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Innovation diffusion and travelling waves.

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In this paper we will examine the dynamics of innovation diffusion, to illustrate the difficulty of using changing frequency distributions to diagnose a particular social learning process. We begin by discussing an influential model of new product diffusion taken from the marketing science literature (Bass 1969). This model estimates the importance of social imitation (or contagion) in the temporal evolution of frequencies of adopters for new consumer durables. This model has some similarities to the social learning components of a model subsequently proposed by Henrich (2001) in anthropology. Limiting our discussion to empirical innovation diffusion in modern market economies, we propose that the patterns of temporal evolution of adoption rates explained by social transmission bias in the Bass and Henrich models can also be explained by economic inequalities that determine optimal adoption timing when the new product shows price decline over time, and/or some improvement in performance. We suggest, further, that these two contrasting explanations (social contagion and economic inequality) can be applied to the situation where social contact frequencies and/or economic inequalities have spatial dependence, and that in such cases, the two explanatory models can produce similar spatiotemporal patterns of adoption diffusion. We illustrate these general observations with data on the diffusion of two recent agricultural innovations in the twentieth century USA. Our conclusion is that temporal and spatiotemporal patterns of innovation diffusion do not, in themselves, enable us to diagnose the importance of different kinds of bias in the decisions of adopters about the optimal timing of adoption. This diagnosis requires additional contextual information about a suite of economic, sociocultural and environmental variables.

The Bass Model and its critics

Typically in the innovation adoption literature, diffusion of an advantageous technology by independent assessment of efficacy is expected to produce an *r*-shaped cumulative growth curve, while diffusion by imitation of prior adopters is expected to produce an *S*-shaped cumulative growth curve (Bass 1969). Bass' influential model (1969) proposes that the population of adopters can be divided into independent adopters ('innovators') and imitators, and that the shape of the cumulative adoption curve will vary as a function of their relative importance. The Bass Model includes an innovation coefficient, p, representing the fraction of the population who will adopt the innovation regardless of the availability of demonstrators, and an imitation coefficient, q, representing the fraction of the population whose choice is determined by the number of previous adopters. The basic model states that:

$$P(t) = p + (q/m)Y(t)$$

where P is the probability of adoption by those who have not yet adopted at time t, p is a constant proportional to the frequency of innovators, q is a constant proportional to the rate of imitation, m is the size of the potential adopting population, and $Y(t)$ is the cumulative proportion of previous adopters at time t. This can then be expressed as a population rate of increase (Van den Bulte 2002):

$$dN(t)/dt = [\, p + (q/m)\, N(t)\,]\, [m - N(t)],$$

where $N(t) = mY(t)$, the cumulative number of adopters.

In cases where $q > p$, adoption will increase to reach an internal peak before declining (Figure 1a), leading to an S-shaped cumulative adoption curve. In cases where $q \leq p$, adoption rates will be at their maximum initially and then tail off (Figure 1b), leading to an r-shaped cumulative adoption curve.

Empirically, the parameters p, q and m can be estimated from discrete time series data by regression techniques (Bass 1969). Taking the adoption rate (S) at time t as:

$$S(t) = pm + (q\text{-}p)Y(t) - q/mY^2(t)$$

and using the analogue form :

$$S(t) = a + bY_{(t\text{-}1)} + cY^2_{t-1}$$

where $Y_{t\text{-}1}$ is the cumulative number of adopters at t-1, and then solving for a, b and c using regression techniques, one can derive p, q and m as follows: m=($-b\pm\sqrt{b^2-4ca}\,/2c$), $p = a/m$, and $q = -mc$ (Bass 1969). The empirical ratio q/p gives an index of the relative importance of initial independent innovation and of subsequent imitation in the diffusion of a particular new cultural trait, and is a shape parameter for the cumulative adoption curve.

