

# Coverage probability of cellular networks using interference alignment under imperfect CSI<sup>☆,☆☆</sup>

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## ARTICLE INFO

### Keywords:

Interference alignment  
Beamforming  
Coverage probability  
Channel errors  
Stochastic geometry

## ABSTRACT

Interference alignment (IA) is well understood to approach the capacity of interference channels, and believed to be crucial in cellular networks in which the ability to control and exploit interference is key. However, the achievable performance of IA in cellular networks depends on the quality of channel state information (CSI) and how effective IA is in practical settings is not known. This paper studies the use of IA to mitigate inter-cell interference of cellular networks under imperfect CSI conditions. Our analysis is based on stochastic geometry where the structure of the base station (BS) locations is considered by a Poisson point process (PPP). Our main contribution is the coverage probability of the network and simulation results confirm the accuracy.

## 1. Introduction

The mobile data traffic demand continues to be increasing at an alarming rate due to more and more services requiring higher data rate and also the rise of device-to-device (D2D) communications. In order to cope with this demand, network operators are moving towards a denser deployment of their networks with multiple tiers (from macro to micro, pico and then femto-cells) sharing the same spectrum [1]. This densification of the networks brings more capacity to local regions but also means that there will be more interference to mitigate in order to truly benefit from this network architecture.

In current wireless networks interference mitigation means that the users are spread across different time slots or frequency bands using techniques such as time-division multiple-access (TDMA), frequency-division multiple-access (FDMA) and etc. The limitation is, however, that the resource available for each user decreases with the number of users. A novel technique, widely known as interference alignment (IA), offers a new way of managing the interference while using the same spectrum. IA was first introduced in [2] and subsequently developed in [3,4]. The beauty of IA is that it can achieve the degree-of-freedom (DoF) of the interference channel, implying capacity achieving at high signal-to-noise ratio (SNR).

The way IA works is by aligning the interference received at every receiver in a reduced dimension space leaving an interference-free space for the desired signal. It was revealed in [4] that a sum-rate of  $\frac{K}{2}$

is achievable by IA in an interference channel with  $K$  users which means that every user can achieve  $\frac{1}{2}$  of the total available resource. This result was massive as this states that the capacity of an interference channel scales with the number of coexisting users. However, such amazing results came with the assumption of infinite frequency or time diversity and that full and perfect channel state information (CSI) is available at all the communicating nodes.

Recent studies have turned the attention to investigate IA with a finite number of dimensions, i.e., coding over a finite number of time or frequency slots and also using the finite number of spatial dimensions offered by the use of multiple antennas to perform IA [5–7]. The effects of imperfect CSI on the IA performance have also been studied in [8–10] and feedback delay was considered in [11,12]. In addition, the feasibility conditions for IA were also studied in [13,14] but even though the feasibility of IA can be determined in many cases, there is no closed-form solution available for the design of the precoders in the general case. In [15–17], several IA algorithms were developed and the effects of CSI errors on these algorithms have also been studied in [18].

Differing from the literature, this paper considers the use of IA in cellular networks under imperfect CSI conditions. While similar objectives have been pursued in [19–21], the novelty of our work lies in that our analysis includes the randomness of base station (BS) deployment and we derive the coverage probability of such IA-assisted cellular network. We note that although coverage probability analysis has been addressed in [22] where the authors developed new general

<sup>☆</sup> This work is supported by EPSRC and BT under grant EP/K504610/1.

Peer review under responsibility of Chongqing University of Posts and Telecommunication.

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<http://dx.doi.org/10.1016/j.dcan.2016.10.007>

Received 2 May 2016; Received in revised form 9 September 2016; Accepted 28 October 2016

Available online 09 November 2016

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models for the multi-cell signal-to-interference-plus-noise ratio (SINR) using stochastic geometry, no ways of mitigating the inter-cell interference was considered in the analysis except by changing the frequency reuse factor in the network and to our knowledge there is no such study including IA with imperfect CSI in their analysis.

