Household Demand in the Presence of Externalities: Model and Applications

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Declaration

I, Sanghmitra Gautam, confirm that the work presented in this thesis is my own. Where information had been derived from other sources, I confirm that this has been indicated in the work.

Abstract

This thesis consists of three chapters on the empirical analysis of demand for a preventive healthcare good in a context where externality or spillover effects are relevant. The application in all three chapters focuses on household adoption of sanitation in the developing world and uses a unique dataset on sanitation take-up from rural India. Research in public health has shown sanitation adoption to generate significant positive externalities that affect health, economic and social 'well-being' of individuals. Failure to internalize such public benefits generates a divergence in the social and private value from sanitation leading to sub-optimal adoption.

In *Chapter 2*, I formulate a simple static model with interdependent adoption decisions to analyse the impact of externalities on household demand for sanitation. I estimate the model and propose an extension to the Hotz & Miller two-stage estimator to account for limitations of using sample survey data with strategic interaction models. *Chapter 3* quantifies the subsequent welfare effects generated by policy interventions in the presence of spillover effects. I extend earlier work by Dagsvik & Karlstrom (2005) for welfare analysis under discrete choice to strategic interaction models, so as to decompose the impact of a subsidy intervention into its direct and indirect effects. I find that between $41\% - 86\%$ effect of a subsidy is propagated through the externality channel. Positive externality effects also imply a welfare gain and an increase in a household's willingness-to-pay for a policy that subsidises sanitation.

Chapter 4 analyses the extent of under-adoption of sanitation and the appropriate choice of policy between sanitation loans and price subsidies to increase sanitation coverage. I formulate a dynamic equilibrium model of household sanitation demand to investigate the role of two distinct sources of market failures: liquidity constraints and externalities, which both lead to under-adoption. I find price subsidies to be more cost effective at increasing sanitation coverage. But the policy effects are heterogeneous with coverage levels, where loans are found to be equally, if not marginally more, effective in villages with no sanitation coverage.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

In the field of development and health economics a large portion of the literature focuses on studying the impact of policy interventions that seek to improve the health outcomes of poor households in developing countries. The provision of preventive healthcare technologies, such as water purification, immunization and sanitation, are especially a challenge in rural developing economies where markets either fail or are missing and as a result goods are under-adopted. In this thesis I study household demand for one such healthcare technology; access to sanitation. Close to 2.5 billion people on the planet do not have access to a basic sanitation facility (WHO-UNICEF 2014) which has made sanitation a topic of concern on the global public health agenda.[1] Research in public health has shown poor sanitation prevalence to contribute to morbidity and mortality, especially among children.

Despite large consensus on the benefits from adoption, there is substantial disagreement on what policies can increase/improve sanitation coverage. The crux of the disagreement on choice of policy stems from differing hypothesis on which market failure drives a household's lack of demand. The lack of sanitation in the developing world is primarily concentrated amongst the poorest sections of urban and rural societies. Thus without institutions that would provide loans for sanitation adoption, a poor liquidity constrained household may otherwise find it difficult or impossible to adopt. In addition to lack of access to credit markets, adoption of sanitation has also been found to generate significant positive health externalities that benefit a larger group of individuals than the individual choosing to adopt. Household are unlikely to fully internalize the benefits their adoption decision generates on others, generating a divergence in the private and social value from

[1] By sanitation facility I mean having access to a toilet and/or a bathroom facility at home

sanitation adoption. With positive health externalities the privately chosen adoption level is below the socially optimal.

In line with this research agenda, this thesis comprises of three chapters that each investigate the role of externalities and borrowing constraints on household demand for sanitation, as well as the impact of different policy interventions to tackle the inefficiencies that generate sub-optimal adoption behaviour. Examples of the policy tools implemented in practice include the use of subsidies: conditional (price) and unconditional (income transfers), sanitation loans (through microfinance credit), direct provision of the good and even provision of information on the health benefits from sanitation adoption. A key feature of such goods is that though in most cases the intervention targets individual agents the benefits that are realized from take-up are not restricted to the targeted individual. The presence of such public good externalities may justify intervening with subsidies but they also play a role in determining the efficacy of the implemented policy. One of the main objectives of this thesis is to understand the impact and role of policy interventions when externalities are relevant.

Chapter 2, analyses the impact of externalities on household demand for sanitation and the subsequent welfare effects generated from a subsidy program towards its provision. I formulate a simple static household demand model with interdependent adoption decisions. A decision making household maximises its utility from adoption, where the adoption of sanitation generates a degree of 'private' utility along with a 'social' utility which in turn depends on the level of sanitation adoption within the village.

Using a dataset on sanitation take-up from rural India, I estimate a static demand model keeping into account the interdependence of household decisions which imply an equilibrium level of adoption. The structure is closely related to a random utility model with the addition of strategic interactions similar to Brock & Durlauf (2001) and allows for possible multiplicity of equilibria. The model is estimated using a Hotz & Miller two-stage procedure that circumvents the computational burden associated with repeatedly solving and estimating the model for the fixed point. However, to obtain consistent estimates the procedure relies on being able to observe the entire vector of household decisions within the relevant reference group. To account for this limitation, common to most datasets from developing countries where data collection is expensive, I propose an extension to the standard two-stage estimator by adapting the correction method from Chesher (1991) to the context of strategic interaction models and games.

The estimated model is used to perform different price and income subsidy simulations. I find that the equilibrium levels attained under a price subsidy regime are substantially higher than those obtained under a pure income transfer. The large demand response to a price subsidy is found to be largely driven by the underlying externality effects. Where the social multiplier effect under a price subsidy is computed to be around 5.6 while for an income subsidy is estimated to be closer to ≈ 1.

To evaluate the welfare impact of subsidy interventions in Chapter 3, I formulate analytical expressions to quantify substitution and income effects generated by a price subsidy and compute Compensating variation (CV) measures for a household's willingness-to-pay for the policy. Specifically, I extend earlier work by Dagsvik & Karlstrom (2005) for welfare analysis under discrete choice to the case of strategic interaction models, so as to decompose the impact of a subsidy intervention firstly along price and income effect components. Furthermore, both price and income effects can be further decomposed into the impact of the policy on private incentives (direct) as well as through the feedback effects generated by underlying externalities (indirect).

Using this tool under different subsidy simulations, I find that substitution effects are significantly larger than income effects, and a substantial amount of this price effect between $41\% - 86\%$ is propagated through the indirect channel. The presence of positive externalities implies a larger welfare gain, while the Deadweight Loss (DWL) generated from the subsidy intervention is realised instead as a Net-Gain ($NetG$), as the society shifts towards a socially optimal level of adoption. A household's willingness-to-pay for the subsidy policy increases once externality effects are accounted for. The analysis shows that policy evaluations that fail to incorporate for the additional spillover effects of the policy would underestimate the impact and true effectiveness of the intervention.

Chapter 4 further analyses the problem of under-adoption of sanitation in the developing world by addressing the current policy debate on the choice between loans and price subsidy policies to increase sanitation coverage in the developing world. I formulate a dynamic equilibrium model of household sanitation demand to investigate the role of two distinct market failures: liquidity constraints and externalities that both lead to under-adoption of sanitation.

The model is also estimated using the dataset from rural India. In addition to the baseline cross section in Chapter 2, I observe household choices over a two period panel. I use the model to compute equilibrium adoption levels under both loans and price subsidy policies to study the optimal design of interventions in an equilibrium setting. Counterfactual analysis reveals existing sanitation level to be below the social planner solution, implying under-adoption of sanitation by as much as 53%. Price subsidies are found to be more cost effective at increasing sanitation coverage. But the policy effects are heterogeneous with coverage levels, where loans are found to be equally, if not marginally more, effective in villages with no sanitation coverage. A price subsidy has a high social rate of return where the presence of externalities accounts for a substantial fraction (between $33\% - 72\%$) of its impact. While a sanitation loan policy generates smaller social returns is found to be cost efficient under targeted delivery.

Chapter 2

Household Demand for Sanitation in the presence of Externalities

2.1 Introduction

In this chapter I analyse the impact of externalities on household demand for sanitation by formulating and estimating a simple static demand model for sanitation. In the model, an individual household's sanitation choice also depends on adoption decisions of other households within the village. The interdependence of choice is modelled as an incomplete information game where a household observes only partial information on the states and decisions of it's neighbours. A pervasive feature of such strategic interaction models is the presence of multiple equilibria which makes solving for equilibrium and estimation computationally burdensome.

The model is estimated using a two-stage method that bypasses the burden associated with having to repeatedly solve the fixed point for each candidate vector of parameters. The chapter also discusses a key limitation of existing two-step estimation methods related to the use of sample data where only a subset of household choices and states are observed by the econometrician. I propose an amended two-step method to account for this problem, which can be treated as a 'presence of measurement error' in the data. The method implemented builds on Chesher (1991, 2000) which proposes a method for bias reduction in estimation of parameters of regression models with covariates subject to classical measurement error in a non linear framework. The correction method is general in its applicability and contributes to the larger literature of two-step estimation of incomplete information games and strategic interaction models under clustered sample data.

The chapter is organized as follows: Section 2.2 presents an overview of the application considered, data and descriptives. Section 2.3 specifies the model. Section 2.4 describes the estimation procedure including the correction method employed. Section 2.5 presents the baseline results and evaluates the model fit. Section 2.6 provides a summary and concluding remarks. The results from the estimated model are further analysed in Chapter 3 where I first specify the tools for counterfactual analysis and welfare measures and then use the estimated model to simulate the impact of hypothetical policy interventions in the context of externalities. All relevant Tables and Figures are provided in the chapter appendix in Section (2.7).

2.2 Background

The application considered in this chapter looks at household sanitation adoption in India. The idea however is more generally applicable to the context of preventive healthcare goods in developing countries when externalities are relevant. The presence and importance of health externalities has previously been identified and documented in the literature. Key examples include, Cohen & Dupas (2010) with bed-nets as prevention against malaria, and Miguel & Kremer (2004) where the provision of de-worming pills was found to generate significant positive spillover effects amongst non-treated students. In most cases, the issue of externality is discussed ex-post policy intervention. In contrast, the idea of incorporating the impact of underlying externalities in investigating the impact of potential subsidy interventions ex-ante has not yet been extensively studied.

2.2.1 Sanitation in India

Understanding the barriers to sanitation adoption (and preventive healthcare in general) has become a critical issue in public health policy. In a recent paper, Dupas (2012) provides a selective review of the recent microeconomic evidence on this issue. The paper provides key stylized facts on household adoption behaviour and health in low-income countries which includes low levels of preventive healthcare expenditures. For example, malaria and diarrhoea[1] account for 8% and 18% of under-5 mortality in India (World Bank 2008-2011).[2] The mortality rates are

[1] disease prevalence largely attributed to poor or lack of basic sanitation infrastructure.

[2] World Bank Development Indicators - http://databank.worldbank.org/ddp/home.do

primarily concentrated in rural/slum regions where under-5 mortality attributed to diarrhoeal diseases is close to 86%. In contrast, the percentage of population in India with access to improved sanitation facilities is only 34% in 2010.[3] India is not only among the top 15 countries, that account for 73% of all under-5 diarrhoeal deaths occurring worldwide, but it also ranks at the top of the list with more than half a million diarrhoeal deaths yearly (WHO 2006).

So far policy interventions in sanitation have had limited success (e.g. Guiteras et al 2015) in creating sanitation densities and social change. Several factors have been put forth as possible explanations – some of them being: (a) the size of the monetary policy being inadequate to construct a functional sanitation facility; (b) the policy beneficiaries are not correctly identified i.e., families living below the poverty line being ambiguous, resulting in low sanitation densities with insignificant improvements in health status; and (c) the sustainability of such interventions given the resources and the potential implementation leakages. A key feature of sanitation like other preventive healthcare goods is that benefits from adoption are not restricted to the household that chooses to adopt. Adoption of sanitation can be viewed as a choice that generates both private as well as social returns, where the social returns capture benefits that display a degree of non-rivalry and non-excludability associated with adoption. This can be thought of as an externality problem, where the returns that a household realises not only depends on its own sanitation adoption but also depends on the adoption behaviour of the overall community in which it lives. Though the exact mechanism of the externality is not clear, for example, it could be a contagion or public good externality, lack of information or simply a preference household have to conform to the norm.

The analysis in this paper is unable to ascertain the precise mechanism that generates the externality, for example, it could be a health externality, information spillovers or simply a preference households have to conform to the norm. However, I am able to characterize the nature and magnitude of such spillover effects i.e., whether the interdependence generates a positive or negative externality for an individual household. The policy implications are consistent with this characterization, and thus are independent of the mechanism or channel of externality itself. One possible approach to disentangle and identify the relevant channels through which the spillover effects propagate, would be through field experiments (RCTs)[4] that are designed to capture the impact of different types of policies, for example provision of

[3]The urban population access is around 58% while the rural population percentage is only 23%

[4]An interesting avenue for future work would be to combine the structural analysis with results from a randomised experiment that could disentangle the various mechanisms that lead to an interdependence in household sanitation decision.

loans, health insurance, health information etc.[5]

2.2.2 Data and Descriptive Statistics

The data used for the estimation comes from a household survey (2009-10) conducted by a policy institution in India called FINISH (Financial Inclusion Improves Sanitation and Health). The institution targets poor rural households with the objective to encourage and provide subsides for the purpose of building sanitation facilities within their home. The data comes from the outlying rural and urban slums around a city in the north of the state of Madhya Pradesh located in central India. This chapter does not aim to assess the impact of any field policy experiment. Instead, I use the cross sectional data to estimate a model of household adoption decision and layout an ex-ante welfare evaluation of hypothetical policy interventions.

The baseline sample comprises of $1,475$ households across 44 villages. Table (2.1) provides a descriptive summary of the sample for the main variables of interest. In the sample on average 37.8% of households have a sanitation facility in their homes. Across the 44 villages, this includes villages were the adoption level is 0% and also where there is 100% sanitation levels. Household characteristics include age, education of household head. In the model the household is assumed to be a single decision making unit. The underlying assumption being that household head is the primary decision maker. On average household heads are in their mid $40s$ this age is slightly lower for female household heads that comprise of 18% of the sample and have at least primary school education. Information on current household wealth, which includes current income and liquid assets which includes savings in the bank account, along with the cost of acquiring sanitation (price) which varies across villages are included as variables in decision making process. Details on construction of the cost of sanitation is provided in Appendix (2.7.3.1)

The impact of externality is assumed to be captured by average prevalence of sanitation in the village. A major limitation of this measure of externality as captured by the average level of adoption within the village is that the data only comprises of a sub sample of total households within the village.[6] This limitation introduces a

[5] As an avenue for future work I aim to combine the analysis with the results from field experiments to first identify the exact mechanism that generate the externality and to further improve of the efficiency of the suggested policy predictions.

[6] Between $10\% - 20\%$ of the total village population was surveyed in the sample

specific source of measurement error; the average adoption level observed by the econometrician may not be an accurate portrayal of the true sanitation coverage level. This mis-measurement introduces a form of classical measurement error that is not restricted to the standard attenuation bias on the parameter estimate. I employ a correction procedure to the estimation methodology described in section (2.4) for the presence of this measurement error. This is crucial as the classical nature of the error under a non linear structure not only implies an attenuation bias but also a potential upward bias resulting in an overestimate of the parameter that captures the impact of the externality.

2.3 Model

In this section, I present a static demand model of sanitation adoption with externalities. The model builds on the interaction structure presented in Brock & Durlauf (2001). I start by defining a simple environment, where a finite number of decision making households, who belong to a village/group, choose whether or not to adopt a health improving technology. In this case, the technology offered is to build a sanitation facility in one's home.[7] The interdependence of sanitation adoption choices and its impact on an individual household's return from adoption is captured by the average sanitation prevalence within the village.

2.3.1 Setup

In order to incorporate the presence of externalities, the interdependent adoption choice of households is modelled as a simultaneous move game of incomplete information. The incomplete information structure provides a tractable framework to model the sanitation adoption behaviour in which a trade-off exists between desirability of having sanitation at home and the social returns from adoption.[8] The externality arises as each household incorporates the actions of others within its payoff, but does not necessarily account for the impact of it's own choice on the well-being of others. At the aggregate, this unaccountability results in a divergence

[7] Sanitation facility refers to a household building a bathroom/toilet facility in their house.

[8] A household may derive social returns from presence of health and/or information externalities. The prevalence of which is driven by the sanitation coverage level within the village.

in the private and socially optimal adoption levels. The model translates the discrete actions of households into a smooth adoption choice probability that represents the likelihood of a household's decision to adopt sanitation.

Environment: A household is assumed to be a single decision making unit, making a discrete decision on whether or not to adopt sanitation. There are a finite number of decision making households $i = 1, ..., N$ each belonging to a group. The population is partitioned into non-overlapping sub-populations (groups/villages) indexed by $g = 1, ..., G$ and represented by I_g such that $\cup_{g=1}^{G} I_g = N$.

Choice: Households face a finite choice set $D_i = \{0, 1\}$ and each household i simultaneously makes a decision which has a realization $d_i \in D_i = \{0, 1\}$ where:

$$d_i = \begin{cases} 1 & Household\, adopts\, sanitation \\ 0 & no\, adoption \end{cases}$$

Let $D = \{0, 1\}^{I_g}$ denote a vector of possible actions for all households in village g and let $d_g = (d_{1g}, d_{2g}, ..., d_{Ig})$ denote a generic element of D. While, $d_{-ig} = (d_{1g}, ..., d_{(i-1)g}, d_{(i+1)g}, ..., d_{Ig})$ denotes the choices of all other households belonging to village g excluding household i.

States & Information: Each household belongs to a group g and is endowed with a set of state variables $(x_{ig}, z_g, \varepsilon_i(d_i))$ that include household x_{ig} and village z_g specific characteristics, $w_{ig} = (x_{ig}, z_g)$.[9] It is assumed that $w_g = (w_{ig}, w_{-ig})$ is a vector of common knowledge variables (information set) observable to all households within village g as well as to the econometrician. In addition, each household is also endowed with a set of taste/preference shocks $\varepsilon_i(d_i)$ that are private information, known only to household i and unobservable to all other households within the group including the econometrician. Let ε_i denote the 1×2 vector of the individual taste shocks $\varepsilon_i(d_i)$. The density of ε_i is denoted by $g(\varepsilon_i)$, $\varepsilon_i = [\varepsilon_i(1), \varepsilon_i(0)]$.

[9]The implicit assumption being all relevant village level characteristics are included in the observable vector z_g and there are no group level unobservables.

Thus the information available to each household is incomplete and restricted to the set of observable variables w_g.

> $[\mathbf{A1}]$: The observable variables w_g is common knowledge possessed by all households i within a village/group g
>
> $[\mathbf{A2}]$: The unobserved taste shocks ε_i are private information possessed only by a household i and are distributed $i.i.d$ across households and choice alternatives

The model allows for village specific observables z_g however there is no allowance for village level unobservable effects. This limitation may lead to an overestimation of the impact of spillovers. A possible solution to account for this limitation would be estimate a village specific fixed effect by estimating the mean of the taste shock ε. This idea is further explored in Chapter 4 to allow for a similar concern of village level unobservables.

2.3.2 Utility and beliefs

The utility function is similar to a standard random utility model with the additional inclusion of d_{-ig} choices of other households within the village. The per period utility function for household i is denoted by:

$$V_i\left(d_i, \overline{d}_{-ig}, w_{ig}, \varepsilon_i; \theta\right) = u\left(d_i, x_{ig}, z_g; \kappa, \beta, \delta\right) + s\left(d_i, \overline{d}_{-ig}; \gamma\right) + \varepsilon_i\left(d_i\right) \tag{2.3.1}$$

which comprises of three additively separable components: private utility $u(\cdot)$, social utility $s(\cdot)$ and an unobserved private preference component $\varepsilon_i(\cdot)$ associated with choice d_i. Utility depends on a vector of state variables (w_{ig}, ε_i), own choice d_i and a function of choice of other households $\overline{d}_{-ig}$. The vector of preference parameters is denoted by $\theta = [\kappa, \beta, \delta, \gamma]$. The social utility is a function of the average level of adoption $\overline{d}_{-ig}$ assumed to capture the presence of externalities within the village.

$$\overline{d}_{-ig} = \frac{1}{I_g - 1} \sum_{j \neq i} \mathbf{1}\left\{d_{jg} = 1\right\}$$

The social utility is parametrized by γ, it captures the importance of the externality or social returns from adoption relative to the private utility from adoption. The externality is generated by the fact that $\overline{d}_{-ig}$ affects the utility derived by household i, but the household does not consider the effect its own choice has on the utility of all other households $j \neq i$. The deterministic component of the utility which includes both private and social utility is assumed to have a linear specification:

$$v_d(d_i, w_{ig}, \overline{d}_{-ig}; \theta) = \kappa + \beta x_{ig} + \delta z_g + \gamma d_i \overline{d}_{-ig} \tag{2.3.2}$$

The partition of the observable state space into household specific and village specific characteristics (x_{ig}, z_g) imposes an exclusion restriction that allows for the identification of the primitive utility function of interest discussed further in section (2.3.3). In addition to assumption $[A2]$, $G(\varepsilon)$ is assumed to have a parametric form of distribution from a known family. The private taste shock terms ε_i are distributed Type 1 Extreme value and the difference $(\varepsilon_i(0) - \varepsilon_i(1))$ is distributed logistic.

Decision rules: Household $i's$ decision rule is a function $\delta_i(w_g, \varepsilon_i) = d_i$. Note that under assumption $[A2]$ $i's$ decision does not depend on ε_{-i} since these taste shocks are private information of the other households in the village and thus are unobservable to household i. The conditional choice probability $p_i(d_i | w_g, \theta)$ is defined as:

$$p_i(d_i = 1 | w_g, \theta) = \int \mathbf{1}\left\{\delta_i(w_g, \varepsilon_i; \theta) = 1\right\} g(\varepsilon_i)\, d\varepsilon_i \tag{2.3.3}$$

where $\mathbf{1}\left\{\delta_i(w_g, \varepsilon_i; \theta) = 1\right\}$ is an indicator function that household $i's$ choice is $d_i = 1$ (adoption) given the vector of state variables (w_g, ε_i). Thus $p_i(d_i = 1 | w_g, \theta)$ denotes the probability that household i chooses to adopt sanitation ($d_i = 1$) conditional on the common knowledge state variables w_g (information set) which is derived by integrating $i's$ decision rule over the region of ε_i for which $\delta_i(w_g, \varepsilon_i; \theta) = 1$. The distribution of d_g given w_g is given by $p(d_g | w_g, \theta) = \prod_{i=1}^{I_g} p_i(d_i | w_g, \theta)$.

Expected Choice-specific Utility: Under incomplete information a household forms expectations about the adoption decision undertaken by its fellow neighbours. The expected utility received by household i from choosing d_i is denoted by:

$$\widetilde{V}_{d_i}(d_i = 1, w_g, \varepsilon_i; \theta) = \underbrace{\sum_{d_{-i}} \left[\kappa + \beta x_{ig} + \delta z_g + \gamma \mathbf{1}\{d_i = 1\} \frac{1}{I_g - 1} \sum_{j \neq i} \mathbf{1}\{d_{jg} = 1\}\right] p_{-i}(d_{-ig} | w_g, \theta)}_{\widetilde{v}_d(d_i, w_g; \theta)} + \varepsilon_i(d_i)$$

$$\textit{where } p_{-i}(d_{-i} | w_g, \theta) = \prod_{j \neq i} p_j(d_j | w_g, \theta) \tag{2.3.4}$$

Since household i does not have information on ε_j taste preference of other households, it constructs beliefs about the expected choice of other households using all relevant observable information. The term $p_{-i}(d_{-ig}|w_g,\theta)$ denotes the conditional probability measure household i places on the choices of others at the time of making its own decision. The first term on the R.H.S of Equation (2.3.4) denotes the expected value of $v_d(d_i,w_{ig},\overline{d}_{-ig};\theta)$ by marginalizing out the decisions of other households using expectations $p_{-i}(d_{-ig}|w_g)$. The deterministic part of the expected utility is defined as:

$$\widetilde{v}_d(d_i,w_g;\theta)=\sum_{d_{-i}} v_d(d_i,w_{ig},\overline{d}_{-ig};\theta)\,p_{-i}(d_{-ig}|w_g,\theta) \tag{2.3.5}$$

Given that the average level of adoption enters into a households utility the corresponding expected social utility depends on the expected average level of adoption:

$$\overline{p}_g=\frac{1}{I_g-1}\sum_{j\neq i} p_{i,j}(d_{jg}=1|w_g,\theta) \tag{2.3.6}$$

where $p_{i,j}(d_j=1|w_g,\theta)$ denotes the expected value from the perspective of household i of household j's choice. Under rational expectations, a household makes a choice so as to maximize its expected utility by following a decision rule which depends on its own choice d_i as well as on the expectation of the choices made by other households.

$$d_i=\begin{cases} 1 & if \quad \widetilde{V}_1\geq\widetilde{V}_0 \\ 0 & \quad otherwise \end{cases} \tag{2.3.7}$$

$$\delta_i(w_g,\varepsilon_i;\theta)=\arg\max_{d\in D}\left[\widetilde{v}_d(d_i,w_g;\theta)+\varepsilon_i(d_i)\right] \tag{2.3.8}$$

$$\widetilde{V}^*(d_i,w_g,\varepsilon_i(d_i);\theta)=\max_{d_i\in D}\left\{\widetilde{V}_1(d_i=1,w_g,\varepsilon_i(1);\theta),\widetilde{V}_0(d_i=0,w_g,\varepsilon_i(0);\theta)\right\} \tag{2.3.9}$$

$\widetilde{V}_1$ and $\widetilde{V}_0$ denote the expected choice-specific utility functions and $\widetilde{V}^*$ in Equation (2.3.9) denotes the optimal indirect utility.[10]

[10]Similar to the random utility model, adding or multiplying all utilities by a constant will not change the probability that a given alternative is chosen. Hence normalizations for level and scale or required. These normalizations are satisfied by setting the deterministic component of utility from non-adoption to zero and further normalizing the variance of the difference of the unobserved taste heterogeneity i.e., $\eta=\varepsilon(0)-\varepsilon(1)$ to $var(\eta)=1$

2.3.3 Equilibrium

Equilibrium: Under the assumption that each household makes its choice given an expectation $\overline{p}_g$ of the average level of sanitation adoption which is independent of the realisations of $\varepsilon_i(d_i)$ for all households i. Under the extreme-value assumption for $\varepsilon_i(d_i)$ and the normalization of the deterministic expected utility from non-adoption to zero, utility maximization by household i implies:

$$p_i(d_i = 1|w_g,\theta) = \frac{exp\left(\kappa + \beta x_{ig} + \delta z_g + \gamma \frac{1}{(I_g-1)} \sum_{j\neq i} p_{i,j}(d_j = 1|w_g,\theta)\right)}{1 + exp\left(\kappa + \beta x_{ig} + \delta z_g + \gamma \frac{1}{(I_g-1)} \sum_{j\neq i} p_{i,j}(d_j = 1|w_g,\theta)\right)} \quad \text{for all } i \text{ and all } w_g \tag{2.3.10}$$

where $p_i(d_i = 1 \mid w_g;\theta)$ denotes the Conditional Choice Probability (CCP) of adoption for household i.[11] Equation (2.3.10) represents a system of equations for each village g that can be solved to determine equilibrium probabilities of adoption. Under a Bayesian Nash equilibrium, households maximize expected utility and household i has consistent beliefs about the choices of other households in the village i.e., the household knows the equilibrium probabilities $\overline{p}_g^*$. The Bayesian Nash equilibrium to the household adoption problem is a collection of beliefs $p_i^*(d_i = 1|w_g,\theta)$ defined by Equation (2.3.10) for each household $i = 1,....,I_g$ belonging to a village $g = 1,...,G$. The term $p_{-i}(d_{-ig}|w_g,\theta)$ in Equation (2.3.5) represents the equilibrium probabilities that choices d_{-ig} are observed, and the summation marginalizes household $i's$ uncertainty about the choices of other households to compute the expected return from choosing $d_i = 1$ given available information w_g. For a fixed w_g, the definition of equilibrium (2.3.10) and the definition of the choice-specific value function in Equation (2.3.5) implies I_g equilibrium probabilities $p_i^*(d_i = 1|w_g,\theta)$ which can be viewed as the solution to the following system of I_g equations per village.

Given that the average level of adoption of other households matter in an individual household's returns from adoption. It is convenient to represent the system of equations in (2.3.10) as a fixed point problem at the village or group level instead.

$$\begin{aligned} \frac{1}{I_g}\sum_{i=1}^{I_g} p_i^*(d_i = 1|x_i, z_g;\theta) &= \frac{1}{I_g}\sum_{i=1}^{I_g}\left\{\frac{exp\left(\kappa+\beta x_{ig}+\delta z_g+\gamma\overline{p}_g^*\right)}{1+exp\left(\kappa+\beta x_{ig}+\delta z_g+\gamma\overline{p}_g^*\right)}\right\} \\ \overline{p}_g^* &= \frac{1}{I_g}\sum_{i=1}^{I_g}\left\{\frac{exp\left(\kappa+\beta x_{ig}+\delta z_g+\gamma\overline{p}_g^*\right)}{1+exp\left(\kappa+\beta x_{ig}+\delta z_g+\gamma\overline{p}_g^*\right)}\right\} \end{aligned} \tag{2.3.11}$$

[11] The Conditional Choice Probability (CCP) denote the reduced form objects of the model observable in the data.

Equation (2.3.11) denotes the equilibrium conditions as a fixed point problem at the village level where $\overline{p}_g^*$ denotes the equilibrium probability of adoption in a village g. In general there maybe more than one solution to the fixed point problem in Equation (2.3.11) further implications of this multiplicity are discussed in (2.3.4).

Identification: Before discussing the properties of the equilibria, I briefly discuss the identification of the utility function. Abstracting away from parametric assumptions, complications induced by the presence of multiple equilibria and by taking the equilibrium choice probabilities $p_i(d_i|w_g)$ as given. The question of identification is whether it is possible to reverse engineer the structural parameters $v_d(d_i, w_{ig}, \overline{d}_{-ig})$ from the observed data. Using Equation (2.3.10) and performing the Hotz & Miller (1993) inversion by taking logs on both sides of the equation, yields the familiar result:[12]

$$\widetilde{v}_d(d_i = 1, w_g) = \ln[p_i(d_i = 1|w_g)] - \ln[1 - p_i(d_i = 1|w_g)] \tag{2.3.12}$$

This equation demonstrates that it is possible to obtain the expected choice-specific value function for any w_g from the observed reduced form choice probabilities. Thus by treating $\widetilde{v}_d(d_i = 1, w_g)$ as known and using Equation (2.3.5) it is possible to derive a system of equations for a fixed w_g:

$$\underbrace{\widetilde{v}_d(d_i, w_g)}_{known} = \sum_{d_{-i}} v_d(d_i, w_g, \overline{d}_{-ig}) \underbrace{p_{-i}(d_{-ig}|w_g)}_{data} \qquad \text{for } i = 1, ..., I_g \tag{2.3.13}$$

In order to highlight the identification issue, I have removed the partition of the state space into (x_{ig}, z_g) household and village specific characteristics in $v_d(d_i, w_g, \overline{d}_{-ig})$ in Equation (2.3.13). Identification requires finding a unique set of $v_d(d_i, w_g, \overline{d}_{-ig})$ primitives that solves this system of Equations (2.3.13). A necessary condition for identification is that there are at least as many equations as free parameters $\widetilde{v}_d(d_i = 1, w_g)$. For a fixed w_g, there are $I_g \times I_g$ unknowns corresponding to the $v_d(d_i, w_g, \overline{d}_{-ig})$ after imposing the normalization for non-adoption and the assumption of average level of adoption.[13] However, there are only I_g equations, which implies that without additional restrictions, the structural parameters of model are not identified. Similar to the approach in simultaneous equation models, I impose exclusion restrictions. The partition of the state space of a household $w_{ig} = (x_{ig}, z_g)$ imposes an exclusion restriction, i.e., it is possible to find a set of covariates x_{ig} that shift the utility of household i independently of the utilities of the other households in

[12]Note: $\widetilde{v}_d(d_i = 0, w_g; \theta) = 0$ and its primitive $v_d(d_i = 0, w_{ig}, \overline{d}_{-ig}; \theta) = 0$ are normalized

[13]Note: If in the model the identity of the households mattered in the adoption decision in this case the degree of under identification would be even greater $I_g \times 2^{I_g - 1}$

the model. Unlike Equation (2.3.13), in Equation (2.3.5) $w_{-ig} = (x_{-ig}, z_g)$ enters expectations p_{-i} but is excluded from $v_d(d_i, w_{ig}, \overline{d}_{-ig})$. Holding x_{ig} fixed, and by varying x_{-ig} it is possible to increase the number of equations that $v_d(d_i, w_{ig}, \overline{d}_{-ig})$ must satisfy. If there are at least I_g points in the support of the conditional distribution of x_{-ig} given x_{ig}, it is possible to generate more equations than free parameters. Furthermore, in the application considered x_{-ig} includes continuous variables with a rich support, thus the requirement of at least I_g points of support is easily met.

2.3.4 Existence and Multiplicity of Equilibria

The existence of equilibria in this setting corresponds to ensuring that the fixed point problem described in (2.3.11) has at least one solution. Given Assumption $[A2]$ household beliefs are monotonic, continuous and strictly bounded inside the set $(0,1)$ and so the existence of a solution to the fixed point follows immediately from Brower's fixed point theorem.[14]

Multiple Equilibria are possible in this model. For a given set of parameters there could be more than one solution for the expression (2.3.11). The multiplicity in the structure arises because of the dependence of a household's choice on the choices made by other households within the village. Intuitively, the stronger the dependence as captured by the social utility component parametrized by γ relative to the private utility component, multiplicity arises. These multiple solutions imply the existence of distinct expected sanitation prevalence levels which are each compatible with individually optimal decisions. With the possibility of multiple equilibria the model is incomplete without the specification of an equilibrium selection mechanism. This "incompleteness" makes it difficult to construct a proper likelihood and objective function which has implications for the estimation of the model.

One way of dealing with the multiplicity would be to develop a theory of the underlying equilibrium selection mechanism and thus complete the model. For example, one can make an assumption about the equilibrium played or more formally model (i.e., parametrically) an equilibrium selection mechanism such as implemented by Bajari, Hong & Ryan (2010). The use of an appropriate equilibrium selection rule assures the existence of a well-defined likelihood function for the entire space of observable outcomes as discussed in the next section. However, the problem with this approach is that the consistency of the estimation procedure crucially depends on the validity of the assumed selection rule, which is not always testable.

[14]See Brock & Durlauf (2001) for a detailed description

An alternative strategy would be to abstract away from imposing an ad-hoc equilibrium selection rule and instead make an assumption on the data observed. In order to obtain consistent parameter estimates I employ assumption $[A3]$ below. In each village, I assume that the data observed is generated from only one of the possible equilibria.

> $[\mathbf{A3}]$: Given a value of primitives of the model $\Psi = (w_g, \theta)$ households in a group g select only one equilibria from the set of possible equilibria and they do not switch to other equilibria and long as Ψ does not change.

Sometimes also referred to as the single-equilibrium-in-data assumption, assumption $[A3]$ provides a $1:1$ mapping between the observed data and the structural primitives of the model. This assumption is frequently employed in the context of estimating incomplete information games and is independent of the choice of estimation method.[15] The assumption is also less restrictive than explicitly assigning ex-ante an equilibrium selection rule the group/village might be at. Under the single-equilibrium-in-data assumption, the multiplicity of the equilibria in the model does not play a role in the identification of the structural parameters.[16]

2.4 Estimation

In this section, I discuss the estimation of the model. In theory, the estimation of the static model of strategic interaction could follow the same methods as the estimation of single agent discrete choice models with the addition of a fixed point condition. The strategic nature of the model imposes an additional restriction, that the strategies or choice probabilities of the households should be in equilibrium. It is possible to use an algorithm similar to the one suggested by Rust (1987). The nested fixed point algorithm could be used to maximize a sample criterion function over the space of structural parameters, and to solve for the equilibrium of the model, for each candidate value of the parameters. For a given equilibria and value of parameters, the remaining

[15] See for example Aguirregabiria & Mira 2007, Bajari et al. 2007, Pakes et al. 2007 & Pesendorfer & Schmidt-Dengler 2008

[16] The single-equilibrium-in-the-data assumption is a sufficient condition for identification but it is not necessary. De Paula (2013), Sweeting (2009) and Aguirregabiria & Mira (2009) present conditions for point-identification of static games of incomplete information when there are multiple equilibria in the data.

structure simplifies to a single agent discrete choice problem for which deterministic decision rules for adoption as a function of state variables and unobserved random shocks can be obtained. Thus under an assumption for the distribution of these shocks, it is possible to derive a likelihood of observing each household's decision conditional on states.

However, with the possibility of multiple equilibria, this full solution maximum likelihood approach would require the repeated solution of the model for each candidate value of the parameters to be estimated. Explicitly solving for the equilibrium proves to be computationally burdensome even for the simple model proposed here. Furthermore, the possible multiplicity of fixed points may render this method impractical without the specification of the underlying equilibrium selection mechanism.

The alternative two-step approach, widely used in the literature, bypasses the computational burden of repeatedly solving for the fixed point. The estimation is broken down into two parts where, under assumption $[A3]$ the equilibrium choice probabilities or beliefs are estimated directly from the observed data in the first stage. While in the second stage, first stage estimates are used to further estimate the structural parameters of the model using maximum likelihood or least squares procedure. The two step method has been shown to preserve the large sample properties of the direct method while being computationally straightforward.[17] The following section introduces a general two-step pseudo log-likelihood estimation procedure and briefly discusses one of its key limitations frequently encountered in practice due to data restrictions. I further propose an amended two-step estimation procedure to account and correct for this source of limitation.

2.4.1 Two-Step Estimator

The first stage of the estimator attempts to recover a consistent estimate $\widehat{p_i}$ for the equilibrium choice probabilities or beliefs $p_i^*(d_i = 1|w_g)$ from the data.[18] This is used to further obtain an estimate $\widehat{\overline{p}_g}$ for the equilibrium average level of adoption in the village $\overline{p}_g^*$.

[17] See Aguirregabiria & Mira 2002, 2007

[18] Ideally, we would like to implement a nonparametric method to estimate choice probabilities in order to reduce the noise from the first stage. However, I employ a parametric approach in order to tackle a more serious source of bias in the resulting parameter estimates explained further.

In this simple model, the first stage simply comprises of computing sample averages of the adoption level in each village as $\overline{p}_g^* = E\left(\overline{d}_{-ig}|w_g\right)$[19] and as $I_g \to \infty \Rightarrow \overline{d}_g \to E\left(\overline{d}_{-ig}|w_g\right)$

$$\widehat{\overline{p}}_g = \overline{d}_{-ig} = \frac{1}{(I_g - 1)} \sum_{j \neq i} \mathbf{1}\{d_j = 1\} \tag{2.4.1}$$

which gives fitted choice probabilities:

$$\widehat{p}_i(d_i = 1|x_{ig}, z_g, \overline{d}_g, \theta) = \frac{exp\left(\kappa + \beta x_{ig} + \delta z_g + \gamma \overline{d}_g\right)}{1 + exp\left(\kappa + \beta x_{ig} + \delta z_g + \gamma \overline{d}_g\right)} \tag{2.4.2}$$

which are substituted, in place of the equilibrium choice probabilities, into the log-likelihood objective function and maximized in the second stage. Estimate $\widehat{\theta}$ as the solution of

$$\widehat{\theta} \ \arg\max \ \mathcal{L}(\theta|\overline{d}_{-ig}, x_{ig}, z_g, d_i) \tag{2.4.3}$$

where the pseudo log Likelihood is given by

$$\begin{aligned} \mathcal{L}(\theta|\overline{d}_{-ig}, x_{ig}, z_g, d_i) &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N}\{d_i \, log\left[\widehat{p}_i(d_i = 1|x_{ig}, z_g, \overline{d}_{-ig}, \theta)\right] \\ &+ (1 - d_i)\, log\left[1 - \left(\widehat{p}_i(d_i = 1|x_{ig}, z_g, \overline{d}_{-ig}, \theta)\right)\right]\} \end{aligned}$$

Note that this estimation procedure does not require the computation of all possible equilibria as a function of θ. The estimator discussed above relies on obtaining consistent estimates of the equilibrium choice probabilities in the first stage. However, the nature of the data raises a key issue in the estimation procedure. The data observed is a small sample of the entire village population and thus the first stage estimate of belief $\overline{d}_g$ may not be a consistent estimate of the true equilibrium levels of average adoption in the village. This not only compromises the consistency of the second stage estimates but also generates a bias in the parameter estimates given the non linear structure. This issue can be treated as the presence of measurement error, where the econometrician observes a value of $\overline{d}_g$ contaminated by noise. In practice, measurement error can result in an imprecise initial first step estimate $\widehat{p}_i$ which can generate large finite sample biases in the two-step estimator of structural parameters in the second stage. The larger the bias in the first step in the estimate $\widehat{p}_i$, the larger the bias of the two-step estimator for θ. In the following

[19] In the case of large groups $\overline{d}_{-ig} \approx \overline{d}_g$

subsection I implement an approximation approach to correct for the presence of 'measurement' error in the choice probabilities given by Equation (2.4.2). I amend the standard two-step procedure to account for this potential source of error and correct for the bias in the resulting second stage parameter estimates.

2.4.2 Measurement Error Correction

The presence of measurement error causes the data generating distribution to differ from the distribution that is of substantive interest. In order to understand the effect of mis-measurement on the information produced by statistical procedures it is necessary to understand the distortions induced by the error.

The two-step estimation procedure above relies on obtaining consistent estimates of $\overline{p}_g$ and p_i in the first step. However, in the presence of measurement error the observed distribution of the estimators may differ from true distributions. The principal source of error in this model lies in the estimate for $\overline{p}_g$ given by the average level of adoption in the village $\overline{d}_g$. The error exists as $\overline{d}_g$ is constructed using only a small sample of households that are observed within each village.[20]

In a linear model, the impact of measurement error, relates to the familiar attenuation effect of the contaminated covariates on the response variates. This is where the presence of measurement error causes covariates to appear to vary over a wider range than they in fact do. Essentially, there is a change in the scale of variation of the covariates but no effect on the variation of the response variate itself. Consequently in a linear model, the effect of a unit marginal change in the error contaminated covariate on a response variate, is smaller than the effect of the same unit change in the error free covariate. In contrast, for a non linear model such as the one considered here, the impact of measurement error is no longer restricted to a simple attenuation of the parameter estimates. There is an added reduction of the curvature of the response function which is driven by the underlying non-linearity.

The bias reduction method outlined below follows from the small error variance approximation to the effect of measurement error on regression functions formulated in Chesher (1991). The approximations provide the basis for the analysis of the properties of estimators when measurement error is present, particularly in the case of a non linear model. Intuitively, the method exploits a small measurement error variance approximation of the conditional density. This allows for a consistent estimation of θ in Equation (2.4.2) in the sense that the inconsistency is of a smaller order than the variance of the measurement error itself.

[20]The survey sample comprises approximately 10%-15% of the entire population of a village

The key intuition is as follows, the presence of measurement error causes the true data generating distribution to differ from the observed data distribution. The small error variance approximation method treats the observed density to be a distorted version of the true density of interest. The method constructs this distorted density by taking a Taylor series approximation of the density around the point of no measurement error. The mean regression function of interest is derived using the distorted density which corrects for the presence of measurement error through correction parameters in the first step of the estimation.

2.4.2.1 Setup & Assumptions

A brief account of the problem is outlined below followed by the main result using the approximation method. While a detailed derivation of the approximations adapted to the context of the model is deferred to the Appendix (2.7.3.2).

Following the model presented in Section (2.3). The outcome variable $d = [d_i]$ is a vector of random variables and $\chi = \chi_{ig} = (x_{ig}, z_g, \overline{d}_g)$ denotes a matrix of covariates for each household. Let $\overline{D} = [\overline{D}_g]$ and $u = [u_{ig}]$ be vectors of error free covariates and measurement error, with absolute continuous joint distribution and with u and $\overline{D}$ independently distributed. As previously stated the observed error contaminated covariate is denoted by $\overline{d}_g$, if $\overline{d}_g = \overline{D}_g + u_{ig}$.

$$var(\overline{d}_g) = var\left(\overline{D}_g + u_{ig}\right)$$

$$var(\overline{d}_g) = var(\overline{D}_g) + var(u_{ig}) + 2cov(\overline{D}_g, u_{ig})$$

with u and $\overline{D}$ independently distributed:

$$var(\overline{d}_g) = var(\overline{D}_g) + var(u_{ig})$$

and $var(\overline{D}_g) = 0$ which gives

$$var(\overline{d}_g) = var(u_{ig})$$

The main result for the mean regression function[21] of interest is given by:

$$E_{d|\overline{d}}\{f(d)|\overline{d}\} = E_{d|\overline{D}}\{f(d)|\overline{d}\} + E^{(1)}_{d|\overline{D}}\{f(d)|\overline{d}\}\Sigma G^{(1)}_{\overline{D}}(\overline{d}) + \frac{1}{2}tr\left[\Sigma E^{(2)}_{d|\overline{D}}\{f(d)|\overline{d}\}\right] + o(\Sigma) \tag{2.4.4}$$

where $o(\Sigma^k) = o(\sigma^2)$ denotes a scalar function of Σ with the property:

$$\lim_{\sigma^2 \to 0} \frac{o(\sigma^2)}{\sigma^2} = 0$$

The function $G^{(1)}_{\overline{D}}(\overline{d})$ is the derivative of the logarithm of the marginal density of $\overline{D}$, in practice however the function is rarely known. However, it is possible to replace $G^{(1)}_{\overline{D}}(\overline{d})$ with $G^{(1)}_{\overline{d}}(\overline{d})$ by knowing the way in which marginal densities of the 'error free' and 'error contaminated' variates are related

$$g_{\overline{d}}(\overline{d}) = g_{\overline{D}}(\overline{d}) + \frac{1}{2}tr\left[\Sigma \nabla_{\overline{dd}'} log\, g_{\overline{D}}(\overline{d})\right] + o(\Sigma)$$

this implies that $E^{(1)}_{d|\overline{D}}\{f(d)|\overline{d}\}\Sigma\left(G^{(1)}_{\overline{D}}(\overline{d}) - G^{(1)}_{\overline{d}}(\overline{d})\right)$ is $o(\Sigma)$ and hence $G^{(1)}_{\overline{D}}(\overline{d})$ can be replaced by $G^{(1)}_{\overline{d}}(\overline{d})$ without affecting the order of approximation error. Since realisations of $\overline{d}$ are observed, it is possible to estimate $G^{(1)}_{\overline{d}}(\overline{d})$ and using the amended mean regression function given by Equation (2.4.4) correct for the detrimental effect of measurement error on the parameter estimates $\hat{\theta}$. In non linear regression models such as the binary choice model considered, measurement error induces an additional non linear effect through the third term on the right hand side of Equation (2.4.4) which depends on the curvature of the regression of $f(d)$ on χ_{ig}, where $\chi_{ig} = (x_{ig}, z_g, \overline{d}_g)$. At values of $\overline{d}_g$ for which the function $E_{d|\chi}\{f(d)|\overline{d}\}$ is convex this term is positive and measurement error causes the regression of $f(d_i)$ on $\overline{d}_g$ to be higher at such points than the regression of $f(d_i)$ on $\overline{D}_g$. At points where the regression of $f(d_i)$ on $\overline{D}_g$ is concave the effect is in the opposite direction (upward bias). It is important to note that the implementation considered in the following section rests on the assumption that the functional form of the regression of d_i on $\overline{D}$ and on χ is known.