Empirical analysis of the shapes of adoption curves in well-documented cases suggests that imitation is a much more influential mechanism of adoption than independent assessment of efficacy (Sultan *et al.* 1990), but the raw data on which such inference is based – a preponderance of S-shaped cumulative curves – could also represent heterogeneity in the economic factors determining individual adoption timing. This is not accounted for in the Bass Model. The assumptions required to generate S-shaped curves in a heterogeneous population using independent assessment are illustrated in Figure 2, after Van den Bulte & Stremersch (2004), who address effectively the estimation of the importance of contagion and of heterogeneity of propensity in empirical cases. Figure 2 shows two theoretical distributions of adoption thresholds for a new product in populations with different levels of income inequality (as indexed by the Gini income concentration coefficient), and the corresponding cumulative adoption curves when the utility value of adoption increases with time (for instance, because of price decline). Both are S-shaped, but the shape reflects income heterogeneity and not contagion-diffusion.

The interpretation of S-shaped curves in the Bass Model only applies in cases where there is population homogeneity in the economic factors determining the utility threshold for adoption subsequent to exposure.

A dual inheritance approach

Predictions of innovation diffusion rates also exist in the dual inheritance literature, and include terms which relate to the strength of cultural selection. S-shaped curves can be simulated by a growth function in which – in the simplest treatment - the intrinsic rate of increase is given by a term $\beta(w_1-\hat{w})$:

$$\Delta p = p(1-p)\,\beta(w_1-\hat{w}),$$

where p is the frequency of the cultural variant, β a rate parameter, and w_1 and $\hat{w}$ denote payoff to the behavior of interest and the average payoff, respectively (McElreath and Henrich, in press). This treatment of the pay-offs omits any social interaction term more complex than random choice of potential models in a perfectly-mixed population. In a well-studied example of the diffusion of hybrid seed-corn in 1940s Iowa, where the new strain brought a 20% gain in yield, the rapid empirical diffusion rate implies a high value for β, β>>1 (*ibid.,* cf. Figure 3). The new high-yielding variant spread more quickly than would have been the case if seed from the old and new strains had been selected for each annual planting proportional to their frequency at each preceding harvest, without any preferential selection by farmers of the new strain. In this case β denotes an assessment bias which amplifies the payoff differential in the randomly mixing situation, and therefore measures the strength of cultural selection.

Henrich (2001) has proposed an explanation of the shape of the Iowa hybrid corn diffusion curve in terms of biased transmission, with particular emphasis on conformist bias. He analyses a curve showing the cumulative frequency of individual adopters in two Iowa farming communities, and his interpretation may well apply. However, in principle, both his curve of Iowa adopters and the curves shown in Figure 3 could be generated if farm size was a factor affecting the time of first contact with seed corn suppliers (because the latter preferentially targeted the larger units, where the greatest gains were to be made), and if farm sizes had the distribution seen in Figure 2. There is a further complicating factor in that the curves in Figure 3 relate to acreage and not to the decisions of individual farmers (which are the focus of explanation for these models). If farm sizes are uniformly distributed, or uncorrelated with adoption timing, then the acreage planted with the new strain can reasonably be taken as a proxy for the frequency with which individual farmers decided to adopt. However, if farm size and adoption timing are correlated, then there will be a systematic error which needs to be controlled for. Specifically, if larger farms adopt earlier, then the cumulative adoption curve for acreage will imply a larger growth coefficient than that of the underlying cumulative frequency of individual farmers' adoption decisions.

Spatial diffusion and travelling waves

Such treatments attempt to estimate the strength of cultural selection from the gradient of the adoption curve, but they omit any spatial dimension. In fact, when several regional adoption curves for the same cultural trait are plotted side-by-side, they can reveal a spatial lag in adoption consistent with a contagion-diffusion process. Figure 3 illustrates this, showing the adoption curves for hybrid corn in five Midwestern and southern states at progressively greater distances from the eastern Iowa diffusion pole (or local origin), and also the overall pattern for the US in two spatial dimensions. Two things are immediately apparent: that lags seen when comparing arrival times in different regions would be consistent with a traveling wave contagion-diffusion explanation, and that the regional (state-level) adoption curves have different shapes and different slopes.

The basis of a contagion explanation of spatial lag is that adoption decisions are based on frequency of exposure to models, and that proximity has an influence on social contact rate. If we imitate from among those whom we see most often, and if our interactions are most frequent with our neighbours, then behaviours spread by imitation will diffuse spatially in a wave-like fashion (like an infective disease epidemic).