This paper is organized as follows. Section 2 will introduce the system model and give some background information on IA. Section 3 then presents the definition of the coverage probability and in Section 4, the expression of the coverage probability is derived assuming a Poisson point process (PPP) for the distribution of the BSs. Section 5 provides the numerical results. Finally, Section 5 concludes the paper.

*Notations*—Throughout, upper-case bold letters denote matrices, while lower-case bold letters denote vectors. In addition,  $(\cdot)^*$  denotes the conjugate transpose operation,  $\mathbb{E}_X\{\cdot\}$  returns the average of an input random entity over random variable (r.v.)  $X$ ,  $\mathbb{P}(\cdot|X)$  gives the probability of an event conditioned on  $X$ ,  $|\cdot|$  computes the modulus of a complex number. Also,  $\Gamma(a, s) = \int_s^{+\infty} x^{a-1}e^{-x}dx$  is the upper incomplete gamma function and  $\gamma(a, s) = \int_0^s x^{a-1}e^{-x}dx$  is the lower incomplete gamma function,  $\mathcal{CN}(\mu, \sigma^2)$  represents a circularly symmetric complex Gaussian r.v. with mean  $\mu$  and variance  $\sigma^2$ .

## 2. System model

We consider a cellular network model that consists of BSs located randomly in a Euclidean plane according to some point process. We also consider multiple user equipments (UE) distributed randomly in the network according to an independent point process. Each UE is assumed to be associated with a serving BS which may or may not be the nearest one. For example, if the association policy is to pair the UE with the BS with the highest SNR, then this BS may or may not be the closest one because of propagation issues (e.g., shadowing). We focus on the downlink and assume that intra-cell interference is non-existent or dealt with perfectly by orthogonalizing the same cell users over time or frequency. As a result, at each time or frequency slot, the network can be modelled as an interference channel with multiple transmitter-receiver pairs. The inter-cell interference is then mitigated using IA.

In particular, IA is applied in the network under the assumption that every node in the network (BSs and UEs) has access to perfect CSI between a BS and its associated UE but imperfect CSI between a BS and the UEs that the BS is creating interference at. Note that a given UE in the network will not suffer interference from every BS in the network due to the high path-loss to some BSs. Therefore, we define a maximum distance  $R_M$  to the UE within which a non-serving BS will be considered as an interferer and above which it is invisible to the UE.

Let us focus our attention onto a circle of radius  $R_M$  in the network and denote by  $G$  the numbers of serving BSs and UEs in each time/frequency slot in that circle. Both the BSs and the UEs are equipped with multiple antennas for spatial IA. The number of antennas at each node is determined so as to fulfil the feasibility conditions for IA [13,23]. We denote by  $\mathbf{H}_{i,j}$  the channel matrix related to the scattering and multipath effect of the environment between the  $j$ th transmitter and the  $i$ th receiver. The estimate of  $\mathbf{H}_{i,j}$ , denoted as  $\hat{\mathbf{H}}_{i,j}$ , is such that

$$\mathbf{H}_{i,j} = \hat{\mathbf{H}}_{i,j} + \Delta\mathbf{H}_{i,j}, \quad \forall i \neq j \quad (1)$$

where  $\Delta\mathbf{H}_{i,j}$  is the estimation error with entries drawn from a complex Gaussian distribution  $\mathcal{CN}(0, \sigma_e^2)$ . The direct links are assumed to be known perfectly since their estimation is easier than that of the interfering links and is typically highly accurate. The path-loss and shadowing will be represented by the quantities  $\ell_i$  for the interfering links and  $\ell_d$  for the direct link. The assumption that we make here is that all the interferers have the same coefficients  $\ell_i$  which in practice is not true but we believe that the study of this case can still provide some meaningful insights. The channel matrices are assumed constant for at least the time of communication over a time or frequency slot. The precoders  $\mathbf{V}_k$  and decoding matrices  $\mathbf{U}_k$  are obtained following the IA

conditions below:

$$\begin{cases} \text{rank}(\mathbf{U}_k^* \mathbf{H}_{k,k} \mathbf{V}_k) = d_k, & \text{for } k = 1, 2, \dots, K, \\ \mathbf{U}_\ell^* \hat{\mathbf{H}}_{\ell,k} \mathbf{V}_k = 0, & \text{for all } \ell \neq k, \end{cases} \quad (2)$$

where  $d_k$  represents the number of streams for the  $k$ th user.

