A Measurement Error Approach to the Study of Poverty

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Abstract

This study investigates the pattern of poverty in Italy in the 1980s and the 1990s, by means of both consumption and income measures, so as to separate the permanent and temporary components. The empirical analysis we conduct addresses not only economic issues, but also those of survey methodology and reliability of data.

The model for the study of poverty is inspired by the literature on measurement error, and allows us not only to identify the two components but also to test if the variances of the shocks vary along the distributions of income and consumption.

We find a slight increase of the transitory component, whereas the permanent one remains constant over the two decades. There is some evidence of non-constancy of the variances of the shocks, and a more precise formal test is under development.

Key-words: Income and consumption poverty; measurement error; permanent and transitory shocks.

JEL classification: D12, D31, I32, J31.

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Executive summary

This paper presents a model for the study of poverty inspired by the literature on measurement errors. The economic theory of the life-cycle, together with the Permanent Income Hypothesis, provide the well-known economic model composed by three stochastic processes for income, permanent income and consumption, where income is the sum of permanent income and a transitory shock, while the innovations in the processes of permanent income and consumption are the same and are regarded as permanent shocks. After observing that the shocks in the income and consumption processes of our model have the same properties as classical measurement errors, we apply measurement error techniques to the three processes for income, permanent income and consumption. We obtain a system of equations where the growth of the headcount ratio index of poverty is related to the variances of the two economic shocks. The empirical estimates of the headcount ratios for Italy in the 1980s and 1990s show an increase in the indices based on income, and a constant profile for those based on consumption, in agreement with the results on inequality indices found in Rosati (2000).

The system of equations provides also a simple test for the assumption of common variance of the shocks, for a specific time period, among all individuals belonging to a same birth cohort. The test is not very sensitive and does not manage to detect non-constancy of the variance within cohorts, but further inspection of the data shows evidence of heteroskedasticity. In particular, the households at the bottom of the distributions of income and consumption show larger variances of the shocks, which then decrease towards the centre and settle around a constant value until the top of the distribution. Therefore we extend the previous poverty model to the case of heteroskedastic shocks, and obtain a rather complex system of equations that allows identification of the variances of the shocks for different values of the household's income or consumption. The system provides also good suggestions for the derivation of two separate tests for the heteroskedasticity of the two shocks.

The present version does not include these last two tests, which are still under development. We are also working at the inclusion of trends in the processes of income and consumption, which will be presented in future versions of the present work.

1 Introduction

The study of welfare needs the careful distinction of its temporary and permanent aspects. Temporary fluctuations of income may hide the real, long-run, level of welfare of a household. Moreover, current income can catch the effects of depth in low-income spells, but may not be able to record differences due to the duration of the poverty period.

The centrality of the distinction of temporary and permanent components has been emphasised in many recent studies on inequality. In particular, Blundell and Preston (1998, 1999) highlight the value of the joint use of income and consumption measures in the identification of the temporary and permanent components of inequality. Following their line, this study aims to investigate further the role of income and consumption in the measurement of temporary and permanent poverty.

The inspiration for the model comes from Chesher and Schluter (1999), where a number of models for different types of errors in the measurement of welfare are derived. The authors observe that "the relationship between income reported over a fixed period and 'permanent' income may have the same structure as the relationship between error contaminated and error free income with transitory income playing the role of measurement error". From this starting point, the standard measurement error techniques can be applied to estimate the true welfare level. Therefore, we specify an economic model for the processes of income and consumption, and then we use measurement error techniques to obtain suitable estimating equations.

The paper is organised as follows. In Section 2, a number of recent Italian studies on poverty are recalled and commented upon, reporting on both empirical findings and on data and methods used. Section 3 describes the two main Italian surveys on income and consumption, and justifies our choice of data. In Section 4 some first descriptive results are presented. Sections 5 and 6 set up the model for the analysis of poverty and present the empirical results. Sections 7 and 8 tackle the issue of heteroskedasticity of the economic shocks, and develop a model for the study of poverty in the case of non-constant variance. Section 9 concludes and presents some issues that deserve further research.

2 Poverty in Italy so far

There are a number of studies on the distribution of incomes and consumption in Italy; most of them are based on the two main Italian surveys, namely the ISTAT (Italian Central Statistical Office) Survey on Family Budgets (SFB) and the Bank of

Italy Survey on Household Income and Wealth (SHIW). The SFB focuses mainly on consumption, while the SHIW clearly deals with incomes, but both surveys record at least one measure of the other variable of interest. Even if, obviously, such a measure is much rougher than that collected in the other survey, nonetheless it allows one to conduct some initial analyses. Most of the previous work has utilised one or other survey, while only recently attention has been paid to the opportunity of exploiting at the same time information coming from both surveys. This, in part, can be due to the features of the surveys, as outlined above; therefore the analyses attempted in this paper are new to the Italian studies on poverty, and can also throw light on a new possible way of utilising jointly our most informative national surveys.

2.1 Consumption measures

One of the main studies is presented in the official report of the Commission for Poverty and Social Exclusion (Commissione d'Indagine sulla Povertà ed Emarginazione, 1996), where the picture of poverty from 1980 to 1994 is drawn. This study utilises the SFB, therefore investigating measures of consumption inequality. The profile of incidence of poverty, obtained referring to the international standard poverty line[1], shows a steady increase over the period 1980-1989 from 8% to 15% (roughly), then a drop in 1990 to around 12%, and a constant or slowly decreasing pattern until 1994.

These figures have been updated until 1997 in Trivellato (1998) [see Table 1]; here the state-of-the-art on inequality studies in Italy is extensively documented and commented upon, not only reporting empirical evidence, but also addressing methodological issues. On the top of the results shown in Table 1, which are based on consumption measures from SFB, analyses of dynamics of inequality are performed, utilising incomes from the panel part of the SHIW to estimate the probabilities of transition into and out of the poverty state between 1989 and 1993. The poor in 1989 have a roughly 50% probability of still being poor in 1993, while only 9% of the non poor in 1989 will be poor in 1993.

To locate Italy in a wider context, we report in Table 2 some figures based on the first wave (1994) of the ECHP (European Community Household Panel), from Eurostat (1997). The information refers to the year 1993; unluckily, these values cannot be compared to those of Table 1, since the poverty line is computed differently, but still give an indication of the phenomenon relative to other European countries. The position of Italy is somewhat intermediate: the incidence indices computed referring to households, individuals and youths aged less than 16, respectively, are only slightly

[1] The poverty line for a 2-component family equals the mean per capita expenditure level.

Table 1: Incidence and intensity of poverty in Italy, 1980-1997. Source: Trivellato (1998).

Year	Poverty line (monthly thousand current Liras)	Incidence(%)				Intensity (%)
		North	Centre	South	Italy	
1980	267	4.6	4.5	16.0	8.3	16
1981	314	5.8	5.6	17.8	9.6	19
1982	368	5.6	5.6	17.7	9.5	19
1983	422	7.1	7.3	17.9	10.6	22
1984	470	7.1	7.5	19.7	11.3	22
1985	549	7.2	6.6	21.0	11.6	22
1986	624	7.7	7.3	22.8	12.6	23
1987	692	9.0	8.1	25.8	14.4	23
1988	749	8.7	9.9	26.0	14.8	23
1989	838	8.2	9.3	26.0	14.4	22
1990	914	7.4	7.7	20.0	11.7	19
1991	1,010	8.0	7.4	19.8	11.8	19
1992	1,042	7.0	7.2	20.7	11.7	19
1993	1,025	5.4	7.8	19.4	10.7	18
1994	1,094	4.4	6.8	20.6	10.2	21
1995	1,143	4.4	6.8	21.9	10.6	22
1996	1,190	3.9	5.7	22.3	10.3	21
1997	1,234	4.3	5.8	24.2	11.2	22

above the EU12 average values.

2.2 Income measures

There are many official publications on income distribution by both Bank of Italy (on SHIW data) and ISTAT (on SFB data). Most of the results are collected and compared in Brandolini (1998); this paper is a comprehensive presentation of all income surveys in Italy after the Second World War, where the different sources are described, the reliability of data assessed, and the time pattern of inequality arising from such data investigated. We will only comment on the SFB and the SHIW data, since the other sources are not of concern to us at the moment.

Brandolini computes Gini index series from SFB and from SHIW in different versions, according to different definitions of income and using different sets of weights. All these series show a similar behaviour, and the author concludes that the pattern of income inequality in the 1980s and early 1990s is "an oscillation around a flattened trend", however remarking that "these are nothing more than working hypotheses that need to be corroborated with further empirical evidence". Beyond the indications on the trends of inequality in the last two decades, the paper gives useful insights on the

Table 2: Incidence of poverty in EU12, 1993. Source: Eurostat (1997).

Country	Poverty line[1] (in PPS)	Incidence(%) Household	Individuals	Youths aged < 16
Belgium	540	13	13	15
Denmark	527	9	6	5
Germany	562	13	11	13
Greece	325	24	22	19
Spain	377	19	20	25
France	516	16	14	12
Ireland	403	21	21	28
Italy[2]	411	18	20	24
Luxembourg	990	14	15	23
Netherlands	516	14	13	16
Portugal	311	29	26	27
United Kingdom	541	23	22	32
EU12	489	17	15	20

[1] The poverty line is set at 50% of the mean after-tax monthly disposable pro capite income, equivalised. The equivalence scale is 1 for the first adult, 0.5 for the following adults and 0.3 for any youth aged less than 14.
[2] In Italian Liras this would be a poverty line at 667.600.

reliability of our two sources of interest, among which is an assessment of underreporting in SHIW relative to national accounts, estimated at around 7% for income from employment and around 50% for income from self-employment.

Table 3: Decile shares in the distribution of post-tax family incomes. Sources: Istat (1996), Banca d'Italia (1997) and Eurostat (1997).

Source	1	2	3	4	5	6	7	8	9	10
SFB (1994)	3.3	5.1	5.9	7.1	7.4	9.8	11.2	12.3	15.3	22.6
ECHP (1994)	2	4	5	7	8	9	11	12	16	26
SHIW (1995)	2.3	3.9	5.2	6.3	7.5	8.9	10.6	12.8	15.9	26.6

Further informations on reliability of data come from official publications. From Eurostat (1997), ISTAT (1996) and Bank of Italy (1997), Table 3 displays the decile shares in the distribution of post-tax family incomes in 1994 from SFB and ECHP, and in 1995 from SHIW. While SHIW and ECHP report substantially identical values, the SFB differs from them, especially in the lower part of the distribution and in the 10^{th} decile. This is a very well-known characteristic of the SFB income data, that are heavily corrected according to the expenditure levels before they are released.

Table 4: Corrections in the SFB data, 1980-1982. Source: ISTAT (1983).

Reported income range (thousand Liras)	**Pre-correction distribution**			**Post-correction distribution**		
	1980	1981	1982	1980	1981	1982
below 400	25.1	17.3	9.90	12.3	8.60	5.90
400-600	28.4	17.9	15.9	20.4	13.2	7.70
600-800	18.9	24.3	21.7	17.7	16.7	14.4
800-1,000	12.7	14.9	17.5	15.3	14.3	13.4
1,000-1,200	7.30	10.0	10.7	11.7	12.6	12.8
1,200-1,500	4.50	8.80	12.0	10.6	14.6	16.3
1,500-2,000	1.60	4.60	8.10	7.30	11.7	16.2
2,000-2,500	0.50	1.20	2.50	2.40	4.40	6.70
2,500-3,000	0.20	0.40	0.80	1.10	1.90	3.00
over 3,000	0.80	0.60	1.10	1.20	1.80	3.50
Total	100.0	100.0	100.0	100.0	100.0	100.0

This correction in the SFB is meant to clean data from values of income not consistent with reported consumption expenditure, but is widely recognised as producing major distortions in the data, that make them unusable in most cases. The correction procedure is published in ISTAT (1983), and the results reported in Table 4, from the same reference, show the distortion in the 1980, 1981 and 1982 data. It has to be mentioned, moreover, that the income data are originally recorded in classes, but the corrected values appearing in the public files are imputed point values. Further results and comments can be found also in Brandolini (1998).

3 Data description

The data available for our study are the SFB and the SHIW over the 1980s and the 1990s. The SFB is a series of cross-sections that covers the period between 1986 and 1995, and collects every year data from more or less 32,000 households. The interviews are conducted quarterly, each quarter collecting data on around 8,000 households. The definitions of the variables over the 10 years are quite homogeneous, so that a direct comparison of the different waves is straightforward.

The SHIW is available from 1977 to 1995, data being collected annually from 1977 to 1987 except in 1985, and then every second year from 1989 to 1995. The number of households interviewed was around 3,000 in the period 1977-1980, rose to around 4,000 in 1981-1984 and finally to around 8,000 from 1986 onwards. The design is substantially a series of cross-sections, but from 1989 a portion of the sample has been followed over time, producing panel data on a restricted number of households. In this study we will only look at the cross-sections.