[21]For derivation refer to Appendix (2.7.3.2)

2.4.2.2 Practical Implementation

For binary choice models, such as in the application considered, where d_i is the binary outcome variable the conditional distribution given covariates $\chi_{ig} = (x_{ig}, z_g, \overline{d}_g)$ is given by:

$$g_{d_i|\chi_{ig}}(d_i|\chi_{ig}, \theta) = p\{u(\chi_{ig}\}^{d_i}[1 - p\{u(\chi_{ig})\}]^{(1-d_i)} \tag{2.4.5}$$

where under the logit specification, $u(\chi_{ig}) = \chi_{ig}^{'}\theta$ is the linear index, and $p(u) = \frac{e^u}{(1+e^u)}$.[22] Using Equation (2.4.4) the approximation under the logit specification reduces to:

$$u(\chi_{ig}) = \kappa + \beta x_{ig} + \delta z_g + \gamma \overline{d}_{-ig} + \gamma(\xi\sigma_g^2)G_{\overline{d}}^{(1)}(\overline{d}_g) + \frac{1}{2}\gamma^2(\xi\sigma_g^2)\left[1 - 2\frac{e^{\chi_{ig}\widehat{\theta}_1}}{\left(1 + e^{\chi_{ig}\widehat{\theta}_1}\right)}\right] \tag{2.4.6}$$

where $\Sigma_g = \xi\sigma_g^2 = \xi\frac{\overline{d}_g(1-\overline{d}_g)}{I_g}$ [23] and $\frac{e^{\chi\widehat{\theta}_1}}{\left(1+e^{\chi\widehat{\theta}_1}\right)}$ is the probability of adoption choice for a given household and $\widehat{\theta}_1$ are the naive first stage parameter estimates from the logit regression on the error contaminated data. The vector of estimates $\widehat{\theta}_1$ comes from the original maximum likelihood estimation. The function $p(.)$ remains unchanged, and $\xi\sigma_g^2 = \xi\frac{\overline{d}_g(1-\overline{d}_g)}{I_g}$. The function $G_{\overline{d}}^{(1)}(\overline{d}_g) = \frac{\partial log f(\overline{d}_g|w_{ig})}{\partial \overline{d}_g}$ which is the derivative of the conditional log density, where $w_{ig} = (x_{ig}, z_g)$ are all co-variates except the error contaminated covariate $\overline{d}_g$. As noted earlier it is possible to produce a non-parametric estimate of $G_{\overline{d}}^{(1)}(\overline{d}_g)$. However, given the small sample size and potential curse of dimensionality implied by the conditional density $f(\overline{d}_g|w_{ig})$ a semi-parametric approach is adopted instead.[24] The method above employs the observed response, the error contaminated covariate data and an assumed functional form for the error free regression function. The approximation method establishes a direct link between the regression functions with conditioning on in turn the error free and the error contaminated covariates and employs as an additional regressor, a nonparametric estimate of derivatives of the logarithm of the joint density of the error contaminated covariates.

[22] If $\Lambda(x)$ denotes the logit function, the first derivative is given by $\Lambda^{(1)}(x)$=$\Lambda(x)[1-\Lambda(x)]$while the second derivative is given by $\Lambda^{(2)}(x)$=$\Lambda(x)[1-\Lambda(x)]\,[1-2\Lambda(x)]$. The logit specification follows from the Type 1 Extreme value assumption of the taste shocks $\varepsilon_i(d_i)$ in the structural model.

[23] ξ is a parameter to account for differences across villages/groups g. The estimate ranges between $\widehat{\sigma}_g^2 = [0.0008, 0.032]$

[24] See Appendix (2.7.3.3) for construction of $G_{\overline{d}}^{(1)}(\overline{d}_g)$

2.4.2.3 Second Stage: Bias Corrected Maximum Likelihood Estimator

The approximate densities (2.7.7) and mean regression function (2.4.4) and (2.4.6) provide the basis for the approximations to the likelihood functions in which there is an allowance for measurement error. By taking one Newton step towards the maximum of such an approximation a 'bias corrected' maximum likelihood estimator is obtained.[25]

In the presence of error contaminated covariates in a non linear model, it is possible to account for the impact of this source of error by incorporating additional correction variables to the original choice probability expression. A new set of estimates is given by $\widehat{\theta}_2$:

$$\widehat{\theta}_2 \ \arg\max \ \mathcal{L}(\theta|\overline{d}_g, x_{ig}, z_g, \Omega_{ig}, d_i) \tag{2.4.7}$$

where the bias corrected log likelihood function[26] is given by:

$$\begin{aligned} \mathcal{L}(\theta|\overline{d}_g, x_{ig}, z_g, \Omega_{ig}, d_i) &= \tfrac{1}{N}\textstyle\sum_{i=1}^{N}\{d_i \, log\left[p_i^{corrected}(d_i = 1|x_{ig}, z_g, \overline{d}_g, \Omega_{ig}, \theta)\right] \\ &+ \ (1-d_i)\, log\left[1 - \left(p_i^{corrected}(d_i = 1|x_{ig}, z_g, \overline{d}_g, \Omega_{ig}, \theta)\right)\right]\} \end{aligned} \tag{2.4.8}$$

and the modified first step choice probability is given by:

$$p_i^{corrected}(d_i = 1|x_{ig}, z_g, \overline{d}_g, \Omega_{ig}, \theta) = \frac{exp\left(\kappa + \beta x_{ig} + \delta z_g + \gamma\overline{d}_g + \mathbf{G}^{(\mathbf{1})}_{\mathbf{d}}(\overline{\mathbf{d}}_{\mathbf{g}})(\xi\sigma^2_{\mathbf{g}})\gamma + \frac{1}{2}\gamma^2(\xi\sigma^2_{\mathbf{g}})\left[\mathbf{1} - \mathbf{2}\frac{\mathbf{e}^{\chi_{\mathbf{ig}}\widehat{\theta}_{\mathbf{1}}}}{\left(\mathbf{1}+\mathbf{e}^{\chi_{\mathbf{ig}}\widehat{\theta}_{\mathbf{1}}}\right)}\right]\right)}{1 + exp\left(\kappa + \beta x_{ig} + \delta z_g + \gamma\overline{d}_g + \mathbf{G}^{(\mathbf{1})}_{\mathbf{d}}(\overline{\mathbf{d}}_{\mathbf{g}})(\xi\sigma^2_{\mathbf{g}})\gamma + \frac{1}{2}\gamma^2(\xi\sigma^2_{\mathbf{g}})\left[\mathbf{1} - \mathbf{2}\frac{\mathbf{e}^{\chi_{\mathbf{ig}}\widehat{\theta}_{\mathbf{1}}}}{\left(\mathbf{1}+\mathbf{e}^{\chi_{\mathbf{ig}}\widehat{\theta}_{\mathbf{1}}}\right)}\right]\right)} \tag{2.4.9}$$

The vector of parameter estimates $\widehat{\theta}_2$ obtained from the second stage are the arguments that maximize the log likelihood given the distribution of variates. The new set of parameters are given by $\widehat{\theta}_2 = [\widehat{\kappa}, \widehat{\beta}, \widehat{\delta}, \widehat{\gamma}, \widehat{\gamma\xi}, \widehat{\gamma^2\xi}, \Sigma_{(2)}]$. The first four parameter estimates are the parameters of interest from the model, followed by two correction term estimates $\widehat{\gamma\xi}$ and $\widehat{\gamma^2\xi}$ controlling for attenuation and the degree of

[25]Chesher, Lancanter & Irish (1985) describe such a procedure in the context of correcting for the effects of random parameter variation.

[26]where $\Omega_{ig} = \mathbf{G}^{(\mathbf{1})}_{\mathbf{d}}(\overline{\mathbf{d}}_{\mathbf{g}})(\xi\sigma^2_{\mathbf{g}})\gamma + \frac{1}{2}\gamma^2(\xi\sigma^2_{\mathbf{g}})\left[\mathbf{1} - \mathbf{2}\frac{\mathbf{e}^{\chi_{\mathbf{ig}}\widehat{\theta}_{\mathbf{1}}}}{\left(\mathbf{1}+\mathbf{e}^{\chi_{\mathbf{ig}}\widehat{\theta}_{\mathbf{1}}}\right)}\right]$

curvature, respectively and $\Sigma_{(2)}$ denotes the variance-covariance matrix after correction. A bootstrap procedure is employed to construct standard errors.

2.5 Empirical Results

2.5.1 Parameter Estimates

Table (2.2) provides the parameter estimates from the two step estimation procedure. Column [1] lists the parameter estimates from the 'naive' two-step estimation while Column [2] lists the parameter estimates obtained after correcting for the presence of measurement error in the first stage. I start by discussing the results for the correction method first captured by the two additional parameter estimates in column [2]. As derived in Section (2.7.3.2) $CorrectionFactor(1)$ denoted by $G^{(1)}_{\bar{d}}(\bar{d}_g)(\xi\sigma^2_g)\gamma$ corrects for the first order effect of measurement error which generates the attenuation effect. While $CorrectionFactor(2)$ denoted by $\frac{1}{2}\gamma^2(\xi\sigma^2_g)\left[1-2\frac{e^{\chi_{ig}\widehat{\theta}_1}}{\left(1+e^{\chi_{ig}\widehat{\theta}_1}\right)}\right]$ captures the second order effect given the non linear structure; the term reduces the curvature of the response curve with the increase in mis-measurement error. It is interesting to note that, for the parameter capturing the externality effect, the estimate indicates a slight upward bias due to the presence of measurement error which, as mentioned in Section (2.4.2), is possible in the case of a non linear model. However, this difference is not found to be significant with the correction terms. Overall, the estimates for the correction term parameters were not found to be significant. This result is checked for robustness to sample size using Monte Carlo simulations and no significant difference was found. To study the accuracy of the approximations, simulation exercises are conducted for different bandwidths and sample sizes.

The estimates in Table (2.2) denote log odds coefficients and thus it is easier interpret the magnitudes by looking at marginal effects in Table (2.3). The average sanitation presence in the village captures the impact of the externality on household demand probability which is found to have a positive and significant impact on an individual household's sanitation adoption decision. The cost of sanitation which varies across villages[27] has a significant impact on the probability of demand. The magnitude of the price effect is six times the effect of wealth (cash-in-hand) that households have available on the sanitation adoption decision. The impact of other household's adoption decision, which captures the externality effect, has the largest impact on adoption probability close to 2.5 times the impact of cost of sanitation and nearly 5 times the impact of cash-in-hand.

In addition, household level characteristics such as age and education of the household head and size of the household are found to have a significant positive impact on the adoption decision. Village level characteristics include presence of facilities that are complements or substitutes to a private household sanitation facility. The presence of drainage facility is found to be complimentary to a household's adoption decision. While, the availability of substitutes captured by availability of public sanitation facility within the village has a negative coefficient though not found to be significant. Using these parameter estimates and the village level fixed point condition, it is possible to simulate the response probability so as to back out the equilibrium levels of sanitation adoption within each village.

2.5.2 Model Fit

To assess if the estimated model captures the essential features of the data, the observed and the predicted choice distributions are compared. Table (2.4) provides results from simulating the sanitation adoption behaviour from the model at baseline. For each village, I compare the model predicted average level of sanitation adoption with the mean observed in the data. The last column in Table (2.4) computes the percent difference of the model predicted equilibria from the data. Noting a few exceptions the model fit is close to the observed data. Figure (2.1) provides a graphical representation of Table (2.4) for four villages. The average prevalence of sanitation in the village is plotted on the horizontal axis. While the vertical axis plots the probability of sanitation adoption for an individual household as a function of sanitation adoption. The vertical dashed black line denotes the observed mean adoption in the data and the diagonal plots the 45^o line. The blue line plots the second order response curve which is determined from the simulation of the village level fixed point described in Equation (2.3.11) for a fine grid of $\overline{d}$ which denotes the

[27]Note: Price of sanitation is a variable constructed using supply side information. It includes the cost of raw materials and the cost of labour for average number of days required to build sanitation. Raw material include bricks, mortar, cement, tin, and tiles. These materials are produced and are readily available as they constitute the basic ingredients in industrial and domestic construction. The demand of these products for the purpose of building sanitation constitutes a very small proportion of the overall demand.

prevalence of sanitation in the village.

2.5.3 Policy Effects

With the structural parameter estimates , it is possible to simulate village level equilibrium response under different subsidy policies that affect the probability of sanitation adoption. Table (2.5) and (2.6) provide counterfactual simulations for provision of unconditional (income) and conditional (price) subsidies at 25% and 50% of the cost of sanitation, respectively. The price subsidy subsidizes the cost of sanitation and is provided conditional on a household adopting while the unconditional subsidy has no such requirement and is essentially an income transfer to each household. Similar to baseline, Figure (2.2) and (2.3) provide graphical representations of the policy effects of both the unconditional and conditional subsidies respectively. The effect of the policy can be viewed as an upward shift of the probabilistic response curves of adoption. The provision of the subsidy increases the probability of adoption at any given level of sanitation coverage in the village.

There are two points of observation to note from Figures (2.2) and (2.3). Firstly, I find the emergence of multiple equilibria in some village under the counterfactual simulated environment. The analysis in this chapter does not extend to determining which one of the three equilibria would the society actually move to under the counterfactual scenario if multiplicity occurs. This is because both the model specification and estimation procedure abstracted away from specifying an Equilibrium Selection Mechanism. A thorough examination of this problem and a possible solution is reserved for proceeding work in Chapter (4).[28] Secondly, I find that the household response to the sanitation price subsidy is significantly larger compared to the case of an income transfer. This lack of demand response under an income transfer is driven by the preference estimates where a household is found to be more responsive to changes in price of sanitation compared to the change in household wealth levels.

[28] When implementing policy simulations, it is important to deal with the potential multiplicity of equilibria that may arise under counterfactual scenarios. Without having specified the equilibrium selection mechanism in the baseline model, it is not possible to determine which equilibria will be selected by the society in the counterfactual scenario. However, in Chapter (4) I propose to bound the set of possible equilibria by an upper and lower limit, under a new policy environment. This approach allows for the impact of the policy to be bounded by a region within which the actual impact of the policy lies determined by the 'true' equilibrium selection mechanism that is otherwise unobserved by the econometrician.

The impact of externalities under a subsidy intervention is depicted clearly in Figure (2.3). Consider the village of Baretha in Figure (2.3) which at baseline is at approximately 19% level of sanitation adoption. Under a price subsidy the response curve shifts up crossing the $45-degree$ line at a new equilibrium that is close to 84% coverage under a 25% price subsidy , and 92% coverage under a 50% price subsidy respectively. In this case the vertical upward shift would be attributable to the direct effect of the subsidy while the subsequent movement along the response curve to the higher equilibrium is generated by the social multiplier driven by the underlying externalities. The social multiplier under a 50% subsidy is close to a factor of 5.6, where in isolation a 50% subsidy generates a 13 percentage point increase in the probability of adoption while in equilibrium generates a 73 percentage point increase ($\frac{73}{13} \approx 5.6$). In contrast the social multiplier under an income transfer for the same village is close to ≈ 1.

In Chapter (3), I focus solely on the price subsidy policy and examine the individual household response due to a change in price that may explain the large changes in response to a price subsidy intervention found in this analysis. In order to achieve this, the objective is to quantify household response in terms of demand elasticities and decompose the subsidy policy effect along four dimensions; specifically, substitution v. income effects and direct v. indirect effects.I also compute a household's willingness-to-pay for a policy that subsidises sanitation adoption.

2.6 Conclusion

There is a large literature on identifying and estimating social multiplier effects in a context where social interactions and/or externalities are relevant. This has included topics of peer effects in classrooms, spillover in the workplace as well as a few examples in healthcare. Most studies have focused on quantifying the magnitude and nature of this effect and conclude by suggesting that policy interventions may seek to exploit such underlying interaction to improve the efficiency of interventions. What has not been clear is, how subsidy interventions affect household choice/demand when externalities are found to be relevant ? In this chapter, I address this question in the existing literature by providing a tractable framework to analyse potential impacts of subsidy policies in the presence of externalities. I formulate a simple model to investigate the impact of subsidy policy interventions on sanitation adoption choices in the presence of demand externalities. The model estimates reveal a presence of a positive externality associated with adoption of sanitation. Using the estimated parameter values I simulate adoption behaviour at village level to generate probability response curves as a function of the average level of adoption at baseline and under different subsidy monetary amounts. I show that, even under provision of a relatively small subsidy amount (25% of cost of sanitation) activates the social multiplier which results in large shifts in the level of sanitation coverage.

Without taking into account the underlying positive externality the ‘actual’ impact of subsidy interventions are likely to be underestimated. In addition, if the policy maker aims to achieve an aggregate level of adoption, it is perhaps possible to achieve this target with more efficient allocation for example, smaller subsidy allocations per household. Conversely, a fixed total subsidy allocation could be distributed amongst many more beneficiary households in order to achieve a higher target level of sanitation coverage. I further tackle the question of optimal policy choice in Chapter 4 to investigate the role of two key market failures: liquidity constraints and externalities. The current model does not fully incorporate the fact that households maybe potentially liquidity constrained in their adoption choice. Thus the large response to price subsidies could be generated from potential relaxation of the liquidity constraints, faced by households that are otherwise unable to borrow funds to finance adoption. I explicitly model the presence of externalities and liquidity constraints within a life-cycle structure where households face a dynamic trade-off between adopting sanitation today or saving for future consumption under uncertain income. The structure is used to address a debate among policy makers on the appropriate choice between loans and price subsidies to increase sanitation coverage.

2.7 Appendix for Chapter 2

2.7.1 Tables

Table 2.1: Descriptives

	Mean		min	max
Panel A: Sample Size and Number of Groups (Villages)				
Sample Size	1475			
Nr. Of groups (Villages)	44			
Average Nr. Of respondents per group	55.9	[37.01]	5	131
Panel B: Variables used in Estimation				
Dependent Variable				
Household has adopted sanitation	0.378	[0.485]		
Individual HH Controls				
Age of Household Head (years)	42.56	[13.22]	20	91
Forward Caste (yes=1)	0.134	[0.34]		
Nr of years of Education	4.61	[4.73]	0	18
Nr. Of Female HH members	2.54	[1.29]	0	9
Dwelling owned by HH (yes=1)	0.889	[0.314]		
Wealth Amount (Rs.)	57,112	[16,167]	12,660	93,142
Savings Amount (Rs.)	4,482	[5,073]	562	15,593
Group level Controls				
Drainage Infrastructure in village (yes=1)	0.428	[0.495]		
Community sanitation Units available (yes=1)	0.507	[0.501]		
Price of Sanitation (Rs.)	8,628	[1,550]	5,712	11,425
Post office in village (yes=1)	0.353	[0.478]		
Externality (Village Average)				
Average sanitation prevalence excl HH(i)	0.378	[0.304]	0.027	0.933

Notes: Standard deviation in parentheses. $Indian Rs. 1000 \approx GBP\, 10$

Table 2.2: Parameter Estimates

Variable	[1] Coef.	Std. Dev	[2] Coef.	Std. Dev
Age of Household Head (yrs)	0.014	[0.006]**	0.014	[0.006]**
Forward Caste	0.760	[0.252]***	0.766	[0.260]***
Education of Household Head (yrs)	0.099	[0.015]***	0.104	[0.018]***
Nr. of Female Household members	0.130	[0.044]***	0.131	[0.044]***
House Owned	0.165	[0.340]	0.108	[0.353]
Wealth Amount (per Rs. 1000)	**0.013**	**[0.005]****	**0.013**	**[0.006]****
Drainage Infrastructure in village	0.242	[0.096]**	0.338	[0.155]**
Community Sanitation available	-0.007	[0.134]	-0.049	[0.146]
Post office in village	0.011	[0.119]	0.032	[0.130]
Price of Sanitation (per Rs. 1000)	**-0.079**	**[0.041]****	**-0.082**	**[0.045]***
Average sanitation presence excl HH(i)	**4.683**	**[0.305]*****	**4.580**	**[0.369]*****
Correction Factor (1)			-1.698	[1.408]
Correction Factor (2)			16.061	[44.049]
Constant	-3.617	[0.523]	-3.589	[0.563]

Notes: Bootstrapped standard error estimates. * denotes signf at 0.10, ** at 0.05, *** at 0.01. $Indian\,Rs.1000 \approx GBP\,10$

Table 2.3: Estimates: Marginal Effects

Variable	Marg Effect	Std. Dev	Odds Coef.	Std. Dev
Age of Household Head (yrs)	0.003	[0.001]**	1.014	[0.006]**
Forward Caste*	0.179	[0.063]***	2.151	[0.558]***
Education of Household Head (yrs)	0.023	[0.004]***	1.110	[0.020]***
Nr. of Female Household members	0.028	[0.010]***	1.139	[0.050]***
House Owned*	0.023	[0.074]	1.114	[0.393]
HH Wealth Amount (per Rs.1000)	**0.003**	**[0.001]****	**1.014**	**[0.006]****
Drainage Infrastructure in village*	0.074	[0.034]**	1.401	[0.217]**
Community Sanitation available*	-0.011	[0.006]*	0.952	[0.119]*
Post office in village*	0.007	[0.028]	1.032	[0.135]
Price of Sanitation (per Rs. 1000)	**-0.018**	**[0.010]***	**0.922**	**[0.041]***
Average sanitation presence excl HH(i)	**0.997**	**[0.073]*****	**97.533**	**[35.954]*****
Predicted probability of adoption	0.320			

Notes: Bootstrapped standard error estimates. * denotes signf at 0.10, ** at 0.05, *** at 0.01. $Indian\,Rs.1000 \approx GBP\,10$

Table 2.4: Baseline Simulation by Village: Fraction of Household Adopters

				Model Simulation		
Village	I_g	$\widehat{\gamma}$ **Estim.**	**Data**	**Equil.1**	**Equil.2**	**Pct Diff.**
vill ID 1	52	4.58	0.81	0.89		-0.08
vill ID 2	47	4.58	0.00	0.05		-0.05
vill ID 3	118	4.58	0.14	0.22		-0.08
vill ID 4	95	4.58	0.26	0.20		0.06
vill ID 5 (Baretha)	27	4.58	0.19	0.17		0.02
vill ID 6	20	4.58	0.35	0.12		0.23
vill ID 7	11	4.58	0.18	0.11		0.07
vill ID 8	15	4.58	0.20	0.21		-0.01
vill ID 9	19	4.58	0.00	0.09		-0.09
vill ID 10	14	4.58	1.00	0.92		0.08
vill ID 11	5	4.58	0.20	0.05		0.15
vill ID 12 (Harirarm Ka Pura)	18	4.58	0.00	0.10		-0.10
vill ID 13	34	4.58	0.71	0.19		0.52
vill ID14	33	4.58	0.45	0.43	0.71	0.02
vill ID 15	16	4.58	0.00	0.06		-0.06
vill ID 16	16	4.58	0.94	0.72		0.22
vill ID 17	32	4.58	0.50	0.24		0.26
vill ID 18	45	4.58	0.00	0.11		-0.11
vill ID 19	21	4.58	0.05	0.05		0.00
vill ID 20	8	4.58	0.63	0.22	0.61	0.41
vill ID 21	42	4.58	0.19	0.17		0.02
vill ID 22	24	4.58	0.00	0.07		-0.07
vill ID 23	16	4.58	0.69	0.26		0.43
vill ID 24	131	4.58	0.11	0.10		0.01
vill ID 25	5	4.58	0.20	0.12		0.08
vill ID 26	62	4.58	0.63	0.83		-0.20
vill ID 27 (Raipur Kala)	12	4.58	0.50	0.55		-0.05
vill ID 28	8	4.58	0.38	0.19		0.19
vill ID 29	68	4.58	0.66	0.16		0.50
vill ID 30	55	4.58	0.65	0.83		-0.18
vill ID 31	12	4.58	0.50	0.35	0.80	0.15
vill ID 32	28	4.58	0.46	0.48		-0.02
vill ID 33	47	4.58	0.89	0.88		0.01
vill ID 34	32	4.58	0.88	0.24		0.64
vill ID 35	38	4.58	0.05	0.11		-0.06
vill ID 36	14	4.58	0.71	0.74		-0.03
vill ID 37	46	4.58	0.15	0.06		0.09
vill ID 38	32	4.58	0.47	0.75		-0.28
vill ID 39	16	4.58	0.88	0.29		0.59
vill ID 40	36	4.58	0.08	0.09		-0.01
vill ID 41	10	4.58	1.00	0.80		0.20
vill ID 42	41	4.58	0.80	0.75		0.05
vill ID 43	7	4.58	0.71	0.87		-0.16
vill ID 44 (Utila)	47	4.58	0.30	0.35		-0.05

Notes: Column (1) & (2) village ID and village sample size, Column (3) social interaction parameter estimate, Column (4) sanitation coverage observed in the data. Column (5) & (6) Model predicted equilibrium sanitation level. Column (7) Percent difference between data and model predicted lowest sanitation level in every village.

Table 2.5: Policy Simulation by Village: Unconditional (Income) Subsidy

		Subsidy: 25% Cost of Sanitation			Subsidy: 50% Cost of Sanitation		
Village	**Data**	**Equil.1**	**Equil.2**	**Equil.3**	**Equil.1**	**Equil.2**	**Equil.3**
vill ID 1	0.81	0.90			0.92		
vill ID 2	0.00	0.06			0.06		
vill ID 3	0.14	0.27	0.61		0.38	0.76	
vill ID 4	0.26	0.23			0.27		
vill ID 5 (Baretha)	0.19	0.19			0.23		
vill ID 6	0.35	0.13			0.14		
vill ID 7	0.18	0.12			0.13		
vill ID 8	0.20	0.77			0.81		
vill ID 9	0.00	0.10			0.11		
vill ID 10	1.00	0.93			0.94		
vill ID 11	0.20	0.05			0.06		
vill ID 12 (Harirarm Ka Pura)	0.00	0.11			0.13		
vill ID 13	0.71	0.21			0.24		
vill ID14	0.45	0.78			0.81		
vill ID 15	0.00	0.07			0.08		
vill ID 16	0.94	0.76			0.80		
vill ID 17	0.50	0.41	0.73		0.78		
vill ID 18	0.00	0.12			0.14		
vill ID 19	0.05	0.06			0.07		
vill ID 20	0.63	0.28	0.64		0.78		
vill ID 21	0.19	0.19			0.22		
vill ID 22	0.00	0.07			0.08		
vill ID 23	0.69	0.49			0.75		
vill ID 24	0.11	0.10			0.11		
vill ID 25	0.20	0.13			0.14		
vill ID 26	0.63	0.85			0.87		
vill ID 27 (Raipur Kala)	0.50	0.67			0.74		
vill ID 28	0.38	0.21	0.64		0.36		
vill ID 29	0.66	0.18			0.20		
vill ID 30	0.65	0.85			0.87		
vill ID 31	0.50	0.83			0.85		
vill ID 32	0.46	0.76			0.80		
vill ID 33	0.89	0.88			0.90		
vill ID 34	0.88	0.50			0.76		
vill ID 35	0.05	0.13			0.15		
vill ID 36	0.71	0.76			0.80		
vill ID 37	0.15	0.07			0.07		
vill ID 38	0.47	0.79			0.82		
vill ID 39	0.88	0.52			0.74		
vill ID 40	0.08	0.10			0.11		
vill ID 41	1.00	0.82			0.85		
vill ID 42	0.80	0.78			0.81		
vill ID 43	0.71	0.89			0.90		
vill ID 44 (Utila)	0.30	0.62			0.74		

Notes: Column (1) village ID and Column (2) sanitation coverage observed in the data. Column (3), (4), (5) Model predicted equilibrium sanitation level under 25% (cost of sanitation) unconditional subsidy. Column (6), (7), (8) Model predicted equilibrium sanitation level under 50% (cost of sanitation) unconditional subsidy.

Table 2.6: Policy Simulation by Village: Conditional (Price) Subsidy

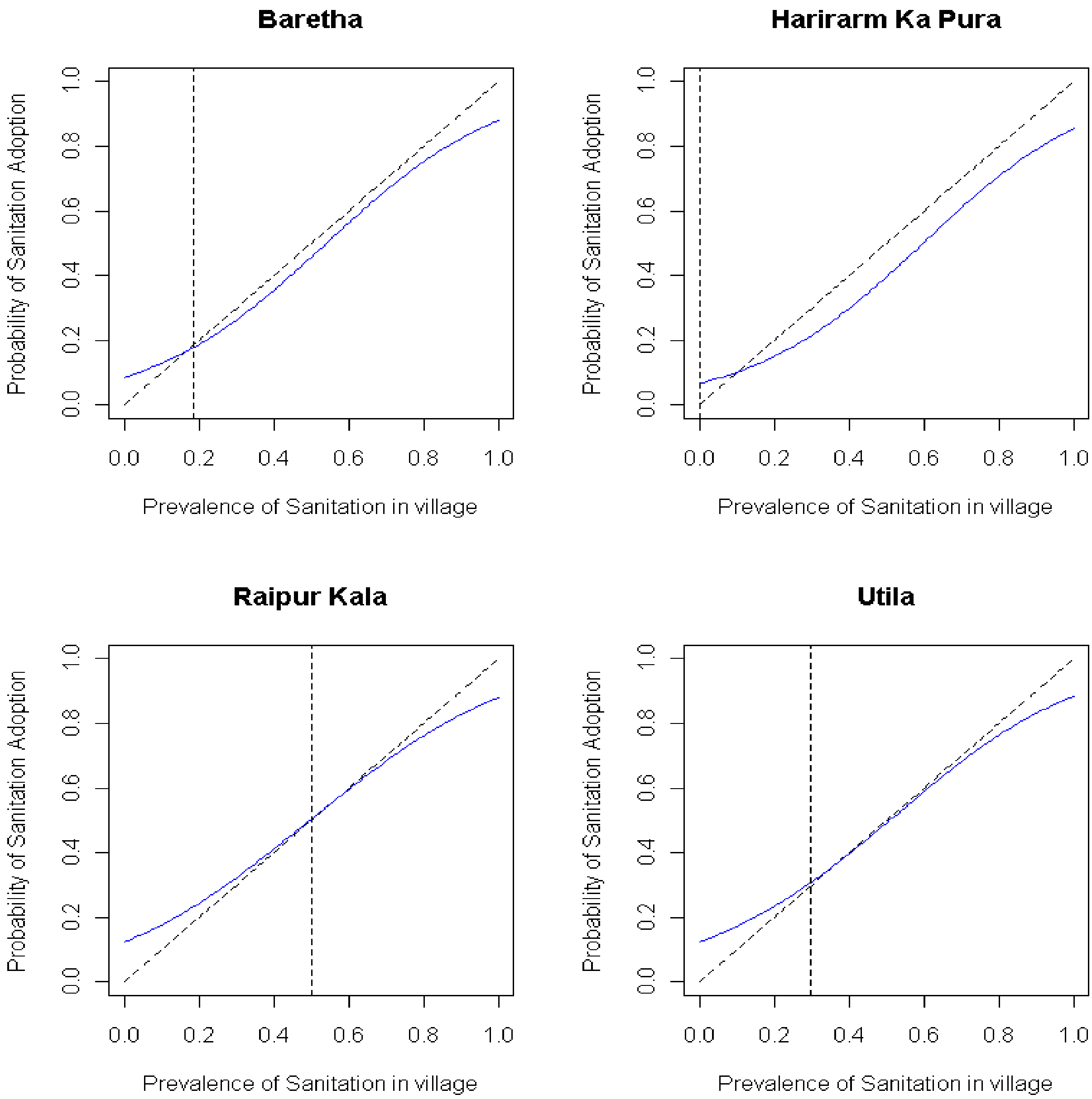
Village	Data	Subsidy: 25% Cost of Sanitation Equil.1	Equil.2	Equil.3	Subsidy: 50% Cost of Sanitation Equil.1	Equil.2	Equil.3
vill ID 1	0.81	0.94			0.96		
vill ID 2	0.00	0.10			0.19	0.68	
vill ID 3	0.14	0.88			0.93		
vill ID 4	0.26	0.83			0.91		
vill ID 5 (Baretha)	0.19	0.84			0.92		
vill ID 6	0.35	0.28	0.61		0.88		
vill ID 7	0.18	0.25	0.56	0.70	0.90		
vill ID 8	0.20	0.89			0.94		
vill ID 9	0.00	0.18			0.84		
vill ID 10	1.00	0.96			0.97		
vill ID 11	0.20	0.09			0.17	0.67	
vill ID 12 (Harirarm Ka Pura)	0.00	0.23	0.63		0.89		
vill ID 13	0.71	0.82			0.91		
vill ID14	0.45	0.89			0.94		
vill ID 15	0.00	0.12			0.28	0.49	0.75
vill ID 16	0.94	0.88			0.93		
vill ID 17	0.50	0.88			0.93		
vill ID 18	0.00	0.24			0.88		
vill ID 19	0.05	0.10			0.21	0.58	0.73
vill ID 20	0.63	0.88			0.93		
vill ID 21	0.19	0.81			0.91		
vill ID 22	0.00	0.12			0.38	0.78	
vill ID 23	0.69	0.87			0.93		
vill ID 24	0.11	0.17			0.83		
vill ID 25	0.20	0.37	0.80		0.91		
vill ID 26	0.63	0.91			0.95		
vill ID 27 (Raipur Kala)	0.50	0.85			0.92		
vill ID 28	0.38	0.88			0.93		
vill ID 29	0.66	0.79			0.90		
vill ID 30	0.65	0.92			0.95		
vill ID 31	0.50	0.91			0.94		
vill ID 32	0.46	0.89			0.93		
vill ID 33	0.89	0.93			0.96		
vill ID 34	0.88	0.87			0.92		
vill ID 35	0.05	0.27			0.87		
vill ID 36	0.71	0.89			0.94		
vill ID 37	0.15	0.11			0.27	0.47	0.76
vill ID 38	0.47	0.89			0.93		
vill ID 39	0.88	0.87			0.92		
vill ID 40	0.08	0.17			0.83		
vill ID 41	1.00	0.91			0.94		
vill ID 42	0.80	0.89			0.93		
vill ID 43	0.71	0.93			0.96		
vill ID 44 (Utila)	0.30	0.86			0.92		

Notes: Column (1) village ID and Column (2) sanitation coverage observed in the data. Column (3), (4), (5) Model predicted equilibrium sanitation level under 25% (cost of sanitation) price subsidy. Column (6), (7), (8) Model predicted equilibrium sanitation level under 50% (cost of sanitation) price subsidy.

2.7.2 Figures

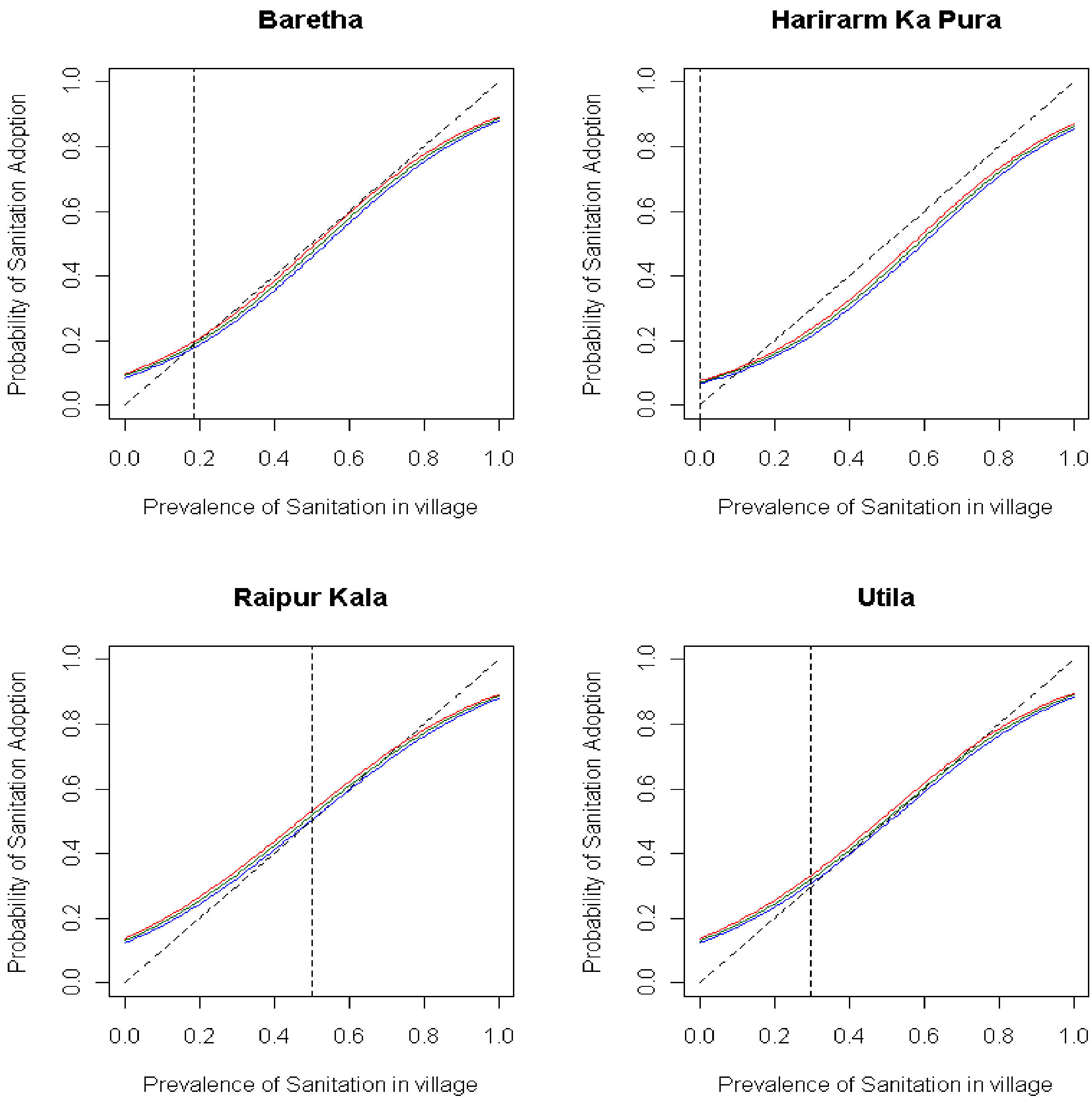
Baseline and Policy Simulations

Figure 2.1: Baseline Simulation: Probabilistic response curves



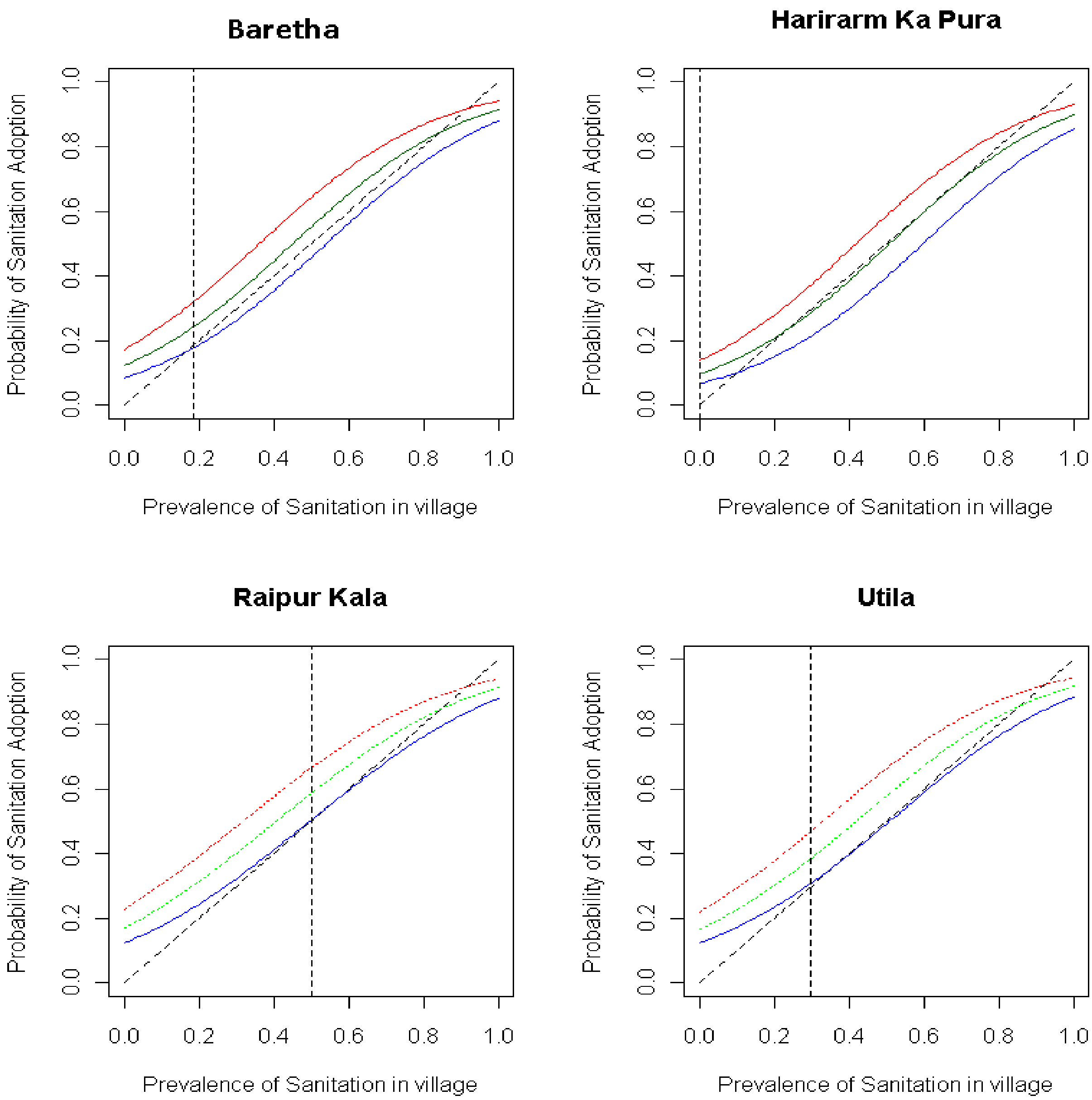
Unconditional (Income) Subsidy: (a) Baseline [Blue] (b) 25% Cost of Sanitation [Green] & (c) 50% Cost of Sanitation [Red]

Figure 2.2: Unconditional Subsidy: Probabilistic response curves



Conditional (Price) Subsidy: (a) Baseline [Blue] (b) 25% Cost of Sanitation [Green] & (c) 50% Cost of Sanitation [Red]

Figure 2.3: Conditional Subsidy: Probabilistic response curves



2.7.3 Estimation

2.7.3.1 Cost of Sanitation

The information used to construct a measure for the cost of sanitation was collected independently of the household survey.[29] The price measure is based on the cost of building the most common type of sanitation facility in the local region i.e., 'Twin Pit Pour Flush' (TPPF) unit. The price measure comprises of two components: the total cost of raw material and the cost of labour required to build the facility itself. The price measure collected at the group level varies across the villages in both cost components. All households within a village face the same price. Following was the formula applied to collect the necessary information based on local knowledge provided by the local municipality authorities..

Price variation across villages g:

- $wage_g$: Approximate daily (informal) wage rate which varies across villages.
- $days$: Approximate time to construct a 'Twin Pit Pour Flush' (TPPF) variation between $3-4$ days. TPPF is the standard and most popular sanitation design unit implemented by the government under the Total Sanitation Campaign (T.S.C)
- $cost_g\,(raw\,materials)$: Approximate cost of raw material (cost of five principle materials used in the construction of a sanitation unit) which include — Bricks, Mortar, Tiles, Ceramic fixtures & Tin sheets.

$$price_g = wage_g * days + cost_g\,(raw\,materials) * quantity(kilogram/piece/unit) \tag{2.7.1}$$

The main source of variation arises from the cost of raw materials which varies across villages and comprises close to 70% of the total cost of sanitation incurred. A point to note here is that the raw materials used in sanitation are widely produced and demanded in the region on a large scale for other domestic and commercial construction. The demand for these products for the purpose of building sanitation constitutes a very small proportion of the overall demand.

[29] Data on the cost measure was collected in July/August 2012 across all villages. I received helpful advice and assistance from the Gwalior Nagar Nigam Seva municipal authorities in the collection process.

2.7.3.2 Measurement Error

Deriving Small Error Variance Approximations to Probability Distributions

Defining the outcome variable d_i as a binary choice of adoption of sanitation and covariates for household i belonging to village g as $\chi_{ig} = \left(x_{ig}, z_g, \overline{D}_g\right)$ where variable $\overline{D}_g$ represents the average choice of all other households excluding household i belonging to group g.

