The relationship between learning mathematics and general cognitive ability in primary school

Richard Cowan, Jane Hurry, and Emily Midouhas

UCL Institute of Education

Author Note

Richard Cowan, Jane Hurry, and Emily Midouhas, Psychology and Human Development, UCL Institute of Education, England.

Correspondence concerning this article should be addressed to Richard Cowan, Psychology and Human Development, UCL Institute of Education, 20 Bedford Way, London WC1H 0AL, England. E-mail: r.cowan@ucl.ac.uk

Abstract

Three relationships between learning mathematics and general cognitive ability have been hypothesised: the educational hypothesis that learning mathematics develops general cognitive skills, the psychometric hypothesis that differences in general cognitive ability cause differences in mathematical attainment, and the reciprocal influence hypothesis that developments in mathematical ability and general cognitive ability influence each other. These hypotheses are assessed with a sample of 948 children from the Twins Early Development Study who were assessed at 7, 9, and 10 years on mathematics, English and general cognitive ability. A cross-lagged path analysis with mathematics and general cognitive ability measures supports the reciprocal influence hypothesis between 7 and 9 and between 9 and 10. A second analysis including English assessments only provides evidence of a reciprocal relationship between 7 and 9.

Statement of contribution

What is already known on this subject?

- The correlations between mathematical attainment, literacy, and measures of general cognitive skills are well established.
- The role of literacy in developing general cognitive skills is emerging.

What does this study add?

- Mathematics contributes to the development of general cognitive skills.
- General cognitive ability contributes to mathematical development between 7 and 10.
- These findings support the hypothesis of reciprocal influence between mathematics and general cognitive ability, at least between 7 and 9.

The relations between learning mathematics and general cognitive ability in primary school

Children develop considerably in mathematical knowledge and skills and general cognitive ability during their time in primary school. Classical educational theory asserts the influence of what they learn on how they think: mathematics learning is supposed to develop reasoning and problem-solving (Smith, 2004). Reasoning and problem-solving are general cognitive abilities that are supposed to affect learning and performance of more than just mathematics. Becoming better at mathematics should therefore improve general cognitive abilities. By contrast, much recent psychological research emphasizes causal influence in the opposite direction: how general cognitive abilities may affect the learning of school mathematics, for example intelligence and speed of processing (Geary, 2011) and reasoning (Fuchs, Geary, Fuchs, Compton, & Hamlett, 2016). A third hypothesis is that the relationship between general cognitive skills and mathematical achievement is reciprocal with each influencing the other (Brody, 1992). The present study uses longitudinal data with repeated measurements to assess the relationships between general cognitive ability and mathematics attainment during the second phase of primary school.

The present study

To investigate the relations between mathematics and cognitive ability in primary school, we use a cross-lagged path analysis approach which includes measurements of mathematics and general cognitive ability at three ages (7, 9, and 10 years). If mathematics does develop cognitive ability, then earlier mathematics should make a statistically significant unique contribution to explaining variance in later cognitive ability even when earlier cognitive ability is included. If general cognitive ability does directly influence later mathematics then it should make a statistically significant unique contribution independently of earlier mathematics ability. If the relationship is reciprocal then both mathematical and general cognitive ability should influence later performance.

We also report a cross-lagged path analysis which includes measures of English at these three ages. This is because both mathematics attainment and general cognitive ability are correlated with reading and spoken language skills (Cowan et al., 2011; Durand, Hulme, Larkin, & Snowling, 2005). Previous studies indicate reading contributes to growth in general cognitive skills (Cain & Oakhill, 2011; Ferrer, McArdle, Shaywitz, Holahan, Marchione, & Shaywitz, 2007; Richie, Bates, & Plomin, 2015; Verhoeven, van Leeuwe, & Vermeer, 2011). The inclusion of the measures of English in the second cross-lagged path model is an attempt to control for the covariation of mathematics and general cognitive ability with reading and language skills.