The more regular pattern of the SFB, and the higher frequency of the observations, make this survey initially more attractive than the SHIW. However, the quality of SFB income data is not very satisfactory, as anticipated in Section 2: first, the correction affects mainly the tails of the distribution, and therefore can affect the incidence of poverty; secondly, the point values we are given are imputed from data collected in classes, and this can seriously bias any measure of intensity, especially in the lowest class. For the purposes of our analyses, this handicap is heavy, and makes the SFB unsuitable. Hence from now on we will focus our attention on the SHIW data.

3.1 Selection of households and variables from SHIW

First of all note that we cannot use the first three waves, namely data from 1977 to 1979, since consumption is not available in these years. Therefore all analyses cover the period from 1980 to 1995.

We dropped those households whose consumption or income fell below the 1st percentile or above the 99th. Moreover, we kept only the households whose head is an employee, therefore excluding self-employed and retired. In fact our main interest is in labour income, hence we only consider heads still in the labour force; but it is also well-known that the measures of incomes from self-employment are not very reliable[2], therefore we choose to restrict our attention to employees only. With this selection, we end up with the sample sizes reported in Table 5, which are roughly 50% of the initial ones[3].

The analyses are conducted within three large cohorts, corresponding to heads of the family born in the 1930s, 1940s and 1950s. Further distinctions have been made according to three geographical areas (North, Centre and South), and according to education (High and Low[4]). Note that in 1980-1983 the age was recorded in ten-year classes, therefore we had to approximate the date of birth in 1981-1983 as follows: in the 1950s cohort fall all households with head aged 21 to 30 in the year they were surveyed (over 1981-1983 this ends up including heads born between 1951 and 1962); in the 1940s those aged 31 to 40 and in the 1930s those aged 41 to 50 (born in 1941-1952 and 1931-1942 respectively). This approximation leads to overlapping of cohorts in those three years, but the only alternative is to drop completely three whole waves.

[2]See, in this regard, the indications reported in section 2.2 from Brandolini (1998).

[3]Self-employed are around 20% of the sample, retired around 30%.

[4]High = High School or above, Low = Secondary School or less.

Table 5: *Sample sizes of utilised data from SHIW, 1980-1995, by birth cohorts of the head of the household.*

Year	**1930s**	**1940s**	**1950s**	**Total**
1980	466	412	165	1043
1981	603	607	303	1513
1982	626	566	262	1454
1983	649	601	199	1449
1984	603	677	396	1676
1986	1065	1253	924	3242
1987	1007	1234	917	3158
1989	869	1278	1067	3214
1991	682	1246	1047	2975
1993	443	1058	1039	2540
1995	299	970	989	2258
Total	7312	9902	7308	24522

3.2 Choice of variables

As regards the variables we use, note that income, as said before, is monthly family labour income from employment. Such measure may under-report family disposable income when the head of the family is an employee, but the family has extra income from self-employment of other members. However the families with head employed and non-zero income from self-employment are few in our sample and their income from self-employment is much smaller than income from employment.

The most appropriate measure of expenditure to use would be that for non-durable goods[5]; unfortunately, two waves (1986 and 1987) do not report this measure, but only total expenditure, therefore we use monthly total consumption. However, we carried out parallel analyses on non-durable consumption only (including housing), leaving out the aforementioned waves, and the results were almost identical, for the years considered, to those obtained with total consumption. For the specific results, see Rosati (2000).

Both income and consumption are rescaled according to an equivalence scale giving weight 1 to the head of the household, 0.7 to extra adults and 0.5 to any household member aged less than 18. Further comments about the choice of the equivalence scale, and a sensitivity analysis are reported in Rosati (2000).

[5]In fact, our interest is in measuring the flow of services enjoyed by a family, and this definition would include expenditure in non-durables and, ideally, a measure of the stock of durable goods owned by the family. This is generally not measured and, moreover, cannot be approximated by expenditure on durables, as it only records purchase of those goods, and not the actual stock. Therefore a reasonable measure of the flow of services can be simply non-durable expenditure.

Once equivalised income and consumption, we transform them into the logarithms, and run all the analyses in the logarithmic scale. The use of the logarithmic scale is common practice, and can be found in several other studies on poverty. Moreover, it is very convenient for us due to some specific features of the models we are going to use in our analyses. In fact, we will use models inspired by the literature on measurement error, which mainly regards additive measurement errors. The measurement error that affects positive variables, like income and consumption, is normally multiplicative, and this yields an additive model in the logarithmic scale, which enables us to use directly the measurement error models available.

4 Descriptive results

The first picture of poverty comes from the computation of the headcount ratios from income and consumption, and their comparison with those obtained from the SFB data in the study by the national Commission on poverty (see Table 1). Our computation on SHIW data makes use of all available households, and not only of those whose head was born between the 1930s and the 1950s, as in the other analyses. This is to make easier the comparison with the official statistics based on SFB, where there is no birth-cohort selection.

The results are reported in Table 6 and shown in Figure 1. For our analyses on SHIW data, we set the poverty lines at the 50% of the mean income and consumption. The indices derived from consumption are lower than those from income (on average 2-3% less), as we expect from the higher volatility of income, given that for our kind of data a larger variance yields a larger headcount ratio.

If we compare the consumption indices from SHIW and SFB, we see that the latter are higher. This may reflect the fact that in SHIW consumption is collected by asking just few recall questions, while in SFB we have much more detailed information, diary-based[6], which yields a higher variance. Notice also that after 1990, when a major revision of the cleaning procedures for the SFB took place, the headcount drops (as also noticed before, see Section 2.1), and settles around the SHIW levels.

The SFB indices are derived by the Commission referring to the poverty line for a 2-components household, reported in the table. We also add a column with the corresponding poverty line for a 1-component family, obtained rescaling the given lines according to the Carbonaro equivalence scale, as suggested by the Commission, i.e. dividing by 1.67. These lines are quite close to those based on SHIW consumption

[6]For further discussion about consumption recall vs diary-based data, see Battistin *et al.*(1999).

Table 6: *Incidence of poverty in 1980-1995. Headcount ratios from SHIW and SFB data.*

	SHIW data				**SFB data**		
	Income		**Consumption**		**(Consumption)**		
Year	**Pov. line**[1]	**HR (%)**	**Pov. line**[1]	**HR (%)**	**Pov. line**[2]	**HR (%)**	**Mean/1.67**[3]
1980	179	10.9	151	7.8	267	8.3	160
1981	196	11.3	187	10.0	314	9.6	188
1982	243	11.4	214	9.1	368	9.5	220
1983	278	10.6	243	8.7	422	10.6	253
1984	316	12.4	281	8.2	470	11.3	281
1986	362	12.1	317	8.4	624	12.6	374
1987	437	15.1	421	12.4	692	14.4	414
1989	500	13.5	477	9.0	838	14.4	502
1991	544	12.2	539	11.5	1010	11.8	605
1993	599	15.1	584	10.4	1025	10.7	614
1995	638	14.8	668	10.8	1143	10.6	684

[1] The poverty line is for a 1-component family, and equals 50% of the mean monthly per capita income/expenditure, equivalised (in thousand Liras).
[2] The poverty line is for a 2-components family, and equals the mean monthly per capita expenditure.
[3] This would be the corresponding poverty line for a 1-component family, if we re-scale according to the Carbonaro equivalence scale.

until 1984 (difference less than 10,000 Liras, around 3 GBP), but become somewhat more distant from 1986 onwards. This reflects a change in the distributions from 1986, as the lines are distribution-dependent, typically a relative increase in the mean consumption if compared to SHIW, i.e. a shift of the SFB distribution towards higher values, since the variance - for what observed before - does not seem to have increased during such period. A comparison of the poverty lines is shown in Figure 2.

5 A formal model

We derive now a model for the study of poverty based on the joint use of income and consumption. After specifying an economic model of consumer's behaviour, we observe that the errors in the income and consumption processes have the same properties as classic measurement error, and therefore we apply measurement error techniques to derive our estimating equations.

5.1 An economic model of consumer's behaviour

As we anticipated at different points of our discussion, one of the main issues in welfare measurement is the distinction of its temporary and permanent aspects. This is particularly important in the policy decision process, where - in turn - we need to take into account only the permanent changes in the welfare level, and ignore any fluctuations of welfare of transitory nature, or vice-versa.

Some studies have tackled this problem looking at (unobservable) permanent income as a measure of permanent welfare; for instance, Abul-Naga and Burgess (1997) develop a model to predict household permanent income making use of both income and expenditure plus an extra set of causal variables for permanent income, such as family size, housing asset ownership, educational and occupational status of the household. Other approaches instead, like the studies by Deaton and Paxson (1994) and Bludell and Preston (1998,1999), add to the PIH an intertemporal model of optimization over the life cycle, which yields an earnings and an expenditure equation that serve as a starting point to develop a complete economic model. We will follow this last approach, and in particular refer to the models used by Blundell and Preston.

Suppose income can be decomposed into a permanent component y^p plus a transitory shock u, namely:

$$y_{it} = y_{it}^p + u_{it} \qquad i \in k \tag{1}$$

for individual i in cohort k at time t, where y_{it}^p and u_{it} are uncorrelated. The u_t's have zero mean and are independent over time. We assume that:

- y_{it}^p evolves according to: $\Delta y_{it}^p = v_{it}$, where v_{it} is a permanent shock uncorrelated with y_{it-1}^p and orthogonal to u_{it}. The v_t's have zero mean and are independent over time[7];

- $\mathrm{Var}_{kt}(u)$ and $\mathrm{Var}_{kt}(v)$ for fixed t are common among all $i \in k$, for all k, but differ across t;

- $\mathrm{Cov}_{kt}(y_{-l}, u) = \mathrm{Cov}_{kt}(y_{-m}, v) = 0 \;\; \forall l, m \geq 1$.

The stochastic component of permanent income, v_{it}, includes all the economic shocks that can be considered permanent, such as an increase in the salary, or an increase in the cost of housing, and so on. It does not include returns on assets, since their riskiness has a more transitory than permanent nature. That would be part of the transitory error u_{it} in the income process.

All-together, we obtain:

$$\Delta y_{it} = \Delta u_{it} + v_{it}$$

i.e. the changes in income are affected by both permanent and transitory shocks.

[7]It would be possible to add a deterministic trend here.

Adding to the above income process a life-cycle model for consumption, we are able to separate the two components of the growth of poverty.

Assuming quadratic preferences, with the discount rate equal to the fixed real interest rate, we obtain the random walk property of consumption as in Hall (1978):

$$\Delta c_{it} = \eta_{it}$$

Here η_{it} is the innovation in the consumption process[8], and is related to the transitory and permanent innovations of income by

$$\eta_{it} = v_{it} + \beta_t u_{it}$$

where $\beta_t = [r/(1+r)](1-(1+r)^{-(T-t+1)})^{-1}$ is a factor annuitising the transitory shock. Notice that $\beta_t = 0$ for large $T-t$ and small r.

The change in consumption is then

$$\Delta c_{it} = v_{it} + \beta_t u_{it}$$

This is approximately equal to the permanent shock when β_t goes to zero, i.e. when we are far from the year of retirement ($T-t$ large), but includes an effect of the transitory shock when we are closer to T. This relation implies that the consumption decisions are mainly driven by the permanent shocks if we are sufficiently far from retirement, while transitory shocks gain a larger and larger role as we approach the last years of participation in the labour force.

All-together, for small r and large $T-t$ the following approximate relation holds:

$$\Delta c_{it} = v_{it}$$

and therefore our model can be summarised by the three equations:

$$\begin{aligned} y_{it} &= y_{it}^p + u_{it} \\ y_{it}^p &= y_{i\ t-1}^p + v_{it} \\ c_{it} &= c_{i\ t-1} + v_{it} \end{aligned}$$

[8] Defined as:

$$\eta_{it} = \frac{r}{(1+r)(1-(1+r)^{-(T-t+1)})} \sum_{k=0}^{T-t} (1+r)^{-k} (E_t - E_{t-1}) y_{t+k}$$

where T is the year of retirement.

5.2 Measurement issues

We present here some crucial issues that need to be considered when studying poverty.

First of all is the definition of poor. In our case, we consider poor any household whose income or consumption falls below a pre-defined line.

Here the second point arises, namely the choice of the poverty line. We use a set of poverty lines, defined each as a certain proportion of the mean of the distribution, for instance 50% or 25% of the mean, and so on.

Finally, once chosen the poverty line, one has to decide which index to use. We develop a model for two very commonly used indices, i.e. the headcount ratio index of poverty and the poverty gap ratio index. We recall here their definitions: the headcount ratio index of poverty is the proportion of households whose income (or consumption, accordingly) is below the poverty line, and is an index of incidence of poverty. Looking instead at the intensity, we have the poverty gap ratio index, which is the average gap (in %) between the poverty line and, for example, income for those below the line. Basically it measures, in percentage, how much lower than the poverty line is the average income among the poors.

It is clear that the poverty lines could be defined in many other ways, and the indices could be others as well. A thorough discussion of these and other issues appears in several studies in the literature, among which, for instance, Atkinson (1987), Foster *et al.* (1984), Sen (1997) and Trivellato (1998).