The joint density function is given by:

$$g_{d_i;\chi_{ig}}(d_i, \chi_{ig}) = g_{d_i|\chi_{ig}}(d_i \mid \chi_{ig}) g_{\chi_{ig}}(\chi_{ig})$$

However, data on $\overline{D}_g$ is not observable, instead realizations on a variate $\overline{d}_g$ are observed:

$$\overline{d}_g = \overline{D}_g + \sigma_{ig} u_{ig} \tag{2.7.2}$$

The variable $\underline{u_{ig}}$ is assumed to be continuously distributed independently of d_i (outcome) and $\overline{D}_g$ (error free covariate) with mean zero and variance one and correlations $corr(u_{ig}, u_{jg}) = \rho_{ij(g)}$ and joint density function $g_u(u)$. Realizations of $\sigma_{ig} u_{ig}$, $\overline{D}_g$ and $\overline{d}_g$ correspond to measurement error, error free covariate and error contaminated covariate, respectively.

For exposition purposes, a simplification in notation is adopted - consider a case where the only covariate is $\overline{D}_g$. Under these assumptions the joint density of $d, \overline{D}, \overline{d}$ is given by

$$g_{d,\overline{D},\overline{d}}(d, \overline{D}, \overline{d}) = g_{d|\overline{D}}(d \mid \overline{D}) g_{\overline{D}}(\overline{D})$$

Using Equation (2.7.2)

$$g_{d,\overline{D},\overline{d}}(d, \overline{D}, \overline{d}) = g_{d|\overline{D}}(d \mid \overline{d} - \sigma u) g_{\overline{D}}(\overline{d} - \sigma u) g_u(u) \tag{2.7.3}$$

where $\sigma = diag(\sigma_i)$ and $\overline{D}$ and $\overline{d}$ are column vectors. The joint density in Equation (2.7.3) and its associated conditional and marginal densities are approximated by taking a Taylor series approximation[30] around the point $\sigma = 0$, retaining the terms up to the second order in the $\sigma_i' s$

[30] The Taylor series of a function $f(x)$ that is infinitely differentiable in a neighbourhood of a real or complex number a is given by the power series $f(a) + \frac{f^{(1)}(a)}{1!}(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots$.

$$
\begin{aligned}
g_{d,\overline{D},\overline{d}}(d,\overline{D},\overline{d}) &= g_u(u)[g_{d|\overline{D}}(d|\overline{d})\,g_{\overline{D}}(\overline{d}) - \sigma_i u_i\{g_{d|\overline{D}}(d|\overline{d})\,g^{(1)}_{\overline{D}}(\overline{d}) + g^{(1)}_{d|\overline{D}}(d|\overline{d})\,g_{\overline{D}}(\overline{d})\} \\
&+ \tfrac{1}{2}\sigma_i\sigma_j u_i u_j\{g_{d|\overline{D}}(d|\overline{d})\,g^{(2)}_{\overline{D}}(\overline{d}) + 2g^{(1)}_{d|\overline{D}}(d|\overline{d})\,g^{(1)}_{\overline{D}}(\overline{d}) + g^{(2)}_{d|\overline{D}}(d|\overline{d})\,g_{\overline{D}}(\overline{d})\}] \\
&+ o(\sigma^2)
\end{aligned}
\tag{2.7.4}
$$

Given $E(u) = 0$ the terms multiplying the linear terms in the u_i disappear and the joint density can be simplified to:

$$
g_{d,\overline{d}}(d,\overline{d}) = g_{d|\overline{D}}(d|\overline{d})\,g_{\overline{D}}(\overline{d}) + \frac{1}{2}\Sigma\left\{g_{d|\overline{D}}(d|\overline{d})\,g^{(2)}_{\overline{D}}(\overline{d}) + 2g^{(1)}_{d|\overline{D}}(d|\overline{d})\,g^{(1)}_{\overline{D}}(\overline{d}) + g^{(2)}_{d|\overline{D}}(d|\overline{d})\,g_{\overline{D}}(\overline{d})\right\} + o(\Sigma) \tag{2.7.5}
$$

While the marginal density is given by the approximation:

$$
g_{\overline{d}}(\overline{d}) = g_{\overline{D}}(\overline{d}) + \frac{1}{2}\Sigma g^{(2)}_{\overline{D}}(\overline{d}) + o(\Sigma) \tag{2.7.6}
$$

The corresponding approximation to the conditional density in Equation (2.7.3) is given by

$$
g_{d|\overline{d}}(d|\overline{d}) = g_{d|\overline{D}}(d|\overline{d}) + \frac{1}{2}\Sigma\left\{2g^{(1)}_{d|\overline{D}}(d|\overline{d})G^{(1)}_{\overline{D}}(\overline{d}) + g^{(2)}_{d|\overline{D}}(d|\overline{d})\right\} + o(\Sigma) \tag{2.7.7}
$$

Using Equation (2.7.7) it is possible to derive the approximation to the mean regression function highlighted in section (2.4.2).

$$
E_{d|\overline{d}}\{f(d)|\overline{d}\} = E_{d|\overline{D}}\{f(d)|\overline{d}\} + E^{(1)}_{d|\overline{D}}\{f(d)|\overline{d}\}\Sigma G^{(1)}_{\overline{D}}(\overline{d}) + \frac{1}{2}tr\left[\Sigma E^{(2)}_{d|\overline{D}}\{f(d)|\overline{d}\}\right] + o(\sigma^2) \tag{2.7.8}
$$

The expression given by Equation (2.7.8) is studied in detail in Chesher (1991). It suffices to note the following

1. The first term on the right hand side of Equation (2.7.8) is just the regression function of $f(d)$ given $\overline{D}$ evaluated at $\overline{d}$. While the second and the third terms give the first order effects of measurement error on the form of the regression function.

2. The second term on the right hand side of Equation (2.7.8) is what generally produces the attenuation effect of measurement error. The behaviour of this term depends crucially on the first derivatives of the logarithm of the density of the error free covariates.

3. The third term on the right hand side of Equation(2.7.8) vanishes when the regression of $f(d)$ on $\overline{D}$ is linear. In other cases it tends to reduce the curvature of the non-linear regressions as conditioning moves from $\overline{D}$ to $\overline{d}$. This term does not depend upon the distribution of the error free covariate.

The small variance approximation method above is used to produce an approximation to the error contaminated regression which, when the error free covariates and the measurement error are independently distributed, is invariant with respect to the distribution of measurement error, depending only on an estimable functional of the marginal density of the error contaminated covariates, namely the derivatives of their log density and the functional form of the error free regression denoted by $G^{(1)}_{\overline{D}}(\overline{d})$.

The first order effect of measurement error on the level of a linear regression function is smaller near the mode of the distribution of the error free variate and the effect on its slope is small where the curvature of the log density of the error free variate is slight. There is an additional non linear effect in non linear regression models, raising the error contaminated regression function when the error free regression function is convex and lowering it where it is concave.

2.7.3.3 Construction of $G^{(1)}_{\overline{d}}(\overline{d}_g)$

1. Regress $\overline{d}_g = \varphi w_{ig} + \upsilon_{ig}$ where w_{ig} are all covariates except $\overline{d}_g$ and υ_{ig} is residual/error
2. Estimate $\widehat{\varphi}$
3. Construct fitted residuals $\widehat{\upsilon}_{ig} = \overline{d}_g - \widehat{\varphi}' w_{ig}$ where $f(\widehat{\upsilon}_{ig}) \approx f(\overline{d}_g | x_{ig}, z_g)$ [31]
4. Construct kernel density for $\hat{f}(\widehat{\upsilon}_{ig}) = \frac{1}{Nh}\sum_{i=1}^{N} K\left(\frac{\widehat{\upsilon}_{ig} - \widehat{\upsilon}_{0g}}{h}\right)$ [32]

---

[31] Alternatively: Plot $\widehat{\upsilon}_{ig}$ and calculate for every point a and a small distance $\triangle$-$\left(\frac{f(a+\triangle)-f(a)}{\triangle}\right)\left[\frac{1}{\left(\frac{f(a+\triangle)+f(a)}{2}\right)}\right]$, Note, this method is problematic at the tails of the distribution.

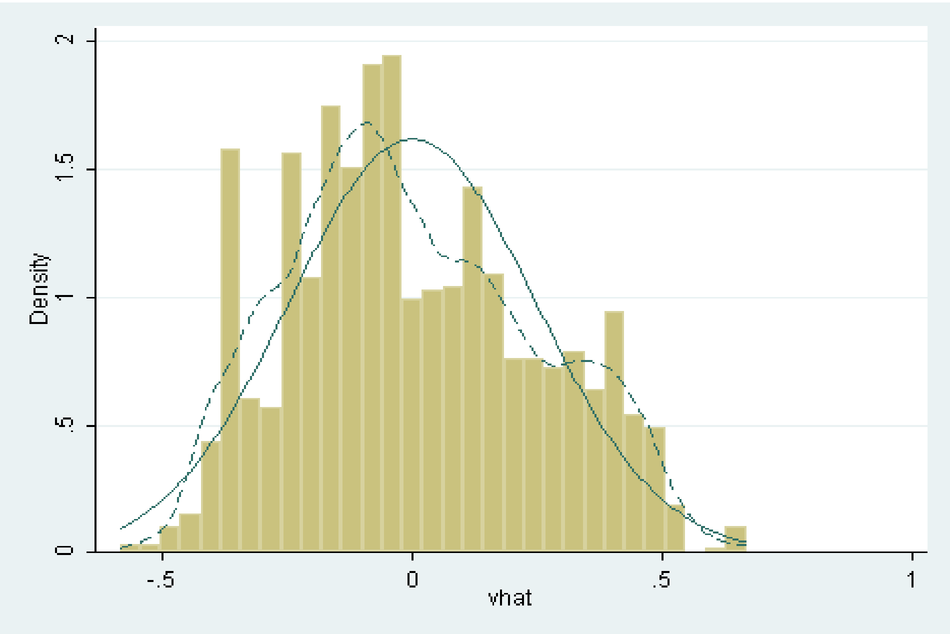
[32] where N is the total sample size and h is the smoothing parameter or bandwidth, Note: the density and its derivative would be sensitive to the bandwidth specification h, see appendix for results that take bandwidth sensitivity into account.

where

$$\frac{\partial \log f(\bar{d}_g|w_{ig})}{\partial \bar{d}_g} = \frac{1}{f(\bar{d}_g|w_{ig})} \cdot \frac{\partial f(\bar{d}_g|w_{ig})}{\partial \bar{d}_g} = \left[\frac{1}{\hat{f}(\widehat{\upsilon}_{ig})}\right] \left(\frac{\partial \hat{f}(\widehat{\upsilon}_{ig})}{\partial \widehat{\upsilon}_{ig}}\right) \tag{2.7.9}$$

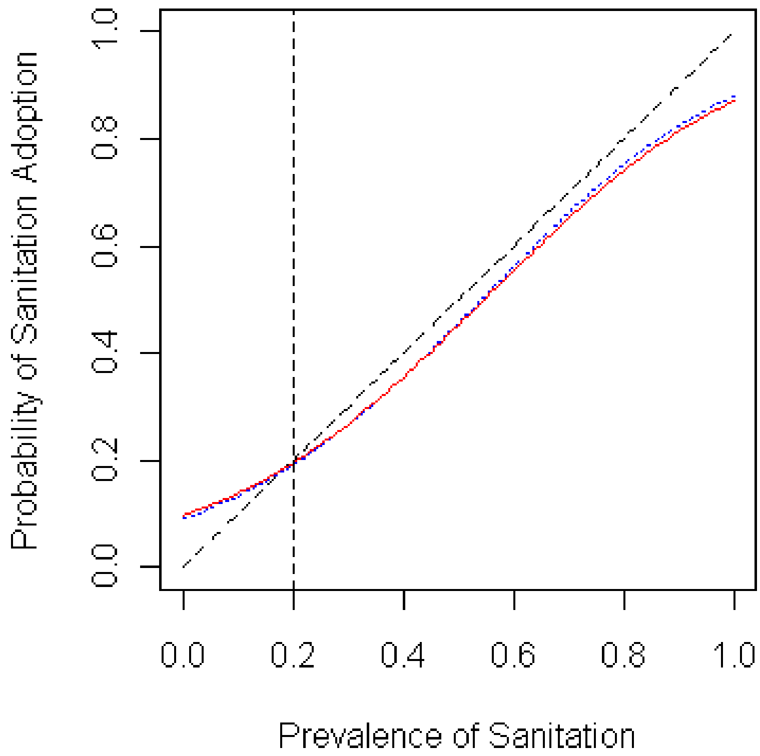
The potential disadvantage of this method that $G_{\bar{d}}^{(1)}(\bar{d}_g)$ is essentially a ratio of kernel estimators and problems may arise when the denominator kernel estimator is close to zero. This is more likely to occur in the tails of the distribution.

Figure 2.4: Density Plot: $\widehat{\upsilon}_{ig}$



Notes: Kernel= Gaussian, bwidth=0.0262

Figure 2.5: Impact of Measurement Error on Response Probability



Notes: Policy simulations are performed on a counterfactual village where the initial sanitation coverage is at 20%. Blue Line (Dashed): Response probability without correction. Red Line (Solid): Response probability with measurement error correction.

Chapter 3

Welfare Analysis with Discrete Choice: An Ex-ante Evaluation of a Sanitation Intervention

3.1 Introduction

In this chapter, I quantify a household's willingness-to-pay for a policy that subsidises the cost of sanitation adoption, by computing Compensating Variation (CV) and Equivalent Variation (EV) welfare measures. To compute welfare I employ the notion of the expenditure function and compensated demand in a discrete choice setting. The structure is then applied to the estimated model and results from Chapter (2) to perform welfare analysis of subsidy policies under different hypothetical interventions with externalities.

Counterfactual policy evaluations are performed by computing demand elasticities for different subsidy policies using the estimated model from Chapter 2. With externalities, the impact of a price subsidy on a household's behaviour can be decomposed into four effects. Firstly, the price subsidy not only changes the relative price of the subsidised good but also changes (increases) the "effective" budget available for consumption. In addition, the subsidy policy has a direct effect on the recipient household as well as an indirect effect driven by the underlying externality. Thus in order to study the demand response to a policy, I first disentangle the substitution or compensated effect of the subsidy from the income effect and further decompose into the direct and indirect effect of the policy. The analysis depends on being able to characterize the compensated or Hicksian choice probability which is also key for computing Compensating and Equivalent Variation measures.[1]

Under a random utility framework it follows that the expenditure function[2] would have a random component and a corresponding distribution. In this case the standard Slutsky equation per se does not exist and therefore the uncompensated (Marshallian) choice probability cannot be used to back out the compensated (Hicksian) effect. The theoretical framework developed by Dagsvik & Karlstrom (2005) first outlined the random expenditure function and its distribution. In this chapter, I adapt their theoretical framework to derive compensated choice probabilities and further extend the analysis to account for additional feedback effects due to the presence of externalities. In addition, the analytical characterization of compensated demand provides a tool to measure welfare through the computation of Compensating Variation (CV), Equivalent Variation (EV) and corresponding Deadweight Loss (DWL). This contribution, provides a formalization to the structural evaluation of the impact of subsidy interventions in the presence of externalities.

I find that the provision of a relatively small price subsidy, 25% of the cost, generates substitution effects that are significantly larger than the income effects. On average, a minimum of 58% and a maximum of 83% of the total effect of a subsidy is attributed to the substitution effect. In addition, a substantial amount of this substitution or 'pure ' price effect is found to be propagated through the indirect channel, from a minimum of 75% to a maximum of 90%. Under positive externalities, the socially optimal level of adoption is larger than the privately chosen equilibrium adoption level. The presence of externalities implies a larger welfare gain computed in terms of both Compensating and Equivalent Variation. Lastly, in the presence of a positive externality the provision of a price subsidy generates a Net Gain ($NetG$) instead of a Deadweight Loss (DWL) which is a gain realised by each household as the village/society shifts towards a socially optimal level of adoption.

The chapter is organised as follows, I first discuss the related literature in Section 3.2. Followed by the theoretical exposition of the problem in Section 3.3, parts of which follow from Dagsvik & Karlstrom (2005). Section 3.4 discusses the analytical derivation of the main components to compute policy effects and extends the results to account for spillover effects generated by interdependent choice. Section 3.5 provides

[1]The area under the compensated or Hicksian demand curve for a change in price with utility held fixed at original level, quantifies the Compensating Variation (CV). The Compensating variation quantifies the maximum amount a household/agent is willing-to-pay of the price change (policy).

[2]The indirect utility function which gives the maximal attainable utility given constraints is also the inverse of the Expenditure function. It is thus possible to derive the expenditure function by inverting the function, or in this case the inverse distribution generated under the random indirect utility.

counterfactual policy results for the sanitation application and Section 3.6 computes welfare by calculating a household's willingness-to-pay for sanitation. Section 3.7 concludes with a discussion on the policy implications resulting from the analysis. All relevant Tables and Figures are provided in the chapter appendix in Section 3.8.

3.2 Related Literature

There exists a large literature on the identification and estimation of welfare measures using individual level data for continuous and discrete choice. Hausman's (1981) paper formulated the nonparametric identification and parametric estimation of exact welfare effects of a price change for a good consumed in continuous quantities. Hausman & Newey (1995) extended the analyses by formulating semiparametric estimation of the welfare effects and developing the corresponding theory of statistical inference. However, these methods cannot directly be implemented in discrete choice settings where the effect of the price change on individual utilities depends in a fundamental way on the discrete nature of the good and the unobserved taste heterogeneity.

Though the main point of reference of this chapter is Dagsvik & Karlstrom (2005) there are many earlier and subsequent papers on the topic that form the existing literature for welfare analysis under discrete choice. Domencich & McFadden (1975) first tackled the welfare measurement problem in the discrete choice setting under the assumption that utility is quasi-linear i.e., additively separable in income. Under this strong assumption that the conditional indirect utility is linear in income, there are no income effects and the Marshallian and Hicksian measures are identical. In subsequent work, Small & Rosen (1981) investigated the measurement of welfare effects of price and quality change for discrete choice with allowance for additive scalar heterogeneity. However in the empirical formulation Small & Rosen (1981) assume that the income effects generated by price changes are infinitesimally small[3] which then allows them to equate the Marshallian and Hicksian measures. With a utility formulation that is non linear in income, obtaining an analytic formula for welfare distributions proved to be a challenge. An alternative approach in the literature proposed different approximations to characterize the welfare distribution. For example, McFadden (1999a) proposed a Monte Carlo simulator for approximating Compensating Variation (CV) in random utility models which would

[3]Under the assumption that the resulting budget share for an agent from the consumption of a discrete good is sufficiently small.

converge to the true distribution of the Compensating Variation (CV). Using this simulation method, Herriges and Kling (1999) investigated the empirical consequences of non linear income effects based on a particular empirical application.

More recently, Dagsvik and Karlstrom (2005) allowed for utility to be non linear in income. They also incorporated additively separable taste heterogeneity that is assumed to follow a known parametric distribution for identifying and estimating welfare effects. In a follow up paper Dagsvik, Strom and Locatelli (2013)[4] employ the structure from Dagsvik and Karlstrom (2005) to quantify income and substitution effects of labour supply under wage rate changes.

The most recent contribution to the literature include Bhattacharya (2015) which establishes nonparametric point-identification of the distribution of the welfare effects of price change in a discrete choice setting, incorporating unobservable heterogeneity in the utility function and assuming no knowledge of the dimension and distribution of the unobservables. The approach undertaken in this chapter significantly differs from Bhattacharya (2015). Specifically, under nonparametric welfare analysis the environment change under a proposed policy intervention, such as different price subsidies would need to be observed in the data. For example, the econometrician would need to observe demand under a price that would equate to being the 'subsidised equivalent' of the original price. In contrast, the approach taken in this chapter as well as by Dagsvik and Karlstrom (2005), does not require for the policy effect to lie within the observed data and thus can be viewed as an extrapolative approach. A nonparametric approach though more flexible on model specification does make demands on observed data. While a more structural approach to welfare reduces data requirement by trading off against additional assumptions on the demand specification, for e.g., additive separability of taste shocks. I would argue that the optimal choice between the two approaches should be made by comparing the gains and losses given the available data at hand for each application.

The existing literature has so far focused on welfare analysis in the context of a single agent/individual framework. One of the main contributions of this chapter to the existing literature is the extension of the framework to a context where household choices are strategic or interdependent. Though the structure does employ specific parametric assumptions to preserve empirical tractability, that are discussed in detail.

[4]The research that forms this chapter was independently developed in a timeline parallel with Dagsvik, Strom and Locatelli (2013)

The gains of having a simple approach is an easily applicable tool to perform welfare analysis in most applied problems which study spillover, peer effects and externalities within a discrete choice setting.

3.3 Theoretical Framework

In this section, I first provide a brief exposition of the theoretical framework described in Dagsvik and Karlstrom (2005). This is followed by a description of the approach I take to make the structure empirically tractable in order to compute substitution and income effects. Lastly, I extend the existing theoretical framework to account for the presence of externalities and to further disentangle the direct and indirect effect of subsidy policies in Section (3.4).

The intuition behind the theory discussed below is as follows, under the random utility model (RUM) framework the standard Slutsky Equation per se does not exist. In which case, the uncompensated (Marshallian) choice probabilities can not be used to back out the corresponding compensated (Hicksian) probability using the Slutsky Equation. Instead, the compensated demand probability is derived from the random expenditure function and its distribution. The joint distribution of expenditure, ex-ante and ex-post policy change choices are used to derive an analytical expression for the compensated (Hicksian) choice probability. Another useful consequence of characterizing the Hicksian choice probability is the ability to quantify standard welfare measures such as Compensating Variation (CV) and Equivalent Variation (EV) as well as the resulting Net Gain that arises from the implementing/providing a subsidy policy.[5]

Using the model outlined in Chapter 2 section (2.3), recall that a household's conditional indirect utility from choosing option j where $D = \{j,k\}$ is assumed to have the form:

$$V_j(y, p_j, w_{ig}, \varepsilon_j) = v_j(y, p_j, w_{ig}) + \varepsilon_j \tag{3.3.1}$$

while the unconditional indirect utility is given by:

$$V_D(y, p_j, w_{ig}, \varepsilon_j) = \max_{r \in D} \left(v_r(y, p_r, w_{ig}) + \varepsilon_r \right) \tag{3.3.2}$$

[5] such welfare computations are further discussed in Section (3.6)

For exposition purposes, in equations (3.3.1) and (3.3.2), component y and p_j which denote household wealth and price of sanitation are explicitly represented in the indirect utility function, and w_{ig} denotes all other household and good (sanitation) characteristics. Just as before the utility function is additively separable in its deterministic $v(y, p_j, w_{ig})$ and random utility components ε_j. If the $\varepsilon_j' s$ are distributed *i.i.d* Type 1 Extreme value for each alternative. The Marshallian (uncompensated) choice probability takes a familiar form:

$$P_D(j, p, y, w_{ig}) = P\left(V_j = \max_{r \in D} V_r\right) = \frac{exp(v_j(p_j, y, w_{ig}))}{\sum_{r \in D} exp(v_r(p_r, y, w_{ig}))} \tag{3.3.3}$$

Dagsvik and Karlstrom (2005) demonstrate that under standard assumptions on the indirect utility function $V_D(y, p_j, w_{ig}, \varepsilon_j)$ the function can be inverted to derive an expenditure function $Y_D(p, u)$

$$u = V_D(p, Y_D(p, u)) \tag{3.3.4}$$

Appendix (3.8.1) provides a proof for the existence of the expenditure function and characterizes its distribution. Using the result in Equation (3.8.5) it is possible to derive the compensated choice probability. The compensated choice probability, given that utility is held constant, is given by:

$$P_D^h(j, p, u) = P\left(v_j(p_j, Y_D(p, u)) + \varepsilon_j = \max_{r \in D} (v_r(p_r, Y_D(p, u)) + \varepsilon_r)\right) \tag{3.3.5}$$

where $P_D^h(j, p, u)$ is the probability of choosing j given that the utility level is held constant and equal to u.

3.4 Implementation and Extension

In this section, I focus on the conditional price subsidy simulations results from Chapter 2 section (2.5.3) and attempt to answer: What impact do subsidy interventions have on household demand for sanitation in the presence of externalities? In order to capture the effect on demand, I compute demand elasticities and decompose the effect of the subsidy along four separate dimensions. First, I divide the total subsidy effect into a substitution and income effect. The provision of the subsidy changes (decreases) the effective price of sanitation that a household faces. The subsequent change in demand is attributable to:

1. **Substitution Effect:** The relative price of sanitation drops compared to the price of other consumption which results in an increase (or decrease) in the probability of adoption of sanitation depending on whether households attribute sanitation to be a normal (or inferior) good.
2. **Income Effect:** The decrease in price generates an increase in available income (wealth) that a household can use to consume more of both sanitation and other consumption.

The computation of such substitution and income effects is not standard in the context of a discrete choice as it requires characterizing the compensated or Hicksian demand. In the section below, I formulate and discuss a structure that allows me to analytically characterize the Hicksian demand and thus separate the two effects in a discrete choice random utility model. Second, I further decompose the substitution and income effects generated by the subsidy into its direct and indirect components. In the presence of externalities, a subsidy intervention generates two additional effects:

1. **Direct Effect:** A primary effect on household demand, characterized by an individual recipient household's isolated response to a subsidy.
2. **Indirect Effect:** A secondary 'feedback' effect generated by the dependence of a household's adoption choice on the adoption behaviour of other households, who in equilibrium also respond to the subsidy.

3.4.1 Compensated Choice Probabilities

I now discuss how I make Equation (3.3.5) empirically tractable in order to compute compensated demand elasticities. The key here is to divide Equation (3.3.5) which represents a marginal probability into its joint probability components. In the case of a binary discrete choice the marginal probability in Equation (3.3.5) comprises of two joint transitional probabilities:

$$P_D^h(j,p,u) = P_D^h(j,j,p,u) + P_D^h(k,j,p,u) \tag{3.4.1}$$

where $P_D^h(j,j,p,u)$ is the probability of choosing choice alternative j both ex-ante and ex-post price change and $P_D^h(k,j,p,u)$ denotes the transitional probability of choosing alternative k ex-ante while moving to alternative j ex-post policy intervention. To characterize each of the joint transitional choice probabilities described in Equation (3.4.1), a counterfactual two period setting is considered. In the first period (ex-ante) the price and wealth are (p^B, y^B). In the second (ex-post) period the price and wealth are given by (p^A, y^A). Given that the analysis focuses on providing a price subsidy this implies, $p^A < p^B$. As above, the present analysis assumes that the random terms $\varepsilon_i(d_i)$ remain unchanged in the ex-post and ex-ante period of intervention.[6]

The intuition behind the steps of derivation is akin to driving a probabilistic version of Shephard's Lemma.[7] Unlike standard consumer theory, both the indirect utility and expenditure functions are instead distributions that can be defined over the joint ex-ante and ex-post choice. To compute each joint transitional probability requires computing an integral over the expenditure distribution. The bounds of the integral are derived by defining the amount of monetary compensation required to maintain the same level of utility ex-ante and ex-post.

STEP 1: Characterize the Joint Distribution

If $P^h(k,j)$ denotes the joint compensated probability of choosing alternative k ex-ante and alternative j ex-post, under the condition that the respective utility levels of the chosen alternatives before and after the policy intervention are the same. The joint distribution is defined by:

$$P^h(k,j) = P\left(\underbrace{V_j^B \leq V_k^B}_{ex-ante\,(p^B,y^B)}, \underbrace{V_k^A(\Upsilon) \leq V_j^A(\Upsilon)}_{ex-post\,\left(p^A,y^A\right)}, max_r V_r^B = max_r V_r^A(\Upsilon) \right) \tag{3.4.2}$$

Since the expenditure is a distribution, the amount of income compensation Υ required to maintain the original utility level is stochastic and can take a set of values over a well defined upper and lower limit.

[6] One particular scenario not explicitly covered in this chapter is with regards to welfare evaluation in a 'dynamic' context i.e., when some time has elapsed from when the policy is introduced. In this case tastes may change from their initial values. Dagsvik (2002) considers discrete choice behaviour in this setting by formulating an explicit representation of the dependence between the error terms at two points of time from an inter temporal version of the IIA (Independence of Irrelevant Alternatives) assumption. This extended version of IIA accommodates serially dependent error terms due to what can be interpreted as taste persistence.

[7] Shephard's Lemma: $\frac{\partial e(p,u)}{\partial p_j} = h_j(p,u)$, the derivative of the expenditure function with respect to price of relevant good equals the compensated demand for that good

STEP 2: Bounds for Stochastic Compensation Υ

The bounds for the stochastic compensation Υ are derived by defining y_r as a deterministic amount of ex-post income that equates the ex-ante and ex-post utility for alternative $r = j$ or k.

$$v_j(p_j^B, y^B) = v_j(p_j^A, y_j) \tag{3.4.3}$$

In Equation (3.4.3) y_j is defined as the deterministic ex-post income that ensures that ex-ante and ex-post utility of alternative j are held equal. Using these deterministic income amounts its possible to derive upper and lower bounds for the stochastic compensation Υ. These bounds provide upper and lower limits of the integral over which the joint distribution defined in Equation (3.4.2) is integrated.

Bounds: Joint Probability $P^h(k, j)$

For k to be the most preferred alternative ex-ante and j to be the most preferred alternative ex-post:

$$v_k(p_k^B, y^B) + \varepsilon_k \geq v_j(p_j^B, y^B) + \varepsilon_j$$

$$v_j(p_j^A, \Upsilon) + \varepsilon_j = v_k(p_k^B, y^B) + \varepsilon_k \geq v_j(p_j^A, y_j) + \varepsilon_j$$

$$\Rightarrow \Upsilon \geq y_j$$

Similarly for j to be the most preferred alternative ex-post

$$v_j(p_j^A, \Upsilon) + \varepsilon_j \geq v_k(p_k^A, \Upsilon) + \varepsilon_k$$

$$v_k(p_k^B, y^B) + \varepsilon_k \geq v_k(p_k^A, \Upsilon) + \varepsilon_k$$

$$v_k(p_k^A, y_k) + \varepsilon_k \geq v_k(p_k^A, \Upsilon) + \varepsilon_k$$

$$\Rightarrow \Upsilon \leq y_k$$

Hence for transitions from k to j to take place holding constant indirect utility level it must be the case that:

$$y_j \leq \Upsilon \leq y_k \tag{3.4.4}$$

Conversely, if j and k are distinct choices and $y_k \leq y_j$

$$P^h(k,j) = 0$$

Using Equation (3.4.2) and (3.4.4) the transition probability of switching is denoted by:

$$P^h(k,j) = \int_{y_j}^{y_k} \frac{exp(v_k(p_k^B, y^B))exp(v_j(p_j^A, y))v_j^{'}(p_j^A, y)dy}{\{\sum_{r=1}^m exp\left(\max\left(v_r(p_r^B, y^B), v_r(p_r, y)\right)\right)\}^2} \tag{3.4.5}$$

$$P^h(k,j) = exp(v_k(p_k^B, y^B)) \int_{y_j}^{y_k} \frac{exp(v_j(p_j^A, y))v_j^{'}(p_j^A, y)dy}{\{\sum_{r=1}^m exp\left(\max\left(v_r(p_r^B, y^B), v_r(p_r, y)\right)\right)\}^2}$$

Bounds: Joint Probability $P^h(j,j)$

Similarly for $P^h(j,j)$, for j to be the most preferred alternative ex-ante and ex-post:

$$v_j(p_j^B, y^B) + \varepsilon_j = v_j(p_j^A, \Upsilon) + \varepsilon_j$$

$$v_j(p_j^A, y_j) + \varepsilon_j = v_j(p_j^A, \Upsilon) + \varepsilon_j$$

$$\Rightarrow \Upsilon = y_j$$

If the ex-post choice is equal to the ex-ante choice the stochastic compensation amount is instead defined as a deterministic amount.

$$\Upsilon = y_j \tag{3.4.6}$$

Using Equation (3.4.2) and (3.4.6) the joint probability of remaining in the same alternative is denoted by:

$$P^h(j,j) = \frac{exp(v_j(p_j^B, y^B))}{\sum_{r=1}^m exp\left(\max\left(v_r(p_r^B, y^B), v_r(p_r^A, y_j)\right)\right)} \tag{3.4.7}$$

$$P^h(j,j) = \frac{exp(v_j(p_j^B, y^B))}{v_j(p_j^B, y^B) + \sum_{r \neq j}^m exp\left(\max\left(v_r(p_r^B, y^B), v_r(p_r^A, y_j)\right)\right)}$$

The marginal compensated choice probability is given by the sum of the two joint probabilities defined above:

$$
\begin{aligned}
P_D^h(j,p,u) = \; & \frac{exp(v_j(p_j^B,y^B))}{v_j(p_j^B,y^B)+\sum_{r\neq j}^m exp\left(\max\left(v_r(p_r^B,y^B),v_r(p_r^A,y_j)\right)\right)} \\
& +exp(v_k(p_k^B,y^B)) \int_{y_j}^{y_k} \frac{exp(v_j(p_j^A,y))v_j'(p_j^A,y)dy}{\left\{\sum_{r=1}^m exp(\max(v_r(p_r^B,y^B),v_r(p_r,y)))\right\}^2}
\end{aligned}
\tag{3.4.8}
$$

It is possible to separate the substitution and income effects of a subsidy policy using the expressions for both compensated (Hicksian) and uncompensated (Marshallian) choice probability in Equation (3.4.8) and Equation (3.3.3) respectively.

3.4.2 Direct and Indirect Policy Effect

STEP 3: Incorporate Externality

In the last step, I modify the compensated choice probability derived in Equation (3.4.8) to take into account the additional impact of price propagated through the externality channel.[8] The 'direct effect' isolates the 'pure price' effect of a subsidy on a household's choice probability. While the indirect effect quantifies the secondary impact of price through the externality channel. If externalities do affect individual household decisions, the computation of the overall substitution effect for a price change must distinguish between the 'pure price' and consequent 'feedback price' effect. Without this distinction, the direct price effect of the policy would be overestimated which could result in an inefficient and expensive allocation of subsidies.

$$P^h(u,p,\overline{d}(p)) = P^m(Y(u,p,\overline{d}(p)),p,\overline{d}(p)) \tag{3.4.9}$$

[8] A similar modification also follows for the uncompensated (Marshallian) choice probability derived in Equation (3.3.3)

Unpacking the direct and indirect effect of a change in price on both sides of Equation (3.4.9) yields:

$$\frac{\partial P^m(p,\overline{d}(p),Y(u,p,\overline{d}(p)))}{\partial p_j} = \underbrace{\overbrace{\frac{\partial P^m}{\partial p_j}}^{A'} + \overbrace{\frac{\partial P^m}{\partial \overline{d}(p)} . \overbrace{\frac{\partial \overline{d}^m(p)}{\partial p_j}}^{D'}}^{B'}}_{C'} + \frac{\partial P^m}{\partial y}\left[\overbrace{\frac{\partial Y}{\partial p_j}}^{A''} + \overbrace{\frac{\partial Y}{\partial \overline{d}(p)} . \frac{\partial \overline{d}^m(p)}{\partial p_j}}^{B''}\right] \tag{3.4.10}$$

from the R.H.S, where $A^{'}$ denotes the direct impact of the price on household uncompensated choice, while $B^{'}$ denotes the indirect effect as a result of interactions within the group. The total uncompensated (Marshallian) price effect, denoted by $C^{'}$, would comprise of this additional feedback effect. Similarly, the L.H.S of Equation (3.4.9) yields:

$$\frac{\partial P^h(u,p,\overline{d}(p))}{\partial p_j} = \underbrace{\overbrace{\frac{\partial P^h}{\partial p_j}}^{A} + \overbrace{\frac{\partial P^h}{\partial \overline{d}(p)} . \overbrace{\frac{\partial \overline{d}^h(p)}{\partial p_j}}^{D}}^{B}}_{C} \tag{3.4.11}$$

where A, B and C constitute the direct, indirect and total compensated response to a price change, which are different from the uncompensated counterparts described in Equation (3.4.10) due to the additional non zero income effect. The computation of object A follows directly from the analytical expressions for the Hicksian choice probabilities derived in Equation (3.4.8). However, in order to compute the indirect effect B and $B^{'}$, the equilibrium condition for each village g has to be taken into account to quantify term D and $D^{'}$respectively. Using the village level equilibrium condition[9] it is possible to back out term D denoted by:

$$\frac{\partial \overline{d}(p)}{\partial p_g} = \frac{\left[\frac{1}{I_g}\sum_{i=1}^{I_g} \Lambda'(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta).\delta_p\right]}{\left[1 - \frac{1}{I_g}\sum_{i=1}^{I_g} \Lambda'(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta).\gamma\right]}$$

where δ_p and γ denote the parameter value for price and externality term respectively.[10] Each household i within a given village g, will face the same $\frac{\partial \overline{d}(p)}{\partial p_g}$ while this effect will differ across different villages g. Also $\frac{\partial \overline{d}(p)}{\partial p_g}$ will differ

[9]For derivation see Appendix (3.8.2)

[10]Note: $\Lambda^{'}(x) = \frac{\partial \Lambda(x)}{\partial x}$

depending on whether $\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)$ denotes a compensated or an uncompensated probability function.

3.5 Application: Sanitation Elasticity Estimates

I use the analytical results derived in Equation (3.4.8) and (3.4.11) to compute demand elasticity estimates for each household affected by the counterfactual policy intervention.

The elasticity estimates are provided in Table (3.1) for a representative household at different points of the wealth and price distribution. For a given price level, the uncompensated and compensated price elasticity are found to be negative with small positive income effects. The estimates reveal that the uncompensated elasticity is larger than the compensated elasticity for a change in price. This fits in with the predictions from a standard demand model where the Slutsky equation implies that substitution and income effects for a change in price of a 'normal' good move in the same direction. Both the uncompensated and compensated own price elasticities tend to increase with the price level and decrease with individual wealth level for a representative household.

The elasticity measures also indicate that both uncompensated and compensated price elasticity measures vary considerably between households. Thus, heterogeneity seems to be an important issue in determining household sanitation choice. Table (3.2) provides elasticity measures under different education levels. For a fixed level of wealth and price demand becomes inelastic for higher levels of education.

Table (3.3) provides the uncompensated and compensated elasticities for a representative household further separated into the direct and indirect effects from the price change respectively. There are two key takeaways from Table (3.3). First, the total compensated and uncompensated response as measured by demand elasticities are significantly larger once the indirect effect of the policy, driven by the underlying externalities, is taken into account. Second, total uncompensated effect is primarily determined by the indirect effect from the substitution channel. The proportion of the impact of subsidy attributable to the indirect effect varies over the distribution of price and wealth. However, at the lower limit it constitutes 41% of the total effect to an upper limit of 86%. This effect is better visualised in Figure (3.1) that plots the absolute value of the elasticities against the average prevalence of sanitation on the horizontal axis for both uncompensated (Marshallian) and compensated (Hicksian) elasticities. The curve for the uncompensated effect lies above the compensated, the positive distance between the two quantifying the remaining income effect of the policy intervention. The relative magnitude of the income effect is significantly smaller than the substitution effect.

The solid black line plots the total effect while the blue and the red plots depict the direct and indirect effect respectively. Though the inverted 'U' shape is driven by the functional form assumptions of the model, the large magnitude still points towards a non linear indirect effect of price change. In contrast, the direct effect is linear and decreases (marginally) as the average prevalence in the village increases. Both set of graphs also demonstrates how the social multiplier mechanism operates. At low levels of adoption prevalence in the village the direct effect is larger than the indirect effect. As each additional household adopts sanitation and the average prevalence of sanitation increases, the indirect effect overtakes the direct effect resulting in the total effect to be strongly driven by the indirect channel.

3.6 Welfare and Willingness-To-Pay

To quantify the welfare effects generated by the subsidy, I continue with the theoretical framework outlined in Section (3.3) to derive Compensating Variation (CV), Equivalent Variation (EV) as well as Net Gain ($NetG$) measures.

Similar to the expenditure function, the calculation of the CV is not straightforward in a random utility model in particular when utility is non linear in household income. A random utility structure implies that CV is also random with a distribution. In contrast to demand for a good consumed in continuous quantities, in a discrete choice setting the effect of a price change on individual utilities depends in a fundamental way on the discreteness of choice possibilities as well as on individual heterogeneity. The method of Dagsvik & Karlstrom (2005) provides tractable formulas for calculating mean and higher order moments of CV for observationally homogeneous populations. However in this application, without an observational homogeneous population, aggregation across different population sub-groups needs to be taken into account. The distributions of the welfare are expressed as simple closed-form transformations of the choice probabilities, enabling easy computation and inference.

The goal is to estimate exact, rather than approximate, impact on individual household welfare, measured in terms of income compensation, of a change in price brought about by subsidies and the associated net gain generated. Results from section (3.5) indicate a small yet positive income effect and by computing exact welfare measures I incorporate this additional effect into the welfare calculation. The model incorporates unobserved individual heterogeneity in utility functions and focuses on recovering the distribution and average values of the impact of price change on individual welfare arising from this heterogeneity distribution.

I briefly review the definition of both Compensating (CV) and Equivalent (EV) variation in the context of random utility models with particular focus on the challenge of calculating the distribution of CV and EV measures. The Compensating Variation is defined as the area under the compensated (Hicksian) demand curves holding the utility level fixed. In order to bring out the essentials of the approach, I shall first go through the argument in a simplified but general setting in the next sub-section.

3.6.1 Compensating and Equivalent Variation

Under a price subsidy, the Compensating Variation (CV) quantifies the maximum amount a household is willing-to-pay for the subsidy provided that utility is held constant at the pre-intervention level. Similarly, the Equivalent Variation (EV) quantifies the maximum amount a household is willing-to-pay for the subsidy with utility fixed at the post-intervention level.[11] The larger is this monetary compensation amount, the larger are the welfare gains realised by a household from the policy. The Compensating Variation (CV) is defined implicitly as the value that solves:

$$\max_{r\in D} V_r(p^B, y^B) = \max_{r\in D} V_r(p^A, y^A - CV)$$

$$\max_{r\in D} \left(v_r(p^B, y^B) + \varepsilon_r\right) = \max_{r\in D} \left(v_r(p_r^A, y^A - CV) + \varepsilon_r\right)$$

From this definition it follows that the CV measure becomes a random variable that may depend on all attributes, initial income and on all random terms $\{\varepsilon_r\}$. From an analytic point of view the difficulty of deriving a formula for the distribution of CV stems from the fact that when price p_r for choice alternative r changes, the alternative that yields maximum utility may be different from the one that maximized utility initially.[12] In other words, the individual household may switch from the alternative chosen initially to a new one, as a result of the change in price or policy introduction. In this analysis, it is assumed that the random terms ε_r which reflect taste shocks are

[11] Under a price drop, subsidy $EV \geq CV \geq 0$

[12] That is, the maximum of the left-hand side of above equation may not be attained at the same discrete alternative as the maximum of the right-hand side except in special cases. Thus the random terms on each side will not cancel

not affected by the policy interventions.[13] Using the distribution of the expenditure function in the previous section it is possible to derive the distribution of CV and its moments:

$$CV = y^A - Y_D(p, V(p^B, y^B)) \tag{3.6.1}$$

where the first moment; the mean compensating variation is given by

$$E[CV] = y^A - EY_D(p, V(p^B, y^B)) \tag{3.6.2}$$

where

$$E\left[P\left(Y_D(p^A, V_D(p^B, y^B)) > y\right)\right] = \sum_{j \in D} \mathbf{1}_j(p_j^B, y^B, p_j^A, y) \int_0^{y_j} \frac{exp(v_j(p_j^A, y_r))}{exp(v_j(p_j^B, y^B)) + exp(v_j(p_j^A, y_r))} \tag{3.6.3}$$

From (3.6.2), the estimated, point-wise average CV ,averaged over heterogeneity, is given by

$$\widehat{CV}(p^A, p^B; y) = \int_{p^A}^{p^B} P^h(p, y) dp \tag{3.6.4}$$

while the EV measure is given by:

$$\widehat{EV}(p^A, p^B; y) = \int_{p^A}^{p^B} P^h(p, y + p - p^A) dp \tag{3.6.5}$$

where price[14] $p^A = (1-\tau)p^B$ and τ denotes the subsidy amount, $P^h(p,u)$ is the compensated probability of households in the sample who would buy at price p and have utility level u.

[13] In general, the initial error terms ε_k^0 may differ from the error terms ε_k after policy. The dependence between ε_k^0 and ε_k will of course depend on the interpretation and modelling assumptions. This extension will be explored further in future work.

[14] Note that the notation p^A and p^B denotes the lower and upper limits of the integral.

3.6.2 Net Gain

The results from Chapter 2 indicate the presence of externality to have a positive effect on a household's sanitation adoption decision. This implies that if households do not internalize the spillover effects while making their decision the socially optimal level of adoption is higher than (to the right of) the privately chosen equilibrium adoption level. With this level of under adoption the provision of subsidy generates a Net Gain ($NetG$) instead of the standard Deadweight Loss (DWL). The Net Gain ($NetG$) can be interpreted as the additional welfare generated for all households within the village. By providing monetary incentives, a subsidy internalises some of the externality by inducing additional households to adopt sanitation. The corresponding average Net Gain from a subsidy amount τ is given by:

$$NetG(CV) = \tau p^B \times P^h(p^B(1-\tau), y) - \int_{p^A}^{p^B} P^h(p, y) dp \tag{3.6.6}$$

$$NetG(EV) = \tau p^B \times P^h(p^B(1-\tau), y) - \int_{p^A}^{p^B} P^h(p, y + p - p^A) dp \tag{3.6.7}$$

Both the Compensating Variation and Net Gain measures are distributions and as such it is possible to compute different moments of the distribution. I present results in this chapter for the first moment of the distribution, computing mean CV and average $NetG$. Table (3.5) provides estimates for both mean Compensating and Equivalent Variation and the corresponding Net Gain values generated once externality effects are taken into account and without. The welfare values are denoted in money metric INR (Indian *Rs*.).