## 3. Coverage probability analysis

In this section, our aim is to derive the coverage probability for a user taken randomly in the network. It is defined as the probability that a given user can achieve some target rate. The coverage probability can be viewed as the complementary of the outage probability given in [9]. Before we can give a mathematical definition of the coverage probability, we will adapt the expressions in [9] to the current model.

### 3.1. Statistics of the achievable rate

Consider a typical user (say user  $o$ ) and the set  $\mathcal{S}_o$  of the BSs that create interference at this user. The received signal at this user can be written as

$$\mathbf{y}_o = \sqrt{\ell_d} \mathbf{H}_{o,o} \mathbf{V}_o \mathbf{x}_o + \sum_{s \in \mathcal{S}_o} \sqrt{\ell_i} \mathbf{H}_{o,s} \mathbf{V}_s \mathbf{x}_s + \boldsymbol{\eta}_o, \quad (3)$$

where  $\boldsymbol{\eta}_o$  is the additive noise drawn from  $\mathcal{CN}(0, \sigma_n^2)$  and  $\mathbf{x}_k$  denotes the data stream for the  $k$ th user. After we apply the decoding matrix, the expression (3) becomes

$$\mathbf{U}_o^* \mathbf{y}_o = \sqrt{\ell_d} \mathbf{U}_o^* \mathbf{H}_{o,o} \mathbf{V}_o \mathbf{x}_o + \mathbf{U}_o^* \sum_{s \in \mathcal{S}_o} \sqrt{\ell_i} \Delta \mathbf{H}_{o,s} \mathbf{V}_s \mathbf{x}_s + \mathbf{U}_o^* \boldsymbol{\eta}_o, \quad (4)$$

where the term  $\mathbf{U}_o^* \sum_{s \in \mathcal{S}_o} \sqrt{\ell_i} \Delta \mathbf{H}_{o,s} \mathbf{V}_s \mathbf{x}_s$  arises from the IA conditions (2) and the CSI error model (1).

If we assume that all the BSs have the same transmit power  $P_o$ , then according to [9], we can express the received interference at any stream of user  $o$  as a r.v.

$$\mathcal{I} = \ell_i P_o \Delta, \quad (5)$$

in which  $\frac{2}{\sigma_e^2} \Delta$  is a r.v. drawn from the  $\chi^2$  distribution with  $2(D - d_o)$  DoFs where  $D$  is the total number of streams sent by the interferers in  $\mathcal{S}_o$  plus that of the serving BS and  $d_o$  is the number of streams sent by the serving BS only.

The probability density function (pdf) of  $\Delta$  is given as

$$f_\Delta(\delta) = \begin{cases} \frac{1}{\sigma_e^2 \Gamma(D - d_o)} \left(\frac{\delta}{\sigma_e^2}\right)^{D-d_o-1} e^{-\frac{\delta}{\sigma_e^2}} & \text{for } \delta \geq 0, \\ 0 & \text{for } \delta < 0, \end{cases} \quad (6)$$

and the achievable rate for the  $l$ th stream of the UE is

$$R_l(P_o, \mathcal{I}) = \log_2 \left( 1 + \frac{\left(\ell_d \frac{P_o}{d_o}\right) |(\mathbf{U}_o^*)_l \mathbf{H}_{o,o} [\mathbf{V}_o]_l|^2}{\mathcal{I} + \sigma_n^2} \right). \quad (7)$$