5.3 Classic multiplicative measurement errors

In this section we consider the simplest form of measurement error that could contaminate a non-negative variable (e. g. income), namely a multiplicative error, independent of the error free variable. The model is given by:

$$W = X\ U^{\sigma} \tag{2}$$

where W is the contaminated variable, X is the error free variable, and U is independent of X and such that $\log U$ has zero mean and unit variance.

This model yields additive measurement error in the log scale:

$$\log W = \log X + \sigma \log U \tag{3}$$

and σ^2 can therefore be interpreted as the variance of the errors in the log scale.

Chesher (1991) derives an approximation for the densities in the case of independent additive measurement error. Call $R = \log W$, $S = \log X$, and $V = \log U$, so that

$R = S + \sigma V$, where V is independent of S. Then:

$$f_R(r) \simeq f_S(r) + \frac{\sigma^2}{2}\, f''_S(r) \tag{4}$$

We now use this result to obtain an approximation for poverty indices.

For example, let us consider the headcount index. It is defined as the cumulative distribution function at the poverty line c: $H_X(c) = F_X(c)$. Notice that $F_X(c) = F_{\log X}(z)$, where $z = \log c$, so we can compute the index using the variables in the logarithmic scale. From (4) we obtain

$$F_R(z) \simeq F_S(z) + \frac{\sigma^2}{2}\, f'_S(z) \tag{5}$$

The approximation for the growth in the headcount index is derived in the next Section, applying this formula to the variables appearing in our economic model.

Similar approximations can be obtained for other indices. For instance, consider the shortfall index of poverty, or poverty gap ratio: $PG(W) = \frac{1}{c}\int_0^c (c - w)\; dF_W(w)$, where as usual c is the poverty line. In this case the following approximation holds:

$$PG(Y) \simeq PG(X) + \frac{\sigma^2}{2} c f_X(c) \tag{6}$$

where $Y = e^{-\sigma^2/2}\, W$, and the other quantities are the same as above (see Chesher and Schluter, 1999). The transformation of W into Y ensures that $E(Y) = E(X)$.

Alternatively, without the above transformation, we obtain:

$$PG(W) \simeq PG(X)\left(1 + \frac{\sigma^2}{2}\right) + \frac{\sigma^2}{2}\left\{c f_X(c) - F_X(c)\right\} \tag{7}$$

Things are much easier if we define the shortfall index for the variables in the logarithmic scale: $PG(R) = \frac{1}{z}\int_{-\infty}^{z} (z - r)\; dF_R(r)$, where as before $z = \log c$. In this case we obtain:

$$PG(R) \simeq PG(S) + \frac{\sigma^2}{2}\,\frac{1}{z}\, f_S(z) \tag{8}$$

under suitable regularity conditions[9]. Approximations for the growth of the poverty gap index in the original and in the logarithmic scale are derived in Rosati (2000); here we only comment on few issues.

The interpretation of $PG(R)$ as an index of poverty may seem not as straightforward as for the usual index obtained in the original scale, since it measures how much the average log-income or log-consumption is smaller than the poverty line (in

[9] $lim_{x \to -\infty}(z - x) f'_X(x) = 0$

percentage). However, to support this index one could argue the following: if the main interest is in the study of the index, then the choice of the logarithmic scale may not be ideal, but if we are mainly concerned about the estimation of the variances of the errors, this choice is useful as it provides much simpler estimating equations. Secondly, an even stronger justification comes from the close relation with the work of Clark, Hemming and Ulph (1981)[10]. They suggest basing a measure of poverty on the average shortfall in utility space, which is exactly our $PG(R)$ if we have logarithmic utility. They make some transformations to arrive at an index which is interpretable as an Atkinson inequality index for the distribution censored at the poverty line - since this is just a monotonic transformation it gives the same poverty ordering as $PG(R)$.

5.4 Application to the income and consumption model: the headcount ratio index

We can apply the approximation (5) to the income and consumption processes introduced earlier[11]. We assumed (omitting the individual's index for simplicity):

$$y_t = y_t^p + u_t \tag{9}$$

$$y_t^p = y_{t-1}^p + v_t \tag{10}$$

where y_t and y_t^p are the logarithms of income and permanent income, respectively. Moreover we assumed that the logarithm of consumption responds only to the permanent shock

$$c_t = c_{t-1} + v_t \tag{11}$$

The use of the approximation in the case of the income process (9) is a natural choice, since we are interpreting current income as an error-ridden measure of permanent income. The same argument does not apply to processes (10) and (11), but the use of the approximation is still legitimate, since v_t has the same stochastic properties as the measurement errors considered above. We obtain:

$$F_{y_t}(z) \simeq F_{y_t^p}(z) + \frac{\sigma_{u_t}^2}{2} f'_{y_t^p}(z) \tag{12}$$

$$F_{y_t^p}(z) \simeq F_{y_{t-1}^p}(z) + \frac{\sigma_{v_t}^2}{2} f'_{y_{t-1}^p}(z) \tag{13}$$

[10] I am indebted to Ian Preston for pointing out the connection with this paper.

[11] Here we shift from levels to logarithms. This is justified in this type of model if we consider Constant Relative Risk Aversion preferences (see Blundell and Preston, 1998), but we expect precautionary savings to introduce a trend into consumption, which we omit for simplicity here. We are working on trends in consumption and income, which will be included in a future version of this study. Since the values of these trends are estimable within cohorts, this in principle should be easy to incorporate.

$$F_{c_t}(z) \simeq F_{c_{t-1}}(z) + \frac{\sigma^2_{v_t}}{2} f'_{c_{t-1}}(z) \tag{14}$$

where z is the logarithm of the poverty line.

In a study where the target is to obtain indices of poverty corrected for measurement error, one would proceed estimating some way the variances of the errors, and then using them to obtain the corrected indices. In our case, however, we are mainly interested in the study of the evolution of poverty over time, so we intend to use the above approximations to obtain an expression for $\Delta F_{y_t}(z)$ and $\Delta F_{c_t}(z)$ as a function of the variances of the shocks. In fact, as the above equations show, the variances of the errors play a central role in the evolution of the indices, so our aim is to exploit those equations to obtain estimates of the variances.

Adding, to (12)-(14), the approximation for y_{t-1}

$$F_{y_{t-1}}(z) \simeq F_{y^p_{t-1}}(z) + \frac{\sigma^2_{u_t}}{2} f'_{y^p_{t-1}}(z) \tag{15}$$

and assuming $f'_{y^p_t}(z) \simeq f'_{y^p_{t-1}}(z) \simeq f'_{c_{t-1}}(z) \simeq f'_{c_{t-2}}(z)$, we have:

$$F_{y_t}(z) - F_{y_{t-1}}(z) \simeq \frac{\sigma^2_{v_t} + \Delta\sigma^2_{u_t}}{2} f'_{c_{t-1}}(z) \tag{16}$$

$$F_{c_t}(z) - F_{c_{t-1}}(z) \simeq \frac{\sigma^2_{v_t}}{2} f'_{c_{t-1}}(z) \tag{17}$$

for one-year differences, and

$$F_{y_t}(z) - F_{y_{t-2}}(z) \simeq \frac{\sigma^2_{v_t} + \sigma^2_{v_{t-1}} + \Delta\sigma^2_{u_t} + \Delta\sigma^2_{u_{t-1}}}{2} f'_{c_{t-2}}(z) \tag{18}$$

$$F_{c_t}(z) - F_{c_{t-2}}(z) \simeq \frac{\sigma^2_{v_t} + \sigma^2_{v_{t-1}}}{2} f'_{c_{t-2}}(z) \tag{19}$$

for two-year differences. We need these last equations as well, as our data are not equally spaced, sometimes observed every year, sometimes every other year, and therefore we have to look at both one- and two-year differences. Denote the time periods by $t = t_1, t_2, \ldots, t_T$, where $t_i - t_{i-1} = 1, 2$. Then we can use the notation $\Delta F_{y_{t_i}}(z)$ for both $F_{y_t}(z) - F_{y_{t-1}}(z)$ and $F_{y_t}(z) - F_{y_{t-2}}(z)$, the context indicating whether we are dealing with one- or two-year differences. The same applies to the measures based on consumption. For simplicity of notation, we will denote from now on $F_{y_{t_i}}$ by F_{yt_i}.

The changes in the headcount can therefore be related to the level of the variance of permanent shocks and the change in the variance of transitory shocks.

Notice that the above results hold for any value of the poverty line z, so a test on the model can be derived from estimation of $\Delta\,\mathrm{Var}_{kt}(v)$ and $\Delta\,\mathrm{Var}_{kt}(u)$ for different values of z. If the estimates change with z, there is an indication that the variances of the shocks (in the log scale) are not common among all individuals in a cohort. This was one of the main assumptions of the model presented in Section 5.1. To investigate this issue, in Section 5.6 we propose a test for heteroskedasticity.

In future research, it may be worth trying to extend this to other poverty measures. As we said earlier, one first extension, to the case of the poverty gap index, is presented in Rosati (2000).

5.5 Computation of the standard errors

We compute now the standard errors for the headcount ratios and for their changes. To do so, we consider two different poverty lines, since as we just anticipated we will compare in Section 5.6 the values obtained at different lines, in order to test for the presence of heteroskedasticity.

Given a fixed time period $t = t_1, \ldots, t_T$, consider two poverty lines c_1 and c_2, and let $z_1 = \log c_1$ and $z_2 = \log c_2$. The estimates of the headcount index are based on income and consumption measures, so that we obtain the following vector of estimates:

$$\hat{F}_t(z_1, z_2) = \begin{bmatrix} \hat{F}_{ct}(z_1) \\ \hat{F}_{yt}(z_1) \\ \hat{F}_{ct}(z_2) \\ \hat{F}_{yt}(z_2) \end{bmatrix}$$

All these are estimates of proportions, so their distribution will be asymptotically normal:

$$\sqrt{n_t}[\hat{F}_t(z_1, z_2) - F_t(z_1, z_2)] \to \mathcal{N}(0, \Pi_t)$$

where the variance-covariance matrix is:

$$\Pi_t = \begin{bmatrix} F_c(z_1)[1 - F_c(z_1)] & & & \\ F_{cy}(z_1, z_1) - F_c(z_1)F_y(z_1) & F_y(z_1)[1 - F_y(z_1)] & & \\ F_c(\min\{z_1, z_2\}) - F_c(z_1)F_c(z_2) & F_{cy}(z_2, z_1) - F_c(z_2)F_y(z_1) & F_c(z_2)[1 - F_c(z_2)] & \\ F_{cy}(z_1, z_2) - F_c(z_2)F_y(z_2) & F_y(\min\{z_1, z_2\}) - F_y(z_1)F_y(z_2) & F_{cy}(z_2, z_2) - F_c(z_2)F_y(z_2) & F_y(z_2)[1 - F_y(z_2)] \end{bmatrix}$$

and all the quantities above refer to year t.

For what concerns the growth in the headcount, we start from the following result:

$$\sqrt{n}\left[\begin{pmatrix} \hat{F}_{t_{i-1}}(z_1, z_2) \\ \hat{F}_{t_i}(z_1, z_2) \end{pmatrix} - \begin{pmatrix} F_{t_{i-1}}(z_1, z_2) \\ F_{t_i}(z_1, z_2) \end{pmatrix}\right] \to \mathcal{N}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \frac{n}{n_{t_{i-1}}}\Pi_{t_{i-1}} & 0 \\ 0 & \frac{n}{n_{t_i}}\Pi_{t_i} \end{pmatrix}\right]$$

since the observations over different years are independent. Here $n = n_{t_{i-1}} + n_{t_i}$. Denote from now on the above variance-covariance matrix by Π_{t_{i-1},t_i}. The growth in the headcount can be expressed as:

$$\Delta\hat{F}_{t_i}(z_1,z_2) = \begin{bmatrix} \Delta\hat{F}_{ct_i}(z_1) \\ \Delta\hat{F}_{yt_i}(z_1) \\ \Delta\hat{F}_{ct_i}(z_2) \\ \Delta\hat{F}_{yt_i}(z_2) \end{bmatrix} = B \begin{bmatrix} \hat{F}_{t_{i-1}}(z_1,z_2) \\ \hat{F}_{t_i}(z_1,z_2) \end{bmatrix}$$

where

$$B = \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix}$$

i.e. $B = [-I_4, I_4]$.

It follows that the asymptotic distribution for the growth in the headcount index is:

$$\sqrt{n}[\Delta\hat{F}_{t_i}(z_1,z_2) - \Delta F_{t_i}(z_1,z_2)] \to \mathcal{N}(0, B\Pi_{t_{i-1},t_i}B')$$

Notice that $B\Pi_{t_{i-1},t_i}B' = n\left(\Pi_{t_{i-1}}/n_{t_{i-1}} + \Pi_{t_i}/n_{t_i}\right)$. Therefore, the standard errors for $\Delta\hat{F}_{t_i}(z_1,z_2)$ are given by the diagonal elements of the matrix:

$$\left(\frac{\Pi_{t_{i-1}}}{n_{t_{i-1}}} + \frac{\Pi_{t_i}}{n_{t_i}}\right)^{\frac{1}{2}}$$

5.6 A simple test for heteroskedasticity

So far, we have assumed independent measurement error; this was because our economic model assumed constant variance of the economic shocks within cohorts. This assumption might be too strict, since it is quite likely that households in different parts of the distribution of income or consumption are affected by the shocks in different ways. Therefore we present here a simple test for heteroskedascticity, derived directly from the results on estimation of the previous section. The test is not very sensitive, but is easy to develop and compute, so it can be used for a first screening.

