12th INTERNATIONAL CONFERENCE ON TRAVEL BEHAVIOUR RESEARCH
Jaipur, India, December 2009

INERTIA AND SHOCK EFFECTS ON MODE CHOICE PANEL DATA: IMPLICATIONS OF THE TRANSANTIAGO IMPLEMENTATION

María Francisca Yáñez
Department of Transport Engineering and Logistics,
Pontificia Universidad Católica de Chile,
Tel: 56-2-354 4270; E-mail: mfyanezc@puc.cl

Elisabetta Cherchi
CRiMM – Dipartimento di Ingegneria del Territorio
Università di Cagliari
Tel: 39-70 675 5274; E-mail: echerchi@unica.it

Juan de Dios Ortúzar
Department of Transport Engineering and Logistics,
Pontificia Universidad Católica de Chile
Tel: 56-2-686 4822; E-mail: jos@ing.puc.cl

Benjamin G Heydecker
Centre for Transport Studies
University College London
E-mail: ben@transport.ucl.ac.uk

ABSTRACT

The mode choice process, especially in the case of commuter trips, reflects the strong tendency people have to simplify the assessment of their options when confronted with successive well-known decisions. Thus, it is common to repeat the "habitual" choice over time involving a potentially important inertia element. However, while inertia effects increase the probability of maintaining the same choice in a stable situation, in a changing environment i.e. one that is disrupted by a radical or significant policy intervention, user behaviour may be affected by a specific response to abrupt changes. Shock effects of this kind could increase the probability of individuals leaving their habitual choices.

Temporal effects have been commonly ignored in practical studies, as most demand models to date have been based on cross-sectional data. A few recent studies dealing with panel data have managed to incorporate inertia effects, but there are no studies that have included both inertia and shock effects. To address this, we started by building a data panel around the introduction of a new and radical policy for the conurbation of Santiago de Chile. The final aim was to develop mode choice models incorporating the effects of three main forces involved in the choice process: (1) the relative values of the modal attributes, (2) the inertia effect, and (3) the shock resulting from and abrupt policy intervention. This paper presents the formulation of an inertia-shock model and its application to each of simulated and real data. The results confirm that changing systems should be modelled respecting the presence of both inertia and shock effects, otherwise serious errors in model estimation may arise.

1. Introduction

The structure of many large cities leads people to commute relatively long distances. These commuter trips constitute a substantial proportion of the total trips in an urban area and mainly take place during peak periods. Most commuter trips have a tendency to be repeated over time, thus acquiring a potentially important inertia component (Lanzendorf, 2003; Pendyala et al., 2001).

Inertia helps taking decisions faster but also increases the probability of maintaining the same choices. Thus, in a stable environment we would expect choices to be influenced greatly by inertia and, as a consequence, it is likely that the effects of travel demand management strategies will be attenuated. On the other hand, a large disincentive (or a large benefit) can create conflicts between economic reasoning and habits. Thus, in a changing environment (i.e. one unsettled by significant travel demand or management policies), the probability that a traveller modifies her choice should be higher and this behaviour may be accelerated by the occurrence of a large and sudden change (i.e. a shock in the system).

The influence of habit (leading to inertia) in the choice process has been discussed in the literature (Goodwin, 1977; Blase, 1979; Williams and Ortúzar, 1982); Daganzo and Sheffi (1979) even proposed a multinomial probit formulation to treat this phenomenon which was later implemented by Johnson and Hensher (1982). More recently, the discrete choice modelling field has seen significant advances in terms of incorporating inertia, examples of that are: a model including prior behaviour on a time-series context (Swait et al., 2004), a model including inertia on a two-wave panel formulation (Cantillo et al., 2007), and a planning-and-action model considering inertia as an effect of previous plans (Ben-Akiva, 2009). All these studies refer to cases where there are no changes in the transport system. As far as we are aware, only three panels have been built around transport supply changes: the before-and-after study carried out in Massachusetts around the introduction of a free bus service (Parody, 1977), the work developed in Amsterdam on an extension of the urban motorway system (Kroes et al., 1996), and our own work on the *Santiago Panel* (Yáñez et al., 2009) gathered before and after the introduction of the Transantiago, a completely new public transport system (Muñoz et al., 2009). Such large interventions would be expected to affect the choice context substantially, reducing or even overcoming the effects of inertia. Notwithstanding this, to our knowledge the only reference to the shock issue itself is our own work (Yáñez et al., 2008; Yáñez and Ortúzar, 2009), while there are appear to be no models dealing with choice processes when both inertia and an abrupt intervention affect the choice context.

In this paper we use data from the *Santiago Panel* to examine this issue. Our aim is to disentangle the effects of three main forces in the choice process: (1) the relative values of the modal attributes; (2) the inertia effect, and (3) the shock resulting from the abrupt implementation of a policy. The remainder of the paper is organized as follows. In section 2, we summarize the main characteristics of the *Santiago Panel*. In section 3, we present the inertia-and-shock model formulation, and analyze the practical estimation issues associated. In section 4, we present and analyze our most important results based on synthetic and real data, and finally in section 5 we summarize our main conclusions.

2. Characteristics of the *Santiago Panel*

The *Santiago Panel* is unique in being a five-day pseudo diary[1] with four waves, one before and three after the implementation of Transantiago (Yáñez et al., 2009). Transantiago is a radical

[1] For budget reasons the panel recorded information for the five working days, but only about the work trips in the morning peak hour.

but, sadly, poorly implemented new public transport system for Santiago de Chile[2]. The new system was designed by a team of Chilean specialists and consultants (Fernández et al., 2008). The aim of the plan was to improve public transport in the city in an effort to stop its decline. However, implementation was delayed by almost two years due to Government and private bodies in making available a series of prerequisites for its implementation (such as the construction of segregated bus-only lanes and the installation of GPS control in all the new buses). Transantiago was then implemented in a “big bang” fashion (i.e. neither pilot studies nor gradual changes were included) during the middle of the summer holidays on February 10th, 2007. The new system is characterised by having integrated fares in a system of feeder and trunk buses in conjunction with the Santiago underground (Metro), which was established as its backbone. Additionally, it features higher quality buses and shorter routes (with substantial reductions in accidents, noise and atmospheric pollution), no competition among buses to gain passengers (hence more service-oriented drivers), and a new payment system featuring a contactless card (Bip! similar to the Oyster card in London and the Octopus card in Hong Kong).