From now on, we define  $z_l \triangleq |(\mathbf{U}_o^*)_l \mathbf{H}_{o,o} [\mathbf{V}_o]_l|^2$  and write the achievable rate of the  $l$ th stream of the UE as

$$R_l(P_o, \mathcal{I}) = \log_2 \left( 1 + \frac{\left(\ell_d \frac{P_o}{d_o}\right) z_l}{\mathcal{I} + \sigma_n^2} \right). \quad (8)$$

This rate expression will be used in the following form:

$$R_l(\rho, \Delta) = \log_2 \left( 1 + \frac{\ell_d z_l}{d_o \left( \ell_i \Delta + \frac{1}{\rho} \right)} \right), \quad (9)$$

where  $\rho = \frac{P_o}{\sigma_n^2}$ .

### 3.2. Outage probability

In [9], the outage probability of a user sending  $d_o$  streams is defined for each of its streams (say for the  $l$  th stream) as the probability that that stream cannot support any rate equal or above a target rate  $R_{out}^l$  at infinite SNR, i.e.,  $\mathcal{P}_{out|D}^l = \mathbb{P}(R_l^\infty < R_{out}^l | D)$ , with  $R_l^\infty(\Delta) \triangleq R_l(\infty, \Delta)$  and

$$R_l^\infty(\Delta) = \log \left( 1 + \frac{\beta z_l}{d_o \Delta} \right), \quad (10)$$

where  $\beta = \frac{\ell_d}{\ell_i}$ .

We define the outage SNR  $\rho_{out}^l$  so that

$$R_{out}^l = \log \left( 1 + \frac{\rho_{out}^l z_l}{d_o} \right). \quad (11)$$

As we will see later on, this definition of the outage SNR allows us to talk about the outage probability without having to worry about the randomness of  $z_l$  on the direct link.

We can now give the outage probability as

$$\mathcal{P}_{out|D,\beta}^l = \mathbb{P}(R_l^\infty < R_{out}^l | D, \beta) \quad (12)$$

$$= \mathbb{P} \left( \log \left( 1 + \frac{\beta z_l}{d_o \Delta} \right) < \log \left( 1 + \frac{\rho_{out}^l z_l}{d_o} \right) \middle| D, \beta \right) \quad (13)$$

$$= \mathbb{P} \left( \Delta > \frac{\beta}{\rho_{out}^l} \middle| D, \beta \right) \quad (14)$$

$$= \int_{\frac{\beta}{\rho_{out}^l}}^{\infty} f_\Delta(x, d_o) dx \quad (15)$$

$$= \frac{1}{\Gamma(D - d_o)} \Gamma \left( D - d_o, \frac{\beta}{\rho_{out}^l \sigma_c^2} \right) \quad (16)$$

Note that  $z_l$  does not appear in this expression, but of course it is needed to link the outage SNR to the outage rate.

### 3.3. Coverage probability

We will only focus on the case where all the users transmit a single stream. In [22] the coverage probability is defined as the probability that a typical mobile user is able to achieve some threshold SINR. Here, we will modify that definition slightly and say that the coverage probability is the probability that a typical user can achieve some threshold target rate  $R_{tar}$ .

We define the target SNR  $\rho_{tar}$  as the SNR needed to transmit at the target rate on an interference-free stream with fading  $z$ . That is,

$$R_{tar} = \log_2(1 + \rho_{tar} z). \quad (17)$$

In the case where we know the number of interferers at the UE,  $S$  and  $\beta$ , we can express the coverage probability as

$$\mathcal{P}_{C|S,\beta} = \mathbb{P}(R^\infty \geq R_{tar} | S, \beta) \quad (18)$$

$$= 1 - \mathbb{P}(R^\infty < R_{tar} | S, \beta) \quad (19)$$

$$= 1 - \mathcal{P}_{out|S,\beta}. \quad (20)$$

Note that here the outage probability  $\mathcal{P}_{out|S,\beta}$  is defined with the target rate as the outage rate. Note also that if there is no interferer ( $S=0$ ),