Method

Participants

Participants were selected from the publicly available SRCD Monograph dataset for children in the Twins Early Development Study (TEDS). Some children with specific medical syndromes such as Down syndrome, extreme low birth weight and autism spectrum disorder had been excluded from this data set (Kovas, Haworth, Dale, and Plomin, 2007). There were two further principles of selection in the analyses reported here: only children with complete data were included and only one child per family was selected. To be included a child had to have complete data on reading, mathematics and general cognitive ability at 7, 9, and 10 years old. The selection of one child per family was made to avoid the risk of clustering at the level of family if both twins were included as clustering can lead to inflated significance levels (Cohen, Cohen, West, & Aiken, 2003). The random variable in the data set for single child selection was used. The resulting sample comprised 948 children, 409 male, 539 female. Of these children, 325 had an MZ twin and 311 had a same sex DZ twin, and 312 had an opposite sex DZ twin.

Measures

Fuller details of the measures are reported in Kovas et al. (2007).

English. The measures of English at 7, 9, and 10 were standardized composites of teacher ratings on five point scales of children's National Curriculum (NC) Levels in the three areas of English: Reading; Speaking and Listening; and Writing (QCDA, 2010).

In rating 7-year-olds, teachers used the Key Stage 1 criteria and guidance. At 7, the children were also assessed using a telephone administered version of the Test of Word Reading Efficiency (TOWRE, Torgesen, Wagner, & Rashotte, 1999). Teacher ratings of NC Levels for English correlated strongly with composite TOWRE scores, r (935) = .62, $p <$ .001. This indicates substantial validity.

For 9- and 10-year-olds, teachers used the Key Stage 2 criteria and guidance. At 10 the children's reading comprehension was also assessed using a web-based version of the reading comprehension subtest of Peabody Individual Achievement Test (Markwardt, 1997) at home. Performance on the web-based reading comprehension test correlated moderately with composite teacher ratings of NC Levels for English at 9 and 10: NC Levels for English at 9, r (948) = .44, $p < .001$; NC Levels for English at 10, r (948) = .47, $p < .001$.

Mathematics. The measures of mathematics at 7, 9, and 10 were standardized composites of teacher ratings on five point scales of children's National Curriculum Levels in the three areas of Mathematics: Numbers (and Algebra for Key Stage 2); Using and Applying Mathematics; Shapes, Space, and Measures. In rating 7-year-olds, teachers used the Key Stage 1 criteria and guidance. For 9- and 10-year-olds, teachers used the Key Stage 2 criteria and guidance. At 10 the children's mathematical skills were also assessed using a web-based measure derived from the NFER 5-14 series. Performance on this web-based measure correlated moderately with composite teacher ratings of NC Levels for Mathematics at 9 and 10: NC Levels for Mathematics at 9, r (948) = .45, $p < .001$; NC Levels for Mathematics at 10, r (948) = .47, $p < .001$.

General cognitive ability. At 7 children were tested over the telephone using the Vocabulary, Similarities, and Picture Completion subtest from the Wechsler Intelligence Scale for Children (WISC-III-UK; Wechsler, 1992) and the Conceptual Grouping subtest from the McCarthy Scales of Children's Abilities (McCarthy, 1972). Test stimuli and instructions to prevent cheating were sent to parents in advance. A validation study of telephone-administered cognitive measures with 6-8 year olds reported a correlation of r = .65 with in person tests. This correlation increased to .72 when corrected for range restriction (Petrill, Rempell, Oliver, & Plomin, 2002).

At 9, children completed a test booklet under parental supervision. The tests comprised adaptations of the Vocabulary and General Knowledge subtests of WISC-III, and adaptations of the Figure Classification and Figure Analogies subtests of the Cognitive Abilities Test 3 (Smith, Fernandes, & Strand, 2001).

At 10, a web-based procedure was used with adaptations of the Multiple Choice Information, Vocabulary, and Picture Completion subtest from WISC-III-UK and Raven's Standard Progressive Matrices (Raven, Court, & Raven, 1996).