Unfortunately, at the time of implementation several problems dominated the system, among others: the buses did not have the necessary technology to allow full use of the contactless card, not many bus-only lanes were constructed (i.e. the commercial speeds assumed when the system was designed could not be achieved), Transantiago had limited human resources, and the operators’ contracts lacked appropriate incentives to attract and transport more passengers. As a consequence, the initial results of the system were not as expected; users had to cope with many more transfers, lower frequency in the feeder services, longer walking times in the suburbs and, particularly at its start, severe overcrowding in the bus and Metro services during peak hours (over 6 passengers/m^2). It was certainly a dramatic shock for large numbers of passengers and made the main news for several months.

The first sample of the *Santiago Panel* consisted of 303 individuals who live in Santiago and work full-time at one of the four campuses and two hospitals of the Pontificia Universidad Católica de Chile. The *Santiago Panel*’s sampling unit is the individual. The information sources used in the panel are:

- Face-to-face interviews with the aid of palm tops. The design of the survey was based on the 2001-2006 Great Santiago Origin-Destination survey (Ampt and Ortúzar, 2004) and considered characteristics of the trip to work during the morning peak hour, socioeconomic characteristics of the respondent and, from the second wave onwards, perceptions about the performance of Transantiago.

- Precise measurement of level-of-service variables using GPS and geocoding of origin-destination pairs.

An interesting feature is that thanks to both the panel’s design and careful maintenance policies (Yáñez et al., 2009) we managed to keep attrition at just 5%, 3% and 7% in waves 2, 3 and 4 respectively, while best experiences previously reported in the literature were around 8% (Duncan, 1992).

A simple statistical analysis of the data gathered allowed us to identify another important feature of the *Santiago Panel* as about 55% of respondents changed their mode of transport (between waves one and four), and for the rest a significant proportion changed either route or the number of transfers in their trips. Therefore, we can say that changes are not an exception here, unlike most other cases such as the Puget Sound Panel (Murakami and Watterson, 1990), where 85% of the workers chose the same mode in waves one and two.

[2] Santiago is the fifth largest conurbation in South America. By December 2002 it had approximately 6 million inhabitants living in 1.5 million dwellings spread over 1,400 km^2. On a typical working day 16.3 million trips were made, 10 million of which used motorised transport modes (DICTUC, 2003).

3. Shock and Inertia Model

3.1 Model Formulation

Our research aim was to identify and study influences on modal choice processes in those cases where a habitual choice situation is perturbed by important changes in its environmental context. In particular, following the introductory discussion, the objective was to estimate a mode choice model that accounts explicitly for the presence of three different forces: (1) the differences in modal attributes, which is considered in any choice model (i.e. even in models for cross-sectional data); (2) the inertia effect, and (3) the shock effect.

Let us consider an initial situation (that corresponds to wave 1 in our panel) where an individual q uses option A_r to travel to work. After this time, a new public policy is introduced, which changes the transport system radically in terms of several attribute values. Given this background, the main hypotheses supporting our formulation are as follows:

- Individuals are utility maximisers, as in traditional random utility theory.
- Individual responses present panel correlation.
- Individuals evaluate each option in the initial situation (first wave) based on their level-of-service (LOS) and socioeconomic (SE) characteristics only. After the initial wave, the utility associated to each option includes also inertia and shock effects.
- The inertia effect is a function of the previous valuation of the options; we assume that each individual q compares the current options A_j^w (i.e. the options available in each wave w) with the option A_r^{w-1} that was chosen in the preceding wave (w-1).
- The inertia effect may vary for each wave and may vary among individuals due to either systematic or purely random effects.
- The effect of inertia might be either positive or negative; the former representing the "typical" inertia effect in absence of changes, the latter indicating the preference for changing that might occur after a significant variation in the transport system.
- A radical intervention may generate a shock effect which in turn may have the power to modify the entire choice process; consequently, individuals may modify their valuation process, altering their utility functions.
- The shock effect is a function of the difference between the utility of option A_j^w, evaluated at the current wave w, and its utility evaluated at the preceding wave (w-1). Hence, the shock effect is expected to be negative when A_j worsens (making its utility lower), and positive when it improves (making its utility higher).
- The perception of the shock may be different for each wave and may vary among individuals due to either systematic or purely random effects.
- The shock effect should have the highest value immediately after the introduction of the new policy, and then its magnitude should attenuate.

According to these assumptions, let the utility associated to each option A_j at wave w=1 (i.e. the base situation) be the sum of observable (V_{jq}^1) and non-observable components (ζ_{jq}^1):

$$U_{jq}^1 = V_{jq}^1 + \zeta_{jq}^1 \tag{1}$$

Thus, the probability of choosing option $A_j \in A^1_{(q)}$ at wave w = 1 will be:

$$P_q\left(A_j^1\right) = P\left(U_{jq}^1 - U_{iq}^1 \geq 0,\ \forall\ A_i^1 \in A_{(q)}^1\right) \tag{2}$$

where $A^1_{(q)}$ is the individual q's choice set in wave w=1. In waves 2 and 3, temporal effects should be also included in order to detect how the choices in one wave (w) are influenced by the choices made in a previous one (w-1). In this paper we will not consider wave 4 of the panel.

Now let us denote by $\tilde{U}^w_{jq}$ the utility that individual q associates to the generic option A_j on wave w (w = 2, 3). This utility will include inertia and shock effects, such that:

$$\tilde{U}^w_{jq} = U^w_{jq} - I^w_{jrq} + S^w_{jq} \tag{3}$$

where:

- $U^w_{jq} = V^w_{jq} + \zeta^w_{jq}$, as in wave w = 1;
- $I^w_{jrq} = f_I(\beta^w_{Ij}, \sigma^w_{Ij}, SE_I, V^{w-1}_{rq} - V^{w-1}_{jq})$ is a general expression of the inertia effect;
- $S^w_{jq} = f_S(\beta^w_{Sj}, \sigma^w_{Sj}, SE_S, V^w_{jq} - V^{w-1}_{jq})$ is a general expression of the shock effect.

where β^w_{Ij} and β^w_{Sj} are the population means, and σ^w_{Ij} and σ^w_{Sj} are the standard deviations of inertia and shock parameters respectively, for option A_j on wave w; SE_I and SE_S are socioeconomic variables that allow for systematic variations of the inertia and shock parameters, and V^w_{jq} is the observable component of the utility function without temporal effects.