From the results of Section 5.4, we have:

$$R_{t_i}(z) = \left(\frac{\Delta F_{yt_i}(z)}{\Delta F_{ct_i}(z)}\right) = 1 + \frac{\Delta\sigma^2_{u_{t_i}}}{\sigma^2_{v_{t_i}}} \tag{20}$$

for annual data and

$$R_{t_i}(z) = 1 + \frac{\Delta\sigma^2_{u_{t_i}} + \Delta\sigma^2_{u_{t_{i-1}}}}{\sigma^2_{v_{t_i}} + \sigma^2_{v_{t_{i-1}}}} \tag{21}$$

for biannual data.

According to our hypotheses, $\sigma^2_{v_t}$ and $\sigma^2_{u_t}$ are constant (within cohorts) for fixed t, and therefore the above ratios are constant as well. It follows that, if we compute $R_{t_i}(z)$, for different values of z, it should be constant.

We can construct a simple test for the null hypothesis:

$$H_0: \quad R_{t_i}(z_1) = R_{t_i}(z_2) \tag{22}$$

Rejection of H_0 implies non-constancy of the variances; however, non-rejection can also occur in the presence of heteroskedasticity, in some particular cases. For instance, this would happen if $\sigma^2_{v_t}$ and $\sigma^2_{u_t}$ are not constant for fixed t, but are such that the ratios above (20) and (21) remain constant.

It is clear that this test is not ideal to test for heteroskedasticity, but as it is very easy to obtain, it can still help to have a general idea about the behaviour of the variances. A specific and precise test for heteroskedasticity is sketched in the following sections.

To test the above hypothesis (22), let us derive the distribution for the quantity $R_{t_i}(z_1, z_2) = R_{t_i}(z_1) - R_{t_i}(z_2)$. Define the function $f(a, b, c, d)' = b/a - d/c$. If we apply it to the vector $\Delta\hat{F}_{t_i}(z_1, z_2)$, we obtain the quantity we are looking for, namely:

$$R_{t_i}(z_1, z_2) = f(\Delta\hat{F}_{t_i}(z_1, z_2))$$

The Jacobian matrix for this transformation is

$$J = \left[-\frac{b}{a^2}, \frac{1}{a}, \frac{d}{c^2}, -\frac{1}{c} \right]$$

In our case we obtain:

$$J_{t_i}(z_1, z_2) = \left[\; -\frac{\Delta F_{yt_i}(z_1)}{(\Delta F_{ct_i}(z_1))^2}, \; \frac{1}{\Delta F_{ct_i}(z_1)}, \; \frac{\Delta F_{yt_i}(z_2)}{(\Delta F_{ct_i}(z_2))^2}, \; -\frac{1}{\Delta F_{ct_i}(z_2)} \; \right]$$

Therefore[12]

$$\sqrt{n} \; \left(\hat{R}_{t_i}(z_1, z_2) - R_{t_i}(z_1, z_2) \right) \to \mathcal{N}(0, \sigma^2_{Rt_i})$$

where $\sigma^2_{Rt_i} = J_{t_i} B \Pi_{t_{i-1}, t_i} B' J'_{t_i}$.

Under the null hypothesis (22), we have $R_{t_i}(z_1, z_2) = 0$, hence

$$\sqrt{n} \; \left(\hat{R}_{t_i}(z_1, z_2) \right) \to \mathcal{N}(0, \sigma^2_{Rt_i})$$

Therefore we can construct the test

$$\chi^2 = \frac{\left[\; \hat{R}_{t_i}(z_1, z_2) \; \right]^2}{\hat{\sigma}^2_{Rt_i}/n}$$

which has an asymptotic Chi-squared distribution with 1 d.f.

[12]This follows from the asymptotic result: if $\sqrt{n}(\hat{\theta} - \theta) \to \mathcal{N}(0, \Sigma)$ then $\sqrt{n}(g(\hat{\theta}) - g(\theta)) \to \mathcal{N}(0, G\Sigma G')$, where $G = [\partial g_i / \partial \theta_j]$.

6 Empirical evidence on poverty

In our analyses, we used both income and consumption to compute headcount indices. According to our model, we need to fix a certain value for the poverty line, and use it for both variables. Since we also want to investigate the effects of the choice of the poverty line, we choose four different values. This, in turn, will enable us to run the simple test for heteroskedasticity as well.

The lines we choose, in each year, correspond to mean consumption, and to 25%, 50% and 75% of mean consumption[13]; they are reported in Table 7. Tables 8-13 present the corresponding headcount indices from income and consumption. All results are computed within the usual three cohorts, and are also displayed in Figures 3-4.

The values of the headcount ratios are clearly different for the four poverty lines, but the profiles over time are quite similar, and this is also reflected in quite small differences in the changes over time. The changes are reported in Tables 14-19[14].

The main findings are in line with the empirical evidence on inequality found in Rosati (2000). In fact, there we found a flat trend for the variances of the permanent shocks and a slightly increasing one for the transitory shocks, and here we notice that the headcount ratios based on consumption are generally constant, while those based on income show an upward trend. According to the theory, the growth of the headcount ratio from consumption is driven by the variance of permanent shocks, and that of the index form income is affected by the variance of both transitory and permanent shocks, and therefore our findings are consistent with the estimates for the variances of the shocks obtained in the inequality context.

Tables 20-22 present the results of the test for heteroskedasticity[15]. The test provides no evidence against our hypothesis, but as we anticipated, it is not very sensitive. In fact, we are testing for constancy of the ratio (21), but this could be true even with heteroskedasticity, as a non-constant variance could be disguised by the fact that we are computing a ratio. Therefore we do not take this as a final answer regarding heteroskedasticity; this issue is investigated further in the next sections.

[13]Note that the lines change with time and with the birth cohort, therefore to a large part they would pick up the effects of trends in consumption.

[14]Notice that these changes are simply the differences between the quantities reported in the previous tables; for instance in the case of consumption at time t, $\Delta HR25 = F_{c_t}(z_t) - F_{c_{t-1}}(z_{t-1})$, where z_t and z_{t-1} are the poverty lines corresponding to 25% of mean consumption at time t and $t-1$, respectively. In fact, in this case, we are only interested in the changes over time, and therefore we do not need to compute those two indices on the same poverty line. This will not be the case when we perform the test for heteroskedasticity: there we need to compute the deltas between indices computed on one poverty line only. Therefore we choose to compute both indices with respect to the line at time $t-1$, so that the deltas that we use for our test are of the type: $\Delta HR25 = F_{c_t}(z_{t-1}) - F_{c_{t-1}}(z_{t-1})$.

[15]See previous note on what poverty lines we used for the test.

Table 7: *Poverty lines: 25%, 50% and 75% of mean consumption, and mean consumption, by birth cohort. Thousand Liras.*

	25% mean			**50% mean**			**75% mean**			**Mean cons.**		
Year	**Cohort**			**Cohort**			**Cohort**			**Cohort**		
	30	**40**	**50**	**30**	**40**	**50**	**30**	**40**	**50**	**30**	**40**	**50**
80	70	81	87	140	161	177	210	241	265	280	322	355
81	88	99	112	176	198	224	264	297	335	352	395	447
82	99	112	130	198	224	261	297	336	390	396	448	521
83	115	130	136	230	260	273	345	391	409	460	521	546
84	133	135	163	265	270	327	398	404	490	530	539	654
86	152	159	163	304	317	326	457	476	489	609	635	653
87	207	203	216	414	405	432	620	608	648	827	810	864
89	226	232	243	452	464	485	677	696	728	903	928	971
91	263	264	275	526	527	550	789	791	825	1051	1054	1099
93	293	290	286	585	579	573	878	869	859	1170	1159	1146
95	315	324	341	630	648	682	946	972	1024	1261	1296	1365

7 Income and consumption processes with dependent errors

In the economic model we adopted so far, the transitory and permanent shocks are assumed to be uncorrelated with permanent income or consumption, and to have constant variance within cohorts. However, as we noticed earlier, it is likely that the shocks have different effects along the distribution of permanent income or consumption. In particular, we expect the shocks to be more volatile in the lower part of the distributions. In fact, here households have - for instance - more difficult access to financial services, or do not have stable job positions, facts that might increase the effects of the fluctuations of the economy.

A first screening has been based on the test derived in Section 5.6. The results we obtained did not show evidence against our hypothesis of constant variance, but as we mentioned earlier the test is not very sensitive, and a non-rejection cannot guarantee that the variances are actually constant. This is why we decided to investigate this issue further.

An indication comes from the regression of the savings (squared) on consumption. In the model $y_t = y_t^p + u_t$, if we look at the simplest case where $y_t^p = c_t$, we obtain $y_t - c_t = u_t$. The regression of the squared savings, therefore, gives an estimate of the conditional variance of the transitory shocks, given consumption. However, even in the more general case where $\Delta y_t^p = \Delta c_t$ but $y_t^p = c_t$ does not necessary hold, such regression still gives information about a likely behaviour of the transitory shocks.

In fact, what we just assumed is that the changes in consumption decisions are only driven by permanent shocks, therefore the changes in the savings are only influenced by transitory shocks. A higher volatility of savings is thus an indication of higher volatility of u_t.

We consider the model

$$s_t^2 = h(c_t) + \varepsilon_t$$

where $s_t = y_t - c_t$, and estimate function $h(.)$ in order to obtain an additive model with constant variance errors. The procedure we use is the so-called additivity and variance stabilising procedure proposed by Tibshirani (1988). Figure 7 shows the results for 1986 in the three cohorts. For each cohort, the left panel shows the plot of s_t^2 against c_t, and the kernel estimate of the regression $E(s_t^2|c_t)$. The right panels show the estimated $h(.)$ functions. The empirical estimates of the regressions show evidence of what we expected: the variance of savings is larger in the bottom of the consumption distribution, and roughly constant after a certain threshold. The cut-off point could indicate the minimum level of consumption of those who have access to financial services, or those who are in stable employment.

8 Dependent measurement errors

After the results obtained in the last section, we develop here a model with non-constant variance.

First of all, we need to clarify what we mean by constant error variance. The multiplicative model we considered in Section 5.3 allows the variance of the errors to vary along the error-free distribution. However, once we transform the data into the log scale, the variance of the errors becomes constant and equal to σ^2. This is what we mean when we use the term constant error variance, since we are working in the logarithmic scale. The model we introduce here, instead, lets the variance of the errors vary with the values of the error-free variables not only in the original scale, but also in the logarithmic one.

The derivation of the results follows the same line as in the previous Sections, namely uses an approximation for the densities in the additive error case, and then applies it to the multiplicative model after taking logs.

In this context we cannot use (4) anymore, since this approximation is only valid for errors independent of the error-free variable. Given:

$$R = S + \sigma V \tag{23}$$

with V and S *not* independent, assume $E(V|S=s)=0$ and $\text{Var}(V|S=s)=g_V(s)$, with $g_V(.)$ positive and bounded. Then we obtain:

$$f_R(r) \simeq f_S(r) + \frac{\sigma^2}{2}\ \{f''_S(r)\ g_V(r) + 2f'_S(r)\ g'_V(r) + f_S(r)\ g''_V(r)\} \tag{24}$$

where the approximation is of order $o(\sigma^2)$. This clearly reduces to formula (4) of Section 5.3 when $g_V(s)=1\ \forall s$.

This approximation for the densities is used now to obtain an approximation for the headcount ratio, but can be applied also to other poverty indices.

Integrating (24), we obtain the following approximation for the cumulative distribution function (and therefore for the headcount ratio index):

$$F_R(z) \simeq F_S(z) + \frac{\sigma^2}{2}\ \{f'_S(z)\ g_V(z) + 2f_S(z)\ g'_V(z) + F_S(z)\ g''_V(z)\} \tag{25}$$

where boundedness of $g_V(.)$ ensures that:

$$\lim_{r\to-\infty} f'_S(r)\ g_V(r) = \lim_{r\to-\infty} f_S(r)\ g'_V(r) = \lim_{r\to-\infty} F_S(r)\ g''_V(r) = 0$$

This model allows identification of the conditional variance function of the error, which was not possible with the approach presented before. This enhanced flexibility of the model helps capture some heterogeneity that goes beyond the multiplicative error specification. In the next Section we present the application of these results to the income and consumption model.

8.1 Identification of the variance functions

We include now in the model the indications that come from the empirical evidence of the previous Sections. In particular, we maintain the uncorrelation of the errors, but allow for non constant variance.

Consider our usual three-equation model:

$$\begin{aligned} y_t &= y_t^p + u_t \\ y_t^p &= y_{t-1}^p + v_t \\ c_t &= c_{t-1} + v_t \end{aligned}$$

where $E(u_t|y_t^p)=0$, $E(v_t|y_{t-1}^p)=0$ and $E(v_t|c_{t-1})=0$.