For each age, standardized composites were formed from the four subtests.

Results

The means for the composite measures ranged from 0.15 to 0.22 and the standard deviations ranged from 0.88 to 0.95. There was no evidence of variation in composite measures with age (7, 9, & 10) or domain (English, Mathematics, General Cognitive Ability) according to a two within –subjects factor ANOVA: age, F (2, 3038) = 0.38, p = .68; domain, F (2, 3038) = 2.49, p = .08; age X domain, F (3.2, 3038) = 2.49, p = .06.

Zero-order correlations between measures are shown in Table 1. Correlations within domains are generally higher than correlations across domains apart from the ratings of English and Mathematics at the same age where the same person is rating both domains.

Cross-lagged path analyses were run in MPlus Version 7.4 (Muthén & Muthén, 1998-2015) using Maximum Likelihood estimation.

Cross-lagged Model 1. The first path model was specified to estimate the cross-lagged effects of mathematics and general cognitive ability (measured as manifest rather than latent variables) between ages 7 and 9, and between ages 9 and 10, while allowing for the stability of both mathematics and general cognitive ability over time. It also allowed for residual covariances of mathematics and general cognitive ability within time point. All coefficients are reported in standardized units. As summarized in Figure 1, all paths between the abilities at adjacent ages were significant. The cross-lagged path between mathematics at 7 and general cognitive ability at 9 is stronger than the cross-lagged path between general cognitive ability at 7 and mathematics at 9. Between 9 and 10 both the cross-lagged paths were of a similar strength, and slightly weaker than the corresponding paths between 7 and 9. This reduction in the predictive power across domains may reflect the increased strength of the within domain paths: the within domain paths between 9 and 10 are stronger than the corresponding paths between 7 and 9. This is consistent both with the shorter time intervals and the greater similarity of the underlying constructs.

Cross-lagged Model 2. This analysis included the measures of English at ages 7 and 9 predicting their adjacent measures of mathematics and general cognitive ability. Therefore, English achievement was adjusted for in each cross-lagged path. The results are summarized in Figure 2. The effect of including English is to reduce the sizes of all the coefficients. The path from mathematics at 7 to general cognitive ability at 9 is still slightly stronger than the path between general cognitive ability at 7 to mathematics at 9. However, there is no longer a significant path between mathematics at 9 and general cognitive ability at 10.

Discussion

As mentioned in the Introduction, evidence exists to support the psychometric hypothesis that general cognitive ability enhances mathematical attainment and this is confirmed in the analyses presented here. The analyses add to our knowledge about the relation between learning in primary school and general cognitive ability in two important ways. They provide support for the claim that learning mathematics improves general cognitive abilities and they indicate that the relationship between general cognitive ability and mathematics learning is reciprocal, at least between the ages of 7 and 9.

Using longitudinal repeated measures data to establish reciprocal relationships is well-established in the investigation of causal relationships, for example in studies of the relationship between academic self-concept and educational achievement (Marsh & Martin, 2011). However, in longitudinal data sets with large sample sizes, measurements of specific variables tend to be time efficient rather than exhaustive. In the present study, phone and web-based testing of cognitive abilities and teacher ratings of mathematics ability offer satisfactory, conventional measurement within the context of large scale studies but having multiple measures of the constructs would improve confidence in the results reported here.

A further limitation of the present study is that the cross-lagged model we utilised does not explicitly consider the passage of time, and that the window of time between time points may be too short to capture the reciprocal effects of mathematics and general cognitive ability. The short period between age 9 and 10 measurements may be a reason why we did not find reciprocal relationships across age 9 to 10.