There is, however, an alternative explanation of such spatial patterns. It may be that heterogeneity in economic factors determining optimal adoption timing has a spatial structure (for instance, with an adoption lag in more economically marginal areas where the fixed capital cost of investment in a new technology would represent a more significant investment burden). People may all be aware of the new technology, but adoption may show a spatial lag due to variation in the economic factors determining optimal timing of adoption.

The case of hybrid corn adoption in the US

In the case of the adoption of hybrid corn, Griliches (1960a) has argued that the observed delay reflects initial preferential targeting of the most profitable regions by seed corn producers, and an additional factor relating to regional variation in the allocation of effort on selective breeding of hybrids adapted to local conditions by agricultural experimental stations in different states. Figure 4 shows the density of corn pickers in 1950, as a measure of corn yield, with the Corn Belt clearly delimited. This region clearly corresponds to the zone of earliest commercial adoption of the hybrid corn, although the major innovation in its development actually took place in Connecticut. Griliches (1960a) also argued that differences in slopes of adoption curves between states (Figure 3) reflected different levels of uncertainty about the gain in yield of the hybrid corn, with gains becoming apparent sooner in regions where absolute yield is higher (and thus where the typical percentage gain in yield from the new crop would be more immediately apparent). Dixon (1980) has since confirmed the correlation of state-specific adoption rates with measures of average farm size and of the agricultural productivity of the land. The additional correlation among states between time of initial adoption and subsequent rate of growth (with late-adopting states also showing a slower rate of substitution by the new strain, see Figure 5) is therefore suggestive of regional heterogeneity in economic thresholds, and not of contagion-diffusion. Indeed, this spatial gradient in within-state substitution rates complicates interpretation of the graph and map of first adoption time, because these actually plot the date at which 10% or more of corn acreage was planted with the hybrid strain, and this will vary with substitution rates as well as with the true time of initial introduction.

Existing versions of the contagion explanation do not predict heterogeneity within and between populations in $\beta(w_1 - \hat{w})$, and this is a limitation of that explanation in its present form. Henrich (2001) does however suggest that social contagion will be more likely to occur where individuals are less able to assess the efficacy of an innovation directly, and this would imply – following Griliches (1960a) - that the later-adopting states should also show a higher value for the q/p index (if we assume that adopters evaluate new products on the basis of information available within-state, and if we make the additional restricting assumption that there is no significant variation between states in internal heterogeneity of economic factors affecting adoption timing). This would therefore merit further investigation.

The case of tractor adoption (and horse devolution) in the US

The diffusion of tractor use in the US is another case which illustrates these issues. A similar set of traveling wave profiles to those for hybrid corn can be recovered for the diffusion of the tractor in selected states in the American Midwest (Figure 6), in this case reflecting increasing distance from an estimated diffusion pole in North Dakota (Casetti and Semple 1969).

The spatial gradient in diffusion of the tractor does not seem immediately compatible with a threshold distribution explanation. A graph of state-level substitution rates shows that their variation does not account for the spatial lag in onset of use (Figure 7): by implication, late-adopting states were not characterized by greater uncertainty about the profitability of adoption. However, according to Duffy-Martini and Silberberg (2004),

> "The diffusion of the tractor was not uniform across the US. As the tractor developed, various farming systems found them useful albeit in different decades. Very roughly, the small grain region and Far West were the earliest adopters. This farming system has an annual cycle of plow, plant and reap. Although the planting was usually done with horses, plowing and reaping could be done with tractors. The huge power needs of these tasks made the tractor a valuable addition to the farm and reduced the number of horses needed. The more complicated farming system of the Corn Belt has a cycle of plow, plant, cultivate and hand harvest for crops. Because of smaller acreages and cultivation between the rows of corn, tractors did not displace many horses until the introduction of the Farmall tractor in 1925. The Cotton Belt did not find tractors competitive until the mechanical cotton picker allowed for the displacement of labor and horses in harvest. The eastern states adoption was more individual as the evolution of the tractor and specific implements became available. Thus it is important to segregate the type of farming by region or even state."