Assume further that the variance function of the permanent shocks is the same in the two processes, namely $\text{Var}(v_t|y_{t-1}^p=z)=\text{Var}(v_t|c_{t-1}=z)$. Denote this common function by $\sigma_{v_t}(z)$. Define also $\sigma_{u_t}(z)=\text{Var}(u_t|y_t^p=z)$.

We obtain the following approximations for the headcount indices:

$$
\begin{aligned}
F_{y_t}(z) &\simeq F_{y_t^p}(z) + \frac{1}{2}\left\{f'_{y_t^p}(z)\ \sigma_{u_t}(z) + 2f_{y_t^p}(z)\ \sigma'_{u_t}(z) + F_{y_t^p}(z)\ \sigma''_{u_t}(z)\right\} \\
F_{y_{t-1}}(z) &\simeq F_{y_{t-1}^p}(z) + \frac{1}{2}\left\{f'_{y_{t-1}^p}(z)\ \sigma_{u_{t-1}}(z) + 2f_{y_{t-1}^p}(z)\ \sigma'_{u_{t-1}}(z) + F_{y_{t-1}^p}(z)\ \sigma''_{u_{t-1}}(z)\right\} \\
F_{y_t^p}(z) &\simeq F_{y_{t-1}^p}(z) + \frac{1}{2}\left\{f'_{y_{t-1}^p}(z)\ \sigma_{v_t}(z) + 2f_{y_{t-1}^p}(z)\ \sigma'_{v_t}(z) + F_{y_{t-1}^p}(z)\ \sigma''_{v_t}(z)\right\} \\
F_{c_t}(z) &\simeq F_{c_{t-1}}(z) + \frac{1}{2}\left\{f'_{c_{t-1}}(z)\ \sigma_{v_t}(z) + 2f_{c_{t-1}}(z)\ \sigma'_{v_t}(z) + F_{c_{t-1}}(z)\ \sigma''_{v_t}(z)\right\}
\end{aligned}
$$

The first two approximations are, respectively, of the order $o(\bar{\sigma}^2_{u_t})$ and $o(\bar{\sigma}^2_{u_{t-1}})$, while the last two are both of order $o(\bar{\sigma}^2_{v_t})$, where $\bar{\sigma}^2_{u_t}$ and $\bar{\sigma}^2_{v_t}$ are the average variances.

Assume now that the variance functions are locally linear around z, so that all the second derivatives in the expressions above are zero. Furthermore assume $f_{y_t^p}(z) = f_{y_{t-1}^p}(z) = f_{c_{t-1}}(z)$, and $f'_{y_t^p}(z) = f'_{y_{t-1}^p}(z) = f'_{c_{t-1}}(z)$. Under these assumptions, we can combine the four equations above and obtain:

$$
\Delta F_{y_t}(z) \simeq \frac{1}{2}\left\{f'_{c_{t-1}}(z)\ [\Delta\sigma_{u_t}(z) + \sigma_{v_t}(z)] + 2f_{c_{t-1}}(z)\ [\Delta\sigma'_{u_t}(z) + \sigma'_{v_t}(z)]\right\} \tag{26}
$$

$$
\Delta F_{c_t}(z) \simeq \frac{1}{2}\left\{f'_{c_{t-1}}(z)\ \sigma_{v_t}(z) + 2f_{c_{t-1}}(z)\ \sigma'_{v_t}(z)\right\} \tag{27}
$$

where the approximations are of the order $o(\max\{\bar{\sigma}^2_{u_t}, \bar{\sigma}^2_{u_{t-1}}, \bar{\sigma}^2_{v_t}\})$ and $o(\bar{\sigma}^2_{v_t})$, respectively.

8.2 Estimation of the conditional variances

Starting form equations (26) and (27), we use now the local linearity of the variance functions to obtain estimating equations. Define:

$$
\begin{aligned}
\sigma_{u_t}(z) &= a_{u_t} + b_{u_t} z \\
\sigma_{v_t}(z) &= a_{v_t} + b_{v_t} z
\end{aligned}
$$

Hence $\Delta\sigma_{u_t}(z) = \Delta a_{u_t} + \Delta b_{u_t} z$, where $\Delta a_{u_t} = a_{u_t} - a_{u_{t-1}}$, and similarly Δb_{u_t}. It follows that $\Delta\sigma'_{u_t}(z) = \Delta b_{u_t}$, and $\sigma'_{v_t}(z) = b_{v_t}$. According to this notation, we can rewrite (26) and (27) as follows:

$$
\Delta F_{y_t}(z) \simeq \frac{1}{2}\left\{f'_{c_{t-1}}(z)\ [(\Delta a_{u_t} + a_{v_t}) + (\Delta b_{u_t} + b_{v_t})z] + 2f_{c_{t-1}}(z)\ [\Delta b_{u_t} + b_{v_t}]\right\} \tag{28}
$$

$$
\Delta F_{c_t}(z) \simeq \frac{1}{2}\left\{f'_{c_{t-1}}(z)\ [a_{v_t} + b_{v_t} z] + 2f_{c_{t-1}}(z)\ b_{v_t}\right\} \tag{29}
$$

Evaluating (28) and (29) at two points z_1 and z_2 not far apart, we can obtain local estimates of the σ functions. A suitable number of (z_1, z_2) pairs will provide a global estimate of the functions. The estimates for a pair (z_1, z_2) come from the solution of the following two systems:

$$\frac{1}{2}\begin{bmatrix} f'_{c_{t-1}}(z_1) & z_1 f'_{c_{t-1}}(z_1) + 2f_{c_{t-1}}(z_1) \\ f'_{c_{t-1}}(z_2) & z_2 f'_{c_{t-1}}(z_2) + 2f_{c_{t-1}}(z_2) \end{bmatrix} \begin{bmatrix} \Delta a_{u_t} + a_{v_t} \\ \Delta b_{u_t} + b_{v_t} \end{bmatrix} = \begin{bmatrix} \Delta F_{y_t}(z_1) \\ \Delta F_{y_t}(z_2) \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} f'_{c_{t-1}}(z_1) & z_1 f'_{c_{t-1}}(z_1) + 2f_{c_{t-1}}(z_1) \\ f'_{c_{t-1}}(z_2) & z_2 f'_{c_{t-1}}(z_2) + 2f_{c_{t-1}}(z_2) \end{bmatrix} \begin{bmatrix} a_{v_t} \\ b_{v_t} \end{bmatrix} = \begin{bmatrix} \Delta F_{c_t}(z_1) \\ \Delta F_{c_t}(z_2) \end{bmatrix}$$

According to the exploratory analyses run on the saving rates, we expect to find a piecewise linear function composed of two parts, a steeper one for small values of z, and a flatter, roughly constant one for values of z greater than a certain threshold $\bar{z}$. The identification of $\bar{z}$, the changing point for the slopes, is of crucial interest for the economic interpretation. We have already tried some possible explanations, related to the access to financial services or to the employment status, but other might arise according to the value we identify.

8.3 Further developments

The empirical estimates of the variance functions, i.e. of the coefficients a_{u_t}, b_{u_t}, a_{v_t} and b_{v_t} have not been obtained yet. They are quite demanding in terms of computation, as they require non-parametric estimation of the density function of consumption and of its derivative at a certain poverty line, and will be implemented in future research.

Another interesting topic is the following. From the model we derived, we can see that it is possible to obtain a precise test for heteroskedasticity, which can actually test separately the heteroskedasticity of the transitory and the permanent shocks. In the first case, in fact, this corresponds to testing the hypothesis $H_0 : b_{u_t} = 0$, while in the second we would have $H_0 : b_{v_t} = 0$. These tests have not been developed in the present study, and will be the object of future research.

9 Conclusions and further developments

This paper offered a new, formalised approach to the study of poverty, and new empirical evidence on Italy to add to the past mainly descriptive studies.

Starting from an economic model of intertemporal optimization, we derived equations that related the growth of poverty to the variances of the transitory and permanent shocks of the economy. This allowed us to identify and estimate separately the permanent and transitory components of the two phenomena by using jointly income and consumption measures. We found evidence of an increase in transitory poverty, but overall constant levels for the permanent component, in line with the findings of other studies on the UK and the US. Separate estimates of the variances of the shocks obtained in earlier studies had shown consistent results, i.e. an increase for the transitory shocks and a flat trend for the permanent.

The model allowed also to derive a tests for one of the assumptions, but the results did not give a final answer and some further research would be appropriate. The assumption we tested was that of common variance of the shocks among all individuals in the same birth cohort. A first very simple test did not detect directly heteroskedasticity, but gave results that can be consistent with both constant and non-constant variance. We therefore performed a preliminary emipical analysis, which confirmed our suspicion, and extended the model to the case of non-constant variance. The further estimates involved in the extended model are quite difficult to compute, and have not been obtained yet. It would certainly be worth dedicating some more research to this computation, as such estimates would also provide a way to test directly for heteroskedasticity, and separately for the two shocks. Moreover, the identification of the shape of the variance functions would give new insight for the economic interpretation, identifying the differences in the impact of the economic shocks along the distributions of income and consumption.

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Table 8: *Headcount ratios*[1] *from consumption for four different poverty lines*[2]*. Cohort of born in the 1930s, SHIW data.*

Year	HR25	SE	HR50	SE	HR75	SE	HR100	SE
80	0.0064	(0.0037)	0.0687	(0.0117)	0.2983	(0.0212)	0.5880	(0.0228)
81	0.0033	(0.0023)	0.0912	(0.0117)	0.3284	(0.0191)	0.5788	(0.0201)
82	0.0000	(0.0000)	0.0879	(0.0113)	0.3115	(0.0185)	0.5927	(0.0196)
83	0.0015	(0.0015)	0.0724	(0.0102)	0.3220	(0.0183)	0.5840	(0.0193)
84	0.0000	(0.0000)	0.0713	(0.0105)	0.3466	(0.0194)	0.6119	(0.0198)
86	0.0009	(0.0009)	0.0986	(0.0091)	0.3380	(0.0145)	0.5775	(0.0151)
87	0.0020	(0.0014)	0.1301	(0.0106)	0.3525	(0.0151)	0.5770	(0.0156)
89	0.0000	(0.0000)	0.0702	(0.0087)	0.3222	(0.0159)	0.5926	(0.0167)
91	0.0015	(0.0015)	0.1041	(0.0117)	0.3504	(0.0183)	0.5806	(0.0189)
93	0.0000	(0.0000)	0.1242	(0.0157)	0.3837	(0.0231)	0.6005	(0.0233)
95	0.0033	(0.0033)	0.1104	(0.0181)	0.3779	(0.0280)	0.6020	(0.0283)

[1] HR25, HR50 and HR75 are the headcount ratio indices computed for a poverty line equal to 25%, 50% and 75% of mean consumption, respectively. HR100 refers to a poverty line equal to mean consumption.

[2] See Table 7.

Table 9: *Headcount ratios*[1] *from income for four different poverty lines*[2]*. Cohort of born in the 1930s, SHIW data.*

Year	HR25	SE	HR50	SE	HR75	SE	HR100	SE
80	0.0064	(0.0037)	0.0644	(0.0114)	0.2425	(0.0199)	0.4635	(0.0231)
81	0.0083	(0.0037)	0.0945	(0.0119)	0.3367	(0.0192)	0.5605	(0.0202)
82	0.0016	(0.0016)	0.0639	(0.0098)	0.2380	(0.0170)	0.4856	(0.0200)
83	0.0046	(0.0027)	0.0570	(0.0091)	0.2219	(0.0163)	0.4592	(0.0196)
84	0.0149	(0.0049)	0.0862	(0.0114)	0.2985	(0.0186)	0.4992	(0.0204)
86	0.0085	(0.0028)	0.0854	(0.0086)	0.2732	(0.0137)	0.4798	(0.0153)
87	0.0040	(0.0020)	0.1291	(0.0106)	0.3476	(0.0150)	0.5412	(0.0157)
89	0.0035	(0.0020)	0.0990	(0.0101)	0.3211	(0.0158)	0.5420	(0.0169)
91	0.0088	(0.0036)	0.1100	(0.0120)	0.3343	(0.0181)	0.5587	(0.0190)
93	0.0158	(0.0059)	0.1535	(0.0171)	0.3679	(0.0229)	0.5576	(0.0236)
95	0.0234	(0.0087)	0.1773	(0.0221)	0.4281	(0.0286)	0.6020	(0.0283)

[1] See note 1 in Table 8.

[2] See Table 7.