Equation (3) represents the utility associated to a generic option belonging to the individual's choice set. In this formulation, both inertia and shock effects are a function of their relative (or conditional) position with respect to the choice on the previous wave; note that in our formulation we will always indicate the option chosen in the previous wave by A_r ; hence when we are on wave w, V^{w-1}_r will indicate the utility associated to option A_r chosen at wave w-1. Note also that if I^w_{jrq} is greater than zero inertia exists; while, if I^w_{jrq} is negative, it implies that the individual has a high disposition to change.

In presence of inertia and shock, the probability to change from the usual option A_r (i.e. option chosen in the previous wave) to A_j (in the paper we will refer to this option as "candidate option") for individual q on wave w, is given by:

$$P_q\left(A^w_j\right) = P\left(\tilde{U}^w_{jq} - \tilde{U}^w_{rq} \geq 0 \quad \wedge \quad \tilde{U}^w_{jq} - \tilde{U}^w_{iq} \geq 0, \quad \forall A^w_i \in A^w_{(q)}, \text{ except } r = j\right) \tag{4}$$

while, the probability to remain with the option (A_r) chosen in the previous wave is given by:

$$P_q\left(A^w_r\right) = P\left(\tilde{U}^w_{rq} - \tilde{U}^w_{jq} \geq 0\right) \tag{5}$$

Following the approach of Cantillo et al. (2007) in order to make the model operational we will assume the following expression for inertia:

$$I^w_{jrq} = (\beta^w_{Ij} + \delta_{Iq} \cdot \sigma^w_{Ij} + \beta_{I\_SE} \cdot SE_I) \cdot \left(V^{w-1}_{rq} - V^{w-1}_{jq}\right)^3 \tag{6}$$

3 Note that this formulation assumes a null inertia effect on wave w for the option chosen on the previous wave (w-1). It means $\tilde{U}^w_{rq} = U^w_{rq} + S^w_{rq}$.

Note that this corresponds to a generalization of Cantillo et al's formulation to account for different inertia parameters across options and over waves; also, according to our assumptions, we postulate the following expression for the shock:

$$S_{jq}^{w} = (\beta_{Sj}^{w} + \delta_{Sq} \cdot \sigma_{Sj}^{w} + \beta_{S\_SE} \cdot SE_S) \cdot \left(V_{jq}^{w} - V_{jq}^{w-1}\right) \tag{7}$$

where inertia and shock vary randomly among individuals according to a certain density function, and δ_{Iq}, δ_{Sq} are the standard factors to introduce panel correlation (note that this could be included either as random parameters or error components).

In this formulation, and as usual in current practice, option attributes and socioeconomic characteristics are associated to parameters that could be either fixed or random; on the other hand, the non-observable component ζ_{jq}^{w} is a random error term formulated as $\zeta_{jq}^{w} = \upsilon_q + \varepsilon_{jq}^{w}$, where υ_q is a random effect specific to the individual and ε_{jq}^{w} is the typical random error distributed independent and identically Gumbel.

Finally, the probability of choosing option A_j on wave w, $(\forall w > 1)$ can be written as:

$$\begin{aligned} P_q\left(A_j^w\right) = {} & \exp\left(V_{jq}^{w} - \left(\beta_{Ij}^{w} + \delta_{Iq} \cdot \sigma_{Ij}^{w} + \beta_{I\_SE} \cdot SE_I\right) \cdot \left(V_{rq}^{w-1} - V_{jq}^{w-1}\right) + \left(\beta_{Sj}^{w} + \delta_{Sq} \cdot \sigma_{Sj}^{w} + \beta_{S\_SE} \cdot SE_S\right) \cdot \left(V_{jq}^{w} - V_{jq}^{w-1}\right)\right) \\ & \cdot \left[\sum_i \left(\exp\left(V_{iq}^{w} - \left(\beta_{Ii}^{w} + \delta_{Iq} \cdot \sigma_{Ii}^{w} + \beta_{I\_SE} \cdot SE_I\right) \cdot \left(V_{rq}^{w-1} - V_{iq}^{w-1}\right) + \left(\beta_{Si}^{w} + \delta_{Sq} \cdot \sigma_{Si}^{w} + \beta_{S\_SE} \cdot SE_S\right) \cdot \left(V_{iq}^{w} - V_{iq}^{w-1}\right)\right)\right)\right]^{-1} \end{aligned} \tag{8}$$

where if $j = r$, then $(V_{rq}^{w-1} - V_{jq}^{w-1}) = 0$ and, as previously discussed, inertia is zero while the shock effect would still be active[4].

Notice that expression (8) is a general formulation. It may accommodate panel correlation either in the representative utility V_{jq}^{w} (using random parameters), error term (as an error component), or in the inertia and shock effects (again using random parameters). But for empirical estimation it is not possible to consider all of these panel correlation forms at the same time. In fact, since the inertia and shock parameters multiply the expressions $\Delta V_I = \left(V_{rq}^{w-1} - V_{jq}^{w-1}\right)$ and $\Delta V_S = \left(V_{jq}^{w} - V_{jq}^{w-1}\right)$ respectively, randomness can not be added in the representative utility and temporal effects at the same time.

Finally, as individual responses present panel correlation, given a sequence of modal choices A_j^w, one for each wave, the probability that a person follows this sequence is given by:

$$P_q\left(A_j^1 \wedge A_j^2 \wedge \ldots A_j^W\right) = \prod_{w=1}^{W} P_q\left(A_j^w\right) \tag{9}$$

As inertia, shock and panel correlation are actually unknown, the probability of this sequence of choices is of Mixed Logit (ML) form (Train, 2003).

4 The shock effect S_{jq}^{w} is null if either the shock parameter is itself null ($\beta_{Sj}^{w} = 0$) or if the utility of option A_j does not change between consecutive waves, i.e. $V_{jq}^{w} = V_{jq}^{w-1}$.

3.2 Identification Issues

As discussed by Walker (2002), a crucial issue in the ML model relates to its theoretical and empirical identification. Due to its great flexibility it is possible to postulate structures that are not identifiable or, even when they are theoretically identifiable, the data do not allow all parameters to be identified. Regarding this issue, the following considerations were made:

- Our model is an ordinary ML model except that it includes the shock and inertia terms. Therefore, theoretical (i.e. satisfaction of order[5], rank and positive-definitiveness conditions) and empirical identification (i.e. related to data richness, see Cherchi and Ortúzar, 2008) must be checked first.
- However, as the attributes of the shock and inertia terms $\Delta V_I = \left(V_{rq}^{w-1} - V_{jq}^{w-1}\right)$, $\Delta V_S = \left(V_{jq}^{w} - V_{jq}^{w-1}\right)$ are continuous and vary among options and individuals, there is no theoretical identification issue *per se* concerning the shock and inertia parameters, and their variances can be estimated.

