeatedly escapes to and retraps from the running state. Our simulations show that the number of escapes and retraps before an eventual switch into the running state may be very large ($\sim$10000). With a large number of escapes of duration $\sim 1/\Gamma_{\rm R}$, this regime is intermediate between the underdamped regime in which a single escape leads to switching, and the overdamped phase-diffusion regime which has a very large number of escapes of very short duration $\sim \omega_{\rm P}$.

The crossover in $\sigma(T)$ is, in detail, described by not one but *two* crossover temperatures $T^*_{\rm low}$ and $T^*_{\rm high}$. The variations in $\bar{I}_{\rm sw}$ and σ for $T^*_{\rm low} < T < T^*_{\rm high}$ are largely determined by the shape of the UTAD; this also determines the T of the maximum in σ, usually identified as the single crossover T. The temperature dependence of the skewness γ_1, parametrising the shape of the switching distribution, indicates $T^*_{\rm low}$ and $T^*_{\rm high}$.

Acknowledgments

This work was supported financially by the UK EPSRC.

[1] Franz A, Koval Y, Vasyukov D, Müller P, Schneidewind H, Ryndyk D *et al.* 2004 *Phys. Rev. B* **69** 014506
[2] Kivioja J, Nieminen T, Claudon J, Buisson O, Hekking F and Pekola J 2005 *Phys. Rev. Lett.* **94** 247002
[3] Männik J, Li S, Qiu W, Chen Q, Patel V, Han S and Lukens J 2005 *Phys. Rev. B* **71** 220509
[4] Krasnov V, Bauch T, Intiso S, Hürfeld E, Akazaki T *et al.* 2005 *Phys. Rev. Lett.* **95** 157002
[5] Fulton T and Dunkleberger L 1974 *Phys. Rev. B* **9** 4760
[6] Kramers H 1940 *Physica* **7** 284
[7] Ben-Jacob E, Bergman D, Matkowsky B and Schuss Z 1982 *Phys. Rev. A* **26** 2805
[8] Fenton J and Warburton P 2008 *Phys. Rev. B* (submitted)