Table (3.5) provides welfare measures for a 25% price subsidy at the village level. This calculation is performed by aggregating over each individual household's welfare measure in the village, thus incorporating the effect of unobserved heterogeneity. The village welfare calculations are in increasing order of sanitation prevalence. I find that CV increases with the average adoption levels in the village, this is driven by the positive externality parameter estimate. Instead the $NetG$ measure follows a non linear shape also seen with the indirect elasticity effect where the largest $NetG$ is associated at the point of inflection of the response curves usually found at mid levels of adoption. The $NetG$ increases with the demand elasticity, where households most responsive to the policy, i.e., at the margin of adopting sanitation, experience the largest gains in welfare. A comparison of Net Gain measures with and without externalities reveals that the effective Net Gain realised is larger once the full extent of the externality effect is taken into account.

The application results reveal that a 25% price subsidy produces an overall average CV of about $Rs.1,238.16$ and a net gain ($NetG$) of about $Rs.85.40$ per household. However, once the feedback effects are taken in account the $NetG$ generated is in fact larger $Rs.138.48$ while the average CV per household increases to $Rs.1,919.15$. With positive externalities a household's willingness-to-pay for a policy increase by 55%, and the effective gain realised by the household is 62% higher. This welfare gain is comparable to 32% of the average monthly income[15] received by the households within the region denoting a non-trivial gain in welfare. I find that in the presence of externalities, the welfare impact of subsides are less 'distortionary' as they generate a larger net gain for each individual household. The use of subsidy intervention can be justified to move the society as a whole to a level of sanitation adoption closer to the socially optimum.

3.7 Conclusion

The consumption of healthcare goods, education and environmental amenities are all examples of goods that comprise non-marketable aspects not reflected in the market price, thus generating externalities and spillover effects. A policy issue of some importance is measuring surplus/welfare realised from such non-marketed aspects, e.g., health and ascertaining whether these goods are consumed optimally, i.e., at or close to a socially optimal level. The contribution of this chapter is to compute the true/effective net gain and welfare from a subsidy policy once the underlying externalities are taken into account.

For the application considered, the adoption of the sanitation is found to generate large positive externalities because households do not necessarily internalise the impact of their own choice on the entire society. In such a scenario, the privately chosen adoption level does not coincide with the socially optimal adoption level which is higher if the adoption of the good generates positive externalities. This is true for most examples for provision of preventive healthcare products e.g., water purification, malaria prevention bed-nets, de-worming pills. If the healthcare policy intervention in question, specifically targets a good with significant externality effects, there is a strong incentive to subsidise adoption of such goods.

The counterfactual analysis above with respect to the compensated demand and its elasticities allows me to quantify the "substitution effect" or pure price effect generated by a subsidy policy. Which maybe of interest to a policy maker who is

[15]equivalent to 2.67% of annual household income

deciding on the appropriate choice of policy to implement for e.g., price or income subsidies. The calculations of the compensated price elasticities for different demographic characteristics reveals a large heterogeneity in response. It maybe possible/desirable to use this heterogeneity to target certain groups of community members. For example, if the objective of the policy is to maximize coverage across the entire village, it maybe worth targeting households who are at the margin of adopting i.e., large price elasticity estimates with small/marginal subsidy amounts. If instead the objective of the policy is to maximize welfare, a larger subsidy amount maybe better targeted to a low income household in the village. I explore and expand the welfare analysis along such different welfare criterion and objectives in Chapter 4

These findings provide important implications towards understanding the impact of policy interventions when externalities are relevant. By including the externality component, I find that a significant amount of the change in response to a subsidy policy is mediated through the indirect feedback effect, between $41\% - 86\%$. If the impact of the externality is not taken into account the overall direct effect of the policy would be overestimated. Failure to take such indirect impacts into account, can cause targeted polices to be unnecessarily expensive or to be incorrectly targeted. This can also lead to inaccurate concerns on the relative impact of health improving policy measures.

The usefulness of this analysis also depends on the policy objectives. In order to address this I would need to ask questions such as: What constitutes a socially optimal adoption level? and, What is the optimal subsidy amount to achieve the social optimal? The answer to these questions entails computing points on the Pareto frontier which would require assumptions on the Pareto weights assigned to each household demographic. Though the policy analysis tools in this chapter allow for computation of welfare at any point on the Pareto frontier the choice of weights is less clear. Therefore, in this chapter I abstract away from making assumptions on the optimal choice of Pareto weights. Having said so, if a policy maker has a target population in mind,[16] the tools developed in this chapter can be used to compute any point on the Pareto frontier given weights and thus the optimal subsidy amount to achieve that socially optimal level.

[16] For example, households at the bottom tail on the wealth distribution that are liquidity constrained and most likely would not able to adopt independently of the presence of externality.

3.8 Appendix for Chapter 3

3.8.1 Expenditure Function

Dagsvik (1995) defines the expenditure function as follows

$$Y_D(p,u) = \{z : V_D(p,z) = u\} \tag{3.8.1}$$

and defines $Y_k(p_k, u-\varepsilon_k)$ as:

$$v_k(p_k, Y_k(p_k, u-\varepsilon_k)) + \varepsilon_k = u \tag{3.8.2}$$

$$v_k(p_k, Y_k(p_k, u-\varepsilon_k)) = u - \varepsilon_k \tag{3.8.3}$$

using $v_k(p_k, y)$ strictly increasing in y, implies that $Y_k(p_k, u-\varepsilon_k)$ is uniquely determined. $Y_k(p_k, u-\varepsilon_k)$ denotes the expenditure required to achieve utility level u, given choice k with price p_k

Expenditure function

$$Y_D(p,u) = \min_{k \in D} Y_k(p_k, u-\varepsilon_k) \tag{3.8.4}$$

With $v_k(p_k, y)$ strictly increasing in y, using (3.8.2) if

$$\begin{aligned} v_k(p_k,y) + \varepsilon_k &< u \\ v_k(p_k,y) + \varepsilon_k &< v_k(p_k, Y_k(p_k, u-\varepsilon_k)) + \varepsilon_k \end{aligned}$$

cancel ε_k on both sides and take inverse holding p_k fixed

$$\begin{aligned} v_k^{-1}(v_k(p_k,y)) &< v_k^{-1}(v_k(p_k, Y_k(p_k, u-\varepsilon_k))) \\ y &< Y_k(p_k, u-\varepsilon_k) \end{aligned}$$

which implies that

$$\{Y_k(p_k, u - \varepsilon_k) > y\} \Leftrightarrow \{v_k(p_k, y) + \varepsilon_k < u\}$$

$$\{Y_k(p_k, u - \varepsilon_k) > y\} \Leftrightarrow \{v_k(p_k, y) + \varepsilon_k < u\}$$

so if $y_k > 0$ $k \in D$

$$P\left(\bigcap_{k\in D} \left(Y_D\left(p_k, u - \varepsilon_k\right) > y_k\right)\right) = P\left(\bigcap_{k\in D} \left(v_k(p_k, y_k) + \varepsilon_k < u\right)\right)$$

$$= F^D\left(u - v_1(p_1, y_1), u - v_2(p_2, y_2), ..., u - v_m(p_m, y_m)\right)$$

if $y_k = y$ for $k \in D$, the distribution of the expenditure function

$$P\left(Y_D(p, u) \le y\right) = 1 - F^D\left(u - v_1(p_1, y_1), u - v_2(p_2, y_2), ..., u - v_r(p_r, y_r)\right)$$

The event $\{Y_k(p_k, u - \varepsilon_k) \le y\}$ implies that the amount y is higher than or equal to the expenditure required to achieve utility u. This is equivalent to the event $\{v_k(p_k, y) + \varepsilon_k \ge u\}$ where the utility implied by income y is higher than or equal to u, using (3.8.4)

$$P(Y_D(p, u) \le y) = P(V_D(p, y) \ge u) \tag{3.8.5}$$

Expenditure Function

$P(Y_D(p, u) \le y) = 1 - F^D\left(u - v_1(p_1, y), u - v_2(p_2, y), ..., u - v_m(p_m, y)\right) = H(x)$

Using integration by parts

$$\int_0^{\overline{y}} 1\left[1 - H(x)\right] dx = x\left(1 - H(x)\right)|_0^{\overline{y}} - \int_0^{\overline{y}} x(-)h(x)\, dx$$

$$\int_0^{\overline{y}} 1\left[1 - H(x)\right] dx = \overline{y}\left[1 - H(\overline{y})\right] - 0\left[1 - H(0)\right] + \int_0^{\overline{y}} x h(x)\, dx$$

$$\int_0^{\overline{y}} 1\,[1-H(x)]\,dx = \overline{y}.0 - 0 + \int_0^{\overline{y}} x\,h(x)\,dx$$

$$\int_0^{\overline{y}} 1\,[1-H(x)]\,dx = \int_0^{\overline{y}} x\,h(x)\,dx$$

$$EY_D(p,u) = \int_0^{\infty} F^D\left(u - v_1(p_1,y), u - v_2(p_2,y), ..., u - v_m(p_m,y)\right) dy \qquad (3.8.6)$$

Using result (3.8.6)

$$\frac{\partial EY_D(p,u)}{\partial p_{1j}} = \frac{\partial\left[\int_0^{\infty} F^D\left(u - v_1(p_1,y), u - v_2(p_2,y), ..., u - v_m(p_m,y)\right) dy\right]}{\partial p_{1j}}$$

$$= \int_0^{\infty} F_j^D\left(u - v_1(p_1,y), u - v_2(p_2,y), ..., u - v_m(p_m,y)\right) - \frac{\partial v_j(p_j,y)}{\partial p_{1j}}\,dy$$

$$= \int_0^{\infty} F_j^D\left(u - v_1(p_1,y), u - v_2(p_2,y), ..., u - v_m(p_m,y)\right) \frac{\partial v_j(p_j,y)}{\partial y}\,dy$$

$$= \int_0^{\infty} F_j^D\left(u - v_1(p_1,y), u - v_2(p_2,y), ..., u - v_m(p_m,y)\right) dv_j(p_j,y) \qquad (3.8.7)$$

$$= P_D^h(j,p,u)$$

3.8.2 Incorporating Externality

The equilibrium condition for a village described in Section (2.3) was given by:

$$\frac{1}{I_g}\sum_{i=1}^{I_g} p_i(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta) = \frac{1}{I_g}\sum_{i=1}^{I_g} \Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta) \qquad (3.8.8)$$

denote $F_i = p_i(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)$ and $\overline{F} = \frac{1}{I_g}\sum_{i=1}^{I_g} F_i$

$$\frac{1}{I_g}\sum_{i=1}^{I_g} F_i = \frac{1}{I_g}\sum_{i=1}^{I_g} \Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)$$

differentiating with respect to price, p_g

$$\frac{\partial \overline{F}}{\partial p_g} = \frac{1}{I_g}\sum_{i=1}^{I_g}\left\{\frac{\partial}{\partial p_g}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right) + \frac{\partial}{\partial \overline{d}(p)}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right).\frac{\partial \overline{d}(p)}{\partial p_g}\right\} \tag{3.8.9}$$

in equilibrium $\overline{F} = \overline{d}(p)$, replacing $\frac{\partial \overline{F}}{\partial p_g}$ with $\frac{\partial \overline{d}(p)}{\partial p_g}$

$$\frac{\partial \overline{d}(p)}{\partial p_g} = \frac{1}{I_g}\sum_{i=1}^{I_g}\frac{\partial}{\partial p_g}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right) + \frac{1}{I_g}\sum_{i=1}^{I_g}\frac{\partial}{\partial \overline{d}(p)}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right).\frac{\partial \overline{d}(p)}{\partial p_g}$$

rearranging

$$\frac{\partial \overline{d}(p)}{\partial p_g}\left[1 - \frac{1}{I_g}\sum_{i=1}^{I_g}\frac{\partial}{\partial \overline{d}(p)}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right)\right] = \frac{1}{I_g}\sum_{i=1}^{I_g}\frac{\partial}{\partial p_g}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right)$$

$$\frac{\partial \overline{d}(p)}{\partial p_g} = \frac{\left[\frac{1}{I_g}\sum_{i=1}^{I_g}\frac{\partial}{\partial p_g}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right)\right]}{\left[1 - \frac{1}{I_g}\sum_{i=1}^{I_g}\frac{\partial}{\partial \overline{d}(p)}\left(\Lambda(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta)\right)\right]}$$

also written as where δ_p and γ denote the parameter for price and externality term

$$\frac{\partial \overline{d}(p)}{\partial p_g} = \frac{\left[\frac{1}{I_g}\sum_{i=1}^{I_g}\Lambda'(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta).\delta_p\right]}{\left[1 - \frac{1}{I_g}\sum_{i=1}^{I_g}\Lambda'(d_i = 1|x_{ig}, z_g, p_g, \overline{d}(p_g), \theta).\gamma\right]} \tag{3.8.10}$$

3.8.3 Tables and Figures

Table 3.1: Elasticities: Representative Household

Wealth	**Elasticity**	**Price: 10th %ile**	**Price:50th %ile**	**Price: 90th %ile**
	M	-0.222	-0.355	-0.452
10th %ile	H	-0.194	-0.308	-0.390
	I	0.006	0.007	0.007
	M	-0.215	-0.345	-0.440
50th %ile	H	-0.188	-0.299	-0.380
	I	0.029	0.032	0.035
	M	-0.199	-0.321	-0.411
75th %ile	H	-0.174	-0.277	-0.354
	I	0.079	0.090	0.097
	M	-0.160	-0.268	-0.341
90th %ile	H	-0.139	-0.226	-0.291
	I	0.171	0.196	0.214

Notes: **M:** Uncompensated Price Elasticity (Marshallian), **H:** Compensated Price Elasticity (Hicksian), **I:** Income Elasticity. Elasticity estimates computed under a 5% change in prices

Table 3.2: Elasticities by Education level: Representative Household

Wealth	El.	Price:10th %ile			Price:50th %ile			Price: 90th %ile		
		NoEduc	Primary	High Sch	NoEduc	Primary	High Sch	NoEduc	Primary	High Sch
10th %ile	**M**	-0.282	-0.216	-0.134	-0.440	-0.347	-0.223	-0.551	-0.442	-0.291
	H	-0.248	-0.189	-0.116	-0.385	-0.300	-0.191	-0.481	-0.382	-0.247
	I	0.008	0.006	0.004	0.008	0.006	0.004	0.009	0.007	0.005
50th %ile	**M**	-0.276	-0.210	-0.129	-0.431	-0.337	-0.214	-0.541	-0.431	-0.281
	H	-0.242	-0.183	-0.112	-0.376	-0.292	-0.183	-0.471	-0.371	-0.238
	I	0.037	0.028	0.017	0.040	0.031	0.020	0.042	0.034	0.022
75th %ile	**M**	-0.259	-0.194	-0.116	-0.407	-0.313	-0.195	-0.513	-0.402	-0.256
	H	-0.227	-0.169	-0.101	-0.355	-0.270	-0.166	-0.446	-0.345	-0.217
	I	0.103	0.077	0.047	0.114	0.088	0.055	0.121	0.095	0.060
90th %ile	**M**	-0.217	-0.155	-0.089	-0.347	-0.256	-0.151	-0.443	-0.332	-0.200
	H	-0.189	-0.135	-0.077	-0.301	-0.219	-0.128	-0.382	-0.283	-0.168
	I	0.231	0.165	0.095	0.259	0.191	0.113	0.278	0.208	0.126

Notes: **M:** Uncompensated Price Elasticity (Marshallian), **H:** Compensated Price Elasticity (Hicksian), **I:** Income Elasticity Elasticity. Elasticity estimates computed under a 5% change in prices

Table 3.3: Direct & Indirect Effect Price Change: Representative Household

Wealth	**El.**	**Price:10th %ile**			**Price:50th %ile**			**Price: 90th %ile**		
		Direct	**Indirect**	**Total**	**Direct**	**Indirect**	**Total**	**Direct**	**Indirect**	**Total**
10th %ile	**M**	-0.222	-0.673	-0.895	-0.355	-2.009	-2.364	-0.452	-4.863	-5.315
	H	-0.194	-0.522	-0.716	-0.308	-1.373	-1.680	-0.390	-2.731	-3.121
50th %ile	**M**	-0.215	-0.573	-0.788	-0.345	-1.626	-1.971	-0.440	-3.562	-4.002
	H	-0.188	-0.449	-0.637	-0.299	-1.138	-1.437	-0.380	-2.135	-2.515
75th %ile	**M**	-0.199	-0.400	-0.599	-0.321	-1.039	-1.360	-0.411	-1.967	-2.379
	H	-0.174	-0.318	-0.492	-0.277	-0.754	-1.031	-0.354	-1.308	-1.661
90th %ile	**M**	-0.160	-0.182	-0.342	-0.268	-0.432	-0.701	-0.341	-0.708	-1.048
	H	-0.139	-0.146	-0.285	-0.226	-0.322	-0.548	-0.291	-0.515	-0.806

Notes: **M:** Uncompensated Price Elasticity (Marshallian), **H:** Compensated Price Elasticity (Hicksian).
Elasticity estimates computed under a 5% change in prices

Table 3.4: Direct & Indirect Effect Price Change: Village

					Marshallian			**Hicksian**	
Village	**Avg level**	**Price (Rs.)**	**Wealth(Rs.)**	**Direct**	**Indirect**	**Total**	**Direct**	**Indirect**	**Total**
vill ID 12	0.000	9,823	34,333	-0.723	-0.394	-1.117	-0.647	-0.388	-1.035
vill ID 15	0.000	10,243	50,625	-0.768	-0.283	-1.051	-0.688	-0.278	-0.966
vill ID 9	0.000	9,725	43,252	-0.720	-0.344	-1.065	-0.645	-0.338	-0.983
vill ID 2	0.000	11,337	48,996	-0.855	-0.284	-1.139	-0.766	-0.282	-1.048
vill ID 18	0.000	10,975	41,442	-0.807	-0.445	-1.252	-0.722	-0.442	-1.163
vill ID 22	0.000	10,016	43,375	-0.749	-0.287	-1.037	-0.671	-0.282	-0.953
vill ID 19	0.048	10,427	28,549	-0.777	-0.255	-1.033	-0.696	-0.252	-0.948
vill ID 35	0.053	10,280	41,130	-0.737	-0.443	-1.180	-0.658	-0.436	-1.094
vill ID 40	0.083	10,273	62,121	-0.738	-0.440	-1.177	-0.659	-0.433	-1.092
vill ID 24	0.115	9,510	49,131	-0.673	-0.466	-1.139	-0.601	-0.456	-1.057
vill ID 3	0.136	7,800	35,849	-0.518	-0.778	-1.297	-0.462	-0.764	-1.226
vill ID 37	0.152	9,788	33,739	-0.699	-0.483	-1.182	-0.624	-0.478	-1.102
vill ID 7	0.182	7,738	43,163	-0.522	-0.722	-1.244	-0.465	-0.710	-1.175
vill ID 5	0.185	9,801	51,061	-0.630	-1.444	-2.074	-0.559	-1.486	-2.045
vill ID 21	0.190	10,475	66,079	-0.673	-1.424	-2.097	-0.596	-1.455	-2.052
vill ID 8	0.200	7,938	64,404	-0.495	-1.496	-1.991	-0.440	-1.517	-1.957
vill ID 25	0.200	11,175	43,640	-0.735	-1.367	-2.102	-0.651	-1.423	-2.075
vill ID 11	0.200	10,055	45,720	-0.716	-0.534	-1.249	-0.639	-0.532	-1.170
vill ID 4	0.263	8,795	89,533	-0.526	-1.979	-2.505	-0.466	-2.045	-2.512
vill ID 44	0.298	9,913	88,217	-0.541	-3.306	-3.847	-0.477	-3.472	-3.949
vill ID 6	0.350	8,313	60,091	-0.482	-3.513	-3.996	-0.427	-3.921	-4.348
vill ID 28	0.375	8,131	121,363	-0.413	-7.175	-7.588	-0.363	-5.151	-5.514
vill ID 14	0.455	8,882	49,082	-0.382	-7.132	-7.514	-0.334	-5.951	-6.285
vill ID 32	0.464	7,155	67,463	-0.305	-5.682	-5.987	-0.268	-4.788	-5.056
vill ID 38	0.469	6,775	86,293	-0.264	-5.682	-5.946	-0.232	-4.788	-5.019
vill ID 27	0.500	8,181	35,873	-0.307	-6.791	-7.098	-0.269	-5.406	-5.675
vill ID 31	0.500	6,900	70,320	-0.276	-2.612	-2.888	-0.242	-2.345	-2.587
vill ID 17	0.500	7,915	49,077	-0.321	-3.875	-4.197	-0.281	-3.418	-3.700
vill ID 20	0.625	8,113	61,725	-0.247	-3.875	-4.122	-0.214	-3.418	-3.632
vill ID 26	0.629	6,030	55,376	-0.160	-1.987	-2.147	-0.140	-1.424	-1.564
vill ID 30	0.655	6,662	63,721	-0.165	-1.651	-1.816	-0.143	-1.166	-1.309
vill ID 29	0.662	7,844	61,159	-0.239	-1.651	-1.890	-0.208	-9.596	-9.803
vill ID 23	0.688	8,875	70,181	-0.234	-2.709	-2.943	-0.202	-1.792	-1.993
vill ID 13	0.706	9,924	48,968	-0.280	-5.831	-6.111	-0.240	-3.283	-3.523
vill ID 26	0.714	6,113	79,764	-0.134	-0.642	-0.776	-0.117	-0.497	-0.614
vill ID 43	0.714	6,113	93,143	-0.113	-0.330	-0.443	-0.098	-0.259	-0.357
vill ID 42	0.805	9,012	78,966	-0.147	-0.279	-0.426	-0.125	-0.212	-0.338
vill ID 1	0.808	5,713	65,686	-0.067	-0.068	-0.135	-0.058	-0.055	-0.113
vill ID 39	0.875	6,350	53,752	-0.085	-0.108	-0.193	-0.074	-0.087	-0.160
vill ID 34	0.875	7,963	52,465	-0.109	-0.143	-0.253	-0.094	-0.112	-0.206
vill ID 33	0.894	7,168	64,653	-0.067	-0.047	-0.114	-0.058	-0.037	-0.095
vill ID 16	0.938	11,425	73,850	-0.125	-0.104	-0.229	-0.104	-0.078	-0.182
vill ID 41	1.000	7,050	58,080	-0.053	-0.026	-0.079	-0.045	-0.021	-0.066
vill ID 10	1.000	6,170	63,711	-0.027	-0.007	-0.034	-0.024	-0.006	-0.029
Total	0.378	8,610	59,208	-0.423	-1.798	-2.220	-0.375	-1.711	-2.086

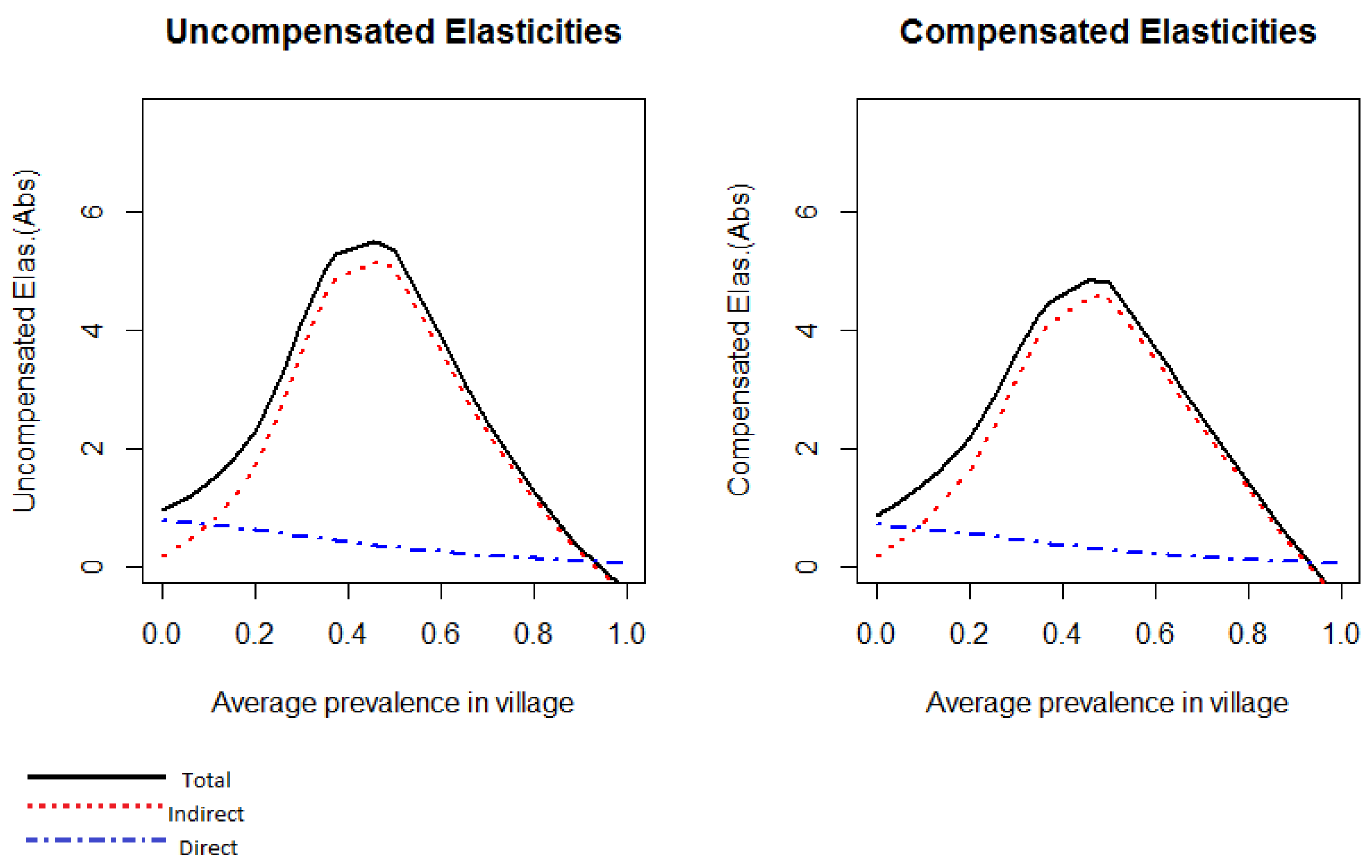
Notes: **M:** Uncompensated Price Elasticity (Marshallian), **H:** Compensated Price Elasticity (Hicksian). Elasticity estimates computed under a 5% change in prices. Rs.1000≈GBP 10

Table 3.5: Welfare Measure: Village

				Equivalent Variation				Compensating Variation			
Village	**Avg Adopt**	**Price (Rs.)**	**Wealth (Rs.)**	**Avg**	**NetG**	**Avg (Ext)**	**NetG (Ext)**	**Avg**	**NetG**	**Avg (Ext)**	**NetG (Ext)**
vill ID 12	0.000	9,823	34,333	197.69	31.60	243.16	39.60	192.31	36.99	236.54	46.34
vill ID 15	0.000	10,243	50,625	151.19	25.35	187.48	32.04	146.90	29.64	182.16	37.47
vill ID 9	0.000	9,725	43,252	185.84	29.24	230.81	37.02	180.85	34.23	224.61	43.34
vill ID 2	0.000	11,337	48,996	141.11	26.78	175.40	33.94	136.66	31.23	169.87	39.58
vill ID 18	0.000	10,975	41,442	215.53	38.35	268.98	48.82	209.08	44.81	260.93	57.03
vill ID 22	0.000	10,016	43,375	155.71	25.36	194.64	32.33	151.40	29.67	189.25	37.83
vill ID 19	0.048	10,427	28,549	167.96	29.04	210.28	37.09	163.07	33.93	204.16	43.34
vill ID 35	0.053	10,280	41,130	286.84	46.07	359.98	58.99	278.96	53.95	350.09	69.08
vill ID 40	0.083	10,273	62,121	330.42	50.25	415.33	64.45	321.74	58.92	404.43	75.58
vill ID 24	0.115	9,510	49,131	324.56	43.40	408.62	55.76	317.05	50.91	399.17	65.41
vill ID 35	0.136	7,800	35,849	430.27	48.35	542.14	62.18	421.65	56.97	531.28	73.27
vill ID 37	0.152	9,788	33,739	285.14	44.43	384.94	61.54	277.53	52.04	374.66	72.08
vill ID 7	0.182	7,738	43,163	453.69	50.42	635.17	72.61	444.69	59.42	622.56	85.57
vill ID 5	0.185	9,801	51,061	580.38	82.91	841.55	123.95	565.90	97.39	820.56	145.59
vill ID 21	0.190	10,475	66,079	737.42	97.91	1143.01	161.46	720.20	115.13	1116.32	189.85
vill ID 8	0.200	7,938	64,404	684.92	69.67	1066.42	115.46	672.30	82.29	1046.77	136.38
vill ID 25	0.200	11,175	43,640	556.93	93.99	868.25	155.99	540.88	110.05	843.22	182.64
vill ID 11	0.200	10,055	45,720	315.20	50.50	492.03	83.94	306.58	59.13	478.57	98.27
vill ID 40	0.263	8,795	89,533	841.31	84.66	1312.44	140.60	826.04	99.93	1288.62	165.97
vill ID 44	0.298	9,913	88,217	1233.46	116.62	1973.53	199.19	1212.32	137.76	1939.71	235.29
vill ID 6	0.350	8,313	60,091	870.43	85.97	1410.10	148.86	854.80	101.60	1384.78	175.93
vill ID 28	0.375	8,131	121,363	1077.87	104.70	1832.38	191.18	1058.67	123.90	1799.74	226.24
vill ID 14	0.455	8,882	49,082	1582.72	133.17	2706.45	244.74	1557.84	158.05	2663.90	290.47
vill ID 32	0.464	7,155	67,463	1401.22	87.02	2424.11	161.99	1384.79	103.45	2395.69	192.56
vill ID 38	0.469	6,775	86,293	1537.40	72.90	2644.33	134.84	1523.52	86.78	2620.45	160.51
vill ID 27	0.500	8,181	35,873	1765.89	111.64	2949.04	199.90	1744.55	132.98	2913.40	238.11
vill ID 31	0.500	6,900	70,320	1424.65	90.67	2307.93	157.00	1407.42	107.89	2280.02	186.83
vill ID 17	0.500	7,915	49,077	1608.55	115.86	2493.26	191.05	1586.66	137.76	2459.32	227.16
vill ID 20	0.625	8,113	61,725	2063.96	127.08	3157.85	206.55	2039.42	151.61	3120.32	246.43
vill ID 26	0.629	6,030	55,376	1782.81	66.19	2638.55	101.77	1769.86	79.14	2619.39	121.69
vill ID 30	0.655	6,662	63,721	2022.09	81.63	2891.59	120.94	2006.07	97.65	2868.68	144.67
vill ID 29	0.662	7,844	61,159	2025.15	115.09	2875.71	169.23	2002.94	137.31	2844.17	201.90
vill ID 23	0.688	8,875	70,181	2551.84	140.85	3496.02	199.21	2524.37	168.31	3458.39	238.06
vill ID 13	0.706	9,924	48,968	2609.79	182.20	3523.21	253.63	2574.59	217.40	3475.70	302.62
vill ID 26	0.714	6,113	79,764	1994.05	64.38	2632.15	87.45	1981.35	77.08	2615.38	104.70
vill ID 43	0.714	6,113	93,143	2159.31	60.65	2763.92	79.67	2147.21	72.75	2748.42	95.57
vill ID 42	0.805	9,012	78,966	3285.22	117.23	4132.80	151.10	3261.53	140.91	4103.01	181.63
vill ID 1	0.808	5,713	65,686	2314.30	36.37	2897.50	46.63	2306.94	43.73	2888.28	56.07
vill ID 39	0.875	6,350	53,752	2466.86	53.34	3083.58	68.28	2456.11	64.09	3070.14	82.04
vill ID 34	0.875	7,963	52,465	3026.76	87.47	3777.40	111.77	3008.98	105.26	3755.21	134.49
vill ID 33	0.894	7,168	64,653	2995.39	54.00	3723.27	68.70	2984.31	65.09	3709.49	82.80
vill ID 16	0.938	11,425	73,850	4506.54	156.63	5588.11	198.73	4473.97	189.20	5547.72	240.06
vill ID 41	1.000	7,050	58,080	3052.10	45.36	3723.56	56.54	3042.71	54.75	3712.10	68.24
vill ID 10	1.000	6,170	63,711	2847.27	20.87	3388.25	25.31	2842.95	25.19	3383.12	30.55
Total	0.378	8,610	129,752	1251.61	71.95	1939.99	116.67	1238.16	85.40	1919.15	138.48

Notes: Compensating Variation (CV) and Equivalent Variation (EV) computed under a 25% (cost of sanitation) price subsidy.
Rs.1000≈GBP 10

Figure 3.1: Direct & Indirect Price Elasticity



Notes: The graphs above plot the absolute value of the average uncompensated (Marshallian) and compensated (Hicksian) elasticity estimates as a function of the average prevalence of sanitation across groups/villages. Elasticity estimates are computed for each individual household within a village which is averaged over each village. Black solid line denotes total price elasticity (absolute value). The blue dashed line denotes the direct effect of the price change while the red dotted line plots the indirect effect generated by the underlying externality.

Chapter 4

Household Demand for Sanitation: Importance of Externalities and Borrowing Constraints

4.1 Introduction

Close to 2.5 billion people on the planet ($\approx 35\%$ of the population) do not have access to basic sanitation (WHO-UNICEF 2014).[1] Lack of sanitation has detrimental effects on individual health, economic and social well-being (Mara, Lane, Scott & Trouba 2010).[2] In recent years there has been significant policy interest in the developing world to increase sanitation adoption amongst households.[3] Finding strategies to tackle under-adoption has become a policy imperative. Governments have used many different policies to incentivize adoption. However, there is substantial disagreement as to which policies are effective at increasing coverage and at an affordable cost to developing countries. The disagreement arises from contrasting views on which market failure generates under-adoption of sanitation.

Two often cited market failures are borrowing constraints and externalities. First, in developing countries with limited or non-existent credit markets, a liquidity constrained household may find it difficult or impossible to purchase sanitation.

[1] Basic sanitation refers access to a toilet/bathroom facility

[2] The World Bank estimates that increase in sanitation coverage can reduce diarrhoeal disease related child mortality by more than a third.

[3] Over the period 2008 – 2015 the Gates Foundation has allocated approx USD. $650 million under the water, sanitation and hygiene (WaSH) program towards increasing sanitation coverage Bill & Melinda Gates Foundation (2011).

Second, sanitation generates positive externalities (Duflo, Greenstone, Guiteras & Clasen 2015). For example, in a similar manner to vaccination; an increase of sanitation adoption amongst neighbouring households reduces the risk of infection and likelihood of being sick for an individual (Geruso & Spears 2015, Hammer & Spears 2013, Spears 2012, 2013, Augsburg & Rodriguez 2015). Such spillover effects are not necessarily internalised by an individual resulting in inefficient adoption.

This paper analyses if there is under-adoption and its extent using data from India, where only 37% of the population has access to sanitation. Having established under-adoption, I ask: If the objective of the policy makers is to maximize coverage subject to government budget constraints, are sanitation loans or price subsidies more effective at increasing adoption? Moreover, I also study the welfare implications of the two policies.

To answer these questions, I develop and estimate a dynamic equilibrium model of sanitation choices of households. The model has two key features. First, it allows for households to be borrowing constrained in their consumption and sanitation adoption choice. Second, to capture the externalities households make interdependent adoption choices. I model the household decisions to adopt sanitation and to save over the life-cycle within a strategic setting of an incomplete information game. Interdependent adoption captures numerous possible channels that can generate externalities.[4]

The challenge of solving and estimating the model arises because agents do not optimise in isolation but instead solve an inter-related system of dynamic programming problems. As is typical of many strategic decision models, multiple equilibria may arise depending on the strength of the externality. This poses challenges in estimation and counterfactual analysis, especially when one is unwilling to specify an arbitrary selection rule.

First, this paper builds on the estimation methodology in Hotz, Miller, Sanders & Smith (1994) and Bajari, Benkard & Levin (2007) by combining a two stage and full solution estimation methods. I demonstrate that by making a small trade-off under the two-step approach; by numerically solving a part of the model, it is possible to have an estimation method that can accommodate a richer set of strategic demand models for a small increase in computational cost.

[4]The framework allows for externalities to arise from different channels. For example, there is a biological contagion channel which affects health but there may also be information spillovers with neighbours sharing knowledge on the benefits from adoption. In addition social norms and peer pressure may also play a role.

Second, this paper proposes a method to conduct counterfactual policy analysis that circumvents the typical issues of multiplicity. Specifically, the burden of solving for all possible equilibria and an additional challenge with simulation ensues when the existing selection rule is not policy invariant and is no longer valid under different counterfactuals. This affects the validity of the policy implications derived which are a function of the underlying selection mechanism. A common approach, undertaken in applied work, is to specify an equilibrium selection rule under estimation with the implicit assumption that the selection rule does not change in counterfactual scenarios.[5] This paper takes an alternative route where I instead bound the set of counterfactual policy outcomes that could be sustained under different selection rules. I do this by first characterizing the conditions under which the model implies strategic complementarity in the sanitation adoption decisions. This allows me to establish an order over the set of equilibria thereby bounding the set using the highest and lowest equilibrium. The equilibrium bounds are used to characterize the region of the policy impact that could be sustained in equilibrium under counterfactual scenarios.

There is an active literature on estimating both static and dynamic games that explicitly address the issue of multiple equilibria. This paper is related to the extensive literature on the estimation of dynamic models using the Hotz & Miller (1993) two-step Conditional Choice Probability (CCP) approach. The literature comprises of numerous extensions to the original two-step estimator for both single-agent and strategic interaction models. More recently, Aguirregabiria & Mira (2007), Bajari, Benkard & Levin (2007), Pakes, Ostrovsky & Berry (2007) and Pesendorfer & Schmidt-Dengler (2008) have developed estimation procedures that allow one to recover the primitives that underlie dynamic choice games. In contrast to the estimation literature, there is a smaller body of work on the problem of conducting counterfactual policy simulations under multiple equilibria. The idea to bound the set of equilibria has previously been exploited in the literature both for the purpose of estimation and simulation. This includes work by Jia (2008), De Paula (2009), Lee & Pakes (2009), Bj*ö*rkegren (2014) and Reguant (2015).[6] In terms of counterfactual simulation, Bj*ö*rkegren's (2014) study of mobile handset adoption in Rwanda is closest to this paper. However, differently from Bjorkegren the simulation method in this paper does not employ the assumption of perfect foresight.

[5]For example in a recent working paper by Fu & Gregory (2016) study household rebuilding behaviour post Katrina hurricane where the neighbour's rebuilding decisions induces amenity spillovers. The paper assumes a specific equilibrium selection rule under which the model is both estimated and simulated for counterfactual policies.

[6]Jia (2008) establishes bounds for the possible set of fixed points under specific selection rules in order to ease the burden of estimating the model.

The model is estimated using a two period household panel data from India. In addition to sanitation adoption, I observe a rich set of household characteristics including asset accumulation, earnings, household demographics and cost of sanitation. The survey covers time periods 2009/10 and 2012/13 and includes information on key demographic features at the village level. Additional information on the cost of sanitation measure was collected at across villages in 2012.

By comparing the existing sanitation levels in the data with the socially optimal level of adoption from the estimated model, I quantify the extent of under-adoption. First, I compare the cost effectiveness of providing sanitation price subsidies differently from loan policies to increase coverage and compute welfare gains under each policy. Second, I decompose the policy effect into its direct and indirect components i.e., externality on an individual household's demand response. Lastly, I use the structural framework to derive implications about optimal policy design and cost efficiency.

An advantage of a structural approach is to be able to simulate the impact of counterfactual policies on adoption decision from changing the direct costs of sanitation, for example through the provision of price subsidies or loans that incentivize adoption. Compared with a utilitarian social planner solution, existing coverage levels are on average 53% below the socially optimal level, implying under-adoption. This wedge is induced by the under valuation by each household of the total benefits derived from adoption. Under cost effectiveness considerations, I find price subsidies to be, in general, more effective at increasing sanitation coverage. However, the policy effects are heterogeneous where loans are found to be equally, if not marginally more, effective in villages with no sanitation coverage. In contrast, I find that price subsidies compared with sanitation loans are more effective at increasing sanitation coverage within villages with mid to low initial sanitation levels. This is because a subsidy policy generates a larger feedback effect in the presence of externalities which propagates through the entire village resulting in a larger shift in the equilibrium sanitation level. Instead sanitation loans are found to be more cost effective in villages with close to zero initial sanitation prevalence.

I find that the additional adoption induced by a price subsidy policy generated positive externalities equivalent to $Rs. 3,181$ (lower bound) and $Rs. 6,253$ (upper bound) of welfare gain for households whose adoption choice was not directly affected by the program. While the policy generated a larger gain equivalent to $Rs. 10,008$ (lower) and $Rs. 12,511$ (upper) for the recipient household directly affected by the policy. The impact of the subsidy policy shifted bounds on net welfare upwards by $Rs. 1,380$ (lower) and $Rs. 3,883$ (upper). A significant proportion of this gain 33% (lower) to 72% (upper) in surplus is accrued indirectly through the spillover effects.

These findings highlight the fact that, with the presence of externalities, accounting for and quantifying the effect of equilibrium interactions among households is essential to understand the impact of policies. This paper argues that subsidies affect household welfare both directly by reducing the relative price for sanitation but also indirectly through an externality that affects the relative cost faced by an individual household. This analysis is consistent with evidence from field experiments on sanitation demand and contributes to the literature on the topic of sanitation. In particular, it complements a recent study by Guiteras, Levinsohn & Mobarak (2015) which analyses the impact of different policy interventions on sanitation take-up behaviour among households in Bangladesh using a randomized experiment. The experiment contrasts between policies that provide information on the benefits of having sanitation at home and policies that directly subsidize the cost of adoption through price subsidies along with a control group. Their analysis finds evidence of positive spillover effects from adoption on households other than the recipient beneficiary both in adoption decisions and health outcomes.

The remainder of the paper is organized as follows. Section 4.2 provides a background and description of the data. Section 4.3 presents the model of household sanitation choice and the identification assumptions on which my results are based. Section 4.4 describes the estimation strategy and discusses the parameter estimates and model fit. Section 4.5 describes how the estimated model is used to simulate the equilibrium sanitation adoption behaviour under counterfactual policy interventions. Results are presented in Section 4.6 with an analysis of counterfactual policies. Section 4.7 concludes. All relevant Table and Figures are provided in the chapter appendix on Section (4.8).

4.2 Context and Data

4.2.1 Sanitation in India

Despite substantial evidence on the importance of sanitation for health and human capital development, progress towards increasing access to sanitation in India has been extremely slow. For example, in rural areas, the fraction of households without a toilet decreased by only 8.8 percentage points between 2001 and 2011, from 78.1% to 69.3% (Ministry of Rural Development 2012). While a number of innovative and successful approaches have increased access to sanitation on a small scale, the national average of 37% sanitation coverage is well below the global average of 64%.[7] The topic of sanitation provision has also garnered important political interest within the country. The current Prime Minister Narendra Modi launched the "Swachh Bharat Abhiyan" (Clean India Mission) initiative which proposed to provide toilets/sanitation to all 110 million rural households that currently do not have one, at a cost of *USD* 22.0 billion (Ministry of Rural Development India, 2014).

Poor sanitation has been linked with causes of intestinal diseases which reduce the absorption of calories and nutrients, and leads to malnutrition and impaired cognitive development among children. There is also a growing body of work within the economic literature, that quantifies the impact of sanitation prevalence on individual health outcomes especially for children. For example, a recent working paper by Geruso & Spears (2015) investigates the impact of poor sanitation coverage on infant mortality in India. By instrumenting for local sanitation prevalence with the religious composition of neighbourhoods to account for endogeneity of sanitation coverage, they find evidence of large infant mortality externalities associated with the lack of sanitation amongst neighbours. Specifically, their analysis finds a decline in $2.6-2.9$ infant deaths per 1000 with a 10% increase in sanitation adoption levels. Augsburg & Rodriguez (2015) use the data on sanitation price variation as an instrument for the sanitation prevalence across village.[8] Their instrumental approach suggest a significant increase in child height for age $z-scores$ by 0.15 standard deviations with a 10% increase in sanitation prevalence.

In contrast, there are few examples that study demand for sanitation and the factors/market failures that affect household choice. An exception to this is a recent paper by Guiteras, Levinsohn & Mobarak (2015) based on a randomized policy experiment conducted in Bangladesh. The experiment measures the impact of different policies: price subsidies, supply-side and information provision, on household sanitation adoption. The findings suggest that lack of information, about the benefits of improved sanitation, or lack of access to markets for sanitation components are not the key deterrents to a household's investment in sanitation. Instead, the significant increase in sanitation ownership and usage among subsidy (price subsidy) recipients suggests that financial constraints might be an important limiting factor in their context. The increased probability of sanitation ownership

[7]The national average of 37% is across rural and urban populations. Sanitation coverage in rural India is estimated at 21.1% while the number for urban population is close to 54% (data.worldbank.org)

[8]The analysis in Augsburg & Rodriguez (2015) also makes use of the dataset employed in the empirical analysis in this paper. They additionally use variation in the cost (price) of sanitation, collected independently in Gautam (2015) and this paper as an instrument. Their analysis provides constructive evidence on the presence of significant positive externalities on child health in the data used.

among non-recipients also suggests that purchasing decisions of one's neighbours affect a household's own purchasing decisions - even without a subsidy incentive. Furthermore, the increase in sanitation adoption rates as the proportion of subsidy voucher recipients suggests the presence of spillover effects and inter-linked adoption decision. My analysis reveals the extent of under-adoption accrued due to externalities as well as induced by binding financial constraints.

4.2.2 Data

The data for the empirical analysis comes from a household panel survey conducted under the FINISH program (Financial Inclusion Improves Sanitation & Health) in India during the periods $2009-10$ and $2012-2013$. The program aims to improve the living standards of poor communities by implementing projects that improve sanitation, hygiene and waste management across the country. The overall objective is to increase sanitation access and coverage and thereby improve the living and economic conditions for poor households that otherwise lack access to basic sanitation. Under its sanitation and hygiene program, FINISH provides sanitation facilities at the household level through a combination of micro-credit lent by Microfinance Institutions (MFIs) or local banks, and/or price subsidies (subsidize the cost of adoption). In addition to the provision of financial incentives, the interventions also include a self-contribution component and health insurance incentives. It is important to mention that though the data was collected under the FINISH program, it does not include the impact of any policy intervention implemented.