This suggests that there may have been a northwest-southeast gradient in tractor adoption timing reflecting a time trend in the utility of adoption for different systems as the technology evolved (or as other input and output values changed), and which could account for the pattern seen in Figure 6 as an alternative to proximity-based contagion-diffusion. Casetti and Semple's data have been re-analysed by several authors, including Cliff and Ord (1975) and Pfeifer and Deutsch (1980), who use more highly parameterised forecasting models. Morrill (1985) found additional effects of economic and environmental variables on adoption lag, including per capita income, percentage of land in cereal crops, and winter temperature, which suggest a role for adoption threshold heterogeneity in space. Griliches (1960b) had already shown that the cumulative frequency curve for tractors on farms is an inadequate indicator of farmers' investment decisions in new machines, and does not take account either of increases in designed use-life during the diffusion period, or of economic factors determining the prolongation of use-life through repair and maintenance. The empirical investment curve for new tractor purchasing (Figure 8) is much less smooth and does not show the normal distribution of investment decisions over time predicted by the simplest form of a social contagion model (e.g. Figure 1a, dashed line). Griliches (1960b) shows that investment decisions were highly sensitive in the short term to cost/benefit ratios, taking account of farm output prices and rates of depreciation of the stock of machinery. This analysis of farmers' investment decisions at the national level suggests that even without any contagion effects we should expect both heterogeneity of adoption timing at the state level, and spatial heterogeneity in optimal adoption timing at larger scales (producing traveling waves).

At the within-state level, Mattingly (1987) interprets the pattern seen in Illinois (Figure 9) as a contagion process, but the spatial lags also correspond broadly to gradients of decreasing average farm sizes and decreasing levels of cash grain farming (Figure 10), both of which may correlate with higher thresholds for investment in a fixed-cost innovation of this kind. It may be therefore that availability of the new technology was simultaneous throughout the state, but that adoption showed a wave-like diffusion pattern because of spatial heterogeneity in economic factors affecting adoption timing. The pattern itself cannot resolve this, and further analysis is required of the spatial variation in adoption times in relation to economic and social factors.

Conclusions

These case studies illustrate the difficulty of interpreting a pattern as evidence of a particular process. *S*-shaped cumulative adoption curves may indicate indirect biased transmission or social contagion, but only where the population is homogeneous with regard to price and utility thresholds for innovation adoption. Where there is heterogeneity, an adoption threshold distribution can produce exactly the same curves without contagion. Spatial traveling waves may also indicate a local contagion-diffusion process, but again, only where the population is homogeneous with regard to adoption thresholds. Where there is spatially-structured heterogeneity, the same spatial lag pattern may be observed without contagion. Neither pattern is sufficient to demonstrate one or other process, in the absence of supplementary information about individual and environmental variation.

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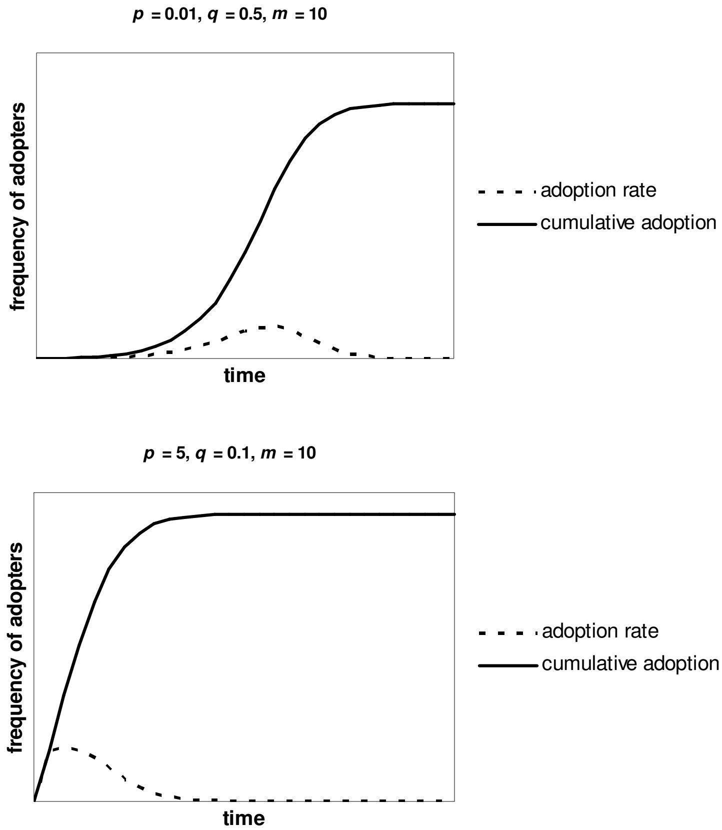