Even if there were multiple measures and knowledge of what intervals between measurements would be appropriate, the evidence would be just longitudinal and it would still be important to establish the relationships with experiments. While maintaining that a reciprocal relationship between education and intelligence was most likely, Brody (1992) pointed to shortcomings in previous experimental evidence. One problem is that the training may not transfer. Early intelligence training did not improve educational achievement (Brody, 1992) and working memory training does not result in improved reading and arithmetic (Melby-Lervåg & Hulme, 2013). Educationists have also acknowledged that some forms of mathematical education may not enhance intellectual development (Hamley, 1939), for example when they concentrate on rote memorizing as the method of learning. Contemporary high stakes testing, where schools are held accountable for their pupils' achievements on tests, is suspected of reducing the benefits of classroom instruction by focusing teaching on the specific skills required to pass the test rather than on deeper conceptual understanding which supports transfer (Popham, 2001).

Another challenge is that individual differences in response to treatments may obscure the overall effects. Reading researchers have identified complex relationships between initial competence and response to intervention (Hurry & Sylva, 2007; Stanovich, 1986). It is likely that similar patterns will be observed in relation to mathematical interventions.

Overall then, while this study does indicate a reciprocal relationship between mathematics learning and general cognitive development at least between 7 and 9, there is much more to be done to establish it and whether it can be used to enhance mathematical achievement or cognitive development.

References

Brody, N. (1992). *Intelligence* (2nd ed.). London: Academic Press.

Cain, K., & Oakhill, J. (2011). Matthew effects in young readers: Reading comprehension and reading experience aid vocabulary development. *Journal of Learning Disabilities, 44*(5), 431-443. doi: 10.1177/0022219411410042

Cohen, J., Cohen, P., West, S. G., & Aiken, L. S. (2003). *Applied multiple regression/correlation analysis for the behavioral sciences* (3rd ed.). Mahwah, NJ: Lawrence Erlbaum Associates.

Cowan, R., Donlan, C., Shepherd, D.-L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology, 103*, 786-803. doi: 10.1037/a0024556

Durand, M., Hulme, C., Larkin, R., & Snowling, M. (2005). The cognitive foundations of reading and arithmetic skills in 7-to 10-year-olds. *Journal of Experimental Child Psychology, 91*, 113-136. doi: 10.1016/j.jecp.2005.01.003

Ferrer, E., McArdle, J. J., Shaywitz, B. A., Holahan, J. M., Marchione, K., & Shaywitz, S. E. (2007). Longitudinal models of developmental dynamics between reading and cognition from childhood to adolescence. *Developmental Psychology, 43*, 1460-1473. doi: 10.1037/0012-1649.43.6.1460

Fuchs, L. S., Geary, D. C., Fuchs, D., Compton, D. L., & Hamlett, C. L. (2016). Pathways to third-grade calculation versus word-reading competence: are they more alike or different? *Child Development*, 87,558–567.doi:10.1111/cdev.12474

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology, 47*, 1539-1552. doi: 10.1037/a0025510

Hamley, H. R. (1939). Memorandum on the cognitive aspects of transfer of training. In Great Britain Board of Education Consultative Committee (Ed.), *Secondary Education* (pp. 439-452). London: HMSO.

Hurry, J., & Sylva, K. (2007). Long-term outcomes of early reading intervention. *Journal of Research in Reading, 30*, 227-248. doi: 10.1111/j.1467-9817.2007.00338.x

Kovas, Y., Haworth, C. M. A., Dale, P. S., & Plomin, R. (2007). The genetic and environmental origins of learning abilities and disabilities in the early school years. *Monographs of the Society for Research in Child Development*, *72*(3), vii-160.

Markwardt, F. C., Jr. (1997). *Peabody Individual Achievement Test–Revised (normative update) manual.* Circle Pines, MN: American Guidance Service.

Marsh, H. W., & Martin, A. J. (2011). Academic self-concept and academic achievement: Relations and causal ordering. *British Journal of Educational Psychology, 81,* 59-77. doi:10.1348/000709910X503501

McCarthy, D. (1972). *McCarthy Scales of Children's Abilities*. New York, NY: Psychological Corporation.

Melby-Lervåg, M., & Hulme, C. (2013). Is working memory training effective? A meta-analytic review. *Developmental Psychology, 49*, 270-291. doi: 10.1037/a0028228

Muthén, L. K., & Muthén, B. O. (1998-2015). *Mplus User's Guide. Version 7.4.* Los Angeles, CA: Muthén & Muthén.