Table 10: *Headcount ratios*[1] *from consumption for four different poverty lines*[2]. *Cohort of born in the 1940s, SHIW data.*

Year	HR25	SE	HR50	SE	HR75	SE	HR100	SE
80	0.0024	(0.0024)	0.0898	(0.0141)	0.3422	(0.0234)	0.5777	(0.0243)
81	0.0033	(0.0023)	0.0956	(0.0119)	0.3427	(0.0193)	0.5964	(0.0199)
82	0.0000	(0.0000)	0.0989	(0.0126)	0.3445	(0.0200)	0.6025	(0.0206)
83	0.0017	(0.0017)	0.0799	(0.0111)	0.3394	(0.0193)	0.6007	(0.0200)
84	0.0000	(0.0000)	0.0694	(0.0098)	0.3117	(0.0178)	0.5968	(0.0189)
86	0.0024	(0.0014)	0.0782	(0.0076)	0.3543	(0.0135)	0.5802	(0.0139)
87	0.0016	(0.0011)	0.1207	(0.0093)	0.3930	(0.0139)	0.5989	(0.0140)
89	0.0000	(0.0000)	0.0822	(0.0077)	0.3466	(0.0133)	0.6041	(0.0137)
91	0.0024	(0.0014)	0.1027	(0.0086)	0.3604	(0.0136)	0.5963	(0.0139)
93	0.0038	(0.0019)	0.0964	(0.0091)	0.3393	(0.0146)	0.6040	(0.0150)
95	0.0041	(0.0021)	0.0856	(0.0090)	0.3381	(0.0152)	0.6031	(0.0157)

[1] See note 1 in Table 8.

[2] See Table 7.

Table 11: *Headcount ratios*[1] *from income for four different poverty lines*[2]. *Cohort of born in the 1940s, SHIW data.*

Year	HR25	SE	HR50	SE	HR75	SE	HR100	SE
80	0.0000	(0.0000)	0.0607	(0.0118)	0.2524	(0.0214)	0.4830	(0.0246)
81	0.0115	(0.0043)	0.0923	(0.0117)	0.3130	(0.0188)	0.5651	(0.0201)
82	0.0018	(0.0018)	0.0777	(0.0113)	0.2827	(0.0189)	0.4947	(0.0210)
83	0.0067	(0.0033)	0.0616	(0.0098)	0.2812	(0.0183)	0.4775	(0.0204)
84	0.0030	(0.0021)	0.0650	(0.0095)	0.2747	(0.0172)	0.4786	(0.0192)
86	0.0120	(0.0031)	0.0726	(0.0073)	0.2650	(0.0125)	0.4709	(0.0141)
87	0.0041	(0.0018)	0.1394	(0.0099)	0.3938	(0.0139)	0.5916	(0.0140)
89	0.0023	(0.0014)	0.1095	(0.0087)	0.3607	(0.0134)	0.5665	(0.0139)
91	0.0048	(0.0020)	0.1083	(0.0088)	0.3628	(0.0136)	0.5738	(0.0140)
93	0.0198	(0.0043)	0.1238	(0.0101)	0.3535	(0.0147)	0.5577	(0.0153)
95	0.0247	(0.0050)	0.1588	(0.0117)	0.3763	(0.0156)	0.5928	(0.0158)

[1] See note 1 in Table 8.

[2] See Table 7.

Table 12: *Headcount ratios*[1] *from consumption for four different poverty lines*[2]. *Cohort of born in the 1950s, SHIW data.*

Year	HR25	SE	HR50	SE	HR75	SE	HR100	SE
80	0.0000	(0.0000)	0.0727	(0.0202)	0.4242	(0.0385)	0.6303	(0.0376)
81	0.0066	(0.0047)	0.1056	(0.0177)	0.3564	(0.0275)	0.5941	(0.0282)
82	0.0076	(0.0054)	0.0802	(0.0168)	0.3435	(0.0293)	0.6298	(0.0298)
83	0.0050	(0.0050)	0.1106	(0.0222)	0.3769	(0.0344)	0.6080	(0.0346)
84	0.0000	(0.0000)	0.1086	(0.0156)	0.3838	(0.0244)	0.6263	(0.0243)
86	0.0011	(0.0011)	0.0779	(0.0088)	0.3290	(0.0155)	0.5952	(0.0161)
87	0.0011	(0.0011)	0.1080	(0.0102)	0.3860	(0.0161)	0.6020	(0.0162)
89	0.0000	(0.0000)	0.1022	(0.0093)	0.3646	(0.0147)	0.6007	(0.0150)
91	0.0038	(0.0019)	0.1337	(0.0105)	0.3935	(0.0151)	0.5912	(0.0152)
93	0.0048	(0.0021)	0.1011	(0.0094)	0.3677	(0.0150)	0.6160	(0.0151)
95	0.0030	(0.0017)	0.1173	(0.0102)	0.3964	(0.0156)	0.6016	(0.0156)

[1] See note 1 in Table 8.

[2] See Table 7.

Table 13: *Headcount ratios*[1] *from income for four different poverty lines*[2]. *Cohort of born in the 1950s, SHIW data.*

Year	HR25	SE	HR50	SE	HR75	SE	HR100	SE
80	0.0000	(0.0000)	0.0727	(0.0202)	0.2727	(0.0347)	0.4061	(0.0382)
81	0.0066	(0.0047)	0.1122	(0.0181)	0.3597	(0.0276)	0.5809	(0.0283)
82	0.0000	(0.0000)	0.0687	(0.0156)	0.2824	(0.0278)	0.5420	(0.0308)
83	0.0000	(0.0000)	0.0905	(0.0203)	0.3266	(0.0332)	0.5276	(0.0354)
84	0.0152	(0.0061)	0.0960	(0.0148)	0.3182	(0.0234)	0.5076	(0.0251)
86	0.0108	(0.0034)	0.0812	(0.0090)	0.2608	(0.0144)	0.5076	(0.0164)
87	0.0065	(0.0027)	0.1123	(0.0104)	0.3501	(0.0158)	0.5972	(0.0087)
89	0.0047	(0.0021)	0.1153	(0.0098)	0.3543	(0.0146)	0.5953	(0.0064)
91	0.0086	(0.0029)	0.1299	(0.0104)	0.4031	(0.0152)	0.5934	(0.0041)
93	0.0202	(0.0044)	0.1319	(0.0105)	0.3667	(0.0150)	0.5914	(0.0017)
95	0.0293	(0.0054)	0.1719	(0.0120)	0.4277	(0.0157)	0.5895	(-0.0006)

[1] See note 1 in Table 8.

[2] See Table 7.

Table 14: *Changes in the headcount ratios*[1] *from consumption for four different poverty lines*[2]*. Cohort of born in the 1930s, SHIW data.*

Year	ΔHR25	SE	ΔHR50	SE	ΔHR75	SE	ΔHR100	SE
81	-0.0031	(0.0060)	0.0225	(0.0234)	0.0301	(0.0403)	-0.0092	(0.0429)
82	-0.0033	(0.0023)	-0.0034	(0.0230)	-0.0169	(0.0376)	0.0139	(0.0397)
83	0.0015	(0.0015)	-0.0154	(0.0215)	0.0105	(0.0369)	-0.0087	(0.0390)
84	-0.0015	(0.0015)	-0.0011	(0.0207)	0.0246	(0.0377)	0.0280	(0.0392)
86	0.0009	(0.0009)	0.0273	(0.0196)	-0.0086	(0.0339)	-0.0345	(0.0350)
87	0.0010	(0.0023)	0.0315	(0.0197)	0.0145	(0.0296)	-0.0005	(0.0307)
89	-0.0020	(0.0014)	-0.0599	(0.0193)	-0.0303	(0.0309)	0.0157	(0.0322)
91	0.0015	(0.0015)	0.0339	(0.0204)	0.0282	(0.0341)	-0.0120	(0.0356)
93	-0.0015	(0.0015)	0.0200	(0.0274)	0.0333	(0.0414)	0.0198	(0.0422)
95	0.0033	(0.0033)	-0.0138	(0.0338)	-0.0058	(0.0511)	0.0016	(0.0516)

[1] The changes refer to the headcount indices as defined in Table 8.

[2] See Table 7.

Table 15: *Changes in the headcount ratios*[1] *from income for four different poverty lines*[2]*. Cohort of born in the 1930s, SHIW data.*

Year	ΔHR25	SE	ΔHR50	SE	ΔHR75	SE	ΔHR100	SE
81	0.0019	(0.0074)	0.0301	(0.0233)	0.0942	(0.0391)	0.0970	(0.0433)
82	-0.0067	(0.0053)	-0.0306	(0.0217)	-0.0986	(0.0363)	-0.0749	(0.0402)
83	0.0030	(0.0043)	-0.0069	(0.0189)	-0.0161	(0.0333)	-0.0265	(0.0395)
84	0.0103	(0.0076)	0.0292	(0.0205)	0.0766	(0.0349)	0.0400	(0.0399)
86	-0.0065	(0.0077)	-0.0008	(0.0200)	-0.0253	(0.0323)	-0.0194	(0.0357)
87	-0.0045	(0.0048)	0.0437	(0.0191)	0.0743	(0.0287)	0.0614	(0.0310)
89	-0.0005	(0.0040)	-0.0301	(0.0207)	-0.0265	(0.0308)	0.0008	(0.0326)
91	0.0053	(0.0056)	0.0110	(0.0221)	0.0133	(0.0339)	0.0166	(0.0359)
93	0.0070	(0.0095)	0.0435	(0.0291)	0.0336	(0.0410)	-0.0011	(0.0426)
95	0.0076	(0.0147)	0.0238	(0.0392)	0.0601	(0.0515)	0.0444	(0.0519)

[1] The changes refer to the headcount indices as defined in Table 8.

[2] See Table 7.

Table 16: *Changes in the headcount ratios*[1] *from consumption for four different poverty lines*[2]*. Cohort of born in the 1940s, SHIW data.*

Year	ΔHR25	SE	ΔHR50	SE	ΔHR75	SE	ΔHR100	SE
81	0.0009	(0.0048)	0.0057	(0.0260)	0.0004	(0.0426)	0.0187	(0.0442)
82	-0.0033	(0.0023)	0.0034	(0.0245)	0.0019	(0.0392)	0.0061	(0.0405)
83	0.0017	(0.0017)	-0.0191	(0.0236)	-0.0051	(0.0393)	-0.0018	(0.0405)
84	-0.0017	(0.0017)	-0.0104	(0.0208)	-0.0278	(0.0371)	-0.0039	(0.0388)
86	0.0024	(0.0014)	0.0088	(0.0174)	0.0427	(0.0313)	-0.0165	(0.0328)
87	-0.0008	(0.0025)	0.0425	(0.0169)	0.0387	(0.0274)	0.0187	(0.0279)
89	-0.0016	(0.0011)	-0.0386	(0.0170)	-0.0464	(0.0272)	0.0052	(0.0276)
91	0.0024	(0.0014)	0.0206	(0.0163)	0.0137	(0.0269)	-0.0078	(0.0276)
93	0.0014	(0.0033)	-0.0063	(0.0177)	-0.0210	(0.0282)	0.0077	(0.0289)
95	0.0003	(0.0039)	-0.0108	(0.0181)	-0.0012	(0.0297)	-0.0009	(0.0307)

[1] The changes refer to the headcount indices as defined in Table 8.

[2] See Table 7.

Table 17: *Changes in the headcount ratios*[1] *from income for four different poverty lines*[2]*. Cohort of born in the 1940s, SHIW data.*

Year	ΔHR25	SE	ΔHR50	SE	ΔHR75	SE	ΔHR100	SE
81	0.0115	(0.0043)	0.0316	(0.0235)	0.0606	(0.0402)	0.0821	(0.0447)
82	-0.0098	(0.0061)	-0.0145	(0.0230)	-0.0303	(0.0377)	-0.0704	(0.0411)
83	0.0049	(0.0051)	-0.0162	(0.0211)	-0.0015	(0.0373)	-0.0172	(0.0414)
84	-0.0037	(0.0054)	0.0034	(0.0193)	-0.0065	(0.0355)	0.0010	(0.0396)
86	0.0090	(0.0052)	0.0076	(0.0168)	-0.0098	(0.0296)	-0.0077	(0.0333)
87	-0.0079	(0.0049)	0.0668	(0.0172)	0.1289	(0.0264)	0.1207	(0.0281)
89	-0.0017	(0.0032)	-0.0298	(0.0186)	-0.0331	(0.0273)	-0.0251	(0.0279)
91	0.0025	(0.0033)	-0.0012	(0.0175)	0.0020	(0.0271)	0.0073	(0.0279)
93	0.0150	(0.0062)	0.0155	(0.0189)	-0.0093	(0.0283)	-0.0162	(0.0293)
95	0.0049	(0.0093)	0.0349	(0.0219)	0.0228	(0.0303)	0.0351	(0.0310)

[1] The changes refer to the headcount indices as defined in Table 8.

[2] See Table 7.