Figure 1. Rate of innovation adoption as a function of time, with different weights for the innovation coefficient (p) and the imitation coefficient (q): (a) $q>p$, and (b) $q \leq p$.

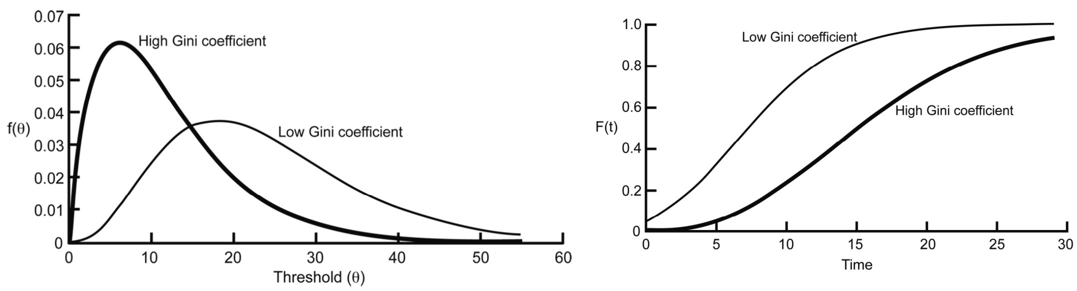

Figure 2. Price threshold distributions for adoption with high and low income inequality (Gini coefficient), and the corresponding cumulative adoption curves with decreasing prices over time (redrawn after Van den Bulte & Stremersch 2004).

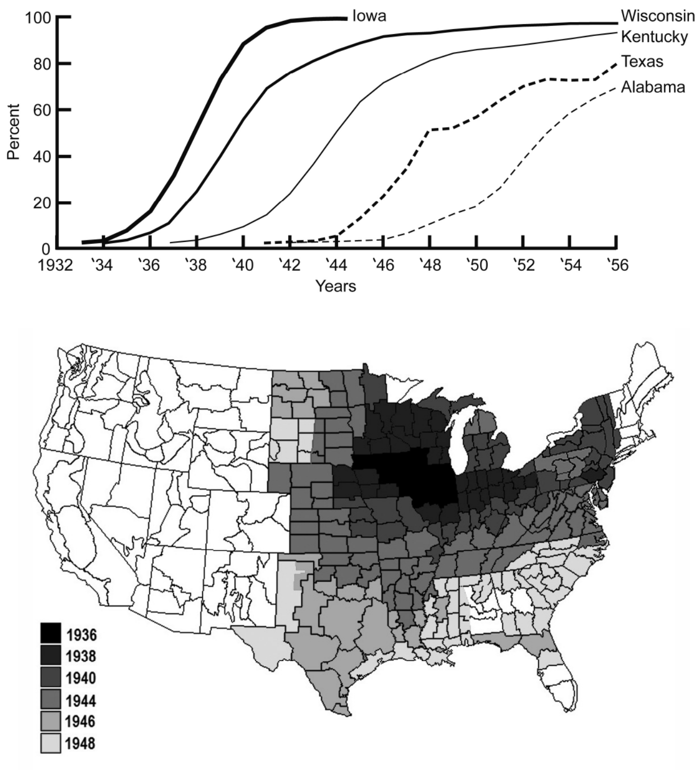

Figure 3. (a) Percent of total corn acreage planted with the hybrid strain, by state (redrawn after Griliches 1957, Figure 1). (b) Diffusion of hybrid corn usage, showing areas that planted 10 or more percent of their corn acreage to hybrid seed at successive time intervals (redrawn after Griliches 1960a, Figure 3).