Descriptive Statistics.

The data used in this project comes from the regional locality of Gwalior located in the state of Madhya Pradesh, India. With only 28% of population with access to sanitation, Madhya Pradesh ranks 30 out of a total of 36 states in the country in order of sanitation coverage. The sample size comprises of 1451 households observed over the two survey periods $2009-10$ and $2012-2013$ from 42 village groups. The FINISH sample comprises of a detailed household survey which includes a rich set of information on household demographics, household members and household head, education, earnings, asset accumulation and consumption values as well as information on sanitation adoption. The dataset also comprises of village level demographics which includes information on daily wage rate for labour, presence of drainage infrastructure and availability of public sanitation facilities within the village.

Table (4.1) provides descriptive statistics for a few variables of interest across the two period panel. The household head is on average 43 years of age with primary school education.[9] A typical family consists of 5 household members half of whom are female. Home ownership rates are high with close to 90% of household heads owning their house. Cash-in-hand refers to the total annual income and liquid assets available to the household for consumption. The stock of assets which also includes savings in the bank is between 8% and 10% of the total available resources for consumption. Income earnings and stock of asset values are deflated to 2010 values.

Cost of Sanitation.

Data on the cost of sanitation was collected in July/August 2012 across all villages.[10] The price measure is based on the cost of building the most common type of sanitation facility in the local region i.e., 'Twin Pit Pour Flush' (TPPF) unit. All households within a village face the same price. The price measure comprises of two components: the total cost of raw material and the cost of labour required to build the facility itself. The price measure varies across villages in both components. The formula applied to construct the price measure is as follows:

Price variation across villages g:

- $wage_g$: Daily (informal) wage rate which varies across villages.
- $days$: Approximate time to construct a 'Twin Pit Pour Flush' (TPPF) variation between $3-4$ days. TPPF is the standard and most popular sanitation design unit implemented by the government under the Total Sanitation Campaign (T.S.C)
- $cost_g\,(raw\,materials)$: Cost of raw material (cost of five principle materials used in the construction of a TPPF unit) which include — Bricks, Mortar, Tiles, Ceramic fixtures & Tin sheets.

$$price_g = wage_g \times days + cost_g\,(raw\,materials) \times quantity(kilogram/piece/unit) \tag{4.2.1}$$

[9]Primary School in India is up to year 5 with a total of 12 years of primary and secondary education.

[10]The information used to construct a measure for the cost of sanitation was collected independently of the household survey. I received helpful advice and assistance from the Gwalior Nagar Nigam Seva municipal authorities in the collection process.

The main source of variation arises from the cost of raw materials which varies across villages and comprises close to 70% of the total cost of sanitation incurred. A point to note here is that the raw materials used in sanitation are widely produced and demanded in the region on a large scale for other domestic and commercial construction. The demand for these products for the purpose of building sanitation constitutes a very small proportion of the overall demand for the goods in the region.[11]

Age profiles. The life-cycle profiles of interest include sanitation adoption and asset accumulation by age of household head. Figure (4.1) depicts the dynamics that the model should be able to replicate. Appendix (4.8.1) describes the way in which life-cycle profiles are obtained using data from different age cohorts. Sanitation adoption varies over the life-cycle of a household head with 37% prevalence among 20 year household heads to just over 70% prevalence by the age of 75. There is a relatively steeper increase in the proportion of adoption between the age of 20 and 26 while adoption tapers off to be flat past the age of 55. The asset stock profile (per Rs. 1000) depicts a hump shaped profile with a steady accumulation of assets up to the age of 55, past which the household de-cumulates to almost it's initial stock of assets by the age of 75.

[11] Table (4.7) in the appendix provides details on variation of price and sanitation prevalence by village.

4.3 Model

This section describes the model which provides a framework to evaluate the impact of different policy interventions on household demand for sanitation. A household is taken to be a single decision making unit where the household head is identified as the primary decision maker. The model closely matches the observed adoption behaviour over the life-cycle, and is designed to capture the key trade-offs faced by a decision making household. Specifically, the structure incorporates features that influence a household's net utility from adoption: (i) the cost of building sanitation, (ii) the impact of binding liquidity constraints, (iii) the strength of the idiosyncratic taste shocks for having sanitation at home and, (iv) the impact of changes in the sanitation coverage within the village. The increase in sanitation coverage within a village can generate spillover effects that are not necessarily internalized by individual households. The impact of such externalities is captured through an interdependence in household demand for sanitation, where the gains derived from having sanitation also depend on the adoption decision of the village as a whole.[12]

The interdependence is modelled as a strategic interaction among households under incomplete information.[13] An individual household evaluates it's private utility from adoption against the social benefits it derives through the spillover effects. The key difference of the structure from a single agent model is that instead of acting in isolation households solve an inter related system of dynamic programming problems given expectations about adoption decision of other households. Under rational expectations, in equilibrium a household's actions must be optimal given their beliefs and their beliefs must be correct on average. In addition, households are also restricted from borrowing against their future income. Thus, the present decision to buy sanitation induces an inter temporal trade-off with savings that could instead be used to insure future consumption against income shocks.

This section describes the primitives of the model, including household's information set and choices, state variable transitions and the timing of choices along with a specification of the household problem. I also describe the Markov Perfect Equilibrium within the model and possible multiplicity of equilibria. Lastly, I discuss the identification of the primitives and the assumptions employed to identify the parameters of interest.

[12]Externalities in the model can be generated from different underlying mechanisms for e.g. health externalities, information spillovers, amenity agglomeration effects as well as peer conformism effects.

[13]Under incomplete information the information set of a decision making household is only partially observed by other households including the econometrician.

4.3.1 Model Specification

There are a finite number of village groups indexed by $g = 1,...,G$ and each village is the relevant reference group for a household. Let N denote the set of households that belong to each village indexed by $i = \{1,2,...,N\}$.

Choice Set. A household head makes decisions based on his or her age a over a finite horizon, where $a = 20,...,75$ and $\mathcal{A}$ denotes terminal decision making age. In addition, within a village different 'aged' households interact with each other where the dynamics evolves over calendar time $t = 1,...,\infty$.[14] At each age a until terminal age $\mathcal{A}$, a household who is alive at time t can choose a pair (d_{it}, c_{it}), where $d_{it} \in \mathcal{D}_{it} = \{0,1\}$ denotes choice to adopt sanitation today or wait until the next period:

$$d_{it} = \begin{cases} 0 & Non adoption \\ 1 & HH adopts sanitation \end{cases}$$

and $c_{it} \in \mathcal{C}_{it}$ denotes the consumption choice today which determines the amount saved for tomorrow A_{it+1}. A household's choice to adopt sanitation is an absorbing state where $k_{it} = k_{it-1} + d_{it}$ denotes status of sanitation adoption.[15] In each period t, different 'aged' households simultaneously decide whether or not to adopt sanitation. The vector of all household actions in period t is given by $d_t = (d_{1t}, d_{2t}, ..., d_{Nt})$ and $c_t = (c_{1t}, c_{2t}, ..., c_{Nt})$.

State variables. Each household i is characterized by a vector of state variables that affect utility: x_{it} and ε_{it}. A household's decisions are based on the age of the household head a_{it}, stock of assets A_{it}, income y_{it}, state of adoption k_{it-1}, cost of sanitation adoption *price* and the level of existing sanitation coverage $\overline{k}_{t-1}$ within the village it resides. Decisions are also based on a household specific idiosyncratic taste for sanitation $\varepsilon_{it} = \left[\varepsilon_{it}^{d=1}, \varepsilon_{it}^{d=0}\right]$ which is a private information shock possessed by household i and unobservable to all other households $-i$ and the econometrician. Household i specific state vector is denoted by $\left(x_{it}, \varepsilon_{it}^{d}\right) = \left(a_{it}, A_{it}, y_{it}, k_{it-1}, \overline{k}_{t-1}, price, \varepsilon_{it}^{d}\right)$. An augmented state space is given by $\left(\tilde{x}_{it}, \varepsilon_{it}^{d}\right) = \left(a_{it}, A_{it}, y_{it}, k_{it-1}, \overline{k}_{t-1}, \xi_{it}, price, \varepsilon_{it}^{d}\right)$ where $\xi_{it} \in x_{it}$ is an allowance for

[14] The village economy can be viewed as an overlapping generations framework of household heads aged between $20-75$ years old.

[15] I do not observe destruction of sanitation units over the two samples in the data. Also treating sanitation adoption as a binary choice is reasonable in the context of rural India where almost all household have at most one toilet/sanitation facility per household.

measurement error in income.[16] Both taste shocks ε_{it} and measurement error ξ_{it} are assumed to be independently distributed ($i.i.d$) across households and time periods.

Information Set, Expectations and Timing. In addition to its own states a household's decision to adopt also depends on the adoption decision of other households in the village. Under private information a household forms expectations about the sanitation adoption behaviour of others based on the common knowledge information set $x_t = (x_{it}, x_{-it})$.[17] A household learns taste shock ε_i prior to making it's own choices, but other households' taste shock ε_{-it} remain unknown to i.

ASSUMPTION COMMON INFORMATION: *The state vector $x_t = (x_{1t}, x_{2t}, ..., x_{Nt})$ denotes the common knowledge information set at time t observable to all households that belong to the same village.*

ASSUMPTION PRIVATE INFORMATION: *The choice specific taste parameter ε_{it}^d are private information shocks and are assumed to be distributed i.i.d across households and time.*

Though a household receives instantaneous utility from its consumption choice in period t, it only enjoys utility from its sanitation decision in the following period. This is because it takes *time to build* a sanitation facility at home which results in a delay between when an adoption choice is made and when the facility can be used and enjoyed at home. Similarly, the level of sanitation coverage $\overline{k}_{t-1}$ which is a state observable to all households at the start of period t captures the impact of underlying externalities generated by the level of adoption up to period $t-1$.

ASSUMPTION TIME TO BUILD: *A household that chooses to adopt sanitation in period t may only realise gains from choice $d_{it} = 1$ in the following period.*

[16]In addition to the taste shock ε the econometrician also observes a noisy measure of income. The notation $(\tilde{x}_{it}, \varepsilon_{it}^d)$ will be used extensively when the model is taken to data and estimated in the following section.

[17]Since the household problem is defined over its life-cycle, where choices are made over age a the state vector denoted by $x_{at} = (x_{it}, x_{-it}|a_{it})$ is used when describing the household problem, where x_{at} indexes the household $i's$ information set at age a.

Household Income. The earnings function is modelled as an exogenous process for the household unit.[18] The log earnings in the current period id given by:

$$\ln y_{it} = f\left(age_{it}, edu_i\right) + z_{it}$$
$$z_{it} = z_{it-1} + u_{it}, \quad u_t \sim N\left(0, \sigma_u^2\right) \tag{4.3.1}$$

where $f(\cdot)$ is a function of the age and education of the household head.[19] The permanent component follows an A.R.(1) process with variance σ_u^2 for innovations. In taking the model to the data measurement error shocks in income ξ_{it} are added to allow for a non degenerate model. The shocks ξ_{it} assumed to be identically distributed across time and households with mean zero and variance σ_ξ^2. See Appendix (4.8.2) for further details on specifications.

Budget Constraint. The main motive for asset accumulation in the model is to finance the purchase of a sanitation facility, insure future consumption against income fluctuations and respond to preference shocks. I assume a standard inter temporal budget constraint augmented for the cost of sanitation (*price*) to relate future assets to the current stock of asset A_{it}, income y_{it}, and consumption c_{it}.

$$A_{it+1} = R\left(A_{it} + y_{it} - c_{it} - price * \mathbf{1}\left[d_{it} = 1\right]\right) \tag{4.3.2}$$

All households in the same village face the same sanitation purchase *price* i.e., the cost of building sanitation at home.[20] The initial stock of assets for household i is assumed to equal $A_{i0} = A_0\left(edu_i\right)$. The real interest rate r, $R = (1+r)$ is the rate at which a household saves and accumulates wealth.[21] Households are allowed to save and accumulate assets but are unable to borrow against their future income.

$$A_{it} \geq 0 \tag{4.3.3}$$

[18]The model does not include the employment decisions of the household head or other members. Each household observed in the data derives a collective annual income amongst all household members where the head is the primary earner.

[19]In northern India the role of household head is culturally assigned to eldest working male (female, if widowed) who is also the primary earner and/or highest educated member within the household. It is assumed that household heads acquire all relevant education by the age 20. Thus edu_i refers to the education level at $a_{it} = 20$ with no further evolution in education attainment.

[20]The model does not allow for dynamics or uncertainty in the price process.

[21]The real rate of interest is set to $r = 0.02$ based on an approximation from interest rate data over the past 50 years from the Reserve Bank of India (*RBI*)

The exogenous borrowing constraint restricts households from holding debt at any age a and time t and thus affects the inter temporal allocation of resources over the lifetime. The constraint may bind for a liquidity constrained household that would otherwise borrow against future income to smooth consumption and/or to purchase sanitation. Given preferences, the constraint may differentially bind over consumption and sanitation adoption choices. In practice the income distribution is modelled as a truncated normal governed by a lower bound parameter. In this case borrowing constraints may bind in a household's decision process.

Sanitation Coverage. Within a village the level of sanitation coverage is denoted by the average level of adoption and evolves according to:

$$\overline{k}_t = \overline{k}_{t-1} + \frac{1}{N}\sum_{j=1}^{N} d_{jt} \qquad j = 1, ..., N \tag{4.3.4}$$

where state $\overline{k}$ denotes the existing level of sanitation each period and $\sum_{j=1}^{N} d_{jt}$ is the sum of the adoption choices made by households in each period. The sanitation coverage $\overline{k}$ captures the level of externality generated by the cumulative actions of all households within the village.

4.3.2 Strategies and Utility

Strategies. The strategy space σ of each household consists of a tuple $\sigma_{it}(x_t, \varepsilon_{it}) = [\delta_{it}(x_t, \varepsilon_{it}), c^o_{it}(x_t.\varepsilon_{it})]$ where δ denotes the adoption decision rule and c^o denotes the policy function for consumption.[22] Let $\sigma_t = \{\sigma_{it}(x_t, \varepsilon_{it})\}_{i=1}^{N}$ be a set of strategy functions which are associated with a set of conditional choice probabilities (*CCPs*) for sanitation adoption $P^{\sigma_t} = \left\{p_i^{\sigma_t}(d_{it}|x_t)\right\}_{i=1}^{N}$ such that:

$$p_i^{\sigma_t}(d_{it}|x_t) = \int 1\{d_{it} = \delta_{it}(x_t, \varepsilon_{it})\}\, g(\varepsilon_{it})\, d\varepsilon_{it} \tag{4.3.5}$$

which represents the expected adoption behaviour of household i from the point of view of other households $-i$, when household i follows strategy profile σ_{it} and other households follow σ_{-it}, and $\sigma_t = [\sigma_{it}, \sigma_{-it}]$ denotes a strategy profile. The choice probability is conditioned on all relevant observable information summarized by x_t at time t.

[22] The policy function for the optimal consumption $c^o_{it}(x_{it}; \sigma_t)$ is given by the maximization of the household problem described in the next section.

Preferences. A household derives utility from consumption c_{it}, state of adoption k_{it-1} and the level of sanitation coverage $\bar{k}_{t-1}$ and evaluates the benefits from sanitation against the cost of purchase. The per period utility for household i below the age $a < \mathcal{A}$ at time t is specified as:

$$u\left(c_{it}, d_{it}, x_{it}, \varepsilon_{it}^{d}; \theta\right) = c_{it}^{\nu}\left[1 + \eta k_{it-1} + \phi \bar{k}_{t-1}\right] + \alpha_{age_i} s\left(age_i, k_{it-1}\right) + \gamma k_{it-1}\bar{k}_{t-1} + \varepsilon_{it}^{d} \tag{4.3.6}$$

The private preference for sanitation ε_{it}^{d} enters as an additively separable shock. The first term on the right hand side of Equation (4.3.6) represents individual utility from consumption (c_{it}) and parameter $1 - \nu$ denotes the coefficient of relative risk aversion. A household also enjoys direct utility from having sanitation at home in the form of convenience and other salient benefits, captured by α_{age} which may vary by the age of household head.[23] The non-separability between sanitation and consumption choice is captured by parameter η. Interacting the utility from consumption with adoption status captures potential complementarities that arise from sanitation adoption that improves latent health status and increases utility from food consumption. In addition to private convenience a household also derives additional benefits from the level of sanitation coverage. For example, a household may derive health benefits from residing in a village with a higher level of sanitation coverage and thus a lower degree of environmental pollution. Sanitation coverage level $\bar{k}_{t-1}$ denotes the fraction of households who have adopted sanitation by the end of the last period. The level of adoption in a village affects an individual household's utility through it's own adoption status captured by γ as well as through the impact on private consumption denoted by ϕ. The utility function $u(\cdot) \to -\infty$ for $c_{it} \to 0$ which restricts adoption for households whose present cash-in-hand does not cover the cost of purchase i.e., $A_{it} + y_{it} < price * \mathbf{1}\left[d_{it} = 1\right]$. The preference specification reflects the *time to build* assumption where utility for i in period t depends on $\left(c_{it}, k_{it-1}, \bar{k}_{t-1}\right)$ instead of present choice d_{it}.

Since the adoption decision is a function of the sanitation prevalence within a village, aggregate village level characteristics may also affect the utility from sanitation and thus are an important feature to incorporate. For example, households in villages with drainage infrastructure and piped water supply are collectively more likely to

[23] A report from the World Bank noted that female members of the household also enjoy a degree of personal safety from having a sanitation facility at home. The outside alternatives e.g. open fields or public sanitation facilities are associated with a higher degree of risk to personal safety especially for women.

adopt, and these villages are also where the existing sanitation coverage is high compared with villages with no drainage or water supply. To capture unobservable group level effects, I allow for a village specific ‘fixed effect’ by allowing the the location parameter μ(mean) of the difference of the taste shock ε_{it} to vary across villages. These location parameters act like ‘fixed effects’ capturing the impact of village specific characteristics not explicitly modelled or observed by the econometrician.[24] Under discrete choice, only the relative flow utility of sanitation adoption relative to non-adoption are identified and the parameter μ_g shifts this difference in values across villages.

4.3.3 Household Problem

Households are forward looking and make decisions so as to maximize the present discounted value of the expected future utility subject to a set of constraints: (4.3.1),(4.3.2),(4.3.3) and (4.3.4). At each age from $a = 20 - 75$ a decision making household who is alive at time t chooses how much to consume c_{it} and save for the future. In addition, households that have not adopted sanitation $k_{it-1} = 0$, also choose whether or not to adopt d_{it} after observing their current period cash-in-hand, given cost of adoption (*price*) and level of sanitation coverage $\overline{k}_{t-1}$. Using the Bellman principle, the dynamic problem of maximizing the expected lifetime utility under a given strategy σ_t for household that have yet to adopt i.e., $k_{it-1} = 0$ can be formulated as:

$$V_i^{k=0}(x_t;\sigma_t) = \int \max_{d_{it}\in\mathcal{D}_{it},c_{it}\in C_{it}} \left\{ v_i^{\sigma_t}(d_{it},c_{it},x_t) + \varepsilon_{it}^d \right\} g(\varepsilon_{it})\, d\varepsilon_{it} \tag{4.3.7}$$

The function V_i denotes $i's$ ex-ante value or EMAX function which reflects expected utility at the beginning of the period before private shocks are realized. While $v_i(\cdot;\sigma_t)$ denotes the choice-specific value functions:

$$v_i(d_{it},x_t;\sigma_t) = \max_{c_{it}\in C_{it}} u_i^{\sigma_t}(c_{it},d_{it},x_{it}) + \beta \underbrace{\sum_{d_{-it}} \sum_{x_{t+1}} V_i(x_{t+1};\sigma_{t+1})\, f_i\left(x_{it+1}|x_{it},d_{it},c_{it},\overline{d}_{-it}\right) \left(\prod_{j\neq i} p_j^{\sigma_t}(d_{-i}[j]|x_t) \right)}_{\sum_{x_{t+1}} V_i(x_{t+1};\sigma_{t+1}) f_i^{\sigma_t}(x_{it+1}|x_t,d_{it},c_{it})}$$

[24]This modelling assumption allows me to relax the independence of ε shocks across households within a village in a specific way which yields as estimable parameter without loosing the tractability of the structure under the *i.i.d* assumption. Details on the variation in the data that identifies μ are discussed in the following section.

For households that have already adopted sanitation i.e., $k_{it-1} = 1$ the value function is given by:

$$V_i^{k=1}(x_t;\sigma_t) = \max_{c_{it}\in C_{it}} u_i^{\sigma_t}(c_{it}, x_{it}) + \beta \sum_{d_{-it}} \sum_{x_{t+1}} V_i(x_{t+1};\sigma_{t+1}) f_i\left(x_{it+1}|x_{it}, c_{it}, \overline{d}_{-it}\right) \left(\prod_{j\neq i} p_j^{\sigma_t}(d_{-i}[j]|x_t)\right)$$

where β is the discount factor and the expectation is over realizations of future states, choices and shocks given the information set available to the household at time t. Also, $f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, c_{it})$ denotes the expected transition probability of observable states x conditional on household i choosing (c_{it}, d_{it}) integrated over the expected adoption decisions of other household $j \neq i$. Households make decisions until terminal age $\mathcal{A}$ with $V(x_t, \varepsilon_{it}|a_{it} = \mathcal{A}, \sigma_t) = 0$.

Adoption Decision Rule. The household decision rule for sanitation adoption is given by:

$$\delta_{it}(x_t, \varepsilon_{it};\sigma_t) = \begin{cases} 1 & if \quad v_i(d_{it}=1, x_t;\sigma_t) - v_i(d_{it}=0, c_{it}, x_t;\sigma_t) - \mu_g \geq \varepsilon_{it}^0 - \varepsilon_{it}^1 \\ 0 & if \quad v_i(d_{it}=1, x_t;\sigma_t) - v_i(d_{it}=0, x_t;\sigma_t) - \mu_g < \varepsilon_{it}^0 - \varepsilon_{it}^1 \end{cases} \tag{4.3.8}$$

Consumption Policy Function. If $k_{it-1} = 0$, the policy function for the optimal consumption is given by:

$$c_{it}^o(x_t, \varepsilon_{it};\sigma_t) = \{\delta_{it}(x_t, \varepsilon_{it};\sigma_t) = 1\} c_{it}^{d=1}(x_t;\sigma_t) + \{\delta_{it}(x_t, \varepsilon_{it};\sigma_t) = 0\} c_{it}^{d=0}(x_t;\sigma_t)$$

$$c_{it}^d(x_t;\sigma_t) = \arg\max_{c_{it}\in C_{it}} \left\{ u_i^{\sigma_t}(d_{it}, c_{it}, x_{it}) + \beta \sum_{x_{t+1}} V_i(x_{t+1};\sigma_{t+1}) f_i^{\sigma_t}(x_{it+1}|x_t, d_{it}, c_{it}) \right\} \tag{4.3.9}$$

Solution. Given a strategy profile $\{\sigma_t\}$, a household's maximization problem can be cast as a finite horizon dynamic programming problem which can be solved via backward recursion from the terminal age $a = \mathcal{A}$. The solution to the household problem would be a function of the underlying strategy profile. However, to ensure consistency with the strategy played i.e., a household's expectation about sanitation prevalence and its future evolution are consistent, also requires the solution to a fixed point. The choice probabilities $p_i^{\sigma_t}(d_{it}|x_t)$ solve the coupled fixed point problem defined by:

If households have yet to adopt sanitation i.e.,$k_{it-1}=0$:

$$V_i^{k=0}(x_t;\sigma_t)=\int \max_{d_{it}\in\mathcal{D}_{it},c_{it}\in C_{it}}\left\{v_i^{\sigma_t}(d_{it},c_{it},x_t)+\varepsilon_{it}^d\right\}g(\varepsilon_{it})\,d\varepsilon_{it}$$

and

$$\Lambda_i(d_{it}|x_t;\sigma_t)=\int 1\left\{d_{it}=\arg\max_{d_{it}\in\mathcal{D}_{it}}\left\{v_i^{\sigma_t}(d_{it},c_{it},x_t)+\varepsilon_{it}^d\right\}\right\}g(\varepsilon_{it})\,d\varepsilon_{it}$$

and for households that have yet to adopt sanitation i.e., $k_{it-1}=1$

$$V_i^{k=1}(x_t;\sigma_t)=\max_{c_{it}\in C_{it}} u_i^{\sigma_t}(c_{it},x_{it})+\beta\sum_{d_{-it}}\sum_{x_{t+1}}V_i(x_{t+1};\sigma_{t+1})\,f_i\left(x_{it+1}|x_{it},c_{it},\overline{d}_{-it}\right)\left(\prod_{j\neq i}p_j^{\sigma_t}(d_{-i}[j]\,|x_t)\right)$$

and

$$\Lambda_i(d_{it}|x_t;\sigma_t)=\int 1\left\{d_{it}=\arg\max_{d_{it}\in\mathcal{D}_{it}}\left\{v_i^{\sigma_t}(d_{it},c_{it},x_t)+\varepsilon_{it}^d\right\}\right\}g(\varepsilon_{it})\,d\varepsilon_{it}$$

Given a set of adoption probabilities $P^{\sigma_t}=\left\{p_i^{\sigma_t}(d_{it}|x_t)\right\}_{i=1}^N$, the value functions $V_i^k(x_t;\sigma_t)$ are solutions to the N Bellman equations and the function $\Lambda_i(d_{it}|x_t;\sigma_t)$ denotes the best response probability function for each household i for a given strategy σ_t.

4.3.4 Equilibrium

Expectations over the adoption decisions of other households, conditional on observed states, allows an individual household to infer how the adoption coverage level will update in the next period. Since the time t states and expectations summarize all relevant information about other households in the village, a household's behaviour depends only on the current state x_t and own current private shock. An equilibrium under this markovian structure can be defined as follows:

The strategy profile $\sigma_t=(\sigma_{1t},\sigma_{2t},...,\sigma_{Nt})$ is a Markov perfect equilibrium if and only if, given the opponents profile σ_{-it}, each household prefers the strategy σ_{it} to all alternative Markov strategies $\sigma_{it}^{'}$. That is, σ_t is a MPE for all households i, at all states x_t and all alternative Markov strategies $\sigma_{it}^{'}$.

$$V_i\left(x_t;\sigma_{it},\sigma_{-it},\theta\right) \geq V_i\left(x_t;\sigma_{it}^{'},\sigma_{-it},\theta\right) \qquad \text{for all } i, x_t, \sigma_{it}^{'} \tag{4.3.10}$$

A household's expected value under an alternative Markov profile $\sigma_{it}^{'}$ given states can be written recursively as:

$$V_i\left(x_t;\sigma_t^{'},\theta\right) = E_{\varepsilon}\left[u_i\left(\sigma_{it}^{'}\left(x_t,\varepsilon_{it}\right),x_{it};\theta\right)+\varepsilon_i\left(d_i\right)+\beta\int V_i\left(x_{t+1};\sigma_{t+1},\theta\right)f\left(x_{t+1}|x_t,\sigma_{it}^{'}\left(x_t,\varepsilon_{it}\right),\sigma_{-it}\left(x_t,\varepsilon_{-it}\right)\right)dx_{t+1}|x_t\right]$$

Existence of this equilibrium is a direct consequence of the finite horizon and finite action-space following Maskin & Tirole (2001). Equation (4.3.10) describe a set of inequalities which form the moment conditions constructed in the estimation procedure in the following section.

Multiplicity of Equilibria. In general, there may exist more than one solution to the system of equations in Equation (4.3.10). The multiplicity of the equilibria arises due to the interdependence of household adoption decisions that are consistent with distinct levels of sanitation coverage in equilibrium. At this stage the model is incomplete without the specification of an equilibrium selection rule (ESR). This 'incompleteness' introduces a challenge with respect to the estimation of the model where without an ESR the objective function is not well defined. This further adds to the computational burden of having to repeatedly solve the model for all possible equilibria under the dual specification above, for each candidate vector of parameters. Even if it would be possible to solve for the entire set of equilibria there is still an open question about the underlying equilibrium selection rule played, on which observed behaviour does not provide any additional information. In order to move forward with the estimation of the model, I impose the following assumption on the observed data.

ASSUMPTION EQUILIBRIUM SELECTION (*SingleMPE*): *The data observed is generated by a single Markov perfect equilibrium profile* σ.

Under this assumption there exists a 1 : 1 mapping between the observed behaviour in the data and the structural objects of the model which is discussed further in the identification section below. The main assumption here is that for each village, the data is generated by the same Markov perfect equilibrium profile. In practice, I pool data from multiple villages in which case Assumption $SingleMPE$, which requires for the equilibrium selection to be consistent across villages, can prove to be restrictive. The assumption that the data observed is generated from one of the possible equilibrium affects the estimation of the reduced form objects i.e., conditional choice probabilities (*CCPs*). In order to consistently estimate the *CCPs*, I divide the villages into subgroups based on village level observables and geographic proximity. The underlying assumption being that villages close in geographic distance and similar in village specific demographics may play the same equilibrium selection rule which is consistent within that subgroup of villages.

4.3.5 Identification

Identification of preference parameters and village shocks. This section discusses how specific elements of the state transitions and flow utility are identified from the empirical moments. In the data, each period t choice and state combination implies a probability distribution over period $t+1$ states and these moments identify parameters that govern state-to-state transitions $f(\cdot)$, including those for the income earnings function.

Parameter vector θ in the flow utility function[25] and the discount factor β are identified through observed state dependent choice distributions. The CRRA coefficient $1-\nu$, which measures the curvature in consumption utility function is identified using the state dependent asset accumulation distribution. The variation in the mean asset accumulation level by age of the household head traces out the marginal utility of consumption. In addition, the change in the asset accumulation (variance) across different age groups helps identify the degree of impatience denoted by the discount factor β.

Similarly, the variation in the proportion of adoption by age of the household head captures $\alpha(age)$. While variation in asset accumulation conditional on sanitation adoption status helps identify η. Across village variation in the sanitation coverage and asset accumulation conditional on adoption status provides identification of parameters that govern the impact of the externality γ and ϕ. The exclusion restriction provides exogenous variation in the sanitation coverage across villages. Unobserved group effects are identified by observing the aggregate adoption choice within a village that is consistently different from the choice distribution in another village

[25]utility function parametrized by $\theta = [\nu, \eta, \phi, \alpha(age), \gamma, \mu_g]$where μ_g denotes mean of the taste shocks capturing village level fixed effects.

given the same set of states. Thus the residual variation from the adoption and asset accumulation distribution of households conditional on coverage levels across villages identifies village level fixed effects denoted by the location (mean) parameter of the taste shock μ_g.

4.4 Estimation

The model is estimated using a two step procedure that closely follows Bajari, Benkard & Levin (2007)[26] which extends the simulation based two-step approach of Hotz, Miller, Sanders & Smith (1994) to the estimation of dynamic games. In addition to the standard discrete choice, BBL (2007) also allows for continuous choices. The estimation is divided into two steps. In the first step, I recover the household's policy functions for adoption and consumption, along with estimates for the state transitions. Under rational expectations, households are assumed to have correct expectations about their environment and the behaviour of other households. As a consequence, by estimating the probability distributions for decisions and states, under the $SingleMPE$ assumption, I effectively recover a household's equilibrium expectation for sanitation adoption in the first stage. In the second stage, I recover the structural parameters that rationalize the observed equilibrium choices as a set of optimal decisions. Following BBL (2007), the conditions for optimality are represented as a system of inequalities that require each household's observed behaviour to be weakly preferred to feasible alternatives at each state.

In this section, I describe the estimation procedure undertaken which differs from BBL (2007) in the first stage. The key difference is the way in which the policy function for the continuous consumption choice is obtained. The approach can be viewed a hybrid of a two-step and full solution method to accommodate the challenges that arise with limited data size.[27] Instead of estimating the policy function of consumption off the observed data, I instead incorporate the single-agent (SA) model dynamic programming numeric solution to back out the consumption (or savings) policy function. This is discussed in further detail below.

4.4.1 First-stage: Policy Functions & State Transitions

Decision Rule. The decision rule for sanitation adoption in Equation (4.3.8) is a function of the choice specific value functions $v(d_{it}, x_t; \sigma_t)$. Using the Hotz and

[26]From now on referred to as BBL (2007)

[27]Under a full solution approach, each household's dynamic problem is numerically solved at all possible states subject to the fixed point equilibrium condition to obtain the policy functions. The two-step procedure bypasses the computational burden associated with solving the model, by directly estimating the policy function off the conditional variation in the choices observed in the data. Though the two-step eases the computational burden it instead places a burden, especially with continuous choice, on the amount of observations required to consistently estimate policy functions from the variation in the data.

Miller (1993) inversion it is possible to recover the choice specific value functions by inverting the observed choice probabilities at each point in the state space. Under the Type 1 extreme value distribution assumption on the taste shocks the inversion takes the familiar form for a binary discrete choice:

$$v_i(d_{it}=1,x_t;\sigma_t)-v_i(d_{it}=0,x_t;\sigma_t)-\mu_g=\ln[p_i(d_{it}=1|x_t)]-\ln[1-p_i(d_{it}=1|x_t)]$$

It is sufficient to recover the difference in the choice-specific value functions to recover the decision rule:

$$\widehat{\delta}_{it}(x_t,\varepsilon_{it};\sigma_t)=\begin{cases}1 & if \quad \ln[\widehat{p}_i(d_{it}=1|x_t;\widehat{\psi})]-\ln[1-\widehat{p}_i(d_{it}=1|x_t;\widehat{\psi})]\geq\varepsilon_{it}^0-\varepsilon_{it}^1\\ 0 & if \quad \ln[\widehat{p}_i(d_{it}=1|x_t;\widehat{\psi})]-\ln[1-\widehat{p}_i(d_{it}=1|x_t;\widehat{\psi})]<\varepsilon_{it}^0-\varepsilon_{it}^1\end{cases}$$

where $\widehat{p}_i(d_{it}=1|x_t)$ is an estimate of the choice probability of adoption conditional on the state variable x_t and $\widehat{\psi}$ denotes a vector of first stage parameters employed in the estimation of the choice probabilities. In general the estimation of conditional choice probabilities (CCPs) would require a fairly flexible specification. However, with a state space that includes continuous variables and restricted sample size, flexibility is difficult to achieve in practice. In addition, the CCP estimates are divided into groups based on village level observables and geographic proximity to account for the underlying equilibrium selection rule. Table (4.5) in the appendix provides first stage CCP estimates. In addition to the chosen parametric specification in Table (4.5) other alternative specifications were tried beforehand.

Consumption Policy Function. In principle, the consumption (or savings) policy function is directly estimable from the observed distribution of consumption (or saving) conditional on the adoption decision.[28] However, to obtain consistent estimates for either consumption or savings from observed conditional variation makes further demands from the existing dataset on the amount of observations within each state partition cell. Instead, I propose to solve for the consumption policy function using the numeric solution for a single-agent dynamic problem.

Hybrid. Given that the strategic element is with respect to the sanitation decision and the consumption decision is affected indirectly through adoption it is possible to decompose the model into smaller individual maximization problems. This parallels

[28]Provided that, for all adoption choice and states, the policy function $c^o(x,\xi)$ is strictly increasing in the measurement error shock in income ξ. Under this monotonicity assumption, the policy function c^o provides a $1:1$ mapping from the space of shocks $\xi\in\mathcal{K}$ to the space of continuous consumption choice $\mathcal{C}$ for all adoption choices and states. For additional details refer to Appendix (**??**)

the second feature of equilibrium models, namely that household's maximize expected discounted utility given their expectations about the decisions of other households and the evolution of the relevant states.

A point of note is that the hybrid procedure follows naturally from the structure of the model which allows me to divide it into smaller parts. Conditional on the probability of adoption the model can be viewed as a single agent dynamic programming problem which can be estimated using a variety of standard techniques such as the nested fixed point approach Rust (1997). Instead of fully solving the model given the fixed point, I only solve an individual household's problem conditional on the strategy played in the village. Given expectations on the adoption decision of other households, an individual household solves its own life-cycle problem of sanitation adoption and consumption. To take an expectation over future income, the integral is approximated by discretizing the income process and approximating it with a discrete state Markov process like in Tauchen (1986).

The single-agent sanitation adoption and consumption problem is solved at all possible state realizations. The consumption (or saving) policy function is obtained by maximizing the value function. In practice, an NLOPT procedure is used under a variant of Nelder-Mead method. The continuation value is obtained using an Interpolation approach. The initial conditions are imputed from the observed data at age $a = 20$. The solution also builds the evolution of the deterministic state transition functions. The inter temporal budget constraint governing asset stock evolution depends on the consumption and adoption decisions made. Both the budget constraint and the liquidity constraint faced by an individual household are built into the SA solution which determines the optimal consumption and saving policy at each point in the state space.

Exogenous State Transitions. The income process described in Equation (4.3.1) is estimated directly from the observed data. Income is modelled to be a function of age, education and education-squared. Table (4.3) lists the parameter estimates for the income process. Income increases with human capital (as measured by education) but at a decreasing rate. Age effects vary, with lower wage predicted for households with older household heads.

Evolution of Sanitation Coverage. In addition, the SA solution is also solved over a grid of sanitation coverage states $\bar{k}$ at all points. I discretize the realizations of sanitation prevalence over a 100 grid points in the numeric solution. The law of motion for sanitation coverage within a village $\bar{k}$ is generated/determined in conjunction with the forward simulation procedure in the next stage. I also assume that an individual household's adoption choice today (t) has an infinitesimal change on the level of sanitation prevalence tomorrow ($t+1$) given that households interact in relatively large village groups.

4.4.2 First-stage: Value Functions

The first stage estimates of the policy and transition functions are used to construct estimates of the value functions which form the moment conditions employed in the second stage. A forward simulation procedure is used to estimate the ex-ante value function $V_i(x_t;\sigma_t,\theta)$ for each household i at state x_t. The procedure allows me to obtain an estimate of the value functions for different strategy profiles σ_t given a parameter vector θ. It is important to note that the forward simulation relies on the assumption that each household i has perfect foresight on the states of its neighbours in the future periods i.e., average assets, income and age.[29] A single simulated path is obtained as follows:[30]

$$\widehat{V}_i(\widetilde{x}_t;\sigma,\theta)=\frac{1}{S}\sum_{s=1}^{S}\sum_{\tau=t}^{T=\mathcal{A}}\beta^{\tau}u_i(\widetilde{x}_\tau,\sigma_\tau(\widetilde{x}_\tau,\varepsilon_{\tau s});\theta)+\varepsilon_{i\tau s}^{d}$$

1. Starting at state $x_0=x_t=(x_{it},x_{-it})$
2. Draw shocks $\left(\varepsilon_{i0s}^{1},\varepsilon_{i0s}^{0},u_{i0s},\xi_{i0s}\right)$ for each household i
   (a) By drawing measurement error shocks in income the state space augments to $\widetilde{x}_{i0s}=(x_0,\xi_{i0s})$
3. Using the policy function estimate $\widehat{\sigma}_i(\widetilde{x}_0,\varepsilon_{i0s};\psi_1)=\left[\widehat{\delta}_i(\widetilde{x}_0,\varepsilon_{i0s};\psi_{11}),\widehat{c}_i^{o}(\widetilde{x}_{i0};\psi_{12})\right]$ compute the specified choices $\left(d_{i0s},c_{i0s}^{o}\right)$ for each household i and the resulting per period utility $u_i\left(d_{i0s},c_{i0s}^{o},x_{i0};\theta\right)+\varepsilon_{i0s}^{d_{i0s}}$
4. Using the estimated transition functions $\widehat{f}\left(\cdot|d_{i0s},c_{i0s}^{o},\widetilde{x}_{i0s},\psi_2\right)$ draw a new state $\widetilde{x}_{i1s}$ for each one of the households i and move forward to the next period
   - **Hybrid:** The evolution for $\bar{k}$ is determined by aggregating the adoption decisions for all households, $\bar{k}_{1s}=\bar{k}_{0s}+\frac{1}{N}\sum_i d_{i0s}$

[29]In future work an idea would be to relax reliance on the somewhat strong assumption of perfect foresight.

[30]The forward simulation to construct V_i for each household is performed village by village using the relevant conditional choice probability estimates.

5. Forward simulation entails repeating steps 2-4 for each household i, $t = T$ periods forward i.e., till each household reaches terminal age $\mathcal{A}$

6. Steps 1-5 generates a single path of play for each one of the households

7. The entire process is repeated for S draws of shocks and average i's discounted sum of utilities over the S simulated paths.

Averaging over the S simulated paths gives an estimate $\widehat{V}_i$ for the value function $V_i(x_t; \sigma_t, \theta)$. Such an estimate can be obtained for any (σ, θ) pair, including the $(\widehat{\sigma}, \theta)$ where $\widehat{\sigma}$ is the policy estimate from the first-stage estimation. It follows that $\widehat{V}_i(x; \widehat{\sigma}, \theta)$ is an estimate of household $i's$ sum of discounted utility from $\widehat{\sigma}_i$ given the strategy of other households $\widehat{\sigma}_{-i}$, where $\widehat{\sigma} = (\widehat{\sigma}_i, \widehat{\sigma}_{-i})$. Note that under the optimal strategy σ^*, $V_i(x; \sigma^*, \theta)$ denotes the solution to the bellman equation while for an arbitrary strategy σ, $V_i(x; \sigma, \theta)$ denotes the value function at an arbitrary strategy σ.

4.4.3 Second-stage: Structural Parameters

The second stage combines first stage estimates with the necessary conditions for equilibrium from the model, to recover the structural parameters that rationalize the observed policies as a set of optimal decisions. The equilibrium inequalities in Equation (4.3.10) define a set of parameters that rationalize the underlying strategy profile σ as a Markov perfect equilibrium (MPE) of the game.[31] Under the assumption of $Assump : SingleMPE$ and $Assump : Excl.Rest$ the second stage estimator discussed in BBL (2007) yields standard point estimates of the parameters. Following the notation of BBL (2007) I define an equilibrium condition as:

$$g_i\left(\left(i, x, \sigma_i^{'}\right)_{\lambda}; \theta, \psi\right) = V_{i(\lambda)}\left(x_{(\lambda)}; \sigma_i, \sigma_{-i}, \theta, \psi\right) - V_{i(\lambda)}\left(x_{(\lambda)}; \sigma_{i(\lambda)}^{'}, \sigma_{-i}, \theta, \psi\right)$$

where $\lambda \in \Lambda$ indexes the equilibrium conditions denoted by a combination $\left(i, x, \sigma_i^{'}\right)$ and ψ denotes parameters from the first-stage process. Each inequality indexed by λ is satisfied at θ, ψ if $g_i\left(\left(i, x, \sigma_i^{'}\right)_{\lambda}; \theta, \psi\right) \geq 0$. The model's parameters are estimated as the solution to this system of inequalities by employing a minimum distance estimator

[31] BBL(2007) denote Θ_0 as the set:

$$\Theta_0(\sigma, f) := \left\{\theta : \theta, \sigma, f \text{ satisfy (4.3.10) for all } x, i, \sigma_i^{'}\right\}$$

The goal of the second stage is to recover Θ_0 using the first stage estimates of the policy functions σ and transitions functions f. Depending on the model and its parametrization, the set Θ_0 may or may not be a singleton.

that minimizes violations of these optimality conditions. The objective function that is minimized is given by:

$$Q(\theta,\psi) = \int \left(\min\left\{g\left(\left(i,x,\sigma_i'\right)_\lambda;\theta,\psi\right),0\right\}\right)^2 dH(\lambda)$$

where $H(\cdot)$ is the distribution over the set Λ of inequalities.[32] Under the assumptions that ensure the model is point identified and that $H(\cdot)$ has a sufficiently large support, $Q(\theta,\psi) > 0$ for all $\theta \neq \theta_0$.

Parameter θ is estimated by minimizing the sample analogue of the objective function at $\psi = \widehat{\psi}$.[33]

$$Q_n(\theta,\widehat{\psi}) := \frac{1}{n_I}\sum_{k=1}^{n_I}\left(\min\left\{\widehat{g_i}\left(\left(i,x,\sigma_{i(k)}'\right)_{\lambda_k};\theta,\widehat{\psi}\right),0\right\}\right)^2$$
$$\widehat{\theta}\ \arg\min_{\theta\in\Theta} Q_n(\theta,\widehat{\psi}) \tag{4.4.1}$$

where $\widehat{g_i}(\cdot)$ is the empirical counterpart to $g(\cdot)$

$$\widehat{g_i}\left(\left(i,x,\sigma_{i(k)}'\right)_{\lambda_k};\theta,\widehat{\psi}\right) = \widehat{V}_{i(\lambda)}\left(x_{(\lambda)};\widehat{\sigma}_i,\widehat{\sigma}_{-i},\theta,\widehat{\psi}\right) - \widehat{V}_{i(\lambda)}\left(x_{(\lambda)};\sigma_{i(\lambda_k)}',\widehat{\sigma}_{-i},\theta,\widehat{\psi}\right)$$

By constructing empirical counterparts to all or a subset of the equilibrium inequalities using the forward simulation procedure described in section (4.4.2) the idea is to search for values of θ that minimizes the violations of these inequalities. Standard errors are computed using a bootstrap procedure. Further details are provided in Appendix (4.8.2).

4.4.4 Parameter Estimates

The model has a total of 74 parameters. I focus here on a subset, in particular on the estimates describing preferences obtained in the second stage in Table (4.2).[34]

Utility in the model is derived from consumption and having sanitation at home. Everything else equal, households derive higher utility from having sanitation at home with the differential effects based on the age of the household head. The estimate of ν indicates a decreasing marginal utility of consumption. While the

[32] The true parameter vector θ_0 satisfies $Q(\theta_0,\psi_0) = 0 = \min_{\theta\in\Theta} Q(\theta,\psi_0)$ where Θ contains θ_0.

[33] The Nelder Mead algorithm was employed to minimize the objective function. Monte Carlo simulations were performed using simulated data to understand the properties of the BBL (2007) estimator.