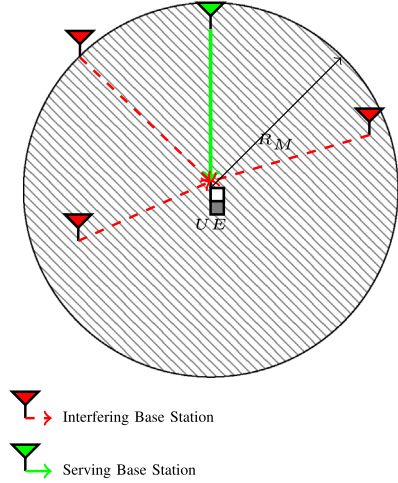


Fig. 1. Local environment of a user taken randomly, with 3 interferers and 1 serving BS.

then  $\mathcal{P}_{out|S,\beta} = 0$  and then  $\mathcal{P}_{C|S,\beta} = 1$ .

However, in the current study, we assume that the number of interfering BS  $S$  is a random number. Therefore, we have

$$\mathcal{P}_C = \mathbb{E}_{S,\beta}[1 - \mathcal{P}_{out|S}] \quad (21)$$

$$= \mathbb{E}_{S,\beta} \left[ 1 - \frac{1}{\Gamma(S)} \Gamma \left( S, \frac{\beta}{\rho_{tar} \sigma_c^2} \right) \right] \quad (22)$$

$$= \mathbb{E}_{S,\beta} \left[ \frac{1}{\Gamma(S)} \gamma \left( S, \frac{\beta}{\rho_{tar} \sigma_c^2} \right) \right], \quad (23)$$

where  $\gamma(a, x)$  is the lower incomplete gamma function and  $\mathbb{E}_{S,\beta}$  means that we average over the number of interfering BS in the vicinity of the UE and  $\beta$ .

## 4. Example

Let us investigate the case where the interfering BSs are distributed according to a PPP  $\Psi$  with intensity  $\lambda$ . We can imagine the vicinity of a UE as in Fig. 1. In this case,

$$\mathbb{P}(S = s | \mathcal{A}) = e^{-\mu} \frac{\mu^s}{s!}, \quad (24)$$

where  $\mu = \int_{\mathcal{A}} \lambda(r) dr$  is the mean of the Poisson process, and  $\mathcal{A}$  is the disc of radius  $R_M$  around the UE.

The coverage probability conditioned on  $\beta$  is then given as

$$\mathcal{P}_{C|\beta} = e^{-\mu} + e^{-\mu} \sum_{s=1}^{\infty} \frac{\mu^s}{s!} \frac{1}{\Gamma(s)} \gamma \left( s, \frac{\beta}{\rho_{tar} \sigma_c^2} \right). \quad (25)$$

We can give this expression in terms of an easy-to-compute integral

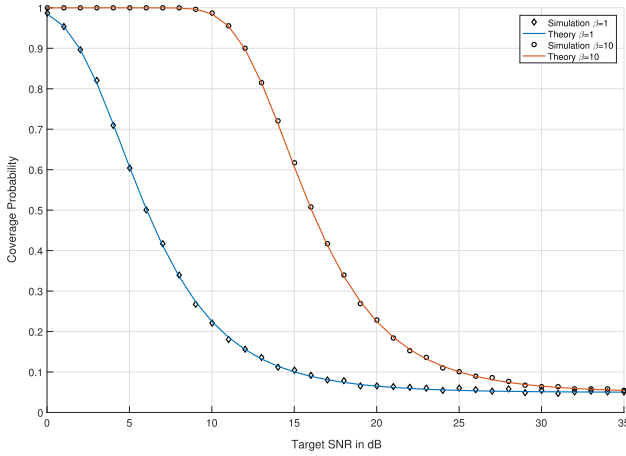
$$\mathcal{P}_{C|\beta} = e^{-\mu} + e^{-\mu} \sum_{s=1}^{\infty} \frac{\mu^s}{s! (s-1)!} \int_0^{\frac{\beta}{\rho_{tar} \sigma_c^2}} t^{s-1} e^{-t} dt \quad (26)$$