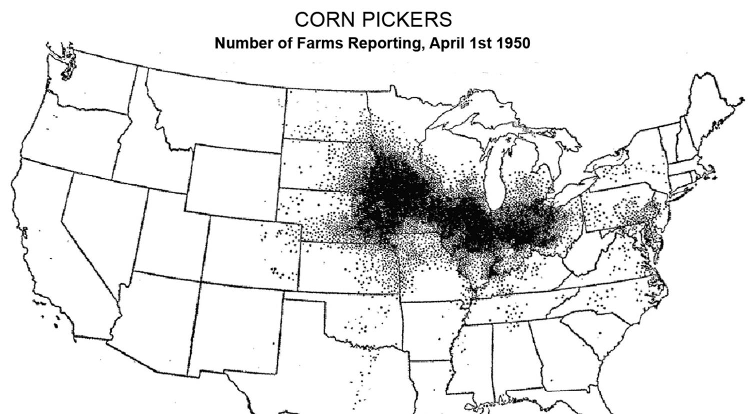

Figure 4. The density of corn pickers in 1950, as a measure of corn yield, with the Corn Belt clearly delimited (redrawn after Griliches 1960a, Figure 5).

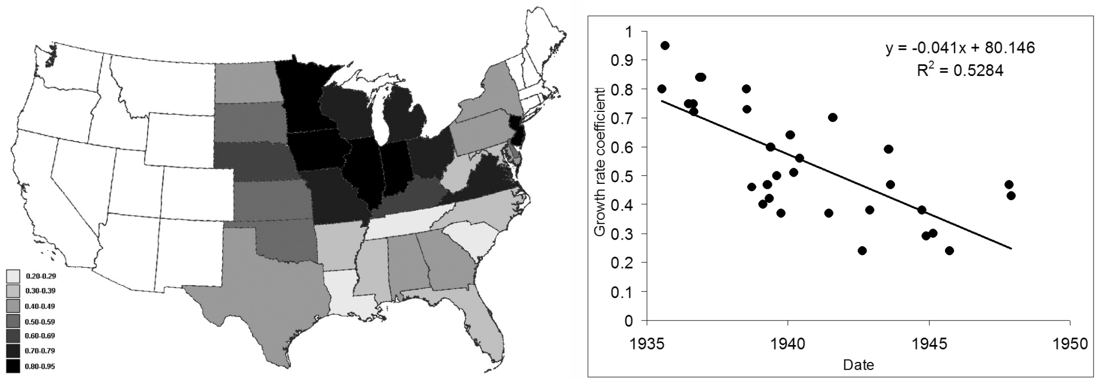

Figure 5. (a) Map showing within-state rates of increase in hybrid corn use. (b) Within-state rate of increase of hybrid corn use, plotted against the date of arrival in each state (the date at which hybrid corn reached 10% of all corn). Data from Griliches (1957) with revised growth coefficient estimates from Dixon (1980; coefficient b_2).