Despite the previous point and considering the high complexity of the proposed inertia-and-shock model, we include here a matrix analysis that allows us to demonstrate clearly how many parameters we can identify. In fact, the inertia-and-shock model (8) has the following five combinations of parameters: $\theta,\ \beta_{\mathrm{I}} \cdot \theta,\ \sigma_I \cdot \theta,\ \beta_{\mathrm{S}} \cdot \theta,\ \sigma_S \cdot \theta$, and we would like to estimate the following five vectors of parameters: $\theta,\ \beta_{\mathrm{I}},\ \sigma_I, \beta_{\mathrm{S}},\ \sigma_S$. In the following matrix, the rows index the combinations while the columns index the parameters (Cantillo, 2004):

$$\Pi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

After some basic operations among the rows, we easily get the identity matrix and so we can conclude that the five parameter vectors are identifiable. Therefore, the whole model is identifiable, except for the scale factor of the error term which has been fixed to unity as usual.

4. Empirical Results

The aim of the work reported in this paper is to estimate models within formulation (8) using data from the *Santiago Panel*. However, because it is quite a complex model, in order to test its empirical performance we first estimate models using synthetic data in a controlled experiment that is free of unknown effects. For both real and synthetic data, models with increasing complexity were estimated, but in all cases the additive errors were assumed to independent and identically distributed (IID) Gumbel.

4.1 Application to Synthetic Data

Simulated data were generated following the approach of Williams and Ortuzar (1982). In particular, for the purpose of our tests, three datasets were built in order to simulate the sequence of singular choices that each individual makes in a three-wave panel. We performed

[5] The order condition states that at most $J\ (J-1)\ /\ 2\ -1$ parameters are estimable in the disturbance, where J is the number of options. Consequently, for the particular case of the *Santiago Panel* which has 12 options, the order condition states that we can estimate at most 65 parameters.

10 repetitions of the data-generation process. The general experiment consisted of three hypothetical options, each described by two generic attributes: cost *C* and travel time *T*. The attributes were built taking random draws following left truncated Normal distributions with arbitrary mean and standard deviation. In particular, as shown in Table 1, we assumed that the LOS characteristics varied after the first wave, but remained stable between waves 2 and 3.

Eventually, 4,000 singular individual choices were generated in each wave, according to the following procedure:

1. Initially at $w = 1$ individuals choose between options according to a compensatory utility maximizing process (i.e. a MNL with scale parameter equal to unity). The systematic utility function was built assuming a linear specification on the attributes. Thus, for each individual we have:

$$U_{jq}^{w} = \beta_T T_{jq}^{w} + \beta_C C_{jq}^{w} + \varepsilon_{jq}^{w} \tag{10}$$

2. In waves 2 and 3, attributes change and individual choices become affected by inertia and shock. Thus, setting the scale parameter to 1 the systematic utility function was built assuming the presence of three terms representing the three forces involved in the choice process:
   - modal attributes: represented by V_{jq}^{w}
   - inertia effect: represented by $(\beta_{Ij}^{w} + \sigma_{Ij}^{w} + \beta_{I\_SE} \cdot SE_I) \cdot \left(V_{rq}^{w-1} - V_{jq}^{w-1}\right)$
   - shock effect: represented by $(\beta_{Sj}^{w} + \sigma_{Sj}^{w} + \beta_{S\_SE} \cdot SE_S) \cdot \left(V_{jq}^{w} - V_{jq}^{w-1}\right)$

Attribute	Wave	Option 1 (mean, st. dev)	Option 2 (mean, st. dev)	Option 3 (mean, st. dev)
Travel Time	1	(200, 100)	(150, 100)	(120, 70)
	2, 3	(220, 100)	(140, 100)	(120, 70)
Cost	1	(490, 100)	(600, 100)	(850, 70)
	2, 3	(350, 100)	(500, 100)	(900, 70)

Table 1. Attributes for the simulated experiment

Shock and inertia effects were generated assuming that the parameters varied for each individual following a Normal distribution (i.e. panel correlation was included as random parameters in the temporal effects) with mean and standard deviation given in Table 2.

In order to analyse a synthetic situation similar to the one experienced by the *Santiago Panel*[6], we assumed a decreasing shock effect and an increasing inertia effect over waves.

As mentioned before, we used the simulated data to understand how models perform when:

- the effects of inertia and shock are not included in the specification (i.e. a classical MNL model), although they are actually present in the data,
- the effects of only considering inertia when the data also includes a shock effect (i.e. an inertia model extending the two-wave formulation developed by Cantillo et al., 2007), and
- the effects of inertia and shock are correctly specified.

The second and third of these models accommodate panel correlation as random parameters for temporal effects. Table 2 presents the results of estimating these three models. It is easy to verify that the three models present high *t* values for tests against zero for each of their

[6] The introduction of Transantiago, which meant a large change (shock) in the transport system, occurred between the first and second waves of the *Santiago Panel*.

parameters. However, the parameters of the MNL and inertia models are significantly different from their targets at the 95% level. A Likelihood Ratio (LR) test (Ortúzar and Willumsen, 2001) allows us to conclude that the MNL is the least appropriate model in terms of fit while, as expected, the proposed model has the best fit. In fact, the LR test for the inertia model against the proposed model LR = 845.5 is much larger than the critical value ($\chi^2_{95\%,4} = 9.488$).

Therefore, considering also the good performance of the inertia-and-shock model in recovering the true parameters (all *t* values for tests against the target parameters are lower than 1.96), we can safely conclude that not only the MNL (an expected result), but also the less restricted inertia model are not appropriate in a rapidly changing environment.