Petrill, S.A.. Rempell, J., Oliver, B. & Plomin, R. (2002). Testing cognitive abilities by telephone in a sample of 6-to 8-year olds. *Intelligence, 30,* 353-360.

Popham, W. J. (2001). Teaching to the test? *Educational Leadership, 58,* 16-20.

QCDA (2010). *The National Curriculum Level descriptors for subjects*. Coventry: Qualification and Curriculum Development Agency.

http://www.fft.org.uk/FFT/media/fft/Events/Dave_T_Descriptions.pdf (accessed 21/4/17).

Raven, J. C., Court, J. H., & Raven, J. (1996). *Manual for Raven's progressive matrices and vocabulary scales*. Oxford, UK: Oxford University Press.

Richie, S. J., Bates, T. C., & Plomin, R. (2015). Does learning to read improve intelligence? A longitudinal multivariate analysis in identical twins from age 7 to 16. *Child Development, 86*, 23-36. doi:10.1111/cdev.12272

Smith, A. (2004). *Making mathematics count: the report of Professor Adrian Smith's inquiry into post-14 mathematics education*. London: The Stationery Office.

Smith, P., Fernandes, C., & Strand, S. (2001). *Cognitive Abilities Test 3 (CAT3)*. Windsor, UK: nferNELSON.

Stanovich, K. E. (1986). Matthew effects in reading: Some consequences of individual differences in the acquisition of literacy. *Reading Research Quarterly, 21*, 360-407.

Torgesen, J. K., Wagner, R. K., & Rashotte, C. A. (1999). *Test of Word Reading Efficiency*. Austin, TX: Pro-Ed.

Verhoeven, L., van Leeuwe, J., & Vermeer, A. (2011). Vocabulary growth and reading development across the elementary school years. *Scientific Studies of Reading, 15*, 8-25. doi: 10.1080/10888438.2011.536125

Wechsler, D. (1992). *Wechsler Intelligence Scale for Children- Third UK edition*. London: The Psychological Corporation.

Table 1

Correlations across ages and domains

	1	2	3	4	5	6	7	8	9
1. Mathematics at 7		.55***	.55***	.73***	.53***	.53***	.37***	.39***	.36***
2. Mathematics at 9			.63***	.47***	.69***	.56***	.38***	.42***	.37***
3. Mathematics at 10				.52***	.57***	.74***	.38***	.40***	.37***
4. English at 7					.62***	.62***	.41***	.38***	.34***
5. English at 9						.67***	.40***	.39***	.39***
6. English at 10							.38***	.39***	.41***
7. General cognitive ability at 7								.41***	.40***
8. General cognitive ability at 9									.55***
9. General cognitive ability at 10									

Note. N = 948. Sample from Twins Early Development Study SRCD Monograph data set.