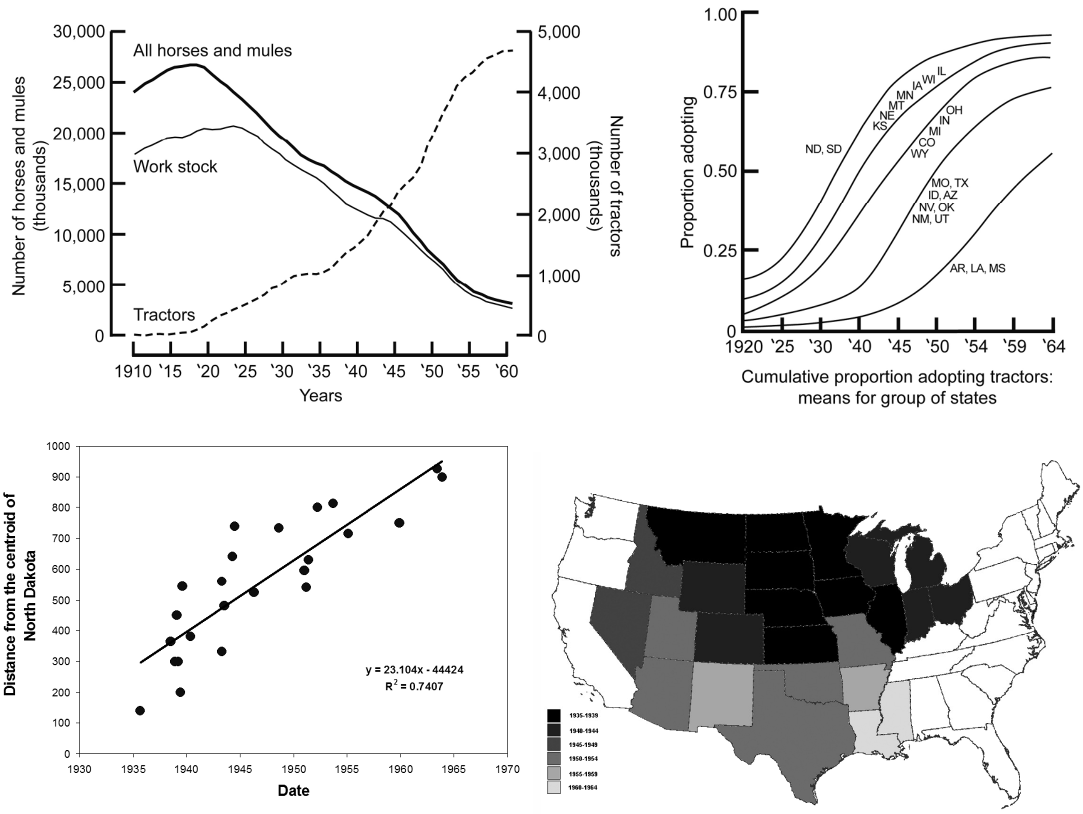

Figure 6. (a) Substitution of the tractor for the horse on farms in the United States, 1910-1960 (redrawn after Olmstead and Rhodes 2001, Figure 1). (b) Mean cumulative adoption curves for the tractor in groups of states in the US central and mountain regions (redrawn after Morrill 1985, Figure 2). (c) Distance from North Dakota against date of peak growth rate in adoption of the tractor in the US central and mountain regions (approximated as the date at which adoption had reached 50% of market saturation), classified by state (distances between centroids of states; data from Casetti and Semple 1969 and Banks 1994). (d) Map of dates of peak growth rate in adoption of the tractor in states in the US central and mountain regions (approximated as the date at which adoption had reached 50% of market saturation; data from Banks 1994).

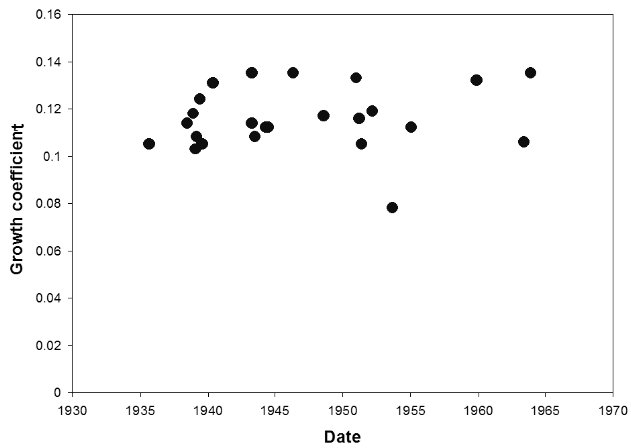

Figure 7. Growth coefficient plotted against date of peak growth rate in adoption of the tractor (approximated as the date at which adoption had reached 50% of market saturation), classified by state (data from Banks 1994).