Parameter	Parameter Target	MNL	Inertia Model	Shock and Inertia Model
Cost	-0.5	-0.29543	-0.375	-0.454
t-tests[(1)]		*(-64.82;44.88)*	*(-44.35;14.78)*	*(-9.13;0.93)*
Travel Time	-0.1	-0.0412	-0.05691	-0.0719
t-tests		*(-8.06;11.50)*	*(-20.56;15.57)*	*(-4.11;1.61)*
Inertia (w_1 - w_2) (mean)	0.1		0.596	0.10982
t-tests			*(36.87;30.68)*	*(21;1.88)*
Inertia (w_1 - w_2) (st. deviation)	0.6		1.02	0.533
t-tests			*(26.8;11.04)*	*(5.61;-0.71)*
Inertia (w_2 – w_3) (mean)	0.7		1.35	0.674
t-tests			*(18.64;8.97)*	*(-7.68;0.30)*
Inertia (w_2 – w_3) (st. deviation)	0.4		1.74	0.361
t-tests			*(25.63;19.74)*	*(3.64;-0.39)*
Shock (w_1 - w_2) (mean)	0.8			0.825
t-tests				*(27.76;0.84)*
Shock (w_1 - w_2) (st. deviation)	0.5			0.612
t-tests				*(5.61;1.03)*
Shock (w_2 – w_3) (mean)	0.2			0.233
t-tests				*(4.59;0.65)*
Shock (w_2 – w_3) (st. deviation)	0.7			0.519
t-tests				*(3.51;-1.22)*
Sample size		12,000	12,000	12,000
Log-likelihood		-7180.709	-5697.021	-5274.26
No. of parameters		2	6	10

(1): t-test against zero; t-test against the true (target) parameter.

Table 2. Model results with simulated data

According to these results, we find that including temporal effects either incorrectly or incompletely is not good enough in terms of the quality of the estimated parameters. This implies an important risk in applications to real data, as the analyst does not know the true parameters and the available statistical tests could lead to wrong specifications.

4.2 Application to the Santiago Panel

Models of increasing complexity were estimated with the data from waves 1 – 3 of the *Santiago Panel*, the main characteristics of which were presented in section 2. In particular, we present the following results: two models without temporal effects, one model that includes only an inertia effect, and two models that accommodate inertia and shock effects together. As the *Santiago Panel* is a five-day panel dataset (i.e. it includes the trips made by each individual during five working days for each wave), all the specifications tested include short panel correlation. This "internal" correlation between the observations of a given individual was tested both via the random parameters and using a specific error component; the first of these approaches gave a superior fit.

A variety of formulations were tested for the model incorporating temporal effects, such as: different inertia effects over waves, different scale over waves, systematic variations for the inertia and shock parameters, and inter-option correlation. But, in all cases, under the usual tests, these models were judged inferior to the models presented below (see summary of results in Table 3 and 4). Indeed, we found a different scale among waves after shock ($\lambda_1 \neq \lambda_{w=2,3}$) in the MNL models, but this difference became non-significant when temporal-effects were included. We found correlation among the options related to bus (bus, bus–Metro, share taxi - bus), but this correlation structure disappeared when we introduced temporal effects. This is in itself a very interesting finding.

Model	**ML3**				**ML4**				**ML5**			
	INERTIA											
	INERTIA EFFECT (mean)	*t-test*	INERTIA EFFECT (st.dev)	*t-test*	INERTIA EFFECT (mean)	*t-test*	INERTIA EFFECT (st.dev)	*t-test*	INERTIA EFFECT (mean)	*t-test*	INERTIA EFFECT (st.dev)	*t-test*
CAR DRIVER	-0.14	*-1.40*	0.408	*4.35*	-0.124	*-1.57*	1.67	*8.24*	0.21	*1.86*	1.25	*6.75*
CAR PASS	-1.32	*-3.39*	1.70	*4.32*	-0.228	*-1.60*	0.53	*2.96*	-0.06	*-1.3*	0.39	*2.90*
METRO	0.91	*5.06*	2.24	*4.50*	0.851	*4.04*	0.28	*1.91*	0.33	*4.34*	0.98	*2.45*
BUS	1.03	*6.73*	1.15	*8.49*	1.44	*7.18*	0.98	*8.29*	1.43	*7.42*	1.19	*8.16*
WALK	-3.53	*-1.97*	15.3	*1.52*	-1.15	*-2.38*	13.1	*2.49*				
SH TAXI-METRO	-0.086	*-1.90*	0.14	*3.32*	0.097	*2.11*	0.11	*1.91*				
BUS-METRO	0.896	*5.05*	0.56	*5.61*	0.124	*2.57*	0.52	*6.17*	1.08	*6.87*	1.26	*6.5*
	SHOCK											
	SHOCK EFFECT (mean)	*t-test*	SHOCK EFFECT (st.dev)	*t-test*	SHOCK EFFECT (mean)	*t-test*	SHOCK EFFECT (st.dev)	*t-test*	SHOCK EFFECT (mean)	*t-test*	SHOCK EFFECT (st.dev)	*t-test*
$W_1 – W_2$	-	-	-	-	0.13	2.34	0.11	*3.63*	0.24	*2.01*	0.198	*5.82*
$W_2 – W_3$	-	-	-	-	-	-	-	-	0.07	*1.95*	0.13	*3.23*

Table 3. Temporal Parameters of Models for the *Santiago Panel*

For space reasons Table 3 only reports the results about the two temporal effects, as they are the object of the paper. Details of all the other attributes included in the specification are given in Appendix 1. The first model (ML1) is the simplest specification as it assumes generic parameters over waves and no temporal effects; the cost parameter was random but specified generic among options and took account of panel correlation; a Log-Normal distribution was used to guarantee that the marginal utility of income was positive[7]. Model ML1 could be valid for common and stable choice environments, but it is at best questionable for changing environments. The second model (ML2) is based on the same assumptions as model ML1, but it allows for a different scale after the shock (i.e. a different scale was estimated for wave 1 and for waves 2-3 [8]).

The third model (ML3) accommodates the inertia effect, the parameters of which are assumed specific across options and Normal distributed across individuals to account also for panel correlation. Models ML4 and ML5 have the same specification of model ML3, but the shock effect is also estimated. The parameter of the shock effect is generic among options, but still randomly distributed and it accounts for panel correlation[9]. Note that although we believe that the shock effect should have a positive parameter, we did not constraint its distribution to be only positive. This is because we would like the model to confirm our assumption. Models ML4 and ML5 differ in that the first assumes that the shock effect is present only between the first two waves (i.e. immediately after the new policy was introduced), while model ML5 allows us to investigate whether or not the shock effect is still significant over later waves.