[34] A complete list of the parameters estimates is provided in Appendix (4.8.4), which includes first stage estimates in Tables (4.3) -(4.5) and the estimates for the village level fixed effects $\widehat{\mu}_g$ in Table (4.6).

parameter η indicates a marginally higher return derived from food/other consumption by having sanitation at home. Households also derive additional utility from the aggregate coverage of sanitation within the village. This effect is captured by the positive parameter estimates for γ and ϕ which capture the non-separable effect of average sanitation coverage on the additional gains a household derives from it own consumption and sanitation.

4.4.5 Model Fit

The fit of the model is evaluated along two dimensions: life cycle profiles and aggregate sanitation coverage observed in the data. The model closely matches the observed behaviour for sanitation adoption choices as well as asset accumulation over the life-cycle. Figure (4.2) plots the empirical and model generated profiles for the fraction of sanitation adoption (4.2.a) and mean assets (4.2.b) by age of the household head. The estimation procedure does not directly employ the empirical age profiles as moments in the estimation of the structural parameters and thus the fit can be viewed as one 'non targeted' moments. Overall the model replicates household behaviour over the life cycle in terms of sanitation adoption fairly well. There exists discrepancy at older ages, the model predicts higher adoption levels than observed in the data past the age of 55. Additional discussion of the model fit with respect to the sanitation coverage across villages observed in the data is included in Section (4.5.2).

Impact of Liquidity Constraints. As a robustness check, I also look at how the simulated sanitation adoption behaviour would change if the underlying model were to be re-estimated without restricting households from borrowing against their future income. I re-estimate the model where under the forward simulation procedure differently from before I allow households to have negative asset holdings (debt) based on optimization of their consumption and sanitation choice subject to the natural borrowing constraint and terminal age conditions.[35] The re-estimated preference parameters are provided in Table (4.4) in the appendix.

Figure (4.3) plots the simulated sanitation adoption profile under the re-estimated model along with the empirical profile. Under relaxation of the liquidity constraints the model simulation predicts a marginal increase in the proportion of sanitation adoption at each age. Though small, the gap between the two model generated profiles decreases with age. This is driven by the model feature where conditional on

[35]Each household must leave terminal age $\mathcal{A}$ without debt i.e., $A_{\mathcal{A}+1} \geq 0$.

not being liquidity constrained households find it optimal to adopt sanitation earlier rather than later in life so as to enjoy the utility from sanitation over a longer time horizon. The maximum difference between the two simulated profiles is less than 7 % and the profile generated under relaxed borrowing constraints is well within the 95 % confidence interval bounds estimated under the model with liquidity constraints. Overall, it is possible to conclude that the household sanitation adoption behaviour under the model where borrowing is fully restricted is robust under this assumption. Though the model where borrowing is fully restricted is taken as a good fit of the data, this exercise does not necessarily conclude that the observed households are indeed liquidity constrained, as the constraint $A_t \geq 0$ may not necessarily bind for all households. Further analysis of this issue is included with the counterfactual policy simulations in Section (4.6).

4.5 Simulation of Sanitation Adoption

In this section, I outline the simulation method to determine sanitation equilibrium levels under different counterfactual environments. Under single agent models the underlying assumption is that each household's outcome varies only with its own policy treatment. To accommodate the presence of externalities, I need to allow a household's outcome to also depend on the outcomes of other households impacted by the policy thereby allowing for multiple equilibria in counterfactual environments. In order to conduct counterfactual experiments in models with multiple equilibria, one approach would be to define the Equilibrium Selection Mechanism (ESM) and to compute the full set of equilibrium under estimated parameter values. However, even if it would be possible to compute the full set equilibria,[36] observed behaviour does not provide any additional guidance on the underlying equilibrium selection rule played.

Most examples in the literature instead impose an equilibrium selection rule ex-ante under which the model is both estimated and simulated for counterfactual policies. A key limitation of this approach is that it does not allow for the possibility that the equilibrium selection mechanism may itself change under counterfactual

[36] Iskhakov, Rust & Schjerning (2016) propose an algorithm, *Recursive Lexiographic Search (RLS)* that attempts to solve for all Markov Perfect Equilibria for a class of Markovian Games that they define as *Directional Dynamic Games (DDG)*. The directional property of the game is defined over the stochastic evolution of certain state variables other than the passage of calendar time or age. Under certain conditions the model in this paper also satisfies directionality in the evolution of the sanitation coverage within a village.

environments. Thus the policy simulation given a selection rule may not be valid under a different counterfactual environment.

My approach here differs, instead of solving and simulating for all possible equilibria I instead bound the set of possible equilibria by an upper and lower limit. To evaluate the effect of counterfactual policies on equilibrium sanitation prevalence, I focus on the resulting shifts in the upper and lower bounds based on changes to the underlying environment.[37] The approach of bounding the equilibrium set is appealing as it allows for changes in the underlying equilibrium selection rules played under counterfactual environments. However, a trade-off for this advantage is that it only allows one to bound the region where the impact of the policy may lie. If the estimated bounds under a policy simulation are too wide this would affect the precision of the policy implications derived.

4.5.1 Strategic Complementarity

To ascertain household behaviour under counterfactual policies, I propose the following approach. First, I characterize the conditions under which the model implies strategic complementarity in the adoption decisions. Specifically, I verify whether the household objective function displays the properties of a supermodular game with respect to the adoption decision, i.e., the household preferences satisfy *Increasing Differences* over the sanitation adoption dimension. Then I derive conditions under which such a condition is sufficient to ensure that the set of equilibria satisfies the ordinal properties that characterize supermodular games, i.e. there is a highest and a lowest pure strategy equilibrium with respect to a household's sanitation adoption. I then exploit the properties of such a structure to characterize the upper and lower bound of the equilibrium sanitation adoption level at village level. The interpretation of the bounds obtained under the iteration procedure depends the properties of the model described in this paragraph. The approach extends a traditional result of supermodular games by employing a notion of separability of the objective function over the choice set similar to Topkis (1978).[38] A detailed derivation of the result is provided in Appendix (4.8.3). The argument is arranged in three steps and is greatly simplified by the choice of timing employed in the model i.e., 'time to build' sanitation.

[37]The idea of bounding the equilibrium set has previously been employed by Jia (2008), De Paula (2009) and Bjorkegren (2014) in different contexts to solve and simulate the model under multiple equilibria.

[38]Ref. Topkis (1978) Theorem (3.3)

The intuition behind the main result is as follows. If household sanitation adoption decisions are strategic complements, then a larger average level of adoption in the village makes adoption - *ceteris paribus* - more attractive for every household. This is the case if the household's objective function satisfies *Increasing Differences* with respect to the adoption decision. I prove that this condition is satisfied by the model under weak assumptions. On the other hand, a larger average level of sanitation adoption may change the trade-off of the consumption and saving decisions of a household and this may, in turn, affect household adoption decisions. I show that, if the parameter capturing the interaction between private consumption and average level of sanitation coverage in the village: ϕ is sufficiently close to zero, then the second channel vanishes. As a consequence, the existence of a highest and a lowest equilibrium with respect of household sanitation adoption is ensured. This result dramatically simplifies the counterfactual analysis, because in order to bound the set of equilibria of the game with respect to the average level of adoption in equilibrium it is sufficient to simulate bounds for two specific upper and lower equilibria.

4.5.2 Simulation Method

To compute the bounds for the set of equilibrium sanitation adoption levels, I use an iterated best response algorithm to search for the fixed points. As explained above the procedure does not attempt to recover all possible equilibria, but instead only the upper and lower limits characterizing the set of possible equilibria. The algorithm can be divided into two steps. I first construct a candidate adoption path using the forward simulation procedure described in section (4.4.2) under estimated parameter values. There are an initial set of households $\{k_{i0}\}_{i=1}^{N}$ who made their sanitation adoption decision before my data begins, I hold their decision fixed. For baseline simulations, the initial adoption level $\bar{k}_0$ is set equal to the sanitation level observed in the first period of the data, such that at the first step of the algorithm households expect the level of sanitation observed in the data.

In the second step, an iterative procedure is used on each candidate adoption path to obtain a fixed point. The index τ denotes an iteration. Each candidate adoption path and equilibrium identified depends on the initial adoption level $\bar{k}_0$ along with the vector of observed states and taste shocks $\varepsilon = \{\varepsilon_{i0}\}_{i=1}^{N}$ drawn. To locate the lower and upper limits $\left(\bar{k}^L, \bar{k}^U\right)$, assumptions are imposed on the future adoption path of all households within a village. For the lower limit, I assume that each household i believes that the level of sanitation in each subsequent period until i reaches terminal age $\mathcal{A}$ remains at the initial level $\bar{k}_0$ under each iteration. Similarly for the upper limit, each household i believes that the level of sanitation in each subsequent period until $\mathcal{A}$ is close to $\approx 100\%$ coverage.

For the lower bound $\overline{k}^L$:

1. Under the assumption that the future level of adoption remains at observed $\overline{k}_0^{\tau=1}$ till terminal age $\mathcal{A}$ (given iteration $\tau = 1$)
2. Draw set of private taste shocks ε_{i0} for each household $i = 1,...,N$ and allow each household to optimize their decision, holding fixed the adoption path of other households given assumptions about the future evolution of $\overline{k}$
3. Compute utility and choices (d_{it}, c_{it}) for each household i using the estimated policy and transitions functions
4. Forward simulate each household's choice problem until each household i reaches $\mathcal{A}$ terminal age
5. With each move one period forward update the sanitation level $\overline{k}$ by averaging over the adoption decisions for all households each period: $\overline{k}_1^{\tau=1}, \overline{k}_2^{\tau=1}, \overline{k}_3^{\tau=1}$ etc.
6. Continue forward simulation for each household updating the sanitation level $\overline{k}$ till $\overline{k}_M^{\tau=1} = 1.0$ i.e., all household have adopted and obtain a candidate adoption path denoted by vector $\overrightarrow{\overline{k}}^{\tau=1} = \left(\overline{k}_0^1, \overline{k}_1^1, ..., \overline{k}_M^1\right)$
7. Repeat steps 1-6 under the same ε_{i0} draws to obtain another candidate adoption vector $\overrightarrow{\overline{k}}^{\tau=2} = \left(\overline{k}_0^2, \overline{k}_1^2, ..., \overline{k}_M^2\right)$
   - Where initial $\overline{k}_0^{\tau=2}$ level (starting point) is obtained by computing the adoption decision rules for all households i under the assumption that $\overrightarrow{\overline{k}}^{\tau=1}$ is the relevant adoption path played.
8. Iterate, using the path from the previous step to form the next adoption path.
9. Repeat until $\overrightarrow{\overline{k}}^{\tau+1} = \overrightarrow{\overline{k}}^{\tau}$ i.e., the adoption path vector in each iteration converges to obtain the fixed point.
   - The first element of this convergent vector $\overrightarrow{\overline{k}}^{\tau+1}$ is obtained as the lower bound $\overline{k}^L$

Similarly for the upper bound $\overline{k}^U$ the convergent vector is obtained by repeating the process starting at the observed level of $\overline{k}_0$ but under the assumption that next period onwards the level of sanitation adoption is $\approx 100\%$. In this way the sanitation adoption path vector converges with each subsequent iteration. The first element of this convergent vector is obtained as the upper bound of the set. Since a candidate adoption path and the fixed point obtained is a function of ε_{i0} shock drawn the iteration procedure is repeated for multiple draws s of $\{\varepsilon_{i0s}\}_{s=1}^{S}$ and the lower and upper bounds are computed as the midpoints of the resulting distributions.[39]

Baseline Simulation

Under the estimated model, I run the simulation procedure on the same environment as the data observed to get a sense of the model 'fit' at the aggregate village level. Using data from the first period the model is simulated one period forward for the predicted upper and lower bounds of the equilibrium level of sanitation. The upper and lower bound simulations are computed for each of the 42 village groups observed in the sample data. Figure (4.6) plots the model simulated bounds for the observed data points in period two $Data : S2$ of the sample panel where the horizontal axis denotes different villages in order of increasing sanitation prevalence.[40] Though the model simulated bounds are wide at certain points in most cases the sanitation prevalence observed in the data lies within the bounds predicted by the model.

Properties of the Model. A key condition for the strategic complementarity in sanitation adoption result in appendix (4.8.3), is that the parameter ϕ which captures the effect of the externality on private consumption in the utility function is sufficiently close to zero, $\phi \approx 0$. The parameter estimates from Table (4.2) show ϕ to be a small positive value yet significantly different from zero.

Figure (4.7) overlays the simulated bounds under $\phi = 0$.[41] The simulated bounds under the model where the value of ϕ is restricted to zero, are very close to the model where ϕ is set equal to the estimated value. The simulated bounds in the figure show a monotonic shift in the bounds, under the estimated model, with a deviation in the value from $\phi = 0 \rightarrow \phi = 0.00514$. Further robustness checks are performed in Table (4.9) Since the bounds simulated under estimated parameter values $\widehat{\theta}$ are close to the bounds simulated under $\widetilde{\theta} : \phi = 0$ for which the theoretical result ensures existence of a highest and lowest equilibria , I use the model estimated values under $\widehat{\theta}$ to simulate policy effects. Given that the upper bound predicted under $\widehat{\theta}$ lies above the upper bound under $\phi = 0$, emphasis is placed on the lower bound of the policy effect.

[39] In practice, the iteration procedure is performed for 250 independent draws for the set of taste shocks ε.

[40] Table (4.8) in the appendix provides a complete list of the simulated bounds at baseline for the observed data in period two.

[41] Table (4.8) also computes the simulated bounds at baseline under the assumption that parameter $\phi = 0$.

4.5.3 Household Valuation of Sanitation

The estimated model can be used to place a valuation on sanitation for a given household over its lifetime under a simulated equilibrium path. This section describes the method used to convert the value of the having sanitation into a monetary valuation by the household (in 2009 INR). To do this, I compute the expected lifetime utility of being in state x_{it} at each point in the state space after having adopted sanitation at the first period. Similarly, it is possible to compute the expected lifetime utility at each point in the state space under the scenario where household does not adopt sanitation. In order to obtain the compensating variation, I add income under the non-adoption scenario as a transfer into available cash-in-hand at each possible state and then recompute the value. This procedure is repeated until the household's expected lifetime utility with the additional income transfer is equal to its expected lifetime utility under sanitation adoption but without the hypothetical income transfer. With externality effects I compute the present value of utility from sanitation under each simulated equilibrium adoption path $\left(\overrightarrow{k}^{L}, \overrightarrow{k}^{U}\right)$. The compensating variation or the willingness-to-pay amounts can be computed at different points of the life cycle for a household given an adoption path.

Figure (4.8) plots the household valuation of sanitation in Indian Rs. (INR) as a function of age of the household head. Figure (4.8) shows that younger household heads place a higher valuation of sanitation since the gains from early adoption persist over time. Similarly older household heads value sanitation less since the time horizon to enjoy the benefits from having sanitation is shorter. The figure also plots a valuation driven by the underlying spillover effects. The solid black line plots the household valuation of sanitation without the endogenous effects driven by the underlying externality. While the dashed plots the household valuation with the endogenous effects incorporated at the upper and lower bound adoption paths respectively. On average household valuation for sanitation ranges between $Rs.2,75,250$ (lower) and $Rs.4,30,500$ (upper) which denotes a non-trivial amount when compared with the average household lifetime income value of $Rs.23,76,000$.[42] The upper and lower bound for the valuation with spillover effects lie above the valuation of sanitation made by a household acting in isolation. On average the difference in valuation is 52% (lower bound) and 71% (upper bound) higher once externality effects are accounted for.

[42]Lifetime income value approximated using survey data from the Gwalior Nagar Nigam Seva information drive 2010-11.

4.6 Policy Experiments

In this section, I examine the impact of different policy interventions on equilibrium sanitation coverage and welfare. In the first exercise, the question of under-adoption of sanitation is addressed by computing the socially optimal level of sanitation and comparing it with observed levels in the data. The second application focuses on the cost effectiveness of two specific policy interventions: sanitation loans and price subsidies. If a policy maker's objective is to maximize sanitation coverage? I examine which of the two policies are more cost effective. The simulations show how policy implications differ once externality effects are taken into account. In order to quantify the effect of the externality, the impact of a policy is decomposed into the private incentives (direct) from adoption and it's impact through the spillover effects (indirect). To contrast the policy implications based on maximizing coverage, I compute changes in bounds of welfare based on maximizing total welfare instead of coverage. Lastly, I study the dynamics of the age effects with potential implications for policy targeting. The impact of a policy is measured by reporting changes in the bounds of equilibrium adoption levels along with the household's willingness-to-pay for the policy under counterfactual scenarios.

4.6.1 Under-adoption of Sanitation

To determine if empirical sanitation coverage levels imply under-adoption, I compute the socially optimal level of sanitation adoption by solving the social planner problem for each village. I consider the problem of a constrained social planner whose objective is to allocate sanitation along to households so as to maximize utility subject to the total fixed endowment of resources. To compute the welfare under the social planner's regime, the following procedure is implemented: the total endowment is computed by aggregating the total consumption and sanitation value within a village. The marginal rate of technical substitution between consumption and sanitation is given by the market cost of sanitation by village ($price_g$). The planner induces households to solve the optimal adoption problem by re-allocating the total endowment between food consumption and sanitation, so as to maximize utility. I assume the planner maximizes a utilitarian Social Welfare Function (SWF) with equal pareto weights assigned to each household within the village.[43] By changing the allocation of sanitation, moving resources between sanitation and

[43] The socially optimal level of sanitation depends on the choice of pareto weights used in the Social Welfare Function (SWF).

consumption, the algorithm searches for the policy functions that maximize total household welfare until there is no other higher value attainable.

Results for a representative village are shown in Table (4.14). The total surplus attained is equal to $Rs.$ 36 million with 81% sanitation coverage. Compared with the baseline this reflects a 295.5% increase in household welfare and a 43.7% increase in sanitation coverage. This exercise reveals that the existing sanitation levels observed in villages are inefficient in the sense that households under-adopt sanitation and instead allocate a larger share of resources to private food consumption. This systematic under-adoption is driven by the under valuation of sanitation made by each household that fails to internalize the total benefit generated from sanitation. The difference in welfare attained between the baseline and social planner's regime reflects the cost of the externality induced by the divergence in the private and social valuation of sanitation.

Table (4.10) computes the utilitarian planner problem for each of the villages based on adoption level and endowment values in the first sample period. Based on the sanitation coverage observed in the data and determined under the social planner solution, the extent to which sanitation is under-adopted is computed for each of the villages. On average, the privately chosen adoption levels in the data are 53% below the socially optimal based on a utilitarian Social Welfare Function (SWF). In the subsequent counterfactual exercises, the equilibrium levels achieved under different policies are compared with the socially optimal adoption levels under the planner's problem.

4.6.2 Cost Effective Policy: Sanitation Loans and Price Subsidies

The social planner solution finds the observed sanitation levels to be below the socially optimal and highlights the potential role of policy interventions to increase sanitation coverage so as to increase total welfare. With a policy maker potentially constrained by the total funds available for allocation it is important to understand if the implemented policy is cost effective. The aim of this exercise is to understand whether specific policies are more cost effective than others at maximizing sanitation coverage. Specifically, I compare the simulated equilibrium coverage levels attained under sanitation loans and price subsidy policies of different sizes for a fixed cost of the policy. The objective under each policy is to correct the suboptimal allocation of sanitation by targeting the underlying market failures faced by individual households.

This analysis also relates to the current debate among policy makers in the field on the appropriate choice of policy to tackle the under-adoption.

To evaluate cost effectiveness I benchmark each loan and subsidy policy against the total cost of the policy to the government to allocate the respective policy. Both loans and price subsidy policies can be allocated as a fraction $(0,1]$ of the cost of sanitation ($price_g$), where 1 denotes a 100% price subsidization or loan allocation. This determines a grid of potential policy structures for which a series of policy cost schedules and corresponding equilibrium adoption levels can be generated for each village.

Figure (4.11) plots the policy cost schedules and sanitation coverage curves for both price subsidies and loans across a grid of counterfactual monetary structures for a representative village. As a conservative estimate of the policy impact the figure plots only the lower bound response. The lower panel displays the relationship between a gird of loans and subsidy policy structures and the corresponding total cost of policy to the government body. The upper panel determines the sanitation coverage levels attained over the same grid of policy structures. To calculate the present cost of a loan I take an estimate of the rate of loan repayment in the local region of $\approx 60\%$ to simulate loan repayment by the village population in my model.

This allows me to interpolate the relationship between the total cost of the policy and the sanitation coverage level attained for that cost of intervention in Figure (4.12). The figure plots the lower bound response. Figure (4.12.a) plots the response for a village that is initially at 0% coverage level and in the next period moves to a coverage level of 9%. I find that in villages with close to zero coverage sanitation loans are as effective, if not marginally more, at achieving maximum coverage for a fixed cost of the policy. As the cost of the policy increases the the effectiveness of the loan declines. The curve for the loan stops past a certain total cost value beyond which households do not find it optimal to adopt sanitation with the take-up of a loan, at which point there is no incentive to provide a loan policy.

Figure (4.12.b) plots the same relationship except that the initial sanitation level is set higher at 22%. The equilibrium cost curves show a very different pattern to panel (4.12.a) where the subsidy policy is found to be more cost effective for all policy cost values. There is also a sharp jump in the response from 0.22 to 0.69 for the price subsidy evaluated at a policy cost value of $Rs.5000\,(\times 1000\,Rs)$. This is driven by the social multiplier generated by the underlying externality effect. With a low initial level of sanitation prevalence a relatively small subsidy amount induces a lot more households to adopt sanitation this effect then multiplies generating further adoption.

In contrast the sanitation loan policy generates a much more modest increase in the adoption levels from 0.22 to 0.31 for the same cost of policy. Using the graph in Figure (4.11) to extrapolate, a $Rs.5000\,(\times 1000\,Rs)$ total cost of policy is associated with providing a price subsidy of 8% subsidization of the cost, to all households within the village. This exercise demonstrates heterogeneity in the impact of the policy with the initial sanitation coverage levels.

4.6.3 Price Subsidies: Direct and Indirect Effects

The policy exercise in the previous section reveals sanitation price subsidies to be in general more cost effective in villages with some existing sanitation coverage. To quantify the importance of externality effects in the demand response for a household, I compare the full equilibrium impact a price subsidy with the impact generated treating the household response in isolation.

Figure (4.9) simulates the equilibrium bounds under different price subsidy amounts as a fraction of the cost for each of the village observed in the data.[44] The impact of the subsides is highly non linear with the initial sanitation coverage and the shape of the response curve changes as a function of the amount of the subsidy given out. Comparing the equilibrium adoption levels under lower bounds with the socially optimal adoption level in Table (4.10), a uniform price subsidy achieves on average 62% (under 5% subsidy), 77% (under 15% subsidy) and 92% (under 25% subsidy) of a social planner's welfare outcome and sanitation allocation.[45]

With positive externalities, the sanitation coverage levels are inefficient where each household does not fully internalize the total benefit derived from adoption. To understand the welfare gain derived for the village economy from a small shift towards the socially optimal level I compute the net welfare gain (loss) generated for a single household that is on the margin of adopting and receives a price subsidy on the full cost of sanitation.[46] I find that provision a subsidy to a single household also produces a marginal increase in the welfare for the entire village community. The present cost of the subsidy is at $Rs.8,628$ but the policy shifts the bounds on welfare for the recipient household by $Rs.10,008$ (lower bound) and $Rs.12,511$ (upper bound) from the combination of direct value of having sanitation as well as the increased utility derived from the spillover effects generated. This results in a

[44] See ref Table (4.11)

[45] Since the true impact of the policy lies between the upper and lower bound, I compare with the lower bound of the policy as a conservative estimate.

[46] The welfare calculations are for a household on the margin of adopting in a representative village.

increase in the bounds on net welfare by $Rs.1,380$ (lower equilibrium) and $Rs.3,883$ (upper equilibrium) of which 33% (lower bound) to 72% (upper bound) are attributed to the indirect effect. In addition to the gain for the recipient household the subsidy also generated a subsequent gain for other households in the village which amounted to $Rs.3,181$ (lower) and $Rs.6,253$ (upper) on aggregate or an equivalent of $Rs.19.1$ (lower) and $Rs.37.2$ (upper) gain per non recipient household. It is important to note that without the presence of externalities, a price distorting subsidy policy would not improve net household welfare relative to an unconditional subsidy policy.

Overall, the impact of the price subsidies on sanitation adoption is consistent with the evidence from Guiteras, Levinsohn & Mobarak (2015) under experimental policy intervention. The subsidy not only induces a greater demand response from targeted households (relative to a pure information provision intervention), but also has a non trivial impact on the adoption decision of non-targeted households within the village. This reinforces the opinion that the design of subsidy policies, for goods with spillover effects, should not be based on targeting individuals but instead be based on targeting groups of households.

4.6.4 Dynamics over age

A price subsidy generates a substitution effect as well as an income effect on the demand response of a recipient household. Figure (4.14) decomposes the impact of a subsidy into its income and substitution effect components. The figure plots the absolute value of the Marshallian (uncompensated) and Hicksian (compensated) price elasticities at a lower equilibrium bound for a representative household over its life-cycle. The Hicksian elasticity measures the pure price effect of the good keeping the utility level fixed. While the vertical difference between the two curves is the residual income effect generated from the increase in the effective budget that a household has available to spend.

Both the Marshallian and Hicksian elasticity (in absolute value) decrease over the life-cycle as marginal utility from adoption decreases with age, this feature is driven by the life-cycle structure subject to terminal value assumptions. The income effect which also diminishes with age is relatively larger at younger ages. The excess sensitivity of the demand response earlier in the life-cycle maybe driven by binding liquidity constraints faced by younger households. This is particularly relevant at younger ages where a larger fraction of the total demand response is attributed to the income effect relative to older ages. Unable to borrow against their future income, younger households who have yet to accumulate sufficient assets respond more on the income effect margin than the price effect, upon receiving the price subsidy. Figure (4.8) also shows that a household's valuation of sanitation decreases with age. With a loan policy a household is able to move resources across time and borrow against future realizations of income to bring forward sanitation adoption. A household's decision to take a sanitation loan compares the value generated from the sanitation between today and tomorrow. Since a household's private valuation from sanitation declines with age, loans would be a preferred policy for targeting younger cohorts who place a higher valuation on sanitation adoption.

4.7 Conclusion

To understand the effectiveness of interventions that aim to maximize sanitation coverage, requires the capability of predicting and comparing outcomes under alternative counterfactual policies. This paper examines the impact of two specific policy interventions: loans v. price subsidies, on sanitation adoption behaviour in a context where household decisions interrelate due to externality effects. I formulate and estimate a dynamic household demand for sanitation that incorporates interdependence of sanitation adoption choice. To identify the model's parameters, I use a combination of household panel dataset along with exclusion restrictions that provide identifying variation at the household and village level. The model is used to compute equilibrium adoption levels and simulate the effect of loans and subsidy policies for sanitation where the recipient household's adoption decision imposes externalities on others.

I illustrate how the framework can be informative about the effectiveness and efficiency of different policies, where otherwise distorting policies instead lead to higher welfare gains when the household decision is no longer treated in isolation. A sanitation adoption subsidy to a single household costing $Rs.8,628$ improved net welfare in a low case by $Rs.1,380$ and a high case by $Rs.3,883$. A large fraction of the impact, between 33% (lower bound) to 72% (upper bound), accrues to non-recipient households. These spillover effects suggest that adoption subsidies for sanitation should not be thought of as targeting individual households, but instead as targeting the whole village or groups of households.

While a significant proportion of sanitation prevalence is driven by price incentives, a small number of households do face binding liquidity constraints for whom targeted loans is a cost effective policy. The ultimate choice of policy is strongly driven by the trade-off between the total cost considerations and the targeting objectives as well as level of sanitation coverage in the targeted village. If the objective of the policy is to 'reach' the maximum number of households, in most cases it is cost effective to provide price subsidies to incentivize adoption of sanitation. In contrast, sanitation loans are found to be cost effective in villages with close to zero initial coverage.

One of the main predictions of the model is that subsidizing the cost of sanitation is a cost effective policy with the presence of externalities. An important extension of this paper would be to combine the structural analysis with field experiment results to disentangle and identify the exact mechanisms that generates this externality. There are still open questions on the precise mechanism that drives the interdependence of sanitation adoption. Other than a health externality other suggested mechanisms include, the presence of information externalities as well as infrastructure and amenity spillovers generated from collective adoption. A better understanding of the mechanisms will not only improve our understanding of the nature of the gains derived from adoption but could also provide insights on improving the efficiency of future policy interventions.

Although the empirical application focuses on the specific topic of sanitation, the structure developed in this paper can be used to study other applications where household/individual decisions interrelate due to spillover effects. The findings in this paper highlight the fact that, when externalities exist, accounting for equilibrium interactions and quantifying its effect has important policy implications. The structure can also be extended to study adoption patterns of other preventive healthcare goods in the developing world for e.g. vaccinations. The size and nature of the externality depends on the specific characteristics of different healthcare goods. The degree of 'social benefit' associated with vaccination adoption may differ from sanitation and thus would provide different policy implications. The analysis in this paper finds subsidies to be a more 'favoured' policy the larger is the impact of the externality or the degree of 'social benefit' associated with the healthcare good. These extensions can provide useful information that can help poor communities as a whole to minimize inefficiencies and absorb the overall benefits thus tackling poverty and mitigating its detrimental effects.

4.8 Appendix for Chapter 4

4.8.1 Data

Estimation of age profiles

I control for cohort effects to obtain the life-cycle profiles using data from different household cohorts. The age profiles of interest such as, sanitation adoption and asset accumulation by age of household head depicts dynamics that the model should be able to replicate. The main concern, when constructing age profiles that account for cohort effects, is with effect of family size, year effects and household specific effects. To account for cohort effects, which is just the average fixed-effect of all households in a single cohort, I follow the approach discussed in French (2005) to obtain age profiles for both sanitation adoption and asset accumulation over a household head's lifetime. The smooth age profiles are obtained by employing a local polynomial regression.

4.8.2 Estimation

Conditional Choice Probability (*CCP*). The conditional choice probabilities are estimated from the observed data. The underlying assumption to obtain consistent equilibrium choice probability estimates relies on the data being generated from the same Markov profile. This assumption however may not hold true when data is pooled across multiple villages. The overall sample of 42 villages are divided into four groups based on village level observables and geographic proximity to one another.

Partially observed sample. To account for the fact that only part of the entire village household behaviour is observed. I implement the correction method from Chesher (1991) extended in Gautam (2015) to account for this source of measurement error in the data and its impact on the choice probability estimates.

Income Process. The function relating age and education to income earnings is given by:

$$\ln y_{it} = f\left(age_{it}, edu_i\right) + z_{it} + \xi_{it}$$

$$f\left(age_{it}, edu_i\right) = \psi_0^y + m_a^y\left(age_{it}\right) + \psi_{edu1}^y edu_i(yrs) + \psi_{edu2}^y\left(edu_i(yrs)\right)^2 + \psi_3^y\left[age_{it} \times edu_i(yrs)\right]$$

$$z_{it} = \rho z_{it-1} + u_{it}, \quad u_t \sim N\left(0, \sigma_u^2\right)$$

where $m_a^y(age_{it})$ are piecewise linear functions in age of the household head with nodes at $20, 25$ and 50. The education of the primary earner (household head) is measured in years $edu_i(yrs)$ along with an interaction term between age and education. The permanent income also includes an A.R.(1) component with persistence parameter calibrated at $\rho = 1$ and the variance of the permanent shocks σ_u^2. Measurement error ξ_{it} shocks are distributed *i.i.d* with mean zero and variance σ_ξ^2.

Equilibrium Condition Inequalities. To estimate $\widehat{g}_i(\cdot)$, defined in sub section (4.4.3), I compute estimates for $\widehat{V}_i\left(x;\sigma'_{i(k)},\widehat{\sigma}_{-i},\theta,\psi\right)$ for a set of alternatives policies σ'_i. To implement this, let $\{\lambda_k\}_{k=1,\ldots,n_I}$ be a set of chosen inequalities from Λ indexed by $\left(i, x, \sigma'_i\right)$ which represent *i.i.d* draws from $H(\cdot)$. BBL (2007) prescribe a variety of ways to choose inequalities. The method of selecting inequalities will have implications for efficiency, but for consistency the only requirement is that $H(\cdot)$ has sufficient support to yield identification.

I draw households i (ε_i) and states x at random and then consider alternative strategies σ'_i that are slight perturbations of the estimated policy $\sigma_i(x,\varepsilon_i;\widehat{\psi})$ i.e., $\sigma'_i(x,\varepsilon_i) = \sigma_i(x,\varepsilon_i;\widehat{\psi}) + \omega$. Given that the strategy σ_i is a tuple consisting of $\left[\delta'_i(x,\varepsilon_i), c_i^{o'}(x_i)\right]$ the perturbation is on both the discrete decision rule $\left\{\delta'_i(x,\varepsilon_i) = 0, if\, \delta_i(x,\varepsilon_i) = 1\right\}$ as well as the continuous consumption policy $c_i^{o'}(x_i) = c_i^{o'}(x_i) + \omega$ for each household. Note that to check for profitable deviations would entail checking one step deviations on either side of the optimal strategy. Given a binary discrete choice there would be four perturbed strategies for each estimated strategy σ. However as the household's maximization problem is not concave it is important to check further than one step deviations. Thus for each chosen inequality, λ_k the next step is to use the forward simulation procedure from section (4.4.2) to construct sample analogues for each of the $V_i\left(x;\sigma'_{i(k)},\sigma_{-i},\theta,\psi\right)$ value functions at the perturbed policy $\sigma'_{i(k)}$, with ω drawn from a normal distribution with mean zero and variance related to the variance of the measurement error shocks in the income process. In practice, two different sizes for the inequality draws was used $n_I = 500$ and $n_I = 1000$. No discernible difference in magnitude of final estimates was found as n_I increased from 500 to 1000.

Standard Errors. The standard errors are computed using bootstrap re-sampling. The villages to which households belong are the unit of re-sampling over which repeated samples of 42 villages are drawn with replacement. Bootstrap is performed over both estimation stages. The first stage elements of the estimation are repeated over each bootstrap sample followed by the second stage. A total of 250 bootstrap samples were drawn to construct standard errors.

4.8.3 Simulation

Supermodular Objective Function.

The objective function of a household i at time t can be expressed as:

$$v_i(d_{it}, c_{it}, x_t; \sigma_t) = u_i^{\sigma_t}(c_{it}, d_{it}, x_{it}) + \beta \sum_{x_{t+1}} V_i(x_{t+1}; \sigma_{t+1}) f_i^{\sigma_t}(x_{t+1} | x_t, d_{it}, c_{it})$$

Define the following notation:

$\hat{c}_{it}^1 = (1+r)A_{it} - A_{it+1} + y_{it} - price_t$

$\hat{c}_{it}^0 = (1+r)A_{it} - A_{it+1} + y_{it}$

$x_{it} = \left(a_{it}, A_{it}, y_{it}, k_{it-1}, \bar{k}_{t-1}, price_t, \xi_{it}\right)$

$\hat{u}_{it,c}^d = \frac{\partial}{\partial c_{it}}\left[u_i^{\sigma_t}(c_{it}, d_{it}, x_{it})\right]$

$\hat{u}_{it,cc}^d = \frac{\partial^2}{\partial c_{it}^2}\left[u_i^{\sigma_t}(c_{it}, d_{it}, x_{it})\right]$

$v_{it}^d = v_i(d_{it}, c_{it}, x_t; \sigma_t)$

Denote the objective functions of an individual household with states x_t, that adopts at time t (i.e $d_{it} = 1$) with $v_{it}^1 = v_i\left(d_{it} = 1, \hat{c}_{it}^1, x_t; \sigma_t\right)$ and of a household that does not adopt (i.e $d_{it} = 0$) with $v_{it}^0 = v_i\left(d_{it} = 0, \hat{c}_{it}^0, x_t; \sigma_t\right)$. The conditional choice-specific value functions are expressed as:

$$v_i\left(d_{it} = 1, \hat{c}_{it}^1, x_t; \sigma_t\right) = \left(\hat{c}_{it}^1\right)^{\nu}\left(1 + \eta k_{it-1} + \phi \bar{k}_{t-1}\right) + \alpha k_{it-1} + \gamma k_{it-1}\bar{k}_{t-1}$$
$$+\beta \textstyle\sum_{x_{t+1}} V_i(x_{t+1}; \sigma_{t+1}) f_i^{\sigma_t}\left(x_{t+1} | x_t, 1, \hat{c}_{it}^1\right)$$

$$v_i\left(d_{it} = 0, \hat{c}_{it}^0, x_t; \sigma_t\right) = \left(\hat{c}_{it}^0\right)^{\nu}\left(1 + \eta k_{it-1} + \phi \bar{k}_{t-1}\right) + \alpha k_{it-1} + \gamma k_{it-1}\bar{k}_{t-1}$$
$$+\beta \textstyle\sum_{x_{t+1}} V_i(x_{t+1}; \sigma_{t+1}) f_i^{\sigma_t}\left(x_{t+1} | x_t, 0, \hat{c}_{it}^0\right)$$

Therefore, the unconditional value function is given by:

$$V_i(x_t;\sigma_t) = \int \max_{d_{it}\in\mathcal{D}_{i,t}, c_{it}\in C_{i,t},} \{v_i(d_{it}, c_{it}, x_t;\sigma_t) + \varepsilon_{it}\} g(\varepsilon_{it}) d\epsilon_{it}$$

$$V_i(x_t;\sigma_t) = p_i^{\sigma_t}(d_{it}=1|x_t)\cdot \left[v_i(d_{it}=1, c_{it}, x_t;\sigma_t) + \varepsilon_{it}^1\right] + [1 - p_i^{\sigma_t}(d_{it}=1|x_t,\sigma_t)]\cdot \left[v_i(d_{it}=0, c_{it}, x_t;\sigma_t) + \varepsilon_{it}^0\right]$$

The expected value of the continuation value at time $t-1$ is:

$$p^{\sigma_t}(d_{it}=1|x_t) = Pr\left(v_i\left(d_{it}=1, \hat{c}_{it}^1, x_t;\sigma_t\right) + \varepsilon_{it}^1 \geq v_i\left(d_{it}=0, \hat{c}_{it}^0, x_t;\sigma_t\right) + \varepsilon_{it}^0\right)$$

$$= \begin{cases} P\left(v_i(d_{it}=1, \hat{c}_{it}^1, x_t;\sigma_t) - v_i(d_{it}=0, \hat{c}_{it}^0, x_t;\sigma_t)\right) & k_{it-1}=0 \\ 0 & k_{it-1}=1 \end{cases}$$

where P is a twice continuously differentiable and weakly increasing function. Integrating over the space of possible states x_{t+1} one gets the expected utility:

$$E_{t-1}[V_i(x_t;\sigma_t)] = \sum_{x_t} \left\{v_i\left(d_{it}=0, \hat{c}_{it}^0, x_t;\sigma_t\right)[1 - p^{\sigma_t}(d_{it}=1|x_t)] + \right.$$

$$\left. + v_i(d_{it}=1, \hat{c}_{it}^1, x_t;\sigma_t)[p^{\sigma_t}(d_{it}=1|x_t)]\right\} f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it})$$

where $p^{\sigma_t}(d_{it}=1|x_t)$ is the probability that a household facing states x_t decides to adopt. The functions $v_i(d_{it}=1, \hat{c}_{it}^1, x_t;\sigma_t), v_i(d_{it}=0, \hat{c}_{it}^0, x_t;\sigma_t)$ - if the solution for A_{it+1} is interior - have derivatives with respect to state A_{it} equal to:

$$\frac{\partial}{\partial A_{it}}\left[v_i\left(d_{it}=1, \hat{c}_{it}^1, x_t;\sigma_t\right)\right] = \nu\left[\hat{c}_{it}^1\right]^{\nu-1}(1+r)(1+\eta k_{it-1}+\phi\bar{k}_{t-1})$$

$$\frac{\partial}{\partial A_{it}}\left[v_i\left(d_{it}=0, \hat{c}_{it}^0, x_t;\sigma_t\right)\right] = \nu\left[\hat{c}_{it}^0\right]^{\nu-1}(1+r)(1+\eta k_{it-1}+\phi\bar{k}_{t-1})$$

because of the Envelope condition. Specifically,

$$\frac{\partial}{\partial A_{it}}\left[v_i\left(d_{it}=0, \hat{c}_{it}^0, x_t;\sigma_t\right)\right] = \nu\left[\hat{c}_{it}^0\right]^{\nu-1}(1+r)(1+\eta k_{it-1}+\phi\bar{k}_{t-1}) +$$

$$+\frac{dA_{it+1}^*}{dA_{it}}\left[-\nu\left[\hat{c}_{it}^0\right]^{\nu-1}(1+\eta k_{it-1}+\phi\bar{k}_{t-1}) + \beta\frac{\partial}{\partial A_{it+1}}\sum_{x_{t+1}} V_i(x_{t+1};\sigma_{t+1}) f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it})\right]$$

Notice that the second line (in the equation above) is equal to zero if the household is in an interior solution for A_{it+1} at time t because of the FOCs with respect to A^*_{it+1}. This is also true in a corner solution, because in such a case $\frac{dA^*_{it+1}}{dA_{it}} = 0$. Thus it is true for any optimal level of A_{it+1}.

$$\frac{\partial}{\partial A_{it}} \sum_{x_t} V_i(x_t;\sigma_t) f_i^{\sigma_{t-1}}(x_t|x_{t-1}, d_{it-1}, \hat{c}_{it-1}) =$$

$$\sum_{x_t} \left\{ \frac{\partial}{\partial A_{it}} \left[v_i\left(d_{it}=0, \hat{c}^0_{it}, x_t;\sigma_t\right)\right] \left[1 - p^{\sigma_t}(d_{it}=1|x_t)\right] + \frac{\partial}{\partial A_{it}} \left[v_i\left(d_{it}=1, \hat{c}^1_{it}, x_t;\sigma_t\right)\right] \left[p^{\sigma_t}(d_{it}=1|x_t)\right]\right.$$

$$\left. + \frac{\partial}{\partial A_{it}} \left[p^{\sigma_t}(d_{it}=1|x_t)\right] \left[v_i\left(d_{it}=1, \hat{c}^1_{it}, x_t;\sigma_t\right) - v_i\left(d_{it}=0, \hat{c}^0_{it}, x_t;\sigma_t\right)\right] \right\} f_i^{\sigma_{t-1}}(x_t|x_{t-1}, d_{it-1}, \hat{c}_{it-1})$$

$$= \sum_{x_t} \left\{ u_c(\hat{c}^0_{it}, x_t)[1 - p^{\sigma_t}(d_{it}=1|x_t)] + u_c(\hat{c}^1_{it}, x_t)\left[p^{\sigma_t}(d_{it}=1|x_t)\right]\right.$$

$$\left. + P'\left(v^1_{it} - v^0_{it}\right)\left[v_i(d_{it}=1, \hat{c}^1_{it}, x_t;\sigma_t) - v_i(d_{it}=0, \hat{c}^0_{it}, x_t;\sigma_t)\right](1 + \eta k_{it-1} + \phi \bar{k}_{t-1})\nu \left[\left(\hat{c}^1_{it}\right)^{\nu-1} - \left(\hat{c}^0_{it}\right)^{\nu-1}\right]\right\}$$

$$\cdot f_i^{\sigma_{t-1}}(x_t|x_{t-1}, d_{it-1}, \hat{c}_{it-1})$$

Now update time to t:

$$\frac{\partial}{\partial A_{it+1}} \sum_{x_{t+1}} V_i(x_{t+1};\sigma_{t+1}) f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it}) =$$

$$\sum_{x_{t+1}} \left\{ \hat{u}^0_c(1+r)[1 - p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})] + \hat{u}^1_c(1+r)\left[p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right]\right.$$

$$+ P'\left(v^1_{it+1} - v^0_{it+1}\right))[v_i(d_{it+1}=1, \hat{c}^1_{it+1}, x_{t+1};\sigma_{t+1}) - v_i(d_{it+1}=0, \hat{c}^0_{it+1}, x_{t+1};\sigma_{t+1})]$$

$$\left. \cdot (1 + \eta k_{it} + \phi \bar{k}_t)\nu \left[\left(\hat{c}^1_{it+1}\right)^{\nu-1} - \left(\hat{c}^0_{it+1}\right)^{\nu-1}\right]\right\} f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it})$$

REQUIREMENTS

4.8.3.1 i-Constant Differences in A_{it+1}, σ_t

Consider two strategy set σ_t' and σ_t such that $\sigma_t' \overset{d_t}{\geq} \sigma_t$ if and only if $d_t'(x_t, \varepsilon_t) \geq d_t(x_t, \varepsilon_t)$ for all x_t, ε_t. Notice that $\sigma_t' \overset{d_t}{\geq} \sigma_t$ implies:

$$\int_0^{\hat{k}_t} f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it}) - f_i^{\sigma_t'}(x_{t+1}|x_t, d_{it}, \hat{c}_{it})\, d\bar{k}_t \geq 0$$

for all $\hat{k}_t \in [0,1]$. This is equivalent to saying that $Pr(\bar{k}_t < \hat{k}_t|x_t, d_{it}, \hat{c}_{it}; \sigma_t) \geq Pr(\bar{k}_t < \hat{k}_t|x_t, d_{it}, \hat{c}_{it}; \sigma_t')$. In other words, the distribution of $\bar{k}_t$ under strategy set σ_t' first order stochastically dominates the distribution of $\bar{k}_t$ under strategy set σ_t. In other words, higher levels of $\bar{k}_t$ become more likely and lower levels of $\bar{k}_t$ become less likely. Notice that no restrictions are imposed on the distribution of the remaining part of vector x_{t+1}.