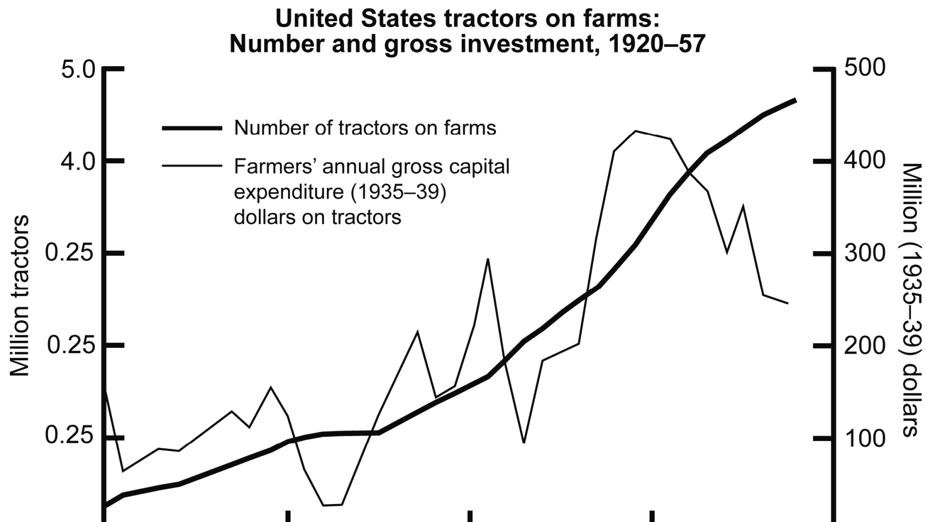

Figure 8. Tractors on US farms: numbers and gross investment, 1920-1957 (redrawn after Griliches 1960b, Chart 1).

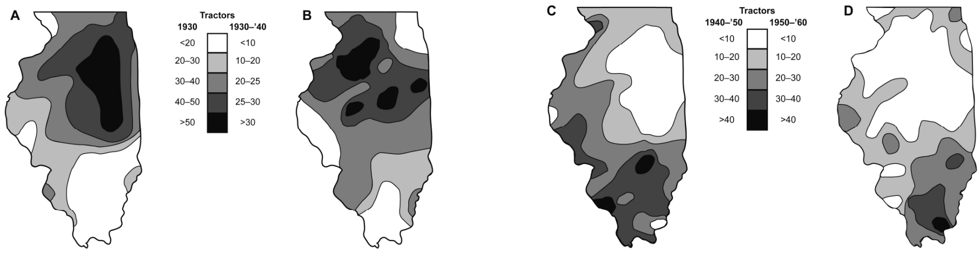

Figure 9. The diffusion of the tractor in Illinois (redrawn after Mattingly 1987).The first (1930) map shows the percentage of farms with tractors; the subsequent maps show the difference in percentages between the beginning and end of the indicated time periods.

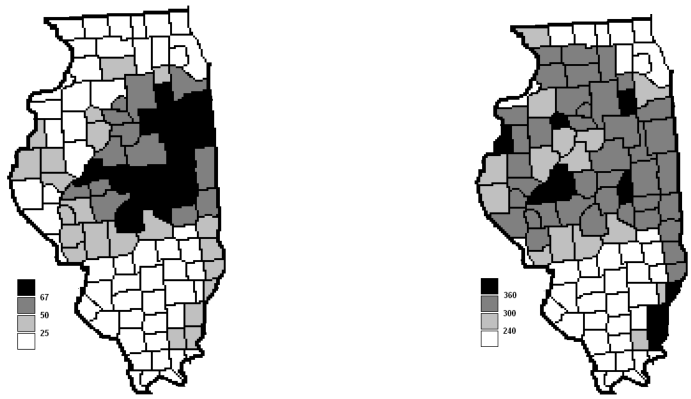

Figure 10. (a) 1949 data on percentage of farms focused on cash-grain cropping, by county (redrawn after Hart 1986, Figure 13). (b) Average farm size in acres, by county, 1982 (redrawn after Hart 1986, Figure 5). Overall in the twentieth century farm size has increased, but the county-by-county pattern of relative sizes for Illinois is conserved: 1939 farm sizes predict 1982 farm sizes with high accuracy (r = 0.94, Hart 1986).