Analysing the results it should first be noted that in all models the estimated parameters are highly significant, with a few exceptions that will be discussed later. However, as expected, the model performance improves when representation of both the inertia and shock effects are introduced to the model. Indeed, according to the LR-test, model ML5 (which allows for inertia and shock, with the latter effect being variable among waves) is the best, while the models without temporal effects present the poorest fit. Moreover, the results obtained with model ML5 confirm our hypotheses, as the shock effect has its highest value immediately after the introduction of the new policy (i.e. between w_1 and w_2), and then its magnitude decreases. Moreover, and especially for complex structures, to evaluate results accurately it is crucial to interpret results carefully in terms of their representation of the real phenomena. For example, although the first two models (ML1 and ML2), which do not include any temporal effect would appear quite good to any seasoned modeller, it is clear that the introduction of temporal effects (ML3, ML4 and ML5) not only improves the fit, but also gives better estimated parameters.

Table 4, in turn, summarizes the main statistics of the models, including the mean values of time and a ranking of the specifications based on their statistical results. The table shows clearly that models without the temporal effects appear to underestimate the SVT substantially.

	ML1	**ML2**	**ML3**	**ML4**	**ML5**
SVT Travel (US$/hr)	1.09	1.16	3.27	2.63	2.03
SVT Waiting (US$/hr)	3.54	4.20	7.27	7.65	8.44
SVT Walking(US$/hr)	2.56	2.83	4.33	6.06	4.92
No. of parameters	19	20	33	35	33
No. of observations	4134	4134	4134	4134	4134
L(max)	-2848.53	-2841.963	-2659.61	-2597.64	-2541.92
Rho adjusted	0.441	0.442	0.475	0.486	0.497
Ranking	**5**	**4**	**3**	**2**	**1**

Table 4. Summary of Models for the *Santiago Panel*

7 We also tested a Normal distribution and checked that the expected proportion of individuals with "incorrect signs" would be minimal (Sillano and Ortúzar, 2005); for this reason and because it gave a slightly superior fit we preferred the log-Normal distribution.

8 We tested different wave-scales, but the difference was not significant for waves 2 and 3 $\lambda_2 = \lambda_3 = \lambda_{w=2,3}$.

9 We also tested, as in models ML1 and ML2, including panel correlation in the cost parameter rather than in the temporal-effects but the models presented a lower fit.

Our empirical results, in particular the population mean of the shock effect from models ML4 and ML5, confirm our prior assumption that the shock parameters should be positive (i.e. if a mode improves, the shock effect should increase its choice probability). However, as the shock parameters have a Normal distribution, in model ML3 12% of individuals would be expected to have a negative shock parameter, while model 4 indicates 12% and 29% of counter-intuitive signs for the first ($w_1 - w_2$) and the last ($w_2 - w_3$) pair of waves respectively. We believe that just 12% of potential counter-intuitive signs is acceptable considering the complexity of the choice scenario and the specification of the related model (Sillano and Ortúzar, 2005). However, 29% seems a substantial and even worrying proportion; but, considering the expected strong fall in shock effect after the second wave, which leads the mean shock parameter to be close to zero, it is not unexpected to obtain a higher percentage of negative signs, even with a low standard deviation (as the case of model ML5).

We also tested shock-effect parameters which are specific across options, but the results were not satisfactory in terms of significance. Additionally, we believe that the shock effect should not vary among options; rather, it should vary among individuals.

Regarding the inertia effect, we decided to keep the non-significant mean inertia parameters in the model when they had large and significant standard deviation parameters associated with them. In this sense, note also that bus has the largest inertia parameter, and contrary to the models with a poorer shock effect specification (ML3 and ML4), car passenger is the only mode with a negative inertia parameter, which represents a greater disposition to change. The model that omits shock effects (ML3) shows also negative inertia parameters for car driver, walk and share taxi - Metro. This may be because the inertia parameters are masking part of shock effects.

Also, as discussed in the introduction, the implementation of Transantiago was a radical shock for the population as it modified in big-bang fashion the whole transport system of the city; it contemplated direct measures over public transport modes, which also had indirect and no less important effects in the private and combined modes. Thus, an initial idea was to consider no-inertia between waves 1 and 2 $I_{jrq}^{2} = 0$ because of the shock. We tested this hypothesis with our best temporal-effect models (ML3, ML4 and ML5) assuming $I_{jrq}^{2} = 0, \forall j \in A_{(q)}$, but the results showed that inertia appeared to be present and with significant effects. Moreover, assuming null inertia does not seem reasonable, especially for private-modes users; the new public transport provided by Transantiago initially had a deficient performance, so there is no reason to re-evaluate (i.e. to remove inertia from the choice process) "usual" choices if the competing options become worse.

Table 4 shows, as expected, positive inertia parameters for less attractive modes (i.e. bus, metro, and bus-metro) and negative inertia parameters for more attractive modes (i.e. car driver, car passenger, walk). Note in fact, that in our notation - see equations (3) and (8) - the inertia parameter is specified with a negative sign. Hence, a positive estimate for the inertia parameter means that the effect of inertia is negative. Therefore, the "candidate" option ($A_j \neq A_r$) is less attractive, as the inertia effect decreases its comparative utility. Contrariwise, a negative estimate for the inertia parameter indicates a positive inertia effect; increasing the comparative utility of the "candidate" option A_j (i.e. an increased disposition to change). In any case, as the inertia parameters have a Normal distribution, different signs among individuals are allowed. This result is consistent with our hypotheses, as different individuals may have different perceptions and behaviour when they face a change in their choice scenarios. Thus, some individuals could be dominated by inertia (maintaining the "usual" choice), while others (perhaps because of dissatisfaction with the previously chosen option) could have a high disposition to change. For example, the best model (ML5) yields only 11.5% of bus users with a negative inertia parameter. This result indicates that there is a fairly general consensus about the low attractiveness of bus, as for most users the inertia effect reduces its utility value when the bus plays the role of "candidate" option. Contrariwise, the car driver mode does not present this clear consensus, as more than 40% of users have a disposition to change their usual choice for this option (positive inertia effect), while the rest of the users present the opposite trend.

In order to clarify the interaction between inertia and shock effects, Table 5 shows an example of how a change in the system affects individual choices. In particular we chose two options with different sign for the inertia effect. Let us start by assuming that individual q chose any option other than bus and car passenger in the previous wave. Thus, bus and car passenger will be candidate options. For each option, the case of an improvement and a worsening are analysed. As can be seen from Table 5, if bus improves between consecutive waves, the probability to change from the usual option to this "candidate" one is affected by two opposing forces: the inertia effect that acts in favour of the usual option (meaning that the effect is negative over the utility of the bus option) and the shock effect that has a positive effect and increases the probability to change from the usual option to bus. On the other hand, if the bus worsens, both temporal effects decrease the probability to choose this candidate option. The analysis is based, of course, on our best model results.