I aim to show that at constant $d_{it} \in \{0,1\}$ and at any possible choice A_{it+1}, for any $\lambda > 0$, there exists a threshold $\varphi > 0$ such that, for $|\phi| \leq \varphi$, then the following inequality holds:

$$\left| \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t'\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)}{\partial A_{it+1}} \right| \leq \lambda$$

for all states x_t and all σ_t', σ_t. Define the vector $z_{t+1} = \left\{a_{t+1,}, A_{-it+1}, y_{t+1}, k_t, \bar{k}_t, price_{t+1}, \xi_{t+1}\right\}$ where $A_{it+1} \notin z_{t+1}$. Now rewrite $v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)$ as follows:

$$v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right) = u_{it}^d + \beta \int_{z_{t+1}} \int_{A_{it+1}} V(x_{t+1}; \sigma_{t+1})\, f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it})\, dA_{it+1}\, dz_{t+1}$$

because A_{it+1} is deterministic, i.e., $f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it}) > 0$ if $A_{it+1} = \hat{A}_{it+1}$ and zero otherwise, and it does not affect the transition probability of any state in z_{t+1}, it is possible to show that:

$$v_i\left(d_{it},c_{it}^d,x_t;\sigma_t\right) = u_{it}^d + \beta \int\limits_{z_{t+1}} V\left(\hat{x}_{t+1};\sigma_{t+1}\right) f_i^{\sigma_t}\left(\hat{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right) dz_{t+1}$$

where $\hat{x}_{t+1} = \{a_{t+1},\hat{A}_{t+1},y_{t+1},k_t,\bar{k}_t,price_{t+1},\xi_{t+1}\}$ and $\hat{A}_{t+1} = \{A_{1t+1},A_{2t+1},...,\hat{A}_{it+1},...A_{n,t+1}\}$. Now taking the partial derivative with respect to A_{it+1} at $A_{it+1} = \hat{A}_{it+1}$:

$$\left.\frac{\partial v_i\left(d_{it},c_{it}^d,x_t;\sigma_t\right)}{\partial A_{it+1}}\right|_{\hat{A}_{it+1},d_{it}} = u_{it,c}^d + \beta \int\limits_{z_{t+1}} \left[\frac{\partial}{\partial A_{it+1}} V\left(\hat{x}_{t+1};\sigma_{t+1}\right)\right] f_i^{\sigma_t}\left(\hat{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right) dz_{t+1}$$

It is useful to show how this formula differs in the case when $d_{it}=0$ and $d_{it}=1$:

$$\left.\frac{\partial v_i\left(d_{it}=0,c_{it}^d,x_t;\sigma_t\right)}{\partial A_{it+1}}\right|_{A_{it+1},d_{it}=0} = -\nu\left[(1+r)A_{it} - A_{it+1} + y_{it}\right]^{\nu-1}\left(1+\eta k_{it-1} + \phi\bar{k}_{t-1}\right)$$

$$+\beta \int_{x_{t+1}} \Big\{\hat{u}_{it+1,c}^0(1+r)[1-p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})] + \hat{u}_{it+1,c}^1(1+r)\left[p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right]$$

$$+P'(v_{it+1}^1 - v_{it+1}^0)[v_i(d_{it+1}=1,\hat{c}_{it+1}^1,x_{t+1};\sigma_{t+1}) - v_i(d_{it+1}=0,\hat{c}_{it+1}^0,x_{t+1};\sigma_{t+1})]$$

$$\cdot(1+\eta k_{it}+\phi\bar{k}_t)(1+r)\nu\left[\left(\hat{c}_{it+1}^1\right)^{\nu-1} - \left(\hat{c}_{it+1}^0\right)^{\nu-1}\right]\Big\} f_i^{\sigma_t}\left(x_{t+1}|x_t,0,\hat{c}_{it}\right) dx_{t+1}$$

and similarly for $v_{it}^1\left(d_{it}=1\right)$:

$$\left.\frac{\partial v_i\left(d_{it}=1,c_{it}^d,x_t;\sigma_t\right)}{\partial A_{it+1}}\right|_{A_{it+1},d_{it}=1} = -\nu\left[(1+r)A_{it} - A_{it+1} - price_t + y_{it}\right]^{\nu-1}\left(1+\eta k_{it-1} + \phi\bar{k}_{t-1}\right)+$$

$$+\beta \int_{x_{t+1}} \hat{u}_{it+1,c}^1(1+r) f_i^{\sigma_t}\left(x_{t+1}|x_t,1,\hat{c}_{it}\right) dx_{t+1}$$

Now consider the derivative of $v_i\left(d_{it},c_{it}^d,x_t;\sigma_t\right)$ with respect to A_{it+1}:

$$\frac{\partial v_i\left(d_{it},c_{it}^d,x_t;\sigma_t\right)}{\partial A_{it+1}} = -\hat{u}_{it,c}^d + \beta \int\limits_{z+1} \frac{\partial}{\partial A_{it+1}}\left[V_i(x_{t+1};\sigma_{t+1})\right] f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right) dz_{t+1}$$

Notice that the 'time to build' assumption makes $\hat{u}_{it,c}$ independent of other households' adoption choice at time t. Moreover, the Markov property implies that, $p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})$ is independent of d_{it}, c_{it}, x_t given x_{t+1}. Lastly, the envelope condition makes $\frac{\partial}{\partial A_{it+1}}\left[V_i(x_{t+1};\sigma_{t+1})\right]$ independent of the value function in period $t+2$ and the subsequent ones. Thus, for a household $\frac{\partial}{\partial A_{it+1}}\left[V_i(x_{t+1};\sigma_{t+1})\right]$ is independent of σ_t. Then σ_t'

$$\frac{\partial v_i\left(d_{it},c_{it}^d,x_t;\sigma_t'\right)}{\partial A_{it+1}}-\frac{\partial v_i\left(d_{it},c_{it}^d,x_t;\sigma_t\right)}{\partial A_{it+1}}=\beta\int\limits_{z_{t+1}}\left\{\frac{\partial V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}}\left[f_i^{\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)\right]\right\}dz_{t+1}$$

Now define the vector $w_{t+1}=\{a_{t+1},A_{-it+1},y_{t+1},k_t,price_{t+1},\xi_{t+1}\}$ where $\bar{k}_t, A_{it+1}\notin w_{t+1}$. Notice that the envelope condition and the Markov Property imply that $\frac{\partial V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}}$ is invariable in all the elements of vector w_{t+1}. It is possible to rewrite:

$$\frac{\partial v_i\left(d_{it},c_{it}^d,x_t;\sigma_t'\right)}{\partial A_{it+1}}-\frac{\partial v_i\left(d_{it},c_{it}^d,x_t;\sigma_t\right)}{\partial A_{it+1}}=\beta\int\limits_{w_{t+1}}\int\limits_{\bar{k}_t}\left\{\frac{\partial V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}}\left[f_i^{\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)\right]\right\}d\bar{k}_t\,dw_{t+1}$$

Using integration by parts we get:

$$=\beta\int_{w_{t+1}}\left\{\left[\int_0^1 f_i^{\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)d\bar{k}_t\right]\frac{\partial}{\partial A_{it+1}}\left[V_i(\bar{x}_{t+1};\sigma_{t+1})\right]\right.$$

$$\left.-\int_0^1\left[\int_0^{\bar{k}_t} f_i^{\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)ds\right]\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial\bar{k}_{t-1}}d\bar{k}_t\right\}dw_{t+1}$$

$$=\beta\int_{w_{t+1}}\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial\bar{k}_{t-1}}\left[\int_0^{\bar{k}_t} f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)ds\right]dw_{t+1}$$

$$=\beta\int_{w_{t+1}}\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial\bar{k}_{t-1}}r_i^{\sigma_t,\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)dw_{t+1}$$

where $\bar{x}_{t+1}=\{a_{t+1},\hat{A}_{t+1},y_{t+1},k_t,\hat{\bar{k}}_t,price_{t+1},\xi_{t+1}\}$ and $r_i^{\sigma_t,\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)=\left[\int_0^{\bar{k}_t} f_i^{\sigma_t}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t'}\left(x_{t+1}|x_t,d_{it},\hat{c}_{it}\right)ds\right]$ Notice that the above is equal to zero if $\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial\bar{k}_{t-1}}$ is equal to zero as well.

CASE 1: $k_{it-1} = 0$.

$$\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial \bar{k}_{t-1}} = \Bigg\{ \Bigg[(1+r)\Bigg[\nu\left(\hat{c}^1_{it+1}\right)^{\nu-1}\phi$$
$$+(\nu-1)\nu \left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=0}\left(\hat{c}^0_{it+1}\right)^{\nu-2}(1+\phi\bar{k}_t)\Bigg]\Bigg]\left[1-p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right]$$

$$+\Bigg[(1+r)\Bigg[\nu\left(\hat{c}^1_{it+1}\right)^{\nu-1}\phi+(\nu-1)\nu\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}\left(\hat{c}^1_{it+1}\right)^{\nu-2}(1+\phi\bar{k}_t)\Bigg]\Bigg]\left[p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right]$$

$$+P'(v^1_{it+1}-v^0_{it+1})\left[\hat{u}^1_{it+1,c}-\hat{u}^0_{it+1,c}\right](1+r)\cdot$$

$$\left\{\phi\left[\left(\hat{c}^1_{it+1}\right)^{\nu}-\left(\hat{c}^0_{it+1}\right)^{\nu}\right]+\left(\hat{u}^1_{it+1,c}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}-\hat{u}^0_{it+1,c}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=0}\right)\right\}$$

$$+P'(v^1_{it+1}-v^0_{it+1})(1+r)\Bigg\{\phi\left[\nu\left(\hat{c}^1_{it+1}\right)^{\nu-1}-\nu\left(\hat{c}^0_{it+1}\right)^{\nu-1}\right]$$

$$+\left(\hat{u}^1_{it+1,cc}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}-\hat{u}^0_{it+1,cc}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}\right)\Bigg\}$$

$$+P'(v^1_{it+1}-v^0_{it+1})\Bigg\{\phi(1+\phi\bar{k}_t)(1+r)\nu^2\left[\left(\hat{c}^1_{it+1}\right)^{\nu-1}-\left(\hat{c}^0_{it+1}\right)^{\nu-1}\right]\left[\left(\hat{c}^1_{it+1}\right)^{\nu}-\left(\hat{c}^0_{it+1}\right)^{\nu}\right]$$
$$+(1+\phi\bar{k}_t)(1+r)\nu\left[\left(\hat{c}^1_{it+1}\right)^{\nu-1}-\left(\hat{c}^0_{it+1}\right)^{\nu-1}\right]\left[\hat{u}^1_{it+1,c}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}-\hat{u}^0_{it+1,c}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=0}\right]\Bigg\}$$

$$+P''(v^1_{it+1}-v^0_{it+1})\cdot[v^1_{it+1}-v^0_{it+1}]\Bigg\{\phi(1+\phi\bar{k}_t)(1+r)\nu\left[\left(\hat{c}^1_{it+1}\right)^{\nu-1}-\left(\hat{c}^0_{it+1}\right)^{\nu-1}\right]\left[\left(\hat{c}^1_{it+1}\right)^{\nu}-\left(\hat{c}^0_{it+1}\right)^{\nu}\right]$$
$$+(1+\phi\bar{k}_t)(1+r)\nu\left[\left(\hat{c}^1_{it+1}\right)^{\nu-1}-\left(\hat{c}^0_{it+1}\right)^{\nu-1}\right]\left[\hat{u}^1_{it+1,c}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}-\hat{u}^0_{it+1,c}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=0}\right]\Bigg\}$$

CASE 2: $k_{it-1} = 1$. The above simplifies because $p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1}) = 0$ thus:

$$\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial \bar{k}_{t-1}} = (1+r)\left\{\nu\left[\hat{c}^0_{it+1}\right]^{\nu-1}\phi+(\nu-1)\nu\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=0}\left[\hat{c}^0_{it+1}\right]^{\nu-2}(1+\eta+\phi\bar{k}_t)\right\}$$

CASE 3: for $d_{it} = 1$ one gets (for any k_{it-1}):

$$\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{it+1}\partial \bar{k}_{t-1}} = (1+r)\left\{\nu\left[c^0_{it+1}\right]^{\nu-1}\phi+(\nu-1)\nu\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}=1}\left[c^0_{it+1}\right]^{\nu-2}(1+\eta+\phi\bar{k}_t)\right\}$$

Now notice that if $\phi = 0$ then the cross derivative for $d_{it} = 0$ becomes:

For **CASE 1:** $k_{it-1} = 0$:

$$\frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t'\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)}{\partial A_{it+1}} = \beta \int_{x_{t+1}} \left[\left[\left[(1+r)(\nu-1)\nu \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=0} \left[\hat{c}_{it+1}^0\right]^{\nu-2}\right] \left[1 - p^{\sigma_{t+1}}(d_{it+1} = 1|x_{t+1})\right]\right.\right.$$

$$+ \left[(1+r)(\nu-1)\nu \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1} \left[\hat{c}_{it+1}^0\right]^{\nu-2}\right] \left[p^{\sigma_{t+1}}(d_{it+1} = 1|x_{t+1})\right]$$

$$+ P'(v_{it+1}^1 - v_{it+1}^0) \left[\hat{u}_{it+1,c}^1 - \hat{u}_{it+1,c}^0\right](1+r) \left[\hat{u}_{it+1,c}^1 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1} - \hat{u}_{it+1,c}^0 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=0}\right]$$

$$+ P'(v_{it+1}^1 - v_{it+1}^0)(1+r) \left[\hat{u}_{it+1,cc}^1 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1} - \hat{u}_{it+1,cc}^0 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1}\right]$$

$$+ P'(v_{it+1}^1 - v_{it+1}^0)(1+r)\nu \left[\left(\hat{c}_{it+1}^1\right)^{\nu-1} - \left(\hat{c}_{it+1}^0\right)^{\nu-1}\right] \left[\hat{u}_{it+1,c}^1 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1} - \hat{u}_{it+1,c}^0 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=0}\right]$$

$$+ P''(v_{it+1}^1 - v_{it+1}^0)[v_{it+1}^1 - v_{it+1}^0](1+r)\nu \left[\left(\hat{c}_{it+1}^1\right)^{\nu-1} - \left(\hat{c}_{it+1}^0\right)^{\nu-1}\right]$$

$$\left.\left. \cdot \left[\hat{u}_{it+1,c}^1 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1} - \hat{u}_{it+1,c}^0 \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=0}\right]\right]\right] r_i^{\sigma_t, \sigma_t'}(x_{t+1}|x_t, 0, \hat{c}_{it})\, dx_{t+1}$$

For **CASE 2:** $k_{it-1} = 1$, the difference $\frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t'\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)}{\partial A_{it+1}}$ becomes:

$$\frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t'\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)}{\partial A_{it+1}} = \beta \int\limits_{x_{t+1}} (1+r)(\nu-1)\nu \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=0} \left[c_{it+1}^0\right]^{\nu-2} (1+\eta)\, r_i^{\sigma_t, \sigma_t'}(x_{t+1}|x_t, 0, \hat{c}_{it})\, dx_{t+1}$$

and similarly for **CASE 3** ($d_{it} = 1$) the difference $\frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t'\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)}{\partial A_{it+1}}$ becomes:

$$\frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t'\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c_{it}^d, x_t; \sigma_t\right)}{\partial A_{it+1}} = \beta(1+r) \int\limits_{x_{t+1}} (\nu-1)\nu \left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{d_{it+1}=1} \left[c_{it+1}^0\right]^{\nu-2} (1+\eta)\, r_i^{\sigma_t, \sigma_t'}(x_{t+1}|x_t, 1, \hat{c}_{it})\, dx_{t+1}$$

These differences are both zero if $\left.\frac{dA_{it+2}^*}{d\bar{k}_t}\right|_{A_{it+1}, d_{it+1}, k_{it}, A_{it}} = 0$ for all $d_{it+1} \in \{0, 1\}$. When is this the case? If A_{it+2}^* is not an interior solution, then the derivative is always zero. If it is interior, consider the FOC w.r.t A_{it+2} in period $t+1$.

When is this the case?

$$\left.\frac{\partial v_i\left(d_{it+1},c^d_{it+1},x_{t+1};\sigma_{t+1}\right)}{\partial A_{it+2}}\right|_{d_{it+1}=0} = -\nu\left[(1+r)A_{it+1}-A_{it+2}+y_{it+1}\right]^{\nu-1}(1+\eta k_{it}+\phi\bar{k}_t)$$

$$+\beta\int_{x_{t+2}}\left\{\hat{u}^0_{it+2,c}(1+r)[1-p^{\sigma_{t+2}}(d_{it+2}=1|x_{t+2})]+\hat{u}^1_{it+2,c}(1+r)p^{\sigma_{t+2}}(d_{it+2}=1|x_{t+2})\right.$$

$$+P'(v^1_{it+2}-v^0_{it+2})[v_i(1,\hat{c}^1_{it+2},x_{t+2};\sigma_{t+2})-v_i(0,\hat{c}^0_{it+2},x_{t+2};\sigma_{t+2})](1+\eta k_{it+1}+\phi\bar{k}_{t+1})$$

$$\left.\cdot(1+r)\nu\left[\left(\hat{c}^1_{it+2}\right)^{\nu-1}-\left(\hat{c}^0_{it+2}\right)^{\nu-1}\right]\right\} f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it+1},\hat{c}_{it+1})\,dx_{t+2}$$

At an interior solution the FOC must be satisfied with equality. Now totally differentiate w.r.t $\bar{k}_t$.

$$\left.\frac{\partial v_i^2\left(d_{it+1},c^d_{it+1},x_{t+1};\sigma_{t+1}\right)}{\partial A_{it+2}\partial\bar{k}_t}\right|_{d_{it+1}=0} = -\nu\left[c^d_{it+1}\right]^{\nu-1}\phi+(1-\nu)\nu\left[c^d_{it+1}\right]^{\nu-2}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}}$$

$$+\beta\int_{x_{t+2}}\frac{\partial V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]$$

$$+\frac{\partial^2 V_i(x_{t+2};\sigma_{t+2})}{\partial A^2_{it+2}}\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}} f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\,dx_{t+2}$$

$$\int_{x_{t+2}}\frac{\partial V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_t}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]dx_{t+2}$$

$$=\int_{w_{t+2}}\int_{\bar{k}_{t+1}}\frac{\partial V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]d\bar{k}_{t+1}\,dw_{t+2}$$

Using this result, it is possible to write:

$$\beta\int_{x_{t+2}}\frac{\partial V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]dx_{t+2}$$

$$=\beta\int_{w_{t+2}}\frac{\partial V_i(\bar{x}_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\int_{\bar{k}_{t+1}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]d\bar{k}_{t+1}$$

$$-\int_{\bar{k}_{t+1}}\frac{\partial^2 V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}\partial\bar{k}_{t+1}}\int_{\underline{k}}^{\bar{k}_{t+1}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]d\bar{k}_{t+1}dw_{t+2}$$

Now notice that, at constant A_{it+2} we get that $\int_{\bar{k}_{t+1}}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]d\bar{k}_{t+1}=s(w_{t+2}|d_{it},\hat{c}_{it})$ that is independent of x_{t+1}. The intuition here is that the Envelope condition and the Markov property imply that $\frac{\partial V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}}$ is invariable in all the elements of vector w_{t+2}. Thus it is possible to write:

$$\beta\int_{w_{t+2}}\frac{\partial V_i(\bar{x}_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\int_{\bar{k}_{t+1}}\frac{\partial}{\partial\bar{k}_t}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]d\bar{k}_{t+1}\,dw_{t+2}$$

$$=\beta\frac{\partial V_i(\bar{x}_{t+2};\sigma_{t+2})}{\partial A_{it+2}}\frac{\partial}{\partial\bar{k}_t}\int_{w_{t+2}}\int_{\bar{k}_{t+1}}\left[f_i^{\sigma_{t+1}}(x_{t+2}|x_{t+1},d_{it},\hat{c}_{it})\right]d\bar{k}_{t+1}=0$$

The above becomes:

$$-\beta \int\limits_{w_{t+2}} \frac{\partial^2 V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}\partial \bar{k}_{t+1}} \int\limits_{\underline{k}}^{\bar{k}_{t+1}} \frac{\partial}{\partial \bar{k}_t} \left[f_i^{\sigma_{t+1}}\left(x_{t+2}|x_{t+1}, d_{it}, \hat{c}_{it}\right)\right] d\bar{k}_{t+1}\, dw_{t+2}$$

Notice that if the optimal solution for A^*_{it+2} is interior, it is possible to calculate the following:

$$\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}} = \frac{\nu\left[c^d_{it+1}\right]^{\nu-1}\phi - \beta\int_{x_{t+2}} \frac{\partial^2 V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}\partial \bar{k}_{t+1}} \int_{\underline{k}}^{\bar{k}_{t+1}} \frac{\partial}{\partial \bar{k}_t}\left[f_i^{\sigma_{t+1}}\left(x_{t+2}|x_{t+1}, d_{it}, \hat{c}_{it}\right)\right] d\bar{k}_{t+1}\, dx_{t+2}}{(1-\nu)\nu\left[c^d_{it+1}\right]^{\nu-2} + (1-\nu)\nu\left[c^d_{it+1}\right]^{\nu-2} + \beta\int_{x_{t+2}} \frac{\partial^2 V_i(x_{t+2};\sigma_{t+2})}{\partial A^2_{it+2}} f_i^{\sigma_{t+1}}\left(x_{t+2}|x_{t+1}, d_{it}, \hat{c}_{it}\right) dx_{t+2}}$$

which is equal to zero if ϕ and $\frac{\partial^2 V_i(x_{t+2};\sigma_{t+2})}{\partial A_{it+2}\partial \bar{k}_{t+1}} = 0$. But under those conditions, one can update $\frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma'_t\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma_t\right)}{\partial A_{it+1}}$ and conclude that $\frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma'_t\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma_t\right)}{\partial A_{it+1}} = 0$ if $\frac{\partial v_i\left(d_{it+1}, c^d_{it+1}, x_{t+1};\sigma'_{t+1}\right)}{\partial A_{it+2}} - \frac{\partial v_i\left(d_{it+1}, c^d_{it+1}, x_{t+1};\sigma_{t+1}\right)}{\partial A_{it+2}}$ at any state vector that can be reached with positive probability from x_t with choices A_{it+1}, d_{it}. This updating process can go on recursively until period $T-1$ (i.e., $\mathcal{A}-1$) at that point notice that $\left.\frac{dA^*_{iT+1}}{d\bar{k}_{T-1}}\right|_{d_{iT-1}} = 0$ because in the last period of life T (i.e., $\mathcal{A}$) households consume all such that $A^*_{iT+1} = 0$. This implies that $\frac{\partial^2 V_i(x_{t+1};\sigma_{t+1})}{\partial A_{iT+1}\partial \bar{k}_T} = 0$ (i.e., household's savings is unaffected by $\bar{k}_T$ because they do not save in any case). Therefore recursively $\frac{\partial^2 V_i(x_{t+1-s};\sigma_{t+1-s})}{\partial A_{iT+1-s}\partial \bar{k}_{T-s}} = 0$ for all $s \in [0, 1, \ldots T]$. Notice that if the solution for A^*_{it+2} is not interior, then $\left.\frac{dA^*_{it+2}}{d\bar{k}_t}\right|_{d_{it+1}} = 0$ and the desired result hold trivially. Lastly, notice that, because $\frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma'_t\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma_t\right)}{\partial A_{it+1}}$ is continuous in ϕ and with finite derivative, then for any $\lambda > 0$ there exists φ such that if $\phi \leq \varphi$ then $\frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma'_t\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c^d_{it}, x_t;\sigma_t\right)}{\partial A_{it+1}} \leq \lambda$. I define $\varphi(\lambda, d_{it}, A_{it+1}, x_t, \sigma_t)$ to be the minimum φ that ensures that the inequality is satisfied at specific values of states and controls $d_{it}, A_{it+1}, x_t, \sigma_t$.

4.8.3.2 i-Increasing Differences in d_{it}, σ_t

To show that $v_t(d_{it} = 1, \hat{c}^1_{it}, x_t;\sigma_t) - v_t(d_{it} = 0, \hat{c}^0_{it}, x_t;\sigma_t)$ is increasing in σ_t. Recall that the value functions are expressed as:

$$v_i\left(d_{it}=1,\hat{c}^1_{it},x_t;\sigma_t\right)=[(1+r)A_{it}-A_{it+1}+y_{it}-price_t]^{\nu}(1+\eta k_{it-1}+\phi\bar{k}_{t-1})+\alpha k_{it-1}+\gamma k_{it-1}\bar{k}_{t-1}$$

$$+\beta\int_{x_{t+1}}V_{t+1}(x_{t+1};\sigma_{t+1})\,f_i^{\sigma_t}\left(x_{t+1}|x_t,1,\hat{c}_{it}\right)dx_{t+1}$$

$$v_i\left(d_{it}=0,\hat{c}^0_{it},x_t;\sigma_t\right)=[(1+r)A_{it}-A_{it+1}+y_{it}]^{\nu}(1+\eta k_{it-1}+\phi\bar{k}_{t-1})+\alpha k_{it-1}+\gamma k_{it-1}\bar{k}_{t-1}$$

$$+\beta\int_{x_{t+1}}V_{t+1}(x_{t+1};\sigma_{t+1})\,f_i^{\sigma_t}\left(x_{t+1}|x_t,0,\hat{c}_{it}\right)dx_{t+1}$$

Thus

$$\left\{\left[v_i\left(d_{it}=1,\hat{c}^1_{it},x_t;\sigma'_t\right)-v_i\left(d_{it}=0,\hat{c}^0_{it},x_t;\sigma'_t\right)\right]-\left[v_i\left(d_{it}=1,\hat{c}^1_{it},x_t;\sigma_t\right)-v_i\left(d_{it}=0,\hat{c}^0_{it},x_t;\sigma_t\right)\right]\right\}_{A_{it+1}}$$

$$=\beta\int_{x_{t+1}}V_i(x_{t+1};\sigma_{t+1})\,f_i^{\sigma'_t}\left(x_{t+1}|x_t,1,\hat{c}_{it}\right)-V_i(x_{t+1};\sigma_{t+1})\,f_i^{\sigma'_t}\left(x_{t+1}|x_t,0,\hat{c}_{it}\right)$$

$$-V_i(x_{t+1};\sigma_{t+1})\,f_i^{\sigma_t}\left(x_{t+1}|x_t,1,\hat{c}_{it}\right)+V_i(x_{t+1};\sigma_{t+1})\,f_i^{\sigma_t}\left(x_{t+1}|x_t,0,\hat{c}_{it}\right)dx_{t+1}$$

because of the assumption that $\bar{k}_t$ is unaffected by d_{it} on the point of view of household i, then for a given A_{it+1}, and the fact that k_{it} is a deterministic state, it is possible to write:

$$\beta\int_{q_{t+1}}\int_{\bar{k}_t}\left[V_i(\tilde{x}'_{t+1};\sigma_{t+1})-V_i(\tilde{x}_{t+1};\sigma_{t+1})\right]\left[f_i^{\sigma'_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)\right]d\bar{k}_t\,dq_{t+1}$$

where $q_{t+1}=\left(a_{t+1},\tilde{A}_{t+1},y_{t+1},price_{t+1},\xi_{t+1}\right)$, $\tilde{x}'_{t+1}=\left(a_{t+1},\tilde{A}_{t+1},y_{t+1},\tilde{k}_t,\bar{k}_t,price_{t+1},\xi_{t+1}\right)$ is the vector of states with $\tilde{k}_t=(k_{1t},k_{2t},...,\tilde{k}_{it},...,k_{nt})$ and $\tilde{k}_{it}=1$ and $\tilde{A}_{t+1}=(A_{1t+1},A_{2t+1},...,\tilde{A}_{it+1},...A_{nt+1})$. Similarly, $\tilde{x}_{t+1}=\left(a_{t+1},\tilde{A}_{t+1},y_{t+1},\tilde{k}_t,\bar{k}_t,price_{t+1},\xi_{t+1}\right)$ is the vector of states with $\tilde{k}_t=(k_{1t},k_{2t},...,\tilde{k}_{it},...,k_{nt})$ and $\tilde{k}_{it}=k_{it-1}$. Notice that the step above follows from the fact that $f_i^{\sigma_t}\left(\tilde{x}'_{t+1}|x_t,d_{it},\hat{c}_{it}\right)=f_i^{\sigma_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)$ at given $A_{it+1}=\tilde{A}_{it+1}$. Then, using integration by parts, the formula above can be written as follows:

$$\Rightarrow\beta\int_{q_{t+1}}\left[V_i(\tilde{x}'_{t+1};\sigma_{t+1})-V_i(\tilde{x}_{t+1};\sigma_{t+1})\right]\int_{\bar{k}_t}\left[f_i^{\sigma'_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)-f_i^{\sigma_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)\right]d\bar{k}_t$$

$$+\int_{q_{t+1}}\frac{\partial\left[V_i(\tilde{x}'_{t+1},\sigma_{t+1})-V_i(\tilde{x}_{t+1},\sigma_{t+1})\right]}{\partial\bar{k}_t}r_i^{\sigma_t,\sigma'_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)dq_{t+1}$$

where $r_i^{\sigma_t,\sigma'_t}\left(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it}\right)\geq 0$ for all $\tilde{x}_{t+1}$

$$= \beta \int_{q_{t+1}} \frac{\partial\left[V_i(\tilde{x}'_{t+1},\sigma_{t+1}) - V_i(\tilde{x}_{t+1},\sigma_{t+1})\right]}{\partial \bar{k}_t} r_i^{\sigma_t,\sigma'_t}(\tilde{x}_{t+1}|x_t,d_{it},\hat{c}_{it})\, dq_{t+1}$$

$$= E_{r_t}\left[\frac{\partial\left[V_i(\tilde{x}'_{t+1},\sigma_{t+1}) - V_i(\tilde{x}_{t+1},\sigma_{t+1})\right]}{\partial \bar{k}_t}\right]$$

Assuming an interior solution at time $t+1$, it is possible to use the Envelope condition to calculate:

$$\frac{\partial\left[V_i(\tilde{x}'_{t+1},\sigma_{t+1}) - V_i(\tilde{x}_{t+1},\sigma_{t+1})\right]}{\partial \bar{k}_t} = \phi\left(\hat{c}^0_{it+1}(d_{it}=1)\right)^{\nu} + \gamma k_{it}(d_{it}=1)$$

$$+\gamma k_{it}(d_{it}=0) - \phi\left(\hat{c}^0_{it+1}(d_{it}=0)\right)^{\nu}\left[p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right]$$

$$-\phi\left(\hat{c}^0_{it+1}(d_{it}=0)\right)^{\nu}\left[1-p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right] - \gamma k_{it}(d_{it}=0)$$

$$-P'(v^1_{it+1}-v^0_{it+1})[v^1_{it+1}-v^0_{it+1}][\phi\left(\hat{c}^0_{it+1}(d_{it}=1)\right)^{\nu} + \gamma k_{it}(d_{it}=1)$$

$$-\phi\left(\hat{c}^0_{it+1}(d_{it}=0)\right)^{\nu}\left[p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right]$$

$$-\phi\left(\hat{c}^0_{it+1}(d_{it}=0)\right)^{\nu}\left[1-p^{\sigma_{t+1}}(d_{it+1}=1|x_{t+1})\right] - \gamma k_{it}(d_{it}=0)$$

For $\phi = 0$ this reduces to:

$$\frac{\partial\left[V_i(\tilde{x}'_{t+1};\sigma_{t+1}) - V_i(\tilde{x}_{t+1};\sigma_{t+1})\right]}{\partial \bar{k}_t} = \left[1 - P'(v^1_{it+1}-v^0_{it+1})[v^1_{it+1}-v^0_{it+1}]\right]\left[\gamma k_{it}(d_{it}=1) - \gamma k_{it}(d_{it}=0)\right]$$

Thus one gets that:

- If $k_{it-1}=1$ then $E_{r_t}\left[\frac{\partial\left[V_i(\tilde{x}'_{t+1};\sigma_{t+1}) - V_i(\tilde{x}_{t+1};\sigma_{t+1})\right]}{\partial \bar{k}_t}\right] = 0$
- If $k_{it-1}=0$ then $E_{r_t}\left[\frac{\partial\left[V_i(\tilde{x}'_{t+1};\sigma_{t+1}) - V_i(\tilde{x}_{t+1};\sigma_{t+1})\right]}{\partial \bar{k}_t}\right] > 0$ for $E_{r_t}\left[P'(v^1_{it+1}-v^0_{it+1})[v^1_{it+1}-v^0_{it+1}]\right] < 1$, which is the case as long as $P(x)$ is "flat" enough.

The latter results implies that, for all households i it must be true that:

$$v_i\left(d'_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right) - v_i\left(d'_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right) \geq v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right) - v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)$$

and the inequality is strict for all households j such that $k_{jt-1} = 0$ i.e., the objective function satisfies *i-Increasing Differences* in d_{it}, σ_t (see. Milgrom and Shannon 1994).

4.8.3.3 i-Highest and i-Lowest Equilibria

I want to show that for all i such that $k_{it-1} = 0$, the following holds:

$$v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma'_t\right) - v_i\left(d''_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma'_t\right) \geq v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma_t\right) - v_i\left(d''_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma_t\right) \tag{4.8.1}$$

holds for any A'_{it+1}, A''_{it+1} and for $d'_{it} > d''_{it}$ and $\sigma'_t \geq \sigma_t$, in the sense defined above.

Proof:

Suppose (4.8.1) is not satisfied i.e., the following instead is true:

$$v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma'_t\right) - v_i\left(d''_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma'_t\right) < v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma_t\right) - v_i\left(d''_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma_t\right) \tag{4.8.2}$$

The fact that $\frac{\partial v_i\left(d_{it}, c^d_{it}, x_t; \sigma'_t\right)}{\partial A_{it+1}} - \frac{\partial v_i\left(d_{it}, c^d_{it}, x_t; \sigma_t\right)}{\partial A_{it+1}} \leq \lambda$ for $0 < \phi \leq \varphi(\lambda, d_{it}, A_{it+1}, x_t \sigma_t)$ implies that:

$$\begin{aligned} & v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma'_t\right) - v_i\left(d'_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma'_t\right) \\ & \quad = v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma_t\right) - v_i\left(d'_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma_t\right) + b(d'_{it}, A'_{it+1}, A''_{it+1}, x_t; \sigma'_t, \sigma_t, \phi) \end{aligned}$$

where $b(\cdot)$ is a continuous function such that $b(d'_{it}, A'_{it+1}, A''_{it+1}, x_t; \sigma'_t, \sigma_t, \phi) \leq \zeta$ if $\phi \leq \varphi(\lambda, d_{it}, A_{it+1}, x_t, \sigma_t)$. Because of the continuity (and finite derivative) of b, for any $\zeta > 0$ there exists $\bar{\phi}(\zeta, d_{it}, A_{it+1}, x_t, \sigma_t)$ such that if $0 < \phi \leq \bar{\phi}(\zeta, d_{it}, A_{it+1}, x_t, \sigma_t)$ then $|b| \leq \zeta$. This implies:

$$\begin{aligned} & v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma'_t\right) = v_i\left(d'_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma'_t\right) \\ & \quad + v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma_t\right) - v_i\left(d'_{it}, \hat{c}^1_{it}(A''_{it+1}), x_t; \sigma_t\right) + b(d'_{it}, A'_{it+1}, A''_{it+1}, x_t; \sigma'_t, \sigma_t, \phi) \end{aligned} \tag{4.8.3}$$

Similarly, one can get:

$$v_i\left(d''_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma'_t\right)-v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right)$$

$$=v_i\left(d''_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma_t\right)-v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)-b(d''_{it},A'_{it+1},A''_{it+1},x_t;\sigma'_t,\sigma_t,\phi)$$

which implies:

$$v_i\left(d''_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma'_t\right)=v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right)$$

$$+v_i\left(d''_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma_t\right)-v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)-b(d''_{it},A'_{it+1},A''_{it+1},x_t;\sigma'_t,\sigma_t,\phi) \tag{4.8.4}$$

Now substituting (4.8.3) and (4.8.4) into (4.8.2) we get:

$$v_i\left(d'_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma'_t\right)-v_i\left(d'_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma_t\right)+b(d'_{it},A'_{it+1},A''_{it+1},x_t;\sigma'_t,\sigma_t,\phi)$$

$$-v_i\left(d''_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma_t\right)+v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)+b(d''_{it},A'_{it+1},A''_{it+1},x_t;\sigma'_t,\sigma_t,\phi)<0$$

Now notice that $\frac{\partial v_i\left(d_{it},c^d_{it},x_t;\sigma'_t\right)}{\partial A_{it+1}}-\frac{\partial v_i\left(d_{it},c^d_{it},x_t;\sigma_t\right)}{\partial A_{it+1}}\leq\lambda$ implies that:

$$v_i\left(d'_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma'_t\right)-v_i\left(d'_{it},\hat{c}^1_{it}(A''_{it+1}),x_t;\sigma_t\right)$$

$$=v_i\left(d'_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right)-v_i\left(d'_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)-b(d'_{it},A'_{it+1},A''_{it+1},x_t;\sigma'_t,\sigma_t,\phi) \tag{4.8.5}$$

Substituting (4.8.5) into (4.8.2) and rearranging to get:

$$v_i\left(d'_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right)-v_i\left(d'_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)$$

$$<v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma'_t\right)-v_i\left(d''_{it},\hat{c}^1_{it}(A'_{it+1}),x_t;\sigma_t\right)-b(d''_{it},A'_{it+1},A''_{it+1},x_t;\sigma'_t,\sigma_t,\phi) \tag{4.8.6}$$

Recall that in the previous section it was shown that for all i such that $k_{it-1} = 0$ the following holds:

$$v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma'_t\right) - v_i\left(d'_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma_t\right) > v_i\left(d''_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma'_t\right) - v_i\left(d''_{it}, \hat{c}^1_{it}(A'_{it+1}), x_t; \sigma_t\right)$$

because v_i satisfies *i-Increasing Differences in* (d_{it}, σ'_t). We also know from the previous paragraph that or $0 < \phi \leq \bar{\phi}(\zeta, d_{it}, A_{it+1}, x_t, \sigma_t)$ we get $|b\left(d''_{it}, A'_{it+1}, A''_{it+1}, x_t; \sigma'_t, \sigma_t, \phi\right) \leq \zeta$. As this is the case for all $d_{it}, A_{it+1}, x_t, \sigma_t$ then there exists $\hat{\zeta}$ such that $|b\left(d''_{it}, A'_{it+1}, A''_{it+1}, x_t; \sigma'_t, \sigma_t, \phi\right)| \leq \hat{\zeta}$ for all $d_{it}, A_{it+1}, x_t, \sigma_t$. Then, if $\phi \leq \bar{\phi}\left(\hat{\zeta}\right)$, the condition (4.8.6) cannot be satisfied. This leads to a contradiction, Q.E.D

Now, because of this result, we know that for any higher beliefs $\sigma'_t \geq \sigma_t$, the best response for any household i implies (weakly) higher d_{it}. Thus with beliefs σ'_t all households play $d_{it} = 1$ with higher probability than under beliefs σ_t. As a result, if σ'_t and σ_t are equilibrium beliefs, then it must be that in equilibrium $\int_0^{\hat{k}_{it}} f_i^{\sigma_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it}) - f_i^{\sigma'_t}(x_{t+1}|x_t, d_{it}, \hat{c}_{it})\, d\bar{k}_t \geq 0$ for all $\hat{k}_t \in [\underline{k}_t, 1]$. It also implies that there exists a highest and a lowest pure strategy N.E. with respect to the distribution of $\bar{k}_t$.

4.8.4 Tables

Table 4.1: Descriptive Statistics

	Data: S1		Data:S2	
Variable	**Mean**	**std. dev**	**Mean**	**std. dev**
Household				
Age of Household Head (yrs)	42.56	(13.22)	43.61	(13.81)
Education level of HH head (yrs)	4.61	(0.34)	4.87	(0.32)
Nr. Of Female HH members	2.54	(1.29)	2.89	(1.40)
Household size	5.20	(1.03)	5.67	(1.10)
Dwelling ownership	0.89	(0.314)	0.90	(0.309)
Cash-in-hand (Rs.)	57,112.32	(16,167)	68,331.69	(18,128)
Savings, Liquid Assets (Rs.)	4,482.13	(5,073)	7,674.34	(4,899)
Village/Group				
Drainage Infratructure	0.43	(0.495)	0.47	(0.461)
Community Sanitation presence	0.51	(0.501)	0.54	(0.489)
Cost of building Sanitation (Rs.)	8,628.00	(1150)	9,281.00	(1256)
Sanitation coverage	0.41	(0.304)	0.61	(0.287)
Nr. of groups	42			
Nr. of observations	1,451			

Notes: This table provides descriptive statistics for key household and group variables across the two sample periods. Monetary values in the second period are deflated to first period values. The GDP (per capita) Rs. 167,600 (2010 estimate). £1 $\approx Rs.100\,(INR)$.

Table 4.2: Structural Estimates: Preference Parameters

Parameter	Estim.	Std Err.	Description
ν	0.3376	(0.014)	$(1-\nu)$ coeff of rel. risk aversion
η	0.00022	(0.0001)	interaction c_t & own sanitation
ϕ	0.00514	(0.002)	interaction c_t & average sanitation prev.
$\alpha_{20\leq a<26}$	4.8155	(0.084)	importance of sanitation at 20≤age<26
$\alpha_{26\leq a<75}$	0.0138	(0.002)	importance of sanitation at 26≤age<75
γ	2.7019	(0.024)	interaction own sanitation & average sanitation prev.
β	0.9436	(0.004)	discount factor

Notes: Model parameters characterizing preferences and discount rate. Bootstrap standard errors computed using 250 bootstrap resamples. Calibrated values: $r = 0.02$ real interest savings rate based on data from the *Reserve Bank of India* (RBI).

Table 4.3: First Stage Estimates: Earnings Function Parameters

Parameter	Coeff.	Std. Err	Variable Description
ψ_0^y	3.831	0.081	Constant
$\psi_{20\leq a<25}^y$	0.431	0.016	HH head Age $20 \leq a < 25$
$\psi_{25\leq a<50}^y$	0.824	0.009	HH head Age $25 \leq a < 50$
$\psi_{50\leq a<75}^y$	-0.106	0.004	HH head Age $50 \leq a < 75$
ψ_{edu1}^y	0.784	0.062	HH head Education (yrs)
ψ_{edu2}^y	-0.082	0.011	HH head Education Sq. (yrs)
$\psi_{age*edu3}^y$	0.110	0.015	HH Age x Education
σ_u^2	0.311	0.012	variance Innovations
σ_ξ^2	0.126	0.018	variance Measurement Error
ρ	1.00	-	presistence (Calibrated)

Notes: Parameter Estimates for the earnings function. Bootstrapped standard errors in parentheses.

Table 4.4: Structural Estimates: Preference Parameters

Parameter	Mod: No Borrowing ($a_t \geq 0$)	Mod: Borrowing Allowed	Description
ν	0.3376	0.4845	$(1-\nu)$ coeff of rel. risk aversion
η	0.00022	0.00012	interaction c_t & own sanitation
ϕ	0.00514	0.00412	interaction c_t & avg sanitation prev.
$\alpha_{20\leq a<26}$	4.8155	5.1259	imp. of sanitation 20≤age<26
$\alpha_{26\leq a<75}$	0.0138	0.00127	imp. of sanitation 26≤age<75
γ	2.7019	2.3452	interaction own sanitation & avg sanitation prev.
β	0.9436	0.9634	discount factor

Notes: Model parameters characterizing preferences and discount rate. Column (2) denotes parameter estimates under model with borrowing restricted. Column (3) denotes parameter estimates under no borrowing restrictions. Calibrated values: $r = 0.02$ real interest savings rate based on data from the *Reserve Bank of India* (RBI).

Table 4.5: First Stage Estimates: Conditional Choice Probability (CCP) Parameters

	Group A (High)		Group B (Low)		Group C (High)		Group D (Low)		
Parameter	**Coeff.**	**Std. Err**	**Coeff.**	**Std. Err**	**Coeff.**	**Std. Err**	**Coeff.**	**Std. Err**	**Description**
$\psi^c_{20\leq a<25}$	0.088	(0.006)	0.032	(0.004)	0.042	(0.004)	0.062	(0.007)	HH head Age $20 \leq a < 25$
$\psi^c_{25\leq a<50}$	0.039	(0.004)	0.021	(0.003)	0.075	(0.008)	0.033	(0.003)	HH head Age $25 \leq a < 50$
$\psi^c_{50\leq a<75}$	-0.022	(0.003)	-0.008	(0.002)	0.001	(0.000)	-0.002	(0.001)	HH head Age $50 \leq a < 75$
ψ^c_{edu}	0.206	(0.013)	0.189	(0.013)	0.244	(0.021)	0.188	(0.018)	HH head Education (yrs)
ψ^c_{asset}	0.071	(0.022)	0.062	(0.014)	0.083	(0.018)	0.058	(0.013)	HH Savings (1000 Rs.)
ψ^c_{inc}	0.331	(0.045)	0.253	(0.042)	0.213	(0.036)	0.268	(0.038)	HH Income (1000 Rs.)
ψ^c_{size}	0.021	(0.004)	0.036	(0.010)	0.041	(0.012)	0.042	(0.010)	Family Size (Nr. Of HH members)
ψ^c_{price}	-0.109	(0.019)	-0.098	(0.010)	-0.116	(0.011)	-0.131	(0.009)	Cost of Sanitation (1000 Rs.)
$\psi^c_{coverage}$	3.811	(0.312)	2.673	(0.264)	3.433	(0.369)	2.851	(0.289)	Sanitation coverage/prevalence
$\psi^c_{\overline{age}}$	0.013	(0.003)	0.021	(0.004)	0.028	(0.003)	0.018	(0.003)	Mean(-i) Age
$\psi^c_{\overline{age}^2}$	-0.001	(0.001)	-0.001	(0.000)	-0.002	(0.001)	-0.002	(0.002)	Mean(-i) Age Sq
$\psi^c_{\overline{edu}}$	0.106	(0.029)	0.181	(0.042)	0.080	(0.018)	0.041	(0.011)	Mean(-i) Education (yrs)
$\psi^c_{\overline{A+y}}$	0.080	(0.012)	0.065	(0.008)	0.101	(0.013)	0.061	(0.010)	Mean(-i) Cash-in-Hand (1000 Rs.)
$\psi^c_{\overline{A+y}^2}$	0.008	(0.002)	0.010	(0.003)	0.011	(0.003)	0.004	(0.001)	Mean(-i) Cash-in-Hand Sq. (1000 Rs.)
ψ^c_0	-3.863	(0.324)	-4.623	(0.481)	-3.920	(0.345)	-4.564	(0.369)	Constant
Drainage Infrastructure	Yes		No		Yes		No		Drainage Infrastructure in village
Community Sanitation	No		Yes		No		Yes		Public sanitation facility in village
Nr. of village in subgroup	9		12		10		11		
Prob. of Adotion (mean)	0.724		0.326		0.867		0.247		

Notes: Parameter Estimates for the conditional choice probabilities by village subgroups. Bootstrapped standard errors in parentheses.