*** $p < .001$.

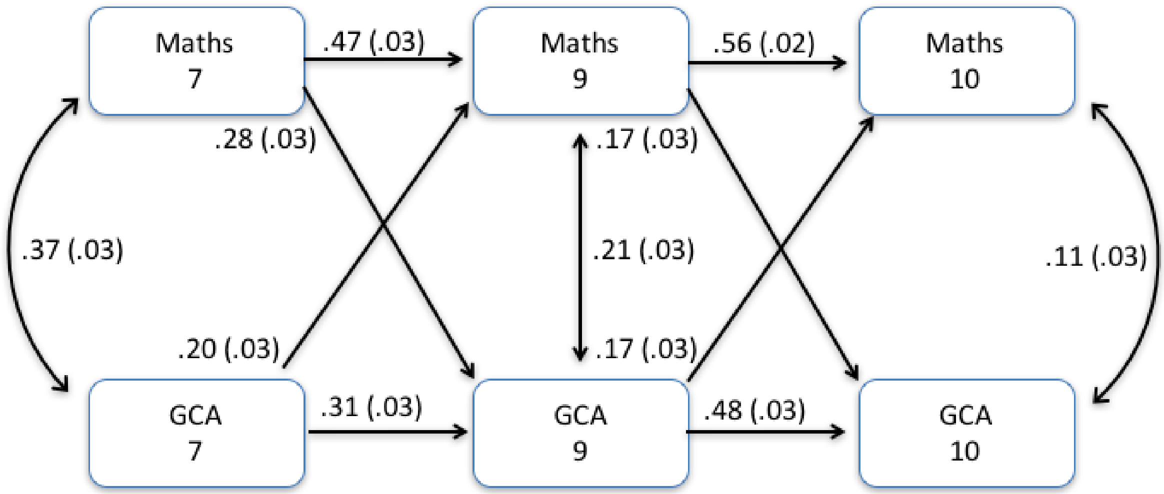

Figure 1. Cross-lagged model 1 showing standardized coefficients and standard errors. GCA = general cognitive ability. All paths were significant at p < .001 except for the residual covariance of mathematics at 10 and general cognitive ability at 10 (*p* < .01).

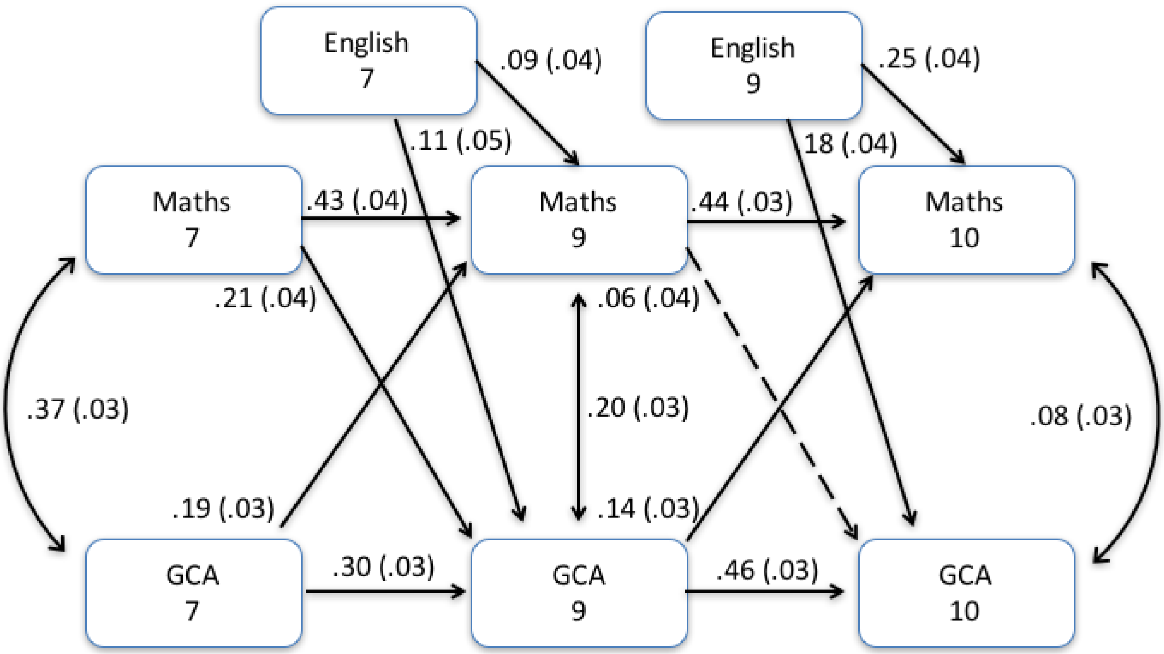

Figure 2. Cross-lagged model 2 showing standardized coefficients and standard errors. GCA = general cognitive ability. Solid lines represent significant paths and dotted lines represent paths that are not significant. All significant paths were significant at $p < .001$ except for the residual covariance of mathematics at 10 and general cognitive ability at 10 ($p < .05$).