$$= e^{-\mu} + \mu e^{-\mu} \int_0^{\frac{\beta}{\rho_{tar} \sigma_c^2}} e^{-t} \left( \sum_{s=1}^{\infty} \frac{(\mu t)^{s-1}}{s! (s-1)!} \right) dt \quad (27)$$

$$= e^{-\mu} + \mu e^{-\mu} \int_0^{\frac{\beta}{\rho_{tar} \sigma_c^2}} e^{-t} \frac{I_1(2\sqrt{t\mu})}{\sqrt{t\mu}} dt \quad (28)$$

$$= e^{-\mu} \left( 1 + \int_0^{\frac{\beta\mu}{\rho_{tar} \sigma_c^2}} \sqrt{\frac{\beta\mu}{\rho_{tar} \sigma_c^2}} e^{-\frac{x^2}{4\mu}} I_1(x) dx \right), \quad (29)$$

where  $I_1(\cdot)$  is the modified Bessel function of the first kind. The final expression (29) can be computed easily knowing only the four



**Fig. 2.** Coverage probability of the network against the target SNR in dB with  $\mu = 3$ ,  $\sigma_e^2 = 10^{-1}$ .

parameters in this expression.

Let us perform a quick analysis of the effects of the different parameters in the expression of  $\mathcal{P}_{C|\beta}$ .

- $\beta$  can be viewed as the signal-to-interference ratio (SIR) at the UE. Increasing it means that the power of the signal at the receiver is getting higher than that of the interference, and therefore it is normal that increasing it would also increase the coverage probability. Looking at (25) we see that if we increase  $\beta$  to infinity we get

$$\mathcal{P}_{C|\beta=\infty} = e^{-\mu} + e^{-\mu} \sum_{K=2}^{\infty} \frac{\mu^{K-1}}{(K-1)!} = e^{-\mu} + e^{-\mu} (e^{\mu} - 1) = 1.$$

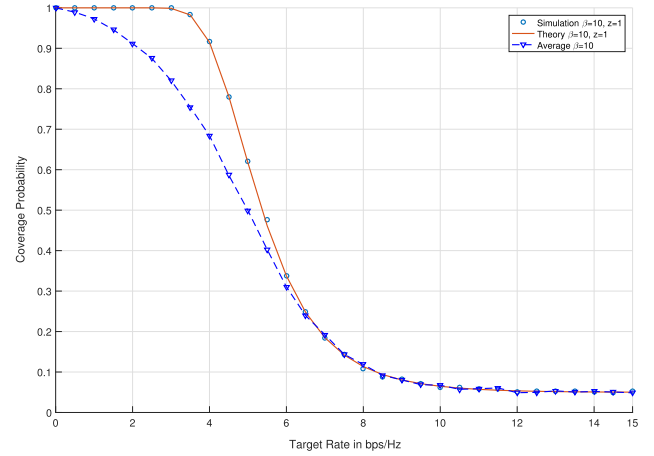
- In the same way decreasing  $\sigma_e^2$  makes  $\mathcal{P}_C$  bigger which makes sense since the channel estimates are more accurate. In the limiting case where  $\sigma_e^2 = 0$ , we have  $\mathcal{P}_C = 1$ .
- Regarding  $\rho_{\text{tar}}$ , increasing its value means that we require a higher rate to be achievable by the UE and thus the coverage probability decreases. The more we decrease the target requirements the better the coverage probability.
- The last parameter  $\mu$  represents the average number of interferers. As a result, when  $\mu$  is close to zero,  $\mathcal{P}_{C|\beta}$  is close to one and as we increase the density of interferers the coverage probability drops.

A closed-form approximation of the coverage probability can be given by truncating the infinite sum in expression (25) after the first few terms. In practice, the number of terms to keep will depend on the value of  $\mu$ .