Table 18: *Changes in the headcount ratios*[1] *from consumption for four different poverty lines*[2]*. Cohort of born in the 1950s, SHIW data.*

Year	ΔHR25	SE	ΔHR50	SE	ΔHR75	SE	ΔHR100	SE
81	0.0066	(0.0047)	0.0329	(0.0379)	-0.0678	(0.0660)	-0.0362	(0.0658)
82	0.0010	(0.0100)	-0.0255	(0.0344)	-0.0129	(0.0569)	0.0357	(0.0580)
83	-0.0026	(0.0104)	0.0304	(0.0390)	0.0334	(0.0637)	-0.0217	(0.0644)
84	-0.0050	(0.0050)	-0.0020	(0.0379)	0.0070	(0.0588)	0.0182	(0.0589)
86	0.0011	(0.0011)	-0.0307	(0.0245)	-0.0548	(0.0399)	-0.0310	(0.0405)
87	0.0000	(0.0022)	0.0300	(0.0191)	0.0570	(0.0315)	0.0067	(0.0323)
89	-0.0011	(0.0011)	-0.0058	(0.0195)	-0.0215	(0.0308)	-0.0012	(0.0312)
91	0.0038	(0.0019)	0.0316	(0.0198)	0.0289	(0.0298)	-0.0095	(0.0302)
93	0.0010	(0.0041)	-0.0327	(0.0199)	-0.0258	(0.0301)	0.0248	(0.0303)
95	-0.0018	(0.0039)	0.0162	(0.0196)	0.0287	(0.0305)	-0.0144	(0.0307)

[1] The changes refer to the headcount indices as defined in Table 8.

[2] See Table 7.

Table 19: *Changes in the headcount ratios*[1] *from income for four different poverty lines*[2]*. Cohort of born in the 1950s, SHIW data.*

Year	ΔHR25	SE	ΔHR50	SE	ΔHR75	SE	ΔHR100	SE
81	0.0066	(0.0047)	0.0395	(0.0383)	0.0870	(0.0622)	0.1748	(0.0666)
82	-0.0066	(0.0047)	-0.0435	(0.0338)	-0.0773	(0.0554)	-0.0389	(0.0591)
83	0.0000	(0.0000)	0.0218	(0.0360)	0.0442	(0.0611)	-0.0143	(0.0662)
84	0.0152	(0.0061)	0.0055	(0.0351)	-0.0085	(0.0567)	-0.0201	(0.0605)
86	-0.0043	(0.0095)	-0.0148	(0.0238)	-0.0574	(0.0379)	0.0000	(0.0416)
87	-0.0043	(0.0061)	0.0312	(0.0194)	0.0892	(0.0302)	0.0377	(0.0329)
89	-0.0019	(0.0048)	0.0030	(0.0202)	0.0042	(0.0304)	0.0143	(0.0316)
91	0.0039	(0.0049)	0.0146	(0.0202)	0.0488	(0.0298)	0.0126	(0.0305)
93	0.0116	(0.0072)	0.0020	(0.0209)	-0.0364	(0.0301)	-0.0360	(0.0308)
95	0.0091	(0.0097)	0.0400	(0.0225)	0.0610	(0.0307)	0.0847	(0.0309)

[1] The changes refer to the headcount indices as defined in Table 8.

[2] See Table 7.

Table 20: *Test for heteroskedasticity*[1]. *Cohort of born in the 1930s, SHIW data.*

Year	R25	R50	R75	T2550	p2550	T2575	p2575	T5075	p5075
81	0.2276	0.6867	0.3649	0.3438	0.5576	0.0305	0.8615	0.7643	0.3820
82	2.0245	1.6370	1.5761	0.0548	0.8150	0.0762	0.7826	0.0071	0.9327
83	-5.6612	0.6793	1.0705	0.0000	0.9945	0.0001	0.9942	1.3771	0.2406
84	-1.3137	0.1097	0.3476	0.1421	0.7062	0.1844	0.6676	0.3948	0.5298
86	-1116.9500	2.2331	1.1473	0.0001	0.9915	0.0001	0.9915	0.4073	0.5233
87	9.0969	0.8677	0.5477	0.2361	0.6271	0.2544	0.6140	2.7672	0.0962
89	1.4278	0.7521	0.7995	0.1513	0.6973	0.1287	0.7198	0.0570	0.8113
91	2.6276	1.7260	1.1836	0.0731	0.7869	0.2205	0.6387	0.2040	0.6515
93	-3.2592	-0.7987	0.7053	0.1618	0.6875	0.3966	0.5289	0.9254	0.3361
95	-0.7224	0.3849	-0.0588	0.0999	0.7520	0.0378	0.8459	0.3816	0.5367

[1] R25, R50 and R75 are the ratios (20) or (21) computed in correspondence of a poverty line equal to 25%, 50% or 75% of mean consumption. With obvious notation, the test T2550 tests the hypothesis that R25 = R50, and similarly the tests T2575 and T5075.

Table 21: *Test for heteroskedasticity*[1]. *Cohort of born in the 1940s, SHIW data.*

Year	R25	R50	R75	T2550	p2550	T2575	p2575	T5075	p5075
81	-2.7262	0.2993	0.5038	0.5292	0.4669	0.5983	0.4392	0.4331	0.5105
82	2.9728	1.0967	1.0206	0.5304	0.4664	0.5811	0.4459	0.0212	0.8843
83	322.4896	0.5251	0.9730	0.0000	0.9952	0.0000	0.9952	2.9624	0.0852
84	2.2380	0.2219	0.6211	0.6565	0.4178	0.4162	0.5188	0.2936	0.5879
86	4.2452	0.5678	0.9821	0.2336	0.6288	0.1852	0.6670	1.0030	0.3166
87	4.3412	0.4700	0.2404	1.3567	0.2441	1.5264	0.2167	1.0361	0.3087
89	1.5440	1.0370	0.8004	0.0808	0.7763	0.1739	0.6767	2.1475	0.1428
91	-0.9135	1.6255	1.1314	0.4996	0.4797	0.3251	0.5685	0.7435	0.3885
93	27.8595	0.3443	0.7613	0.0350	0.8516	0.0339	0.8539	1.4730	0.2249
95	0.1528	0.2270	0.4730	0.0004	0.9840	0.0072	0.9323	0.7092	0.3997

[1] See note in Table 20.

Table 22: *Test for heteroskedasticity*[1]. *Cohort of born in the 1950s, SHIW data.*

Year	R25	R50	R75	T2550	p2550	T2575	p2575	T5075	p5075
81	0.9970	0.6677	0.3305	0.0442	0.8335	0.2213	0.6380	0.2598	0.6102
82	-6.3290	1.4294	1.4133	0.0301	0.8623	0.0299	0.8627	0.0007	0.9784
83	0.0000	1.2655	-2.7921	0.0328	0.8562	0.0160	0.8994	0.0333	0.8551
84	-2.0141	0.4970	0.9434	1.0973	0.2949	1.5122	0.2188	1.0281	0.3106
86	-3.9634	0.4824	1.0671	0.2972	0.5856	0.3906	0.5320	0.7125	0.3986
87	9.0762	1.0655	0.5835	0.3846	0.5351	0.4318	0.5111	2.8547	0.0911
89	1.7187	0.6407	0.4546	0.0912	0.7627	0.1248	0.7239	0.7030	0.4018
91	8.9514	2.3623	0.5241	0.0000	0.9967	0.0000	0.9958	1.0239	0.3116
93	221.4405	0.2938	0.9533	0.0002	0.9890	0.0002	0.9891	3.1421	0.0763
95	1.2611	0.9091	0.5823	0.0657	0.7977	0.2359	0.6272	0.8064	0.3692

[1] See note in Table 20.

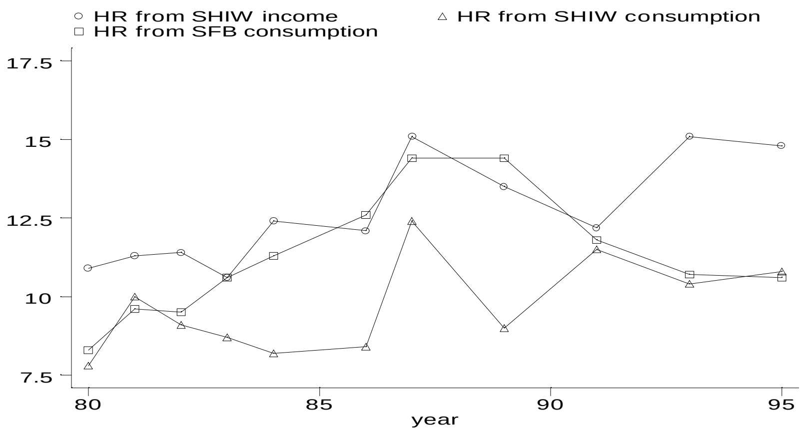

Figure 1: *Headcount ratios based on income and consumption, SHIW and SFB data.*

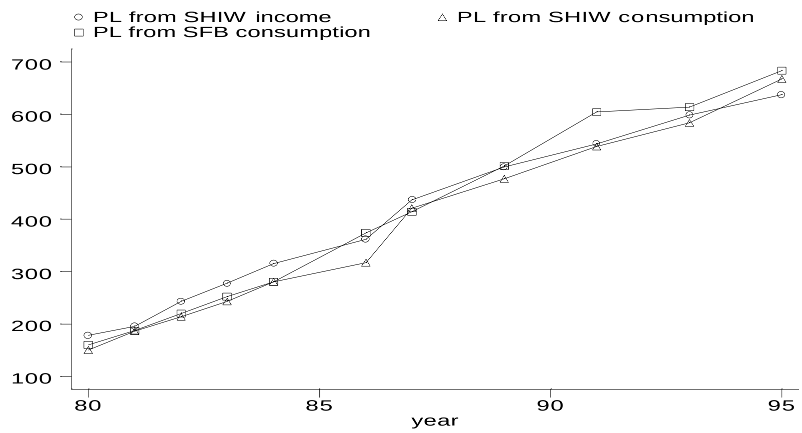

Figure 2: *Poverty lines based on income and consumption, SHIW and SFB data.*

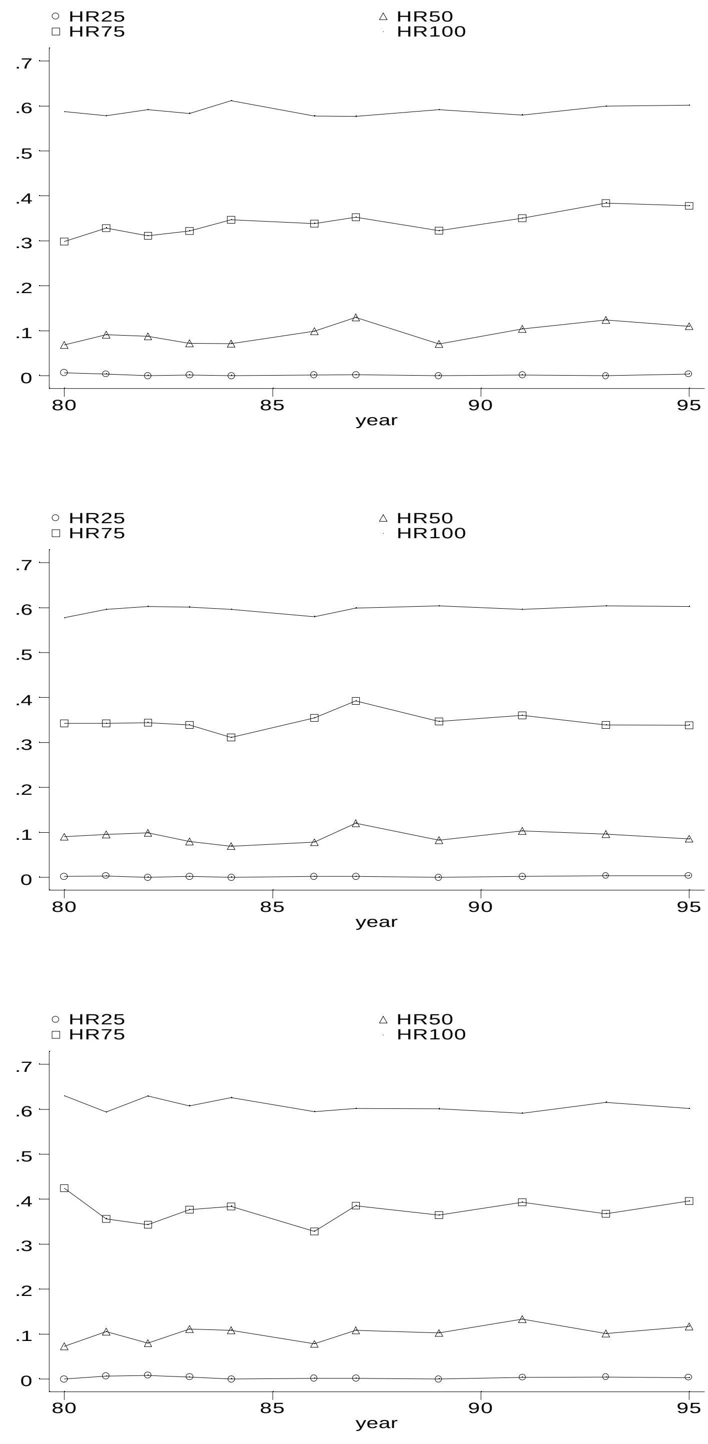

Figure 3: *Headcount ratios from consumption for four different poverty lines. HR25, HR50 and HR75 refer to poverty lines equal to 25%, 50% and 75% of mean consumption, respectively, and HR100 to mean consumption. The values of the poverty lines are displayed in Table 7. SHIW data, by birth cohort. Top: born in the 1930s, centre: born in the 1940s, bottom: born in the 1950s.*