Forces Involved in the Choice Process			Option Attributes	Inertia Effect I^w_{jrq}			Shock Effect S^w_{jq}		
Options	Performance	Total Utility		Inertia Parameter β^w_{Ij}	$\Delta V_{Inertia,q}$	Meaning for Candidate Option[10]	Shock Parameter β^w_{Sj}	$\Delta V_{Shock,q}$	Meaning for Candidate Option
Bus	Improve	$\tilde{U}^w_{Bus,q}$	$\tilde{V}^w_{Bus,q}$	(+)	+	Decrease Utility (-)	+	(+)	Increase Utility (+)
	Worsen							(-)	Decrease Utility (-)
Car Passenger	Improve	$\tilde{U}^w_{CarPass,q}$	$\tilde{V}^w_{CarPass,q}$	(-)		Increase Utility (+)		(+)	Increase Utility (+)
	Worsen							(-)	Decrease Utility (-)

Table 5. Analysis of the temporal effects for the best model

For car passengers, both of the temporal effects act in the same direction (increasing the probability to change) when this option improves, while inertia and shock have different sign when car passenger worsens. As the inertia effect of car passenger always increases its utility, this effect is reinforced (when this candidate option improves) or contra-rested (when this candidate option worsens) by the shock.

Table 5 is also useful to show that the sign (i.e. the interpretation) of the inertia effect is defined by the inertia parameter, as $\Delta V_{Inertia,q}$ should be normally non-negative, while the shock effect sign is defined by $\Delta V_{Shock,q}$.

5. Conclusions

When usual and repetitive choices are disturbed by the abrupt introduction of a new policy, it is necessary to consider that both inertia and shock effects may exist. In this paper we discuss an inertia approach which is appropriate for modelling with multi-wave panel data, including the possibility of dealing with strong policy changes (i.e. shock effects). We proposed a general

[10] Including the minus sign in front of I^w_{jrq}; see equation (3).

model formulation allowing for temporal effects that can differ among individuals, waves and options. The shock and inertia model was applied to simulated and real data, and our results provide evidence about the presence of both inertia and shock effects in common decisions made in an unstable choice environment.

The experiment with simulated data showed that a traditional model (without temporal effects), might lead to bias in the estimated coefficients. Moreover, we found that an incorrect specification of temporal effects could be risky, as it may conceal a bad performance (in terms of recovering the true parameters) behind apparently good statistical tests of fit.

Our best models, estimated with real data from the *Santiago Panel*, reveal that the shock effects vary among individuals, are constant over options, present a peak immediately after the introduction of the new policy (in our case between waves 1 and 2), and then their magnitude decreases. Additionally, the whole shock effect could be either positive (if the option improves) or negative (if the option worsens), but the estimated shock parameter should always be positive.

On the other hand, the inertia effects vary among individuals and over options, but not over the waves. The model allows positive and negative inertia effects, and the sign is actually determined by the sign of the estimated inertia parameter (attractive options show a negative inertia parameter). This means that the inertia effect represents a disposition to change (increasing utility value); contrariwise, the positive inertia parameter of unattractive options increases the probability of maintaining the usual choice.

These findings reinforced our belief that systems should be modelled with data that can capture the effect of new policies as well as both habit and inertia effects in the individuals' choice processes.

Finally, this paper presents the formulation and estimation of models that can accommodate shock and inertia effects together. Thus, we showed empirically the advantages of a correct temporal-effects inclusion in terms of a better explanation of real phenomena, and we are currently studying the appropriate use of these models for prediction purposes.

References

Ampt, E.S. and Ortúzar, J. de D. (2004) On best practice in continuous large-scale mobility surveys. *Transport Reviews*, 24, 337-363.

Ben-Akiva, M. (2009) Planning and action in a model of choice. In S. Hess and A. Daly (eds.), *Choice Modelling: the State-of-the-Art and the State-of-Practice, Proceedings from the Inaugural International Choice Modelling Conference*. Emerald, Bingley (forthcoming).

Blase, J.H. (1979) Hysteresis and catastrophe theory: a demonstration of habit and threshold in travel behaviour. *Proceedings 7th PTRC Summer Annul Meeting*, University of Warwick.

Cantillo, V. (2004) Modelación de Demanda Incorporando Umbrales Mínimos de Percepción y Valoración de Atributos. PhD Thesis. Department of Transport Engineering and Logistics, Pontificia Universidad Católica de Chile (in Spanish).

Cantillo, V., Ortúzar, J. de D. and Williams, H.C.W.L. (2007) Modelling discrete choices in the presence of inertia and serial correlation. *Transportation Science*, 41, 195-205.

Cherchi, E. and Ortúzar, J. de D. (2008) Empirical identification in the mixed logit model: analysing the effect of data richness. *Networks and Spatial Economics*, 8, 109-124.

Daganzo, C.F. and Sheffi, Y. (1979) Estimation of choice models from panel data. *Proceedings 26th Annual Meeting of the Regional Science Associatio*n, Los Angeles.

DICTUC (2003) Encuesta de Movilidad 2001 de Santiago. Informe final para el Ministerio de Planificación, Departamento de Ingeniería de Transporte, Pontificia Universidad Católica de Chile, Santiago (in Spanish).

Duncan, G.J. (1992) Household panel studies: prospects and problems. Working Paper 54, European Scientific Network on Household Panel Studies, University of Essex.

Fernández J.E., de Cea, J. and Malbrán, H. (2008) Demand responsive urban public transport system design: methodology and application. *Transportation Research*, 42, 951-972.

Goodwin, P.B. (1977) Habit and hysteresis in mode choice. *Urban Studies*, 14, 95-98.

Johnson, L.W. and Hensher, D.A. (1982) Application of multinomial probit to a two-period panel data set. *Transportation Research*, 16A, 457-464.

Kroes, E.P., Daly, A.J., Gunn, H.F. and van der Hoorn, A.I.J.M. (1996) The opening of the Amsterdam ring road: a case study on short-term effects of removing a bottleneck. *Transportation*, 23, 71-82.

Lanzendorf, M. (2003) Mobility biographies: a new perspective for understanding travel behaviour. *10th International Conference on Travel Behaviour Research.* Lucern.