## 5. Numerical results

In this section, we provide numerical results based on Monte Carlo simulations of the model studied in this paper and compare the results with the theory. The channel estimation errors are generated randomly from a complex Gaussian distribution of mean 0 and variance  $\sigma_e^2 = 10^{-1}$ . This value corresponds to the performance of channel estimation techniques with training SNR  $\approx 20$  dB and up to 8 antennas at the UE and the BS [24]. The number of interfering BS is drawn from a Poisson distribution with expected value  $\mu = 3$ . In order to consider the case where the user is at the cell edge, we choose  $\beta = 1$  so that the strength of the desired signal is comparable to that of the interfering signals. We will also show the results of  $\beta = 10$  for comparison. The fading coefficient  $z$  on the direct link is assumed exponentially distributed with rate 1 according to [25], Lemma 1.

Fig. 2 shows the evolution of the coverage probability against the target SNR. The simulation curves are obtained by first computing the coverage probability for a given target rate and realization of  $z$ , then we



**Fig. 3.** Coverage probability of the network against the target rate in bps/Hz with  $\mu = 3$ ,  $\sigma_e^2 = 10^{-1}$  for  $z=1$ .

link the target rate to the target SNR using Eq. (17). In so doing, we have removed the effects of the randomness of  $z$ . Every point is averaged over ten thousand iterations. We see a perfect match between the theoretical predictions and the simulations. As we would expect, the coverage probability drops faster when  $\beta = 1$  than when  $\beta = 10$  because the received SIR is much stronger with  $\beta = 10$ . This result can be used to obtain the coverage probability for a given target knowing the value of the fading on the communication link.

Fig. 3 shows the coverage probability against the target rate. In this case, we look at two different scenarios in order to see the effect of the fading on the coverage probability. The first scenario is with a fixed fading coefficient  $z=1$  and the second scenario is with  $z$  drawn according to the exponential distribution with rate 1. In this second scenario we plot the average of the coverage probability over the fading coefficient. We see that the curve for the average coverage probability is lower, which means that on average there is some performance loss due to fading.

## 6. Conclusion

This paper presented the study of cellular networks using IA to mitigate inter-cell interference. The structure of practical BS deployments is taken into account by considering a random point process to model the BS locations. The coverage probability is derived and given as an easy-to-compute integral. Simulation results are provided and agree with the theoretical results. Throughout this analysis it was assumed that the interfering BSs had the same path-loss towards the UE in consideration. Modifying this assumption to consider different path-loss per BS is left for future work.

## References

- [1] A. Damnjanovic, J. Montojo, Y. Wei, T. Ji, T. Luo, M. Vajapeyam, T. Yoo, O. Song, D. Malladi, A survey on 3GPP heterogeneous networks, *IEEE Wirel. Commun.* 18 (3) (2011) 10–21.
- [2] M. Maddah-Ali, A. Motahari, A. Khandani, Communication over MIMO X channels: interference alignment, decomposition, and performance analysis, *IEEE Trans. Inf. Theory* 54 (8) (2008) 3457–3470.
- [3] S. Jafar, S. Shamai, Degrees of freedom region of the MIMO X channel, *IEEE Trans. Inf. Theory* 54 (1) (2008) 151–170.
- [4] V. Cadambe, S. Jafar, Interference alignment and spatial degrees of freedom for the k user interference channel, in: Proceedings of the IEEE International Conference on ICC'08 Communications, May 2008, pp. 971–975.
- [5] M. Razaviyayn, G. Lyubeznik, Z.-Q. Luo, On the degrees of freedom achievable through interference alignment in a MIMO interference channel, *IEEE Trans. Signal Process.* 60 (2) (2012) 812–821.
- [6] S. Jafar, M. Fakhereddin, Degrees of freedom for the MIMO interference channel, *IEEE Trans. Inf. Theory* 53 (7) (2007) 2637–2642.
- [7] G. Bresler, D. Cartwright, D. Tse, Feasibility of interference alignment for the MIMO interference channel, *IEEE Trans. Inf. Theory* 60 (9) (2014) 5573–5586.