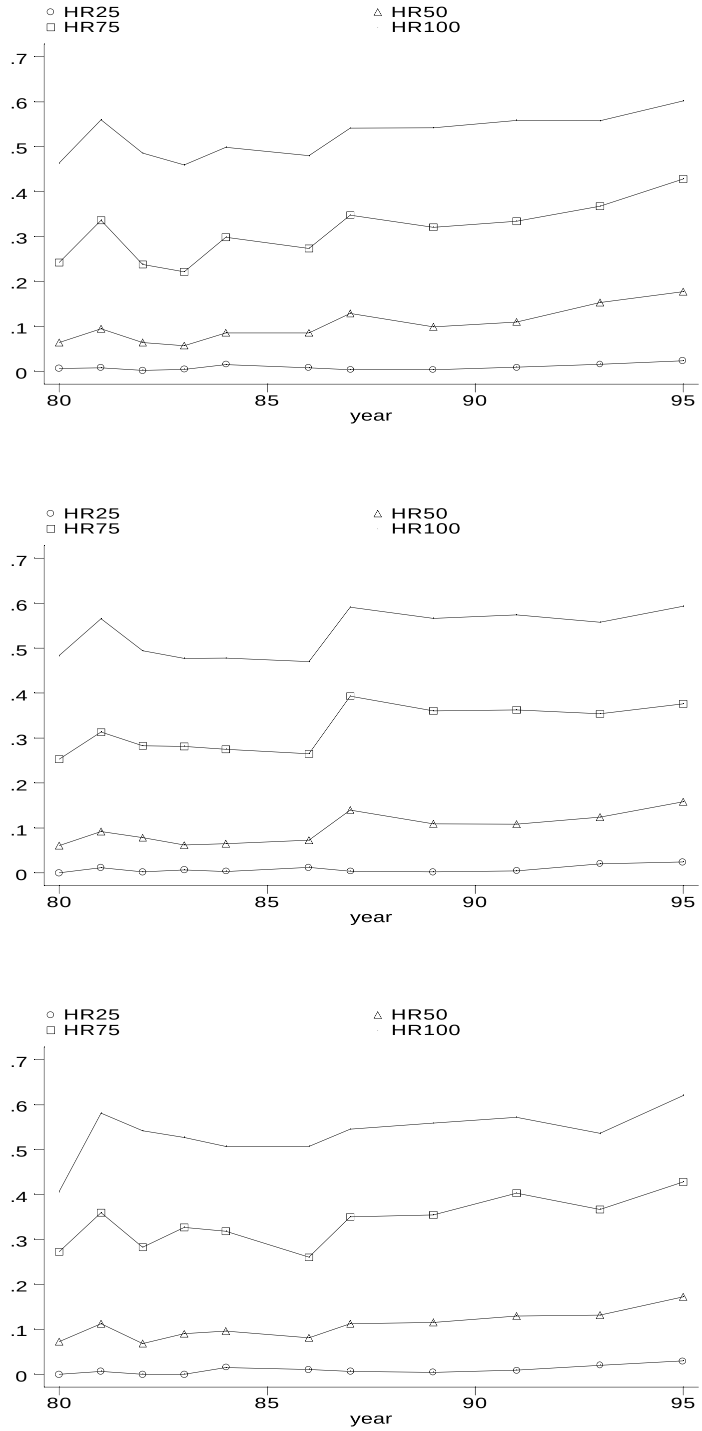

Figure 4: *Headcount ratios from income for four different poverty lines (as defined in Figure 3). SHIW data, by birth cohort. Top: born in the 1930s, centre: born in the 1940s, bottom: born in the 1950s.*

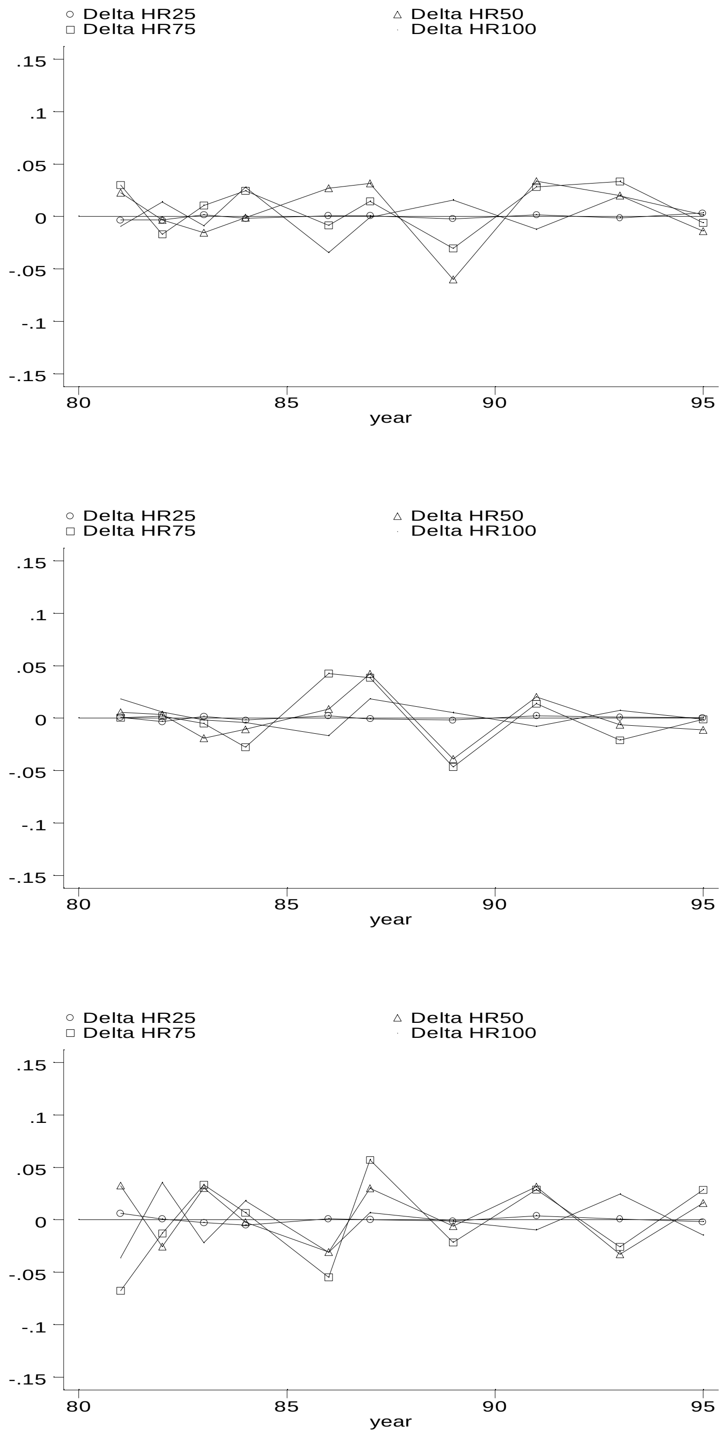

Figure 5: *Changes in the headcount ratios from consumption for four different poverty lines (as defined in Figure 3). SHIW data, by birth cohort. Top: born in the 1930s, centre: born in the 1940s, bottom: born in the 1950s.*

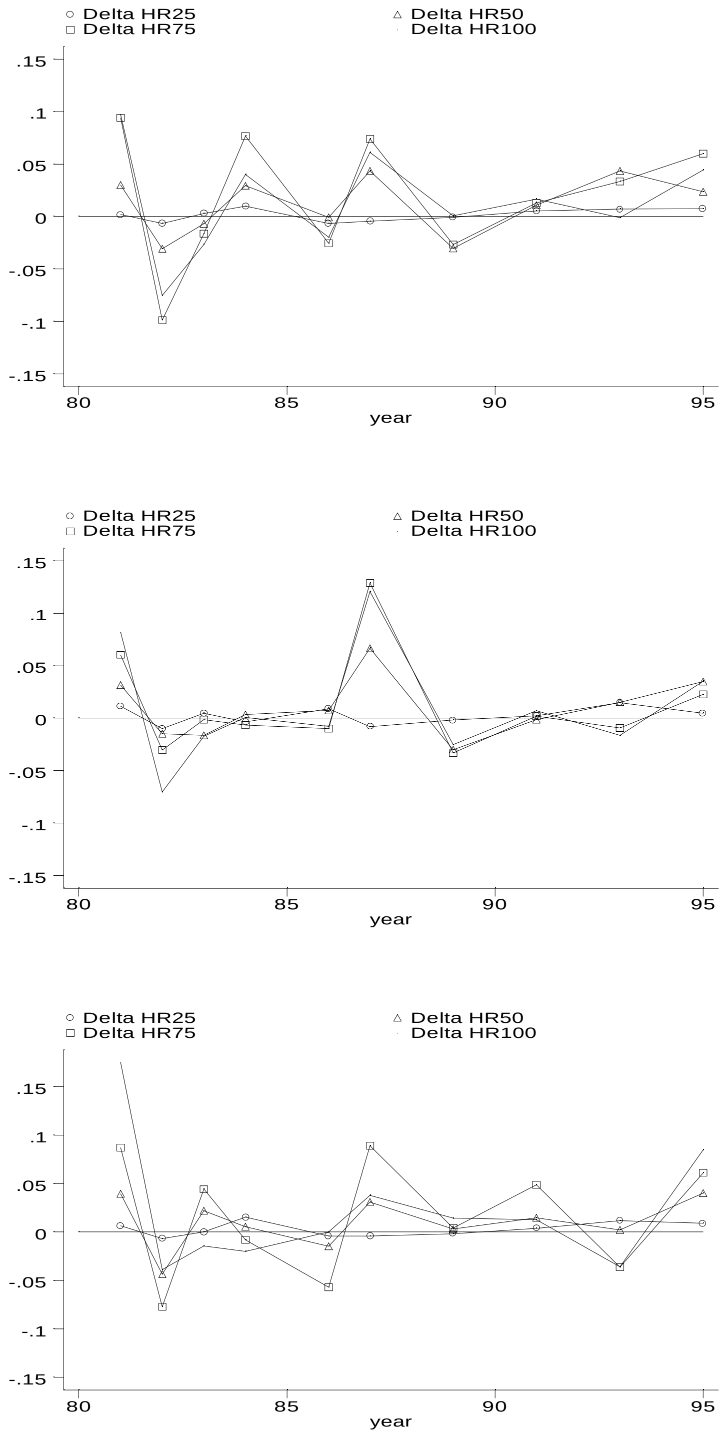

Figure 6: *Changes in the headcount ratios from income for four different poverty lines (as defined in Figure 3). SHIW data, by birth cohort. Top: born in the 1930s, centre: born in the 1940s, bottom: born in the 1950s.*

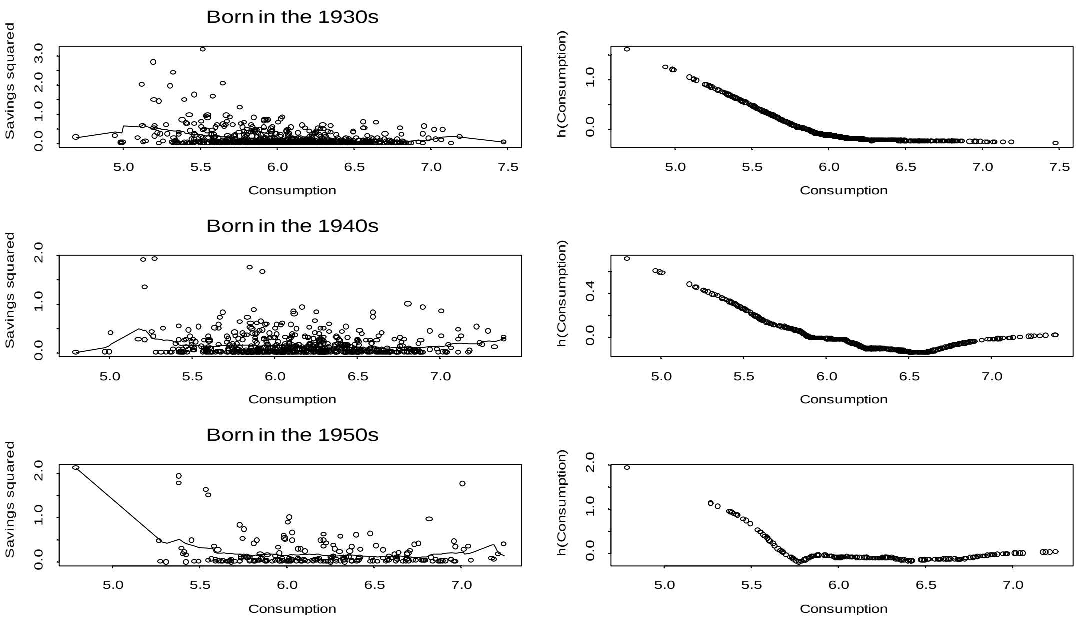

Figure 7: *Regression of saving rates on consumption for the year 1980, cohort born in the 1930s.*