Table 4.6: Structural Estimates: Village "Fixed Effects"

Village	**Data**		**Coeff.** $\widehat{\mu}_g$	**Std. Err**
	S1	**S2**		
(mean)			2.24	(0.03)
vill ID 9	0.00	0.00	1.56	(0.07)
vill ID 15	0.00	0.00	1.24	(0.04)
vill ID 18	0.00	0.09	1.42	(0.01)
vill ID 22	0.00	0.13	0.98	(0.05)
vill ID 12	0.00	0.28	0.33	(0.01)
vill ID 19	0.00	0.31	2.12	(0.02)
vill ID 2	0.00	0.42	1.19	(0.02)
vill ID 35	0.06	0.54	0.96	(0.03)
vill ID 40	0.08	0.19	0.67	(0.01)
vill ID 24	0.12	0.35	1.62	(0.02)
vill ID 3	0.14	0.57	0.88	(0.01)
vill ID 37	0.15	0.53	0.45	(0.01)
vill ID 5	0.19	0.26	1.43	(0.03)
vill ID 21	0.19	0.46	1.45	(0.04)
vill ID 11	0.20	0.60	1.69	(0.01)
vill ID 8	0.21	0.50	2.23	(0.05)
vill ID 7	0.25	0.50	1.97	(0.01)
vill ID 4	0.27	0.63	2.35	(0.01)
vill ID 44	0.31	0.47	3.23	(0.04)
vill ID 25	0.33	1.00	2.31	(0.05)
vill ID 6	0.37	0.63	1.41	(0.01)
vill ID 28	0.38	0.38	2.49	(0.02)
vill ID 17	0.45	0.59	2.69	(0.04)
vill ID 14	0.47	0.78	3.91	(0.04)
vill ID 32	0.47	0.89	1.3	(0.03)
vill ID 31	0.50	0.67	2.48	(0.01)
vill ID 27	0.50	0.75	1.86	(0.01)
vill ID 38	0.50	0.86	3.84	(0.08)
vill ID 26	0.61	0.85	4.21	(0.04)
vill ID 20	0.63	0.75	3.64	(0.08)
vill ID 30	0.63	0.79	2.13	(0.02)
vill ID 43	0.67	0.83	0.97	(0.05)
vill ID 29	0.67	0.93	4.12	(0.03)
vill ID 13	0.71	0.94	4.67	(0.04)
vill ID 23	0.71	0.86	3.96	(0.04)
vill ID 36	0.78	0.89	2.34	(0.02)
vill ID 1	0.82	0.90	1.49	(0.05)
vill ID 42	0.82	0.89	2.67	(0.02)
vill ID 39	0.88	0.88	2.57	(0.03)
vill ID 34	0.88	0.96	4.96	(0.02)
vill ID 33	0.88	0.95	3.51	(0.04)
vill ID 16	0.91	1.00	2.88	(0.01)

Notes: Parameter estimates for location (mean) of taste shocks ε_i^d. The villages are listed in increasing order of sanitation coverage. Data:S1 and Data:S2 denote sanitation coverage over the two sample periods. Bootstrapped standard errors in parentheses.

Table 4.7: Village: Price & Coverage variation

Village	**Cost of Sanitation per unit (Rs.)**	**Data S1**	**Data S2**
vill ID 9	9,725	0.00	0.00
vill ID 15	10,243	0.00	0.00
vill ID 18	10,975	0.00	0.09
vill ID 22	10,016	0.00	0.13
vill ID 12	9,823	0.00	0.28
vill ID 19	10,427	0.00	0.31
vill ID 2	11,337	0.00	0.42
vill ID 35	10,280	0.06	0.54
vill ID 40	10,273	0.08	0.19
vill ID 24	9,510	0.12	0.35
vill ID 3	7,800	0.14	0.57
vill ID 37	9,788	0.15	0.53
vill ID 5	9,801	0.19	0.26
vill ID 21	10,475	0.19	0.46
vill ID 11	10,055	0.20	0.60
vill ID 8	7,938	0.21	0.50
vill ID 7	7,738	0.25	0.50
vill ID 4	8,795	0.27	0.63
vill ID 44	9,913	0.31	0.47
vill ID 25	11,175	0.33	1.00
vill ID 6	8,313	0.37	0.63
vill ID 28	8,131	0.38	0.38
vill ID 17	7,915	0.45	0.59
vill ID 14	8,882	0.47	0.78
vill ID 32	7,155	0.47	0.89
vill ID 31	6,900	0.50	0.67
vill ID 27	8,181	0.50	0.75
vill ID 38	6,775	0.50	0.86
vill ID 26	6,030	0.61	0.85
vill ID 20	8,113	0.63	0.75
vill ID 30	6,662	0.63	0.79
vill ID 43	6,113	0.67	0.83
vill ID 29	7,844	0.67	0.93
vill ID 13	9,924	0.71	0.94
vill ID 23	8,875	0.71	0.86
vill ID 36	6,113	0.78	0.89
vill ID 1	5,713	0.82	0.90
vill ID 42	9,012	0.82	0.89
vill ID 39	6,350	0.88	0.88
vill ID 34	7,963	0.88	0.96
vill ID 33	7,168	0.88	0.95
vill ID 16	11,425	0.91	1.00

Table 4.8: Village: Simulation Bounds

	Data		Mod: $\phi = 0$	Mod: $\phi > 0$
Village	**S1**	**S2**	**LB-UB**	**LB-UB**
vill ID 9	0.00	0.00	(0.00,0.19)	(0.02,0.20)
vill ID 15	0.00	0.00	(0.00,0.25)	(0.00,0.25)
vill ID 18	0.00	0.09	(0.02,0.29)	(0.03,0.29)
vill ID 22	0.00	0.13	(0.10,0.65)	(0.10,0.66)
vill ID 12	0.00	0.28	(0.05,0.29)	(0.05,0.31)
vill ID 19	0.00	0.31	(0.00,0.34)	(0.02,0.35)
vill ID 2	0.00	0.42	(0.05,0.36)	(0.07,0.39)
vill ID 35	0.06	0.54	(0.35,0.68)	(0.36,0.71)
vill ID 40	0.08	0.19	(0.12,0.66)	(0.15,0.68)
vill ID 24	0.12	0.35	(0.18,0.71)	(0.21,0.76)
vill ID 3	0.14	0.57	(0.20,0.76)	(0.26,0.77)
vill ID 37	0.15	0.53	(0.32,0.72)	(0.34,0.78)
vill ID 5	0.19	0.26	(0.19,0.71)	(0.21,0.74)
vill ID 21	0.19	0.46	(0.23,0.63)	(0.61,0.82)
vill ID 11	0.20	0.60	(0.26,0.75)	(0.29,0.76)
vill ID 8	0.21	0.50	(0.24,0.81)	(0.24,0.82)
vill ID 7	0.25	0.50	(0.30,0.82)	(0.32,0.81)
vill ID 4	0.27	0.63	(0.28,0.80)	(0.31,0.85)
vill ID 44	0.31	0.47	(0.33,0.64)	(0.33,0.67)
vill ID 25	0.33	1.00	(0.40, 0.72)	(0.41,0.73)
vill ID 6	0.37	0.63	(0.37,0.78)	(0.37,0.79)
vill ID 28	0.38	0.38	(0.38,0.72)	(0.39,0.75)
vill ID 17	0.45	0.59	(0.56,0.89)	(0.58,0.90)
vill ID 14	0.47	0.78	(0.47,0.82)	(0.75,0.83)
vill ID 32	0.47	0.89	(0.58,0.92)	(0.58,0.93)
vill ID 31	0.50	0.67	(0.62,0.89)	(0.64,0.90)
vill ID 27	0.50	0.75	(0.68,0.83)	(0.68,0.84)
vill ID 38	0.50	0.86	(0.76,0.94)	(0.76,0.94)
vill ID 26	0.61	0.85	(0.72,0.92)	(0.76,0.93)
vill ID 20	0.63	0.75	(0.72,0.93)	(0.72,0.95)
vill ID 30	0.63	0.79	(0.74,0.90)	(0.76,0.94)
vill ID 43	0.67	0.83	(0.78,0.93)	(0.79,0.96)
vill ID 29	0.67	0.93	(0.73,0.95)	(0.73,0.99)
vill ID 13	0.71	0.94	(0.78,0.97)	(0.81,1.00)
vill ID 23	0.71	0.86	(0.81,0.98)	(0.81,0.99)
vill ID 36	0.78	0.89	(0.81,0.96)	(0.81,0.97)
vill ID 1	0.82	0.90	(0.83,0.97)	(0.86,0.98)
vill ID 42	0.82	0.89	(0.83,0.94)	(0.83,0.94)
vill ID 39	0.88	0.88	(0.89,0.99)	(0.89,1.00)
vill ID 34	0.88	0.96	(0.88,0.98)	(0.89,1.00)
vill ID 33	0.88	0.95	(0.88,0.99)	(0.88,1.00)
vill ID 16	0.91	1.00	(0.91,0.98)	(0.95,0.99)

Table 4.9: Simulation Bounds (Perturbation ϕ)

	Data			**Mod:** $\phi = 0$					**Mod:** $\widehat{\phi} =$ 0.00514
Village	**S1**	**S2**		**P1**	**P2**	**P3**	**P4**	**P5**	**P6**
vill ID 24	0.120	0.350	**LB**	0.181	0.185	0.190	0.194	0.197	0.208
			UB	0.712	0.727	0.733	0.744	0.751	0.758
vill ID 3	0.140	0.570	**LB**	0.201	0.206	0.222	0.237	0.258	0.263
			UB	0.764	0.765	0.765	0.767	0.769	0.770
vill ID 21	0.190	0.460	**LB**	0.230	0.272	0.357	0.484	0.590	0.611
			UB	0.626	0.691	0.734	0.798	0.820	0.820
vill ID 14	0.470	0.780	**LB**	0.470	0.549	0.581	0.628	0.675	0.754
			UB	0.817	0.818	0.820	0.821	0.823	0.827
vill ID 13	0.710	0.940	**LB**	0.776	0.781	0.787	0.790	0.799	0.808
			UB	0.973	0.976	0.984	0.991	0.999	1.000

Table 4.10: Village: Social Planner Problem

Village	Total #HH (approx)	Total Endowment value (x1000 Rs.)	Cost of Sanitation per unit (x1000 Rs.)	Data S1	Utilitarian Social Planner	Under-adoption (%)
vill ID 9	190	8,217.88	9.725	0.00	0.72	100%
vill ID 15	162	8,201.25	10.243	0.00	0.73	100%
vill ID 18	301	12,474.04	10.975	0.00	0.78	100%
vill ID 22	240	10,410.00	10.016	0.00	0.75	100%
vill ID 12	121	4,154.29	9.823	0.00	0.74	100%
vill ID 19	210	5,995.29	10.427	0.00	0.73	100%
vill ID 2	470	23,028.12	11.337	0.00	0.77	100%
vill ID 35	762	31,341.06	10.280	0.06	0.62	91%
vill ID 40	360	22,363.56	10.273	0.08	0.74	89%
vill ID 24	873	42,891.36	9.510	0.12	0.66	82%
vill ID 3	786	28,177.31	7.800	0.14	0.58	77%
vill ID 37	306	10,324.13	9.788	0.15	0.70	79%
vill ID 5	270	13,786.47	9.801	0.19	0.76	76%
vill ID 21	282	18,634.28	10.475	0.19	0.81	77%
vill ID 11	100	4,572.00	10.055	0.20	0.72	72%
vill ID 8	308	19,836.43	7.938	0.21	0.82	74%
vill ID 7	226	9,754.84	7.738	0.25	0.78	68%
vill ID 4	633	56,674.39	8.795	0.27	0.92	70%
vill ID 44	313	27,611.92	9.913	0.31	0.89	65%
vill ID 25	109	4,756.76	11.175	0.33	0.78	57%
vill ID 6	200	12,018.20	8.313	0.37	0.77	52%
vill ID 28	164	19,903.53	8.131	0.38	0.94	60%
vill ID 17	324	15,900.95	7.915	0.45	0.84	46%
vill ID 14	220	10,798.04	8.882	0.47	0.75	38%
vill ID 32	187	12,615.58	7.155	0.47	0.76	38%
vill ID 31	127	8,930.64	6.900	0.50	0.74	32%
vill ID 27	120	4,304.76	8.181	0.50	0.74	32%
vill ID 38	328	28,304.10	6.775	0.50	0.91	45%
vill ID 26	413	22,870.29	6.030	0.61	0.88	31%
vill ID 20	169	10,431.53	8.113	0.63	0.87	28%
vill ID 30	366	23,321.89	6.662	0.63	0.86	26%
vill ID 43	140	13,040.02	6.113	0.67	0.91	27%
vill ID 29	453	27,705.03	7.844	0.67	0.88	24%
vill ID 13	340	16,649.12	9.924	0.71	0.96	26%
vill ID 23	168	11,790.41	8.875	0.71	0.91	22%
vill ID 36	280	22,333.92	6.113	0.78	0.94	17%
vill ID 1	347	22,793.04	5.713	0.82	0.91	10%
vill ID 42	273	21,557.72	9.012	0.82	0.85	3%
vill ID 39	163	8,761.58	6.350	0.88	0.78	-12%
vill ID 34	215	11,279.98	7.963	0.88	0.89	2%
vill ID 33	314	20,301.04	7.168	0.88	0.91	3%
vill ID 16	167	12,332.95	11.425	0.91	0.92	1%

Notes: This table show the socially optimal level of sanitation coverage Social Planner calculations are performed using endowment level from observed villages in period $S1$. Column (5) and (6) denote the proportion of sanitation adoption observed in the data and under the social planner solution respectively. On average the extent of under-adoption of sanitation is close to 53% with respect to a utilitarian SWF, where the planner assigns equal pareto weights to each household in the village. £1 $\approx Rs.100\,(INR)$.

Table 4.11: Village: Price Subsidy Simulated Bounds

Village	Cost of Sanitation (Rs.)	Data S1	Pol: Price Subsidy (LB-UB) No Subsidy	Sub: 5%	Sub: 15%	Sub: 25%
vill ID 9	9,725	0.00	(0.02,0.20)	(0.05,0.25)	(0.18,0.72)	(0.64,0.98)
vill ID 15	10,243	0.00	(0.00,0.25)	(0.02,0.26)	(0.12,0.42)	(0.72,0.88)
vill ID 18	10,975	0.00	(0.03,0.29)	(0.05,0.30)	(0.15,0.48)	(0.65,0.80)
vill ID 22	10,016	0.00	(0.10,0.66)	(0.10,0.66)	(0.17,0.67)	(0.81,0.90)
vill ID 12	9,823	0.00	(0.05,0.31)	(0.05,0.31)	(0.23,0.62)	(0.84,0.91)
vill ID 19	10,427	0.00	(0.02,0.35)	(0.03,0.35)	(0.08,0.41)	(0.68,0.75)
vill ID 2	11,337	0.00	(0.07,0.39)	(0.09,0.39)	(0.12,0.43)	(0.62,0.88)
vill ID 35	10,280	0.06	(0.36,0.71)	(0.38,0.71)	(0.65,0.84)	(0.72,0.91)
vill ID 40	10,273	0.08	(0.15,0.68)	(0.17,0.69)	(0.19,0.72)	(0.59,0.78)
vill ID 24	9,510	0.12	(0.21,0.76)	(0.28,0.79)	(0.31,0.83)	(0.66,0.90)
vill ID 3	7,800	0.14	(0.26,0.77)	(0.35,0.79)	(0.84,0.96)	(0.91,0.99)
vill ID 37	9,788	0.15	(0.34,0.78)	(0.36,0.79)	(0.71,0.86)	(0.76,0.92)
vill ID 5	9,801	0.19	(0.21,0.74)	(0.30,0.78)	(0.64,0.82)	(0.70,0.86)
vill ID 21	10,475	0.19	(0.61,0.82)	(0.62,0.84)	(0.66,0.89)	(0.69,0.90)
vill ID 11	10,055	0.20	(0.29,0.76)	(0.29,0.78)	(0.32,0.79)	(0.62,0.84)
vill ID 8	7,938	0.21	(0.24,0.82)	(0.32,0.83)	(0.63,0.85)	(0.68,0.88)
vill ID 7	7,738	0.25	(0.32,0.81)	(0.34,0.82)	(0.70,0.88)	(0.76,0.93)
vill ID 4	8,795	0.27	(0.31,0.85)	(0.37,0.89)	(0.68,0.92)	(0.75,0.96)
vill ID 44	9,913	0.31	(0.33,0.67)	(0.36,0.69)	(0.76,0.92)	(0.86,0.95)
vill ID 25	11,175	0.33	(0.40, 0.72)	(0.43,0.74)	(0.48,0.76)	(0.62,0.91)
vill ID 6	8,313	0.37	(0.37,0.79)	(0.38,0.79)	(0.61,0.83)	(0.66,0.84)
vill ID 28	8,131	0.38	(0.39,0.75)	(0.41,0.76)	(0.60,0.79)	(0.65,0.82)
vill ID 17	7,915	0.45	(0.58,0.90)	(0.72,0.91)	(0.75,0.92)	(0.78,0.98)
vill ID 14	8,882	0.47	(0.75,0.83)	(0.78,0.84)	(0.80,0.89)	(0.88,0.97)
vill ID 32	7,155	0.47	(0.58,0.93)	(0.70,0.94)	(0.73,0.95)	(0.80,1.00)
vill ID 31	6,900	0.50	(0.64,0.90)	(0.67,0.91)	(0.72,0.94)	(0.81,0.97)
vill ID 27	8,181	0.50	(0.68,0.84)	(0.72,0.85)	(0.81,0.92)	(0.84,0.93)
vill ID 38	6,775	0.50	(0.76,0.94)	(0.78,0.94)	(0.81,0.94)	(0.82,0.95)
vill ID 26	6,030	0.61	(0.76,0.93)	(0.78,0.94)	(0.82,0.96)	(0.88,0.99)
vill ID 20	8,113	0.63	(0.72,0.95)	(0.75,0.96)	(0.79,0.96)	(0.86,0.99)
vill ID 30	6,662	0.63	(0.76,0.94)	(0.80,0.96)	(0.83,0.96)	(0.88,0.97)
vill ID 43	6,113	0.67	(0.79,0.96)	(0.79,0.96)	(0.84,0.97)	(0.88,0.99)
vill ID 29	7,844	0.67	(0.73,0.99)	(0.75,0.99)	(0.78,0.99)	(0.82,1.00)
vill ID 13	9,924	0.71	(0.81,1.00)	(0.84,1.00)	(0.88,1.00)	(0.89,1.00)
vill ID 23	8,875	0.71	(0.81,0.99)	(0.84,1.00)	(0.85,1.00)	(0.88,1.00)
vill ID 36	6,113	0.78	(0.81,0.97)	(0.82,0.97)	(0.86,0.98)	(0.94,0.99)
vill ID 1	5,713	0.82	(0.86,0.98)	(0.87,0.98)	(0.92,0.99)	(0.93,0.99)
vill ID 42	9,012	0.82	(0.83,0.94)	(0.86,0.96)	(0.88,0.96)	(0.92,0.97)
vill ID 39	6,350	0.88	(0.89,1.00)	(0.89,1.00)	(0.90,1.00)	(0.90,1.00)
vill ID 34	7,963	0.88	(0.89,1.00)	(0.91,1.00)	(0.92,1.00)	(0.93,1.00)
vill ID 33	7,168	0.88	(0.88,1.00)	(0.88,1.00)	(0.90,1.00)	(0.90,1.00)
vill ID 16	11,425	0.91	(0.95,0.99)	(0.95,1.00)	(0.95,1.00)	(0.96,1.00)

Notes: Policy simulations are performed on observed villages. £1 $\approx Rs.\,100\,(INR)$.

Table 4.12: Simulated Bounds under different policies

Initial Sanitation		**Policy Effect: Equilibrium Sanitation (LB-UB)**	
Coverage (Fraction)	**Uncond Loan**	**Sanitation Loan (100% of cost)**	**Price Subsidy (25% of cost)**
0	(0.07,0.28)	(0.02,0.39)	(0.16,0.42)
0.05	(0.11,0.35)	(0.05,0.46)	(0.21,0.58)
0.15	(0.28,0.49)	(0.16,0.58)	(0.39,0.66)
0.25	(0.38,0.60)	(0.26,0.69)	(0.47,0.78)
0.35	(0.60,071)	(0.42,0.75)	(0.66,0.81)
0.45	(0.71,0.77)	(0.69,0.84)	(0.75,0.88)
0.55	(0.75,0.81)	(0.81,0.90)	(0.82,0.91)
0.65	(0.82,0.86)	(0.88,0.93)	(0.90,0.96)
0.75	(0.85,0.91)	(0.94,0.98)	(0.96,0.98)
0.85	(0.91,0.96)	(0.95,0.98)	(0.98,0.98)
0.95	(0.95,0.98)	(0.97,0.99)	(0.98,1.00)

Notes: Policy simulations are performed on a counterfactual village where the initial distribution of all state variables: age, assets, income and cost of sanitation $Rs.8628$ excluding the initial sanitation coverage are held constant. The initial sanitation coverage is determined by generating a random allocation of sanitation for different households in the village holding fixed all other characteristics.

Table 4.13: Household Valuation of Sanitation

	Compensation Amount (x 1000 Rs.)	
Age	**No Ext**	**With Ext (LB-UB)**
20	259.8	(578.4,884.6)
25	242.3	(557.1,834.6)
30	185.3	(483.8,732.4)
35	158.8	(389.6,632.5)
40	140.2	(326.5,561.4)
45	128.4	(259.8,438.5)
50	110.8	(224.0,328.5)
55	89.3	(163.4,267.2)
60	72.3	(125.4,189.2)
65	56.2	(96.4,136.1)
70	33.45	(63.2,97.5)
74	12.6	(35.7,63.5)

Notes: Compensation amount denotes the valuation of sanitation made by a household. The amounts are computed for a representative household at different ages £1 $\approx Rs.100\,(INR)$.

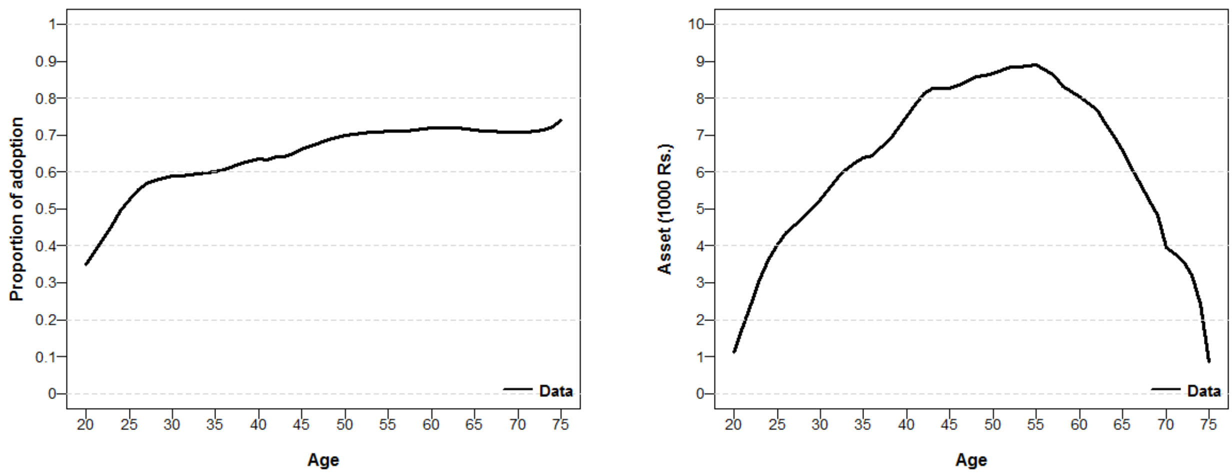
Table 4.14: Estimated Welfare Change: Social Planner's Solution

	Total Welfare (x1000. Rs)	**Sanitation Coverage**
Baseline S1	9,247.2	0.370
Social Planner	36,569.6	0.807
Change	+295.5%	+118.1%

Notes: This table shows the change in the welfare for a representative village from enacting the social planner's solution where the total endowment is calculated with respect to the first sample period. The social planner induces households to solve the optimal adoption problem by re-allocating the total endowment between food consumption and sanitation, so as to maximize utility. A utilitarian Social Welfare Function (SWF) is maximized with equal pareto weights for each household within the village

4.8.5 Figures

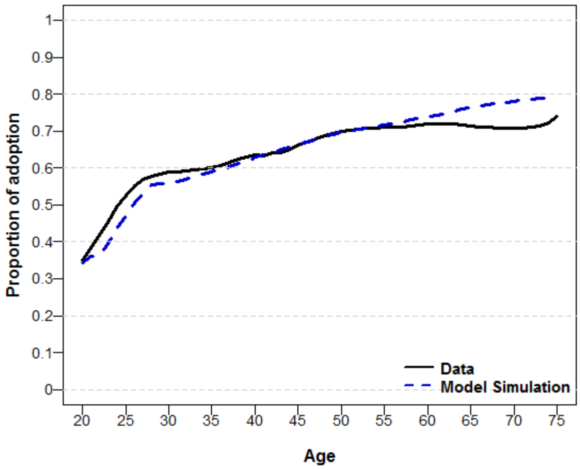
Figure 4.1: Life Cycle Profiles



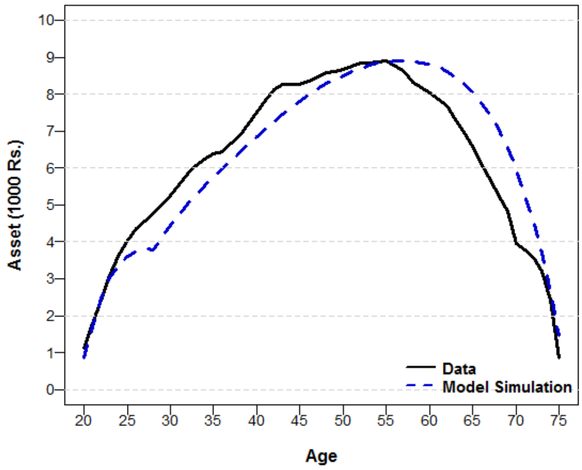
(a) Proportion of Sanitation Adoption

(b) Assets over the Life Cycle $(1000 Rs.)$

Figure 4.2: Model Fit: Life Cycle Profiles



(a) Proportion of Sanitation Adoption



(b) Assets over the Life Cycle $(1000\, Rs.)$

Figure 4.3: Impact of Liquidity Constraints

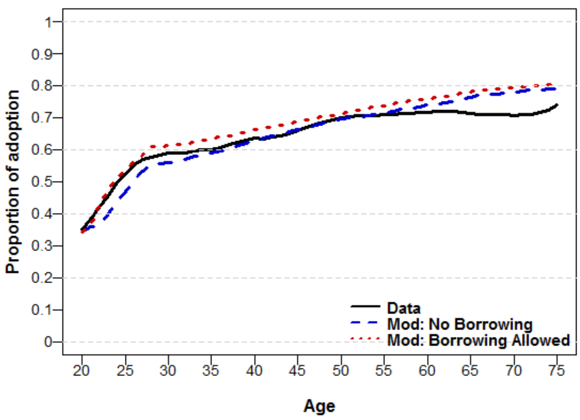
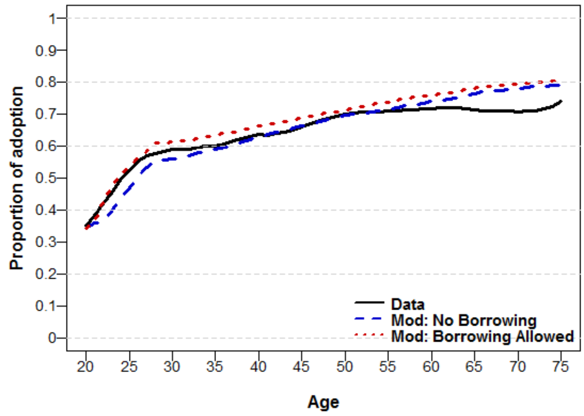
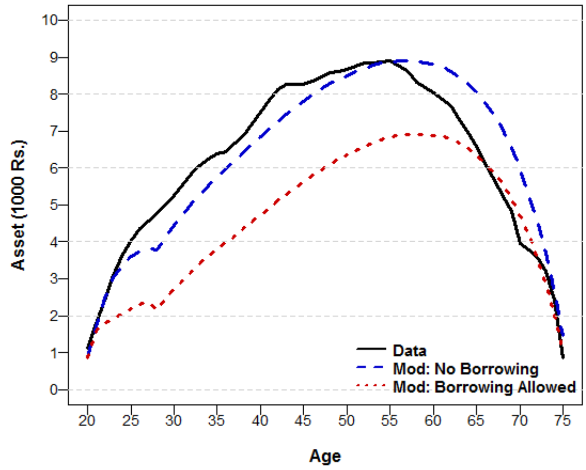


Figure 4.4: Model Fit: Impact of Liquidity Constraints



(a) Proportion of Sanitation Adoption



(b) Assets over the life cycle $(1000\,Rs.)$

Figure 4.5: Model: Household Income

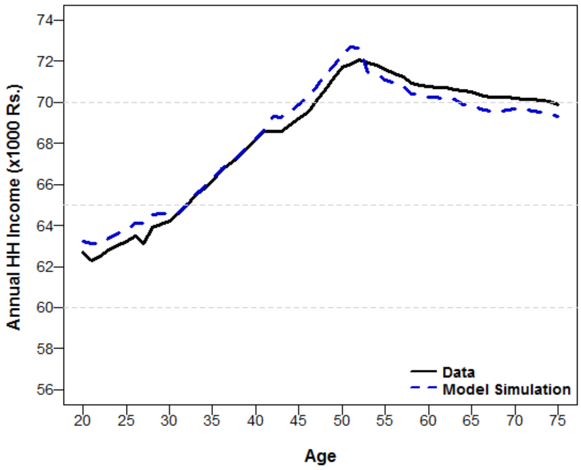


Figure 4.6: Model Fit: Simulation by Village

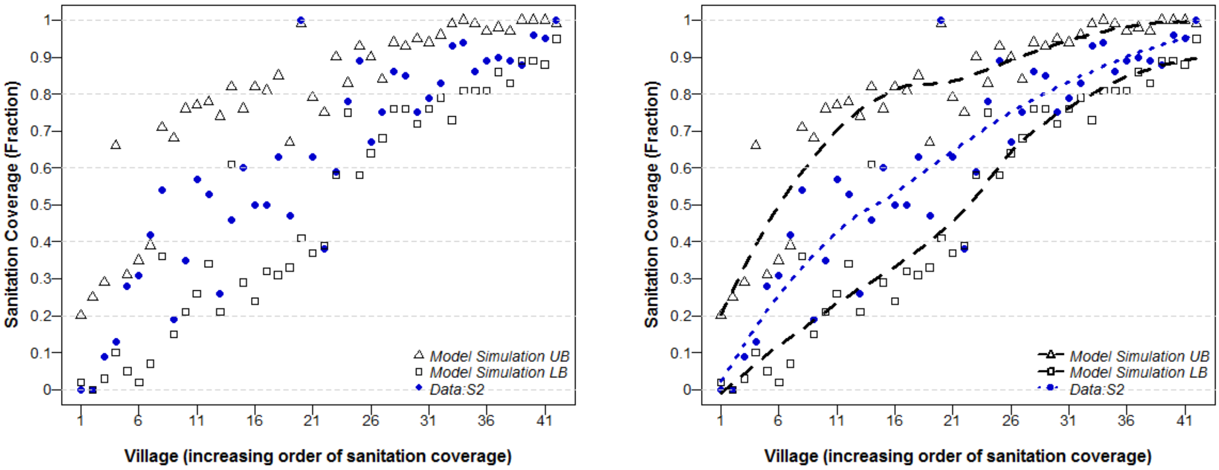


Figure 4.7: Model Fit: Model $\phi = 0$

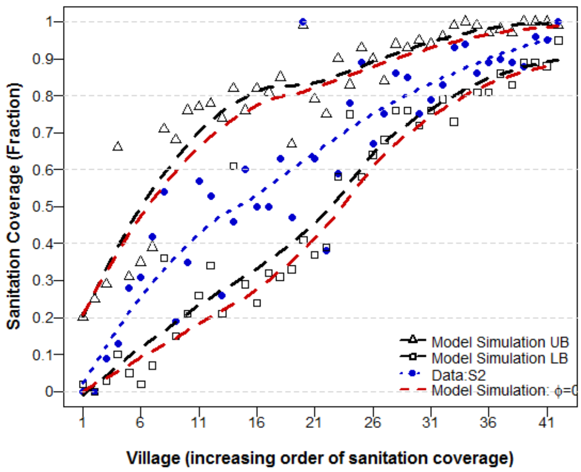


Figure 4.8: Household Valuation of Sanitation

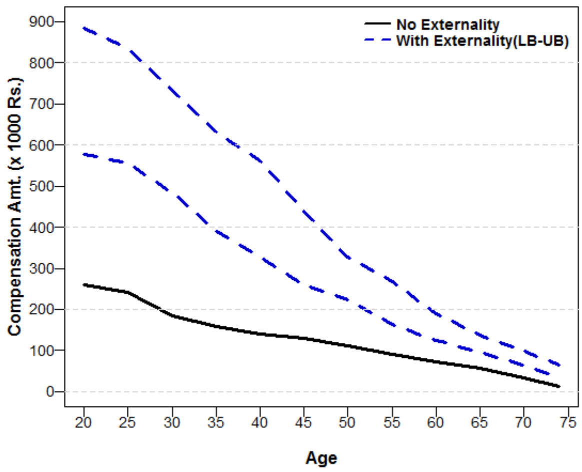


Figure 4.9: Model simulation: Price Subsidy

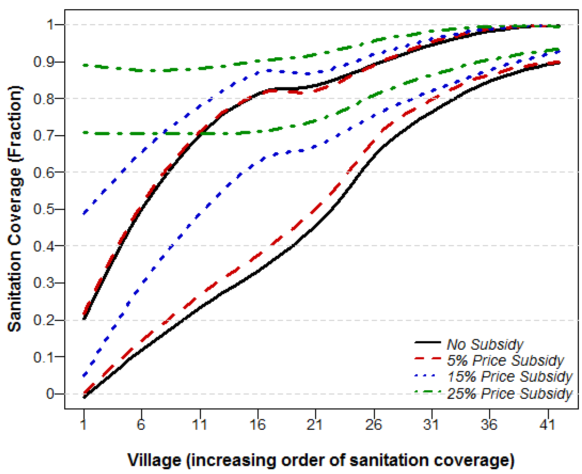
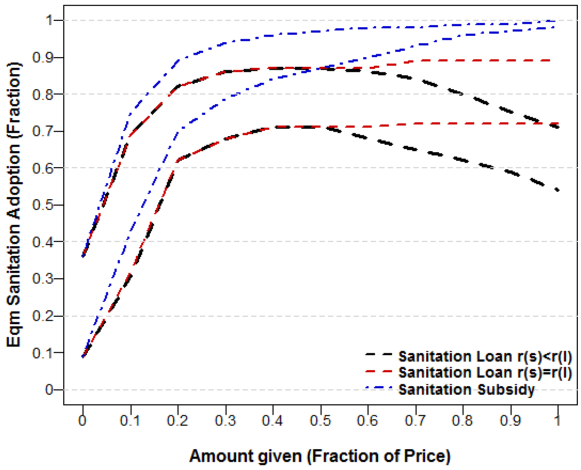
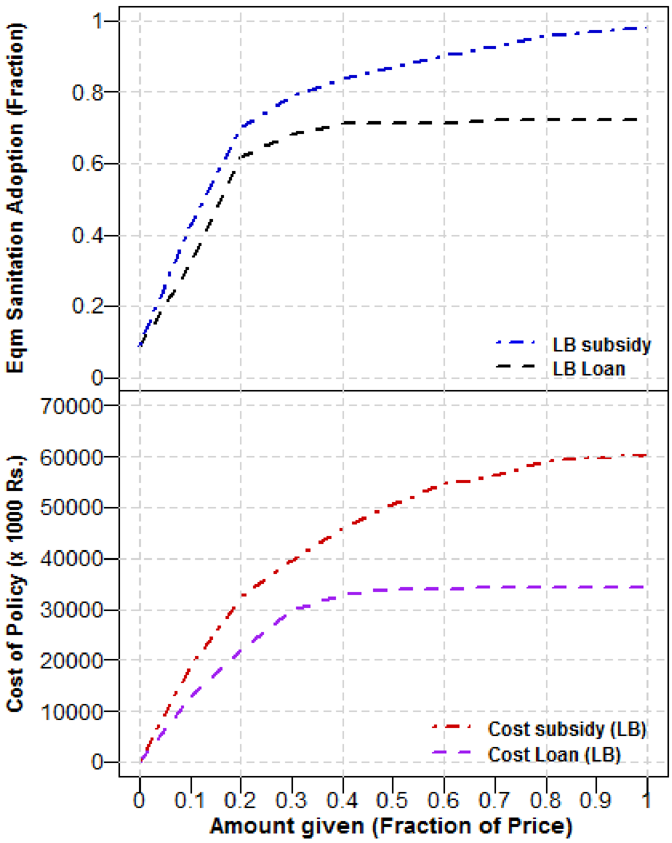


Figure 4.10: Equilibrium Adoption: Size of Loans & Subsidies



Notes: The simulations plot the upper and lower bound for the predicted equilibrium sanitation level one period ahead. Policy simulations are performed on a counterfactual village where the initial distribution of all state variables: age, assets, income and cost of sanitation *Rs.*8628 are held constant and the initial sanitation coverage is fixed at 0%.

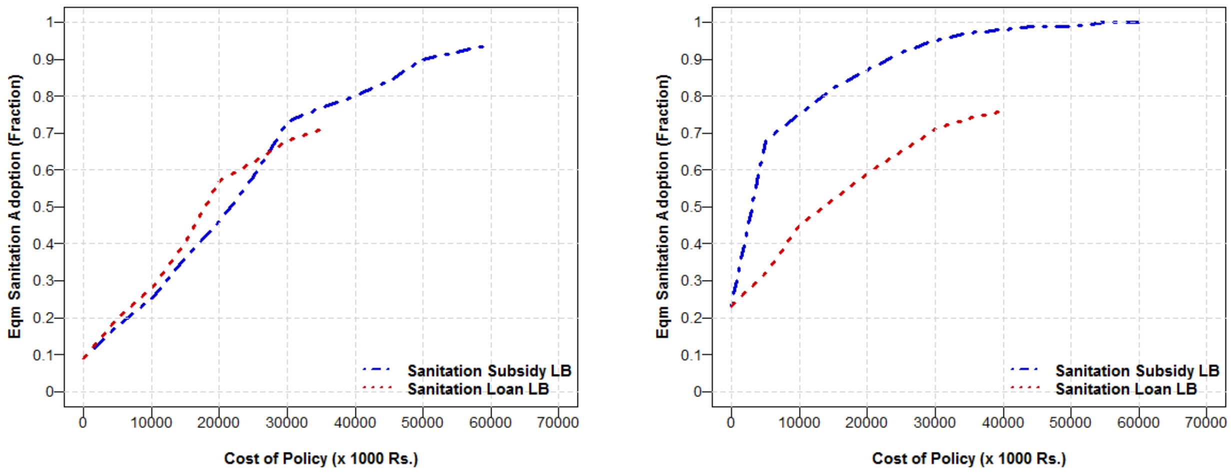
Figure 4.11: Equilibrium Adoption: Cost of Policy & Size of Policy (lower bound)



Notes: The simulations plot the upper and lower bound for the predicted equilibrium sanitation level one period ahead. Policy simulations are performed on a counterfactual village where the initial distribution of all state variables: age, assets, income and cost of sanitation $Rs. 8,628$ are held constant and the initial sanitation coverage is fixed at 0%.

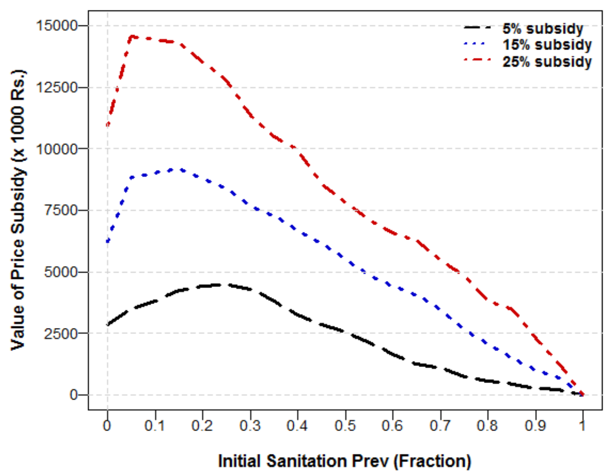
Figure 4.12: Equilibrium Adoption: Cost and Allocation of Policy (lower bound)

(a) ZERO INITIAL COVERAGE (b) LOW INITIAL COVERAGE



Notes: The simulations plot the lower bounds for the predicted equilibrium sanitation level one period ahead. Policy simulations are performed on a counterfactual village where the initial distribution of all state variables: age, assets, income and cost of sanitation $Rs.8628$ are held constant and the initial sanitation coverage is fixed at 0%.

Figure 4.13: Value of price subsidy: lower bound



Notes: The simulations plot the lower bound for the value of subsidy one period ahead. Policy simulations are performed on a counterfactual village where the initial distribution of all state variables: age, assets, income and cost of sanitation $Rs.8628$ excluding the initial sanitation coverage are held constant. The initial sanitation coverage is determined by generating a random allocation of sanitation for different households in the village holding fixed all other characteristics.

Figure 4.14: Price Elasticity: Substitution & Income Effects

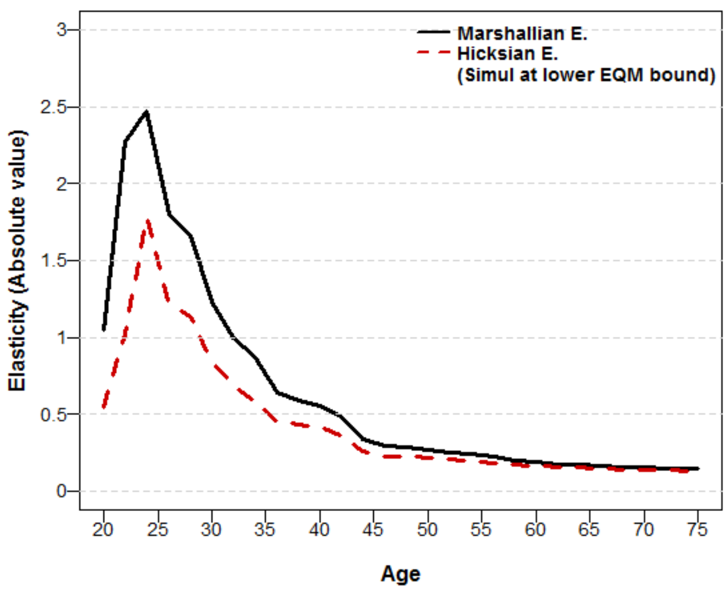
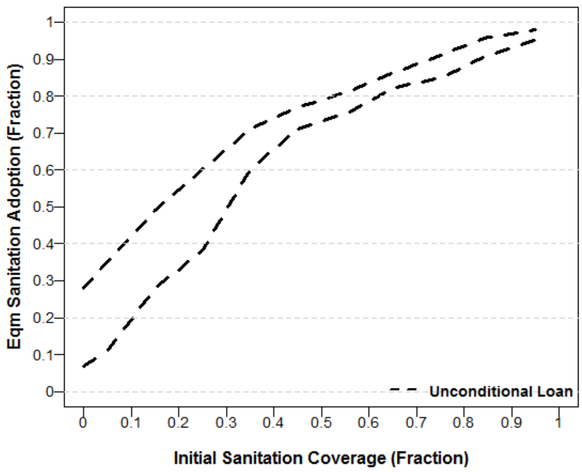
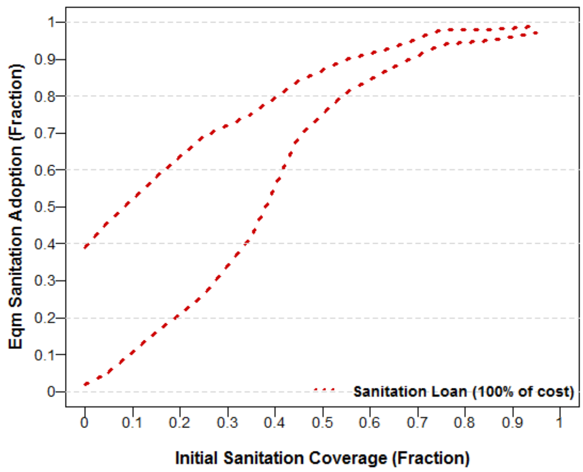


Figure 4.15: Simulated Policy Bounds

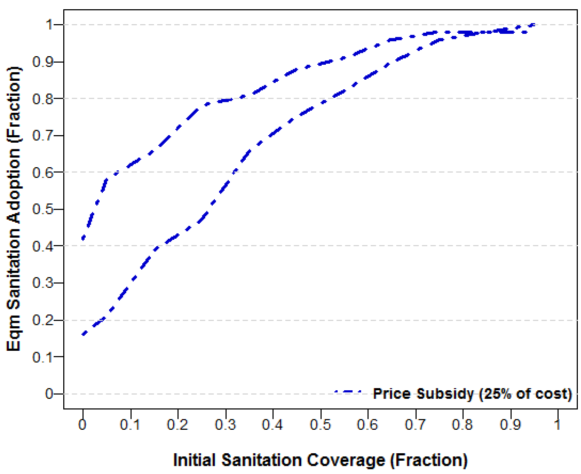
(a) UNCONDITIONAL LOAN



(b) SANITATION LOAN (100% OF COST)



(c) PRICE SUBSIDY (25% OF COST)



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