- [8] R. Guiazon, K.-K. Wong, D. Wisely, Capacity analysis of interference alignment with bounded CSI uncertainty, *IEEE Wirel. Commun. Lett.* 3 (5) (2014) 505–508.
- [9] R. Guiazon, K.-K. Wong, M. Fitch, Capacity distribution for interference alignment with CSI errors and its applications, *IEEE Trans. Wirel. Commun.* PP (99) (2015) (1–1).
- [10] —, Evolution of capacity lower bound of interference alignment with least-square channel estimation, in: *Proceedings of the IEEE China Summit and International Conference on Signal and Information Processing (ChinaSIP)*, July 2015, pp. 582–585.
- [11] O. El Ayach, A. Lozano, R. Heath, On the overhead of interference alignment: training, feedback, and cooperation, *IEEE Trans. Wirel. Commun.* 11 (11) (2012) 4192–4203.
- [12] R. Krishnamachari, M. Varanasi, Interference alignment under limited feedback for MIMO interference channels, *IEEE Trans. Signal Process.* 61 (15) (2013) 3908–3917.
- [13] O. Gonzalez, C. Beltran, I. Santamaria, A feasibility test for linear interference alignment in MIMO channels with constant coefficients, *IEEE Trans. Inf. Theory* 60 (3) (2014) 1840–1856.
- [14] O. El Ayach, S. Peters, R. Heath, The feasibility of interference alignment over measured MIMO-OFDM channels, *IEEE Trans. Veh. Technol.* 59 (9) (2010) 4309–4321.
- [15] K. Gomadam, V. Cadambe, S. Jafar, Approaching the capacity of wireless networks through distributed interference alignment, in: *Proceedings of the IEEE GLOBECOM 2008. IEEE Global Telecommunications Conference*, Nov 2008, pp. 1–6.
- [16] K. Gomadam, V. Cadambe, S. Jafar, A distributed numerical approach to interference alignment and applications to wireless interference networks, *IEEE Trans. Inf. Theory* 57 (6) (2011) 3309–3322.
- [17] H. Shen, B. Li, Y. Luo, F. Liu, A robust interference alignment scheme for the MIMO X channel, in: *Proceedings of the 15th Asia-Pacific Conference on Communications, APCC*, Oct 2009, pp. 241–244.
- [18] P. Aquilina, T. Ratnarajah, Performance analysis of ia techniques in the MIMO IBC with imperfect CSI, *IEEE Trans. Commun.* 63 (4) (2015) 1259–1270.
- [19] C. Suh, D. Tse, Interference alignment for cellular networks, in: *Proceedings of the 46th Annual Allerton Conference on Communication, Control, and Computing*, Sept 2008, pp. 1037–1044.
- [20] R. Tresch, M. Guillaud, Cellular interference alignment with imperfect channel knowledge, in: *Proceedings of the IEEE International Conference on ICC Workshops, Communications Workshops*, June 2009, pp. 1–5.
- [21] B. Zhuang, R.A. Berry, M.L. Honig, Interference alignment in MIMO cellular networks, in: *Proceedings of the 2011 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, May 2011, pp. 3356–3359.
- [22] J.G. Andrews, F. Baccelli, R.K. Ganti, A tractable approach to coverage and rate in cellular networks, *IEEE Trans. Commun.* 59 (11) (2011) 3122–3134.
- [23] G. Bresler, D. Cartwright, D. Tse, Feasibility of interference alignment for the MIMO interference channel: the symmetric square case, in: *Proceedings of the IEEE, Information Theory Workshop (ITW)*, Oct 2011, pp. 447–451.
- [24] M. Biguesh, A. Gershman, Training-based MIMO channel estimation: a study of estimator tradeoffs and optimal training signals, *IEEE Trans. Signal Process.* 54 (3) (2006) 884–893.
- [25] O. Ayach, R. Heath, Interference alignment with analog channel state feedback, *IEEE Trans. Wirel. Commun.* 11 (2) (2012) 626–636.