Muñoz, J.C., Ortúzar, J. de D. and Gschwender, A. (2009) Transantiago: the fall and rise of a radical public transport intervention. In W. Saleh and G. Sammer (eds.), *Travel Demand Management and Road User Pricing: Success, Failure and Feasibility*. Ashgate, Farnham.

Murakami, E. and Watterson, W.T. (1990) Developing a household travel survey for the Puget Sound Region. *Transportation Research Record*, 1285, 40–48.

Ortúzar, J. de D. and Willumsen L.G. (2001) *Modelling Transport*. Third Edition, John Wiley and Sons, Chichester.

Parody, T. (1977) Analysis of predictive qualities of disaggregate modal-choice models. *Transportation Research Record*, 637, 51-57.

Pendyala, R.M., Parashar, A. and Muthyalagari, G.R. (2001) Measuring day-to-day variability in travel characteristics using GPS data. *80th Annual Meeting of the Transportation Research Board*, Washington D.C.

Sillano, M. and Ortúzar, J. de D. (2005) Willingness-to-pay estimation with mixed logit models: some new evidence. *Environment and Planning*, 37A, 525-550.

Swait, J., Adamowicz, W. and Buren, M. (2004) Choice and temporal welfare impacts: incorporating history into discrete choice models. *Journal of Environmental Economics and Management*, 47, 94-116.

Train, K.E. (2003) *Discrete Choice Methods with Simulation*. Cambridge University Press, Cambridge.

Walker, J.L. (2002) Mixed logit (or logit kernel) model: dispelling misconceptions of identification. *Transportation Research Record*, 1805, 86–98.

Williams, H.C.W.L. and Ortúzar, J. de D. (1982) Behavioural theories of dispersion and the mis-specification of travel demand models. *Transportation Research*, 16B, 167-219.

Yáñez, M.F., Heydecker, B.G. and Ortúzar, J. de D. (2008) A panel data model to forecast the effect of a radical public transport innovation. *4th International Symposium on Travel Demand Management (TDM 2008)*. Vienna.

Yáñez, M.F., Mansilla, P. and Ortúzar, J. de D. (2009) The Santiago Panel: measuring the effects of implementing Transantiago. *Transportation* (Online first).

Yáñez, M.F. and Ortúzar J. de D. (2009) Modelling choice in a changing environment: assessing the shock effects of a new transport system. In S. Hess and A. Daly (eds.), *Choice Modelling: the State-of-the-Art and the State-of-Practice, Proceedings from the Inaugural International Choice Modelling Conference*. Emerald, Bingley (forthcoming).

Appendix 1

Parameters/Models	**ML1**	**ML2**	**ML3**	**ML4**	**ML5**
Number of cars	2.32	2.85	2.13	3	1.96
t-test vs 0	*8.71*	*5.73*	*9.64*	*11.61*	*8.87*
CW (mean)	-0.154	-0.19	-0.16	-0.0825	-0.106
t-test vs 0	*-20.12*	*-7.33*	*-21.2*	*-18.45*	*-19.95*
CW (st.dev.)	0.256	0.31			
t-test vs 0	*21.95*	*6.39*			
TRAVEL	-0.0316	-0.04	-0.09803	-0.0407	-0.0404
t-test vs 0	*-8.49*	*-4.61*	*-7.23*	*-9.7*	*-10.4*
WAITING	-0.102	-0.15	-0.218	-0.1182	-0.1677
t-test vs 0	*-11.98*	*-4.40*	*-13.12*	*-10.66*	*-11.09*
WALKING	-0.0739	-0.10	-0.1297	-0.0937	-0.0977
t-test vs 0	*-12.92*	*-4.95*	*-14.13*	*-15.83*	*-16.85*
INTERCHANGES	-0.721	-0.88	-0.628	-0.62	-0.496
t-test vs 0	*-8.92*	*-6.29*	*-8.4*	*-8.13*	*-4.83*
COMFORT	1.3	1.58	1.17	1.24	0.869
t-test vs 0	*6.28*	*4.67*	*8.45*	*9.3*	*8.33*
CAR DRIVER	0.154	-0.03	0.237	-0.0579	0.143
t-test vs 0	*1.02*	*-0.12*	*1.55*	*-0.4*	*1.16*
CAR PASSENGER	-1.83	-2.68	-1.7	-1.79	-1.45
t-test vs 0	*-10.69*	*-3.75*	*-10.37*	*-10.69*	*-8.46*
SHARE TAXI	-0.896	-1.11	-0.465	-1.15	-0.767
t-test vs 0	*-5.11*	*-4.80*	*-3.02*	*-6.45*	*-4.64*
METRO	0.363	0.46	0.324	0.267	0.414
t-test vs 0	*3.64*	*3.26*	*3.47*	*2.71*	*4.07*
WALK	0.49	0.41	0.272	0.287	0.59
t-test vs 0	*2.24*	*1.37*	*1.53*	*1.62*	*4.54*
BICYCLE	-2.69	-3.70	-2.9	-3.3	-3.24
t-test vs 0	*-10.95*	*-4.70*	*-14.76*	*-12.11*	*-13.08*
PARK’N’RIDE	-0.898	-1.44	-0.862	-2.32	-1.46
t-test	*-4.6*	*-3.20*	*-4.47*	*-8.92*	*-6.43*
KISS’N’RIDE	-0.806	-1.08	-0.739	-0.71	-0.497
t-test	*-4.96*	*-4.20*	*-4.68*	*-4.48*	*-2.61*
SHARE TAXI-METRO	0.758	1.00	1.12	0.692	0.514
t-test vs 0	*5.07*	*3.93*	*8.04*	*4.63*	*3.46*
BUS-METRO	0.476	0.49	0.631	0.396	0.328
t-test vs 0	*5.82*	*4.57*	*8.35*	*4.96*	*3.16*
BUS-SHARE TAXI	-0.111	-0.26	0.162	-0.22	-0.476
t-test vs 0	*-0.46*	*0.73*	*0.71*	*-0.92*	*-2.08*
Scale $\lambda_{w=1,2,3}$		0.702			
t-test		*5.36, 2.27* [11]			
L(max)	*-2848.53*	*-2841.963*	*-2659.61*	*-2597.64*	*-2541.92*
Rho adjusted	*0.441*	*0.442*	*0.475*	*0.486*	*0.497*

[11] t-test against cero, t-test against one.